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ECONOMICALLY RATIONAL EXPECTATIONS

AND THE DEMAND FOR MONEY

BY

JAMES R. SCHMIDT

A Thesis Submitted
In Partial Fulfillment of the
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DOCTOR OF PHILOSOPHY

Thesis Director's Signature:

[Signature]

Houston, Texas

May, 1978
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I. OVERVIEW</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Nature of the Study</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Outline</td>
<td>3</td>
</tr>
<tr>
<td><strong>II. PRICE EXPECTATIONS AND THE DEMAND FOR MONEY:</strong></td>
<td>5</td>
</tr>
<tr>
<td>THEORETICAL ISSUES</td>
<td></td>
</tr>
<tr>
<td>2.1 The Quantity Theory</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Price Changes as an Explicit Return on Money</td>
<td>12</td>
</tr>
<tr>
<td>2.3 Transactions Models of Money Demand</td>
<td>14</td>
</tr>
<tr>
<td>2.4 The Threshold Hypothesis</td>
<td>19</td>
</tr>
<tr>
<td><strong>III. REVIEW OF PREVIOUS STUDIES AND EMPIRICAL RESULTS</strong></td>
<td>21</td>
</tr>
<tr>
<td>3.1 Hyperinflation</td>
<td>21</td>
</tr>
<tr>
<td>3.2 Post-War Studies of the U.S.</td>
<td>31</td>
</tr>
<tr>
<td><strong>IV. ECONOMICALLY RATIONAL EXPECTATIONS</strong></td>
<td>40</td>
</tr>
<tr>
<td>4.1 Information Sets and Forecasting</td>
<td>40</td>
</tr>
<tr>
<td>4.2 Time Series Analysis and Forecasting</td>
<td>42</td>
</tr>
<tr>
<td>4.3 Previous Expectation Mechanisms and the ARIMA Model</td>
<td>48</td>
</tr>
<tr>
<td>4.4 Leading Indicators</td>
<td>55</td>
</tr>
<tr>
<td><strong>V. EXPECTATION MODELS AND CAUSALITY TESTS</strong></td>
<td>62</td>
</tr>
<tr>
<td>5.1 ARIMA Models of Inflation, the Price Level, and Income</td>
<td>62</td>
</tr>
<tr>
<td>5.2 ARIMA Models of Leading Indicators</td>
<td>69</td>
</tr>
<tr>
<td>5.3 Intervention Analysis</td>
<td>78</td>
</tr>
<tr>
<td><strong>VI. MONEY DEMAND MODELS</strong></td>
<td>90</td>
</tr>
<tr>
<td>6.1 Inflationary Expectations and the Demand for Money</td>
<td>90</td>
</tr>
<tr>
<td>6.2 Income Expectations and the Demand for Money</td>
<td>107</td>
</tr>
<tr>
<td>6.3 Interest Rate Expectations and the Demand for Money</td>
<td>118</td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>VII. EXPECTED INFLATION AND THE HOUSEHOLD SECTOR</td>
<td>133</td>
</tr>
<tr>
<td>7.1 Overview</td>
<td>133</td>
</tr>
<tr>
<td>7.2 The Linear Allocation of Spending Power System</td>
<td>140</td>
</tr>
<tr>
<td>7.3 Expected Inflation and the LASS System</td>
<td>149</td>
</tr>
<tr>
<td>7.4 Estimation of the Modified LASS System</td>
<td>150</td>
</tr>
<tr>
<td>VIII. SUMMARY</td>
<td>168</td>
</tr>
<tr>
<td>APPENDIX 1. SOURCES OF THE DATA USED IN CHAPTERS V AND VI</td>
<td>172</td>
</tr>
<tr>
<td>APPENDIX 2. ESTIMATES OF THE AGGREGATE MONEY DEMAND MODELS USING AN ALTERNATIVE FUNCTIONAL FORM</td>
<td>173</td>
</tr>
<tr>
<td>APPENDIX 3. SOURCES AND DESCRIPTION OF THE DATA USED TO ESTIMATE THE MODIFIED LASS SYSTEM</td>
<td>186</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>194</td>
</tr>
</tbody>
</table>
CHAPTER I

OVERVIEW

1.1 Nature of the Study

The main objective of this study is to determine the influence, or lack thereof, of price expectations upon the demands for selected financial assets. The study will focus primarily upon the demand for money balances, but other financial assets which serve as substitutes for money will also be given attention. Recognition of the effects of price or inflationary expectations upon financial and real decisions made by economic agents does not begin with this study. Rather, the potential relationships to be examined were hypothesized at an early date. During a response to the Royal Commission on the Depression of Trade and Industry in 1886, Alfred Marshall noted:

"I will confine myself to some remarks as to what may be called the law of hoarding, viz. that the demand for a metal for purposes of hoarding is increased by a continued rise in its value and diminished by a continued fall, because those people who hoard believe that what has been rising in value for some time is likely to go on rising and vice versa."

If we consider contemporary forms of money in place of Marshall's notion of "metals", the implication is quite clear. Expectations of an increase in the general price level will cause a decrease, ceteris paribus, in the quantity of money demanded.

1Throughout this study, price expectations will refer to expectations of the rate of change in price. Hence, the expression may refer to either inflationary or deflationary expectations.
When households and firms anticipate increases in the price level, additional expected costs are associated with their money holdings over and above those which result from the fact that money provides no explicit return. The magnitude of these costs will vary directly with the expected rate of inflation and stock of money being held. Attempts to economize on these balances are undertaken since a loss in the purchasing power of the holdings is implied. One of the results of such behavior would be to increase the income velocity of money.

The methodology to be utilized in this study is largely empirical, but the various analyses are consistent with existing theoretical constructions. Initially, the effects of price expectations will be examined in single equation models of the demand for money. No new theoretical models will be offered. Instead, we shall use models which have previously appeared in the literature and adjust them to take account of the price expectations influence. In the case of the model given in Friedman's [31] restatement of the quantity theory, explicit consideration of price expectations is consistent with the model's current form. Thus, it serves as a convenient theoretical base.

Even though important results could be derived from single equation models, they would not indicate which alternative assets or variables would be affected by a significant relationship between price expectations and holdings of money. If the hypothesis of a negative relationship is true, presumably the holdings of other financial assets will be affected. Substitutions away from money may appear as increments to holdings of various interest-earning assets. Hence, the demands for these assets should also be considered in a larger model which describes asset choice.
Spending also constitutes an alternative to holding money balances. Substitutions away from money may appear in the form of increased levels of expenditures upon physical goods. In the case of households, the substitutions would imply increasing the level of expenditures upon nondurables and services, or adding to their stocks of durable goods.

To account for the above interactions of money balances, interest-earning assets, and expenditures, a second model is needed. A system of demand equations will be employed to deduce the effects of price expectations upon a set of financial assets and expenditure components. Note that the possibility of a negative relationship between price expectations and the holdings of interest-earning assets has not been ruled out.

1.2 Outline

In Chapter II, we examine the theoretical foundations of the relationship between price expectations and the demand for money. While the existence of a negative relationship remains as the principal concern, we also consider an additional hypothesis of major importance, the threshold hypothesis.

Chapter III contains a review of several studies which have dealt with price expectations and money demand. In particular, we focus upon the empirical results given by these studies and the structures of the models which were employed. This discussion serves as a point of departure for the present study.

In Chapter IV, we discuss the process of expectation formation and present a set of forecasting techniques which may be used for
modeling such processes. Chapter V presents estimates of the expectation models. The inflation rate, the price level, income, and an interest rate are all considered. Various sets of information are entertained in the course of constructing acceptable models for these economic series.

In Chapter VI, several specifications of the aggregate demand for money are entertained and estimated. Our primary focus is upon the empirical relationship between expected inflation and money demand. Several other issues are also examined, including the role of expected income and an expected interest rate in the determination of money demand.

Chapter VII presents a more detailed model which considers the impact of expected inflation upon household choices among consumption and portfolio alternatives. A system of consumer demand equations is proposed and estimated in an attempt to measure the influence of expected inflation upon money demand while allowing for the interdependencies between money balances and other assets or commodity groups. A summary of the results from the study is offered in Chapter VIII.
CHAPTER II

PRICE EXPECTATIONS AND THE DEMAND

FOR MONEY: THEORETICAL ISSUES

2.1 The Quantity Theory

Among competing theories of the demand for money, the quantity theory, as presented by Friedman [31], provides a convenient focus for examining the influence of price expectations upon the demand for money. The general notion of a "Quantity Theory of Money" has appeared in various forms, most notably in the writings of Fisher [30] and the Cambridge school. However, our discussion of the quantity theory, and our use of the term itself, will refer to Friedman's exposition.

Friedman's restatement of the quantity theory treats money balances as one type of asset in which wealth may be held. Determination of the amount of money balances which are held can be treated as a problem of utility or profit maximization. In the case of households, the demand for money is analogous to the demand for a commodity or its services. For firms, a convenient analogy is the demand for a factor of production. The demand for money should therefore be considered as one aspect of the more general problem of asset choice. In Friedman's [34] terms, money constitutes a "temporary abode for generalized purchasing power". Thus, the definition of money may be expanded to include other liquid assets in addition to the narrowly defined money stock of currency in the hands of the public plus demand deposits. Liquid assets which have narrow variations in their market values and relatively small transactions costs of conversion are the
most logical candidates for inclusion. In his subsequent empirical work, Friedman [33] includes time deposits at commercial banks, currency in the hands of the public, and demand deposits in his operational definition of money. Any discrepancy between the Keynesian definition of money and Friedman's is attributable to the latter's view of money as a temporary abode of purchasing power. Specifically, the Keynesian concept of money is based upon the services which money provides as a means of payment and its role as a reserve of funds for speculative purposes. Hence, interest earning assets are excluded. Yet money, using this narrow definition, still constitutes a store of generalized purchasing power.

The demand for money balances will depend upon the return from holding money and the returns from assets which are alternative forms of holding wealth. Such assets have nontrivial transactions costs associated with converting them into money and have real values which are subject to significant fluctuations with respect to the rate of interest or price level. Attention is restricted to bonds, equities, and physical goods in Friedman's analysis. The nominal rates of return on bonds and equities are represented simply by their respective interest rates. Consideration is also given to the possibility of capital gains or losses on the assets due to changes in the levels of interest rates and the price level. However, an assumption that interest rates are relatively stable over time allows Friedman to arrive at a model which utilizes only the current levels of the respective rates. The level of money holdings is inversely related to the rates of return on alternative financial assets.
Within the context of the quantity theory, our particular interest lies in the expected rate of return on physical goods, an additional alternative to holding money. Assume there is a fixed volume of physical goods in the economy, $\bar{Q}$. The nominal value of the goods may be expressed as $P \cdot \bar{Q}$, where $P$ is the aggregate price level. Differentiating with respect to time yields

$$\frac{d(P\bar{Q})}{dt} = \bar{Q} \frac{dP}{dt} + P \frac{d\bar{Q}}{dt}.$$  \hspace{1cm} (2.1.1)

Since $\bar{Q}$ denotes the constant stock of physical goods, we obtain

$$\frac{d(P\bar{Q})}{dt} = P\bar{Q} \left( \frac{dP}{dt} \cdot \frac{1}{P} \right).$$  \hspace{1cm} (2.1.2)

The rate of price change represents a rate of return on physical goods. The total return on $P\bar{Q}$ due to increases in the price level is of course a nominal return.\(^2\) Regardless of whether our frame of reference is continuous or discrete time, the appropriate rate of return is the expected rate of price change since agents are operating in a given time period without information about the actual rate.\(^3\) This is consistent with the familiar distinction between ex ante and ex post values of an economic variable. Since physical goods represent an alternative way of holding wealth, a negative relationship should exist between the expected rate of price change and the amount of

---

\(^{1}\)The notation which follows is taken from Rousseas [67].

\(^{2}\)We have assumed a zero rate of depreciation on the volume of physical goods since concern over depreciation obscures the main point. The rate of price change constitutes a return on holdings of physical goods, despite the fact that the total return (total change in nominal value) during a market period may still be negative due to depreciation.

\(^{3}\)Specification of the relevant information sets used in the formation of expectations will be considered in a subsequent chapter.
money balances agents wish to hold. In a deflationary environment, the substitutions which are implied will be from physical goods to money and vice versa in the context of inflation. Of course, these patterns of substitution may not occur if the rate of price change is not large enough to offset the transactions costs involved in making the conversion.

In addition to the rates of return on alternative assets, the demand for money balances will depend on total wealth. In Friedman's theory, wealth is not restricted to the various classes of financial assets and tangible capital, but rather is defined to include nonhuman wealth as well. Problems immediately arise if attempts are made to measure the broader concept of total wealth. As an alternative, the wealth aggregate can be expressed as the ratio of income to a general rate of return, implying that the flow of income is the expected return on the total stock of wealth. This view of income corresponds to Friedman's [32] concept of permanent income. Hence, permanent income, divided by a general interest rate which reflects the expected rate of return on total wealth, is proposed as a constraint in the demand function for money balances.

Friedman also considers other factors that may exert an influence on money demand. For the individual agent, the larger the proportion of wealth held in nonhuman form, the larger will be his income stream in future periods. The potential effect on money demand from expectations of higher income levels in the future is measured by the division of total wealth between human and nonhuman forms. Lastly, the tastes and preferences of the particular agent are recognized as a determinant of the demand for money. The need for including these "utility"
factors in the demand function for money is not as apparent as the inclusion of the above variables. Presumably, the demand function of a commodity or asset is derived, either implicitly or explicitly, from the exercise of maximizing a given utility function.\footnote{Aigner [1] presents a utility function which yields demand equations for financial assets. The general form of these equations is broadly consistent with the form of the demand equation yielded by the quantity theory.}

On the basis of the above discussion, we write the demand function for money as

\[
M = f(P, i_b, i_e, \pi^e; w; \frac{\overline{Y}^p}{r}; u),
\]

where

- \(M\) = nominal money balances
- \(P\) = aggregate price level
- \(i_b\) = bond rate
- \(i_e\) = equity rate
- \(\pi^e\) = expected rate of price change
- \(w\) = ratio of nonhuman to human wealth
- \(\overline{Y}^p\) = nominal permanent income
- \(r\) = expected rate of return on total wealth
- \(u\) = utility factors.

The expected rate of return on total wealth will be closely related to the bond and equity rates, allowing us to drop \(r\) from (2.1.3). Rewriting, we obtain

\[
M = f(P, i_b, i_e, \pi^e; w; \overline{Y}^p; u).
\]
Assuming that the demand function is homogeneous of degree one in price and permanent income, the demand for money can be expressed in real terms as

\[
\frac{M}{P} = f(b, e, \pi^e; w, \frac{y^p}{P}; u).
\]

In a later discussion, Friedman [33] introduces the concept of permanent price and proposes that this is the appropriate price variable in the demand function for money. The notion of a permanent price level is basically an application of the ideas surrounding permanent income to the aggregate price level.\(^5\) Any observed price level can be decomposed into a permanent element and a transitory element. Given a sufficient number of observations, the sum of the transitory elements will be approximately zero. Agents may consider the real value of their money balances on the basis of long-term price movements rather than upon the current level of prices. Hence, the "normal" or expected price level is the relevant measure. Introducing the permanent price variable into the model of (2.1.5) is accomplished by substitution for the current price, P. The new version is then

\[
\frac{M}{P^p} = f(b, e, \pi^e; w, \frac{y^p}{P^p}; u).
\]

The income constraint is now the ratio of nominal permanent income and permanent price. This ratio is not the same as permanent real income, \(\frac{y^p}{P}\). In general, the two measures will not be identical since the expectation mechanisms generating them will differ. The basic issue is whether agents form expectations of the real value of their incomes

\(^5\)We need not preclude the existence of permanent prices in individual markets.
or, alternatively, form separate expectations of their nominal incomes and the general price level.

Several functional forms of the money demand function are implied by the alternative methods of expectation formation. The model of (2.1.5) describes the case where agents only form expectations of nominal income whereas in (2.1.6), expectations of the price level are formed in addition to nominal income. Also admissible is the following model

\[(2.1.7) \quad \frac{M}{p^p} = f(i_b, i_e, \pi^e; w; y^p; u).\]

Here, expectations are formed for real income and the price level.

Finally, we could consider

\[(2.1.8) \quad \frac{M}{P} = f(i_b, i_e, \pi^e; w; y^p; u).\]

In the above model, the relevant price variable is the observed price level but permanent real income, or expectations of real income, is the appropriate income constraint. 6

The quantity theory, as presented by Friedman, has been discussed for two reasons. First, it gives a concise statement of the wealth or utility maximizing approach to the demand for money. This framework appears throughout the literature dealing with monetary issues. Secondly, it offers a behavioral model which explicitly recognizes the role of price expectations as a causal factor in the demand for money.

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6 The models of (2.1.7) and (2.1.8) might be a bit more contrived than (2.1.5) or (2.1.6) in the sense that their homogeneity properties are less apparent.
2.2 **Price Changes as an Explicit Return on Money**

In his restatement of the quantity theory, Friedman chooses to consider the rate of price change as only a return on physical goods. However, we need not conclude that all substitutions away from money due to a positive expected rate of price change will be in favor of physical goods. As an alternative, we can view the expected rate of price change as a general rate of return on real money balances. Hence, changes in money balances may appear as increments or decrements to any or all of a number of alternative asset classes. Interest-earning assets are prime candidates.\(^7\) Physical goods are certainly not precluded as substitutes under this alternative view. Indeed, increased consumption of nondurable goods and services, or increased expenditures upon durable goods may be observed as agents economize on their holdings of money. Irrespective of the actual character of substitution, the latter interpretation of price expectations allows the variable a more flexible, or general, role. The notion of the expected rate of price change functioning as a return on money balances can be easily demonstrated.

For purposes of simplicity, assume a constant nominal money stock over a given period of time. The total return on the real stock of money may be regarded as the change in its real value between the beginning and end of the period.\(^8\) Algebraically, we have

---

\(^7\) Johnson [44] notes that expectations of inflation may also be viewed as a cost of holding other assets whose money values are fixed. This would imply that substitutions among interest-earning assets might be observed in response to inflationary expectations.

\(^8\) A return on money may also be earned in the form of services provided by the given money stock. See Klein [46]. This return would be added to the expected return generated by price changes.
(2.2.1) \[ R = \frac{\bar{M}}{P_{t_1}} - \frac{\bar{M}}{P_{t_0}}, \]

where \( R \) is total return. Since \( P_{t_1} = P_{t_0} + \Delta P \), we may drop the time subscripts and find

(2.2.2) \[ R = \frac{\bar{M}}{P + \Delta P} - \frac{\bar{M}}{P}, \]

where \( \Delta P \) is the change in the price level during the discrete time period. Rewriting we obtain

\[ R = \frac{\bar{M} - (\bar{M}/P)(P + \Delta P)}{P + \Delta P} \]
\[ = \frac{-\bar{M}(\Delta P/P)}{P + \Delta P} \]
\[ = -\frac{\Delta P}{P} \cdot \bar{M} \frac{1}{P + \Delta P}. \]  

(2.2.3)

Hence, the rate of price change is analogous to an interest rate (negative in sign during inflation) with continuous compounding over the time period. Agents do not have perfect information concerning the change in the price level during the period. Thus, the expected, or ex ante, rate of price change is actually the relevant variable to consider.

The fact that price expectations result in an expected cost of holding money does not conflict with the presence of a nominal interest rate found in most formulations of the demand for money. According to the Fisherian hypothesis, nominal interest rates may fully reflect, or at least partially reflect, the expected rate of price change. However, the nominal interest rate on an asset considered to be a representative substitute for money is commonly viewed as an opportunity cost while the expected rate of price change should just be viewed as an additional cost. Both costs are relevant to the actions of agents
who hold money. If the price level is expected to increase, each cost has simply increased. The reaction of agents to the respective costs need not be identical. While the sacrifice of interest income is easily understood and accounted for by agents, the loss implied by inflationary expectations need not be perceived as readily. In terms of the demand function, the elasticities of money demand with respect to a representative interest rate and the expected rate of price change may well differ.

2.3 Transactions Models of Money Demand

The general model of money demand offered by Friedman's restate-
ment of the quantity theory does not ascribe any special importance to a transactions motive for holding money. The technology of trans-
actions and the level of balances required to facilitate exchange is determined in solving the more general problem of allocating wealth among alternative forms of assets or expenditures. This is consistent with the belief that the stream of services yielded by money stems from more than its function as a medium of exchange. In contrast, Keynesian models of money demand are more explicit in their consideration of balances maintained for transactions purposes. An extensive review of studies utilizing the Keynesian model will not be undertaken here, but we may write the basic model in linear form as

\[
(2.3.1) \quad \frac{M}{P} = g_1 \left( \frac{Y}{P} \right) + g_2(y),
\]

indicating that the demand for real money balances is a function of real income and an interest rate which represents the return on an
alternative asset. These variables reflect the Keynesian notions of the transactions, precautionary, and speculative motives for holding money.

The transactions motive for holding money has been stressed in models advanced by Baumol [8] and Tobin [73]. In the transactions approach, money holdings are not determined within the context of utility maximization. Instead, the level of holdings is the result of minimizing costs, either real or opportunity, associated with maintaining money balances. There have been numerous applications and extensions of the inventory approach. However, Baumol's original model can be used to demonstrate the role of price expectations in a transactions model of money demand.

Assume that an individual receives a real income of T and spends this total in a steady stream over a given period of time. Hence, T will also be the real value of transactions undertaken by the individual. For convenience, we assume a stable price level throughout the period. The real income total is initially placed in some interest-earning asset and conversions back to cash for making transactions are assumed to be evenly spaced throughout the time period. The amount of the sequential conversions, denoted by C, is constant in real terms. Interest income is earned on the portion of real income maintained in the earning asset and we let i denote the nominal interest rate. Converting the earning asset into money involves a transactions cost, denoted by b, the brokerage fee incurred during each conversion which

---

9The Keynesian model does not include price expectations as a functional argument. This convention follows from the usual assumption of stable prices in Keynesian models.
10Tobin [73] proves that this timing sequence for transactions is optimal for minimizing costs.
is independent of transaction size. Given the above assumptions, the average cash balance held by the individual during the period is \( C/2 \).

Two costs of making transactions may now be identified. First, the total number of conversions from the earning asset to money is \( T/C \), implying a transactions cost of \( b(T/C) \). Secondly, an interest cost is involved in maintaining the average level of money balances, \( i(C/2) \).

The total cost of transactions over the period may be written as

\[
(2.3.2) \quad L = b\left(\frac{T}{C}\right) + i\left(\frac{C}{2}\right).
\]

The individual seeks to minimize this total cost. Differentiating with respect to \( C \) and setting the result equal to zero yields

\[
(2.3.3) \quad \frac{d(L)}{dC} = -\frac{bT}{C^2} + \frac{i}{2} = 0.
\]

Hence,

\[
\frac{bT}{C^2} = \frac{i}{2},
\]

or,

\[
\frac{C}{2} = \frac{1}{2 \cdot 2bT/i}.
\]

Substituting real money balances, \( \frac{M}{P} \), for \( \frac{C}{2} \) gives

\[
(2.3.4) \quad \frac{M}{P} = \frac{1}{2 \cdot 2bT/i},
\]

which is the familiar square root formula for money demand.

We may also construct a simple transactions model for a time period in which prices are rising. Following the above analysis, assume that the individual receives a nominal income of \( T \) and spends it over the given time period \( t_n - t_0 \), where \( n \) is the number of conversions between the asset and cash. The amount of each conversion, \( C \),
is assumed to be constant in nominal terms and the respective conversions are equally spaced during the period. Thus, the average cash balance in nominal terms is \( C/2 \). The average real cash balance is approximated by \( C/2\bar{P} \) where \( \bar{P} \) is the average price level over the period.\(^1\) As in the original model, the individual must consider the transactions costs involved in making conversions and the interest cost of maintaining a cash balance. The rate of price change must also be considered. As shown in (2.2.3), it implies an additional cost of holding cash balances. We approximate this cost by \( \pi(C/2\bar{P}) \) where \( \pi \) is the rate of price change observed over the entire period. The total real cost of transactions is

\[
(2.3.5) \quad L = b\left(\frac{\pi}{C/\bar{P}}\right) + i\left(\frac{C}{2\bar{P}}\right) + \pi\left(\frac{C}{2\bar{P}}\right). \quad \text{\(^{12}\)}
\]

The brokerage fee, \( b \), is assumed to be fixed in real terms.

To minimize the total real cost, we differentiate (2.3.5) with respect to \( (C/\bar{P}) \) and set the result equal to zero. This yields

\[
(2.3.6) \quad \frac{d(L)}{d(C/\bar{P})} = -b\left(\frac{T}{\bar{P}}\right) + \frac{i}{2} + \frac{\pi}{2} = 0.
\]

\(^1\)We do not consider the explicit form of \( \bar{P} \). The average real cash balance over the entire period is some function of the respective averages of real cash balances in the sub-periods between conversions. The real value of the cash balance is not independent of time once movements in the price level are observed. Positive price changes imply that the real value of the "lot size" of conversion, \( C \), is decreasing over the time period. But this is consistent with the notion that a larger volume of transactions, in real terms, are undertaken during the early part of the period, when real values are highest, rather than at the end of the period. Barro [5] has developed a model in which equally spaced transactions through time are optimal when prices, income, and transactions costs grow at the same rate.

\(^{12}\)This expression is actually an approximation but we shall assume an equality for purposes of discussion.
Rearranging, we have

\[(C/F)^2 = \frac{2b(T/F)}{1+\pi},\]

or,

\[\frac{C}{2F} = \frac{1}{2}\left[\frac{2b(T/F)}{1+\pi}\right]^{\frac{1}{2}}.

Substituting average real money balances, \(M/F\), for \(C/2F\) results in

\[(2.3.7) \quad \frac{M}{F} = \frac{1}{2}\left[\frac{2b(T/F)}{1+\pi}\right]^{\frac{1}{2}},\]

where the equality should be regarded only as an approximation. The expression summarizes the functional relationships among real balances, the volume of transactions, a representative interest rate, and the rate of price change. For the purposes of this study, the most salient feature is the presence of the rate of price change in a money demand model whose construction was based upon the transactions motive.

The hypothesis of a negative relationship between real money balances and the expected rate of price change has now been derived from two models of money demand, each having been constructed under very different assumptions.\(^{13}\) Debate over the merits of the transactions and wealth approaches, typified by the quantity theory, has taken place mainly through comparisons of empirical evidence. The relative performance of current income and permanent income, or wealth, in estimations of money demand functions has been the most popular testing ground.

\(^{13}\) The negative relationship is also apparent after expressing (2.3.7) in terms of natural logarithms.
2.4 The Threshold Hypothesis

Given the hypothesis that the rate of price change is functionally related to money demand, several questions naturally arise. What is the strength of the relationship? Is the rate of price change always an important variable to consider? In examining this last question, we may offer an additional hypothesis. The magnitude of the expected cost, or loss in purchasing power, of holding money during a period of inflation will vary directly with the expected rate of price change and the stock of money being held. If the expected rate is positive and relatively high, there should be a significant desire to economize money holdings. When the expected rate of inflation is low, the stimulus will be smaller. Johnson [44] describes this situation as the threshold effect, i.e., expected inflation must be sufficiently high in value before it has a noticeable effect upon financial and real decisions. Agents may not be cognizant of the costs, or their actions may not be affected, if the rate is relatively low. Also, substitutions away from money will involve certain transactions costs, both observed and unobserved. Thus, avoidance of a portion of the cost associated with expected inflation may not compensate the agents for the transactions costs incurred.\textsuperscript{14} The existence of the threshold effect is an additional hypothesis that should be tested. Unfortunately, defining a threshold level or rate itself is not as practical as conducting empirical tests to establish its existence. With respect to the American experience of the post-war era, the last decade of relatively high inflation rates would seem to be the period in which

\textsuperscript{14} The cost can never be fully avoided. Holders of money will always incur a "tax" of some magnitude during inflation.
price expectations were most likely to have exerted a causal influence. A significant empirical relationship between expected inflation and money demand is anticipated for this particular time period. If, in addition, the relationship in a time period characterized by low rates of expected inflation is not significant, then there would be evidence to support the existence of a threshold effect. These issues are examined below in the course of estimating various models of money demand.
CHAPTER III

REVIEW OF PREVIOUS STUDIES
AND EMPIRICAL RESULTS

3.1 Hyperinflation

According to the threshold hypothesis, the expected rate of price change must be above a given level before a causal relationship is established with money demand. Therefore, hyperinflation should provide an environment in which an effect of expected inflation upon money demand is most likely to be observed. The definition of hyperinflation is somewhat arbitrary but, in general, the term refers to a sustained period of extremely high rates of inflation. Hyperinflation may be viewed as a sufficient condition for the existence of a functional relationship between the expected rate of price change and money demand, but it need not be necessary.

In hyperinflation, the rate of price change completely overshadows other costs of holding money such as those represented by interest rates on alternative assets. Substitutions away from money and toward fixed-value assets with fixed nominal returns are impractical as a hedge against the inflation. Agents also hesitate to assume creditor positions given the degree of uncertainty associated with the rate of price change. This leaves only physical goods as the asset capable of serving as an alternative to money. If interest-earning assets are not practical substitutes for money, the role of the expected rate of price change is identical with that prescribed in the quantity theory. It represents a cost of holding money via its role as the rate of return.
on physical goods.

Cagan [18] analyzed the relationship between the expected rate of price change and the demand for money using data from several European hyperinflations. After a preliminary examination of the pattern of fluctuations in the real money balance series, Cagan advanced the hypothesis that the expected rate of price change was the principal variable which could account for the variation. He noted that real income and wealth appeared to be relatively stable during periods of hyperinflation. Hence, these variables were omitted in the construction of the model of money demand. The wide fluctuations which were observed in the real money series can be interpreted in two distinct ways. First, the observations could have been generated by shifts in the demand function for money. Alternatively, they could be viewed as changes in the quantity of money demanded, having been generated by movements along a stable demand function. This latter view is consistent with specifications of the demand function which include the expected rate of price change as an argument. Cagan's model thus follows the spirit of the quantity theory.

Using Cagan's notation, the basic model is written as

\[
(3.1.1) \quad \ln \frac{M}{P} = -\alpha E - \delta,
\]

where \( M \) is the quantity of money, \( P \) is the price level, \( E \) is the expected rate of price change, and \( \alpha \) and \( \delta \) are constants.\(^1\) Since \( E \) is unobservable, the following mechanism of expectation formation is

\(^1\)The demand for money is actually a demand for a desired level of money balances. It is assumed that the actual level of balances equals the desired level.
(3.1.2) \( \frac{dE_t}{dt} = \beta(C_t - E_t) \),

where \( C_t \) is the observed rate of price change in time \( t \) and \( \beta \geq 0 \). The above mechanism has subsequently become known as the "adaptive expectations" hypothesis and has been used extensively in a wide variety of applications. Expectations are adaptive in the sense that the expected rate of price change for future periods is constantly revised according to the difference between observed rates and expected rates for the preceding period. Therefore, \( \beta \) is termed the coefficient of expectation and indicates the speed at which the expected rate of price change adjusts to the observed rate. The series of past price changes is implicitly assumed to be the only set of information used in the formation of expectations.

Solution of the differential equation in (3.1.2) yields the expression

(3.1.3) \( E_t = \frac{\beta}{e^{\beta t} - \frac{t}{T}} \int_{-T}^{t} C_x e^{\beta x} dx \),

where \( e \) is the natural number.\(^2\) Hence, the expected rate of price change is a function of past rates with the weights constrained to follow the familiar pattern of exponential decline.

The model of (3.1.3) has thus far been developed in a continuous time framework whereas observations of money balances and the actual rate of price change are available periodically. A useful approximation of (3.1.3) is provided by

\(^2\) The constant of integration in the solution is set equal to zero.
(3.1.4) \[ E_t = \frac{(1-e^{-\beta})}{e^{\beta t}} \sum_{x=-T}^{t} C_x e^{\beta x}. \]

The above expression is applicable in the context of discrete time and retains the characteristics of the continuous time version. Substituting (3.1.4) into (3.1.1) and adding a stochastic error term gives

(3.1.5) \[ \ln \left( \frac{M_t}{P_t} \right) = -a \frac{(1-e^{-\beta})}{e^{\beta t}} \sum_{x=-T}^{t} C_x e^{\beta x} - \delta + \varepsilon_t. \]

Cagan estimated this model using monthly data from the respective countries which had experienced hyperinflation. The periods of hyperinflation were defined as "beginning in the month the rise in prices exceeds 50 percent and as ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year", [18]. The empirical results support the existence of a relationship between the quantity of money demanded and the expected rate of price change. For the seven hyperinflations which were considered, the total correlation coefficients yielded by the model were in the range of .926 to .992, indicating a strong association between the two variables. The high percentage of variation in real money balances explained by the expected rate of price change would appear to suggest that it is the variable of principal importance in hyperinflation.

Obviously, there is a sizeable continuum of inflation rate values between price stability and hyperinflation. Although the relative importance of the expected inflation rate, as compared with other variables, may decline as we approach price stability, the possibility exists that a significant relationship still remains at lower rates. Hyperinflation merely provides an environment in which the relationship is most likely to be observed and also be the strongest.
Barro [5] has also conducted an empirical examination of hyperinflation, using data on four of the hyperinflations considered by Cagan. The basic premise is familiar. The inflation rate functions as a cost of holding money and the desire to economize money holdings increases with the rate of inflation. In what form does this economizing action take place? Barro considers two possibilities, decreases in the time interval between payments and decreases in the amount of transactions which are accomplished through the use of money. We do not repeat the exact derivation of the money demand model here, but instead focus upon the general form of the model and the assumptions made in its construction. Even though tests were conducted using data from hyperinflations, the analysis is applicable in the context of lower inflation rates.

The model is of the inventory type and thus focuses on the transactions motive for holding money. First, we will consider the optimizing problem of an employer (firm) in a steady state situation. Assume that prices and the nominal income received by the firm (its receipts) are growing at a continuous rate, $\pi$,

\[ P(t) = P_0 e^{\pi t}, \]

\[ Y(t) = Y_0 e^{\pi t}, \]

where $e$ is the natural number and $t$ is a time index. The firm may choose to convert its flow of income to assets or claims in the form of physical goods, wage payments to employees, or foreign exchange. Interest-earning assets are assumed to be unavailable for use as a store of value. As in most models which deal with the transactions motive, there is a cost associated with the conversion of assets.
Consistent with the steady state form of the model, assume that these costs may be written as

\[ a(t) = a_0 e^{\pi t}. \]  

Hence, transfer costs grow at the same rate as prices and nominal income. The problem facing the firm consists of optimally selecting dates in the future, \( t_1, \ldots, t_{n-1} \), to convert assets back into cash holdings given the fact that cash was initially transferred into the asset(s) at time \( t_0 \). Some transfer pattern will exist that balances the cost of making transfers with the interest cost (the depreciation of money due to inflation) of holding money. Firms desire to minimize these costs of using money for transactions purposes.

Consider a time \( T \) during which \( n \) transfers are to be made. The sum of the discounted interest and transfer costs incurred during the interval \( T \), where \( T = t_n - t_0 \), is

\[ C_T = Y_0 \{ t_n - t_0 - \frac{1}{\pi} \left[ n - e^{\pi (t_0 - t_1)} \ldots - e^{\pi (t_{n-1} - t_n)} \right] \} + a_0 \cdot n. \]

The firm must choose \( n \) and the transfer dates \( t_1, \ldots, t_{n-1} \) such that the total discounted cost, \( C_T \), is minimized. The first order conditions of the minimization problem yields the following expression which relates the time period between transactions to nominal income and the inflation rate:\(^3\)

\[ \frac{T}{n} = \frac{2a_0}{\pi Y_0}. \]

\(^3\)The first order conditions given by minimizing \( C_T \) with respect to the \( t_i \) indicates that the optimal times of transfers are spaced evenly throughout the entire time interval \( T \).
Barro proceeds to show that the firm's average money balance is given by

\[(3.1.11) \quad \left( \frac{M}{P} \right)_F \approx \frac{1}{2} \cdot \frac{Y}{P} \frac{T}{n}. \]

The implication of (3.1.10) and (3.1.11) is clear. The inflation rate is negatively related to the volume of real money balances held by the firm. Average holdings are economized by reducing the amount of time between successive transactions.

Thus far only the demand for money balances by firms or employers has been considered. To construct an aggregate demand function, we must now consider employees as well. Barro shows that the average real money balance held by an employee is

\[(3.1.12) \quad \frac{M}{P} \approx \frac{1}{2} \frac{W}{P} \frac{T}{n}, \]

where \(W\) is the nominal wage rate. Since \(\frac{T}{n}\) is the length of the payments period, \(t_i - t_{i-1}\), the real wage payment in the \(i^{th}\) period is \(\frac{W}{P} \left( \frac{T}{n} \right)\). Employees incur two types of costs from using money as a transactions medium. First, there is a lag between wage accruals, which are continuous, and the receipt of the wage payment at the end of the period. By not receiving instantaneous payment, employees are incurring a cost due to the delay of payment. Considering the time span of \(T\), the average time between wage accruals and the expenditure of wage payments by employees will be \(\frac{T}{n}\). Hence, the cost of receiving

---

4 Employees are assumed to spend their wage payments at a uniform rate. A single payment of wages is entirely spent during the space of a payment period, \(T/n\). It should also be noted that Barro treats wage payments as the total flow of funds out of the firm. This includes payments to all factors of production including the owners. Hence, the receipts or income of the firm, denoted by \(Y\), is equal to the wage payments. The growth rate of nominal wages is therefore equal to the growth rate of income of the firm.
a delayed payment of wages is \( r(\frac{W}{F})(\frac{T}{n}) \) and the total cost of delay over the \( n \) time periods, \( t_1, \ldots, t_n \), is

\[
(3.1.13) \quad C_r = r(\frac{W}{F})(\frac{T}{n})(T),
\]

where \( r \) is the rate of discount which employees apply to the delays in payment.\(^5\) The second source of costs is the loss in purchasing power of the average level of money balances held during the \( n \) time periods. Using (3.1.12), this cost over the entire interval \( T \) is given by:

\[
(3.1.14) \quad C_{\pi} = \frac{1}{2}(\frac{W}{F})(\pi)(\frac{T}{n})(T).
\]

The total cost incurred by employees is therefore

\[
(3.1.15) \quad C_{\text{employee}} = \frac{1}{2}(\frac{W}{F})(\pi)(T)(\pi+2r).
\]

Having derived the respective costs of holding money which apply to firms and employees, we may write the expression for the total cost of holding money over time interval \( T \) as

\[
(3.1.16) \quad C_{\text{total}} = T\left[\frac{1}{2}(\frac{Y}{F} + \frac{W}{F})\pi + \frac{W}{F} \cdot r + \frac{\pi n}{n} + \frac{a}{P} \cdot \frac{n}{T}\right],
\]

To find the optimal payments period, (3.1.16) is minimized with respect to \( T/n \). The solution yields

\[
(3.1.17) \quad \frac{T}{n} = [(a/F)/(Y/P)(\pi+r)]^{\frac{1}{2}},
\]

where \( \frac{W}{F} = \frac{Y}{F} \) has been assumed. The expression demonstrates the negative relationship between the length of the payments period, \( T/n \), and the rate of inflation, \( \pi \).

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\(^5\) If the delays in payment force employees to assume debt as a means of completing transactions, then \( r \) is the borrowing rate.
The aggregate money demand function is obtained by combining the respective demand functions of the employers (firms) and employees.

Summing (3.1.11) and (3.1.12) gives

\[ M = \frac{1}{2} \frac{Y}{P} n + \frac{1}{2} \frac{W}{P} n. \]  

By substituting (3.1.17) into the above expression while assuming that \( \frac{Y}{P} = \frac{W}{P} \), we have

\[ M = \left(\frac{a}{P}\right) \frac{Y}{P} / (\pi + \tau) \right)^{1/2}, \]

or, alternatively

\[ M = \frac{A(Y/P)}{\sqrt{\pi + \tau}}, \]

where \( A = \left(\frac{a}{P}\right) \frac{1}{2} \). The aggregate money holdings of employers and employees is negatively related to the inflation rate. Economizing on money holdings in response to inflation is accomplished through decreases in the payments period or, equivalently, through an increase in the rate at which transactions are made.\(^6\)

Use of alternative assets as a medium of exchange is another method which may be used to economize on money balances, especially during high rates of inflation. Physical goods are well suited for this function since their real values, net of depreciation, may either remain stable or perhaps increase. Regardless of the specific surrogate which is chosen, we may write the demand function for money as

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\( ^6 \) Since the optimal length of the payments period, (3.1.17), was determined by accounting for the costs incurred by employees and firms, both agents are affected by decreases in the payments period. Alternatively, we might say that both the goods market and labor market are affected. Barro does not discuss the relative gains or losses implied for employees and firms.
(3.1.21) \( \frac{M}{P} = \frac{[1-\Gamma] \cdot AY/P}{\sqrt{\pi+r}} \),

where \( \Gamma \) is the fraction of transactions accomplished by use of the money substitute. We do not present the exact form of \( \Gamma \) in this survey but note that it is a function of the inflation rate, \( \pi \), and the rate of discount, \( r \), relevant to employees.

The money demand function of (3.1.21) was derived under the assumption of a continuous rate of price change over time. However, the model can be generalized to apply to situations where the observed inflation rate is not constant. We assume that agents make their economic decisions on the basis of an "effective" rate of inflation, i.e., the rate which they believe will prevail in the current and future periods. Barro calls this rate the "perpetual rate" and we denote it by \( \pi^e \). Implicitly, the perpetual rate is an expectation of the inflation rate where expected future values are constant, regardless of the lead time of the expectation.\(^7\) By incorporating \( \pi^e \) in the model of (3.1.21) we have, assuming equality,

(3.1.22) \( \frac{M}{P} = \frac{[1-\Gamma(\pi^e,r)] \cdot AY/P}{\sqrt{\pi^e+r}} \),

or, in logarithmic form,

(3.1.23) \( \ln \left( \frac{M}{P} \right) t = \alpha_1 - \alpha_2 \ln(\pi^e+r) t + \ln(1-\Gamma) t \),

where \( \alpha_1 = \ln(AY/P) \). The "perpetual rate" of inflation is unobservable so the above equation cannot be estimated directly. Barro offers

\(^7\)The optimization problems which yield the series of money demand functions depend upon the assumption of a constant inflation rate, which may be either actual or perceived. Use of the "perpetual" inflation rate allows Barro to extend the applicability of his basic model while retaining its essential characteristics.
the following variant of Cagan's [18] adaptive expectations mechanism to describe the relationship between the "perpetual rate" and the actual rate of inflation

\[(3.1.24) \quad \sqrt{\pi_t^e} = \beta_t \sqrt{\bar{\pi}_t} + (1-\beta_t) \sqrt{\pi_{t-1}^e}, \]

where \( \bar{\pi}_t \) is the average inflation rate during the time period.

The model of (3.1.23) was subsequently estimated using data from four European hyperinflations. The employee discount rate, \( r \), was assumed to be zero and real income was assumed to be constant over the respective periods. As in Cagan's [18] study, the results indicate a strong association between the demand for money and the measure of the inflation rate. The estimated coefficient \( \hat{\alpha}_2 \) was significantly different from zero in all cases and the percentage of variation explained by the model ranged from .935 to .990. These results, when considered along with Cagan's, seem to dispel any doubts about the relevance of inflation as a cost of holding money during hyperinflation. Our task is to determine whether the cost is relevant in an environment of lower rates of inflation or deflation. We have discussed the Barro model at length because it demonstrates this cost of inflation during periods when prices are increasing, irrespective of a particular rate of increase. Thus, it provides an additional theoretical base for the empirical work undertaken in later chapters.

3.2 Post-War Studies of the U.S.

In this section, we shall review two empirical studies of the relationship between price expectations and the demand for money that do not rely upon data from hyperinflations. Each study demonstrates
a methodology which may be used in examining the issue and both use quarterly data from the post-war period in the U.S. Thus, they provide a source which may eventually be used for comparing the results of the present investigation with previous evidence.

Goldfeld [37] considered the influence of price expectations in the context of a model which also includes the expected values of real income and a nominal interest rate. The model may be written as

\[(3.2.1) \quad m = \alpha(y^e)^\beta(i^e)^\delta e^\gamma e,\]

where

- \(m\) = real money stock
- \(y^e\) = expected real income
- \(i^e\) = expected interest rate
- \(\pi^e\) = expected inflation rate
- \(e\) = natural number

By taking natural logarithms, and attaching time subscripts, the model may be rewritten in log-linear form as

\[(3.2.2) \quad \ln m_t = \alpha^*_t + \beta \ln y^e_t + \delta \ln i^e_t + \gamma \pi^e_t.\]

At the theoretical level, the log-linear specification cannot be extended to the price expectations variable because of the possibility of negative values. However, if the stream of price expectations was known, a priori, to consist only of positive values, the specification of (3.2.1), and thus (3.2.2), could be extended.

\[8\] Goldfeld actually considered two interest rates in his development of the model. We shall limit our discussion to one rate for ease of exposition.
In order to estimate (3.2.2), Goldfeld adopts Cagan's adaptive expectations mechanism to describe the process of expectation formation. The mechanism is applied to each of the variables and the resulting expectation models are

\[(3.2.3) \quad \ln y^e_t - \ln y^e_{t-1} = \lambda (\ln y_t - \ln y^e_{t-1}), \]
\[\ln i^e_t - \ln i^e_{t-1} = \lambda (\ln i_t - \ln i^e_{t-1}), \]
\[\pi^e_t - \pi^e_{t-1} = \lambda (\pi_t - \pi^e_{t-1}), \]

where \(y^e_t, i^e_t, \) and \(\pi^e_t\) are the expected values of real income, the given interest rate, and the rate of price change in time \(t\), respectively. The same process of expectation formation is assumed to apply to each of the three variables. This is easily seen by the common value of the coefficient of expectation, \(\lambda\).

By combining the mechanisms of (3.2.3) with the log-linear model of (3.2.2) and then applying the familiar Koyck transformation, the final model of money demand is

\[(3.2.4) \quad \ln m_t = a + b \ln y_t + c \ln m_{t-1} + d \ln i_t + f \pi_t + \epsilon_t, \]

where \(\epsilon_t\) is a stochastic disturbance with zero mean and the parameters are functions of \(\lambda\) and the corresponding parameters of (3.2.2). By assuming identical coefficients of expectation in (3.2.3), Goldfeld avoids the problem of non-linear relationships between parameters in the final model and those in the original, (3.2.2). \(^9\) Estimation of (3.2.4) using quarterly data from the post-war period indicated a

\(^9\) See Feige [26] for an extensive discussion of this problem.
significantly negative relationship between price expectations and the demand for real money balances. A second estimation of the model was undertaken using observed price expectations measures constructed by de Menil [24]. The estimated coefficient of the variable was negative but barely significant.

As an alternative to the adaptive expectation mechanism, Goldfeld continued his analysis by offering the following proxies for the expected values of real income, the interest rate and the rate of price change:

\[
\ln y_t^e = \frac{n_1}{j=0} w_j \ln y_{t-j},
\]

\[
\ln i_t^e = \frac{n_2}{j=0} w_j \ln i_{t-j},
\]

\[
\pi_t^e = \frac{n_3}{j=0} w_j \pi_{t-j}.
\]

Substituting into (3.2.2) yields

\[
\ln m_t = \alpha + \frac{n_1}{j=0} \phi_j \ln y_{t-j} + \frac{n_2}{j=0} \phi_j \ln i_{t-j} + \frac{n_3}{j=0} \phi_j \pi_{t-j} + \epsilon_t,
\]

where the stochastic disturbance term has been added. Estimation of the above model was accomplished by using the techniques associated with Almon [2] distributed lags. The coefficient of the rate of price change was negative and significantly different from zero. Once again, the observed price expectations series of de Menil were substituted for the proxy in equation (3.2.6) and the model was reestimated. Two series were used and while the coefficient was negative in both cases,

10. Note the apparent contradiction in the formulation of the weighting scheme. The variables of interest are expected values in time t but the mechanisms which yield these expected values include the observed values in time t.
only one provided a significant relationship.

Models of the demand for money may alternatively be regarded as
descriptions of the demand for a desired stock. The models considered
above have implicitly assumed equality between the actual stock of
money held and the desired stock. Studies of the demand for money and,
in general, studies of the demands for other stocks frequently relax
this assumption. They admit the possibility of a discrepancy between
the actual and desired stock by postulating an adjustment mechanism
which relates the former, an observable quantity, with the latter,
usually unobserved. Goldfeld employs the following mechanism in his
third, and final, approach to the price expectations issue

\[(3.2.7) \quad \ln M_t - \ln M_{t-1} = \theta(\ln M^*_t - \ln M_{t-1}) ,\]

where \(M^*_t\) is the desired nominal stock of money and \(\theta\) is the coefficient
of adjustment. In essence, the mechanism assumes that the change in
the stock of money held reflects a partial adjustment of previous
holdings to the current level of desired holdings. The parameter \(\theta\)
merely represents the speed of the adjustment process.

Consider the following model,

\[(3.2.8) \quad \left(\frac{M^*_t}{P_t}\right) = \alpha + \beta \ln y_t + \delta \ln i_t + \gamma \pi^e_t .\]

Real income and the interest rate are now represented by their observed
values while the rate of price change retains its expectational form.
Substituting (3.2.7) into (3.2.8) and applying the Koyck transformation
yields

\[(3.2.9) \quad \ln \left(\frac{M_t}{P_t}\right) = a + b \ln y_t + c \ln \left(\frac{M_{t-1}}{P_t}\right) + d \ln i_t + f \pi^e_t + \varepsilon_t ,\]
where \( \varepsilon_t \) is the stochastic disturbance term. The above model was estimated using the adaptive expectation approach for measuring \( \pi^e \) as well as the two observed expectations series of de Menil. The calculated coefficient of the price expectations variable was insignificant in all three cases. Since the stock adjustment hypothesis has strong intuitive appeal in short-term situations, these results would appear to cast some doubt upon the importance of price expectations. However, the results from estimating (3.2.9) are contingent upon the assumptions concerning the expectation mechanisms or measures, and the form of the adjustment model. We should also note that Goldfeld's empirical tests do not treat the hypothesis of a threshold effect. The sample period employed was 1952II - 1972IV. Thus, the insignificance of the price expectations variable in (3.2.9) may be due to the comparatively small number of observations from the period when rates of price change were relatively high. Splitting the sample would have provided information on the effect.

Shapiro [69] has studied the relationship of interest by using a money demand model which is similar to Goldfeld's version in (3.2.6). The expected values of real income, the interest rate, and the rate of price change are expressed as

\[
(3.2.10) \quad y^e_t = f_1(y_t, y_{t-1}, \ldots, y_{t-n}),
\]

\[
i^e_t = f_2(i_t, i_{t-1}, \ldots, i_{t-m}),
\]

\[
\pi^e_t = f_3(\pi_t, \pi_{t-1}, \ldots, \pi_{t-\nu}).
\]

These formulations of the expectation mechanism are quite general, indicating that the expected values of the respective variables are
functions of their present and past values. The pure expectations version of the money demand model, with time subscripts added, may be written in linear form as

\[(3.2.11) \quad m_t = \alpha + \beta y_t^e + \delta i_t^e + \gamma \pi_t^e,\]

where \( m_t \) is the real stock of money at time \( t \). Expectations of the respective variables are assumed to be given by the following weighting functions

\[(3.2.12) \quad y_t^e = \sum_{j=0}^{n} \phi_j y_{t-j}, \]
\[i_t^e = \sum_{j=0}^{m} \phi_j^i i_{t-j}, \]
\[\pi_t^e = \sum_{j=0}^{v} \phi_j^\pi \pi_{t-j}. \]

Substituting these mechanisms into \( (3.2.11) \) yields

\[(3.2.13) \quad m_t = \alpha + \sum_{j=0}^{n} b_j y_{t-j} + \sum_{j=0}^{m} c_j i_{t-j} + \sum_{j=0}^{v} d_j \pi_{t-j} + \varepsilon_t, \]

where \( \varepsilon_t \) is a stochastic disturbance. Shapiro notes that finding the lag coefficients to be significant in an estimation of the above model would not allow us to conclude that the expectational version of the money demand model is appropriate. The independent variables consist of current and lagged values and no attempt has been made to specify an adjustment model for the real money stock. Hence, significant coefficients on individual (or all) lagged variables could indicate the existence of expectation mechanisms which are based upon past values as well as a stock adjustment effect.

With the above caveat in mind, the model in \( (3.2.13) \) was estimated in first differenced form using the methods associated with Almon [2].
lags. Quarterly observations over the period of 1950IV - 1970II comprised the data set. Rates of price change were derived from the implicit price deflator for GNP. The price variable was found to be significant in the estimation. Shapiro also estimated (3.2.13) using time deposits in place of the narrowly defined money stock, M1. As we might expect, the price variable was not significant in the formulation since the demand for interest-earning assets will not be as sensitive to the cost of inflation due to the positive return which may still be obtainable during mild inflations.

The studies of the post-war experience in the U.S. which we have reviewed provide a useful foundation for the present study. First, they demonstrate that the general problem which we are examining is not trivial nor worthy of investigation. Secondly, they illustrate the empirical nature of the issue. Finally, the results appear to indicate, although only tentatively, that a relationship exists between price expectations and the demand for money. However, this does not mean that we should unconditionally accept the results or the methodologies and model forms which are employed. In particular, the expectation mechanisms may be questioned. We have already noted that the mechanism which is based upon an unconstrained weighted sum of present and past values of a variable conflicts with the intuitive notion of an expectation.\footnote{Examples are given by (3.2.5) and (3.2.12).} Agents presumably operate with a certain set of information and are able to form an expectation of the value for a variable in the current period. Hence, the observed value in the current period is irrelevant. The mechanism should be modified and
presented in a form such as

\[ (3.2.14) \quad x_t^e = \sum_{j=1}^{n} \phi_j x_{t-j}, \]

where \( X \) represents an arbitrary economic variable.

In each model that was estimated, calculation of the parameters in the expectation mechanism was accomplished by regression procedures. In the case of adaptive expectations, the coefficient of expectation was the relevant parameter while in the general lag model, the respective weights formed the parameter set. The estimated coefficients are unique to the context of the model which is being examined. The same theoretical mechanism might be used in other models, such as consumption or investment. Subsequent estimation would yield coefficients which are again unique to these contexts. According to this approach, agents must form an expectation of the same variable within each behavioral relationship where the variable is hypothesized to exert an influence. This unduly complicates the nature and effects of expectation formation.
CHAPTER IV

ECONOMICALLY RATIONAL EXPECTATIONS

4.1 Information Sets and Forecasting

If expectations of an economic variable could be modeled, or generated, outside the context of a given behavioral relationship, a certain degree of flexibility would be achieved. One mechanism could be used to generate expectations for use in any model which requires the expected value of the variable in question. Of course, expectations of a variable peculiar to a given market should be generated by using the information in that particular market. For example, as we disaggregate from the general price level to the prices of various markets, the individual markets (and their models which we construct) will provide the necessary information for forming expectations of the market's price. Returning to the macro level, it is obvious that a number of broad information sets could exist, representing some type of reduced form model which explains the movement of the macro variable. The aggregate price level or the level of income are convenient examples.

Muth's [55] concept of rational expectations is based upon the above considerations. In this scheme, expectations are rational if they are formed by using the economic model which determines the variable of interest. Using Muth's notation, consider an isolated market which is described by the following equations

\[(4.1.1) \quad C_t = -\beta p_t \quad \text{(demand)}\]
\[ P_t = \gamma p^e_t + u_t \quad \text{(supply)} \]
\[ P_t = C_t \quad \text{(market equilibrium)} \]

where \( p_t \) is the observed market price, \( p^e_t \) is the expected price in time \( t \), and \( u_t \) is a stochastic error term which follows a linear process.

Muth shows that the solution of the above model implies the following expectation mechanism for prices

\[ p^e_t = \sum_{i=1}^{\infty} \psi_i p_{t-i}. \tag{4.1.2} \]

The expectation of market price in time \( t \) is a weighted sum of past prices, a model form which was advocated in (3.2.14). Thus, the relevant information set given by the market for purposes of expectation formation is restricted to the past history of prices. This result is somewhat paradoxical since we would expect the market structure itself to provide additional information.\(^1\)

Once we leave the realm of a single market and consider expectations of variables at the macro level such as the price level or income, specification of the relevant model becomes a more difficult task. A useful alternative is provided by the concept of "economically rational expectations" developed by Feige and Pearce [28]. The basic assumption of this approach is that expectations are forecasts of future values of the economic variable(s) under consideration. Various information sets will be available for use in forming the expectations or forecasts but they will differ in their degree of applicability and in their respective costs. Costs of information sets are difficult to quantify

\(^1\) Nelson [58] has considered an extended form of (4.1.1) whose solution implies that additional information sets are relevant.
but we assume that the use of the past history, or values, of an economic variable involves a relatively low cost. Hence, this set will be used extensively by agents in forming their expectations.

Other information sets may be evaluated in terms of the additional precision in forecasting which may be obtained. If a given set will reduce the error or variance of the forecast then it should be incorporated into the mechanism of expectation formation, assuming that the benefits from the reduction outweigh the costs of obtaining the information. When examining forecasts of the rate of price change and income, we might, for example, consider monetary aggregates or measures of fiscal policy as possible information sets that would improve forecasting efficiency.

4.2 Time Series Analysis and Forecasting

Given our assumption that expectations are forecasts of future values, forming expectations in an accurate fashion implies that the forecasts must be optimal, irrespective of the specific information sets which are employed. There are a number of criteria on which to base the notion of optimality. This section proposes a particular definition of optimality and describes the procedures and methodology which will eventually be used to formulate expectation mechanisms.

We initially restrict our attention to the past history of an

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2 Barro and Fisher [6] would classify such expectations as rational. Specifically, they state that "expectations are rational if they are optimal predictions based on the information available to the individual".

3 Note that there is an association between the formation of economically rational expectations and the rational expectations framework as given by Muth. The proper economic model may be a reduced form where, for example, the rate of price change is a function of a particular monetary or fiscal aggregate.
economic variable. The value of the variable in the current period, 
\( Y_t \), is assumed to be generated by a linear stochastic process which 
characterizes the series as it moves through time. Hence, \( Y_t \) is re-
garded as a random variable. In terms of a linear filter, we may 
write the general form of this representation as

\[(4.2.1) \quad Y_t = a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots,\]

\[
Y_t = (1 + \psi_1 B + \psi_2 B^2 + \ldots) a_t ,
\]

\[
Y_t = \psi(B) a_t ,
\]

where \( \psi(B) \) is an infinite polynomial in the backshift (or lag) operator 
\( B \), \( B Y_t = Y_{t-1} \), and the \( a_{t-j} \) are independent and identically distrib-
uted error terms, commonly referred to as "white noise" disturbances. 
The sequence of the \( Y_t \) is stationary if, for any integers \( k, n, t_1, \ldots, t_n \), 
the joint probability distribution of \( Y_{t_1}, \ldots, Y_{t_n} \) is identical to that 
of any other set \( Y_{t_1+k}, \ldots, Y_{t_n+k} \). Stationarity of the stochastic 
process is also assured if the infinite polynomial \( \psi(B) \) converges, for 
all \( B \), on or within the unit circle \( |B| \leq 1 \).\(^4\)

If the polynomial \( \psi(B) \) is invertible, we may rewrite \((4.2.1)\) and 
show that \( Y_t \) is a function of the past values of \( Y 
\[(4.2.2) \quad \Gamma(B) Y_t = a_t ,\]

\[(1 - \gamma_1 B - \gamma_2 B^2 - \ldots) Y_t = a_t ,\]

or

\(^4\)For a proof, see Jenkins and Watts [43, chap. 6]. An equivalent 
condition for stationarity is that the roots of \( \psi(B) \) must lie outside 
the unit circle.
\[ Y_t = \gamma_1 Y_{t-1} + \gamma_2 Y_{t-2} + \ldots + a_t, \]

where \( \Gamma(B) = [\psi(B)]^{-1} \). The polynomial \( \psi(B) \) is invertible if \( \Gamma(B) \) is stationary, or equivalently, the roots of \( \Gamma(B) \) lie outside the unit circle.

The representation of the stochastic process in (4.2.1) may also be written as

\[ (4.2.3) \quad Y_t = \frac{\theta(B)}{\phi(B)} a_t, \]

where \( \phi(B) \) and \( \theta(B) \) are polynomials of order \( p \) and \( q \), respectively. The \( \psi(B) \) polynomial has been replaced by the ratio of two finite polynomials. Alternatively, we have

\[ (4.2.4) \quad \phi(B) Y_t = \theta(B) a_t, \]

or

\[ Y_t - \phi_1 Y_{t-1} - \ldots - \phi_p Y_{t-p} = a_t - \phi_1 a_{t-1} - \ldots - \theta_q a_{t-q}. \]

Following the terminology of Box and Jenkins [12], this mixed form is known as an ARMA\((p,q)\) model. It consists of an autoregressive process, AR\((p)\), as represented by the polynomial \( \phi(B) \), and a moving average process, MA\((q)\), as represented by \( \theta(B) \). The roots of the respective polynomials must lie outside the unit circle to satisfy the stationarity requirement.

In some cases, the model of (4.2.4) will not be sufficient to characterize the time series \( Y_t \) as a stationary stochastic process. However, finite differences of the series may satisfy the stationarity property. The model may therefore be extended to
(4.2.5) \[ \phi(B)(1-B)^d Y_t = \theta(B) a_t , \]

where \( d \) is the order of differencing applied to the raw series. Box and Jenkins refer to this general form as the autoregressive integrated moving average model, or ARIMA\((p,d,q)\). Certain economic time series display consistent upward movements through time and differencing is frequently needed to achieve a stationary series. The appropriately differenced series might also have a "level" or trend associated with it. This possibility can be accounted for by rewriting (4.2.5) as

(4.2.6) \[ \phi(B)(1-B)^d Y_t = \mu + \theta(B) a_t . \]

Our interest in the ARIMA model stems from its forecasting capabilities. Box and Jenkins have shown that forecasts based on the ARIMA model are optimal in the sense that they are minimum mean square error forecasts. To demonstrate this attribute, let us write the ARIMA model of (4.2.5) as

(4.2.7) \[ \Psi(B) Y_t = \theta(B) a_t , \]

where \( \Psi(B) = \phi(B)(1-B)^d \). The model may be expressed in terms of a finite difference equation by

(4.2.8) \[ Y_{t+\ell} = \psi_1 Y_{t+\ell-1} + \cdots + \psi_{p+d} Y_{t+\ell-p-d} - \theta_1 a_{t+\ell-1} \]

\[ - \cdots - \theta_q a_{t+\ell-q} + a_{t+\ell} , \]

or, alternatively, by

(4.2.9) \[ Y_{t+\ell} = \sum_{j=0}^{\infty} \psi_j a_{t+\ell-j} , \]
where \( \psi(B) = \frac{\theta(B)}{\varphi(B)} \). Our problem is to forecast the value of \( Y_{t+\ell} \) given values of the series up to and including \( Y_t \). Hence, using the ARIMA model for forecasting purposes assumes that the time series of interest is the only relevant information set.

Let us denote the forecast of \( Y_{t+\ell} \) made at time \( t \), as \( \hat{Y}_{t}(\ell) \), where \( \ell \geq 1 \) is known as the lead time. Assume that the "best" forecast, in terms of mean square error, is

\[
(4.2.10) \quad \hat{Y}_{t}(\ell) = \psi^* a_t + \psi^*_{\ell+1} a_{t-1} + \ldots
\]

By subtracting (4.2.10) from (4.2.9), the error of the forecast is

\[
(4.2.11) \quad Y_{t+\ell} - \hat{Y}_{t}(\ell) = \sum_{j=0}^{\ell-1} \psi_j a_{t+\ell-j} + \sum_{j=0}^{\infty} (\psi_{\ell+j} - \psi^*_{\ell+j}) a_{t-j}.
\]

Squaring the forecast error and taking expectations yields

\[
(4.2.12) \quad \text{E}[Y_{t+\ell} - \hat{Y}_{t}(\ell)]^2 = \sum_{j=0}^{\ell-1} \psi_j \sigma_a^2 + \sum_{j=0}^{\infty} (\psi_{\ell+j} - \psi^*_{\ell+j})^2 \sigma_a^2.
\]

The above expression is the mean square error of the forecast, \( \hat{Y}_{t}(\ell) \), and is minimized when \( \psi^*_{\ell+j} = \psi_{\ell+j} \). From (4.2.10), the optimal forecast is given by

\[
(4.2.13) \quad \hat{Y}_{t}(\ell) = \psi_{\ell} a_t + \psi_{\ell+1} a_{t+1} + \ldots,
\]

where the \( \psi \) weights are provided by the ARIMA model applied to \( Y_t \).

The forecast error, from (4.2.11), is

\[
(4.2.14) \quad e_t(\ell) = Y_{t+\ell} - \hat{Y}_{t}(\ell) = \sum_{j=0}^{\ell-1} \psi_j a_{t+\ell-j}.
\]

---

This notation is taken from Box and Jenkins [12, chap. 5].
and the variance of the forecast error is therefore

\[(4.2.15) \quad \text{Var}(e_t(\ell)) = \sum_{j=0}^{\ell-1} \psi_j \sigma^2 \cdot j.\]

Since the \(a_t\) are white noise disturbances with zero mean, the conditional expectation of \(e_t(\ell)\) at time \(t\) is zero. Hence, the forecast is unbiased. It follows from (4.2.14) that the expectation of \(Y_{t+\ell}\) conditional on the values of \(Y_i\) up to and including time \(t\) is

\[(4.2.16) \quad E[Y_{t+\ell} | Y_{t}, Y_{t-1}, \ldots] = \hat{Y}_t(\ell).\]

Thus, the conditional expectation of \(Y_{t+\ell}\) is the minimum mean square error forecast of \(Y_{t+\ell}\), made at time \(t\).

The forecast for lead time \(\ell\) may also be written in the form of the difference equation containing the autoregressive and moving average parameters

\[(4.2.17) \quad \hat{Y}_t(\ell) = \psi_1 [Y_{t+\ell-1}] + \ldots + \psi_p [Y_{t+\ell-p-d}] + \theta_1 [a_{t+\ell-1}] - \ldots - \theta_q [a_{t+\ell-q}] + [a_{t+\ell}],\]

where the brackets denote conditional expectations. For example, the forecast for a lead time of one period from an ARIMA(1,1,0) model is

\[(4.2.18) \quad \hat{Y}_t(1) = \psi_1 Y_{1_t} + \psi_2 Y_{2_t-1},\]

where \(\psi_1 = (1+\phi_1)\) and \(\psi_2 = -\phi_1\). The disturbance term does not appear since \(E[a_{t+1}] = 0\).

A result of particular significance is provided by the case where
the forecasts are for a lead time of one period. The forecast error is given by

\[(4.2.19) \quad e_t(1) = Y_{t+1} - \hat{Y}_t(1),\]

or, from \((4.2.14),\)

\[e_t(1) = a_{t+1}.\]

Hence, the disturbances of the stochastic process are also one step ahead forecast errors. In practice, the true parameters of the ARIMA model are not known but must be estimated. If the least squares criterion of minimizing the sum of squared residuals is used, then we are minimizing the sum of squared errors of the one step ahead forecasts.

4.3 Previous Expectation Mechanisms and the ARIMA Model

If the past history of an economic series is the only information set to be utilized in forming expectations of future values, the ARIMA model which describes the series will be, for our purposes, the expectation mechanism or model. The main advantage of using this procedure lies in the fact that the economic series itself will determine the appropriate ARIMA model and thereby the proper expectation mechanism. We will not have to constrain or specify the form and then estimate the parameters within the context of a particular economic model. Rather, a single expectation mechanism can be employed in the estimation of various economic relationships which contain the variable of interest.

Several mechanisms of expectation formation which have been used in the past are actually special cases of ARIMA models. The most
obvious example is provided by the "static" expectations mechanism, in which the expected value of a variable in the next period is identical with the current observed value. Using the notation adopted above, this equality is written as

\[(4.3.1) \quad \hat{Y}_t(1) = Y_t.\]

Consider the ARIMA(0,1,0) model,

\[(4.3.2) \quad (1-B)Y_t = a_t,\]

which indicates that the \(Y\) series follows a random walk through time. By expanding the model, we have

\[(4.3.3) \quad Y_{t+1} = Y_t + a_{t+1}.\]

Taking expectations yields

\[(4.3.4) \quad \hat{Y}_t(1) = Y_t,\]

which is the familiar model of static expectations, expressed in the forecasting framework.

The adaptive expectations model of Cagan [18], in terms of discrete time, is also a special case of an ARIMA model. Expressing the Cagan model of (3.1.2) in discrete time gives

\[(4.3.5) \quad Y_{t+1}^e - Y_t^e = \delta(Y_t^e - Y_t^e),\]

where \(Y_{t+1}^e\) is the expected value of \(Y\) in time \(t+1\) and \(\delta\) is the coefficient of expectation. After rearranging terms, the model becomes

\[(4.3.6) \quad Y_{t+1}^e = Y_t^e + \delta(Y_t^e - Y_t^e).\]
If the expected values are considered to be forecasts, the above form may be expressed as

\[(4.3.7) \quad \hat{Y}_t(1) = \hat{Y}_{t-1}(1) + \delta(y_t - \hat{Y}_{t-1}(1)),\]

where $\hat{Y}_t(1)$ is the one period ahead forecast based upon information available through time period i. Thus, the adaptive expectation mechanism yields a forecast of the future period on the basis of the forecast made for the previous period and the absolute error of that forecast. As a result, the mechanism is sometimes referred to as an "error learning" model.

Consider the ARIMA(0,1,1) model,

\[(4.3.8) \quad (1-B)Y_t = (1-\theta B)a_t,\]

or, equivalently,

\[(4.3.9) \quad Y_t = Y_{t-1} + a_t - \theta a_{t-1}.\]

From (4.2.18), the error term $a_{t-1}$ equals the observed value in time $t-1$ less the forecast of the value, $Y_{t-1} - \hat{Y}_{t-2}(1)$. Substitution into (4.3.9) yields

\[(4.3.10) \quad Y_t = Y_{t-1} + a_t - \theta(Y_{t-1} - \hat{Y}_{t-2}(1)),\]

and via the stationarity property,

\[(4.3.11) \quad Y_{t+1} = Y_t + a_{t+1} - \theta(Y_{t-1} - \hat{Y}_{t-1}(1)).\]

Taking conditional expectations at time t yields

\[(4.3.12) \quad \hat{Y}_t(1) = y_t - \theta Y_{t+1} + \theta \hat{Y}_{t+1}(1).\]
Now let $\theta_1 = 1 - \delta$. Via substitution, we have

\begin{equation}
(4.3.13) \quad \hat{Y}_t(1) = Y_{t-1}(1) + \delta(Y_t - \hat{Y}_{t-1}(1)) ,
\end{equation}

which is identical to the forecasting formula given by the adaptive expectations mechanism in (4.3.7). Thus, Cagan's expectation mechanism, when expressed in terms of discrete time, is an ARIMA(0,1,1) model for forecasting purposes.

To examine the weights for past observations of $Y_t$, note that the ARIMA(0,1,1) model can be written as

\begin{equation}
(4.3.14) \quad \Gamma(B)Y_t = a_t ,
\end{equation}

or,

\begin{equation}
(1-\gamma_1 B - \gamma_2 B^2 - \ldots)Y_t = a_t .
\end{equation}

The weighting polynomial $\Gamma(B)$ is the ratio of the differencing operator and the moving average polynomial, $\theta(B)$,

\begin{equation}
(4.3.15) \quad (1-\gamma_1 B - \gamma_2 B^2 - \ldots) = \frac{1-B}{1-\theta_1 B} ,
\end{equation}

or,

\begin{equation}
(1-\theta_1 B)(1-\gamma_1 B - \gamma_2 B^2 - \ldots) = 1 - B .
\end{equation}

To find the weights, equate the coefficients of the powers of the backshift operator, $B$. We have,

\begin{align*}
\gamma_1 &= 1 - \theta_1 = \delta \\
\gamma_2 &= \theta_1 \gamma_1 = \delta(1-\delta) \\
\gamma_3 &= \theta_1 \gamma_2 = \delta(1-\delta)^2 \\
&\quad \vdots \\
\gamma_j &= \theta_1 \gamma_{j-1} = \delta(1-\delta)^{j-1} .
\end{align*}
The weights, $\gamma_j$, decline exponentially, implying that the forecast (or expectation as we have assumed), $\hat{Y}_t(1)$, is an exponentially weighted average of past values of $Y_t$. This is a familiar result given by the adaptive expectation mechanism.

The empirical study of the demand for money in subsequent chapters will utilize ARIMA models for determining expectation mechanisms. Expectations of the rate of price change is obviously one area where the forecasting methodology is appropriate. However, there are other, less obvious, cases where the forecasting approach may be of value. Permanent income is found in theoretical models of the demand for money, particularly those associated with the quantity theory. Since permanent income is an unobservable variable, some type of model or mechanism must be used to obtain an operational measure. In the course of his study of consumption, Friedman [32] offered three definitions of permanent income. First, permanent income is an expectation or "the expected value of a probability distribution", [32, page 21]. Secondly, permanent income is equal to the return on the sum of human and non-human wealth. Thirdly, permanent income is an exponentially weighted average of past incomes. This latter definition is an attempt to offer empirical content to the expectational definition. Mayer [52, page 36] points out that "the three definitions can coexist peacefully because they are on different levels of abstraction". We interpret Friedman's efforts of measurement as support for the expectational definition at the theoretical level. Hence, permanent income could be equated with the expectation of income conditional on past values of income. Since we are assuming expectations to be forecasts made by agents in the economy, the method of measurement used by Friedman can
be cast in a forecasting framework.

Specifically, the adaptive expectations mechanism was used by Friedman to measure permanent income. The mechanism, in a continuous time framework, is analogous to (3.1.2) and may be written as

\[(4.3.17) \quad \frac{dY^p_t}{dt} = \delta(Y_t - Y_t^p),\]

where \(Y_t^p\) is permanent income in time \(t\).\(^6\) In discrete time, the mechanism is

\[(4.3.18) \quad Y_{t+1}^p - Y_t^p = \delta(Y_t - Y_t^p).\]

Following the previous discussion which examined the forecasting characteristics of the adaptive expectation mechanism, we may rewrite (4.3.18) as

\[(4.3.19) \quad Y_{t+1}^p = \hat{Y}_t(1) = \hat{Y}_{t-1}(1) + \delta(Y_t - \hat{Y}_{t-1}(1)).\]

Permanent income in time \(t+1\) is a forecast, or expectation, of income in time \(t+1\) based on the information contained in past values of income. Forecasts based upon the formula of (4.3.19) imply the use of an ARIMA(0,1,1) as shown by (4.3.13). The exponentially declining weights applied to past values of income are assured under that particular ARIMA form.

Measuring permanent income by the adaptive expectations scheme assumes that the stochastic process generating observed values of income is an ARIMA(0,1,1) model. We have argued that expectations are forecasts from a particular stochastic process and that they will

\(^6\)Friedman also allowed for the possibility of a constant to capture the drift in the income series. We abstract from this term in the following discussion.
satisfy the mean square error criterion of optimality. To this end, one of the requirements is that the ARIMA model describing the variable in question must be identified correctly. There is no guarantee that an ARIMA(0,1,1) is the proper description of the stochastic process generating income. Use of such a model when a different \((p,d,q)\) parameter set is correct would yield non-optimal forecasts or expected future values.\(^7\) This point can also be considered in terms of the weights applied to past observations. If the ARIMA(0,1,1) model is incorrect, the set of weights given by Friedman, and used in subsequent studies, are incorrect. Errors of measurement are automatically introduced into models which employ deficient measures of permanent income to test various hypotheses.

The present study avoids the problems outlined above by identifying and estimating the proper ARIMA model which describes income.\(^8\) The one period ahead forecasts generated by the estimated model will be our measured series of expected income. The same procedures may be applied to the price level in order to construct a measure of the expected price level. The ARIMA model fitted to the income series need not be identical with the model for the price level, as indicated in Chapter II. Although the adaptive expectations mechanism is a special case of an ARIMA model, we do not rule out the possibility that it may

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\(^7\) The forecasts would be non-optimal with respect to the alternative model whose coefficients are known. However, estimating an incorrectly identified or misspecified model will result in a larger forecast error variance. The implications could even be worse. If the residuals from estimating a grossly misspecified model do not pass a test of randomness, then the model does not provide a stationary representation of the series under examination.

\(^8\) Lee [49] also used this strategy to study consumption and the permanent income hypothesis.
be the appropriate model for a given series. The identification stage of model construction may indeed indicate an ARIMA(0,1,1).

4.4 Leading Indicators

The previous sections of this chapter have outlined a procedure which may be used to forecast future values of an economic time series given only the information contained in the past values of the series. Placing such a restriction on the process of expectation formation may prove to be costly. Other information sets may exist which would substantially increase the accuracy, over time, of forecasts made by economic agents. There are costs involved in overestimating or underestimating the future values of economic variables and agents presumably desire to minimize these costs. Identifying possible information sets is aided by economic theories which hypothesize relationships among the variables of interest. For example, Cagan [19] contends that there exists a direct relationship between secular increases in the money stock and secular increases in the price level. In general, supporters of the monetarist position would extend the argument to short-term movements. Keynesians, on the other hand, would place primary emphasis upon a fiscal measure as the main determinant in the movement of prices. Similar arguments would apply to the determination of the level of income. A more eclectic position would recognize the potential influence of both the monetary aggregate and the fiscal measure. We restrict our attention to information sets which reflect, or are directly provided by, the structures of reduced form models.

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The auxiliary information sets which we shall consider consist of the present and past values of variables which may improve forecasts of the variable of interest. A given variable $X$ will be used in forecasting future values of $Y$ if the forecasts so obtained are "better" than forecasts based only upon the present and past values of $Y$ itself. This is also a definition of causality between variables $X$ and $Y$ which has been advanced by Granger [39]. The mean square error criterion is used to compare the forecasts from the respective methods. A more formal statement of Granger's definition of causality may be summarized.

Let $\{A_t, t=0,1,2,\ldots\}$ be the given information set which is of potential use in forecasting future values of $Y$. This set contains at least $\{(X_t, Y_t)\}$, the time series of the $X$ and $Y$ variables, respectively. Let $\tilde{A}_t = \{A_s | s< t\}$ and $\tilde{\tilde{A}}_t = \{A_s | s \leq t\}$. Define $\tilde{X}_t, \tilde{Y}_t, \tilde{\tilde{X}}_t, \tilde{\tilde{Y}}_t$ in a similar manner. Finally, let $\sigma^2(Y|B)$ be the minimum mean square error of $P_t(Y|B)$, the single step predictor of $Y_t$ given information set $B$.

The Granger definition is now

1. $X$ causes $Y$

$$\sigma^2(Y|\tilde{X}) < \sigma^2(Y|\tilde{\tilde{X}})$$

2. $X$ causes $Y$ instantaneously

$$\sigma^2(Y|\tilde{A}, X) < \sigma^2(Y|\tilde{A})$$

We could also consider causality from $X$ to $Y$ by adapting the above definition. If both $X$ causes $Y$ and $Y$ causes $X$, there is feedback between the two variables. Obviously, a large number of potential relationships could exist. Pierce and Haugh [62] have classified 256 forms of causality relationships within the above framework.
If there exists a one-way causal relationship between $X$ and $Y$ (no feedback or reverse causality), we may describe it in terms of a dynamic regression model

\[(4.4.1) \quad Y_t = V(B)X_{t-b} + N_t,\]

where $V(B)$ is an infinite polynomial in the backshift operator, $b$ is the lag time of the system, and $N_t$ is a disturbance term but not necessarily white noise. The above model is also known as the transfer function relating an input series, $X_t$, to the output series, $Y_t$. When dealing with economic series, $X_t$ could be called a leading indicator or $Y_t$. Following Box and Jenkins [12], the transfer function can be decomposed as

\[(4.4.2) \quad Y_t = \frac{\omega(B)}{\delta(B)}X_{t-b} + \frac{\delta(B)}{\psi(B)}a_t,\]

where $a_t$ is a white noise disturbance, and $\omega(B)$ and $\delta(B)$ are finite polynomials of degree $s$ and $r$, respectively. Rearranging, we have

\[(4.4.3) \quad \psi(B)\delta(B)Y_t = \psi(B)\omega(B)X_{t-b} + \delta(B)\theta(B)a_t.\]

Combining the products of the respective polynomials yields

\[(4.4.4) \quad \delta^*(B)Y_t = \omega^*(B)X_{t-b} + \theta^*(B)a_t.\]

The forecasting formula from (4.4.4), for an arbitrary lead time $\lambda$, is given by

\[(4.4.5) \quad \hat{Y}_t(\lambda) = \delta^*_{l}[Y_{t+\lambda-1}] + \ldots + \delta^*_{p^*+r}[X_{t+\lambda-p^*+r}] + \omega^*_{0}[X_{t+\lambda-b}] - \ldots - \omega^*_{p^*+s}[X_{t+\lambda-b-p^*+s}] - \ldots\]
\[ \theta^*[a_{t+k-1}] - \ldots - \theta^*[a_{t+q-r}] + \]
\[ [a_{t+k}] , \]
where \( p^* = p + d \). Thus, forecasts obtained from the transfer function model use the information contained in the histories of both \( X \) and \( Y \).

A testing procedure for the existence and direction of causality between two variables has been developed by Haugh [42]. Such a procedure will indicate whether the construction of a transfer function model is justified. In the context of the univariate model of (4.2.1), the disturbance (or innovation) \( a_t \) represents that part of \( Y_t \) that cannot be explained by the past values of \( Y \). It would appear that some measure of association between the \( a_t \) and leading indicator series \( X_t \) would give an indication of the potential improvement in forecasting. However, the raw \( X_t \) series may contain autoregressive or moving average elements which would obscure the true contribution of \( X_t \) to improving forecasts of \( Y_t \). To guarantee that the above influences have been eliminated, the \( X_t \) and \( Y_t \) series are "prefiltered" by the appropriate linear filters

\[ (4.4.6) \]
\[ Y_t = \psi_1(B)a_t \]
\[ X_t = \psi_2(B)b_t , \]

or,

\[ \Pi_1(B)Y_t = a_t \]
\[ \Pi_2(B)X_t = b_t , \]

\[ ^{10} \text{This effect is similar to the spurious regression problem discussed in Granger and Newbold [40].} \]
where \( a_t \) and \( b_t \) are the prewhitened \( Y_t \) and \( X_t \) series, respectively.

The cross-correlations between the two prewhitened series, at various lead times, may be used to detect the nature of causality between \( X_t \) and \( Y_t \). \(^{11}\) Pierce [61, page 15] has summarized the principal causality events according to the values of the cross-correlations. For convenience, we reproduce his table below.

**TABLE 4-1. CAUSALITY PATTERNS INDICATED BY CROSS-CORRELATIONS**

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Values of ( \rho_{ba}(k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. X causes Y</td>
<td>( \rho_{ba}(k) \neq 0 ) for some ( k &gt; 0 )</td>
</tr>
<tr>
<td>2. Y causes X</td>
<td>( \rho_{ba}(k) \neq 0 ) for some ( k &lt; 0 )</td>
</tr>
<tr>
<td>3. Instantaneous Causality</td>
<td>( \rho_{ba}(k) \neq 0 )</td>
</tr>
<tr>
<td>4. Feedback</td>
<td>( \rho_{ba}(k) \neq 0 ) for some ( k &gt; 0 ) and for some ( k &lt; 0 )</td>
</tr>
<tr>
<td>5. Y does not cause X</td>
<td>( \rho_{ba}(k) = 0 ) for all ( k &lt; 0 )</td>
</tr>
<tr>
<td>6. Unidirectional causality from ( X ) to ( Y )</td>
<td>( \rho_{ba}(k) \neq 0 ) for some ( k &gt; 0 ) and ( \rho_{ba}(k) = 0 ) for all ( k &lt; 0 )</td>
</tr>
<tr>
<td>7. ( X ) and ( Y ) are related instantaneously but in no other way</td>
<td>( \rho_{ba}(k) = 0 ) for all ( k \neq 0 ) and ( \rho_{ba}(0) \neq 0 )</td>
</tr>
<tr>
<td>8. ( X ) and ( Y ) are independent</td>
<td>( \rho_{ba}(k) = 0 ) for all ( k )</td>
</tr>
</tbody>
</table>

In practice, the true parameters of the linear filters in (4.4.6) are not known but must be estimated. Thus, the cross-correlations of the

\(^{11}\) The theoretical cross-correlation between \( b_t \) and \( a_t \) for lead time \( k \) is

\[
\rho_{ba}(k) = \frac{v_{ba}(k)}{[v_a v_b]^2},
\]

where \( v_{ba}(k) \) is the cross-covariance between \( a_t \) and \( b_t \) for lead time \( k \).
residuals from the estimated models are used to assess the causality events in Table 4-1. \(^{12}\)

Several tests, based upon the residual cross-correlations, are available for examining the pattern of causality. Haugh \([42]\) showed that the set of residual cross-correlations, \(\{\hat{r}_{ba}\}\), are asymptotically normally distributed and independent with zero means and standard deviations of \(1/\sqrt{N}\) where \(N\) is the sample size. This distribution is valid under the null hypothesis of independence between \(X_t\) and \(Y_t\). Hence, one test would consist of comparing the respective \(\hat{r}_{ba}(k)\) with their standard deviations. Haugh \([42]\) also derived the following test statistic for checking the independence of two time series

\[
(4.4.7) \quad S_M^* = N^2 \sum_{k=-M}^{M} (N-|k|)^{-1} \hat{r}_{ba}^2 (k).
\]

This statistic is distributed as \(\chi^2(2M+1)\) under the null hypothesis of independence. If independence between a theoretical leading indicator and a variable which we desire to forecast cannot be rejected, then the transfer function model is inappropriate. Finally, we can consider the analog of \((4.4.7)\) for only negative or positive lags.\(^{13}\) The case where

\[
(4.4.8) \quad S_M^*(+) = N^2 \sum_{k=1}^{M} (N-|k|)^{-1} \hat{r}_{ba}^2 (k) > \chi^2_M\]

\(^{12}\) The residual cross-correlations are given by

\[
\hat{r}_{ba}(k) = \frac{\hat{E}_{t-k} \hat{E}_{t-k}^*}{[\hat{E}_{t}^2 \hat{E}_{t-k}^2]^{1/2}}
\]

\(^{13}\) Pierce \([61]\) uses a variant of this statistic in testing for causality among a number of economic variables.
at a given \( \alpha \) level would indicate support for an interpretation that \( X \) causes \( Y \). Alternatively, the case where

\[
S^k_N(-) = N^2 \sum_{k=-1}^{-M} (N-|k|)^{-1} \hat{c}_{ba}(k)^2 > \chi^2(\alpha)
\]

would support the interpretation that \( Y \) causes \( X \). Both statistics could be greater than the selected critical value, indicating feedback. Note that the contemporaneous cross-correlation, \( k = 0 \), is neglected in the statistics since it has no bearing on the issue of predictability.

The above test may be combined sequentially to provide a convenient procedure for detecting causality between two variables. After obtaining the residuals from the estimated filters of (4.4.6) and calculating the residual cross-correlations, the statistic in (4.4.7) is used to test for independence. If the null hypothesis of independence is rejected, (4.4.8) is used to detect causality between the leading indicator, \( X_t \), and the variable to be predicted, \( Y_t \). If the value of \( S^k_M(+) \) exceeds the critical value, \( S^k_M(-) \) must be considered. One-way causality from \( X_t \) to \( Y_t \) must be present before the transfer function model of (4.4.1) can be utilized [13]. Finally, the individual cross-correlations may be used to gain information concerning the lag structure contained in the transfer function specification.
CHAPTER V

EXPECTATION MODELS AND CAUSALITY TESTS

5.1 ARIMA Models of Inflation, the Price Level, and Income

In this section we present ARIMA models which were constructed for
the respective series of interest: the inflation rate, the aggregate
price level, nominal income, and real income. The inflation rate is
measured as

\[(5.1.1) \quad \frac{P_t - P_{t-1}}{P_{t-1}} \times 100,\]

where \(P_t\) is the GNP deflator in time \(t\). The deflator was selected as
the measure of the price level since the money demand functions con-
sidered pertain to aggregate holdings. Alternative measures, such as
the Consumer Price Index, would be more appropriate if we were dealing
with disaggregated holdings of specific sectors. Forecasting models
of the C.P.I. will be utilized in conjunction with the analysis of
household money holdings in Chapter VII.

Income is measured by GNP, following a practice that is found in
many earlier studies. Both the GNP series and the GNP deflator series
are seasonally adjusted due to the absence of a seasonally unadjusted
series for the GNP deflator. All the series reflect the recent revi-
sion in the National Income Accounts [74]. Specific data sources are
summarized in Appendix 1.

The identification stage of constructing an ARIMA model follows
a standard set of procedures or guidelines. First, the degree of
differencing necessary to ensure time stationarity of the series is determined. The behavior of the sample autocorrelation and partial autocorrelation functions associated with the varying degrees of differencing serve as the major guidelines in determining the exact degree. In addition, the behavior of the sample variance as successive differences are taken can be of use in the analysis.

Once the degree of differencing has been established, the orders of the autoregressive and moving average polynomials are determined from the sample autocorrelation and partial autocorrelation functions calculated for the differenced series. The model which is identified by using the above procedures should be considered, at best, as a tentative form for characterizing the time series of interest. A unique model is not always indicated by the identification procedure.

Estimation of the model is accomplished by using an iterative technique to minimize the sum of squared residuals which results from the specific parameterization.\(^1\) This procedure will not be discussed in detail here, but one aspect must be noted, the use of backforecasting in conjunction with model estimation. Assume, for simplicity, that the model to be estimated contains a first-order moving average term,

\[(5.1.2) \quad y_t = (1-\theta_1 B)a_t\]

or

\[y_t = a_t - \theta_1 a_{t-1},\]

\(^1\text{For an extended discussion of the methodology, see Box and Jenkins [12, chap. 7].}\]
where \( y_t \) is a suitably differenced series. For \( t = 1 \), the relationship is

\[
y_1 = a_1 - \theta_0 a_0.
\]

Since \( y_1 \) is the first observation in the time series, \( a_0 \) cannot be estimated from the above form, but \( a_0 \) is obviously an important factor in determining \( y_1 \). Ignoring \( a_0 \) by setting it equal to zero will introduce a "transient" into the series which may have noticeable effects upon the estimation process. Alternatively, the value of \( a_0 \) may be "backforecasted" by applying the model of the stochastic process to previous observations. In essence, use of the backforecasting technique results in minimizing an "unconditional" sum of squared residuals. Neglect of the backforecasted residuals implies minimization of a "conditional" sum of squared residuals.\(^2\) Estimates based upon the latter procedure may be viewed as suspect. All of the ARIMA models presented in this study were estimated by an algorithm which incorporates the backforecasting technique.

As noted above, the model form indicated by the identification stage is only tentative. After estimating the model, diagnostic checks are conducted using the residuals to determine whether an adequate representation of the time series has indeed been found. One type of diagnostic check is provided by the autocorrelation function of the residuals which allows one to check the possibility of incorrect parameterization of the model. A more general test of model adequacy is provided by the Box and Pierce [14] "Q" statistic. This statistic is

\(^2\)A comprehensive discussion of backforecasting and the difference between the unconditional and conditional sums of squares is given in Box and Jenkins [12, pages 209-220].
used as an overall test for the randomness of the residuals obtained from fitting an ARIMA model.

Recall that a crucial assumption underlying the ARIMA model form is that the error terms associated with the process being modeled are white noise or, equivalently, independent and identically distributed random variables. Formally, the Q statistic is

\[(5.1.3) \quad Q = n \sum_{i=1}^{k} r_i^2(\hat{\alpha}) ,\]

where \(n\) is the number of observations in the differenced series and \(r_i(\hat{\alpha})\) is the \(i^{th}\) order autocorrelation of the residuals. \(Q\) is approximately distributed as \(\chi^2(k-g)\) where \(g\) is the number of parameters estimated in the ARIMA model. A calculated value of \(Q\) which exceeds the selected critical value of the distribution would indicate that the model is inadequate. Subsequent action usually takes the form of changing the degree of differencing or re-specifying the orders of the polynomials in the ARIMA specification.

The ARIMA models presented in Table 5-1 below were estimated using quarterly data over the sample period of 1953III - 1976II. Presumably, effects upon the respective series stemming from the Korean War were avoided, or minimized, by selecting this period. The Q statistic, the variance of the model residuals (excluding the backforecasted residuals), and \(2\hat{\sigma}\) limits for the coefficients, appearing below the coefficients, are reported for the respective models. Natural logarithms of the GNP deflator, nominal GNP, and real GNP were used in the estimations.\(^3\)

\(^3\)Feige and Pearce [27] also used this transformation when modeling the GNP series.
TABLE 5-1. ARIMA MODELS, 1953III - 1976II

<table>
<thead>
<tr>
<th>Description</th>
<th>Equation</th>
<th>Standard Error</th>
<th>Constant</th>
<th>Q</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>$(1-B)\pi_t = (1-.495B)a_t$</td>
<td>±.185</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{\sigma}^2 = .168$</td>
<td></td>
<td>±.185</td>
<td>Q = 19.9</td>
<td>(23 d.f.)</td>
</tr>
<tr>
<td>GNP Deflator</td>
<td>$(1-B)^2p_t = (1-.491B)a_t$</td>
<td>±.182</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{\sigma}^2 = .000016$</td>
<td></td>
<td>±.182</td>
<td>Q = 17.6</td>
<td>(23 d.f.)</td>
</tr>
<tr>
<td>Nominal GNP</td>
<td>$(1+.250B^5)(1-.436B)(1-B)Y_t = .012 + a_t$</td>
<td>±.207</td>
<td>±.185</td>
<td>±.005</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{\sigma}^2 = .000098$</td>
<td></td>
<td>±.207</td>
<td>Q = 17.4</td>
<td>(22 d.f.)</td>
</tr>
<tr>
<td>Real GNP</td>
<td>$(1-B)y_t = .008 + (1-.259B^5)a_t$</td>
<td>±.002</td>
<td>±.203</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{\sigma}^2 = .000185$</td>
<td></td>
<td>±.002</td>
<td>Q = 12.8</td>
<td>(23 d.f.)</td>
</tr>
</tbody>
</table>

The forecasting formula derived from the inflation model in Table 5-1 is

\[ \hat{\pi}_t^{(\ell)} = \pi_{t+\ell-1} - .495a_{t+\ell-1} \]

A lead time of one period is used in the above formula to generate an observed series of expected inflation rates. Forecasting commences with the first observation of the differenced series and continues until the last observation. The one-step ahead forecast is given by

\[ \hat{\pi}_t^{(1)} = \pi_t - .495(\hat{\pi}_t - \hat{\pi}_{t-1}^{(1)}) \]

As shown in (4.3.13), this expression for the one-step ahead forecast corresponds to the forecasting formula yielded by the adaptive expectations model. If the past history of inflation is the only information set relevant for forecasting future rates, economically rational ex-
expectations of the inflation rate are adaptive. We must note, however, that this result is dependent upon the method of measuring the inflation rate and the nature of the data being used to construct the forecasting model.

Since the major concern of this study lies with the influence of expected inflation upon the demand for money, it seems natural to construct the series of expected inflation rates by using a forecasting model which is based upon observed rates. However, expectations of future inflation rates could also be formed from expectations of the future levels of the aggregate price level. In essence, inflationary expectations may only be the consequence, or a derivative, of forming expectations for the price level. Consider the money demand model given in (2.1.6) where both the expected price level and the expected inflation rate are present. If we assume that the expected price level is given by a suitable forecasting model, we are explicitly requiring that agents form two separate expectations, one of the future inflation rate and one of the future price level. Thus, forecasting future rates of inflation from a model of the inflation rate itself may be redundant. Agents would be incurring needless costs in the form of gathering information and the "value of time" which is lost in conducting the forecasting exercise. The expectation of the present period's inflation rate, formed last period using the expected price level is

\[ \frac{p_t^e - p_{t-1}}{p_{t-1}} \times 100, \]

(5.1.6)

where \( p_t^e \) is the one-step ahead forecast obtained from the ARIMA model which has been constructed for the price level.
Some criterion must be adopted to select the appropriate method of generating expectations of the inflation rate from the two options outlined above. Recall that forecasting accuracy is the prime consideration in constructing the expectation models. Hence, the sum of squared forecast errors from the estimated ARIMA model of the inflation rate will be compared to the corresponding sum of squared forecast errors yielded by the inflation forecast of (5.1.6). This is identical to comparing the sum of squared residuals since the residuals from an estimated ARIMA model are one-step ahead forecast errors.

The ARIMA model given in Table 5-1 for the price level is used to construct the series of one-step ahead forecasts or, on the basis of our assumptions, the expectations of next period's price level. Since natural logarithms of the price series were used in estimating the model, the one-step ahead forecasts of the log values must be converted back to the scale of the raw price series.\(^4\) Note that the transformation involves more than a simple antilog operation.

The forecasts of the raw price level derived from this transformation were used to construct the inflation forecasts given by (5.1.6). The observed values of inflation were then subtracted from the forecasts, giving the forecast errors to be used in constructing the appropriate sum of squares. Two observations of the price series are lost due to the differencing operator in the ARIMA model. Also, the use of backforecasted residuals in calculating the respective sum of squares was avoided. This resulted in the use of 89 forecast errors to calculate the respective sums of squares:

\(^4\)This procedure may be referenced in Nelson [57, pages 161-163].
Sum of Squares from \( \frac{P_t^e - P_{t-1}}{P_{t-1}} \times 100 = 12.781 \)

Sum of Squares from ARIMA (0,1,1) model of inflation rate = 12.833.

The method of forming inflationary expectations from forecasts of the absolute price level is marginally superior to using the ARIMA model constructed for the inflation series itself. However, the difference is not large enough to effectively discriminate between the two procedures. The exact method to be used will have to be determined within the theoretical context or behavioral model being used to describe the role of inflationary expectations. If we are considering a money demand model with expected price as an argument, then formation of inflationary expectations will be viewed as an action which is secondary to, or merely accompanies, the formation of expectations for the price level. In money demand models where only expected inflation appears, the choice is not so straightforward. Both alternatives must be considered in this case. Hence, estimates of the money demand relationship will be offered for both methods of forming expectations.

5.2 ARIMA Models of Leading Indicators

In Chapter IV, we discussed the possibility that forecasts from the ARIMA model constructed for a variable may be improved by using information contained in a second time series. A testing procedure to determine whether such a relationship exists was also described in detail. The first step of the procedure involves estimating an ARIMA model for the time series which is being entertained as a leading indicator of the variable of interest. The potential leading indicators which we consider consist of various monetary and fiscal aggregates,
or their rates of change. Selection of this set is consistent with
the aggregate nature of the variables for which we are attempting to
obtain efficient forecasts, namely the inflation rate, the price level,
nominal income, and real income.

All the indicator series are seasonally adjusted and are therefore
consistent with the form of the data used for the series in Table 5-1.
The ARIMA models which were estimated for the indicators of the infla-
tion rate are presented in the following table.

| TABLE 5-2. ARIMA MODELS FOR POTENTIAL LEADING
<table>
<thead>
<tr>
<th>INDICATORS OF INFLATION, 1953III - 1976II</th>
</tr>
</thead>
</table>

\[
\begin{align*}
\hat{M}_1 &= (1-B)M_1 = (1-\cdot475B^2)a_t \\
& \pm .188 \\
\hat{\sigma}^2 &= .254 \quad Q = 24.4 \quad (23 \text{ d.f.}) \\
\hat{M}_2 &= (1-B)M_2 = (1-\cdot407B^2-\cdot245B^4)a_t \\
& \pm .207 \pm .206 \\
\hat{\sigma}^2 &= .261 \quad Q = 16.0 \quad (22 \text{ d.f.}) \\
\hat{M}_B &= (1-B)MB = (1-\cdot581B)(1-\cdot357B^4)a_t \\
& \pm .171 \pm .203 \\
\hat{\sigma}^2 &= .142 \quad Q = 19.5 \quad (22 \text{ d.f.}) \\
\hat{M}_1 &= (1-\cdot260B-\cdot524B^2)(1-B)^2M_1 = a_t \\
& \pm .193 \pm .192 \\
\hat{\sigma}^2 &= .979 \quad Q = 19.8 \quad (22 \text{ d.f.}) \\
\hat{M}_2 &= (1-B)^2M_2 = (1-\cdot564B^2)a_t \\
& \pm .182 \\
\hat{\sigma}^2 &= 3.701 \quad Q = 21.5 \quad (23 \text{ d.f.}) \\
\hat{M}_B &= (1-B)^2MB = (1-\cdot463B)(1-\cdot231B^4)a_t \\
& \pm .192 \pm .220 \\
\hat{\sigma}^2 &= .075 \quad Q = 18.0 \quad (22 \text{ d.f.})
\end{align*}
\]
\[
(1-B)^2 E_t = (1-.788B)(1-.250B^5+.371B^1) a_t \\
\pm .135 \pm .213 \pm .223 \\
\hat{\sigma}^2 = 13.827 \quad Q = 13.2 \quad (21 \ d.f.)
\]

\[
(1-B)R_t = 3.165 + a_t \\
\pm 1.369 \\
\hat{\sigma}^2 = 42.662 \quad Q = 19.3 \quad (24 \ d.f.)
\]

NOTES:  MB = Monetary Base

FEE = Full Employment Expenditures

FER = Full Employment Receipts

The \( \dot{M}_1, \dot{M}_2, \) and \( \dot{MB} \) series are the percentage rates of change of the corresponding monetary aggregates. Data sources may be referenced in Appendix I.

ARIMA models for the leading indicators of nominal GNP and the GNP deflator are presented in Table 5-3. While these indicators are also found in the previous table, natural logarithms of the data were used in estimation to ensure compatibility with the models for nominal GNP and the GNP deflator.

**TABLE 5-3. ARIMA MODELS FOR POTENTIAL LEADING INDICATORS OF NOMINAL GNP AND THE GNP DEFlator, 1953III - 1976II**

\[
(1-B)^2 M_1_t = (1-.476B^2) a_t \\
\pm .189 \\
\hat{\sigma}^2 = .000025 \quad Q = 24.3 \quad (23 \ d.f.)
\]

\[
(1-B)^2 M_2_t = (1-.402B^2-.249B^4) a_t \\
\pm .208 \quad \pm .207 \\
\hat{\sigma}^2 = .000026 \quad Q = 16.0 \quad (22 \ d.f.)
\]

\[
(1-B)^2 MB_t = (1-.574B)(1-.306B^4) a_t \\
\pm .174 \quad \pm .203 \\
\hat{\sigma}^2 = .000014 \quad Q = 21.6 \quad (22 \ d.f.)
\]
Finally, we present the estimated models of the leading indicators being considered for real GNP. The indicators are merely the real values of those found in Table 5-3. The GNP deflator was used in calculating the real values. All of the series were converted to natural logarithms before estimation to be consistent with the form of the data used to estimate the model for real GNP.

**TABLE 5-4. ARIMA MODELS FOR POTENTIAL LEADING INDICATORS OF REAL GNP, 1953III - 1976II**

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Model</th>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t-statistic</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real M1</td>
<td>((1-B)^2M_{t} = (1-.270B^2)a_{t})</td>
<td>(0.204)</td>
<td>(0.00043)</td>
<td>19.4</td>
<td>(23 d.f.)</td>
</tr>
<tr>
<td>Real M2</td>
<td>((1-B)^2M_{t} = (1-.400B^2)a_{t})</td>
<td>(0.196)</td>
<td>(0.00044)</td>
<td>18.5</td>
<td>(23 d.f.)</td>
</tr>
<tr>
<td>Real MB</td>
<td>((1-B)^2M_{t} = (1-.515B)a_{t})</td>
<td>(0.186)</td>
<td>(0.00031)</td>
<td>19.8</td>
<td>(23 d.f.)</td>
</tr>
<tr>
<td>Real FEE</td>
<td>(M_{t} = 0.0084 + a_{t})</td>
<td>(0.0046)</td>
<td>(0.0048)</td>
<td>27.6</td>
<td>(24 d.f.)</td>
</tr>
<tr>
<td>Real FER</td>
<td>(M_{t} = 0.0091 + a_{t})</td>
<td>(0.0050)</td>
<td>(0.0058)</td>
<td>10.4</td>
<td>(24 d.f.)</td>
</tr>
</tbody>
</table>
Following the Haugh [42] procedures outlined in Chapter IV, both
the residuals from the leading indicator models of Tables 5-2 - 5-4
and the residuals from the "output" series in Table 5-1 were calculated.
The appropriate pairs of the prewhitened series were then cross-corre-
lated. Once these residual cross-correlations are obtained, the set
of test statistics given in (4.4.7), (4.4.8), and (4.4.9) can be cal-
culated. Recall that the null hypothesis of this testing procedure is
that the input series (i.e. the leading indicator) and the output
series are independent. Failure to reject the null hypothesis implies
that the input series will not improve the forecasts of the output
series. If independence cannot be rejected, the use of the term
"leading indicator" is no longer appropriate to characterize the input
series. Table 5-5 contains the test statistics for the relationships
between the inflation rate and the indicators which are being considered.

<table>
<thead>
<tr>
<th></th>
<th>S* +</th>
<th>S* (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>M1</td>
<td>25.35</td>
<td>8.69</td>
</tr>
<tr>
<td>M2</td>
<td>26.23</td>
<td>12.03</td>
</tr>
<tr>
<td>MB</td>
<td>19.09</td>
<td>9.00</td>
</tr>
<tr>
<td>M1</td>
<td>27.64</td>
<td>12.59</td>
</tr>
<tr>
<td>M2</td>
<td>25.31</td>
<td>10.48</td>
</tr>
<tr>
<td>MB</td>
<td>19.92</td>
<td>10.35</td>
</tr>
<tr>
<td>FEE</td>
<td>27.27</td>
<td>11.76</td>
</tr>
<tr>
<td>FER</td>
<td>22.10</td>
<td>7.93</td>
</tr>
</tbody>
</table>

NOTES: The degrees of freedom for the χ² distribution used in
testing with $S_{12}^+$, $S_{12}^+(-)$, and $S_{12}^-(+)$ are 25, 12, and 12, respectively. All tests utilize the 5% level for the decision rule. The corresponding critical values are $\chi^2(25) = 37.7$ and $\chi^2(12) = 21.0$.

The statistics presented above indicate that none of the potential leading indicators will be of use in improving forecasts of the inflation rate. In terms of the Granger definition of causality, and given our methods of measurement, none of the variables which were considered have a causal relationship with inflation. While these results may be surprising, they confirm the findings of Feige and Pearce [28]. Hence, the ARIMA model of the inflation rate will be used for forecasting inflation and, by assumption, for generating a measure of the expected inflation rate.

The test statistics for the leading indicators of the price level, as measured by the GNP deflator, are presented in Table 5-6. It was shown above that no improvement in the forecasts of the inflation rate can be realized by using the indicators of Table 5-5 in conjunction with the inflation rate series. However, if the forecasts of the price level can be improved by using one or more indicators, it may be possible to improve the forecasts of inflation given by (5.1.6). Recall that the expectation of next period's price level, $P^e_t$, is assumed to be the one-step ahead forecast of the price level made in time $t-1$. 
TABLE 5-6. TEST STATISTICS FOR THE POTENTIAL LEADING INDICATORS OF THE PRICE LEVEL, 1953III - 1976II

<table>
<thead>
<tr>
<th></th>
<th>S*</th>
<th>S* (+)</th>
<th>S* (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>23.08</td>
<td>8.81</td>
<td>14.27</td>
</tr>
<tr>
<td>M2</td>
<td>25.49</td>
<td>12.22</td>
<td>13.23</td>
</tr>
<tr>
<td>MB</td>
<td>23.33</td>
<td>12.25</td>
<td>11.06</td>
</tr>
<tr>
<td>FEE</td>
<td>15.54</td>
<td>6.43</td>
<td>8.88</td>
</tr>
<tr>
<td>FER</td>
<td>21.98</td>
<td>6.38</td>
<td>6.73</td>
</tr>
</tbody>
</table>

None of the statistics exceed the selected critical value of the appropriate $\chi^2$ distribution. These results fail to support the use of the indicators to improve the forecasts obtained from the ARIMA model constructed for the price level. Inflation forecasts derived from (5.1.6) will also not be improved by using the indicators. Hence, economically rational expectations of the price level and the inflation rate, given by (5.1.6), may be generated from the ARIMA model of the price level. Since the indicators entertained for the price level are of no use, we have no additional information on which to distinguish between the two methods of forecasting inflation, forecasting with the ARIMA model for inflation or forecasting with the ARIMA model for the price level. Both techniques will be used in the study of the money demand functions.

Economically rational expectations of income, measured by GNP, will also be used in the course of our study of money demand. As in the case of inflation and the price level, potential leading indicators of income should be incorporated into a forecasting model if the precision or efficiency of forecasting can be improved. The appropriate
test statistics from the cross-correlation analysis are presented in
the following table.

<table>
<thead>
<tr>
<th></th>
<th>$S^*_12$</th>
<th>$S^*_12(+)</th>
<th>S^*_12(-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML</td>
<td>28.72</td>
<td>9.70</td>
<td>16.51</td>
</tr>
<tr>
<td>M2</td>
<td>34.79</td>
<td>17.67</td>
<td>17.04</td>
</tr>
<tr>
<td>MB</td>
<td>18.86</td>
<td>7.69</td>
<td>11.16</td>
</tr>
<tr>
<td>FEE</td>
<td>23.22</td>
<td>13.58</td>
<td>8.13</td>
</tr>
<tr>
<td>FER</td>
<td>27.17</td>
<td>4.38</td>
<td>21.51**</td>
</tr>
</tbody>
</table>

**significant at .05 level

The statistics indicate that none of the monetary and fiscal aggreg-
gates can be considered as leading indicators of income. The only
significant relationship in the table is one of reverse causality from
income to full employment receipts. Hence, the respective series con-
sidered in the above table will not be used in an attempt to improve
upon the forecasts obtained from the ARIMA model of income. 5

5These results have definite implications for reduced form models which
attempt to explain the level or movement of income. The most popular
model in this class is, of course, the St. Louis equation [3]. Let us
suppose that the $S^*_M$ and $S^*_M(+) statistics for ML, full employment
expenditures, and full employment receipts had exceeded the critical
values of the relevant $\chi^2$ distributions. On that basis, we would have
attempted to build a forecasting model based upon the following
specification,

\[(5.2.1) \quad Y_t = V_1(B) \cdot ML_t + V_2(B) \cdot FEE_t + V_3(B) \cdot FER_t + N_t ,\]

where the $V_i(B)$ are infinite polynomials in the backshift operator and
$N_t$ is a noise, or error, process. This multiple input transfer func-
tion is identical, in spirit, to the St. Louis equation. Our results
show that forecasts from models like (5.2.1) will not be superior to

(continued)
By constructing forecasting models for nominal income and price, one measurement of economically rational expectations of real income may be generated, namely $Y^e_t / P^e_t$. However, agents may form expectations of real income by using a forecasting model based upon the real income series itself. A decision as to which method takes precedence will be dictated by the theoretical model in which the expectation of real income appears. To ensure that an efficient forecasting model for real income has been found, the cross-correlation analysis was conducted for real income and several leading indicators. The results are given in the following table.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>$S^{*}_{12}$</th>
<th>$S^{*}_{12}(+)$</th>
<th>$S^{*}_{12}(-)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real M1</td>
<td>32.11</td>
<td>10.13</td>
<td>20.23</td>
</tr>
<tr>
<td>Real M2</td>
<td>39.47**</td>
<td>14.07</td>
<td>25.03**</td>
</tr>
<tr>
<td>Real MB</td>
<td>37.25</td>
<td>15.91</td>
<td>20.99</td>
</tr>
<tr>
<td>Real FEE</td>
<td>20.91</td>
<td>11.05</td>
<td>7.41</td>
</tr>
<tr>
<td>Real FER</td>
<td>20.30</td>
<td>8.78</td>
<td>10.08</td>
</tr>
</tbody>
</table>

**significant at .05 level

The $S^{*}_{12}$ statistic for real M2 and real income exceeds the selected critical value, indicating that we fail to accept the hypothesis of independence. However, the $S^{*}_{12}(+)$ statistic does not exceed the forecasts derived from a simple ARIMA model of income. However, models like the St. Louis equation may still be useful for examining structural or contemporaneous relationships between income and its possible determinants. Presumably, past values of income embody the influences of such determinants.
relevant critical value and this latter statistic is more appropriate to the detection of one-way causality from real M2 to real income. Hence, we do not pursue the construction of a forecasting model which incorporates the monetary aggregate. It can easily be seen from the $S_{12}^*$ statistic that the cross-correlations for negative lags are dominating the value of $S_{12}^*$. The remaining series considered as potential leading indicators do not appear to be relevant for forecasting values of real income. Hence, the ARIMA model for real income will be used to generate the series of economically rational expectations.

In summary, application of the Haugh procedure for testing causality has failed to reveal any causal relationship among the series which we desire to forecast and the potential leading indicators considered for them. If these results seem surprising, recall that the rejection of any causal relationships is made within the scope of a particular definition advanced by Granger [39]. We do not wish to become involved in a philosophical argument over appropriate definitions of causality. The major concern in this study is whether one economic variable can be used to improve forecasts of another. Our results indicate that the forecasts will not be improved in any of the cases examined. Thus, the ARIMA models of the individual series can be used for forecasting, and, by assumption, for generating economically rational expectations.

5.3 Intervention Analysis

The previous section dealt with the hypothesis that a forecasting model of an economic variable may utilize other information sets in addition to the past history of the variable in question. The economic
series selected as potential leading indicators were found to be irrelevant for forecasting purposes. However, there is another type of information set which may be used by economic agents in the course of constructing efficient forecasting models. This set is composed of events or institutional changes which significantly affect the values of the economic variable being studied. These events may be viewed as "interventions" in the series of values observed for the variable. The interventions of interest in this study are the various phases of the wage and price controls which were in effect during the early 1970's. Economically rational agents might use the information sets represented by the respective phases to form expectations of inflation rates. However, some of the control periods may not have been seen to have a significant influence and therefore would not be used by agents in forming expectations. It would not be economically rational to use worthless sets of information.

To assess the influence of interventions upon a variable, we follow the work of Box and Tiao [15] by specifying the following stochastic model,

\[(5.3.1) \quad z_t = f(\tau, I) + N_t,\]

where \(z_t\) is the series of interest, \(\tau\) is a set of exogenous variables, \(I\) is a set of interventions, and \(N_t\) is noise or a stochastic disturbance. The results of Section 5.2 indicated that no leading indicators or input variables would be of use in a forecasting model for either inflation or the price level. Hence, the set of exogenous variables, \(\tau\), is dropped from (5.3.1), giving
(5.3.2) \[ z_t = f(I) + N_t. \]

The function describing the effects of the interventions is written as

\[ (5.3.3) \quad f(I) = \sum_{j=1}^{k} \left[ \omega_j(B)/\delta_j(B) \right] I_{tj}, \]

where \( k \) is the number of interventions being considered. The polynomials in the backshift operator, \( \omega_j(B) \) and \( \delta_j(B) \), have degrees of \( r_j \) and \( s_j \), respectively. Further, we assume that the noise variable in (5.3.2) \( N_t \), may be decomposed into an autoregressive-moving average process

\[ (5.3.4) \quad \psi(B) \cdot N_t = \theta(B) \cdot a_t, \]

where \( a_t \) is white noise. Substituting (5.3.3) and (5.3.4) into the basic model of (5.3.2) yields the general form of the intervention model

\[ (5.3.5) \quad z_t = \sum_{j=1}^{k} \left[ \omega_j(B)/\delta_j(B) \right] I_{jt} + \frac{\theta(B)}{\psi(B)} a_t. \]

Note the similarity between this form and the transfer function model of (4.4.2). The model of (5.3.5) is actually a multiple-input transfer function with interventions serving as the inputs. Specification of the degrees of the \( \omega_j(B) \) and \( \delta_j(B) \) will determine the patterns of response in the \( z_t \) that result from the interventions. A catalog of these patterns is available for reference in Box and Tiao [15].

The phases of the wage and price controls may be treated as interventions and then analyzed in a model like (5.3.5) for the inflation rate or the price level itself. These interventions are represented by indicator variables which have a value of 1 when the particular phase, or intervention, is in effect, and a value of 0 otherwise.
Thus, the interventions are input "pulses" in the raw series being modeled. Since we want to obtain a numerical measure of the direct effect upon the level of the raw series, the $\delta_j(B)$ polynomials will be set equal to unity and the $\omega_j(B)$ polynomials will be specified as constants, $\omega_j$. For simplicity, consider a model in which two interventions are present. Given the above specifications for the $\delta_j(B)$ and $\omega_j(B)$, we have

\begin{equation}
(5.3.6) \quad z_t = \omega_1 I_{1t} + \omega_2 I_{2t} + \frac{\theta(B)}{\psi(B)}a_t.
\end{equation}

The $\omega_1$ and $\omega_2$ parameters represent the effects of the respective interventions upon the series, $z_t$. If we were restricting our attention to Phases I and II, the above model would be used to deduce their effects upon either the inflation rate or the price level.

The first step in estimating the model of (5.3.6) is specification of the noise model given by (5.3.4). This is accomplished by finding an acceptable ARIMA model for the raw series, $z_t$. The results presented in Table 5-1 show that an ARIMA(0,1,1) may be used as the noise model for the inflation rate. Therefore, the intervention model for the inflation rate is written as

\begin{equation}
(5.3.7) \quad \pi_t = \omega_1 I_{1t} + \omega_2 I_{2t} + \frac{(1-\theta B)}{(1-B)}a_t.
\end{equation}

Rearranging, we have

\begin{equation}
(5.3.8) \quad (1-B)\pi_t = \omega_1 (1-B)I_{1t} + \omega_2 (1-B)I_{2t} + (1-\theta B)a_t.
\end{equation}

In the case of the price level, an ARIMA(0,2,1) was found to be an adequate representation of the time series. Thus, the intervention model for the price level is identical, in form, to the inflation model.
given above with the exception that the differencing operator is squared.

Since an estimate of $\theta_1$ is available from the noise model, the indicator variables and the raw series could be suitably transformed to yield a linear model capable of being estimated by linear regression procedures. However, the parameters in (5.3.8) should be jointly estimated. This is accomplished by minimizing

$$S(\beta) = \sum_{t=-\infty}^{n} a_t^2,$$

where $n$ is the number of observations in the differenced series and $\beta$ is the $m \times 1$ vector of parameters in the model. Minimizing $S(\beta)$ by a suitable nonlinear algorithm will produce estimates of $\beta$ that are approximately maximum likelihood.\(^6\) As in the case of estimating ARIMA models, the backforecasting procedure for the $a_t$, where $t \leq -Q$, should be employed. Thus, the sum of squares which we are actually minimizing is

$$S(\beta) = \sum_{t=-Q}^{n} a_t^2.$$

Only $a_0$ is backforecasted for the inflation model of (5.3.8) since the noise model is of the moving average type and contains one parameter.\(^7\) Autoregressive models or mixed models will usually require additional backforecasts. Once the minimum of $S(\beta)$ has been found, the variance-covariance matrix for the parameters is given by

---
\(^6\)For details, see Box and Tiao [15, pages 72-73].
\(^7\)A complete discussion of backforecasting within moving average models is available in Box and Jenkins [12, page 213]. Thirty backforecasted residuals, $a_0, \ldots, a_{-30}$, were estimated at each iteration of the nonlinear algorithm.
(5.3.11) \[ V(\beta) = 2\hat{\sigma}^2 \cdot (S_{ij})^{-1}, \]

where \( \hat{\sigma}^2 = S(\hat{\beta})/(n-m) \) and \( S_{ij} \) is a matrix of the derivatives 
\[ \frac{\partial^2 S(\beta)}{\partial \beta_i \partial \beta_j}, \]
evaluated at \( \beta = \hat{\beta} \). The roots of the diagonal elements of \( V(\beta) \) represent the approximate standard errors of the parameter estimates. Inferences about the significance of individual parameters may be based upon these values.

Intervention models for the inflation rate and the price level were estimated by a standard nonlinear algorithm [11]. The backforecasting procedure was employed in the routine to provide an estimate of \( a_0 \) and to ensure that the unconditional sum of squares was the function being minimized. Initially, the intervention models were specified to include the effects of all the respective phases, Phase I through Phase IV and the Second Freeze on prices. Thus, these models contained five interventions. Quarterly data for the two series was used in the estimation along with the following indicator variables which represent the interventions:

<table>
<thead>
<tr>
<th>( I_{1t} )</th>
<th>( I_{2t} )</th>
<th>( I_{3t} )</th>
<th>( I_{4t} )</th>
<th>( I_{5t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 0 )</td>
<td>( t = 0 )</td>
<td>( t = 0 )</td>
<td>( t = 0 )</td>
<td>( t = 0 )</td>
</tr>
<tr>
<td>( t = 1 )</td>
<td>( t = 1 )</td>
<td>( t = 1 )</td>
<td>( t = 1 )</td>
<td>( t = 1 )</td>
</tr>
<tr>
<td>elsewhere</td>
<td>elsewhere</td>
<td>elsewhere</td>
<td>elsewhere</td>
<td>elsewhere</td>
</tr>
</tbody>
</table>

(Phase I) (Phase II) (Phase III) (Second Freeze) (Phase IV).
The use of quarterly data obviously caused some problems in assigning values of the indicator variables to the actual time periods covered by the respective phases.

Continued high values of the inflation series during the Phase III, Second Freeze, and Phase IV periods should prompt one to question the influence of that set of controls. Estimates of the inflation and price models containing all of the interventions indicated that these controls were ineffective. A sequential estimation procedure was then adopted in an effort to find the subset of interventions which yielded significant effects.

In the first step, the Phase IV intervention was excluded and the resulting intervention model was estimated. Once again, several phases were found to be insignificant. Next, the intervention for the Second Freeze was excluded. This model, when estimated, contained insignificant parameters. The procedure of excluding interventions was continued until all of the estimated parameters in the final model were significant. For the inflation rate, the final model contained the interventions reflecting Phases I and II. For the price level, only Phase I proved to be a significant influence upon the series. Natural logarithms of the price series were used in estimating the price model. The final models are given in the following table, accompanied by the approximate standard errors of the parameters which are in parentheses.

\[
\begin{align*}
\text{Inflation} \ (1-B)\pi_t &= -0.662(1-B)I_{1t} - 0.534(1-B)I_{2t} + (1-0.546B)a_t \\
&\quad (\pm 0.271) \quad (\pm 0.252) \quad (\pm 0.096) \\
\delta^2 &= 0.1601 \\
\text{GNP Deflator} \ (1-B)^2p_t &= -0.0038(1-B)^2I_{1t} + (1-0.466B)a_t \\
&\quad (\pm 0.0022) \quad (\pm 0.095) \\
\delta^2 &= 0.00016
\end{align*}
\]

The parameter estimates indicate that the interventions included in the models had a negative influence upon the respective series. The impact of Phase I upon the inflation rate is stronger than that of Phase II, a result which we would expect given the characteristics of those two control periods. \(^8\)

Economically rational agents would use the forecasting models of Table 5-9 to form expectations of inflation and the price level if they recognized the interventions as useful information sets. \(^9\) The cost of acquiring these information sets would be very close to zero so a strong case could be made for selecting the intervention model as

---

\(^8\) Our finding of a significant effect attributable to Phase II differs from the results of Box and Tiao [15]. Their study found Phase I to be the only period of importance. This difference in results is probably attributable to differences in the measurement of price and the number of observations used in our study.

\(^9\) Note that the significance of an intervention parameter is being equated with using the information in making forecasts of future values. We can only judge the significance, and therefore the prospective use, of an intervention by modeling the entire time series which includes observations past the intervention. Thus, we cannot view significance as implying that the agent knows whether or not to use the intervention in making forecasts at the time of its occurrence. In essence, our modeling procedures are quantifying the effect of interventions after the fact.
forecasting devices. However, the ARIMA models for inflation and the price level given in Table 5-1 will also reflect the influence of the price controls when they are used for forecasting.

Consider the ARIMA model for the inflation rate,

\[ (1-B)\pi_t = (1-.495B)a_t. \]

Rewriting, we have

\[ \pi_{t+1} = \pi_t + a_{t+1} - .495a_t. \]

To derive the one-step ahead forecasting model, we take conditional expectations which gives

\[ \hat{\pi}_t(1) = \pi_t - .495a_t. \]

The residual, \( a_t \), is the error of the one-step ahead forecast of \( \pi_t \) made in time \( t-1 \). Thus, we have

\[ \hat{\pi}_t(1) = \pi_t - .495(\pi_t - \hat{\pi}_{t-1}(1)). \]

Even though the forecast of inflation for the initial period of wage and price controls could be much too high, subsequent forecasts will take this into account.

Turning once again to the intervention model of the inflation rate in Table 5-9, the forecast from the model for 1971III, the quarter in which the Phase I intervention first appeared, will be too high if we assume that the intervention significantly lowered the inflation rate. The high forecast simply reflects the fact that agents did not anticipate the controls. When the forecast for 1971IV is made, the presence
of Phase I will affect it in three ways, by directly incorporating the inflation rate of 1971IIII, by using the intervention parameter of Phase I, and by the use of the forecast error incurred in 1971IIII.\footnote{None of this information is redundant. Each of these components of the effect due to Phase I, or other interventions, comes from either the parameterization of the model or its moving average character.} This forecast error will presumably be high. A similar situation is easily seen for the case of Phase II. The above comments also apply to the ARIMA model for the price level since it is also of the moving average type. The ARIMA and interventions models for both series will be used to generate expectation series for the study of money demand.

Forecasting formulas derived from intervention models will obviously differ from those yielded by simple ARIMA models. To illustrate this procedure, we again confine our attention to the intervention model for inflation. From Table 5-9, we have,

\[
(1-B)\pi_t = -0.662(1-B)I_{1t} + 0.534(1-B)I_{2t} + (1-0.546B)a_t,
\]

or,

\[
\pi_{t+1} = \pi_t - 0.662I_{1t,t+1} + 0.662I_{1t} - 0.534I_{2t,t+1} + 0.534I_{2t}
\]

\[+ a_{t+1} - 0.546a_t.\]

Taking conditional expectations at time $t$ in the above expression yields the formula for the one-step ahead forecasts

\[(5.3.12) \quad \hat{\pi}_t(1) = \pi_t - 0.662[I_{1,t+1}^I] + 0.662[I_{1t}^I] - 0.534[I_{2,t+1}^I]
\]

\[+ 0.534[I_{2t}^I] - 0.546a_t,\]

where the square brackets denote conditional expectations. Obviously,
$[I_{1t}]$ is equal to $I_{1t}$ since the intervention has occurred in time $t$.

The value to be used for $[I_{1,t+1}]$ is not as apparent since the forecast for inflation in time $t+1$ is being made in time $t$. We might be inclined to set the value equal to zero but the implications of doing so are not desirable, given the form of the data being used in the intervention model. Suppose the inflation rate for 1971IV is being forecast and $[I_{1,t+1}]$ or equivalently, $[I_{1,1971IV}]$, is set equal to zero. The forecasting formula in this case is

$$\hat{\pi}_{1971III}^{(1)} = \pi_{1971III} + 0.662 I_{1,1971III} - 0.546 a_{1971III},$$

where $I_{1,1971III}$ equals one. Recall that the effect of the Phase I intervention upon the inflation rate was found to be negative in sign by the model in Table 5-9. However, the intervention, as it appears in the above forecasting model, will cause the forecast for inflation in 1971IV to increase. This does not seem appropriate. Thus, we will not set $[I_{1,t+1}]$ equal to zero but rather will use the value of the indicator variable that corresponds with the particular intervention, denoted by $i$, and time index $t+1$. To continue our example, the forecast for 1971IV will be

$$\hat{\pi}_{1971III}^{(1)} = \pi_{1971III} - 0.662 I_{1,1971IV} + 0.662 I_{1,1971III} - 0.546 a_{1971III},$$

or,

$$\hat{\pi}_{1971III}^{(1)} = \pi_{1971III} - 0.546 a_{1971III}. $$

Treating $[I_{1,t+1}]$ in this way implies that agents will revise their forecasts for a given time period once the intervention has been
observed in that same period. This action would appear to be rational.

As in the case of simple ARIMA models, the intervention models for inflation and the price level may both be used to forecast future values of the inflation rate. To determine whether one method is superior, the sum of squared residuals was calculated from forecasts of the inflation rate given by the intervention model of the price level. The sum of squares for the intervention model of inflation was also calculated. The respective values, based on 89 residuals, are:

\[
\text{SS inflation model} = 12.018 \\
\text{SS price model} = 12.297.
\]

The intervention model of inflation appears to outperform the price level model in terms of the sums of squares but the difference in the values is very small. Therefore, both will be used in our subsequent studies of money demand.
CHAPTER VI

MONEY DEMAND MODELS

6.1 Inflationary Expectations and the Demand for Money

Many theoretical forms of the aggregate money demand equation have been advanced in the course of studying this important relationship. These forms range from the simple Keynesian equation, which specifies money holdings as a function of income and an interest rate, to the elaborate specifications contained in multi-equation models of macro-economic activity. Our analysis is not conducted within the framework of such large systems. Rather, we shall deal only with the money demand equation itself. This limitation appears warranted given that our emphasis is upon a particular variable, inflationary expectations, which is relatively new to the study of money demand. The task of incorporating the variable into larger models is left to others.

We shall adopt the following model of money demand for the first stage of our investigation into the influence provided by inflationary expectations,

\[ m_t^* = f(y_t, i_t, \pi_t^e) \]

In this model \( m_t^* \) is the desired quantity of real money balances, \( y_t \) is real income, \( i_t \) is a nominal interest rate, and \( \pi_t^e \) is the expected rate of inflation expressed in percentage terms. The relationship between the actual and desired stock of real money balances is specified according to the popular stock-adjustment model,
\[(6.1.2) \quad m_t - m_{t-1} = \theta (m^t - m_{t-1}) \cdot\]

Hence, we assume that agents may only partially adjust their actual holdings of real balances to the desired level in a given time period. The parameter \(\theta\) describes the speed of this adjustment process. Combining the above mechanism with a linear specification of the model in (6.1.1), and adding a stochastic disturbance term, yields

\[(6.1.3) \quad m_t = a + b \cdot y_t + c \cdot m_{t-1} + d \cdot i_t + f \cdot \pi^e_t + \varepsilon_t ,\]

where \(c = 1 - \theta.\)

Other versions of the money demand function which include income expectations and interest rate expectations will be considered in later sections of the chapter. Since the number of expectations series to be considered is rather sizeable, the models used to generate the respective series are summarized in the following table. Recall that the actual values of the expectations are the one-step ahead forecasts generated from the forecasting versions of these time series models.

<table>
<thead>
<tr>
<th>TABLE 6-1. ARIMA MODELS USED TO GENERATE EXPECTED INFLATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation (\ldots \ldots \cdot (1-B)\pi^t = (1-.495B)a^t)</td>
</tr>
<tr>
<td>Inflation with Intervention (\ldots \cdot (1-B)\pi = -0.662I^* - 0.534I^* + (1-.546B)a^t)</td>
</tr>
<tr>
<td>Price (\ldots \ldots \cdot (1-B)^2P^t = (1-.491B)a^t)</td>
</tr>
<tr>
<td>Price with Intervention (\ldots \cdot (1-B)^2P = -0.0038I^* + (1-.466B)a^t)</td>
</tr>
</tbody>
</table>

\(^1\)Valentine [75] has used this model to study the influence of inflationary expectations upon the demand for money in Australia. However, his measures of inflationary expectations are not estimated by forecasting methods but rather by inclusion of parameters in the money demand model itself.
NOTES: $T^*$ refers to the interventions which have been suitably differenced.

In estimating the money demand model of (6.1.3), an effort was made to select a data set or time period which would reflect the prediction given by the threshold hypothesis. Specifically, expected inflation is most likely to exert a significant influence upon money demand in a period of relatively high inflation. As a result, the time period of 1968I - 1976II was selected for estimation. While the particular quarter chosen for the initial observation may appear to be somewhat arbitrary, it provides an approximate boundary between a period of low inflation and a period of distinctly higher inflation. It seems logical to first test the relationship using a data set suggested by theory. If the influence of expected inflation upon money demand is found to be significant, then other periods which are characterized by lower values of inflation should be examined.

The money stock variable is measured by M1, defined as demand deposits and currency held by the public. The choice of M1, rather than M2, as the measure of money balances was made for the following reason. One of the effects of inflationary expectations may be to cause agents to hedge by substituting interest-earning assets, such as time deposits at banks, for money defined as M1. While the possibility of this action is remote in periods of extremely high inflation, it must be considered for the moderate levels of inflation encountered in the U.S. experience. A significant relationship between expected inflation and the demand for money as measured by M2 would be less likely, or meaningful, than for money measured by M1. Portfolio adjustments between M1 and M2 might obscure the real effects of expected inflation.
Before estimation, the money stock was deflated by the GNP deflator to obtain real money balances. Real income, \( y_t \), is measured by real GNP and the interest rate, \( i_t \), is measured by the 3-month Treasury bill rate. Two alternative functional forms for the model of (6.1.3) were entertained. In the first specification, all of the variables were converted to natural logarithms which yields the familiar log-linear form of the demand function. This functional form is valid since the values of expected inflation from the forecasting models were positive over the entire sample period. However, we must recognize the possibility that negative values for this variable could occur outside of our sample period. Thus, expected inflation will enter a second specification in linear form while the remaining variables will still be measured by natural logarithms.

The results obtained from estimating the complete logarithmic specifications by ordinary least squares are presented in Table 6-2. \(^2\) There are four equations to be considered, the result of using the different expectations models for inflation. The parameter for the expected inflation variable is significantly different from zero in each of the four equations. Thus, the conclusion that inflationary expectations affect the demand for money is robust with respect to the alternative models employed to generate these expectations. However, the value of the parameter for expected inflation does vary among the alternative forecasting models. While the model fit appears to be excellent in each case with the values of adjusted \( R^2 \) varying from .949 to .963, this narrow range of values for the measure of explanatory

\(^2\) All computations reported in this chapter were done using SHAZAM [77], an econometrics computer program.
### TABLE 6-2. MONEY DEMAND EQUATIONS USING QUARTERLY RATE OF EXPECTED INFLATION, 1968I - 1976II

<table>
<thead>
<tr>
<th>Equation</th>
<th>Source of $\pi^e$</th>
<th>Constant</th>
<th>$m_{t-1}$</th>
<th>$y$</th>
<th>$i$</th>
<th>$\pi^e$</th>
<th>$h$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Inf.</td>
<td>.206 (.870)</td>
<td>.946 (19.410)</td>
<td>.016 (.755)</td>
<td>-.014 (-2.296)</td>
<td>-.018 (-3.410)</td>
<td>.14</td>
<td>.955</td>
</tr>
<tr>
<td>6.2</td>
<td>Inf.</td>
<td>.070 (.245)</td>
<td>.971 (19.070)</td>
<td>.017 (.661)</td>
<td>-.015 (-2.198)</td>
<td>-.014 (-2.551)</td>
<td>1.00</td>
<td>.949</td>
</tr>
<tr>
<td></td>
<td>Int.</td>
<td>(.245)</td>
<td>(19.070)</td>
<td>(.661)</td>
<td>(-.517)</td>
<td>(-.483)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.3</td>
<td>Price</td>
<td>.203 (.237)</td>
<td>.947 (19.477)</td>
<td>.016 (.736)</td>
<td>-.014 (-2.295)</td>
<td>-.017 (-3.403)</td>
<td>.14</td>
<td>.955</td>
</tr>
<tr>
<td></td>
<td>(.237)</td>
<td>(19.477)</td>
<td>(.736)</td>
<td>(-.264)</td>
<td>(-.321)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.4</td>
<td>Price</td>
<td>.207 (.214)</td>
<td>.944 (22.124)</td>
<td>.018 (.953)</td>
<td>-.014 (-2.662)</td>
<td>-.019 (-4.431)</td>
<td>-.10</td>
<td>.963</td>
</tr>
<tr>
<td></td>
<td>Int.</td>
<td>(.214)</td>
<td>(22.124)</td>
<td>(.953)</td>
<td>(-.250)</td>
<td>(-.339)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTES:**

Numbers in the first set of parentheses are t-ratios.
Numbers in the second set of parentheses are long-run elasticities.
This format for presenting the estimated equations will be used in future tables as well.

*Source of $\pi^e$*

- Inf. - ARIMA Model of Inflation Rate
- Inf.Int. - Intervention Model of Inflation Rate
- Price - ARIMA Model of Price Level
- Price Int. - Intervention Model of Price Level
power does not permit us to judge one equation as being preferable to the others.

Of particular interest are the estimates of the parameters for the lagged real money stock. The speed of adjustment in the underlying stock adjustment mechanism is given by subtracting the parameter from one. With these parameter estimates lying in the range of .947 to .971, a very slow speed of adjustment between the actual stock of real money balances and the desired stock is implied. Other writers have reported this slow adjustment for sample periods which differ from the period being used here [54].

The insignificance of the real income variable in the estimated equations differs from the findings of most studies of money demand. However, our finding for the sample period of 1968I - 1976II could indicate that a structural change in the money demand function has taken place within the post-war period. We should also recall the procedures and results in the studies of hyperinflation which were surveyed in Chapter III. Income was neglected as an explanatory variable in estimating models of money demand during hyperinflation, with no great loss in explanatory power. It may be the case that income is not a relevant variable for determining money demand in periods of extremely high inflation, due to the dominant influence of inflationary expectations. But hyperinflation need not be a necessary condition for the expected inflation rate to overshadow income as a determinant. Our results may be indicating that the cost considerations attendant with holding money, as represented by the nominal interest rate and the expected rate of inflation, dominate the influence of income at inflation rates such as those during 1968I - 1976II.
Other evidence also limits our concern over an insignificant income term. First, the explanatory power of the respective equations in Table 6-2 does not appear to be adversely affected by the insignificance of income. Second, in the course of the cross-correlation analysis of money and income conducted in the previous chapter, the contemporaneous correlation between the pre-whitened versions of the respective series was calculated. Regression models such as (6.1.3) are essentially based upon contemporaneous correlations among the dependent and independent variables. The contemporaneous correlation between money and income was .175 and the 2\(\sigma\) limits for the value were \(\pm .211\). Thus, it is not surprising that income was insignificant in the estimation of the money demand equation. In fact, a finding of significance could indicate the presence of the "spurious regression" problem discussed by Granger and Newbold [40].

The estimated parameters for the interest rate variable are significantly different from zero in all of the equations of Table 6-2 and are not much affected by the model used to measure expectations. This finding also holds in the specifications presented in Appendix 2 where expected inflation enters linearly. If the expected inflation rate is a factor in determining the level of nominal interest rates, then it has an indirect influence upon money demand in addition to its direct influence. Further examination of the table reveals that the coefficient of the expected inflation variable exceeds the coefficient of the interest rate in three of the four specifications where expected inflation entered logarithmically. The lone exception is in the equation where inflationary expectations were generated by the intervention model of the inflation rate. Since the respective parameters are quite
close, the 95% confidence intervals were constructed for the coefficients of the expected inflation rate. None of the interest rate coefficients of equations 6.1, 6.3, and 6.4 were smaller than the lower limits of the appropriate intervals. Hence, in the context of these three estimated equations, expected inflation yields a negative influence upon money demand which is not significantly different from the negative influence provided by the interest rate. While the strength of expected inflation is a bit surprising, it is additional evidence which provides justification for the presence of an expected inflation term in the demand function for money.

The results also provide information about the elasticities of money demand with respect to the independent variables. The measures of elasticity which are reported in Table 6-2 and subsequent tables are "long-run" elasticities. These are computed by dividing the estimated coefficients of the independent variables by the estimate of the adjustment coefficient or, equivalently, by the value obtained from subtracting the coefficient of the lagged dependent variable from one. The long-run elasticities for the variables in the logarithmic specifications of Table 6-2 appear in the second set of parentheses. For the expected inflation variable, the demand elasticity ranges from -.321 to -.483. Care should be exercised in interpreting these values since the expected inflation variable is measured in percentage points. An elasticity of -.321 implies that a one percent increase in the expected inflation rate, for example from 2.0 percent to 2.02 percent, will cause a .321 percent decrease in the demand for money, ceteris paribus. If an increase in the expected inflation rate contributes to an increase in the nominal interest rate, the total effect from the
inflation variable will of course be greater. Since the point estimates for the interest rate parameters were less than those for the expected inflation variable in equations 6.1, 6.3, and 6.4, the long-run interest elasticity of the interest rate is also less than the long-run elasticity of expected inflation.

As previously noted, the equations in Table 6-2 were estimated by ordinary least squares. However, this technique may be inadequate due to the presence of autocorrelated errors in the structural model being estimated. This is not the case for the models and sample period being used above. To check for the possibility of autocorrelated errors, Durbin's [25] h statistic was calculated for each of the estimated equations. The hypothesis of zero autocorrelation among the errors in the respective equations could not be rejected.

The expected inflation rates used in the estimation of the models reported in Table 6-2 are measured on a quarterly basis. This follows from our use of a forecasting model which is constructed using quarterly rates of inflation. The same is true for the alternative measures of expected inflation rates derived from one-quarter ahead forecasts of the actual price level. However, the interest rate variable, as represented by the 3-month Treasury bill rate, is measured on an annual basis. Likewise, the real money stock and real income are represented by annual totals. Some question might arise concerning the compatibility of the quarterly rate of expected inflation and the remaining variables in the model. In particular, it would be convenient to have similar time frames of measurement to compare the demand elasticities of the interest rate and expected inflation. To examine this issue, the quarterly rates of expected inflation, i.e., the one-step ahead forecasts of
inflation from the forecasting models, were annualized using the following formula

\[ \pi^e_{\text{annualized}} = \left[ \left(1 + \frac{\hat{\pi}_t(1)}{100}\right)^4 - 1 \right] \times 100 \]

where \( \hat{\pi}_t(1) \) is the one-step ahead forecast derived from any of the four forecasting models.

The money demand model of (6.1.3) was estimated using the annualized rates of expected inflation derived by (6.1.4). Once again, all the variants of the expectation measures, reflecting the respective forecasting models for inflation, were utilized. Both functional forms of the money demand model were also considered. The results from estimating the complete logarithmic specifications of the model are presented in Table 6-3 while the results for the alternative functional form where expected inflation enters linearly may be referenced in Appendix 2. In general, comparison of the results in Table 6-2 with those of Table 6-3 indicates that they are very similar. A relatively high degree of explanatory power and the absence of autocorrelated errors, as evidenced by the \( h \) statistic, characterize both sets of equations. In addition, the parameter estimates for the lagged real money stock, real income, and the interest rate are either identical or nearly so across the respective combinations of equations.

Under the complete logarithmic specification, the parameter estimates for the expected inflation rate are equal in the equations which employ the intervention model of inflation and the price model to generate expectations. The estimates differ by only .001 under the two remaining schemes of expectation measurement. Most importantly, the annualized measures of expected inflation are significant determinants
<table>
<thead>
<tr>
<th>Equation</th>
<th>Source of $\pi^e$</th>
<th>constant</th>
<th>$m_{t-1}$</th>
<th>$y$</th>
<th>$i$</th>
<th>$\pi^e$</th>
<th>$h$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>Inf.</td>
<td>.230</td>
<td>.947</td>
<td>.016</td>
<td>-.014</td>
<td>-.017</td>
<td>.14</td>
<td>.955</td>
</tr>
<tr>
<td></td>
<td>(.963)</td>
<td>(19.440)</td>
<td>(.757)</td>
<td>(-2.304)</td>
<td>(-3.419)</td>
<td>(-.264)</td>
<td>(-.321)</td>
<td></td>
</tr>
<tr>
<td>6.6</td>
<td>Inf.</td>
<td>.089</td>
<td>.971</td>
<td>.017</td>
<td>-.015</td>
<td>-.014</td>
<td>1.00</td>
<td>.949</td>
</tr>
<tr>
<td></td>
<td>Int.</td>
<td>(.362)</td>
<td>(19.089)</td>
<td>(.670)</td>
<td>(-2.198)</td>
<td>(-2.569)</td>
<td>(-.517)</td>
<td>(-.483)</td>
</tr>
<tr>
<td>6.7</td>
<td>Price</td>
<td>.227</td>
<td>.948</td>
<td>.016</td>
<td>-.014</td>
<td>-.017</td>
<td>.14</td>
<td>.955</td>
</tr>
<tr>
<td></td>
<td>(.952)</td>
<td>(19.507)</td>
<td>(.737)</td>
<td>(-2.304)</td>
<td>(-3.411)</td>
<td>(-.269)</td>
<td>(-.327)</td>
<td></td>
</tr>
<tr>
<td>6.8</td>
<td>Price</td>
<td>.232</td>
<td>.944</td>
<td>.018</td>
<td>-.014</td>
<td>-.018</td>
<td>-.10</td>
<td>.963</td>
</tr>
<tr>
<td></td>
<td>Int.</td>
<td>(1.077)</td>
<td>(22.144)</td>
<td>(.953)</td>
<td>(-2.669)</td>
<td>(-4.436)</td>
<td>(-.250)</td>
<td>(-.321)</td>
</tr>
</tbody>
</table>
of the demand for money. This result holds for the complete logarithmic specification and for the specifications where expected inflation enters linearly.

The long-run elasticities of money demand with respect to the expected inflation rate vary from -.321 to -.483 in the complete logarithmic specification, exactly the same range as was observed in the estimates based upon quarterly rates. Interest rate elasticities in the complete logarithmic specifications range from -.250 to -.517. While the parameter estimates of the interest rate are less than those of the expected inflation rate in three of the four equations in Table 6-3, they are not significantly different. The evidence suggests that the strength of the negative relationship between money demand and expected inflation is roughly the same for the quarterly and annualized series of expectations. The relationship between the respective demand elasticities for the interest rate and expected inflation is also similar for the estimates obtained using the two types of series. These results lend further support to a conclusion that expected inflation is important in determining the demand for money.

We now address the threshold hypothesis. The above discussion demonstrated the effect of inflationary expectations upon money demand in the context of inflation rates which were relatively high for the post-war period. It remains to be seen whether the negative relationship is significant in a period of lower inflation rates. The threshold hypothesis predicts that a significant relationship will not be found. Agents will only react to expected inflation, by attempting to minimize their losses in real purchasing power, if the inflation rate is relatively high. To examine these issues, the money demand model in
(6.1.3) was estimated over the sample period of 1954II - 1967IV. The initial period was chosen according to the availability of forecasts from the time series models for inflation and the price level. Once again, real GNP was used as the measure of income and the 3-month Treasury bill was used for the interest rate. Recall that the expected inflation rates are the one-step ahead forecasts from the time series models in Table 6-1. Both the complete logarithmic specification and the specifications where expected inflation enters linearly were estimated.

The residuals obtained from estimating the respective equations by ordinary least squares exhibited some degree of autocorrelation. Hence, the equations were re-estimated using a maximum likelihood technique designed to account for first-order autocorrelation. The results are presented in Table 6-4. Care must be exercised when assessing the significance of the variables in a specific equation. The t-ratios are asymptotic and inferences should be based upon a $N(0,1)$ distribution.

As in Tables 6-2 and 6-3, there are four equations to be considered, the consequence of entertaining the respective forecasting models for inflation and the price level. The most important feature of the results is the insignificance of the expected inflation variable in all of the equations. Selection of a particular forecasting model for inflation seems to have no bearing on this issue of significance.

The explanatory power of the respective equations, as indicated by the adjusted $R^2$, is extremely high with all values equal to .999.

---

3 See Beach and MacKinnon [9] for a full discussion.
<table>
<thead>
<tr>
<th>Equation</th>
<th>Source of $\pi^e$</th>
<th>constant</th>
<th>$m_{t-1}$</th>
<th>$y$</th>
<th>$i$</th>
<th>$\pi^e$</th>
<th>$\hat{\rho}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.9</td>
<td>Inf.</td>
<td>.050</td>
<td>.927</td>
<td>.054</td>
<td>-.014</td>
<td>-.00002</td>
<td>.34</td>
<td>.999</td>
</tr>
<tr>
<td></td>
<td>(.259)</td>
<td>(21.899)</td>
<td>(4.861)</td>
<td>(-.501)</td>
<td></td>
<td>(-.010)</td>
<td>(2.50)</td>
<td></td>
</tr>
<tr>
<td>6.10</td>
<td>Inf.</td>
<td>.045</td>
<td>.928</td>
<td>.053</td>
<td>-.013</td>
<td>-.0001</td>
<td>.34</td>
<td>.999</td>
</tr>
<tr>
<td></td>
<td>Int.</td>
<td>(.235)</td>
<td>(21.903)</td>
<td>(4.816)</td>
<td>(.736)</td>
<td>(-.181)</td>
<td>(2.47)</td>
<td></td>
</tr>
<tr>
<td>6.11</td>
<td>Price</td>
<td>.051</td>
<td>.927</td>
<td>.054</td>
<td>-.014</td>
<td>.00001</td>
<td>.35</td>
<td>.999</td>
</tr>
<tr>
<td></td>
<td>(.266)</td>
<td>(21.965)</td>
<td>(4.875)</td>
<td>(-.494)</td>
<td>(.740)</td>
<td>(-.192)</td>
<td>(2.51)</td>
<td></td>
</tr>
<tr>
<td>6.12</td>
<td>Price</td>
<td>.052</td>
<td>.927</td>
<td>.054</td>
<td>-.014</td>
<td>.00004</td>
<td>.35</td>
<td>.999</td>
</tr>
<tr>
<td></td>
<td>Int.</td>
<td>(.274)</td>
<td>(22.007)</td>
<td>(4.898)</td>
<td>(.740)</td>
<td>(-.192)</td>
<td>(2.52)</td>
<td></td>
</tr>
</tbody>
</table>
The autoregressive parameter, $\hat{\rho}$, is significantly different from zero and therefore confirms the applicability of the technique used for estimation. Examining the coefficients for the lagged money stock reveals that the speed of adjustment between actual and desired balances is a bit higher in the 1954II - 1967IV sample period than in the 1968I - 1976II period. However, tests of significance indicated that the difference between the coefficients for the lagged money stock in corresponding equations for the two periods was not statistically significant.

In contrast with the results for 1968I - 1976II, the estimated coefficient for real income is significantly different from zero in the 1954II - 1967IV period. The value is stable across the respective methods of inflation forecasting and functional forms assumed for the money demand model. Long-run elasticities of money demand with respect to real income assume only two values, .736 and .740, in the equations employing the complete logarithmic specification.\footnote{These values are similar to those reported by Goldfeld [37] even though his sample period differs from the one employed here.} Coefficients for the interest rate variable are almost identical to those which were estimated for the 1968I - 1976II period. However, point estimates of the long-run elasticities for the interest rate are less due to the lower values of the coefficients on the lagged money stock variable. These elasticity estimates range from -.181 to -.192 across the respective equations, indicating that money demand was relatively insensitive to changes in the interest rate during the sample period.

Support for the threshold hypothesis is provided by the significance of the coefficient for expected inflation in the 1968I - 1976II
period and the insignificance of the coefficient in the 1954II - 1967IV period. In addition, statistical tests were conducted to determine whether the estimated coefficients for expected inflation in 1968I - 1976II were significantly different from those of the 1954II - 1967IV period. These t statistics indicated that the coefficients were indeed significantly different, for all measures of expected inflation and in the context of both functional forms.

This finding is not surprising given the extremely low values of the parameter estimates from the earlier period.\(^5\) On the basis of the statistical evidence presented here, we may conclude that a threshold effect exists in the demand for money with respect to the expected inflation rate. The observed inflation rate must be relatively high before expectations of future rates exert a significant influence. High rates of observed inflation are translated into high expected rates of inflation by the very nature of the forecasting models used by agents to form expectations. A review of the integrated moving average character of our models for inflation and the price level would reveal this characteristic.

Results obtained from estimating the money demand equations over 1954II - 1967IV while using the annualized measure of expected inflation are presented in Table 6-5. Inspection of the table reveals almost no differences between these results and those obtained from using the quarterly rates of expected inflation for the same time period. Both sets of results offer evidence confirming the prediction

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\(^5\)The Chow test is not applicable to our problem. The error structures of the models differ in the two sample periods and we are only interested in the performance of one variable in the model.
TABLE 6-5. MONEY DEMAND EQUATIONS USING ANNUALIZED RATE OF EXPECTED INFLATION, 1954II - 1967IV

<table>
<thead>
<tr>
<th>Equation</th>
<th>Source of $\pi^e$</th>
<th>constant</th>
<th>$m_{t-1}$</th>
<th>$y$</th>
<th>$i$</th>
<th>$\pi^e$</th>
<th>$\hat{\rho}$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.13</td>
<td>Inf.</td>
<td>.049</td>
<td>.927</td>
<td>.054</td>
<td>-.014</td>
<td>-.00002</td>
<td>.34</td>
<td>.999</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.259)</td>
<td>(21.897)</td>
<td>(4.860)</td>
<td>(-4.500)</td>
<td>(-.011)</td>
<td>(2.50)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(.740)</td>
<td>(-.192)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.14</td>
<td>Inf.</td>
<td>.045</td>
<td>.928</td>
<td>.053</td>
<td>-.013</td>
<td>-.001</td>
<td>.34</td>
<td>.999</td>
</tr>
<tr>
<td></td>
<td>Int.</td>
<td>(.236)</td>
<td>(21.904)</td>
<td>(4.814)</td>
<td>(-4.429)</td>
<td>(-.075)</td>
<td>(2.46)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(.736)</td>
<td>(-.181)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.15</td>
<td>Price</td>
<td>.051</td>
<td>.927</td>
<td>.054</td>
<td>-.014</td>
<td>.00001</td>
<td>.35</td>
<td>.999</td>
</tr>
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<td></td>
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<td>(-.192)</td>
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</table>
of the threshold hypothesis.

6.2 Income Expectations and the Demand for Money

In this section we consider the influence of income expectations upon the demand for money. The use of expected income, rather than current or observed income, is not new to the study of money demand. Feige [26] and Koot [48] have dealt with models containing expected income, as have Goldfeld [37] and Shapiro [69] in the studies which were surveyed in Chapter III.

With the exception of Shapiro, the dominant feature of these treatments of expected income is a reliance upon the adaptive expectations mechanism to describe expectation formation. Therefore, coefficients of expectation, or the adjustment parameter of the mechanism, were estimated by regression methods. We have previously shown that such a procedure is equivalent to forcing an integrated moving average model upon the series of observed income. Since the IMA(1,1) model need not be an acceptable forecasting model for income, errors in measurement may plague studies that employ it.

The present study adopts a different approach. Instead of forcing an expectation model for income upon the money demand relationship, we find an acceptable forecasting model based upon sets of information which are useful to the exercise. As shown in Chapter V, the only set relevant for the forecasting of future income values is the past history, or observed values, of the income series itself. One-step ahead forecasts from the time series model for income are then used as measures of expected income over the appropriate time period. Hence, our approach views expectation formation as an exercise which is distinct
from the determination of money demand. In fact, this measure of expected income could be integrated into behavioral models describing other facets of consumer behavior.

We noted earlier that the concept of permanent income, as developed by Friedman [32], may be viewed as being identical to an expectation of future income, but only in the context of a particular version of permanent income's definition. Some writers would disagree with equating expected income and permanent income. While Meyer and Neri [53] offer an expectation mechanism for income which incorporates the weighting structure used by Friedman to calculate permanent income, their expectation mechanism differs from Friedman's measurement method. We shall not attempt to resolve the question of whether measuring permanent income by time series models is the correct method. The abstract nature of permanent income precludes a definitive answer. However, forecasts derived from ARIMA models of the income series can be considered as one particular method of measuring permanent income. We do not require a logical equivalence between the two since our emphasis is upon expected income, not permanent income.

Two regimes describing the formation of income expectations will be entertained for this study. In the first, we assume that expectations of real income are formed by using an acceptable forecasting model derived from the observed series of real income. This forecasting model was developed and discussed in Chapter V. The money demand model

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6 Expressing the Friedman weighting scheme in terms of a time series forecasting model yields an \(IMA(1,1)\) representation for the income series, as shown in Chapter IV. A bit of algebra reveals that Meyer and Neri's [53] expectation mechanism is an ARIMA \((1,1,1)\) model when expressed in its forecasting form.
consistent with forming expectations of real income is given by:

\[(6.2.1) \quad m^*_t = f(y^e_t, i_t, \pi^e_t),\]

where \(m^*_t\) is the desired real money stock, \(i_t\) is an interest rate, and \(\pi^e_t\) is the expected rate of inflation. This model is a variant of (6.1.3), the only difference being the presence of expected real income, \(y^e_t\), instead of observed income, \(y_t\). We assume that the relationship between desired and actual real money balances is again given by the stock adjustment mechanism of (6.1.2). After substitution of the adjustment mechanism into a linear specification of (6.2.1), and adding a stochastic disturbance term, we have

\[(6.2.2) \quad m_t = a + b \cdot y^e_t + c \cdot m_{t-1} + d \cdot i_t + f \cdot \pi^e_t + \varepsilon_t,\]

where \(c = 1 - \theta\).

The second regime assumes that expectations of nominal income, rather than real income, are formed by agents using an acceptable forecasting model for the nominal income series. This regime will be used in conjunction with the formation of expectations of the price level, \(P^e_t\). We have already considered expectation models for the price level and found that inflationary expectations could be derived directly from the expectations of the price level. Hence, we assume that the income constraint in the money demand model which utilizes an expected inflation measure derived from the expected price level is given by \(y^e_t/P^e_t\).

We can use the term "real expected income" to distinguish the above variable from \(y^e_t\), which shall still be referred to as expected real income. The appropriate money demand model consistent with the second
regime of income expectations is given by:

\[(6.2.3) \quad m^* = f \left( \frac{Y_t^e}{p_t^e}, i_t, \pi_t^e \right), \]

where \( \pi_t^e \) is expected inflation derived from expectations of the price level. By using the stock adjustment mechanism and adopting a linear specification with a stochastic disturbance, we have\(^7\)

\[(6.2.4) \quad m_t = a + b \cdot \frac{Y_t^e}{p_t^e} + c \cdot m_{t-1} + d \cdot i_t + f \cdot \pi_t^e + \varepsilon_t. \]

The time series models used to generate expectations of real income, nominal income and the price level are presented once again in Table 6-6 for convenient reference. Recall that these models were examined in detail in Chapter V.

**TABLE 6-6. ARIMA MODELS FOR GENERATING INCOME AND PRICE EXPECTATIONS**

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
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<td>Real Income</td>
<td>((1-B)y_t = .008 + (1-.259B^5)a_t)</td>
</tr>
<tr>
<td>Nominal Income</td>
<td>((1+.250B^5)(1-.436B)(1-B)Y_t = .012 + a_t)</td>
</tr>
<tr>
<td>Price</td>
<td>((1-B)^2 P_t = (1-.491B)a_t)</td>
</tr>
<tr>
<td>Price with</td>
<td></td>
</tr>
<tr>
<td>Intervention</td>
<td>((1-B)^2 P_t = -.381t + (1-.466B)a_t)</td>
</tr>
</tbody>
</table>

Following the same sequence of procedures used in Section 6.1 of this chapter, the models of (6.2.2) and (6.2.4) were estimated by ordinary least squares using the sample period of 1968I - 1976II. This

\(^7\)The specification of (6.2.4) could be modified to describe the demand for real money balances valued in terms of the expected price, \(M_t/p_t^e\), where \(M_t\) is the nominal money stock. However, the results obtained from estimating such a model were extremely poor. Thus, we shall employ real money balances valued in terms of observed prices, as in (6.2.4).
period is chosen so as to allow an examination of the threshold hypothesis in the context of a money demand model which contains income expectations. We must admit the possibility that substituting expected income for the observed income series will alter the estimation results presented in Section 6.1. The remaining variables in (6.2.2) and (6.2.4), real money balances, the interest rate, and expected inflation, are measured in the same manner as before.

The results obtained from estimating the models are presented in Table 6-7. Both the functional form in which all variables are in logarithms and the form in which expected inflation enters linearly were utilized. Results from the latter form are presented in Appendix 2. Each of the four models used to generate expected inflation are entertained. However, we do not consider the money demand model where expected inflation is derived from expectations of price and income expectations are based upon the real income series. Our neglect does not imply that such a model is incorrect.

The estimation results clearly show that our measures of expected income are insignificant in the sample period. This is not surprising given that the forecasts of income which are used as the expectation series are derived from observed values of income. Since the observed income variable in the models presented in Table 6-2 were insignificant determinants of money demand, a conclusion that our measures of expected income are inappropriate is not justified.

Expected inflation, expressed in quarterly rates, is significant across all of the methods of measurement, thereby reaffirming the role of the variable in the demand for money. The actual estimates of the parameters differ very little from the corresponding equations in
<table>
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<th>$y^e$</th>
<th>$\pi^e$</th>
<th>i</th>
<th>$\pi^e$</th>
<th>h</th>
<th>$R^2$</th>
</tr>
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<td>-.016</td>
<td>.15</td>
<td>.954</td>
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<td>-.012</td>
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<td>(-1.231)</td>
<td>(-.923)</td>
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<td>(-.372)</td>
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<td></td>
<td></td>
<td>(-.292)</td>
<td>(-.375)</td>
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</tbody>
</table>
Table 6-2 which employed observed income. With the exception of Equation 6.18, long-run elasticities of expected inflation in the complete logarithmic specifications vary from -.372 to -.375. This spread is quite close to the range of elasticities calculated for the models containing observed income. Long-run interest rate elasticities yielded by the complete logarithmic specifications using expected income range from -.292 to -.349, with the exception of Equation 6.18. These values are a bit higher than the values associated with the specifications using observed income, due mainly to the slower adjustment speeds which are implied by the coefficients of the lagged money stock.

With respect to the 1968I - 1976II period, the results obtained from estimating the models containing income expectations are quite similar to the results obtained from employing observed income. The same observation holds true for estimation based upon annualized rates of expected inflation. Results from estimating these versions of the equations may be referenced in Table 6-8.

The respective versions of the money demand model which contain income expectations were also estimated using the sample period of 1954II - 1967IV. The estimation procedure which incorporates first-order autocorrelated errors was used once again to examine this specific sample period. Results obtained from this exercise are given in Table 6-9.

In general, the same differences between the two sample periods, 1954II - 1967IV and 1968I - 1976II, which appeared while using observed income in the models are present in the results which use expected income as the constraint. The expected income variable is
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<th>constant</th>
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<th>( y^e )</th>
<th>( \frac{y^e}{p^e} )</th>
<th>( i )</th>
<th>( \pi^e )</th>
<th>( h )</th>
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<td>(-.372)</td>
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<td>$i$</td>
<td>$n^e$</td>
<td>$\hat{\rho}$</td>
<td>$\bar{R}^2$</td>
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<td>(.836)</td>
<td>(-.213)</td>
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significant across all the methods used to measure expected inflation. Long-run elasticities for the expected income variable range from .836 to .926 while the elasticities for the interest rate lie between -.213 and -.241 in the complete logarithmic specifications.

While the performance of expected income improved in the 1954II - 1967IV period, the coefficient of the expected inflation variable is insignificant across all the methods of expectation measurement. In addition, the coefficients of the expected inflation variable which were estimated for the 1968I - 1976II period are significantly different from the corresponding estimates from the 1954II - 1967IV period.\footnote{The only possible exception to this statement might be found in the specification where expected inflation enters linearly and is generated by the intervention model of the inflation rate. In this case, the test statistic, or t-ratio, is 1.501. However, this exceeds the critical value for a one-sided test at the 10% level of significance. Given our theoretical justification for negative signs of the coefficients, a one-sided test may be the preferable one.}

The results indicate that there is substantial evidence from the models incorporating income expectations to support the threshold hypothesis. Annualized measures of inflation were also used to estimate the money demand models over 1954II - 1967IV. The results are presented in Table 6-10 and are very similar to those obtained from the specifications which used quarterly rates. Once again, support for the existence of the threshold hypothesis is provided by these results. The coefficients of the expected inflation variables are insignificant and are significantly less than the corresponding values from the 1968I - 1976II period.\footnote{As in the case where quarterly rates were used, the only exception may be the specification where expected inflation enters linearly and is measured by the intervention model of inflation. Once again, the t-statistic still exceeds the critical value for a one-sided test at the 10% level of significance.} In brief, estimation of the money demand
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<tr>
<td>6.32</td>
<td>Price</td>
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<td>.939</td>
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<td>-.0003</td>
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<tr>
<td></td>
<td>Int.</td>
<td>(.008)</td>
<td>(22.601)</td>
<td></td>
<td>(4.577)</td>
<td>(-4.279)</td>
<td>(-.210)</td>
<td>(2.36)</td>
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</tbody>
</table>
models containing expected income has provided more evidence favoring the threshold hypothesis.

6.3 Interest Rate Expectations and the Demand for Money

The final area of our investigation into the aggregate demand for money is concerned with the role of interest rate expectations. We have considered, in sequence, the effects of inflationary expectations and income expectations upon money demand. Those versions of the money demand specification did not incorporate an expected interest rate, but instead relied upon observed values to represent the cost, in terms of interest earnings foregone, associated with holding money balances. Generalizing the previous theoretical models by substituting an expected interest rate for the observed rate results in a form which has been labeled the "multiple expectations" or "pure expectations" model of the demand for money. This model form is given by:

\[ m^*_t = f(y^e_t, i^e_t, \pi^e_t) \]

where \( i^e_t \) is the expected nominal interest rate in time \( t \) while the remaining variables retain their previous identities. Studies which have considered this form, or variants thereof, include those by Feige [26], Goldfeld [37], Koot [48], and Shapiro [69]. As in the case of modeling expected income, these studies have modeled interest rate expectations by imbedding a generating mechanism, typically the adaptive expectations version, in the demand equation. Shapiro is again the lone exception. Hence, the value of the parameter in the mechanism is estimated by regression procedures. Expectations measured in this fashion need not be economically rational or even weakly rational.
They simply do not come from an efficient forecasting model.

To deal with the issue of interest rate expectations, we adopt once again a modeling strategy based upon the construction of suitable time series models. The procedure to be followed is identical to that pursued in Chapter V for modeling inflationary and income expectations. First, an acceptable ARIMA model must be found for the observed series of the interest rate, in this case the 3-month Treasury bill. Second, the causality relationship between the interest rate series and potential leading indicators must be assessed. If the indicators selected appear to have no value in terms of improving forecasts of future rates, then the ARIMA model of the interest rate will be used as the forecasting model to generate expectations. If the indicators do have some value, then agents would be economically rational in employing them through the use of some transfer function specification. 10

Monthly observations of the 3-month Treasury bill rate, from July, 1953 through June, 1976, form the data set used in constructing the ARIMA model. The period was chosen to insure that the time span used for interest rate modeling would be equal in length to the time span used for the inflation and income series. Quarterly averages of the interest rate were rejected as a possible data set due to the degree of variation in the rate that may occur in a three month period. The data source is given in Appendix I.

The model identification stage of building the ARIMA model indicated the presence of seasonality in the interest rate series. Seasonal

10 Nelson [56] has used ARIMA model representations of interest rate series for this purpose, as has Watson [76] in his study of bank portfolio selection.
differencing proved to be inappropriate, but a seasonal parameter was included in the specification as dictated. The estimation stage resulted in the following representation for the series:

\begin{equation}
(6.3.2) \quad (1 + 0.264B^6)(1 - 0.263B)(1 - B)i_t = a_t \\
(\pm 1.17) \quad (\pm 1.17) \\
\hat{\sigma}^2 = 0.0874 \quad Q = 30.5 \quad (22 \text{ d.f.})
\end{equation}

where $i_t$ is the rate on 3-month Treasury bills, $a_t$ is the white noise disturbance, and B is the familiar backshift operator. The relatively high value of Q is due to the residual autocorrelation at lag 20 lying outside of its $2\hat{\sigma}$ limit. However, there appears to be no economic justification for including a parameter in the ARIMA model to capture this effect. The remainder of the residual autocorrelations are inside their $2\hat{\sigma}$ limits so we accept the model of (6.3.2) as being valid. Note the similarity between the parameter values contained in the regular and seasonal autoregressive polynomial. This might cause us to consider the possibility that the ARIMA model has been misspecified. To resolve this question, the roots of the higher order polynomial were calculated. None of the lengths of the roots matched, or was close to, the real root of the first order autoregressive polynomial.\footnote{For a discussion of the relevance of polynomial roots in ARIMA models, see Box and Jenkins [12].} Thus, we view (6.3.2) as an acceptable model for the interest rate series. One-step ahead forecasts of the interest rate from the autoregressive model of (6.3.2) are given by:

\begin{equation}
(6.3.3) \quad \hat{t}(1) = 1.263i_t - 0.263i_{t-1} - 0.264i_{t-5} + 0.333i_{t-6} + 0.069i_{t-7},
\end{equation}
where \( \hat{z}_t(1) \) denotes the forecast of the rate in period \( t+1 \) made at the end of period \( t \).

The set of variables selected as possible leading indicators of the interest rate is composed of \( M1 \), the monetary base (MB), and the rates of change in the two aggregates. Rates of change were calculated as

\[
(6.3.4) \quad \frac{X_t - X_{t-1}}{X_{t-1}} \cdot 100,
\]

where \( X_t \) is the observed value of the indicator series in time \( t \).

The first step in assessing the relationship between the respective variables and the interest rate is to construct acceptable ARIMA models from the time series of each. Seasonally unadjusted observations, gathered on a monthly basis, for the period of July, 1953 to June, 1976 were utilized in constructing these models. The time period covered by the ARIMA models for potential leading indicators should match the period covered by the model for the prospective output series, which is the interest rate in this case. By applying the usual identification and estimation sequence to the respective indicator series, the models given in Table 6-11 were found to be acceptable.

**TABLE 6-11. ARIMA MODELS FOR POTENTIAL LEADING INDICATORS OF THE INTEREST RATE, 1953(7) - 1976(6)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>ARIMA Model</th>
<th>( \hat{z}_t(1) ) Coefficients</th>
<th>( \hat{z}_t(2) ) Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M1 )</td>
<td>( (1-B)(1-B^2)M1_t = (1+.194B+.136B^3+.135B^6)(1-.378B^12)a_t )</td>
<td>( \pm .121 ), ( \pm .126 ), ( \pm .128 )</td>
<td>( \pm .120 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \hat{\sigma}^2 = .753 )</td>
<td>( Q = 26.1 ) (20 d.f.)</td>
</tr>
<tr>
<td>( MB )</td>
<td>( (1-B)(1-B^2)MB_t = (1-.189B+.163B^3+.228B^6)(1-.485B^12)a_t )</td>
<td>( \pm .120 ), ( \pm .123 ), ( \pm .123 )</td>
<td>( \pm .121 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \hat{\sigma}^2 = .080 )</td>
<td>( Q = 29.1 ) (20 d.f.)</td>
</tr>
</tbody>
</table>
\[ M_1 \ldots (1+.430B^{12})(1-.127B-.218B^3+.166B^7)(1-B^{12}) = a_t \pm .114 \pm .121 \pm .122 \pm .122 \]

\[ \hat{\sigma}^2 = .175 \quad Q = 26.5 \quad (20 \text{ d.f.}) \]

\[ M_B \ldots (1-B^{12}) = (1-.123B+.226B^3)(1-.685B^{12}) = a_t \pm .119 \pm .119 \pm .091 \]

\[ \hat{\sigma}^2 = .118 \quad Q = 23.4 \quad (21 \text{ d.f.}) \]

The residual series were calculated for the models in Table 6-11 and then each was cross-correlated with the residuals obtained from the ARIMA model for the interest rate. This exercise is simply an application of the Haugh [42] procedure which was utilized extensively in Chapter V. Summary statistics yielded by the procedure are given in Table 6-12.

**TABLE 6-12. TEST STATISTICS FOR THE POTENTIAL LEADING INDICATORS OF THE INTEREST RATE, 1953(7) - 1976(6)**

<table>
<thead>
<tr>
<th></th>
<th>$S^*_{30}$</th>
<th>$S^*_30 (\pm)$</th>
<th>$S^*_{30} (-)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>120.72**</td>
<td>44.03**</td>
<td>73.39**</td>
</tr>
<tr>
<td>$M_B$</td>
<td>112.01**</td>
<td>32.98</td>
<td>78.94**</td>
</tr>
<tr>
<td>$\hat{M}_1$</td>
<td>100.42**</td>
<td>48.88**</td>
<td>48.29**</td>
</tr>
<tr>
<td>$\hat{M}_B$</td>
<td>87.68**</td>
<td>37.84</td>
<td>49.84**</td>
</tr>
</tbody>
</table>

**significant at the 5% level.**

**NOTES:** The degrees of freedom associated with these Chi-square tests are, respectively, $S^*_{30} \sim 61, S^*_30 (\pm) \sim 30$.

Independence of the monetary indicators and the interest rate is clearly rejected based upon the values of the $S^*_{30}$ statistic. However, the values of the $S^*_30 (\pm)$ statistic for positive lead times do not exceed the appropriate critical value in the case of the monetary base.
and its rate of change. This latter result casts considerable doubt upon the propriety of constructing a transfer function for either of the two series and the interest rate. Note that the values of the $S^*_{30}(-)$ statistic for the monetary base and its rate of change do exceed the critical values of the test distribution. This indicates reverse causality between the two series and the rate of interest. A relationship of this type could arise from the monetary authority using the interest rate as a policy target and, simultaneously, using the monetary base or its rate of change as an instrument. Hence, changes in the rate of interest would lead changes in the monetary base.

For the money stock and its rate of change, the test statistics indicate that a feedback relationship exists with respect to the interest rate. The values of $S^*_{30}(+) \text{ and } S^*_{30}(-)$ both exceed the selected critical value of the test distribution for each series. On the basis of these values, we might conclude that the information set consisting of present and past values of the money stock series, or its rate of change, could be used for forecasting future values of the interest rate. However, the opposite statement could also be made by simply reversing the respective roles and considering the money stock as the output series while viewing the interest rate as an input series. One-way causality from these monetary variables to the interest rate does not exist. Unfortunately, accurate identification and estimation of a transfer function which would quantify a specific relationship requires that causality be one-way between the prospective input series and the output series [13].

On the basis of the above discussion, we shall utilize the ARIMA model of the interest rate series given in (6.3.2) to obtain the
one-step ahead forecasts of the rate. Following our usual assumption, these forecasts will be used as numerical measurements of the expected rate of interest. It is debatable whether these expectations can correctly be called economically rational, given the ambiguities indicated in the causality tests. However, it would be appropriate to term them weakly rational since they are formed from the efficient use of an important information set, the past values of the variable itself.\footnote{Essentially, we are equating the notion of weakly rational expectations with the structure of rational expectations given by Muth's market model. In each case, expectations are formed using only the past history of the variable which the agents wish to forecast.}

Since the observations of the interest rate are on a monthly basis, the forecasts provided by the ARIMA model of (6.3.2) are also monthly. This time frame is at odds with the quarterly observations of the money stock, expected income, and expected inflation used in estimating the demand for money models. To obtain a measure of the expected interest rate on a quarterly basis, two procedures were applied to the monthly forecasts of the rate. First, the one-step ahead forecasts for each of the three months contained in a particular quarter were averaged in order to yield a consistent measure. In the second scheme, the one, two, and three-step ahead forecasts made at the last month of the preceding quarter were averaged. Algebraically, the two measures are given by:

\begin{align*}
(6.3.5) \quad \hat{i}_{\text{quarter } j} &= \left( \hat{i}_t(1) + \hat{i}_{t+1}(1) + \hat{i}_{t+2}(1) \right) / 3 \\
(6.3.6) \quad \hat{i}_{\text{quarter } j} &= \left( \hat{i}_t(1) + \hat{i}_t(2) + \hat{i}_t(3) \right) / 3
\end{align*}

where $t$ is the last month in quarter $j-1$. Both measures were used in
estimating the pure expectations model of money demand. However, the results obtained under the respective schemes were very similar so we shall only report those associated with the first technique of measurement.

To estimate the pure expectations model, we combine the stock adjustment mechanism for the desired real money stock, \( m_t^e \), with a log-linear specification of (6.3.1). After adding a stochastic disturbance, the model becomes

\[
(6.3.7) \quad m_t = a + b \cdot y^e_t + c \cdot m_{t-1} + d \cdot i^e_t + f \cdot n^e_t + \epsilon_t.
\]

Alternatively, the expected income variable could be of the form \( y^e_t/P^e_t \) if we were considering the regime where inflationary expectations are formed from expectations of the aggregate price level and income expectations are based upon the nominal income series. As written, the model in (6.3.7) refers to the regime where inflationary expectations are formed from past values of the inflation rate and income expectations are for real income. The method used by agents to form interest rate expectations is identical in each of the respective regimes.

Following the procedures adopted in sections 6.1 and 6.2 of this chapter, the model was estimated by ordinary least squares using quarterly observations over the period of 1968I - 1976II. All of the methods used to construct the series of inflationary expectations were considered, as well as the two methods used for generating income expectations. Only the results obtained from the complete logarithmic specification will be reported here. These results may be referenced in Table 6-13 while the results from estimating the alternative specification in which expected inflation enters linearly may be viewed
<table>
<thead>
<tr>
<th>Equation</th>
<th>Source of $\pi^e$</th>
<th>constant</th>
<th>$m_{t-1}$</th>
<th>$y^e$</th>
<th>$\frac{y^e}{\bar{P}}$</th>
<th>$i^e$</th>
<th>$\pi^e$</th>
<th>$h$</th>
<th>$\bar{R}^2$</th>
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</thead>
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<tr>
<td>6.33</td>
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<td>.962</td>
<td>.002</td>
<td>-.017</td>
<td>-.014</td>
<td>.10</td>
<td>.957</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.993)</td>
<td>(20.108)</td>
<td>(.086)</td>
<td></td>
<td>(-2.824)</td>
<td>(-2.567)</td>
<td></td>
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<td>(-.368)</td>
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<tr>
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<td>-.004</td>
<td>-.019</td>
<td>-.010</td>
<td>.99</td>
<td>.952</td>
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<tr>
<td></td>
<td>(.499)</td>
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<td>(-3.030)</td>
<td>(-1.747)</td>
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<td>(-1.000)</td>
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<tr>
<td>6.35</td>
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<tr>
<td></td>
<td>(.990)</td>
<td>(20.120)</td>
<td>(.083)</td>
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<td></td>
<td></td>
<td>(-.459)</td>
<td>(-.378)</td>
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<tr>
<td>6.36</td>
<td>Price</td>
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<td>.954</td>
<td>.007</td>
<td>-.016</td>
<td>-.016</td>
<td>-.10</td>
<td>.964</td>
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<tr>
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<td>(.161)</td>
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<td>(-3.834)</td>
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<td>(-.348)</td>
<td>(-.348)</td>
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</tbody>
</table>
in Appendix 2.

Inspection of Table 6-13 reveals that estimation by ordinary least squares is appropriate, as evidenced by the values of Durbin's h statistic. The values of $\bar{R}^2$ for the respective models indicate a relatively good fit to the data. Expected inflation is a significant variable and the parameter values have the anticipated sign, a result that characterizes all the money demand specifications which have been considered for the 1968I - 1976II period. Little doubt should remain that expected inflation provides a negative influence upon the demand for money. This influence is fairly strong, as has been shown by the long-run elasticities of expected inflation. In the pure expectations models, these elasticities vary from -.348 to -.378, excluding the model where expectations are generated by the intervention model of inflation.

The parameter estimates for the expected interest rate are significant in all the specifications and are a bit larger, in absolute value, than the estimates for the observed interest rate in the corresponding equations of Tables 6-2 and 6-7. Long-run elasticities for the rate range from -.348 to -.459, again excluding the equation which uses the intervention model to generate expected inflation. Both types of the expected income variable are insignificant over the sample period. Overall, the results summarized in Table 6-13 are very similar to those found for the corresponding equations in Tables 6-2 and 6-7 which excluded expected interest rates in favor of the observed rate. This indicates that the particular form of the interest rate and income constraints, whether observed or expected, has little influence upon the performance of the general model of money demand over the sample
period being used. This conclusion may also be inferred from the estimation of the respective equations using annualized measures of expected inflation. These results are given in Table 6-14. Examination of the table reveals that the parameter estimates are nearly identical to those given in Table 6-13. Indeed, parameter estimates for the expected interest and inflation rate are exactly the same, as are the long-run elasticities.

The pure expectations model was also estimated using quarterly observations over the period 1954II - 1967IV. As in the previous sections of this chapter, the reason for this second estimation phase is to assess the possibility of a threshold effect on the part of expected inflation. The Beach-MacKinnon [9] technique which allows correction for autocorrelated errors was utilized to estimate the respective equations. We present the results for the complete logarithmic specifications in Table 6-15. Long-run elasticities of the expected interest rate are lower in this time period than in 1968I - 1976II, due mainly to the faster speed of adjustment between desired and actual real balances. The values range from -.213 to -.225 which represent a rather mild influence upon money demand. Expected income is a significant variable in these equations, with the elasticities lying between .773 and .817. These values are close to those calculated for the model form which contained the observed rate of interest. Expected inflation is insignificant as a determinant of money demand for the 1954II - 1967IV period, a finding which is consistent with the results gained from estimating the models in previous sections which did not contain the expected interest rate. The coefficients for expected inflation from the corresponding equations estimated over 1968I - 1976II
<table>
<thead>
<tr>
<th>Equation</th>
<th>Source of $\pi^e$</th>
<th>$\text{constant}$</th>
<th>$m_{t-1}$</th>
<th>$y^e$</th>
<th>$\frac{y^e}{p^e}$</th>
<th>$i^e$</th>
<th>$\pi^e$</th>
<th>$h$</th>
<th>$R^2$</th>
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<td>-.017</td>
<td>-.014</td>
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<td>(-2.831)</td>
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<td>.952</td>
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<tr>
<td>6.39</td>
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<td>.002</td>
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<td>-.014</td>
<td>.10</td>
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<td></td>
<td></td>
<td>(-.348)</td>
<td>(-.348)</td>
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</tr>
<tr>
<td>Equation</td>
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<td>$m_{t-1}$</td>
<td>$y^e$</td>
<td>$\frac{y^e}{p^e}$</td>
<td>$i^e$</td>
<td>$\pi^e$</td>
<td>$\hat{\rho}$</td>
<td>$\bar{R}^2$</td>
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</tr>
<tr>
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<td>-0.0001</td>
<td>0.30</td>
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</tr>
<tr>
<td></td>
<td>(.124)</td>
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<td>(5.357)</td>
<td>(.806)</td>
<td>(5.294)</td>
<td>(-.080)</td>
<td>(2.20)</td>
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<td>0.929</td>
<td>0.058</td>
<td>-0.016</td>
<td>-0.0002</td>
<td>0.30</td>
<td>0.999</td>
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</tr>
<tr>
<td></td>
<td>(.110)</td>
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<td>(.817)</td>
<td>(5.205)</td>
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<td>(24.237)</td>
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<td></td>
<td>(5.583)</td>
<td>(-5.460)</td>
<td>(.076)</td>
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are significantly different from those in Table 6-15 for 1954II - 1967IV, thereby giving further evidence to support the existence of the threshold hypothesis. Thus, a conclusion that the threshold effect, due to expected inflation, is present in the demand for money is applicable to all of the model forms we have investigated, irrespective of whether income and the interest rate are represented by observed or expected values. We could make the same statement for the equations which utilize annualized measures of expected inflation. The results obtained from the pure expectations model when annualized rates were used may be referenced in Table 6-16.

In summary, this chapter has considered three forms of the aggregate money demand function, each reflecting a particular combination of expected inflation and observed or expected values of an interest rate and income. Multiple interest rates were not considered in an effort to avoid expanding our presentation of results which is already formidable in its coverage. An important goal of the study has been achieved nonetheless. It has been demonstrated that expected inflation does provide a significant influence upon money demand and that the presence of a nominal rate of interest does not dilute its independent impact. Furthermore, the effect of expected inflation is not always significant but is sensitive to the magnitude of the variable. This evidence lends strong support to the threshold hypothesis and firmly establishes a conclusion that inflationary expectations have a real effect in terms of decreasing the demand for real money balances.
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<th>$m_{t-1}$</th>
<th>$y^e$</th>
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CHAPTER VII

EXPECTED INFLATION AND THE HOUSEHOLD SECTOR

7.1 Overview

The results of the previous chapter have shown that the expected rate of inflation is inversely related to the aggregate demand for money during time periods when the actual rate of inflation is relatively high. In this chapter we shall restrict our attention to the household sector and examine the effects of expected inflation in greater detail. Two issues which are closely related will be analyzed simultaneously.

First, the effect of expected inflation upon the holdings of money balances of households will be determined. Even though our results at this juncture have indicated an inverse relationship between the expected inflation rate and the aggregate demand for money, there is no guarantee that this relationship holds for the household sector. Households may not recognize or adjust to the costs, in terms of declines in purchasing power, which are associated with high rates of inflation. This could also apply to the anticipated costs which accompany high rates of expected inflation. Alternatively, households may actually recognize these costs but not adjust to them due to the transactions costs involved in doing so.¹

¹The possibility of a positive relationship between expected inflation and the money holdings of households must also be admitted although it is contrary to all of the theoretical foundations we have surveyed. Such a finding could lend support to the theory that money is held by households solely for the purposes of transactions. The positive (continued)
The second issue to be analyzed concerns the effect of expected inflation upon consumer spending and on holdings of a diverse set of financial and real assets, including money. The household's decision to hold a given amount of money balances is only one phase in the larger choice problem of dividing an existing wealth total and income stream among current expenditures and asset stocks. Thus, the effect of expected inflation upon money holdings will be dependent upon the interrelationships among the components to which wealth and the income stream can be allocated. The demands, or allocations, of the respective components should be examined simultaneously within a general model of consumer choice.

The effect of expected inflation upon consumer expenditures has been studied in some detail by Juster and Wachtel (J-W) [45], and by Springer [70]. Both of these studies examined two broad components of consumer spending, expenditures on durable goods and expenditures on nondurables and services. The theories which purport to describe the relationship between expected inflation and spending on these components are largely intuitive and lack a rigorous foundation. Basically, there are two arguments which may be offered, one put forth by J-W and the other having been summarized by Springer. The former theory views expectations of high inflation rates in the future as creating a general climate of uncertainty which in turn will affect decisions concerning saving, spending, and asset stocks. In general, it is felt that such uncertainty will make consumers pessimistic about the future (continued) relationship could indicate that agents increase their money holdings to keep pace with the rising nominal values of commodities which they wish to purchase.
and provide a negative influence upon current spending. A critical assumption in this line of reasoning stipulates that consumers will put more weight on the prospect of declining real incomes than on the prospect that nominal incomes will keep pace with the rising price level. Recognizing that the theory lacks rigor, J-W note that this argument has psychological and sociological roots, as well as economic.

The alternative theory, summarized by Springer, predicts that expected inflation will cause a reallocation of expenditures to the current period. Consumers anticipate paying higher prices for commodities and also foresee a decline in the real value of returns from saving and from their stocks of interest-earning assets. Current spending upon durable goods is most likely to be affected by this intertemporal substitution effect. Such goods have longer periods of use and, in general, command higher prices than other components of consumption expenditures. While a positive influence of expected inflation upon expenditures for nondurables and services is not ruled out, the theory predicts that the strongest effect will be upon durables.²

Both J-W and Springer estimated spending equations which contained an expected inflation variable. J-W found that their measure of expected inflation was negatively related to expenditures on durable goods and positively related to expenditures on nondurables and ser-

²In the case of fully anticipated inflation, there will be no real effects upon spending decisions. This state of affairs is, for the most part, a theoretical curiosity. The empirical results in Chapter V have demonstrated that there will usually be errors associated with forecasts of future inflation rates. Hence, the values of future inflation rates are not anticipated exactly. We should thus observe real effects upon the components of consumer spending in response to expected inflation.
vices. Springer's empirical results indicated a positive relationship between expected inflation and purchases of durable goods while non-durables and purchases of services exhibited a negative relationship with expected inflation. These conflicting results are probably due to the different methods used to measure expected inflation and the different specifications of the spending equations. J-W used subjective measures, derived from survey data, while Springer used an objective measure derived from applying a weighting scheme to a series of observed inflation rates. In any event, the issue of how expected inflation affects consumer spending has not been resolved. A preferable methodology for studying this problem is to incorporate the spending decisions of consumers into a general model of consumer choice which includes assets as well as commodities. This is carried out below.

Expected inflation will also affect the process of asset allocation within the household's portfolio. This influence is most easily recognized with respect to assets which have fixed nominal values. Time deposits at commercial banks and accounts at savings institutions are prime examples. For such assets, expected inflation will depress the expected real value of interest returns and also erode the amounts of purchasing power which are being stored in these forms. Thus, we would expect to find that expected inflation provides a negative influence upon the totals of these assets in the household's portfolio. The magnitude of the effect should depend upon the value of the expected inflation rate. The rate would have to be sufficiently high in order for the decline in an asset's real value and in the real value of its return to be noticeable. In addition, there would have to be enough compensation from substituting away from the asset to overcome the
transactions costs involved in doing so. There could even be a mechanism at work in this situation that is similar to the threshold effect for expected inflation and money balances. The expected inflation rate may have to be sufficiently high to prompt substitutions away from assets which have fixed nominal values. Recall that near-monies such as time deposits at commercial banks or accounts at savings institutions yield implicit services to the depositor in the form of liquidity and decreased risk. The expected real value of interest returns from the assets may have to fall substantially before a significant amount of substitution in response to expected inflation takes place.

Substitutions away from assets whose values are fixed in nominal terms could go in two directions. First, households might reallocate a portion of these assets to expenditures. However, consumers may not desire to increase expenditures but instead may prefer to substitute toward assets whose prices are variable, such as common stocks. There need not be a decline in the real values of such assets or in the real value of their returns in an inflationary period.

In essence, we have described two strategies for hedging the value of wealth against inflation. But there are other possible strategies that may be followed by households. For example, the household could assume a higher risk position by substituting common stocks, a variable price asset, for time deposits at a commercial bank, a fixed nominal valued asset. As an alternative, the household may substitute one fixed nominal valued asset for another that offers a higher nominal return.

A convenient illustration is given by substitutions away from money balances to time deposits that earn a positive interest return.
While such an action might not optimize the growth in the real value of purchasing power represented by the portfolio and its nominal returns, it may optimize the combination of pecuniary and nonpecuniary returns. This behavior suggests a hierarchy of substitutions in response to expected inflation. If the expected inflation rate is relatively low, the bulk of substitutions, or hedging, may be from money balances (whose expected return is negative) to assets which are highly liquid but earn a modest nominal return. Time deposits would be a likely candidate. But as the expected inflation rate increases, perhaps to the point where the growth in the real value of the asset and its return was zero, substitutions toward assets with higher nominal interest returns would be undertaken. At high rates of expected inflation, such as the 11% or 12% rates in 1975 suggested by our forecasting models, we may observe substitutions away from assets like time deposits as well as from money.

To examine the possible effects of inflationary expectations, we shall employ a complete set of consumer demand equations for commodities and asset stocks. Two objectives must be met in the course of our modeling procedures. First, the set of demand equations must be derived from the utility maximizing behavior of households. If this requirement is not satisfied, then any demand equations which may be offered would be merely conjectural and would lack any element of consistency. Second, the balance sheet restriction, or "adding-up" constraint, stressed by Brainard and Tobin [16] must be satisfied to qualify the model as a description of portfolio allocation.

Multiequation models of consumer demand which are based upon utility theory have been used extensively to describe and quantify the
allocation of income among various expenditure components. A partial list of these models would include the linear expenditure system (L.E.S.)\textsuperscript{3}, the Rotterdam model\textsuperscript{4}, the direct and indirect addilog models\textsuperscript{5}, the equation system generated by the S-branch utility tree\textsuperscript{6}, and the system generated by the transcendental logarithmic (translog) utility function\textsuperscript{7}. Since expected inflation may be of importance in determining the level of spending upon commodities, the system of demand equations which we employ must include specifications for commodity demands. However, the dependency of commodity and asset demands on expected inflation must also be accounted for by incorporating specifications of the asset demands in the model. This may not be done in an ad hoc manner by simply adopting specifications which appear to be relevant.\textsuperscript{8} Alternatively, the demand equations for assets can be derived from, or based upon, an acceptable utility function. To ensure consistency, the demand equations for both the commodities and the assets being considered must be derived from a common form of the utility function.

While the models mentioned above have been applied extensively in the study of consumer expenditure, there has been much less work done on asset demands that are consistent with an underlying specification of a utility function. The concept of assets, either interest earning or not, generating utility is essentially an extension of the ideas of Friedman [31] and Patinkin [60] which focus upon real money balances

\textsuperscript{3}See Stone [71], Goldberger [35], or Goldberger and Gamaletos [36].
\textsuperscript{4}See Theil [72].
\textsuperscript{5}See Theil [72].
\textsuperscript{6}See Brown and Heien [17] and Barth, Kraft, and Weist [7].
\textsuperscript{7}See Christensen, Jorgenson, and Lau [21].
\textsuperscript{8}The survey paper of Feige and Pearce [29] contains several examples.
as providing utility to agents. Chetty [20] pursued this concept and specified a constant elasticity of substitution (CES) utility function for holdings of money and time deposits. The purpose of the study was to examine the substitutability of the two assets but commodities and other assets which are alternatives to money were neglected. Saito [68] has extended the use of utility theory in studying asset selection by considering seven assets and debt held by the household sector. After specifying an exponential utility function for asset holdings, Saito derives demand equations which are of the same form as those of the linear expenditure system. Estimation of this system gives measurements of the own and cross-interest elasticities for the assets.

7.2 The Linear Allocation of Spending Power System

Clements [22, 23] has recently proposed an equation system that integrates the selection of both commodities and assets by the household sector. Known as the linear allocation of spending power system (LASS), the model is a description of how the consumer allocates an existing asset total plus his income stream among commodities and specific assets. Hence, current consumption, current saving, and the allocation of saving and the reallocation of existing spending power among assets are all subsumed in the model. Essentially, LASS is an extension or generalization of the linear expenditure system with the inclusion of assets being its outstanding feature. After specifying the commodity and asset groups to be considered, the resulting system of equations can be estimated and, as in the study by Saito, measurements of various own and cross elasticities can be obtained. Clements estimated the LASS system using Australian data. Our concern lies not with describing
the degree of substitutability among assets or commodities, but rather with measuring the impact of expected inflation upon asset and commodity selection. Thus, the LASS system will be modified to meet the specific needs of this study. For ease of reference, the general form of the notation used by Clements will be adopted.

The household's task of allocating its holdings of assets and current income stream among commodities and specific asset stocks is an intertemporal problem or decision. If the household saves during one time period, it is essentially storing purchasing power which may be used to finance consumption in future periods. Liquidation of this savings total may occur in the following time period or be spread over successive periods. The acquisition of durable goods is similar since these commodities yield services, i.e., a return, over a given period of time until obsolescence. Even the level of spending upon nondurable commodities in the present period is the result of an intertemporal decision since present consumption could be postponed in favor of future consumption. We begin by specifying the following utility function for the household in time period t,

\[
U_t = U(x_t, ..., x_{t+T}, y_t, ..., y_{t+T})
\]

where,

\(x_t\) is the \(n\)-vector of commodity amounts consumed in period \(t\), \(t = t, ..., t+T\),

\(y_t\) is the \(m\)-vector of asset amounts held at the end of period \(t\).

\(T\) is the number of time periods in the planning horizon. There are \(\ell = n+m\) commodities and assets entering the household's utility function. The utility enjoyed by the household in any given time
period, \( t \), is dependent upon the commodity and asset amounts which are consumed or held in the present period as well as the corresponding amounts in future periods. Following the standard view of the household's choice problem, (7.2.1) must be maximized with respect to a constraint that summarizes the availability of spending power in the present period, \( t \), and in all future periods of the planning horizon.

The constraint upon maximization, in any time period \( \tau \), is written as

\[
(7.2.2) \quad p^{\tau} x^{\tau} + \tilde{a}^{\tau} v^{\tau} = Y^{\tau} - Q^{\tau} + R^{\tau} - D^{\tau} + \tilde{z}^{\tau} v^{\tau-1},
\]

where,

- \( p \) is the \( n \)-vector of commodity prices,
- \( \tilde{a} \) is the \( m \)-vector of asset prices (termed "pure" asset prices),
- \( Y \) is labor income,
- \( Q \) is the amount of taxes paid by the household,
- \( R \) is non-labor income,
- \( D \) is the amount of depreciation of the stock of durable goods, and
- \( \tilde{z} \) is the \( m \)-vector of current period prices of the assets held in the previous period (termed "pure" realization prices).

Thus, the budget or wealth constraint stipulates that consumption plus the current value of the asset portfolio must equal the current flow of after-tax income plus the current value of the asset portfolio held at the end of the preceding period. To simplify the form of the con-

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The notion of a realization price is employed so that the stock of assets held in the previous period may be valued in terms of prices prevailing in the current period.
straint, it is assumed that changes in the asset portfolio are uniformly distributed over time. As a result of this assumption, we may write the following expression for non-labor income

\[
R^\tau = \frac{1}{2} \hat{\Sigma} \hat{\xi} \nu^\tau \nu_{\tau-1}^\tau + \frac{1}{2} \hat{\Sigma} \hat{\xi} \nu^\tau \nu_{\tau-1}^\tau ,
\]

where

\( \hat{\Sigma} \) is the vector of market yields or nominal interest rates earned on the interest-earning assets,

\( \hat{\xi} \) is a diagonal matrix of pure realization prices of the interest-earning assets,

\( \nu \) is the vector of the amounts of the interest-earning assets at period end, and

\( \hat{\xi} \) is a diagonal matrix of pure prices of the interest-earning assets.

The timing assumption used above is also applied to the depreciating assets in the portfolio, giving

\[
-D^\tau = \frac{1}{2} \hat{\Sigma} \hat{\xi} \nu^\tau \nu_{2\tau-2,\tau-1}^\tau + \frac{1}{2} \hat{\Sigma} \hat{\xi} \nu^\tau \nu_{2\tau-2,\tau}^\tau ,
\]

where

\( \xi \) is the vector of (negative) depreciation rates,

\( \hat{\xi} \) is a diagonal matrix of pure realization prices of the depreciating assets,

\( \nu \) is the vector of the amounts of the depreciating assets at period end,

\( \hat{\xi} \) is a diagonal matrix of pure prices of the depreciation assets.

Combining (7.2.3) and (7.2.4) yields

\[
R^\tau - D^\tau = \frac{1}{2} \hat{\Sigma} \hat{\xi} ^\tau \nu_{\tau-1}^\tau + \frac{1}{2} \hat{\Sigma} \hat{\xi} ^\tau \nu_{\tau}^\tau ,
\]
where

\[ \Sigma_T = [\Sigma^\tau_1 | \Sigma^\tau_2] \], a partitioned \( m \)-vector of nominal interest rates and depreciation rates,

\[ \nu_T = [\nu^\tau_1 | \nu^\tau_2] \], a partitioned \( m \)-vector of the holdings of interest-earning and depreciable assets at period end,

\[ \hat{\Sigma} = \begin{bmatrix} \hat{\Sigma}^\tau_1 & 0 \\ 0 & \hat{\Sigma}^\tau_2 \end{bmatrix} \]

\[ \hat{\alpha} = \begin{bmatrix} \hat{\alpha}^\tau_1 & 0 \\ 0 & \hat{\alpha}^\tau_2 \end{bmatrix} \]

By substituting (7.2.5) into (7.2.2), the constraint can be simplified to

(7.2.6) \[ \frac{\delta^\tau \Sigma_T}{\delta^\tau X_T} = s_T \]

where

\[ \Sigma_T = [\Sigma^\tau | \nu^\tau] \], a partitioned \( \ell \)-vector of the \( n \) commodity amounts and the \( m \) asset amounts,

\[ \Sigma_T = [\Sigma^\tau | \alpha^\tau] \], a partitioned \( \ell \)-vector of the \( n \) commodity price and \( m \) "adjusted" asset prices,

\[ \alpha_T = [I_m^{-1/2} \Sigma^\tau] \hat{\alpha}_T \], an \( m \)-vector of the "adjusted" asset prices with \( I_m \) the identity matrix,

\[ s_T \] the "spending power" available to the household.

The spending power total can also be written as

(7.2.7) \[ s_T = Y_T - Q_T + \frac{Z^\tau X_T}{\delta^\tau X_T} \]

where

\[ Z_T = [0 | Z^\tau] \], a partitioned \( \ell \)-vector with the number of leading zeros representing the number of commodities.

\[ \bar{Z}_T = [I_m^{1/2} \Sigma^\tau] \bar{Z}_T \], an \( m \)-vector of "adjusted" realization prices for the assets.
Although the maximization problem of the household is intertemporal in nature, a collapsibility theorem demonstrated by Hadar [41] indicates that such a problem may be reduced to a single-period problem. Essentially, the theorem notes the existence of a one-period utility function and budget constraint that will yield demand equations which are consistent with the intertemporal problem. Following Clements, the form of the one-period utility function and budget constraint will not be derived for LASS. Instead, we will assume the following form

\[(7.2.8) \quad u_t = u(x_t, y_t),\]

where \(x_t\) and \(y_t\) are the vectors of commodity and asset amounts in time \(t\), respectively. For the maximization constraint, we have

\[(7.2.9) \quad P_t x_t = s_t,\]

where \(P_t\) and \(x_t\) are the partitioned vectors of commodity and asset prices, and quantities, respectively. Thus, we have reduced the rather complicated allocation problem, which is intertemporal in nature, to a one-period problem.

The Klein-Rubin [47] utility function is assumed as the particular form for the function in (7.2.8). Two considerations dictated this choice. First, the utility function can be maximized subject to a constraint and the solution will yield exact demand equations for the arguments in the function. Second, the resulting demand equations or allocation system is flexible and will allow us to incorporate the

\(^{10}\)Powell [65] has offered a concise proof of the theorem. It may also be referenced in an appendix to the Clements [23] paper.
expected inflation rate as an additional variable. The utility function may be written in vector form as

$$U_t = B'\ln(X_t - G)$$

(7.2.10)

with the conditions,

$$\sum_{i=1}^\ell B_i = 1$$

$$B_i \geq 0 (i=1,\ldots,\ell)$$

$$X_{it} - G_i \geq 0 (i=1,\ldots,\ell) .$$

The $B_i$ are the elements of the $\ell$-vector $B$ while the $X_{it}$ are the elements of the $\ell$-vector of commodity and asset amounts, $X_t$. The $G_i$ are elements of the $\ell$-vector $G$ and are commonly referred to as the "subsistence" quantities of the commodities and assets in the utility function.

After allocating a subtotal of spending power to ensure these subsistence quantities, the household then allocates the remainder among the set of commodities and assets. The $B_i$ describe the manner in which this remainder, or "supernumerary" spending power, is divided among the set. Thus, the $B_i$ are referred to as marginal spending power shares of the respective commodities and assets.

Having described the utility function, the maximization problem must now be solved. We have

$$\text{Maximize } U_t = B'\ln(X_t - G)$$

$$\text{s.t. } P_tX_t = s_t .$$

(7.2.11)

Since the problem is essentially the same as that present in dealing
with the linear expenditure system, we shall follow the solution inGoldberger [35]. The Lagrangian and its associated conditions, after dropping the time subscript, are

\begin{align*}
(7.2.12) \quad U^* &= B' \ln(X-G) - \lambda (P'X - s) \\
\frac{\partial U^*}{\partial X} &= (X-G)^{-1} B - \lambda P = 0 \\
&= -P'X + s = 0
\end{align*}

where the hat denotes a diagonal matrix. Rearranging the first condition yields

\begin{align*}
(7.2.13) \quad B &= \lambda \hat{P}(X-G) .
\end{align*}

Recalling the condition imposed upon the marginal spending-power shares provides

\begin{align*}
1 &= \hat{P}' = \lambda \hat{P}'(X-G) ,
\end{align*}

where \( \hat{\cdot} \) is the transposed unit vector of dimension \( \lambda \) and the hat is used to denote a diagonal matrix. Proceeding, we have

\begin{align*}
1 &= \lambda \hat{P}X - \lambda \hat{P}G ,
\end{align*}

or,

\begin{align*}
1 &= \lambda s - \lambda P'G \\
\lambda &= (s-P'G)^{-1} .
\end{align*}

Substitution of the above expression into (7.2.13) yields

\begin{align*}
B &= (s-P'G)^{-1} \hat{P}(X-G) ,
\end{align*}
whence,

(7.2.14) \[ \hat{P}_X = \hat{P}_G + (s-P^G)B \]

or,

(7.2.15) \[ X = G + (s-P^G)\hat{P}^{-1}B \]

The expression in (7.2.15) is the complete set of demand equations, in vector form, for the LASS system.\(^{11}\) A specific demand equation from the set may be written as

(7.2.16) \[ x_i = g_i + \frac{b_i}{p_i} (s- \sum_{i=1}^{n} p_i g_i) \]

These demand equations will eventually be used to derive elasticity expressions, but for the purpose of estimation we shall deal with the allocation system given by (7.2.14). It is easily seen that the LASS system is closely related to the linear expenditure system, with the latter actually being a special case of the former when no assets enter the system. A specific equation in the LASS allocation system may be written as

(7.2.17) \[ p_i x_i = p_i g_i + b_i (s- \sum_{i=1}^{n} p_i g_i) \]

The term in parenthesis represents the "supernumerary" spending power available to the household while \( b_i \) is the marginal spending power share of the \( i \)th commodity or asset.

\(^{11}\) See Goldberger [35], Theil [72], or Powell [66] for descriptions of the characteristics of these demand equations.
7.3 Expected Inflation and the LASS System

The allocation system given by (7.2.14) does not incorporate a description of how inflationary expectations might affect the household's allocation decisions. Yet, the results presented in previous chapters of this study have indicated that expected inflation has an influence upon money demand and, quite possibly, upon the demands for other assets and commodities. To allow for the influence of inflationary expectations upon the allocation of spending power among the respective commodities and assets, we shall assume that expected inflation alters only the marginal spending-power shares and that this effect can be modeled in a linear fashion. The subsistence totals of commodities and assets, i.e., the elements of the $G$ vector in (7.2.14), are assumed to be unaffected by expected inflation.\(^{12}\)

In terms of one specific equation of LASS, the system in (7.2.14) is modified to

\[(7.3.1) \quad P_{it} X_{it} = P_{it} G_{it} + (B^* + B^{**} \cdot \pi^e_t)(s - \sum_{i=1}^{L} P_{it} G_{it}), \]

where $t$ is the time subscript and $\pi^e_t$ is the expected rate of inflation in time $t$. In accordance with the condition imposed upon the $B_i$ in the original LASS system,

\[\sum_{i=1}^{L} B_i = 1,\]

we have,

\[(7.3.2) \quad \sum_{i=1}^{L} (B^* + B^{**} \cdot \pi^e_t) = 1,\]

\(^{12}\) The assumption might be invalid in an environment of extremely high rates of inflation or deflation but it seems acceptable for the U.S. experience of the last decade.
or,
\[
\sum_{i=1}^{2} b^*_i + \pi^e \cdot \sum_{t=1}^{2} b^{**}_t = 1.
\]

By adopting the normalization conventions

(7.3.3) \[
\sum_{i=1}^{2} b^*_i = 1
\]

and

(7.3.4) \[
\sum_{i=1}^{2} b^{**}_i = 0
\]

the condition given in (7.3.2) is satisfied. The marginal spending-power shares will differ in each time period as a result of adopting the modified LASS system of (7.3.1). The normalization convention given in (7.3.4) is particularly interesting. In effect, it means that the declines in commodity and asset allocations which result from inverse relationships with expected inflation will be exactly matched by increases in the allocation of spending power to the commodities and assets that are positively related to expected inflation.¹³

7.4 Estimation of the Modified LASS System

To analyze the impact of expected inflation upon the demands for various commodities and assets, the parameters in the modified LASS

¹³In essence, we have imposed the "adding-up" constraint upon the effects of expected inflation. The inclusion of additional variables in the linear expenditure system, which is a special case of LASS, has been undertaken by several authors. In his study comparing various equation systems, Parks [59] specified the share and subsistence parameters to be a linear function of a time index. Pollack and Wales [63] entertained several specifications that made the subsistence parameters depend upon time, past consumption, and the growth rate of consumption, respectively. Our modification of the LASS system is thus in the spirit of these previous studies although the problem being studied here would appear to be more complicated.
system must be estimated. The complete system is written as

\[(7.4.1) \quad \hat{P}_t X_t = \hat{P}_t G + (s_t \hat{P}_t G)(B^{**} + B^{***} \cdot u^e_t)\]

where the vectors have been underscored. It is easily seen that the equations of the system are nonlinear. Estimation is accomplished by maximizing the likelihood function for the system which results from a specified error structure.

After adding an \( \lambda \)-vector of disturbance terms to \((7.4.1)\), we have

\[(7.4.2) \quad \hat{P}_t X_t = \hat{P}_t G + (s_t \hat{P}_t G)(B^{**} + B^{***} \cdot u^e_t) + u_t \cdot\]

Further, we assume that there are \( T \) observations available on the variables contained in the system. Estimating the parameters in \((7.4.2)\) by using the complete system of equations will be accompanied by a covariance matrix of disturbances which is singular. Therefore, one equation is dropped to avoid this problem and only \( \lambda - 1 \) equations need be estimated. This set of equations may be referred to as the "reduced" system. Powell [64] and Pollack and Wales [63] have shown that the maximum likelihood estimates of the parameters in a linear expenditure system are invariant to the selection of the equation to be dropped. This result also applies to the complete LASS system. Note that estimates of \( B^{**}_\lambda \) and \( B^{***}_\lambda \) in the omitted equation will be provided by the normalization conventions adopted in \((7.3.3)\) and \((7.3.4)\).

The disturbance vector for the \( \lambda - 1 \) equations in the reduced system, in time \( t \), becomes

\[(7.4.3) \quad \tilde{u}_t = \left[ u_{1t} \ u_{2t} \ldots \ u_{\lambda-1,t} \right]^\top.\]
The covariance matrix of disturbances is specified by the following

\[ E(u_{it}) = 0, \]

and

\[
E(u_{it}u_{jt}) = \sigma_{ij} \quad \text{if} \quad t = t \\
= 0 \quad \text{if} \quad t \neq t
\]

for \( i, j = 1, \ldots, k \). Thus, heteroskedasticity and contemporaneous correlation across equations has been allowed for in the error structure.

In vector notation, we have,

\[ E(\tilde{u}_t) = 0, \]

and

\[
E(\tilde{u}_t\tilde{u}_t') = (k-1)\Omega(k-1) = \{\sigma_{ij}\}
\]

where \( \Omega \) is the covariance matrix of the reduced system.

Under the assumption that \( \tilde{u}_t \) has a multivariate normal distribution with a zero mean and covariance given by (7.4.4), the likelihood function of \( \tilde{u}_1, \ldots, \tilde{u}_T \) is written as

\[
(7.4.5) \quad L(\tilde{u}_1, \ldots, \tilde{u}_T) = \prod_{t=1}^{T} \frac{(k-1)}{2} \frac{1}{2} e^{-\frac{1}{2} \tilde{u}_t' \tilde{\Omega}^{-1} \tilde{u}_t}.
\]

The log-likelihood function is

\[
(7.4.6) \quad \ell(\tilde{u}_1, \ldots, \tilde{u}_T) = -\frac{(k-1)T}{2} \ln 2\pi - \frac{T}{2} \ln |\Omega| - \frac{1}{2} \sum_{t=1}^{T} \tilde{u}_t' \tilde{\Omega}^{-1} \tilde{u}_t.
\]

We concentrate the log-likelihood by noting that

\[
\hat{\Omega} = \frac{1}{T} \sum_{t=1}^{T} \tilde{u}_t \tilde{u}_t',
\]

which is the maximum likelihood estimator of \( \Omega \). Substituting the
expression into (7.4.6) gives

\[(7.2.7) \quad 1(\tilde{\mu}_1, \ldots, \tilde{\mu}_T) = -\frac{(T-1)T}{2} \ln 2\pi - \frac{T}{2} \ln |\tilde{\mu}| - \frac{T}{2} \text{tr}(\tilde{\mu}^{-1} \tilde{\mu}) \].

Since \( \tilde{\mu} = \tilde{\mu} \), we have \(^{14}\)

\[1(\tilde{\mu}_1, \ldots, \tilde{\mu}_T) = C \frac{T}{2} \ln |\tilde{\mu}| ,
\]

or,

\[(7.4.8) \quad 1(\tilde{\mu}_1, \ldots, \tilde{\mu}_T) = D \frac{T}{2} \ln |\frac{T}{T} \sum_{t=1}^{T} \tilde{\mu}_t \tilde{\mu}_t|\]

where \(C\) and \(D\) are additive constants. Thus, maximizing (7.4.8) is equivalent to maximizing the complete log-likelihood function in (7.4.6).

Two commodity categories and six asset categories were considered for the modified LASS system. Since the system contains \(3\) \(k\) parameters, where \(k\) is the number of equations, the estimation problem can become very tedious as larger numbers of commodities and assets are entertained. There are \(3k - 2\) parameters to be estimated under the normalization conventions adopted above, the subsistence totals for all \(k\) members of the complete system, the share parameters for the \(k - 1\) members of the reduced system, and the expected inflation parameters for the \(k - 1\) members. Our modified LASS system contains eight equations so 22 parameters must be estimated.

The commodity and asset categories of the system are:

1. Nondurables
2. Services
3. Savings Accounts

\(^{14}\)See Malinvaud [51], especially pages 338-341, or MacKinnon [50].
4. Durables
5. Equities
6. Bonds
7. Demand Deposits and Currency
8. Debt.

Debt is treated as a negative asset and retains its negative sign throughout the analysis. Data for the end-of-period holdings of the respective assets by households was taken from the Flow of Funds Accounts for the U.S.\textsuperscript{15} A complete description of the data appears in Appendix 3. The sample period consists of quarterly observations from the first quarter of 1968 through the second quarter of 1976. This period matches that used for our examination of inflationary expectations and the aggregate demand for money. Our rationale for this selection is the same as was used in the previous context. Specifically, an effect upon household holdings of money, or other assets and commodities, stemming from expected inflation should be strongest in a period of relatively high inflation rates.

The forecasting techniques employed in Chapter V were used to generate the series of expected inflation rates for the modified LASS system. A quarterly series of observations on the Consumer Price Index formed the basic data set. The observations were seasonally unadjusted and covered the period from the third quarter of 1953 through the second quarter of 1976. This matches the span of the time series of observations used in Chapter V. Since the modified LASS system is concerned only with household behavior, the C.P.I. is used in this

\textsuperscript{15}See Clements [22, chap. 6] for a discussion of aggregation in the context of LASS.
context instead of the GNP deflator. Several procedures were examined in the course of constructing the series of expected inflation rates. Initially, the observations on the price level were used to construct an observed inflation rate series. Application of the identification, estimation, and diagnostic checking sequence indicated a problem with the stationarity of the rate series. The price level itself was then selected and the same sequence of modeling procedures was applied. Nonstationarity in the series was again found to be present. Finally, natural logarithms of the price level were used and an ARIMA(0,2,1) model was found to provide an adequate representation of the time series. The estimated model was

\[(7.4.9) \quad (1-B)^2 CP_t = (1-.332B) a_t \]

\[\hat{a}_t \pm .199 \]

\[\delta^2 = .000014 \quad Q = 34.3 \quad (23 \text{ d.f.}) \]

where B is the backshift operator and CPₜ denotes the natural log of the price level in time t. One-step ahead forecasts from (7.4.9) are given by

\[(7.4.10) \quad \hat{CP}_t(1) = 2CP_t - CP_{t-1} - .332(\hat{CP}_t - \hat{CP}_{t-1}(1)) , \]

with the notation following the conventions used in Chapter V. These forecasts are of the logarithm of the price level whereas the goal of this forecasting procedure is to provide forecasts of the inflation rate. Two additional steps are required. First, the logarithmic forecasts were converted to forecasts of the raw price level by using the transformation outlined in Nelson [57]. The forecasts of the price level were then used to generate one-step ahead forecasts of the
inflation rate via

\[
(7.4.11) \quad \frac{\hat{P}_t(1) - P_t}{P_t} \times 100
\]

where \(\hat{P}_t(1)\) is the one-step ahead forecast of the price level, made in
time \(t\). The series of measurements given by (7.4.11) composed the
expected inflation series to be used in the estimation of the modified
LASS system.

In Chapter V, the concept of interventions in an economic time
series was discussed and a procedure of modeling their effects was
employed. If the interventions represented by the wage and price con-
trols were entertained in the context of modeling the GNP deflator,
then they should also be considered in modeling the Consumer Price
Index. After estimating the univariate model for the C.P.I., which
appears in (7.4.9), the estimation technique associated with the
analysis of these interventions was applied.\(^{16}\) As before, the entire
set of interventions representing the wage and price controls were
built into a time series model of the form shown in (5.3.5). Individ-
ual interventions were then deleted on successive rounds of estimation
until a subset whose effects were significant could be found. The
asymptotic \(t\)-ratios of the individual parameters representing the
respective interventions were used as criteria in the selection process.
Surprisingly, only the moving average parameter of the univariate, or
noise, model was significant. No cases were found where either one
intervention parameter or a subset of them was significant. This
result contrasts with those obtained in the work with the GNP deflator.
Recall that significant effects were attributable to interventions in

\(^{16}\) See Chapter V for a review of the estimation procedure.
the case of the deflator. Any apparent conflicts are probably due to the basic nature or construction of the respective indices.

The data for the consumption and asset values was seasonally un-
adjusted and expressed in per capita terms. Spending power totals were also converted to per capita observations. As noted above, the sample period contained 34 quarterly observations spanning the first quarter of 1968 through the second quarter of 1976. Seven equations are in-
cluded in the reduced form of the modified LASS system. Two parameters of the full system need not be directly estimated because of the dropping of one equation. Hence, there are 22 parameters to be estimated. The allocation equation for services was the one which was dropped.

Estimation is accomplished by maximizing the simple form of the concentrated likelihood function, given in (7.4.8), with respect to the parameters of the system. A numerical method which accounts for the nonlinear character of the equation system was used for the esti-
mation. Specifically, the Gauss-Newton method was used to maximize the likelihood function. This algorithm is contained in a computer program originally written by Bard [4] and subsequently modified by Clements [22]. Further modifications were made by the present author to accommodate the equation system under consideration. The principal modification consisted of supplying analytic derivatives of the equa-
tions as required by the basic program.

Two sets of constraints were imposed upon the parameters during estimation of the modified LASS system. First, each of the share parameters was constrained to be positive, as required by the Klein-

\footnote{See Goldfeld and Quandt [38] for a general discussion of nonlinear estimation.}
Rubin utility function. Second, the sum of the share parameters for the commodities and assets in the reduced system was constrained to be less than or equal to one. This constraint ensures that the share parameter for the omitted commodity or asset will be positive and allows the normalization convention of (7.3.3) to be imposed upon the complete system. No constraints were imposed upon the parameters which measure the influence of expected inflation. The results obtained from the estimation procedure are presented in Table 7-1.\textsuperscript{18}

Asymptotic standard errors of the parameters appear in the parenthesis below the estimates. The standard errors for the parameters directly estimated by the maximum likelihood technique are obtained, as usual, by consulting the negative inverse of the Hessian associated with the concentrated log-likelihood function. Estimates of the share parameter and expected inflation parameter in the equation for services are provided by the normalization conventions in (7.3.3) and (7.3.4), respectively. The standard error of the share parameter may be obtained by noting that

\[
\text{Var}(B^*_\lambda) = \text{Var}(1-\lambda^T B^*) ,
\]

where $\lambda$ is the $k - 1$ unit vector and $B^*$ is the $k - 1$ vector of share parameters included in the reduced system. Hence,

\textsuperscript{18}The system was also estimated without imposing the constraints. However, the share parameter for debt had a negative sign which is inconsistent with the conditions of the Klein-Rubin utility function and also implies that total utility increases with the level of debt. The likelihood ratio test of the constraints indicated that they are invalid. Hence, our statistical results apply only to the case where the utility function is assumed, a priori, to be of the Klein-Rubin form. Further study should be concerned with the applicability of more general functional forms, such as the translog utility function.
<table>
<thead>
<tr>
<th>Commodity/Asset</th>
<th>Subsistence Parameters (C)</th>
<th>Share Parameters (R*)</th>
<th>Expected Inflation Parameters (R**)</th>
<th>Quasi-R²</th>
<th>Quasi-Durbin-Watson</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Nondurables</td>
<td>89.296</td>
<td>.019</td>
<td>.00483</td>
<td>.777</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td>(163.132)</td>
<td>(.018)</td>
<td>(.00359)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Services</td>
<td>228.296</td>
<td>.005</td>
<td>-.00873</td>
<td>.193</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>(296.367)</td>
<td>(.035)</td>
<td>(.00589)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Savings Accounts</td>
<td>-74.333</td>
<td>.225</td>
<td>.06112</td>
<td>.726</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(452.281)</td>
<td>(.059)</td>
<td>(.01524)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Durables</td>
<td>-46.616</td>
<td>.139</td>
<td>.01696</td>
<td>.891</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>(495.088)</td>
<td>(.077)</td>
<td>(.00417)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Equities</td>
<td>150.202</td>
<td>.500</td>
<td>-.09039</td>
<td>.592</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>(1877.93)</td>
<td>(.167)</td>
<td>(.01409)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Bonds</td>
<td>50.594</td>
<td>.038</td>
<td>.00914</td>
<td>.789</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(299.998)</td>
<td>(.024)</td>
<td>(.00206)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Demand Deposits and Currency</td>
<td>113.419</td>
<td>.074</td>
<td>.01146</td>
<td>.798</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>(339.170)</td>
<td>(.047)</td>
<td>(.00465)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Debt</td>
<td>-658.701</td>
<td>0</td>
<td>-.00439</td>
<td>.659</td>
<td>1.46</td>
</tr>
<tr>
<td></td>
<td>(160.120)</td>
<td>(.016)</td>
<td>(.00439)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ \text{Var}(B^*_k) = \text{\(\sum\)} \tilde{M} \text{\(\sum\)} , \]

where \(\tilde{M}\) is the covariance matrix of \(B^*\). For the inflation parameter in the excluded equation we have

\[ \text{Var}(B^{**}_k) = \text{Var}(\text{\(-\sum\)} B^{**}) , \]

where \(B^{**}\) is the \(l - 1\) vector of expected inflation parameters included in the system. Hence,

\[ \text{Var}(B^{**}_k) = \text{\(\sum\)} \tilde{\Gamma} \text{\(\sum\)} , \]

where \(\tilde{\Gamma}\) is the covariance matrix of \(B^{**}\).

Table 7-1 also reports two summary statistics calculated for each of the equations in the system. The quasi-\(R^2\) statistic attempts to measure the explanatory power of the estimated equations while the quasi-Durbin-Watson statistic may be used to indicate the presence of autocorrelated errors. These statistics are only approximations to their counterparts in classical regression analysis since the residuals from each of the equations do not have zero means. Examination of the table reveals that only the estimated equation for services seems to be deficient in terms of explanatory power. This poor performance of the services equation is probably the consequence of the relatively low share parameter. The estimated equation for demand deposits and currency is our main concern in this analysis and it has a fairly high \(R^2\) value. The Durbin-Watson statistics seem to indicate the possible presence of autocorrelation in the equations. However, no attempt is made here to allow for autocorrelated errors in the
system. 19

Consulting the results in Table 7-1, we find that several of the subsistence totals have negative signs. While this is unusual, it is not inconsistent with the existence of the Klein-Rubin utility function which was assumed to generate the allocation system. Only the total for debt exceeds its estimated standard error, indicating that households do have a minimum level of debt which must be held. Also of interest is the subsistence total for services. The value is quite high with respect to the observed values which were used in the estimation. In view of the small share parameter, the high subsistence value would indicate that households view the amount of services consumed as a necessity, with a very small portion of supernumerary spending power being allocated toward the commodity. While this interpretation would appear logical for consumers in an advanced economy such as the U.S., we must qualify the results given for the services equation by recalling the low R² value which was obtained for it.

Turning to the share parameters for the remaining commodity and assets, we see that equities and savings accounts are dominating the secondary, or marginal, allocation of spending power. 20 These estimates are not surprising since the totals of observed spending power

---

19 The possible procedures which could be followed to correct for non-random errors are, at this writing, somewhat nebulous. Several problems would plague any attempt. First, the errors need not follow an autoregressive process. Rather, they may follow a moving average process and the Durbin-Watson statistic cannot indicate this. Second, if the process is autoregressive, we still have no indication of its order. Third, the prevailing technique which is used, once a first order process is assumed, involves forcing the same process upon all equations in the system. See Berndt and Savin [10] or MacKinnon [50] for a discussion of this technique.

20 We use the phrase "secondary allocation" since initial allocations of spending power are in terms of the subsistence values.
are dominated by the equity and savings account values. If we neglect the effects of expected inflation, it is seen that consumption, composed of expenditures on nondurables and services, and additions to the stock of durables accounts for only 16.3% of the secondary allocation. This illustrates the dominance of the asset accounts once again. The share parameter for debt lies at the lower bound of the nonnegativity constraint forced upon it during estimation. Thus, the high absolute value of the subsistence total for debt is not surprising. These estimates suggest that debt does not provide utility, or disutility, to the household in a direct manner. Rather, only the expenditures or additions to asset holdings which are facilitated by holding debt yield utility. Such an interpretation would appear to be plausible.

Of greater importance in the present study are the estimates of the parameters which reflect the influence of expected inflation. In contrast with our theoretical expectations, the expected rate of inflation is positively related to the allocation of spending power to money balances, represented by demand deposits and currency in the system. As the expected rate of inflation increases, the household substitutes toward money balances. Substitutions are also made in favor of nondurables, durables, savings accounts, and bonds. Hence, all substitutions in response to expected inflation are not in favor of physical goods. There is some hedging done against expected inflation in the form of substituting toward interest-earning assets. Savings accounts enjoy the highest increase from these substitutions, as indicated by its having the highest parameter with a positive sign. Durables have the next highest increase followed by money balances.

The allocation of spending power to equities is negatively related
to expected inflation, a result that is unexpected since equities are commonly thought of as a convenient hedge against inflation. The relationship is quite strong as evidenced by the value of the expected inflation parameter. Services also have an expected inflation parameter which is negative in sign but the poor performance of the services equation tends to discount this result. The negative sign of the parameter for debt is also a bit dubious since the standard error is high in relation to the estimated value. Thus, equities is the most important commodity or asset having a negative relationship with expected inflation. Substitutions toward the commodities and assets which have expected inflation parameters with positive signs are mainly at the expense of equities held in the household's portfolio.

Overall, the results imply that households do not manage their asset portfolios in a manner that is consistent with the theory of portfolio selection. It appears that declines in the real value of money holdings are of secondary importance to households. Substitutions toward physical goods, represented here by nondurables and durables, and toward savings accounts are indicated by the estimates and do constitute hedging actions. However, substitution in favor of money balances in response to positive rates of expected inflation implies that households are more than willing to incur costs in terms of reduced purchasing power and declines in the real value of this portfolio item. Furthermore, substitution away from equities provides the main source of funds used for increasing the holdings of money balances.

Of the assets considered in the household portfolio, equities rank high in terms of the volatility of their prices and returns. If expectations of high inflation rates create a general climate of
uncertainty, an effect postulated by Juster and Wachtel [45], then the attractiveness of equities in the household portfolio may indeed be adversely affected. In addition, the variance of expected rates of inflation is typically higher in a period of relatively high and unstable inflation rates. This introduces increased uncertainty directly into calculations of the expected real returns on earning assets like equities. Such an influence will increase the degree of risk associated with a given equity position, perhaps prompting substitutions away from equities. Even though a decline in the real value of money balances during inflation is assured, the security offered by holding those balances may be preferred to the increased risk associated with holding equities as a hedge. These arguments provide a rationale for the empirical results and indicate that substitutions away from equities in response to expected inflation is not an implausible action on the part of households.

Although the estimates of the expected inflation parameters provide information on the direction and amounts of substitution, we do not have measurements which describe the relative strength of the effect from expected inflation upon the individual assets and commodities. Elasticities of demand with respect to expected inflation provide such a measure. Consider a typical equation of the system,

(7.4.12) \[ p_{it} x_{it} = p_{it} g_i + (B^{**}B^{**} \pi_{it}) \left( s - \sum_{t=1}^{T} \frac{p_{it} g_i}{p_{it}} \right) \]

Dividing both sides of the allocation equation by the adjusted price term gives

(7.4.13) \[ x_{it} = g_i + \frac{1}{p_{it}} (B^{**}B^{**} \pi_{it}) \left( s - \sum_{t=1}^{T} \frac{p_{it} g_i}{p_{it}} \right) \]
which is the demand equation for an arbitrary commodity or asset in time $t$. Hence, the expected inflation elasticity is given by

$$
\frac{\partial x_{it}}{\partial \pi_t^e} \cdot \frac{\pi_t^e}{X_{it}} = \frac{B_i^{**}}{P_{it}} \cdot \frac{\pi_t^e}{X_{it}} \left( s_{t-1} - \sum_{i=1}^{2} P_{it} F_i \right).
$$

(7.4.14)

Numeric values of the elasticities are time dependent so mean values of the variables were used in the computations reported in Table 7-3.

**TABLE 7-3. EXPECTED INFLATION ELASTICITIES IN THE MODIFIED LASS SYSTEM**

<table>
<thead>
<tr>
<th>Commodity/Asset</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nondurables</td>
<td>.185</td>
</tr>
<tr>
<td>Services</td>
<td>-.321</td>
</tr>
<tr>
<td>Savings Accounts</td>
<td>.309</td>
</tr>
<tr>
<td>Durables</td>
<td>.162</td>
</tr>
<tr>
<td>Equities</td>
<td>-.368</td>
</tr>
<tr>
<td>Bonds</td>
<td>.245</td>
</tr>
<tr>
<td>Demand Deposits and Currency</td>
<td>.227</td>
</tr>
<tr>
<td>Debt</td>
<td>.068</td>
</tr>
</tbody>
</table>

**NOTES:** The elasticity for debt is positive in sign since debt is treated as a negative asset in the system.

Within the set of commodities and assets that are positively related to expected inflation, the elasticity calculations indicate that savings accounts are the most sensitive to changes in the expected inflation rate. The elasticity value, together with the relatively high expected inflation parameter estimated for the asset, shows that households believe savings accounts to be a convenient hedge against future inflation. While the asset may be convenient, especially in
terms of low transactions costs, substituting toward it may not be the best strategy to pursue in terms of avoiding or minimizing the costs associated with inflation. On several occasions during the sample period, the expected rate of inflation exceeded 12%, when expressed on an annual basis. Hence, the real value of dollars held in savings accounts was declining as was the real value of the interest returns.

Money balances have an expected inflation elasticity of .227, somewhat smaller than that for savings accounts. This elasticity illustrates, once again, that households substitute toward money balances in response to expected inflation. In fact, the response is stronger than those of either nondurables or durables. Households are apparently willing to suffer the losses in the real value of their purchasing power if they undertake such substitution. While the elasticity value for money balances is not overly high, it indicates that expected inflation has a significant effect upon the demand for money by households.

The results which have been found for the household sector conflict with our earlier finding that the aggregate demand for money and expected inflation have an inverse relationship. This disparity may be due to the different modeling techniques which were used to study the aggregate demand for money and the household sector's demand for money. In the former case, the model forms were based upon macroeconomic theory while the analysis at the level of the household was based upon a microeconomic theory of behavior. We did not estimate a model form for the household sector that is similar to the form used for aggregate money demand.

Another explanation of the difference in results could lie with
the business sector's response to expected inflation. If businesses practice more efficient techniques of managing monetary accounts, a negative relationship between their holdings of money balances and expected inflation might be observed. The business sector's negative response to expected inflation, in terms of substitutions away from money balances, perhaps outweighs the positive response that we have estimated for households. This relationship between the two sectors would then explain the result obtained earlier for the aggregate demand for money. An analysis of these propositions is beyond the scope, and goals, of this study. However, if a negative relationship between expected inflation and money balances held by the business sector does exist, then businesses could be shifting an inflation tax, which exists for money balances at the aggregate level, to the household sector.
CHAPTER VIII

SUMMARY

This study has presented a detailed examination of the effects of inflationary expectations upon the demand for money. Although money demand was the topic of primary interest, other features of the study are worthy of note. In modeling the process of expectation formation, a methodology was introduced that treats expectations as forecasts derived from suitable forecasting models. Expectation formation is thus freed from the context of a particular structural economic model, such as a consumption or investment function. Rather, the expectations stream can be considered in conjunction with any number of structural models containing an expectations variable as an argument.

Expectations of the future values of three economic variables were considered in the course of the study; the inflation rate, income, and an interest rate. In each case an attempt was made to incorporate other economic variables into forecasting models for the variable of interest. Statistical tests indicated that the information contained in those other variables were of no use in constructing acceptable models. Only the past history of the variable being forecast proved useful. However, the forecasting models for both the price level and the inflation rate could be used to quantify the influence of the wage and price control periods.

The influence of inflationary expectations was first analyzed in the context of the aggregate demand for money. After specifying a theoretical model for aggregate money demand, three different versions
or variations of the basic model were considered. Primary emphasis was
given to the versions where expected inflation appeared, alternatively,
with observed values of income and an interest rate and with expecta-
tions of future income and the interest rate. The latter model is
popularly known as the pure expectations model of money demand. Quar-
terly observations from the post-war period were used in the estimation
of the respective models. The observations were split into two sample
periods in an effort to examine the threshold hypothesis of expected
inflation's effect. The hypothesis predicts that observed rates of
inflation must be relatively high for a significant relationship be-
tween expected inflation and money demand to be found. Thus, the
second sample period was composed of observations from 1968I - 1976II,
a period of relatively high inflation rates for the U.S.

Results obtained from the estimations tended to support the
existence of a threshold effect. Expected inflation was statistically
significant as an explanatory variable for money demand in the later
time period. It proved insignificant in estimates of the models using
data from 1954II - 1967IV. Tests were also conducted on the equality
of regression coefficients obtained for the respective periods. In
sum, the analysis conducted for the aggregate demand for money estab-
lishes the importance of expected inflation upon decisions to hold
money balances if the rate is relatively high. This finding has
definite implications for macroeconomic activity. A decrease in the
demand for money in response to expected inflation implies an increase
in the velocity of a given money stock. It is well known that velocity
increases may lead to increases in the general price level, particular-
ly when the economy is approaching full employment. Thus, there is a
potential for inflation to become self-feeding since higher rates of observed inflation will cause upward revisions in expectations of future rates.

The second area of investigation in the study sought to determine the influence of expected inflation upon the demand for money by the household sector. Recognizing that the choice to hold money is made within the context of other portfolio decisions and spending decisions, the demands for consumption commodities and assets in the household's portfolio were integrated into a model of consumer choice. The model described the allocation of available funds among competing commodities and assets. Utility theory provided the basis for the model, placing the analysis on firm theoretical grounds. After deriving allocation equations from a popular utility function, the system was estimated using data from the 1968I - 1976II period.

Several interesting results were implied by the estimates of the system's parameters. First, expected inflation was found to be positively related to the household sector's demand for money. This result means that households substitute toward money balances in response to positive rates of expected inflation, perhaps in an effort to continue financing the purchases of commodities which are rising in nominal value. Apparently, the household sector disregards the expected losses in the purchasing power of the dollars they hold. Substitutions toward money come at the expense of decreased equity holdings, an effect which is possibly due to the uncertainty over returns introduced by high rates of expected inflation.

Holdings of savings accounts were also found to be positively related to expected inflation. Calculation of the demand elasticities
of expected inflation for the members in the allocation system indicated that savings accounts were more responsive to movements in the expected inflation rate than the other assets which have positive elasticities. This group included both nondurable and durable goods. Hence, the results show that substitutions in response to expected inflation are not all in favor of physical goods. In fact, the elasticities for both durables and nondurables were less than the elasticity found for money balances. It is apparent that financial assets, and money balances in particular, play an important role in the household sector's reaction to expectations of future inflation.
APPENDIX 1

SOURCES OF THE DATA USED IN CHAPTERS V AND VI


**M1, M2** - Data for the years up to and including 1958 were taken from *Banking and Monetary Statistics, 1941 - 1970*, Board of Governors of the Federal Reserve System. The data for 1959 - 1976 were taken from a publication supplied by the Board, "Historical Money Stock Data", dated February, 1977. The yearly revisions in the seasonally adjusted series of monetary aggregates dictates the use of both sources. See the latter publication for the definition of M1 and M2.

**Monetary Base (MB)** - Research Department, Federal Reserve Bank of St. Louis.


**3-Month Treasury Bill Rate** - *Federal Reserve Bulletin*, various issues.
APPENDIX 2

ESTIMATES OF THE AGGREGATE MONEY DEMAND MODELS

USING AN ALTERNATIVE FUNCTIONAL FORM

The tables in Chapter VI presented the results obtained from estimating the aggregate money demand models of (6.1.3), (6.2.2), (6.2.4), and (6.3.7). Natural logarithms for all of the variables in the models were used in the estimations. In this appendix, we present estimates of the above models using a specification where all of the variables, with the exception of expected inflation, are measured by the natural logarithms of their values. These specifications must be considered since the expected inflation rate could be negative, even though the series constructed with our forecasting models contained no negative values. The results, and their interpretation, are consistent with those obtained from the complete logarithmic specifications. Thus, they will be presented without further discussion.

All of the symbols, abbreviations, and variable definitions are identical to those found in the corresponding tables of Chapter VI. Numbers in parenthesis are the t-ratios for the respective parameters.
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APPENDIX 3

SOURCES AND DESCRIPTION OF THE DATA USED TO
ESTIMATE THE MODIFIED LASS SYSTEM

The modified LASS system is written as

\[ \hat{P}_t X_t = \hat{P}_t C + (s_t - \hat{P}_t G)(B^{**} + H^** \cdot n_t^b) \]

where \( \hat{P}_t \) is a diagonal matrix containing the \( n \) observed prices of the consumption commodities and the \( m \) adjusted prices of the assets included in the system. Recall that the adjusted prices are determined from the pure, or observed, prices of the respective assets by

\[ a_t = [I_m - \hat{f}_t \cdot \hat{a}_t^c] \hat{a}_t, \]

where \( a_t \) is the \( m \)-vector of adjusted prices, \( \hat{a}_t \) is the \( m \)-vector of pure prices, and \( \hat{f}_t \) is the diagonal matrix of interest yields or negative depreciation rates. Hence, there must be three data items for each asset: the pure price, the interest rate or yield, and the end-of-period holdings. For commodities, the interest yield is zero and only data on price and expenditure is needed. We shall consider the sources for each commodity and asset individually.

Nondurables

The nondurables component index of the Consumer Price Index was used for the pure price series. The index was taken from 1975 Business Statistics and subsequent issues of the Survey of Current Business, U.S. Department of Commerce. Expenditure totals, not seasonally
adjusted, were taken from various issues of the *Survey of Current Business*.

**Services**

The services component index of the C.P.I. formed the pure price series. The source is identical to those noted in the case of nondurables. Expenditure totals, not seasonally adjusted, were also taken from the *Survey of Current Business*.

**Savings Accounts**

The C.P.I. was used as the pure price of savings accounts. A weighted average of several interest rates was constructed and used for the interest rate, or yield, on savings accounts. The asset components used in the weighting scheme included:

1. Mutual Savings Bank Passbook Deposits
2. Mutual Savings Bank Certificates of Deposit
3. Savings and Loan Passbook Deposits
4. Savings and Loan Certificates of Deposit
5. Commercial Bank Passbook Deposits
6. Other Deposits at Commercial Banks.

Amounts and interest rates for the above were taken from the Federal Reserve Board - M.I.T. - Penn (FMP) data deck. The deck was supplied by the Banking Section, Board of Governors of the Federal Reserve System.

End-of-period holdings of savings accounts were taken from the Flow of Funds Accounts maintained by the Federal Reserve System. Holdings of savings bonds issued by the federal government were added to the savings accounts total. The quantity of savings accounts was
represented by end-of-period holdings deflated by the C.P.I. This
convention is necessitated by the form of the household's budget con-
straint used in the maximization problem.

**Durables**

The durables component index of the C.P.I. was used for the pure
price series. This index was taken from the same sources listed for
nondurables and services.

A series for the seasonally unadjusted stock of durable goods was
unavailable but a seasonally adjusted series is contained in the FMP
deck. In addition, the Flow of Funds data bank reports the seasonally
unadjusted expenditures on durable goods and the depreciation of the
existing stock. A stock series was constructed by taking the 1951IV
value for the stock of durables from the FMP deck and cumulating the
seasonally unadjusted expenditures, net of depreciation, given by the
Flow of Funds data. Since the series which enters the estimation of
the equation system begins with 1968I, the effect of starting the
accumulation procedure with the seasonally adjusted stock of 1951IV
should not have an adverse influence. The quantity variable for
durables is represented by the nominal value of the stock deflated by
the durables component of the C.P.I.

Calculation of the depreciation rate for durables was patterned
after the assumption that depreciation charges are uniformly distrib-
uted over each time period. We have, for time \( t \),

\[
\ell_t = \frac{D_t}{\frac{1}{2} v_t + \frac{1}{2} v_{t-1}},
\]

where
\[ r_t = \text{depreciation rate} \]
\[ D_t = \text{depreciation charge} \]
\[ \bar{a}_t = \text{pure price of durables} \]
\[ v_t = \text{quantity of durables} \]
\[ \bar{z}_t = \text{realization price of durables}. \]

The realization price was taken to be equal to the pure price.

**Equities**

The pure price of equities was measured by Standard and Poor's Combined Index of 500 stocks, taken from various issues of the *Survey of Current Business*. This index was expressed in terms of the base period which is compatible with the bases of the price indices used for the other assets and commodities.

End-of-period holdings of security credit were added to the holdings of equities while the liability account of security credit, or security debt, was subtracted. All of these items were taken from the *Flow of Funds* data. The quantity variable for the resulting equity total was represented by the holdings deflated by the pure price adopted above.

A proxy for the interest rate, or yield, of equities was constructed using a weighted average of the dividend-price ratios for common and preferred stocks. The ratios were taken from various issues of the *Federal Reserve Bulletin*. Weights for the averaging process were given by the respective ratios of the market value of common and preferred stocks to the market value of securities listed on all exchanges. The latter total and its components are available in the *Annual Report of the Securities and Exchange Commission* and it
constitutes approximately 80% of the market value of all stocks.

**Bonds**

Six asset classifications given in the Flow of Funds data were aggregated to form the bonds category. These included:

1. Short-term marketable government securities
2. Other Treasury and agency issues, excluding savings bonds
3. State and local obligations
4. Corporate and foreign bonds
5. Open market paper
6. Money market fund shares.

The pure price of the short-term governments was measured by the inverse of one plus the quarterly yield on 3-month Treasury bills. This same procedure was applied to the sum of open market paper and fund shares, with the commercial paper rate being used for the yield after conversion to a quarterly basis. Data for the yields appear in the FMP deck. The pure prices of the remaining three components of the bonds category were taken from various issues of the Federal Reserve Bulletin.

The pure price series for the entire category was constructed by using a weighted average of the five individual price series. Ratios of the face value of each component to the total face value of all bond holdings served as the weights.

A weighted average of market yields was used as a proxy for the interest rate of the bonds category. The yields which entered the calculation correspond with the asset components listed above. These yields, along with their sources, were
1. 3-month Treasury bill rate - FMP deck
2. Long-term U.S. government bond rate - Federal Reserve Bulletin
3. Standard and Poor's domestic municipal rate (15 bonds) - Survey of Current Business
4. Moody's Aaa bond rate - FMP deck
5. Commercial paper rate - FMP deck.

A separate rate was not included for the fund shares since they were aggregated with the open market paper classification. The weights for the averaging calculation were the market values of the respective classifications, expressed as ratios of the total market value of bonds. Market values were given by the product of the pure prices and face values of each bond classification.

**Demand Deposits and Currency**

The C.P.I. was used for the pure price of money balances and end-of-period holdings were taken from the Flow of Funds data. The quantity variable was represented by real balances, with the C.P.I. being used for the deflator. Money balances earn no explicit return so an interest rate was not needed. The pure price then equals the adjusted price and the demand function in the system describes the demand for real balances.

**Debt**

The pure price of debt was measured by the C.P.I. Several debt classifications listed in the Flow of Funds data were aggregated to form the debt category. Those included were

1. Installment consumer credit
2. Other consumer credit
3. Bank loans, not elsewhere classified
4. Other loans
5. Trade credit.

The quantity variable was the sum of end-of-period holdings of the above items, deflated by the C.P.I.

A proxy for the interest rate of the debt total was constructed using several steps. Initially, the classifications of installment consumer credit and other consumer credit were combined to form one debt grouping. The remaining three classifications in the above list formed a second debt grouping. The commercial loan rate, given in the FMP deck, was used as the interest rate for the second group. Data for the interest rates of various components of the first group, which we shall term consumer credit, were available only from 1972 through 1976. The components included

1. New automobile loans at commercial banks
2. Credit card plans at commercial banks
3. Other consumer goods loans at commercial banks
4. Personal loans at commercial banks
5. New automobile loans at finance companies
6. Other consumer goods loans at finance companies
7. Personal loans at finance companies.

A weighted average of these rates, to be used for the rate on consumer credit, was constructed using the amounts of the respective components. All the series were taken from the Federal Reserve Bulletin.

The interest rates of consumer credit for the years 1968 through 1971 were obtained by an interpolation scheme based upon the relationship between the commercial loan rate and the weighted average rate
for consumer credit during 1972 through 1976. Finally, the overall interest rate of the debt category was calculated by a weighted average of the rate for the consumer credit group and the commercial loan rate.
BIBLIOGRAPHY


