INFORMATION TO USERS

This was produced from a copy of a document sent to us for microfilming. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help you understand markings or notations which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure you of complete continuity.

2. When an image on the film is obliterated with a round black mark it is an indication that the film inspector noticed either blurred copy because of movement during exposure, or duplicate copy. Unless we meant to delete copyrighted materials that should not have been filmed, you will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., is part of the material being photographed the photographer has followed a definite method in "sectioning" the material. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.

4. For any illustrations that cannot be reproduced satisfactorily by xerography, photographic prints can be purchased at additional cost and tipped into your xerographic copy. Requests can be made to our Dissertations Customer Services Department.

5. Some pages in any document may have indistinct print. In all cases we have filmed the best available copy.

University Microfilms International
300 N. ZEEB ROAD, ANN ARBOR, MI 48106
18 BEDFORD ROW, LONDON WC1R 4EJ, ENGLAND
HILL, JOHN KENTON

CAPITAL MARKETS AND HOUSEHOLD SAVING: EMPIRICAL TESTS AND SIMULATIONS FOR URBAN COLOMBIA.

RICE UNIVERSITY, PH.D., 1979
RICE UNIVERSITY

CAPITAL MARKETS AND HOUSEHOLD SAVING:
EMPIRICAL TESTS AND SIMULATIONS FOR URBAN COLOMBIA

by

John Kenton Hill

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

Thesis Director's signature:

Ronald Soligo

Houston, Texas

May, 1979
Abstract

CAPITAL MARKETS AND HOUSEHOLD SAVING: EMPIRICAL TESTS AND SIMULATIONS FOR URBAN COLOMBIA

BY

John Kenton Hill

In the conventional life-cycle model, households are assumed to base their current consumption and savings decisions upon lifetime projections for labor incomes and family size. Further, capital markets are assumed to be perfect. It then follows that current consumption will depend upon an arithmetic average of the lifetime income distribution of the consuming unit, and that inelastic saving responses to changes in incomes or interest rates necessarily reflect upon the tastes of the consuming unit.

This thesis develops a long-range consumption planning problem which recognizes a disparity between borrowing and lending rates of interest. Optimal consumption is shown to depend upon current income and higher-order moments of a household's income distribution, as well as an arithmetic average of that distribution. It is also demonstrated that inelastic consumer responses to external shocks which are generally associated with an acceleration in trading activity may be market-induced.

The consumption function which follows from the above planning problem is then estimated using the 1967-68 CEDE budget survey of the
four major Colombian cities. An assumption of perfect capital markets significantly reduces the explanatory power of the consumption function. Estimates of household taste parameters are sensitive to assumptions about prevailing capital markets.

Simulations are also made of household reactions to changes in incomes and interest rates. An average Colombian household exhibits a marginal propensity to consume transitory income of .15. These propensities are relatively age-insensitive. For a permanent change in income, households in upper, middle, and lower income brackets consume at the margin .70, .78, and .84, respectively. Simulations for changes in lending rates place the interest elasticity of Colombian saving between .3 and .5. Upper income groups exhibit an elasticity of 1.4. The largest responses to changes in interest rates are found for changes in borrowing rates. Interest elasticities for variations in borrowing rates are four times as large (in absolute value) as those computed for changes in lending rates.
ACKNOWLEDGEMENTS

Thanks are first due to the members of my dissertation committee: Professors Ronald Soligo, Gordon Smith, and George Marcus. I would also like to thank Professor Wayne Thirsk for his helpful suggestions during the formative stages of the thesis. I am also grateful to the Program of Development Studies at Rice University for providing financial support and the very competent secretarial services of Lois Thomas. The data used in this thesis was made available by the Centre de Estudios de Desarrollo of the Universidad de los Andes, Bogotá. Finally, I wish to acknowledge the constant encouragement given by my wife, Robin, and by my parents, Shirley and Jack.
### TABLE OF CONTENTS

**CHAPTER I.** INTRODUCTION ........................................... 1

**CHAPTER II.** CAPITAL MARKET OPPORTUNITIES AND

CONSUMPTION PLANNING ........................................... 5

II.1 Introduction .................................................. 5

II.2 Background: Conventional Analysis a la Fisher, Modigliani, Brumberg, et al. .................................................. 6

II.2.a General set-up ............................................. 6

II.2.b Properties of the solution ................................ 9

II.2.c Empirical evidence ......................................... 12

II.3 The Importance of the "Perfect Capital Markets" Assumption .................................................. 17

II.3.a General remarks ............................................ 17

II.3.b A simplified analysis .................................... 23

II.3.c Consumption analysis with market imperfections .................................................. 28

II.4 Summary ....................................................... 32

**CHAPTER III.** CONSUMER BEHAVIOR IN IMPERFECT FINANCIAL

ENVIRONMENTS: AN APPROXIMATION ..................................... 34

III.1 Introduction .................................................. 34

III.2 General Framework ........................................... 35

III.2.a The constraint set ........................................ 35

III.2.b The utility function ...................................... 43

- iii -
CHAPTER I

INTRODUCTION

It is a long-standing proposition of economic analysis that the level, composition, and rate of saving are primary determinants of economic growth and development. Abstention from current consumption releases resources which may be used to produce human and non-human capital and thereby expand future consumption possibilities. Poor economic performance is very often diagnosed as a failure to accumulate and/or properly allocate domestic capital. Rather than improving domestic capital markets, however, the post-war strategy in most developing countries has been one of circumvention. Development strategies have emphasized foreign aid and intervention in commodity and factor markets. Resident firms are encouraged to internally finance their investment outlays. Funds which are available for loan are channelled into export and import competing sectors to hasten the adoption of large-scale, capital intensive production techniques. The foreign sector is viewed as the engine of development.

The effects of these policies have been aptly referred to as "financial repression" and "market fragmentation."\(^1\) Inflation and ceilings on nominal interest rates have stunted the accumulation of

---

\(^1\) Edward Shaw [48] and Ronald McKinnon [37]. These books contain a more thorough development of the ideas presented above.
financial assets. Low income groups seek shelter from monetary un-
certainty by investing in durables, while households with internal
investment opportunities reinvest at low rates of return. Ceilings
on borrowing rates force nonmarket forms of rationing scarce funds.
Those who are fortunate enough to obtain credit borrow at relatively
low interest and employ production techniques which are often socially
inefficient in view of the number of high yield projects which are
left unsponsored. The lack of uniformity in interest rates induces
a wide variety of technologies and discriminates in favor of upper
income groups. In short, it is claimed that conventional develop-
ment strategies have lowered domestic saving rates, promoted a poor
allocation of existing capital, and encouraged income inequality.

An old approach, and yet one with some recent experience, urges
a more self-reliant, non-interventionist method of attack. Shaw,
McKinnon, et al. stress financial reform—a revival of the market
and a faith in relative prices. Monetary authorities are to offer
a broader menu of real rates of interest with more uniform invest-
ment opportunities across income groups. The responsibility of
raising domestic capital moves from the public sector to the private
sector.

The purpose of this thesis is to estimate the effects of capital
market reforms, and other policies oriented toward improving the
distribution of income, on household consumption and saving in urban
Colombia. More specifically, we will address the following questions:

(i) How responsive is the demand for assets to changes in interest
rates? Does this sensitivity depend upon the distribution of income?

(ii) How responsive are debt holdings to changes in borrowing rates?

(iii) Would a redistribution of income alter the rate of saving?

(iv) What would be the effects on consumption and saving of a social security system?

All evidence is derived from a unified analytical framework. A household's current consumption-savings decision is viewed as a solution to an intertemporal utility maximization problem. However, in contrast to the conventional life-cycle model developed by Fisher, Modigliani, Brumberg, and Ando (henceforth referred to as the FMBA model), capital markets are not assumed to be perfect. Differences in borrowing and lending rates of interest are explicitly recognized, as are the differences in borrowing and lending opportunities across income groups.

Chapter II summarizes the fundamental propositions and results of empirical testing pertaining to the FMBA model, and then examines the sensitivity of these propositions to a relaxation in the assumption of perfect capital markets. In Chapter III we then present a model which, in contrast to a "current income" theory of consumption, admits to a time dimension in the consumer's problem, but which, unlike the conventional life-cycle model, does not impose equality between borrowing and lending rates of interest. The model is empirically estimated in IV.1-IV.3 using the 1967-68 budget survey of four Colom-
bian cities. In Chapter IV.4 we then present estimates of household responses to a transitory income change, a change in income which is expected to persist for three years, a permanent income change, the introduction of a social security system, and a variety of changes in interest rates. The thesis is summarized and conclusions are drawn in Chapter V.
CHAPTER II

CAPITAL MARKET OPPORTUNITIES AND CONSUMPTION PLANNING

II.1 Introduction

In this thesis we view a household's current consumption-savings decision as arising from a solution to an intertemporal utility maximization problem. An explicit recognition of capital market imperfections will differentiate this work from the conventional analysis of Modigliani and Brumberg [39], Modigliani and Ando [40], and, before them, I. Fisher [18]. This chapter summarizes the fundamental propositions and results of empirical testing pertaining to the FMBA model, and then presents the implications for consumption analysis of a relaxation in the assumption of perfect capital markets.

In section II.2.a we review the general structure of the life-cycle model. A brief examination of its theoretical properties follows in II.2.b. Section II.2.c is then devoted to a summary of the results from empirical testing of FMBA hypotheses in both developed and underdeveloped countries. With this background we define and illustrate the roles of capital market imperfections in household consumption decisions. Section II.3.a is a general discussion of capital markets and consumption choice. Section II.3.b provides illustrations of some of the propositions mentioned in the previous
section. We conclude in II.3.c with a discussion of the methodological issues involved in recognizing imperfect capital markets.

II.2 Background: Conventional Analysis à la Fisher, Modigliani, Brumberg, et al.

II.2.a General set-up

There are several reasons why a household's current consumption may not equal its current resources. First, an individual's preferred consumption path may not coincide with his earnings profile. There is then an incentive to synchronize income receipts with desired consumption by saving or incurring debt. Middle-aged individuals accumulate to provide for retirement, while younger cohorts often dissave when their current income is below a lifetime average.

A second motive for saving emerges in the context of household production. Rural households and those with self-employed earners save in order to accumulate the physical capital required in their profession. To the extent that this form of saving is used to supplement whatever labor income might accrue in its absence, these first two motives are indistinct. The distinction is made, however, because households which are more production-oriented, wherein the savings-investment decision is so crucial for income propagation, are likely to react to a very different set of market stimuli than those with relatively exogenous income profiles. The availability of complementary inputs, systems of land tenure, and expected market prices are of considerable importance to rural farmers. Retirement savings,
on the other hand, will be more sensitive to rates of interest on financial assets.

A third reason for saving is the estate motive--the accumulation for bequest. While not particularly relevant for the majority of individuals, this motive is responsible for a significant fraction of aggregate personal saving in both developed and underdeveloped countries.

The last two motives for saving are due to the presence of uncertainty in economic environments. Randomness in prospective real incomes (due to price fluctuations and other macroeconomic phenomena) and in compulsory expenditures (certain medical expenses, for example) promotes the accumulation of liquid assets to insure against such contingencies. Risk of default and excessive depreciation are responsible for quantitative restrictions on debt and high rental rates on housing and other durables. Relatively expensive terms for durable rental induce many households to acquire these services by purchasing the durable. Because of quantitative restrictions on debt finance, such as a downpayment requirement for home purchases, the household is not, however, able to borrow the entire amount of the purchase price and pay off the principal as the durable is consumed. Equity is acquired in the durable, and this increases the net worth of the consuming unit.

The FMBA model is based upon an intertemporal planning problem wherein the individual household maximizes its well-being by selecting an optimum lifetime distribution of consumption expenditures, subject to the financial constraints imposed by its non-property income endowment and the capital market opportunities which are available. We make
the following assumptions:¹/

(i) Consumption in each time period consists of a single, non-
durable commodity.

(ii) The labor income to be earned in future periods is known
with certainty and is exogenous to the household's con-
sumption planning problem.

(iii) Households make no bequests and anticipate receiving no
inheritances.

(iv) Capital markets are perfect. The potential for inter-
temporal trading is summarized by the existence of one-
period loans and investments which bear the same rate of
interest. There are no quantitative restrictions on the
holding of debt. The household can then select any dis-
tribution of consumption which is consistent with non-
negative net worth at the end of its existence.

(v) The current rate of interest is expected to prevail in all
future time periods.

The sole motive for saving or incurring debt is to synchronize labor
income receipts with a more preferred consumption path.²/ Consumption

¹/ Most of these assumptions can be relaxed without a signifi-
cant amount of additional complexity. See, for example, Somermeyer and
Bannik [51]. However, the model is quite sensitive to the assumption
of perfect capital markets as we demonstrate in subsequent sections.

²/ Note that a potentially significant instrument for con-
sumption smoothing has been assumed away. By varying its work effort,
a household is able to attain stable consumption levels without borrow-
ing or lending. Of course, the assumption of perfect capital markets
renders this choice variable somewhat redundant.
is nondurable and capital markets are perfect, so that the FMBA model fails to recognize the accumulation of wealth induced by imperfect rental markets and downpayment requirements for durable purchases. There is no uncertainty and hence no precautionary motive for holding assets. But despite its rather limited scope, the model remains a cornerstone in the theory of intertemporal consumer behavior.

With assumptions (i)-(v), the household's planning problem can be formulated as

\[ \text{Maximize } U(c_1, \ldots, c_T) \]

\[ \text{subject to } PV = \sum_{i=1}^{T} (1+r)^{1-i}y_i^L + (1+r)w_0 \geq \sum_{i=1}^{T} (1+r)^{1-i}c_i, \]

where \( c_i \) denotes consumption in time period \( i \), \( T \) the length of the planning period (as measured by the number of years the household head expects to live), \( r \) the rate of interest on saving and debt, \( y_i^L \) the labor income expected in time period \( i \), and \( w_0 \) the household's initial non-human wealth. The household selects a stream of consumption to maximize utility with the constraint that the present value of its consumption not exceed its human and non-human wealth.

II.2.b Properties of the solution

The planning problem stated above can be solved, at least conceptually, for current consumption in terms of its exogenous determinants:

\[ c_1 = f(PV, \text{ age, } r, \text{ tastes}). \]

\[ ^1/ \text{See Mosak [41, pp. 116-117] for a derivation of the financial constraint written below.} \]
Optimal consumption depends *inter alia* upon the household's total wealth and is, therefore, independent of the time distribution of income receipts. All current and expected resources can be collapsed into a single index, the capitalized value of its labor and property income. The fraction of PV which is allocated to present consumption is determined, in part, by the age of the household planner. If tastes and the rate of interest are consistent with constant lifetime consumption, \( c_1 \) will equal normal income, \( \left[ \sum_{i=1}^{T} (1+r)^{1-i} \right]^{-1} PV \). Whether or not the household pursues a rising or declining lifetime consumption strategy, depends upon the difference between its subjective rate of time preference (one minus the marginal rate of substitution of present for future consumption evaluated for a constant consumption plan) and the market rate of time preference \( r \). A disparity between the two rates will lead to an adjustment of \( c_1 \) away from normal income \( (y_N)^1 \), with the magnitude of the required adjustment reflecting the elasticity of substitution.

It is instructive to illustrate the nature of the solution by focusing upon the class of utility functions which are additive and homothetic in the \( c_i \) 's. This leads to the following specification for optimal current consumption:

\[
c_1 = \left\{ \frac{\sum_{i=1}^{T} (1+r)^{1-i}}{\Sigma_{i=1}^{T} (1+r)^{1-i} \left( \frac{1+\delta y(1-i)}{1+r} \right)} \right\} y_N^n
\]

As a function \( f:R^n \rightarrow R \) is said to be homothetic if whenever \( f(x^1) = f(x^2) \), then \( f(\alpha x^1) = f(\alpha x^2) \) for each \( \alpha > 0 \). The level sets of \( f(x) \) are radial extensions of one another. In the static theory of consumer behavior, this property implies that all commodities exhibit an income elasticity of demand equal to unity.
where \( \delta \) is a pure rate of time preference and \( \gamma \) is the partial elasticity of substitution between consumption in any two time periods.

The validity of the following remarks is then immediate:

(a) Current consumption is independent of the time distribution of income receipts. A lifetime redistribution of income which leaves \( y_N \) unchanged, such as the introduction of a social security system with a rate of return \( r \), will not alter \( c_1 \).

(b) If \( r \) equals \( \delta \), consumption is equalized over time at the level \( y_N \). The optimal saving rate is \( (y_1-y_N)/y_1 \), so that, with age-related variations in labor income, saving rates exhibit considerable variation over the life cycle.

(c) The marginal propensity to consume out of transitory income is roughly \( 1/T \). An aggregate consumption function must then account for the age distribution of the population in order to predict the macroeconomic effects of fiscal stabilization policies.

(d) If all present and future resources increase by an amount \( \Delta y_p \), the resulting change in \( c_1 \) is

\[
\Delta c_1 = \left\{ \frac{\sum_{i=1}^{T} (1+r)^{1-i}}{\sum_{i=1}^{T} (1+r)^{1-i} \left[ \frac{1+\delta}{1+r} \right]^{\gamma(1-i)}} \right\} \Delta y_p.
\]

Hence, the marginal response to a permanent income change will exceed, equal, or fall short of unity as \( \delta \) is greater than, equal to, or less than \( r \). The effects on aggregate saving of a permanent redistribution of income are
determined by the differences in δ and r for the donor and recipient classes.

(e) An increase in the interest rate on future transactions will change \( c_1 \) for two reasons. First, a rise in \( r \) will lower \( PV \) as future incomes are discounted at a higher rate. The present purchasing power of future resources is reduced, and this will tend to lower current consumption. Secondly, a change in \( r \) will generally alter the fraction of total wealth devoted to \( c_1 \). From equation (2.2), it follows that a rise in \( r \) will increase, decrease, or leave unchanged \( c_1/PV \) as the elasticity of substitution \( γ \) falls short of, exceeds, or equals unity. A sufficient condition for a positive savings response to an increase in the rate of interest is that \( γ \) exceed or equal unity. Conversely, a zero or negative interest elasticity of saving implies that \( γ \) is less than one.

II.2.c **Empirical evidence**

The life-cycle model enjoyed immediate success in explaining the empirical "facts." It provides a reconciliation of the current income studies from cross-section and short- and long-term aggregate data. First, if it is assumed that households seek to maintain roughly constant levels of consumption over their lives, and that labor income typically rises in the early years and declines toward the end of the worker's life, then in a cross-section we would expect to find a relatively large proportion of households in high-income brackets with their current income above a lifetime average and hence saving a portion
of their income. Low-income groups, on the other hand, will contain a large number of households whose current income is below its norm and who are, consequently, either borrowing or depleting assets. Thus, cross-sectional evidence should show that the average propensity to consume falls with income.

If tastes and the age and wealth distributions of the population (and interest rates when γ ≠ 1) are constant over time, equation (2.2) provides a stable aggregate consumption function. Modigliani and Ando assume that income expectations are related to current income by the equation

$$βy_1^L = (T-1)^{-1} \sum_{i=2}^{T} (1+r)^{1-i} y_i^L.$$  \hspace{1cm} (2.3)

A substitution of (2.3) into (2.2) then yields

$$c_1 = K[1 + β(T-1)]y_1^L + Kw_0(1+r),$$

where K is the (constant) fraction $c_1/PV$. In the short run, consumption is not proportional to aggregate income. If, however, the ratios of labor income and wealth to total income are roughly constant over the long term, consumption will bear a proportional relationship to national income.

Thus, the FMBA model is compatible with all three of the observed relations between current income and current consumption. It also receives support from several empirical studies which indicates a predictive superiority for models embodying long-term income concepts. For example, in a comparative analysis of savings functions using
quarterly and annual U.S. data, Craig [8] concludes that "the overall good performance of the Normal Income Model, both with quarterly and annual data, supports the hypothesis that saving, or alternatively consumption, is functionally related to normal rather than measured income (p. 1137)"; and that "present income and lagged consumption are the two most important variables to include in savings functions (p. 1144)."[1]

Testing of the model has not proved conclusive, however. One of its more salient implications is that a household's marginal consumption response to a windfall income receipt is roughly 1/T, and hence is extremely low for the great majority of the population. While virtually all studies suggest lower propensities to consume income identified as transitory, there is little evidence which corroborates the theory in its strict version. Using time series data for Canada, Laumas [36] estimates a propensity for nondurable consumption out of transitory income of .368, implying a national planning horizon as short as 2.7 years. These relatively large MPCs are present in cross-section data as well. For example, Bodkin [5] finds a propensity to spend of .7-.9 for the U.S. 1950 Urban Consumption Survey, which covers the receipts of National Service Life Insurance dividends. After including several socio-demographic variables as proxies for permanent income, Bird and Bodkin [4] obtain an MPC of .383 from windfall income

[1] Craig's results support the life-cycle model only to the extent that two assumptions are satisfied: (1) that stable aggregate savings relations can be deduced from a life-cycle framework; and (2) that normal income, which is generated by expected future incomes at the micro level, can be measured at the aggregate level as a weighted average of past incomes.
using the same data base. The estimate is substantially reduced, but it does not support a strict version of the theory.

The MPC out of wealth or windfall income should also increase with the age of the household planner. In one of the more supportive tests of this proposition, Landsberger [35], using data from two Israeli household surveys, can corroborate this tendency for only four out of the eight cases considered.

There is some evidence which suggests that the FMBA model severely understates the demand for wealth. In his simulations of a golden age U.S. population whose savings behavior is based upon the life-cycle model, Tobin [53, p. 255] finds that for $\delta=(r-g) = .02$ and an (constant) elasticity of output with respect to capital of .33, the aggregate ratio of wealth to labor income is .21. This contrasts with Goldsmith's estimate of 4 for the ratio of productive tangible wealth to net national income.

While the testing of life-cycle models has produced mixed results for developed countries, the evidence from LDCs can be construed as indicating an irrelevance of long-term income variables for such environments. There appears to be no pronounced variation in saving rates over the life cycle which can be ascribed to intertemporal trading. In a study of household savings behavior in Indonesia, Kelley and Williamson [33] find that "the variation in average age-specific household savings predicted by the [FMBA] model is far greater than the actual mean savings level (p. 396)," and that "the predictions for the 20-29 age group are especially poor (p. 397)." Consumption out of windfall income is too high to be consistent with a strict version of
the life-cycle theory. Even in a relatively successful application of the theory to Colombian data, Crockett and Friend [9] obtain an MPC out of transitory income of .127.

But perhaps the most striking characteristic of the empirical results from LDCs is that there is not a clear-cut superiority of normal or permanent income variables over current income in explaining household consumption behavior. In a Brookings study of consumption and saving in Latin America, Musgrove et al. [45] conclude:

Life-cycle models of consumer behavior imply that the rate of saving should depend upon the level of lifetime or permanent income but should be fairly or completely independent of how that income is distributed through time. We find very little evidence for such long-term relations. There is some variation of the saving rate over the life-cycle, but it appears to be due simply to variation in family needs as children are born and grow up. Even if households in Latin America share the long horizons and preferences assumed by such models, they are largely prevented from acting on them by the difficulty of anticipating their real incomes and by the inability to borrow and lend freely to adjust their consumption streams. (Ch. 7, p. 14.)

In an application of the extended linear expenditure system to Colombian data, Howe's [29] estimates of MPCs show only moderate increases when current income is replaced by normal income, and the coefficients are more dispersed according to their higher standard errors. Howe, in fact, concludes that "on the basis of frequency of violation of the theoretical structure of the expenditure system, the current income ELES is judged superior to the normal income ELES (p. 209)." Finally, in a study of savings behavior in India, Ramana-
than [46] finds an invariance, in both predictive accuracy and in individual coefficients, of his results when normal income is measured
as the mean income of households in a given education- or occupation-age class rather than a discounted average of these cell means over future periods. The first technique is consistent with short time horizons, so that his results do not establish a particular importance of permanent income variables to household savings decisions.

The persistent anomalies to life-cycle savings behavior exposed in recent studies have been attributed to the presence of a number of realities unrecognized by the conventional FMBM model: uncertainty in both real incomes and rates of return, imperfect funds markets, imperfect information, and consumer irrationality. This thesis examines the sensitivity of life-cycle properties and empirical tests to a recognition of imperfect financial markets. The next sections are devoted to a development of the implications of such imperfections for the theory, analysis, and empirical testing of household consumption planning.

II.3 The Importance of the "Perfect Capital Markets" Assumption

II.3.a General remarks

A perfect financial market is one wherein an individual can exchange goods across time at a constant rate. Imperfections in funds markets consist of those features present in markets for intertemporal

\[ ^1/ \] Variations in rates of exchange across individuals would constitute a market imperfection as viewed from the aggregate, but not necessarily from the perspective of an individual consuming unit. These distortions are, of course, essential to any aggregative analysis of savings behavior, and they will be recognized in subsequent chapters. In this chapter, however, we focus upon the behavior of an individual household and do not emphasize the effects of fragmented market situations upon aggregate saving.
trading which truncate a household's opportunity locus between present and future consumption. There are several forms of capital market imperfections which may be distinguished: a disparity between the rate of return on financial assets and interest rates on debt, quantitative restrictions on the holding of debt, and imperfect markets for the rental of durable services. In each situation, present and future goods are less than perfect substitutes in the market place. We now proceed to discuss, within the context of long-range consumption planning, the implications of imperfect funds markets for household consumption theory.

Consider first the simple case of constant, but unequal borrowing and lending rates. Let \( r^a \) denote the rate of interest on assets, \( r^b \) the rate on debt, and, as before, \( \delta \) the rate of impatience for a constant intertemporal consumption plan. Figure 2.1 illustrates the optimal time paths of consumption when capital markets are perfect (\( \hat{c} \)) and when \( r^b > \delta > r^a (c^\star) \). A relatively high penalty rate discourages the acquisition of debt during the early years. As a consequence, once the debt is retired, the household consumes at a higher level than that selected in a perfect financial environment. A declining consumption plan toward the end of the planner's life would be expected if \( \delta > r^a \).

It is worth noting that the variation in saving rates associated with plan \( c^\star \) are much less pronounced than those corresponding to \( \hat{c} \). The presence of a disparity between \( r^a \) and \( r^b \) provides one possible explanation for the age-specific rigidity in saving rates found in many empirical studies.

When borrowing and lending rates differ, the rate at which an individual can transform present into future consumption, or vice versa,
Figure 2.1: Optimal Consumption and Capital Market Opportunities
depends upon the status of his capital account. If he is holding debt, the rate of transformation equals \((1+r^b)\). If his non-human wealth is positive, the rate equals \((1+r^a)\). And when the individual is neither borrowing nor lending, the rate equals \((1+r^b)\) for an increase and \((1+r^a)\) for a decrease in present consumption. The life cycle becomes segmented into intervals defined by the different phases of an optimal capital account. Consumption within any given segment is responsive to income variations which occur in neighboring periods, but it is (locally) invariant to income receipts or losses during periods which lie outside the segment. This phenomena is, no doubt, partially responsible for the high correlation between consumption and current income, and the rather poor performance of long-term income concepts, which typify empirical estimation of consumption and savings relations for LDCs.

The above remarks can be illustrated by considering the consequences of a windfall income receipt for individuals currently borrowing on their future resources. To the extent that income profiles exhibit the kind of structure portrayed in figure 2.1, there is a presumption, and indeed frequent empirical verification, that this class contains a relatively large number of young households. Thus, we can associate net debtors with individuals whose current income and, as a result of high borrowing costs, current consumption are below their respective lifetime averages. The marginal benefit of having an extra unit of consumption today will then exceed the incremental benefit of increasing future consumption. If a household's most preferred consumption path is relatively stable, it is not unreasonable to expect a high marginal consumption response to windfall income from households whose
current income and consumption are below their lifetime norms. A disparity between $r_b^b$ and $r_a^a$ accentuates the importance of current income for liquidity-constrained households. Their effective time horizon can be extremely short, and yet this result is consistent with long-range foresight. We then have a way of reconciling the fundamental structure of the FMBA model with the empirical observations of high and age-uniform marginal propensities to consume transitory income. Households with the longest planning horizons are most affected by funds markets imperfections, and they can react by consuming a large fraction of windfall receipts. The effects of fiscal stabilization policies may depend not only upon the age distribution of the population, but upon the distribution of liquidity-constrained and liquidity-unconstrained households within the economy, and hence upon the monetary policy in effect at the time of the fiscal action.

The variations in marginal rates of transformation which occur during various phases of the life cycle have implications for the effects on consumption and saving of changes in interest rates. Families may be insensitive to a change in either $r_a^a$ or $r_b^b$. Inelastic consumption responses can then be the result of imperfections in capital markets, and they need not indicate a lack of substitutability between present and future consumption or a high rate of household impatience.

The presence of quantitative restrictions on debt and imperfect rental markets for durable services strengthen the conclusions drawn thus far. Both serve to increase the probability of a household finding itself liquidity constrained during a phase of its life cycle.
The decision as to whether purchase or rent the services of a durable revolves around the difference between its rental rate \( r^R \) and the sum of its economic rate of depreciation \( d \) (physical depreciation plus capital gains or losses) and the cost of capital \( r \). For households currently holding liquid assets, durable ownership is more attractive as \( r \) equals \( r^a \) and generally \( r^R > d + r^a \).\(^1\) For liquidity-constrained individuals, \( r \) equals \( r^b \). Rates of interest on durable-specific debt are usually lower than the rates for consumption loans. However, what prevents a household from acquiring equity in durables is the downpayment requirement. In the absence of second mortgages, or their availability at a higher rate of interest, the decrease in consumption of other goods required to finance the downpayment can make durable rental an attractive method of obtaining durable services. This explains the viability of durable rental for some households, but high rental rates further influence the household's allocation decisions by creating an incentive to accumulate wealth to meet future downpayment requirements. As stated by Dolde [11, p. 8], "such market-induced accumulation may be absolutely necessary to explain observed wealth-age profiles." Furthermore, there is generally an incentive when purchasing a durable to borrow up to the quantitative limit on debt holding, resulting in more fragmentation of the life cycle.

\(^1\) We abstract from transactions costs which can be prohibitive for highly mobile individuals.
II.3.b A simplified analysis

In this section we provide a simplified analytical demonstration of the propositions stated in II.3.a. All of the assumptions (i)-(v) made in II.2.a continue to hold with the exception of the equality between borrowing and lending rates. For expository convenience, we focus upon a two-period consumption planning problem.

Letting $A_i$ denote the quantity of financial assets held at the end of period $i$ and $B_i$ the stock of debt held at the end of $i$, the financial constraints within which a household optimizes can be written as

$$y_1^L + (1+r^a)A_0 + B_1 - A_1 - (1+r^b)B_0 - c_1 = 0$$

$$y_2^L + (1+r^a)A_1 + B_2 - A_2 - (1+r^b)B_1 - c_2 = 0$$

$$(1+r^a)A_2 - (1+r^b)B_2 = 0.$$

The objective is to maximize utility by choosing non-negative feasible values for $c_1, c_2, A_1, A_2, B_1,$ and $B_2$.

What distinguishes the case of perfect funds markets from the imperfect case, is the existence of a kink in the constraint set located at the household's endowment $[(1+r^a)A_0^L + (1+r^b)B_0^L + y_1^L, y_2^L]$. Begin where consumption equals the endowment and $A_i = B_i = 0$ for $i = 1, 2$. If consumption in the present period is to be increased by an amount $\Delta c_1$, the household must borrow and, therefore, foregoes future consumption in an amount equal to $(1+r^b)\Delta c_1$. If, however, $\Delta c_1 < 0$, then $\Delta c_2$ equals $(1+r^a)\Delta c_1$. Hence, when capital markets are imperfect, the rate at which an individual can exchange present and
future goods depends upon his capital account and the type of transaction he is interested in making.

A two-period constraint is depicted in figure 2.2. The point E corresponds to the household's endowment, i.e., the consumption path which could be selected by not participating in external funds markets. Note that any change in the endowment will change the opportunity set and will, therefore, generally alter the household's consumption choice. If, on the other hand, \( r^a \) were to equal \( r^b \), the constraint set would be linear (see dashed line in 2.2) and a redistribution of resources from say E to E' would not change consumption.

When funds markets are imperfect, a household may be in equilibrium at a point where the rate at which it would indifferently exchange present and future goods is unequal to the market rate of exchange. Such equilibria are not infrequent, particularly when there is a large differential between \( r^a \) and \( r^b \), and when present and future goods are viewed as close substitutes. But an analysis of this case is most useful in demonstrating how a high marginal propensity to consume transitory income is consistent with long-range consumption planning.

Suppose that, upon receipt of a windfall gain, a household spends the entire sum on present goods. Is this an optimal distribution? An increase in current consumption, without a corresponding increase in future consumption, will lower the marginal rate of substitution of \( c_1 \) for \( c_2 \). Let \( MRS^* \) denote this new rate and \( MRS \) the equilibrium rate before the income receipt. Then if \( MRS \) were equal to the market rate of exchange between \( c_1 \) and \( c_2 \), the hypothetical distribution would not be optimal. The household could increase its well-being by exchanging
Figure 2.2: Consumption Possibilities with Less Than Perfect Markets: \( \tan \alpha = (1+r^a) \) and \( \tan \beta = (1+r^b) \).
present goods for future goods. But if a household is neither borrowing nor lending before the windfall gain, \(\text{MRS}^*\) can lie between \((1+r^a)\) and \((1+r^b)\). A marginal propensity to consume transitory income of unity is consistent with a life-cycle model when capital markets are imperfect. This case is illustrated in figure 2.3.

We conclude this section with a brief discussion of the interest elasticity of consumption when funds markets are imperfect. When a household is in equilibrium, its marginal rate of substitution of present for future consumption is a weighted average of \((1+r^a)\) and \((1+r^b)\). The first term receives no weight in households which are borrowing on prospective resources, so that their consumption will be perfectly inelastic with respect to changes in \(r^a\). Similarly, individuals who are saving a portion of their current income will be unresponsive to variations in borrowing rates. Families neither lending nor borrowing may be insensitive to a change in either \(r^a\) or \(r^b\) (see figure 2.4). When capital markets are imperfect, inelastic consumption responses to changes in interest rates need not suggest an unwillingness to substitute present and future goods on the part of the population. Indeed, a high elasticity of substitution will ceteris paribus increase the probability of a household finding itself in equilibrium at a kink in its financial constraint set, at which point the household is (locally) insensitive to variations in both \(r^a\) and \(r^b\).
Figure 2.3: Distribution of Windfall Income at an Autarchic Equilibrium—All of the Increase in Current Income Is Consumed in the Present Period.

Figure 2.4: Effects of Changes in Interest Rates on Consumption—Consumption is Unresponsive to Variations in Lending and Borrowing Rates Which Rotate the Opportunity Locus through the Cones ACB and DCF.
II.3.c Consumption analysis with market imperfections

We have argued that the conventional life-cycle model is sensitive to its assumption of perfect markets, and that a recognition of imperfections in funds markets provides a partial reconciliation of the empirical anomalies to a strict version of the life-cycle model with its most fundamental assumption, that of long-range foresight on the part of the household planner. However, an explicit allowance for differences between borrowing and lending rates, or other forms of capital market imperfections, introduces a significant amount of complexity into long-range consumption modelling. When funds markets are less than perfect, optimal consumption depends upon the distribution of income receipts. There is no single index of permanent income into which the current and prospective resources of a household can be collapsed.\footnote{It is tempting to ascribe Friedman's [20] imprecise method of defining permanent income to a recognition of capital market imperfections and the irrelevance of a discounted value of expected future incomes. But the existing theory on consumption decisions in imperfect financial environments points toward an insurmountable index number problem. Moreover, if we interpret permanent income as a long-term measure of household resources and permanent consumption as the level of consumption attained in a world of perfect markets, the assumption of zero correlation between transitory income and transitory consumption would be untenable. When current income fails below permanent income, indicating a negative transitory component, an illiquid capital account may induce a level of consumption below its permanent counterpart.} And yet to more accurately depict prevailing financial environments, whether for purposes of statistical testing or for improving econometric specifications and policy forecasts, we require a tractable analytic structure. This section examines the analytical possibilities of long-range planning models when markets are less than perfect.
Imperfections in funds markets truncate the household’s opportunity locus between present and future consumption. In contrast to the simple linear structure in (2.1), the constraint set exhibits variable marginal rates of transformation. Consider an idealized environment wherein a single commodity is consumed, and where single-period loans and securities are available at constant, but unequal rates of interest. If the current rates \( r^a \) and \( r^b \) are regarded as permanent, and if there are no planned bequests, the constraints within which the household must optimize are

\[
\begin{align*}
\sum_{i=1}^{T} \left(1 + r^a\right) A_{i-1} + B_i &= \left(1 + r^b\right) B_{i-1} - A_i - c_i = 0 & \text{for } i = 1, \ldots, T; \\
\left(1 + r^a\right) A_T &= \left(1 + r^b\right) B_T = 0; & A_i, B_i &\geq 0
\end{align*}
\] (2.4)

The constraint set is a \( T \)-dimensional polyhedron, which can be geometrically represented by the intersection of \( 2^{T-1} \) half-spaces. Each half-space corresponds to a given sign pattern of the state variables, \( A_i \) and \( B_i \). For example, the set of (efficient) consumption vectors consistent with \( B_i > 0 \) (i=1, \ldots, J) and \( A_i > 0 \) (i=J+1, \ldots, T) can be derived recursively from (2.4) as:

\[
\begin{align*}
\sum_{i=1}^{J} \left(1 + r^b\right)^{1-i} (y_{i-1} - c_i) + \sum_{i=J+1}^{T} \left(1 + r^a\right)^{1-j} (1 + r^a)^{j-i} (y_i - c_i) &= 0,
\end{align*}
\]

where \( w^*_{0} = (1 + r^a) A_0 - (1 + r^b) B_0 \). This equation defines one of a number of \( (T-1) \)-dimensional hyper-surfaces which together form the boundary of (2.4). There are \( 2^{T-1} \) combinations of efficient sign patterns in

\[
\frac{1}{1/}
\]

It can be shown that, for any feasible and efficient solution, \( A_i B_i = 0 \). An individual expecting to hold both financial assets and debt can maintain his wealth at the end of period \( i \), and increase consumption in periods beyond \( i \), by retiring the debt.
the state variables, and hence $2^{T-1}$ linear equations which locally describe the constraint set.  

Equilibrium may occur in the interior of a $(T-1)$-dimensional facet or along an edge of the polyhedron. In the first instance, a locally accurate solution for optimal consumption follows by substituting the appropriate discount factors into the constraint equation in (2.1), and then optimizing with a particular utility function. The solution routine is simplified even further if the equilibrium occurs in a facet of smaller dimension. There are, in general, \[ \frac{(T-1)!}{(T-i)!(i-1)!} 2^{T-1} \] facets of dimension $(T-i)$ for $i = 1, \ldots, T$. Each such facet corresponds to a subset of the constraint set where $(i-1)$ of the state

---

1/ For example, there are four possible sign patterns in three dimensions: $(A_1, A_2) > 0$, $(A_1, B_2) > 0$, $(B_1, A_2) > 0$, and $(B_1, B_2) > 0$. Using (2.4), we can generate four half-spaces

\[
\begin{align*}
&w^k_0 + (y_1^L - c_1) + (1+r^a)^{-1}(y_2^L - c_2) + (1+r^a)^{-2}(y_3^L - c_3) \geq 0 \\
&w^k_0 + (y_1^L - c_1) + (1+r^b)^{-1}(y_2^L - c_2) + (1+r^b)^{-2}(y_3^L - c_3) \geq 0 \\
&w^k_0 + (y_1^L - c_1) + (1+r^a)^{-1}(y_2^L - c_2) + (1+r^a)^{-2}(y_3^L - c_3) \geq 0 \\
&w^k_0 + (y_1^L - c_1) + (1+r^b)^{-1}(y_2^L - c_2) + (1+r^b)^{-2}(y_3^L - c_3) \geq 0
\end{align*}
\]

which, when intersected, provide an alternative representation of the household's constraints.

2/ The household's endowment $[y_1^L + (1+r^a)A_0 - (1+r^b)B_0, y_2^L, \ldots, y_T^L]$ is the only facet of 0-dimension. It is the vertex of the polyhedron defined by the intersection of all $(T-1)$-dimensional facets.
variables are zero. If the utility function is additive, the household's planning problem can be reformulated as a group of separable sub-problems of lower dimensions. Suppose, for example, that \( A_j = B_j = 0 \). Then (2.4) can be written as a collection of two independent constraints.

\[
y_i^L + (1+r^a)A_{i-1} + B_i - (1+r^b)B_{i-1} - A_i - c_i = 0 \quad i=1,\ldots,J; \\
(1+r^a)A_j - (1+r^b)B_j = 0; \quad A_i, B_i \geq 0 \quad i=1,\ldots,J.
\]

\[
y_i^L + (1+r^a)A_{i-1} + B_i - (1+r^b)B_{i-1} - A_i - c_i = 0 \quad i=J+1,\ldots,T; \\
A_j = B_j = 0; \quad (1+r^a)A_T - (1+r^b)B_T = 0; \quad A_i, B_i \geq 0 \quad i=J+1,\ldots,T.
\]

Optimal current consumption can be obtained by maximizing a utility function with arguments \((c_1,\ldots,c_J)\) subject to (2.5).

Thus, given the optimal sign values of the state variables, it is a straightforward exercise to express optimal consumption as function of its exogenous determinants. However, such a function will be valid for data in a neighborhood of their initial values. Moreover, the optimal sign values can only be ascertained through computer solutions to a full-scale planning problem. There appear to be two alternative modes of analysis: (1) Simulate household responses to changes in incomes and interest rates through actual computer solutions to consumption planning problems. Microeconomic simulation has been used by Tobin and Dolde [54] and Dolde [11] in the context of U.S. financial environments. The primary advantage of this approach is that it is possible to incorporate a high degree of realism into the constraint
set. The drawback is that the method is often too expensive to be used for statistical testing and estimation; (2) Limit the scope for funds markets imperfections to the case of constant, but unequal rates of interest on debt and financial assets. Then approximate the truncated constraint set by a smooth transformation frontier which, when substituted into the planning problem, will yield an analytical solution for the optimal consumption path. This is accomplished in Chapter III of the thesis. The approach is useful in a statistical context where a large number of solutions are required and where the ranges in the exogenous data are sufficiently broad to preclude the employment of consumption functions of local validity.

II.4 Summary

In this chapter we first reviewed the assumptions and general structure of the conventional life-cycle model. Several propositions were then deduced, e.g., (i) that it is not current income which is most crucial in the household's planning problem, but rather its human and non-human wealth; (ii) the marginal propensity to consume out of transitory income is roughly 1/T; and (iii) the interest elasticity of consumption depends *inter alia* upon the elasticity of substitution, and a low consumption response implies a low $\gamma$. We then argued that much of the empirical testing of life-cycle hypotheses did not support them, particularly with data from developing countries. However, in sections II.3.a - II.3.b it was demonstrated that propositions (i)-(iii) would not generally be valid under a relaxation of the "perfect capital markets" assumption. By recognizing imperfections in funds markets,
it is possible to reconcile the empirical anomalies with models of long-range consumption planning. We concluded by suggesting two methods of analyzing household consumption and saving in the presence of market imperfections: a smoothing of the truncated constraint set with subsequent application of standard econometric techniques, and computer simulations of household decisions using more detailed specifications of financial constraints.
CHAPTER III

CONSUMER BEHAVIOR IN IMPERFECT FINANCIAL ENVIRONMENTS:
AN APPROXIMATION

III.1 Introduction

Conventional studies of household consumption behavior assume
that either (a) effective planning horizons are extremely short and,
consequently, that current income is the sole resource variable which
is pertinent to the household's consumption-savings decision; or
(b) that households are free to borrow and lend at a single rate of
interest and, hence, that the crucial determinant of their behavior
is the capitalized value of their labor and property income stream.
Individuals do, however, save a portion of windfall income receipts.
Assumption (a) cannot be justified on the grounds of an innate dis-
regard for the future. Moreover, some form of intertemporal trading
exists in every economy, and a current income model cannot be deduced
from such financial environments. These observations are responsible
for the emergence of permanent income theories of consumer behavior.
But an assumption of perfect capital markets is patently unrealistic
even for developed countries. Indeed, most empirical studies which
employ a permanent income framework do not corroborate the theory in
its strict version.
In this chapter we formulate an intertemporal planning problem whose financial constraints permit an accumulation of debt or assets, but do not impose equality between borrowing and lending rates of interest. The scope for intertemporal trading is summarized by a transformation frontier which depends upon the household's labor income endowment, an interest rate defining the marginal rate of transformation at the endowment, and a parameter \( \sigma \) designed to index the severity of imperfections in capital markets. When \( \sigma \) equals 0, capital markets are perfect and one interest rate prevails on the market. As \( \sigma \) becomes positive, there is a disparity between the interest rates on savings and debt. In the limit, as \( \sigma \) approaches \( +\infty \), the household's endowment is its only feasible consumption point.

In section III.2 we develop the constraint set and utility function which together define the household's intertemporal planning problem. Section III.3 is then devoted to a presentation and description of the optimal consumption function which is generated by this planning problem and which will serve as a theoretical basis for all subsequent analysis.

III.2 General Framework

III.2.a The constraint set

When capital markets are perfect, a household can exchange goods across time at a constant rate. The marginal rate of transformation between present and future consumption is independent of the status of the household's capital account. Imperfections in funds markets, on the other hand, truncate the consumption possibilities frontier about
the income endowment point. The constraint set exhibits variable marginal rates of transformation, and it is precisely this feature of capital market imperfections which complicates the household planning problem. The severity of the additional complications naturally depends upon the nature of the financial environment modelled through the constraints. As argued in section II.3.c, equations (2.4), where loans and securities are assumed to bear different but constant rates of interest, are too complex to be used in a statistical context.

The strategy adopted here is to introduce a constraint equation which is endowed with a relatively simple (yet meaningful) analytic structure, and whose parameters can be estimated through an approximation of (2.4). The following criteria are established in order to restrict the class of acceptable formulations:

(a) The elasticity of substitution between consumption in any two periods is assumed constant and equal. This facilitates a solution for the optimal consumption path and minimizes the number of parameters to be dealt with in the empirical estimation.

(b) The transformation equation is to be time additive in the $c_i$'s. This assumption appears crucial if an optimal consumption function is to be deduced.

(c) The constraint equation is to be normalized about the household's income endowment. That is, in addition to the case of perfect markets, the smoothed constraint should provide an exact approximation to the other polar extreme, where capital markets are nonexistent and where the household's
choice set is a cube with \((y_1^L + w_0)^*\) as the first coordinate and \((y_2^L, \ldots, y_T^L)\) the remaining coordinates of its vertex. Such a restriction supports the interpretation of the elasticity of substitution as an index of the severity of imperfections in funds markets.

These three conditions are sufficient to dictate the following constraint equation:

\[
\sum_{i=1}^{T}(1+r)^{1-i}(c_i/y_i)^\sigma c_i = PV, \tag{3.1}
\]

where, for notational convenience, we have defined \((y_1^L + w_0)^*\) as \(y_1\) and \(y_i^L\) as \(y_i\) for each \(i = 2, \ldots, T\).

The household’s opportunity set depends upon its endowment vector \((y_1, \ldots, y_T)\), an interest parameter \(r\), and an exponential parameter \(\sigma\). The interest rate is the marginal rate of transformation between consumption in two successive periods when in a neighborhood of the endowment. The parameter \(\sigma\) reflects the degree of difficulty in trading intertemporally. When \(\sigma\) equals 0, equation (3.1) reduces to the case of perfect capital markets. If \(\sigma\) is positive, the term \((c_i/y_i)^\sigma\) penalizes consumption in the \(i^{th}\) period when \(c_i\) exceeds \(y_i\), while it lowers the marginal price when \(y_i\) exceeds \(c_i\). This creates a disparity between the rates of interest on loans and saving. When \(\sigma\) equals \(+\infty\), the household’s consumption possibilities are restricted to its income endowment.

The above arguments can be algebraically demonstrated by calculating the marginal rate of transformation between \(c_{i+1}\) and \(c_i\).
\[
- \frac{\partial c_{i+1}}{\partial c_i} = (1+r)(c_i/y_i)\sigma (c_{i+1}/y_{i+1})^{-\sigma}
\]  

(3.2)

When consumption is evaluated at the endowment, the marginal rate of transformation is \((1+r)\). Hence, if \(r\) is assumed constant across households, autarchic consumption possibilities are equivalent up to a scalar multiple. From (3.2) it is also clear that \(\sigma\) denotes the percentage change in the marginal rate of transformation associated with a one percent change in the ratio \((c_{i+1}/c_i)\). It is the reciprocal of the elasticity of substitution around the transformation frontier. When \(\sigma\) equals 0, the constraint set is linear with slope \(-(1+r)\). As \(\sigma\) grows positive, the constraint becomes truncated about the endowment. All of these properties of (3.1) are summarized and illustrated in figure 3.1.

One caveat is in order. Equation (3.1) is defined for strictly positive endowment vectors. If a household optimizes within the constraints (2.4), it may accumulate sufficient debt to render \(y_1\) negative. While a real theoretical possibility, such a circumstance is not likely to arise in empirical applications. We have ignored any constraints on the net worth of the consuming unit, aside from the terminal condition; and yet it is extremely difficult, especially in developing countries, for an individual to obtain a net debt position in excess of his current income. The remaining \(y_i\)'s denote non-property incomes, and the assumption that they be strictly positive might, at first glance, seem to rule out retirement. However, social security receipts, an important source of retirement income, should
Figure 3.1: Approximation of Capital Market Opportunities
also be included in $y_1$. They, like future wages, are regarded as exogenous and cannot be costlessly liquidated to finance current consumption. Secondly, for many economic classes in less developed countries, there is no retirement decision. The household head merely works until he dies or becomes an appendage to another primary unit.

The accuracy with which the smoothed constraint (3.1) captures the financial opportunities of any given household depends upon the parameters $r$ and $\sigma$. A natural way of evaluating these parameters is to first construct a constraint set from (2.4) and observed mean values for $r^a$ and $r^b$, and then to choose $r$ and $\sigma$ so as to minimize a measure of the set difference between the constraints (2.4) and (3.1). The purpose of the remainder of this section is to provide solutions to a number of such approximation problems and thereby illustrate the nature of the relationships between $(r^*,\sigma^*)$ and the data $(r^a,r^b,y_1,...,y_T)$. Details on the approximation techniques can be found in Appendix B.

The geometric structure of the reference constraints (2.4) complicates not only a solution to the household's utility maximization problem, but also the approximation procedure. Some notion as to the relation between $(r^*,\sigma^*)$ and the dimension parameter $T$ can be had by comparing the best-approximating solutions for two- and three-dimensional problems under a constant resource distribution. The solutions for various interest rate regimes are reported in Table 3.1.
TABLE 3.1: Approximations for Selected Values of \( \rho(\rho^a, \rho^b) \) and a Constant Resource Distribution

<table>
<thead>
<tr>
<th>( T )</th>
<th>( (\rho^a, \rho^b) )</th>
<th>( \rho^* )</th>
<th>( \sigma^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(0.0,.30)</td>
<td>.141</td>
<td>.144</td>
</tr>
<tr>
<td>2</td>
<td>(.10,.30)</td>
<td>.197</td>
<td>.088</td>
</tr>
<tr>
<td>2</td>
<td>(0.0,.50)</td>
<td>.227</td>
<td>.230</td>
</tr>
<tr>
<td>3</td>
<td>(0.0,.30)</td>
<td>.134</td>
<td>.156</td>
</tr>
<tr>
<td>3</td>
<td>(.10,.30)</td>
<td>.187</td>
<td>.096</td>
</tr>
<tr>
<td>3</td>
<td>(0.0,.50)</td>
<td>.207</td>
<td>.262</td>
</tr>
</tbody>
</table>

The interdimensional differences in \( \rho^* \) and \( \sigma^* \) are consistent. The optimal values for the interest parameter \( \rho \) tend to decrease with \( T \). However, there is an inverse relationship between \( \sigma^* \) and \( T \) which suggests not only a limit to which \( \rho^* \) depends upon \( T \), but that the nonlinearities introduced into (2.4) through a wedge between borrowing and lending rates are of (at least) equal importance (as measured by the exponential parameter \( \sigma \)) in constraints of higher dimension.

We now turn to the sensitivity in \( (\rho^*, \sigma^*) \) to the interest rates \( \rho^a \) and \( \rho^b \). Note first that the optimal values for \( \rho \) are roughly equal to one minus a geometric average of \( (1+\rho^a) \) and \( (1+\rho^b) \) --i.e., .140 for (0.0,.30), .196 for (.10,.30), and .225 for (0.0,.50). The optimal values for \( \sigma \) vary directly with the differential between \( \rho^a \) and \( \rho^b \). When \( (\rho^b-\rho^a) \) equals .2, .3, and .5, \( \sigma^* \) equals .1, .15, and .25 respectively. The rather low \( \sigma^* \)'s (.25 at an interest rate
differential of .5) are due to the fact that approximation errors are measured with respect to all non-negative and feasible consumption vectors. When \((r^a, r^b) = (0.0, 0.0)\) and the error is measured over a region defined by the intersection of \((2.4)\) and \(\{c_i \mid c_i \geq ay_i\}\), the two-dimensional solutions for \((r^*, \sigma^*)\) are \((.144, .339)\) for \(\alpha = .5\) and \((.145, .702)\) for \(\alpha = .75\). The significance of the kink in the constraint set increases in measure as consumption choices are restricted to smaller neighborhoods of the income endowment.

The solutions for \((r^*, \sigma^*)\) depend upon the distribution of exogenous resources, i.e., whether income receipts are concentrated in the current period, in future periods, or are distributed evenly across time. This much is clear from a geometric inspection of a two-period approximation problem. For example, as the ratio of future to current resources decreases, the kink in the constraint rotates toward the horizontal axis, and the linear segment with slope \((1+r^a)\) becomes more dominant in the set of feasible consumption points. Thus, as \((y_f/y_1)\) approaches 0, the best-approximating values for \((r^*, \sigma^*)\) will tend toward \((r^a, 0)\). Conversely, as \((y_f/y_1)\) approaches \(+\infty\), \((r^*, \sigma^*)\) should approach \((r^b, 0)\). Table 3.2 presents optimal solutions to several approximation problems where \(T=2\), \((r^a, r^b) = (0.0, 0.0)\), and \(y_2 = (1+RG)y_1\).

The results in Table 3.2 are generally supportive of the above propositions. As \(RG\) approaches \(-1(+\infty)\), \(r^*\) tends toward 0.0(.30); and as \(|RG|\) deviates from 0, \(\sigma^*\) tends toward 0. The more skewed the distribution of household resources, the greater is the linearity in the constraint set, and \(r^*\) and \(\sigma^*\) should be adjusted accordingly.
TABLE 3.2: Approximations for $T = 2$, $(r^a, r^b) = (0.0, .30)$, and $y_2 = (1 + RG)y_1$

<table>
<thead>
<tr>
<th>RG</th>
<th>$r^*$</th>
<th>$\sigma^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.50</td>
<td>.126</td>
<td>.105</td>
</tr>
<tr>
<td>-.25</td>
<td>.142</td>
<td>.135</td>
</tr>
<tr>
<td>0.00</td>
<td>.141</td>
<td>.144</td>
</tr>
<tr>
<td>.25</td>
<td>.138</td>
<td>.145</td>
</tr>
<tr>
<td>.50</td>
<td>.135</td>
<td>.142</td>
</tr>
<tr>
<td>1.00</td>
<td>.136</td>
<td>.131</td>
</tr>
<tr>
<td>2.00</td>
<td>.157</td>
<td>.098</td>
</tr>
<tr>
<td>3.00</td>
<td>.189</td>
<td>.067</td>
</tr>
<tr>
<td>5.00</td>
<td>.239</td>
<td>.031</td>
</tr>
</tbody>
</table>

III.2.b Utility function

The objective function to be maximized subject to (3.1) is

$$U = -\rho^{-1} \sum_{i=1}^{T} (1 + \delta)^{1-i} N_i c_i^{-\rho},$$  (3.3)

where $N_i$ denotes the number of equivalent adults expected to reside within the household during period $i$, and $\delta$ and $\rho$ are parameters of the utility function. Household utility is a discounted sum of utility levels pertaining to present and future time periods.\(^1/\)

\(^1/\) The additivity assumption is commonplace in a dynamic context. It is restrictive only in that consumption in each time period will be a normal good.
utility, in turn, depends upon total consumption and the size and composition of the family.

The functional specifications used for the arguments $c_i$ and $N_i$ have several convenient properties.\(^1\) First, the partial elasticity of substitution ($\gamma$) between (per capita) consumption in any two time periods is constant and equal to $(1+\rho)^{-1}$. The CES form is consistent with a broad spectrum of indifference curves. The real drawback is that utility is constrained to be homothetic.\(^2\) In a world of perfect capital markets, the household would allocate a fraction of its human and non-human wealth to $c_i$ which is independent of the level of its wealth. This proportionality assumption is common to the work of Friedman [20] and Modigliani and Brumberg [39].

As noted by Landsberger [35], the time path of optimal consumption is sensitive to the specification of utility when family size and composition are included as arguments. Some allowance must be made for family size, and indeed Landsberger suggests that its conventional omission may explain many non-supportive tests of the life-cycle model. In (3.3) the number of equivalent adults enters with an exponent of $(n+1)$, a procedure which differs from the convention of merely substituting $(c_i/N_i)$ into the utility function. The formulation in (3.3) gives the household an incentive to equalize per capita consumption

\(^1\)The exponential form of (3.3) has been used extensively in the literature on choice under uncertainty due to its property of constant relative risk aversion. It is also well established in the literature on consumption functions. See, for example, Heien [26] and Weber [57].

\(^2\)See fn. 1, page 10 for a definition of homotheticity.
over time. It is marginal utility which determines the optimal consumption path, and (3.3) yields an expression for marginal utility which is homogeneous of degree zero in family consumption and family size.

III.3 Solution and Its Properties

A maximization of (3.3) subject to (3.1) yields the following expression for optimal current consumption: \[ \log c_t = -\frac{1}{1+\sigma} \log \left( \sum_{i=1}^{T} (1+r)^{1-i} \left( \frac{1+i}{1+r} \right)^{r_{i-1}} \frac{1}{(1+\sigma)^{y_i}} \cdot \left( \frac{1+\sigma}{1+r} \right)^{\frac{N_i}{N_i^{1+\sigma}}(1+\sigma)^{y_i}} \right) + \frac{\sigma}{1+\sigma} \log y_i + \frac{1}{1+\sigma} \log PV. \]

To facilitate an understanding of this consumption function, we will now enumerate and discuss several of its properties.

The extent to which a household participates in external funds markets depends upon its need and its ability to do so. A desire to trade goods across time arises whenever the distribution of family needs \( (N_i) \) is poorly synchronized with the distribution of its income receipts \( (y_i) \). But the volume of transactions eventually undertaken by the household is also related to the nature of financial markets, here measured by the parameters \( r \) and \( \sigma \). What we wish to emphasize in this section is the relationship between truncations in the household's opportunity set and its optimal consumption path.

This is most readily accomplished by ignoring the demographic terms in

\[ ^{1/} \text{See Appendix A for a derivation of (3.4).} \]
(3.4). However, before proceeding, we first illustrate the role of the $N_i$'s for the case of perfect financial markets. All of the points now made will hold with only a marginal reinterpretation under more general financial conditions.

The expression for optimal current consumption under perfect markets follows from (3.4) by setting $\sigma$ to 0.

$$\log c_1 = \log PV - \log \left\{ \sum_{i=1}^{T} \frac{(l+r)^{1-i}}{1+r} \frac{(1+i)^{\gamma(1-i)}}{(N_i/N_1)} \right\}$$

(3.5)

Notice first that what is crucial is the family's current size and composition relative to their future distribution. If, for example, family size is constant over time, optimal consumption is independent of its magnitude. The consumption function is misspecified if current family size is included as an independent variable, but the distribution of $N_i$ is ignored. Secondly, as with the household's monetary resources, it is a discounted sum of the $N_i$'s which is relevant in consumption decisions. However, the discount factors in the demographic terms involve both $\delta$ and $r$. Indeed, if $\gamma$ equals unity, the household's impatience rate is the appropriate discount factor.

A nonconstant distribution of family size may serve to increase or decrease the need to exchange goods, depending upon the interactions between $N_i$ and $y_i$. Generally speaking, there is a rough correspondence between the two distributions, and this diminishes the need to save or dissave.\footnote{The distribution of $N_i$ is regarded as exogenous in this model. It would be interesting to determine the extent to which an observed demographic profile is a reaction to prevailing capital markets. A household can partially circumvent borrowing and lending markets by planning its family size to coincide with its expected labor income distribution.} For example, if both $N_i$ and $y_i$ are
expected to grow at the same geometric rate, and if \( \delta \) equals \( r \), then it follows from (3.5) that the optimal consumption path is the household's resource endowment. This is an important observation. Whenever there is a positive correlation between the distributions of \( n_i \) and \( y_i \), a failure to recognize family needs which vary over the life cycle places undue stress on financial markets to account for any apparent sluggishness on the part of the household to participate in external markets.

We now examine equation (3.4) to determine the relationships between an optimal consumption path and the parameter \( \sigma \), an index of financial market imperfections. In comparison to the results of Chapter II, where the interest rates \( r^a \) and \( r^b \) were assumed constant, the properties of (3.4) are qualitatively consistent and differ only by a degree of continuity. Special cases are used only to expedite the communication and they do not fundamentally alter the results.

In a perfect financial environment, normal income \( y_N \) is a sufficient statistic for the \( y_i \) distribution. Together with the rate of interest, an arithmetic average of current and expected consumable resources, with weights \( (1+r)^{1-i} \left[ \sum_{i=1}^{T} (1+r)^{1-i} \right]^{-1} \), defines the financial opportunities of the household and hence determines its optimal consumption path. As argued in Chapter II, such is not the case when loans and securities bear different rates of interest. The distribution of \( y_i \), and not merely its first moment, is of potential significance to the planning decision. Equation (3.4) conveys this message. Assuming \( \delta = r \), ignoring the demographic variables, and using a first-order Taylor approximation (about \( \sigma = 0 \)) of the first logarithmic
term in (3.4) we have

\[ \log c_1 = \sigma \gamma (1+\sigma)^{-1} \log y_1 + (1+\sigma)^{-1} \log y_N + \sigma (1-\gamma (1+\sigma)^{-1}) \log y_G, \]  

(3.6)

where \( y_G \) is the geometric mean of the \( y_1 \) distribution with weights identical to those in \( y_N \). Even after using the smoothed constraint set, there are a multiplicity of pertinent income concepts. There is no single index of permanent income when capital markets are imperfect. A household's consumption choice depends \textit{inter alia} upon its current resources \( y_1 \) and first and higher order moments of the \( y_1 \) distribution.

The consumption function in (3.4) is easily manageable when \( \gamma = 1 \) and \( N_1 = N_1 \).

\[ \log c_1 = (1+\sigma)^{-1} \log \left( \frac{\sum_{i=1}^{T} (1+r)^{1-i}}{\sum_{i=1}^{T} (1+\delta)^{1-i}} \right) + \sigma (1+\sigma)^{-1} \log y_1 + (1+\sigma)^{-1} \log y_N. \]  

(3.7)

Aside from a time preference factor, consumption is a geometric average of \( y_1 \) and the income concept used in conventional life-cycle studies. The weights in this average depend upon the ease with which goods can be exchanged across time. Figure 3.2 illustrates optimal consumption paths for various values of \( \sigma \).

In Chapter II we developed several propositions concerning market-induced rigidities in consumer responses to external shocks which are generally associated with an acceleration in trading activity. This was most clear in the case of an equilibrium which occurs at a "kink" in the constraint set and where, consequently, an individual would save a small fraction of windfall income or would exhibit an inelastic response to changes in borrowing and lending.
Figure 3.2: Optimal Consumption Paths
for Selected Values of $\sigma$: Eq.(3.7)

$\log y$ and $\log c^*$
rates. These rigidities are attributable to the truncated opportunity set and are also present in the continuous version of the model. Consider the impact of a transitory income change. From (3.7)

\[ \frac{\partial c_1}{\partial y_1} = \sigma (1+\sigma)^{-1} (c_1/y_1) + (1+\sigma)^{-1} (c_1/PV) = \sigma (1+\sigma)^{-1} + (1+\sigma)^{-1}T^{-1} \]  

(3.8)

If households can borrow and lend at a single interest rate, windfall income is distributed evenly over the remainder of the planning period. The presence of impediments to intertemporal trading, however, can lead to a marginal propensity to consume out of transitory income as large as unity. The nature of the financial environment also influences an individual's responsiveness to changes in interest rates. Let \( \eta_{cr} \) denote the interest elasticity of consumption computed from (3.7) and \( \eta^*_{cr} \) the elasticity which would obtain under perfect markets. Then it follows from (3.7) that

\[ \eta_{cr} = (1+\sigma)^{-1} \eta^*_{cr}. \]  

(3.9)

The response of an individual to a change in interest rates varies inversely with the index of imperfections in funds markets. This result echoes the proposition developed in Chapter II that a low interest elasticity of consumption need not reflect upon the willingness with which individuals would indifferently trade across time, but that it may be a consequence of prevailing financial environments.

A final property of (3.4) which should be noted is that the consumption function exhibits an elasticity of unity with respect to permanent income. That is, if the resources expected in all time
periods increase by the same proportion $\Delta \log y_P$, then

$$\frac{\Delta \log c_1}{\Delta \log y_P} = 1.$$  \hspace{1cm} (3.10)

This property is due to the homotheticity assumed for both the utility function and the constraint equation. It holds irrespective of the value of $\sigma$. 
CHAPTER IV

ECONOMETRIC TESTING

IV.1 Preliminaries: Measurement of Variables and Parameter Identification

The consumption function in (3.4) requires several types of information: the length of the planning period T, income and demographic profiles, two financial parameters \( r \) and \( \sigma \), and the two taste parameters \( \gamma \) and \( \delta \). In this section we first discuss the measurement of exogenous data, and then raise some methodological and econometric questions concerning the appropriate designation of parameters in (3.4).

**Dependent variable**

The household derives its well-being from a lifetime distribution of current and expected consumption. The dependent variable in (3.4) should then measure current family consumption. For many categories, the period between acquisition and exhaustion of product services is contained in the period of reference used in the data collection. In these cases, expenditures coincide with services rendered. There are, however, a number of durables which generate services beyond the reference period and, consequently, whose consumption is erroneously measured by periodic outlays. Adequate consumption data does exist for the two durables which absorb the largest fractions of income,
clothing and housing. Clothing expenditures are sufficiently regular and frequent to be used as proxies for the depreciation of existing and newly acquired stocks. And, for households with owned dwellings, imputed rent is used as a measure of housing services. The troublesome categories are educational expenditures, home furnishings and appliances, and vehicle outlays.

Given the dearth of information on the age and market value of durable stocks, any division of the above expenditures into consumption and saving is arbitrary. The following two concepts of household consumption were regarded as adequate to test for sensitivity in parameter estimates to changes in the definition of $c_1$:

$$c^*_1 = \text{expenditures on food and beverages, education, medical care, recreation,}^{1/} \text{transportation (exclusive of vehicle purchases), all income in kind, cleaning and paper products, insurance, personal hygiene, imputed or paid rent on occupied dwelling, utilities, home maintenance and repair;}$$

$$c^{**}_1 = c^*_1 \text{ plus expenditures on home furnishings and appliances plus vehicle purchases.}$$

The first consumption concept, $c^*_1$, includes nondurable expenditures, imputed or paid rent, and educational outlays. The second concept, $c^{**}_1$, is somewhat broader and includes home furnishings and vehicle purchases.

---

$^{1/}$Taylor [52] has argued that lotteries have been a viable investment alternative for many Colombians. These expenditures were, therefore, treated as saving and were excluded from recreational outlays.
Planning period

The planning horizon was measured, for all households in the CEDE sample, as the difference between 71 and the age of the primary earner. The household then dissolves after the 70th year of the head. Selection of the age of the primary earner is, of course, arbitrary; and there are significant differences in life expectancy across age and socioeconomic classes which we ignore. Computational costs and a priori notions of sensitivity, however, discouraged experimentation with this parameter.

Expected labor income

The variables $y^L_{i}(i=2,\ldots,T)$ denote the labor and labor-related receipts expected by the household in $(i-1)$ periods. They were quantified using a procedure analogous to the "cell mean" approach suggested by Watts [56]. To measure the expected labor income of a given worker, first obtain a sample of individuals, from all age groups, which may be assumed to share a common underlying income profile. The grouping variables used were education and occupation (see Appendix C). Differences in earnings between individuals of the same age bracket are attributed to differences in ability or other permanent characteristics. They are expected to persist over the life cycle. Let $\bar{y}^L_{i}$ denote the average earnings of individuals $(i-1)$ years older than the individual under consideration. Then the income expected in $(i-1)$ periods by our earner is measured by

$$\frac{1}{y_{i}^{L}}\quad \text{The assumption of permanency in the differential } \left(\frac{y_{i}^{L}}{\bar{y}^{L}_{i}}\right) \quad \text{is somewhat arbitrary. It is possible to parameterize the expectation process:}$$

$$y_{i}^{L} = \left(\frac{y^{L}_{i}}{\bar{y}^{L}_{i}}\right)\bar{y}^{L}_{i(l+g)}(i-1)$$

where differentials are permanent
$y_i^L = (y_i^L / \bar{y}^L_i) y_i^L (1 + g)^{i-1}$,

where $g$ is the rate of labor-augmenting technological progress for the income group.

The "cell mean" procedure allows for transitory income which is age-related, but fails to adjust for windfall elements of a macroeconomic nature. As a result, all households whose primary earner was unemployed during the interview period were not used in the estimation, and $y_i^L$ was reduced by the amount of overtime wages. The term $y_i^L$ consisted of the following components:

$y_i^L = \text{salaried and self-employed earnings less personal income taxes less social security payments plus social security receipts.}$

The average income profile was obtained by regressing household labor income ($y_i^L$) on a quadratic function of the age of the primary earner.

The results for various education-occupation groups are presented in Appendix D. Expected labor incomes were then generated using the estimated $y_i^L$ profile and a value of .03 for the rate of technical progress.  \(^1\)

---

when $\tau = 1$ and transitory when $\tau = 0$. Such an adjustment may indeed be sensible if $y_i^L$ includes income from supplementary members with nonpermanent work status. Computational considerations, however, precluded the additional parameter search.

\(^1\)The growth rate of 3 percent represents an average increase in real wages over the 1967-68 period. This assumes not only that these wage changes are expected to be permanent, but that they are uniform across occupation classes.
Current resources

The first coordinate of the household's endowment \( y_1 \) consists of its current labor income net of personal taxes and certain transitory elements \( y^L_1 \), transitory nonproperty income \( y^T_1 \), initial net worth \( A_0-B_0 \), and current capital income. The first two components were measured as

\[
y^L_1 = \text{salaried and self-employed earnings less personal taxes less social security payments plus social security receipts},
\]

\[
y^T_1 = \text{inheritances plus income in kind plus transfer payments (exclusive of social security) plus nonclassified income}.
\]

The stock-related components proved more difficult to quantify.

First, data on the capital account of the consuming unit is limited to the estimated value of owned dwellings and remaining mortgage payments. Estimates of household net worth were made by converting observed capital income \( +y^K_1 \) and interest and principal payments on outstanding debt \( -y^K_1 \) into their respective stock counterparts through two nominal interest rates, RA and RB. Secondly, the appropriate composition of \( +y^K_1 \) and \( -y^K_1 \) is not altogether clear.

The abstract model with which we have been dealing assumes the existence of a perfectly liquid and divisible asset which provides no direct services. By liquidating its assets, a household can exchange future for present goods at a rate \( (1+r^a) \). There are, however, several forms of property (including human capital) which have a special time distribution of returns and which are erroneously treated as securities of single-period maturity. Social security, for example,
represents an extremely illiquid investment whose returns should be captured in future periods and not included in $y^1$ at their stock value. But the controversial asset of greatest statistical significance is the owner-occupied home. With high penalty rates for housing rental, consumption possibilities are overstated if imputed rent is included in $y^{+K}_1$. Indeed, it is often more efficient when increasing current expenditures to borrow at a rate $r^b$ rather than liquidate the dwelling and rent housing services for present and future periods. Thus, imputed rent is perhaps more appropriately treated as an exogenous stream of services and not solely reflected in the $y^1$ term.

With this caveat, current household consumables were measured as

$$ y^1 = y^1_L + y^1_T + [(1+RA)(RA)^{-1}y^{+K}_1] - [(1+RB)(RB)^{-1}y^{-K}_1] $$

where

$y^{+K}_1 =$ imputed rent on owned housing less property taxes
plus interest and dividends plus capital gains or losses plus other capital income.

$y^{-K}_1 =$ interest payments on outstanding loans (excluding mortgages).\(^1/\)

Demographic profiles

Measurement of the $N_i$ distribution poses two problems: the weighting of family members in computing equivalent adults, and a method of evaluating expected family size and composition. Howe [29, __________]

\(^1/\) The household’s outstanding home mortgage payments are accounted for in $y^K_1$ where imputed rent applies to the family's net equity in housing.
p. 290], in an application of the linear expenditure system to the
CEDE data set, provides a set of weights for each city and for family
members of ages 0-7, 8-17, and 18 or over. The inter-city estimates
varied considerably and, consequently, were averaged according to the
fraction of the total sample represented by the relevant city. The
household's current number of equivalent adults, irrespective of city
of residence, was computed as

\[ N_1 = (0.33) \times (\# \text{ members of age 0-7}) + (0.59) \times (\# \text{ members of age 8-17}) + (1.0) \times (\# \text{ members 18 or over}). \]

In measuring expected family size and composition, we first
regressed, within education-occupation groups, \( N_1 \) on a quadratic
function of the age of the household planner (see Appendix E). The
question then arose to what extent a differential between \( N_1 \) and the
value predicted by the regression \( \hat{N}_1 \) should be regarded as a pro-
portionate deviation within all age brackets, hence calling for a
multiplicative scaling of \( N_1 \) to arrive at the number of equivalent
adults expected by the given household in (i-1) periods; and to what
extent the difference between \( N_1 \) and \( \hat{N}_1 \) reflects a phase problem, a
more fundamental difference between the compositions of the given
household and that represented by the \( \hat{N}_1 \) profile? It was felt that
the data set partition into education-occupation groups would par-
tially control for phase differences in the family cycle. Equation
(3.4) is invariant to a multiplicative shift of the \( N_1 \) distribution,
so that the predicted profile was used for \( N_1(i=1,\ldots,T) \).
Financial and taste parameters

Equation (3.4) contains four parameters--two relating to the financial constraints of the household \((r, \sigma)\) and two which describe family tastes \((\delta, \gamma)\). In conventional studies of life-cycle behavior, the constraints are generally assumed known and exogenous to the econometric problem. Cross-section and/or time series data are then exploited to effectively estimate the tastes of the household sample. However, households very often face a myriad of investment opportunities, with different expected rates of interest. There may be several forms of debt--each with its own interest rate, quantitative restrictions, and provisions regarding permissible commodity purchases. The constraints (2.4), (3.1) and certainly (2.1), are themselves approximations to the financial opportunities of any given household. Two questions then arise: (1) Which parameters should be determined independent of the data base and which should be estimated given the consumption choices of Colombian households? and (2) Which parameters can we identify from a cross-section sample, given the analytical structure of (3.4)?

(1) There are two ways of evaluating the constraint parameters. First, \(r\) and \(\sigma\) could be estimated simultaneously, together with the taste parameters \(\delta\) and \(\gamma\), at values which minimize the sum of squared errors obtained from (3.4) and the observed Colombian consumption decisions. A methodological defense of this procedure could be made by appealing to data deficiencies in observed financial parameters and to the inherent error involved in using (2.1), (2.4), or (3.1) to approximate a more complex set of financial restrictions. Second,
r and σ could be determined independently of the CEDE data by using "observed" lending and borrowing rates to construct the constraint locus described by (2.4), and then searching over r and σ to minimize the discrepancy between (2.4) and (3.1). Observations on r^a and r^b are imperfect and multiple, but it is information which could improve the estimates of δ and γ.

(2) The above arguments, of course, are academic if the analytical structure of (3.4) and/or the Colombian data base do not permit an effective identification of all four parameters. In a non-linear context, a set of parameters is identified if the problem of minimizing the sum of squared errors by choice of these parameters has a unique global solution. A sufficient condition for identification is then that the residual function be strictly convex in its parameter arguments. Unfortunately, the Hessian of (3.4) is data dependent and too complex to be globally tested for positive definiteness.

There are special cases in which conclusive answers can be given to the identification question. For example, if σ = 0, household consumption can be expressed as

\[ \log c_1 = \log PV - \log \left( \sum_{i=1}^{T} (1+r)^{1-r} N_i (N_1/N_1) \right), \]

where \( v = \left( \frac{1+\delta}{1+r} \right)^{\gamma} \). Given the interest rate r, a minimum sum of squared errors can be obtained by choice of the single parameter v. There are, however, multiple combinations of δ and γ which yield \( v^* \). Under perfect capital markets, the utility function (3.3) cannot be completely identified without exogenous variation in r. A trivial
case of underidentification occurs if capital markets are perfectly imperfect, i.e., \( \sigma = +\infty \). The predicted consumption of a household is its consumable resources \( y_1 \), and this is independent of taste considerations. Finally, consider a sample where \( y_1 = y_i \) and \( N_1 = N_i \) for each \( i = 1, \ldots, T \). Equation (3.4) then reduces to

\[
\log \left( \frac{c_i}{y_1} \right) = (1+\sigma)^{-1} \log \frac{\sum_{i=1}^{T} (1+r)^{1-i} \gamma(1-i) \left( \frac{1+\sigma}{1+\sigma \gamma} \right)}{\sum_{i=1}^{T} (1+r)^{1-i} \left( \frac{1+\delta}{1+r} \right)}.
\]

Given \( r \) and \( \sigma \), we may again proceed to define a single parameter \( \left( \frac{1+\sigma}{1+\sigma \gamma} \right) \) which, when appropriately evaluated, will yield a minimum sum of squared errors, but for which there are multiple \((\gamma, \delta)\) solutions.

These examples leave a rather pessimistic feeling as to the possibility of identifying all of the constraint and taste parameters. Lest the reader come away with a conclusive viewpoint on this question, consider equation (3.4) when \( \gamma = 1 \).

\[
\log c_1 = \sigma(1+\sigma)^{-1} \log y_1 + (1+\sigma)^{-1} \log \left\{ \sum_{i=1}^{T} (1+r)^{1-i} y_i \right\} - (1+\sigma)^{-1} \log \left\{ \sum_{i=1}^{T} (1+\delta)^{1-i} \left( N_i / N_1 \right) \right\}
\]

This consumption function permits an identification of all three of the remaining parameters \( r, \sigma, \) and \( \delta \). Indeed, it appears conceptually possible, given appropriate true values of the exogenous data and the parameters themselves, to identify all of the constraint and taste parameters in (3.4).
Even if there exists a unique solution to the sum of squares problem, parameter estimates may be so highly correlated that the system is effectively underidentified. There is a natural suspicion of this problem given the task of estimating from a cross-section the tastes and constraints of the sampled households. The next section is devoted to a test for the presence of underidentification and to a test of the sensitivity in parameter estimates under the two procedures outlined earlier for evaluating $r$ and $\sigma$.

IV.2 Sensitivity Results for Experimental Group

The consumption function in (3.4) was independently estimated for six groups of Colombian households defined by the educational and occupational characteristics of their primary earners.\(^{1/}\) Ideally, sensitivity tests could be made for each group. However, there proved to be at least four areas of controversy--those involving the measurement of family consumption, the specification of demographic variables, the measurement of wealth, and the designation of parameters to be estimated using the CEDE data. As few as two tests per area implies the possibility of 16 regressions for each group. Such an endeavor

\(^{1/}\)This data set partition was motivated by two considerations: (1) Families within groups are likely to be more homogeneous and will presumably exhibit a more stable set of taste parameters. (2) Due to the complex algebraic structure of (3.4), computational considerations favor the use of relatively small numbers of observations per regression. A majority of the computer costs of estimating (3.4) are incurred in the evaluation of household residuals. Each iteration requires one or more evaluations for every observation. Each household, in turn, carries a data array with over 100 arguments (for example, in a group with a primary earner 16 years of age, there would be 55 dimensions corresponding to the income endowment and an additional 55 for current and expected family size). Computer storage requirements are then roughly 100 times the number of observations in the regression sample, which, for the entire CEDE sample of 2949 households, is excessive.
is beyond the financial constraints of this research project. Instead, a limited amount of sensitivity analysis was performed within the single group of households whose primary earners have been exposed to secondary education and are employed in white collar occupations (see Appendix C). Conclusions drawn from this experimental group are assumed (with some reservation) to apply with similar validity to all other groups. This section reports the econometric results for group 3 under various regimes of data measurement and parameter specification.

One issue raised in section IV.1 centers around the constraint parameters \((r, \sigma)\), i.e., whether they should be estimated independently of taste considerations through an approximation of (2.4), or estimated simultaneously with the taste parameters by using consumption data. For real rates of interest on assets and debt of 0% and 30%, \(y_i = y_{11}\), and \(T = 2\), the values of \(r\) and \(\sigma\) which minimize the discrepancy between (2.4) and (3.1) are .14 and .14, respectively. The regression results presented in Table 4.1 were obtained from a nonlinear search for the taste parameters with both constraint parameters extraneously evaluated at .14.\(^1\) Each cell in the table corresponds to a regression where RA (the nominal interest rate used in converting observed capital income to its stock counterpart) takes on its column value and the demographic variables their row index (NF denotes the absence of

\(^1\) All parameter estimates reported in Chapter IV were made with the assistance of two computer programs designed for unconstrained extremum problems: FLETCH and GRADX (on file at the University of British Columbia). Gradients of the likelihood function were approximated through finite differencing. References to convergence criterion and the presence of multiple optima are contained in the commentary below.
family size considerations, and F denotes their inclusion at the predicted values given in Appendix E).

In order to gauge the sensitivity in estimates to the procedure used to determine the constraint parameters, a three parameter search for \((\sigma, \gamma, \delta)\) was made for each cell category with \(r\) evaluated at .14. These results are presented in Table 4.2.

Finally, Table 4.3 contains the results of a \((\gamma, \delta)\) search for two competing definitions of household consumption (see p. 53). The following exogenous data apply to these regressions: \(r = .14, \sigma = .5,\) and RA = .25.

### TABLE 4.1: Estimates of \((\gamma, \delta)\) for Group 3 (468 observations)*

<table>
<thead>
<tr>
<th></th>
<th>RA = .12</th>
<th>RA = .33</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\hat{\gamma} = .117)</td>
<td>(\hat{\gamma} = 2.06)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.121)</td>
<td>(.316)</td>
</tr>
<tr>
<td>R = .828</td>
<td>R = .840</td>
<td></td>
</tr>
<tr>
<td>(\hat{\delta} = -.152)</td>
<td>(\delta = .129)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.255)</td>
<td>(.002)</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\hat{\gamma} = .116)</td>
<td>(\hat{\gamma} = 1.78)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.064)</td>
<td>(.342)</td>
</tr>
<tr>
<td>R = .837</td>
<td>R = .846</td>
<td></td>
</tr>
<tr>
<td>(\hat{\delta} = -.105)</td>
<td>(\delta = .130)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.117)</td>
<td>(.003)</td>
</tr>
</tbody>
</table>
TABLE 4.2: Estimates of ($\sigma, \gamma, \delta$) for Group 3
(468 observations)*

<table>
<thead>
<tr>
<th></th>
<th>RA = .12</th>
<th>RA = .33</th>
</tr>
</thead>
<tbody>
<tr>
<td>NF</td>
<td>$\hat{\sigma} = 1.43$</td>
<td>$\hat{\sigma} = 2.96$</td>
</tr>
<tr>
<td></td>
<td>(1.512)</td>
<td>(.907)</td>
</tr>
<tr>
<td>$\hat{\gamma}$</td>
<td>$\gamma = .136$</td>
<td>$\gamma = .194$</td>
</tr>
<tr>
<td></td>
<td>(.028)</td>
<td>(.046)</td>
</tr>
<tr>
<td>$\hat{\delta}$</td>
<td>$\delta = -.140$</td>
<td>$\delta = -.066$</td>
</tr>
<tr>
<td></td>
<td>(.049)</td>
<td>(.046)</td>
</tr>
<tr>
<td>F</td>
<td>$\hat{\sigma} = 1.79$</td>
<td>$\hat{\sigma} = 1.61$</td>
</tr>
<tr>
<td></td>
<td>(1.863)</td>
<td>(.522)</td>
</tr>
<tr>
<td>$\hat{\gamma}$</td>
<td>$\gamma = .099$</td>
<td>$\gamma = .291$</td>
</tr>
<tr>
<td></td>
<td>(.023)</td>
<td>(.075)</td>
</tr>
<tr>
<td>$\hat{\delta}$</td>
<td>$\delta = -.164$</td>
<td>$\delta = .032$</td>
</tr>
<tr>
<td></td>
<td>(.064)</td>
<td>(.028)</td>
</tr>
</tbody>
</table>

|        | R = .834               | R = .844               |
|        | R = .841               | R = .850               |

TABLE 4.3: Estimates of ($\gamma, \delta$) for Group 3
(468 observations)*

<table>
<thead>
<tr>
<th></th>
<th>$c_1^*$</th>
<th>$c_1^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\gamma} = .455$</td>
<td>$\hat{\gamma} = .448$</td>
</tr>
<tr>
<td></td>
<td>(.084)</td>
<td>(.085)</td>
</tr>
<tr>
<td>$\hat{\delta}$</td>
<td>$\delta = .075$</td>
<td>$\delta = .084$</td>
</tr>
<tr>
<td></td>
<td>(.100)</td>
<td>(.010)</td>
</tr>
<tr>
<td></td>
<td>R = .845</td>
<td>R = .841</td>
</tr>
</tbody>
</table>

*The R statistics are simple correlation coefficients between observed and predicted consumption. Figures in parentheses are asymptotic standard errors calculated from the inverse of the Hessian of the likelihood function. The first consumption concept $c_1^*$ was utilized in all regressions reported in Tables 4.1-4.2.
One of the most notable features of the empirical findings is the responsiveness in all parameter estimates to the method used to evaluate the constraint parameters. The values of \( r \) and \( \sigma \) which minimize the discrepancy between (2.4) and (3.1) are respectively .14 and .14. However, these values are not consistent with a minimum sum of squared errors in predicting family consumption. Estimates of \( \sigma \) are well above unity for all regressions in Table 4.2. And despite the fact that \( \hat{\sigma} \) is measured with considerable imprecision, these results are statistically significant. In Table 4.4 we present the likelihood ratios pertaining to the null hypothesis "\( \sigma = .14 \)" and confidence intervals for \( \sigma \) based on the asymptotic standard errors and a 95% level of significance.

| TABLE 4.4: Likelihood Ratios and Confidence Intervals for \( \hat{\sigma} \) |
|-----------------------------------|-----------------------------------|
| RA = .12                          | RA = .33                          |
| NF                                |                                  |
| \( L = 16.5 \)                     | \( L = 14.8 \)                     |
| \( \sigma \in (0.42, 2.43) \)     | \( \sigma \in (1.19, 4.74) \)     |
| F                                 |                                  |
| \( L = 18.0 \)                     | \( L = 11.6 \)                     |
| \( \sigma \in (0.09, 4.65) \)     | \( \sigma \in (0.59, 2.63) \)     |

The critical value for a \( \chi^2 \)-test with 1 degree of freedom and a 99% confidence level is 6.63, so that the hypothesis "\( \sigma = .14 \)" is firmly rejected in all cases according to the likelihood ratio test. This conclusion is corroborated by the t-tests for all but the FxRA = .12 cell.
There are several explanations for the high estimates of \( \sigma \).

First, we have ignored any uncertainty a household may face in its future labor income, in (real) rates of interest it receives on investments, or in its life expectancy. And yet each form of randomness, depending upon the decision maker's attitude toward risk, can induce household behavior which is consistent with short effective planning horizons. As demonstrated in section III.3, this phenomenon may be reconciled with the structure of our model through severe truncations in a family's consumption possibilities set.

Secondly, the cross-section data base may not permit a separation of consumption characteristics which are attributable to the tastes of the household from those which are due to its financial environment. There did not appear to be multiple optima in any of the regressions in Tables 4.1-4.3. However, the parameter behavior between iterations of the regressions in Table 4.2 was very similar to a problem reported by D. Smallwood for a highly nonlinear, vintage model of production.\(^1\)

Successive iterations produced extremely small increments in the likelihood function, and the paths followed by the algorithms were unstable. The percentage change in the coefficients during the final iterations of the \((\sigma, \gamma, \delta)\) regressions was generally \(10^{-2}\), as compared to an order \(^1\)The problem is known as "indeterminacy." It is defined by Smallwood [49, p. 51] as follows: "...parameters are indeterminate (or indeterminate to within \(\delta\)) for some point in the parameter space, given values for the exogenous variables, if the same (or nearly the same) values for the endogenous variables are generated by another point in the parameter space, for all possible disturbances." The concepts of indeterminacy, underidentification, and multiple maxima of the likelihood function are related, but they are not equivalent. A complete account of Smallwood's estimations is contained in [23, Ch. 6].
of magnitude of $10^{-5}$ for the $(\gamma, \delta)$ regressions in Table 4.3 and in the second column of Table 4.1. The order of magnitude of the final gradients was also $10^1$ for the first group of regressions as opposed to $10^{-4}$ for the second group. These considerations lend support to a preference for extraneous evaluation of $r$ and $\sigma$.

A final explanation is that the values for $r$ and $\sigma$ which minimize the approximation error are valid only if the error is measured over all possible consumption choices, i.e., all non-negative and financially feasible points. When the error is measured over a region defined by the intersection of the above constraint and $\left[ C_{i1} \geq C_{i2} \right]$, the optimal values $(r^*, \sigma^*)$ are respectively $(.14, .34)$ and $(.14, .70)$ for $\sigma = .5$ and $\sigma = .75$. It can therefore be argued that the high estimates of $\sigma$ partly reflect the fact that households eventually trade within a neighborhood of their endowments, and that a higher $\sigma$ provides a better approximation over a subset of the constraint set. This is a valid argument, and it will be recognized in the final regressions.

Estimates of both the taste and constraint parameters are conditional upon the manner in which $r$ and $\sigma$ are determined. The elasticities of substitution ($\gamma$) in Table 4.2 are all less than .3, and, with the exception of the FxRA=.33 cell, their upper confidence limits are also less than .3 for a 95% level of significance. In contrast, the estimates of $\gamma$ in the second column of Table 4.1 (where $\sigma = .14$) are significantly above unity, and those in Table 4.3 (where $\sigma = .5$) are roughly .45. The household impatience rates are also generally lower in Table 4.2 than in Tables 4.1 and 4.3. There is
then a negative correlation between $\sigma$ and the taste parameters $(\gamma, \delta)$. This is consistent with the asymptotic covariances obtained from the 3-parameter regressions (see Table 4.5).

<table>
<thead>
<tr>
<th>NF</th>
<th>RA = .12</th>
<th>RA = .33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cov($\hat{\sigma}, \hat{\gamma}$) = -.011</td>
<td>Cov($\hat{\sigma}, \hat{\gamma}$) = -.036</td>
<td></td>
</tr>
<tr>
<td>Cov($\hat{\sigma}, \hat{\delta}$) = -.021</td>
<td>Cov($\hat{\sigma}, \hat{\delta}$) = -.039</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Cov($\hat{\sigma}, \hat{\gamma}$) = -.015</td>
<td>Cov($\hat{\sigma}, \hat{\gamma}$) = -.034</td>
</tr>
<tr>
<td></td>
<td>Cov($\hat{\sigma}, \hat{\delta}$) = -.048</td>
<td>Cov($\hat{\sigma}, \hat{\delta}$) = -.013</td>
</tr>
</tbody>
</table>

The high correlations between parameters (less than -.75 in all regressions) are again indicative of underidentification or indeterminacy. But the case for exogenous evaluation of the constraint parameters is strengthened further by the suspiciously low estimates of the utility parameters. This is particularly true for the impatience rates. Estimates of $\delta$ in the first column of Table 4.2 are significantly negative, with upper confidence limits of roughly -.04 at the 95% level. The estimates of $\delta$ for RA = .33 are somewhat larger, but they are nevertheless small at -.066 and .032.

There are two sources of bias which partially explain these results. First, the model's theoretical structure does not provide for bequests or late accumulation as insurance against longevity. Any such behavior is "explained" in (3.4) by a low rate of time preference. It is doubtful, however, that this omission significantly alters the
estimates. Second, a single-good model cannot account for market-induced accumulation which is often associated with the acquisition of durable services. This point has some validity as evidenced by the direct relationships between \( \delta \) and RA. As we shall shortly argue, an increase in RA effectively reduces the role of durables in the estimation.

The most notable result within tables is the sensitivity in parameter estimates to the interest factor RA. The elasticities of substitution (\( \gamma \)) in the first column of Table 4.1 are roughly .116 with upper confidence limits less than .36 at the 95% level. When RA = .33, the estimates of \( \gamma \) are significantly above unity. And the household impatience rates are less than -.1 when RA = .12, but significantly greater than .1 when RA = .33. The results in Table 4.2 are analogous, although less pronounced. Estimates of both \( \gamma \) and \( \delta \) show modest increases as RA varies from .12 to .33. There is also a tendency for \( \delta \) to increase with RA. In the first row of Table 4.2, the estimates are respectively 1.43 and 2.96 for RA = .12 and RA = .33. The estimates within the second row are not statistically differentiable.

For households with positive capital income, a decrease in RA necessarily increases measured wealth (see p. 57) and hence the \( y_1 \) coordinate of their resource endowments. Measured consumption, however, is independent of RA, so that a decrease in RA increases observed saving and reinvestment. This acceleration in trading is responsible for the decrease in \( \delta \) between columns of 4.2. The lower impatience rate provides additional incentive to save or reinvest by accentuating the differences between \( r \) and the autarchic rates of impatience.
Families without observed capital income, on the other hand, are unaffected by the change in RA and remain within a neighborhood of their endowments. A lower $\gamma$ offsets the decline in $\delta$ by maintaining a fairly constant rate of impatience at their equilibrium positions. In summary, a change in RA affects the estimates of all parameters by changing the distributions of current to future resources.

The covariation between the parameters and RA is not surprising in view of the theoretical importance of wealth in the life-cycle model. A household's stock of assets reflects its past desires and abilities to redistribute resources through time, and it is this variable which often contributes substantially to differences between $y_1$ and $c_1$. It is then most unfortunate that we do not have access to more detailed information on the capital accounts of Colombian households. Wealth estimates were made by converting observed flows of capital income into stocks through the interest rates RA and RB. The inaccuracy in this procedure is compounded by the very imperfect nature of observed capital income. Flows of services from all durables other than housing are ignored, as are the stocks of assets which do not generate observable income (for example, currency holdings within the household or demand deposits in the banking system).

Capital income is dominated by imputed rent on owner-occupied dwellings (80%), with rentals (arrendamientos) and interest and dividends constituting the second and third most significant components (12% and 4%, respectively). The average ratio of annual rent to market value is .12 for housing in Colombia, so that this value for RA is somewhat appropriate considering the domination of imputed rent in observed
capital income. However, there are two problems with such an evaluation. First, a low RA magnifies the asymmetric manner in which wealth is measured. Households with owned dwellings are endowed with positive (and often extremely large) amounts of wealth not necessarily because of past accumulation, but simply because this portion of their wealth is in an observable form. Second, it is disturbing that the presence of a durable good should so greatly influence the testing of a model which is not particularly suited to an explanation of durable acquisition. A household's net equity position in housing may be the result of imperfect rental markets, and it need not signal an expected decline in future resources as implied by the life-cycle model.

One way of dealing with these difficulties is to exclude imputed rent on owner-occupied housing from the wealth computations. As argued in section IV.1, when rental markets are imperfect, consumption possibilities may be more accurately represented if the distribution of services from owned housing is viewed as exogenous. Estimates of the utility parameters are less sensitive to the magnitude of RA when imputed rent is removed from $Y_i^K$ and additively appended to $y_i(i=1, ..., T)$. For RA = .25, the estimates are

$$
\hat{Y} = .415 \quad \hat{\delta} = .076 \quad R = .846 ,
$$

where the demographic variables were included in (3.4) and $c_{i1}$ was used to measure family consumption.

The experimental results are quite conclusive with regard to the specification of demographic variables and the measurement of family consumption. The most striking feature of the regressions in
Tables 4.1-4.2 is that an incorporation of current and future demographic terms consistently provides a higher correlation between observed and predicted consumption. The estimates of \( \gamma \) and \( \delta \) change by less than .01 when \( c_{t-1} \) is replaced by a broader concept of household consumption. In conclusion, the experimental results for group 3 suggest that the real areas of controversy involve the specification of the constraint parameters and the measurement of wealth.

IV.3 Final Results

In the previous section we performed several sensitivity tests for the class of households associated with a secondary level of education and white collar occupations. We now summarize and assimilate these results and present final regression estimates for six groups of Colombian families.

**Consumption concept**

The parameter estimates in Table 4.3 are only marginally sensitive to the definition of household consumption. However, this result may not carry over to the upper income groups due to a higher frequency and magnitude of durable goods purchases. The dependent variable in (3.4) was, therefore, measured as \( (c_{i-1} + c_{i-1}^{*})/2 \), i.e., nondurable expenditures plus imputed or paid rent plus educational outlays plus \( (1/2)(\text{home furnishings and appliances plus vehicle purchases}) \).

**Demographic specification**

The parameter estimates in Tables 4.1-4.2 are rather insensitive to the specification of family size and composition. The \( N_{i} \) profile
was incorporated in the final regressions on the basis of the higher
correlations between observed and predicted consumption for the demo-
graphic equations.

**Evaluation of financial parameters**

One issue raised in section IV.1 dealt with whether the constraint
parameters should be determined simultaneously with the taste parameters
at values which minimize the sum of squared errors obtained from (3.4)
and the observed Colombian consumption decisions, or whether they should
be evaluated independently of the budget data through an approximation
of the constraints (2.4). The test results for group 3 tend to favor
the second approach. When $\sigma$ is endogenous to the CEDE search, the
estimates of $\gamma$ and $\delta$ are extremely low (see Table 4.2) and the $\delta'$s are
well above the values which provide a minimum approximation error for
any reasonable differences between borrowing and lending rates. It is
recognized that the majority of Colombians consume within a subset of
their constraints and that a higher $\sigma$ better approximates (2.4) over a
neighborhood of the income endowment. However, it is suspected that
the estimates of $\sigma$ in Table 4.2 may also reveal an inability to dis-
tinguish behavior which is due to the tastes of the family from that
induced by imperfect financial conditions. This is consistent with the
high correlations between parameters and poor convergence exhibited by
these regressions. Therefore, $r$ and $\sigma$ were extraneously determined
in the final analysis.

The best approximating values $(r^*, \sigma^*)$ depend upon the time
distribution of a household's resources. As lifetime resources become
more concentrated in current (future) periods, the optimal solutions
for \( r \) and \( \sigma \) tend toward \( r^a \) and \( 0 \) (\( r^b \) and \( 0 \)). While it is impractical to solve a multi-dimensional approximation problem for each sampled \( y_i \) distribution, some adjustment must be made in \( (r^*, \sigma^*) \) for the timing of income receipts. To this end, five sets of financial parameters were computed for every income group. A household is assigned a parameter set in accordance with its ratio of current to average future resources. The assignment procedure is summarized in figure 4.1.

It can be argued that those subsets of the constraint set which correspond to extreme ratios of current to future consumption are irrelevant for most households, and that the area over which the set difference between (2.4) and (3.1) is measured should be appropriately restricted. All approximation errors were, therefore, computed over the region AOB (see figure 4.1).

**Resource measurement**

The major difficulty encountered in the measurement of household endowments deals with the treatment of imputed rent on owner-occupied housing (\( y^R \)). First, due to a predominance of \( y^R \) in observed capital incomes, estimates of the utility parameters are overly sensitive to the interest factor \( RA \) which was used to compute household wealth (e.g., see Table 4.1). Families with owned dwellings are endowed with large amounts of wealth simply because this portion of their wealth is in an observable form. Second, the model developed in Chapter III assumes that consumption in each period consists of a single, non-durable commodity. It is then disturbing that the presence of a durable good (such as housing) should significantly influence the testing of a model which is not well-suited to an explanation of durable
Figure 4.1: Assignment of Financial Parameters

If \( y_{x/y_1} \in (2.41, +\infty) \), unit h is assigned parameter set 1

1. \( [1.5, 2.41] \)
2. \( (.67, 1.5) \)
3. \( [.41, .67] \)
4. \( [0.0, .41] \)

\((r^*, \sigma^*)^1\) is computed for \( y_{x/y_1} = 2.41 \)
\((r^*, \sigma^*)^2\) = 2.00
\((r^*, \sigma^*)^3\) = 1.00
\((r^*, \sigma^*)^4\) = .50
\((r^*, \sigma^*)^5\) = .41
acquisitions. Finally, it is not clear whether a household's net equity in housing should be captured at its stock value in the \( y_1 \) coordinate of the resource endowment, or whether it should be distributed more evenly over the endowment at the flow level \( y^R \). An argument can be made that, because of imperfections in housing rental markets, consumption possibilities are more accurately represented by viewing imputed rent as an exogenous stream of services to be enjoyed in future (as well as present) time periods and not to be liquidated to finance current expenditures.

In the final regressions, wealth in the form of owner-occupied housing was dispersed evenly over the life cycle. Household endowments were measured as

\[
y_1 = y^L_1 + y^T_1 + y^R_1 + [(1+RA)(RA)^{-1}(y^K_1)] - [(1+RB)(RB)^{-1}(y^K_1)]
\]

and

\[
y_i = y^L_i + y^R_i \quad i = 2, \ldots, T
\]

with \( y^R_1 \) now removed from \( y^K_1 \). The procedure is very much ad hoc, but it has the positive qualities of reducing asymmetries in wealth measurement, of deemphasizing the role of durables in the estimation, and of recognizing imperfections in housing rental markets.

Financial data

To compute \((r^*, c^*)\) and household wealth, information is required on the capital market opportunities available to Colombians during the 1967-68 period. Table 4.7 presents nominal interest rates for various savings and debt instruments which were available to upper, middle, and lower (permanent) income classes.
TABLE 4.7: Financial Opportunities for Income Classes*

<table>
<thead>
<tr>
<th>Assets</th>
<th>Nominal Interest Rate (%)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial assets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>currency (U,M,L)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>savings deposits (U,M,L)</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>time deposits (U,M,L)</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>BCH cedula bonds (U,M)</td>
<td>9.25**</td>
<td>Hanson, pp. 14-15</td>
</tr>
<tr>
<td>stocks (U)</td>
<td>15.6</td>
<td>Musalem, p. 88</td>
</tr>
<tr>
<td>corporate bonds (U)</td>
<td>12</td>
<td>Taylor, p. 11</td>
</tr>
<tr>
<td>government bonds (U)</td>
<td>10***</td>
<td>Hanson, p. 20</td>
</tr>
<tr>
<td><strong>Productive assets (U,M,L)</strong></td>
<td>20-30</td>
<td>Berry</td>
</tr>
<tr>
<td>Housing, Real Estate (U,M,L)</td>
<td>15-25</td>
<td>Berry</td>
</tr>
<tr>
<td><strong>Debt</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Institutional consumption loans(U)</td>
<td>15-25</td>
<td>Berry</td>
</tr>
<tr>
<td>Extrabank lending (U,M,L)</td>
<td>30-35</td>
<td>Taylor, p. 11</td>
</tr>
<tr>
<td></td>
<td>24-90</td>
<td>Wai, p. 304</td>
</tr>
<tr>
<td>Housing finance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BCH credit (U,M)</td>
<td>13</td>
<td>Seers, p. 400</td>
</tr>
<tr>
<td>ICT credit (M,L)</td>
<td>14</td>
<td>Seers, p. 400</td>
</tr>
</tbody>
</table>

* U, M, and L denote whether or not the instrument is available to upper, middle, or lower income groups.

** The reported rate represents an average of returns on 1967 issues (7.5%) and 1968 issues (11%).

*** The reported rate represents an average of returns on 1967 issues (8.5%) and 1968 issues (11.5%).
The first group of assets listed in the table consists of financial assets. While households in upper income brackets had access to a broad menu of financial savings instruments (stocks, bonds, etc.), opportunities for smaller savers were restricted to currency holdings and savings and time deposits (earning negative real rates of interest in view of an inflation rate of 8% during the period). The second group includes all productive assets (plant, equipment, inventories, etc.). Rates of return for this category vary considerably due to the heterogeneous nature of these asset-types and due to economies of scale within investment categories.\(^1\) The range of 20-30% in Table 4.7 was suggested by A. Berry. The third group of assets consists of housing and real estate. Investment opportunities are perhaps most uniform for these assets as all income groups have at least limited access.\(^2\)

Borrowing opportunities can be classified into pure consumption loans and credit specifically designed for housing finance. Consumption loans from institutional sources are available only to the wealthier segments of the population. Middle and lower income groups are generally restricted to extrabank lending markets (professional moneylenders, traders, shopkeepers, landlords, friends and relatives).

\(^1\) Compare, for example, the rate of return earned by the small fruitstand operator who lives at or near a subsistence level with a mean rate of return of 241.6% reported by Huddle [32, p. 32] for a small group of Colombian artisans.

\(^2\) Housing is a remarkably divisible form of investment. The small saver can periodically increase his equity by purchasing and stockpiling individual bricks.
Interest rates on these transactions are notoriously high.\textsuperscript{1} The range cited by Taylor [52, p. 11] is, at best, indicative of median interest rates, while Wai's [55, p. 304] range perhaps more accurately reflects both the variation and potential magnitude of extrabank lending costs. The interest rates used to summarize credit conditions for housing finance were those offered by the Banco Central Hipotecario and the Instituto de Credito Territorial. However, the problem faced by most Colombians was credit availability rather than cost.\textsuperscript{2} Even the ICT loans, which were originally designed for low-income housing, were available to a small fraction of the poor due to limited funds and greater profitability of high-income loans.

The values for \( r \) and \( \sigma \) which best approximate the capital market conditions faced by any given household depend \textit{inter alia} upon the (permanent) income class of the sampled unit. The educational and occupational variables provided a clean way of sorting households.

\textbf{Upper income}--Groups 1 and 2 were assigned the financial opportunities available to upper income Colombians. These groups correspond to households with primary earners who were employed in technical or administrative occupations (see Appendix C) and who had been exposed to secondary or university level education. Over 75\% of the families sampled by CEDE which were in the top occupational class were also in the top income quartile (Musgrove [45, p. II-50]). The

\textsuperscript{1}Wai [55, p. 302] reports a mean interest rate of 40\% for non-institutional credit in a large sample of less developed countries.

\textsuperscript{2}Indicative of the shortage of home loans is that only 29.9\% of homes built in 1968 were financed with any kind of credit (Seers [47, p. 397]).
mean labor incomes for groups 9 and 13 were respectively 14,230 and 21,930 pesos per quarter, as compared with a figure of roughly 15,000 which defines the top decile.

**Middle income**--Groups 3 and 4 were designated as middle income. These were households with secondary education and with white and blue collar occupations (see Appendix C). Over 70% of the sample with a secondary level of education was in the top 50% of the income distribution (Musgrove [45, p. II-46]). The mean labor incomes of the two groups were 9,280 and 6,440 respectively, which compares with 5000 as defines the top 50%.

**Lower income**--Groups 5-7 were assigned lower income financial opportunities. Their education was limited to the primary level, and their occupational characteristics were respectively white collar, blue collar, and other (see Appendix C). Roughly 65% of the families which had primary education were in the bottom 50% of the income distribution (Musgrove [45, p. II-46]). The mean labor incomes of the three groups were respectively 4,400, 3,680, and 3,110.

Each household is now assigned a (permanent) income class and hence a particular set of instruments with which it could save and dissave.

To estimate the consumption function in (3.4), the interest rates associated with all types of assets and debt must be averaged into a single rate of return on assets and a single rate of interest on debt. Turning first to the asset side, investments were classified into three
categories--financial, productive, and housing and related investments. The nominal interest rates assigned to each category are presented below by income class.

<table>
<thead>
<tr>
<th>Income Class</th>
<th>$R^F_A$</th>
<th>$R^P_A$</th>
<th>$R^H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper</td>
<td>.12</td>
<td>.25</td>
<td>.20</td>
</tr>
<tr>
<td>middle</td>
<td>.07</td>
<td>.20</td>
<td>.20</td>
</tr>
<tr>
<td>lower</td>
<td>.04</td>
<td>.20</td>
<td>.20</td>
</tr>
</tbody>
</table>

A rate of 12% was used to index the rates of return on financial assets available to upper income Colombians. This rate is characteristic of the interest earned on corporate bonds, and it lies between the interest rates on government and BCH bonds and the yields on corporate stocks. Households in middle income brackets were assumed to earn a rate of return of 7% on financial assets, an average of the rates pertaining to savings deposits and BCH cedula bonds. Financial saving instruments available to low income groups were limited to savings deposits with commercial banks, earning a nominal return of 4%. Productive investments were assumed to earn a rate of return equal to 25% for upper income groups and 20% for households in middle and low income brackets. The differential of 5% is in recognition of economies of scale for large savers in this asset category. Investment opportunities in housing were regarded as uniform across income classes. An interest rate of 20% was used.

The next step involves a weighting of the interest rates which apply to the three types of assets. Unfortunately, no data exists at
the micro level on the capital accounts of Colombian households. Information on the composition of household assets is restricted to estimates of the ratios of average asset holdings within a given income class to the mean stock of wealth for that class. The following range estimates of global mean shares were communicated to the author by A. Berry:

<table>
<thead>
<tr>
<th>Income Class</th>
<th>FA/W</th>
<th>PA/W</th>
<th>H/W</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper</td>
<td>10-25%</td>
<td>40-60%</td>
<td>30-40%</td>
</tr>
<tr>
<td>middle</td>
<td>10-20%</td>
<td>20-50%</td>
<td>40-50%</td>
</tr>
<tr>
<td>lower</td>
<td>10-20%</td>
<td>20-50%</td>
<td>40-50%</td>
</tr>
</tbody>
</table>

A more appropriate weighting scheme, however, is phrased in terms of mean individual shares, i.e., an equally weighted sum of individual asset shares for all members in the group. Differences in the two share concepts are more pronounced the more dispersed are the individual shares, and the importance of the difference naturally depends upon the variations in rates of return across types of assets. But, for low income groups in particular, rates of interest vary substantially between assets, and investment opportunities are non-uniform. The self-employed, small-scale entrepreneur may receive a relatively high rate of return on his savings because of his "own" investment options. His share of wealth in productive assets is above average and, if his stock of wealth is also above average due to the high returns he earns, the composition of his capital account is
overrepresented by the global shares.\footnote{Consider two individuals, the first of whom owns a small business and, consequently, has accumulated a stock of wealth equal to 100 units. One-tenth of his assets are financial and the remainder are productive. The second individual, on the other hand, can only save in the form of currency holdings or savings deposits with a commercial bank. His stock of wealth is 10 units, consisting entirely of financial assets. The two measures of central tendency in the share of financial assets are then

\[
global\ share = \frac{(10+10)}{(100+10)} = .18
\]

\[
individual\ share = (.5)(10/10) + (.5)(10/100) = .55.
\]

The global share concept is a poor measure of central tendency because of the strong correlation between household wealth and its share in the form of productive assets.}

The occupational grouping variable provides one way of correcting for the bias introduced by the global mean shares. The groups classified as blue collar contain all artisans and a relatively large number of self-employed earners (see Appendix C). It was then decided to use a high share in productive assets for these groups and a high share in financial assets for households in all other occupational groups. The final shares used in computing RA are reproduced below.

<table>
<thead>
<tr>
<th>Income Group</th>
<th>$v^q_{FA}$</th>
<th>$v^q_{PA}$</th>
<th>$v^q_{H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>.2</td>
<td>.4</td>
<td>.4</td>
</tr>
<tr>
<td>3,4</td>
<td>.6</td>
<td>.2</td>
<td>.2</td>
</tr>
<tr>
<td>4</td>
<td>.2</td>
<td>.4</td>
<td>.4</td>
</tr>
<tr>
<td>5,6,7</td>
<td>.6</td>
<td>.2</td>
<td>.2</td>
</tr>
<tr>
<td>6</td>
<td>.2</td>
<td>.4</td>
<td>.4</td>
</tr>
</tbody>
</table>
For comparative purposes, two sets of weights were used for each of the groups 4 and 6.

Borrowing rates were computed as a weighted average of the interest rates on consumption and housing loans. As borrowing opportunities are specific to commodities, the weights should reflect the individual shares of housing and nonhousing consumption in total consumption. For the entire CEDE sample, imputed or paid rent constituted 25% of total consumption expenditures. Thus, the weights used for each household, irrespective of its (permanent) income class, were .75 for pure consumption loans and .25 for home-related debt.\footnote{This assumes a unitary elasticity to consume housing out of permanent income. Given the inaccuracy with which effective borrowing rates were measured, the assumption is rather innocuous.} Individual interest rates are summarized below by income class.

**Upper income**—Consumption loans from institutional sources are only available to upper income Colombians. Due to quantitative restrictions on and limited availability of these loans, their effective rate of interest is probably higher than the range of 15-25% cited in Table 4.7. An interest rate of 30% was used to characterize the cost of institutional consumption loans. As for housing finance, upper income groups had access to credit from the BCH. However, because of downpayment requirements (30%) and maximum loan values, the effective interest rate is closer to 20%.

**Middle income**—Consumption loans to middle and lower income groups are limited to extrabank sources. An interest rate of 50% is...
indicative of the high penalty rates involved in these transactions. Middle income households did have occasional access to BCH and ICT home loans. But loan availability was more of an impediment than cost. An interest rate of 30% was used.

Lower income--Home loan shortages were most acute for the poorest segment of the population. Interest rates of 40% and 50% were used to reflect the costs to these groups of obtaining home and pure consumption loans.

All financial data used in the estimation of (3.4) are presented in Table 4.8.

Final regressions.

The consumption function in (3.4) was independently estimated for six groups of Colombian households. To facilitate an interpretation of the results, the educational and occupational characteristics associated with the primary earners in each group are reproduced below. The regression results are presented in Table 4.9. Two

<table>
<thead>
<tr>
<th>Group</th>
<th>Education</th>
<th>Occupation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>secondary, university, or post-graduate</td>
<td>professionals</td>
</tr>
<tr>
<td>3</td>
<td>secondary</td>
<td>white collar</td>
</tr>
<tr>
<td>4</td>
<td>secondary</td>
<td>blue collar</td>
</tr>
<tr>
<td>5</td>
<td>primary</td>
<td>white collar</td>
</tr>
<tr>
<td>6</td>
<td>primary</td>
<td>blue collar</td>
</tr>
<tr>
<td>7</td>
<td>primary</td>
<td>other</td>
</tr>
</tbody>
</table>

1/ Households in groups 1 and 2 were pooled to increase sample size. For more detail on the grouping variables, see Appendix C.
TABLE 4.8: Financial Parameters for All Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>RA</th>
<th>RB</th>
<th>((r,\sigma)^1)</th>
<th>((r,\sigma)^2)</th>
<th>((r,\sigma)^3)</th>
<th>((r,\sigma)^4)</th>
<th>((r,\sigma)^5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>.204</td>
<td>.275</td>
<td>.195</td>
<td>.172</td>
<td>.159</td>
<td>.151</td>
<td>.124</td>
</tr>
<tr>
<td></td>
<td>.000</td>
<td></td>
<td>.000</td>
<td>.028</td>
<td>.050</td>
<td>.025</td>
<td>.000</td>
</tr>
<tr>
<td>3,4</td>
<td>.122</td>
<td>.450</td>
<td>.370</td>
<td>.239</td>
<td>.195</td>
<td>.157</td>
<td>.042</td>
</tr>
<tr>
<td></td>
<td>.000</td>
<td></td>
<td>.000</td>
<td>.148</td>
<td>.236</td>
<td>.116</td>
<td>.000</td>
</tr>
<tr>
<td>4</td>
<td>.174</td>
<td>.450</td>
<td>.370</td>
<td>.259</td>
<td>.226</td>
<td>.189</td>
<td>.094</td>
</tr>
<tr>
<td></td>
<td>.000</td>
<td></td>
<td>.000</td>
<td>.124</td>
<td>.190</td>
<td>.089</td>
<td>.000</td>
</tr>
<tr>
<td>5,6,7</td>
<td>.104</td>
<td>.475</td>
<td>.395</td>
<td>.244</td>
<td>.198</td>
<td>.156</td>
<td>.024</td>
</tr>
<tr>
<td></td>
<td>.000</td>
<td></td>
<td>.000</td>
<td>.170</td>
<td>.265</td>
<td>.132</td>
<td>.000</td>
</tr>
<tr>
<td>6</td>
<td>.168</td>
<td>.475</td>
<td>.395</td>
<td>.269</td>
<td>.234</td>
<td>.193</td>
<td>.088</td>
</tr>
<tr>
<td></td>
<td>.000</td>
<td></td>
<td>.000</td>
<td>.140</td>
<td>.210</td>
<td>.098</td>
<td>.000</td>
</tr>
</tbody>
</table>
TABLE 4.9: Final Regressions

<table>
<thead>
<tr>
<th>Group</th>
<th># Observations</th>
<th>Gamma</th>
<th>Delta</th>
<th>R-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>368</td>
<td>.336</td>
<td>-.085</td>
<td>.709</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.171)</td>
<td>(.113)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>469</td>
<td>.238</td>
<td>-.126</td>
<td>.847</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.034)</td>
<td>(.038)</td>
<td></td>
</tr>
<tr>
<td>4\textsuperscript{a}</td>
<td>243</td>
<td>.349</td>
<td>.024</td>
<td>.837</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.082)</td>
<td>(.047)</td>
<td></td>
</tr>
<tr>
<td>4\textsuperscript{b}</td>
<td>243</td>
<td>.289</td>
<td>-.028</td>
<td>.836</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.050)</td>
<td>(.036)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>.204</td>
<td>-.103</td>
<td>.844</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.045)</td>
<td>(.055)</td>
<td></td>
</tr>
<tr>
<td>6\textsuperscript{a}</td>
<td>629</td>
<td>.484</td>
<td>.115</td>
<td>.336</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.055)</td>
<td>(.014)</td>
<td></td>
</tr>
<tr>
<td>6\textsuperscript{b}</td>
<td>629</td>
<td>.334</td>
<td>.046</td>
<td>.839</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.034)</td>
<td>(.015)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>256</td>
<td>.173</td>
<td>-.036</td>
<td>.838</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.067)</td>
<td>(.078)</td>
<td></td>
</tr>
</tbody>
</table>
estimates of the utility parameters were made for each of the blue collar groups. The regressions $4^a$ and $6^a$ correspond to interest rates computed for the high shares of wealth in productive assets (rows three and five in Table 4.8), while $4^b$ and $6^b$ correspond to interest rates computed for the high shares of wealth in financial assets (rows two and four in Table 4.8). The R-statistics are simple correlation coefficients between observed and predicted consumption. Figures in parentheses are asymptotic standard errors calculated from the inverse of the information matrix.

With the exception of regression $6^a$, all correlation coefficients are greater than .7, and for six out of the eight cases, the R-statistics exceed .8. Group 6 is composed of households with primary education and blue collar occupations. Regression $6^a$ assumes a relatively favorable return on invested savings--a nominal return of 17% as compared with 10% in $6^b$. The R-statistics generally conform to the coefficients of determination obtained in cross-section studies which employ a normal or current income framework.\(^1\) However, it should be noted that the regressions in Table 4.9 presuppose a very specific set of capital market opportunities. The only parameters which are estimated with the CEDE data set are the utility parameters $\gamma$ and $\delta$.

The elasticity of substitution $\gamma$ measures the degree to which a household will indifferently substitute between present and future

\(^{1/}\)For the case of Colombia, see Crockett and Friend [9] and Musgrove [45].
consumption. A low $\gamma$ indicates that the family will maintain a constant per capita consumption plan, irrespective of prevailing capital markets and the distribution of its monetary resources. Other things being equal, the lower the elasticity of substitution, the smaller will be a household's consumption response to a change in the rates of interest on assets and/or debt. The point estimates of $\gamma$ in Table 4.9 range from .17 to .48, with a mean value of .3. Although none of the estimates are statistically distinguishable at a 90% level of confidence, they do tend to increase with the permanent income class of the sampled units. Ignoring the regressions 4 and 6, which assume relatively high rates of interest on savings, the mean values are .34 for the upper income group (1-2), .26 for the middle income groups (3 and 4), and .24 for the poorest groups (5, 6 and 7). The influence of the assumed values for the interest rate RA on the elasticities of substitution can be discerned by comparing regressions 4 to 4 and 6 to 6. For families exposed to secondary education and employed in blue collar occupations, a five percentage point decrease in the return on savings lowers the estimate of $\gamma$ by .06. For households with primary education and blue collar occupations, a decrease in RA of seven percentage points lowers the elasticity of substitution by .15.

Unfortunately, there is no research on consumption and saving in less developed countries which provides estimates of utility functions comparable to (3.3). Four independent estimates of the elasticity of substitution derived from U.S. data are summarized below.
With the exception of the study by Heien, the above results place $\gamma$ between .4 and .5 for U.S. households. The estimates reported in Table 4.9 are broadly consistent, although somewhat smaller. The mean value for $\gamma$ over all regressions excluding $4^a$ and $6^a$ is .26. The intercountry differences conform to the tendency revealed in the Colom- bian regressions for $\gamma$ to increase with the standard of living enjoyed by the sampled units.

The impatience rate $\delta$ denotes the excess over one unit of future consumption which a household requires if it is to forego one unit of present consumption and yet remain at the level of utility defined by an initial lifetime distribution of constant per capita consumption. It was the contention of the classical capital theorists (Böhm-Bawerk, Wicksell, Fisher, et al.) that $\delta$ is positive and that it varies inversely with the real income of the consuming unit. Given these a priori standards as to what constitutes a plausible rate of impatience, the estimates in Table 4.9 are rather disappointing. The estimates of $\delta$ are highly dispersed, both between and within groups of households. The range is from -.13 to .11, and an average of the absolute values of the asymptotic t-ratios equals 1.71 for all regressions excluding $4^a$ and $6^a$ -- a relatively small number in view of the

\[1/\text{Range in point estimates for five interest rate series.}\]
large sample size. Only one of the estimates has an upper confidence limit which is negative at a 95% level. However, the point estimates of δ are negative in five out of the eight cases, and their mean value over all regressions except 4^a and 6^a equals -.055. There is a tendency for δ to decrease with permanent income. Excluding 4^a and 6^a, the mean values are -.09 for the upper income group, -.08 for the middle income groups, and -.03 for the poorest groups. However, given the inaccuracy with which each estimate is measured, no firm conclusion can be reached regarding the relationship between δ and permanent income.

The negative estimates of δ suggest that life-cycle motives for saving are relatively unimportant to Colombian households. Because of imperfections in capital markets, the need to save or borrow for purposes of smoothing per capita consumption may have been largely circumvented through adjustments in work effort and/or family planning. Early accumulation, as a precautionary fund, to meet downpayment requirements on future home purchases, to acquire materials for self-constructed dwellings, to finance outlays on household durables, or for purposes of social status, can be erroneously interpreted as a negative rate of impatience. The effects of such accumulation are twofold. First, the wealth of the consuming unit exceeds that level which would be predicted by a strict life-cycle model of saving. In estimating (3.4), we partially controlled for such accumulation by distributing imputed rent on owner-occupied housing evenly over the income endowment. But the presence of other motives for saving further influence our estimates through the flow variables.
Unfortunately, it is impossible to associate each peso of observed saving with the motive which inspired it.

Similar difficulties have been encountered in studies attempting to measure U.S. impatience rates. Depending upon which interest rates were assumed to characterize U.S. financial markets, Weber's [57] estimates of $\delta$ range from -.03 to .46. Heien's [26] estimate equals -.08 with an asymptotic $t$ ratio of 1.47. Precise and plausible estimates of household impatience rates have continued to elude researchers using data from both developed and underdeveloped countries.

IV.4 Consumption Responses to Policy Variables: Own Estimates and Inter-study Comparisons

The primary focus of this research is not upon the estimates of the taste parameters per se, but rather upon household responses to changes in policy variables which follow from our modelling of the consumer's economic problem. The fundamental axiom which pervades our analysis is that a household's reaction to a windfall income receipt, to the introduction of a new financial security, or to any exogenous shock, can be deduced from an integrated framework—a consumption planning problem consisting of a preference function and a constraint set. Household preference functions are first estimated by exploiting information on demographic profiles and on observed responses to a subset of possible perturbations in the constraints. When combined with a carefully specified planning problem, the estimated taste parameters can then be used to simulate the effects on consumption of a change in any exogenous variable, independent of whether or not such a change has actually occurred. This methodology contrasts with the
conventional approach of writing consumption as an implicit function of the policy variables, taking a linear or log-linear approximation to this function, and then using historical incidents of policy changes to estimate their respective marginal impacts. Little or no attention is paid to the specification of these impact parameters or to the underlying taste and constraint parameters which determine them.

In this section we provide estimates of the consumption responses of Colombian households to a purely transitory income receipt, a change in income which is expected to persist for three years, a permanent income change, the introduction of a social security system, and an assortment of changes in borrowing and lending rates of interest. Whenever possible, these estimates are compared with those obtained in other studies of the CEDE budget data.

Two sets of estimates were made for each experiment. The first set was derived from the taste parameters presented in Table 4.9 and from the constraint parameters used in their estimation. The regression results in 4a and 6a were used for households in blue collar occupations. The second set of estimates were derived from a more plausible set of taste parameters. The values used for each group are shown below. Small, but positive rates of impatience were assumed for each group. The differences in $\gamma$ and $\delta$ between income classes reflect the nature of the relationships between the utility parameters and permanent income suggested by the regressions in Table 4.9. Except for groups 4 and 6, the financial parameters used in the second set of estimates are identical to those of the first set. For the blue collar groups, the two sets of interest rates presented in Table 4.8 were
averaged and new approximating values calculated for \( r \) and \( \sigma \).

<table>
<thead>
<tr>
<th>Group</th>
<th>Gamma</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>.35</td>
<td>.02</td>
</tr>
<tr>
<td>3</td>
<td>.25</td>
<td>.03</td>
</tr>
<tr>
<td>4</td>
<td>.30</td>
<td>.03</td>
</tr>
<tr>
<td>5</td>
<td>.20</td>
<td>.05</td>
</tr>
<tr>
<td>6</td>
<td>.30</td>
<td>.05</td>
</tr>
<tr>
<td>7</td>
<td>.20</td>
<td>.05</td>
</tr>
</tbody>
</table>

**Income effects**

One source of variation in a household's consumption decision is the income endowment vector. We now present, for representative households in 24 age-income cohorts, estimates of marginal consumption responses to a transitory income receipt (MPCI), to a change in income which is expected to persist for three years (MPC3), and to a permanent income change (MPCP). The cohorts correspond to the six educational-occupational groups used previously and to families with primary earners of ages 21, 30, 40, and 55. Each MPC was computed through finite differencing of the optimal consumption choices which pertain to the initial and perturbed income endowments. A perturbation factor of 100 was used in all computations.\(^1\) The demographic and labor income profiles used are those described in Appendices D and E. The wealth variables assigned to households of ages 21, 30, 40, and 55 are averages over all families in the relevant group and in age brackets 16-25, 26-35, 36-50, and 51-65, respectively.

\(^1\) Because the income changes are non-infinitessimal, the above propensities to consume represent averages of the MPCs which
Estimates of the marginal propensities to consume are presented in Table 4.10. Those denoted by an "*" correspond to the second set of taste and constraint parameters. The MPCs within each cell are arranged by age. Thus the first entry applies to a household with a primary earner of age 21, the second to a household of age 30, and so on. For comparative purposes, we also summarize in Table 4.11 the MPCs out of various income concepts obtained in independent studies of the CEDE data.

MPC1: According to the conventional life-cycle model with perfect capital markets, a household's marginal propensity to consume transitory income is roughly equal to the reciprocal of the number of years remaining in the life of the decision maker. Hence, for a population with a median age of 40 and a life expectancy of 70 years, the aggregate MPC out of windfall income should equal approximately .03. However, in Chapter II it was demonstrated that larger MPCs could be expected from a population operating in imperfect financial markets. The results in Table 4.10 are consistent with this remark. The MPCs in the first two rows range from .06 to .21 with a mean value of .14 for the first parameter set and a mean of .16 for the second set. These values conform remarkably to the propensities to consume windfall income reported by Crockett and Friend (rows 7 through 9 in Table 4.11). Their estimates range from .11 to .19 with a mean of .14.

A subsidiary proposition to that stated above is that, in a world of perfect markets, a household's marginal propensity to consume apply to the initial and new consumption points. However, experimentation with the perturbation factor did not significantly alter the propensities.
<table>
<thead>
<tr>
<th>GROUP</th>
<th>1-2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HPC1</td>
<td>.06</td>
<td>.12</td>
<td>.15</td>
<td>.13</td>
<td>.18</td>
<td>.15</td>
</tr>
<tr>
<td></td>
<td>.07</td>
<td>.11</td>
<td>.15</td>
<td>.13</td>
<td>.19</td>
<td>.15</td>
</tr>
<tr>
<td></td>
<td>.00</td>
<td>.12</td>
<td>.16</td>
<td>.13</td>
<td>.20</td>
<td>.16</td>
</tr>
<tr>
<td></td>
<td>.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HPC1*</td>
<td>.10</td>
<td>.17</td>
<td>.16</td>
<td>.17</td>
<td>.16</td>
<td>.16</td>
</tr>
<tr>
<td></td>
<td>.10</td>
<td>.15</td>
<td>.17</td>
<td>.17</td>
<td>.17</td>
<td>.17</td>
</tr>
<tr>
<td></td>
<td>.11</td>
<td>.16</td>
<td>.18</td>
<td>.17</td>
<td>.18</td>
<td>.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HPC3</td>
<td>.16</td>
<td>.24</td>
<td>.32</td>
<td>.29</td>
<td>.36</td>
<td>.32</td>
</tr>
<tr>
<td></td>
<td>.17</td>
<td>.24</td>
<td>.35</td>
<td>.30</td>
<td>.38</td>
<td>.34</td>
</tr>
<tr>
<td></td>
<td>.20</td>
<td>.27</td>
<td>.39</td>
<td>.45</td>
<td>.44</td>
<td>.44</td>
</tr>
<tr>
<td></td>
<td>.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HPC3*</td>
<td>.22</td>
<td>.33</td>
<td>.33</td>
<td>.37</td>
<td>.32</td>
<td>.35</td>
</tr>
<tr>
<td></td>
<td>.24</td>
<td>.33</td>
<td>.36</td>
<td>.37</td>
<td>.35</td>
<td>.37</td>
</tr>
<tr>
<td></td>
<td>.27</td>
<td>.34</td>
<td>.39</td>
<td>.41</td>
<td>.37</td>
<td>.39</td>
</tr>
<tr>
<td></td>
<td>.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HPCP</td>
<td>.37</td>
<td>.52</td>
<td>.68</td>
<td>.66</td>
<td>.69</td>
<td>.71</td>
</tr>
<tr>
<td></td>
<td>.45</td>
<td>.70</td>
<td>.72</td>
<td>.72</td>
<td>.72</td>
<td>.77</td>
</tr>
<tr>
<td></td>
<td>.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HPCP*</td>
<td>.57</td>
<td>.72</td>
<td>.72</td>
<td>.83</td>
<td>.68</td>
<td>.77</td>
</tr>
<tr>
<td></td>
<td>.65</td>
<td>.76</td>
<td>.74</td>
<td>.85</td>
<td>.74</td>
<td>.88</td>
</tr>
<tr>
<td></td>
<td>.72</td>
<td>.81</td>
<td>.77</td>
<td>.87</td>
<td>.78</td>
<td>.86</td>
</tr>
<tr>
<td></td>
<td>.84</td>
<td>.89</td>
<td>.82</td>
<td>.91</td>
<td>.86</td>
<td>.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Author</td>
<td>Sample</td>
<td>Consumption Concept</td>
<td>MPC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>---------------------</td>
<td>-----</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Crockett and Friend(^b/) [9, p. 52]</td>
<td>Colombia</td>
<td>$c_d Y_N$</td>
<td>.791</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. &quot;</td>
<td>&quot;</td>
<td>$c_d Y_N$</td>
<td>.735</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. &quot;</td>
<td>&quot;</td>
<td>$c_m Y_N$</td>
<td>.662</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. &quot;</td>
<td>&quot;</td>
<td>$c_d Y_R$</td>
<td>.546</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. &quot;</td>
<td>&quot;</td>
<td>$c_a Y_R$</td>
<td>.463</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. &quot;</td>
<td>&quot;</td>
<td>$c_m Y_R$</td>
<td>.412</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. &quot;</td>
<td>&quot;</td>
<td>$c_d Y_T$</td>
<td>.111</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. &quot;</td>
<td>&quot;</td>
<td>$c_a Y_T$</td>
<td>.186</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. &quot;</td>
<td>&quot;</td>
<td>$c_m Y_T$</td>
<td>.127</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Howe(^c/) [29, pp. 188-213]</td>
<td>Bogotá</td>
<td>$c_d Y_N$</td>
<td>.847</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. &quot;</td>
<td>Barranquilla</td>
<td>$c_d Y_N$</td>
<td>.898</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. &quot;</td>
<td>Cali</td>
<td>$c_d Y_N$</td>
<td>.885</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. &quot;</td>
<td>Medellín</td>
<td>$c_d Y_N$</td>
<td>.783</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14. &quot;</td>
<td>Bogotá</td>
<td>$c_d Y_T$</td>
<td>.850</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15. &quot;</td>
<td>Barranquilla</td>
<td>$c_d Y_T$</td>
<td>.711</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16. &quot;</td>
<td>Cali</td>
<td>$c_d Y_T$</td>
<td>.873</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17. &quot;</td>
<td>Medellín</td>
<td>$c_d Y_T$</td>
<td>.736</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19. &quot;</td>
<td>low-young</td>
<td>$c_L Y_T$</td>
<td>.970</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20. &quot;</td>
<td>low-old</td>
<td>$c_L Y_T$</td>
<td>.939</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21. &quot;</td>
<td>middle-young</td>
<td>$c_L Y_T$</td>
<td>.914</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22. &quot;</td>
<td>middle-old</td>
<td>$c_L Y_T$</td>
<td>.846</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23. &quot;</td>
<td>upper-young</td>
<td>$c_L Y_T$</td>
<td>.860</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24. &quot;</td>
<td>upper-old</td>
<td>$c_L Y_T$</td>
<td>.954</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25. Musgrove [45, p. LIII-23]</td>
<td>Colombia</td>
<td>$c_{d/2} Y_T$</td>
<td>.878</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
a/
Income and consumption variables are defined as follows:

$y$ equals total current income--including taxes, imputed rent, capital gains, income in kind, inheritances, and lotteries.

$y_N$ was estimated by Crockett and Friend through a regression of household disposable income on the mean income of members of the same group--groups defined by education, occupation, tenure of dwelling, and housing expense class.

$y_N$ was estimated by Howe through a regression for each city of log $y$ on age and dummy variables for education, occupation, and socio-economic stratum.

$y_p$ was estimated through a simultaneous regression of log $c$ and log $y$ on dummies for age, education, occupation, city, life-cycle stage, and employment (see Musgrove [44]).

$y_T$ equals ex ante transitory income--including lotteries and inheritances.

$y_R$ equals $(y-y_N-y_T)$.

c equals total expenditures--including taxes, imputed rent, vehicle and durable purchases.

$c_d$ equals $c$ less taxes.

$c_a$ equals $c_d$ except that imputed rent is replaced by half of mortgage payments.

$c_m$ equals $c_a$ less household durables.

$c_{d/2}$ equals $c_d$ less half of household durables.

b/
Regressions included family size, age, and the ratio of dividends plus interest to current income.

c/
Estimates derived from the ELES. Family-size variables were included.

d/
Estimates derived from the ELES. The samples in 19-24 consisted of households in three socio-economic strata (defined ex ante by neighborhood), all residents of Bogotá. Separate regressions were made for young (head under 45) and old households.
windfall income should vary directly with the age of the planner. The
MPCs reported in Table 4.10 do increase with age, but the relationship
is not pronounced. Differences in the propensities between the ages
21 and 55 are no greater than .03 for all groups except 1-2, and for
households in professional and technical occupations the differences
are less than .06. Age rigidity in marginal propensities to consume
transitory income, however, is consistent with a life-cycle theory
tailored to imperfect financial environments (see Chapter II). Group
1-2, which exhibits the strongest relationship between age and the
propensity to consume, consists of families assumed to face the most
perfect of the capital market opportunities modelled.

Finally, it is also worth noting a tendency for the estimated
MPCs to vary inversely with the permanent income class of the represen-
tative household. The mean propensities over all age cohorts for
upper, middle, and lower income groups are respectively .08, .14, and
.16 for the first set of taste and constraint parameters, and .11, .16,
and .17 for the second parameter set. As is the case with propensities
to consume income of any degree of permanency, interclass differences
reflect not only differences in tastes and family size, but also
differences in capital market opportunities.

MPC3: An income gain (loss) which is expected to continue for n
periods uniformly increases (decreases) a household's consumption possi-
bilities relative to the increase (decrease) for a gain (loss) which
is expected to continue for n-i periods. Hence, if present consumption
is normal, the marginal propensity to consume associated with the
first change should exceed the propensity of the second. This simple, but important proposition has received frequent empirical support,\(^1\) and it is consistent with the results in Table 4.10. The mean propensities over rows three and four are respectively .31 and .35. The differences between the three- and one-year MPCs are .17 for the first parameter set and .19 for the second.

The estimated marginal propensities to consume are also more dispersed when computed for income changes of longer duration. The three-year MPCs range from .14 to .44 for the first set of taste and constraint parameters, and from .22 to .44 for the second parameter set. The ranges in the estimates reflect the importance of age and income group upon the magnitude of MPC3. There is a direct relationship between the propensity to consume and the age of the household planner. The mean spreads in the MPCs over age cohorts are .09 and .07 for rows three and four respectively. The MPCs are inversely related to the permanent income class of the representative unit. The mean values for households in upper, middle, and lower income groups are respectively .20, .31, and .35 for the first parameter set, and .27, .35, and .38 for the second set.

MPCP: Consumption is most responsive to income receipts which are anticipated for the remainder of the life of the household planner. For the first set of utility and constraint parameters, the propensities to consume permanent income range from .37 to .99, with a mean

\(^1\)For the case of Colombia, Crockett and Friend find that, irrespective of the manner in which consumption is measured, the propensity to consume normal income exceeds the propensity out of residual income which, in turn, exceeds the MPC out of windfall income (see \#1 through \#9 in Table 4.11). The three income concepts were
of .69. The MPCs derived from a more plausible set of parameters, with positive rates of impatience for all cohorts, generally exceed those from the first set. The estimates in row six of Table 4.10 range from .57 to 1.04, and their mean value equals .79.

These propensities to consume conform fairly well to the normal income coefficients obtained by Crockett and Friend (see #1-3 in Table 4.11).\footnote{See note a in Table 4.11 for a description of the normal and permanent income variables used by Crockett and Friend, Howe, and Musgrove.} Their estimates, for three definitions of household consumption, range from .66 to .79 with a mean value of .73. However, the MPCs in Table 4.10 are generally smaller than the propensities to consume normal or permanent income estimated by Howe (#10-13) and by Musgrove (#25). Howe's estimates, for the four cities surveyed, range from .78 to .90, with a mean of .85. The MPC reported by Musgrove equals .88.

The strongest relationships between a household's MPC and its age and income group are found for income changes of permanent duration. Once again, age is directly related, and income class inversely related, to the estimated MPCs. For the first set of taste and constraint parameters, the mean spread over all age cohorts equals .24, and the mean values for units in upper, middle, and lower income classes are respectively .51, .67, and .77. For the second parameter set, the mean spread over all age cohorts equals .20, and the mean MPCs for households in upper, middle, and lower income classes are respectively .70, .78, and .84.
Unfortunately, there is no research involving the CEDE data which can be compared with the age-income relationships estimated here. There are several studies which delineate household MPCs by age, education, and socioeconomic group. However, these propensities are all derived from current income models of saving behavior. Current income is composed of both a permanent and a transitory component. Evidence pertaining to the effects of age, education, etc. upon the propensity to consume current income does not necessarily suggest a corresponding relationship between these variables and a household's marginal response to a permanent income change.

Evidence which does relate a household's propensity to consume out of current income to the age of its planner is contained in two sources. First, Musgrove et al. [45, p. III-51] estimate the following relation between current consumption and current income:

$$\log c = \alpha_0 + \alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3 + \alpha_4 \log y,$$

where the $A_i$'s are age dummies for the brackets 35-49, 50-64, and over 64 (households with primary earners of ages 12-34 compose the base class). The resulting estimates of the age coefficients are

$$\hat{\alpha}_1 = .014^* \quad \hat{\alpha}_2 = .038 \quad \hat{\alpha}_3 = -.010^*$$

Hence, there is a slight tendency for households with older heads to consume a larger fraction of an increment in current income. However, these results may merely reflect a positive correlation between age and

$^{1/}$An "$^*"$ denotes a parameter estimate which is not significantly different from zero at a 95% confidence level.
permanent income, and they need not imply a direct relationship between age and a household's MPC out of permanent income.1/

The second source of evidence is a study by Howe and Musgrove[30]. In an application of the extended linear expenditure system, separate estimates of MPCs out of current income were made for six groups of households defined by age and socio-economic stratum (see #19-24 in Table 4.11). For any given stratum, households with primary earners under 45 consumed a larger fraction of increments in current income than did households with primary earners 45 or over. The mean differential across the three strata was .135. However, these results do not necessarily imply a similar relationship between age and an MPC out of permanent income. It can be shown that if transitory and permanent income are uncorrelated, then

$$\text{MPC}(y) = t\text{MPC}(y_T) + (1-t)\text{MPC}(y_p),$$

where $t$ equals the ratio of the variance in $y_T$ to the variance in $y$.

Hence, sufficient conditions for an observed inverse relationship between MPC$(y)$ and age to imply an analogous relationship between

\[ ^{1/} \text{It can be shown that if consumption is a linear function of permanent income (yp), if transitory income (yt) and permanent income are uncorrelated, and if x and yT are also uncorrelated, then an estimation of "c = a1y + a2x" will yield the following estimate of } \beta: \]

$$\hat{\beta} = \frac{\text{Cov(x,y_p)}\text{Var(y_T)}}{|V_{xy}|},$$

where $|V_{xy}|$ denotes the determinant of the variance-covariance matrix for the variables $x$ and $y$. Thus, even though age and consumption are assumed to be unrelated, a direct relationship between the two will be observed if age and permanent income are positively correlated.
MPC(y_p) and age are that MPC(y_T) be a non-decreasing function of age and that t be a non-increasing function of age. The first condition, while true for a conventional life-cycle model with perfect capital markets, does not necessarily hold in an imperfect financial environment. There is no available evidence which attests to the validity of the second condition.

There is also no evidence with which to corroborate the relations exhibited in Table 4.10 between an MPC out of permanent income and the income class of the representative household. Musgrave et al. [45, p. III-39] find that, when dummy variables are included for education, a double-log regression of current income on current consumption yields parameter estimates for the dummy variables which increase with level of education. However, these results may again reflect a positive correlation between education and permanent income. Howe and Musgrove [30] obtain MPCs out of current income which decrease with higher socioeconomic strata (see #19-24 in Table 4.11). However, sufficient conditions for these results to imply an inverse relationship between socioeconomic stratum and an MPC out of permanent income are that t be a nonincreasing function, and MPC(y_T) a nondecreasing function, of socioeconomic stratum. There is some evidence which supports the first condition.1/ But the second condition, while consistent with the results in Table 4.10, has no independent support.

An application to social security

The consumption responses presented in the previous section were computed for uniform displacements in the first n coordinates of the

1/ See Musgrove [45, p. III-18].
income endowments, n denoting the degree of permanency in the income change. However, it is of some interest to investigate household reactions to asymmetric perturbations in their endowments. We now simulate the effects of social security, which lowers the income profile during the contributing years and raises it during retirement.

To this end, each income group was assigned a tax rate to be applied to the labor incomes of its household members. The tax rates were computed so that, for an individual entering the system at age 16, a discounted sum of his contributions through age 65 equals a discounted sum of the benefits he receives for the remaining five years of his life. The benefits for each retirement period were assumed to equal the labor income expected at age 65. Thus, each tax rate \( t \) was computed from the following equation:

\[
\frac{\sum_{i=1}^{50} (1+r^s)^1 - i \cdot y_i}{\sum_{i=1}^{51} (1+r^s)^1 - i \cdot y_0} = \frac{\sum_{i=51}^{55} (1+r^s)^1 - i \cdot y_i}{y_0},
\]

where the rate of return \( r^s \) was taken to be .03 for all groups. The resulting annual benefits and tax rates are presented below.

<table>
<thead>
<tr>
<th>Income Group</th>
<th>Benefits</th>
<th>Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>7410</td>
<td>.098</td>
</tr>
<tr>
<td>3</td>
<td>4562</td>
<td>.107</td>
</tr>
<tr>
<td>4</td>
<td>3835</td>
<td>.123</td>
</tr>
<tr>
<td>5</td>
<td>2156</td>
<td>.109</td>
</tr>
<tr>
<td>6</td>
<td>1377</td>
<td>.085</td>
</tr>
<tr>
<td>7</td>
<td>1314</td>
<td>.096</td>
</tr>
</tbody>
</table>
The income profile of each age-income cohort was first reduced by applying the appropriate tax rate to the labor incomes earned through the age of 65. Retirement benefits were then computed by discounting the tax contributions. Annual benefits were generated by the following formula:

$$\text{BEN} = \frac{\sum_{i=1}^{T-5} (1+r_s)^{1-i} y_i L}{\sum_{i=T-4}^{T} (1+r_s)^{1-i}}$$

Households entering the system after the age of 16, which includes all of the age cohorts modelled here, will accumulate benefits which are insufficient to maintain a retirement income equal to the salaries earned initially at 65. It was assumed that each family would seek employment during the ages 66 to 70 so as to compensate for the differential. However, this leads to an income profile which, relative to the original profile, is consistently lower during the contributing years and identical during retirement. Any additional leisure provided by the benefits would be ignored. Therefore, the last five coordinates of the endowment were further increased by one-half of the annual benefits as a proxy for the leisure enjoyed during retirement. This solution is admittedly rather ad hoc.

Household consumption responses to the introduction of social insurance are presented in Table 4.12. Each entry equals the ratio of the change in optimal current consumption to the change in current income. Separate propensities were computed for each age-income cohort. Rows one and two correspond to the first set of utility parameters and rows three and four to the second set. In rows two and
<table>
<thead>
<tr>
<th>GROUP</th>
<th>1-2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MPC</strong></td>
<td>.89</td>
<td>.75</td>
<td>.80</td>
<td>.84</td>
<td>.90</td>
<td>1.05</td>
</tr>
<tr>
<td><strong>Parameters I and No Reform</strong></td>
<td>.66</td>
<td>.62</td>
<td>.62</td>
<td>.62</td>
<td>.90</td>
<td>.87</td>
</tr>
<tr>
<td><strong>Parameters I and Reform</strong></td>
<td>.66</td>
<td>.62</td>
<td>.62</td>
<td>.62</td>
<td>.90</td>
<td>.87</td>
</tr>
<tr>
<td><strong>Parameters II and No Reform</strong></td>
<td>1.34</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td><strong>Parameters II and Reform</strong></td>
<td>1.34</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
</tr>
</tbody>
</table>

TABLE 4.12: Marginal consumption responses to social security.
four, the introduction of social security was assumed to be concurrent with capital market reform. All households were given the financial opportunities available to the upper income group.

Inaction of a social insurance program shifts the consumption possibilities set inward during the years in which contributions are made, and it will reduce or increase a household's future consumption opportunities as the return \( r^s \) is less than or greater than the real rates of interest on private investments. The results in Table 4.12 suggest that the increase in tax liabilities for households in all age cohorts is generally associated with a decrease in current consumption. Units facing high penalty rates on debt are discouraged from borrowing to maintain their present levels of consumption. Households accumulating wealth view the return to their contributions as sufficiently unattractive to warrant an increase in current consumption expenditures.

An MPC equal to unity means that, for each peso contributed to the fund, current expenditures on household consumables are reduced by one peso, and private saving remains unchanged. The mean over all entries in row one equals .80. Hence, in the absence of capital market reforms, the first set of taste parameters predict an average reduction of .80 in household consumption for each peso collected in taxes. For the second parameter set, the mean equals .94, so that private saving is reduced at the margin by .06. The ease in credit conditions associated with reforms in capital markets enables households to better maintain their present levels of consumption. The means over all entries in rows two and four are respectively .47 and .61. And for
households in group six, assumed to possess relatively high rates of impatience, the reforms are significant enough to induce a positive consumption response to the increase in taxes.

The effects upon the propensities of age and income group are most readily ascertained by comparing the mean MPCs computed over all income and age cohorts respectively. These statistics are presented in Table 4.13 for each parameter set and financial regime.

Households with young primary earners reduce their present consumption by a greater amount at the margin than do those with older heads. In the face of high interest rates on debt, young families are discouraged from borrowing to support their present expenditures. On the other hand, families with older primary earners can reduce their saving and/or wealth to maintain their consumption levels. The spreads in the mean MPCs between cohorts of age 21 and 55 are .17 for the first set of utility parameters and .28 for the second set. Reforms in capital markets, however, destroy this relationship.

In the absence of capital market reforms, upper income groups tend to reduce their present consumption expenditures by a smaller amount at the margin than do the lower income groups. The spreads in the mean MPCs between cohorts of upper and lower income status are -.19 for the first parameter set and -.02 for the second set. These results, however, primarily reflect the poorer financial opportunities faced by lower income households. When coupled with financial reform, the introduction of social security induces a larger reduction in consumption from upper income households than from those in the lower income brackets.
TABLE 4.13: Averages of Marginal Consumption Responses to Social Security

<table>
<thead>
<tr>
<th>Age</th>
<th>Financial Regime</th>
<th>MPC for Parameters</th>
<th>Age</th>
<th>Financial Regime</th>
<th>MPC for Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>.87 .1.06</td>
<td>21</td>
<td>Reform</td>
<td>.37 .49</td>
</tr>
<tr>
<td>30</td>
<td>No</td>
<td>.83 .98</td>
<td>30</td>
<td>Reform</td>
<td>.48 .60</td>
</tr>
<tr>
<td>40</td>
<td>No</td>
<td>.81 .94</td>
<td>40</td>
<td>Reform</td>
<td>.45 .58</td>
</tr>
<tr>
<td>55</td>
<td>No</td>
<td>.70 .78</td>
<td>55</td>
<td>Reform</td>
<td>.60 .76</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income Class</th>
<th>Financial Regime</th>
<th>MPC for Parameters</th>
<th>Income Class</th>
<th>Financial Regime</th>
<th>MPC for Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper</td>
<td>No</td>
<td>.68 .94</td>
<td>upper</td>
<td>Reform</td>
<td>.68 .94</td>
</tr>
<tr>
<td>middle</td>
<td>No</td>
<td>.78 .90</td>
<td>middle</td>
<td>Reform</td>
<td>.61 .56</td>
</tr>
<tr>
<td>lower</td>
<td>No</td>
<td>.87 .96</td>
<td>lower</td>
<td>Reform</td>
<td>.36 .52</td>
</tr>
</tbody>
</table>
**Interest rate effects**

A household's optimal consumption path depends not only upon its income endowment, but also upon the interest rates available on assets and debt. In Table 4.14 we present, for each age-income cohort, estimates of interest elasticities relating changes in consumption to a change in the rate of return on saving, a change in the borrowing rate of interest, and an equal and opposite change in both the borrowing and the lending rate. The estimates were made by first calculating new values for \( r \) and \( \sigma \) which correspond to the perturbed values for \( r^a \) and/or \( r^b \). The interest changes used in all experiments are reproduced below.

<table>
<thead>
<tr>
<th>Income class</th>
<th>Change in ( r^a )</th>
<th>Change in ( r^b )</th>
<th>Change in ( r^a ) and ( -r^b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper</td>
<td>.05</td>
<td>-.05</td>
<td>.03</td>
</tr>
<tr>
<td>middle</td>
<td>.05</td>
<td>-.15</td>
<td>.05</td>
</tr>
<tr>
<td>lower</td>
<td>.05</td>
<td>-.15</td>
<td>.05</td>
</tr>
</tbody>
</table>

Given new values for the constraint parameters, interest elasticities were computed by taking the ratio of the percentage change in optimal household consumption to the percentage change in \( r^a \) and/or \( r^b \). \(^{1/}\) To facilitate an analysis of these elasticities, averages over age and income group are also presented in Table 4.15.

**Changes in lending rates:** The effects of changes in interest rates are traditionally decomposed into two parts: a substitution effect and an income effect. For an increase in \( r^a \), the substitution

\(^{1/}\)The denominator for a simultaneous change in \( r^a \) and \( r^b \) was taken as an equally weighted average of the two percentage changes.
<table>
<thead>
<tr>
<th>GROUP</th>
<th>ELASTICITY</th>
<th>1-2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters I</td>
<td>Change in $r^a$</td>
<td>-.08</td>
<td>-.03</td>
<td>.00</td>
<td>.01</td>
<td>-.05</td>
<td>-.06</td>
</tr>
<tr>
<td>Parameters I</td>
<td>Change in $r^b$</td>
<td>-.03</td>
<td>-.03</td>
<td>.00</td>
<td>.00</td>
<td>-.06</td>
<td>-.01</td>
</tr>
<tr>
<td>Parameters II</td>
<td>Change in $r^a$</td>
<td>.12</td>
<td>.03</td>
<td>-.02</td>
<td>-.06</td>
<td>.15</td>
<td>.05</td>
</tr>
<tr>
<td>Parameters II</td>
<td>Change in $r^b$</td>
<td>.01</td>
<td>.00</td>
<td>.00</td>
<td>.04</td>
<td>.12</td>
<td>.04</td>
</tr>
<tr>
<td>Parameters II</td>
<td>Change in $r^a$</td>
<td>-.02</td>
<td>-.02</td>
<td>.00</td>
<td>-.01</td>
<td>.00</td>
<td>-.01</td>
</tr>
<tr>
<td>Parameters II</td>
<td>Change in $r^b$</td>
<td>-.03</td>
<td>-.03</td>
<td>-.01</td>
<td>-.01</td>
<td>.00</td>
<td>-.01</td>
</tr>
<tr>
<td>Parameters II</td>
<td>Change in $r^a$</td>
<td>-.22</td>
<td>-.15</td>
<td>-.11</td>
<td>-.06</td>
<td>-.03</td>
<td>-.05</td>
</tr>
<tr>
<td>Parameters II</td>
<td>Change in $r^b$</td>
<td>-.33</td>
<td>.32</td>
<td>.12</td>
<td>.18</td>
<td>.20</td>
<td>.28</td>
</tr>
<tr>
<td>Parameters II</td>
<td>Change in $r^a$</td>
<td>.03</td>
<td>.03</td>
<td>.03</td>
<td>.03</td>
<td>.03</td>
<td>.03</td>
</tr>
<tr>
<td>Parameters II</td>
<td>Change in $r^b$</td>
<td>-.04</td>
<td>-.04</td>
<td>-.02</td>
<td>-.02</td>
<td>-.02</td>
<td>-.02</td>
</tr>
</tbody>
</table>
TABLE 4.15: Averages of Interest Elasticities

<table>
<thead>
<tr>
<th>Age</th>
<th>$\overline{\text{ETA}(r^a)}$ for Parameters I</th>
<th>$\overline{\text{ETA}(r^a)}$ for Parameters II</th>
<th>$\overline{\text{ETA}(r^b)}$ for Parameters I</th>
<th>$\overline{\text{ETA}(r^b)}$ for Parameters II</th>
<th>$\overline{\text{ETA}(r^a,r^b)}$ for Parameters I</th>
<th>$\overline{\text{ETA}(r^a,r^b)}$ for Parameters II</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>-.04</td>
<td>-.07</td>
<td>.16</td>
<td>.28</td>
<td>-.01</td>
<td>-.01</td>
</tr>
<tr>
<td>30</td>
<td>-.03</td>
<td>-.05</td>
<td>.10</td>
<td>.21</td>
<td>-.02</td>
<td>-.02</td>
</tr>
<tr>
<td>40</td>
<td>-.03</td>
<td>-.05</td>
<td>.10</td>
<td>.17</td>
<td>-.02</td>
<td>-.02</td>
</tr>
<tr>
<td>55</td>
<td>-.02</td>
<td>-.02</td>
<td>.02</td>
<td>.04</td>
<td>-.02</td>
<td>-.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income Class</th>
<th>$\overline{\text{ETA}(r^a)}$ for Parameters I</th>
<th>$\overline{\text{ETA}(r^a)}$ for Parameters II</th>
<th>$\overline{\text{ETA}(r^b)}$ for Parameters I</th>
<th>$\overline{\text{ETA}(r^b)}$ for Parameters II</th>
<th>$\overline{\text{ETA}(r^a,r^b)}$ for Parameters I</th>
<th>$\overline{\text{ETA}(r^a,r^b)}$ for Parameters II</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper</td>
<td>-.04</td>
<td>-.14</td>
<td>.04</td>
<td>.17</td>
<td>-.02</td>
<td>-.02</td>
</tr>
<tr>
<td>middle</td>
<td>-.03</td>
<td>-.04</td>
<td>.06</td>
<td>.18</td>
<td>-.02</td>
<td>-.02</td>
</tr>
<tr>
<td>lower</td>
<td>-.03</td>
<td>-.03</td>
<td>.14</td>
<td>.18</td>
<td>-.02</td>
<td>-.01</td>
</tr>
</tbody>
</table>
effect discourages present consumption, which has become relatively more expensive for households currently holding assets and for young debtors who expect to hold assets in the future. The income effect, on the other hand, favors present consumption, as real incomes have also increased. Virtually all elasticities in rows one and four of Table 4.14 are nonpositive, indicating a predominance of the former over the latter. The means across all age-income cohorts equals -.03 for the first set of taste and constraint parameters, and -.05 for the second set. The ranges for both sets are respectively -.10 through .03 and -.22 through .03. Neither age nor income class significantly influences the magnitudes of the elasticities. The groups which exhibit the most responsiveness (in absolute value) to a change in $r^a$ are households with primary earners in professional, technical, or blue collar occupations. The lower interest elasticities reflect the higher elasticities of substitution used for these groups.

The only other evidence on the interest elasticity of Colombian saving or consumption is contained in a study by Hanson [25]. Hanson's estimates are derived from the 1968 rise in the yield on BCH cedula bonds. The BCH bonds were reissued at a nominal rate of 9.5%. They sold at a 14% discount and were tax exempt, so that a comparable market rate for a 40% marginal tax bracket would be 15.5%. When compared with the effective yield of 10.5% on the old cedulas, this represents a 47% increase. Hanson estimates an elasticity of financial saving with respect to the rate of interest on the aggregate Colombian portfolio of 3.8. After adjusting for the rise in the yields on government bonds which followed in 1968 and 1969, his estimate is still high at 7.1.
Hanson [25, p. 22] concludes that "the interest elasticity of saving, or at least saving in the form of financial assets, is extremely high. Of course, these calculations are based upon an historical incident and are somewhat upward biased, because of the contemporary changes in exchange policy and financial institutions which increased (measured) saving from its previous low level."

The estimates in Table 4.14 are significantly lower than those reported by Hanson. Multiplying each entry by -10 (for a rough conversion from a consumption to a saving elasticity), our results place the interest elasticity of Colombian saving between .3 and .5. Even the upper income groups, which hold the majority of aggregate wealth, exhibit an elasticity of only 1.4 when averaged over the second parameter set. Our estimates are much more in line with those obtained by Wright [58] and Weber [57] for U.S. data. Wright finds that a one percent rise in interest rates will stimulate an increase in private saving of .19-.24 of one percent, while Weber reports a negative savings response to a higher interest rate.

Changes in borrowing rates: The largest elasticities in Table 4.14 are those computed for changes in interest rates on debt. The average increase in household consumption associated with a one percent decrease in $r^b$ equals .10 of one percent for the first parameter set and .18 for the second. A majority of the variation in the estimates between rows two and five occurs for groups 1-2, 3, and 5. These are also the groups whose impatience rates from the second parameter set exceed those from the first set by at least .10. The elasticities are closely related to age. A household with a primary earner of age 21
increases its consumption by .16-.28 of one percent in response to a one percent decrease in the borrowing rate. A household with a head 55 years of age, on the other hand, exhibits an interest elasticity of .02-.04. The higher elasticities for young households reflect the greater illiquidity and greater relevance of borrowing opportunities associated with the early stages of a life cycle.

**Changes in borrowing and lending rates:** Given a simultaneous increase in $r_a$ and decrease in $r_b$, the percentage increase in consumption predicted for an average Colombian household equals -.02. Consumption elasticities are uniform across age-income cohorts, with a range of 0.0 to -.04. Virtually all responses are negative due to the fact that the absolute values of the percentage changes in lending rates exceeded those used for the borrowing rates.

**Welfare analysis**

It is of some interest to measure the welfare costs or benefits which accrue to households having to trade in one set of financial markets rather than in another. Lifetime consumption plans of representative families with head 16 years of age were computed for the second set of utility parameters and for various assumptions about capital markets and income profiles. The consumption profile of each income group was then discounted and weighted by the appropriate demographic variables to arrive at the lifetime utility produced by the profile. The results are presented in the upper halves of the cells in Table 4.16. To compare utility levels between regimes, we also present the compensations (in **tens** of pesos) to household
<table>
<thead>
<tr>
<th>GROUP</th>
<th>1-2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anarchy</td>
<td>-24x10^-3</td>
<td>-24x10^-7</td>
<td>-27x10^-5</td>
<td>-23x10^-8</td>
<td>-20x10^-2</td>
<td>-11x10^-7</td>
</tr>
<tr>
<td></td>
<td>194</td>
<td>2</td>
<td>32</td>
<td>2</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>First Interest Set</td>
<td>-89x10^-5</td>
<td>-24x10^-7</td>
<td>-24x10^-5</td>
<td>-23x10^-8</td>
<td>-20x10^-2</td>
<td>-88x10^-8</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Second Interest Set</td>
<td>-11x10^-6</td>
<td>-24x10^-7</td>
<td>-24x10^-5</td>
<td>-23x10^-8</td>
<td>-20x10^-2</td>
<td>-88x10^-8</td>
</tr>
<tr>
<td></td>
<td>85</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Third Interest Set</td>
<td>-87x10^-5</td>
<td>-24x10^-7</td>
<td>-24x10^-5</td>
<td>-23x10^-8</td>
<td>-20x10^-2</td>
<td>-87x10^-8</td>
</tr>
<tr>
<td></td>
<td>-11</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>SS and First Interest Set</td>
<td>-11x10^-4</td>
<td>-34x10^-7</td>
<td>-33x10^-5</td>
<td>-36x10^-8</td>
<td>-22x10^-2</td>
<td>-13x10^-7</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>81</td>
<td>82</td>
<td>45</td>
<td>25</td>
<td>22</td>
</tr>
<tr>
<td>SS and Second Interest Set</td>
<td>-13x10^-4</td>
<td>-34x10^-7</td>
<td>-33x10^-5</td>
<td>-36x10^-8</td>
<td>-22x10^-2</td>
<td>-13x10^-7</td>
</tr>
<tr>
<td></td>
<td>165</td>
<td>81</td>
<td>85</td>
<td>45</td>
<td>26</td>
<td>22</td>
</tr>
<tr>
<td>SS and Third Interest Set</td>
<td>-11x10^-4</td>
<td>-34x10^-7</td>
<td>-33x10^-5</td>
<td>-36x10^-8</td>
<td>-22x10^-2</td>
<td>-13x10^-7</td>
</tr>
<tr>
<td></td>
<td>86</td>
<td>81</td>
<td>80</td>
<td>45</td>
<td>25</td>
<td>22</td>
</tr>
</tbody>
</table>
consumption necessary to equate utilities for the consumption path computed under a given set of assumptions about capital markets and expected labor incomes, and for the path generated under the second set of constraint parameters and under the income profiles in Appendix D. Each entry represents the permanent (absolute) increase or decrease in consumption required to achieve indifference between the given and base regimes.

The entries in the first row of Table 4.16 denote utilities for consumption paths which are optimal given a complete absence of borrowing and lending markets. Households in upper income brackets require a permanent adjustment in their consumption of 1,940 pesos as compensation for a loss of their current trading opportunities. However, the only other significant compensation is for households with secondary education and blue collar occupations. The peso equivalents for groups 3, 5, 6, and 7 are all less than 90 pesos. This indicates not only that the capital market opportunities available to these groups are relatively poor, but that their demographic and income profiles are well synchronized.

The entries in the second row of Table 4.16 correspond to the base regime. Entries in the third row were computed under more pessimistic assumptions about the way in which capital market imperfections might impinge upon optimal household consumption plans. The values for r used in row three are identical to those used in row two. However, the parameter σ was changed to .5 for all cohorts.\(^1\) Only the

\(^1\)Recall that σ indexes the severity of imperfections in capital markets. Larger values for σ imply a more truncated set of consumption possibilities.
upper income group suffers badly from additional truncations in its opportunity set. The average compensation for all other groups equals six pesos, again indicating that their demographic and income profiles are relatively well synchronized. The utilities presented in row four were computed under the assumption that $\sigma = 0$, i.e., that the interest rate on assets equals the interest rate on debt. A familiar result emerges. The average annual adjustment in household consumption necessary to achieve indifference between current utilities and those associated with perfect capital markets equals six pesos for all groups but 1-2. Households in upper income brackets, however, would benefit substantially from a situation of perfect markets.

Welfare estimates were also made for consumption plans computed under the social security system described earlier. These results are presented in rows five through seven of Table 4.16. The interest parameters for each of these rows correspond one-to-one with the parameters used in rows two through four. The largest reductions in lifetime utilities occur for consumption paths which are optimal under a system of social insurance. With the exception of the upper income group, these losses are also independent of the nature of capital markets. Even for a set of perfect financial opportunities, the decrease in liquidity and forced saving which occur during the contributing years lead to a substantial decrease in real incomes. The size of each loss varies directly with the permanent income class of the given household. Average compensations over row seven are respectively 860, 810, and 310 pesos for households in upper, middle, and lower income groups.
CHAPTER V

SUMMARY AND CONCLUSIONS

The assumptions which are most fundamental to the conventional life-cycle model are (1) that a household bases its current consumption and savings decision upon lifetime projections of labor incomes and family size, i.e., planning horizons are extremely long; and (2) that a household is free to synchronize its income and preferred consumption profiles by borrowing and lending at a single rate of interest, i.e., capital markets are perfect. If both assumptions are valid, then effective planning horizons will also be long. Current consumption will depend upon the human and non-human wealth of the consuming unit, and not upon its current income. Windfall income will be consumed over the entire lifespan of the household. And inelastic responses to changes in interest rates necessarily suggest a lack of substitutability between present and future consumption.

Empirical testing of the model, however, has produced evidence, especially for developing countries, which is consistent with very short effective planning horizons. Current income is often as powerful in predicting consumption expenditures as are long-term income concepts. Marginal propensities to consume windfall income are not particularly sensitive to age, and they are too high to corroborate a strict version of the life-cycle model.
It was argued in Chapter II that the appearance of these empirical anomalies did not necessarily invalidate the model's first assumption. The origin of behavior which is often misconstrued as indicating short innate planning horizons may be the imperfections which pervade many financial markets. Expected family needs and incomes may be largely irrelevant to households facing high penalty rates on borrowing, quantitative restrictions on debt, and imperfect home rental markets. Current income can take on a significant role of its own, even for families which are very conscious of the future.

To empirically test the usefulness of an intertemporal planning problem as a theoretical basis for household consumption analysis, the conventional life-cycle model must be suitably tailored when used in the context of an imperfect financial environment. In Chapter III we formulated a utility maximization problem whose constraints permit an accumulation of debt and assets, but do not impose equality between borrowing and lending rates of interest. The scope for intertemporal trading was summarized by a transformation frontier which depends upon the household's labor income endowment, an interest rate defining the marginal rate of transformation at the endowment, and a parameter $\sigma$ designed to index the severity of imperfections in funds markets. When $\sigma$ equals 0, capital markets are perfect and one interest rate prevails on the market. As $\sigma$ becomes positive, there is a disparity between the interest rates on savings and debt. In the limit, as $\sigma$ approaches infinity, the household's endowment is its only efficient consumption point.
An optimal consumption function was then deduced from the above planning problem. Its qualitative properties are consistent with the propositions outlined in Chapter II concerning consumer behavior in imperfect financial environments. The importance of current income as a determinant of current consumption varies directly (and normal or life-cycle income inversely) with the parameter \( \sigma \), as does the household's marginal consumption response to windfall income. The interest elasticity of saving varies inversely with \( \sigma \).

The consumption function was estimated using the CEDE budget data collected from the four principal Colombian cities. A limited number of regressions were made in which the constraint parameter \( \sigma \) was endogenous to the statistical search. An assumption of perfect capital markets consistently and significantly reduced the explanatory power of the consumption function. In the final regressions, both constraint parameters were extraneously evaluated at values for which the smoothed constraint best approximates a constraint set constructed from observed borrowing and lending rates of interest. The sensitivity tests performed indicate that a one percentage point decrease in the lending rate leads to a change in the estimates of the elasticity of substitution and impatience rate of \(-4.5\% \) and \(-30\% \) respectively. Estimates of household taste parameters are very sensitive to assumptions about prevailing capital markets.

Separate regressions were made for six groups of households with various educational backgrounds and occupations. The mean estimate of the elasticity of substitution \( \gamma \) was .3, a value which is broadly consistent with (although slightly smaller than) estimates which have
been made for U.S. households. Although none of our estimates were statistically distinguishable between groups for a 90 percent confidence level, they did tend to increase with the permanent income class of the sampled unit. The mean values were .34 for the upper income group, .26 for the middle income groups, and .24 for the poorest groups.

Estimates of the impatience rate $\delta$ were less satisfactory. Only one had an upper confidence limit which was negative at a 95 percent level. However, the point estimates were negative in five out of eight cases, and their mean value was -.055. The negative estimates of $\delta$ suggest that life-cycle motives for saving are relatively unimportant to Colombian households. Because of imperfections in capital markets, the need to save or borrow for purposes of smoothing per capita consumption may have been largely circumvented through adjustments in work effort and/or family planning. Early accumulation, as a precautionary fund, to meet downpayment requirements on future home purchases, to acquire materials for self-constructed dwellings, to finance outlays on household durables, or for purposes of social status, can be erroneously interpreted as a negative rate of impatience.

The primary focus of this research is not upon the estimates of the taste parameters per se, but rather upon household responses to changes in policy variables which follow from our modelling of the consumer's economic problem. In Chapter IV estimates were presented for a purely transitory income change, a change in income which is
expected to persist for three years, a permanent income change, the introduction of a social security system, and an assortment of changes in borrowing and lending rates of interest. The results are sensitive to the kind of financial environment within which a family is assumed to operate. All estimates in Chapter IV were computed for realistic differences in borrowing and lending opportunities across income classes. Separate estimates were made for two sets of utility parameters—those obtained from the CEDE data search and those for which small, but positive rates of impatience were assumed. On the whole, the results were only marginally sensitive to changes in these parameters. We now summarize the principal findings of Chapter IV.4.

1. An average Colombian household exhibits a marginal propensity to consume transitory income of .15. The propensities are relatively age-invariant. Differences between households with primary earners of ages 21 and 55 are generally no greater than .03. In contrast to the conventional life-cycle model, the effects of fiscal stabilization policies are invariant to the age distribution of the population if funds markets are imperfect.

2. Marginal consumption responses to income receipts which are expected to continue for n periods exceed the propensities computed for income receipts which are expected to continue for (n-1) periods. For one-year, three-year, and permanent income changes, the mean propensities were .15, .33, and .74 respectively. Hence, the net effect on aggregate saving of a redistribution of income depends not only upon differences in tastes between the recipient and donor classes,
but upon differences in the degree of permanency associated with
the transfer receipt or payment.

3. An average Colombian household exhibits a marginal propensity
to consume permanent income of .69-.79. Higher propensities were
obtained for households with old primary earners and in lower income
brackets. For the parameter set with positive impatience rates, the
mean spread over all age cohorts was .20, and the mean propensities
for families in upper, middle, and lower income classes were .70,
.78, and .84 respectively.

4. The increase in tax liabilities associated with an introduction
of social security generally leads to a decrease in current consumption.
Households facing high penalty rates on debt are discouraged from
borrowing to maintain their present consumption levels. Households
accumulating wealth view a return to their contributions of 3% as
sufficiently unattractive to warrant an increase in their current
consumption expenditures. On average, a Colombian family will de-
crease its consumption by .87 for each peso collected in taxes.
Private saving is reduced at the margin by .13 pesos.

Households with young primary earners reduce their present con-
sumption by a greater amount than do those with older heads who can
reduce their saving and/or wealth to maintain consumption levels.
Upper income groups tend to decrease their consumption by a smaller
amount at the margin than do the lower income groups. These results,
however, primarily reflect the poorer financial opportunities faced
by lower income households. When coupled with financial reform, the
introduction of social insurance induces a larger reduction in consumption from upper income families than for those in the lower income brackets.

5. Simulations of household responses to changes in the lending rate place the interest elasticity of Colombian saving between .3 and .5. These estimates are significantly smaller than Hanson's estimate of 7.1, and they are much more in line with those obtained by Weber and Wright with U.S. data. Upper income groups are most responsive to changes in the lending rate. Their elasticity is roughly 1.4 based upon the second set of utility parameters.

6. The largest responses to changes in interest rates were found for changes in borrowing rates. The average increase in household consumption associated with a one percent change in $r^b$ was .14 of one percent. The highest elasticities were obtained for young families, reflecting the greater illiquidity and greater relevance of borrowing opportunities during the early stages of a life-cycle.

7. Estimates were also made of the welfare costs or benefits which accrue to households having to operate in one set of financial markets rather than in another. For all but the upper income group, the average annual adjustment in household consumption necessary to achieve indifference between current utilities and those associated with perfect capital markets was six pesos. Households in professional and technical occupations require an additional 85 pesos in their annual consumption as compensation for having to trade in imperfect funds markets,
The largest reductions in lifetime utilities occurred for consumption paths which are optimal under a system of social insurance. Even for a set of perfect financial opportunities, the decrease in liquidity and forced saving during the contributing years lead to an annual welfare loss of 860, 810, and 310 pesos for households in upper, middle, and lower income groups.
APPENDIX A

SOLUTION OF MODEL IN CHAPTER III

The consumption function presented in equation (3.4) of section III.3 is obtained from the Kuhn-Tucker conditions which characterize an optimal solution to the household's planning problem. Before proceeding with this derivation, we first note the following properties of the utility function and constraint equation:

(a) The utility function $U(c)$ is positively monotonic in its arguments, i.e.

$$\frac{\partial U}{\partial c_i} = (1+\delta)^{1-i} N_i c_i^{p-1} > 0.$$ 

(b) The marginal utility of $c_i$ approaches $\pm \infty$ as $c_i$ approaches 0.

(c) $U$ is a concave function of $c$.

(d) The constraint equation $[\sum_{i=1}^{T} (1+r)^{1-i} (c_i/y_i)^{\sigma} c_i]$ is a convex function of $c$.

(e) It is assumed that $\Pi \nu > 0$, so that the constraint set obeys Slater's condition.

Assumption (a) ensures that, at an optimum, all resources will be fully utilized and that the Lagrange multiplier $\lambda$ associated with the constraint will be positive. Assumption (b) guarantees that consumption will be positive in every period. The remaining assumptions (c)-(e) establish both necessity and sufficiency for the Kuhn-Tucker conditions.
Given that the $c_i$'s and $\lambda$ are positive, the first-order conditions for the household's problem are

\[ (1+\delta)^{1-i} N_i c_i^{1-\sigma} - \lambda (1+\sigma)(1+r)^{1-i} \left( \frac{c_i}{y_i} \right)^\sigma = 0 \quad i=1, \ldots, T \]  
(A.1)

\[ PV - \sum_{i=1}^{T} (1+r)^{1-i} \left( \frac{c_i}{y_i} \right)^\sigma c_i = 0. \]

Eliminating $\lambda$, this system of $T+1$ equations can be reduced to a system of $T$ equations in as many unknowns.

\[ N_i^{\sigma+1} \left( \frac{y_i}{c_i} \right)^{\sigma-1} \left( \frac{c_i}{y_i} \right)^{\sigma-1} - \left( \frac{1+r}{1+\delta} \right) N_{i+1}^{\sigma+1} \left( \frac{y_{i+1}}{c_{i+1}} \right)^{\sigma-1} c_{i+1} = 0 \quad i=1, \ldots, T \]  
(A.2)

\[ PV - \sum_{i=1}^{T} (1+r)^{1-i} \left( \frac{c_i}{y_i} \right)^\sigma c_i = 0 \]

Finally, after some straightforward exponential manipulation, these $T$ equations can be written in matrix form as

\[
\begin{bmatrix}
\varphi_1 & \varphi_2 & \varphi_3 & \cdots & \varphi_T \\
N_1 y_1 & -RDN_2 y_2 & 0 & \cdots & 0 \\
0 & \varphi_1 & \varphi_2 & \cdots & \varphi_T \\
0 & N_2 y_2 & -RDN_3 y_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & -RDN_T y_T \\
y_1^{-\sigma} (1+r)^{1-\sigma} y_2^{-\sigma} (1+r)^{-2} y_3^{-\sigma} \cdots (1+r)^{1-T} y_T^{-\sigma} & \cdots & \cdots & \cdots & (1+r)^{-T} y_T^{-\sigma} \\
\end{bmatrix} \begin{bmatrix}
c_1^{\sigma+1} \\
c_2^{\sigma+1} \\
c_3^{\sigma+1} \\
\vdots \\
c_T^{\sigma+1} \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
PV \\
\end{bmatrix}
\]

where $\varphi_1 = -\frac{(1+\sigma)(1+\sigma)}{(\sigma+1+\sigma)}$, $\varphi_2 = -\frac{\sigma(1+\sigma)}{(\sigma+1+\sigma)}$, and $RD = \left( \frac{1+r}{1+\delta} \right)^{-\frac{(1+\sigma)}{(\sigma+1+\sigma)}}$. 


An application of Cramer's rule yields the following expression:

\[
S_i^\sigma + 1 = \frac{\gamma(T-1) \left( \frac{1+\sigma}{1+\sigma\gamma} \right) \left( \frac{\sigma\gamma(1+\sigma)}{1+\sigma\gamma} \right) \left( \frac{1+\sigma}{1+\sigma\gamma} \right)}{\sum_{i=1}^T (1+r)^{T-i} \left( \frac{1+\sigma}{1+\sigma\gamma} \right) \left( \frac{\sigma\gamma-1}{1+\sigma\gamma} \right) \left( \frac{1+\sigma}{1+\sigma\gamma} \right)} \cdot y_i \cdot N_i ,
\]

where \( \gamma = (1+\rho)^{-1} \). Equation (3.4) in the text follows by taking logarithms of (A.3).
APPENDIX B

CONSTRAINT APPROXIMATIONS

$T = 2$

For a two-dimensional problem, the error in approximating the constraints (2.4) with a class of functions defined by (3.1) is measured through the $c_2$-coordinates of the constraint surfaces for (feasible and efficient) consumption vectors which are consistent with $A_1 > 0$, and through the $c_1$-coordinates for vectors associated with $B_1 > 0$ (see figure B.1). First generate a grid $[\tilde{c}_1(1),...,\tilde{c}_1(P_1),\tilde{c}_2(1),...,\tilde{c}_2(P_2)]$ where the $\tilde{c}_1(i)$ coordinates provide an equal density partition of the interval $[0,y_1]$, and the $\tilde{c}_2(i)$ coordinates similarly partition the interval $[0,y_2]$. The densities of each subdivision are governed by the relationships $(P_1/P_2) = (y_1/y_2)$ and $P_1 + P_2 = T$. For every point in the grid, now use the constraints (2.4) and the data $(r^a, r^b, y_1, y_2)$ to generate an actual consumption vector $[c_2(1),...,c_2(P_1),c_1(1),...,c_1(P_2)]$. Thus $c_2(i)$ denotes the maximum level of future consumption which can be achieved given a present consumption level of $\tilde{c}_1(i)$, and $c_1(i)$ denotes the efficient level of present consumption given $\tilde{c}_2(i)$. A vector of predicted consumption is then formed from the $\tilde{c}$ grid, the values for $(r,\sigma,y_1,y_2)$ and the approximating constraint (3.1). Denote this vector by $[\hat{c}_2(1),...,\hat{c}_2(P_1),\hat{c}_1(1),...,\hat{c}_1(P_2)]$. The approximation problem is formulated as

$$\min_{(r,\sigma)} E = \sum_{i=1}^{P_1} [c_2(i)-\hat{c}_2(i)]^2 + \sum_{i=1}^{P_2} [c_1(i)-\hat{c}_1(i)]^2 \quad (B.1)$$
Figure B.1: Approximation for $T = 2$
The optimal values for $r$ and $\sigma$ were obtained by solving (B.1) given a value for $\tilde{P}$ at which $(r^*, \sigma^*)$ stabilized.

$T = 3$

The constraint set in a three-dimensional problem can be represented by a sequence of two-dimensional constraints in the $(c_1, c_2)$-plane which correspond to feasible values for consumption in the third period. Hence, the approximation error for $T = 3$ can be measured by integrating over the two-dimensional errors generated by all feasible values for $c_3$. To this end, first construct a grid $[\tilde{c}_3(1), \ldots, \tilde{c}_3(\tilde{P})]$ which provides an equal density partition of the interval $[0, c_3^{\text{max}}]$, where $c_3^{\text{max}} = (1 + r^a)^2 y_1 + (1 + r^a) y_2 + y_3$. Each $\tilde{c}_3(i)$ leads to an actual and predicted constraint locus in the $(c_1, c_2)$-plane. In contrast to the earlier process, however, a measurement of two-dimensional errors is now complicated by the absence of a point at which (2.4) and (3.1) necessarily coincide. For $T = 2$, this point occurred at the income endowment $(y_1, y_2)$. We therefore measure the two-dimensional errors through an angular partition of the $(c_1, c_2)$-plane (see figure B.2). Let $c(i, j)$ denote the efficient values for $c_1$ and $c_2$ which are consistent with the constraints (2.4), the grid point $\tilde{c}_3(i)$, and a ratio $(c_2/c_1) = \sigma(j)$. Similarly, let $\hat{c}(i, j)$ denote the predicted consumption vector which is derived from the constraint (3.1), $\tilde{c}_3(i)$, and the $(c_2/c_1)$ ratio $\sigma(j)$. The approximation problem is written as
Figure B.2: Approximation for $T = 3$

\[ y_2^K = y_2 + (y_3 - c_3)(1 + r^{a,b})^{-1} \]
\[ \min_{P} \tilde{E} \tilde{P} \]

\[ E = \sum_{i=1}^{\tilde{P}} \sum_{j=1}^{\tilde{P}} |c(i,j) - \hat{c}(i,j)|, \quad (B.2) \]

where \( |\cdot| \) is the Euclidean norm and where \( \tilde{P} \) corresponds to the density of the two-space partition.
APPENDIX C

THE CEDE HOUSEHOLD BUDGET DATA¹

The household data examined in this study was collected during 1967-68 by the Centro de Estudios de Desarrollo (CEDE, Center for Development Studies) of the Universidad de los Andes, Bogotá. Households in the four major cities (Bogotá, Barranquilla, Cali, and Medellín) were interviewed during the months of February, August, and November of 1967 and May of 1968. The survey contained a variety of information on expenditures, incomes, and socio-demographic variables.

The basic sampling unit was the household, defined by a group of people sharing the same kitchen and a sizable portion of food expenses and not restricted to members related by blood or marriage. Households were selected first by neighborhoods, then by blocks, and finally by dwellings. Sampling by neighborhoods was conducted proportionately to the population. High income families were overrepresented during the later stages to obtain a more representative sample of their income and expenditure patterns. The data base was originally designed to include several panels of budget data across quarters. However, due to a high mortality rate, few households were interviewed in all periods. For purposes of this study, households were pooled over time and treated as independent observations.

¹This section provides a brief overview of the CEDE data, its collection and preparation. The reader is referred to Musgrove [43] for a thorough description of sampling procedures, to Howe and Villaveces [31].
The CEDE survey was conducted as part of a program for Joint Studies in Latin American Economic Integration (ECIEL). The data analyzed here were obtained from the ECIEL data file. Information in the file was cleaned to correct for invalid, inconsistent, or extreme values, and the file contains several income and expenditure aggregates and dummy variables.

In the original survey, three reporting periods were used: weekly, monthly, and quarterly (with smaller items recorded over shorter periods and larger items recorded for the previous quarter). All of the income and expenditure data used in this study were in tens of pesos colombianos per quarter.

The analysis of Chapter IV is performed for groups of households sharing particular educational and occupational characteristics. Education classes were defined as follows:

\[ \begin{align*}
E_1 &= \text{some university or post-graduate education} \\
E_2 &= \text{some secondary education} \\
E_3 &= \text{some primary education} \\
E_4 &= \text{illiterate or no education}.
\end{align*} \]

Occupation was originally coded into 96 separate categories. The aggregated categories used here are those provided in the ECIEL file:

\[ \begin{align*}
0_1 &= \text{professional, technical, managerial} \\
0_2 &= \text{clerical, sales, office employees} \\
0_3 &= \text{vehicle operators, craftsmen, artisans} \\
0_4 &= \text{agriculture, mining, personal services, unskilled labor.}
\end{align*} \]

for a detailed account of the data preparation, and to Musgrove [45] for summary statistics.
In Table C.1 we present the size of the original sample by education-occupation class. For this study, households were not analyzed if their current labor income \( y_1 \) or current resources \( y_1 \) were non-positive or if their primary earners were unemployed, less than 16, or over 65. Groups of households were further restricted to education-occupation classes with more than 100 remaining observations. The sizes and definitions of the final groups are summarized in Table C.2.

**TABLE C.1: Original Sample Size by Education-Occupation**

<table>
<thead>
<tr>
<th></th>
<th>( E_1 )</th>
<th>( E_2 )</th>
<th>( E_3 )</th>
<th>( E_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0_1</td>
<td>213</td>
<td>167</td>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>0_2</td>
<td>38</td>
<td>482</td>
<td>320</td>
<td>32</td>
</tr>
<tr>
<td>0_3</td>
<td>10</td>
<td>246</td>
<td>649</td>
<td>31</td>
</tr>
<tr>
<td>0_4</td>
<td>17</td>
<td>177</td>
<td>462</td>
<td>70</td>
</tr>
</tbody>
</table>

**TABLE C.2: Final Groups**

<table>
<thead>
<tr>
<th>Group</th>
<th>Education-Occupation</th>
<th># Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((E_1, 0_1))</td>
<td>204</td>
</tr>
<tr>
<td>2</td>
<td>((E_2, 0_1))</td>
<td>164</td>
</tr>
<tr>
<td>3</td>
<td>((E_2, 0_2))</td>
<td>469</td>
</tr>
<tr>
<td>4</td>
<td>((E_2, 0_3))</td>
<td>243</td>
</tr>
<tr>
<td>5</td>
<td>((E_3, 0_2))</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>((E_3, 0_3))</td>
<td>629</td>
</tr>
<tr>
<td>7</td>
<td>((E_3, 0_4))</td>
<td>256</td>
</tr>
</tbody>
</table>
APPENDIX D

ESTIMATION OF LABOR INCOME PROFILES

There are several factors which can influence the current and expected labor income of a household: the educational and occupational characteristics of its working members, their ages and number, attitudes toward work effort, and innate abilities which can generate income differentials within skill classes. The estimates of expected labor earnings used in Chapter IV were made by regressing, within education-occupation groups (see Appendix C), household nonproperty income (see p. 55) on a quadratic function of the age of the primary earner, i.e.,

\[ y_1^L = A0 + (A1)(\text{age}) + (A2)(\text{age})^2. \]

The regression results reported in Table D.2 are interpreted as determining the (expected) labor income of an "average" household during a period of employment stability and zero labor-augmenting growth.\(^1\) To facilitate an evaluation of the results, we provide in Table D.1 statistics concerning observed labor incomes and, in Table D.3, estimates of family earnings for selected ages of the primary earner.

\(^1\)In an attempt to purge the predicted income profiles of macroeconomic transitory elements, overtime wages were deducted from labor income, and households with a nonpositive \(y_1^L\), or with an unemployed primary earner less than 60 years of age, were removed from the test samples.
The first variables which are used to account for income inequality are the education and occupation of the household head. Differences in the means and standard deviations of group earnings (see Table D.1) suggest a joint statistical significance for these variables. Groups 1, 2-4, and 5-7 correspond respectively to families whose head has been exposed to university, secondary, and primary education. The importance of education in explaining family income is clear from the marked differences in the $y_1^L$ distribution as we move from groups 5-7 to those with more advanced educational training. The effects of occupational class, however, appear conditional upon the education variable. A significant reduction in mean incomes occurs for groups 2-4, where education remains at the secondary level and occupations vary from technical to white collar to blue collar. On the other hand, variations in mean incomes are rather vague within primary education groups.

While the educational-occupation stratification appears to contribute significantly to an explanation of labor income inequality, there remains considerable variation within groups. This variation is further reduced by including the age of the household head as an explanatory variable. The results are presented in Table D.2. Adjusted coefficients of determination are generally low. In no case did age explain more than 13% of the variation in $y_1^L$, and, in five out of the seven groups, less than 3% was explained. The low $R^2$'s, however, do not indicate an absence of age-related differences in household earnings. We should expect within age brackets income
inequality which is due to differences in ability, work effort,¹ and skills, considering the broad aggregates used in occupational classification. The F-tests do establish a statistical significance for age as an explanatory variable in groups 1, 2, 3, and 6. But the effect of age on labor earnings is most transparent from Table D.3. The ratio $\hat{y}_1^L(a)/\hat{y}_1^L(20)$ is over 1.5 for all groups, with the exception of 4 and 5, and it is roughly 4 for groups with primary earners in technical and administrative occupations. For the majority of groups, the regression estimates predict at least 50% variation in household labor earnings over the life cycle.

Turning to the structural properties of the regression equations, note first that all of the coefficients A2 are negative (except in the case of group 4 where the A2 estimate is less than half its standard error). There is then a general tendency for age-related increases in income to diminish at older age levels. However, in only four out of the seven groups did $\hat{y}_1^L$ peak before the age of 65 (see Table D.3). Finally, note that the groups which exhibit the most age variation are associated with upper-income occupational characteristics.²

¹ Family size, while significant, appears to be relatively unimportant in explaining family income. Musgrove, et al.[45, Ch.II] find a positive correlation between family size and income, but they also report an inverse relationship between income per person and household size and a negligible difference between per capita income concentration and the dispersion of income per family.

² The reader is referred to Musgrove, et al. [45, Ch. II] for a thorough description of the distribution, structure, and determinants of household incomes in Latin America,
TABLE D.1: Descriptive Statistics for Observed Labor Incomes

<table>
<thead>
<tr>
<th>Group</th>
<th>Number of observations</th>
<th>Mean ( y_{IL} )</th>
<th>Standard deviation ( y_{IL} )</th>
<th>Range ( y_{IL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>211</td>
<td>2193</td>
<td>1276</td>
<td>210 - 6500</td>
</tr>
<tr>
<td>2</td>
<td>167</td>
<td>1423</td>
<td>1105</td>
<td>45 - 6000</td>
</tr>
<tr>
<td>3</td>
<td>478</td>
<td>928</td>
<td>872</td>
<td>60 - 9000</td>
</tr>
<tr>
<td>4</td>
<td>245</td>
<td>644</td>
<td>612</td>
<td>47 - 6000</td>
</tr>
<tr>
<td>5</td>
<td>317</td>
<td>440</td>
<td>325</td>
<td>15 - 2562</td>
</tr>
<tr>
<td>6</td>
<td>637</td>
<td>368</td>
<td>284</td>
<td>9 - 3524</td>
</tr>
<tr>
<td>7</td>
<td>271</td>
<td>311</td>
<td>443</td>
<td>2 - 6000</td>
</tr>
</tbody>
</table>
### TABLE D.2: Regression Estimates for Labor Income Profiles

<table>
<thead>
<tr>
<th>Group</th>
<th>A0(GR)</th>
<th>A1(GR)</th>
<th>A2(GR)</th>
<th>$R^2$</th>
<th>F-Statistic*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2881.</td>
<td>217.0</td>
<td>-2.108</td>
<td>.125</td>
<td>16.0**</td>
</tr>
<tr>
<td></td>
<td>(951.0)</td>
<td>(43.20)</td>
<td>(.4667)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-2195.</td>
<td>164.3</td>
<td>-1.728</td>
<td>.066</td>
<td>6.88**</td>
</tr>
<tr>
<td></td>
<td>(1132.)</td>
<td>(56.62)</td>
<td>(.6784)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-142.9</td>
<td>44.15</td>
<td>-.3917</td>
<td>.022</td>
<td>6.29**</td>
</tr>
<tr>
<td></td>
<td>(408.2)</td>
<td>(19.92)</td>
<td>(.2313)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>548.1</td>
<td>-2.430</td>
<td>.1209</td>
<td>.005</td>
<td>1.65</td>
</tr>
<tr>
<td></td>
<td>(531.3)</td>
<td>(26.69)</td>
<td>(.3245)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>221.1</td>
<td>6.772</td>
<td>-.0366</td>
<td>.012</td>
<td>2.98</td>
</tr>
<tr>
<td></td>
<td>(181.5)</td>
<td>(8.180)</td>
<td>(.0879)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-69.56</td>
<td>20.32</td>
<td>-.2196</td>
<td>.016</td>
<td>6.16**</td>
</tr>
<tr>
<td></td>
<td>(127.4)</td>
<td>(6.192)</td>
<td>(.0728)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-135.1</td>
<td>19.58</td>
<td>-.1962</td>
<td>.004</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td>(259.5)</td>
<td>(12.11)</td>
<td>(.1353)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The F-statistics reported below correspond to tests of the joint hypothesis that $A1 = A2 = 0$.

**Statistics are significant at a 95% confidence level or better.
TABLE D.3: Predicted Labor Incomes for Selected Ages

<table>
<thead>
<tr>
<th>Group</th>
<th>$\hat{y}_1^{L}(20)$</th>
<th>$\hat{y}_1^{L}(\bar{a})$</th>
<th>$\hat{y}_1^{L}(65)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>617</td>
<td>2465</td>
<td>2320</td>
</tr>
<tr>
<td>2</td>
<td>399</td>
<td>1608</td>
<td>1181</td>
</tr>
<tr>
<td>3</td>
<td>583</td>
<td>978</td>
<td>1072</td>
</tr>
<tr>
<td>4</td>
<td>548</td>
<td>633</td>
<td>901</td>
</tr>
<tr>
<td>5</td>
<td>342</td>
<td>446</td>
<td>507</td>
</tr>
<tr>
<td>6</td>
<td>249</td>
<td>391</td>
<td>324</td>
</tr>
<tr>
<td>7</td>
<td>178</td>
<td>340</td>
<td>309</td>
</tr>
</tbody>
</table>

*The entries in the second and fourth columns correspond to the predicted earnings for ages 20 and 65. The term $\hat{y}_1^{L}(\bar{a})$ denotes the predicted labor income for the mean age of the group * (where mean ages ranged from 38 to 43).
APPENDIX E

ESTIMATION OF DEMOGRAPHIC PROFILES

The analysis in Chapter IV utilizes data on time profiles of household size and composition. Estimated profiles were obtained by regressing, within education-occupation groups (see Appendix C), the number of equivalent adults currently residing in a household (see p. 58) on a quadratic function of the age of the primary earner, i.e.,

\[ N_1 = B_0 + (B_1)(\text{age}) + (B_2)(\text{age})^2. \]

The regression results are reported in Table E.1.

All adjusted coefficients of determination are low, so that there remains substantial variation in family size within age brackets. However, the F-tests support a significant relationship between \( N_1 \) and age for all but the 5th group. The predicted profiles exhibit a similar concave structure. Family size initially increases with age, reflecting household formation and the birth and aging of offspring. But, for households with older primary earners, the \( B_2 \)-coefficient, which is negative in all groups, begins to dominate. Family size decreases as family members die or leave the sampled unit.

For purposes of intergroup comparisons, we have reproduced in Table E.2 the predicted values of \( N_1 \) at ages 20, 65, and the age at which family size reaches a maximum. Note first that there is a tendency for groups with less educated primary earners to have larger
families early in their life cycles (compare 3 to 5 and 1 to 2). This may represent early family formation or a greater incidence of parental dependency upon the primary earners in these groups. Second, there is a remarkable uniformity across groups in both the age at which family size is maximal and the maximum value of \( N_1 \). Differences in \( \hat{N}_1 \) (65) between groups are significant, however, and may correspond to different lengths of time offspring remain with the household and/or to different frequencies of elderly dependence. The differences seem to be most closely associated with education (compare 3 to 5, 4 to 6, and 1 to 2).
TABLE E.1: Regression Estimates for Family Size Profiles

<table>
<thead>
<tr>
<th>Group</th>
<th>B0(GR)</th>
<th>B1(GR)</th>
<th>B2(GR)</th>
<th>R^2</th>
<th>F-Statistic*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.541</td>
<td>.261</td>
<td>-.0026</td>
<td>.181</td>
<td>24.5**</td>
</tr>
<tr>
<td></td>
<td>(.861)</td>
<td>(.039)</td>
<td>(.0004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.007</td>
<td>.120</td>
<td>-.0012</td>
<td>.028</td>
<td>3.4**</td>
</tr>
<tr>
<td></td>
<td>(1.436)</td>
<td>(.072)</td>
<td>(.0009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>.462</td>
<td>.141</td>
<td>-.0014</td>
<td>.072</td>
<td>19.6**</td>
</tr>
<tr>
<td></td>
<td>(.588)</td>
<td>(.029)</td>
<td>(.0003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.095</td>
<td>.078</td>
<td>-.0007</td>
<td>.020</td>
<td>3.5**</td>
</tr>
<tr>
<td></td>
<td>(1.264)</td>
<td>(.063)</td>
<td>(.0008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.468</td>
<td>.062</td>
<td>-.0006</td>
<td>.002</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>(.916)</td>
<td>(.041)</td>
<td>(.0004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-.097</td>
<td>.182</td>
<td>-.0017</td>
<td>.073</td>
<td>26.6**</td>
</tr>
<tr>
<td></td>
<td>(.733)</td>
<td>(.036)</td>
<td>(.0004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-.037</td>
<td>.179</td>
<td>-.0020</td>
<td>.101</td>
<td>26.9**</td>
</tr>
<tr>
<td></td>
<td>(.679)</td>
<td>(.029)</td>
<td>(.0003)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The F-statistics reported below correspond to tests of the joint hypothesis that B1 = B2 = 0.

** Statistics are significant at a 95% confidence level or better.
<table>
<thead>
<tr>
<th>Group</th>
<th>(\hat{N}_1(20))</th>
<th>(\hat{N}<em>1(a</em>{\text{max}}))</th>
<th>(\hat{N}_1(65))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6</td>
<td>4.0</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>(50)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.9</td>
<td>4.0</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>(50)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.7</td>
<td>4.0</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>(50)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.4</td>
<td>4.3</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>(56)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
<td>4.1</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>(52)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.9</td>
<td>4.8</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>(53)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2.7</td>
<td>4.0</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>(45)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The entries in the second and fourth columns correspond to predicted family size at ages 20 and 65. The term \(\hat{N}_1(a_{\text{max}})\) denotes the maximum household size which is predicted to occur over the life cycle, with \(a_{\text{max}}\) given in parentheses.*
BIBLIOGRAPHY


   paper.


   Dividends of 1950 and Consumption: A Further Test of
   the 'Strict' Permanent-Income Hypothesis," *J. Polit. Econ.,*
   October 1965.

   September 1959.

6. Brown G. "The Impact of Korea's 1965 Interest Rate Reform on
   Savings, Investment, and Balance of Payments," paper pre-
   sented at the CENTO Symposium on Central Banking Monetary

7. Chandavarkar, A. J. "Some Aspects of Interest Rate Policies in
   Less Developed Economies: The Experience of Selected Asian

8. Craig, G. D. "Predictive Accuracy of Quarterly and Annual Aggre-
   gative Savings Functions," *J. Am. Statist. Assn.,* September
   1970.

9. Crockett, J. and Friend, I. "Consumption and Saving in Economic
   Development," presented at the International Conference on
Income, Consumption and Prices, October 1973.


30. Howe, H. and Musgrove, P. "The Extended Linear Expenditure System: Estimates from ECI EL Household Budget Data for


47. Seers, D. *Toward Full Employment: A Program for Colombia.*


LIST OF SYMBOLS

A  stock of assets
B  stock of debt
RA  nominal interest rate on assets
RB  nominal interest rate on debt
r^a  real interest rate on assets
r^b  real interest rate on debt
w_0  initial net worth
w^*  initial net worth plus interest receipts (payments)
w_0  human and non-human wealth
PV  household consumption
N  number of equivalent adults
L  labor income
y^T  transitory income
y^R  imputed rent on owner-occupied housing
+K  current capital income
-K  current interest and principal payments
y_L  current resources
y_N  normal income
y_P  permanent income
T  length of planning horizon
g  rate of labor-augmenting technical progress
\gamma, \delta  utility parameters
\tau, \sigma  constraint parameters