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RICE UNIVERSITY, P.H.D., 1979
RICE UNIVERSITY

HF-INDUCED PLASMA WAVES IN IONOSPHERIC SPORADIC E

by

FRANK THOMAS DJUTH

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

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HOUSTON, TEXAS

MAY 1979
ABSTRACT

HF-INDUCED PLASMA WAVES IN IONOSPHERIC SPORADIC E

by

Frank Thomas Djuth

The 430 MHz backscatter radar at Arecibo, Puerto Rico is used to diagnose the interaction of a powerful HF radio wave in a sporadic-E plasma. Resonant enhancements in the plasma line spectrum are observed to peak at the purely growing mode, that is, at 430 MHz ± f_{HF}, where f_{HF} is the HF frequency. Altitude profile measurements of the scattered radar signal reveal sporadic-E enhancements at the ion line as well as at the two plasma lines. On the few occasions when plasma line excitation is observed outside of sporadic E in the upper E region between 130 km and 140 km altitude, only decay mode enhancements are evident.

The sporadic-E plasma line enhancements along with the accompanying ion line enhancements are attributed to an HF-induced oscillating two-stream instability, which is parametrically driven near minimum threshold. Possible saturation mechanisms are examined. The dominance of the oscillating two-stream instability in sporadic E is consistent with theoretical predictions for parametric excitation in a collisionally dominated plasma containing steep electron density gradients in the vertical direction. The development of the oscillating two-stream instability is, however, most simply understood if the
sporadic-E plasma exhibits horizontal patchiness over spatial scales much less than 300 m.

Continuous monitoring of the plasma line and ion line power profiles during the observations allows studies of the strength and altitude of the sporadic-E enhancements to be made over time scales of several hours. HF-induced enhancements are detected only when the blanketing frequency, \( f_b E_s \), exceeds \( f_{HF} \). Asymmetries in the scattered power at the two plasma lines are usually present. The power asymmetries are viewed in terms of two geometrical models for plasma wave propagation and generation in sporadic E.
ACKNOWLEDGEMENTS

Like most enterprises of this nature, the dissertation presented herein represents a cooperative effort entailing the assistance of several individuals. In this regard, I would like to extend a special thanks to Dean William E. Gordon, my thesis advisor, for providing me with a research opportunity in ionospheric plasma physics subsequent to my Master's dissertation in gamma-ray astronomy. Dr. Gordon's guidance and constant encouragement created a relaxed atmosphere for independent research and innovative thought. I am also indebted to Dr. Herbert C. Carlson, who introduced the subject matter of this dissertation to me and supplied data from the 1976 ionospheric modification experiment conducted at Arecibo Observatory. Data taking during the experimental observations was greatly facilitated by the superb computer programming contributed by Cesar LaHoz. Discussions with Dr. Donald B. Muldrew dealing with the theoretical aspects of the present study proved to be quite instructive as did discussions with Drs. Lewis M. Duncan, Ivan J. Kantor and Robert L. Showen. Ms. Ruth Parks deserves my utmost praise after typing this massive thesis. Finally, I would like to take this opportunity to express my heartfelt appreciation to Marlene Dzikzmaniec, whose support, aid and reassurance made the writing of this dissertation possible.
In general, the Arecibo experiments would not be possible without a concerted effort on the part of the staff at Arecibo Observatory. I am grateful for their help as well as the assistance provided by fellow scientists participating in the Arecibo heating experiments. The research encompassed by this dissertation was supported in part by the Atmospheric Research Section, National Science Foundation, through NSF Grant No. ATM76-15550. The Arecibo Observatory is part of the National Astronomy and Ionosphere Center, which is operated by Cornell University under contract with the National Science Foundation.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. THE EXPERIMENT</td>
<td>6</td>
</tr>
<tr>
<td>2.1 The Facility</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Incoherent Scatter Theory</td>
<td>8</td>
</tr>
<tr>
<td>2.3 The Experimental Design</td>
<td>14</td>
</tr>
<tr>
<td>3. THE SPORADIC-E PLASMA</td>
<td>27</td>
</tr>
<tr>
<td>3.1 Plasma Structure</td>
<td>27</td>
</tr>
<tr>
<td>3.2 E-Region Temperatures</td>
<td>33</td>
</tr>
<tr>
<td>3.3 E-Region Collision Frequencies</td>
<td>34</td>
</tr>
<tr>
<td>4. THE HF ELECTRIC FIELD OF THE MODIFYING WAVE</td>
<td>41</td>
</tr>
<tr>
<td>5. PARAMETRIC INSTABILITY THEORY</td>
<td>52</td>
</tr>
<tr>
<td>5.1 General Coupled Mode Formalism</td>
<td>52</td>
</tr>
<tr>
<td>5.2 The Coupled Wave Equations in a Plasma</td>
<td>60</td>
</tr>
<tr>
<td>5.3 Parametric Instabilities in Homogeneous Plasmas</td>
<td>62</td>
</tr>
<tr>
<td>5.4 Parametric Instabilities in Inhomogeneous Plasmas</td>
<td>65</td>
</tr>
<tr>
<td>5.5 Linear Theory Applied to the Ionosphere</td>
<td>67</td>
</tr>
<tr>
<td>5.6 Saturation of Parametric Instabilities in the Ionosphere</td>
<td>70</td>
</tr>
<tr>
<td>5.7 Parametric Excitation in Sporadic E</td>
<td>78</td>
</tr>
<tr>
<td>5.7.1 Metallic Ions</td>
<td>79</td>
</tr>
<tr>
<td>5.7.2 The Decay Instability</td>
<td>82</td>
</tr>
<tr>
<td>5.7.3 The Oscillating Two-Stream Instability</td>
<td>88</td>
</tr>
<tr>
<td>5.8 Frequency Matching Conditions</td>
<td>90</td>
</tr>
<tr>
<td>5.9 Magnetic Field Effects</td>
<td>91</td>
</tr>
</tbody>
</table>
6. EXPERIMENTAL RESULTS ................................................................. 97
   6.1 Terminology ........................................................................... 98
   6.2 Prior Investigations ............................................................... 98
   6.3 The 1976 Investigations .......................................................... 101
      6.3.1 Brief Summaries ............................................................... 102
         (A) July 17, 1976 ................................................................. 102
         (B) July 19, 1976 ................................................................. 103
         (C) July 21, 1976 ................................................................. 104
      6.3.2 Extended Observations ....................................................... 105
         (A) July 17, 1976 ................................................................. 105
         (B) July 20, 1976 ................................................................. 112
         (C) July 25, 1976 ................................................................. 117
   6.4 Spectral Information ............................................................... 122
   6.5 The 1977 Observations ............................................................ 125
      6.5.1 June 3, 1977 ................................................................. 125
      6.5.2 June 7, 1977 ................................................................. 128
      6.5.3 June 9, 1977 ................................................................. 129
      6.5.4 June 13, 1977 ............................................................... 131
      6.5.5 June 14, 1977 ............................................................... 133
   6.6 Summary ............................................................................. 135

7. DISCUSSION .............................................................................. 169
   7.1 The HF Electric Field Strength in Sporadic E .................... 169
   7.2 Linear Absorption ................................................................. 171
   7.3 Langmuir Wave Ray Tracing ................................................ 173
   7.4 Plasma Wave Generation ....................................................... 175
   7.5 Threshold Considerations ...................................................... 178
   7.6 Time Variability of the Enhanced Plasma Line ................... 185
   7.7 Propagation of OTSI Plasma Waves ...................................... 189
   7.8 Sporadic-E Models ................................................................. 195
   7.9 The Power Asymmetry .......................................................... 199
   7.10 Plasma Line Power Profile .................................................. 205
   7.11 Sporadic-E Spectra .............................................................. 206
8. SUMMARY AND CONCLUSIONS................................................. 224
REFERENCES........................................................................ 230
TABLE OF SYMBOLS................................................................. 241
APPENDIX............................................................................. 247
  A.1 Barker Coding and Power Measurements......................... 247
  A.2 Spectral Estimates......................................................... 250
  A.3 Autocorrelation Function Measurements.......................... 251
  A.4 Discrete Fourier Transform Measurements....................... 252
CHAPTER 1
INTRODUCTION

Since the mid 1960's the nature of parametric interactions in plasmas has been a topic of considerable interest in plasma research (Silin, 1965; DuBois and Goldman, 1965, 1967a). The attention focused upon the parametric processes has been stimulated in large part by problems related to plasma heating in fusion research (Thomson et al., 1974; Nuckolls et al., 1972). In both inertial (laser) and magnetic confinement approaches to nuclear fusion, one is concerned with the efficient heating of a plasma by electromagnetic radiation. Although optical frequencies are involved in the former case while radio frequencies are important in the latter, the essential plasma physics is the same in both cases.

One of the fundamental processes by which electromagnetic waves interact with plasmas is through the parametric excitation of new waves. In this respect resonant three-wave interactions are of primary importance. The incident electromagnetic wave may couple either to two of the electrostatic normal modes in the plasma or to one electrostatic and one electromagnetic mode (Drake et al., 1974). The former process is commonly referred to as anomalous absorption of the incident wave while the latter interaction is termed stimulated scattering. Stimulated scattering is distinguished by the fact that electromagnetic
radiation downshifted in frequency may leave the plasma without further interacting with it. In either situation the electrostatic oscillations produce a degree of plasma turbulence which serves to heat the plasma. However, all of the effects associated with heating are not desirable. In laser fusion applications, for example, high frequency Langmuir waves undergo Landau damping and in so doing produce suprathermal electrons. These electrons then become an obstacle to fusion since they seriously preheat the pellet core. Stimulated scattering, on the other hand, allows a large portion of the incident electromagnetic energy to escape from the plasma. The overall complexity of experimental problems has made the study of parametric instability growth rates, thresholds, and their control an area of intensive investigation.

Many illustrations of parametric absorption processes may be found in electronics, simple electrical and mechanical systems, and throughout most of physics. Elementary examples include the parametric amplifier used at the front-end of the receiver system in the present experiment, a child on a swing, and a stretched string attached to a tuning fork prong. This latter problem was studied by Rayleigh (1883). In addition, stimulated scattering processes have well known analogues in nonlinear optics termed Raman and Brillouin scattering.

It is clear that parametric processes are not unique to plasma physics. However, the dynamics of the plasma medium itself are noteworthy. An incident "pump" wave may act to simultaneously couple a variety of different mode combinations. In the linear approximation, the coupling processes are regarded as independent, each exhibiting
its own instability threshold. Once a threshold is exceeded, nonlinear saturation theory must be applied to the particular instability in order to determine the means by which the plasma attains a new steady state.

Typically many competing processes are available to the plasma in order to saturate a given instability. The process which dominates often depends upon many of the system variables. These include the degree to which the threshold was exceeded, the wavelengths of the excited modes and their damping rates, and the plasma pressures and temperatures to name a few. The richness in variety of parametric phenomena in plasmas appears to be unlimited.

The parametric excitation of plasma waves in the ionospheric F region by a powerful HF (4-12 MHz) radio wave was first experimentally detected at Arecibo Observatory in 1971 by Carlson et al. (1972). Since that time many characteristics of the interaction processes have been reported (see e.g. Kantor, 1974; Carlson and Duncan, 1977; Muldrew and Showen, 1977; Showen and Behnke, 1978; and Muldrew, 1978a). In the F region the low frequency ion acoustic waves, which are also excited, are heavily damped by ion Landau damping, whereas the high frequency Langmuir waves are lightly damped by Coulomb collisions with $O^+$ ions.

Initial observations of plasma oscillations excited in blanketting sporadic E by an HF wave were described by Gordon and Carlson (1976). This work provided the impetus for the present study. The mere detection of amplified Langmuir oscillations in sporadic E is of interest since it is indicative of a resonant interaction induced in a plasma environment completely different from that of the F region.
In the E region collisions play a significant role in the damping of the electrostatic modes of the plasma. Although the ion mode is still severely damped by a combination of ion Landau damping and ion-neutral collisions, the high frequency electron fluctuations in the E region are heavily damped by electron-neutral collisions. This latter mode may in fact become more highly damped than the ion mode. Furthermore, typical electron density scale heights in the F region (50-75 km) are sufficiently large to allow the plasma to be regarded as homogeneous. The homogeneous plasma approximation, however, is not necessarily valid in sporadic E where large density gradients are present.

Because of its uniqueness, the sporadic-E plasma constitutes a very interesting medium in which to study wave-plasma interactions. This dissertation encompasses an experimental investigation into the excitation of plasma fluctuations in sporadic E by a powerful HF pump wave. The principle plasma diagnostic employed in the observations of the sporadic-E plasma was the incoherent scatter radar at Arecibo, Puerto Rico. In the following chapter the associated experimental procedures utilized in the present study will be described. Subsequent chapters deal with the nature of the unmodified sporadic-E plasma and the HF electric field structure in a stratified plasma. Parametric instability theory is then applied to a model sporadic-E plasma. This provides a meaningful basis for the presentation of experimental results and their interpretation. In the final chapter suggestions for a continued program of research into sporadic E are included.
In concluding this introduction, we note that the MKS system of units is employed throughout this work. In addition, the symbols of variables are defined in the text only upon first usage. As a convenient reference, however, a Table of Symbols has been inserted at the end of the dissertation wherein symbols used in sections subsequent to the one in which they are defined are listed.
CHAPTER 2
THE EXPERIMENT

2.1 THE FACILITY

The observations reported in this dissertation were made at the Arecibo Observatory, which is located approximately 15 km south of the coastal city of Arecibo, Puerto Rico. The geodetic coordinates of the Observatory are 18° 20' 46.2" north latitude and 66° 45' 10.5" west longitude. Local time is Atlantic Standard Time which is Greenwich mean time minus 4 hours.

The most outstanding feature of Arecibo Observatory is a fixed spherical reflector which is used in conjunction with an incoherent backscatter radar centered at 430 MHz. The antenna dish is 305 m in diameter and has a radius of curvature of 265 m. At 430 MHz the radar beam width is 1/6°. The sky coverage of the antenna is ± 20° in zenith angle and 360° in azimuth. The 430 MHz transmitter has a maximum duty cycle of 6% and may be operated at a peak pulsed power of 2.5 MW.

The magnetic dip angle at Arecibo is 50° while the magnetic declination is 8° 30' west. In the E region at 110 km the magnetic field strength, B, translates into an electron gyrofrequency, \( f_{ec} = eB/2 \pi m_e \) of 1.11 MHz where e and \( m_e \) are the electron charge and mass respectively. During ionospheric modification experiments
an HF log-periodic antenna is mounted at the center of a feed arm which is suspended above the dish. This restricts the 430 MHz line feed to zenith angles \( \gtrsim 3^\circ \). Typically, the feed is pointed in the direction of magnetic north at a zenith angle of \( \sim 4^\circ \). As a result observations are made at \( \sim 44^\circ \) with respect to the magnetic field.

The incoherent scatter radar is complemented by two conventional sweep frequency ionosondes. One (a modified C-3) is located at Arecibo Observatory; the second (a C-2/4) is positioned at Los Caños, which lies approximately 11 km to the north of the observatory. In addition, air-glow sensors centered at 5577 \( \AA \) and 6300 \( \AA \) are operated whenever feasible during the nighttime.

Two separate ionospheric modification experiments provided the observations presented in this dissertation. The first experiment took place over the time period July 14, 1976 to July 29, 1976; the second, more recent experiment extended from June 2, 1977 through June 17, 1977.

Ionospheric modification is accomplished with the aid of an HF (4-12 MHz) transmitter set at a frequency \( f_{HF} \). The associated experiments are nonstandard in that they involve special arrangements with Arecibo Observatory in order to make the HF transmitter operational and to raise and attach the HF antenna to the feed arm. Although the HF transmitter is designed for a maximum power output of 160 kW, it is more realistically operated between 50 kW and 100 kW. Arcing at the HF antenna feed is usually the limiting factor.
2.2 INCOHERENT SCATTER THEORY

Gordon (1958) was first to note that the detection of electromagnetic scattering from individual electrons in the ionosphere (incoherent scattering) was within the capabilities of modern day radar technology. Because of the random thermal motion of the electrons, the total scattered signal was expected to represent the sum of individual contributions having a random distribution of phases.

The detectability of incoherent scattering was first verified by Bowles (1958). Since that time, incoherent scatter theory has evolved to the point where a very precise understanding of the plasma phenomena responsible for the scattering is possible. The confirmation of the theory by experiment has made incoherent scatter radar a powerful diagnostic tool for probing the ionosphere.

Detailed discussions of the theory of incoherent scatter may be found in excellent reviews by Evans (1969) and Farley (1971). In the following discussion only the salient features of the theory relevant to the present observations will be described. In this summary physical mechanisms are emphasized rather than mathematical rigor.

Incoherent scattering is an example of the more general case of scattering from a diffuse medium. In such a medium scattering results from changes in the dielectric constant brought about by electron fluctuations which occur naturally in a plasma in thermal equilibrium. Radars operated in the backscatter mode single out the Fourier components of irregularities having wave normals that are aligned either parallel or antiparallel to the radar wave vector \( \vec{k} \) where \( |\vec{k}| = 4 \pi/\lambda \) and \( \lambda \) is the radar wavelength. Partial reflection is possible only
when the spacing between phase fronts in the medium equals $\lambda/2$. This is simply the Bragg condition for constructive interference of the scattered radar signals when the returned signal is averaged over the scattering volume. Irregularities having wave vectors $\pm \vec{k}$ generate Doppler shifted radar signals at $\pm f$ where $f = 2v/\lambda$ and $v$ is the phase velocity of the irregularity wave front.

A scale length of interest when discussing the ionospheric plasma is the Debye length which is defined as

$$\lambda_D \equiv (\epsilon_0 k T_e / n_e e^2)^{1/2} = 69 (T_e / n_e (m^{-3}))^{1/2} \text{ m}$$

where $T_e$, $n_e$, $e$, $\epsilon_0$, and $k$ are respectively the electron temperature, density and charge, the permittivity of free space, and Boltzmann's constant. Organized wave motion within the plasma is not maintained whenever the Debye length is much greater than the radar wavelength, that is whenever $\alpha >> 1$ where $\alpha \equiv 4\pi \lambda_D / \lambda$. When $\alpha$ is very large, the radar wave interacts independently with individual electrons, each of which may be assigned the Thomson scattering cross section, $\sigma_e = 10^{-28} \text{ m}^2$. The resultant incoherent scatter spectrum occupies a broad bandwidth and is Gaussian in shape. The spectrum is interpretable in terms of Doppler shifts arising from the thermal motion of electrons.

Typical values of $\lambda_D$ for the F region and sporadic E are 3 mm ($T_e = 1500^{\circ}K$, $n_e = 8 \times 10^{11} \text{ m}^{-3}$) and 2 mm ($T_e = 270^{\circ}K$, $n_e = 3 \times 10^{11} \text{ m}^{-3}$), respectively. Consequently $\alpha = 0.05$ (F region) and $\alpha = 0.04$ (sporadic E), and in both instances $\alpha << 1$. Under these conditions, scattering
occurs as a result of longitudinal oscillations in the plasma. The incoherent scatter spectrum contains two components, a low frequency ion component near the ion normal mode in a collisionless plasma, and a high frequency electron component near the Langmuir normal mode. The attendant plasma oscillations bring a degree of coherence to the scattering process, which is nevertheless euphemistically called incoherent.

In Figure 2.1a the incoherent scatter power spectrum is sketched for F-region conditions where collisions with neutral molecules are negligible (see e.g. Hagfors, 1961). The Doppler shift from 430 MHz is normalized by dividing by the ion thermal velocity \( v = (2kT_i/m_i)^{1/2} \) where \( T_i \) and \( m_i \) are the ion temperature and mass respectively. The electron resonance frequency \( \omega_r \) is discussed below and is approximately equal to the electron plasma frequency, \( \omega_p \), where \( \omega_p = (n_e e^2/\varepsilon_0 m_e)^{1/2} = 2\pi f_p \) and \( m_e \) is the electron mass. For typical F-region parameters \( \theta(\omega) = 1 \) and \( \theta(\omega_r) \) correspond to Doppler shifts \( \Delta f_o \approx 3-4 \) kHz and \( \Delta f_o = \pm (5-10 \) MHz) respectively. If neither the plasma as a whole nor any of its components exhibit bulk motion, the spectrum will be symmetric about zero frequency.

The ion component, or "ion line," represents the organizing influence of ion pressure waves in controlling the motion of electrons. The electrons follow the movement of the ions in order to maintain charge neutrality. When the radar viewing angle, \( \phi \), with respect to the earth's magnetic field, is \( 0 \leq \phi < 85^\circ \) (\( \phi = 44^\circ \) at Arecibo), the ion line spectrum is virtually unaffected by the magnetic field. The two shoulders in the ion spectrum at \( \Theta = \pm 1 \) represent the remnants of spikes due to ion acoustic waves. In the fluid approximation ion
acoustic waves denote the low frequency normal mode of the plasma. For comparable electron and ion temperatures, these waves are severely damped by ion Landau damping in the kinetic description. The resultant coupling with thermal ions destroys the organized wave motion and broadens the resonance to widths characteristic of thermal ion velocities.

The power present in the electron component, or "plasma line," is smaller than that of the ion component by a factor of $\alpha^2$ (Farley, 1971). Plasma lines are caused by weakly damped electrostatic oscillations near the plasma frequency. These lines appear in the incoherent scatter spectrum as narrow features, upshifted and downshifted in frequency by the resonance frequency, $f_r$. For $f_{ec} << f_e$, $f_r$ may be written as (Yngvesson and Perkins, 1968)

$$f_r^2 = f_e^2 + \frac{12KTe}{\lambda^2 m_e} + f_{ec}^2 \sin^2 \phi$$

where $f_e$ is the plasma frequency, $f_{ec}$ is the electron cyclotron frequency. The resonance peak is sharp since in the absence of electron collisions the width is determined by electron Landau damping which is very small for $\alpha << 1$. When the line is measured experimentally, however, the inhomogeneity of the ionosphere spreads the observed bandwidth by an amount commensurate with the altitude resolution of the experiment.

The presence of a second narrow electron line near the lower hybrid resonance frequency of the plasma (not shown in Figure 2.1a) was predicted by Salpeter (1961). At Arecibo, this line occurs at a
frequency, $f_w$, given by

$$f_w = \sin \phi \cdot f_{ec}$$

A resonance peak at this frequency has been experimentally detected only recently by Behnke and Hagen (1978), who termed it the "whistler line."

In the limit of small $\alpha$ only the ion line contribution to the total incoherent scatter cross section is significant. For $T_e/T_i \lesssim 3$ the cross section per electron, $\sigma_T$, may be approximated by

$$\sigma_T = \sigma_e/(1 + T_e/T_i)$$

Thus, when $T_e = T_i$, $\sigma_T = \sigma_e/2$, which is half the cross section found for the $\alpha >> 1$ regime.

In the E region and below, electron-neutral and ion-neutral collisions must be included into the description of incoherent scattering. Collisions play a significant role whenever the ion mean free path is of the order of $1/k$. The neutral gas then acts as a viscous medium which inhibits ion motion. In responding to a charge imbalance the ions undergo diffusion. Consequently, ion fluctuations become correlated for longer periods of time. This is equivalent to saying that the ion power spectrum narrows and maximizes at zero frequency.

A hypothetical spectrum dominated by collisions is sketched in Figure 2.1b (see e.g. Dougherty and Farley, 1963). For typical E-region conditions $\Theta(\omega) = 1$ and $\Theta(\omega_p)$ correspond to $\Delta f_0 = 1$ kHz and $\Delta f_0 = \pm (3-4$ MHz$)$ respectively. In contrast to the example in Figure 2.1a, spectral "shoulders" in the ion line spectrum are
completely absent. This situation is brought about by neutral collisions and ion Landau damping which combine to attenuate ion acoustic waves so strongly that it is hardly appropriate to describe them as waves at all. As neutral collisions increase, the wave is completely damped well within one cycle of oscillation. This is an important consideration when one attempts to parametrically drive oscillations at the ion acoustic frequency (section 5.7).

For $T_e = T_i$ the total cross section is unchanged by collisions. Thus, collisions serve only to redistribute energy in the power spectrum of the scattered signal. Although the plasma line resonance still occurs at the frequency $f_r$, the resonance becomes broadened by collisions to widths of the order of the electron collision frequency.

The goal of the present experiment was to investigate the excitation of plasma waves in sporadic-E plasmas. These observations entail enhancements in the incoherent scatter spectrum which are at times several orders of magnitude above ambient levels. Although the most notable increases occur for the plasma lines, ion line enhancements are also predicted and observed.

Extensive studies of the parametrically enhanced backscatter spectrum in the F region have been conducted by many investigators using the Arecibo facility (see e.g. Kantor, 1972; Showen, 1975; and Duncan, 1977). Figure 2.2 illustrates the enhanced F-region spectrum schematically along with a typical sporadic-E spectrum. The terminology originally introduced by Kantor (1972) is indicated in Figure 2.2 and will be adopted whenever applicable in characterizing sporadic-E
spectra. For the moment any further discussion of the sporadic-E \( E_S \) observations will be postponed until a detailed description of the parametric process and its application to sporadic-E plasmas is presented in Chapter 5.

2.3 THE EXPERIMENTAL DESIGN

As described above, \( E_S \) observations were made during two different ionospheric modification experiments, one in July 1976, the other in June 1977. The two sets of observations differ in the type of data gathered over the approximate two-week duration of each experiment. Both experiments, however, employ the 430 MHz incoherent scatter radar as the primary diagnostic tool for plasma phenomena.

The experimental procedures involved in the two experiments will be briefly outlined below. The Appendix (A) provides a fuller description of some of the commonly used analysis techniques and will be referenced in order to avoid prolonged discussions.

During the 1976 experiment, incoherent scatter power profiles (power versus altitude) were measured at frequencies centered on the ion line (430 MHz), the upshifted plasma line (\( \sim 430 \text{ MHz} + f_{HF} \)), and the downshifted plasma line (\( \sim 430 \text{ MHz} - f_{HF} \)). Power enhancements above ambient values are expected at these frequencies as a result of parametric excitation of ion acoustic waves and Langmuir waves. In addition to the power measurements, the ion line autocorrelation function (section A.3) was determined as a function of altitude for two specific time delays, or time lags. The two measured points on the
autocorrelation function provided enough spectral information to allow a direct estimate of the $T_e/T_i$ profile in the F region. In the E region, altitude dependencies of $v_i$, $T_e$ ($= T_i = T_n$), and the ion mass were derivable with the aid of assumptions based on E-region models.

In order to obtain the best possible altitude resolution, the transmitted 430 MHz radar pulses were phase coded using a 13-baud Barker code (section A.1). A detailed account of the pulse-compression techniques commonly employed at Arecibo is given by Ioannidis and Farley (1972) and Gray and Farley (1973). The altitude resolution, $\Delta h$, of the power measurements is given by $\Delta h = c\Delta b/2$, where $\Delta b$ is the baud length of the Barker code. The baud length used in the experiment was 4 $\mu$sec which implies an altitude resolution of 600 m. The interpulse period (IPP), or the spacing between transmitted pulses, was usually set at 11.3 msec during the observations. For autocorrelation function measurements, two phase-coded pulses separated by the lag time were transmitted within the above IPP.

The final stages of the receiver system used in the 1976 experiment are shown in Figure 2.3. Prior to arriving at one of three 430 MHz receivers (REC), the scattered signal from the ionosphere is amplified, filtered and split into three separate receiver channels.

The data taking process, as well as all other operational aspects of the 1976 experiment, were computer controlled by a multipurpose program written by Cesar LaHoz. During an observation, the HF transmitter was repeatedly cycled on and off for fixed time intervals. The time unit relevant to the data taking process was the "subinterval," which was the time required to make a complete series of measurements.
with the 430 MHz radar. The measurements made sequentially during the
course of a subinterval included the noise at 430 MHz, the noise at
430 MHz plus the noise due to a calibration diode, the ion line power,
the plasma line power at 430 MHz - $f_{HF}$ + 3 kHz (approximate location
of the decay line in the F region), the plasma line power at 430 MHz +
$f_{HF}$ - 3 kHz, the ion autocorrelation function at the first preselected
lag, $\tau_1$, and finally the autocorrelation function at the second lag,
$\tau_2$. The integration times (in IPP's) for individual measurements were
selectable. Since the accumulated data were recorded on a subinterval
by subinterval basis, the on and off times of the HF transmitter were
always an even number of subintervals.

Power measurements were efficiently made by summing the abso-
lute values of the digitally sampled voltage, $V(t)$, (see Farley, 1969a),
while for autocorrelation measurements the hybrid quantity
$V(t) \cdot \text{sign}(V(t + \tau))$ was accumulated (section A.3). The gate width,
or the time between voltage samples within an IPP, was set equal to
4 $\mu$sec and thus matched the baud length of the Barker code. Conse-
quently, independent power and autocorrelation function estimates were
available at 600 m altitude intervals. Since 600 consecutive samples
of the signal were obtained within each IPP, the range of altitudes
sampled was 360 km.

Since the power profile measured just after the receivers are
gated open may exhibit a "recovery profile," tests were performed for
this system related difficulty. No recovery problems, however, were
evident in the data. As a result, the electron density profile could
be deduced directly from the unenhanced ion line power profile by
correcting the power profile for the approximate inverse square range
dependence of radar power, applying a near field correction factor
(Rowe, 1974), and then multiplying by a first order correction term
\( \alpha (1 + \frac{T_e}{T_i}) \) to account for the temperature dependence of the
incoherent scatter cross section.

In the 1977 series of observations, attention was directed
toward measuring the power spectra of the enhanced ion line and the
two plasma lines in \( E_s \). In order to accomplish this, single pulse
experiments were performed wherein the scattered signal from the
ionosphere was Fourier analyzed for spectral content (section A.2).
Figure 2.4 illustrates the height-time arrangement schematically. In
the F region the observed scattering region is a thin slab of thickness \( \lesssim 300 \text{ m} \). The gate width, \( \Delta \), dictated the frequency bandwidth,
\( w \), of the Fourier transformed spectrum. In general \( w = 1/\Delta \).

The discrete Fourier transform of the sampled time series was
performed using a Fast Fourier Transform (FFT) algorithm (see e.g.
Cochran et al., 1967). The FFT employed in the experiment had a
"radix of two," which required that the number of samples, \( N \), inputted
to the transform equal \( 2^n \), where \( n \) is an integer. Typically, 128
samples were used. The resultant power spectra contained power esti-
mates at frequency intervals of \( 1/(N\Delta) \).

The "frequency resolution" of the power spectrum is an ill-
deﬁned concept since the spectral windowing (section A.4) arising
from the finite sampling interval, \( T_s \), of the scattered signal extends
over all frequencies. Nevertheless, the resolution, \( \Delta f \), is often
defined in terms of the approximate width of the largest peak in the 
\[ \frac{\sin(\pi f T_s)}{\pi f T_s} \] windowing function. Thus, \( \Delta f \equiv 1/T_s \).

The frequency resolution coincides with the frequency spacing of the power spectrum estimates provided the scattered signal is sampled throughout the entire sampling interval \( T_1 \). When this is not the case (i.e. when \( T_s < T_1 \)), all of the power estimates are not "independent" in the sense that their separation distances are less than the frequency resolution. The frequency resolution may be increased by transmitting longer pulses. However, in the present experiment the longest pulse, \( T_{\text{max}} \), that may be transmitted and analyzed is \( T_{\text{max}} = 2h/c \), where \( h \) is the height of the scattering region. In practice the longest experimental sampling period is somewhat less than \( T_{\text{max}} \) since ground clutter increases the required gate delay for the ion line. One might be inclined to accept a limited amount of ground clutter on the ion line channel in order to gain increased resolution in the plasma line measurements, which are unaffected by ground clutter. Unfortunately, ion line "leakage" of the large amplitude ground clutter signal into the plasma line channels severely hampered this effort. The exact source of this leakage has yet to be determined.

The final stages of the 1977 receiver system are shown in Figure 2.5. The three independently operated receiver channels were centered on the ion line and the two plasma line frequencies. After the initial gate delay, the three channels were alternately sampled at 2 \( \mu \text{sec} \) intervals until 128 samples were obtained per channel. Thus, a voltage sample on a given channel was recorded every 6 \( \mu \text{sec} \). The alias free bandwidth follows as \( 1/6 \mu \text{sec} = 167 \text{ kHz} \). Therefore, the
125 kHz filters on the plasma line channels and the 50 kHz filter on
the ion line channel were sufficiently narrow in bandwidth to prevent
aliasing.

In addition to the usual computer control of the data taking
procedure, the 1977 experimental design entailed on-line processing of
the data sampled during each IPP. This required that the 128 point
time series recorded for each receiver channel be Fast Fourier trans-
formed, and that the magnitude of the complex vector output be calcu-
lated and accumulated on an IPP by IPP basis. In addition, the 128
point power profiles were computed and accumulated for each receiver
channel. The integration of power spectra and power profiles was per-
formed for a predetermined number of IPP's whereupon the data were
recorded on magnetic tape while selected data were visually displayed.
The above computations could be performed in a time effective manner
only with the aid of the AP-12OB Array Processor (Floating Point
Systems, Inc.) which was fitted into the data-taking minicomputer (a
Harris 6024/4). The minimum IPP was determined by the smallest amount
of time required by the computer to process the data and regulate the
experiment. This IPP (≈ 28 msec) was used throughout most of the
experiment. Since the transmitted pulses were limited to times ≲ 800
μsec for E-region observations, the maximum 6% duty cycle of the 430
MHz radar was easily satisfied.

During a given observation period, the incoherent scatter
spectra were calibrated by measuring spectra while a 50 ohm dummy load
was connected to the front end of the receiver system. Subsequently,
a calibration noise source of known temperature was inserted and the measurement repeated. At the end of an observation the 430 MHz radar was disabled and similar measurements were performed with the receiver system connected to the antenna.
FIGURE 2.1 Arecibo incoherent scatter power spectrum illustrated for (a) the F region using the calculations of Hagfors (1961), and (b) the E region using the calculations of Dougherty and Farley (1963). The normalized Doppler frequency, θ, is defined as $\theta(\omega) \equiv (\omega/k)(m_1/2kT_i)^{1/2}$. In case (a) it is assumed that $T_e = T_i$ and $\alpha \ll 1$, while in (b) the conditions are $T_e = T_i$, $\alpha \ll 1$, and $\psi_i = 10$, $\psi_e = 0.3$ where $\psi_B \equiv (v_B/k)(m_B/2kT_B)^{1/2}$. The parameters used in (b) approximate E-region conditions near 106 km.

FIGURE 2.2a A schematic illustration of the parametrically enhanced backscatter spectrum in the F region over Arecibo (Kantor, 1972).

FIGURE 2.2b A schematic representation of a "typical" HF-enhanced sporadic-E spectrum measured during the 1977 experiment. The structure displaced by $\sim 2-4$ kHz from the growing mode was not resolved in the measurement.
FIGURE 2.1
FIGURE 2.3 Receiver arrangement during the 1976 experiment.
FIGURE 2.4 Sampling procedure for the single pulse experiment.
FIGURE 2.5 Receiver arrangement during the 1977 experiment.
CHAPTER 3
THE SPORADIC-E PLASMA

In this chapter the essential features of midlatitude sporadic E ($E_s$) will be summarized along with some of the general characteristics of E region-dynamics. The goal of this discussion is to provide a brief description of the $E_s$ parameters relevant to the instability processes that will be detailed in subsequent chapters. Indeed, the study of sporadic E in the temperate zone along with its morphology, structure, and physical mechanisms is a discipline unto itself. Since sporadic E is obviously a phenomenon of great complexity, no effort will be made here to survey the vast amounts of observational and theoretical work devoted to the subject. A general review of observed properties and physical mechanisms associated with sporadic E is given by Whitehead (1970), while more detailed treatments of specific phenomena may be found in Radio Science (1972). An excellent review of the lower thermosphere dynamics relevant to midlatitude sporadic E is provided by Evans (1978). E-region dynamics are discussed at length in the large compendium of papers assembled by Hines (1974).

3.1 PLASMA STRUCTURE

Although $E_s$ is generally observed to occur in the 90-120 km altitude range, the observed heights of most of the $E_s$ regions reported
in the present study were between 105 km and 117 km. In large part this may be attributed to a selection effect stemming from the fact that the majority of the observations were conducted during the daytime in Es regions where the blanketing frequency, \( f_b E_s \), was \( \geq 4 \) MHz. During the nighttime, the Es over Arecibo is observed to occur at lower altitudes in a preferred region extending from 90-93 km (Rowe, 1973).

The thickness of a typical Es region, as determined from rocket observations, ranges from 1-3 km (see e.g. Miller and Smith, 1976; Smith and Mechtly, 1972; Layzer, 1972; Smith, 1970, 1966; Bowhill, 1966). The rocket electron density profiles are universally characterized by steep vertical gradients, while multiple Es peaks and irregular vertical structure may or may not be present in a given profile. Based upon these measurements, an average value for the vertical distance over which the electron density rises from the ambient E-region level to the peak Es level may be arbitrarily set at 500 m. In a specific Es region, however, this scale length may vary appreciably as a function of position along the Es altitude profile.

Electron density profiles measured by means of incoherent scatter radar do not at present possess the altitude resolution necessary to resolve details within Es regions. A typical electron density profile measured during the 1976 experiment is illustrated in Figure 3.1. In this particular case the Es is located near 111 km altitude and is detectable in three consecutive 600 m range gates.
This implies that the total width of the $E_S$ is $\leq 1.8$ km. On the whole the vertical dimensions of the $E_S$ observed during the 1976 experiment ranged from $< 600$ m to $\sim 3$ km.

Despite the increased sophistication of many of the modern techniques used to examine $E_S^*$, the vast majority of $E_S^*$ measurements are still performed using conventional sweep frequency ionosondes. Historically, the existence of "abnormal $E$" was first noted by Sir Edward Appleton (1930) using the frequency change method of ionospheric radio wave sounding. Since that time "abnormal $E$" has become known as "sporadic $E$" and a standard terminology has been developed in order to identify the various features on $E_S^*$ ionograms. Figure 3.2 depicts an idealized ionogram along with some of the standard nomenclature. The blanketing frequency, $f_bE_S^*$, is the frequency below which no O-mode traces are visible at virtual heights ($h^*$) above the $E_S$.

The critical frequency, $fE_S^*$, is the greatest frequency at which a more or less continuous $E_S$ echo is detected. When $fE_S$ is distinguishable for individual O-mode and X-mode traces, the critical frequencies are denoted $fO_{E_S}$ and $fX_{E_S}$ respectively. If the critical F-region frequency, $f_0F_2$, exceeds $f_bE_S^*$, then the E region is by definition partially transparent to radio waves in the frequency interval $f_bE_S^* - f_0E_S^*$.

The correct interpretation of the observed $E_S$ ionograms traces has been a subject of considerable controversy. In this regard attention is focussed upon the physical properties of $E_S$ which serve to reflect and scatter incident radio waves. In most $E_S$ models $f_bE_S$ is interpreted as the smallest value of the plasma frequency at the $E_S$ peak lying within the cone of angles defined by the first Fresnel zone.
of the blanketing frequency in the F region. This association is supported by rocket observations, wherein $f_{bE_s}$ values are typically within 10% of the measured plasma frequency at the peak of the $E_s$ region (Reddy and Rao, 1968). On the other hand, the interpretation of $f_{oE_s}$ (or $fE_s$) usually marks the point of divergence of structural $E_s$ models.

Of the several models proposed, two have emerged as the most likely explanations of the experimental observations. In one model, the peak plasma frequency is assumed to vary appreciably along the horizontal dimensions of the $E_s$. It is assumed that the plasma exhibits a patchy spacial structure and that reflection within a given patch is always total. On the basis of this model, it is inferred that $f_{oE_s}$ represents the maximum plasma frequency of any of the reflecting ionization clouds lying within the ionosonde field of view. Two variations of the patchy $E_s$ model exist. In the first variation one assumes that small intense patches having peak plasma frequencies above $f_{bE_s}$ are embedded in an otherwise horizontally stratified plasma layer of maximum frequency $f_{bE_s}$, while in the second the local peak plasma frequency is permitted to fluctuate equally both above and below a spacially average value of the peak plasma frequency (Whitehead, 1972a).

In the second model, weak (1-5%) density irregularities in the vertical direction act as a source of partial reflections by scattering the incident radio waves (Reddy, 1968). Among others, a Gaussian spectral distribution of irregularities centered near the scale size $c/(2f_{bE_s})$ is usually assumed. In this model, it is the size of the irregularity rather than the amplitude of the density fluctuation that
determines the maximum detectable scattered frequency (Booker and Gordon, 1950). More specifically, an irregularity of scale size L would scatter all radio waves having a wavelength in the medium greater than 2L.

A third model in which partial reflection is attributed to gradient reflections from thin layers containing steep gradients does not appear to be applicable to the majority of experimental observations. Calculations by Chessell (1971 a,b) and Miller and Smith (1977) indicate that a partial transparency \( f_{0s} - f_{bs} \) of only about 1 MHz may be explained in this manner, while transparencies much greater than this are commonly observed.

Experimentally, Whitehead (1972a) concluded that, on the basis of radio sounding data, a patchy \( E_s \) plasma containing spacial variations above and below the mean peak plasma frequency was consistent with the observations. At the same time, Whitehead indicated that small scale irregularities capable of scattering radio waves might also be present. Miller and Smith (1975) used the Arecibo 430 MHz radar to scan \( E_s \) regions by sweeping the beam in azimuth. Their study revealed that considerable horizontal structure may at times be present in the \( E_s \) plasma and that the deduced horizontal scale lengths range from hundreds of kilometers to less than 300 m. The ionosonde measurements of Chessell et al. (1973) support a patchy \( E_s \) model containing horizontal density variations over scales of 3-7 km. These results also imply that the density irregularities lead to refractive focusing of the incident HF beam.
On the other hand, vertical irregularities possessing scale sizes of \( \sim 25 \) m were detected in the rocket experiment of Bowhill (1966). Furthermore, Tao (1966) found that certain types of oblique VHF (50 MHz) propagation in \( E_s \) may be attributed to scattering from density irregularities having vertical and horizontal scale sizes of 50 m and 200 m respectively. As we shall see later, the present study suggests that a hybrid \( E_s \) model incorporating the features of a patchy plasma along with small scale vertical and horizontal irregularities (\(< 300 \) m) probably provides the best physical description of \( E_s \).

Although many variations of the wind shear theory exist, it is generally accepted that the convergence of ions by shears in the neutral wind is the primary driving force in the formation of \( E_s \). The theory behind the wind shear mechanism has been developed and extended by many authors (e.g. Whitehead, 1961, 1966; Axford, 1963; Axford and Cunnold, 1966; MacLeod, 1966; Kato, 1966). Above \( \sim 120 \) km north-south wind shears serve to converge ionization by driving ions up and down magnetic field lines, while at lower altitudes it is the Lorentz force due to east-west winds that compresses the ionization. The wind shear theory is not, however, without its problems as Whitehead (1972b) and Layzer (1972) are quick to point out. One of its chief difficulties arises when one attempts to account for the sporadic nature of \( E_s \), since the wind shears necessary to converge ions into thin layers are almost always present.

In the daytime E region \( \text{NO}^+ \) and \( \text{O}_2^+ \) are the dominant ions, both of which have comparable concentrations (Strobel, 1974). These ions, however, have too fast a recombination rate to allow regions of high
electron density to form within the context of the wind shear theory (Layzer, 1964; Harper et al., 1975). Nevertheless, diffusion limited regions brought about by the convergence of metallic ions are consistent with the theory (Whitehead, 1967). Furthermore, the presence of metallic ions in the altitude regions where \( E_s \) forms is confirmed by rocket-borne mass spectrometer measurements (Narcisi, 1968; Zbinden et al., 1975). The metallic ions most common are \( \text{Fe}^+ \), \( \text{Mg}^+ \), and \( \text{Si}^+ \), but traces of \( \text{Al}^+ \), \( \text{Na}^+ \), \( \text{Ca}^+ \), and \( \text{K}^+ \) are also detected. Metallic ions are therefore assumed to compose the bulk of the \( E_s \) ionization. This is supported by mass spectrometer data recorded in \( E_s \) wherein \( \text{Fe}^+ \) and \( \text{Mg}^+ \) in comparable quantities appear to be the dominant ions (Young et al., 1967). The metallic ions are presumably meteoritic in origin.

3.2 E-REGION TEMPERATURES

In general, the ion temperature at Arecibo in the 100-130 km region may be roughly estimated on the basis of established incoherent scatter data. However, barring a direct experimental measurement at the time of interest, an accurate temperature value may not be deduced. At E-region altitudes \( > 100 \text{ km} \) temperature oscillations with periods of \( \sim 12 \text{ hours} \) are most prominent in the Arecibo observations. These oscillations have been tentatively associated with a dominant semi-diurnal tidal mode (Harper et al., 1976; Wand, 1976). Although the average phase of the semi-diurnal oscillation undergoes a gradual seasonal variation (see e.g. Evans, 1978), the day to day shift of the phase and period may be considerable (Harper, 1977). This variability
is thought to result from local effects which include temperature fluctuations due to gravity waves as well as variations in the atmospheric medium (Wand, 1976).

Any temperature prediction is uncertain to the extent that the temperature oscillations described above must be included as an unknown. Because of this difficulty, assumed temperature values are reliable only to within 50-70°K. This uncertainty is reflected in Figure 3.3a which illustrates a range of possible temperature values derived from typical Arecibo measurements.

At altitudes <\sim 130 km the neutral temperature, $T_n$, and electron temperature, $T_e$, are simply given in terms of the ion temperature, $T_i$. We may write $T_e = T_n = T_i$. This follows since particle collisions are frequent and a close thermal coupling exists within the gas (Wand and Perkins, 1970).

3.3 E-REGION COLLISION FREQUENCIES

In the limit that $\omega_{HF} \gg \nu_{en}$, where $\nu_{en}$ is the electron-neutral collision frequency, the high frequency conductivity in a plasma becomes independent of $\omega_{HF}$ and the concept of an effective collision frequency is meaningful (Ginzburg, 1970). The value of $\nu_{en}$ may then be calculated using the displaced Maxwellian distribution method of Banks and Kockarts (1973). Given the cross sections of Banks and Kockarts applied to the lower ionosphere, one obtains
\[ \nu_{en} \text{ (sec}^{-1}) = 2.33 \cdot 10^{-11} n(N_2)T_n + 6.5 \cdot 10^{-12} n(O_2)T_n \]
\[ + 1.8 \cdot 10^{-10} n(O_2)T_n^{1/2} + 8.2 \cdot 10^{-10} n(O)T_n^{1/2} \]

(3.1)

where \( n(N_2) \), \( n(O_2) \), and \( n(O) \) are molecular and atomic number densities (cm\(^{-3}\)), and \( T_n \) is the neutral temperature (°K), which has been set equal to the electron temperature.

At characteristic E-region temperatures (≤ 300°K), ion-neutral collisions occur primarily as a result of induced dipole attraction. If one assumes that polarization attraction is most important and allows for an average ion mass of 31 amu, one arrives at (Hill and Bowhill, 1977)

\[ \nu_{in} (31^+) = 4.3 \cdot 10^{-10} n(N_2) + 4.2 \cdot 10^{-10} n(O_2) \]
\[ + 2.4 \cdot 10^{-10} n(O) \]

(3.2)

The corresponding result for Fe\(^+\) ions is

\[ \nu_{in} (Fe^+) \propto 0.6 \nu_{in} (31^+) \]

At E-region altitudes (≤ 120 km) the electron-ion collision frequency, \( \nu_{ei} \), may be expressed as (Eq.(6.14) of Ginzburg, 1970)

\[ \nu_{ei} = 3.8 \cdot 10^3 \text{ sec}^{-1} \frac{n_5}{T_2^{3/2}} \]

where \( n_5 \) is the electron density in units of 10\(^5\) cm\(^{-3}\) and \( T_2 \) is the electron temperature in units of 100°K. For typical E-region parameters \( \nu_{ei} \) is approximately 1 kHz. This value may of course increase
considerably in $E_s$, where electron densities are often much larger than ambient levels.

The shaded area in Figure 3.3b illustrates the range of $v_{in}$ values commonly measured at Arecibo by incoherent scatter techniques. Estimates of $v_{in}$ and its variability were made from the experimental results of Wand (1976), Harper et al. (1976), Zamlutti and Farley (1975), Dines (1974), and Wand and Perkins (1970). A calculated range of acceptable $v_{en}$ values is indicated by the shaded region in Figure 3.3c. In deriving the zone of $v_{en}$ values, the relative $O_2$, $N_2$, and $O$ abundances in the E region were approximated by the CIRA (1972) values. The range of $v_{in}$ values in Figure 3.3b were subsequently used in conjunction with Eq. (3.2) to obtain an absolute range of neutral abundances. These abundances, combined with the mean temperatures in Figure 3.3a, were then substituted into Eq. (3.1) in order to produce the displayed $v_{en}$ regions.

The derived values of $v_{en}$ are consistent with the measurements of Showen (1972). However, Showen's measured $v_{en}$ values, which have rather large statistical uncertainties attached to them, suggest that the lower boundary of acceptable $v_{en}$ values be set closer to the dashed line in Figure 3.3c. The electron-neutral collision frequencies reported by Thrane and Piggot (1966) are in general slightly larger than those calculated here. An additional experimental estimate of $v_{en}$ may be obtained from the plasma line decay time in $E_s$ measured by Gordon and Carlson (1976). If one assumes that $v_{ei} \approx 3.3$ kHz in the observed instability region then one finds $v_{en} \approx 7.3$ kHz near 113 km. This is just outside of the shaded area in Figure 3.3c.
Finally we note that on the basis of incoherent scatter data, Zamlutti and Farley (1975) reported experimental values for the proportion of Fe$^+$ ions, $T_i$, and $\nu_{in}$ in $E_S$. Equally good fits to their data were possible for parameter values of 70% or 90% Fe$^+$ ions, 240°K or 290°K, and 3363 sec$^{-1}$ or 5118 sec$^{-1}$, respectively. The $E_S$ was centered at 104.1 km and was observed in late afternoon. A similar experiment conducted by Behnke and Vickery (1975) on midafternoon $E_S$ located near 100 km indicated a 70% relative abundance of Fe$^+$ ions.
20 JULY 1976
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FIGURE 3.1 An electron density profile exhibiting sporadic E measured by incoherent scatter radar. The altitude resolution of the data is 600 m.
FIGURE 3.2 Idealized ionogram containing sporadic E. The nomenclature of URSI (1972) has been adopted.
FIGURE 3.3 Ranges of parameter values (shaded areas) for the ion temperature (a), the ion-neutral collision frequency (b), and the electron-neutral collision frequency (c) in the E region over Arecibo. The dashed line in (c) is discussed in the text.
CHAPTER 4
THE HF ELECTRIC FIELD OF THE MODIFYING WAVE

The modifying HF beam is launched vertically into the ionosphere from a crossed log-periodic antenna located along the focal line of the 300 meter reflecting dish. A single frequency radio wave in the range of 4-12 MHz is transmitted with typical powers of order 100 kW. Either ordinary (O) mode or extraordinary (X) mode polarization may be selected. In the present experiment, the O mode is of primary interest since a wave of this polarization propagates to altitudes where the electron densities are sufficiently large to allow the parametric decay instabilities to be excited. At vertical incidence the ray path (direction of energy flow) of the O mode deviates in the direction of magnetic north before reflecting at an altitude where the HF frequency, \( f_{HF} \), equals the local plasma frequency, \( f_e \). At this point the ray direction is perpendicular to the magnetic field. Ray paths are strongly influenced by the electron density gradient near the point of reflection and are therefore dependent upon the location of the reflection point along the electron density profile. For a wave reflecting in the F region, where a representative scale height near reflection might be 50 km, the deviation to the north is approximately 25 km. If one assumes that sporadic E is a horizontally
stratified plasma layer and that scale heights within the layer are 1 km or less, then the amount of deviation expected is not greater than a few hundred meters.

It is of interest to estimate the magnitude of the HF electric field near the height of reflection where the coupling of the electromagnetic field to the plasma is strongest. In the region below reflection the altitude dependence of the field takes the form of a standing wave pattern. If the electric field perturbs the plasma only weakly, the field strength may be determined from Maxwell's equations accompanied by a "cold plasma" approximation. An exact solution to the resulting wave equation may be obtained by numerically integrating the "full-wave" solution described by Budden (1966). Miller and Smith (1977) have applied this technique to $E_s$ density profiles measured by Langmuir probes aboard sounding rockets in order to determine the transmission and reflection coefficients of the $E_s$ "layers." In the work of Miller and Smith the calculated $E_s$ electric fields were not reported.

Since incoherent backscatter techniques cannot at present resolve details within the electron density profile of $E_s$, a full-wave treatment of the HF electric field problem is unwarranted. In fact, this approach is not strictly correct because the cold plasma approximation excludes effects due to nonlinear wave-plasma interactions which are relevant to the present study. In light of these difficulties and others, the wave equation will be solved analytically for a few simple profiles. The goal of this exercise is to qualitatively
describe the standing electric field pattern of the modifying HF wave given the large density gradients assumed to be present in $E_s$.

We begin by examining the case of a wave vertically incident upon a linear plasma layer which is horizontally stratified in space. Effects due to absorption and the presence of an external magnetic field will be neglected. For steep density gradients, linear absorption due to ohmic losses near reflection is expected to be small. This absorption may be estimated using Eq.(31.13) of Ginzburg (1970) in conjunction with Ginzburg's Figure 31.1. If a typical gradient in $E_s$ is approximated by the density corresponding to $f_bE_s$ divided by the estimated layer half width, then the absorption may be calculated to be at the 1.0 db level or less. Here we have assumed that $f_{HF}/f_bE_s < 0.9$. The introduction of a uniform magnetic field into the calculations for vertical incidence does not qualitatively affect the final results. This is particularly true when one is considering a linear density profile (Ginzburg, 1970).

In general, the wave equation for propagation in an inhomogeneous, isotropic plasma may be written in terms of electric field $E(z)$ as
\[
\frac{d^2E}{dz^2} + \frac{\omega_{HF}^2}{c^2} \varepsilon E = 0
\]
The permittivity, $\varepsilon$, for a linear density profile, $N(z)$, is given by
\[
N(z) = N_r \left( \frac{z}{z_r} \right)
\]
\[
N_r = \frac{m e \omega_{HF}^2}{4 \pi e^2}
\]
\[
\varepsilon = 1 - \frac{N(z)}{N_r} = 1 - \frac{z}{z_r}
\]
where $z$ equals zero at the base of the layer and $z_r$ is the point of reflection. By defining the new variable, $\zeta$,

$$\zeta = \left( \frac{2 \omega_{HF}}{c^2 z_r} \right)^{1/3} (z - z_r) \quad (4.1)$$

The wave equation may be written (Ginzburg, 1970)

$$\frac{d^2 E}{d\zeta^2} - \zeta E = 0$$

Solutions to this equation are given in terms of linear combinations of the Airy functions, $Ai(\zeta)$ and $Bi(\zeta)$ (Abramowitz and Stegun, 1972). However, only the solution $Ai(\zeta)$ is physically meaningful since $Bi(\zeta)$ monotonically increases as a function of $z$ for $z > z_r$. After normalizing $E$ in terms of the incident electric field, $E_{inc}$, the following solution is obtained.

$$S \equiv \left( \frac{E(z)}{E_{inc}} \right)^2 = 4\pi \left( \frac{\omega_{HF} z_r}{c} \right)^{1/3} Ai^2(\zeta)$$

This function is plotted in Figure 4.1a for a scale length, $z_r$, of 0.5 km and $f_{HF}$ equal to 5.94 MHz. A maximum amplitude of 14.3 is achieved for $\zeta = -1.02 \equiv z_m$ (0.032 km below $z_r$). At $\zeta = 3.8$ (0.12 km above $z_r$), $S \approx 10^{-4}$.

As a simple extension of the above model, one may examine the electric field structure in the double linear layer shown in Figure 4.1b. As in the case of the single layer, an analytical solution to the wave equation can be obtained. After applying the appropriate boundary conditions at $z = 0$, the solution may be written (Budden, 1966)
\[
\frac{E(\xi)}{E_{\text{inc}}} = \begin{cases} 
A \xi + B \xi; & p = +1 \text{ for } 0 \leq \xi \leq \xi_1 \\
-A \xi + B \xi; & p = -1 \text{ for } -\xi_1 \leq \xi \leq 0
\end{cases}
\]

\[
\xi = -\left(\frac{\omega_{\text{HF}}^2}{c^2 \xi_1 X}\right)^{1/3} (pz_1 + \xi_1 X), \quad X \equiv \left(\frac{f_{\text{HF}}}{f_{\text{max}}^2}\right)^2
\]

where \(f_{\text{max}}\) is the peak plasma frequency of the layer, and 

\[
g(\xi) = A_i(\xi) B(\xi) - B(\xi) A_i(\xi)
\]

\[
h(\xi) = -A_i(\xi) B^*(\xi) + B(\xi) A^*(\xi)
\]

In the above equations the primes indicate differentiation with respect to \(\xi\) and \(\xi_0\) denotes \(\xi(0)\). \(A\) and \(B\) are constants determined by wave continuity conditions at \(z = \xi_1\) and \(z = -\xi_1\).

For \(X \lesssim 0.9\), \(f_{\text{HF}} \gtrsim 4\) MHz, and \(\xi_1 \gtrsim 0.1\) km, the above solution essentially reduces to that of a single linear layer and radio wave penetration through the layer is negligible (see e.g. Figure 16.8 of Budden, 1966). In Figure 4.1b the double layer solution is depicted for the parameter values \(\xi_1 = 0.5\) km, \(f_{\text{HF}} = 5.94\) MHz, and \(X = 0.95\). In this figure \(N_0 = (m_e w_{\text{max}}^{2}/4 \pi e^2)\). The HF frequency equals the local plasma frequency, \(f_e\), at \(z = \pm 24\) m. As a result radio wave propagation in the altitude range \(-24 < z < +24\) m is evanescent in nature. The topside \((z > 0)\) electric field strength at \(z = 24\) m is due to evanescent tunneling and is approximately equal to \(E_{\text{inc}}\). The first maximum of the bottomside electric field indicates
a "swelling factor," $S$, of 12.9. This is slightly less than the factor 14.3 calculated using a single linear layer with the same density gradient. As $X$ increases the net effect is to decrease the bottom-side swelling and to increase the leakage to the topside.

Figure 4.1c illustrates the effect of increasing the density gradient (decreasing $z_1$) on the solution to the double layer problem. The parameters used in the calculation are the same as those used in the preceding example except that $z_1$ has been reduced from 0.5 km to 0.1 km. The region of evanescence extends from $-4.8 \text{ m} < z < +4.8 \text{ m}$. Although an increase in the density gradient tends to limit tunneling, this effect is more than compensated for by the decrease in layer thickness. The swelling factor at the first bottomside maximum is 5.8 compared to 8.4 obtained for the single layer analogue.

In the calculations outlined above, we have not taken into account possible nonlinear interactions near the point of reflection. However, a sufficiently intense electric field may give rise to Langmuir waves which are parametrically driven unstable. The resultant flow of energy from the incident electromagnetic radiation into electrostatic oscillations depletes the strength of the electromagnetic pump wave. Thus, in the nonlinear regime one must take into account this "anomalous" source of absorption in addition to the linear collisional absorption discussed above. DuBois et al. (1973) have examined the effects of both linear and nonlinear absorption on the Airy structure produced in an infinite linear layer. For F-region conditions and a scale height of 75 km they conclude that the overall standing
wave pattern remains intact but that the power maxima of the Airy structure are reduced 20 to 30%. Approximately 60% of the total absorption is nonlinear. This is consistent with the nonlinear absorption estimate of $\sim 16\%$ made by Perkins et al. (1974) for F-region conditions at Arecibo.

Given comparable power densities near reflection, nonlinear absorption in $E_S$ is expected to be considerably less than in the F region. This is because the instability region in $E_S$ is narrowed by the smaller characteristic scale lengths present ($\ll 1$ km). If one assumes that only deviative absorption is important, then the electric field structure may be estimated using the absorption free solution scaled down by $\ll 25\%$.

As noted earlier, the presence of an external magnetic field causes the ray path of a vertically incident radio wave to refract near the point of reflection. However, the HF beam has a finite beam width due to diffraction at the antenna aperture. For typical HF frequencies used in this experiment, the half-power beam width, $\theta_{HF}$, is of the order of $10^\circ$. Oblique propagation within the magnetic meridian plane is easiest to consider. In this case reflection occurs at the same altitude as for vertical incidence provided that the incident ray directions are within a certain critical angle, $\theta_S$, of vertical (Ginzburg, 1970). This is the so-called "Spitze" phenomenon. The value of $\theta_S$ for a 5 MHz signal is $\sim 18^\circ$. For off-meridian propagation the reflection altitudes are only slightly less than the meridional values.
An important effect associated with oblique incidence involves the direct conversion of electromagnetic waves into electrostatic waves near the critical point, \( z_c \), where \( \omega_e(z_c) = \omega_{HF} \) (Stenzel et al., 1974a). In the absence of a magnetic field reflection occurs at an altitude, \( z_r \), determined by \( \omega_e(z_r) = \omega_{HF}\cos(\theta_o) \) where \( \theta_o \) is the angle of incidence measured from the vertical. Below reflection, an Airy type electric field structure similar to that at vertical incidence is set up (see Ginzburg, 1970). However, if the incident electromagnetic wave contains a component in the plane of incidence (i.e. parallel to the gradient) then the electric field patterns will also exhibit a resonant singularity at the critical altitude, \( z_c \). Near \( z_c \), where the field component parallel to the gradient is particularly large, the oscillating electric field may give rise to charge bunching along the gradient. In a finite temperature plasma, the charge accumulations lead to the propagation of plasma waves down the gradient towards regions of lower density. Upward propagating plasma waves are not possible since the plasma wave dispersion relationship implies that such waves are immediately reflected. It is noteworthy that the measurements of Stenzel et al. (1974a) indicate that linear conversion into electrostatic waves is more efficient at steep density gradients. In section 7.4 linear conversion will be examined further as a possible interaction process for an HF wave in \( E_s \).

In concluding the discussion of the HF electric field, it is important to recognize that the assumption of a stratified ionosphere may find only limited application in reference to \( E_s \). As described in
Chapter 3 many of the characteristics of $E_s$ are interpretable in terms of both small and large scale irregularities contained within a plasma of minimum plasma frequency $f_p E_s$. When a radio wave propagates through such an irregular medium a considerable amount of diffraction and scattering takes place. As a result it is not at all unlikely that large distortions may be created in the Airy structure derived above for a stratified ionosphere (see e.g. Pitteway, 1958). Furthermore, even in a smooth ionosphere, instability produced irregularities may affect the propagation of HF waves. The variety of possible effects includes the scattering of the HF wave into Langmuir waves by field-aligned inhomogeneities (Graham and Fejer, 1976), scattering of O-mode radiation into trapped X-mode radiation (Ryzhov, 1977), and the self-action of the HF wave (Vas'kov and Gurevich, 1973).
FIGURE 4.1 Calculated electric field structure for a single linear layer (a) and two double linear layers (b) and (c). The transmission coefficients squared, $\left(\frac{E(\infty)}{E_{\text{inc}}}\right)^2$, for double layers b and c are 0.18 and 0.54 respectively. The arrows in the plots bracket the regions that are overdense at $f_{\text{HF}}$. 
FIGURE 4.1
CHAPTER 5
PARAMETRIC INSTABILITY THEORY

Theoretical studies of the parametric amplification of plasma waves under F-region conditions have been performed by a number of authors (e.g. Perkins and Kaw, 1971; Fejer and Leer, 1972a; Harker, 1972; Fejer and Kuo, 1973; Arnush et al., 1974; Perkins et al., 1974; and references therein). These results, however, were derived for plasma parameters much different than those found in $E_s$. Consequently, it is necessary to delineate the limits of applicability of the F-region results to the $E_s$ plasma. Furthermore, in developing theoretical models for the interaction of an HF wave in $E_s$, one must give consideration to processes that are recessive in the F region, but may become dominant in $E_s$. Thus, the goal of the investigation presented below is to describe the various wave-plasma interactions that might be feasible in an $E_s$ plasma.

5.1 GENERAL COUPLED MODE FORMALISM

Mode coupling is a very common phenomenon associated with the interaction of one or more different types of disturbances (modes) in a medium. Of special relevance to the parametric processes in plasmas considered here is the coupling of two normal modes, one of which has
a frequency much greater than the other. In a plasma the high frequency mode corresponds to propagating Langmuir waves while the low frequency oscillation is associated with the motion of the ions (e.g. ion acoustic waves). Parametric excitation results in the transfer of energy from the electromagnetic pump to electrostatic plasma oscillations which subsequently grow in amplitude. When this process is viewed as a type of nonlinear wave-wave interaction in the plasma, the plasma motion is adequately described by the fluid equations. This allows the whole interaction process, commonly referred to as the decay instability, to be formulated as a mode coupling problem. Given the broad nature of this problem, it is simplest to develop the general formalism for mode coupling and then proceed to show that the response of a driven plasma can be cast into the proper mathematical form.

In order to examine the parametric instability in a homogeneous, unmagnetized plasma, we first outline the relevant formalism for a pair of coupled harmonic oscillators. Many different treatments of the coupled mode problem may be found in the parametric instability literature (e.g. DuBois and Goldman, 1967a; Nishikawa, 1968a,b; Lashmore-Davies, 1975; Nishikawa and Liu, 1976; Franklin, 1977). The approach which is adopted below was suggested by Wolf (1976) and is somewhat different than the formalism developed by Nishikawa and Liu (1976) in that a solution by Laplace transforms rather than by Fourier transforms is employed. This derivation is included because of its inherent simplicity and subsequent use in illustrating parametric processes in $E_s$ plasmas. The notation of Nishikawa and Liu (1976) is maintained whenever possible.
In formulating the equations of motion for two coupled harmonic oscillators, we begin by introducing a spacially homogeneous external pump field, \( Z(t) \), which may be written as

\[
Z(t) = 2Z_0 \cos \omega_0 t
\]  

(5.1)

where \( Z_0 \) is a constant. This field is assumed to be undamped. With the introduction of the coupling operators, \( L_1 \) and \( L_2 \)

\[
L_1(X(t)) = \left[ \frac{d^2}{dt^2} + 2\Gamma_1 \frac{d}{dt} + \omega_1^2 \right] X(t)
\]

\[
L_2(Y(t)) = \left[ \frac{d^2}{dt^2} + 2\Gamma_2 \frac{d}{dt} + \omega_2^2 \right] Y(t)
\]

the coupled mode equations for the oscillator amplitudes \( X(t) \) and \( Y(t) \) follow as

\[
L_1(X(t)) = \lambda Z(t) Y(t)
\]  

(5.2)

\[
L_2(Y(t)) = \mu Z(t) X(t)
\]  

(5.3)

where the \( \Gamma \)'s are the amplitude attenuation coefficients, and \( \lambda \) and \( \mu \) are coupling constants. Eqs.(5.2) and (5.3) employ a linear approximation in that they do not contain products of the small quantities \( \lambda \) and \( \mu \). Without loss of generality we specify that \( \omega_1^2 \ll \omega_2^2 \). The quantity \( \omega_1(\omega_2) \) corresponds to the low (high) frequency oscillation in the unpumped plasma.
A solution to Eqs.(5.2) and (5.3) may be obtained by first taking their Laplace transforms. This yields

\[
(s^2 + 2\Gamma_1 s + \omega_1^2)\psi(s) - \lambda Z_o [\xi(s + i\omega_o) + \xi(s - i\omega_o)] = C_1(s)
\]

(5.4)

\[
(s^2 + 2\Gamma_2 s + \omega_2^2)\xi(s) - \mu Z_o [\psi(s + i\omega_o) + \psi(s - i\omega_o)] = C_2(s)
\]

where

\[
\psi(s) = \int_0^\infty e^{-st}x(t)dt, \quad \xi(s) = \int_0^\infty e^{-st}y(t)dt
\]

(5.5)

\[
C_1(s) = \dot{x}(0) + x(0)(s + 2\Gamma_1), \quad C_2(s) = \dot{y}(0) + y(0)(s + 2\Gamma_2)
\]

and \(C_1\) and \(C_2\) are everywhere analytic.

In order to bring the above results into the form of Nishikawa and Liu (1976), and to make the above expressions easily factorable, we assume \(\Gamma_1^2 \ll \omega_1^2\), \(\Gamma_2^2 \ll \omega_2^2\) (i.e. the waves oscillate many times before being damped) and add \(\Gamma_1^2\psi(s)\) and \(\Gamma_2^2\xi(s)\) to the left hand sides of Eqs.(5.4) and (5.5) respectively. This approximation introduces no serious errors into the present calculations. In situations where \(\Gamma_1 (\Gamma_2)\) is large, the plasma response at \(\omega_1 (\omega_2)\) cannot adequately be described as a wave and the whole idea of a wave-wave interaction loses its meaning.

Eqs.(5.4) and (5.5) become

\[
D_1(s)\psi(s) - \lambda Z_o [\xi(s + i\omega_o) + \xi(s - i\omega_o)] = C_1(s)
\]

(5.6)

\[
D_2(s)\xi(s) - \mu Z_o [\psi(s + i\omega_o) + \psi(s - i\omega_o)] = C_2(s)
\]

(5.7)

where

\[
D_n = (s + \Gamma_n + i\omega_n)(s + \Gamma_n - i\omega_n) \quad n = 1, 2
\]
Resonant transfer of energy is expected to occur when the frequency matching condition \( \omega_0 = \omega_1 + \omega_2 \) is met. Let \( \omega_0 = \omega_2 + \delta \). To first order we may calculate the dispersion relationship by using Eqs.(5.6) and (5.7) to locate the lowest order poles, \( s_p \), of \( \psi(s) \) and \( \xi(s) \). If one assumes that \( \delta \ll \omega_1, \omega_2 \) then the resonance condition indicates that we should look for the poles of \( \psi(s) \) near \( s_p \approx 0 \), while a search near \( s_p \approx i\omega_0 \) should be made for \( \xi(s) \). To solve for the poles, the substitution \( s \rightarrow s \pm i\omega_0 \) is made in Eq.(5.7), which subsequently becomes

\[
\xi(s \pm i\omega_0) D_2(s \pm i\omega_0) - \mu Z_0 [\psi(s \pm i2\omega_0) + \psi(s)] = C_2(s \pm i\omega_0)
\]  \quad (5.8)

We may neglect any contribution from \( \psi(s \pm i2\omega_0) \) since the poles at \( s_p \approx 2i\omega_0 \) are off-resonant. The initial boundary conditions are set such that \( C_1(s) = C_2(s) = 0 \). After substituting Eq.(5.8) into Eq.(5.6) and applying the resonance approximation,

\[
D_2(s \pm i\omega_0) \approx \pm 2i\omega_2 (s \pm i\omega_0 + \Gamma_2 \mp i\omega_2),
\]

one may separate out the real and imaginary parts of the final result using \( s \equiv y + ix \). The following relations are then obtained.

**REAL**

\[
(y + \Gamma_1)^2 + \omega_1^2 - x^2 = \frac{-\mu \lambda Z_0^2}{2\omega_2} \left[ \frac{x - \delta}{(y + \Gamma_2)^2 + (\delta - x)^2} - \frac{x + \delta}{(y + \Gamma_2)^2 + (\delta + x)^2} \right]
\]  \quad (5.9)

**IMAGINARY**

\[
(y + \Gamma_1)(-2x) = \frac{-\mu \lambda Z_0^2}{2\omega_2} \left[ \frac{y + \Gamma_2}{(y + \Gamma_2)^2 + (\delta - x)^2} - \frac{y + \Gamma_2}{(y + \Gamma_2)^2 + (\delta + x)^2} \right]
\]  \quad (5.10)
The two possible solutions to Eq.(5.10) are

\[ x = 0 \quad \text{and} \quad x \neq 0 \quad y + \Gamma_1 = \frac{(y + \Gamma_2)}{(\omega_2/\delta u_\lambda Z_0^2)[(x+\delta)^2 + (y+\Gamma_2)^2][(x-\delta)^2 + (y+\Gamma_2)^2]} \]  

(5.11a)  

\[ (5.11b) \]

Solution (a) is often referred to as the purely growing mode instability or, in relation to plasma physics, the oscillating two-stream instability. Solution (b) is termed the decay instability since it involves the decay of the external pump wave, Z, into two lower frequency waves.

At this point we note that there is an essential difference between the problem of three interacting harmonic oscillators and the three-wave decay process in a plasma. All three-wave decay processes must satisfy the phase matching conditions in wave number, \( \vec{k}_0 = \vec{k}_1 + \vec{k}_2 \), as well as the frequency matching condition, \( \omega_0 = \omega_1 + \omega_2 \), where \( \omega_1 \) (\( \equiv \chi \)) and \( \omega_2 \) denote the low and high frequency responses in the pumped plasma respectively. For the infinite wavelength pump considered above, \( \vec{k}_0 = 0 \), and consequently \( \vec{k}_1 = -\vec{k}_2 \). Given either \( \omega_1 \) or \( \omega_2 \), the other quantity is immediately obtainable from the frequency matching condition.

Because products of oscillating quantities were neglected in the formulation of the coupled mode problem, solutions (a) and (b) represent completely independent modes of excitation. Each mode may therefore be examined separately.
Purely Growing Mode

A growing, unstable solution for case (a) may be found by setting \( x = 0 \) in Eq. (5.9) and requiring that \( y > 0 \). For \( \Gamma_1^2 \ll \omega_1^2 \), the threshold criterion follows as

\[
K_{th}(\delta) = \frac{-\omega_2}{\delta} (\Gamma_2^2 + \delta^2) \omega_1^2
\]  
(5.12)

where we have defined \( K \equiv \mu \lambda Z_0^2 \). The minimum threshold occurs at \( \delta = \delta_m \) where

\[
\delta_m = -\text{sign}(\lambda \mu) \Gamma_2,
\]  
(5.13)

\[
K(\delta_m) = K_m = Z_m^2 |\mu \lambda| = 2 \omega_1^2 \omega_2 \Gamma_2
\]  
(5.14)

and

\[
\omega_o = \omega_2 - \Gamma_2 \text{sign}(\lambda \mu)
\]  
(5.15)

In the physical situation of interest, \( \text{sign}(\lambda \mu) \) is +1.

The condition for a maximum growth rate, \( y_m \), may be obtained by differentiating Eq. (5.9) \( (x = 0) \) with respect to \( \delta \) and setting \( (dy/d\delta) = 0 \). We find

\[
\omega_o = \omega_2 - (y_m + \Gamma_2) \text{sign}(\lambda \mu)
\]  
(5.16)

Decay Mode

Given the solution in Eq. (5.11b), the threshold power, \( K_{th}(\delta) \), and frequency mismatch, \( x_{th}(\delta) \), are easily derivable from Eq. (5.9) by putting \( y = 0 \) and solving for \( K(\delta) \) and \( x(\delta) \). The desired results are (see e.g. Nishikawa and Liu, 1976)
\[ K_{th}(\delta) = \frac{\Gamma_1 \Gamma_2 \omega_2}{\delta} \left[ 4\delta^2 + \frac{\Gamma_2^2 + 2\Gamma_1 \Gamma_2 + \omega_1^2 + \Gamma_1^2 - \delta^2}{\Gamma_1 + \Gamma_2} \right] \]  
(5.17)

\[ x_{th}(\delta) = \pm \left[ \frac{1}{\Gamma_1 + \Gamma_2} \left( \Gamma_2^2 \omega_1^2 + \Gamma_1 \delta^2 + \Gamma_1^2 \Gamma_2 + \Gamma_1 \Gamma_2^2 \right) \right]^{1/2} \]  
(5.18)

The minimum threshold, \( K_m \), and the frequency shift \( \delta_m \), are determinable from the equation \( dK_{th}(\delta)/d\delta = 0 \) which yields

\[ 3\delta_m^4 + 2\delta_m^2 \left[ (\Gamma_1 + \Gamma_2)^2 - \omega_1^2 + \Gamma_1^2 \right] - \left[ (\Gamma_1 + \Gamma_2)^2 + \omega_1^2 - \Gamma_1^2 \right] = 0 \]  
(5.19)

Of particular interest for ionospheric applications is the solution to Eq.(5.19) in the limiting case where the ion acoustic frequency is much greater than the Langmuir wave damping rate (i.e. \( \omega_1 >> \Gamma_2 \)). In this limit, one finds

\[ | K_m | = 4\omega_1 \omega_2 \Gamma_1 \Gamma_2 \text{ at } \omega_0 = \omega_2 + \text{sign} (\lambda \mu) \omega_1 , \; x_m = \pm \omega_1 \]  
(5.20)

where terms of order \( (\Gamma_1/\omega_1)^2 \) have been neglected compared to unity.

In general, a more accurate solution to the coupled mode problem than the one given above in Eq.(5.11) may be obtained by solving for the higher order poles in Eqs.(5.6) and (5.7). When this is done the above results are once again recovered provided that \( \omega_1 << \omega_2 \) and \( K << 8\delta \omega_2^3 \). These criteria are easily satisfied in the ionosphere.
In situations where they do not hold \( \delta \) should be replaced by

\[
\delta \rightarrow \delta - \frac{K}{2\omega_2 (4\omega_0^2 - \omega_1^2)}
\]

to provide a first order correction to all of the above equations.

5.2 THE COUPLED WAVE EQUATIONS IN A PLASMA

As noted earlier, the parametric processes of importance to the present experiment occur when an incident electromagnetic wave near the plasma frequency couples together low frequency ion oscillations and high frequency plasma waves. If a fluid description of the plasma is used, the parametric coupling may be expressed in terms of a pair of coupled wave equations similar in form to Eqs. (5.2) and (5.3). Once this is accomplished, the threshold and growth rate results of the preceding section may be used after making the appropriate variable substitutions.

A particularly straightforward derivation of the coupled wave equations in a plasma may be found in Nishikawa (1968b). In this treatment a hydrodynamic description of the plasma is employed. The pump electric field, \( \mathcal{E}_p \equiv 2 \mathcal{E}_1/2 \cos(\omega_0 t) \) is assumed to have an infinite wavelength (the dipole approximation). In addition, an adiabatic pressure law is presumed to hold

\[
\dot{\psi}_{p\alpha} = \gamma_{\alpha} T_{\alpha} \dot{\psi} n_{\alpha} \quad \text{or} \quad p_{\alpha} \gamma_{\alpha} n_{\alpha}
\]

where \( \kappa \) is Boltzmann's constant, \( p_{\alpha} \) and \( T_{\alpha} \) are the pressure and temperature respectively, \( n_{\alpha} \) is the spatially averaged number density,
\( \gamma_\alpha \) is a constant (usually 1 or 3) and \( \alpha \) is used to denote either the ion (i) or electron (e) species. Since only singly ionized ions are considered, one has \( n_e = n_i \equiv n_0 \).

Additional assumptions include \( \omega_o \gg \omega_e \gg \nu_i, \nu_e, \nu_i \) where \( \nu_i \) and \( \nu_e \) are the total ion and electron collision frequencies respectively. The plasma frequency \( \omega_\alpha \) and the unpumped plasma resonance frequency, \( \omega_{\alpha k} \), are given by

\[
\omega^2_\alpha = \frac{n_0 e^2}{\varepsilon_0 m_\alpha}, \quad \omega_{\alpha k} = \omega^2_\alpha + \frac{\gamma_\alpha k^2}{m_\alpha}
\]

The above criterion suggests a resonance condition of \( \omega_o \approx \omega_{ek} + \Omega_k \), where the ion acoustic frequency in the absence of the pump field, \( \Omega_k \), may be expressed as

\[
\Omega^2_k = \frac{k}{m_i} \{ \gamma_e T_e + \gamma_i T_i \} k^2.
\]

One further assumes that \( \varepsilon_0 E_{1/2}^2/(n_0 k T_e) \ll 1 \), which requires that the electric field energy density be much smaller than the kinetic pressure of the particles. Finally, static currents are omitted from consideration.

Given the above assumptions, Nishikawa's derivation leads us to the formulation of a pair of differential equations, one of which details the high frequency electron density fluctuations while the other characterizes the low frequency ion response. The coupled mode equations may be expressed as
\[
\frac{d^2}{dt^2} n_e^- + \omega_{ek} n_e^- + v_e \frac{d}{dt} n_e^- = \frac{ie}{m_e} n_i^- \mathbf{k} \cdot \mathbf{E}_{1/2}
\]
\[
\frac{d^2}{dt^2} n_i^- + \Omega_k^2 n_i^- + v_i \frac{d}{dt} n_i^- = -\frac{ie}{m_i} n_e^- \mathbf{k} \cdot \mathbf{E}_{1/2}
\]

where the primes indicate perturbed quantities. Thus, the physical quantities to be substituted for the variables in the preceding section are

\[n_i^- = X \quad n_e^- = Y \quad \mathbf{k} \cdot \mathbf{E}_{1/2} = Z_0 \quad \omega_{ek} = \omega_2\]

\[v_i = 2\Gamma_1 \quad v_e = 2\Gamma_2 \quad \frac{-ie}{m_i} = \lambda \quad \frac{ie}{m_e} = \mu \quad \Omega_k = \omega_1\]

5.3 PARAMETRIC INSTABILITIES IN HOMOGENEOUS PLASMAS

The derivation of parametric instability formalism for a homogeneous plasma in the preceding two sections provides a starting point for a more comprehensive treatment of the problem. Inspection of the above results allows for an initial description of the decay instability and the oscillating two-stream instability in a plasma. When the equations of section 5.1 are applied in the ensuing discussion, the variable substitutions itemized at the end of the preceding section will automatically be made.

In the three-wave decay interaction the incident electromagnetic pump wave decays into an ion acoustic wave of frequency \(\omega_i^- = \Omega_k\) and a Langmuir wave (Stokes component) at \(\omega_2^- = \omega_o - \omega_i\) where \(\omega_o\) is the pump frequency. The anti-Stokes component at \(\omega_2^+ = \omega_o + \omega_i\) will be ignored.
since it is off-resonant. The phase matching conditions are of course satisfied in all three-wave decay processes. Thus,

\[ \omega_0 = \omega_1 + \omega_2 \]

\[ \hat{k}_0 \sim 0 = \hat{k}_1 + \hat{k}_2 \]

\[ \hat{k} \equiv \hat{k}_1 = - \hat{k}_2 , \]

where \( \hat{k}_0, \hat{k}_1 \) and \( \hat{k}_2 \) are the pump wave vector, ion acoustic wave vector and plasma wave vector, respectively. This implies that the Langmuir wave and ion acoustic wave are oppositely directed.

For \( \Omega_k \gg \nu_e / 2 \), the minimum electric field threshold, \( E_0 \), of the decay instability is (Eq.(5.20))

\[ \left( \hat{k} \cdot \hat{E}_0 \right)^2 = \frac{4}{\varepsilon_0} \left( 1 + \frac{3 T_i}{T_e} \right) \frac{\nu_i \nu_e}{\Omega_k \omega_0} \]

(5.21)

where \( n_0, \omega_0, \varepsilon_0 \) are respectively the electron density, the HF pump frequency, and permittivity of free space and \( \hat{k} \) is a unit vector in the direction of \( \hat{k} \). The other quantities are defined in section 5.2. For convenience, the homogeneous plasma thresholds are also listed in Table 5.1. In deriving the above equation the approximation \( \omega_{ek} \sim \omega_0 \) has been employed. The constant \( \nu_e \) has been set equal to one since the phase velocity \( \Omega_k / \hat{k} \) is less than the mean electron thermal speed, \( \nu_e \) (isothermal approximation), while \( \nu_i \) was set equal to three since \( \Omega_k / \hat{k} > \nu_i \), where \( \nu_i \) is the mean ion speed (adiabatic approximation). Minimum threshold corresponds to a frequency mismatch, \( \delta = \Omega_k \), that is (Eq.(5.20)) \( \omega_0 = \omega_{ek} + \Omega_k \). For constant \( \hat{k} \), the associated frequency bandwidth of the resonance response at \( \omega_1 \) or \( \omega_2 \) is of the order of \( \Omega_k \).
The oscillating two-stream instability (OTSI) produces zero frequency ion fluctuations along with Langmuir waves having the same frequency as the pump. This instability may appropriately be thought of as a five-wave process wherein the electromagnetic wave decays into two oppositely directed Langmuir waves and two frequency shifted acoustic waves of zero frequency. A four plasmon description of the interaction is necessitated by the fact that both the Stokes ($\xi(i\omega_0)$) and the anti-Stokes ($\xi(-i\omega_0)$) components in Eq.(5.8) are simultaneously resonant. This is also in keeping with the absolute nature of the instability. An absolute instability is one that grows in time rather than in space. For any purely growing mode, the presence of a low frequency component at $\omega'_1 = 0$ is indicative of a local amplification of a perturbation which continues until nonlinear effects finally limit the growth. From the frequency matching conditions it is easy to see that the group velocity of the high frequency response is also zero. Thus, two oppositely directed Langmuir waves must be present in order to form the requisite standing wave pattern of zero group velocity. By way of contrast, the decay instability as observed in the ionosphere undergoes non-local convective growth (Arnush et al., 1974). This means that attendant electrostatic waves grow as they propagate through the plasma.

The minimum threshold for the oscillating two-stream instability is (Eq.(5.14))

$$\left( \frac{k \cdot \hat{\xi}_0}{n_0 \omega_0} \right)^2 = \frac{4}{\varepsilon_0} \left( 1 + \frac{T_i}{T_e} \right) \frac{\nu_e}{\omega_0}$$

(5.22)
where once again it has been assumed that $\omega_{ek} \sim \omega_0$. In addition, $\gamma_i$ has been set equal to one since $\omega_i/k = 0 < \nu_i$. The minimum threshold corresponds to a mismatch $\delta = -\nu_e/2$ or $\omega_0 = \omega_{ek} - \nu_e/2$. Thus, $\omega_0 < \omega_{ek}$ for the OTSI, whereas $\omega_0 > \omega_{ek}$ for the decay instability. In the fluid approximation, the frequency width of the resonance response for the purely growing mode is infinitely narrow. This follows since the condition $\omega_i = 0$ must be precisely satisfied if there is to be a solution to the eigenmode problem (Eqs.(5.9) and (5.10)).

5.4 PARAMETRIC INSTABILITIES IN INHOMOGENEOUS PLASMAS

In this section we introduce the results of theoretical calculations of parametric instability thresholds in an inhomogeneous plasma medium. These results, itemized in Table 5.2, will be of subsequent use when applied to a model $E_s$ plasma in section 5.7.

Parametric instabilities excited in an isothermal plasma with a density variation given by $n_e = N_r \exp(-x/H) \sim N_r (1 - x/H)$, where $x$ is the distance below the point of Langmuir wave reflection, have been discussed by Fejer and Leer (1972b) and Perkins and Flick (1971). In these studies the pump field, $E_p$, is assumed to be perpendicular to the density gradient. The gradient has the effect of spatially limiting the instability region and thus allows for energy loss due to the propagation of plasma waves away from the interaction region.

Both of the above treatments give the same threshold for the OTSI. This threshold is listed as Eq.(5.23) in Table 5.2 where $k_{ii}$ is the component of the plasma wave (or ion acoustic wave) wave number.
parallel to \( \hat{\mathbf{E}}_p \) (and normal to the gradient) and \( \theta^\circ \) is the angle between \( \hat{\mathbf{E}}_p \) and \( \hat{\mathbf{k}} \). The OTSI maintains its status as an absolute instability even with the inclusion of the density gradient. In the limit \( k_\parallel H \to \infty \) the inhomogeneous plasma threshold reduces to the result for a homogeneous medium (Table 5.1, Eq.(5.22)). The validity criterion for Eq.(5.23) is discussed in detail by Fejer and Leer (1972b). Outside of the validity region, the WKB approximations used in deriving the threshold break down and numerical methods are required.

In the presence of a gradient, the decay instability transforms into a convective instability. The calculated decay threshold in this case is listed as Eq.(5.24) in Table 5.2, where \( k_\perp \) is the component of the ion acoustic wave vector parallel to the density gradient. The convective amplification factor of the initial plasma wave amplitude is defined as \( \exp(A) \). Once again the homogeneous result is recovered in the limit \( k_\parallel H \to \infty \).

The decay threshold of Eq.(5.24) is generally valid only for a certain range of wave vectors with \( k_\perp > 0 \). However, the minimum threshold occurs at \( k_\perp = 0 \). In this particular case the threshold may be written according to Eq.(5.25).

In calculating all of the above decay thresholds, Perkins and Flick (1971) assume moderate ion damping, that is (see Liu, 1976)

\[
\frac{\nu_i}{\omega_i} < \left( \frac{\lambda_D}{H} \right)^{2/3} \frac{1}{k\lambda_D}
\]

If the ion damping is heavy (i.e. \( T_e \sim T_i \)), then the threshold is determined primarily by ion nonlinear Landau damping. In this case
the relevant threshold is given by Eq.(5.26). Contained within the validity criterion of Eq.(5.26) is a working definition of "heavy damping."

5.5 LINEAR THEORY APPLIED TO THE IONOSPHERE

Even in the subthreshold regime of a pumped but stable ionospheric plasma, significant modifications in the plasma density fluctuations are expected due to the presence of the HF electric field. The spectra of these density variations in the F region over Arecibo have been calculated by Hagfors and Gieraltowski (1972) who solved the Boltzmann equation to first order for an unmagnetized plasma. The computed incoherent scatter spectra exhibit resonant enhancements in both the ion and electron components. These enhancements occur at frequencies close to those predicted by the fluid treatment for the growing and decay modes and are very narrow in band width. For low electric field strengths, the resonant enhancement of the ion line is not significant in comparison to the frequency band averaged power scattered at the ion line in the unpumped plasma. Resonant contributions are important only for electric field values close to threshold. For this reason enhanced low frequency fluctuations are a good diagnostic for determining whether the parametric instability threshold has been exceeded.

On the other hand, the subthreshold plasma line enhancements relative to the electron thermal fluctuations are considerably larger than the corresponding ion enhancements. In addition, the total power in the electron component increases rapidly with increasing electric
field strength. Consequently, large plasma line enhancements may occur even in a pumped plasma that is well below threshold.

In the ionosphere, where $T_e \sim T_i$, the ion Landau damping time of ion acoustic waves, is of the order of one period of oscillation. In light of this, it is no longer physically meaningful to describe the low frequency response of the three-wave decay process as a wave. Instead, the interaction is often referred to as a decay into a resistive quasi-ion mode (Liu and Kaw, 1976). In this case, the parametric process is more aptly described as a nonlinear wave-particle process rather than a nonlinear wave-wave interaction. The latter process falls under the collective heading of coherent mode coupling while the former process is termed nonlinear Landau damping. Since nonlinear Landau damping involves interactions with particles, it requires the full Vlasov treatment. Coupling within the F-region plasma occurs primarily as a result of ion nonlinear Landau damping (Perkins et al., 1974 and references therein). This process entails resonant interactions of ions with the low frequency beat mode of the pump and a Langmuir wave. Ion nonlinear Landau damping is similar to its linear counterpart in that resonant interactions take place whenever the thermal ion velocity is close to the phase velocity of the beat mode. Thus, the pump wave beats with a Langmuir wave giving rise to a heavily damped ion mode at the difference frequency. The ion mode in turn reacts with the pump field producing a new high frequency Langmuir wave.

Perkins et al. (1974) developed the linear theory of the decay instability in the F region based upon a fluid description of the electrons and a Vlasov description of the ions. Only the dominant nonlinear
Landau damping terms were included in the calculation. The results, valid for a magnetoactive plasma, yield a nonlinear intensity damping increment, $\nu_{nl}$, for the plasma waves of

$$
\nu_{nl} = \nu_e \left[ 1 - \frac{\omega_0}{\nu_e} \left( \frac{k \cdot \mathbf{E}_0}{4n_0 k T_e/e_0} B(x_2) \right)^2 \right], \quad x_2 = \frac{\omega_0 - \omega_2}{k} \left( \frac{m_i}{2 \kappa T_i} \right)^{1/2}
$$

(5.27)

where $B(x_2)$, the coupling function shown in Figure 1 of Perkins et al. (1974), is reproduced in Figure 5.1a. The parametric instability, which occurs whenever $\nu_{nl} < 0$, drives electron oscillations near the resonance frequency in the unpumped plasma, that is, near

$$
\omega_r^2 = \omega_e^2 + \frac{k_{\perp}^2}{k^2} \omega_{ec}^2 + \frac{3k^2 \kappa T_e}{m_e}
$$

(5.28)

where $\omega_{ec}$ is the electron cyclotron frequency and $k_{\perp}$ is the component of $\mathbf{k}$ perpendicular to the magnetic field. The nonlinear Landau damping threshold is listed as Eq.(5.29) in Table 5.1.

Although ion nonlinear Landau damping has been shown to be the dominant mechanism associated with the growth rate of the decay instability in the F region, it should be remembered that several other interaction mechanisms operate simultaneously but at a much lower level (e.g. coherent mode coupling). Furthermore, when one is interested in the subthreshold regime, it is important to consider the sources of plasma wave emission in addition to the nonlinear damping rates (Harker, 1972). In this regard, the Cerenkov emission term in the wave kinetic equation plays an essential role in determining the level of plasma wave enhancement (see e.g. Eq.11, DuBois and Goldman, 1972). In a pumped plasma frequency-shifted ion Cerenkov emission.
dominates any contribution from spontaneous Cerenkov emission (DuBois and Goldman, 1967a; Hagfors and Gieraltowski, 1972; Valeo et al., 1972). The latter emission occurs naturally in an unpumped plasma and may be thought of as a noisy source of Langmuir waves. The frequency-shifted ion emission is the result of the mixing of spontaneously emitted ion acoustic waves with the pump wave.

The above theoretical treatment was in large part developed in response to ionospheric modification experiments carried out in the F region. In general, the conclusions drawn for the F-region plasma are not directly applicable to even a homogeneous sporadic-E plasma since plasma waves are strongly damped in the E region by electron-neutral collisions. Despite this, all of the physical processes that occur in the F region are expected to be present in Eₜ. As will be discussed in section 5.7, the question is one of the relative importance of the OTSI which may compete with decay interactions stemming from ion nonlinear Landau damping.

5.6 SATURATION OF PARAMETRIC INSTABILITIES IN THE IONOSPHERE

Up to this point only the linear theory of parametric instabilities has been presented. In the linear approximation a wave propagating in the plasma medium does not influence any of the other waves that may be present. Plasma oscillations are therefore decoupled. Once an instability threshold is exceeded, the linear theory predicts that the amplitudes of the excited waves will grow exponentially. Saturation theory deals with the ways by which the plasma restricts this growth.
One means of stabilization has already been considered in reference to parametric excitation of decay instabilities in inhomogeneous plasmas. A density gradient serves to limit the instability region spatially. Since the decay instability in the ionosphere is convective (Arnush et al., 1974), the time interval over which the propagating waves will be amplified before leaving the instability region is also limited.

A second type of saturation process, termed pump depletion, requires a self-consistent description of the pump field wherein the pump induced waves are allowed to react back upon the pump field. In this mechanism the flow of pump energy into the parametrically driven plasma waves constitutes a nonlinear contribution to the high frequency plasma conductivity (see DuBois et al., 1973). The subsequent decrease in the pump intensity leads to a steady state situation in which the pump amplitude is just below threshold.

Finally, if the anomalous absorption of the pump is not severe, a third saturation process may dominate wherein the nonlinear damping rates increase leading to an increase in the instability threshold. In this case, the growth of plasma oscillations is ultimately limited by nonlinear effects which saturate the instability by means of wave-wave and wave-particle interactions. The resultant spreading in frequency and wave number of the plasma fluctuations produces a state of plasma turbulence that is subject to collisional dissipation. In the present study, the so-called weak turbulence theory applies (Kruer and Valeo, 1973) since instability thresholds are not greatly exceeded and
\[
\frac{e E_p^2}{n_e k T_e} \approx 3.3 \cdot 10^{-3} \approx \frac{16 \gamma_{ia}}{\omega_r} \approx 4.1 \cdot 10^{-3}
\]

where \( \gamma_{ia} \) is the ion acoustic damping rate and \( E_p \), the pump field, has been set equal to 0.4 V^2/m^2. Typical \( E_s \) parameters of 240°K and \( 3.2 \cdot 10^{11} \text{m}^{-3} \) have been assumed for \( T_e \) and \( n_e \) respectively.

Before discussing the possible saturation mechanisms that may be operative in \( E_s \), we shall first summarize the preexisting theory that has been proven applicable to the F region. In this regard, prior theoretical considerations have been directed primarily towards decay mode saturation since, in the F region, the decay instability completely dominates the OTSI. Convective saturation has only a minor effect on the decay instability because of the large scale heights (\( \sim 75 \text{ km} \)) present at F-region altitudes (see Eq. 94 of Perkins et al., 1974). In this case the propagation times of plasma waves within the instability region are long enough to allow the development of other saturation processes. In addition, pump depletion is not a controlling factor since only an estimated 15-20% of the pump power is nonlinearly absorbed (Perkins et al., 1974; DuBois et al., 1973). It is generally agreed that the most prominent saturation mechanism in the F region is ion nonlinear Landau damping (Perkins et al., 1974; Fejer and Kuo, 1973; Krue and Valeo, 1973; Kuo and Fejer, 1972; DuBois and Goldman, 1972). Experimental measurements at Arecibo tend to support this conclusion (Duncan, 1977; Carlson and Duncan, 1977; Kantor, 1974; Kantor, 1972).

As a saturation mechanism for the decay instability, ion nonlinear Landau damping is a natural extension of the linear theory of
the preceding section. Initially, high frequency plasma waves are
driven unstable by the electromagnetic pump. These waves grow in
amplitude until they exceed threshold as pump waves and couple to lower
frequency daughter waves via nonlinear Landau damping. The daughter
waves may subsequently acquire sufficient amplitudes to act as pumps,
etc. Thus, the growth of plasma waves produced in the decay process is
limited by the destabilizing effect that these waves have upon a series
of satellite waves. The resulting cascade of plasma waves toward lower
frequencies and smaller wave numbers acts to redistribute plasma wave
energy in phase space. Stabilization ultimately occurs whenever the
energy flow into plasma waves is balanced by the transfer of energy to
electrons through collisional losses and the Landau damping of plasma
waves.

The maximum nonlinear Landau damping rate for a plasma wave
pump \((\omega_0, k_0)\) occurs when the beat velocity at the high frequency
response \((\omega_1, k_1)\) is near the thermal ion velocity, \(v_i\). That is, when

\[
\frac{\omega_0(k_0) - \omega_1(k_1)}{|k_0 - k_1|} \sim v_i
\]

Since this interaction process peaks for counterstreaming plasma waves
(Kruer and Valeo, 1973), the position of spectral maxima are determined
by the condition \(k_1 \sim -k_0\). The magnitude of \(k_1\) must be slightly less
than \(|k_0|\) because of momentum and energy conservation requirements.
This yields

\[
\omega_0 - \omega_1 \sim 2 k_0 v_i \sim 2 k_0 c_s = 2 \Omega_k
\]
where \( c_s \) and \( \Omega_k \) are the ion acoustic velocity and frequency respectively. As a result, nonlinear Landau damping leads to spectral peaks in the incoherent scatter spectrum near \( f_{\text{HF}} - (n + 1)\Omega_k \) where \( n = 0, 1, 2 \ldots \).

Perkins et al. (1974) and Fejer and Kuo (1973) have independently developed the two-dimensional theory of nonlinear Landau damping. These calculations were performed specifically for F-region conditions where the assumption \( v_e \ll v_i \) is valid. The peaked nature of the saturation spectrum predicted for the Arecibo experiment is illustrated in Figures 5.1b and 5.1c.

At low pump powers, a related saturation mechanism, termed coherent mode coupling, may also be operative in the F region (see Duncan, 1977). Mode coupling produces the same avalanching effect in wave number space as nonlinear Landau damping. However, mode coupling saturation would presumably take place either as a result of the three-wave decay of electrostatic daughter waves or because of a nonlinear four-wave coupling process. The former process is expected to be effective only when \( k\lambda_D < 10^{-2} \) (DuBois and Goldman, 1972) or alternatively when \( T_e \gg T_i \) (Yuen, 1975). In the latter mechanism, two of the parametrically excited Langmuir waves couple to produce two new Langmuir waves possessing different wave vectors (Yuen, 1978). This process is more appropriate for the ionospheric plasma since it operates when \( T_e \sim T_i \) and competes favorably with nonlinear Landau damping. The saturation spectrum expected to accompany a mode coupling process would entail a continually decreasing cascade from the initially unstable decay mode towards other nonresonant modes at lower frequencies (Godfrey
et al., 1973). Duncan (1977) associated mode coupling with several of the plasma wave spectra observed in the F region over Arecibo.

Additional saturation mechanisms that may operate in conjunction with the decay instability include the coupling saturation of Bezzerides and DuBois (1976) and the perturbed orbit saturation discussed by Weinstock and Bezzerides (1973). Modifications in the electron and ion orbits, however, are predicted to be a dominant saturation mechanism only in a regime where electron Landau damping is important or where the instability thresholds are greatly exceeded.

In the present experiment, frequency spectra taken of the HF-induced plasma line enhancements in $E_s$ often contain only narrow band components shifted upwards and downwards from 430 MHz by very nearly $f_{HF}$. On the basis of the linear theory presented above, a displacement of exactly $f_{HF}$ may be interpreted in terms of the OTSI. In addition to the observations, there are strong theoretical arguments for believing that the OTSI overshadows the decay instability in $E_s$ (section 5.7). In light of this it is of interest to investigate the means by which the OTSI saturates in an $E_s$ plasma when and if the threshold is exceeded.

Since the OTSI is an absolute instability, convective saturation is not directly applicable. In addition, pump depletion by the instability in $E_s$ is expected to be even less than the depletion that occurs in the F region (Chapter 4). Under F-region conditions the OTSI threshold is estimated to be between 2.1 and 2.5 times the decay threshold due to nonlinear Landau damping (Perkins et al., 1974; Fejer and Kuo, 1973). In the F-region plasma the OTSI is expected to saturate
via a nonlinear Landau damping cascade in much the same manner as the
decay instability (Perkins et al., 1974; Fejer and Kuo, 1973). Once the
OTSI exceeds threshold, a large increase in the spectral line at the
pump frequency is predicted as well as attendant increases in the
strength of the decay line and the entire spectrum of cascading lines
due to the nonlinear Landau damping of the decay instability.

On the basis of the above discussion, one would expect the
OTSI in $E_s$ to saturate by means of nonlinear Landau damping if the
decay instability is also present and has a lower threshold. On the
other hand, if only the OTSI is present, then other saturation mechan-
isms may become competitive with nonlinear Landau damping. Since the
beat frequency of the pump with either of the two daughter waves of
the OTSI falls either at $\omega_0$ or at zero frequency, nonlinear Landau
damping may not proceed within the framework of the OTSI itself. How-
ever, the plasma waves excited by the OTSI represent electrostatic
pump waves, which, nevertheless, may still initiate a nonlinear Landau
damping cascade.

Nishikawa et al. (1973) examined the problem of nonlinear sta-
bilization of the OTSI within the context of weak turbulence theory.
They pointed out that the lowest order nonlinear effect consists of a
downshifting in the natural (unpumped) resonant frequency of the long
wavelength electron plasma oscillations. Stabilization occurs when the
natural plasma frequency becomes less than the pump frequency. At this
point, the OTSI converts into the usual decay instability which may
then be saturated via nonlinear Landau damping.
In the above scenario, the very long wavelength fluctuations (i.e., $k\lambda_D \ll 1$) are the first to undergo a nonlinear frequency shift. When $\delta = \omega_0 - \omega^*$ becomes positive these modes become unstable against the decay instability while the shorter wavelength oscillations may simultaneously become unstable against the OTSI. As in any saturation process of this nature, the plasma stabilizes whenever the collisional losses of plasma waves balance the energy input into the OTSI by the pump.

In a more complete treatment of OTSI saturation, Weinstock and Bezzerides (1975) determined the saturation spectrum of the plasma turbulence due to the nonlinear upshifting of $\delta$. Nonlinear mode coupling terms were introduced into the fluid equation which effectively reduced particle pressure. It is this reduction in pressure that enables the unstable waves to extend toward shorter wavelengths. The results of Weinstock and Bezzerides will be discussed in greater detail in section 7.10.

The OTSI saturation theory described above is most important for low pump powers. Plasma phenomena associated with the OTSI driven by large electric field strengths include the formation of regions having large density depletions. Plasma is forced out of these regions by radiation pressure (i.e., the pondermotive force). In one dimension, the growth of these cavities may continue until the radiation pressure is balanced by the increased pressure exerted by regions of enhanced density.

The fully evolved system contains a strong electric field structure describable in terms of a stable, spatially varying envelope (Weiland and Wilhelmsson, 1977). Each wave packet within the envelope, termed
a soliton, may travel at a small group velocity due to nonlinear effects. Solitons often appear in computer simulations performed in laser fusion research (Valeo and Kruer, 1974) where they exhibit interesting particle properties. Two solitons, for example, are often seen to fuse together leading to the annihilation of one along with the emission of ion acoustic waves.

In general soliton formation appears to be the most important in plasmas where \( \frac{\varepsilon_0 E_p^2}{n_e K_T e} \sim 1 \) (Valeo and Kruer, 1974). Since electric field density encountered in the present experiment are much less than thermal pressures, processes related to the evolution of solitons will not be considered here.

In summary, the two OTSI saturation mechanisms that appear to be most relevant to the \( E_s \) plasma include the nonlinear Landau damping of parametrically excited plasma waves and stabilization resulting from a nonlinear downshifting in the natural plasma frequency. To these one might add four-wave mode coupling since this type of mechanism applies equally well to either the decay instability or the OTSI (Yuen, 1978).

5.7 PARAMETRIC EXCITATION IN SPORADIC E

Most of the theoretical developments given up to this point have had varying degrees of application to the F region. However, even without a knowledge of the precise nature of the plasma, it is apparent that the parametric instability will proceed somewhat differently in sporadic E. In order to approach the sporadic-E problem, we shall make some initial threshold estimates for a model \( E_s \) plasma. In an
oversimplification the plasma will be assumed to be horizontally stratified. This allows for limited application of the inhomogeneous plasma thresholds discussed in section 5.4. In general, any stabilizing effect due to horizontal boundedness in the $E_S$ plasma (Pesme et al., 1973; Fuchs and Beaudry, 1978) may be ignored in comparison to the limitations placed upon the instability by large vertical density gradients.

In all of the threshold estimates we shall use a "mean" temperature value of $T_e = T_i = 240^\circ$K which is representative of $E$-region temperatures near 110 km where most of the $E_S$ observations in the present experiment were made. There exists of course a range of equally acceptable temperature values that might have just as well been chosen (Figure 3.3a). Finally, $\omega_o = \omega_{HF}$ will be set equal to 5.1 MHz, which is indicative of the other HF pump frequencies used during the 1976 and 1977 experiments.

5.7.1 Metallic Ions

The experimental observations discussed in Chapter 3 imply that a significant fraction of the ions present in $E_S$ are metallic. In reference to the decay instability it is important to examine how the low frequency ion-acoustic mode(s) of the plasma change in the presence of a mixed ionic species. In order to consider the greatest ion mass range we shall assume that the $E_S$ plasma is a two-component mixture of lighter $N_0^+$ and $O_2^+$ ions having an average mass of 31 amu and heavier $Fe^+$ ions (56 amu).
In the fluid approximation (neglecting collisions) the general plasma dispersion relationship may be written as (Schulz and Koons, 1970)

\[ l = \sum_{j} \omega_{j}^{2} \left[ (\omega_{k} - V_{j} k)^2 - c_{j}^{2} k^2 \right]^{-1} \]

where the summation over \( j \) includes electron and ion species and

\[ \omega_{j}^{2} = \frac{n_{j} q_{j}^{2}}{\varepsilon_{0} m_{j}} \quad \quad c_{j}^{2} = \frac{\gamma_{j} kT_{j}}{m_{j}} \]

The quantities \( n_{j}, q_{j}, m_{j}, T_{j} \) and \( V_{j} \) represent respectively the density, charge, mass, temperature and mean velocity. We assume \( V_{j} = 0 \) and set \( \gamma_{j} \) equal to 3 for ions and 1 for electrons. For a two-ion electrically neutral plasma we have

\[ l = \frac{\omega_{e}^{2}}{\omega_{k}^{2} - c_{e}^{2} k^2} + \frac{\omega_{1}^{2}}{\omega_{k}^{2} - c_{1}^{2} k^2} + \frac{\omega_{2}^{2}}{\omega_{k}^{2} - c_{2}^{2} k^2} \]

\[ n_{e} = n_{1} + n_{2} \]

where \( j = e, 1, 2 \) represent the electron and the heavier and lighter ion components respectively. To simplify the above relation, let us assume \( \omega_{1}^{2} = \omega_{2}^{2} \), that is

\[ \frac{n_{1}}{n_{2}} = \frac{m_{1}}{m_{2}} = 1.8 \approx 2 \]

for singly ionized \( \text{Fe}^{+} \) and \( (\text{NO}^{+}, \text{O}_{2}^{+}) \) ions. Thus, the plasma contains 67\% \( \text{Fe}^{+} \) ions by number, which is consistent with \( E_{s} \) observations (section 3.3). For \( \omega_{k}^{2} << c_{e}^{2} k^2 \) the above dispersion relation has two branches given by
\[
\left( \frac{\omega_{k2}}{k} \right)^2 = 1.5 \, c_1^2 + \frac{2\omega_1^2 \lambda_D^2}{1 \! + \! k^2 \lambda_D^2} + \frac{c_1^2}{4} \left( \frac{k^2 c_1^2}{\omega_1^2} \right)
\]

\[
\left( \frac{\omega_{k1}}{k} \right)^2 = 1.5 \, c_1^2 - \frac{c_1^2}{4} \left( \frac{k^2 c_1^2}{\omega_1^2} \right)
\]

where \( \omega_{k2}/k \) and \( \omega_{k1}/k \) are the high and low frequency branches corresponding to \( \Omega_k/k \) in a single ion plasma and \( \lambda_D \) is the Debye length. Neglecting terms of order \( kc_1/\omega_1 \ll 1 \) and \( k\lambda_D \ll 1 \) we obtain

\[
\left( \frac{\omega_{k2}}{k} \right)^2 \sim 1.5 \left( \frac{3\kappa T_1}{m_1} \right) + \frac{2\kappa T_e}{m_e} \left( \frac{n_i}{n_e} \right)
\]

\[
\left( \frac{\omega_{k1}}{k} \right)^2 \sim 1.5 \left( \frac{3\kappa T_1}{m_1} \right)
\]

For \( E_s \) parameters of \( T_e = T_1 = T_2 = 240^\circ \text{K} \) and a wave number \( k = 18 \text{ m}^{-1} \) this yields

\( \text{(NO}^+, \text{ O}_2^+) \)

\[
\omega_{k2} = 0.85 \, \Omega_{k2} = 1.24 \text{ kHz} \cdot 2\pi
\]

\[
\Omega_{k2} = \left( \frac{\kappa T_e}{m_2} + \frac{3\kappa T_2}{m_2} \right)^{1/2} \quad k = 1.45 \text{ kHz} \cdot 2\pi
\]

\( \text{(Fe}^+) \)

\[
\omega_{k1} = 1.06 \, \Omega_{k1} = 1.15 \text{ kHz} \cdot 2\pi
\]

\[
\Omega_{k1} = \left( \frac{\kappa T_e}{m_1} + \frac{3\kappa T_1}{m_1} \right)^{1/2} \quad k = 1.08 \text{ kHz} \cdot 2\pi
\]

where \( \Omega_{kj} \) is the ion acoustic frequency in a single ion plasma containing species \( j \).
The calculated differences between $\Omega_{kj}$ and $\omega_{kj}$ are too small to be measurable given the frequency resolution of the present experiment. In addition, the frequency shifts have a negligible effect on the evaluation of parametric thresholds and growth rates since the uncertainties in the other parameters involved in the calculation are proportionally much larger. Finally, we note that the maximum separation between the two new low frequency modes is $(\Omega_{k2} - \Omega_{k1})/2\pi = 0.37$ kHz which is much less than their mean resonance width, $\sim (\Omega_{k1} + \Omega_{k2})/4\pi$. The coupling between ion modes in the plasma tends to narrow the frequency separation of the single ion modes. Consequently, the $(\text{NO}^+, \text{O}_2^+)$ and $\text{Fe}^+$ ion modes, as well as the other metallic ion modes (i.e. $\text{Mg}^+$ and $\text{Si}^+$) may be collectively treated as a single mode.

5.7.2 Decay Instability

As discussed earlier, the parametric coupling responsible for the decay instability in the F region contains one mode, the ion mode, which is heavily damped in the absence of the pump. As a result, the initial assumptions of an F-region model usually entail (Perkins et al., 1974)

$$\nu_e \sim 2\gamma \ll \nu_i \sim kc_s$$

where $\gamma$ is the instability growth rate and $c_s$ is the ion acoustic velocity. The ion Landau damping rate is approximately equal to the ion acoustic frequency. Because of the heavy ion damping, nonlinear effects associated with the ion waves are usually ignored (see e.g. DuBois and Goldman, 1972). The above assumptions allow the decay instability to be treated as a nonlinear Landau damping process.
In contrast to F-region conditions, optimum $E_s$ instability parameters near 110 km yield

$$\nu_i \approx \nu_{in} + \Omega_k \approx \Omega_k + 0.9 \text{ kHz} \approx \Omega_k$$

$$\nu_e \approx \nu_{en} + \nu_{ei} \approx 11 \text{ kHz} + 3 \text{ kHz} = 14 \text{ kHz}$$

In this case both the electron and the ion decay mode are heavily damped. The decay instability thresholds for a homogeneous $E_s$ plasma are presented in Table 5.1, Eqs.(5.21) and (5.29). The angle $\theta$ represents the excitation angle with respect to the pump field, which is taken to be parallel to the magnetic field. For the Arecibo observing geometry $\theta = 44^\circ$ while the magnitude of the observed wave vector $|\vec{k}|$ is $18 \text{ m}^{-1}$. Even though the decay thresholds are not strictly applicable to $E_s$ since $\nu_e << \nu_i$ is not satisfied in the E region, the two thresholds do give some indication of the relative importance of the three-wave decay compared to nonlinear Landau damping. It is clear, for example, that if the decay instability proceeds in $E_s$, it does so mainly through a nonlinear Landau damping process.

If large vertical density gradients are present in the plasma then the decay threshold will be increased beyond the homogeneous plasma value. The extent of this increase, which will subsequently be denoted $E_+^2$, may be seen in Eqs.(5.24), (5.25), and (5.26). In these threshold expressions, we have assumed a plasma wave amplification factor of $\exp(A) = \exp(5)$ for $A = 5$, which is consistent with observed plasma line enhancements in $E_s$ over Arecibo. Within the limits of validity of Eq.(5.24), the minimum threshold is obtained for
\[ k_\perp = 16.7 \text{ m}^{-1} (|\mathbf{k}| = 18 \text{ m}^{-1}), \theta^- = 68^\circ. \] In this case one has
\[ E_+^2 = 2108/H(\text{m}) (V^2/\text{m}^2), \text{ or } E_+^2 = 4.2 \ V^2/\text{m}^2 \text{ for } H = 500 \ \text{m}. \]

It is worth noting that the thresholds computed using Eq.(5.24) are valid only for \( H \gtrsim 522 \ \text{m}. \) At smaller values of \( H \) the anti-Stokes component of the decay instability resonance at \( \omega_0 + \Omega_k \) enters into the threshold calculation (see Perkins and Flick, 1971). This leads to thresholds that are generally greater than those predicted by the above equation. It will be shown below that the anti-Stokes component may come into play even in the threshold calculation for a homogeneous \( E_s \) plasma. In light of this, estimates made using Eq.(5.24) (and Eq. (5.25)) should be viewed as lower limits.

Since the inhomogeneous plasma threshold for a three-wave decay increases as the angle of ion wave propagation with respect to the pump increases, a minimum value of \( E_+^2 \) is expected for \( k_\perp = 0 (\theta^- = 0^\circ) \). If one could in fact make radar observations looking parallel to the pump then the threshold would be given by Eq.(5.25) with \( \theta^- = 0^\circ \), that is \( E_+^2 = 213/H(\text{m}) (V^2/\text{m}^2) \). Thus, even under optimum conditions contributions to the decay threshold that are large in comparison to the homogeneous plasma values in Eqs.(5.21) and (5.29) (\( \theta = 0 \)) arise when \( H \lesssim 500 \ \text{m}. \)

It is important to recognize that the three-wave decay thresholds for an inhomogeneous plasma considered above were derived assuming moderate ion damping (section 5.4) and \( v_e \ll \Omega_k \). The former assumption insures the propagation of weakly damped ion acoustic waves in the plasma. This, in effect, requires that \( T_e \gg T_i \). However, \( T_e \sim T_i \) in
the ionosphere, and, as pointed out above, the decay instability in $E_\parallel$ is expected to proceed primarily as a result of nonlinear Landau damping, if indeed it is excited at all. Notwithstanding this, the three-wave decay process is worth considering because the implications of the second assumption ($\nu_e \ll \Omega_k$), which is employed in Eqs.(5.21), (5.29), (5.24), (5.25) and (5.26) are easier to illustrate within the framework of a fluid description of the plasma. This will be discussed below. When nonlinear Landau damping is dominant, the decay threshold increase, $E_\parallel^2$, is given by Eq.(5.26''). In this case the threshold is significantly increased whenever $H < 1$ km.

In general, the decay instability occurs over an altitude range that extends at most from the point of HF reflection down to an altitude where strong electron Landau damping limits the growth of Langmuir waves. The lower boundary is commonly fixed at the height at which $k\lambda_D = 0.25$ (Perkins and Flick, 1971). For a linear density gradient, this altitude lies at a distance $H/6$ below reflection (Perkins et al., 1974).

Close to the point of HF reflection in the F region (and above the 430 MHz radar observation altitude) parametric decay is controlled by four-wave processes. Four-wave processes dominate whenever $\nu_e \gtrsim 2\Omega_k$ where $\nu_e$ is the total intensity damping rate of the Langmuir waves (DuBois and Goldman, 1972). When $\nu_e \ll 2\Omega_k$ only the Stokes frequency component ($\omega_0 - \Omega_k$) of the high frequency response is on resonance. The parametric coupling is then appropriately described as a three-wave decay process. However, once the resonance width of the Langmuir waves
(\sim v_e) becomes greater than the separation between the Stokes and anti-Stokes components, both components become simultaneously resonant \cite{DuBoisGoldman67b}. The coupling then behaves as a four-mode process, wherein the dragging due to the inclusion of the anti-Stokes satellite wave causes a rapid increase in the threshold. The increased importance of the anti-Stokes component as \( v_e \) becomes large is evident from Eq.(5.8) wherein the ratio of the Stokes to anti-Stokes components, \(|\xi(\omega_o - \Omega_k)/\xi(\omega_o + \Omega_k)|^2\), may be computed to be \((1 + 16(\Omega_k/v_e)^2)\).

In the \( E \) region, the greater Langmuir wave damping rates resulting from electron-neutral collisions has the net effect of extending the four-mode coupling region down to observational heights. Since \( v_e > \sim \Omega_k \), the threshold is no longer adequately represented by Eq.(5.20). A more accurate calculation using Eqs.(5.17) and (5.19) indicates that four-mode coupling increases the threshold above that predicted by Eq.(5.20) by about 25\%, 50\%, and a factor of 2 respectively for \( v_e = \Omega_k \), \( v_e = 2\Omega_k \), and \( v_e = 4\Omega_k \).

The threshold discussions for an inhomogeneous plasma put forth in this section provide some guidelines for determining the point at which gradients become an important factor in the decay threshold calculation. The specific results, however, are not directly applicable to ionospheric modification experiments since they are based on the assumption that the pump field is perpendicular to the density gradient, an assumption which will in general not be valid in \( E_s \). Fejer \cite{Fejer78} points out that a more representative value for the increase in the decay instability threshold in the ionosphere may be
expressed as

\[ E_+^2 = \frac{4n_0 k T_i}{\varepsilon_0} \frac{A}{kH} \frac{1}{B_{\text{max}}(x_2)} \]

where vertical propagation of the Langmuir wave and a vertical pump wave have been assumed. In this case, one has \( E_+^2 = 223/H(m) \left( V^2/m^2 \right) \) for the same parameter values as those used in Eq.(5.25*).

In summary we note that \( E_s \) decay thresholds are considerably greater than those typically encountered in the F region (see e.g. Muldrew, 1978a). This is in part due to the increase in the homogeneous plasma threshold which scales directly in proportion to \( v_e \). In addition, the large electron density gradients found in \( E_s \) increase the decay threshold and may in fact control the threshold determination for scale lengths \( H \lesssim 500 \text{ m} \). Four-mode processes represent a correction to the usual three-wave threshold that revises the threshold upwards. In addition, recent Langmuir ray tracing calculations performed by Muldrew (1978b) indicate that the convective growth of Langmuir waves in \( E_s \) is insufficient to produce the observed plasma line enhancements of Gordon and Carlson (1976). Muldrew's calculations may in fact even overestimate the convective growth since the nonlinear damping decrement of Perkins et al. (1974) used in the calculations was derived assuming \( v_e \ll v_i \sim \Omega_k \). At any rate it is evident from these results that plasma wave convection is an effective means of saturating the decay instability in \( E_s \).
5.7.3 Oscillating Two-Stream Instability

The minimum OTSI threshold for a homogeneous plasma is given in Eq. (5.22'). When the threshold in Eq. (5.22) is compared with the corresponding decay threshold in Eq. (5.21) for $\nu_e \ll \Omega_k$, $\nu_i \ll \Omega_k$, and $T_e = T_i$, one has

$$\frac{E_o^2 \text{ OTSI}}{E_o^2 \text{ DECAY}} = \frac{8 \nu_e}{\omega_0} \frac{\Omega_k \omega_0}{16 \nu_i \nu_e} = \frac{\Omega_k}{2 \nu_i} \gg 1$$

For $\nu_e \gg \Omega_k$, where four-mode decay is important, we have (Nishikawa and Liu, 1976)

$$\frac{E_o^2 \text{ OTSI}}{E_o^2 \text{ DECAY}} \propto 2.6 \frac{\Omega_k^2}{\nu_i \nu_e}$$

The above equation indicates that the OTSI threshold is the smaller threshold whenever $\nu_e \gtrsim 2.6 \Omega_k^2/\nu_i \sim 15$ kHz. This condition is almost certainly satisfied in the E region over Arecibo for altitudes $\lesssim 105$ km and probably holds even at greater altitudes (Figure 3.3c). The minimum value of $\nu_e$ necessary for a dominant OTSI might increase somewhat if nonlinear Landau damping is included in the decay threshold calculation. Nevertheless, the above estimates imply that, in a homogeneous plasma, the OTSI becomes competitive with the decay instability for $\nu_{en}$ values found in the E region (see e.g. Eqs. (5.22') and (5.29')).

The OTSI threshold for an inhomogeneous plasma is evaluated in Eq. (5.23'). For $\theta' = 0$, the threshold is significantly increased only when $H \lesssim 200$ m. It is worth noting that gradients have a much greater influence upon the decay instability threshold than the OTSI threshold.
(Table 5.2). When gradients dominate (i.e. when $H$ is small), the three-wave decay threshold is of the order of $1.6 \, \text{A}$ times larger than the OTSI threshold (Eqs. (5.23) and (5.25)) for the case where the pump field is perpendicular to the density gradient. In comparing Eqs. (5.23) and (5.26) one finds that the decay instability has the larger threshold and that the decay threshold ($\propto H^{-2/3}$) increases more rapidly with decreasing $H$ than the corresponding OTSI threshold ($\propto H^{-1}$). From this, one concludes that, when large density gradients are present, the OTSI is easier to excite than the decay instability.

Once again, we note that the inhomogeneous plasma thresholds referred to above are based on the assumption of a pump field that is perpendicular to the density gradient. More realistically, the OTSI threshold is not expected to be affected at small plasma wave numbers (Fejer, 1978, Eq.45). Fejer's criterion for neglecting inhomogeneities requires that $k < 1.1 \times 10^{-3} \, H(m) \, \text{m}^{-1} = 0.6 \, \text{m}^{-1}$ for $H = 500 \, \text{m}$ and the parameters used in Table 5.2. Consequently, at the shorter plasma wavelengths observed in the present experiment ($k \gtrsim 18 \, \text{m}^{-1}$) one would still conclude that the large density gradients found in $E_s$ are an important factor in the threshold determination.

In summary, on the basis of the theoretical considerations outlined above, one would predict that the OTSI is excited more readily than the decay instability in an $E_s$ plasma containing steep density gradients. This contrasts with F-region results wherein the decay instability is found to be dominant. The increased importance of the OTSI in $E_s$ comes about because electron collisions and density
gradients are much more effective at limiting the convective decay instability than the purely growing OTSI.

5.8 FREQUENCY MATCHING CONDITIONS

In a homogeneous medium the frequency matching condition for the OTSI at minimum threshold is

\[ f_{HF} = f_r - \nu_e/(4\pi) \quad \text{or} \quad f_{HF} = f_r - 1.1 \, \text{kHz} \quad (5.30) \]

for \( \nu_e = 14 \, \text{kHz} \). For a linear density profile of scale length \( H \) the minimum threshold occurs at a distance, \( z \), down from the point of HF reflection given by

\[ z = \left( \frac{H}{f_{HF}^2} \right) \left( f_{ec}^2 \sin^2 \theta + 3.73 \cdot 10^{-4} T_e - \left( \frac{\nu_e}{2\pi} \right) f_{HF} - \nu_e^2/(4\pi)^2 \right) \]

where \( T_e \) is in \(^{\circ}\)K and where a wave number \( k = 18 \, \text{m}^{-1} \) has been assumed. For \( T_e = 240^{\circ}\)K, \( \nu_e = 14 \, \text{kHz} \), \( f_{ec} = 1.1 \, \text{MHz} \), \( \theta = 44^{\circ} \), and \( f_{HF} = 51 \, \text{MHz} \), this becomes

\[ z = 0.0255 \, H, \quad f_{HF} - f_e(z) = 65.3 \, \text{kHz} \]

or \( z = 12.7 \, \text{m} \) for \( H = 500 \, \text{m} \).

The OTSI results may be compared with those predicted for the decay instability. In this case, the condition for minimum threshold is (Perkins et al., 1974)

\[ \omega_{HF} = \omega_r + 2\pi\delta = \omega_r + x_2(B_{\text{max}}) k (2\kappa T_i/m_i)^{1/2} \]

where \( x_2(B_{\text{max}}) \) denotes the value of \( x_2 \) at the maximum of the resonance coupling function, \( B(x_2) \), in Eq.(5.27). For \( \text{Fe}^+ \), \( T_e = T_i = 240^{\circ}\)K, and \( k = 18 \, \text{m}^{-1} \), we obtain
\[ x_2(B_{\text{max}}) = 1.2 \quad \delta = 0.92 \text{ kHz}. \]

For NO\textsuperscript{+} ions \( \delta = 1.25 \) kHz. Using the above frequency matching condition for Fe\textsuperscript{+} ions one finds that the minimum decay threshold occurs at

\[ z = 0.0263 \text{ H}, \quad f_{\text{HF}} - f_e(z) = 67.5 \text{ kHz} \]

or at \( z = 13.1 \) m for \( H = 500 \) m. Thus, at minimum threshold the OTSI occurs at a slightly greater altitude than the decay instability for a fixed wave number.

In practice, plasma waves having a range of wave numbers are generated throughout the entire instability region. Consequently, plasma waves may propagate through a sizable portion of the unstable region above or below the observation height before being detected in the narrow altitude interval where \( k = + k_r \) or \( - k_r \), where \( k_r \) is the radar wave vector. In Chapter 7 the role of convective propagation in an \( E_s \) plasma will be discussed in greater detail.

5.9 MAGNETIC FIELD EFFECTS

With the exception of the nonlinear Landau damping calculation of the decay threshold (Perkins et al., 1974), the presence of an external magnetic field has not been directly accounted for in any of the OTSI and decay instability calculations reported above. However, the results of Fejer and Leer (1972b) and Perkins and Flick (1971) indicate that the magnetic field does not appreciably affect the threshold of either instability. Perkins and Flick (1971) argue that no change in the thresholds is to be expected when the magnetic field is parallel to
the electric field and perpendicular to the density gradient. The OTSI threshold has been calculated by McBride (1970) for a homogeneous plasma in which \( \mathbf{E} \) is parallel to \( \mathbf{E}_p \), and \( \mathbf{E}_p \) is perpendicular to the magnetic field, \( \mathbf{B} \). Within this geometry one would expect electron gyroresonance effects to be greatest since plasma waves would be generated perpendicular to \( \mathbf{B} \). The magnetic field configuration of McBride (1970) leads to an increase in the field-free plasma threshold given in Eq.(5.22) by a factor of \( (\omega_{ec}^2 + \omega_{UH}^2)/(\omega_e \omega_{UH}) \), where \( \omega_{UH} \), the upper hybrid frequency, is defined as \( \omega_{UH} = (\omega_e^2 + \omega_{ec}^2)^{1/2} \). In the present experiment, where \( \omega_{ec} = 1.1 \) MHz and \( \omega_e \approx 5 \) MHz, the threshold increase is only \( \approx 7\% \).
<table>
<thead>
<tr>
<th>INSTABILITY</th>
<th>THRESHOLDS</th>
<th>VALIDITY CRITERIA</th>
<th>EQUATION NUMBER*</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillating Two-stream Instability</td>
<td>$E_0^2 = \frac{4}{n_0^2 k T_e} \left( 1 + \frac{T_i}{T_e} \right) \frac{\nu_e}{\omega_0 \cos^2 \theta}$</td>
<td>$\omega_{ek} \approx \omega_0$</td>
<td>Eq.(5.22)</td>
<td>Eq.(5.14)</td>
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<tr>
<td>For $T_e = T_i$</td>
<td>$E_0^2 = \frac{0.42}{\cos^2 \theta} \left[ \frac{v^2}{m^2} \right]$</td>
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<td>Eq.(5.22')</td>
<td></td>
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<tr>
<td>Three-wave Decay Instability</td>
<td>$E_0^2 = \frac{4}{n_0^2 k T_e} \left( 1 + \frac{3T_i}{T_e} \right) \frac{\nu_i \nu_e}{\Omega_i \omega_0 \cos^2 \theta}$</td>
<td>$\Omega_i \gg \frac{\nu_e}{2}$</td>
<td>Eq.(5.21)</td>
<td>Eq.(5.21)</td>
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<tr>
<td>For $T_e = T_i$, $\nu_i = \Omega_i$</td>
<td>$E_0^2 = \frac{0.84}{\cos^2 \theta} \left[ \frac{v^2}{m^2} \right]$</td>
<td>$\omega_{ek} \approx \omega_0$</td>
<td>Eq.(5.21')</td>
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</tr>
<tr>
<td>Ion Nonlinear Landau Damping</td>
<td>$E_0^2 = \frac{4}{n_0^2 k T_e} \frac{\nu_e}{\omega_0 \cos^2 \theta B(x_2)}$</td>
<td>$\nu_e \ll \nu_i$</td>
<td>Eq.(5.29)</td>
<td>Perkins et al. (1974)</td>
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<td>$x_2 = \frac{\omega_e - \omega_i}{k} \left( \frac{m_i}{2k T_i} \right)^{1/2}$</td>
<td>$\omega_{ic} \ll \Omega_i$</td>
<td>$\Omega_i \ll \omega_i \ll \omega_{ec}$</td>
<td>$\omega_{ec} &lt; \omega_e$</td>
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<tr>
<td>For $T_e = T_i$, $F^+_e$ ions,</td>
<td>$E_0^2 = \frac{0.35}{\cos^2 \theta} \left[ \frac{v^2}{m^2} \right]$</td>
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<td>Eq.(5.29')</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>k</td>
<td>= 18 \text{ m}^{-1}$</td>
<td>$B_{max}(x_2) = 0.6$</td>
<td>$x_2 = 1.2$</td>
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</table>

* Thresholds are evaluated in the primed equations for $T_e = 240^\circ K$, $\nu_e = 14 \text{ kHz}$, $f_0 = 5.1 \text{ MHz}$. 


<table>
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<th>INSTABILITY</th>
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<th>EQUATION NUMBER*</th>
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<tbody>
<tr>
<td>Oscillating Two-stream Instability</td>
<td>$E_0^2 \frac{n_0}{n_0 T_T e} = \frac{2}{T} \left( 1 + \frac{T_i}{T_e} \right) \frac{1}{k H} \left( \frac{k}{k_t} \right)^2 \left( \frac{3HkT_T e}{\omega_0 \omega} \right)^{2/3}$</td>
<td>$k \gg 1$ \begin{align*} &amp;\Sigma &amp; \left( \frac{3HkT_T e}{\omega_0 \omega} \right)^{2/3} \end{align*}</td>
<td>Eq.(5.23)</td>
<td>Perkins and Flick (1971)</td>
</tr>
<tr>
<td></td>
<td>+ $4 \frac{\nu_e}{\nu_0}$ \left( 1 + \frac{T_i}{T_e} \right) \frac{v_e}{\omega_0}$</td>
<td>$k \equiv \cos \theta k$</td>
<td></td>
<td>Fejer and Leer (1972b)</td>
</tr>
<tr>
<td>For $T_e = T_i$</td>
<td>$E_0^2 = \frac{26.7}{\cos \theta H(m)} + 0.42 \frac{v^2}{m^2}$</td>
<td>$H &gt;&gt; 15 \text{ m}$</td>
<td>Eq.(5.23°)</td>
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<tr>
<td>Three-wave Decay Instability</td>
<td>$E_0^2 \frac{n_0}{n_0 T_T e} = 2.4A \left( 1 + \frac{3T_i}{T_e} \right) \frac{1}{k H} \left( \frac{k}{k_t} \right)^2 \left( \frac{\Omega_k}{c_s^2} \right)^{1/2}$</td>
<td>$\Sigma \frac{\nu_e}{\nu_0} &lt;&lt; \Omega_k$ \begin{align*} &amp;\Sigma &amp; \left( \frac{3HkT_T e}{\omega_0 \omega} \right)^{2/3} \end{align*}</td>
<td>Eq.(5.24)</td>
<td>Perkins and Flick (1971)</td>
</tr>
<tr>
<td>(Convective)</td>
<td>+ $4 \frac{\nu_e}{\nu_0}$ \left( 1 + \frac{3T_i}{T_e} \right) \frac{v_e}{\omega_0}$</td>
<td>$k_L &gt; \left( \frac{\Omega_k}{c_s^2} \right)^{1/2}$ and $H &gt; \lambda_D \frac{1}{k^2 \lambda_D^2} \left( \frac{m_i}{m_e} \right)^{1/2} \left( \frac{\Omega_k}{\nu_i} \right)^{1/2}$</td>
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<tr>
<td>For $T_e = T_i$, $\nu_i = \Omega_k$, $A = 5$</td>
<td>$E_0^2 = 319 \tan \theta \frac{\nu^2}{\cos \theta} \frac{1}{H} + 0.84 \left( \frac{\nu^2}{m^2} \right)$</td>
<td>$k_L &gt; 16.7 \text{ m}^{-1}$ \begin{align*} &amp;\Sigma &amp; \left( \frac{3HkT_T e}{\omega_0 \omega} \right)^{2/3} \end{align*}</td>
<td>Eq.(5.24°)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$H &gt;&gt; 522 \text{ m}$</td>
<td>$\Sigma \frac{\nu_e}{\nu_0} &lt;&lt; \Omega_k$ \begin{align*} &amp;\Sigma &amp; \left( \frac{3HkT_T e}{\omega_0 \omega} \right)^{2/3} \end{align*}</td>
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TABLE 5.2 (continued)

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<tbody>
<tr>
<td>Three-wave Decay</td>
<td>( E_0^2 = \frac{1.6A}{n_0 k T_e} e^{-\frac{1}{3T_e}} \left(1 + \frac{3T_i}{T_e} \right) \left( \frac{v_i}{\Omega_k} \right)^{1/2} )</td>
<td>Assumes ( v_e &lt; \Omega_k ) ( k_L = 0 )</td>
<td>Eq.(5.25)</td>
<td>Perkins and Flick (1971)</td>
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<td>Instability (Convective)</td>
<td>+ ( \frac{0.2}{\epsilon_0} \left( \frac{v_i}{\Omega_k} \right) \left( \frac{v_e}{\omega_0} \right) )</td>
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<td></td>
</tr>
<tr>
<td>For</td>
<td>( v_i = \Omega_k ), ( T_e = T_i ), ( A = 5 )</td>
<td>( E_0^2 = \frac{213}{H(m) \cos \theta} + 0.16 \left( \frac{v^2}{m} \right) )</td>
<td>Eq.(5.25')</td>
<td></td>
</tr>
<tr>
<td>Ion Nonlinear Landau Damping</td>
<td>( E_0^2 = \frac{1}{n_0 k T_e} \left( \frac{v_i}{\omega_i} \right)^{2/3} \left( 1 + \frac{\lambda_D}{\lambda_D} \right)^{2/3} \left( 3A \right)^{2/3} )</td>
<td>Valid for ( v_e &lt; \Omega_k ) and ( \frac{v_i}{\omega_i} &gt; \left( \frac{\lambda_D}{H} \right)^{1/4} \left( k \lambda_D \right)^{3/8} ) (near threshold)</td>
<td>Eq.(5.26)</td>
<td>Liu (1976)</td>
</tr>
<tr>
<td>Convective</td>
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<tr>
<td>For</td>
<td>( v_i = \Omega_k ), ( T_e = T_i ), ( A = 5 )</td>
<td>( E_0^2 = 0.38 \left( \frac{v_i}{\Omega_k} \right)^{2/3} \left( \frac{v^2}{m} \right) )</td>
<td>Valid for ( H &gt; 1 ) m</td>
<td>Eq.(5.26')</td>
</tr>
<tr>
<td>( Fe^+ ) ions</td>
<td>( E_0^2 = \frac{24}{(H)^{2/3}} \left( \frac{v^2}{m} \right) )</td>
<td></td>
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</tbody>
</table>

* Thresholds are evaluated in the primed equations for \( T_e = 240^\circ K \), \( v_e = 14 \) kHz, \( f_0 = 5.1 \) MHz, \( |k| = 18 \) m\(^{-1}\).
FIGURE 5.1. (a) The coupling function $B(x_2)$ versus $x_2$ (Perkins et al., 1974). The various curves are labeled by electron-to-ion temperature ratio. (b) and (c) The parametric decay saturation spectrum for non-linear Landau damping predicted by Perkins et al. (1974) and Fejer and Kuo (1974), respectively, for plasma waves propagating at 45° with respect to the HF pump electric field. The results are given for various pump intensities, $P$, in units of the minimum threshold power for plasma waves traveling parallel to the pump. The excited waves are due to frequency-shifted ion Cerenkov emission from both the pump and the non-linearly stabilized plasma waves, which are generated primarily at small angles with respect to the pump field. In (c) the plasma wave number is normalized to the Debye wave number. The wave number $A$ is that of a plasma wave at the pump frequency; the wave number $B$ is the frequency-matched wave number.
CHAPTER 6
EXPERIMENTAL RESULTS

In this chapter, the salient features of the $E_s$ observations made during the 1976 and 1977 ionospheric modification experiments are presented. Particular attention is focussed upon the properties of the $E_s$ modification process that are basic to the wave-plasma interaction. Phenomena that appear to be peculiar to the detailed structure and evolution of the $E_s$ region are not discussed at length. Thus, the goal of the data presentation is to stress the general characteristics of the excitation process that tend to repeat. These properties offer a common framework within which the physics of the interaction process may be discussed.

The 1976 and 1977 observations are especially amenable to the study of basic plasma processes since they provide a large data base under varying $E_s$ conditions. These observations are remarkable in that they offer an extended and continual viewing of the modified $E_s$ plasma. Typically, observations were conducted for an hour or more. This contrasts with previous observations of this nature which were commonly performed over time intervals of 15 minutes or less.
6.1 TERMINOLOGY

Before discussing the experimental results it is necessary to include a brief note on terminology. The expression "enhanced plasma line" will be used below in reference to increases in plasma line power resulting from the modifying effects of the HF transmitter. These enhancements are in addition to "photoelectron enhancements" due to Cerenkov emission by daytime photoelectrons (Yngvesson and Perkins, 1968). The "natural plasma line," which is the plasma line predicted by incoherent scatter theory (section 2.2), is too weak to be observed within the integration times (≈ 10 sec) typically used in the observations. The terms upshifted plasma line PL(+) and downshifted plasma line PL(-) denote the Doppler shifted echoes measured by the 430 MHz radar at frequencies near (430 MHz + f_{HF}) and (430 MHz - f_{HF}) respectively. Finally, when times are specified, they refer to the local time at the Arecibo Observatory, that is, Atlantic Standard Time (AST).

6.2 PRIOR INVESTIGATIONS

In their initial report describing the enhancement of plasma lines in sporadic E, Gordon and Carlson (1976) noted a relation between the excitation of plasma lines and the value of the blanketing frequency, f_{DE}, exceeding f_{HF}. The plasma line observations were made at Arecibo Observatory using a 5.7 MHz HF pump wave that delivered ≈ 50 μW/m² to the base of the E_s region. In the experiment 500 μsec pulses were transmitted using the 430 MHz radar and the scattered signal was Fourier analyzed for spectral content. The experiment employed
observational techniques similar to those described in section 2.3 for the June 1977 measurements.

The radar scattering volume was located 0.7 km south of the perimeter of the first Fresnel zone sampled by the Los Caños ionosonde, while the corresponding perimeter for the Arecibo ionosonde aligned about 6 km to the south of the scattering volume.

At the beginning of the observation strong enhancements at the downshifted and upshifted plasma lines were continuously monitored. The intensity of the upshifted plasma line was stronger than its downshifted counterpart by $\sim 15$ db. This power asymmetry was accompanied by a large frequency spread between $f_b E_s$ and $f_{HF}$. At the beginning of the observation, $f_b E_s$ measured at Los Caños was 8.8 MHz, but $f_b E_s$ subsequently dropped to 3.4 MHz near the time that the enhancements disappeared. Although, on the basis of the results of Gordon and Carlson alone, there is no reason to believe that a correlation between $(f_b E_s - f_{HF}) \gtrsim 1$ MHz and the dominance of the upshifted plasma line exists, such an association will become clearer after the 1976 observations are presented.

At the start of Gordon and Carlson's measurements, no fluctuations $\gtrsim 3$ db were evident in the plasma line strength within the 30 sec time resolution of the data. After $\sim 20$ min of observations, ionograms recorded at Los Caños indicated that $f_b E_s$ had fallen below $f_{HF}$. At least 6 min elapsed before a similar event occurred over Arecibo. Of special importance is the fact that less than 3 min before the decline in $f_b E_s$ was recorded at Arecibo, the observed plasma line enhancements
abruptly ceased. Over a 7 min time interval prior to the cessation of plasma line echoes, large 10 db - 15 db fluctuations appeared in the downshifted plasma line power but not in the upshifted power.

The above results suggest that $f_b E_s$ is associated with either a continuously overdense plasma or overdense patches which are continuously present within the area of $E_s$ viewed by the 430 MHz radar (a circle $\sim 320$ m in diameter). The large fluctuations in the downshifted plasma line power were interpreted by Gordon and Carlson in terms of overdense patches passing through the radar field of view.

Many of the features of $E_s$ excitation that were noted during the course of the 20 min observation of Gordon and Carlson also appeared to varying degrees in the 1976 and 1977 measurements. On the basis of the present observations, for example, the postulated requirement that $f_b E_s$ be greater than $f_{HF}$ in order for excitation to occur becomes a well established criterion. On the other hand, while there are times during the observations when the upshifted plasma line is stronger than its downshifted counterpart, the reverse asymmetry appears to be present almost as often.

Since the 1976 and 1977 data offer two different types of information about the $E_s$ plasma processes, it is convenient to examine the two data sets individually. Summaries of the results of these two experiments follow.
6.3 THE 1976 OBSERVATIONS

As discussed in section 2.3 the 1976 measurements most important to the present study provided altitude profile information at radar frequencies centered on the ion line and both plasma lines. These observations were highlighted by the exceptional altitude resolution (600 m) and the good time resolution (\(\sim 6\) sec) which were incorporated into the experimental design.

Unless otherwise specified, the 430 MHz radar was pointed during a given observation in the direction of magnetic north (353° azimuth) at a zenith angle of 4°. These coordinates are typically used to orient the diagnostic radar toward the middle of the HF beam in the \(F_2\) region. Although one would like to reorient the radar beam closer to the vertical during the \(E_s\) observations, this is not possible since the 430 MHz line feed is restricted to zenith angles \(\gtrsim 4°\). (This restriction will be removed in future heating experiments at Arecibo Observatory.) In certain instances, however, useful information may be obtained by pointing the radar beam towards the center of the observing volume of the Los Caños ionosonde. This will be discussed later in reference to a specific observing period.

The two radar beam geometries used during the 1976 and 1977 observations are illustrated in Figure 6.1. The HF beam widths for various frequencies are shown as well as the first Fresnel zones of the Arecibo and Los Caños ionosondes for a frequency of 4.6 MHz. At \(E\)-region altitudes (\(\sim 110\) km) the half-power beam width of the diagnostic radar is adequately represented by a circle approximately 320 m in diameter.
In general, the 1976 series of measurements are noteworthy because of the duration of some of the $E_s$ observations. Indeed, over 10 hrs of data containing plasma line enhancements in $E_s$ were recorded during the 1976 experiment. As a result the 1976 $E_s$ data will be presented below in a condensed format wherein the shorter $E_s$ observations will be briefly summarized while the data recorded during the three longest observing periods will be described in greater depth.

6.3.1 Brief Summaries

6.3.1(A) July 17, 1976

During an evening observing period extending from 182150 to 213036 AST on July 17, 1976, Arecibo ionograms indicated the $f_{E_s}$ maintained a value of $\sim 6.0$ MHz from the beginning of the observation until 2041 AST when it fell to $\sim 4.5$ MHz. The HF transmitter was operated at 50 kW power and was tuned to a frequency of 5.1 MHz. Los Caños ionograms were not available during the observation period, although several Arecibo ionograms were. Unfortunately, none of the Arecibo ionograms cover the period 182150-182900 AST. During this interval strongly enhanced downshifted plasma lines were observed at 109 km altitude within the $E_s$, while upshifted lines were barely visible during only a few 4 sec subintervals.

Rough estimates of $f_b E_s$ were made from the 430 MHz radar ion line profiles, based on the assumption that the plasma frequency of the derived peak $E_s$ density corresponds to $f_b E_s$. These estimates yield $f_b E_s$ values that are near 6.2 MHz $\pm$ 0.5 MHz at 182150 but which decline
to 5.0 MHz ± 0.5 MHz at 182900 when the plasma lines were no longer excited. At the beginning of the observation no large fluctuations in the plasma line strengths were apparent within the 4 sec time resolution of the experiment. However, just before disappearing completely, the downshifted plasma line enhancements faded away for ~ 50 sec and then returned again for ~ 50 sec.

After 182900 AST the values of $f_b E_S$ estimated from the radar measurements ranged from 3-4 MHz except for a few minutes surrounding 183637 AST when $f_b E_S$ was estimated to be between 4.6 MHz and 5.6 MHz. No enhanced $E_S$ plasma lines were observed on the 430 MHz radar after 182900. However, from 184306-185908 AST enhancements of both the downshifted and upshifted plasma lines were evident in the F region. During this time Arecibo ionograms indicated that $f_o F_2$ was between 7 MHz and 8 MHz.

The above results are consistent with the requirement that $f_{HF}$ be greater than $f_b E_S$ for excitation to take place. In addition, the dominance of the downshifted plasma lines occurs in conjunction with values of $(f_b E_S - f_{HF}) \lesssim 1$ MHz.

6.3.1(B) July 19, 1976

In an afternoon observation of $E_S$ performed on July 19, 1976 the HF heating frequency was initially set at 5.425 MHz while the transmitter was operated at 52 kW power. The observing period began at 135501 AST and ended at 143446 AST. Ionograms recorded at Los Caños revealed that $f_b E_S$ was 5.8 MHz at 1355 AST but dropped to 5.2 MHz at
1400 AST. Following this $f_mE_s$ quickly fell to 4.2 MHz at 1405 AST and remained between 4.4 MHz and 3.65 MHz for the duration of the observation. Values of $f_0F_2$ ranged from 7 MHz - 8 MHz during the experiment while $fE_s = f_mE_s$.

$E_s$ enhancements of both the downshifted and upshifted plasma lines appeared in the data only from 140028-140228 AST. During this period it is reasonable to assume that $f_mE_s$ was greater than $f_{HF}$. The relative plasma line strengths were approximately equal. Ion line profiles indicated that the $E_s$ was located near 113 km. After 140228 AST no further $E_s$ enhancements were seen, but enhanced F-region plasma lines were briefly visible between 141219 and 141319 AST.

6.3.1(C) July 21, 1976

No enhanced $E_s$ plasma lines were evident during $E_s$ observations conducted on July 21, 1976 from 113100-121420 AST. Except for a brief time period ($\leq$ 10 min) surrounding 1140, Los Caños ionograms indicated that $f_mE_s$ was always above $f_{HF}$ during the experiment. The $E_s$ was located near an altitude of 100 km. The blanketing frequency ranged from 5.5 MHz to 6.6 MHz except at 1140 AST when it briefly dropped to 5.2 MHz. Values of $fE_s$ were never more than 1.0 MHz above $f_mE_s$. At the beginning of the experiment the HF transmitter was operated at a frequency of 5.425 MHz, but this was later changed to 4.6 MHz at 115825 AST. Although the HF transmitter power was initially set at 75 kW, the power was subsequently lowered to 50 kW at 120110 AST.
The July 21 observation represents the only time during the 1976 experiment when $E_s$ plasma lines were not excited when $f_{HF}$ was less than $f_bE_s$. This anomalous result may be explained by the fact that, in this case, the $E_s$ altitude (100 km) was much lower than in any of the other observations ($\sim 110$ km). This will be discussed further in Chapter 7.

6.3.2 Extended Observations

6.3.2(A) July 17, 1976

Figure 6.2 summarizes the plasma line observations and the Arecibo and Los Caños ionosonde data recorded between 050000 and 090000 AST on July 17, 1976. During this time the Los Caños ionograms were missing a time code. As a result, the absolute ionogram time is only an estimate accurate to ± 10 min. The ionogram times relative to one another are, however, correct. The solid horizontal line in Figure 6.2 labeled "$f_{HF}$" performs the dual function of indicating the HF transmitter frequency (5.1 MHz) and the times at which radar observations were conducted with the HF transmitter in operation. Also listed is the operating power of the transmitter. At the bottom of the figure, the time intervals over which the upshifted (PL(+)) and downshifted (PL(-)) plasma lines were observable in the $E_s$ and the $F_2$ region are denoted by solid horizontal lines.

Throughout the observing period the HF transmitter was set at 50 kW power and operated in a pulsed mode. The pulsing allowed for the collection of data samples that were representative of a "modified" and an "unmodified" ionosphere. The "on" and "off" time periods were
54.4 sec and 6.1 sec respectively. It should be noted, however, that the ionosphere may remain in an altered state during "HF off" times if modifications to the ambient plasma conditions decay over times \( \gtrsim 6 \) sec (e.g. thermal time scales of 15 sec - 30 sec in the F region). The enhanced \( E_s \) plasma lines, on the other hand, decay over time scales of the order of 100 \( \mu \)sec (Gordon and Carlson, 1976).

The time at which the enhanced \( E_s \) plasma lines terminate (072700 AST) correlates well with the sudden drop in \( f_d E_s \) below \( f_{HF} \). Any such association is not expected to be extremely accurate in time since the ionosondes and the 430 MHz radar sample different ionospheric volumes (Figure 6.1). At any rate the time at which the \( E_s \) plasma lines disappear is bracketed by the times at which \( f_{oF2} \approx f_{HF} \) becomes visible at Los Caños and Arecibo. Given the geometrical arrangement outlined in Figure 6.1 this is consistent with a southerly movement of the \( E_s \) ionization.

In Figure 6.3 the \( E_s \) ion line and plasma line power is displayed for the time periods when \( E_s \) plasma line enhancements were observed. The time resolution of the data is 6.1 sec while the altitude resolution is 600 m. The power values plotted are the sum of three consecutive altitude samples. The altitude of the middle sample was selected by searching a narrow altitude region (\( \sim 30 \) km) surrounding the \( E_s \) region for the maximum normalized signal power. The peak altitudes used for the power summations are graphed immediately above the signal power. In the case of plasma line power measurements, the selected altitude maxima are random (and therefore meaningless) whenever plasma line
enhancements (either photoelectric or HF-induced) are not present. When \( E_s \) is continually present the peak ion line altitudes generally remain the same or change slightly when the HF transmitter is switched off depending upon the degree of ion line enhancement.

The power values plotted represent residual signal strengths after noise subtraction and are normalized to the mean noise level. The signal-to-noise ratio is abbreviated as S/N. The upshifted plasma line power has been increased by 3.0 db relative to the downshifted power in accordance with the absolute two-way gain calibration of the 430 MHz antenna performed by Duncan (1976). The one standard deviation (1 \( \sigma \)) noise levels after background noise subtraction are marked on the power axis for each plasma line. The plasma line noise level was calculated by averaging the scattered power in 10 range gates surrounding the peak plasma line range but beyond the sidelobes of the Barker code.

The apparent "noise level" for the plasma lines, as determined by the power displayed in Figure 6.3 while the HF is off, is consistently above the 1 \( \sigma \) level. At least in part this reflects the fact that power maxima were selected from 50 independent samples within the 30 km search region. Furthermore, in this particular case, photoelectron enhancements of the \( E_s \) plasma lines are also partially responsible for the increase in the noise level.

All data shown in Figure 6.3 have been duly compensated for any changes in gain or noise level which may have occurred during the experiment. Data is not shown at times when external sources of interference were known to be present. At these times gaps appear in the plotted power and the peak altitude selection is random. The vast majority of
such interference in the data stems from arcing at the HF antenna feed. When arcing occurs, it is most likely to take place immediately after the transmitter is switched on.

Figure 6.3 illustrates the same type of power asymmetry in the enhanced plasma lines as that reported by Gordon and Carlson (1976). The upshifted line strengths are generally larger than their downshifted counterparts by 10 db - 15 db. Prior to 0630 AST large-scale fluctuations are apparent in both of the plasma line intensities. The sudden increases and drops to and from the noise level are correlated with increases in the ambient (HF off) ion line power (i.e. electron density). For example, at times near 0540, 0610, and 0725 AST periods of plasma line enhancements are accompanied by rises in the unenhanced ion line power above a S/N level of 10.0. Since the ion line power is proportional to electron density, this suggests that the plasma lines were excited whenever dense plasma patches passed through the radar field of view.

Near 0610 AST plasma line enhancements are present even though the Los Caños ionograms show $f_{bE_s} < f_{HF}$. Nevertheless, if one allows an ~10 min displacement in the ionogram scalings towards earlier times because of the uncertainty in the time code, then fair agreement between $f_{bE_s} > f_{HF}$ and plasma excitation is obtained.

During most of the observation it is not possible to determine whether the observed plasma line enhancements correlate with $f_{bE_s} > f_{HF}$ since the F region was completely blanketed. However, a comparison of $f_0F_2$ values near 0500 and 0730 AST indicates that it is reasonable to
believe that \( f_{bE_S} \) was below \( f_{HF} \) at times when no enhancements appeared in the data. More quantitatively, one may use the ion line power profile in order to obtain an estimate of the plasma frequency, \( f_{e} \), at the \( E_S \) peak. A representative value of \( f_{bE_S} \) may then be inferred by assuming \( f_{bE_S} \sim f_{e} \). When this is done, one finds that \( (f_{bE_S} - f_{HF}) \) is for the most part \( \gtrsim 1.4 \text{ MHz} \) on those occasions when plasma line enhancements appear in the data. In this regard it is interesting to note that the upshifted plasma line is at the same time consistently the dominant plasma line.

The critical electron density necessary for plasma line enhancements to occur may be deduced from the power in the peak of the \( E_S \) ion line profile observed just before plasma line enhancements appear or just after they disappear. The power profiles recorded near 0727 AST, when the plasma line enhancements terminated, are especially useful since they may be transformed into electron density profiles using ionograms as a means of calibration. When this is done and a correction for signal clutter is applied (section A.1), the critical \( E_S \) density is found to be \( 3.2 \pm 0.2 \cdot 10^{11} \text{ m}^{-3} \), which corresponds to a plasma frequency of \( 5.1 \pm 0.1 \text{ MHz} \). Thus, at 0727 AST the measured critical plasma frequency lies quite close to \( f_{HF} \). This is just what one expects if \( f_{HF} \geq f_{bE_S} \) is a necessary condition for \( E_S \) enhancements, and if the ion line power is an indicator of the average plasma frequency within the radar field of view.

The fact that the measured critical frequency for \( E_S \) excitation approximated \( f_{HF} \) (\( = 5.1 \text{ MHz} \)) as well as it did is, in part, fortuitous
since the power scattered at the ion line is subject to volume averaging, both in altitude and laterally across the radar beam. In this particular case, the averaging in altitude was probably minimal since the $E_s$ region appeared to be centered at an altitude corresponding to an integer number of baud lengths.

In addition to asymmetries in plasma line power, slight differences are occasionally observed in the altitudes of the PL (+) and PL (-) peaks within the plasma line power profiles. Several representative altitude profiles of relative ion and plasma line power are presented in Figure 6.4 for times when the altitude asymmetry was and was not observed. In Figures 6.4b, d and e the downshifted plasma line maxima are displaced by between 0.3 and 0.6 range gates towards altitudes greater than the corresponding peak altitudes for the upshifted line (1 range gate = 600 m). Figures 6.4a and 6.4c represent times when no asymmetry was observed.

A statistically significant asymmetry is present in approximately 10% of the July 17 data. The asymmetry is always directed in the same sense wherein the downshifted lines appear at greater altitudes than upshifted lines. In general the altitude asymmetry tends to preferentially occur at times when the downshifted plasma line power is comparable to or greater than the upshifted plasma line power. However, cases do exist where the altitude asymmetry appears in the data in conjunction with a dominant upshifted plasma line.

Instrumental effects cannot be totally ruled out as a possible cause of the observed asymmetry. However, it should be pointed out that
any ambiguity in range due to the Doppler shifts at the plasma lines would produce an asymmetry opposite to that observed. Furthermore, the magnitude of any such ambiguity in range would be very small and probably undetectable. This follows since the plasma line observations involve frequency offsets from the operating frequency of the Barker decoder of \( \sim 3 \) kHz, whereas the inverse of the effective pulse length (i.e. the baud length) is 250 kHz.

Experimental tests of the hardwired Barker decoder used within the receiver system in Figure 2.3 were carried out by Muldrew and Showen (1977) during the 1976 experiment. These tests were designed to determine the response of the decoder to a variable frequency signal. During the tests, the input signal was set at varying frequency displacements from the operating frequency of the decoder and the principal and secondary maxima of the Barker pattern were noted. The results of these tests (Muldrew, 1978c) imply that little distortion in the idealized Barker pattern is to be expected for the spectral widths and Doppler shifts typically encountered in the measured \( E_s \) plasma line spectra.

Finally, one might add that Muldrew (1978c) has noted an asymmetry in the heights of the two plasma lines recorded in the F region during the 1975 Heating Experiment. These data were obtained using a Barker coding scheme similar to that of the 1976 experiment. Muldrew finds the mean altitude asymmetry to be \( \sim 65 \) m, wherein the upshifted plasma line appears at greater heights than the downshifted plasma line. On the other hand, the 1976 plasma line data recorded in the F region indicate a small altitude asymmetry in the reverse direction (i.e. the
downshifted plasma line is at higher altitudes). Thus the direction of the altitude asymmetry in the 1976 data is the same for both F-region and $E_s$ excitation.

Figure 6.5 displays the ion and plasma line (signal minus noise) power as a function of time for several consecutive range gates. Data is shown for a time interval over which plasma line intensities were particularly strong. The 0 db base lines for the plasma line plots are set at the $+1 \sigma$ noise level. The corresponding base line for the ion plot is $+4.5 \sigma$. From this figure it is evident that plasma line and ion line enhancements in excess of the sidelobes of the Barker code pattern are confined to an altitude interval of $\leq 2$ range gates (1.2 km) and at times appear in only one 600 m range gate.

6.3.2(B) July 20, 1976

The observations conducted from 125331-172532 AST on July 20, 1976 provide the best documented example of the association between $f_bE_s > f_{HF}$ and the excitation of $E_s$ plasma lines. Figure 6.6 displays the relevant ionogram data recorded during the observation period along with the observed times of enhanced plasma lines. The presence of downshifted $E_s$ plasma lines between 1333 and 1524 AST is indicated by a dotted line in order to signify that the enhancements were very small and only intermittently present. Over the course of the observation the altitude of the $E_s$ gradually descended from 111 km down to 107 km.

Data gathered between 125331 and 134729 AST were recorded in an HF pulsing mode wherein the transmitter was on for 266 sec and off for 30 sec. The time resolution of these data was 30 sec. From
135206-172532 AST the HF transmitter was cycled on for 54.4 sec and off for 6.1 sec. In this case the time resolution was 6.1 sec. The altitude resolution was determined by the standard 600 m separation between range gates.

From 1405 to 1535 AST the F-region trace was absent from the recorded ionograms due to blanketing by the $E_s$. However, by carefully monitoring $f_b E_s$ after 1535 AST one was able to anticipate the decline of the blanketing frequency and adjust $f_{HF}$ accordingly. Thus, $f_{HF}$ was initially set at 4.6 MHz but was later stepped up in frequency to 5.94 MHz, then down to 5.425 MHz and finally back to 4.6 MHz.

At the start of the observation the 430 MHz line feed was pointed at 4° zenith angle and 353° azimuth. However, from 1357 AST onwards the feed was positioned at 5.54° zenith angle and 14.44 azimuth. This observing geometry is illustrated in Figure 6.1. After the feed was repositioned, the sampling volume of the diagnostic radar was located at the center of the first Fresnel zone over the Los Caños ionosonde. However, it should be noted that the difference between the ionosonde and radar sampling areas is quite large. For reflection within an $E_s$ region at 110 km altitude, the diameter of the first Fresnel zone at 4.6 MHz is 3.8 km while the half power beam width of the 430 MHz radar is only $\approx 320$ m. If one assumes that the Los Caños ionosonde samples an area roughly the size of the first Fresnel zone, then it is clear the diagnostic radar samples a considerably smaller area of the $E_s$. 
In Figure 6.6 the relation between $f_b E_s$ and plasma line excitation in $E_s$ is quite striking. Although the times at which $f_b E_s$ dips below $f_{HF}$ do not always coincide exactly with the cessation of $E_s$ plasma line enhancements, the small mismatches may at least in part be attributed to the differences in the observing areas of the radar and ionosonde as well as the finite time resolution of the ionosonde measurements.

Oftentimes the disappearance (appearance) of $E_s$ enhancements is coincident with the appearance (disappearance) of enhancements in the F region. During times of transition enhanced $E_s$ and F-region plasma lines may simultaneously appear within a 6 sec integration period of the radar data. One such example is illustrated in Figure 6.7. In general, Figure 6.7 serves to indicate the content of the power profile data gathered during the 1976 experiment.

It is tempting to associate the dual excitation of plasma lines in $E_s$ and the $F_2$ region with leakage of the HF beam through the $E_s$ volume observed by the diagnostic radar. However, this interpretation is probably not correct. The HF beam bends very quickly under the influence of the earth's magnetic field just before reflecting in the steep $E_s$ electron density gradients. Thus, waves radiated approximately 6° from the center of the HF beam pattern are responsible for the $E_s$ excitation in the observed radar volume. On the other hand, prior to reflecting in the F region, the HF beam is refracted (magnetic) northwards a distance of the order of 25 km. Enhancements in the F region come about as a result of electric fields that are carried along ray paths that start out close to vertical but subsequently bend into the
observing volume near the point of reflection. Consequently, the observed F region excitation is controlled by the amount of the HF beam which is able to penetrate the Es almost immediately above the Observatory. The dual excitation illustrated in Figure 6.7 could come about as a result of the passage through the region of observation of either an overdense Es boundary or an area of patchy ionization.

Figures 6.8 and 6.9 depict selected ion line and plasma line power time histories in a format similar to that used in displaying the intensities in Figure 6.3. Antenna gain corrections (~3 db) have been applied to the measured PL (+) intensities in order to make them directly comparable with the downshifted intensities.

The time development of the enhanced Es plasma line power displays an extremely interesting morphology. Prior to 1524 AST, the enhanced upshifted plasma line power clearly dominates the enhanced downshifted power. In fact, the downshifted enhancements are at times either completely absent or very small and undetectable within the statistics of the experiment. Although the F region is completely blanketed during this period, one may assume that fbEs is greater than 7.0 MHz - 7.5 MHz on the basis of foF2 values measured at 1330 and 1530 AST. Given this, one obtains (fbEs - fHF) ~ 2.4 MHz for this portion of the observation.

After the HF frequency is increased from 4.6 MHz to 5.94 MHz at 1530 AST the reverse power asymmetry is immediately apparent. In this case (fbEs - fHF) ~ 0.6 MHz. Downshifted plasma line intensities are on the average 5 db - 10 db greater than their downshifted counterparts.
This power asymmetry is maintained until 1600 AST when the upshifted power once again dominates. After the HF frequency is reduced to 5.425 MHz at 161412 AST the upshifted power is greater at first but this trend gives way to the reverse situation where the downshifted power is larger.

The $E_s$ plasma line enhancements were intermittently present between 1700 and 1711 AST. When $E_s$ plasma lines were visible during this time interval, the PL (-) power was greater than the PL (+) by $\sim 10$ db. Thus, on the basis of the above discussion, one may conclude that the July 20 observations provide additional evidence for an association between $(f_b E_s - f_{HF}) > \zeta$, or $\zeta < 1$ MHz, and the dominance of the upshifted (downshifted) plasma line.

As in the case of the July 17 observations, there exists a general tendency for the plasma line power to track in a fashion similar to the ambient ion line level. Furthermore, large ($\sim 10$ db) fluctuations in the enhanced plasma line power on short time scales ($\zeta 3$ min) are all but nonexistent. Comparatively speaking, however, the downshifted plasma line power is seen to undergo larger fractional variations over short time scales ($\zeta 30$ sec) than the upshifted power. Over longer time scales ($\zeta 3$ min) both enhanced plasma lines and the enhanced ion line tend to map out similar fractional intensity variations.

Figures 6.10 and 6.11 display plasma line and ion line time histories recorded in consecutive range gates over two selected time periods. The 0 db base lines for the plasma lines are set at $+1 \sigma$. 
above noise level while the ion line base lines are + 35 σ and + 23 σ respectively in Figures 6.10 and 6.11. Prior to 1530 AST an altitude asymmetry in the peak plasma line power is statistically significant only in an occasional 6 sec integration of the plasma line power profiles. Small asymmetries (< 0.5 range gates) appear most frequently during the time interval 1430-1445 AST. These slight asymmetries are difficult to detect in Figure 6.10 barring a very careful examination of the data. After 1530 AST, altitude asymmetries of as much as 1 range gate are consistently present in the data (Figure 6.11). The appearance of the altitude asymmetry between 1530 and 1600 AST is consistent with earlier observations that the asymmetry is most prominent whenever the downshifted plasma line power becomes comparable to the upshifted power.

6.3.2(C) July 25, 1976

During the morning of July 25, 1976 very strong $E_s$ was observed on both the Los Caños and Arecibo ionosondes. Figure 6.12 illustrates the variation of the scaled ionogram parameters as well as the times at which plasma line observations were made. The Los Caños ionograms indicate that $f_b E_s$ extended beyond $f_0 F_2$ over the time period 0815-1300 AST while $f E_s$ ranged from 6 MHz - 16 MHz. On the basis of $f_0 F_2$ values recorded just before the F region was completely blanketed at 0810 AST and just after the F region reappeared at 1300 AST, it seems reasonable to assume that $f_0 F_2$ was near 5.5 MHz during the intervening period. This allows one to initially set a lower
limit on $f_b E_s$ of 5.5 MHz. Throughout the observing period, the HF transmitter power and frequency were 80 kW and 5.1 MHz respectively.

Several different $E_s$ plasma line observations were conducted between 083630 and 124112 AST. However, only the observations made from 083630-095159 AST and from 104050-121808 AST employed Barker-coded pulses to obtain an altitude resolution of 600 m for the ion and plasma line power profiles. All other observations involved spectral measurements and will be discussed later in Chapter 7.

The time resolution of the Barker data was 6.1 sec. During both of the data taking runs using the Barker code the HF transmitter was repeatedly cycled on for 356 sec and off for 356 sec.

All ion line and plasma line observations conducted between 083630 and 124112 AST were performed concurrently with an experiment designed to investigate the interaction of the transmitted HF wave in the D region. This experiment is relevant to the observations presented here only to the extent that it entailed additional pulsing of the HF transmitter beyond the 12 min cycles described above for the Barker runs.

When the HF transmitter was cycled "on" it was operated at a 98% duty cycle wherein the transmitter was turned off for 120 μsec at regular 6 msec intervals. When the HF transmitter was "off" it was run at a 2% duty cycle wherein the transmitter was cycled on for 120 μsec every 6 msec. These pulsing sequences are illustrated in Figure 6.13 along with the Fourier transformed HF "on" and HF "off" power spectra of the transmitted HF wave. During the "on" periods the pulsing had the net effect of smearing $\sim 2\%$ of the HF power over a
bandwidth of \( \sim 10 \) kHz centered at \( f_{HF} \). When the HF transmitter was operated in the "off" mode all of the transmitted power (2\% of the CW power) was spread over the same 10 kHz bandwidth.

The above pulsing was permitted during the plasma line observations on the assumption that a 2\% change in the HF on or HF off duty cycle would do little to affect the parametric instabilities. The rise times of parametrically enhanced plasma lines are of the order of a few msec (Showen, 1975; Muldrew, 1978a) while decay times measured in \( E_s \) are of the order of 0.1 msec (Gordon and Carlson, 1976). Thus, no enhancements of the plasma line were anticipated in the HF "off" mode. Significant changes in the plasma line intensity level due to the pulsing was expected only when the 430 MHz diagnostic radar sampled the \( E_s \) enhancements at the time or shortly after the 120 usec HF "on" notch was present in the \( E_s \) region. During the \( E_s \) observations, the HF pulsing sequence was shifted with respect to the radar IPP so that this did not occur.

Despite the above precaution, occasional enhancements of the upshifted and downshifted plasma lines appeared in the Barker data while the HF transmitter was in the "off" mode. The altitudes of the enhancements were typically within 1 range gate of the HF "on" enhancements measured at about the same time. Furthermore, the HF "off" enhancements were comparable to and sometimes greater than the HF "on" enhancements. At present the HF "off" enhancements lack an adequate physical explanation. Given the limited amount of documentation that is available concerning the exact circumstances under which the 6 msec pulsing was performed, instrumental effects cannot be ruled out. The
observed enhancements, for example, can be completely explained if the HF transmitter were intermittently left on during HF "off" cycles for time periods of a few seconds.

The HF "on" Barker data exhibited no unusual features and is presumed to be unaffected by the 6 msec pulsing. During the first observation period extending from 083630-095159 AST the altitude of the E\textsubscript{s} as determined from the ion line profiles ranged from 107 km to 113 km. The E\textsubscript{s} appeared in the unenhanced ion line measurements as a sharp power increase in 2-6 consecutive range gates. The previously described altitude asymmetry of the plasma lines was occasionally present but the extent of the asymmetry was always \textless 0.2 range gates.

In Figure 6.14 the plasma line and ion line power is plotted as a function of time. Plasma line power is not graphed at times when the HF transmitter is cycled "off." Figure 6.14 clearly illustrates a systematic power asymmetry between the upshifted and downshifted plasma lines. On time scales of the order of a minute the downshifted power is observed to undergo slightly larger functional power fluctuations than the corresponding upshifted line. In addition, the data exhibits a spectacular rise in both the enhanced downshifted and upshifted plasma line power at 0916 AST. This is accompanied by a similar increase in the ion line power.

The value of f\textsubscript{b}E\textsubscript{s} could not be determined during the first observation period on July 25 since the F region was completely blanketed. However, estimates of f\textsubscript{b}E\textsubscript{s}, made on the assumption that the E\textsubscript{s} peak plasma frequency obtained from the ion line profile approximates f\textsubscript{b}E\textsubscript{s}, yielded f\textsubscript{b}E\textsubscript{s} > 8 MHz during the period 083630-095159 AST.
Thus, a large separation between \( f_b E_S \) and \( f_{HF} \) \((f_b E_S - f_{HF}) > 3 \text{ MHz}\) once again appears to occur in conjunction with a power asymmetry wherein the PL (+) line strength dominates the PL(-) intensity.

During the second Barker observation (104050-121808 AST), the altitude of the \( E_S \) ranged from 117 km to 114 km. The \( E_S \) was observed in 1-3 range gates of the unenhanced ion line power profile. This implies that the thickness of the \( E_S \) region varied from \( \leq 600 \text{ m} \) to \( \sim 1.2 \text{ km} \). No altitude asymmetry in the heights of the enhanced plasma lines is apparent in these data.

Figure 6.15 illustrates the time histories of the upshifted and downshifted plasma line power. Once again the dominance of the upshifted plasma line power is clearly visible. Although the downshifted plasma line power undergoes slightly larger fluctuations than the upshifted line, both plasma lines map out the same intensity trends over time scales of \( \sim 2 \text{ min} \). A similar mapping in plasma line power is also evident in the preceding Barker run. In addition, there is a tendency during both observations for the enhanced ion line to track in a fashion similar to the plasma lines.

During the second Barker observation, \( f_b E_S \) could not be determined from the Los Caños ionograms since the F region was still blanketed. In addition, only \( f_0 F_2 \) and \( f E_S \) values were available from the Arecibo ionograms. However, if \( f_b E_S \) is estimated from the ion power profiles as in the earlier observation, then one finds that \( f_b E_S > 7 \text{ MHz} \). Thus, on yet another occasion, the dominance of the PL (+) line may be identified by the presence of a large frequency spread between \( f_b E_S \) and \( f_{HF} \) \((f_{HF} = 5.1 \text{ MHz})\).
Finally, it is interesting to examine the sudden drop in the intensities of both plasma lines at 1055 AST. This loss in plasma line signal is accompanied by an abrupt lowering of the ion line intensity below the ambient (HF "off") level that was observed before and after 1055 AST. The implication is that an \( E_s \) region of very low electron density, or "\( E_s \) hole," passed through the diagnostic radar field of view. This conclusion is borne out by the observed ion line power profiles which indicate that the intensity of the \( E_s \) faded and at one point completely disappeared between 105514 and 105544 AST.

6.4 SPECTRAL INFORMATION

The 1977 observations consisted of measurements of plasma line and ion line power spectra (section 2.3) and therefore complemented the power profile measurements made during 1976. The single pulse radar techniques used for the spectral measurements resulted in an altitude resolution of \( cT_o/2 = 0.75 \text{ km} - 105 \text{ km} \) for typical radar pulse lengths, \( T_o \), of 500 \( \mu \text{sec} \) - 700 \( \mu \text{sec} \). The altitude resolution was extremely poor compared to the 600 m resolution achieved during 1976. Nevertheless, since the observed instability region is confined to a very narrow altitude region the observation altitude may be determined from the time delay of the leading edge of the radar pulse, or to lesser accuracy from the trailing edge of the pulse. In general, the altitude of the plasma line enhancements may be obtained to within 1 km accuracy.

In presenting the spectral observations, individual spectra will be initially examined for evidence of the decay mode and the oscillating two-stream instability (OTSI). As discussed in Chapter 5, in
the absence of any instability saturation, the spectrum of the decay mode should exhibit a single peak Doppler shifted from 430 MHz by 
±(f_{HF} - f_{ia}) where f_{ia} is the ion acoustic frequency, which ranges from 0.9 kHz - 1.3 kHz in E_{s} (section 5.8). The resonance width is expected to be of the order of f_{ia}. In contrast to the decay instability, the OTSI leads to a frequency component of width \ll f_{ia} displaced from 430 MHz by \pm f_{HF}. Additional structure may of course be present in the spectrum as a result of instability saturation.

Although the separation between the data points in the experimental power spectra is given by 1/\Delta = 1/(128 \cdot 6 \mu sec) = 1.30 kHz (section 2.3), the frequency resolution, \Delta f, is determined by the sampling interval, T_{s}, of the scattered radar pulse. Typically T_{s} = 500 \mu sec and \Delta f = 2 kHz. Thus, to a certain extent, individual spectral estimates reflect the influence of the convolution of the \sin(x)/x spectral window (section A.4) with nearby spectral features. Despite the \sim 2 kHz frequency resolution, the position of a narrow spectral peak may be estimated to an accuracy of less than 2 kHz by examining the relative values of 3 or 4 of the highest points which serve to define a spectral peak. These estimates are of course subject not only to statistical errors but also to uncertainties associated with the \sin(x)/x convolution of a feature having a finite bandwidth. In fact a complete deconvolution is not possible. Nevertheless, the spectral peaks examined below have sufficiently narrow bandwidths to allow one to distinguish between a plasma line spectrum dominated by the decalv line and a spectrum which is mainly characteristic of the OTSI.
The time $T_s$ over which the plasma line echo is sampled varied within the experiment from one observation to the next. Therefore, in order to compare the observed spectra for a given $T_s$ value with the spectral signature expected for a single frequency it is most convenient to generate theoretical spectra directly by performing a Fast Fourier Transform of a sinusoidal wave train boxed by a unit step function, $g(t)$. As a function of time, $t$, $g(t)$ is defined by
\[
g(t) = \begin{cases} 
1 & 0 \leq t \leq T_s \\
0 & t < 0, \ t > T_s 
\end{cases}
\]
This allows adequate comparisons to be made provided that the frequency response of the receiver system does not significantly broaden or distort the received signal.

In order to determine the receiver response, a sinusoidal test signal generated by a stable oscillator was inserted into the receiver system at the Butterworth filter stage in Figure 2.5. The receiver system in front of the filter has a negligible effect upon the signal because of its broadband characteristics. The baseband mixer output was digitally sampled and Fourier analyzed using the standard data taking procedures. Figure 6.16 illustrates a representative discrete Fourier transformed spectrum calculated for a signal offset of -10 kHz from d.c. Also shown is the spectrum expected for a perfect $(\sin(x)/x)^2$ windowing function. Inside a 40 kHz frequency band centered on the spectral peak, the test results match the $(\sin(x)/x)^2$ function to within 3%.

This is well within the statistical error bars of the spectral measurements presented here. As a result the calculated spectra provide
a sufficient degree of accuracy to allow comparisons to be made with the observed spectra. Outside the 40 kHz band the variance is only slightly larger than 3%. The probable cause of the small peaks observed in the test spectrum near -50 kHz, -30 kHz, 10 kHz, and 30 kHz is harmonic generation within the test signal oscillator.

6.5 THE 1977 OBSERVATIONS

The 1977 spectral data of $E_s$ plasma line enhancements consisted of five separate observations. In general, these observations reaffirmed the association between plasma line excitation and $f_{HF} \leq f_B E_s$. Within the 5 min time resolution of the recorded Los Caños ionograms, the commencement (cessation) of all plasma line enhancements observed during 1977 were coordinated with increases (decreases) in $f_B E_s$ above (below) $f_{HF}$. On two occasions when observations were conducted when $f_B E_s$ was below $f_{HF}$, no enhancements were observed even though $fE_s$ was slightly greater than $f_{HF}$.

The 1977 spectral observations of enhanced $E_s$ plasma lines are summarized below. The five occasions on which enhancements were visible in the data are presented in chronological order.

6.5.1 June 3, 1977

During observations conducted on July 3, 1977 from 1735-1946 AST F-region plasma line enhancements were continuously observed, except for a brief period of time from 1947-1857 AST when $E_s$ enhancements suddenly appeared and then disappeared. The HF pump frequency was set at 5.185
MHz, while the transmitter power was 40 kW. Los Caños ionograms recorded at 5 min time intervals indicated that \( f_b E_s \) ranged from 2.7 MHz - 4.1 MHz throughout the observing period, except for a brief time period at 1850 AST when a single ionogram displayed an \( f_b E_s \) value of 5.3 MHz.

Figures 6.17 and 6.18 depict the upshifted and downshifted plasma line spectra measured during consecutive 20 sec integration periods near 1850 AST. The power in each 1.3 kHz frequency cell is expressed as a signal-to-noise ratio (S/N). The spectra presented from 184318-184725 AST and 185727-185924 AST represent F-region enhancements at an altitude of approximately 230 km. These spectra exhibit some of the saturation characteristics of the decay mode and plasma wave propagation effects which have been previously examined by Kantor (1972, 1974), Showen (1975) and Duncan (1977). The large decay line peak located at a \( \sim 3 \) kHz displacement from \((430 \text{ MHz} \pm f_{\text{HF}})\) is associated with frequency-shifted ion Cerenkov emission from the pump wave (Harker, 1972; Perkins et al., 1974). If this displacement is denoted \( f_{\text{ia}} \), then it is clear that a secondary peak is consistently present in the upshifted and downshifted plasma line spectrum at a displacement of \( \sim 3 f_{\text{ia}} \). The location of this peak conforms with theoretically predicted spectral enhancements (Perkins et al., 1974 and Fejer and Kuo, 1973) due to Cerenkov emission from plasma waves travelling along the magnetic field. These waves are generated at the first satellite peak produced as a result of nonlinear Landau damping saturation (section 5.6). In addition, the small peak observed in the downshifted plasma line spectrum at \((430 \text{ MHz} - f_{\text{HF}})\) is too large to be an
artifact of the spectral windowing and is presumably due to either a weakly driven OTSI, Cerenkov emission in a pumped but stable plasma (Hagfors and Gieraltowski, 1972), or to the scattering of the pump wave into Langmuir waves by plasma density striations (Fejer, 1978). A detailed description of the enhanced F-region spectrum and several possible explanations for many of the observed features may be found in a review by Carlson and Duncan (1977) and the study of Muldrew (1978a).

The $E_s$ enhancements present from 184902-185413 AST offer a marked contrast to those found in the F region. The altitude of the enhancements is 110 km. While the F-region spectra exhibit considerable structure, the $E_s$ spectra are dominated by a single peak located near 430 MHz ± $f_{HF}$. Thus, these spectra are indicative of the presence of the OTSI. All of the secondary $E_s$ peaks in Figure 6.18 are attributable to the "ringing" associated with the $(\sin(x)/x)^2$ windowing function centered on the primary peak. In Figure 6.19 representative spectra are plotted in greater detail. To facilitate comparisons, the spectrum expected for a boxed sinusoidal wave at 0 kHz displacement is included in the plots by normalizing the power of the peak point in the calculated spectrum to the power at the experimentally observed peak. This windowing is depicted by the shaded area in the figure.

From Figure 6.19a it is evident that a very narrow single frequency spike at 0 kHz cannot reproduce the observed spectrum in every detail. However, reasonable agreement is found for data points on the side of the peak directed away from 430 MHz. This is consistent with
the presence of a strong narrow feature located at 0 kHz displacement. The data points on the 430 MHz side of the peak are generally greater than the calculated windowing function. This result may be attributed to the presence of unresolved spectral structure located within ~ 5 kHz of 0 kHz displacement. The spectral intensity of any such structure is of the order of 5 db - 10 db less than the peak intensity. Furthermore, it is apparent that the growing mode is much stronger than any contribution from a decay line. If the decay line were dominant in the E_s spectra, one would expect the data point D3 in Figure 6.19a to be at least 2.5 times larger than D2, and point U3 would have to exceed U2 by at least a factor of 3.5. The above ratios assume that the decay line is narrow and is displaced towards 430 MHz by 1 kHz or more.

The spectra displayed in Figure 6.19 have not been corrected for the asymmetry in antenna gain at 430 MHz ± f_HF. For the June 3 data this correction requires that the upshifted plasma line power be increased by a factor of 2.1 with respect to the downshifted power. Even after applying this correction, however, the downshifted power is still consistently greater than the upshifted power by more than 5 db. On the basis of the 1976 observations, this power asymmetry might have been predicted since (f_bE_s - f_HF) < 0.1 MHz.

6.5.2 June 7, 1977

The E_s observations of June 7, 1977 were performed between 1330 and 1410 AST using a pump frequency of 4.07 MHz and a transmitter power of 50 kW. The Los Caños ionograms recorded during this period indicated that f_bE_s ranged from 4.0 MHz - 6.1 MHz. The value of f_oE was
approximately 3.5 MHz. The observed plasma line intensities were highly variable, often intensifying to a signal-to-noise ratio, S/N, of 100 before relaxing to a weak level (S/N \sim 2) or disappearing entirely. No plasma line excitation was observed at altitudes other than that of the \text{E}_s, which appeared at 109 km. The termination of plasma line enhancements at 1400 AST was followed by a decline in \text{f}_b\text{E}_s at 1405 AST to a value below \text{f}_\text{HF} (\text{f}_b\text{E}_s = 4.0 MHz).

Figure 6.20a illustrates a typical upshifted/downshifted spectral pair measured in the presence of strong \text{E}_s excitation. Once again the spectra appear to be consistent with a sharp spike at \( (430 \text{ MHz} \pm \text{f}_\text{HF}) \) with little or no unresolved structure on the 430 MHz side of the peak. The absence of any oscillation in the \( \sim 10 \) data points surrounding the observed peak may be accounted for if the width of the deconvolved central peak were \( \sim 1 \text{ kHz} \).

The antenna gain correction necessitates that the upshifted plasma line power be increased by a factor of 2.0 with respect to the downshifted power. After making this correction the upshifted and downshifted plasma line intensities appeared to be comparable throughout the observing period.

6.5.3 June 9, 1977

The observations of June 9, 1977 were conducted from 1130-1232 AST with the HF transmitter frequency set at 4.07 MHz. The transmitter power was 50 kW. Throughout the data taking period \text{f}_b\text{E}_s hovered around \text{f}_\text{HF} and ranged from 3.8 MHz - 5.0 MHz. As a result \text{E}_s plasma line
enhancements were intermittently present at an altitude of \( \approx 114 \text{ km} \), while at other times excitations in the upper E region (131 km – 136 km) were observed.

Figure 6.21b depicts a representative upper E-region enhancement that occurred at an altitude of 136 km. Weak \( E_s \) enhancements (S/N \( \approx 2 \)) appeared to be present in both the upshifted and downshifted plasma line spectra at times prior to (Figure 6.21a) and subsequent to this spectral integration period. However, between 115500 and 115511 AST the normal E-region spectrum underwent a sudden intensification and then quickly subsided to its previous level. The phenomenon occurred several times throughout the observing period. Each time the spectrum took on a similar appearance.

It is noteworthy that the upper E-region spectrum exhibits the same type of diffusive cascade from the decay line as that observed by Duncan (1977) and Showen (1975) in the F region. Duncan (1977) attributed the diffusive cascade of plasma wave energy away from the decay line to saturation of the decay wave at low pump powers. If the cascade is used as an indicator that the decay mode is above threshold, then this result implies that the decay instability may be excited at altitudes as low as 131 km in the absence of any enhancements indicating the presence of the OTSI. This is in sharp contrast to the \( E_s \) excitations, which were observed to occur near 114 km. In these spectra a peak at the growing mode (0 kHz displacement) is the dominant feature.

In Figure 6.20b the plasma line spectra of a particularly strong \( E_s \) enhancement is shown. These spectra are representative of
the other $E_s$ spectra measured on June 9. The spectra in Figure 6.20b follow the general outline of the $(\sin(x)/x)^2$ windowing function located at 0 kHz displacement. However, deviations from this pattern, as evidenced by the smooth descent of the data points leading away from the peak are indicative of a spectral feature having a finite bandwidth. This bandwidth may be roughly estimated to be $\lesssim 1$ kHz.

Finally, we note that even after an antenna gain correction of 2.0 is applied to the upshifted plasma line power, the downshifted $E_s$ line still appears to be, on the average, slightly ($\sim 2$ db) stronger than the corresponding upshifted line. The plasma lines in the upper $E$ region are approximately equal in intensity.

6.5.4 June 13, 1977

The spectral observations of June 13, 1977 were performed from 1134-1258 AST using an HF pump frequency of 4.07 MHz and a transmitter power of 63 kW. During the observing period $f_B E_s$ ranged from 4.0 MHz - 4.95 MHz while the corresponding values of $f E_s$ ranged from 4.0 MHz - 5.4 MHz. The observation of enhanced plasma lines in the $E_s$ was well correlated with $f_B E_s > f_{HF}$. Plasma line enhancements were continuously observed from the start of the data taking period until $\sim 1225$ AST, when $f_B E_s$ declined to $\sim 4.0$ MHz. Thereafter, plasma line enhancements were only occasionally observed.

Some representative spectra recorded during the June 13 observing period are shown in Figures 6.22, 6.23 and 6.24. During the observations the level of $E_s$ plasma line enhancement varied appreciably. The plasma lines appeared to maintain at least a minimal level of
excitation of $S/N \sim 3$ (Figure 6.22b and 6.22c), while oftentimes intensifying to levels of order $S/N \sim 30$ or more (Figures 6.22a and 6.23). Within the 5 min time resolution at which the ionograms were recorded, a correlation was apparent between increases in plasma line strength and increases in $f_b E_s$.

The majority of $E_s$ plasma line spectra exhibited a single, sharp peak located near (430 MHz $\pm f_{HF}$) and displayed little or no structure on the 430 MHz side of the peak. The widths of the stronger peaks were $\lesssim 1$ kHz, while the weaker plasma lines were only slightly broader. During one brief period (1143-1150 AST), the measured enhancements intermittently displayed a small secondary peak at the downshifted line in addition to the peak at the growing mode. Figure 6.24b illustrates a representative spectrum in which the smaller peak appears at a displacement of $\sim 2.6$ kHz from the growing mode.

Near the end of the observation period $f_b E_s$ hovered about $f_{HF}$. In this situation one would expect the HF beam, on occasion, to penetrate the $E_s$ region. Not surprisingly, there is evidence in the data that during this period HF beam intensities in the upper E region were at times strong enough to excite the decay instability. A typical spectrum recorded in this altitude region is displayed in Figure 6.24a. During the integration period of the spectrum, $f_b E_s$ was measured to be 4.0 MHz. The spectrum resembles other upper E-region spectra described in section 6.5.3.

The antenna gain correction for the June 13 data requires the upshifted plasma line power be increased approximately by a factor of 2.0 relative to the downshifted power. After making this correction,
the downshifted power in \( E_s \) is on the average 3 db - 5 db greater than the upshifted power. The intensities of the upper E-region plasma lines were nearly equal.

6.5.5 June 14, 1977

The \( E_s \) observations of June 14, 1977 were conducted from 1704-1745 AST using an HF frequency of 5.185 MHz. The transmitter power was initially set at 50 kW but was later increased to 100 kW. During the observation period enhanced plasma lines were alternately excited in the \( E_s \) and the F regions. \( E_s \) plasma lines were initially observed between 1704 and 1729 AST, over which time the recorded values of \( f_b E_s \) and \( f E_s \) ranged from 8.5 MHz - 6.4 MHz and 11.5 MHz - 7.0 MHz respectively. The ionosonde, operated at Los Caños, produced soundings at 5 min intervals.

The E-region excitation halted abruptly at 1729 AST and was accompanied by a decline in \( f_b E_s \), which fell from 6.4 MHz at 1725 AST to 4.3 MHz at 1730 AST. The corresponding values of \( f E_s \) were both 7.0 MHz. F-region plasma line enhancements were visible from 1731-1737 AST. At 1738 AST enhanced \( E_s \) plasma lines suddenly reappeared.

This reappearance was coordinated with changes in \( f_b E_s \) and \( f E_s \), which increased from 3.8 MHz to 5.5 MHz and from 6.7 MHz to 9.2 MHz respectively between 1735 and 1740 AST. During the time interval 1738-1743 AST strong \( E_s \) enhancements were observed. At 1745 AST, F-region enhancements once again reappeared and were observed until 1822 AST. The ionogram recorded at 1745 AST revealed that \( f_b E_s \) and \( f E_s \) had fallen to 4.4 MHz and 7.0 MHz respectively. Thereafter, the \( E_s \) gradually weakened and finally disappeared completely at 1800 AST.
Figures 6.25 and 6.26 illustrate several representative $E_s$ spectra measured on June 14. In this case the frequency resolution and the spacing of the Fourier transformed spectral points are 2.23 kHz. The windowing function is displayed at smaller, 1.12 kHz intervals in order that the instrumental broadening of a finite bandwidth signal may be better gauged. During the two time intervals over which excitation within $E_s$ was observed, spectral enhancements at both the upshifted and downshifted plasma lines were continually present. The level of enhancement was generally above $S/N = 10$.

The spectral peaks in the data were centered very close to $(430 \text{ MHz} \pm f_{HF})$. Although there are times when the observed widths of the spectral peaks are narrow ($\lesssim 2$ kHz) (Figure 6.25c) unresolved spectral structure on the 430 MHz side of the peak was typically visible within 5 kHz - 6 kHz of the main peak. On a few occasions a secondary peak was resolved within the 2.23 kHz frequency resolution of the experiment. One such case is shown in Figure 6.26a, where a small peak displaced $\sim 4.5$ kHz from the growing mode may be discerned in the downshifted plasma line spectrum.

Finally it is interesting to note the power asymmetries that appear in the data. (The required antenna gain correction entails increasing the upshifted plasma line strength relative to the downshifted power by a factor of 2.1.) One observational result that is implicit in the data is that the upshifted plasma line power is always greater than the downshifted power from 1704-1729 AST, when $f_b E_s$ exceeded $f_{HF}$ by $\gtrsim 1$ MHz. On the other hand, during the time interval 1738-1742
AST, when \((f_b E_s - f_{HF})\) was \(< 0.3\) MHz, the downshifted plasma line was dominant. During the final minute of \(E_s\) observations (1742-1743 AST) the upshifted plasma line was the stronger line by \(\sim 6\) db.

6.6 SUMMARY

The experimental results of 1976 and 1977 indicate the presence of strong HF-induced plasma line enhancements in \(E_s\). On most occasions the plasma line enhancements were accompanied by readily detectable enhancements in the ion line power. The ion line enhancements are characteristic phenomena associated with the parametric instability (section 5.5).

Within the data, a correlation is established between the excitation of plasma lines and the relative values of \(f_{HF}\) and \(f_b E_s\). In order for excitation to occur, \(f_b E_s\) must be equal to or exceed \(f_{HF}\). During observational periods when \(f_b E_s\) was known to be below \(f_{HF}\) while \(fE_s\) was greater than \(f_{HF}\), no plasma lines were observed. With the exception of the June 21, 1976 observations, during which time the \(E_s\) was located near 100 km altitude, \(E_s\) plasma line enhancements were consistently observable whenever \(f_b E_s\) was greater than \(f_{HF}\).

Several interesting asymmetries between the upshifted and downshifted plasma lines are often noted. In terms of the strength of the scattered signal at the plasma lines, there are times when the upshifted line is dominant, while at other times the downshifted line clearly contains more power. The direction of the asymmetry appears to be associated with the value of \((f_b E_s - f_{HF})\). In Figure 6.27 the ratio
of upshifted plasma line power to downshifted plasma line power is plotted as a function of \((f_{bE_S} - f_{HF})\) for all of the data accumulated during the 1976 and 1977 experiments. From this figure it is evident that when \(f_{bE_S}\) exceeds \(f_{HF}\) by more than approximately 1 MHz, the upshifted plasma line is the stronger line. On the other hand, whenever \(f_{bE_S}\) is within 1 MHz of \(f_{HF}\) the reverse asymmetry is observed.

A second plasma line asymmetry, associated with the peak altitude of the observed plasma line enhancements, is intermittently present in the data. When the altitude asymmetry is observed, the downshifted plasma line peak always appears at greater altitudes than the corresponding upshifted peak. The largest observed separation in the peak altitudes is approximately 1 range gate, or 600 m. An instrumental origin for this asymmetry, however, cannot be completely ruled out in the absence of careful tests conducted on the receiver system (including the Barker decoder) at the time of the observation.

The variations in the enhancement levels of the plasma lines with time constitute an additional point of interest. On those occasions when one was assured that \(f_{bE_S}\) was above \(f_{HF}\), the \(E_S\) plasma line power often appeared to be relatively constant. Most of the large plasma line power variations were irregular in time and usually occurred over periods of 2 minutes or less. At least in certain instances, these fluctuations are attributable to variations in the detailed structure of the \(E_S\) at the altitude of excitation. There is no evidence in the data of large amplitude regular fading in the plasma line signal strength that one might associate with the formation of
field-aligned striations within the plasma. When plasma line fading is present, it is small in amplitude and tends to have a period of 5 min - 10 min (e.g. Figures 6.8 and 6.9).

All of the measured power spectra of the $E_S$ plasma lines contain a dominant narrow band peak at the growing mode ($430 \text{ MHz} \pm f_{HF}$). The width of the peak is generally estimated to be $< 1 \text{ kHz}$. Additional unresolved structure is typically present on the 430 MHz side of the peak located within $\sim 5 \text{ kHz}$ of the growing mode. On a few occasions secondary peaks were resolved at frequency displacements of $\sim 2.6 \text{ kHz}$ and $\sim 4.5 \text{ kHz}$. Finally, at those times when excitation occurred above the $E_S$ in the normal $E$ region between 130 km and 140 km altitude, only the decay mode was observed.
FIGURE 6.1 The two 430 MHz radar geometries used during the experimental observations. In the first, the radar beam was directed at 4° zenith angle and 353° azimuth, while in the second the beam coordinates were 5.54° zenith angle and 14.44° azimuth. The 430 MHz beam width at 110 km is approximately 320 m in diameter. The first Fresnel zones are illustrated for an ionosonde frequency of 4.6 MHz and are 3.8 km in diameter. Also shown in the figure are the HF half-power beam widths at frequencies of 4 MHz, 5 MHz and 6 MHz, which represent angular extents of 16°, 13° and 11° respectively.
FIGURE 6.7 Experimental profile data illustrating simultaneous HF-induced enhancements in parts of the F region and $E_s$. The topside plasma line at 359 km is photoelectron enhanced. The PL (+) S/N scale has been adjusted for antenna gain in order to make these intensities directly comparable with those of PL (-). The altitude resolution is 600 m. The displayed profiles were recorded during one 6 sec "HF on" subinterval from 155921 - 155927 (AST) on July 20, 1976. The letters A and B mark "HF off" ion line levels in $E_s$ that were measured 30 sec after and 30 sec before the plotted profiles, respectively. Thus, small ion line enhancements are indicated in the $E_s$ region, which is located at an altitude of 108 km.

The electron density scale, $N_e$, is only approximate since the profile has not been corrected for the $(1 + T_e/T_i)$ dependence on cross section. However, measured $T_e/T_i$ ratios (top scale) are represented at three altitudes by the bold dots. The $T_e/T_i$ values at 240 km and 359 km were derived from the altitudes of the observed plasma lines (Evans, 1969; Wand, 1970), while the point at 297 km was calibrated using ionosonde measurements of $f_0F_2$. The $N_e$ scale was set using $f_0E$ and assuming $T_e$ = $T_i$.
FIGURE 6.8
FIGURE 6.9
FIGURE 6.14
SPECTRUM OF INPUT TEST SIGNAL
CENTERED AT -10 kHz

\((\sin(x)/x)^2\)

POWER (db)

BASEBAND FREQUENCY OFFSET (kHz)

FIGURE 6.16
FIGURE 6.17  Upshifted plasma line spectra measured during consecutive 20 sec integration periods on June 3, 1977.
FIGURE 6.18  Downshifted plasma line spectra measured during consecutive 20 sec integration periods on June 3, 1977.
FIGURES 6.19 - 6.26 Representative power spectra of HF-enhanced sporadic-E plasma lines measured during the 1977 experiment. The observed altitude of the excitation is listed in each figure along with the transmitter frequency and power, and the time period of integration. The frequency resolution is determined by the sampling period of the scattered signal (section 2.3). Powers are expressed as signal-to-noise ratios, S/N, and are displayed on a logarithmic scale. The peak powers are labelled in each spectrum. The data points in the spectra recorded on June 3 through June 13 represent frequency cells that are 1.3 kHz wide. The June 14 data, obtained in cooperation with Robert L. Showen, were frequency analyzed at 2.23 kHz intervals. The spectral windowing function (section A.4) centered at zero displacement is depicted as a shaded region in each spectrum.
3 JUNE 1977
$f_{hf} = 5.185$ MHz  40 kW
2.00 kHz FREQUENCY RESOLUTION
110 km ALTITUDE

**FIGURE 6.19**
Figure 6.20

7 June 1977
134714 to 134726 (AST)
S/N = 237

109 km Altitude
1.85 kHz Frequency Resolution
f_{H} = 4.07 MHz
50 kW

9 June 1977
120220 to 120233 (AST)
S/N = 1494

114 km Altitude
1.75 kHz Frequency Resolution
f_{H} = 4.07 MHz
50 kW
9 JUNE 1977
\( f_{\text{hf}} = 4.07 \text{ MHz} \), 50 kW

115131 TO 115448 (AST)
1.75 kHz FREQUENCY RESOLUTION
114 km ALTITUDE

\( S/N = 2.3 \)

115500 TO 115511 (AST)
1.43 kHz FREQUENCY RESOLUTION
136 km ALTITUDE

\( S/N = 22 \)

FIGURE 6.21
13 JUNE 1977
\( f_{uf} = 4.07 \ \text{MHz} \ 63 \ \text{kW} \)
1.72 kHz FREQUENCY RESOLUTION
109 km ALTITUDE

\begin{align*}
\text{a} & & \text{S/N} = 34 & \quad 115823 \ \text{TO} \ 115831 \ (\text{AST}) \\
\text{b} & & \text{S/N} = 3.7 & \quad 120430 \ \text{TO} \ 120513 \ (\text{AST}) \\
\text{c} & & \text{S/N} = 5.9 & \quad 120637 \ \text{TO} \ 120725 \ (\text{AST}) \\
\text{S/N} & \quad \text{DISPLACEMENT FROM} & \quad \text{DISPLACEMENT FROM} & (430 - 4.07) \ \text{MHz} & (430 + 4.07) \ \text{MHz} \\
\text{DISPLACEMENT FROM} & \quad (\text{kHz}) & \quad (\text{kHz}) & \\
\end{align*}

FIGURE 6.22
13 JUNE 1977

\[ f_{\text{HF}} = 4.07 \text{ MHz} \quad 63 \text{ kW} \]

1.75 kHz FREQUENCY RESOLUTION
109 km ALTITUDE

**a**

\[ S/N = 41 \]

120919' TO 120927 (AST)

\[ S/N = 5.7 \]

**b**

\[ S/N = 60 \]

121222 TO 121230 (AST)

\[ S/N = 14 \]

FIGURE 6.23
13 JUNE 1977

\[ f_{HF} = 4.07 \text{ MHz} \quad 63 \text{ kW} \]

\begin{itemize}
  \item \textbf{a} \hspace{5cm} S/N = 11 \hspace{5cm} 124052 \text{ TO } 124113 \text{ (AST)}
  \hspace{5cm} \text{1.43 kHz FREQUENCY RESOLUTION}
  \hspace{5cm} \text{137 km ALTITUDE}
  \hspace{5cm} S/N = 5.4

  \item \textbf{b} \hspace{5cm} S/N = 10 \hspace{5cm} 114347 \text{ TO } 114352 \text{ (AST)}
  \hspace{5cm} \text{1.72 kHz FREQUENCY RESOLUTION}
  \hspace{5cm} \text{109 km ALTITUDE}
  \hspace{5cm} S/N = 3.1
\end{itemize}

**FIGURE 6.24**
14 JUNE 1977
2.23 kHz FREQUENCY RESOLUTION
110 km ALTITUDE

**a**

170410 TO 170425 (AST)

\[ f_{HF} = 5.185 \text{ MHz} \quad 50 \text{ kW} \]

S/N = 33

**b**

170647 TO 170702 (AST)

\[ f_{HF} = 5.185 \text{ MHz} \quad 50 \text{ kW} \]

S/N = 89

**c**

172639 TO 172655 (AST)

\[ f_{HF} = 5.185 \text{ MHz} \quad 100 \text{ kW} \]

S/N = 36

FIGURE 6.25
14 JUNE 1977
f_{HF} = 5.185 MHz  105 kW
2.23 kHz FREQUENCY RESOLUTION
110 km ALTITUDE

173819 TO 173833 (AST)
S/N = 462

174008 TO 174022 (AST)
S/N = 1413

174008 TO 174022 (AST)
S/N = 215

FIGURE 6.26
Figure 6.27  The ratio of antenna gain corrected upshifted plasma line power to downshifted plasma line power plotted as a function of $(f_{bE_s} - f_{HF})$ for the data accumulated during the 1976 and 1977 observations. Each point represents an integration in time of 3 to 6 min. For the solid points $f_{bE_s}$ was scaled from ionogram traces. When the F region was completely blanketed the value of $f_{bE_s}$ was set equal to the peak plasma frequency in the $E_s$ region as determined by the 430 MHz ion line profile. Data points determined in this manner are shown as open circles.
CHAPTER 7
DISCUSSION

7.1 THE HF ELECTRIC FIELD STRENGTH IN SPORADIC E

In order to gain a general idea of the HF pump strength in sporadic E ($E_s$), one may calculate the electric field strength assuming a horizontally stratified plasma. This of course is a rather large oversimplification if significant horizontal structure is present. Nevertheless, the stratified plasma approximation lends a degree of tractability to the electric field problem which allows comparisons with estimated thresholds to be made.

The analytical solutions to the wave equation in Chapter 4 indicate that evanescent tunneling through the layer is negligible whenever $f_{HF}$ is $\lesssim 95\%$ of the peak plasma frequency. When this condition is met the electric field for a linear layer resembles the Airy function sketched in Figure 4.1a. For representative $E_s$ values of $f_{HF} = 5.1$ MHz and $H \equiv z_r = 500$ m, the maximum swelling factor is $\sim 14$. An increase in the density gradient serves to compress the altitude scale of the Airy pattern since the normalized distance below HF reflection, $\zeta \propto (H)^{-1/3}$ (Eq.(4.1)).

Whenever the pump frequency is $\gtrsim 95\%$ of the peak plasma frequency of the layer, appreciable leakage of the radio wave through the
layer begins to occur. In this case the electric field pattern contains a travelling wave component, and the swelling factor, \( S \), is reduced in a fashion commensurate with the amount of leakage (Figures 4.1b and 4.1c).

The electric field intensity near reflection may be written as

\[
E^2 \left( \frac{V}{m} \right)^2 = 2 \, S \, Z \, P_{inc}
\]

where \( Z = \frac{1}{c \varepsilon_0} = 377 \) ohms, and \( P_{inc} \) is the power flux incident upon the sporadic \( E \). \( P_{inc} \) may be expressed in terms of the total transmitter power, \( P_{HF} \), as

\[
P_{inc} = \frac{P_{HF} G \varepsilon}{4 \pi r^2}
\]

where \( r \) is the range and the gain, \( G \), is given by

\[
G = \frac{4 \pi}{\lambda_{HF}^2} \left( \pi \frac{d^2}{4} \right).
\]

In the above expression for the gain, "d" is the diameter of the spherical reflector (305 m), and \( \varepsilon \) is the frequency dependent efficiency factor. For Arecibo \( \varepsilon \) ranges from 0.25 at 5 MHz to 0.50 at 10 MHz.

In Table 7.1 representative values of \( E^2 \), \( P_{inc} \), and \( S \) are presented for the HF frequencies, \( f_{HF} \), and powers used during the 1976 and 1977 \( E_s \) observations. In calculating \( P_{inc} \), the \( E_s \) was assumed to be at a range of 110 km, while the swelling factors were computed for \( H = 500 \) m. Also listed in the table are the half-power beam width of the HF beam, \( \theta_{HF} \), and the critical Spitze angle, \( \theta_s \).

It is important to remember that the electric field described above in terms of the Airy structure is for a very idealized density profile. If one introduces horizontal density irregularities into the
plasma with scale sizes of the order of or greater than the radio
wavelength in the medium, $\lambda_m$, then a considerable amount of refractive
focussing and defocussing of the HF beam would be expected. Since re-
fraction becomes important for O-mode propagation whenever the index of
refraction, $\mu$, is $\sim 0.3$ (Ginzburg, 1970), a representative value of $\lambda_m$
for $f_{HF} = 5$ MHz may be chosen near $\lambda_m \approx \lambda_{HF}/\mu \approx 3 \lambda_{HF} = 180$ m. Obser-
vationally, there is evidence that HF waves used in sounding experiments
conducted on $E_s$ undergo at least a limited amount of focussing within
the $E_s$ plasma. (Chessell et al., 1973). Furthermore, if irregularities
having scale sizes $\sim \lambda_m$ are present in $E_s$ then additional modifications
in the electric field structure might be anticipated as a result of
Fresnel diffraction and/or scattering.

In order to facilitate comparisons with calculated thresholds
in subsequent sections, the electric field swelling factor will, to
first approximation, be estimated by the stratified plasma result. In
addition, the direction of the electric field vector will be assumed to
be aligned with the earth's magnetic field. At all times, however, it
should be borne in mind that the magnitude of the HF electric field
may vary appreciably from place to place within the $E_s$ plasma and that
its direction need not be precisely aligned with the magnetic field.

7.2 LINEAR ABSORPTION

If linear absorption is sufficiently great, then the resulting
electron temperature increase may become large enough to invalidate the
assumption of thermal equilibrium, that is $T_e = T_i = T_n$. A rough
estimate of the increase in $T_e$ may be made using elementary energy balance arguments (see e.g. Ginzburg, 1970, Eq. 38.15). The change in electron temperature may be expressed as

$$\frac{3}{2} \propto n_e(T_e - T_i) = \frac{E^2 e_0 \nu_{eff}}{2} \propto \frac{E^2 e_0 \nu_{e}}{4}$$

where $E^2$ is the time averaged electric field intensity, $\tau$ is the relaxation time of the temperature increase in the absence of $E$, and

$$\nu_{eff} \propto \nu_e(T_e/T_i)^{1/2} \propto \nu_e$$

Based on the D and E-region measurements of Showen (1972), the relaxation time at 110 km may be estimated at $\sim 20$ msec. Using this value of $\tau$ along with $E^2 = 0.4(V/m)^2$, $\nu_e = 13$ kHz, $T_i = 275$°K and $n_e = 3 \cdot 10^{11} m^{-3}$, one obtains

$$\left(\frac{T_e}{T_i} - 1\right) = \frac{2.3 \times 10^{-10}(J/m^3)}{n_e \kappa T_i} = 0.13$$

Given the span of $\nu_{en}$ and $\tau$ values generally encountered in the 105 km - 115 km altitude range, $T_e/T_i$ may be expected to increase by between 10% and 20% near the point of reflection in $E_s$. Such increases do not significantly alter any of the previously derived results arrived at by assuming $T_e/T_i = 1$.

In addition to losses due to deviative absorption, the incident radio wave is attenuated as a result of nondeviative absorption in passing through the D region. The severity of the nondeviative absorption is proportional to the amount of D-region ionization present, which is
dependent upon, among other factors, the extent of solar and/or magnetic activity. Consequently, the exact amount of absorption is indeterminable without a direct absorption measurement. Nevertheless, nondeviative absorption is generally negligible at night and in the absence of solar disturbances obeys an \( \sim (\cos \chi)^{0.7} \) dependence on the solar zenith angle, \( \chi \), during the daytime (Al'pert, 1973). Even during magnetically quiet times, however, the maximum daytime absorption may undergo day to day variations of a factor of two or more (Wratt, 1977). An "average" one-way absorption coefficient may be estimated using the empirical formulation of Davies (1965). At a frequency of 5 MHz the computed daytime absorption maxima are 2.9 db and 3.3 db for the 1976 and 1977 experiments, respectively. In calculating these absorption coefficients, the average sunspot numbers during the 1976 and 1977 observing periods were taken to be 0 and 34 respectively.

7.3 LANGMUIR WAVE RAY TRACING

In detailing the observing geometry within the \( E_s \) plasma, it is important to consider the propagation paths of the Langmuir waves which may, for example, be generated in a parametric interaction. In this regard, it is instructive to examine the Langmuir ray paths computed by Muldrew (1978b) for \( E_s \) conditions over Arecibo. The ray paths were calculated using Snell's law in conjunction with the dispersion relationship for plasma waves in a warm magnetoactive plasma. The dispersion relationship may be written
\[ \omega^2 = \omega_e^2 + \omega_{ec}^2 \sin^2 \beta + 3 \kappa^2 \frac{T_e}{m_e} \] (7.1)

where \( \beta \) is the angle between the wave vector, \( \vec{k} \), and the magnetic field. Collisional damping is included in the calculation by setting \( \omega = \omega_r + i\nu_e \) where \( \omega_r \) is the real part of the frequency term. During the actual ray tracing, \( \omega_r \) was fixed at \( \omega_r = \omega_{HF} \) while \( \nu_e \) was set equal to \( 8 \cdot 10^3 \text{ sec}^{-1} \).

Figure 7.1a illustrates a ray path calculated by Muldrew (1978b) (Muldrew's Figure 1) for an angle, \( \phi \), between the constant density gradient, \( \nabla n_e \), and the magnetic field, \( \vec{B} \), of 80°. The continuous line indicates the group, or ray, path while the short lines extending from this path denote the relative magnitude of the plasma wave normal, \( \vec{k} \), as well as its direction. Arrows along the ray path show the direction of group propagation. The plasma wave normals are given at 0.1 msec intervals as determined from the propagation time at the group velocity.

The ray tracing depicted in Figure 7.1a was carried out by setting the magnitude of the wave vector at the origin, \( \vec{k}_o \), equal to 18 m\(^{-1}\) while orienting it 4° off vertical in the magnetic meridian plane. This is the proper observing geometry for the detection of plasma waves over Arecibo using the 430 MHz incoherent scatter radar. Plasma wave normals pointed downwards at the origin, as in Figure 7.1a, would give rise to upshifted plasma lines in the data while upward pointed normals would lead to downshifted plasma lines. The ray paths for upshifted and downshifted plasma lines are completely symmetric. Thus, the ray path appropriate for a downshifted plasma line may be
obtained from one designated for an upshifted line simply by reversing
the ray direction and rotating the wave normals by 180°.

Figures 7.1b and 7.1c illustrate two more examples of ray paths
computed by Muldrew for \( \phi = 20° \) and \( \phi = -20° \). The Langmuir ray paths
shown in Figure 7.1 will prove to be a useful aid in subsequent dis-
cussions of the OTSI thresholds and growth rates relevant to the \( E_s \)
observations. The specific conditions for which the ray paths have
been calculated are \( T_e = 269°K, H = 500 \text{ m}, f_{ec} = 1.11 \text{ MHz}, \text{ and } f_{HF} = 5.7 \text{ MHz}. \) If the value of \( H \) were chosen to be less than (greater than)
500 m, then the shapes of the ray paths would be preserved. However,
the times and distances would be decreased (increased) directly in
proportion to \( H \).

7.4 PLASMA WAVE GENERATION

In Chapter 5 linear conversion of electromagnetic waves into
plasma waves near HF reflection was discussed in relationship to radio
wave propagation at oblique angles in an isotropic inhomogeneous plasma.
The applicability of this process is plasma wave generation in \( E_s \)
plasmas will now be discussed within the context of the present obser-
vations.

In the absence of an external magnetic field, the electric field
structure above the "reflection height" of an HF wave at oblique inci-
dence is evanescent at first, but subsequently approaches a singular,
or critical point \( z_c \), at \( \mu = 0 \), that is at \( \omega_e(z_c) = \omega_{HF} \). At the criti-
cal point the electric field component parallel to the gradient undergoes
a rapid peaking. If one neglects magnetic field effects, then the maximum electric field peaking occurs when the angle of incidence of the HF wave with respect to the gradient is 12°. In calculating this angle, we have used parameter values of $f_{HF} = 5$ MHz and $H = 500$ m in conjunction with Figure 20.3 of Ginzburg (1970). Although the peak electric field at $z_c$ may be substantial in an isotropic plasma (Ginzburg's Eq. 20.21), the actual increase in an ionospheric plasma is considerably reduced because of the presence of the earth's magnetic field (Ginzburg's Eq. 27.25). It is, however, difficult to solve for the electric field near $z_c$ in the presence of the earth's magnetic field. Nevertheless, even without an exact solution to the problem, it may be shown that only an X-mode wave has a resonance in the ionospheric plasma. Thus, in order for linear conversion to proceed, the transmitted O-mode wave must first couple into an X-mode wave which then must propagate to an X-mode singularity point (Stenzel et al., 1974b). This process by itself is not expected to be very efficient, although plasma turbulence generated by parametric interactions near O-mode reflection may enhance the linear coupling somewhat (Stenzel et al., 1974b; Fejer, 1979).

Even if a significant electric field did exist at a critical point, there are physical reasons why its importance to plasma wave generation in the present experiment should be minimal. If, for example, plasma waves were produced via linear conversion near $\omega_e(z_c) = \omega_{HF}$, then the plasma wave vectors at $z_c$ must be small in order to be detectable in the present experiment. After substituting $E_s$
parameters of $H = 500 \text{ m}$ and $v_e = 8000 \text{ sec}^{-1}$ into Eq. (7.1), one finds that, near $z_c$, $k_r < k_i$, where $k_r$ is the real part of the plasma wave vector ($k_r = 1.26 \text{ m}^{-1}$ for Arecibo) and $k_i$ is the imaginary component (see Muldrew, 1978b). Thus, plasma waves are heavily damped as they propagate away from the critical point down to the observation altitude. Under these conditions, plasma waves at the observed intensities ($\approx e^{10}$ above thermal levels) are unlikely to survive the attenuation in transit to the point of detection.

Observationally, linear conversion would preferentially give rise to upshifted plasma lines rather than downshifted lines. The plasma line power asymmetry, $PL(+) > PL(-)$, which is observed at times in the data, is thus consistent with the presence of a linear conversion process. Nevertheless, as will be shown in section 7.7, this is not the only possible mechanism that may be responsible for such an asymmetry. There is, at the same time, experimental evidence in support of the fact that linear conversion alone cannot be the only physical mechanism responsible for the observed $E_s$ plasma line enhancements. A linear conversion process would produce only plasma line enhancements. However, ion line enhancements often accompany the plasma line enhancements in the data. As discussed in section 5.5, observable ion line enhancements are expected whenever the parametric instability is very near or above threshold.

Parametric processes may occur either below the point of HF reflection or at a resonant singularity near $\omega_e = \omega_{HF}$. Parametric excitation at the resonant singularity results whenever the transformed
electrostatic waves moving parallel to the density gradient have sufficient amplitudes to drive either the decay instability or OTSI (Liu, 1976). However, it is difficult to drive the parametric instabilities along the density gradient because of convective losses, while plasma wave generation at angles nearly perpendicular to the electric field is subject to high instability thresholds and slow growth rates. On the basis of threshold considerations alone, a parametric process driven directly by the HF wave below reflection would appear to be a more likely mechanism for plasma wave and ion wave generation. Moreover, many of the arguments presented above that weigh against a linear conversion process in $E_s$ apply equally well to parametric excitation at $z_c$. Consequently, we shall restrict attention to parametric excitation below the point of HF reflection by the transmitted O-mode wave.

7.5 THRESHOLD CONSIDERATIONS

In Chapter 5 a comparative study of the expected decay mode and OTSI thresholds in a model $E_s$ plasma was made. The results of this study indicated that the OTSI and decay instability thresholds for a homogeneous plasma are approximately equal at altitudes near 110 km where most of the $E_s$ plasma line enhancements were observed to occur. At lower altitudes the decay threshold becomes larger due to the increased importance of electron-neutral collisions. In an inhomogeneous plasma such as $E_s$ large gradients were shown to limit the decay instability more severely than the OTSI. In all, the OTSI was predicted to be the dominant instability if electric field strengths above threshold
are indeed attained in $E_s$. The predicted dominance of the OTSI is in general borne out by the spectra presented in the preceding chapter. As a result, the OTSI will be discussed below to the exclusion of the decay instability.

The homogeneous plasma threshold for the OTSI is directly proportional to $T_e$ and $v_e$ (Table 5.1). Both $T_e$ and $v_e$, however, may undergo rather large diurnal and semidiurnal variations in the E region as a result of upper atmospheric tides. Consequently, barring any direct measurements of these parameters at the time of the observation, thresholds may not be precisely determined. Indeed, it was just this uncertainty in E-region parameters that necessitated the construction of possible parameter ranges in Figure 3.3.

In order to determine whether it is reasonable to believe that the OTSI is above threshold in $E_s$, one might begin by comparing the experimental electric field values in Table 7.1 with the OTSI threshold for a homogeneous plasma, $E_0 = 0.42 \, V^2/m^2$, given in Eq. (5.22). (The electric field strengths should be viewed as upper limits for a stratified plasma since D-region absorption has not up to this point been taken into account.) Since the observation altitude is located 10 m - 15 m down from the point of HF reflection (section 5.8), electric field strengths near the observation point are expected to be midway between those at $\zeta = 0$ and $\zeta_m(H) = 36 \, m \, (H = 500 \, m)$, where $\zeta_m$ is the location of the first Airy maximum. Thus, Table 7.1 indicates that near the Airy maximum the electric field power estimates for a stratified $E_s$ plasma are for the most part within a factor of two of the minimum OTSI threshold.
In determining the $E_o^2$ and $E^2$ values appropriate for a specific sporadic-E observation one should of course use parameter values consistent with the experimental conditions. In Table 7.2 the minimum OTSI threshold for a homogeneous plasma is listed for $E_s$ altitudes representative of individual measurements made during the 1976 and 1977 experiments. $E_o^2$ was determined using $v_{en}$ values at the lower boundary of the shaded region in Figure 3.3c and mean $T_e$ values taken from Figure 3.3a. Also listed are the estimated electric field strengths, $E^2(\zeta)$, for the experimental HF powers and frequencies. The deduced values of $E^2$ include a correction for D-region absorption which was made using calculated attenuation coefficients (section 7.2) and by assuming a $\cos(\chi)^{0.7}$ dependence of these coefficients on solar zenith angle, $\chi$. For observations in which the altitude of the plasma line excitation varied significantly, threshold and electric field values are given for the widest range of plasma parameters characterizing the measurement.

From Table 7.2 it is evident that all of the $E_s$ electric field estimates for a stratified plasma are below the minimum OTSI threshold. However, during most of the observations (rows 1, 2, 4, 5, 8-12, 16 and 17) the computed electric fields are within a factor of 2 or 3 of threshold in at least a portion of the region between the Airy maximum ($\zeta = \zeta_m$) and the point of reflection ($\zeta = 0$). When comparing $E^2$ with the estimated thresholds, it should be remembered that there are rather large uncertainties attached to some of the parameters that enter into the calculation of $E_o^2$. If, for example, the $v_{en}$ values lying along the dashed line in Figure 3.3c (section 3.3) are more nearly correct, then
the threshold estimates would be lowered by $\sim 30\%$. In addition, the HF electric field strengths in $E_s$ are always somewhat uncertain since the amount of D-region absorption during the daytime (nominally $\sim 3 \text{ db}$) is not precisely known. Nevertheless, the uncertainties involved in the above electric field and threshold estimates are generally not large enough to account for a factor of 2-3 difference in the calculated values of $E^2$ and $E_0^2$.

The $E^2$ estimates in Table 7.2 assume a horizontally stratified model of the $E_s$ plasma. However, as noted in Chapter 3, a patchy plasma containing horizontal density irregularities provides a more accurate description of $E_s$. On the basis of experimental observations of the amplitude of radio wave reflections from $E_s$ and the F region, Chessell et al. (1973) conclude that on the whole the concept of an $E_s$ layer horizontally stratified and embedded in the E region does not apply. This conclusion was prompted by the rapid fading rate of the $E_s$ and F-region echoes observed over time scales of 1.5 sec - 2.0 sec (the time resolution of Chessell's experiment) as well as the large oscillations in the energy of the reflected signal recorded as a function of frequency. Chessell et al. point out that the fading is interpretable in terms of radio wave focusing by plasma patches within the $E_s$ region, while the amplitude variations with frequency may be attributed to Fresnel diffraction of radio waves propagating through regions of patchiness.

If diffraction and/or refractive focusing (and defocusing) of the incident HF wave takes place in $E_s$, then one would expect the HF field strengths at certain locations within the $E_s$ plasma to exceed the
values calculated for a stratified plasma. Although the exact amount of increase is not determinable without a detailed knowledge of the $E_s$ structure, a factor of $\approx 3$ increase is not excessive if irregularities of order $\lesssim 10\%$ are present (see e.g. Singleton, 1964). Thus, while many of the HF electric field strengths calculated for a stratified plasma are a factor of 2 to 3 below the OTSI threshold, it is not unlikely that fields above threshold would at times be present in a patchy $E_s$ plasma.

Prior to 1500 AST during the July 20, 1976 observation, Table 7.2 (rows 6 and 7) indicates that a factor of 4 to 5 increase in $E^2$ is needed to bring the HF electric field above threshold. While the PL (+) line was strongly excited on this occasion (Figure 6.8), the PL (-) line was barely detectable. On July 21, 1976 (row 3) no plasma line enhancements were observed in the data. In this case $E^2$ is at least a factor of 5 below $E_0^2$. If radio wave focusing within $E_s$ is a valid concept, then the July 21 observation may be used to set an upper limit on the electric field increase that might be anticipated as a result of focusing.

During the observations of June 7, June 9 and June 13, 1977, large plasma line enhancements appeared only intermittently in the data. At the same time $f_b E_s$ hovered near $f_{HF}$. On these three occasions, one is inclined to believe that periods of sudden intensification in plasma line power represent times when abnormally large pump fields happened to be present in the radar observing volume. The large electric field strengths could have come about as a result of the focusing of the HF beam. In addition, the electric field swelling, $S$, might have occasionally increased due to increases in $H$. Values of $H$ greater than 500 m
are expected near the peak of an $E_s$ region or in the vicinity of a large scale vertical irregularity.

In summary, one might conclude that during most of the 1976 and 1977 observations discussed in Chapter 6, it is not unlikely that the OTSI threshold was exceeded in portions of the $E_s$ plasma. However, it is unlikely that the threshold was ever greatly exceeded.

In the process of examining the threshold conditions of the parametric instability in $E_s$, it is useful to do a comparative study of the relative plasma line and ion line enhancement levels in both the pumped and unpumped plasma. In this regard, theoretical predictions are readily obtainable for the case in which only ion Cerenkov emission is important, that is, in the situation where the plasma is pumped but linearly stabilized. Perkins et al. (1974) show that the amplitude of the electron density fluctuations, $n_{k_2}$, in a linearly stabilized plasma is larger than the amplitude of the ion acoustic fluctuations, $n_{k_1}$, in the unpumped plasma by a factor of

\[
\left| \frac{n_{k_2}}{n_{k_1}} \right|^2 = \frac{E^2}{4\epsilon_0 n_e T_e} \left( \frac{\omega_p}{v_e} \right) \left( \frac{2 \lambda_D}{k_2} \right) \left( \frac{m_i}{m_e} \right)^{1/2}
\]

\[
= \frac{E^2}{E_0^2} \left( \frac{k_1}{k_2} \right) \left( \frac{m_i}{m_e} \right)^{1/2} k_1 \lambda_D \tag{7.2}
\]

where $E_0$ is the minimum homogeneous plasma threshold, and $k_1$ and $k_2$ are the ion and electron plasma wave numbers respectively. In order to compute an upper limit for this ratio in a subthreshold plasma, the pump electric field, $E$, may be set equal to $E_0$ in the above equation.
In addition, if we put \( k_1 = k_2 \), and set \( m_i = 56 \text{ amu} \) and \( k_1 \lambda_D = 0.04 \), the result \( |n_{k_2}|^2 / |n_{k_1}|^2 = 13 \) is obtained.

In order to compare the above estimate of \( |n_{k_2}|^2 / |n_{k_1}|^2 \) with typical experimental values based upon the observations of Chapter 6, we define

\[
S_{\text{ion}} = G(0) \sigma_{\text{ion}} \Delta h
\]

\[
S_{\text{pl}} = G(f_{\text{HF}}) \sigma_{\text{pl}}^* \Delta h^*
\]

where \( S \) is the observed signal power at the ion line or plasma line, \( G \) is the 430 MHz radar gain, \( \sigma \propto |n_k|^2 \) is the scattering cross section, and \( \Delta h \) is the width of the altitude region over which density fluctuations are detected (Kantor, 1972). A * is used to denote the quantities associated with HF enhancements. For representative experimental parameter values in \( E_s \) of \( S_{\text{ion}} = 30 \text{ S/N} \), \( S_{\text{pl}} = 10 \text{ S/N} \), \( \Delta h = 600 \text{ m} \), \( \Delta h^* = 10 \text{ m} \), one has

\[
\frac{\sigma_{\text{pl}}^*}{\sigma_{\text{ion}}} = \frac{S_{\text{pl}}}{S_{\text{ion}}} \frac{\Delta h}{\Delta h^*} \frac{G(0)}{G(f_{\text{HF}})} = 360
\]

which is much greater than the estimated value of 13. One would expect the ratio of density fluctuations calculated using Eq.(7.2) to be exceeded whenever the parametric instability is above threshold since all of the nonlinear effects related to the instability are introduced by the electron fluid (see e.g. Perkins et al., 1974). The ion motion on the other hand is essentially a highly damped low frequency response to the electron fluctuations.
In addition to the above arguments, there is another, more compelling reason for believing that the parametric instability is above threshold in $E_s$. This latter argument is based upon the fact that observable (HF-induced) ion line enhancements are readily apparent in the ion line profile data presented in Chapter 6. As noted in section 5.5 such enhancements are expected whenever the parametric instability is very close to or above threshold (Hagfors and Gieraltowski, 1972; Perkins et al., 1974). Despite the fact that the parametric instabilities, which are excited locally in the $E_s$ plasma, occur over only a very narrow ($<< 600$ m) altitude region, ion line enhancements of a factor of 2 to 5 are commonly observed in the data. This, taken together with the measured $E_s$ spectra described in section 6.3.4, supports the idea that the OTSI is above threshold in $E_s$.

7.6 TIME VARIABILITY OF THE ENHANCED PLASMA LINE

To a certain extent the intensity of the observed $E_s$ plasma line excitation reflects the degree to which the minimum threshold is exceeded in the plasma. (An additional factor related to the geometry of plasma wave propagation will be discussed in section 7.8.) Thus, variations in the plasma line intensity during observation periods when $f_b E_s$ is consistently above $f_{HF}$ may at least in part be attributed to changes in $E^2$, $E_0^2$, or both.

One might attempt to associate changes in the enhanced plasma line intensity with changes in the altitude of excitation since $E_0$ is directly proportional to $\nu_{en}$, and $\nu_{en}$ is a quickly decreasing function
of altitude (Figure 3.3c). There are in fact some indications in the July 25, 1976 data that the plasma line enhancement level on the average decreases with decreasing excitation altitude, or equivalently, with increasing threshold. This is illustrated by the gradual decline in the plasma line power over the time interval 0836-0952 AST in Figure 6.14. When observations are resumed at 1041 AST (Figure 6.15) the higher $E_s$ excitation altitudes are accompanied by significant increases in the level of plasma line enhancement. The above association between changes in plasma line power and changes in the altitude of the enhancements is not consistently observed in all of the data. This may, however, simply be a reflection of the fact that the level of HF enhancements is at times more strongly influenced by variations in the pump electric field power than by changes in the threshold.

As noted earlier, fluctuations in the enhanced ion line and plasma line levels could come about as a result of variations in electric field strength due to the presence of irregularities. In order for the HF ray paths to be altered near reflection, the scale size of an irregularity must be of the order of or greater than the HF wavelength in the medium, that is, $\gtrsim 180$ m for $f_{HF} = 5$ MHz (section 7.1). If one assumes that the $E_s$ moves at neutral wind velocities near 50 m/sec, then these irregularities would produce power fluctuations in the plasma lines over time scales $\gtrsim 4$ sec, which is close to the 4 sec - 6 sec time resolution of the 1976 data.

The data in Figures 6.3, 6.8, 6.9, 6.14 and 6.15 provide several examples of times when plasma line enhancements were continuously present
for periods \( \gtrapprox 15 \) min. It is interesting to note that during periods of continual plasma line enhancement (i.e. times when one is assured that the frequency matching conditions in Eq.(5.30) were continuously met in the observed plasma), the signal strengths of the enhanced plasma lines fade in and out over time scales that typically range from 5 min - 10 min. These short term fluctuations are often superimposed upon longer term fading of 30 min or more (e.g. Figure 6.8). Only occasionally do sudden intensifications in the level of excitation appear in the data (e.g. July 17, 1976 at 0632 and 0647 AST on PL (-); July 25, 1976 at 0917 AST). (The abrupt drop in the plasma line intensity at 1055 AST on July 25, 1976 is discussed in section 6.3.2(C)). At the times when the plasma line power suddenly intensifies for a short period of time one is inclined to believe that an irregularity having a size of at most a few kilometers along the \( E_s \) drift direction passed through the radar field of view. Such an irregularity might lead to either a large increase in the \( E_s \) pump field as a result of focusing effects or to an exceptionally good viewing geometry for the parametrically excited plasma waves. On the other hand, if variations in the enhancement level over time scales \( \gtrapprox 5 \) min are interpreted in terms of changes in the pump field and/or viewing geometry due to irregular structure within the \( E_s \), then for \( E_s \) drift velocities of \( \sim 50 \) m/sec the irregularities must have scale sizes \( \gtrapprox 30 \) km.

Although variations in the electric field strength may account for some of the plasma line power fluctuations during periods of continual plasma line enhancements, abrupt plummets and rises in the plasma line power from the noise level are almost certainly associated
with the dipping of $f_{HF}$ above and below $f_{D}E_{S}$ (section 6.3.1(A)). On the basis of frequency matching conditions required for parametric excitation (Eq.(5.30)), it is clear that the OTSI condition $\omega_{HF} = \omega_{r} - \nu_{e}/2$ must be precisely satisfied within the $E_{S}$ plasma if parametrically enhanced plasma waves are to be present in the radar observing volume. If OTSI enhancements are to be continuously visible, then the necessary frequency matching condition must be met at all times in at least part of the $\sim 320$ m circular viewing area of the radar. In this regard it is interesting to note that large plasma line power increases accompanied by subsequent fading to the noise level are not very commonplace in the data over time scales $\lesssim 1$ min. For $E_{S}$ drift velocities near 50 m/sec the implication is that the scale sizes of irregularities in $E_{S}$ that affect parametric excitation the most are $>> 300$ m or unresolved and $<< 300$ m.

The plasma line enhancements observed during the July 17, 1976 measurements (Figure 6.3), particularly those between 0520 and 0630 AST, provide a good illustration of the passage overhead of plasma patches that are overdense at $f_{HF}$ and have dimensions $>> 300$ m. The electron density profiles measured by incoherent scatter during the July 17 observation support the idea that the periods of plasma line enhancement represent those occasions on which the peak $E_{S}$ plasma frequency exceeds $f_{HF}$ (section 6.3.2). In addition, the large fluctuations in the PL (+) and PL (-) powers at the beginning of the July 25, 1976 observation (1041-1045 AST in Figure 6.15) may also be the result of overdense plasma regions passing through the radar beam.
One possible \( E_s \) model described in Chapter 3 associates \( f_b E_s \) values with the smallest plasma frequency within an observational beam width defined by the first Fresnel zone in the F region. This model is consistent with the observed correlation between \( E_s \) excitation and \( f_{HF} \leq f_b E_s \). In addition, the above model is often extended to include a description of the non-blanketing \( E_s \) ionogram trace, which is explained in terms of HF reflection from sparsely distributed overdense patches lying within the ionosonde field of view.

There is a limited amount of observation time contained within the 1976 and 1977 experimental data wherein one is confident that \( f_b E_s \) was below \( f_{HF} \) during the observation period while \( f_E \) was above \( f_{HF} \). No plasma line enhancements were apparent within the available data. The detection of such enhancements would serve to verify the above model since total reflection within an \( E_s \) patch is necessary in order to create electric field strengths large enough for parametric excitation. No conclusion may be drawn from the failure to detect such patches since the radar beam width is much smaller than the HF beam width of the ionosonde. Because of the differences in beam widths, it is possible that plasma wave enhancements occurred in isolated patches but that these patches simply did not pass through the beam of the diagnostic radar.

7.7 PROPAGATION OF OTSI PLASMA WAVES

Having discussed the OTSI in the homogeneous plasma approximation, we shall now focus attention upon those elements of the instability
process that are dependent upon the inhomogeneous nature of $E_s$. Notwithstanding any limits that may be placed upon the application of the homogeneous approximation to the experimental observations, it is clear that by including the effects of plasma wave convection, one may only increase the instability threshold (section 5.7.3). Thus, the thresholds presented in Table 7.2 should be viewed as minimal thresholds since they do not take into account any of the geometrical constraints necessary in order to insure the detectability of the instability generated plasma waves. Furthermore, since the calculated electric field strengths are at best close to the homogeneous plasma thresholds, the effects of plasma wave convection must be carefully introduced into any model of $E_s$ excitation so as to avoid large increases in the instability threshold.

The OTSI thresholds for an inhomogeneous plasma discussed in section 5.7.3 were derived assuming that the pump field, $\vec{E}$, was perpendicular to the density gradient. If $\vec{E}$ is taken to be parallel to $\vec{B}$ and a vertical density gradient is assumed, then it is clear that the geometry employed for the derived threshold is not the same as that in a stratified sporadic E-plasma over Arecibo. Nevertheless, Eq.(5.23) may be used to gain some insight into increases in the minimum homogeneous plasma threshold due to plasma wave convection.

Let us assume that $E_s$ consists of a stratified plasma containing a vertical density gradient. Using Eq.(5.23) one may express the threshold increase, $E_+^2$, due to the gradient as $E_+^2 = 27/(\cos\theta \circledast H(m)) \ (V/m)^2$. Care must be exercised in choosing $\theta$, since $\vec{E}$ is not perpendicular to the gradient. However, regardless of whether or not $\vec{E}$ has
a component parallel to the gradient, it is most important to consider the rate at which $\mathbf{\mathbf{\hat{k}}}$ rotates away from $\mathbf{\mathbf{\hat{E}}}$ and thereby decouples from the pump field. This rate is determined primarily by the component of $\mathbf{\mathbf{\hat{k}}}$ perpendicular to the gradient. Hence, for Arecibo, where the viewing angle with respect to a vertical gradient is $4^\circ$, we choose $\theta^\circ = 90^\circ - 4^\circ = 86^\circ$. Assuming $H = 500$ m one obtains $E_+^2 = 0.8 \ (V/m)^2$.

In light of the estimated homogeneous plasma thresholds and electric field strengths given in Table 7.2, this is a substantial increase in the instability threshold. If $0.8 \ (V/m)^2$ must be added to the homogeneous threshold values then it is difficult to see how the minimum threshold for observable nonlinear plasma waves is ever exceeded in $E_s$. Thus, the threshold increase is large enough to lead one to examine $E_s$ models other than a horizontally stratified plasma in order to determine whether there might be other geometries more conducive to the development of the OTSI. Prior to doing this, however, one must adopt a more versatile means of estimating the OTSI threshold in an inhomogeneous plasma.

In a physical derivation of the OTSI threshold, Perkins and Flick (1971) indicated that the threshold condition might be generalized to

\[
\frac{\varepsilon_0 E_0^2}{n e kT_e} = 4 \left( 1 + \frac{T_i}{T_e} \right) \left( \frac{\gamma}{\omega_0} \right)
\]

where $\gamma = ν_e + τ^{-1}$ and $τ$ is the time required for plasma wave packets to traverse the instability region. The value of $τ$ may be computed using

\[
τ = 2 \int_0^{X_0} \frac{dx}{v_{gx}}
\]
where \( x_0 \) is the lower boundary of the instability region and \( V_{gx} \) is the group velocity parallel to the electron density gradient \( \nabla n_e \). In order to arrive at the inhomogeneous threshold in Eq. (5.23) the distance \( x_0 \) is fixed at the height at which the component of \( \mathbf{k} \) parallel to the gradient equals the invariant component, \( k_{\parallel} \), perpendicular to the gradient (i.e. \( \mathbf{k} \) is rotated 45° away from \( \mathbf{E} \)).

We shall now examine the OTSI for the ray paths shown in Figure 7.1 using the threshold formulation in Eq. (7.3). As noted above, if one assumes that the density gradients in \( E_s \) have a distribution of horizontal components, then the inhomogeneous threshold given in Eq. (5.23) is not directly applicable since \( \mathbf{E} \) will not in general be perpendicular to \( \nabla n_e \). In addition, the introduction of an external magnetic field complicates the ray paths and alters the development of wave normals. As a result, the threshold determination is somewhat dependent upon the detailed ray geometries.

Figure 7.1a illustrates the Langmuir ray path for the case where \( \mathbf{E} \) (assumed parallel to \( \mathbf{B} \)) is almost perpendicular to \( \nabla n_e \). In order to determine the relative importance of convective losses within this geometry, one may begin by estimating \( \tau \) for an instability region having an upper boundary near the reflection point of the Langmuir waves. In describing the manner in which this estimate is obtained, exact locations along the ray path will be specified in terms of the propagation time, \( t \), along the ray path measured from the origin. A "+" ("-") sign will be used to indicate distances measured parallel (antiparallel) to the ray direction. In Figure 7.1a, for example, plasma waves are reflected at \( t = -0.35 \) msec.
In order to estimate $x_0$ for use in Eq. (7.4) let us require that the $\vec{k}$ direction be within $45^\circ$ of $\vec{B}$ in order that coupling to the electric field may be maintained. The largest acceptable angle is of course dependent upon the actual strength of $\vec{E}$. At $45^\circ$ the homogeneous plasma threshold, which is a measure of the coupling efficiency, is $E_0^2 / \cos^2(45^\circ) = 2 E_0^2$. The portion of the ray path in Figure 7.1a to be integrated therefore extends roughly from $t = -0.6$ msec to $t = -0.1$ msec, or approximately $x_0 = 8$ m down from reflection. Within this altitude region one has $v_{gX} \approx v_g = 0.4 v_e = 26$ km/sec, where $v_e = (2kT/e/m_e)^{1/2}$. Substituting $v_e$ into Eq. (7.4) leads to the result $\tau = 6 \cdot 10^{-4}$ sec or $1/\tau = 1.6 \cdot 10^3$ sec$^{-1}$. In order to assess the comparative effects of convective losses and collisions on the threshold, one should compare $1/\tau$ with $v_e$. Assuming $v_e \approx 1.0 \cdot 10^4$ sec$^{-1}$, one has $v_e \gg 1/\tau$. Hence, the collisional term in Eq. (7.3) is more important in determining the $E_s$ threshold.

Outside of the region of Langmuir reflection in Figure 7.1a, plasma wave normals cannot be effectively coupled to $\vec{E}$. This follows since at large distances away from reflection, Snell's law requires that $\vec{k}$ be parallel to $\vec{v}_e$ and, as a result, $\vec{k}$ is almost orthogonal to $\vec{E}$ ($\phi = 80^\circ$). Thus, the ray path sketched in Figure 7.1a primarily gives rise to downcoming plasma waves at the origin, which would lead to upshifted plasma lines in the data. The amount of attenuation that these plasma waves undergo in reaching the observation point is dependent upon the extent of the instability region. If this region extends down to $t = -0.1$ msec then the plasma waves will be attenuated 4 db,
while if the instability is confined to a narrow region near $t = -0.35$ msec, the collisional losses would be 12 db (Muldrew, 1978b).

Figure 7.1b illustrates the case where the gradient is $20^\circ$ off vertical. In the immediate vicinity of the origin (e.g. $|t| \leq 0.5$ msec), this geometry would preferentially lead to upgoing plasma waves if the coupling of $\mathbf{k}$ at $45^\circ$ to $\mathbf{E}$ were sufficient to drive the OTSI. The instability should become easier to drive as $t$ increases beyond 0.0 msec since the direction of $\mathbf{k}$ approaches that of $\nabla n_e$ leading to an angle between $\mathbf{k}$ and $\mathbf{E}$ of $20^\circ$. In this region collisions should once again dominate the threshold determination since $1/\tau = 1/0.5$ msec = 2000 sec$^{-1} \ll v_e$. The observation of large parametric enhancements in the upgoing plasma wave intensity is therefore dependent upon whether non-linear wave intensities can be driven at $20^\circ$ with respect to $\mathbf{E}$.

In addition to the region near $t = 0.0$ msec, the OTSI might also be driven whenever $t \lesssim -0.7$ msec. If one assumes that a plasma wave packet takes at least 0.5 msec to traverse this instability region, then collisions will once again dominate the threshold. An instability driven in this region would give rise only to upshifted plasma lines. However, in traversing the distance from $t = -0.7$ msec to $t = 0.0$ msec the plasma waves would be attenuated by $\sim 30$ db (Muldrew, 1978b).

Finally, Figure 7.1c depicts the situation where $\mathbf{E}$ is nearly parallel to $\nabla n_e$. Within such a geometry, the instability region extends from $t = 0.0$ msec downwards in altitude in the direction of negative $t$. As a result, the instability generated plasma waves would propagate to the origin travelling in the upgoing direction. Thus, such a geometry would produce primarily downshifted plasma lines.
In summary, we note that if density irregularities are introduced into the $E_s$ plasma, then the associated horizontal density gradients perform the important function of creating plasma wave geometries that are favorable for the detection of nonlinear plasma waves. When the proper ray paths are obtained, the contribution of convective losses to the OTSI threshold may be reduced to the point where the threshold is given by the homogeneous plasma result.

7.8 SPORADIC-E MODELS

Given the Langmuir ray geometries described in the preceding section, one may formulate theoretical models to account for the observed plasma wave enhancements in $E_s$. In this regard, a viable model would be expected to explain, among other features in the data, the observed plasma line power asymmetries.

In one possible model one might require that excitation occur only when the plasma wave normals are very closely aligned with $\hat{E}$, or alternatively, with $\hat{B}$. This model, which will be denoted as Model A, has been proposed by Muldrew (1978b). In Model A, geometries with $180^\circ > \phi > 0^\circ$ would preferentially create upshifted plasma lines (Figures 7.1a, 7.1b), while $-180^\circ < \phi < 0$ geometries would be required for the generation of strong downshifted lines (Figure 7.1c).

Let us assume that small scale density irregularities are present within the $E_s$ plasma and that the gradient vectors are distributed more or less evenly over a cone of angles centered on the vertical. In the presence of such irregularities there will be, on the average,
more cases of parametric excitation with $\phi > 0$ than with $\phi < 0$. On the basis of this sort of statistical averaging of favorable instability conditions, one would expect the total plasma line intensities generated by $\phi > 0$ geometries (i.e. upshifted plasma lines) to dominate those created in $\phi < 0$ geometries (downshifted plasma lines). Thus, Muldrew's model succeeds in explaining the dominance of the upshifted plasma line observed by Gordon and Carlson (1976). In its present form, however, this model does not account for all of the observations presented in Chapter 6 since there are cases where the downshifted line is the dominant plasma line. Nevertheless, if small scale $E_s$ irregularities are at some point invoked in order to lower the threshold below that predicted for a stratified plasma, then the scale size of the irregularities must be $\ll 300$ m. This requirement is dictated by the fact that a particular power asymmetry is consistently present throughout a given observation period and that an asymmetry is not seen to reverse direction within the 4 sec – 6 sec time resolution of the data. If the scale size of irregularities most important to the instability process were of the order of the dimension of the radar beam width ($\sim 320$ m) or greater, then on the basis of the ray paths in Figure 7.1 one would expect the direction of the power asymmetry to alternate as geometries with $\phi < 0$ and $\phi > 0$ alternately passed through the field of view.

Model A is applicable principally when electric field strengths are very close to the minimum threshold. In this case parametric excitation occurs only when $\hat{\kappa}$ is very nearly aligned with $\hat{E}$. In the
presence of a pump electric field just above $E_0$ coupling between $\hat{k}$ and $\hat{E}$ might, for example, be possible only when the two vectors are oriented within $10^\circ$ of one another. The requirement that $\hat{k}$ and $\hat{E}$ be closely aligned has the additional effect of increasing the relative importance of convective losses in determining the threshold. This follows since the transit times of plasma waves through the instability region decreases as the maximum allowable coupling angle decreases. Convective and collisional losses become equally important when plasma wave transit times are of the order of $0.1$ msec, that is, whenever $1/\tau \sim 10$ kHz $\sim v_e$.

In a second, slightly different model, Model B, one might assume that the OTSI can be driven when $\hat{k}$ is within as much as $30^\circ$ of $\hat{E}$. As in Model A an angular distribution of gradients is assumed to be present in the plasma as a result of small scale density irregularities. Gradients such as those in Figures 7.1a and 7.1c would once again lead to dominant upshifted and downshifted plasma lines. However, gradients near vertical (Figure 7.1b) would give rise to both types of plasma lines.

In Model B it is assumed that the OTSI is driven strongly enough to allow an extended instability region to develop in the $E_S$ plasma. This becomes possible since the increased plasma wave coupling to the pump permits plasma wave growth to occur even when $\hat{k}$ is not precisely aligned with $\hat{E}$. A typical instability region would extend from a point near HF reflection down to some lower boundary, below which the instability would cut off. Several means by which the instability might cut off at lower altitudes are discussed in section 5.7.3. To these,
one might add the decoupling of the pump wave from the plasma waves that occurs whenever the angular separation between $\vec{k}$ and $\vec{E}$ becomes very large.

Since the OTSI produces oppositely directed plasma waves of equal magnitude locally, one would expect the electrostatic electric field structure to maintain some semblance of a standing wave pattern within the instability region. However, because plasma waves are produced only at instability heights, the upgoing wave intensity must decrease as one moves towards the lower boundary of the instability region. Below this region only downcoming waves are possible. Thus, the dominance of the upshifted plasma line is interpreted in terms of observational altitudes that are near the lower boundary of the instability region.

In the absence of collisional damping, the downshifted plasma line intensity would be at most equal to that of the upshifted plasma line. This would occur near the plasma wave reflection point where an electrostatic standing wave resembling the Airy structure is likely to form to good approximation. However, if as discussed in section 7.4 the collisional absorption rate becomes very large near reflection, then the upgoing waves will not be totally reflected. Consequently, a travelling upgoing plasma wave component would dominate in the vicinity of reflection. As a result, either the upshifted or the downshifted plasma line may be dominant within Model B.

It should be noted that Model B differs from Model A only in that the former model allows for a larger degree of coupling to be
maintained between the pump field and the plasma waves. Model A is most applicable for electric field strengths near $E_0$, while Model B is of interest whenever $E > 1.5 E_0$.

7.9 THE POWER ASYMMETRY

Having developed theoretical models for the excitation and propagation of plasma waves in $E_S$, we shall now examine their applicability to the experimental observations. In Chapter 6 the power asymmetry of the enhanced plasma lines was discussed and its relationship to the relative values of $f_b E_S$ and $f_{HF}$ was noted. Of greatest importance is the observation that either the downshifted or upshifted plasma line may be the dominant line. Dominant upshifted (downshifted) plasma lines appear to be associated with $(f_b E_S - f_{HF})$ values that are greater than (less than) $\sim 1$ MHz.

In general, one might interpret small differences between $f_b E_S$ and $f_{HF}$ as being indicative of the fact that the point of HF reflection lies close to the "peak" of the $E_S$ region. When $f_b E_S$ is much greater than $f_{HF}$, reflection presumably takes place further down along the lower "ledge" of the $E_S$ region. As discussed in Chapter 3, the vertical density gradient is generally observed to be very large along the bottom-side (or topside) of the $E_S$ region. Here, we refer to an average gradient since small scale irregularities may be present in the $E_S$ and, consequently, local density gradients may vary considerably from the mean gradient. The average vertical gradient must of course go to zero and eventually reverse direction at the $E_S$ peak.
Within the framework of the first model outlined in the preceding section, Model A, there are two viable means by which one might increase the downshifted plasma line strength relative to the upshifted plasma line strength (Muldrew, 1978b). First, one might postulate the presence of a tilted $E_s$ region. If the sideways tilt is in the proper direction, that is, if there is a gradient component directed towards magnetic south, then the number of irregularities with $\phi < 0^\circ$ will be increased at the expense of those with $\phi > 0^\circ$. The $E_s$ tilt must, however, be very large ($\gtrsim 40^\circ$) if the downshifted plasma line power is to surpass the upshifted plasma line power. Rocket observations, on the other hand, indicate that $E_s$ regions have slopes that are $\lesssim 1^\circ$ (Smith and Mechtly, 1972).

Nevertheless, if $E_s$ regions do exhibit tilts on specific occasions (e.g. near the edge of a large $E_s$ cloud), then at times one would expect the $E_s$ "layer" to have the proper orientation necessary to increase the relative strength of the downshifted plasma line. However, given the variety of circumstances under which a dominant downshifted plasma line was noted in the data and the duration of these observations, it does not seem reasonable to postulate that the $E_s$ regions had the proper northward tilt throughout every observation of this kind. During the July 20, 1976 observation, for example, the plasma line power asymmetry reversed direction at 1530 AST, immediately after the HF frequency was increased from 4.6 MHz to 5.94 MHz. More than anything else this suggests that the power asymmetry is associated with the location of parametric excitation within the $E_s$. 
The second possibility for increasing the downshifted power in Model A hinges upon the decrease in the mean vertical gradient in the vicinity of the $E_S$ peak (Muldrew, 1978b). At smaller gradients, the local distribution of gradients within the plasma is expected to become more isotropic in nature. Given the magnetic field geometry over Arecibo, this means that the range of possible $\phi$ values, initially centered at the vertical, or $\phi = 40^\circ$, begins to extend up to $\pm 180^\circ$ as one approaches the $E_S$ peak. Implicit within this application of Model A is the conclusion that, at best, equal downshifted and upshifted plasma line intensities may be accounted for near the $E_S$ peak. What is needed, however, is an adequate explanation for the dominance of the downshifted plasma line.

Within the context of Model B, a dominant upshifted or downshifted plasma line is possible depending upon the relative location of the observation altitude in the instability region. In applying Model B to $E_S$ one might imagine a situation where the mean vertical scale length, $H$, increases from 500 m on the bottomside of the $E_S$ to 2 km just below the $E_S$ peak. In the approximation of a horizontally stratified plasma, the electric field swelling corresponding to $H = 2$ km is uniformly increased by a factor of $(4)^{1/3} = 1.6$ while the location of the first Airy maximum descends from 36 m to 58 m below the point of HF reflection. At the same time the observation altitude changes from $\sim 12$ m to $\sim 49$ m below the reflection point.

Let us assume that the OTSI coupling of plasma waves to the pump electric field is maintained down to an area just past the first Airy
maximum. In this case it is clear that increases in H serve to extend the lower boundary of the instability region while simultaneously moving the relative location of the observing point closer to the lower boundary. Thus, if the degree of OTSI excitation is more or less uniform over the instability region, Model B predicts that the relative power of the upshifted plasma line power should become even stronger near the peak of a stratified layer. In order to account for the increased importance of the downshifted plasma line near the $E_s$ peak, one must disregard the Airy description of the electric field structure and allow the portion of the instability region below the observation height to become proportionally larger as the peak is approached.

In an alternative explanation for the dominance of the downshifted plasma line, one might invoke an electric field dependence upon altitude as a means of increasing the relative power in the downshifted plasma line. Within the scope of an Airy treatment of the electric field, it may be shown that for the scale lengths $H \lesssim 3$ km assumed to be present in $E_s$ the observation altitudes will be located between the first Airy maximum and the point of HF reflection. Therefore, in the observed instability region the electric field strength will increase with decreasing altitude. This has the net effect of counterbalancing the preference given to the upshifted plasma line in Model A, since the larger field strengths at lower altitudes will increase the coupling to the pump field of plasma waves propagating upwards from below the observation point relative to the coupling above the observation point.
The results of the preceding discussion may be summarized in the following way. When \((f_bE_s - f_{HF})\) is large implying that HF reflection occurs well down along the lower ledge of the \(E_s\) region, the power asymmetry \((PL(+) > PL(-))\) may be explained either by Model A or Model B. The relevant model depends upon pump field strength in sporadic E. Given the near threshold electric field strengths estimated in section 7.5 for experimental conditions, one is inclined to choose Model A for application to the present experiments.

The greatest difficulties arise when one seeks to explain the appearance of the reverse power asymmetry \((PL(-) > PL(+)\)) at small values of \((f_bE_s - f_{HF})\). In the event that excitation takes place near the \(E_s\) peak, Model B may be applicable if the electric field swelling is large enough. While a dominant downshifted plasma line is not inconsistent with Model B, the relative importance of the upshifted plasma line is expected to increase near the \(E_s\) peak for a stratified plasma.

In the weak field limit, one may use Model A to explain the dominance of the downshifted plasma line if the pump field increases with decreasing altitude is significant. The altitude dependence must be strong enough to compensate for the tendency of the PL (+) line to dominate in an \(E_s\) plasma containing small scale density irregularities and a uniform electric field. When reflection occurs along the sporadic-E bottomside far away from the peak, the increase in electric field strength with decreasing altitude is presumably not large enough to remove the bias given to upshifted plasma line generation as a result of the distribution of \(\phi\) values within the plasma.
In concluding the discussion of the power asymmetry, we note that the introduction of a plasma containing irregularities is more than a matter of peripheral interest to the description of the OTSI. If, as suggested in section 7.5, electric field strengths at reflection in $E_s$ are at best close to the homogeneous plasma threshold, then the presence of irregularities is necessary in order to insure the detection of non-linear plasma waves. This comes about since in a horizontally stratified layer containing a large vertical density gradient, plasma waves exhibiting the proper detection geometries quickly decouple from the pump field. In this case the OTSI is not effectively driven at weak field strengths.

In estimating the scale sizes of the postulated irregularities one requires that they be $\ll 300$ m. This is indicated by the relative constancy of the upshifted and downshifted plasma line strengths over time scales of $\sim 12$ sec. The implication is that many irregularities are continuously present within the radar field of view. In addition, the observed power asymmetry does not undergo rapid reversals within the 4 sec - 6 sec time resolution of the data. One would expect such reversals to occur if individual irregularities drifted through the observing volume at velocities of the order of 50 m/sec. Further evidence that any irregularities present in the plasma are much smaller than the beam width is afforded by the fact that the PL (+) power, the PL (-) power, and, when observable, the enhanced ion line power trace out very similar intensity variations in time. The variations usually occur in the data over time scales $\gtrsim 5$ min and may be in part caused by
changes in the number of irregularities and/or changes in the size and
magnitude of the density fluctuations.

7.10 PLASMA LINE POWER PROFILE

In Chapter 6 it was noted that the $E_s$ plasma line profiles
occasionally exhibited an altitude asymmetry wherein the PL (+) excita-
tion peak appeared at lower altitudes than the corresponding PL (-) peak.
In a stratified plasma the observed interaction region is confined to a
narrow altitude region $\sim 10$ m below the point of HF reflection. If
plasma waves are excited only in this region, then any altitude asym-
metry in the heights of the plasma lines would be undetectable within
the 600 m altitude resolution of the present experiment. On those
occasions when an altitude asymmetry appeared in the plasma line data,
the $E_s$ was confined to $\lesssim$ three 600 m range gates along the ion line
power profile. Consequently, a detectable altitude asymmetry is indi-
cative of plasma wave generation in a sizable portion of the $E_s$ region.

It is interesting to inquire into how the observed altitude
asymmetry, if real, might come about. On the basis of the electric
field discussion in Chapter 4, it is evident that the electric field
strength rapidly declined after the critical density point at $\omega_{HF} = \omega_e$
is reached. The electric field strengths of radio waves propagating
through to the topside of an $E_s$ "layer" are significant only for very
thin layers possessing thicknesses $\lesssim 100$ m, or whenever $f_{HF}$ is within
$\sim 95\%$ of the peak plasma frequency of the layer. In either case the
extent of the parametric interaction region would be much less than 100 m.
As an alternative means of extending the excitation region one might consider the situation where parametric interactions occur only on the bottomside of the $E_s$ region but allow for the transference of energy to the $E_s$ topside. The convection of instability generated plasma waves through the layer is an extremely unlikely process since plasma waves are attenuated quite strongly in regions where the local plasma frequency is close to or exceeds the critical HF frequency. A more viable mechanism might take into account the acceleration of electrons as a result of the Landau damping of unstable plasma waves on the $E_s$ bottomside. The high energy electrons would in turn penetrate the layer and excite plasma waves in much the same manner as daytime photoelectrons. In addition to extending the observed interaction region, this process would also lead to an altitude asymmetry wherein PL (-) lines would occur at greater altitudes than PL (+) lines. This is in agreement with the observed direction of the altitude asymmetry in the data.

7.11 SPORADIC-E SPECTRA

As noted in section 6.5, the prominent feature in all of the $E_s$ spectra measured during the 1977 experiment is a sharp peak at the growing mode. Since the growing mode is distinguishable from the decay line within the frequency resolution of the experiment, one is led to the important conclusion that the OTSI is the dominant parametric instability in an $E_s$ plasma. This is in agreement with the theoretical predictions of Chapter 5.
Regardless of the detailed plasma wave geometries responsible for the observed plasma line enhancements, it is of interest to examine the means by which the nonconvective OTSI saturates within the $E_s$ plasma. This of course presupposes that the instability threshold is indeed exceeded. Nevertheless, one may interpret the large ion line enhancements observed in association with plasma line excitation as one indication that the OTSI is very close to or above threshold (section 7.5). In addition, the threshold calculations of section 7.5, taken together with the corresponding estimates of the HF electric field strength in sporadic $E_s$ indicate that it is reasonable to believe that the OTSI threshold was exceeded (at least locally within the plasma) during most of the observations reported in Chapter 6. On the other hand, it is unlikely that the OTSI threshold was at any time greatly exceeded.

In Chapter 5 two prominent means of OTSI saturation were discussed; one entailed ion nonlinear Landau damping saturation while in the other stabilization was achieved by a nonlinear downshift in the natural frequency of the medium. Both saturation mechanisms were derived assuming a homogeneous plasma and are therefore directly applicable to an $E_s$ plasma only to the extent that the homogeneous approximation applies locally. As discussed in section 7.7, this is likely to be the case whenever electron-neutral collisions rather than convective losses dominate the threshold determination.

Saturation by means of ion nonlinear Landau damping is especially favorable whenever the decay mode destabilizes before the OTSI.
In this case plasma wave energy is redistributed into the same series of spectral lines as those initially generated by ion nonlinear Landau damping of the decay instability (Fejer and Kuo, 1973). However, as pointed out earlier, the experimental spectra do not exhibit a strong decay line which one would expect if the decay instability were present.

Even in the absence of a decay mode that is readily driven unstable, saturation might still occur as a result of the generation of nonlinear Landau damped satellite waves located near \( f_{HF} - 2\cdot m\cdot f_{ia} \), \( m = \) positive integer. In this process the primary beat mode consisting of an OTSI-generated plasma wave and a daughter wave of frequency \( \omega_r(k_2) \) would interact with resonant ions having velocities, \( v_i \), near

\[
v_i \propto \frac{\omega_{HF} - \omega_r(k_2)}{|\hat{k}_1 - \hat{k}_2|} \propto \frac{2 f_{ia}}{2|\hat{k}_1|} = c_s
\]

where \( k_1 \) and \( k_2 \) are the wave vectors of the electrostatic pump wave and first satellite wave respectively. We have assumed that the interaction peaks for counterstreaming waves, that is for \( \hat{k}_2 \propto -\hat{k}_1 \) (Kruer and Valeo, 1973, Perkins et al., 1974). For an \( E_s \) plasma the first satellite wave would be located at frequency displacements, \( \Delta f = 1.8 \) kHz (for \( Fe^+ \)) or \( \Delta f = 2.6 \) kHz (for \( NO^+ \)), assuming \( k_1 = 18 \) m\(^{-1}\) (section 5.8).

Upon examination of the experimental spectra presented in Chapter 6 one finds that although there may occasionally be a peak in the data near \( (f_{HF} - 2.6 \) kHz) (Figure 6.24b), peaks at this displacement are generally not resolved. One of course cannot rule out the possibility that some of the unresolved structure often present on the 430 MHz side of the growing mode might be due to a small satellite peak.
The 2 $f_{ia}$ displacement estimated above for the first satellite peak presupposes that the pump-daughter interaction occurs near the observation height and that the wave vectors are directed parallel to $\vec{E}$. If, in most cases, $\vec{E}$ is assumed to be aligned with $\vec{B}$, then the observed offset will not be exactly 2 $f_{ia}$ since the Arecibo observations are made at $\theta = 44^\circ$ with respect to the magnetic field. In general, the frequency displacement will vary depending upon the location and orientation of the nonlinear interaction within the plasma. The dependence upon plasma wave geometries is a propagational effect that comes about because of the $k$ and $\theta$ dependences in the dispersion relationship for $\omega_r(\theta, k_2)$ (Eq.(5.28)). Geometric effects of this nature are responsible for small additional shifts in the positions of the satellite peaks observed in the parametric decay saturation spectrum in the F region over Arecibo (Perkins et al., 1974; Duncan, 1977).

For the present purposes we shall restrict the discussion to magnetic field effects. In this case one may relate the wave numbers of plasma waves travelling parallel to $\vec{B}$ (i.e. parallel to $\vec{E}$) to those observed at Arecibo ($\theta = 44^\circ$) by means of the relationship

$$k_2^2 = k^2 \left[ 1 + \frac{\omega_{ec}^2 \sin^2 \theta m_e}{3kT_e k^2} \right] = 6.7 k^2$$

where $k_2$ and $k$ refer to waves propagating at $\theta = 0^\circ$ and $\theta = 44^\circ$ respectively. In the above calculation we have assumed $T_e = 280^\circ$K and $k = 18$ m$^{-1}$. Thus, if plasma waves are created at $\theta = 0^\circ$ and propagate to $\theta = 44^\circ$, or if the HF pump wave is scattered into plasma waves at $\theta = 44^\circ$ as a result of frequency-shifted ion Cerenkov emission, then
the frequency offsets of the first and all subsequent satellite peaks increase by a factor of $(6.7)^{1/2}$. This follows since $\Delta f = f_{ia} = k_2 c_s$. Consequently, depending upon the ion composition of $E_s$, the frequency displacement of the first satellite peak is predicted to occur in the range $\Delta f = 4.4$ kHz to 6.0 kHz. In addition, it is clear that in an irregular $E_s$ plasma containing varying plasma wave geometries, the widths of the observed satellite peaks will be broadened as a result of plasma waves that are generated outside of the immediate vicinity of the observation altitude and/or as a result of Cerenkov emission from angles other than $\theta = 0^\circ$.

It is possible that unresolved structure near the frequency offsets estimated above for the satellite waves may be present at times in the experimental spectra. As noted earlier, secondary spectral peaks were resolved only occasionally in the 1977 data. However, the measured spectra shown in Figures 6.24b and 6.26a, which exhibit small peaks at displacements of $\sim 2.6$ kHz and $\sim 4.5$ kHz, are consistent with the offsets calculated for the two geometries above. The peaking at 4.5 kHz, for example, might arise as a result of Cerenkov emission due to the mixing of the pump field with nonlinear plasma waves generated along the pump direction as a result of nonlinear Landau damping. An analogous process is observed in the F region in conjunction with the decay instability (Perkins et al., 1974).

A much better example of spectral peaking near $\Delta f = 5$ kHz may be found in the spectral observations of $E_s$ plasma line enhancements made by Kantor (1977). These measurements were conducted on July 25,
1976 during the time intervals 1006-1019 AST and 1227-1242 AST. $E_s$ spectra obtained during the first time period consisted of a single peak displaced from 430 MHz by $\pm f_{HF}$. The peaks possessed very narrow bandwidths and were similar in structure to many of those measured during the 1977 experiment (e.g. Figure 6.20). The estimated sporadic-E electric field strength and threshold during the above time period are approximately those in row 10 of Table 7.2.

The estimated threshold during the second observation period (row 11 of Table 7.2) was approximately 30% less than the one estimated for the first period. The spectra recorded during the second observing period exhibited rather strong secondary peaking. Two representative spectra are presented in Figure 7.2. The secondary peaks in the spectra occur at frequency displacements from $(430 MHz \pm f_{HF})$ of approximately 4.5 kHz to 5.0 kHz. Taken in the context of the magnetic field effects described earlier, the spectral observations are consistent with the formation of a satellite peak near $f_{HF} - 2f_{ia}$ as a result of a nonlinear Landau damping saturation mechanism. The fact that the $2f_{ia}$ displacement lies closer to 4.4 kHz than to 6.0 kHz implies that $f_{ia}$ is closer to values expected for Fe$^+$ ions than for NO$^+$ or O$_2^+$ ions.

Because of the large relative amplitude of the secondary peak, one is inclined to believe that this peak is due to nonlinear daughter waves formed parallel to the magnetic field but which subsequently propagated into the radar observing geometry. Effects similar to this are expected on the basis of the $E_s$ models presented in section 7.8, since it is the nonlinear plasma waves that are presumably detected in $E_s$ over
Arecibo following a sizable amount of attenuation enroute to the observation point.

Finally it is noteworthy that the upshifted plasma line in the spectrum displayed in Figure 7.2 exhibits a small additional peak at a displacement of \( \sim -9.5 \text{ kHz} \). Spectral peaking near this frequency offset is expected if daughter waves at \( f_{HF} - 4f_{1a} \) are produced in the nonlinear Landau damping cascade.

In light of the above observations one may conclude that there is evidence on at least some occasions that the OTSI is saturated by means of nonlinear Landau damping. Furthermore, the fact that secondary peaking was observed only during the second spectral data taking run on July 25, 1976 suggests that the threshold for the destabilization of the first daughter wave was exceeded only during the second observation and not in the first when the estimated OTSI threshold was greater.

A second OTSI saturation mechanism that may be operative in \( E_s \) entails the nonlinear conversion of the OTSI into the decay instability (section 5.6). This is accomplished by means of a nonlinear shift in the natural resonance frequency of the plasma. The amount of frequency shift, \( \Delta \omega \), may be expressed as (Nishikawa et al., 1973)

\[
(\omega_0 + \Delta \omega)^2 = 1 - \frac{T_e}{(T_e + T_i)} + 1/2 \left[ W/n_e \kappa T_e \right]
\]

where \( W \) is the total energy density of the plasma waves. For \( T_e \sim T_i \) and assuming \( W \sim \varepsilon_0 E^2 \) (Kruer et al., 1970) where \( E \) is the pump field strength, one obtains \( W/n_e \kappa T_e \sim 10^{-3} \) and \( \Delta f \sim 2.2 \text{ kHz} \) for a pump frequency, \( f_0 \), of 5.0 MHz.
A similar calculation may be performed using the developments of Weinstock and Bezzerides (1975). In this case the expression for the frequency shift is given by

$$\Delta \omega = \omega_o - [\omega^2_r - \text{Re}(G_e(k))]^{1/2} \quad (7.5)$$

where \(\text{Re}(G_e(k))\) is the real part of the resonant electron mode coupling term specified by Weinstock and Bezzerides' Eqs.(11) and (23). In the absence of contributions from parametric decay waves, \(\text{Re}(G_e(k))\) may be approximated by

$$\text{Re}(G_e(k)) = \frac{\kappa T_e}{2} \frac{\langle (\delta E)^2 \rangle}{n_e \kappa T_e} \frac{\varepsilon_o \omega_e^2}{m_e c_s^2}$$

where \(\langle (\delta E)^2 \rangle\) is the mean electric field density of the plasma waves which may be taken to be \(\sim E^2\). After evaluating \(\text{Re}(G_e(k))\) for the same parameters as above and substituting into Eq.(7.5), one obtains

$$\Delta f \approx 2.6 \text{ kHz}, \text{ which is in agreement with the value of 2.2 kHz derived previously.}$$

Whenever the OTSI develops in a nonuniform medium, the frequency matching conditions of the resonance may be met over a wide range of altitudes since \(k\) may be adjusted in the dispersion relationship for \(\omega_r\) in order to compensate for changes in electron density (i.e. \(\omega_e\)). In order to simplify the ensuing discussion, we restrict our attention to examining the OTSI saturation in the vicinity of the observed instability height where \(|\vec{k}| = 18 \text{ m}^{-1}\). The minimum threshold for the OTSI occurs whenever the condition \(\delta = (f_{HF} - f_r) = -\nu_e/4\pi\) is satisfied. For \(\nu_e = 14 \text{ kHz}\) one has \(\delta = -1.1 \text{ kHz}\). The minimum threshold in \(E_s\) for the
decay instability is obtained at \( \delta = (f_{HF} - f_r) \sim + 0.9 \text{ kHz (for Fe}^+ \text{)} \sim f_{ia} \). On the basis of the frequency matching conditions, the two instabilities are distinguishable by the necessary requirement that \( \delta \) be less than zero for the OTSI while \( \delta \) must be greater than zero for the decay instability.

Let us now consider the situation where the value of \( f_r \) determined for the 430 MHz radar wave number undergoes a downshifting in frequency. If the downshift is of sufficient magnitude a transformation from \( \delta < 0 \) to \( \delta > 0 \) may occur, thereby converting the OTSI into the decay instability. The amount of downshift in \( f_r \) required to saturate the OTSI in an \( E_s \) plasma was estimated above to be between 2.2 kHz and 2.6 kHz. This is just enough of a shift to convert the OTSI at minimum threshold into the decay instability at minimum threshold for the \( E_s \) conditions considered above.

The development of the OTSI may be described in the following manner. After the HF pump is turned on the long wavelength plasma perturbations \( (k \lambda_o \ll 1) \), which have the largest growth rate, undergo the fastest exponential growth. As the plasma wave amplitudes become large, the plasma resonance frequency at these wavelengths undergoes a downshifting in frequency thereby transforming the OTSI into the decay instability. The decay instability then saturates and in so doing spreads plasma wave energy to smaller wave numbers. At the same time, the downshifting of the natural plasma frequency causes the shorter wavelength oscillations to become unstable against the OTSI. This comes about since the change in the natural resonance frequency brings
plasma waves at previously stable wave numbers closer to the optimum frequency matching conditions. As in any saturation process, the migration of the OTSI towards shorter and longer wavelengths is halted whenever there is a balance between energy input to the OTSI and the energy dissipated by collisions, convective losses, etc.

The saturation of the OTSI described above offers a marked contrast to ion nonlinear Landau damping of the decay instability. While the OTSI saturation energy may spread to both larger and smaller wave numbers, saturation of the decay instability spreads plasma wave energy only towards smaller wave numbers. The spread of OTSI-generated plasma waves towards smaller wave numbers is accomplished by the nonlinear Landau damping saturation of the transformed decay instability. Collisional damping of these waves is an important source of energy loss for the OTSI. This damping serves to stabilize the OTSI in much the same manner as it serves to stabilize the decay instability when driven directly by the pump field. In addition, convective losses of waves associated with the decay instability may also act to stabilize the OTSI. This may be particularly important in an $E_s$ plasma since such losses are facilitated by steep density gradients.

The diffusion of plasma wave energy towards larger wave numbers takes place within the framework of the OTSI itself (i.e. $\delta(k) < 0$). In general, the cascade to larger wave numbers is commonly observed in computer simulations in the strong turbulence regime. The fluctuation spectrum may be estimated using the solutions of Weinstock and Bezerides (1975). According to these results, the spectrum extends from a critical
wave number, $k_c$, up to a maximum wave number $k_m$. $k_c$ is essentially determined by the condition that $\delta(k_c) = 0$ in the steady state plasma. Let us assume that the OTSI is stabilized in a homogeneous plasma specified by parameters appropriate for the observation point in the geometry shown in Figure 7.1b. In this case a typical value of $k_c$ as determined from Weinstock and Bezzerides' Eq.(23) is 18.0 m$^{-1}$. Starting at $k_c$, the wave number spectrum falls off approximately as $k^{-2}$ until it reaches a value of $k_m = 20.8$ m$^{-1}$, after which it abruptly goes to the thermal level (Weinstock and Bezzerides' Eq. 27 and subsequent comments). Thus, for the experimental pump intensities very little spreading of the spectrum towards large $k$ values is expected. This is simply a reflection of the fact that the above saturation mechanism (or any other saturation process) has a minimal impact upon the spectrum of plasma turbulence for electric fields just above minimum threshold.

In Chapter 5 it was pointed out that nonresonant mode coupling saturations, which apparently operates in the F region (Duncan, 1977), might also be applicable to OTSI saturation in $E_s$. Indeed, observations during the 1977 experiment of parametric excitation near 130 km altitude in the normal E region indicate the presence of a spectral cascade extending from the decay line towards 430 MHz (Figures 6.21b and 6.24a). However, there is no evidence for a nonresonant cascade in any of the plasma line spectra recorded in $E_s$.

We shall now offer a general description of some of the physical mechanisms that may be responsible for the $E_s$ spectra presented in Chapter 6. In this regard, the most important observation to be made
is that all of the spectra measured during the 1977 experiment contain a single dominant peak at 430 MHz $\pm f_{\text{HF}}$. Furthermore, in many cases a small amount of unresolved structure appears to be present within $\sim 5$ kHz of this peak.

The sharp line at the growing mode is indicative of the presence of the OTSI. The secondary peaking occasionally observed in the 1977 data may be attributed to nonlinear Landau damping saturation of the OTSI plasma waves. This same mechanism may also be responsible for a part or all of the unresolved structure in the spectra. In addition, the secondary peaking in the spectra displayed in Figure 7.2 is almost certainly due to an ion nonlinear Landau damping cascade.

In cases where little or no structure outside of the growing mode is present, it is possible that saturation occurs as a result of a nonlinear shift in the natural plasma frequency. Given the minimum threshold conditions under which this mechanism would operate, one would expect, at most, a small amount of diffuse spectral structure to be present within $\sim 2 f_{\text{i}}$ of the growing mode. This structure would presumably be due to the conversion of the OTSI into the decay instability at longer wavelengths.

The unresolved structure within $\sim 1 f_{\text{i}}$ of the growing mode might also arise as a result of frequency-shifted ion Cerenkov emission at the decay mode in a pumped but stable plasma (Hagfors and Gieraltowski, 1972). The generation of a weak decay component in this manner would be indistinguishable from a weak decay mode component driven by the OTSI. One may, however, be reasonably certain that the
decay mode is not strongly excited since none of the saturation features commonly observed in enhanced F-region plasma line spectra (Duncan, 1977) are present in the $E_S$ data.

Finally, it is interesting to note the variability in the plasma line enhancements observed in the June 9, 1977 and June 13, 1977 data. During these two observations the upshifted and downshifted plasma line intensities remained at a constant, very low level for the greater portion of the observing period ($S/N = 1$ to $6$; Figures 6.21a, 6.22b and 6.22c). Occasionally, however, sudden intensifications in plasma line power do appear in the data. While all of the spectra contain a narrow peak at the growing mode, the spectra at low $S/N$ tend to be slightly broader than those at larger ratios. Although it cannot be firmly established, it is tempting to interpret the weak enhancements as Cerenkov emission in a pumped but stable $E_S$ plasma below threshold, while attributing the larger enhancements to the OTSI. If this is the case, then the slight broadening at low $S/N$ is interpretable in terms of the altitude averaging of linear plasma waves created at the decay and growing modes (Hagfors and Gieraltowski, 1972).

In summary, the most important point to be made with respect to the $E_S$ spectral observations is that a peak at the growing mode is the dominant feature in all spectra. This we have interpreted in terms of the OTSI driven near minimum threshold. There is evidence in the spectral data that ion nonlinear Landau damping of OTSI generated plasma waves is an important saturation mechanism in the $E_S$ plasma. During most of the 1977 observations, however, the HF pump field did
not exceed the OTSI threshold to an extent great enough to allow the
development of a large amount of saturation-related spectral structure.
At the same time the large ion line enhancements observed in the power
profile data indicate that the HF electric field strengths in $E_s$ are
large enough to excite the OTSI. When the HF pump field is near
threshold, the signatures of the OTSI saturation mechanisms proposed
in Chapter 5 are hard to distinguish within the $\sim 1.7$ kHz frequency
resolution of the present experiment. On those occasions when unreso-
solved structure appears in the $E_s$ spectra near the growing mode,
either ion nonlinear Landau damping or a downshifting in the natural
plasma frequency of the medium may be responsible for the unresolved
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FIGURE 7.1 Langmuir ray paths in a model sporadic-E plasma computed by Muldrew (1978b).
FIGURE 7.2  Sporadic-E power spectra measured by Kantor (1977). The vertical scale is linear and is normalized to the peak power in each plasma line.
CHAPTER 8
SUMMARY AND CONCLUSIONS

In the present investigations the interaction of a powerful HF wave in sporadic E (E_s) is systematically studied. The majority of the E_s regions were located between 105 km and 115 km altitude. The enhanced plasma line spectra measured in E_s are found to exhibit sharp peaks at 430 MHz ± f_{HF}, that is, at the purely growing mode. Additional unresolved spectral structure is often present within ~ 5 kHz of the growing mode. On occasion secondary peaking in the spectra is resolved in the data. The large peaks at the growing mode are interpretable in terms of the oscillating two-stream instability. Thus, in contrast to F-region observations wherein the decay instability is typically the primary means of excitation, spectral measurements reveal that the oscillating two-stream instability is the dominant mode of parametric interaction in the E_s plasma. It is interesting to note, however, that in the few cases where plasma line excitation was observed outside of E_s in the upper E region between 130 km and 140 km altitude, only the decay mode was evident.

The leading role played by the oscillating two-stream instability in E_s is consistent with theoretically predicted thresholds and growth rates derived for a model E_s plasma. The increased electron-
neutral collision frequencies present at lower E-region altitudes along with the steep electron density gradients anticipated in $E_s$ regions are found to inhibit the decay instability to a much greater extent than the oscillating two-stream instability. Possible saturation mechanisms for the oscillating two-stream instability that may be operative in the $E_s$ plasma include nonlinear Landau damping of the unstable Langmuir waves (Fejer and Kuo, 1973; Perkins et al., 1974) and the nonlinear downshifting in the natural electron plasma frequency of the medium (Nishikawa et al., 1973).

On the whole, the amount of structure observed in the enhanced plasma line spectrum outside of the growing mode does not permit a determination of the dominant saturation mechanism. The absence of a large amount of saturation structure is understandable since estimates of the pump electric field in the plasma indicate that the oscillating two-stream instability threshold was never greatly exceeded. Nevertheless, the secondary peaking is clearly exhibited in enhanced plasma line spectra recorded during one observation period, when the estimated pump field was comparatively large. These secondary peaks, taken together with their observed frequency displacements from the growing mode, are indicative of the satellite wave generation that is characteristic of nonlinear Landau damping saturation.

In addition to the spectral observations, power profile measurements were made at the ion line as well as the upshifted $(430 \text{ MHz} + f_{HF})$ and the downshifted $(430 \text{ MHz} - f_{HF})$ plasma lines. Plasma line and ion line enhancements are observed only when $f_D E_s$ exceeds $f_{HF}$. This is
consistent with the interpretation of $f_bE_s$ as the minimum plasma frequency in the plasma. From the standpoint of the plasma physics involved in the interaction process, the ion line enhancements detected during the observations are interpretable in terms of a parametric interaction process that is above threshold.

Although sudden fluctuations in plasma line power are occasionally observed during times of continuous monitoring of the enhanced plasma line power, the plasma line enhancement level, on the whole, tends to fade gradually over time scales of $\sim 5$ minutes or more. At times, a more or less regular fading in the enhanced plasma line power is observed at periods of 5 to 10 minutes. The fractional intensity variations associated with this fading are not, however, very large.

In general there is evidence in the data that $E_s$ is basically patchy in structure. Typical irregularity scale lengths deduced for the patches range from a few km to $\sim 30$ km.

On several occasions, the $E_s$ maxima in the downshifted plasma line power profiles appeared to be located at slightly greater altitudes than the corresponding upshifted plasma line maxima. This may be indicative of plasma wave generation on both the topside and the bottomside of the $E_s$ region. Furthermore, large asymmetries in the relative powers of the upshifted and downshifted plasma lines are a common feature of the data. The upshifted plasma line power appears to dominate whenever $(f_bE_s - f_{HF})$ is $\gtrsim 1$ MHz while the reverse asymmetry is observed whenever $f_bE_s$ is within $\sim 1$ MHz of $f_{HF}$. The power asymmetries are interpretable within the context of two geometrical
models of plasma wave propagation and generation in \( E_s \). Dominant up-shifted plasma lines may arise whenever the oscillating two-stream instability is driven only in a very narrow altitude region extending down from the point of HF reflection and/or whenever small scale (\(< 300 \) m) irregularities are present in the \( E_s \) plasma. In order to account for the dominance of the downshifted plasma line power, an altitude dependence of the HF electric field strength is suggested wherein the field amplitude increases with decreasing altitude near the observation height of the 430 MHz radar. Finally, we note that the introduction of small scale horizontal irregularities in plasma density is necessary if the oscillating two-stream instability is to be efficiently driven given the HF pump field strengths estimated in \( E_s \) for the present experiment.

Most of the observations discussed in the present \( E_s \) study were examined on the basis of their merit in facilitating a better understanding of the plasma physics involved in the HF interaction process. These same data have yet to be viewed in terms of their application to the aeronomy of the lower thermosphere. Questions related to the times of occurrence of the \( E_s \), its rate of descent, the variability in time of the structure, altitude, and intensity of the \( E_s \), as well as the proportion of metallic ions in the plasma composition, all remain to be addressed. On July 17, 1976, for example, sporadic \( E \) was observed on two occasions separated in time by nearly 12 hours. This, taken together with the observed rate of descent of the \( E_s \) (\(~ 2 \) km/hr) may be viewed as evidence for the \((2,4)\) tidal mode.
On the other hand, the variation in the height of the $E_s$ region in Figure 6.10 might be interpreted in terms of the convergence of ionization by gravity waves. Furthermore, photoelectron enhancements of $E_s$ plasma lines were apparently detected on several occasions during the 1976 experiment. These observations should yield information about a slice of the photoelectron energy spectrum that is not usually accessible in the $E$ region. Finally, the limited amount of temperature information available in the 1976 data may be of use in investigations of the theoretically predicted phenomenon of thermal runaway in the lower $E$ and $D$ regions (Perkins and Roble, 1978; Duncan and Coco, 1978).

In the future an ongoing program of $E_s$ spectral and profile observations should be maintained. It is especially important to coordinate these observations with background measurements of ion temperature and the ion-neutral collision frequency. When feasible, the decay time of the $E_s$ plasma lines should be measured in order that a direct estimate of the electron collision frequency may be obtained for use in threshold calculations.

Constant monitoring of the enhanced plasma line power profile under conditions where $f_d E_s \propto f_{HF}$ would provide a valuable means for mapping plasma patches within $E_s$. Furthermore, it would be very useful to sweep the 430 MHz radar beam in azimuth and zenith angle during the mapping process, especially within the region of the first Fresnel zone of an ionosonde. In addition, the electron density distribution of the patches may be obtained by varying $f_{HF}$ between individual radar sweeps.
As a better understanding of parametric excitation in $E_s$ is achieved, the instability process should find wide ranging applications as a diagnostic tool for examining the dynamical properties of the lower E region. At the same time the interaction process itself will continue to be extremely interesting from the viewpoint of the non-linear plasma physics involved. Studies focusing upon the saturation of the oscillating two-stream instability in $E_s$ have direct applications to laser fusion research, particularly in the regime of a strongly turbulent plasma. While HF pump fields in the present study were near threshold, the anticipated order of magnitude increase in electric field strengths to be provided by the new Arecibo HF facility (now under construction) promises to make future investigations into nonlinear interactions in $E_s$ plasmas an extremely exciting area of research. In this regard, spectral measurements, perhaps with increased frequency resolution using pulse-to-pulse correlation techniques, should be made as in the past at the ion line and two plasma lines. However, frequency searches throughout a range of Doppler shifts are recommended in an effort to uncover any other unstable coupling process that may be occurring within the plasma. Such coupling might arise as a result of the presence of the earth's magnetic field which introduces additional normal modes into the plasma (e.g. modes at the lower hybrid frequency and harmonics of the electron cyclotron frequency).
REFERENCES


Farley, D. T., "Incoherent scatter power measurements; a comparison of various techniques," Radio Science, 4, 139-142, 1969a.


Kantor, I. J., private communication, 1977.


<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>A</td>
<td>convective amplification factor of initial plasma wave amplitude $= \exp(A)$</td>
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<tr>
<td>AST</td>
<td>Atlantic Standard Time</td>
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<tr>
<td>$\mathbf{B}$</td>
<td>external magnetic field</td>
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<tr>
<td>$B(x_2)$</td>
<td>parametric decay coupling function for nonlinear Landau damping</td>
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<tr>
<td>$B_{\text{max}}(x_2)$</td>
<td>maximum value of $B(x_2)$ for a given $T_e/T_i$</td>
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<tr>
<td>$\Delta b$</td>
<td>baud length of Barker code</td>
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<tr>
<td>c</td>
<td>speed of light</td>
</tr>
<tr>
<td>$c_s$</td>
<td>ion acoustic velocity $= \Omega_k/k$</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>e</td>
<td>electron charge</td>
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<tr>
<td>$\mathbf{E}$</td>
<td>electric field of modifying HF wave</td>
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<tr>
<td>$E_p$</td>
<td>pump electric field (section 5.2)</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Sporadic E</td>
</tr>
<tr>
<td>$E_0$</td>
<td>minimum threshold electric field</td>
</tr>
<tr>
<td>$E_{1/2}$</td>
<td>half amplitude of pump electric field (section 5.2)</td>
</tr>
<tr>
<td>$E_t^2$</td>
<td>threshold increase in an inhomogeneous plasma (section 5.7.2)</td>
</tr>
<tr>
<td>$f_e$</td>
<td>electron plasma frequency $= \omega_e/2\pi$ (section 2.2)</td>
</tr>
<tr>
<td>$f_r$</td>
<td>electron resonance frequency of the plasma $= \omega_r/2\pi$ (section 2.2)</td>
</tr>
<tr>
<td>$f_o$</td>
<td>pump frequency $= \omega_{\text{HF}}/2\pi$</td>
</tr>
<tr>
<td>$f_{ec}$</td>
<td>electron gyrofrequency $= \omega_{ec}/2\pi$ (section 2.1)</td>
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\( f_{HF} \) frequency of modifying HF wave = \( \omega_{HF}/2\pi \)

\( f_{ia} \) ion acoustic frequency = \( \Omega_{k}/2\pi \) (section 5.2)

\( f_{ES} \) top frequency of the Sporadic-E ionogram trace (section 3.1)

\( f_{bES} \) blanketing frequency of Sporadic E for 0 mode (section 3.1)

\( f_{oES}, f_{xES} \) top frequency of distinguishable 0-mode and X-mode traces (section 3.1)

\( f_{oF2} \) critical F2 region frequency for 0 mode (section 3.1)

\( \Delta f_0 \) 430 MHz radar Doppler shift (section 2.2)

\( h \) altitude

\( H \) electron density scale length (section 5.4)

\( HF \) High Frequency band (4-12 MHz)

\( \Delta h \) altitude resolution of radar measurement (section 2.3)

\( IPP \) Interpulse Period (section 2.3)

\( \hat{k} \) plasma wave vector or radar wave vector

\( \hat{k} \) unit vector in the direction of \( \hat{k} \)

\( K \) defined as \( \mu \lambda Z_o^2 = e^2/(m_i m_e) \cdot (\hat{k} \cdot \hat{E}_p)^2/4 \); proportional to the pump field intensity (section 5.1)

\( K_m \) minimum threshold value of \( K \) (section 5.1)

\( K_{th}(\delta) \) threshold value of \( K \) (section 5.1)

\( k_{II} \) component of plasma or ion acoustic wave vector parallel to pump electric field and perpendicular to density gradient (section 5.4)

\( k_{\perp} \) component of plasma or ion acoustic wave vector perpendicular to pump electric field and parallel to density gradient (section 5.4)

\( m_e \) electron mass

\( m_i \) ion mass
\( n_e \) electron number density

\( n_i \) ion number density

\( N_r \) electron density at the point of HF or plasma wave reflection (Chapter 4 and section 5.4)

\( n_o \) for singly ionized ions \( n_e = n_i = n_o \) (section 5.2)

\( n(M) \) molecular or atomic number density of species \( M \) (section 3.3)

\( \nabla n_e \) electron density gradient (section 7.3)

OTSI Oscillating Two-Stream Instability

\( P_{HF} \) HF transmitter power

\( P_{inc} \) power flux incident upon sporadic \( E \)

\( P_\alpha \) pressure of species \( \alpha \)

\( \text{PL}(+) \) upshifted plasma line; located at 430 MHz + \( f_{HF} \)

\( \text{PL}(-) \) downshifted plasma line; located at 430 MHz - \( f_{HF} \)

\( S \) swelling factor of HF modifying wave near reflection (Chapter 4)

\( S/N \) signal-to-noise power ratio

\( T_e \) electron temperature

\( T_i \) ion temperature

\( T_n \) neutral temperature

\( T_p \) Barker code length (section A.1)

\( T_s \) sampling time interval of scattered signal (section 2.3)

\( T_0 \) radar pulse length (Figure 2.4)

\( T_1 \) total sampling time interval (section 2.3)

\( v_e \) mean electron thermal speed \( (2\kappa T_e/m_e)^{1/2} \)

\( V_g \) magnitude of plasma wave group velocity (section 7.7)
\( v_i \) mean ion thermal speed; \((2kT_i/m_i)^{1/2}\)

\( V_gx \) plasma wave group velocity parallel to the electron density gradient \( \nabla n_e \) (section 7.7)

\( V(h,t) \) digitally sampled voltage corresponding to an altitude \( h \) at time \( t \) (section A.2)

\( x_m \) low frequency resonance response at minimum threshold (section 5.1)

\( x_{th}(\delta) \) low frequency resonance response at threshold (section 5.1)

\( x_0 \) lower boundary of oscillating two-stream instability region (section 7.6)

\( x_2 \) velocity in the acoustic range normalized to \( v_i \) (section 5.5)

\( X(t) \) amplitude of perturbed low frequency oscillator = \( n_i^* \), the amplitude of the ion fluctuations (sections 5.1 and 5.2)

\( y_m \) maximum growth rate (section 5.1)

\( Y(t) \) amplitude of perturbed high frequency oscillator = \( n_e^* \), the amplitude of the electron density fluctuations (sections 5.1 and 5.2)

\( z_c \) critical altitude where \( \omega_e(z_c) = \omega_{HF} \) (Chapter 4)

\( z_r \) point of reflection of HF wave incident on a plasma layer or scale length of a linear density profile, \( = H \) (Chapter 4)

\( Z(t) \) spacially homogeneous external pump field; 
\( Z(t) = 2Z_0 \cos \omega t \) (section 5.1)

\( Z_0 \) half amplitude of \( Z(t) \); \( = \hat{k} \cdot \vec{E}_{1/2} \) (section 5.1)

\( \alpha \) \( i \) or \( e \); used to denote ion or electron species, respectively; also \( \alpha = 4\pi\lambda_D/\lambda \) (section 2.2)

\( \beta \) angle between the wave vector, \( \hat{k} \), and the external magnetic field, \( \vec{B} \) (section 7.3)

\( \Gamma_1, \Gamma_2 \) amplitude attenuation coefficients of \( X(t) \) and \( Y(t) \), respectively; \( = 1/2 \ v_i \) and \( 1/2 \ v_e \)
\( y_\alpha \) ratio of the specific heat at constant pressure to the specific heat at constant volume for species \( \alpha \) (section 5.2)

\( \delta \equiv \omega_0 - \omega_2 \) (section 5.1)

\( \Delta \) gate width; time between voltage samples (section 2.3)

\( \delta_\text{m} \) frequency shift, \( \delta \), at minimum threshold

\( \varepsilon \) plasma permittivity

\( \varepsilon_0 \) permittivity of free space

\( \zeta \) normalized distance along Airy pattern;
\( \zeta \equiv (\omega^2_{HF}/c^2 z_r)^{1/3}(z - z_r) \) (Chapter 4)

\( \zeta_\text{m} \) location of first Airy maximum; = -1.02 (Chapter 4)

\( \theta \) angle of plasma wave excitation with respect to the pump field (section 5.7.2)

\( \theta(\omega) \) normalized Doppler frequency (see Figure 2.1 caption)

\( \theta_S \) Spitze angle (Chapter 4)

\( \theta_{\text{HF}} \) half-power beam width of HF beam (Chapter 4)

\( \theta_0 \) angle of incidence of HF wave measured from vertical (Chapter 4)

\( \theta^- \) angle between pump field, \( \mathbf{E}_p \), and \( \mathbf{k} \) when \( \mathbf{E}_p \) is perpendicular to the electron density gradient (section 5.4)

\( \kappa \) Boltzmann constant

\( \lambda \) radar wavelength; \( |\mathbf{k}| = 4\pi/\lambda \) or coupling constant of \( Y(t) \) to \( X(t) = -ie/m_i \)

\( \lambda_D \) Debye length (section 2.2)

\( \lambda_{\text{HF}} \) wavelength of transmitted HF wave

\( \mu \) index of refraction (section 7.1) or coupling constant of \( X(t) \) to \( Y(t) = ie/m_e \) (section 5.1)
\( \nu_e \) total electron collision frequency (section 5.2);
total intensity damping rate of Langmuir waves
(section 5.7.2)

\( \nu_i \) total ion damping rate

\( \nu_{ei} \) electron-ion collision frequency (section 3.3)

\( \nu_{en} \) electron-neutral collision frequency (section 3.3)

\( \nu_{in} \) ion-neutral collision frequency (section 3.3)

\( \nu_{nl} \) nonlinear intensity damping increment (section 5.5)

\( \rho(h, \tau) \) autocorrelation function at altitude \( h \) for time lag \( \tau \)
(section A.2)

\( \sigma \) one standard deviation

\( \phi \) angle between the electron density gradient, \( \nabla n_e \),
and the external magnetic field \( B \) (section 7.3)

\( \Omega_k \) ion acoustic frequency (section 5.2)

\( \omega_{ec} \) electron gyrofrequency, \((eB/m_e)\)

\( \omega_r \) electron resonance frequency of the plasma (section 5.5)

\( \omega_{HF} \) frequency of modifying HF wave

\( \omega_o \) pump frequency; \( = \omega_{HF} \)

\( \omega_1 \) low frequency resonance in an unpumped plasma; \( = \Omega_k \)
(section 5.1)

\( \omega_2 \) high frequency resonance in an unpumped plasma; \( = \omega_{ek} \)
(section 5.1)

\( \omega_\alpha \) plasma frequency of species \( \alpha \) (section 5.2)

\( \omega_{\alpha k} \) unpumped plasma resonance frequency of species \( \alpha \) in the
absence of an external magnetic field (section 5.2)

\( \omega'_1 \) low frequency resonance response in a pumped plasma
(section 5.1)

\( \omega'_2 \) high frequency resonance response in a pumped plasma
(section 5.1)
APPENDIX

A.1 BARKER CODING AND POWER MEASUREMENTS

Since the radar pulses used to measure the power profile during the 1976 observations were Barker coded (Chapter 2), one needs to consider the effects of clutter, or signals from unwanted altitudes, on the profile measurements. In this regard one may follow the developments of Gray and Farley (1973). We shall initially assume that the transmitted pulse is scattered from a thin slab of ionosphere and that the received signal passes through an ideal filter perfectly matched to the rectangular transmitted pulse. In this case the decoded signal, \( G(t) \), may be represented as

\[
G(t) = \int_{-\infty}^{\infty} b(t_1) \cdot f(t_1) \cdot b(t_1 - t) \, dt_1
\]

In the present experiment, \( b(t) \) is a 13-baud Barker code pattern having a baud length, \( \Delta b \), of 4 \( \mu \)sec (Figure A.1 a). The quantity \( b(t_1) \cdot f(t_1) \) represents the modifications made on the transmitted signal as a result of ionospheric scattering. If \( f(t_1) = 1 \), that is, if there is no ionospheric modification, one obtains that autocorrelation function of the pattern itself (Figure A.1 b).

The received power as a function of time may be represented as

\[
G(t)^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b(t_1) \cdot b(t_2) \cdot \langle f(t_1) \cdot f^*(t_2) \rangle \cdot b(t_1 - t) \cdot b(t_2 - t) \, dt_1 \, dt_2
\]
or \( G(t)^2 = \int_{-\infty}^{\infty} \rho(\tau) \ F(t, \tau) \ d\tau \) \hfill (A.1)

where \( F(t, \tau) = \int_{-\infty}^{\infty} b(t_1) \cdot b(t_1 + \tau) \cdot b(t_1 - t) \cdot b(t_1 + \tau - t) \ dt_1 \)

and \( \rho(\tau) \) is the incoherent scatter autocorrelation function.

If the signal returned from the ionosphere had perfect correlation, then \( \rho(\tau) \) would be constant and the received signal would depend only upon the ionospheric scattering profile, which would in turn be convolved with the correlation pattern in Figure A.1 c. However, the incoherently scattered signal is a random signal possessing a finite coherence time. This implies that the autocorrelation function goes to zero after a finite period of time and that the scattering medium changes while the pulse is being scattered. If the random phase changes that occur in the Barker coded pulse during the scattering process are large, then the central peak of the perfect correlation pattern will be broadened and the sidelobes will increase in strength. Consequently, as a minimal requirement for use of a Barker coded pulsing scheme, one usually specified that \( T_p / T_x \leq 1 \), where \( T_p \) is the Barker code length and \( T_x \) is the first zero crossing of the autocorrelation function of the medium.

The value of \( T_p \) for the 1976 experimental arrangement is 52 \( \mu \)sec. In the E region the incoherent scatter power spectrum is narrowed by the increased ion-neutral collision frequency and the autocorrelation function approaches the Lorenzian limit. Typically, \( T_x \gtrsim 300 \mu \)sec and the condition \( T_p / T_x \leq 1 \) is easily satisfied. In the F region, where \( T_x \) is
of the order of 100 µsec, this condition is still satisfied but to a much lesser extent.

In general, the measured power profile will be a convolution of the true density profile with \( |G(h)|^2 \), where the time, \( t \), in Eq. (A.1) has been replaced by an equivalent altitude, \( h = ct/2 \), and \( T_p/T_x \) (i.e. \( \rho(\tau) \)) is now a function of altitude. In practice one does not normally attempt to deconvolve the power profile since this is a formidable task. Instead, when the ionosphere exhibits a smooth density profile over altitude intervals \( \leq cT_p/2 \), one may assume that the scattered power is proportional to the measured values of \( S + C \) where \( S \) and \( C \) are the signal and clutter powers respectively. The systematic errors involved in this approximation for a 52 µsec Barker code have been calculated by Gray and Farley (1973). On the average these errors represent only a 1% effect in going from the normal E region to the F region. This is usually much less than the statistical error bars in the present work, and consequently correction factors need not be applied to the data.

In the case of sporadic E, the assumption of a smooth density profile is not valid since the half-power widths, \( h^\prime \), of the layers are typically 1-2 km (\( h^\prime \ll cT_p/2 = 7.8 \) km). Under these circumstances, the observed layer thickness will be broadened as a result of the altitude averaging over \( \sim 600 \) m, which is implicit in the \( \langle |G(h)|^2 \rangle \) convolution for a 4 µsec baud length. In addition, the peak power recorded in the layer will be reduced an average of \( (1 - (S/S+C)) = 0.08 \) (assuming a layer thickness of at least \( c\Delta b/2 = 600 \) m) due to the presence of the sidelobes in the Barker pattern (Gray and Farley, 1973). Thus, the
measured power at the sporadic-E peak must be increased by \( \sim 8\% \) in order to compensate for the reduction of \( S \) relative to \( S + C \).

A.2 SPECTRAL ESTIMATES

The spectral information contained within a scattered radar signal that is digitally sampled is available in two basic forms. One may form the discrete Fourier transform (DFT) of the measured complex voltage, \( V(t) \), according to the relation

\[
A_r = \sum_{k=0}^{N-1} V_k \exp(-2\pi i rk/N) \quad r = 0,1, \ldots, N-1
\]

where \( A_r \) is the \( r^{th} \) coefficient of the DFT, \( V_k = V(k\Delta t) \), \( \Delta t \) is the time interval between sampled points, and \( N \) is the number of samples. The \( r^{th} \) frequency component, \( r/N \), of the power spectrum, \( P \), then follows as

\[
P_r = A_r \cdot A_r^* \quad (A.2)
\]

Alternatively, one may calculate the autocorrelation function, \( \rho(h,\tau) \), defined as

\[
\rho (h, \tau) = \frac{\langle V(h,t) \cdot V^*(h,t + \tau) \rangle}{\langle |V(h,t)|^2 \rangle} \quad \tau = k\Delta t
\]

where \( V(h,t) \) is the voltage corresponding to an altitude \( h \) at time \( t \), and \( \tau \) is a delay time, or time lag. The power spectrum and \( \rho(h,\tau) \) constitute a Fourier transform pair and are therefore related by

\[
P_r = \frac{1}{2\pi} \sum_{k=0}^{N-1} \rho_k \exp(-2\pi i rk/N) \quad r = 0,1, \ldots, N-1
\]

where \( \rho_k = \rho(h,k\Delta t) \).
A.3 AUTOCORRELATION FUNCTION MEASUREMENTS

The simple two-pulse scheme used for autocorrelation measurements in the present experiment is illustrated in Figure A.2. Both pulses have the same polarization and are at the same frequency. Correlation is obtained only between signals having a common scattering volume, that is, between signals returning from the same altitude region. In general, correlation measurements involve a convolution of the autocorrelation function of the medium with functions of the transmitted pulse shape and the receiver impulse response, or alternatively, the receiver gating (Farley, 1969b). These effects may be kept at a minimum by matching the receiver response to the transmitted pulse shape.

By varying the spacing between transmitted pulses, it is possible to obtain a correlation function estimate, \( \rho(h, \tau) \), for any value of the lag time \( \tau = t_2 - t_1 \). The altitude resolution of this estimate may be approximated by the height interval over which \( \rho \) is averaged, namely \( c \Delta t^* / 2 \). Here, \( \Delta t^* \) is the receiver gate width or the pulse width, whichever is larger. Calculations of \( \rho \) using a two-pulse scheme introduce signals from unwanted altitudes as indicated by the shaded areas in Figure A.2. Although signals from these two altitudes are in general uncorrelated and therefore offer no net contribution to \( \rho \), they do act as an additional source of noise (i.e. clutter).

Experimental estimates of \( \rho(h, \tau) \) were efficiently made using a one-bit by multi-bit hybrid correlation technique (see e.g. Farley, 1969b). During each IPP the real part of the unnormalized lag product, \( \text{Re}(V(h,t) \cdot \text{sign}(V(h,t + \tau))) \) was calculated and accumulated. Since only
the real component of $\rho$ was computed, information about the plasma drift velocity, which is contained in the imaginary part of $\rho$, was discarded.

Finally, we note that both pulses in the double pulse scheme used in the present experiment were Barker coded. The minor ($< 2\%$) distortions in the measured autocorrelation function caused by the sidelobes of the code are discussed in detail by Mathews (1975).

A.4 DISCRETE FOURIER TRANSFORM MEASUREMENTS

In single pulse measurements of the type made during the 1977 experiment, spectral information is obtained by Fourier analyzing the scattered signal received within each IPP (Figure 2.4). Depending upon the pulse length and gate delay, the echo from the instability region may or may not be present during the entire sampling period $T_1$. The two limiting factors in the spectral analysis of the time series stem from (1) the finite time period, $T_s$, over which the scattered signal is sampled, and (2) the discrete nature of the sampling process within $T_1$.

The estimation of power spectra based upon a time-limited signal is a routine problem in digital signal processing and as such has been examined in detail by many investigators (see e.g. Jenkins and Watts, 1968; Blackman and Tukey, 1959; Schwartz and Shaw, 1975). In the present experiment power spectra were obtained by taking the discrete Fourier transform (DFT) of the signal time series and then multiplying the DFT by its complex conjugate (Eq.(A.2)).
The finite nature of the signal record has the effect of automatically introducing a convolution term, $W(f)$, into the measured spectral estimate. For a continuously sampled signal, one has

$$\overline{P}(f) = \int_{-\infty}^{\infty} W_B(y)P(f-y)dy$$

where $W_B(f) = T_s(\sin(\pi f T_s)/(\pi f T_s))^2$, $P(f)$ is the spectral density function of the instability region, and $\overline{P}(f)$ is the available estimate of $P(f)$. $W_B(f)$ is often referred to as the Bartlett spectral window. Because of the windowing, spectral structure must be interpreted with a considerable degree of care. It is clear, for example, that a strong peak in the true spectrum may produce symmetric oscillations in the observed spectrum as a result of leakage through the sidelobes of the spectral window.

Since the variances in a single spectral estimate are large, one typically integrates spectra over many IPP's in order to obtain a more accurate representation of the true spectrum. These estimates may be shown to approximately obey a chi-square probability distribution (Jenkins and Watts, 1968). For a Bartlett spectral window and integrations over more than ~10 IPP's, this distribution approaches a Gaussian. Given this and assuming that $P(f)$ varies smoothly over the width of the spectral window, one may estimate the variance using the relation

$$\text{Var} \{ \overline{P}(f) \} \approx \frac{2\overline{P}(f)^2}{3L}, \quad L = \text{number of spectra integrated}$$

(A.3)
A generalized expression for the variance as derived by Jenkins and Watts (1968) may be written

\[
\text{Var} \{ \overline{P}(f) \} = \frac{1}{LT_s} \int_{-\infty}^{\infty} p^2(x) W_B(f-x) \{ W(f+x) + W(f-x) \} \, dx
\]

(A.4)

In the actual experiment the signal voltage, \( V(t) \), was not continuously sampled, but digitally sampled at \( N \) regular time intervals of length \( \Delta \). A DFT was thus performed. Since the DFT is frequency band-limited, spectral estimates were susceptible to the aliasing of higher frequencies as lower frequencies. A frequency, \( f \), for example, may be aliased by frequencies at \( f \pm r/\Delta \), where \( r \) is a positive integer. To avoid aliased contributions from outside the DFT bandwidth, \( V(t) \) was electronically filtered so that its bandwidth, \( w \), was less than \( 1/\Delta \).

Typical experimental values were

\[ \Delta = 6 \, \mu \text{sec} \quad \text{and} \quad w = 125 \, \text{kHz} < 1/\Delta = 167 \, \text{kHz}. \]

It should be remembered, however, that while \( V(t) \) may be band-limited, the Bartlett windowing cannot be.

In order to efficiently perform a DFT on the data, a Fast Fourier Transform (FFT) algorithm of radix two was used with decimation in time (see e.g. Brigham, 1974; Cochran et al., 1967). The errors in the FFT spectral estimate arising from roundoff errors in the digitizing of \( V(t) \) (Welch, 1969) and roundoff errors in the arithmetic operations used to compute the FFT (Weinstein, 1969) were found to be at the 0.4% and 1·10⁻⁷% levels respectively. As a result, random errors due to the stochastic nature of the sampled signal dominated. Under these
conditions, the variance of the spectral intensity estimate may be obtained by using Eqs. (A.3) and (A.4) with \( f \) replaced by \( n/N \Delta \), where \(-N/2 \leq n \leq N/2\), \( n \) is an integer, and \( N \) is an even number of samples (Jenkins and Watts, 1968).
FIGURE A.1 A 13-bit Barker code, $b(t)$, and its autocorrelation function, $G(t)$. 

$G(t) = \int b(t_1)b(t_1-t)dt_1$ 

$T_p/T_x = 0$
FIGURE A.2  Geometry of a double pulsing scheme for two pulses of width $\Delta t$. 