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CAVITATION INCEPTION ON DECELERATING SURFACES.

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CAVITATION INCEPTION ON
DECELERATING SURFACES

by

Toshimasa Tokuno

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

Thesis Director's Signature:

William J. Walker

Houston, Texas

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NOTATIONS

$A_p$  
cross sectional area of piston (cylinder)

$A_R$  
cross sectional area of reservoir

$a$  
acceleration of piston

$a_c$  
critical acceleration

$a_R$  
acceleration of fluid in the reservoir

$H$  
reservoir level

$H^*$  
fluid length in motion in the reservoir, $H = 2.0$ cm

$L$  
cylinder length

$L^*$  
instantaneous cylinder length

$L_c^*$  
cylinder length at critical acceleration

$N$  
constant for fixed mass of particular gas

$P$  
pressure at bubble wall

$p_a$  
ambient pressure, system static pressure

$p_c$  
critical pressure for cavitation inception

$p_i$  
pressure at the end of the cylinder

$p_o$  
pressure offset for the pressure history equation

$p_p$  
pressure on the piston face

$p_{peak}$  
peak pressure on the piston face

$p_s$  
surface tension pressure

$p_v$  
vapor pressure

$p_{\infty}$  
pressure at infinity in the liquid

$R$  
bubble radius

$R_i$  
initial bubble radius

$R_{max}$  
maximum bubble radius

$R_0$  
initial radius of empty cavity

$r$  
radial distance from bubble center
S  surface tension
T  temperature
T_{1/3}  time from the point where the acceleration was 2/3 a_c to the peak acceleration point
T_{1/3}^*  time from the point where the pressure is 1/3 (p_a - p_{peak}) to the peak pressure point
t  time
\tau  time from p_a to p_{peak} in the pressure history equation
U  bubble wall velocity
u  radial velocity in the liquid
V_P  piston velocity
V_R  fluid velocity in the reservoir

GREEK SYMBOLS

\beta  R/R_0
\theta  contact angle
\theta_A  advancing contact angle
\theta_R  receding contact angle
\mu  viscosity
\rho  liquid density
\sigma_{ij}  stress tensor
\phi  velocity potential
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENT</td>
<td>1</td>
</tr>
<tr>
<td>NOTATION</td>
<td>11</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>HISTORICAL BACKGROUND</td>
<td>5</td>
</tr>
<tr>
<td>Tensile Strength of Liquids</td>
<td>5</td>
</tr>
<tr>
<td>Cavitation Nuclei</td>
<td>6</td>
</tr>
<tr>
<td>Cavitation Inception</td>
<td>7</td>
</tr>
<tr>
<td>Bubble Dynamics</td>
<td>9</td>
</tr>
<tr>
<td>EXPERIMENTAL APPARATUS</td>
<td>14</td>
</tr>
<tr>
<td>EXPERIMENTAL PROCEDURE</td>
<td>18</td>
</tr>
<tr>
<td>Experimental Preparation</td>
<td>18</td>
</tr>
<tr>
<td>&quot;Critical Point&quot;</td>
<td>19</td>
</tr>
<tr>
<td>Calculation of Pressure on Piston</td>
<td>20</td>
</tr>
<tr>
<td>Parameters</td>
<td>22</td>
</tr>
<tr>
<td>High Speed Motion Pictures</td>
<td>23</td>
</tr>
<tr>
<td>EXPERIMENTAL RESULTS AND DISCUSSION</td>
<td>24</td>
</tr>
<tr>
<td>Effects of Cylinder Size</td>
<td>24</td>
</tr>
<tr>
<td>Effects of Reservoir Level</td>
<td>24</td>
</tr>
<tr>
<td>Effects of Pressure History Shape</td>
<td>25</td>
</tr>
<tr>
<td>Effects of System Static Pressure</td>
<td>26</td>
</tr>
<tr>
<td>Effects of Temperature</td>
<td>27</td>
</tr>
<tr>
<td>Effects of Piston Material</td>
<td>28</td>
</tr>
<tr>
<td>Effects of Surface Tension</td>
<td>29</td>
</tr>
<tr>
<td>Effects of Viscosity</td>
<td>30</td>
</tr>
<tr>
<td>High Speed Motion Pictures</td>
<td>30</td>
</tr>
</tbody>
</table>
THEORETICAL MODEL

Bubble Dynamics

Calculation Results

COMPARISON BETWEEN EXPERIMENTAL RESULTS AND THEORETICAL MODEL

CONCLUSIONS

TABLES

FIGURES

APPENDICES

Appendix I: Surface Wettability and Cavitation Nuclei

Appendix II: Theoretical Calculation

REFERENCES
INTRODUCTION

Because of harmful effects to hydraulic machines, cavitation phenomena have been studied by a number of investigators. Knapp et al [24] summarized those works and defined cavitation as the growth and rapid collapse of vapor or gas and vapor-filled bubbles in liquids. The phenomenon is caused by dynamic reduction and recovery of local static pressure at essentially constant temperature. In contrast, if the bubble growth is caused by heating, it is called boiling. Some places where cavitation can be observed easily are: the throat of a venturi tube, the propeller of ships, an underwater object such as a rapidly moving hydrofoil, and an oscillating surface of an underwater ultrasonic device.

Cavitation characterized by bubble growth can be classified into two major categories as follows:

1) vaporous cavitation
2) gaseous cavitation

In vaporous cavitation, the bubble growth is the result of vaporization (conversion of liquid to vapor) so that the cavity is filled primarily with vapor. Consequently, the bubble growth occurs "explosively" and the bubble collapse is "implosive".

Gaseous cavitation is caused by diffusion of dissolved gas into the cavity (degassing), so that the rate of growth and collapse is much more moderate. Pseudo cavitation could be categorized between the limiting cases of vaporous and gaseous cavitation. In this case, the bubble growth is caused by the combination of vaporization and degassing. Obviously, the less diffusion that takes place into the cavity, the more
violent the collapse that takes place. In the strict sense, cavitation usually seen in hydro-machines cannot be called vaporous cavitation but instead is actually pseudo cavitation.

In hydraulic machinery, the local static pressure can be reduced from ambient pressure because of fluid motion or moving surfaces. For example, it is well known that the pressure at a venturi throat is below ambient, which can be shown clearly by Bernoulli's theorem for steady flow. A great deal of pressure reduction is possible near an accelerating surface even if the motion of the surface and/or fluid is very small. In this case, the pressure reduction is caused by the inertia of fluid that tends to separate the fluid from the surface. Cavitation takes place, if the pressure is reduced and maintained for sufficient duration below a certain level. This level is determined by the physical properties and conditions of the liquid and/or contacting surfaces. Study about this level, i.e., study to analyze the mechanism of cavitation inception, is one of the major areas of interest in cavitation work and is the main subject in the dissertation.

Once cavitation takes place, it has some undesirable effects with respect to hydraulic machines. They are: the modified hydrodynamics of the flow due to the interruption of the continuity of the liquid phase as cavities appear, the lowered efficiency due to the power loss spent on the gas phase, and the most serious one—the significant material damage of contacting surfaces. The material "erosion" is caused by the violent collapse of cavities. Although the detailed mechanism of the damage has not been understood thoroughly, it is believed that the damage is caused by a "shock wave" and/or "micro jet". A shock wave is produced at the final moment of cavity collapse when the pressure at the
collapse center becomes extremely large due to the inertia of the fluid. In this study, over 3,000 mmHg positive pressure spike with very short time duration (within 50 μsec.) has been recorded near the cavitation region. A microjet is created only when the cavity is located adjacent to solid boundary. While the cavity is collapsing, the cavity wall near the boundary deforms faster than the other side due to the pressure difference created by the existence of the boundary. This results in the formation of microjet of liquid toward the boundary. It has been stated [38] that the velocity of the microjet can exceed 100 m/sec.

Cavitation in the blood circulatory system has been studied because it may cause blood cell destruction long before material damage becomes significant. The mechanism of this blood cell destruction is not totally understood, but can be due to either shock wave or microjet, or more likely a combination of these two produced by cavity collapse. Walker [41, 42] reported cavitation observed near the moving diaphragm of an artificial heart in a mock circulatory loop. Cavitation on a tilting disk prosthetic heart valve and the amount of red blood cell destruction have also been reported by the author, Walker, and Dube [39]. This cavitation is created due to the sudden stopping (decelerating) of the moving surface wherein the liquid tends to separate because of its inertia. In this study, a piston and cylinder system was used to model these situations. In the system, the moving piston driven by a spring stops suddenly, creating a pressure reduction in the cylinder and, as a result, cavitation on the piston face.

This dissertation details cavitation inception in this system using water as a working fluid. However, cavitation inception in blood was found to very similar to that in water quantitatively as well
as qualitatively.

Questions to be answered are: under what conditions cavitation takes place, what are the important parameters affecting cavitation inception, and how important are these parameters. Through the experiments, various methods for detecting cavitation were tried. These experiments provided acceleration-pressure recordings and high speed motion pictures. The "critical point" indicating whether or not cavitation takes place then had to be defined and was measured in terms of peak acceleration. Because of difficulties with equipment for measuring the pressure on the piston face directly, the procedure to calculate this pressure using measured values of acceleration was developed and the "critical pressure" for the incipient cavitation was calculated. Then parameters affecting this "critical pressure", such as system static pressure, temperature, and piston material, were investigated.

Theoretical calculations to analyze cavitation inception were performed. Since it was found from high speed motion pictures that the "critical point" occurs when a few small hemispherical cavities (1-2 mm dia.) appear on the piston, the "critical pressure" could be calculated by using an existing bubble dynamics theory. Parameters affecting this "critical pressure" were then examined by means of this calculation and were correlated with the experimental results.
HISTORICAL BACKGROUND

Tensile Strength of Liquids

As the volume of a given mass of gas is increased at constant temperature, the pressure exerted by the gas decreases; but this pressure remains positive, however large the volume may be. The same is not true of liquids. For homogeneous liquids, the forces tending to hold liquid "particles" together are external pressure and intermolecular cohesive forces. Just as solid materials whose intermolecular forces are much larger than those of liquids can support large tension, liquids should be expected to support tensile stresses because of intermolecular forces. In the fluid at rest, the stress tensor is everywhere isotropic and only normal stresses act. Fluids at rest are normally in a state of compression, therefore, for convenience the stress tensor in a fluid at rest is written as;

$$\sigma_{ij} = -p\delta_{ij}$$

(1)

where

$$p = -\frac{1}{3} \sigma_{ii}$$

(2)

the quantity p may be termed the fluid static pressure. It is clear that when the fluid is experiencing normal tensile stresses, the pressure (not gage pressure) has a negative value.

It is well known that in ideal conditions, a homogeneous liquid can hold a large tensile stress before it is "ruptured". The largest stress that can be measured is the tensile strength of the liquid at that temperature. Depending upon the equipment and condition of the liquid and its container, measured data show rather wide scatter. Briggs [3] reported the largest values for water. Using a centrifugal
method he measured a tensile strength over 250 atmospheres for the temperature range of 5-32°C.

A number of theoretical calculations have also been tried. Among them, the simplest one is that due to Frenkel [10] who assumed that tensile strength is equal to the surface tension pressure. The surface tension pressure \( P_s \) is given by:

\[
P_s = \frac{2S}{R}
\]  

for a bubble of radius \( R \) and the surface tension \( S \). This pressure force acts in compression and tends to collapse the bubble. Frenkel estimated a tensile strength of 14,000 atmospheres for water by assuming a hole with a radius of one-half the mean intermolecular spacing. More sophisticated calculations by other investigators show lower values. For example, Finch [6] introduced the hole theory with pressure gradients where holes tend to drift and accumulate. He calculated an even lower tensile strength than the experimental values reported by Briggs.

Among other theoretical works, a very interesting calculation was done by Kuper and Trevena [25]. They showed that the reduction in the tensile strength of water caused by saturating it with air at a pressure of one atmosphere is less than 0.5%.

**Cavitation Nuclei**

While very large values of tensile strength for pure liquids have been reported, in ordinary liquids and their containers cavitation starts near the vapor pressure. This lack of significant tension is because of the existence of "weak spots" or cavitation nuclei. As Holl
[17] reviewed, there are various types of proposed nucleation sites. They are gas bubbles, gas pocket in a crevice, gas bubbles with organic skin, and hydrophobic solid.

In ordinary hydraulic systems, free gas bubbles are the most feasible nuclei for cavitation. However, according to Epstein and Plesset [5], small bubbles would dissolve in a short time due to surface tension. For example, in air-saturated water at 22°C, an air bubble of 10μ diameter should dissolve completely in less than 7 seconds, while it takes approximately 100 minutes for an air bubble of 100μ diameter. Nevertheless, there is considerable evidence for persistence of nuclei and various mechanisms have been proposed to explain this persistence. Harvey et al [11] have suggested that gas can be trapped in small crevices on hydrophobic solid surfaces. This idea has been accepted widely and extended by many investigators such as Apfel [1] and Miller and Rylander [28]. This model is discussed in the Appendix. Another well-known mechanism for nuclei persistence was proposed by Fox and Herzfeld [8]. They suggested that the nuclei are very small bubbles stabilized by an organic skin which mechanically prevents loss of gas by diffusion and can support the pressure difference between the inside and outside of the bubble. The other type of "weak spot" is a hydrophobic, i.e. non-wetted, solid in the case of a gas free system. Plesset [33] shows that the hydrophobic solid causes a weak spot in the liquid and the tension is considerably less than that of the pure liquid.

**Cavitation Inception**

In order to verify those nucleation models, numerous experiments to measure the tension (lowest pressure) required to initiate cavitation have been performed. A classical experiment is that due to Parkin and
Kermeen [31]. In this experiment, a 2 inch diameter hemispherical nosed body was tested in a 14 inch water tunnel in which ordinary tap water was used. The liquid tension was measured directly by means of a pressure transducer at the point of minimum pressure of the hemispherical nose. They reported liquid tensions of -20 to -180 mmHg depending upon the free stream velocity. By prepressurizing the system, Harvey et al [12,13] obtained higher tensions than in the untreated case. Obviously this suggests that the liquid tension required to initiate cavitation depends upon the size of nuclei. However, it is very difficult to quantitize the size and the number of nuclei. Keller [21] attempted to use a scattered light counting method and used a Coulter counter to verify his method [22]. Acoustic methods have also been investigated by several investigators such as Davids and Thurston [4], Silberman [36], and Straub et al [37], in which sound velocity or sound attenuation was measured in liquids containing bubbles. Obviously those methods are good only for bubbles suspended in liquids so that they are not applicable to nuclei on solid surfaces.

Since cavitation takes place easily on an object moving continually in water, many inception studies have been performed by using water tunnels. Holl [15] investigated the effect on hydrofoils of dissolved air content on inception of gaseous and vaporous cavitation. The effects of surface irregularities or surface roughness, which are important in turbulent boundary layers, have been reported by Holl [15] and more recently by Arndt and Ippen [2].

Cavitation can also be seen easily in underwater high power ultrasonic devices, and is called vibratory cavitation. Several investigations on this type of cavitation have also been done. However,
the mechanism of cavitation inception is quite different from that in a
flowing system because of the nucleation bubble growth due to so called
"rectified diffusion". During subcavitation level vibration, at the
wall of small nucleation bubbles in a vibrating pressure field, air
diffusion from the liquid into the bubbles occurs when the pressure
decreases and the local liquid is oversaturated. On the other hand, air
is redissolved when the pressure is high. Since the rate of air dif-
sion into the bubble is much larger than that of redissolving, the
diffusion seems to be "rectified" and the nucleation continues to grow
until cavitation starts.

The system of interest in the present study is a decelerating piston
and cylinder system in which cavitation bubbles appear mainly on the
piston face. Consequently, the piston surface is though to be a main
source of nuclei. Interestingly enough, this type of cavitation has
seldom if ever been investigated.

**Bubble Dynamics**

In general, cavitation induced growth and collapse of individual
cavities or bubbles is a very transient motion. The typical life time
of cavities is on the order of milliseconds. This transient motion,
especially the very rapid collapse of bubbles, is closely related to
noise, vibration, blood cell destruction, and material damage caused by
cavitation in hydraulic machines. As a result, there is great concern
about so called "bubble dynamics". If the spatial density of cavitation
bubbles is low enough, each bubble will behave independently of its
neighbors. Consequently, the situation can be treated as a simple
episode of expansion and collapse of a single bubble. This is felt to
be the case even though that episode is (especially the collapse) is not
so simple. If the cavitation bubble is located adjacent to a solid wall, which is a common occurrence, the bubble cannot keep its spherical shape in collapsing. Furthermore, it is not unusual that the collapse includes "rebounds" several times after the first collapse. "Rebound" is a regrowth and recollapse of cavitation bubbles.

Several investigators have tried to analyze the bubble motion theoretically as well as experimentally. According to Knapp [24], the first theoretical work was done by W. H. Besant in 1859. He introduced and formulated the concept of the empty cavity which is defined as follows: An infinite mass of homogeneous incompressible fluid acted upon by no forces is at rest, and a spherical portion of the fluid is suddenly annihilated. It is required to find the instantaneous alteration to pressure at any point of the mass, and the time in which the cavity will be filled. The pressure at an infinite distance is assumed to remain constant. In 1917, Lord Rayleigh proceeded to solve this problem as follows: For spherical symmetry the radial flow is irrotational, and there exists a velocity potential $\phi$ given by

$$
\phi = \frac{UR^2}{r}
$$

(4)

where $U$ is the cavity-wall velocity, $R$ is the radius of the cavity at time $t$, and $r$ is the radial distance which must be greater than $R$.

From equation (4), the velocity $u$ at position $r$ can be given by

$$
\frac{u}{U} = \frac{R^2}{r^2}
$$

(5)

Then, assuming that the fluid is inviscid as well as incompressible, the
kinetic energy of the entire body of liquid at time \( t \) is equal to the work done on the entire body of fluid as the cavity is collapsing from the initial radius \( R_0 \) to \( R \); i.e.,

\[
\frac{\rho}{2} \int_{R}^{\infty} u^2 \pi^4 r^2 dr = \frac{4\pi p_\infty}{3} (R_0^3 - R^3)
\]  

(6)

where \( \rho \) is the density of liquid and \( p_\infty \) is the pressure at infinity.

Since the pressure is zero inside and at the wall of the cavity, no work is done at the cavity wall. Using equation (5), equation (6) becomes

\[
U^2 = \frac{2p_\infty}{3\rho} \left( \frac{R_0^3}{R^3} - 1 \right).
\]  

(7)

Since \( U = \frac{dR}{dt} \), an expression for time \( t \) required for a cavity to collapse from \( R_0 \) to \( R \) is

\[
t = R_0 \sqrt{\frac{3\rho}{2p_\infty}} \int_{\beta}^{1} \frac{3\beta^2}{\sqrt{(1 - \beta)^3}} d\beta
\]  

(8)

where

\[
\beta = \frac{R}{R_0}.
\]

Although this Rayleigh solution has been well accepted and agrees rather well with experiment, in the real case the pressure inside the cavity is not zero because of the bubble contents and the surface tension. Recalling that cavitation is caused by a dynamic reduction of local static pressure, it follows that the pressure field may also vary.
Plesset [32] solved the following equations numerically and tried to analyze the cavitation observed on the head of the 1.5" caliber ogive projectile in a water tunnel.

The following relation was given by Lamb [26]: Assuming the spherical symmetry (irrotational), equations (4) and (5) are still valid. With constant density and gravity neglected, the equation of motion for the liquid is

\[
\frac{\partial \phi}{\partial t} + \frac{u^2}{2} + \frac{p}{\rho} = \frac{p_\infty}{\rho}
\]  

(9)

where \( p = p(r,t) \) = local pressure in the liquid, and \( p_\infty = p_\infty(t) \) = pressure at infinity in the liquid, where \( u = 0, \phi = 0 \). Hence, the motion of the bubble wall becomes

\[
-R \frac{dU}{dt} - \frac{3}{2} U^2 = \frac{p_\infty - p}{\rho}
\]  

(10)

where \( P = P(t) \) = pressure in the liquid at the bubble wall. Let the bubble be filled with gas and vapor, then assuming a perfect gas inside the bubble, the pressure at the bubble wall is given by

\[
P = p_v - \frac{2S}{R} + \frac{NT}{R^3}
\]  

(11)

where \( p_v \) = vapor pressure

\( S \) = surface tension

\( T \) = temperature

\( N \) = constant for fixed mass of particular gas

Plesset [32] neglected the gas content in the bubble and calculated the
cavitation bubble motion in his system by integrating equation (10) numerically from the maximum bubble radius to the positive time direction for the bubble collapse and to the negative time direction for the growth.

In equation (9), there is no viscous term because of assumed irrotationality and incompressibility. However, Poritsky [34] pointed out that the resultant of the viscous stresses may not vanish at the bubble wall and that equation (11) should be

\[ P = p_\infty - \frac{2S}{R} + \frac{NT}{R^3} - 4\mu \frac{U}{R} \quad (12) \]

More recent and sophisticated works on the spherical bubble dynamics were done by Trilling [40], Hickling and Plesset [14], Florshnetz and Chao [7], Ivany and Hammit [19], and Hsieh [18]. They investigated the effects of compressibility and thermodynamics on the bubble motion including rebound.

The nonspherical collapse of cavities near solid boundaries was studied experimentally by Kling and Hammit [23] and theoretically by Mitchell and Hammit [29]. Jones and Edwards [20], and Shutler and Mesler [35] later studied the dynamics of hemispherical bubbles attached to solid boundaries, in which their interests were on the material damage of solid boundaries. Nande and Ellis [30] then reported more complete bubble dynamics, in which they examined the nonhemispherical bubble collapse theoretically and experimentally.
EXPERIMENTAL APPARATUS

A piston-cylinder system, in which the piston driven by a compression spring stops suddenly, was used to create cavitation. When the piston stops, the inertia of fluid inside the cylinder causes a pressure reduction. Obviously the lowest pressure is achieved on the piston face; and if this low pressure exceeds a certain limit (below vapor pressure), cavitation takes place.

Figure 1 shows a schematic of the system used in this study. A clear plexiglas cylinder is shown attached to the front and the rear reservoirs. The piston is attached to a piston rod and can move back and forth smoothly inside the cylinder. Outside the rear reservoir, there is a driving unit attached to the piston rod. The solenoid can hold and then release the piston with some initial predetermined displacement. These three units, the cylinder, reservoirs unit, the driving unit, and the solenoid, are attached separately to the base. The cylinder, made of clear polished plexiglas, has flat machined outside surfaces which make it easier to film piston motion and cavitation. With a round-outside cylinder, there was some difficulty in getting enough lighting which is necessary for high speed photography. Two different sizes of cylinders have been used. These are 15 mm I.D. x 70 mm long and 17 mm I.D. x 90 mm long.

Reservoirs attached on both ends of the cylinder are also made of plexiglas. The dimensions of these reservoirs are as follows: for the front reservoir, 40 mm wide, 40 mm long, and 60 mm high; for the rear reservoir, 30 mm wide, 25 mm long, and 60 mm high. Due to the reservoirs on both ends, the piston can be submerged completely so that there is little chance for air to be introduced into the cylinder.
Both reservoirs can be sealed such that the system pressure can be controlled. Instead of the front reservoir, a flexible balloon could be used at the end of the cylinder. This has an advantage of reducing the system volume, which is convenient in some cases; however, this makes the liquid handling more difficult.

The piston is one of the most important components in this system. Teflon, Delrin, and pyrolytic carbon have been tried as piston materials. Each of these materials gives different nucleation site characteristics. Teflon and Delrin pistons were machined carefully in order to get the surface as smooth as possible. A pyrolytic carbon disk was removed from a Lillehei-Kaster prosthetic heart valve and glued to an aluminum base. At the early stage of this work, an aluminum piston with Teflon piston rings was tried. Although Teflon is the ideal material for piston rings, because of the minimum friction against plexiglas cylinder, it was found that cavitation occurs on the Teflon ring before it is seen on the piston face; in other words, Teflon has more or larger nucleation sites than aluminum. Hence, that type of piston was abandoned. The clearance between piston and cylinder was controlled within 0.05 mm in most cases in order to minimize the effect of leakage.

The driving unit consists of two plates attached to the base, a compression spring (usually a combination of two or three springs), a stop ring attached to the piston ring, and a damper between the piston ring and the front stop plate. As the piston is pulled back from its resting position, the spring is compressed between the stop ring and the rear plate. The solenoid can then hold the piston at the predetermined position. Upon release, the piston is driven forward until the stop ring hits the stop plate. By placing a damper between the stop ring and
the stop plate, smooth acceleration histories can be realized. Several combinations of spring and damper have been tried in order to get different shapes of acceleration histories.

An accelerometer is attached to the end of the piston rod thus enabling the acceleration of the piston to be recorded on a storage scope or on high speed motion pictures. The accelerometer used in this work is ENDEVCO Model 2225, which is a piezoelectric transducer and has a capability of up to 20,000 g measurement and an 80,000 Hz frequency response. A high frequency response is desired in this study because of the transient motion of cavitation bubbles. The measurements were taken with a charge amplified, ENDEVCO Model 2721, and the charge sensitivity of the particular accelerometer used was 0.733 pC/g.

Pressures have been measured with a Millar Micro-Tip pressure transducer, which is commonly used to measure pressures in the human circulatory system. The advantages of this pressure transducer are its compactness (1.6 mm O.D. flexible catheter) and the high frequency response. It is so compact that it can be installed almost anywhere without disturbing the flow. For example, it could be set on the piston face and could measure the critical pressure directly. Actually this has been tried but several of these expensive pressure transducers have been demolished by the violent collapse of cavitation bubbles. Even when the pressure transducer is located downstream, it functions as a good microphone due to the high frequency response. In this position it senses pressure waves caused by bubble collapse, which are used to determine whether or not cavitation takes place. These recordings of acceleration and pressure provide useful information about cavitation, but the most powerful method observing cavitation phenomena
is with the use of high speed motion pictures. In this study, a Hycam
16 mm high speed motion picture camera (Model 41-004 by Redlake Co.)
has been used. The maximum film speed of this camera is 11,000
frame/sec. The camera has the additional capability of imposing
oscilloscope tracings simultaneously on the film. The detailed operation
of the camera will be explained in the next section.

In the front reservoir, there is a 5 MHz ultrasonic probe (5 mm dia.
quartz) on a 1/4" stainless steel bar facing toward the piston. The
probe releases a short 5 MHz ultrasonic pulse every half millisecond and
receives echoes from objects located on its path. This device was to
detect the piston motion, the cavitation bubble existence, and small gas
bubbles left after cavitation bubble collapse. Unfortunately, it did not
work very well. The reasons are as follows: the 2 KHz pulse-echo signal
is not frequent enough to follow the stopping piston motion, the
boundary between bubbles and piston is not clear, and it is very diffi-
cult to correlate the echo counting with the size and the population of
small gas bubbles.

At the early stage of this study, a scattered light method for
detection of cavitation was tried. The system consisted of a laser
beam unit and a photo-diode aligned across the cylinder in front of the
piston face. The concept of this method is that if there exist bubbles
on the path, the laser beam is scattered on the bubble wall and the
intensity measured by the photo-diode is reduced. Although this method
is very popular in cavitation studies, it was found to be inappropriate
due to the uncertain position of cavitation bubbles on the moving
piston.
EXPERIMENTAL PROCEDURE

As noted earlier, the purposes of this study were to find the "critical point" of cavitation inception and to examine parameters affecting it. In the present section, methods used for those purposes will be explained.

Experimental Preparation

Water used in the experiments was distilled in the laboratory and stored at room temperature (24-26°C). The water was then poured slowly into the front reservoir along the side wall. The purpose of this procedure was to avoid air bubbles in the system. During this time, the piston was positioned in the rear reservoir. After the piston was submerged completely, air bubbles adhering to the piston and the cylinder were removed by using a syringe. At least 15 minutes were taken to begin the experiment in order that tiny air bubbles could be dissolved and/or conditions of nucleation sites could be stabilized. Great care was exercised so that any visible bubble would be eliminated in the cylinder and on the piston. If air (gas) bubbles were introduced in the system during the course of the experiment, they were removed or the water was changed. After filling the system and setting the pressure transducer, the driving unit, the accelerometer, and the solenoid were connected and the piston was set at a predetermined position to give the desired "cylinder length". After setting the reservoir water level, the solenoid was clamped into position. Initial tests were always in the subcavitation region, and it was checked that smooth acceleration histories were achieved. If it was not smooth, all of the units were realigned correctly or another damper tried until smooth curves were achieved. In every shot, the piston was pulled back by hand and held by
the solenoid at an initial displacement position. The piston was then released by switching off the solenoid. Two damper thicknesses were used to obtain different acceleration histories. They are herein described as the thin damper and the thick damper.

"Critical Point"

From the subcavitation level, where both the acceleration and the pressure history are smooth (shown in Figure 2), the initial displacement is increased little by little until cavitation is detected on the acceleration and the pressure recording. A typical record of a shot in the cavitation region is shown in Figure 3. On the recordings, cavitation is recognized by disturbances seen on the acceleration history after the peak and by a bottom flat period of pressure followed by high frequency signals which correspond with disturbances on the acceleration. High frequency signals on the pressure and disturbances on the acceleration records are due to violent collapses of cavitation bubbles. Cavitation bubbles exist during that flat pressure period. For a small cavitation episode, however, the flat period is not quite flat but high frequency signals can be seen as long as it cavitates.

When the initial displacement of the piston is increased little by little, the peak value of deceleration gets larger and in the transition from subcavitation level to cavitation level there is an initial condition which gives a sharp positive spike on the pressure and a sharp negative spike on the acceleration. This is herein defined as "the critical point" and the peak value of acceleration is defined as the "critical acceleration". A typical recording of the critical point shot is shown in Figure 4. Using this critical acceleration, the "critical
pressure" for this "critical point" can be calculated.

Calculation of Pressure on Piston

As mentioned earlier, while the piston is stopping, the pressure inside the cylinder is reduced due to the inertia of the liquid. Because of some difficulties in equipment, the direct measurement of pressure on the piston was not possible. However, from the momentum equation of fluid motion, the instantaneous pressure on the piston, where the minimum pressure is achieved, can be calculated in terms of fluid mass in motion, fluid motion (acceleration), and the ambient pressure.

First of all, let it be assumed that the liquid is incompressible and that the viscous forces are negligible along the cylinder wall and reservoir walls. As shown in Figure 5, the volume of fluid in motion is divided into two parts, control volume 1 and control volume 2. Then, assuming uniform velocity distributions across the cross section, the flow in each control volume can be treated as a simple one-dimensional flow. For control volume 1, the momentum equation becomes

\[
(P_p - P_i) A_p = \rho L A_p a_p 
\]

(13)

where no flow through the gap between piston and cylinder was assumed. For control volume 2, the momentum equation becomes

\[
(P_i - P_a) A_R = \rho H A_R a_R 
\]

(14)

where the gravitational force was neglected. From the continuity equation,
\[ A_P V_P = A_R V_R \]  

(15)

Hence,

\[ A_P a = A_R a_R \]  

(16)

Eliminating \( P_i \) from equations (13) and (14), and using equation (16), the pressure on the piston \( P_p \) is given by

\[ P_p = P_a + (L^* + \frac{A_P}{A_R} H^*) a \]  

(17)

Obviously, the interface of these two control volume systems is not so simple. Here, it was assumed that the pressure at the end of cylinder, \( P_i \), is uniform over the cross section of the cylinder and is the same as the pressure at the cylinder-center level in the reservoir, where that pressure was also assumed to be uniform over the horizontal cross section of the reservoir. The validity of this calculation will be discussed throughout this dissertation. However, it is clear from equation (17) that the smaller value of \( A_P/A_R \) and or the larger value of \( L^* \) would decrease the effect of the fluid motion in the reservoir.

Using the measured critical acceleration in equation (17), the critical pressure, \( P_c \), is calculated by

\[ P_c = P_a + (L_c^* + \frac{A_P}{A_R} H^*) a_c \]  

(18)

where \( L_c^* \) is the cylinder length at the instant the critical acceleration is recorded. Since the damper is compressed more at that instant, \( L_c^* \) is
shorter than the cylinder length, L, set initially at the experiments. The difference, \((L - L_c^*)\), can be a function of many variables, such as force applied by the spring, damper thickness and hardness, and friction in the system. From observations of piston motion on high speed motion pictures, \((L - L_c^*)\) can be approximated in terms of peak acceleration, which should be within the range of \(-150\) to \(-350\) g, as follows:

\[
\text{For "thin damper":} \quad L - L_c^* = 0.06 - 0.0002 a_c
\]

\[
\text{For "thick damper":} \quad L - L_c^* = 0.11 - 0.0002 a_c
\]

where \(a_c\) is measured in g's (gravitation acceleration) and length is presented in cm.

In the present system, values of \(A_p/A_R\) are: for 15.2 mm I.D. cylinder \(A_p/A_R = 0.1133\); and for 17.0 mm I.D. cylinder \(A_p/A_R = 0.1419\); and \(H^*\) is given by \(H^* = H - 2.0\) cm.

Parameters

This "critical point" measurement including the "critical acceleration" measurement and the "critical pressure" calculation, has been done under various test conditions. Parameters which have been examined are:

1. cylinder size (length and bore)
2. water level in reservoir
3. shape of acceleration history (pressure history)
4. system static pressure
5. temperature
6. piston material
7. surface tension
8. viscosity

High Speed Motion Pictures

High speed motion pictures have been taken in order to determine the piston motion (position and speed) and also to obtain general information about cavitation phenomena in the system. For example, it is necessary to know how and where cavitation bubbles grow and how and when they collapse. For most cases, a Pentax 100 mm Macro-lens (F4.0) with Pentax Close-up Extension Tube No. 2 was used as a main lens. The light source was a 250 w flood lamp located behind the cylinder facing toward the camera in order to obtain enough light. For simultaneous oscilloscope recordings, a Pentax 55 mm lens (F1.8) was set as an auxiliary lens on the other side of the camera where the eye piece was usually located and the scope directed toward this lens. Since there is no shutter between lens and film, it is a streak recording and the signal on the scope was not swept. Actually, two dots are positioned at the center of the screen with some distance. One of them is for the acceleration signal and the other one has no input and is used as a base line. Because the image from the auxiliary lens falls behind the film, no backing film, such as Kodak RAR Film 2484 and 2479, should be used. In most cases, a film speed of 3,500 to 7,000 frame/sec has been used; and the exact film speed is given by lines marked every one millisecond on the side of the film.
EXPERIMENTAL RESULTS AND DISCUSSION

Effects of Cylinder Size

Experiments were performed with two different sizes of cylinders. In the 15.2 mm I.D. cylinder, the critical point was measured for lengths of 3, 4, and 5 cm. In the 17 mm I.D. cylinder, the critical point was measured for lengths of 5, 6, and 7 cm. In all cases, a Teflon piston was used and experiments were conducted at room temperature and at atmospheric pressure. Results are shown in Table 1. It is clear that the longer the cylinder length, the less the absolute value of critical acceleration. In other words, the larger the mass of fluid in motion, the more easily cavitation takes place. This is also obvious in equation (17) or (18), where the larger value of \(-a\) is necessary in order to create the same value of pressure on the piston in the shorter cylinder. Since the variation of cylinder bore was only 2 mm, there was no significant effect on the critical acceleration. The cylinder bore appears in equation (18) in terms of cross sectional area ratio, \(A_p/A_R\). Hence, the effects of cylinder bore should be discussed with those of reservoir size and/or fluid motion in the reservoir.

Although effects of cylinder size (length) strongly influence the critical acceleration, the cylinder size (system size) itself should affect neither properties of the liquid nor characteristics of cavitation inception of the system. The obvious variation of the critical pressure data in Table 1 could be due to various pressure histories. Those effects are explained later.

Effects of Reservoir Level

Due to the dimension of the reservoir (6 cm high, and the center of cylinder is 2 cm from the bottom), the reservoir level, \(H\), could be
controlled only from 4 cm to 6 cm. Within this range, the effect of gravitational force is negligible. The experiments were done in the 15.2 mm I.D. cylinder for L = 5 cm and a "thick damper". Results are shown in Table 2. Effects of reservoir level on the critical acceleration were small but not negligible. In the experiments, the acceleration difference of 5 g for a 1 cm reservoir level difference was barely measurable. From equation (18), it is clear that the reservoir level becomes less important for longer cylinder length and smaller values of $A_p/A_R$. Like cylinder size, the reservoir size itself does not affect the critical pressure. However, the critical pressure may not be calculated by equation (18) if the level is low and assumptions made to derive the equation cannot be held. For this reason, the reservoir level was kept above 4 cm.

**Effects of Pressure History Shape**

In order for a cavitation nucleus to grow into a cavitation bubble filled mainly with vapor, the local static pressure should drop below the vapor pressure for a certain time duration. Hence, for such pressure histories in this system, the peak pressure and the slope (steepness) of the pressure history are very important factors to determine the size (magnitude) of cavitation. Here, the measured critical point, which is the case of small cavitation, is dependent upon the slope of the pressure history. In order to describe the steepness of pressure histories, a time duration, $T_{1/3}$, was defined on the acceleration recording as the time from the point where the acceleration was $2/3 a_c$ to the peak acceleration point.

Some of the data are shown in Table 1 and plotted in Figure 6. As shown clearly in Figure 6, the relation between $T_{1/3}$ and $P_c$ is almost
linear. A smaller value of $T_{1/3}$ means a steeper pressure history; i.e., it is more difficult to cavitate with a steeper pressure history. It is also seen in Figure 6 that there is more scatter of data in the steeper pressure history region. The dotted line represents the correlation of theoretical modeling, which will be discussed later.

**Effects of System Static Pressure**

Effects of system static pressure, $P_a$, were examined for a pressure range of -200 to +200 mmHg. Again, a Teflon piston was used in the 15.2 mm I.D. cylinder. First of all, the effect of system pressure appears on the critical acceleration. It is obvious from equation (18) that if the critical pressure does not change, more deceleration (negative acceleration) is necessary for a higher value of $P_a$. Data shown in Figure 7 were obtained by several series of experiments of $L = 3, 4, \text{ and } 5$ and with a thin damper. This effect on the critical pressure can be thought due to the change of nucleation size and/or the shape of the pressure history. When the system pressure is raised, the nuclei, usually in the shape of small air bubbles suspended in the liquid or air pockets in surface crevices, decrease their size due to simple compression and (maybe more importantly) due to increased air solubility. For a decreased system pressure, the phenomena are opposite. Obviously smaller nuclei give a higher critical point. In Figure 7, the curve shows a rapid drop in the higher system pressure region. This could be due to the rapidly increasing surface tension force as the bubble gets smaller. By the nature of this system, the pressure history becomes steeper for larger deceleration, which is needed for higher system pressure. In addition, even if the acceleration history is the same, which means that $T_{1/3}$ is identical, and even
if the calculated peak pressure is the same, the resulting pressure histories for different system pressures may not have the same slope.

Hence, for higher system pressure, the pressure history gets steeper and the critical pressure should be lower. Consequently, data shown in Figure 7 should have this effect also. It should also be noted that more scattering of data is seen in the higher static pressure region; and this could coincide with the scattering in the steeper pressure history region of Figure 6.

Effects of Temperature

Effects of temperature were examined for the temperature range of 4°C to 47°C by using a Teflon piston in a 15.2 mm I.D. cylinder. Figure 8 shows two series of experiments, L = 4 and 5 cm, with thin damper. Since variations of critical acceleration are within 30 g for both cases, the effect of steepness of pressure history can be neglected. The system temperature can be thought as affecting the critical pressure in two ways. First of all, the vapor pressure is a strong function of temperature, and the higher the temperature, the higher the vapor pressure. Hence, it is easier to vaporize the water and easier to cavitate. The system temperature also affects the surface tension, but its effect can be neglected because of the relatively small variation of surface tension over the temperature range of interest. The effect of surface tension will be discussed in a later section. Another important effect of temperature is on the state of nuclei. Although the simple expansion of air bubble by heating is small, there could occur some diffusion of air into the bubble so that the critical point is lowered. The two solid lines in Figure 8 were given by theoretical modeling and will be explained later.
**Effects of Piston Material**

Since the lowest pressure is achieved at the piston face and small air bubbles have a relatively short lifetime in the body of water, the piston face can be considered as the main source of nucleation sites. From the high speed motion picture study, which will be discussed later, it was confirmed that usually cavitation bubbles appear on the piston face first. Hence, the surface condition of the piston should be very important on cavitation inception.

In most cases of experiments, the Teflon piston was used to examine various parameters, and it gave very repeatable results on the critical point measurement. Two different kinds of piston material, rather than Teflon, were also examined.

A Delrin piston was tried in the 15.2 mm I.D. cylinder and a pyrolytic carbon piston was tried in the 17 mm I.D. cylinder. Table 3 shows a series of experiments with the Delrin piston. The critical point in Run #2 was noticeably higher than that for the Teflon piston and shortly after the cavitation episode (Run #3) the critical point was in the level for Teflon piston. Actually the critical point measurements for the Delrin piston are not repeatable at all. The same tendency was seen for the pyrolytic carbon piston. This higher critical point (lower critical pressure) is evidence of the fact that Delrin and pyrolytic carbon have smaller and/or fewer nucleation sites than Teflon. This could also be shown by scanning electron microscopic studies and by the crack model shown in the Appendix. The SEM pictures in the Appendix show that the Teflon surface machined by lathe has many crevices along with machining grooves. Machining grooves are rather shallow so that they may not become nucleation sites. However, crevices (15 – 20μ wide)
look very deep and due to the large contact angle of Teflon (less wettable), air inside the crevice could not be dissolved completely and may become a nucleation site for cavitation. On the other hand, Delrin and pyrolytic carbon surfaces do not have such crevices and the contact angles of these materials are much less (more wettable); hence, the nucleation sites, if they exist, are much smaller than those on Teflon. The number of nucleation sites on the material can be checked by decreasing the static pressure of material submerged under water in a vacuum chamber. Teflon and Delrin pistons were submerged for 24 hours before testing, and it was found that several bubbles appeared on Teflon while no bubble expansion occurred on Delrin.

The critical pressure increase in Run #4 in Table 3 can be explained by the existence of small bubbles left after a cavitation episode (Run #3). During cavitation phenomena, a small amount of air diffusion into a cavity is expected due to the low pressure. Then, after the cavity collapses, it takes a certain time for that air to be redissolved. That dissolving time is dependent upon the size of air bubbles. And, if the system experiences another pressure reduction (inducer of cavitation) before all air bubbles are redissolved completely, they can be nucleation sites. In the case of Delrin, remaining small bubbles were greater than nucleation sites on the surface and lower the critical point. On the other hand, since the measurements were very repeatable for Teflon, those remaining bubbles may be less than the gas pockets in surface crevices.

Effects of Surface Tension

Effects of surface tension were examined by using water with dishwashing detergent. The surface tension was measured by capillary method and decreased up to half of that of water alone. The experiments
were carried out very carefully, avoiding any visible air bubble in the system. However, a noticeable difference on the critical point was not found. Hence, effects of surface tension are negligible in this system.

**Effects of Viscosity**

The viscosity of the fluid was changed from approximately 1 centipoise for normal water up to 3.6 centipoise by adding glycerol to the water. Within this range, effects on the critical point measurement were negligible. If nuclei are in the form of gas pockets in surface crevices and if they are small and located deep in the crevices, viscosity would affect the cavitation inception in the way that higher viscosity increases the time for the interface of gas and liquid in the crevice to reach the piston surface, resulting in a higher critical point. This effect was not seen in the experiments.

Some effects of viscosity were observed with large clearance between the piston and cylinder. As shown in Table 4, the critical deceleration increased considerably for larger clearance. This is because for larger clearance the flow through it cannot be neglected. Hence, equation (18) cannot be applied to calculate the pressure on the piston. Although it is very difficult to account for this effect in the pressure calculation, it is obvious that higher viscosity would decrease the flow through the gap. This can be seen in Table 4, where the critical deceleration increase for larger clearance was less in higher viscosity liquid. Since there is no effect of viscosity change for small clearance, it can be said that the flow through the gap is negligible and equation (18) can be applied in this case.

**High Speed Motion Pictures**

A number of high speed motion pictures have been taken in order to
collect general information about piston motion as well as cavitation bubble motion. Data of piston motion (piston position and acceleration) were used to calculate the pressure history. The bubble motion appearing on the film was then correlated with the theoretical model of bubble motion under the simulated pressure history and will be explained in later sections.

Figure 9 shows a typical cavitation episode in the system. After the piston was released with an initial displacement of 6.3 mm (the releasing point is not shown in the Figure), the piston speed reached a constant speed of approximately 1.8 m/sec, which was maintained until the stop ring hit the damper. From the pictures, the piston starts decelerating in Frame 4 and it reaches the peak deceleration point in Frame 15. In Frame 14, several small bubble formations can be seen on the piston. As they continue to grow, some of the neighboring bubbles are combined together, forming larger bubbles. Naturally, if the piston deceleration is so great that the bubbles continue to grow, they are combined further, and then the piston is separated from the column of water. In general, two kinds of bubble forms can be observed. One is the hemispherical bubble usually seen on the piston face near the center. Another form of cavitation bubble can be seen at the edge of the piston. In Figure 9, a formation of "vapor cloud" is shown around the edge of the piston. This vapor cloud seems to remain on the cylinder wall and keeps its position while the piston moves backwards due to the compressed damper action. The bubble growth appears to start just before the peak acceleration and continues after the peak acceleration. Then, after reaching the maximum bubble size, the bubbles start collapsing. In Figure 9, the collapsing starts in Frame 19 or 20 and the first collapse
takes place in Frame 21, followed by two rebounds (the second collapse in Frame 25 and the third collapse in Frame 27). The acceleration recording during the collapse shows some disturbance from Frame 22 to Frame 27. The acceleration loses its trace on the pictures. This disturbance can be seen more clearly on the scope (shown in Figure 3) and also appears on the pressure recording. This can be explained by the same mechanisms proposed to explain material damage due to cavitation. These mechanisms are the "microjet" formation due to the existence of the solid boundary and the shock wave created by regrowth of bubbles (rebound). Since the cavitation bubbles have variations in sizes, the collapsing time is different such that the disturbance appears in high frequency.

Figures 10, 11, and 12 show the high speed motion pictures (Film #7810, 7814, and 7815) taken near the "critical point". Observed cavitation bubbles are hemispherical for most cases and have maximum sizes of 0.4 - 0.8 mm radius. In these cases, the acceleration and the piston position were read on each frame and the pressure history was calculated by equation (17). Figures 13, 14, and 15 show the results for each film. For Film #7810, the calculated peak pressure was -66.2 mmHg, while the critical pressure for the same setup was -22.2 mmHg (shown in Table 1). Hence, the cavitation episode in this case seems above the critical point. For Film #7814, the peak pressure was -44 mmHg and the critical pressure should be -33.5 mmHg; and for Film #7815, they are -55 mmHg and -42.9 mmHg. Hence, these cases were taken almost at the critical point. It may be noted that the "critical point" which was defined on the acceleration-pressure recording is the small cavitation case in which several hemispherical bubbles appear whose
maximum size is about 1 mm in diameter. Presumably, the number of bubbles seen on the piston face is directly related to the number of nucleation sites and does not affect the critical point. While only two bubbles can be seen in Film #7810, several bubbles are observed in Film #7814 and Film #7815. Since Film #7814 and 7815 were taken consecutively with the same piston, the similar bubble distribution can be seen on the piston.
THEORETICAL MODEL

Bubble Dynamics

From high speed motion pictures taken at or near the "critical point", such as those shown in Figures 10, 11, and 12, it was found that at the "critical point" which was defined by the acceleration-pressure recordings (explained in Experimental Procedure) a few hemispherical bubbles appear on the piston face. Consequently, this critical point could be redefined in terms of bubble dynamics; i.e., in terms of the maximum bubble size and the bubble collapsing speed. It is obvious that the number of bubbles shown in the pictures is directly related to the number of nuclei. Then, if it is assumed that the number of nuclei does not affect the critical point at the "critical pressure", this critical point can be simulated theoretically by the motion of a single bubble.

Several simplifying assumptions have been made. First of all, by neglecting all of the effects due to a solid boundary (piston face), the bubble can be treated as a spherical bubble. It was then assumed that the nucleus is a small bubble filled with air and vapor and that the bubble keeps its spherical shape throughout the motion, growth, and collapse. Further, assuming that the effects of cylinder wall and other bubbles are negligible; i.e., the distances from the bubbles to the cylinder wall or to the other bubbles are large compared with the bubble size, and also assuming that the pressure gradient along the cylinder length is neglected, the flow becomes spherically symmetric and irrotational. Thus, equations (10) and (11) are valid.

\[-\frac{dU}{dt} - \frac{3}{2} U^2 = \frac{P_{\infty} - P}{\rho}\]  

(10)
\[ P = p_v - \frac{2S}{R} + \frac{NT}{R^3} \]  \tag{11}

Viscous forces have been neglected in equation (10) and (11). Equation (10) is a second order non-linear ordinary differential equation and cannot be integrated analytically. As shown before, Plesset [32] performed the integration numerically for the bubble whose inside pressure is constant. He started the integration at a maximum bubble radius and calculated the bubble growth and collapse by integrating for both directions in time.

Here, equation (10) is integrated numerically by using the "Runge-Kutta" method [9]. The differences from the work by Plesset are: the calculation is started from a stable nucleation gas bubble and performed continuously through the bubble growth and collapse. The initial conditions are the radius of the nucleation gas bubble and zero velocity of the bubble wall, while Plesset used a maximum radius of cavitation bubble and zero velocity of bubble wall as initial conditions. Due to the assumption of a stable gas bubble before the pressure reduction starts, the pressure at the bubble wall, \( P \) should be equal to the pressure at infinity, \( p_\infty \left( p_a \right) \). Hence, from equation (11), the constant quantity NT can be calculated, where air is assumed to be a perfect gas. Here, the whole bubble motion is assumed to occur isothermally, so that \( P \) becomes a function of \( R \) only.

The quantity \( p_\infty \) was defined as the pressure at infinity in the liquid. Due to the assumptions made previously, the pressure on the piston, \( p_p \), can be used for \( p_\infty \). From the acceleration recording along with the piston position from each frame of high speed motion picture, the pressure on the piston face, \( p_p \), can be calculated for each frame by
using equation (17), which gives a pressure history such as those shown in Figures 13, 14, and 15. In this theoretical modeling, the pressure history can be approximated by the equation

\[ p_\infty(t) = -Ae^{-B(t-t_0)^2} + p_a + p_o \]  

(21)

where \( p_\infty(0) = p_a \) and \( p_\infty(t_0) = p_{\text{peak}} = -A + p_a + p_o \).

The computer program was written in Fortran IV and PDP-03 microcomputer has been used to solve the bubble motion and to examine some parameters.

The computer program and the typical printout of the results are shown in the Appendix.

**Calculation Results**

Calculation results are shown in Figure 16 through Figure 26. Figure 16 shows time histories of bubble radius (the initial radius is 0.05 mm) and bubble wall velocity under the pressure history of \( p_\infty = -794.65 e^{-2(t-1.5919)^2} + 765 \) in which the peak pressure, \( p_{\text{peak}} \), is 

-29.65 mmHg and \( p_\infty \) at \( t = 0 \) is 760 mmHg, atmospheric pressure. The surface tension was set at 72 dyn/cm and the vapor pressure was set at 26 mmHg, which are the values of water at room temperature (24-26°C). The actual computer printout of this case is shown in the Appendix. In this particular bubble motion, the bubble starts growing rapidly just before the peak pressure, then 0.4 m/sec after the peak pressure the bubble reaches its maximum size and 0.543 m/sec after the peak pressure the bubble collapses completely. The wall velocity has a positive (growing) maximum value of 2.09 m/sec just after the peak pressure and at the last step of collapsing the wall velocity exceeds 50 m/sec.
Bubble motions for various initial size bubbles under a similar pressure history are shown in Figure 17. Naturally, the larger nucleation bubble will grow more and collapse later. The point when a rapid bubble growth occurs is more clear for smaller nuclei. Figure 19 shows the relation between initial bubble size, \( R_i \), and maximum bubble size, \( R_{\text{max}} \), under four different pressure histories. This relation is not quite linear, especially in the smaller \( R_i \) region (less than 0.05 mm), which can be explained by the rapidly increasing surface tension force as the radius of the initial bubble gets smaller.

It is obvious that a nucleation bubble will behave differently for various pressure histories. Figure 18 shows bubble motions under various shapes of pressure histories. A 0.05 mm radius air bubble grows to a 1 mm radius cavitation bubble. It is seen that the steeper the pressure history, the greater the pressure reduction should be. Since the steeper pressure history means faster pressure recovery after the peak pressure, the cavitation bubble has a shorter lifetime.

Figure 16 shows the relation between the initial nucleation bubble size, \( R_i \), and the maximum bubble size, \( R_{\text{max}} \), for five different pressure histories which was set in Figure 18 such that a 0.05 mm radius bubble grows to a 1 mm radius. It is found that for a steeper pressure history, the maximum bubble radius is less sensitive to the initial bubble size.

Figure 20 shows the maximum bubble sizes given under various pressure histories. Two cases of initial bubble size \((R_i = 0.02, 0.05 \text{ mm})\) were examined for three values of \( B \) (0.5, 1.0, and 5.0) which determines the slope of the pressure histories. In Figure 20, it is clearly shown that the greater the pressure reduction \((-p_{\text{peak}})\), the
larger the bubble grows, and it is also clear that the steeper the pressure history (the greater value of B), the less the bubble grows. The most interesting fact in this Figure is that there is a pressure reduction level under which the value of B (steepness of pressure history) does not affect the bubble growth. Those levels are $p_{\text{peak}} = 33 \text{ mmHg}$ for $R_1 = 0.05 \text{ mm}$ and $p_{\text{peak}} = 23 \text{ mmHg}$ for $R_1 = 0.02 \text{ mm}$. These points can be considered as a kind of "critical point"; however, they are not practical because the bubble growth for these points are still very small (less than five times $R_1$) and there is almost no way to detect that kind of bubble motion in experiments.

As discussed earlier, the "critical point" defined in the experiments by means of acceleration-pressure recordings could be redefined by using this theoretical model. Here the "critical pressure" is redefined as the peak pressure, $p_{\text{peak}}$, such that a small nucleation bubble grows to a certain maximum size. Assuming that this size is 1 mm radius, which is evaluated later, the following results were found: Figure 21 shows the relation between the initial bubble size and various pressure histories (given by value of B and peak pressure), which represent the critical point. It is clear that the smaller the initial bubble size and the steeper the pressure history, the harder to cause cavitation; i.e., the greater pressure reduction is necessary ($p_c$ gets smaller). The relation between the initial bubble size and the critical pressure is almost linear for the same value of B except the small initial size region, where the surface tension effect gets larger as the initial size decreases. The dotted lines in Figure 21 are the cases in which the surface tension is one-half (36 dyns/cm). Obviously the effect of surface tension is rather small.
2 to 5 mmHg difference on the critical pressure). Judging from slopes of the curves in Figure 21, the critical pressure is more sensitive to the initial bubble size for the steeper pressure history (higher value of B). This is more clear in Figure 22, where the steepness is described by the time duration, $T_{1/3}^*$. $T_{1/3}^*$ is defined as the time duration from the point at which the pressure is $\frac{1}{3}(p_a - p_\text{peak})$ to the peak.

Effects of ambient pressure, $p_a$, were examined for the range of ±200 mmHg from atmospheric pressure. Figure 23 shows the relation between initial bubble size and critical pressure for various pressure histories. Effects of ambient pressure on the critical pressure look rather serious for smaller nucleation bubbles in this Figure. However, it should be noted that from the nature of equation (21), pressure histories near the peak for the same values of B and $p_o$ but different $p_a$ are not the same, even though the peak pressures are set equal. The higher ambient pressure gives steeper pressure history and the steeper pressure history gives higher critical point as shown in Figures 17 and 21. In order to eliminate this effect, the value of $p_o$ was controlled such that $p_o = 5$, for $p_a = 960$, $p_o = 205$ for $p_a = 760$, and $p_o = 405$ for $p_a = 560$. Hence, the pressure histories for these ambient pressures are identical. The difference in calculations is the starting point where the initial bubble size is set and the equilibrium pressure inside the bubble is calculated. Results are shown in Figure 24. The effect of ambient pressure gets smaller as the initial bubble size decreases. It should be noted that the higher ambient pressure gives a lower critical point for the same initial bubble size, which is opposite from the results in Figure 23.

Effects of temperature are shown in Figure 25 and 26. Values of
vapor pressure and surface tension used for each temperature are:

for $5^\circ C - p_v = 6.53 \text{ mmHg}; S = 74.9 \text{ dyn/cm}$, for $15^\circ C - p_v = 12.8 \text{ mmHg};$

$S = 73.5 \text{ dyn/cm}$, for room temperature $- p_v = 26 \text{ mmHg}; S = 72 \text{ dyn/cm},$

and for $40^\circ C - p_v = 55.3 \text{ mmHg}; S = 69.6 \text{ dyn/cm}$. As shown in Figure 25, the relations between the initial bubble size and the critical pressure are almost parallel for each value of $B$. Obviously the higher the temperature, the larger the critical pressure. For the case of $40^\circ C$ and $B = 1$, the critical pressure is positive for almost the entire range of initial bubble size, which means that no tension in the liquid exists in this situation. Figure 26 shows the effects of temperature on the critical pressure for fixed initial bubble size and fixed value of $B$. 

COMPARISON BETWEEN EXPERIMENTAL RESULTS AND THEORETICAL MODEL

From high speed motion pictures with acceleration data superimposed, the pressure history on the piston face can be calculated by using the instantaneous acceleration and piston position data in equation (17). The results of this calculation is shown in Figure 13, 14, and 15 for Film #7810, 7814, and 7815. For each pressure history, the best fitted curve presented by equation (21) was found. Here, the fit for the lower pressure part is more important since the major bubble growth starts near the vapor pressure. Then, under the given pressure history, the motion of the bubble with a certain value of initial size was calculated and compared with the bubble motion observed on pictures.

Figure 27 shows the correlation for Film #7810. It is seen that the simulated pressure curve fits the experimental points under 200 mmHg excluding the pressure recovery side. Here, for convenience, a symmetric pressure curve was used and the bubble motion was calculated for \( R_1 = 0.005 \) mm (10\( \mu \) diameter). As shown in Figure 27, even from this small initial size, the calculated maximum size is almost twice as large as the one observed. The dotted line represents the bubble motion for higher peak pressure and a more reasonable bubble size. It agrees with the experimental point for bubble growth, although the bubble collapse took place much later in the experiment. If this is because of the effects of viscosity and/or adhesion on the piston, the bubble motion in Film #7810 is simulated by the dotted line rather than the original calculation. It should be noted that the actual peak pressure was higher than the calculated value. This can be explained by the existence of cavitation bubbles. Before discussing the peak pressure, it is instructive to consider the pressure history after the peak.
In Figure 13, equation (17) was used to obtain experimental points on the pressure recovery side. However, while cavitation bubbles exist, equation (17) cannot be applicable because the pressure inside the bubble is almost constant near the vapor pressure and for large cavitation the liquid motion is different from the piston motion. For cavitation above the critical point, an essentially constant pressure was recorded in the cylinder while cavitation bubbles exist (shown in Figure 3). For the critical case, shown in Figure 4, the pressure recovery looks much slower than that calculated by equation (17). Actually, the existing bubble effect on the pressure on the piston becomes significant as soon as the bubble grows from its submicroscopic size to visible size. In Figure 10 small bubbles appear on the film before the peak acceleration, so that the actual pressure from that point, including the peak pressure, should be higher than the values calculated by equation (17). Although the calculated critical pressure is not a true value, it is still convenient to present the critical point in terms of critical pressure.

Figure 28 and 29 show the correlation for Film #7814 and 7815. Disagreements on the bubble motion are less than Film #7810, possibly because of smaller bubbles in this case. However, the maximum sizes calculated are still larger than those observed. For the more precise comparison of bubble motions, higher film speed and higher magnification are necessary, which are difficult in the current apparatus.

In the theoretical modeling, the critical point was redefined in terms of maximum bubble size and it was set at 1 mm radius. For those pressure histories simulated in Figure 28 and 29, the initial radii to grow to 1 mm are 0.053 mm and 0.06 mm respectively. From SEM pictures
(shown in the Appendix), several surface crevices were found which have typical dimensions of 15μ - 20μ wide and 50μ - 60μ long. Hence, even though there is a difference of shape between spherical bubble and gas pocket in a crevice, the initial bubble size of 0.01 - 0.03 mm radius may be more appropriate for the theoretical modeling. Using the critical pressures given in Table 1 as the peak pressures in the cases of Film #7814 (L = 5 cm, H = 5 cm, and thin damper) and Film #7815 (L = 3 cm, H = 5 cm, and thin damper), the maximum bubble radius for both cases becomes 0.65 mm for $R_1 = 0.01$ mm and 0.72 mm for $R_1 = 0.02$ mm. Although the radius of 1 mm used in most of the theoretical calculations was somewhat large, very little effect on the following comparisons can be expected since there is a very linear relation between the peak pressure and the maximum radius above $R_{max} = 0.5$ mm (shown in Figure 20).

Effects of some parameters on the critical pressure, which were examined experimentally, can be explained rather well by the results from the theoretical model. First of all, the effects of pressure history steepness was compared in Figure 6. The dotted line was taken from theoretical calculation for $R_1 = 0.05$ mm shown in Figure 25, assuming $T_{1/3} = T_{1/3}^*$. While the experimental result is almost linear, the theoretical result is curved and obviously deviates from the experimental results for higher values of $T_{1/3}$. This can be explained by the difference between $T_{1/3}$ and $T_{1/3}^*$. $T_{1/3}$ was defined on the acceleration history as the time duration between the point where the acceleration was $\frac{2}{3}a_c$ and the peak acceleration point. Since the instantaneous cylinder length for the former point is longer than that of the latter, the pressure at the former point is not equal to $\frac{1}{3}(p_a - p_{peak})$, where $T_{1/3}^*$ was started measuring. Actually $T_{1/3}$ is shorter than $T_{1/3}$ and the
difference is greater for larger value of $T_{1/3}$. This can explain the relatively large disagreement for higher value of $T_{1/3}$ in Figure 6.

The effects of system static pressure are rather complicated. As shown in the theoretical calculation in Figure 24, the system pressure change has little effect on the critical pressure as far as the lower part of pressure history is the same and the initial bubble size is unchanged. However, in general, the higher system pressure requires greater deceleration and in experiments, higher deceleration makes the acceleration history steeper. Furthermore, even if the identical acceleration histories are given for different system pressure, the calculated pressure history for higher system pressure becomes steeper than that for lower system. Neglecting the initial bubble size change, which can be considered small for the pressure change of ±200 mmHg, the experimental data shown in Figure 7 may be presenting the effects of steepness of pressure history. Comparing Figure 7 with Figure 22, this statement makes more sense. From Figure 21, it is clear that the effects of surface tension can be neglected, which was proved in the experiments.

In the experimental data and discussion section, it was noted that there was larger scattering of data for smaller values of $T_{1/3}$ (steeper acceleration history) in Figure 7 and for the higher system static pressure in Figure 7. This can be explained by using Figure 21. It is shown that for steeper pressure histories, the lines decline more. Hence, the critical pressure becomes more sensitive to the initial bubble size for steeper pressure histories, resulting in a larger scattering of data. It is also seen in Figure 21 that for the very small initial bubbles (less than 0.01 mm radius) the critical pressure
becomes very sensitive. This can explain the fact that the critical point measurements were not very repeatable for Delrin and pyrolytic carbon pistons on which smaller or no nucleation sites are expected.

In Figure 26, the calculated effect of temperature for fixed initial bubble size are shown. Two of these theoretical lines were drawn in Figure 8. Basically, the experimental data agree with the theoretical lines; however, a small effect of initial bubble size change can be seen. Obviously, as the temperature increases, the nucleation bubble becomes larger and the critical point is lowered.
CONCLUSIONS

A totally new method to study cavitation inception has been developed. It includes detection of cavitation, definition of the critical point for cavitation inception, measurement of the critical pressure, and possibility of theoretical simulation.

Among the several ways tried to detect cavitation, observations of pressure in the cylinder and acceleration of piston were done successfully as well as taking high speed motion pictures. By applying these methods, the critical point of cavitation inception was defined. Although the direct measurement of critical pressure was not possible, it was calculated rather accurately by solving the momentum equation of the liquid in motion.

In experiments, several parameters affecting the critical point have been examined and it was found that the shape of pressure reduction, the temperature, and the piston material have significant effects on the critical pressure while the system size, the system static pressure, surface tension, and viscosity have negligible effects.

By using an existing theory of bubble dynamics, the critical point defined experimentally was modeled successfully. The parameters affecting the critical point have been simulated by that modeling. In spite of some disagreement in bubble motion, those simulations were successful. However, that disagreement in bubble motion made it impossible to characterize the nucleation sites which was intended.
TABLES
<table>
<thead>
<tr>
<th>CYLINDER BORE</th>
<th>DAMPER</th>
<th>L(cm)</th>
<th>$a_c$(g)</th>
<th>$L^*_c$(cm)</th>
<th>$p_c$(mmHg ab.)</th>
<th>$T_{1/3}$(msec)</th>
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<tbody>
<tr>
<td>15.2mm I.D.</td>
<td>&quot;Thin&quot;</td>
<td>3.0</td>
<td>-340</td>
<td>2.872</td>
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<td></td>
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<td>0.39</td>
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<td>7.0</td>
<td>-142</td>
<td>6.862</td>
<td>-0.9</td>
<td>0.64</td>
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TABLE 1: Critical Point Measurements

Liquid: Water, Piston Material: Teflon
H: 5 cm, Temperature: Room temperature (25-26°C)
System Pressure: atmospheric pressure
TABLE 2: Critical Point Measurement, Effect of Reservoir Level

Liquid: Water, Cylinder: 15.2mm I.D., L = 5 cm
Piston Material: Teflon, Damper: "Thick"
Temperature: Room, System Pressure: Atmospheric

<table>
<thead>
<tr>
<th>H(cm)</th>
<th>H*(cm)</th>
<th>a_c (g)</th>
<th>L_c*(cm)</th>
<th>p_c (mmHg ab.)</th>
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<td>4.0</td>
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<td>-23.88</td>
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<tr>
<td>5.0</td>
<td>3.0</td>
<td>-205</td>
<td>4.85</td>
<td>-22.30</td>
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<tr>
<td>6.0</td>
<td>4.0</td>
<td>-200</td>
<td>4.85</td>
<td>-19.88</td>
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TABLE 3: Critical Point Measurement on Delrin Piston

Liquid: water, Cylinder: 15.2mm I.D., L = 4 cm
H: 5 cm, Damper: thick
Temperature: room, System Pressure: Atmospheric

<table>
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<tr>
<th>Run #</th>
<th>a_c (g)</th>
<th>p_{peak} (mmHg)</th>
<th>Notes</th>
</tr>
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<tr>
<td>1</td>
<td>-280</td>
<td>-102.6</td>
<td>No cavitation and no cavitation episodes before this run</td>
</tr>
<tr>
<td>2</td>
<td>-290</td>
<td>-133.4</td>
<td>Critical point</td>
</tr>
<tr>
<td>3</td>
<td>-320</td>
<td>(-225.9)</td>
<td>Cavitation</td>
</tr>
<tr>
<td>4</td>
<td>-265</td>
<td>-56.4</td>
<td>Critical point taken approximately 15 seconds after Run #4</td>
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TABLE 4: Effects of Piston Clearance and Viscosity of Liquid

Cylinder: 15.22mm I.D., L = 5 cm, H: 5 cm
Piston: Teflon, Damper: thick
Temperature: Room, System Pressure: Atmospheric

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<tr>
<th>Liquid</th>
<th>Piston Clearance (mm)</th>
<th>$a_c$ (g)</th>
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<tr>
<td>Water</td>
<td>0.02</td>
<td>-205</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>-230</td>
</tr>
<tr>
<td>25% glycerol solution</td>
<td>0.02</td>
<td>-205</td>
</tr>
<tr>
<td>(Approx. 2 centipoise)</td>
<td>0.12</td>
<td>-215</td>
</tr>
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</table>
FIGURES
FIGURE 1: Schematic of the Piston-Cylinder Cavitation System
FIGURE 2: Acceleration-Pressure Recording in Sub-cavitation Level

cylinder: 15.2mm I.D., L = 5 cm, H = 5 cm
piston: Teflon
damper: thick
FIGURE 3: Acceleration-Pressure Recording in Cavitation Level

cylinder: 15.2 mm I. D., \( L = 5 \) cm, \( H = 5 \) cm
damper: thick Teflon piston
FIGURE 4: Acceleration-Pressure Recording at the Critical Point

cylinder: 15.2mm I.D., L = 5 cm, H = 5 cm
piston: Teflon
damper: thick
FIGURE 6: Effects of Steepness of Pressure History on the Critical Pressure Based Upon the Data Shown in Table 1
FIGURE 7: Effects of System Static Pressure on the Critical Pressure
FIGURE 8: Effects of Temperature on the Critical Pressure
FIGURE 9: High Speed Motion Pictures for Cavitation Episode

(Film #7805) (See next page-61a)

Experimental Conditions

cylinder: 15.2 mm I.D., L = 4 cm, H = 5 cm
piston: Teflon, initial displacement: 6.3 mm
damper: thin
liquid: water, temperature: room (25°C)
pressure: atmospheric
film speed: approx. 5000 frame/sec
FIGURE 10: High Speed Motion Pictures
(Film #7810)

**Experimental Conditions:**

- **Cylinder:** 15.2 mm I.D., L = 5 cm, H = 5 cm
- **Piston:** Teflon, initial displacement: 8 mm
- **Damper:** thick
- **Liquid:** water, temperature: room
- **Pressure:** atmospheric
- **Film Speed:** approx. 5600 frame/sec
FIGURE 11: High Speed Motion Pictures (Film #7814)

Experimental Conditions:

- **cylinder**: 15.2mm I.D., L = 5 cm, H = 5 cm
- **piston**: Teflon, initial displacement: 5.6 mm
- **damper**: thin
- **liquid**: water, temperature: room
- **pressure**: atmospheric
- **film speed**: 6400 frame/sec
FIGURE 12: High Speed Motion Pictures (Film #7815)

Experimental Conditions:

- cylinder: 15.2 mm I.D., L = 3 cm, H = 5 cm
- piston: Teflon, initial displacement: 7.1 mm
- damper: thin
- liquid: water, temperature: room
- pressure: atmospheric
- film speed: 6400 frame/sec
FIGURE 13: Piston Motion (Acceleration and Position) and Calculated Pressure History for Film #7810

- $a_{peak} = -217 \text{g}$
- Piston speed $\approx 1.25 \text{m/sec}$
- $P_{peak} = -66.2 \text{ mmHg}$
FIGURE 14: Piston Motion (Acceleration and Position) and Calculated Pressure History for Film #7814
FIGURE 15: Piston Motion (Acceleration and Position) and Calculated Pressure History for Film #7815

- $a_{peak} = -345g$
- Piston speed $\approx 1.8\text{m/sec}$
- $p_{peak} = -55\text{mmHg}$
FIGURE 16: Motion of Bubble ($R_i = 0.05 \text{ mm}$)

Under the Pressure History of

$$p_{\infty}(t) = -794.65 e^{-2(t-1.5919)^2} + 765$$
FIGURE 17: Bubble Motion Under the Pressure History of

\[ p_\infty = -794.65 \, e^{-2(t-1.5919)^2} + 765 \]

for

1. \( R_i = 0.15 \, \text{mm} \), 2. \( R_i = 0.05 \, \text{mm} \), 3. \( R_i = 0.005 \, \text{mm} \)
FIGURE 18: Motion of Bubble ($R_i = 0.05$ mm, $R_{\text{max}} = 1.0$ mm)

Under Various Pressure Histories: $p_\omega(t) = Ae^{-Bt^2} + 765$

a. : $A = 869.3$, $B = 10$ ; $p_{\text{peak}} = -104.3$ mmHg
b. : $A = 829.0$, $B = 5$ ; $p_{\text{peak}} = -64.0$ mmHg
c. : $A = 794.7$, $B = 2$ ; $p_{\text{peak}} = -29.7$ mmHg
d. : $A = 777.8$, $B = 1$ ; $p_{\text{peak}} = -12.8$ mmHg
e. : $A = 766.2$, $B = 0.5$ ; $p_{\text{peak}} = -1.2$ mmHg
FIGURE 19: $R_1$ vs. $R_{max}$ Under Various Pressure Histories, a, b, c, d and e.

Given in Figure 18
FIGURE 20: Bubble Growth ($R_{max}$) vs. Pressure Reduction for Initial Bubble Sizes of 1. $R_i = 0.05$ mm, and 2. $R_i = 0.02$ mm

where $p_\infty = -Ae^{-Bt^2} + 765$; $p_{peak} = -A + 765$
FIGURE 21: Relation Between Initial Bubble Size ($R_1$) and the Critical Pressure Which Grow the Bubble to $R_{\text{max}} = 1 \text{ mm}$. 

Pressure Histories are Given by $p_\infty = -Ae^{-Bt^2} + 765$,

where $p_a = 760 \text{ mmHg}$, and $p_c = -A + 765$.
**FIGURE 22: Effects of Steepness of Pressure History on the Critical Pressure**

- $R_1 = 0.05 \text{ mm}$
- $R_1 = 0.02 \text{ mm}$

$P_c$ (mmHg)

$B = 15, 10, 5, 3, 2$ 
$t_{1/3}^{*} (\text{msec}) = 1, 0.5$
FIGURE 23: Effects of Ambient Pressure, $p_a$, on the Initial Bubble Size—Critical Pressure Relation. Pressure Histories are Given by $p_w = - Ae^{-Bt^2} + p_a + p_o$,

where $p_o = 5$ and $p_a$ are: 1. 960(+200), 2. 760(0), and 3. 560 (-200)
FIGURE 24: Effects of Ambient Pressure

Pressure Histories are Given by \( p_0 = Ae^{-Br^2} + 965 \)

and Ambient Pressures, \( p_a \), are:

1. 960(+200), 2. 760(0), and 3. 560(-200)
FIGURE 25: Temperature Effects on the Initial Bubble Size--Critical Pressure Relation

Temperatures: a = 5°C, b = 15°C, c = room (24-26°C), and d = 40°C

a, b, c, d for B = 5; and a', b', c', d' for B = 1
\( p_\infty = -831.2e^{-1.35t^2} + 765 \)

\( R_1 = 0.005 \text{ mm} \)

\( R_1 = 0.01 \text{ mm}; \, p_{\text{peak}} = -22 \text{ mmHg} \)

**FIGURE 27:** Correlation with Theoretical Model for Film #7810
\[ p_\infty = -806e^{-3.2t^2} + 762 \]

\( R_i = 0.005 \text{ mm} \)

FIGURE 28: Correlation with Theoretical Model for Film #7814
FIGURE 29: Correlation with Theoretical Model for Film #7815

$p_\infty = -817e^{-4.5t^2} + 762$

$R_i = 0.005 \text{ mm}$

$\bullet$ = experimental points
APPENDICES
APPENDIX I

SURFACE WETTABILITY AND CAVITATION NUCLEI

Contact Angle

The existence of gas on solid surfaces involves the degree of wetting of the solid by the liquid. This degree of wetting (wettability) is determined by the relative strengths of tensions acting on the interfaces between solid and liquid, between solid and gas, and between liquid and gas. The relation of these are shown in Figure A-1. Suppose that the local solid surface is normally a plane and that the line of contact is free to move only in a direction parallel to the solid surface. For equilibrium, there exists a single scalar condition, which is

\[ \gamma_{SG} = \gamma_{SL} + \gamma_{LG} \cos \theta \]

The angle, \( \theta \), is called the contact angle and is a measure of wetting. Alternative definitions are: wetting \( 0^\circ < \theta < 90^\circ \); nonwetting \( 90^\circ < \theta < 180^\circ \).

If the surface is wettable, it is termed hydrophilic; otherwise, it is hydrophobic. The contact angle can be determined by observing a drop of liquid or a gas bubble attached to a solid surface. The contact angle is usually measured in the liquid from the solid surface.

If the solid surface is dry and wetting imperfect, \( \theta \) depends on whether the liquid approaches an equilibrium position by advancing or receding over the surface (giving angles, \( \theta_A \) and \( \theta_R \)). These angles can be measured by tipping the surface, where a liquid drop or a gas bubble attached in the solid surface deforms. Just before it moves, a large advancing contact angle, \( \theta_A \), is measured at the front of the drop or at
the rear of the bubble and a small receding contact angle, $\theta_R$, is measured at the rear of the drop or at the front of the bubble. It should be noted that $\theta_A > \theta_R$ always, and that whenever the actual contact angle is less than $\theta_R$ or greater than $\theta_A$ the gas moves rapidly along the surface.

Contact angle measurements were done for Teflon, Delrin, pyrolytic carbon, and plexiglas which are the materials of interest in this study. The contact angles were measured on the photographs in which either a water drop or an air bubble was attached on the surface. For Teflon, Delrin, and pyrolytic carbon, the same pistons used in the cavitation system were used. Plexiglas was machined in the same manner as the pistons and was polished. The results are shown in Table A-1. It was found that Teflon is the most nonwettable and pyrolytic carbon is the most wettable. Plexiglas and Delrin are between and Delrin is slightly more wettable than plexiglas.

**Gas Pockets in Hydrophobic Cracks**

Harvey et al [11] proposed that the undissolved gas nuclei could exist as pockets in submicroscopic, hydrophobic (nonwetting) cracks and interstices on solid surfaces. This became the widely accepted model of cavitation nuclei because of its great advantage of providing a physically conceivable explanation for the existence of cavitation nuclei.

Consider the hydrophobic conical crack of apex angle of $2\alpha$. As shown in Figure A-2, a liquid-gas interface, which usually has a concave shape, can be set inside the crack with the contact angle $\theta$. It was explained in the previous section that $0^\circ < \theta < 90^\circ$ for wetting ($\theta = 0^\circ$ for perfect wetting) on a hydrophilic surface and $90^\circ < \theta < 180^\circ$
for nonwetting ($\theta = 180^\circ$ for complete nonwetting) on a hydrophobic surface. If the contact angle is $\theta > 90^\circ + \alpha$, the liquid should be on the concave side of the interface. Hence, the surface tension is acting to oppose the advance of the interface into the crack. Due to this action of surface tension, the gas in a hydrophobic crack cannot be dissolved completely but can remain in the gaseous phase as a possible nucleus for cavitation.

Even if the liquid is undersaturated with the gas, it is difficult to dissolve all of the gas. In this case, by dissolving some of the gas the liquid will advance into the crack with an advancing contact angle, $\theta_A > \theta$, so that the radius of interface, $R$, gets smaller, resulting in a reduced gas pressure in the cavity. Approaching the equilibrium condition, the interface is stabilized at a new position with smaller radius than the initial one.

From the contact angle measurements, it is clear that only Teflon has a chance to have permanent gas pockets if the surface cracks have an apex angle of $10^\circ - 28^\circ$.

**SEM Study of Material Surface**

A scanning electron microscope was used to study surfaces of Teflon, Delrin, and plexiglas. The samples were machined with the same machining conditions as those used for the pistons. The plexiglas sample was machined and polished. Then, all samples were vacuum coated with aluminum.

As shown in Figures A-3 and A-4, the Teflon surface has some cracks (or crevices) approximately 20$\mu$ wide and 60$\mu$ long. The depth of those cracks cannot be measured from these pictures, but judging from the shadow in the crack in Figure A-4, it can be larger than the width.
It is not clear what made the cracks. Since the pistons and SEM samples were machined by lathe, there are many machining grooves. However, they are rather shallow so that they could not be cavitation nucleation sites. Figure A-5 and A-6 show a Delrin surface, where many shallow machining grooves are seen, but surface cracks such as those on the Teflon surface cannot be seen. Polished plexiglas provides the smoothest surface among these three surfaces (shown in Figure A-7). In Figure A-7, there is a rough spot made by machining. Actually this spot was a rotation center of the lathe and there should not be such a spot in the plexiglas cylinder wall used in the cavitation system.
TABLE A-1

CONTACT ANGLE MEASUREMENT

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<th>$\theta$</th>
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<th>$\theta_R$</th>
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<td>105° - 120°</td>
<td>87° - 92°</td>
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<tr>
<td>Delrin</td>
<td>65° - 80°</td>
<td>85° - 90°</td>
<td>40° - 55°</td>
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<tr>
<td>Pryolytic Carbon</td>
<td>35° - 55°</td>
<td>----</td>
<td>----</td>
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<tr>
<td>Plexiglas</td>
<td>70° - 85°</td>
<td>104° - 108°</td>
<td>68° - 80°</td>
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</table>
FIGURE A-1: Liquid-Gas Interface in Contact with a Solid Face
FIGURE A-2: Gas Pocket in a Hydrophobic Conical Crack
FIGURE A-3: SEM Picture; Teflon (50X)

FIGURE A-4: SEM Picture; Teflon (300X)
FIGURE A-5: SEM Picture; Delrin (100X)

FIGURE A-6: SEM Picture; Delrin (300X)
FIGURE A-7: SEM Picture; Plexiglas (50X)
APPENDIX II
THEORETICAL CALCULATION

Computer Program

The equation to be solved in equation (10),

\[-R \frac{dv}{dl} + \frac{3}{2} U^2 = \frac{p_\infty - P}{\rho}\]

where \(P\), pressure at the bubble wall, is given by equation (11)

\[P = p_v - \frac{2S}{R} + \frac{NT}{R^3}\]

and \(p_\infty\) is expressed by equation (21)

\[p_\infty(t) = -Ae^{-B(t - t_0)^2} + p_a + p_o\]

There are initial conditions, \(R = R_i\) and \(U = 0\) at \(t = 0\).

This problem was solved numerically by the "Runge-Kutta" method. The computer program to solve this problem is as follows.
SOLN FOR MOTION OF CAVITATION BUBBLE
R=MM, RDOT=CM/SEC, T=MSEC
F=MMHG

DIMENSION R(350), RDOT(350), PT(350), T(350)
DIMENSION PG(350), PS(350), P0(350), Q(13)
REAL*4 M1, M2, M3, M4, K1, K2, K3, K4, NT, NH, MO, MM

WRITE(6,300)
300 FORMAT(/, 39H ENTER A B PA PQ TO R H HH NH NV S D RD//)
READ(5,400) (Q(I), I=1,13)

400 FORMAT(13F12.4)
A=Q(1)
B=Q(2)
PA=Q(3)
PO=Q(4)
T0=Q(5)
R(1)=Q(6)

13
H=Q(7)*1.0E-6
HH=Q(8)*1.0E-6
NH=Q(9)
HO=H*0.05
PV=Q(10)*1333.224
S=Q(11)
D=Q(12)
RDOT(T)=Q(13)
T(I)=0.0
PD(I)=0.0
BO=1.0
MO=0.0
MM=0.0
I=1

II=349
III=349
ID=349

NT=(PA*1333.224-PV)*R(I)**3/1000.+2.*S*R(I)**2/100.-
PT(I)=-A**2.*7.1828**(-B*T0**2)+PA+PD
PP=PA+PD-A
PG(I)=NT*R(I)**3/1333.224
PS(I)=20.*S/R(I)/1333.224
PR=PV/1333.224+PS(I)

WRITE(6,325) R(I), PP, PR
325 FORMAT(2X,'R0=', F6.4,'xPT=', F6.4,'xPP=', F6.4,'xPR=', F6.4,'xR0=', F6.4)

K1=H*RD(T(I))
M1=10.*H*(PV+NT)*R(I)**3-R(I)**3-20.*S/R(I)
K2=H*(RD(T(I))+0.5*M1)
K3=H*(PV+NT)/RA**3-2.*S/RA-PTA*1333.224)/D/RA
K3=H*(RD(T(I))+0.5*M2)
K5=K3**2

WRITE(6,400) (Q(I), I=1,13)
1  
-H*1.5*(RDOT(I)+0.5*M2)**2/R8
K4=H*(ROOT(I)+M3)

RC=R(I)*0.1+K3
T(I+1)=T(I)+1000.*H

PT(I+1)=-A*2.71828**(-B*(T(I+1)-T))**2+PA+PD

IF(PT(I+1),GE,PR) GO TO 40
MM=MM+(PR-PT(I+1)*0.5-PT(I)*0.5)*H*1000.
GO TO 41

40 IF(PR(I),GE,FR) GO TO 41

MM=MM+(PR-PT(I+1)*0.5-PT(I)*0.5)*H*1000.

41 M4=H*(PV+NT/RC**3-2.*S/RC-PT(I+1)**1333.224)/D/RC

1  
-H*1.5*(RDOT(I)+M3)**2/RC

RD(I+1)=RDOT(I)+(M1+2.*M2+2.*M3+M4)/6.

IF(IO.LT.349) GO TO 15
IF(RDOT(I+1).LT.0.) IO=I

15 PG(I+1)=NT*1000./R(I+1)**3/1333.224
PS(I+1)=20.*S/R(I+1)/1333.224
PD(I+1)=PG(I+1)-PS(I+1)+PV/1333.224-PT(I+1)
IF(PD(I+1),LE,0.0) GO TO 30
MO=M0+(PD(I+1)+PD(I))*H*500.
GO TO 31

30 IF(T(I+1),LT,TO) GO TO 31
IF(PD(I),LE,0.0) GO TO 31
MO=M0+(PD(I+1)+PD(I))*H*500.

31 IF(III.LT.349) GO TO 4
IF(R(I+1).GE.3.*R(I)) III=I

4 IF(III.LT.349) GO TO 7
IF(RDOT(I+1).LE.-100.) II=I

7 IF(R(I+1).GE.10.) GO TO 3
IF(R(I+1).LE.0.1*R(I)) GO TO 3
IF(I,GE,3.49) GO TO 3

IF(I,LE,1) GO TO 10
IF(T(I+1).GE,T) GO TO 10
IF(R(I+1).GE.3.*R(I)) GO TO 2

IF(R(I+1).GE.B0*R(I)) GO TO 10
IF(H.GE,H0*NH) GO TO 6
IF(RDOT(I+1),LT,RDOT(I)) GO TO 6

H=H+HO

6 RC(R)=R(I+1)
RD(I)=RDOT(I+1)

T(I+1)=T(I+1)
PT(I)=PT(I+1)
PG(I)=PG(I+1)

PS(I)=PS(I+1)
GO TO 5

5 H=HH

10 I=I+1
BO=BO+0.1
GO TO 1

3 WRITE(6,350) I
350 FORMAT(I4,/)  
WRITE(6,675) MO
675 FORMAT(2X,M0='.F10.4)
WRITE(6,680) MM

680 FORMAT(2X,'MM=','F10.4/) WRITE(6,625)

625 FORMAT(/9X,'T','/12X,'PT','/12X,'R','/12X,'RDOT','/12X,'PG','/12X,'PS'/)
J=1

14 WRITE(6,650)T(J),PT(J),R(J),RDOT(J),PG(J),PS(J)
650 FORMAT(5X,F8.4,4X,F10.3,5X,F8.4,4X,F12.3,4X,F10.3,4X,F10.4/)
IF(J.EQ.1) GO TO 11

IF(J.EQ.10) GO TO 12
J=10
GO TO 14

12 J=I
GO TO 14

11 WRITE(6,600)

600 FORMAT(/11H PRINT ALL?/) READ(5,450) IFLAG

450 FORMAT(I2)
IF(IFLAG.EQ.1) GO TO 19
IF(IFLAG.EQ.2) GO TO 8
IF(IFLAG.EQ.3) GO TO 18
IF(IFLAG.EQ.4) GO TO 20
19 IF(I.GE.30) GO TO 9
DO 200 J=1,I+1

200 WRITE(6,385)T(J),PT(J),R(J),RDOT(J),PG(J),PS(J)
385 FORMAT(5X,F8.4,4X,F10.3,5X,F8.4,4X,F12.3,4X,F10.3,4X,F10.4/)
GO TO 8:

9 DO 250 J=1,III

250 WRITE(6,550)T(J),PT(J),R(J),RDOT(J),PG(J),PS(J)
550 FORMAT(5X,F8.4,4X,F10.3,5X,F8.4,4X,F12.3,4X,F10.3,4X,F10.4/)
600 FORMAT(5X,F8.4,4X,F10.3,5X,F8.4,4X,F12.3,4X,F10.3,4X,F10.4/)
DO 225 J=III,II+5

225 WRITE(6,550)T(J),PT(J),R(J),RDOT(J),PG(J),PS(J)
550 FORMAT(5X,F8.4,4X,F10.3,5X,F8.4,4X,F12.3,4X,F10.3,4X,F10.4/)
DO 275 J=II,I+1,2

275 WRITE(6,525)T(J),PT(J),R(J),RDOT(J),PG(J),PS(J)
525 FORMAT(5X,F8.4,4X,F10.3,5X,F8.4,4X,F12.3,4X,F10.3,4X,F10.4/)
GO TO 8:

18 WRITE(6,700)
700 FORMAT(/13H ENTER NEW RO/) READ(5,725) R(1)

725 FORMAT(F12.4)
GO TO 13

20 WRITE(6,750)
750 FORMAT(/19H ENTER NEW A AND TO/) READ(5,775) A,TOL

775 FORMAT(2F12.4)
GO TO 13

8 CONTINUE

STOP
END
A Sample Case

The following case was solved.

Initial bubble radius, $R_i$: 0.005 mm

Pressure history: $p_{\infty} = -794.65 e^{-2(t-1.5919)^2} + 765$

$p_{\text{peak}} = -29.65 \text{ mmHg}$

$p_a = 760 \text{ mmHg}$

Vapor pressure: 26 mmHg

Surface tension: 72 dyn/cm

Liquid density: 1.0 gm/cm$^3$
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<th>R (mm)</th>
<th>K (cm/sec)</th>
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PRINT ALLY

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REFERENCES


