INFORMATION TO USERS

This material was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or “target” for pages apparently lacking from the document photographed is “Missing Page(s)”. If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in “sectioning” the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from “photographs” if essential to the understanding of the dissertation. Silver prints of “photographs” may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

5. PLEASE NOTE: Some pages may have indistinct print. Filmed as received.

University Microfilms International
300 North Zeeb Road
Ann Arbor, Michigan 48106 USA
St. John's Road, Tyler's Green
High Wycombe, Bucks, England HP10 9HR
BIEGERT, EDWARD KARL
A STUDY OF BERYLLIUM-7: POLARIZED HELIUM-3
EXPERIMENTS, R-MATRIX ANALYSIS.

RICE UNIVERSITY, PH.D., 1978
RICE UNIVERSITY

A Study of $^7$Be: Polarized
$^3$He Experiments, R Matrix Analysis

by

Edward Karl Biegert

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

Doctor of Philosophy

THESIS DIRECTOR'S SIGNATURE:

[Signature]

HOUSTON, TEXAS

May, 1978
A Study of $^7$Be: Polarized $^3$He Experiments, R Matrix Analysis

ABSTRACT

Edward Karl Biegert

This thesis reports on a systematic study of the compound nucleus $^7$Be. An extensive multi-channel R matrix analysis of the $^7$Be system up to 8 MeV excitation energy has successfully fit all the extant data for four different reactions sharing $^7$Be as a compound nucleus. This is the first such investigation to directly fit the data instead of fitting a smoothed set of empirical phase shifts. We have calculated energy levels and their widths and compare these to previous measurements. The low energy $^6$Li destruction cross section has been determined at energies of astrophysical interest (1 eV to 100 keV) and is larger than earlier estimates by 10%. The R matrix analysis was extended to include the $^7$Li system, requiring only two additional parameters to simultaneously fit the data of both the $^7$Be and $^7$Li systems.

Extending available polarization data to 28 MeV excitation energy in $^7$Be, we have measured analyzing powers for $^3$He-$^4$He elastic scattering using a polarized $^3$He target. The R matrix analysis at lower energies provided the calibration for a $^4$He gas polarimeter to monitor the $^3$He beam polarization in measurements of analyzing powers and relative differential cross sections for the elastic channel up to 45 MeV (lab). We have made the first phase shift analysis in this energy range to incorporate polarization data as well as cross sections. The results are described qualitatively by resonating group calculations. No positive parity levels in $^7$Be are observed.
TABLE OF CONTENTS

Outline
List of Appendices
List of Tables
List of Figures
Text
Appendices
Bibliography
A Study of $^7$Be: Polarized $^3$He Experiments, R Matrix Analysis

I. Introduction
   A. Organization of this Thesis 2
   B. Why Polarized $^3$He?
      1. Spin Dependent Forces in Nuclear Physics 5
      2. Elastic Scattering 7
      3. Few Nucleon Systems 9
      4. Transfer Reactions 11

II. $^7$Be System: Low Energies
   A. Introduction 12b
   B. R Matrix Analysis
      1. Formalism 14
      2. Sum Rules 20
      3. Boundary Conditions and Background Levels 23
   C. Energy Dependent Analysis
      1. Methodology 27
      2. Data Selection 33
      3. Data Normalizations 36
      4. Low Energy $^6$Li Destruction Cross Section 38
      5. Analysis and Discussion 44
   D. Mass 7 Charge Independent Analysis 46
   E. Summary 50
III. $^{7}$Be System: Intermediate Energies

A. Introduction 52

B. Polarized $^{3}$He Target Experiment
1. Experimental Details: Polarized Target 53
2. Results of Polarized Target Experiments 54

C. Polarized $^{3}$He Beam
1. Experimental Details
   a. Polarized Beam 56
   b. Data Handling 58
   c. Normalization of Data 60
      1) Absolute Calibration Points 62
      2) Particle Identification 64
      3) Polarimeter Design 65
      4) Polarimeter Calibration 67
      5) Polarimeter Operation 68
      6) Polarimeter Data Analysis 68
      7) Crosstalk 69
2. Results of Polarized Beam Experiments 70

D. Phase Shift Analysis 71
1. Simplex Search Routine 73
2. Results and Discussion 74

IV. Summary: Polarized $^{3}$He and the Compound Nucleus $^{7}$Be 76
APPENDICES

A. Topics Related to the R Matrix Analysis

1. Data Sets and Normalizations
2. R Matrix Parameters: $^7\text{Be}$
3. R Matrix Parameters: Mass 7
4. Phase Shifts: $^7\text{Be}$
5. Figures: Fit to $^7\text{Be}$ Data
6. Figures: Fit to Mass 7 Data

B. Topics Related to the Experiments

1. Coincidence Geometry
2. Polarimeter and Crosstalk
3. Calculation of Relative Cross Sections
4. Empirical Phase Shifts
## LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>S(E) Values</td>
<td>43</td>
</tr>
<tr>
<td>Level Parameters: $^7$Be</td>
<td>51b</td>
</tr>
<tr>
<td>Polarized Target Analyzing Powers</td>
<td>54a</td>
</tr>
<tr>
<td>Normalizations - 38 MeV $^3$He Data</td>
<td>69</td>
</tr>
<tr>
<td>Polarized Beam Analyzing Powers and Cross Sections</td>
<td>70a</td>
</tr>
<tr>
<td>Circle of Normalizations</td>
<td>A1.1</td>
</tr>
<tr>
<td>R Matrix Normalizations: $^7$Be</td>
<td>A1.4</td>
</tr>
<tr>
<td>R Matrix Channels: $^7$Be</td>
<td>A2</td>
</tr>
<tr>
<td>R Matrix Parameters: $^7$Be</td>
<td>A2</td>
</tr>
<tr>
<td>R Matrix Parameters: Mass 7</td>
<td>A3</td>
</tr>
<tr>
<td>R matrix Phase Shifts: $^7$Be</td>
<td>A4</td>
</tr>
<tr>
<td>Polarimeter Specifications</td>
<td>B2.11a</td>
</tr>
<tr>
<td>Cross Section Formulae</td>
<td>B2.12</td>
</tr>
<tr>
<td>Empirical Phase Shifts: $^7$Be</td>
<td>B4</td>
</tr>
<tr>
<td>NUMBER</td>
<td>CAPTION</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>2.0</td>
<td>7\textsuperscript{Be} Term Diagram</td>
</tr>
<tr>
<td>2.0a</td>
<td>R Matrix Formalism</td>
</tr>
<tr>
<td>2.1</td>
<td>Resonant Phase Shifts</td>
</tr>
<tr>
<td>2.2</td>
<td>R Matrix Elements</td>
</tr>
<tr>
<td>2.3</td>
<td>Data Sets and 7\textsuperscript{Be} Levels</td>
</tr>
<tr>
<td>2.4</td>
<td>S(E) for \textsuperscript{6}Li(p,\textsuperscript{3}He)\textsuperscript{4}He</td>
</tr>
<tr>
<td>2.5</td>
<td>Contour Plots - Polarization</td>
</tr>
<tr>
<td>2.6</td>
<td>n-\textsuperscript{6}Li 260 KeV Resonance</td>
</tr>
<tr>
<td>2.7a</td>
<td>R Matrix Levels of 7\textsuperscript{Be}</td>
</tr>
<tr>
<td>2.7b</td>
<td>Summary of 7\textsuperscript{Be} Level Parameters</td>
</tr>
<tr>
<td>3.1</td>
<td>Polarized Target Analyzing Powers</td>
</tr>
<tr>
<td>3.2</td>
<td>Excitation Function 47°</td>
</tr>
<tr>
<td>3.3</td>
<td>Angular Distributions</td>
</tr>
<tr>
<td>3.4</td>
<td>Polarized Target Spectra</td>
</tr>
<tr>
<td>3.5</td>
<td>Polarized Beam Experiment</td>
</tr>
<tr>
<td>3.6</td>
<td>Polarized Beam Spectra</td>
</tr>
<tr>
<td>3.7</td>
<td>Scattering Geometry</td>
</tr>
<tr>
<td>3.8</td>
<td>Analyzing Powers &amp; Cross Sections</td>
</tr>
<tr>
<td>3.9</td>
<td>Resonating Group Phase Shifts</td>
</tr>
<tr>
<td>A1.1</td>
<td>Circle of Normalizations</td>
</tr>
<tr>
<td>A5.1</td>
<td>Fit to the 7\textsuperscript{Be} Data</td>
</tr>
<tr>
<td>A6.1</td>
<td>Fit to the Mass 7 Data</td>
</tr>
<tr>
<td>B1.0</td>
<td>Coincidence Geometry</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>CAPTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1.1</td>
<td>SAEF Singles vs. Coincidence</td>
<td>B1.6a</td>
</tr>
<tr>
<td>B1.2</td>
<td>SAEF and Beam Spread</td>
<td>B1.6b</td>
</tr>
<tr>
<td>B1.3</td>
<td>Angular Error - One Detector</td>
<td>B1.7a</td>
</tr>
<tr>
<td>B1.4</td>
<td>Angular Shift in Polarimeter</td>
<td>B1.8a</td>
</tr>
<tr>
<td>B1.5</td>
<td>SAEF and Longitudinal Errors</td>
<td>B1.8b</td>
</tr>
<tr>
<td>B2.1</td>
<td>Polarimeter Statistics</td>
<td>B2.11a</td>
</tr>
<tr>
<td>B2.2</td>
<td>Polarimeter Spectra</td>
<td>B2.11b</td>
</tr>
<tr>
<td>B2.3</td>
<td>Fractional Error in Asymmetry</td>
<td>B2.11c</td>
</tr>
<tr>
<td>B2.4</td>
<td>Average Polarimeter Analyzing Power</td>
<td>B2.11d</td>
</tr>
</tbody>
</table>
"It is reasonable," he admitted. "And therefore I suspect it. Whenever you find a perfectly reasonable explanation of anything in nature or human conduct, look for something else. Things in the real universe don't all fit together like the pieces of a child's puzzle."

John Taine (Eric Temple Bell)

The Time Stream
1. INTRODUCTION

1A. ORGANIZATION OF THIS THESIS

We have performed several experiments with polarized 3He beams and targets in attempts to broaden our understanding of polarization phenomena in the elastic scattering of few nucleon systems. In the introduction to this thesis, we indicate a few of the reasons for using polarized particles in nuclear scattering experiments, and especially for using polarized 3He.

The simplest case with which to begin this study is the scattering of the spin $1/2$ 3He from a spin zero nucleus, and we measured target analyzing powers for $4\text{He}(3\text{He},3\text{He})4\text{He}$ elastic scattering for effective incident 3He energies from 11 MeV to 35 MeV (Ba75). It was originally planned to incorporate these data in a comprehensive R matrix analysis of the 7Be system. We began to work on this ambitious project, but the complexity of the problem forced us to restrict the R matrix analysis to bombarding energies below 11 MeV. The section of this thesis following the introduction describes our R matrix study (Bi77) of the 7Be system for incident 3He energies below 11 MeV. Energy levels and widths of the compound nucleus, scattering phase shifts, and normalizations
of the existing data are generated by this analysis. In addition, a better estimate of the $^6$Li destruction cross section factor for energies of astrophysical importance (up to 50 keV) is calculated. Using the powerful R matrix analysis techniques, we have made a charge independent study of the mass 7 system incorporating this $^7$Be analysis and G.M. Hale's similar analysis of the $^7$Li system (Ha76). The results are remarkably simple and require only two more Coulomb shift parameters to fit the combined data of the two mirror nuclei ($^7$Be and $^7$Li) than are needed to fit the data of one system alone.

Between 11 MeV and 35 MeV, polarization data for the $^{4}$He$(^3$He,$^3$He)$^{4}$He reaction have been made with a polarized $^3$He target (this experiment) and a polarized $^3$He beam (Birmingham, Ka76). My phase shift analysis reported in this thesis covering this energy range is the first to incorporate polarization data. For $^3$He bombarding energies between 35 and 45 MeV, we have measured elastic scattering analyzing powers (Bi76b) and relative differential cross sections with a polarized $^3$He beam (Ma76) at the Texas A and M Variable Energy Cyclotron (TAMVEC). Our data and phase shift analysis are compared to resonating group calculations in the third section of this thesis. The third section also
includes the design, characteristics, and operation of a polarimeter (To77) we had to build to monitor the beam polarization. It was also necessary to renormalize our earlier data (Bi76b), which were taken with an incorrectly calibrated polarimeter (Ju76).

The fourth section of this thesis is a summary of what we have learned from analysis of our data.

Several important topics are relegated to the appendices. Appendix A contains all the parameters, results, and fits to the data of our R matrix analysis. Only a few randomly chosen figures are shown (21 pages) to illustrate the scope and complexity of the analysis. The appendix on data normalizations contains an interesting example of the confusion abounding in the literature. Errors we have discovered in the literature are reported. Phase shifts generated by the R matrix analysis are tabulated in an appendix for the use of other investigators who may want to compare our results with their models.

Appendix B contains topics related to our new polarimeter. The oft neglected effects of geometry errors and coincidence requirements on gas target detector systems are discussed in detail. Another portion of this appendix describes the polarimeter and the problem of crosstalk (overlap of particle peaks). My formulae
for the extraction of relative differential cross sections are presented. Finally, the results of our higher energy phase shift analysis are tabulated and fits to the data are shown.

1B. WHY POLARIZED 3He?

1B.1 SPIN DEPENDENT FORCES IN NUCLEAR PHYSICS

The discovery by Oxley (Ox53) that high energy protons are polarized after elastic scattering from complex nuclei gave direct experimental evidence for a noncentral spin dependent part of the nucleon-nucleus interaction. Since then, polarization effects have been studied in a large number of investigations. The oscillatory behaviour of the polarization in angular distributions with corresponding oscillations in the elastic scattering differential cross sections and the large total reaction cross sections intimate that the polarization is associated with diffraction effects due to shadow scattering from the surface of the interaction region.

Fermi gave a lucid description of the polarization phenomenon in Fe55. He added a noncentral spin orbit coupling term to the central part of the nuclear potential.
\[ V(r) = V_c(r) + V_{s0}(r) \vec{S} \cdot \vec{L} \]

\( \vec{S} \) and \( \vec{L} \) are the spin and orbital angular momentum operators of the projectile. The spin orbit potential \( V_{s0}(r) \) is assumed to be real and negative. Fermi argued that the potential of a projectile with spin \( \vec{S} \) inside a uniform nucleon distribution cannot depend on \( \vec{L} = \vec{r} \times \vec{p} \) since the projectile cannot locate the center of the nucleus to determine its angular momentum. However, near the nuclear surface the gradient of the nucleon density supplies a preferred direction \( \vec{\nabla} f(r) \). The easiest way to construct an \( \vec{L} \) and \( \vec{S} \) dependent scalar potential is the Thomas form factor of the spin orbit well shape:

\[ V_{s0}(r) \vec{S} \cdot \vec{L} = \vec{S} \cdot \vec{\nabla} f(r) \times \vec{p} = \frac{1}{r} \frac{df(r)}{dr} \vec{S} \cdot \vec{L} \]

Thus the noncentral interaction is confined to the surface of the interaction region. Particles with different spin orientations will see different potentials at slightly different radii. Interference between the scattered waves causes a left-right asymmetry in the scattered intensity. For \( V_{s0} \) negative, spin up particles
are scattered preferentially to the left.

The scattering of 3He nuclei is dominated by strong surface absorption even more pronounced than that of alpha particles (4He) because of the smaller binding energy of the 3He nucleus. This makes 3He ideally suited as a nuclear surface probe. If in addition the spin 1/2 3He nuclei are polarized, a wealth of information can be obtained about the spin orbit interaction near the nuclear surface. Polarization studies with 3He are currently of interest and can be classified into three main groups: elastic scattering, transfer reactions, and light nucleus studies.

1B.2 ELASTIC SCATTERING

Elastic scattering of polarized 3He from a broad range of target nuclei at various energies is important to help to determine the spin contribution to the nuclear potential. A nuclear spin orbit \((I \cdot L)\) term in the optical model potential has been investigated by Taylor (Ta65) and a spin-spin term in the 3He optical potential by Khan (Kh66) and Hafele (Ha69). Cross section data alone are too insensitive to the spin orbit potential to permit an unambiguous determination of its depth and shape.
Even for the case of nucleons, which have been the subjects of extensive study, optical model analyses have shown that it is not generally possible to determine even the central potential parameters uniquely from the cross section data. Due to the surface nature of the interaction, the ambiguities in the helium optical potentials are even more pronounced. Polarization measurements have helped to resolve these ambiguities (Fu73). In a study with polarized tritons, the Los Alamos group has reported spin orbit potentials for mass three projectiles considerably higher than those predicted from a folding model (Ha75). Studies with 3He also raise the question of how the spin orbit term of a composite nucleus can be larger than the sum of the spin orbit terms of its constituents.

The semiclassical strong absorption theory (SAM) of Frahn (Fr64) for the elastic scattering of spin 1/2 particles can be tested with a polarized 3He beam. The theory predicts polarizations and spin orbit potential depths where the assumptions of the model are valid, i.e., there is not too small a number of participating partial waves and there is sufficiently strong absorption. Our experimental values of the polarization in 3He + 12C elastic scattering at 45 MeV (To76) can be compared to
strong absorption model predictions in this energy domain where the model should yield good results \((\text{Fr65})\).

DWBA analyses of stripping reactions may also benefit by the use of \(^3\text{He}\) elastic scattering polarization data to improve the wavefunctions generated by solving the Schroedinger equation. Of course, here one really wants to prepare a target in an excited state to perform a scattering experiment \((\text{Va76})\).

Polarization measurements of elastic scattering from light nuclei will be extremely useful in selecting among different sets of phase shifts which reproduce the cross section data but predict different values of the polarization. Polarized \(^3\text{He}\) beams now make it relatively easy to search the regions predicted by Plattner and Bacher \((\text{Pl71})\) for the points where the analyzing power reaches unity, its maximum value. These points provide absolute calibration standards of spin 1/2 polarization.

1B.3 FEW NUCLEON SYSTEMS

The light nucleus measurements are also of interest because of the increasing capabilities of theorists to make ab initio calculations. The method of the resonating group structure \((\text{Wh37})\) and \(R\) matrix theory \((\text{Wi47})\) have
been used to calculate observables for scattering systems involving light nuclei. Comparison with experimental data for several systems over a broad range of energies suggests that these methods yield satisfactory results.

Resonating group calculations have been found to be useful in providing starting points for phase shift analyses (Bi76), predicting the presence of resonance levels in light nuclei, obtaining a better understanding of the importance of the Pauli principle, and in constructing effective two-body potentials between clusters of nucleons. Recent calculations employing an effective nucleon-nucleon spin orbit interaction have resulted in qualitative agreement between observed and predicted values of the polarization in $^{3}$He + $^{4}$He scattering (Ba75).

$R$ matrix analysis techniques have already resolved several controversies in few nucleon systems (see Bi77 and Ca75). The global nature of this investigative technique involves a comprehensive study of the entire problem; it is not based only on data directly related to the pertinent reaction. It is possible to calculate accurate estimates of the errors of all predicted observables as well as of the model parameters, energy levels, and their reduced widths. In these studies,
polarization data are at the very least tantamount to
cross section data for the few nucleon systems. In fact,
any model is adequately constrained if a complete
experiment has been made, that is, if all possible types
of observables have been measured.

1B.4 TRANSFER REACTIONS

The improvement of the understanding of reaction
mechanisms leading to appropriate models and theories
and the application of the j-dependence of the analyzing
power to nuclear spectroscopy are two important reasons
for studying 3He induced reactions. The one nucleon
transfer reactions (3He,d) and (3He,4He) are analogous
to the (d,n) stripping reaction and the pickup reaction
(p,d), respectively. The two nucleon transfer reactions
are also of interest. For example, the (3He,p) and
(3He,n) reactions lead to proton rich residual nuclei.
The use of 3He particles is also advantageous in the
formation of excited states in the residual nuclei
(Ro75).

Strong j-dependent effects in the analyzing power
of the (3He,d) and (3He,4He) reactions are expected since
the j-dependence of the (d,p) and (d,t) reactions are
associated with spin orbit forces acting on a nucleon in the bound state. These 3He induced reactions have been studied for a few nuclei with striking results. Roman (Ro75) observed large, j-dependent analyzing powers for 33 MeV 3He's. These studies, when coupled with DWBA analyses, will be valuable spectroscopic tools.
Figure 2.0  
$^7$Be Term Diagram
2. 7Be SYSTEM: LOW ENERGIES

2A. INTRODUCTION

Previous analyses of reaction cross sections and polarizations in terms of resonances in the 7Be compound nucleus have been less than successful. In 1956 Marion (Ma56) measured the excitation function at four angles and deduced the angular distributions for the 6Li(p,3He)4He reaction. Fitting the angular distributions with compound nucleus theory, he postulated a 3/2+ and a 5/2- level at excitation energies of 6.5 MeV and 7.19 MeV, respectively. Five years later in a similar experiment on the same reaction, Khanh (Kh61) obtained results in agreement with Marion's levels. The next year, however, McCray's elastic scattering of protons by 6Li (Mc62) could not be fit with the 3/2+ level. Instead, McCray found his data could be fit by a 5/2- level at 7.58 MeV excitation and a broad interfering 1/2+ level at 8 MeV above ground state. The following year, Jeronymo's 6Li(p,3He)4He reaction data (Je63) were fit by Marion's levels (3/2+ at 6.5 MeV and 5/2- at 7.8 MeV). The elastic scattering data published concurrently with Jeronymo's data [Tombrello and Parker (To63), 4He(3He,3He)4He; Harrison and Whitehead (Ha63), 6Li(p,p)6Li] did not lend support to any positive parity level assignments. To
complicate matters, Beaumovieille's 1963 analysis of the
data from his 6Li(p,3He)4He experiment (Be63) was
incompatible with Marion's level assignments (3/2+,5/2-)
but was in agreement with McCray's (5/2-,1/2+). More
recent experiments do not provide confirmation of the
existence of positive parity levels: Spiger and Tombrello
(1967-Sp67), 4He(3He,3He)4He and 4He(3He,p)6Li; and the
1968 6Li(p,3He)4He analyzing power measurements of Brown
and Petitjean (Br68). Two particle pick-up theory was
successfully used to analyze the 1974 measurement of the
6Li(p,3He)4He cross sections of Johnston and Sargood
Jo74) without recourse to postulating positive parity
levels. However, just this year positive parity states
in 7Be have again been postulated, this time at 17 MeV
excitation (Sn77) by workers at Duke.

In an attempt to simultaneously fit all the previous
data with a single set of levels in the compound nucleus
7Be, we have undertaken an R matrix analysis of the 7Be
and mass 7 systems at low energies (below 8 MeV excitation).
This covers the energy range of dispute and avoids the
complications involved with many-particle final states.

Other experimenters have provided polarized proton
and polarized 3He data in the 7Be system at low energies.
Polarization data for 3He bombarding energies below
11 MeV have been taken using polarized 3He targets (Ha70, Bo72, and Ha72) and in double scattering experiments (Ar69 and Mc70) to aid in searches for the $A_y = 1$ points predicted by Plattner and Bacher (P171). We have analyzed these and all previous data with our R matrix search. Our analysis produces the set of energy levels of 7Be and their widths that best describe all the extant data for four reactions leading to the 7Be compound nucleus, not just a small sampling of data.

As a result of this R matrix analysis, we have made absolute normalizations of the scattering experiments in this energy range and compare them to those suggested by Harrison (Ha67). In addition, we have suitably normalized relative measurements for low energy scattering of protons by 6Li. These data allow us to make the best determination of the 6Li destruction cross section factor.

2B. R MATRIX ANALYSIS

2B.1 FORMALISM

The overwhelming amount of information already collected in the scattering and reaction processes involving very few nucleons still seems to be only
marginally sufficient to describe the complex behaviour of these systems. A phenomenological understanding of these processes appears to call for an approach with two main features (Do75). First, all reaction channels and complementary experiments should be described simultaneously. Second, an energy dependent parameterization should be used, if only because "complete experiments" have not been made at individual energies.

The simultaneous description of all reaction channels allows one to use the important unitarity (conservation of probability) principle of the collision matrix $S$. The $S$ matrix operates in spin-angle space, transforming the spin-angle vector of the unperturbed outgoing wave into the spin-angle vector of the actual outgoing wave. (Spin-angle space is the composite space formed by combining configuration space and spinor space.) Time reversal invariance means the matrix can be made symmetric, while conservation of angular momentum means that the $S$ matrix can be written as a block diagonal matrix with a block for each value of $J$ that is filled out with complex matrix elements. Parity conservation subdivides this into a separate block for each $J^\pi$. Unitarity places restrictions on the complex matrix elements so that they are not all independent. In fact, the $N(N + 1)/2$ complex
elements for each \( N \times N \) block of the general matrix are equivalent to half this number of independent parameters for a unitary matrix. It is often convenient to make this reduction by using the real symmetric reactance matrix \( K \), \( iK = (S+1)/(S-1) \). The \( K \) matrix then completely describes the scattering process (see B152). The \( K \) and \( S \) matrices can be written in terms of the energy dependent \( R \) matrix.

The \( R \) matrix formalism of Wigner and Eisenbud (Wi47) as developed by Lane and Thomas (La58) parameterizes the energy dependence of a unitary description of a collision process. In this theory, configuration space is divided into two parts by a multidimensional surface \( S \): an inner region within which the separations of all nucleons are of the order of nuclear dimensions and the nuclear interactions occur, and an outer region where it is assumed only two-body states are present and where only long range nonnuclear forces (Coulomb and centrifugal) are important. If \( S_c \) is the channel surface for a pair of particles \( c \), the totality of all such surfaces is

\[
S = \sum_c S_c
\]

Sachs (Sa53) suggests that the surface \( S \) may be visualized
as a polyhedron, each hypersurface of which is a channel entrance. The channels themselves are then cylinders normal to these planes. The surface $S_c$ is defined by a channel radius $R_c$ for the compound nucleus in each channel or alternative reaction $c$. For values of the channel coordinate $r_c < R_c$ corresponding to the inner region of configuration space, the nucleus and particle combine into a compound system about whose dynamical behaviour and wavefunction we know little. In the outer region $r_c > R_c$, the wavefunction describes the motion of the separated target and projectile under the influence of known nonnuclear forces. The value of the wavefunction and its derivative at each channel entrance to the surface at $r_c = R_c$ defines the complete behaviour of the wavefunction in each channel. Thus, once the wavefunction is known at the surface $S$, the requirement of continuity across the surface completely determines the wavefunction everywhere in the open channels outside $S$. With this knowledge of the stationary states of the system, the elements of the collision matrix $S_{ij} = \exp(2i\delta_{ij})$ can be readily obtained. An essential condition for this method to be realistic is that the surface $S$ between the two regions be at small enough distances between particle pairs that the Coulomb barrier and phase space effects
Figure 2.0a  R MATRIX FORMALISM
occur mainly in the external region (Do75).

There are several derivations for the partial fraction expansion of the derivative matrix $R$. One relies on a causality principle (Ka53), another on a variational principle (Ko48 and Ja51), and several match logarithmic derivatives of the wavefunctions on the surface $S$ and then apply Green's theorem (La58 and Wi47). These are discussed in detail in Lane and Thomas' review article (La58). Basically, a complete set of orthonormal eigenfunctions $\chi_{\lambda}$ of the Hamiltonian $H$ are obtained for real energy eigenvalues $E_{\lambda}$ by imposing real boundary conditions on the logarithmic derivative of the wavefunction at the channel radii $R_c$. The projection of an eigenfunction $\chi_{\lambda}$ on a channel surface $S_c$ is designated a reduced width amplitude, denoted by $\gamma_{\lambda c}$, and is a real transition strength. If $\mu_c$ is the reduced mass for channel $c$,

$$\frac{2\mu_c}{\hbar^2} \gamma_{\lambda c}^2$$

is the probability that the stationary state $\chi_{\lambda}$ consists of the pair $c$ on the surface $S$. $\gamma_{\lambda c}^2$ is called the reduced width of the level. The partial
widths $\Gamma_{\lambda_c}$ are related to the $\chi_{\lambda_c}$ by

$$\Gamma_{\lambda_c} = 2 \chi_{\lambda_c}^2 P_c$$

where for charged particle pairs $P_c$ is the Coulomb penetration factor. In the interaction region $r_c < R_c$, the eigenfunctions are complete and the actual scattering wavefunction $\psi_c$ can be expanded in terms of them. If

$$H \psi_c = E \psi_c,$$

then

$$\psi_c = \sum_{\lambda} A_{\lambda_c} \chi_{\lambda_c}.$$

The expansion coefficients $A_{\lambda_c}$ are found to be

$$A_{\lambda_c} = \frac{\chi_{\lambda_c}}{(E_{\lambda} - E)}$$

where

$$\chi_{\lambda_c} = \int \chi_{\lambda_c}^* \nabla \psi_c \cdot \sigma S$$

The integral is to be extended over the boundary surface $S$ where $\text{grad} \psi_c$ is the normal derivative. Since $\chi_{\lambda_c}$ depends only on the normal derivative of $\psi_c$ on $S$, there is a relation between the value and normal derivative of $\psi_c$ on $S$

$$\psi_c = \sum_{\lambda} \frac{\chi_{\lambda_c}}{E_{\lambda} - E} \chi_{\lambda_c}$$

It can be shown that if both $\psi_c$ and $\text{grad} \psi_c$ are
expanded in an orthogonal system, the Green's function which relates the value of the wavefunction in the internal region to its derivative on the surface is the derivative or R matrix

\[ R_{cc} = \sum_{\lambda} \frac{\gamma_{\lambda c} \gamma_{\lambda c}}{E_{\lambda} - E} \]

2B.2 SUM RULES

A "compound nucleus" is formed by combining the projectile nucleus with the target nucleus. If the energy of the incident nucleus is rapidly redistributed among the nucleons of the compound nucleus, no single nucleon has sufficient energy to escape. When the probability is small for a statistical fluctuation in the energy distribution to give enough energy to a nucleon so that it is emitted from the compound nucleus, the compound nucleus may have time to undergo a radiative transition to a bound state. Elastic and inelastic scattering occur when the projectiles' energy dissipates more slowly so that a nucleon or nucleons are re-emitted.

The partial width \( \Gamma_{\lambda} \) provides a direct measure of the transition probability for the emission of a particle from a virtual state \( \lambda \) in the compound nucleus.
These energy states \( E_\lambda \) are not stationary states and have an associated energy uncertainty \( \Gamma_\lambda \), inversely related to the lifetime \( t_\lambda \) of the compound nucleus state by

\[
\Gamma_\lambda = \frac{\eta}{t_\lambda}
\]

If several types of particles may be emitted from a given state, the total width \( \Gamma_\lambda \) is the sum of the partial widths \( \Gamma_{\lambda c} \) for each open channel \( c \),

\[
\Gamma_\lambda = \sum_c \Gamma_{\lambda c}
\]

Furthermore, normalization of the stationary state wavefunctions cause the partial widths to satisfy a sum rule (Sa53)

\[
\sum_c \frac{\Gamma_{\lambda c}}{\Gamma_p^g (E_\lambda)} = 1
\]

The index \( c \) refers to every state of excitation and of relative orbital angular momentum \( l \) of a given pair of fragments (channel). The energy dependent width for the scattering cross section with a broad resonance on a single particle (pure) potential is \( \Gamma_p^g \).

The reduced widths are the quantities of real
interest, however. The relative signs of the reduced widths produce interference and cancellations between submatrices corresponding to various resonances, hence polarization phenomena are particularly sensitive to them. The reduced widths are often normalized by the Wigner limit (Te52), since this ratio gives some sort of estimate of the degree to which the wavefunctions are described by the independent particle model. Since the interior wavefunctions are normalized,

\[ \sum_c \left[ \frac{2\mu_c}{\hbar^2} \right] \gamma_{\lambda c}^2 = \int_S \left[ \sum_{\text{spin}} \chi_\lambda^* \chi_\lambda \right] dS \]

The R function, of course, has an infinite number of poles at the eigenenergies \( E_\lambda \) so that a complete description requires knowledge of the infinitely many parameters \( \gamma_{\lambda c} \). Fortunately the R function is often dominated by a single isolated pole and the Breit-Wigner one-level dispersion formula may be used. In this one-channel approximation, the probability density \( \chi_\lambda^* \chi_\lambda \) has the same density on \( S_c \) as it does throughout the volume \( S \). The integral is then the ratio of the area \( S_c \) to the volume of \( S \), or
\[ \int \left[ \sum_{\text{spin}} \chi^*_\lambda \chi_\lambda \right] dS = \frac{4\pi R_c^2}{\sqrt{3} \pi R_c^3} = \frac{3}{R_c} \]

The "first sum rule" of Teichmann and Wigner (Te52) becomes:

\[ \sum_c \chi^2_{\lambda c} = \frac{3\hbar^2}{2\mu_c R_c} \equiv \theta^2 \]

If the wavefunction \( \chi_\lambda \) can be written as a product of one-particle wavefunctions (independent particle model), the integral vanishes for all but one channel \( c \), and the Wigner limit \( \theta^2 \) is reached.

2B.3 BOUNDARY CONDITIONS AND BACKGROUND LEVELS

In the Wigner-Eisenbud \( R \) matrix theory (Wi47), eigenfunctions of the Hamiltonian are obtained for real energy eigenvalues \( E_\lambda \) by imposing real boundary conditions \( B_c \) on the homogeneous logarithmic derivative of the radial wavefunction at the channel radii \( R_c \). The choice of boundary conditions is nonrestrictive, that is, all of the observables are independent of the boundary condition. Lane and Thomas (La58) and Barker (Ba72)
show that once a set of R matrix parameters have been
found with a specified set of boundary conditions, the
observables can be reproduced exactly for a different
set of boundary conditions by appropriate transformations
of the R matrix parameters. Three common choices are
\( B_c = 0, \quad B_c = -1, \) and \( B_c = S(E) \), where \( S(E) \) is the shift
function of Wigner and Eisenbud (Wi47). In this analysis
all three choices were used.

The boundary condition \( B_c = 0 \) is useful when the
transformation of R matrix parameters to sets of parameters
with different boundary conditions is to be made. With
\( B_c = 0 \), the transformations are in particularly simple
forms, as shown in La58 and Ba72. Computer codes now
exist (Ha76) to transform one set of R matrix parameters
into another set with specified boundary conditions.
Care must be taken when interpreting the correlation
between the transformed parameters and the original R
matrix parameters, however, since the new set of boundary
conditions makes the calculation of observables more
sensitive to small changes in different parameters. It
is also possible to take a given set of R matrix parameters,
channel radii, and boundary conditions and transform
them directly into physically observable resonance level
energies and partial widths of the compound nucleus.
A common choice of boundary conditions when only the centrifugal potential is present in the asymptotic (outer) region (the particles are uncharged) is \( B_c = -l \), where \( l \) is the relative orbital angular momentum of the two particles in channel \( c \). This choice is made so that certain factors in the expansion of the Breit-Wigner resonance formula in terms of \( R \) matrix parameters have simple forms. (See La58.) For a square well potential with no Coulomb interaction, this particular choice of boundary conditions makes the reduced level widths independent of the relative orbital angular momentum. The reduced level widths then correspond to the single particle widths. The selection \( B_c = -l \) is sometimes made to determine if there is a simple model capable of describing the behaviour of the reduced level widths. This was the case in a recent \( R \) matrix analysis of the \( p \)-alpha and \( n \)-alpha systems, when the choice of \( B_c = -l \) as boundary conditions showed the reduced level widths to depend only slightly on \( l \) (Ha76).

Often if the boundary conditions are properly selected, at certain resonant energies only a few terms in the expansion of the scattering wavefunction may be large. One then hopes to identify the \( E_\lambda \) with physical resonances of the system. Since the energy dependence
of the widths and resonant energy of a resonance can be accounted for mostly by the behaviour of the two-particle wavefunctions in the external Coulomb and centrifugal potentials, it is common to define a shift function $S(E)$ which depends only on Coulomb wavefunctions (Wi47 and La58) evaluated at the channel radius. If the boundary conditions are chosen so that the level shift

$$
\Delta_{\lambda c} = -(S-B_c) \chi_{\lambda c}^2
$$

vanishes at the peak of a resonance, $B_c = S(E_r)$, then often one can identify $E_r = E_\lambda$, where $E_r$ is the resonant energy.

$$
E_r = E_\lambda + \Delta_{\lambda c}
$$

The level shift represents the difference between the energies associated with the boundary conditions and that which results in a resonance of the reaction. Another choice often made is $B_c = S(0)$, which reduces to $B_c = -i\lambda$ if there is no Coulomb field.

For example, in this analysis we observe two resonances at $E_r = E_\lambda$ by the $5/2^-$ state in 7Be (Bi77). Figure 2.1 shows the characteristic $R$ matrix parameters and phase shifts from the analysis of $3\text{He} + 4\text{He}$ and $p + 6\text{Li}$ scattering and reactions. Here the $E_\lambda$ 's and $\chi_{\lambda}^2$'s correspond to two physical resonances in 7Be (Sp67). Compare to figure 2.0 for the energy levels and resonances.
Figure 2.1  Resonant Phase Shifts in $^{7}$Be
2C. ENERGY DEPENDENT ANALYSIS

2C.1 METHODOLOGY

The derivative R matrix is the symmetric matrix of inverse logarithmic derivatives with respect to radial separations of the various two-particle states. The derivatives are evaluated at the channel surfaces $S_c$ at specified values of the radial separations $R_c$. These channel radii $R_c$ can be among the parameters to be determined and can be different for different channels. The term channel refers to a distinct two-particle state with specific particle identities, relative orbital angular momentum $l$, total spin $S$, and total angular momentum $J$. Because of conservation of angular momentum and parity, the R matrix separates into submatrices of given $J$ and parity $\pi$. In this analysis, for example, the R matrix has dimension 16 (Appendix A2 lists the 16 entrance and exit channels) with 9 blocks on the diagonal. Refer to figure 2.2. All other off-diagonal elements are zero.

A small group at Los Alamos Scientific Laboratory has been working on R matrix analyses of light nuclei
R Matrix element:

\[ R_{cc} = \sum_{\lambda} \frac{\chi_{\lambda c'} \chi_{\lambda c}}{E_{\lambda} - E} \]

Figure 2.2 R Matrix Elements

The R matrix is Block diagonal.
for several years. Don Dodder, Gerry Hale, and Kay Witte have developed the largest non-weapons computer code at Los Alamos (and very probably the largest non-weapons scientific research code in the country). Their prodigious computer program EDA (Energy Dependent Analysis) handles any possible observable in spin space for any number of two-particle channels for particles of any spin (Do77). It is an energy dependent extension of EIA, their general purpose single energy multiple channel partial wave analysis program (Do77). In EDA the submatrices (one for each J and parity) of the R matrix may be expressed in either of two forms. In this analysis, the familiar form (La58)

\[ R_{c'c} = \sum_{\lambda} \frac{\chi_{\lambda c'} \chi_{\lambda c}}{E_{\lambda} - E} \]

was used. Here c' and c refer to initial and final channels, respectively, and \( \lambda \) is a label specifying a particular energy level. Any number of \( E_{\lambda} \)'s for a given submatrix are allowed in EDA. Within a submatrix, transitions occur between channels whenever both channels have nonzero reduced width amplitudes \( \gamma_{\lambda c} \neq 0 \) for a given value of \( \lambda \). Such transitions can involve not only changes of two-particle states, but also changes
of total spin S and orbital angular momentum. For example, Appendix A2 contains a list of allowed transitions in this analysis.

EDA is a Hessian matrix minimization procedure capable of making global searches on R matrix parameters (channel radii \( R_c \), the \( \gamma_{\lambda c} \)'s, and the \( E_\lambda \)'s) and normalization parameters for data covering a system's whole observable space over a large energy range. Normalization factors are used when there is a set of data that has an overall normalization as well as relative errors. An extreme example of this is the case where only relative values of an observable have been measured, as in Bo60, Kh61, and Be63, as well as our own relative differential cross sections at higher energies. Any of the parameters can be fixed at definite values at any time during the analysis, and linear constraints can be imposed on sets of parameters. Provisions are made for varying the values of the energy and angle for the observables in instances where the energy and angular resolution are not well known. The limitations on the total number of data, sizes of the matrices, etc., are imposed by the total storage capacity of the computer system at Los Alamos.

The search routine is described in Do76. It uses
analytic first derivatives of chi-square with respect to the parameters in a succession of steps that build up an approximation to the Hessian matrix, $\mathbf{H}$. The Hessian (curvature) matrix of second derivatives of chi-square with respect to the parameters $a$ is

$$H_{ij} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

A more functional matrix to use is the covariance matrix $\mathbf{H}$ (Be69) which is related to the Hessian matrix $\mathbf{H}$ by

$$\mathbf{H} = (2\mathbf{H})^{-1}$$

The squares of the errors of the parameters $a_k$ are simply the diagonal elements of $\mathbf{H}$ when a solution is found. The parameter step $\Delta a_k$ necessary to move to a solution is

$$\Delta a_k = \sum_i \sum_j \langle v_{ki} \parallel \lambda_i \parallel v_{ij} \rangle \frac{\partial \chi^2}{\partial a_j}$$

where $\lambda_i$ is an eigenvalue of $\mathbf{H}$ and $V(ji)$ the corresponding eigenvector. The eigenvalues $\lambda_i$ provide much information concerning the surface of chi-square in the vicinity of the parameters $a(i)$. The eigenvalues are large when chi-square space is broad and flat for a given parameter, while small eigenvalues correspond to steep valleys.
Negative eigenvalues indicate saddle points in chi-square space. Successive approximations differ by a rank one matrix in an algorithm based on equations (31) and (32) of Powell (Po70). The method differs from those suggested by Powell in not requiring a positive definite $H$ at intermediate stages of the search, but instead in modifying the direction of parameter changes for the subsequent step whenever $H$ has negative eigenvalues (saddle points). The errors on the parameters are found by inversion of the Hessian matrix. The errors are correlated and are defined by the $\chi^2 + 1$ rule (Be69), that is, when a parameter is changed by its error and all the remaining parameters are searched on until a minimum of chi-square is found, the chi-square increases by one. The error in a parameter is the square root of the diagonal element of the error (covariance) matrix obtained by inverting the Hessian matrix (Be69). The function which is minimized is chi-square,

$$\chi^2 = w(i)(N(i)X(i)-Y(i))^2 + w(j)(N(j)-1)^2 + w(k)(E(k)-e(k))^2$$

where

$Y(i) =$ measured observable with relative error $\Delta Y(i)$

$w(i) =$ experimental weight for $Y(i), = (\Delta Y(i))^{-2}$
\[ X(i) = \text{calculated observable} \]

\[ N(j) = \text{normalization for } j\text{th set of data} \]

with experimental scale error \( \Delta N(j) \)

\[ N(i) = \text{the appropriate } N(j) \text{ for measurement } Y(i) \]

\[ w(j) = \text{experimental weight for } N(j), = (\Delta N(j))^{-2} \]

\[ E(k) = \text{experimental nominal energy with error } \Delta E(k) \]

\[ e(k) = \text{adjusted energy} \]

\[ w(k) = \text{experimental weight for } E(k) = (\Delta E(k))^{-2} \]

The first sum runs over all data points (index i), the second over all sets of data that have common normalizations (index j), and the third over those energies to be adjusted in the fitting procedure (index k). The second sum in chi-square accounts for adjustments in the normalization (or scale) of data sets included in the first sum \((N(i) = N(j) \text{ for all points } i \text{ in the } j\text{th data set})\), implying that relative and scale errors can be provided separately. The third sum accounts for adjustments in the experimental energies.

The actual goodness of fit indicator used was the weighted variance (chi-square per degree of freedom), which has an expected value of 1.00 when a solution has been reached. The number of degrees of freedom is given by \( \text{Nd} - \text{Np} \) in which Nd is the number of data points and Np is the number of independent parameters.
2C.2 DATA SELECTION

S.D. Baker and I compiled a set of 2254 data for four reactions in the 7Be system for this analysis. All significant data published by December, 1974, that lie below the 7.79 MeV excitation energy in the compound nucleus 7Be corresponding to the p' + 6Li* threshold were included. Above this excitation level it becomes difficult to model exactly all the allowed reaction channels. Below 7.79 MeV excitation energy most open channels are two-body channels which can be easily described by an appropriate choice of R matrix parameters. Since the center of mass energy available to the three particles will not be sufficient to allow the particles to penetrate the Coulomb barrier, the p-d-alpha breakup channel at 7.080 MeV 7Be excitation energy is not prominent and is accounted for by background levels and widths. The four reactions for which data were gathered are

\[
\begin{align*}
4\text{He}(3\text{He},3\text{He})4\text{He} & \quad E(3\text{He}) \leq 10.86 \text{ MeV} \\
4\text{He}(3\text{He},p)6\text{Li} & \quad E(3\text{He}) \leq 10.86 \text{ MeV} \\
6\text{Li}(p,3\text{He})4\text{He} & \quad E(p) \leq 2.55 \text{ MeV} \\
6\text{Li}(p,p)6\text{Li} & \quad E(p) \leq 2.55 \text{ MeV}
\end{align*}
\]

No data were accepted for incident lab energies of
10.86 MeV for 3He and 2.55 MeV for protons corresponding to the 7.79 MeV 7Be excitation energy. The term diagram of 7Be and the regions spanned by the different data sets are illustrated in figure 2.3.

The selection of criteria for the input data and the process of applying these criteria to the extant data is critical because of the danger of biasing the results of the analysis. The criteria used were similar to those described in detail in Do76.

1. It is important to use data that reflect accurately the quantity actually measured by the experimenters. Numerical values of the experimenters' best estimate of the observable and as complete as possible statement of the errors involved were sought whenever possible. In several instances, the experimenters no longer could retrieve the data in numerical form and the data were obtained from published graphs and figures. Only "raw" data were used. Manipulated data in the form of legenbre polynomial coefficients or Gamow plots were ignored.

2. Data superseded by a later work of the same group of investigators were not used. This criterion was applied to the Tombrello and Parker 4He(3He,3He)4He data (To63, graphical form) after talking with Tombrello. Newer data were not necessarily given preference over
Figure 2.3  Data Sets and $^7\text{Be}$ Levels

Various levels of $^7\text{Be}$ and reaction thresholds are shown. The energy range spanned by the different data sets for the 4 possible reactions are indicated by the shaded areas.
older data. However, data taken during the 1930's and 1940's often exist only as published figures and were not included in this analysis.

3. Redundant data were not culled to save computation time.

4. After letting EDA search on the basic data set until the chi-square per degree of freedom decreased from an initial value of 40 to 3.21, we deleted individual data points if their contribution to chi-square in the analysis was greater than 1300. Under this stringent criterion, five data from the $^4\text{He}(^3\text{He},p)^6\text{Li}$ reaction of Sp66 were discarded. Errors of 20% were assigned to eighteen points whose contributions to chi-square were greater than ten and which had relatively small values of the cross section (less than 9 mb). Errors of 50% were assigned to ten points with small cross sections (less than 9 mb) and whose individual contributions to chi-square ranged in the hundreds. All of these were isolated points, that is, their poor fit was not associated with poor fits at nearby angles and energies. Appendix A contains a list of the data that were reassigned larger errors.

5. We distinguished two types of errors for each data set, absolute (scale) and relative errors. An absolute
error is the uncertainty in the normalization of an entire data set. Relative errors are associated with each datum of a data set and reflect the accuracy of individual measurements including effects such as counting statistics, angle dependent errors, background subtraction, etc. The assignment of absolute and relative errors was often difficult and was the result of careful study of the literature and correspondence with several of the original authors.

Appendix A gives a list of each of the various data sets, with relevant information about each.

2C.3 DATA NORMALIZATIONS

In the several reactions under consideration which form 7Be as a compound nucleus, different investigators have used different schemes to normalize their data. This analysis attempts not only to find a consistent relative normalization for the different experiments, but also to determine the overall absolute (scale) normalization.

There is some confusion involving the alleged normalizations of the various data sets. Appendix A1 illustrates the confusion of the earlier data
normalizations.

To permit the most general set of normalizations, the data set of each investigator was broken into subsets according to the way the experiment was performed. Each subset corresponded to a particular angle if excitation functions were made or to a given energy if the data were taken in the form of angular distributions. Every subset had its own normalization (scale) factor which was varied by the search routine as a free parameter. However, in the cases that the experimenters measured both reaction and elastic scattering cross sections, we constrained the normalizations of the scattering and reaction cross sections to be equal when they were measured by identical apparatuses. These include the data sets of Sp66, Ba51, and Ha66.

The data set normalizations that we obtain are generally consistent with those of Harrison (Ha67). Appendix A contains a list of the data sets, the contribution to chi-square of the normalization factors, and the derived consistent set of normalizations of this analysis. The normalizations of Harrison to those of this analysis are compared in the appendix below.
2C.4 LOW ENERGY 6Li DESTRUCTION CROSS SECTION

The 6Li(p,3He)4He reaction is of special interest as an energy source in very young stars of low density and temperature (Sa55). It may play an important role in fusion processes and be an intimate part of stellar nucleosynthesis affecting the universal abundance of helium and lithium. In particular, it creates a 3He and an alpha particle which may be re-used in either the p-p chain or triple-alpha process (first stage of helium burning which forms carbon), respectively. These reactions occur at only a few million degrees (Sa55). For a 6Li-d fueled fusion reactor, the 6Li(p,3He)4He reaction is particularly important. Since the d(3He,p)4He reaction has a Q value of 18.4 MeV, the protons that are produced can regenerate energetic 3He to continue the cycle via the 6Li(p,3He)4He reaction (Q = 4.0 MeV). Accurate absolute cross sections are needed at all energies that may be encountered in a thermonuclear reactor (proton energies less than about 15 MeV) (Mc71).

In astrophysical work (Fo67) it is common to define the cross section factor $S(E)$ by

$$\sigma = (S(E)/E) \exp(-2 \pi \eta).$$
At low energies charged particle reactions are dominated by the Coulomb barrier penetration factor (Gamow factor), the exponential in the expression above. The cross section factor $S(E)$ includes the energy dependence of Coulomb scattering not contained in the Gamow exponential in addition to the energy dependence of the nuclear factors affecting the cross section. Far from a nuclear resonance, $S(E)$ is a slowly varying function of the energy $E$. It is customary to express the cross section factor as a Maclaurin series in the center of mass energy $E$

$$S(E) = S(0) \left[ 1 + \frac{S'(0)}{S(0)} E + \frac{1}{2} \frac{S''(0)}{S(0)} E^2 + \ldots \right]$$

Audouze and Reeves (Au69) give cross section factors for the destruction of $^6\text{Li}$ by protons calculated by a best-fit curve to the data of Sawyer and Phillips (Sa53b), Fiedler and Kunze (Fi67), Gemeinhardt et al. (Ge66), Bertrand et al. (Be68), and the results of Beaumeville as normalized by MacCray (Be64). The data show considerable scatter, even after renormalization. They find $S(0) = 2.4$ MeV-barns, which is considerably lower than the value $S(0) = 6 \pm 3$ MeV-barns of Salpeter (Sa55) using the data of Sawyer and Phillips. Their results are shown
Figure 2.4  $S(E)$ for $^6\text{Li}(p,^3\text{He})^4\text{He}$

Our $^7\text{Be}$ and Mass 7 predictions of $S(E)$ are plotted with the unnormalized data of Gemeinhardt, Spinka, and Fiedler. The predictions by Spinka and Audouze are also shown. The numbers in parentheses are the extrapolation to zero energy $S(0)$. 
in figure 2.4 and are tabulated in a summary at the end of this section.

Spinka and Tombrello (Sp71) have made a careful absolute measurement of the total reaction cross section for \( 6\text{Li}(p,4\text{He})3\text{He} \) between 151 and 560 keV. Their cross section data were combined with the data of Gemeinhardt et al. and extrapolated to energies of astrophysical interest. They found the cross section factor to be \( S(0) = 3 \text{ MeV-barns with zero slope.} \) There is an important error in the Spinka paper that we have discovered. The cross sections in figure 4 of their paper (Sp71) are for the \( \text{H}(6\text{Li},4\text{He})3\text{He} \) reaction where the angle is the 4He cm angle, not the 3He center of mass angle in the \( 6\text{Li}(p,3\text{He})4\text{He} \) reaction, as claimed.

The \( R \) matrix analysis of the \( 7\text{Be} \) system has several features that make it an attractive tool for studying the behaviour of the lithium destruction cross section at energies of astrophysical interest. The search routine builds up the covariance matrix by an iterative procedure so that once a solution is found, the errors in the \( R \) matrix parameters are well-defined. Since these errors are correlated, the covariance matrix can be used to make good predictions of the error in any of the predicted observables as well as the error of any \( R \) matrix parameter.
In particular, it is possible for us to calculate the $6\text{Li}(p,3\text{He})4\text{He}$ cross section at arbitrarily small energies (less than one eV) and to quote an error which indicates the consistency of the low-energy extrapolation with the experimental measurements at higher energy. This procedure is reminiscent of least squares adjustments of the physical constants.

Another important attribute of this analysis is the global nature of the study. We collected over two thousand data (2254) for four pertinent reactions up to a $7\text{Be}$ excitation energy of about 8 MeV. The inclusion of $R$ matrix parameters and resonances far above the energy range of interest is important because these levels provide background phase shifts at very low energies. Unlike previous $R$ matrix analyses which fit parameters to experimentally determined phase shifts, our investigation is the first such analysis to fit experimental data by observables calculated directly from $R$ matrix parameters which are searched on simultaneously. The importance of the global aspect of the study is evident in the Los Alamos group's investigation of the $5\text{Li}$ system (Ca75). They made the discovery of an inaccuracy in the accepted value of the $d-t$ reaction
cross section in the kilovolt energy range in a similar R matrix analysis. The \( J = 3/2^+ \) resonance (first excited state at 16.66 MeV excitation energy) caused a significant deviation from the Gamow penetration factor curve at lower energies than were expected. All prior calculations had missed this fact because they were based only on information directly related to one reaction rather than on a comprehensive study of the entire problem. Experimenters had even noticed the deviation from the Gamow plot, but they had dismissed it as an unexplained error (Ca75).

Nine different sets of data for the \( ^6\text{Li}(p,3\text{He})^4\text{He} \) reaction with incident proton energies below 1 MeV were included in my analysis in addition to the data up to 8 MeV excitation level of \( ^7\text{Be} \). See Appendix A for a list of all the data sets. Differential cross sections have been measured by Sp71, Be63, Ma56, and Ba51. Relative measurements of the differential cross section in the form of angular distributions have been made by Bo60, Kh61, and Be63. Ge66 and Fi67 have absolute measurements of the total reaction cross sections at several energies, while Br68 has angular distributions of the polarization analyzing efficiency. The data of Sawyer and Phillips (Sa53b) exist only in graphical form
as Gamow plots (not cross section data) and therefore were not included in this analysis. Despite attempts to contact the authors, we have not been able to transcribe the data described by Johnston and Sargood (Jo74). Their low energy angular distributions for the $6\text{Li}(p,3\text{He})4\text{He}$ reaction exist only in graphical form "normalized to Marion as corrected by Harrison." Once again the problem of normalizations rears its head. Their data are not included in the analysis, but do not disagree with our results.

---

**SUMMARY OF PREVIOUS MEASUREMENTS**

<table>
<thead>
<tr>
<th>AUTHOR</th>
<th>$S(0)$</th>
<th>$S'(0)/S(0)$</th>
<th>$S''(0)/2S(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salpeter</td>
<td>$6 \pm 3$</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Audouze and Reeves</td>
<td>2.4</td>
<td>-0.8</td>
<td>0.42</td>
</tr>
<tr>
<td>Spinka and Tombrello</td>
<td>3.0</td>
<td>0.0</td>
<td>...</td>
</tr>
<tr>
<td>Biegert (this work)</td>
<td>3.352</td>
<td>-2.043</td>
<td>-1.345</td>
</tr>
</tbody>
</table>
2C.5 ANALYSIS AND DISCUSSION

The R matrix parameters giving the best fit (a chi-square per degree of freedom of 1.583) to the 7Be system data are tabulated in Appendix A.2. The channel radii were kept fixed during the analysis at 4.02 fm (3He + 4He cluster) and 4.20 fm (p + 6Li configuration). The ground state (3/2-) and first excited state (1/2-) of 7Be both lie below the 3He + 4He threshold (refer to figure 2.2) and were kept fixed during the search on the other R matrix parameters. Levels far removed from the energy domain spanned by the data provided background phase shifts; nonzero reduced width amplitudes were chosen for certain levels to allow all possible transitions between levels with the fewest number of parameters. Additional levels were allowed in the 1/2+, 3/2-, 5/2+, 3/2+, 7/2-, and 5/2- submatrices. We observe energy levels at 4.56 MeV (7/2-), 6.46 MeV (5/2- F wave), and 7.08 MeV (5/2- P wave) 7Be excitation energies.

Background $E_\lambda$'s for many levels were set at $\pm$ 100 MeV. The search routine was unable to determine these parameters when they were allowed to vary as free parameters. The effect of placing the background $E_\lambda$'s
very high is to make the contribution to the background phase shifts essentially constant over the energy range of interest.

The values of $E_\lambda$ are sometimes arbitrary and may appear to be resonance levels if they lie in the range of energies covered by the data and search. These $E_\lambda$'s may represent a compromise between the effects of the next $E_\lambda$ in the infinite expansion and the sum of the effects of the more distant $E_\lambda$'s. They can be distinguished from real resonance levels by assiduous study of the associated phase shifts. A true resonance corresponds to a pole in the S matrix, which causes the associated phase shift to increase dramatically from a small value, pass through 90°, and plateau at a value near 180°. (The resulting plot of phase versus energy looks much like a titration curve.) The full width of the level is the distance the pole is from the real axis in the complex energy plane. For example, we observe a 3/2- level at 9.20 MeV, a 5/2+ level at 11.7 MeV, and a 3/2+ level at 8.81 MeV in our 7Be analysis, but none of the associated phases exhibit resonance-like behaviour. Compare these to the physical resonances (5/2-) in figure 2.1.

The phase shifts for these levels are shown in
figure 2.1 and are tabulated in Appendix A.4. In Appendix A.2 the R matrix parameters are converted to resonant energies and partial widths. Contour plots of the polarization for the $4\text{He}(3\text{He},3\text{He})4\text{He}$ and $6\text{Li}(p,p)6\text{Li}$ elastic scattering reactions are shown in figure 2.5.

The $4\text{He}(3\text{He},3\text{He})4\text{He}$ polarizations we obtain show considerable structure, especially near $E(3\text{He}) = 5.22$ MeV where the F waves resonate at the $7/2^-$ level of $7\text{Be}$. A sharp maximum near $120^\circ$ center of mass is observed in agreement with the predictions of Spiger and Tombrello (Sp67). A much broader maximum at $120^\circ$ is observed in the isobaric analogue, $7\text{Li}$. In fact, Gerry Hale's R matrix analysis of $7\text{Li}$ shows the anomaly to be a double maximum (Ha76).

2D. MASS 7 CHARGE INDEPENDENT ANALYSIS

Evidence for the equivalence of the neutron-neutron and proton-proton forces is prevalent in the study of the structure of complex nuclei. The energy differences of mirror nuclei can be ascribed largely to the Coulomb energy, and once a correction for this Coulomb energy shift has been made, the energy levels and angular momentum properties ($J$ and parity assignments) of the
Figure 2.5a  Polarizations for $^6\text{Li}(p,p)^6\text{Li}$

The contour map generated by the $^7\text{Be}$ R matrix analysis for proton polarizations in the elastic scattering of protons by $^6\text{Li}$ is shown.
Figure 2.5b  Polarizations for $^4\text{He}(^3\text{He}, ^3\text{He})^4\text{He}$

The contour map generated by the R matrix analysis for the $^3\text{He}$ polarizations in the elastic scattering of $^3\text{He}$ by $^4\text{He}$ is shown. Notice the ridge near $80^\circ$ cm where $P=-1$ ($E=9-11$ MeV). Also observe the resonance region near 5 MeV where the $F$ 7/2 waves resonate.
lower excited states of the light isobaric analogue
nuclei are the same.

This observation can be aptly demonstrated by this
combined R matrix analysis of the mass 7 system. Using
the best-fit R matrix parameters of the 7Be system, only
two additional parameters are necessary to fit the
combined data of both mirror nuclei. An energy level
that is a pure 3He + 4He (3H + 4He) cluster has a Coulomb
energy shift that is different from a level that is a
pure p + 6Li (n + 6Li) cluster. Denoting these two
Coulomb energy differences by $\Delta E_2$ and $\Delta E_1$,
respectively, the 7Be levels lie higher in energy by an
amount

$$E(\gamma \text{Be})_c = E(\gamma \text{Li})_c + \Delta E_c$$

The subscripts $c = 1, 2$ refer to the open reaction channels
in this analysis; 1 corresponds to 3He+4He or 3H+4He,
and 2 corresponds to the particle pair p+6Li or n+6Li.
The Coulomb energy shifts include mass differences, and
these can be removed to find the energy shift $\Delta C$ due
to the electromagnetic interaction:

$$\Delta E_c = \Delta M_c - \Delta C_c$$

where
\[ \Delta M_i = M(\text{^3H}) - M(\text{^3He}) = 0.530 \text{ MeV} \]
\[ \Delta M = M(n) - M(p) = 1.293 \text{ MeV} \]

are the nuclear mass differences. Since each state of the compound nucleus in this model is a superposition of the two clusters, in this analysis each level had a Coulomb energy shift that was proportional to the admixture of each cluster defining the level:

\[
E_\lambda(^7\text{Li}) = E_\lambda(^7\text{Be}) - \left[ \frac{\chi_\lambda^2}{\chi_{\lambda1}^2 + \chi_{\lambda2}^2} \Delta E_1 + \frac{\chi_{\lambda2}^2}{\chi_{\lambda1}^2 + \chi_{\lambda2}^2} \Delta E_2 \right]
\]

The values of \( \chi_{\lambda c} \) (c=1,2) in the above expression were not searched on in the analysis but were fixed at their values for the best-fit 7Be solution. (This was necessary because the computer code can handle only linear constraints between parameters. Kay Witte is modifying the code so that when this analysis is extended to higher energies, the values of the reduced width amplitudes in the above expression will be allowed to vary.

The R matrix parameters which gave the best fit to the 7Be system were used as initial values in our analysis.
of the mass 7 system. We obtained data for the $^7$Li compound nucleus from Gerry Hale's R matrix analysis of the charge conjugate system. We allowed the $^7$Be parameters to vary as before, but constrained the $^7$Li parameters as described below. The energy levels were shifted from the $^7$Be levels by Coulomb energy shift factors described above, and the $^7$Li reduced width amplitudes were set equal to those of $^7$Be.

The results are remarkably simple and require the addition of only two more Coulomb energy shift parameters to fit the combined data of both isobaric analogue nuclei than to fit the data on one system alone. We find

$$\Delta c_1 = -0.334 \text{ MeV}$$

$$\Delta c_2 = -0.159 \text{ MeV}$$

to produce a weighted variance (chi-square per degree of freedom) of 3.4 for the combined data. One parameter is not enough to fit both the 5.21 MeV $^3$He-$^4$He resonance and the n-$^6$Li resonance at 260 KeV since these two sharp resonances have different cluster configurations and different Coulomb shifts. Appendix A.3 contains the R matrix parameters that give the best fit to the mass 7 system. These can be compared to the $^7$Be parameters in Appendix I.2. The fit to the data are shown in Appendix A.6, which contains excitation functions of the mass 7
Figure 2.6 \( n^6\text{Li} \) Resonance at 260 keV

The \( ^6\text{Li}(n,n)^6\text{Li} \) 260 keV resonance is also observed in the \( ^4\text{He}(t,t)^4\text{He} \) channel near \( E(t)=8.7 \text{ MeV} \). The mass 7 R matrix fit to some cross section data in this region is shown.
data and predicted values of the observables.

The n-6Li reaction is currently of considerable interest. The high reaction cross sections, small mass, and high nuclear density (greater than iron, water, or uranium) of LiH have important implications for the design of mobile reactor shields (Ro56). The deuterium-tritium exothermic fusion reaction $t(d,n)\alpha\text{He}$ ($Q = 17.6$ MeV) is a promising energy source. For such a d-t reactor, tritium must be regenerated; the $6\text{Li}(n,t)\alpha\text{He}$ reaction (slow neutron, $Q = 4.8$ MeV) is especially favored (Ro71). The charge independent analysis was not only able to reproduce very well the behaviour of the n-6Li 260 keV resonance, but of all the other resonances in both systems as well. Figure 2.6 illustrates the fit to the data over the 260 keV resonance region.

2E. SUMMARY

We have calculated energy levels and their widths of both 7Be and 7Li by including all the extant data of both systems in a comprehensive R matrix analysis. Our analysis is the first major R matrix investigation to fit the data directly, rather than fitting R matrix parameters to a set of smoothed empirical phase shifts.
Our results shown in figure 2.7 are in qualitative agreement with the results published in the literature. Our 7Be analysis and the combined analysis have resolved ambiguities in the normalizations of published measurements. A consistent set of normalizations for all the experiments is a direct product of our study. A survey of our empirical normalizations and fit to the data indicates the "good" and "poor" experiments.

This analysis indicates that it may be possible to obtain a better fit to the data (and thus to know the total cross section more accurately and to make reasonable choices concerning the validity of various experiments) by measuring observables obtained in charged particle experiments on mirror nuclei.
Figure 2.7a  R Matrix Levels of $^7$Be

The energy levels for $^7$Be generated by the $^7$Be and mass 7 analyses are compared. The excitation energies (MeV above ground state) and widths in MeV (number on right side of each level) are indicated.
### $^7$Be Level Parameters

<table>
<thead>
<tr>
<th>Level</th>
<th>Investigator</th>
<th>$E_h$ (MeV)</th>
<th>$\gamma_1$ (MeV)</th>
<th>$\gamma_2$ (MeV)</th>
<th>$\gamma_3$ (MeV)</th>
<th>$\gamma_4$ (MeV)</th>
<th>$\gamma_5$ (MeV)</th>
<th>$R_\alpha$ (fm)</th>
<th>$R_\beta$ (fm)</th>
<th>$\gamma$ (MeV)</th>
<th>Boundary Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/2$^-$</td>
<td>Boykin</td>
<td>0.0</td>
<td>1.18</td>
<td>0.11</td>
<td>3.1</td>
<td>3.1</td>
<td>3.1</td>
<td>4.02</td>
<td>-1.351</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Barnard</td>
<td>0.0</td>
<td>0.38</td>
<td>0.04</td>
<td>3.8</td>
<td>3.8</td>
<td>3.8</td>
<td>4.02</td>
<td>-1.203</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/2$^-$</td>
<td>Boykin</td>
<td>0.43</td>
<td>0.87</td>
<td>0.29</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
<td>4.02</td>
<td>-1.351</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bisgert</td>
<td>0.43</td>
<td>0.856</td>
<td>0.379</td>
<td>4.02</td>
<td>4.02</td>
<td>4.02</td>
<td>-1.203</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7/2$^-$</td>
<td>Ivanovitch</td>
<td>4.566</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>0.162</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Spigel</td>
<td>4.57</td>
<td>1.6</td>
<td>4.3</td>
<td>4.3</td>
<td>4.3</td>
<td>4.3</td>
<td>4.4</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Barnard</td>
<td>4.57</td>
<td>0.7</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tombrello</td>
<td>4.54</td>
<td>2.99*</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td></td>
<td>1.252</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>Bisgert</td>
<td>4.56</td>
<td>0.724</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td>0.162</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>5/2$^-$</td>
<td>Spigel</td>
<td>6.73</td>
<td>3.1</td>
<td>0.01</td>
<td>1.36</td>
<td>0.0</td>
<td>0.0</td>
<td>4.0</td>
<td>4.0</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tombrello</td>
<td>6.51</td>
<td>3.96*</td>
<td>5.02*</td>
<td>5.02*</td>
<td>5.02*</td>
<td>5.02*</td>
<td>4.4</td>
<td>4.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bisgert</td>
<td>6.462</td>
<td>0.441</td>
<td>0.088</td>
<td>0.195</td>
<td>0.021</td>
<td>0.021</td>
<td>4.02</td>
<td>4.2</td>
<td>1.252</td>
<td>-1</td>
</tr>
<tr>
<td>5/2$^-$</td>
<td>Marion</td>
<td>7.80</td>
<td>0.75*</td>
<td>4.70*</td>
<td>4.70*</td>
<td>4.70*</td>
<td>4.70*</td>
<td>4.02</td>
<td>4.0</td>
<td>4.0</td>
<td>4.08 0.72</td>
</tr>
<tr>
<td></td>
<td>McCray</td>
<td>7.18</td>
<td>0.101*</td>
<td>5.02*</td>
<td>5.02*</td>
<td>5.02*</td>
<td>5.02*</td>
<td>4.08</td>
<td>4.0</td>
<td>4.0</td>
<td>4.08 0.816</td>
</tr>
<tr>
<td></td>
<td>Spigel</td>
<td>7.21</td>
<td>0.023</td>
<td>1.2</td>
<td>0.010</td>
<td>0.020</td>
<td>0.020</td>
<td>4.0</td>
<td>4.0</td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>Tombrello</td>
<td>7.18</td>
<td>0.11*</td>
<td>5.00*</td>
<td>5.00*</td>
<td>5.00*</td>
<td>5.00*</td>
<td>4.0</td>
<td>4.0</td>
<td></td>
<td>0.836</td>
</tr>
<tr>
<td></td>
<td>Bashkin</td>
<td>7.16</td>
<td>0.03</td>
<td>0.40</td>
<td>0.03</td>
<td>0.40</td>
<td>0.40</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.00 0.43</td>
</tr>
<tr>
<td></td>
<td>Bashkin</td>
<td>7.20</td>
<td>0.033</td>
<td>0.734</td>
<td>0.015</td>
<td>0.176</td>
<td>0.176</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.00 0.500</td>
</tr>
<tr>
<td>7/2$^-$</td>
<td>Spigel</td>
<td>9.27</td>
<td>1.6</td>
<td>1.3</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* - Units of MeV·fm

---

Figure 2.7b  
$^7$Be Level Parameters

The $^7$Be level parameters for this thesis and previous investigations are compared.
3. 7Be SYSTEM: INTERMEDIATE ENERGIES

3A. INTRODUCTION

The experimental evidence for the existence of highly excited levels in 7Be is somewhat weaker than for 6Be (3He + 3He). Below an excitation energy of 12 MeV in 7Be, the 3He + 4He elastic scattering differential cross sections (Sp67) provide clear evidence for the expected shell model states with p-shell configurations. Only minor changes in the phase shifts determined from the cross section data are required to fit polarized 3He measurements (Ha72) up to 9 MeV excitation. In higher energy differential cross section measurements (Ja70 and Sc69) up to 42 MeV excitation some broad structure is apparent, but attempts to extend the phase shifts to excitation energies above 12 MeV have not been successful. Resonating group calculations (Ko74) employing an $\ell$-dependent phenomenological imaginary potential reproduce the cross section data over a wide energy range and predict a broad $\ell = 2$ level at 11.6 MeV and both $\ell = 4$ and $\ell = 5$ levels near 25 MeV excitation.

In order to extend the range of available polarization data beyond the present limit of about 9 MeV excitation,
we have measured the polarization analyzing power for
the elastic scattering of 4He from a polarized 3He target
at bombarding energies from 24.5 to 42.0 MeV, corresponding
to excitation energies in 7Be between 12 and 20 MeV.
We have made a phase shift analysis. The results are
compared with the predictions of the resonating group
theory.

3B. POLARIZED 3He TARGET EXPERIMENT

3B.1 EXPERIMENTAL DETAILS: POLARIZED TARGET

A 4He beam from the TAMVEC was incident on a polarized
3He target that I built at Rice (Bi76). The 4He bombarding
energy for the 4He-3He elastic scattering was 24.5 to
42.0 MeV, corresponding to an equivalent 3He bombarding
energy range from 18.38 to 31.5 MeV. The energy referred
to in this paper concerning the 4He-3He elastic scattering
will be the equivalent 3He lab energy. The experimental
arrangement is described in detail in Bi76. Detectors
were placed symmetrically on either side of the beam at
laboratory angles 20°, 35°, and 50° to cancel certain
instrumental asymmetries. For the 4He-3He elastic
scattering, detection of both 3He and 4He at laboratory
angles of 20° and 35° and of 3He at 50° provided
measurements of the analyzing power at five center of mass angles. Particle identification at the smallest angle was necessary to distinguish the two particles. A section describing the particle identification methods follows below. The rms angular resolution of 1.03° for each angle was calculated by taking into account the beam collimation, scattered particle collimation, and multiple scattering in the entrance foil. Details of the construction of my target and of the determination of the 3He polarization (typically 0.13 in this experiment) are given in Ha72 and Ba69 as well as Bi76. The parameter f, which is defined in Ba69 and which appears in the optical determination of the 3He polarization, was taken as 0.7. The analyzing power A_y was calculated using the equations in Bi76. Repeated measurements at each 3He bombarding energy reproduced satisfactorily.

3B.2 RESULTS OF POLARIZED TARGET EXPERIMENTS

Table 3.1 gives our target analyzing power in 3He-4He elastic scattering at equivalent 3He bombarding energies. The errors include counting statistics and estimates of the precision with which the target polarization is measured, but they do not include any
<table>
<thead>
<tr>
<th>E</th>
<th>47.0°</th>
<th>80.0°</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.36</td>
<td>+0.1180±.0128</td>
<td>-.1397±.0424</td>
</tr>
<tr>
<td>22.0</td>
<td>+0.1789±.0410</td>
<td>-.2024±.0408</td>
</tr>
<tr>
<td>23.5</td>
<td>+0.0673±.0851</td>
<td>-.0303±.0838</td>
</tr>
<tr>
<td>26.0</td>
<td>+0.3316±.0655</td>
<td>-.1724±.0316</td>
</tr>
<tr>
<td>27.93</td>
<td>-.0081±.1025</td>
<td>-.2399±.0407</td>
</tr>
<tr>
<td>30.0</td>
<td>-.2897±.0485</td>
<td>-.1414±.0291</td>
</tr>
<tr>
<td>31.85</td>
<td>-.2369±.0353</td>
<td>-.1584±.0365</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E</th>
<th>84.6°</th>
<th>110.0°</th>
<th>140.0°</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.36</td>
<td>-.1900±0.422</td>
<td>+0.0390±0.0654</td>
<td>-.1718±0.0350</td>
</tr>
<tr>
<td>22.0</td>
<td>-.2801±0.0444</td>
<td>-.0291±0.0687</td>
<td>+0.0342±0.0460</td>
</tr>
<tr>
<td>23.5</td>
<td>-.3831±1.069</td>
<td>+.2943±1.538</td>
<td>+.1640±1.340</td>
</tr>
<tr>
<td>26.0</td>
<td>-.2858±0.0337</td>
<td>+.1551±0.0566</td>
<td>+.1860±0.0700</td>
</tr>
<tr>
<td>27.93</td>
<td>-.2741±0.0430</td>
<td>+.2432±0.0683</td>
<td>+.1703±1.015</td>
</tr>
<tr>
<td>30.0</td>
<td>-.2998±0.0314</td>
<td>+.2899±0.0529</td>
<td>+.0477±0.0801</td>
</tr>
<tr>
<td>31.85</td>
<td>-.2856±0.0343</td>
<td>+.2444±0.0632</td>
<td>+.1821±0.0859</td>
</tr>
</tbody>
</table>

Table 3.1  ANALYZING POWERS

The analyzing powers for \(^4\text{He} + ^3\text{He}\) elastic scattering with a polarized \(^3\text{He}\) target are tabulated by equivalent \(^3\text{He}\) lab energy and center of mass angle for the detection of the \(^3\text{He}\).
contribution from systematic errors in the determination of the target polarization. Consequently all of the analyzing powers may be multiplied by a single factor between 0.85 and 1.15. The excitation function at 47° in the center of mass is shown in figure 3.2. The range of variation of the analyzing power at the other angles is indicated by two representative angular distributions shown in figure 3.3. The variations in the values of Ay at the forward and backward angles presumably are related to the structure that appears in the differential cross sections in this same energy range.

3C.1 POLARIZED 3He BEAM

Our demonstration of a beam of polarized, singly charged 3He ions at the TAMVEC facility (Ma76b) with current as high as 150 nA and measured beam polarization of 0.08 has enabled us to perform several scattering experiments with beam energies up to 45 MeV. The first measurements of analyzing powers with this beam (presently the highest attainable energy of polarized 3He beams) have extended the range of available polarization data in 3He + 4He scattering to about 28 MeV 7Be excitation energy (Bi76b and Ma76).
Figure 3.2 \(^4\text{He}(^3\text{He},^3\text{He})^4\text{He}\) Analyzing Powers

The analyzing power excitation function for 47° cm is shown for the elastic scattering of \(^3\text{He}\) by \(^4\text{He}\). The Rice and Birmingham data are in qualitative agreement.
Figure 3.3  

ANGULAR DISTRIBUTIONS

Angular distributions for cross sections and polarizations in $^3\text{He} + ^4\text{He}$ elastic scattering are shown for two energies. The solid lines are fits by my empirical phases. Our relative differential cross sections at 38 MeV were added after the polarization data had been fit. Notice the fit to the polarizations gives the correct structure to the other data.
Figure 3.4 Polarized Target Spectra

Typical spectra from the polarized target in the elastic scattering of $^4\text{He}$ from polarized $^3\text{He}$ are shown.
3C.1A EXPERIMENTAL DETAILS: POLARIZED BEAM

We used the polarized 3He beam developed by Don May and described by him in Ma76 to measure analyzing powers in the elastic scattering of 3He by 4He at center of mass energies of 21.7 and 26 MeV. A view of the collimation and detector layout is given in figure 3.5. Our experiment at 21.7 MeV is described in detail in Ma76. The higher energy measurement was made in the same way, briefly described below. The scattering cell containing 4He gas at a pressure of 50 cm of Hg, the polarimeter cell, and all detectors were placed in the old Yale scattering chamber, an evacuated chamber on the TAMVEC beam line (Ju76). Before entering the target cell through a 2.2 mg/cm² aluminum entrance foil, the beam was collimated by two rectangular apertures in 0.203 cm tantalum and by large lead bricks.

Two silicon surface barrier detectors were mounted on either side of the target cell in the horizontal scattering plane. Both were mounted on rotatable arms set at symmetric angles on either side of the beam axis. The angles were reproducible within 0.05°. Each detector had its own collimating telescope 1.5 cm from the
Figure 3.5 POLARIZED BEAM EXPERIMENT
6.1 mg/cm² Ni exit foil of the 4He scattering cell. The total angular resolution for scattering, including effects of entrance and exit collimation and straggling in the foils, was estimated to be 1.7° rms in the lab for 3He particles detected and 1.8° rms in the lab for alpha particles detected.

The unscattered beam traveled downstream approximately 75 cm into a polarimeter built by David Judd which continuously monitored the beam polarization. This polarimeter is described in detail in Ju76. Inside the polarimeter the beam was degraded to 13.0 MeV by a series of interchangeable aluminum foils. It was collimated and entered a cell filled with 4He gas near atmospheric pressure. Scattered particles were detected by two silicon surface barrier detectors on either side of the beam axis at 25° lab in the horizontal scattering plane. Due to energy straggling of the beam in the degrading foils, the kinematics of the reaction, and the angular resolution of the detection system, the 3He and 4He spectra are merged into one peak. The 3He particles are scattered forward at 45° cm and the alphas are back-scattered at 128° cm.

The incident beam was polarized (typically 0.07) normal to the scattering plane. Since the beam polarization
was reversed by rotating a linear polarizer in the optical pumping system of the source, the geometry was not affected by a polarization reversal. The current produced at the scattering cell by the beam was measured by an electrometer. This measurement remained stable throughout the experiment, but it is not expected that this gave an accurate measure of the total beam current on target. By sweeping the beam across the entrance collimators and measuring the current on these also, we estimated that the total current incident on target exceeded 50 nA.

3C.1B DATA HANDLING

Bi76 gives the electronics flowchart for acquiring counts from each pair of detectors. Counts were taken from the four detectors and accumulated in four spectra. Typical spectra are shown in figure 3.6. The electronics were arranged to give equal amounts of dead time to each pair of particle detectors. In the spectra from the scattering cell, both peaks due to elastically scattered 3He and elastically scattered 4He could be discerned. Yields were taken from both peaks and the subtraction of background was performed in a consistent manner. A straight line representing an estimate of the background
Figure 3.6  Polarized Beam Spectra

Spectra for the elastic scattering of $^3$He by $^4$He are shown for both transverse spin orientations and left and right detectors. The $^4$He peak is indicated. In Judd's polarimeter, the $^4$He and $^3$He peaks are merged and lie on the shoulder of the proton background. Compare to figure B2.2.
was drawn between two points chosen as representative of the background on either side of the peaks. For mathematical details of this calculation, see Bi76. The regions used for estimating the background and the integration limits for the 3He and 4He elastic peaks were changed only when the angle was changed. The peaks for the elastically scattered particles could not be separated in the spectra taken by the polarimeter detectors due to the large angular acceptance of this device (5.1° rms). The background subtraction was performed as for the spectra for the scattering cell, and the regions for background subtraction and the two integration limits were not changed throughout the data taking period.

A complete discussion of the calculation of the analyzing powers and asymmetries is given by May (Ma76) and is briefly summarized here. For each angular setting, the total accumulated right and left counts $R$ and $L$ in a peak were used to form the ratio

$$RHO = \frac{L(+R(-)}{L(-)R(+)}$$

The (+) and (-) subscripts refer to the sense of the beam polarization, up or down, with respect to the scattering plane. See figure 3.7. The experimental asymmetry is obtained
Figure 3.7 SCATTERING GEOMETRY

The geometry (Basel Convention) and coordinate system for left and right scattering are illustrated.
\[ pA = \frac{\text{RHO}-1}{\text{RHO}+1} \]

where \( p \) is the beam polarization and \( A \) is the target analyzing power (Basel convention). The above expression is independent of detector efficiency, target geometry, and instrumental asymmetry effects.

Under the assumption that the pressure of the 4He target gas in the scattering cell and polarimeter remained constant throughout the experiment, it is possible to extract relative differential cross sections from the yields in the counters. In this type of calculation, the polarimeter acts as a monitor to normalize the flux and target pressures between runs. In Appendix B3 I derive explicitly the formulae for the cross sections and the associated errors. The final results are center of mass relative differential cross sections \( \sigma/\sigma_m \) for the elastic scattering of 3He by 4He, that is, the overall normalization is unknown. However, if one were to use a monitor for which the differential cross section \( \sigma_m \) and the relative solid angle efficiency factor were known, absolute cross sections could be determined.

3C.1C NORMALIZATION OF DATA

The polarimeter described by Judd (Ju76) and used
in this experiment to continuously monitor the beam polarization was incorrectly calibrated. Furthermore, it suffered from problems of severe background, resolution of particle peaks, and alignment difficulties. Judd's polarimeter spectra (figure 3.6) show one peak consisting of both 3He and 4He particle counts superimposed on the shoulder of a large background due to protons passing through the detectors. Since this polarimeter could only be used for relative measurements, it was decided to build a new polarimeter that could be calibrated by an absolute calibration point or the R matrix analysis.

The Birmingham collaboration has also had difficulties in calibrating a polarimeter for their polarized 3He beam. They originally had a p-3He (polyethylene sheet) polarimeter based on McSherry and Baker's data taken with a polarized 3He target and two p-3He phase shift analyses (Tombrello and Haeberli) (Fn71). This polarimeter was not efficient or accurately calibrated, so the p-3He polarimeter was recalibrated (Ka74) by 3He+4He elastic scattering using Armstrong's double scattering data (Ar69). The Birmingham collaboration claim (B175) that their p-3He polarimeter was thus calibrated by Armstrong's double scattering measurement to within +10% and -4% (Bu75). Recently (Ka77) they have switched to a more
efficient (higher yield, larger asymmetry) d-3He polarimeter. This recoil deuteron polarimeter (deuterated polyethylene sheet) for their polarized 3He beam was calibrated by their p-3He polarimeter in a 1% measurement.

By calibrating our polarimeter with 3He + 4He analyzing powers that are calculated more accurately than they can presently be measured, we hope to avoid these difficulties. The analyzing powers, of course, come from our R matrix analysis.

3C.1C1 ABSOLUTE CALIBRATION POINTS

The absolute calibration of the polarization in the scattering of spin 1/2 particles from particles without spin can be obtained by observing the scattering of polarized 3He from 4He near an angle and energy where the analyzing power is predicted to achieve its maximum value, unity. Such absolute calibration points have been predicted by a theorem of Plattner and Bacher (Pl71). The analyzing power, or polarization, can be expressed in terms of the non-spin-flip and spin-flip components of the scattering amplitude f and g as

\[ P = 2\text{Im}(f^*g)/(f^*f + g^*g) \]

where * denotes complex conjugation. The complex functions
f and g are readily expressed in terms of partial wave phase shifts. A necessary and sufficient condition that f and g must satisfy for the polarization to take its maximum value is to require

\[ g = \pm i \quad \text{for} \quad P = \pm 1. \]

The non-spin-flip amplitude f can be normalized by an arbitrary complex phase angle so that f becomes real and equal to unity for all angles and energies. Then the required renormalized spin-flip amplitude

\[ g = \pm i \quad \text{for} \quad |P| = 1. \]

Since g must be a smooth function of energy and angle with no sudden discontinuities, if at some energy \( E_1 \) the g-trajectory describing the scattering in question includes one of the points \( g = \pm i \), and if at some energy \( E_2 \) the corresponding g-trajectory excludes the point \( g = \pm i \), then between \( E_1 \) and \( E_2 \) there must exist a point where \( P = \pm 1 \). Since the absolute value of P must always be less than unity, one can perform scattering experiments to search for the largest value of P in that energy interval and find the absolute calibration point.

For 3He-4He scattering, Plattner and Bacher have calculated three points where \( |P| = 1 \) using the phase shifts of Hardy, et al (Ha72). Analysis indicates that there may be three additional points where \( P = -1 \) between
13 and 15.5 MeV 3He lab energy (P171). Ware and Smythe (Wa74) scattered 4He from a polarized 3He target in an attempt to search for the calibration point at 11.5 MeV 3He lab energy. They found \( P = -1 \) at 11.81 MeV and a lab scattering angle of 45.5°. This corresponds to a center of mass angle of 81° (3He detected) or a lab angle of 49.5° if the alpha particle is detected. However, the precision of this experiment suffers from a lack of small angle data due to the inability to distinguish between the 3He and 4He.

3C.1C2 PARTICLE IDENTIFICATION

A means of identifying particles is necessary because of reaction products (or light recoil nuclei) coming from the target with energies near the energy of the elastically scattered projectiles. A common method of particle identification is the Goulding system (Go64) which uses a detector telescope combination (transmission and stopping detectors). Particles are identified by the product of their mass and charge \( mZ^2 \) which is an exponential function of the energy lost in the counters. Electronic modules are available which use this method to uniquely identify particles in the range of energies
from 10 to 45 MeV. Outside this range, the identification is no longer independent of energy and other methods must be used.

Often a range-lookup system of particle identification is used when the detected particles are in an energy region where the Goulding method fails (Hi69). This method makes explicit use of the range-energy relationship for a given particle in silicon (detector) to calculate the thickness of the transmission detector in a detector telescope. This method, which requires a fast computer and considerable interfacing of the electronics to the data acquisition computer, slows down the rate at which data can be handled because of the many necessary calculations for each event detected.

3C.1C3 POLARIMETER DESIGN

We carefully designed the polarimeter used in this experiment to monitor the beam polarization to avoid the necessity of particle identification inherent in previous polarimeters. I used the mass 7 R matrix analysis to generate the ridge (see contour plots) in angle and energy (85° cm, 9-13 MeV) where the analyzing power for the reaction 4He(3He,3He)4He is maximum (A = -1) and is
insensitive to changes in angle or energy. In this
region, $I F^2$, the figure of merit which determines the
time necessary to achieve a certain fractional error in
the measurement, is maximum. The laboratory scattering
angle can be chosen so that both the scattered $^3$He and
the recoil $^4$He emerge at the same angle ($48.5^\circ_{\text{lab}},$
$82.92^\circ_{\text{cm}}$ $^3$He scattering angle). A coincidence measurement
between both particles with detectors placed at this
angle on either side of the beam eliminates the background
due to the alpha continuum and protons from the reactions
of $^3$He on the foils and scattering chambers. Both
particles have the same large analyzing power but with
opposite sign. If the peaks are not resolved, the total
analyzing power will be zero. At 10 MeV $^3$He incident
energy, the kinematics separate the recoil and scattered
particle peaks enough for an unfolding routine to
completely resolve the peaks. The coincidence requirement
also reduces the angular acceptance of the collimation
system. The effects of beam size, construction errors,
and misalignment on the angular resolution of the
coincidence geometry are discussed in Appendix B1. The
polarimeter built by C.W. Towsley at TAMVEC is described
in Appendix B2.
3C.1C4 POLARIMETER CALIBRATION

The energy calibration for the data acquisition system was made with an alpha source (241Am 5.477 MeV) and calibrated pulser-decade attenuator. Energy loss calculations for the passage of particles through the $^4$He gas and polarimeter exit foils for all peaks resulted in a mean scattering energy of 9.65 MeV in the center of the polarimeter. The average polarimeter analyzing power was obtained by integration:

$$A_y = \frac{\int_{\theta_0 - \delta}^{\theta_0 + \delta} \left[ \frac{d\sigma}{d\Omega}(\theta) \right] \Omega(\theta) A_y(\theta) \ d\theta}{\int_{\theta_0 - \delta}^{\theta_0 + \delta} \left[ \frac{d\sigma}{d\Omega}(\theta) \right] \Omega(\theta) \ d\theta}$$

$A_y(\theta)$ is the analyzing power as determined by the R matrix calculation, $\Omega(\theta)$ is the solid angle efficiency function for the coincidence geometry described in Appendix B1, and $\frac{d\sigma}{d\Omega}$ is the laboratory scattering differential cross section calculated from the $R$ matrix analysis. The limits of integration are defined by the angular acceptance $\delta = 2.45^\circ \text{ lab}$ and nominal scattering angle $\theta_0 = 48.5^\circ \text{ lab}$. Folding in these weighting
functions, the average analyzing power for the polameter was determined to be -0.968 within 2% at 82.9°cm 3He scattering angle for an incident 3He energy of 9.65 MeV.

3C.1C5 POLARIMETER OPERATION

A beam of 38.2 MeV polarized 3He++ from the Texas Variable Energy Cyclotron was incident on a scattering cell in the ORTEC scattering chamber. The scattering cell was filled to 50.8 cm of Hg with 4He gas (0.67 atm). To normalize the 3He-4He scattering data at 38 MeV, the detector arms were set to 21.5° lab, where a maximum in the asymmetry had been observed (Ma76) and the counting rate was large. The exiting beam was collimated, degraded to about 10 MeV by a 171 mg/cm² packet of aluminum foils, and then entered the polarimeter cell. Data were collected for approximately equal beam integrations (measured by a Faraday cup at the end of the beam line) for both transverse spin orientations of the beam.

3C.1C6 POLARIMETER DATA ANALYSIS

The left and right counter spectra were first summed over all runs to produce two spectra with good statistics
to ensure smooth peak shapes. The double peaks in each spectrum were fitted with two gaussian curves to determine the shape parameters for the peaks in the left and right detectors. See figure B2.2 in Appendix B2 for typical spectra and the fit to them. The fitting routine used a Hessian matrix calculated with analytical derivatives to find the optimum parameters. The areas of the peaks for each run were then determined by a least-squares fit of the amplitudes for the gaussian shapes to the data, holding the shape parameters constant during the fitting.

The coincidence requirement was strictly enforced by requiring the areas of the appropriate peaks in the correct spectra to be equal, since a 3He detected in the left counter corresponded to a 4He detected in the right, and vice versa. The asymmetries were then calculated in the usual manner. The normalizations for the 38 MeV data are thus determined by two points (21.5° lab):

<table>
<thead>
<tr>
<th>DETECTED</th>
<th>CM ANGLE</th>
<th>3He</th>
<th>ANALYZING POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>3He</td>
<td>37.5°</td>
<td></td>
<td>-0.428 ± 0.012</td>
</tr>
<tr>
<td>4He</td>
<td>137.0°</td>
<td></td>
<td>-0.370 ± 0.013</td>
</tr>
</tbody>
</table>

3C.1C7 CROSSTALK

This polarimeter could be used more effectively if
the data could be analyzed entirely during data acquisition. The main problem with on-line analysis is the separation of the 3He and 4He peaks in the spectra. Because of the overlap (crosstalk) between the peaks (about 17% of the total peak area), the asymmetry calculated by setting symmetric limits on the peaks and channel by channel integration will generally be smaller than the asymmetry calculated by deconvolution of the peaks. The average deviation in the asymmetry calculated by these two methods was 5.7%, lying in a range from 0.2% to 23.2%. The optimum position for placing limits for on-line integration can be calculated by minimizing the fractional error in the asymmetry. This balances the effects of increasing the accuracy of the measurement by including more counts and decreasing the asymmetry because of crosstalk. This calculation and the effects of crosstalk are discussed in detail in Appendix B2.

3C.2 RESULTS OF POLARIZED BEAM EXPERIMENTS

Table 3.8 lists the analyzing powers and relative differential cross sections for 3He-4He elastic scattering at 38 MeV and 45 MeV obtained in this experiment. The errors include counting statistics and estimates of the
<table>
<thead>
<tr>
<th>$\theta_{cm}$</th>
<th>$(\frac{\sigma}{\sigma_m})$</th>
<th>$\Delta(\frac{\sigma}{\sigma_m})$</th>
<th>$A_y$</th>
<th>$\Delta A_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.20</td>
<td>13.06</td>
<td>0.23</td>
<td>-0.1459</td>
<td>0.0240</td>
</tr>
<tr>
<td>37.20</td>
<td>12.00</td>
<td>2.00</td>
<td>-0.2565</td>
<td>0.0365</td>
</tr>
<tr>
<td>43.56</td>
<td>21.36</td>
<td>0.30</td>
<td>-0.1605</td>
<td>0.0234</td>
</tr>
<tr>
<td>52.13</td>
<td>16.69</td>
<td>0.24</td>
<td>-0.0036</td>
<td>0.0209</td>
</tr>
<tr>
<td>60.60</td>
<td>11.15</td>
<td>0.12</td>
<td>+0.1149</td>
<td>0.0218</td>
</tr>
<tr>
<td>65.00</td>
<td></td>
<td></td>
<td>+0.1094</td>
<td>0.0208</td>
</tr>
<tr>
<td>68.96</td>
<td>13.58</td>
<td>0.12</td>
<td>+0.0547</td>
<td>0.0137</td>
</tr>
<tr>
<td>77.19</td>
<td>13.98</td>
<td>0.13</td>
<td>-0.0337</td>
<td>0.0167</td>
</tr>
<tr>
<td>80.00</td>
<td>8.06</td>
<td>0.08</td>
<td>-0.0939</td>
<td>0.0252</td>
</tr>
<tr>
<td>85.25</td>
<td>8.29</td>
<td>0.08</td>
<td>-0.1158</td>
<td>0.0251</td>
</tr>
<tr>
<td>90.00</td>
<td>3.03</td>
<td>0.03</td>
<td>-0.2554</td>
<td>0.0488</td>
</tr>
<tr>
<td>100.00</td>
<td>0.838</td>
<td>0.01</td>
<td>-0.0337</td>
<td>0.0539</td>
</tr>
<tr>
<td>104.80</td>
<td>1.55</td>
<td>0.02</td>
<td>+0.2673</td>
<td>0.0469</td>
</tr>
<tr>
<td>110.00</td>
<td>5.35</td>
<td>0.06</td>
<td>+0.1751</td>
<td>0.0293</td>
</tr>
<tr>
<td>120.00</td>
<td>9.42</td>
<td>0.14</td>
<td>+0.0583</td>
<td>0.0252</td>
</tr>
<tr>
<td>130.00</td>
<td>8.06</td>
<td>0.12</td>
<td>-0.0793</td>
<td>0.0274</td>
</tr>
<tr>
<td>137.00</td>
<td>4.36</td>
<td>0.08</td>
<td>-0.1514</td>
<td>0.0441</td>
</tr>
<tr>
<td>142.00</td>
<td>3.76</td>
<td>0.05</td>
<td>-0.0675</td>
<td>0.0352</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta_{cm}$</th>
<th>$(\frac{\sigma}{\sigma_m})$</th>
<th>$\Delta(\frac{\sigma}{\sigma_m})$</th>
<th>$A_y$</th>
<th>$\Delta A_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.93</td>
<td>1.074</td>
<td>.0202</td>
<td>-0.1746</td>
<td>0.0364</td>
</tr>
<tr>
<td>43.56</td>
<td>2.270</td>
<td>.0554</td>
<td>-0.0612</td>
<td>0.0280</td>
</tr>
<tr>
<td>52.13</td>
<td>1.180</td>
<td>.019</td>
<td>+0.0427</td>
<td>0.0372</td>
</tr>
<tr>
<td>60.60</td>
<td>0.439</td>
<td>.0090</td>
<td>+0.3028</td>
<td>0.0616</td>
</tr>
<tr>
<td>68.96</td>
<td>0.402</td>
<td>.3095</td>
<td>+0.2117</td>
<td>0.0586</td>
</tr>
<tr>
<td>100.00</td>
<td>0.0637</td>
<td>.0020</td>
<td>-0.9113</td>
<td>0.1889</td>
</tr>
<tr>
<td>110.00</td>
<td>0.157</td>
<td>.0037</td>
<td>+0.4044</td>
<td>0.0943</td>
</tr>
<tr>
<td>120.00</td>
<td>0.348</td>
<td>.0065</td>
<td>+0.1719</td>
<td>0.0625</td>
</tr>
<tr>
<td>130.00</td>
<td>0.548</td>
<td>.0150</td>
<td>-0.1508</td>
<td>0.0528</td>
</tr>
<tr>
<td>140.00</td>
<td>0.352</td>
<td>.00764</td>
<td>-0.3414</td>
<td>0.0627</td>
</tr>
</tbody>
</table>

Table 3.8 ANALYZING POWERS AND CROSS SECTIONS

Analyzing powers and relative differential cross sections for the elastic scattering of polarized $^3$He by $^4$He reported in this thesis are tabularized here.
precision with which the beam polarization is measured. The variations in the values of the analyzing power and differential cross section at 38 MeV are shown in figure 3.3.

3D. PHASE SHIFT ANALYSIS

I have performed a phase shift study of $^3$He + $^4$He elastic scattering data at eight energies between 28 MeV and 45 MeV (lab). Cross section data (Sc69,Ja70), polarization data (Ba74,Ka76,Ma76,Bi77), and total reaction cross sections (Ko77) were included to make a set of approximately 75 data at each energy. Complex phase shifts for partial waves through $l = 5$ were allowed to vary while the $l = 6$ and $l = 7$ complex phase shifts were kept fixed during the searches. The resonating group phase shifts of Furber (Fu76) were used as starting values in this analysis to see how close the fitted phases would be to the resonating group values.

This procedure has been used before on studies of the 7Be system. Schwandt (Sc69) performed such an analysis on $^3$He-$^4$He elastic scattering differential cross sections from 27 to 43 MeV (lab). In his analysis the agreement of the empirical phases with the calculated
ones was qualitative, but the calculated phases served as useful starting points. A phase shift analysis of elastic scattering differential cross sections at higher energies [49-77 MeV (lab)] was made by Fetscher (Fe73). Comparison of his results with phases from resonating group calculations (Fe71) showed systematic differences similar to those found by Schwandt. However, a detailed comparison at the highest energy with a resonating group calculation employing a phenomenological imaginary potential (Ta71) showed a great similarity between the two sets of phase shifts, especially in the pronounced odd-even dependence on the orbital angular momentum exhibited by both the real and imaginary parts of the phases and arising from exact treatment of the Pauli exclusion principle (Br74). None of these analyses used polarization data or included the spin orbit interaction.

For the first time a phase shift analysis has been made with polarization data included in the basic data sets. The phases used as starting points in my analysis were from Furber's resonating group calculations on the mass 7 (7Li and 7Be) system which included a complex spin orbit potential and a phenomenological imaginary potential. The depth and shape of the spin orbit potential was determined from low energy 3He-4He, 3H-4He, p-4He,
and n-4He phase shifts, although no polarization for the
7Be system was included in the resonating group analysis.
The Hamiltonian for the calculation contained a superposition
of nucleon-nucleon (N-N) potentials consistent with low
ergy N-N scattering data.

3D.1 SIMPLEX SEARCH ROUTINE

My modification (Bi76) of the simplex algorithm
developed by Nelder and Mead (Ne64) based on the simplex
method of Spendley, Hext, and Himsworth (Sp62) was used
as the main search routine of this analysis. A simplex
of n+1 vertices in the n-dimensional parameter space of
the phases was formed and chi-square evaluated at each
vertex. New simplexes were formed by replacing the
vertex having the worst response (chi-square) with a
reflection, contraction, or expansion in the hyperplane
of the remaining points. The simplex thus changed shape
to conform to the chi-square surface in factor space,
expanding when a favorable change of parameters brought
about a decrease in chi-square and contracting when near
a minimum. Provisions were made to vary the number of
parameters searched in, to fix various phases at constant
values, and to use equations of constraint between
parameters.

No assumptions were made about the surface of chi-square except that it was continuous and had a unique minimum in the vicinity of the initial simplex. (This method converges even when the initial simplex straddles several minima.) The least possible information was used at each step to vary the parameters and past moves were not recorded. In addition, each calculation of chi-square resulted in a movement toward an optimum set of phase shifts. The convergence criterion at a minimum was that suggested by Nelder and Mead (Ne64).

3D.2 RESULTS AND DISCUSSION

The empirical phase shifts giving the best fit to the data are tabulated in Appendix B4. Figure 3.9 shows some of the resonating group predictions and the phases of this analysis. The real parts of the phases lie close to the predicted values and show the odd-even structure observed before (Br74). The imaginary parts of the phases are less well determined, although the addition of total reaction cross sections to the data sets would help to provide some of the needed constraints. Only the F wave phases exhibit resonance behaviour in this
energy range. No indication of positive parity levels at 17 MeV excitation (28 MeV 3He lab energy) is present for the elastic channel.
Figure 3.9  RESONATING GROUP PHASE SHIFTS

The real parts of the resonating group phase shifts (Fu76) used in this analysis are shown.
4. SUMMARY: POLARIZED $^3$He and the COMPOUND NUCLEUS $^7$Be

This thesis reports on a systematic study of the compound nucleus $^7$Be. Our extensive multi-channel $R$ matrix analysis on the $^7$Be system up to 8 MeV excitation energy has successfully fit all the extant data for four different reactions sharing $^7$Be as a compound nucleus. This is the first $R$ matrix investigation to directly fit the data instead of fitting a smoothed set of empirical phase shifts. We have calculated energy levels and their widths more accurately than before, differing only slightly from previous measurements. The low energy $^6$Li destruction cross section has been determined at energies of astrophysical interest (1 eV to 100 keV) and is larger than earlier estimates by 10%.

To extend this analysis to higher energies (up to 28 MeV excitation), we have measured analyzing powers for $^3$He-$^4$He elastic scattering using a polarized $^3$He target. Our $R$ matrix analysis at lower energies provided the calibration for a new $^4$He gas polarimeter to monitor the $^3$He beam polarization in measurements of analyzing powers and relative differential cross sections for the elastic scattering channel up to 45 MeV (lab). I have
made the first phase shift analysis in this energy range to incorporate polarization data as well as cross sections. The results are described qualitatively by resonating group calculations. No positive parity levels of 7Be are observed.
APPENDIX A  R MATRIX ANALYSIS

1. R Matrix Data Sets and Normalizations
2. R Matrix Parameters: $^7$Be
3. R Matrix Parameters: Mass 7
4. Phase Shifts: $^7$Be
5. Figures: Fit to the $^7$Be Data
6. Figures: Fit to the Mass 7 Data
Figure Al.1 NORMALIZATIONS

The circle of alleged normalizations is shown. Our normalizations are compared to those of Harrison.
APPENDIX A.1 NORMALIZATIONS

There is some confusion involving the alleged normalizations of the various data sets. McCray (Mc63) made an absolute measurement of $^6\text{Li}(p,p)^6\text{Li}$ scattering by normalizing to Rutherford scattering. His $^6\text{Li}(p,^3\text{He})^4\text{He}$ reaction data were then normalized to his elastic scattering data. Tombrello and Parker (To63) measured both elastic and reaction cross sections for $^4\text{He}(^3\text{He},^3\text{He})^4\text{He}$ and $^4\text{He}(^3\text{He},p)^6\text{Li}$. They claim their reaction data were normalized to those of McCray (Mc63). "Since extensive data were available for the $^6\text{Li}(p,^4\text{He})^3\text{He}$ reaction in this energy region, these data were related to the $^4\text{He}(^3\text{He},p)^6\text{Li}$ reaction by means of reciprocity relations." (To63). Fasoli (Fa64b) normalized his $^6\text{Li}(p,p)^6\text{Li}$ scattering data to the yield of $^3\text{He}$ from the $^6\text{Li}(p,^3\text{He})^4\text{He}$ reaction (Fa64a) which were then normalized by using the reciprocity theorem to the $^4\text{He}(^3\text{He},p)^6\text{Li}$ data of To63. However, Fasoli claims there is a 17% difference between the elastic scattering data of Fa64b and Mc63.

To complicate the situation, both Jeronymo (Je63) and Harrison (Ha67) claim that To63 made an absolute measurement of the $^4\text{He}(^3\text{He},p)^6\text{Li}$ reaction cross sections.
Jeronymo used detailed balance to normalize his $^{6}\text{Li}(p,3\text{He})^{4}\text{He}$ data to that of To63, while Harrison normalized his $^{6}\text{Li}(p,p)^{6}\text{Li}$ elastic scattering data to that of McCray (Mc63).

Harrison (Ha67 and Ha66) attempted to find a consistent set of relative and absolute normalizations for the sets of data described above. There is still some ambiguity in the normalizations, however, since in the circle of alleged normalizations, Fasoli claims (Fa64) his data differ by 17% with the data of McCray (Mc63) when the data should be identical.
APPENDIX A.1

This Appendix contains data set information for the 7Be system.

NORM Data set normalization which multiplies each datum.
Chi The contribution to chi-square of NORM.
RXN \[ \text{[Type of Data]}(\text{Pair1}, \text{Pair2}) \]
Type of data C = differential cross section
\ P = polarization
\ I = integrated cross section
Particle pairs 1 = 3He + 4He
\ 2 = p + 6Li

COMMENTS
\ A data was taken from published graphs
\ B lab observable converted to center of mass
\ C relative measurements only
\ D(n) contains n data points that had large chi-squares and whose errors were altered
\ E average over energy
\ F normalized at 0.200 MeV to Sawyer and Phillips
\ G(tag) ganged to normalization at tag of same reference
<table>
<thead>
<tr>
<th>NORM</th>
<th>CHI</th>
<th>RXN</th>
<th>TAG</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chuang Ch71</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.969</td>
<td>0.026</td>
<td>C(1,1)</td>
<td>1.72 MeV</td>
<td></td>
</tr>
<tr>
<td>1.057</td>
<td>0.072</td>
<td>C(1,1)</td>
<td>2.46 MeV</td>
<td></td>
</tr>
<tr>
<td>1.274</td>
<td>1.159</td>
<td>C(1,1)</td>
<td>2.98 MeV</td>
<td></td>
</tr>
<tr>
<td>Ivanovitch Iv67</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.067</td>
<td>0.098</td>
<td>C(1,1)</td>
<td>65.3°c.m.</td>
<td>D(2)</td>
</tr>
<tr>
<td>1.017</td>
<td>0.007</td>
<td>C(1,1)</td>
<td>90°c.m.</td>
<td></td>
</tr>
<tr>
<td>1.024</td>
<td>0.013</td>
<td>C(1,1)</td>
<td>125°c.m.</td>
<td></td>
</tr>
<tr>
<td>Boykin Bo72</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.166</td>
<td>0.897</td>
<td>P(1,1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardy Ha72</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.968</td>
<td>0.049</td>
<td>P(1,1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Armstrong Ar69</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.196</td>
<td>0.027</td>
<td>P(1,1)</td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>Barnard Ba64</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.987</td>
<td>0.018</td>
<td>C(1,1)</td>
<td>54.733°c.m.</td>
<td>D(4)</td>
</tr>
<tr>
<td>1.006</td>
<td>0.003</td>
<td>C(1,1)</td>
<td>63.45°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.998</td>
<td>0.001</td>
<td>C(1,1)</td>
<td>73.95°c.m.</td>
<td></td>
</tr>
<tr>
<td>1.002</td>
<td>0.000</td>
<td>C(1,1)</td>
<td>90.03°c.m.</td>
<td></td>
</tr>
<tr>
<td>1.001</td>
<td>0.000</td>
<td>C(1,1)</td>
<td>104.633°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.986</td>
<td>0.020</td>
<td>C(1,1)</td>
<td>106.6°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.992</td>
<td>0.006</td>
<td>C(1,1)</td>
<td>116.617°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.986</td>
<td>0.021</td>
<td>C(1,1)</td>
<td>125.3°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.983</td>
<td>0.030</td>
<td>C(1,1)</td>
<td>140.83°c.m.</td>
<td></td>
</tr>
<tr>
<td>Spiger Sp66</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.932</td>
<td>0.133</td>
<td>C(1,1)</td>
<td>39.2°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.962</td>
<td>0.040</td>
<td>C(1,1)</td>
<td>47.0°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.923</td>
<td>0.171</td>
<td>C(1,1)</td>
<td>54.7°c.m.</td>
<td></td>
</tr>
<tr>
<td>Value 1</td>
<td>Value 2</td>
<td>Atom</td>
<td>Angle</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>0.924</td>
<td>0.170</td>
<td>C(1,1)</td>
<td>63.4°c.m.</td>
<td>D(1)</td>
</tr>
<tr>
<td>0.917</td>
<td>0.208</td>
<td>C(1,1)</td>
<td>70.1°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.910</td>
<td>0.247</td>
<td>C(1,1)</td>
<td>77.0°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.897</td>
<td>0.329</td>
<td>C(1,1)</td>
<td>80°c.m.</td>
<td>D(5)</td>
</tr>
<tr>
<td>0.913</td>
<td>0.226</td>
<td>C(1,1)</td>
<td>85.26°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.909</td>
<td>0.252</td>
<td>C(1,1)</td>
<td>90°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.917</td>
<td>0.203</td>
<td>C(1,1)</td>
<td>98.4°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.905</td>
<td>0.274</td>
<td>C(1,1)</td>
<td>106.4°c.m.</td>
<td>D(2)</td>
</tr>
<tr>
<td>0.891</td>
<td>0.373</td>
<td>C(1,1)</td>
<td>116.8°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.872</td>
<td>0.540</td>
<td>C(1,1)</td>
<td>125.2°c.m.</td>
<td></td>
</tr>
<tr>
<td>0.859</td>
<td>0.669</td>
<td>C(1,1)</td>
<td>135.0°c.m.</td>
<td></td>
</tr>
<tr>
<td>1.066</td>
<td>0.096</td>
<td>C(1,2)</td>
<td>15° lab</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>20° lab</td>
<td>G(39.2°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>25° lab</td>
<td>G(47° c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>30° lab</td>
<td>G(54.7°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>35° lab</td>
<td>G(63.4°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>40° lab</td>
<td>G(70.1°c.m.), D(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>45° lab</td>
<td>G(70.1°c.m.), D(4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>50° lab</td>
<td>G(80°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>55° lab</td>
<td>G(85.26°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>60° lab</td>
<td>G(90°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>65° lab</td>
<td>G(98.4°c.m.), D(2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>70° lab</td>
<td>G(106.4°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>75° lab</td>
<td>G(116.8°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>80° lab</td>
<td>G(116.8°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>85° lab</td>
<td>G(125.2°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>22.5° lab</td>
<td>G(39.2°c.m.), D(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>31.6° lab</td>
<td>G(63.4°c.m.), D(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>36.8° lab</td>
<td>G(63.4°c.m.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C(1,2)</td>
<td>40.8° lab</td>
<td>G(70.1°c.m.)</td>
</tr>
</tbody>
</table>

Spinka Sp71
<table>
<thead>
<tr>
<th>Source</th>
<th>20° lab helium</th>
<th>20° lab alpha</th>
<th>60° lab helium</th>
<th>60° lab alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marion Ma56</td>
<td>1.00</td>
<td>0.700</td>
<td>0.14</td>
<td>0.013</td>
</tr>
<tr>
<td>Spieler</td>
<td>2.00</td>
<td>1.20</td>
<td>0.20</td>
<td>0.010</td>
</tr>
<tr>
<td>Brown Br68</td>
<td>0.790</td>
<td>0.677</td>
<td>0.14</td>
<td>0.013</td>
</tr>
<tr>
<td>Fiedler Pl81</td>
<td>3.00</td>
<td>2.00</td>
<td>0.20</td>
<td>0.010</td>
</tr>
<tr>
<td>Gemeinhart</td>
<td>1.093</td>
<td>0.677</td>
<td>0.14</td>
<td>0.013</td>
</tr>
<tr>
<td>Bouchet Bo60</td>
<td>0.829</td>
<td>0.677</td>
<td>0.14</td>
<td>0.013</td>
</tr>
<tr>
<td>Khanh Kh61</td>
<td>0.797</td>
<td>0.677</td>
<td>0.14</td>
<td>0.013</td>
</tr>
<tr>
<td>Beaumeville Be63</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Bashkin Ba51</td>
<td>6.748</td>
<td>3.958</td>
<td>0.34</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Excited fcn.

A,B,C

20° lab helium

A,B,D(5)

60° lab helium

A,B,D(11)

C(2,1)

G(164°) lab A,B,D(3)

C(2,2)

164° lab A

C(2,3)

7.217

2.160
<table>
<thead>
<tr>
<th>Fasoli  Fa64</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.903</td>
<td>0.185</td>
<td>C(2,2)</td>
<td>90°c.m.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>0.910</td>
<td>0.156</td>
<td>C(2,2)</td>
<td>110°c.m.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>0.990</td>
<td>0.001</td>
<td>C(2,2)</td>
<td>125°c.m.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>1.085</td>
<td>0.097</td>
<td>C(2,2)</td>
<td>147°c.m.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>1.105</td>
<td>0.144</td>
<td>C(2,2)</td>
<td>150°c.m.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>1.120</td>
<td>0.183</td>
<td>C(2,2)</td>
<td>169°c.m.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>McCray Mc63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.155</td>
<td>7.203</td>
<td>C(2,2)</td>
<td>70°c.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.104</td>
<td>3.532</td>
<td>C(2,2)</td>
<td>90°c.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.085</td>
<td>2.434</td>
<td>C(2,2)</td>
<td>110°c.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.120</td>
<td>4.598</td>
<td>C(2,2)</td>
<td>125°c.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.159</td>
<td>7.517</td>
<td>C(2,2)</td>
<td>140°c.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.196</td>
<td>10.78</td>
<td>C(2,2)</td>
<td>160°c.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harrison Ha63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.098</td>
<td>0.127</td>
<td>C(2,2)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### 7Be R Matrix Channels

<table>
<thead>
<tr>
<th>CHANNEL</th>
<th>PARTICLES</th>
<th>$l_{\text{max}}$</th>
<th>$s_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$^3\text{He} + ^4\text{He}$</td>
<td>4</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>$^p + ^6\text{Li}$</td>
<td>1</td>
<td>3/2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>INDEX</th>
<th>CHANNEL</th>
<th>$(2S+1)L$</th>
<th>$S$</th>
<th>$l$</th>
<th>$j^*$</th>
<th>RADIUS (fm)</th>
<th>BOUNDARY COND.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$2S$</td>
<td>1/2</td>
<td>0</td>
<td>1/2+</td>
<td>4.0206452</td>
<td>-0.0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$2S$</td>
<td>1/2</td>
<td>0</td>
<td>1/2+</td>
<td>4.2026757</td>
<td>-0.0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>$2P$</td>
<td>1/2</td>
<td>1</td>
<td>3/2−</td>
<td>4.0206452</td>
<td>-1.351</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>$4P$</td>
<td>3/2</td>
<td>1</td>
<td>3/2−</td>
<td>4.2026757</td>
<td>-1.601</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>$2P$</td>
<td>1/2</td>
<td>1</td>
<td>3/2−</td>
<td>4.2026757</td>
<td>-1.601</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>$2P$</td>
<td>1/2</td>
<td>1</td>
<td>1/2−</td>
<td>4.0206452</td>
<td>-1.203</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>$4P$</td>
<td>3/2</td>
<td>1</td>
<td>1/2−</td>
<td>4.2026757</td>
<td>-1.531</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>$2P$</td>
<td>1/2</td>
<td>1</td>
<td>1/2−</td>
<td>4.2026757</td>
<td>-1.531</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>$2D$</td>
<td>1/2</td>
<td>2</td>
<td>5/2+</td>
<td>4.0206452</td>
<td>-2</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>$2D$</td>
<td>1/2</td>
<td>2</td>
<td>3/2+</td>
<td>4.0206452</td>
<td>-2</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>$4S$</td>
<td>3/2</td>
<td>0</td>
<td>3/2+</td>
<td>4.2026757</td>
<td>-0</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>$2F$</td>
<td>1/2</td>
<td>3</td>
<td>7/2−</td>
<td>4.0206452</td>
<td>-3</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>$2F$</td>
<td>1/2</td>
<td>3</td>
<td>5/2−</td>
<td>4.0206452</td>
<td>-3</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>$4P$</td>
<td>3/2</td>
<td>1</td>
<td>5/2−</td>
<td>4.2026757</td>
<td>-1</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>$2G$</td>
<td>1/2</td>
<td>4</td>
<td>9/2+</td>
<td>4.0206452</td>
<td>-4</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>$2G$</td>
<td>1/2</td>
<td>4</td>
<td>7/2+</td>
<td>4.0206452</td>
<td>-4</td>
</tr>
</tbody>
</table>
### Appendix A2

#### $^7$Be $\Xi$-Matrix Parameters

<table>
<thead>
<tr>
<th>$J^P$=1/2$^+$</th>
<th>$J^P$=3/2$^-$</th>
<th>$J^P$=1/2$^-$</th>
<th>$J^P$=3/2$^-$</th>
<th>$J^P$=5/2$^-$</th>
<th>$J^P$=7/2$^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>0.285</td>
<td>-100 $^P$</td>
<td>100 $^P$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$ 1 $2S$</td>
<td>1.021</td>
<td>4.030</td>
<td>6.934</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$ 2 $2S$</td>
<td>0.097</td>
<td>0.0 $^P$</td>
<td>-8.401</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_2$</td>
<td>9.200</td>
<td>-1.586 $^P$</td>
<td>100 $^P$</td>
<td>100 $^P$</td>
<td>-100 $^P$</td>
</tr>
<tr>
<td>$\gamma$ 1 $2P$</td>
<td>0.990</td>
<td>-0.302 $^P$</td>
<td>-1.077 $^P$</td>
<td>0.702 $^P$</td>
<td>-4.21 $^P$</td>
</tr>
<tr>
<td>$\gamma$ 2 $4P$</td>
<td>-0.443 $^P$</td>
<td>0.0 $^P$</td>
<td>-3.787 $^P$</td>
<td>-4.525 $^P$</td>
<td>0.0 $^P$</td>
</tr>
<tr>
<td>$\gamma$ 2 $2P$</td>
<td>-1.165 $^P$</td>
<td>0.0 $^P$</td>
<td>4.174 $^P$</td>
<td>0.0 $^P$</td>
<td>0.0 $^P$</td>
</tr>
<tr>
<td>$E_3$</td>
<td>-1.157 $^P$</td>
<td>100 $^P$</td>
<td>100 $^P$</td>
<td></td>
<td>-100 $^P$</td>
</tr>
<tr>
<td>$\gamma$ 1 $2P$</td>
<td>0.925</td>
<td>3.421</td>
<td>0.0 $^P$</td>
<td>0.0 $^P$</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ 2 $4P$</td>
<td>0.0 $^P$</td>
<td>-4.164 $^P$</td>
<td>5.225 $^P$</td>
<td>0.0 $^P$</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ 2 $2P$</td>
<td>0.0 $^P$</td>
<td>-7.431 $^P$</td>
<td>-10.279 $^P$</td>
<td>-17.795 $^P$</td>
<td></td>
</tr>
<tr>
<td>$E_4$</td>
<td>11.747</td>
<td>100 $^P$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$ 1 $2D$</td>
<td>1.079</td>
<td>0.293</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### $^7$Be $\Xi$-Matrix Parameters

<table>
<thead>
<tr>
<th>$J^P$=3/2$^+$</th>
<th>$J^P$=7/2$^-$</th>
<th>$J^P$=5/2$^-$</th>
<th>$J^P$=7/2$^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>6.811</td>
<td>21.342</td>
<td>-121.95 $^P$</td>
</tr>
<tr>
<td>$\gamma$ 1 $2D$</td>
<td>0.383</td>
<td>-2.395</td>
<td>-4.717 $^P$</td>
</tr>
<tr>
<td>$\gamma$ 2 $4S$</td>
<td>-2.044 $^P$</td>
<td>-.108 $^P$</td>
<td>0.0 $^P$</td>
</tr>
<tr>
<td>$E_2$</td>
<td>3.517</td>
<td>145.99 $^P$</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ 1 $2P$</td>
<td>0.851</td>
<td>7.649</td>
<td></td>
</tr>
<tr>
<td>$E_3$</td>
<td>6.192</td>
<td>5.749</td>
<td>100 $^P$</td>
</tr>
<tr>
<td>$\gamma$ 1 $2P$</td>
<td>0.664</td>
<td>0.182</td>
<td>-0.648 $^P$</td>
</tr>
<tr>
<td>$\gamma$ 2 $4P$</td>
<td>-0.297 $^P$</td>
<td>0.857 $^P$</td>
<td>2.848 $^P$</td>
</tr>
<tr>
<td>$E_4$</td>
<td>-100 $^P$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$ 1 $2G$</td>
<td>5.098</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_5$</td>
<td>-100 $^P$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$ 1 $2G$</td>
<td>8.172</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### $^7$Be $\Xi$-Matrix Parameters

<table>
<thead>
<tr>
<th>$J^P$=9/2$^-$</th>
<th>$J^P$=7/2$^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_6$</td>
<td>-100 $^P$</td>
</tr>
<tr>
<td>$\gamma$ 1 $2G$</td>
<td>8.172</td>
</tr>
</tbody>
</table>
## APPENDIX A3

### $^{7}$Be - $^{7}$Li $\beta$-MATRIX PARAMETERS

#### MASS 7 ANALYSIS

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\lambda$</th>
<th>$\lambda$</th>
<th>$\lambda$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}^+$</td>
<td>$\frac{3}{2}^-$</td>
<td>$\frac{1}{2}^-$</td>
<td>$\frac{5}{2}^+$</td>
<td>$\frac{3}{2}^+$</td>
</tr>
<tr>
<td>$E_1$</td>
<td>$E_2$</td>
<td>$E_3$</td>
<td>$E_4$</td>
<td>$E_5$</td>
</tr>
<tr>
<td>$1S$</td>
<td>$2S$</td>
<td>$2P$</td>
<td>$2D$</td>
<td>$2F$</td>
</tr>
<tr>
<td>$Y_1$</td>
<td>$Y_2$</td>
<td>$Y_2$</td>
<td>$Y_1$</td>
<td>$Y_2$</td>
</tr>
</tbody>
</table>

### $^{7}$Be - $^{7}$Li $\beta$-PARAMETERS

#### $\lambda=5$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\lambda$</th>
<th>$\lambda$</th>
<th>$\lambda$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{7}{2}^+$</td>
<td>$\frac{5}{2}^-$</td>
<td>$\frac{3}{2}^-$</td>
<td>$\frac{5}{2}^+$</td>
<td>$\frac{3}{2}^+$</td>
</tr>
<tr>
<td>$E_1$</td>
<td>$E_2$</td>
<td>$E_3$</td>
<td>$E_4$</td>
<td>$E_5$</td>
</tr>
<tr>
<td>$1S$</td>
<td>$2S$</td>
<td>$2P$</td>
<td>$2D$</td>
<td>$2F$</td>
</tr>
<tr>
<td>$Y_1$</td>
<td>$Y_2$</td>
<td>$Y_2$</td>
<td>$Y_1$</td>
<td>$Y_2$</td>
</tr>
<tr>
<td>$\Delta E_1 = -0.8649$</td>
<td>$\Delta E_2 = 1.4708$</td>
<td>$\gamma_{AC}(^{7}Li) = \gamma_{AC}(^{7}Be)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX A4  $^7$Be PHASE SHIFTS

$^7$Be phase shifts from the R matrix analysis are tabulated for the four reactions studied. Real parts of the phases and the amplitudes (absorption coefficients of the imaginary parts) are listed as a function of incident $^3$He energy for several energies in the analysis.
<table>
<thead>
<tr>
<th>E(3He)</th>
<th>5/2⁺</th>
<th>P₃/₂⁻</th>
<th>P₁/₂⁻</th>
<th>D₃/₂⁺</th>
<th>D₃/₂⁻</th>
<th>F₃/₂⁻</th>
<th>F₅/₂⁻</th>
<th>G₉/₂⁺</th>
<th>G₇/₂⁺</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.72</td>
<td>0.085 1.0</td>
<td>-3.567 1.0</td>
<td>-7.065 1.0</td>
<td>-0.097 1.0</td>
<td>-0.152 1.0</td>
<td>0.030 1.0</td>
<td>0.021 1.0</td>
<td>-0.001 1.0</td>
<td>-0.001 1.0</td>
</tr>
<tr>
<td>2.493</td>
<td>-16.011 1.0</td>
<td>-8.488 1.0</td>
<td>-13.088 1.0</td>
<td>-0.359 1.0</td>
<td>-0.532 1.0</td>
<td>0.197 1.0</td>
<td>0.125 1.0</td>
<td>-0.004 1.0</td>
<td>-0.009 1.0</td>
</tr>
<tr>
<td>2.994</td>
<td>-20.937 1.0</td>
<td>-12.175 1.0</td>
<td>-17.096 1.0</td>
<td>-0.641 1.0</td>
<td>-0.921 1.0</td>
<td>0.510 1.0</td>
<td>0.294 1.0</td>
<td>-0.011 1.0</td>
<td>-0.022 1.0</td>
</tr>
<tr>
<td>3.495</td>
<td>-25.538 1.0</td>
<td>-16.011 1.0</td>
<td>-21.027 1.0</td>
<td>-1.011 1.0</td>
<td>-1.409 1.0</td>
<td>1.197 1.0</td>
<td>0.599 1.0</td>
<td>-0.024 1.0</td>
<td>-0.048 1.0</td>
</tr>
<tr>
<td>4.001</td>
<td>-29.796 1.0</td>
<td>-19.910 1.0</td>
<td>-24.865 1.0</td>
<td>-1.466 1.0</td>
<td>-1.984 1.0</td>
<td>2.772 1.0</td>
<td>1.114 1.0</td>
<td>-0.040 1.0</td>
<td>-0.092 1.0</td>
</tr>
<tr>
<td>4.493</td>
<td>-31.304 1.0</td>
<td>-21.376 1.0</td>
<td>-26.280 1.0</td>
<td>-1.658 1.0</td>
<td>-2.219 1.0</td>
<td>3.872 1.0</td>
<td>1.383 1.0</td>
<td>-0.060 1.0</td>
<td>-0.115 1.0</td>
</tr>
<tr>
<td>4.905</td>
<td>-32.900 1.0</td>
<td>-22.980 1.0</td>
<td>-27.816 1.0</td>
<td>-1.879 1.0</td>
<td>-2.486 1.0</td>
<td>5.758 1.0</td>
<td>1.738 1.0</td>
<td>-0.076 1.0</td>
<td>-0.145 1.0</td>
</tr>
<tr>
<td>4.607</td>
<td>-34.351 1.0</td>
<td>-24.489 1.0</td>
<td>-29.252 1.0</td>
<td>-2.099 1.0</td>
<td>-2.747 1.0</td>
<td>8.828 1.0</td>
<td>2.142 1.0</td>
<td>-0.094 1.0</td>
<td>-0.178 1.0</td>
</tr>
<tr>
<td>4.807</td>
<td>-35.723 1.0</td>
<td>-25.961 1.0</td>
<td>-30.647 1.0</td>
<td>-2.324 1.0</td>
<td>-3.009 1.0</td>
<td>14.742 1.0</td>
<td>2.615 1.0</td>
<td>-0.115 1.0</td>
<td>-0.215 1.0</td>
</tr>
<tr>
<td>5.604</td>
<td>-37.010 1.0</td>
<td>-27.387 1.0</td>
<td>-31.396 1.0</td>
<td>-2.552 1.0</td>
<td>-3.270 1.0</td>
<td>29.312 1.0</td>
<td>3.165 1.0</td>
<td>-0.139 1.0</td>
<td>-0.258 1.0</td>
</tr>
<tr>
<td>5.202</td>
<td>-38.239 1.0</td>
<td>-28.794 1.0</td>
<td>-33.325 1.0</td>
<td>-2.786 1.0</td>
<td>-3.534 1.0</td>
<td>79.437 1.0</td>
<td>3.815 1.0</td>
<td>-0.166 1.0</td>
<td>-0.306 1.0</td>
</tr>
<tr>
<td>5.400</td>
<td>-39.403 1.0</td>
<td>-30.171 1.0</td>
<td>-34.628 1.0</td>
<td>-3.023 1.0</td>
<td>-3.797 1.0</td>
<td>137.146 1.0</td>
<td>4.578 1.0</td>
<td>-0.197 1.0</td>
<td>-0.361 1.0</td>
</tr>
<tr>
<td>5.619</td>
<td>-40.614 1.0</td>
<td>-31.657 1.0</td>
<td>-36.038 1.0</td>
<td>-3.288 1.0</td>
<td>-4.086 1.0</td>
<td>155.416 1.0</td>
<td>5.577 1.0</td>
<td>-0.237 1.0</td>
<td>-0.429 1.0</td>
</tr>
<tr>
<td>5.942</td>
<td>-42.251 1.0</td>
<td>-33.770 1.0</td>
<td>-38.059 1.0</td>
<td>-3.681 1.0</td>
<td>-4.504 1.0</td>
<td>163.695 1.0</td>
<td>7.419 1.0</td>
<td>-0.304 1.0</td>
<td>-0.564 1.0</td>
</tr>
<tr>
<td>6.445</td>
<td>-44.422 1.0</td>
<td>-36.848 1.0</td>
<td>-41.062 1.0</td>
<td>-4.285 1.0</td>
<td>-5.123 1.0</td>
<td>168.072 1.0</td>
<td>11.682 1.0</td>
<td>-0.435 1.0</td>
<td>-0.763 1.0</td>
</tr>
<tr>
<td>6.948</td>
<td>-46.026 1.0</td>
<td>-39.624 1.0</td>
<td>-43.878 1.0</td>
<td>-4.859 1.0</td>
<td>-5.681 1.0</td>
<td>169.964 1.0</td>
<td>17.790 1.0</td>
<td>-0.600 1.0</td>
<td>-1.034 1.0</td>
</tr>
<tr>
<td>7.450</td>
<td>-46.498 0.9391</td>
<td>-42.023 0.9998</td>
<td>-26.445 0.9991</td>
<td>-5.390 1.0</td>
<td>-6.155 1.0</td>
<td>171.129 1.0</td>
<td>27.832 1.0</td>
<td>-0.804 1.0</td>
<td>-1.361 1.0</td>
</tr>
<tr>
<td>7.953</td>
<td>-47.420 0.9145</td>
<td>-43.978 0.9313</td>
<td>-48.787 0.9931</td>
<td>-5.825 1.0</td>
<td>-6.523 0.9998</td>
<td>172.066 1.0</td>
<td>44.001 1.0</td>
<td>-1.051 1.0</td>
<td>-1.757 1.0</td>
</tr>
<tr>
<td>8.455</td>
<td>-49.438 0.8534</td>
<td>-45.419 0.9328</td>
<td>-51.020 0.9803</td>
<td>-6.168 1.0</td>
<td>-6.753 0.9963</td>
<td>172.977 1.0</td>
<td>67.598 0.9954</td>
<td>-1.342 1.0</td>
<td>-2.193 1.0</td>
</tr>
<tr>
<td>8.957</td>
<td>-51.883 0.8128</td>
<td>-46.260 0.9866</td>
<td>-53.210 0.9619</td>
<td>-6.383 1.0</td>
<td>-6.825 0.9981</td>
<td>173.971 1.0</td>
<td>93.731 0.99234</td>
<td>-1.681 1.0</td>
<td>-2.701 1.0</td>
</tr>
<tr>
<td>9.459</td>
<td>-54.398 0.79028</td>
<td>-46.372 0.9739</td>
<td>-55.416 0.9392</td>
<td>-6.440 1.0</td>
<td>-6.731 0.99461</td>
<td>175.117 1.0</td>
<td>114.388 0.93653</td>
<td>-2.069 1.0</td>
<td>-3.273 1.0</td>
</tr>
<tr>
<td>9.961</td>
<td>-56.797 0.78267</td>
<td>-45.588 0.9559</td>
<td>-57.687 0.9137</td>
<td>-6.608 1.0</td>
<td>-6.848 0.98794</td>
<td>176.462 1.0</td>
<td>125.206 0.91835</td>
<td>-2.507 1.0</td>
<td>-3.906 1.0</td>
</tr>
<tr>
<td>10.462</td>
<td>-58.965 0.78779</td>
<td>-43.718 0.93034</td>
<td>-60.098 0.8876</td>
<td>-5.956 1.0</td>
<td>-6.119 0.87764</td>
<td>178.040 1.0</td>
<td>134.354 0.96123</td>
<td>-2.996 1.0</td>
<td>-4.600 1.0</td>
</tr>
<tr>
<td>10.880</td>
<td>-60.257 0.79763</td>
<td>-41.737 0.90802</td>
<td>-61.696 0.87059</td>
<td>-5.574 1.0</td>
<td>-5.829 0.96876</td>
<td>179.249 1.0</td>
<td>139.560 0.97323</td>
<td>-3.355 1.0</td>
<td>-5.101 1.0</td>
</tr>
</tbody>
</table>
### 7 Be PHASE SHIFTS

\[ \langle \text{He} (^3\text{He}, p) \rangle L_1 \] and \[ L_1(p, ^3\text{He}) \langle \text{He} \rangle \]

<table>
<thead>
<tr>
<th>( E(^3\text{He}) )</th>
<th>( 1/2^+ )</th>
<th>( 2P-4P )</th>
<th>( 3/2^- )</th>
<th>( 2P-2P )</th>
<th>( 1/2^- )</th>
<th>( 2P-4P )</th>
<th>( 2P-2P )</th>
<th>( 3/2^+ )</th>
<th>( 2D-4S )</th>
<th>( 5/2^- )</th>
<th>( 2F-4P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.450</td>
<td>-68.51</td>
<td>0.1706</td>
<td>-66.03</td>
<td>0.0115</td>
<td>-66.04</td>
<td>0.0127</td>
<td>-68.21</td>
<td>0.0248</td>
<td>-68.37</td>
<td>0.0276</td>
<td>41.58</td>
</tr>
<tr>
<td>7.953</td>
<td>-70.50</td>
<td>0.4045</td>
<td>-67.17</td>
<td>0.0428</td>
<td>-67.31</td>
<td>0.0436</td>
<td>-69.19</td>
<td>0.0878</td>
<td>-70.88</td>
<td>0.0802</td>
<td>-50.12</td>
</tr>
<tr>
<td>8.455</td>
<td>-73.50</td>
<td>0.5212</td>
<td>-68.15</td>
<td>0.0812</td>
<td>-68.66</td>
<td>0.0762</td>
<td>-69.74</td>
<td>0.1577</td>
<td>-74.39</td>
<td>0.1203</td>
<td>-51.55</td>
</tr>
<tr>
<td>8.957</td>
<td>-76.72</td>
<td>0.5825</td>
<td>-68.83</td>
<td>0.1263</td>
<td>-70.05</td>
<td>0.1083</td>
<td>-69.89</td>
<td>0.2287</td>
<td>-78.50</td>
<td>0.1999</td>
<td>-52.28</td>
</tr>
<tr>
<td>9.459</td>
<td>-79.83</td>
<td>0.6128</td>
<td>-69.08</td>
<td>0.1788</td>
<td>-71.42</td>
<td>0.1396</td>
<td>-69.77</td>
<td>0.2965</td>
<td>-82.95</td>
<td>0.1730</td>
<td>-52.29</td>
</tr>
<tr>
<td>9.961</td>
<td>-82.70</td>
<td>0.6224</td>
<td>-68.78</td>
<td>0.2393</td>
<td>-72.69</td>
<td>0.1701</td>
<td>-69.51</td>
<td>0.2393</td>
<td>-87.56</td>
<td>0.1929</td>
<td>-51.67</td>
</tr>
<tr>
<td>10.462</td>
<td>-85.24</td>
<td>0.6159</td>
<td>-67.88</td>
<td>0.3074</td>
<td>-73.78</td>
<td>0.1999</td>
<td>-69.26</td>
<td>0.4092</td>
<td>-92.09</td>
<td>0.2114</td>
<td>-50.60</td>
</tr>
<tr>
<td>10.800</td>
<td>-86.74</td>
<td>0.6031</td>
<td>-66.90</td>
<td>0.3568</td>
<td>-74.36</td>
<td>0.2196</td>
<td>-69.16</td>
<td>0.4382</td>
<td>-95.08</td>
<td>0.2237</td>
<td>-49.72</td>
</tr>
</tbody>
</table>

### \( L_1(p, p) \langle L_1 \rangle \)

<table>
<thead>
<tr>
<th>( E(^3\text{He}) )</th>
<th>( 1/2^+ )</th>
<th>( 2P-4P )</th>
<th>( 3/2^- )</th>
<th>( 2P-2P )</th>
<th>( 1/2^- )</th>
<th>( 2P-4P )</th>
<th>( 2P-2P )</th>
<th>( 3/2^+ )</th>
<th>( 4P-4S )</th>
<th>( 5/2^- )</th>
<th>( 4P-4P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.450</td>
<td>0.521</td>
<td>0.9391</td>
<td>0.007</td>
<td>0.9993</td>
<td>-46.500</td>
<td>0.00140</td>
<td>-0.018</td>
<td>0.9992</td>
<td>0.065</td>
<td>0.99969</td>
<td>-52.35</td>
</tr>
<tr>
<td>7.993</td>
<td>0.579</td>
<td>0.91454</td>
<td>0.096</td>
<td>0.9987</td>
<td>-46.774</td>
<td>0.01550</td>
<td>-0.190</td>
<td>0.99893</td>
<td>0.765</td>
<td>0.99605</td>
<td>-53.36</td>
</tr>
<tr>
<td>8.455</td>
<td>0.566</td>
<td>0.85360</td>
<td>0.322</td>
<td>0.99570</td>
<td>-47.107</td>
<td>0.04449</td>
<td>-0.530</td>
<td>0.99610</td>
<td>2.325</td>
<td>0.98682</td>
<td>-54.70</td>
</tr>
<tr>
<td>8.959</td>
<td>0.315</td>
<td>0.81282</td>
<td>0.704</td>
<td>0.98839</td>
<td>-47.450</td>
<td>0.08447</td>
<td>-0.990</td>
<td>0.99052</td>
<td>4.643</td>
<td>0.97135</td>
<td>-56.02</td>
</tr>
<tr>
<td>9.459</td>
<td>2.127</td>
<td>0.79208</td>
<td>1.239</td>
<td>0.97507</td>
<td>-47.931</td>
<td>0.13145</td>
<td>-1.537</td>
<td>0.98145</td>
<td>7.528</td>
<td>0.95027</td>
<td>-57.21</td>
</tr>
<tr>
<td>9.961</td>
<td>1.810</td>
<td>0.78267</td>
<td>1.906</td>
<td>0.95372</td>
<td>-48.458</td>
<td>0.18210</td>
<td>-2.159</td>
<td>0.96846</td>
<td>10.742</td>
<td>0.92543</td>
<td>-58.31</td>
</tr>
<tr>
<td>10.462</td>
<td>2.519</td>
<td>0.78779</td>
<td>2.661</td>
<td>0.92243</td>
<td>-49.104</td>
<td>0.23373</td>
<td>-2.853</td>
<td>0.95152</td>
<td>14.033</td>
<td>0.89054</td>
<td>-59.40</td>
</tr>
<tr>
<td>10.800</td>
<td>2.220</td>
<td>0.79763</td>
<td>3.182</td>
<td>0.89483</td>
<td>-49.625</td>
<td>0.26829</td>
<td>-3.366</td>
<td>0.93798</td>
<td>16.188</td>
<td>0.88275</td>
<td>-60.17</td>
</tr>
</tbody>
</table>

Note: The values represent partial wave phase shifts in radians for the indicated reactions at various energies (in MeV).
APPENDIX A5    FIT TO THE $^7$Be DATA

The fit of the R matrix analysis to the $^7$Be data is illustrated. Figures were chosen at random to give an unbiased sampling of the types, amount, and fits to the data. All four reactions are represented. Angles and energies are indicated. All experimental data which are plotted are plotted as normalized by this analysis. Normalizations may be found in Appendix A1.
APPENDIX A6 FIT TO THE MASS 7 DATA

The fit of the R matrix analysis to the mass 7 (7\text{Li} and 7\text{Be}) data is illustrated. Figures were chosen at random, and only a small fraction of the data are presented. All data are plotted as normalized by the analysis. All the reactions in the analysis are represented. Angles and energies are indicated.

The fit to the \(^3\text{He} + \(^4\text{He}\) elastic scattering at 106° cm is incorrectly plotted at the lower energies. The actual fit to the low energy data at this angle is much better.
APPENDIX B PHASE SHIFT ANALYSIS

1. Coincidence Geometry
2. Polarimeter and Crosstalk
3. Cross Section Formulae
4. $^7$Be Phase Shifts
APPENDIX B1

COINCIDENCE GEOMETRY

Little is known about geometric corrections to the Silverstein "G factor" (Si59) for gas scattering experiments (Wa66). The G factor relates the yield to the beam intensity, target density, and scattering cross section. Silverstein calculates this geometry factor for several types of aperture boundaries for gas scattering experiments in which a beam traverses the target gas and scattered particles enter the detector through a system of two apertures in parallel planes. For two rectangular slits of widths \( w_1 \) (\( w_2 \)) and height \( h \) a distance \( R_1 \) (\( R_2 \)) from the scatter, the zeroth order G factor is

\[
G = \frac{w_1 w_2 h}{R_2 d \sin \theta_0}
\]

where \( d \) is the distance between slits. It is easily shown that to maximize the counting rate (yield \( Y \)) in a counter given by

\[
Y = I n \sigma G
\]

(symbols are described earlier in the text), the slits should be symmetric, i.e.,

\[
w_1 = w_2 = w
\]
The angular acceptance $\delta$ relates the other parameters

$$\delta = \arctan(w/d)$$

Various combinations of slit sizes and separations give the same angular acceptance, although some constraints can be imposed by the size of the apparatus. For a given angular acceptance, the counting rate is maximized by moving the collimating apertures as close to the scattering cell as possible.

In an experiment in which particles are detected in coincidence, the yield depends on the solid angle efficiency function $\Omega(\theta) (SAEF)$ instead of the $G$ factor. This function gives the effective solid angle for the entire system of detectors and changes the effective angular acceptance from that of the scattering without the coincidence requirement. If $z_1$ and $z_2$ refer to the boundaries of the overlap of the penumbras for the two detectors in a coincidence experiment (refer to figure Bl.1), $\theta$ is the actual scattering angle of the detected particle, and $\theta_0$ is the nominal scattering angle of the detectors, we define the solid angle efficiency function $(SAEF)$

$$\Omega(\theta) = \int_{z_1}^{z_2} H(z, \theta) \, dz$$
The "hit function" \( H(Z, \Theta) \) is defined below. We restrict ourselves to the geometry of the polarimeter used in this experiment, although the general case will be discussed in a forthcoming paper. In this instance, both detectors are at symmetric angles \( \Theta_o \) with respect to the beam (Z-axis). As discussed in the text, both the 3He and 4He emerge from the scattering at this lab scattering angle and can be detected in coincidence. If the 3He scatters at a different lab angle \( \Theta_3 \) other than \( \Theta_o \), the 4He lab scattering angle \( \Theta_4 \) is defined by the kinematics. For a given point Z on the beam line in the penumbra of each detector, the "hit function" is

\[
H(Z, \Theta) = H_1(Z, \Theta_3) H_2(Z, \Theta_4)
\]

where \( H_i(Z, \Theta_j^i) = 1 \) if a particle scattered at angle \( \Theta_j^i \) from point Z lands in detector \( i \), and \( H_i(Z, \Theta_j^i) = 0 \) otherwise. Before calculating \( \Omega(\Theta) \), we carry out some simplifications. Since large corrections originate from the overlap of penumbras and intensity distribution of the beam in the reaction plane, we neglect the deviation of the target point to the direction of the y-axis (out of the scattering plane).

In this approximation, the SAEF is the area of the beam that will produce counts in the detectors as a
function of scattering angle. The integration of the SAEF over the angular acceptance gives the Silverstein $G$ factor.

$$G(\theta) = \int_{\theta_{0}-\delta}^{\theta_{0}+\delta} \Omega(\theta) \, d\theta$$

We wish to investigate the effects of the coincidence requirement, finite beam size, and longitudinal as well as angular misalignment of the detectors on $\Omega(\theta)$. The effects of different beam profiles (gaussian, linear, homogeneous, etc) are relegated to a later paper. In our experiment the beam was uniform in intensity and rectangular in shape due to the homogenizing foils and tantalum collimation apertures. This is the only case we shall study here. Monte Carlo techniques are unnecessary since the SAEF can be calculated analytically if the proper parameterization is chosen.

Refer to figure B1.0 for the details of the geometry used in the calculation. Two identical collimator systems were placed at symmetric angles $\theta_{0}$ on either side of the beam ($Z$-axis). The right ($4\text{He}$) detector was allowed to have an angular error $\Delta \theta$ ($\theta_{4} = \theta_{0} + \Delta \theta$) and could be displaced along the $Z$-axis so that the umbrae of it and the left ($3\text{He}$) detectors were shifted. The
Figure B1.0  COINCIDENCE GEOMETRY
beam was taken to be rectangular in shape with a total width $\Delta x$ centered at $X = 0$; during the experiment the measured width was 0.5 cm.

EFFECTS OF COINCIDENCE REQUIREMENT

Assuming a line beam and no geometric alignment errors, figure B1.1 illustrates the sharpening of the angular acceptance due to the coincidence requirement. In both cases, the SAEF is strictly triangular in shape. This is in contrast to Halbert, Mason, and Northcliffe (Ha68), who claim that for singles the detection efficiency is linear along the beam axis while in the penumbra, but that a coincidence requirement makes it become quadratic.

EFFECTS OF FINITE BEAM SIZE

With the coincidence requirement and no misalignment, the effect of a homogeneous beam of width $\Delta x = 0.5$ cm ($\Delta x = 0.07R_2$) was studied. Figure B1.2a shows the contributions to the SAEF of each part of the beam, while figure B1.2b shows the integrated SAEF over the width of the beam. Notice that the function is no longer triangular but is quadratic, and that the effective
Figure B1.1  SAEF Singles vs. Coincidence

The SAEF for singles (S) and coincidence (C) geometries are compared for a line beam.
Figure Bl.2  SAEF and Beam Spread

The effect of the finite beam size on the SAEF is shown in figure A. The beam is centered about $x=0$. Figure B is the SAEF integrated over $x$. 
angular acceptance is larger than that of a line beam but is still smaller than if the coincidence requirement were removed.

**EFFECT OF ANGULAR ERRORS ON ONE DETECTOR**

Keeping the coincidence requirement and correction for finite beam size, we explore the distortion in the SAEF due to an angular error in aligning one of the detectors. Figure B1.3 depicts the effects on the SAEF (integrated over the width of the beam) caused by angular errors $\Delta \theta$ in the 4He detector. Notice the "flat-top" effect and smearing of the angular acceptance as a function of $\Delta \theta$. An angular error in the 3He detector can be translated to an equivalent angular error in the 4He detector, but they will not be the same because of the kinematics. In the experiment reported in this thesis, the angular error was a maximum of 0.5°.

**EFFECTS OF SHIFTING BOTH DETECTORS BY A SMALL ANGLE**

It is possible for both detectors to be shifted in angle by the same amount, that is,

$$\theta_3 = \theta_0 + \Delta \theta, \quad \theta_4 = \theta_0 - \Delta \theta$$
Figure B1.3  Angular Error One Detector

The effects of angular errors of 0°, 0.5°, and 1.0° in the right detector on the SAEP are shown.
Figure Bl.4 illustrates the effect of rotating the entire collimation system for both detectors in coincidence by an amount $\Delta \theta$ with respect to the beam axis. As before, the beam is assumed to have a finite width. Notice the shift of the maximum of SAEF away from the nominal detector angle and the distortion as the angular shift becomes large.

EFFECTS OF LONGITUDINAL ERRORS

Figure Bl.5 shows the effects on the SAEF as a function of displacements of one of the detectors (4He) along the beam axis. There are dramatic changes in the SAEF which are antisymmetric about the center of the 3He detector's umbra.

CONCLUSIONS

The main sources which contribute to the corrections are the overlap of the penumbrae of the left and right defining apertures and the intensity distribution of the beam (for our experiment, the distribution is constant so that the beam width becomes the crucial factor).

The coincidence requirement sharpens the angular
Figure B1.4  Angular Shift in Polarimeter

The effects of angular errors of 0°, 0.5°, and 1.0° in the angular alignment of the entire polarimeter on the SAEF are shown. The SAEF has been integrated over the width of the beam.
Figure B1.5 SAEF and Longitudinal Errors

The effect of longitudinal errors of -0.25, 0.0, 0.25, and 0.50 cm in the alignment of the right detector on the SAEF are demonstrated.
resolution of the system even when both detectors are misaligned. The effects of misalignment and construction errors are minimized for geometries having large umbrae and small penumbrae. This is achieved by using identical detectors, each with large symmetric slits placed as close to the scatterer as possible, with the distance between slits defined by the desired angular acceptance. This configuration also maximizes the counting rate.
Figure 3.5 depicts the layout of the experiment with the polarized 3He beam including polarimeter and scattering cell, while Table B2.1 contains the specifics of the apparatus.

Polarimeter coincidence spectra chosen at random are displayed in figure B2.2. Both spin orientations (up and down with respect to the scattering plane) of the beam are indicated for the left and right polarimeter detectors. Notice the significant amount of overlap between the 3He and 4He peaks. The solid curve through the data is the least squares fit of gaussian shapes to the data as described in the text. The area under the 3He peak in the left detector is equal to the area under the 4He peak in the right detector, and vice versa. This added constraint increases the statistics, thus decreasing the error in the determination of the asymmetry for each run.

If one performs on-line analysis with this polarimeter, the asymmetry obtained is a function of the limits of integration due to crosstalk. (On-line analysis is made by setting symmetric limits on a peak and summing the...
counts in the intervening channels. Background regions are picked on either side of the peaks so that an interpolated background under the peaks is automatically subtracted.) Figure B2.3 shows the relation between the fractional error in the asymmetry and the position of the limits of integration. The limits of integration which optimize the fractional error in the asymmetry are labelled "A" in figure B2.2. It is not obvious from inspection of the spectra that this is the optimum location for the limits of integration. Since the limits would remain fixed for an entire experiment, only one run need be completely analyzed (least squares peak deconvolution) to determine the overall normalization of the on-line asymmetries.
TABLE B2.1

Beam Collimation

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Tantalum (0.040&quot;) circular aperture (r = 1.00 cm)</td>
</tr>
<tr>
<td></td>
<td>38 cm from center of scattering cell</td>
</tr>
<tr>
<td>C2</td>
<td>Tantalum (0.040&quot;) circular aperture (r = 0.64 cm)</td>
</tr>
<tr>
<td></td>
<td>26.67 cm from C1</td>
</tr>
<tr>
<td>S1</td>
<td>Aluminum shielding barrel (r = 5.72 cm, L = 26.67 cm)</td>
</tr>
</tbody>
</table>

Scattering Cell

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Havar foil 1.6 mg/cm² P max = 60 cm Hg</td>
</tr>
<tr>
<td>r</td>
<td>4.45 cm Height = 1.27 cm</td>
</tr>
</tbody>
</table>

Detector Collimation

<table>
<thead>
<tr>
<th>Width</th>
<th>Height</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>0.32 cm</td>
<td>1.59 cm 7.30 cm from center of cell</td>
</tr>
<tr>
<td>AS</td>
<td>0.32 cm</td>
<td>2.38 cm 6.35 cm from W1</td>
</tr>
<tr>
<td>W2</td>
<td>0.32 cm</td>
<td>0.95 cm 6.35 cm from AS</td>
</tr>
</tbody>
</table>

S2  Lead shielding barrel 12.70 cm Long
Angular acceptance arctan (W/S) = 1.44°

Beam Collimation and Degrading Foils

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C3</td>
<td>0.040&quot; Tantalum 3.18 cm from center of polarimeter w = 0.50 cm, height = 0.95 cm</td>
</tr>
<tr>
<td>C4</td>
<td>0.050&quot; Stainless steel w = 1.3 cm, length = 20.32 cm 2</td>
</tr>
<tr>
<td>FP</td>
<td>Aluminum Foil Packet 171 mg/cm²</td>
</tr>
</tbody>
</table>

Polarimeter

| Havar foil 1.6 mg/cm² P max = 90 cm Hg |
| r = 3.18 cm                           |
| height = 1.91 cm                      |

Detector Collimation

<table>
<thead>
<tr>
<th>Width</th>
<th>Height</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0.30 cm</td>
<td>1.59 cm 3.00 cm from center of cell</td>
</tr>
<tr>
<td>S</td>
<td>aluminum shielding barrel</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r = 1.27 cm length = 7.00 cm</td>
<td></td>
</tr>
</tbody>
</table>
Angular acceptance arctan (W/S) = 2.45°
Figure B2.2  Polarimeter Spectra
Figure B2.3  FRACTIONAL ERROR IN ASYMMETRY

The fractional error in the polarimeter asymmetry as a function of symmetric peak limits is shown. The limits corresponding to the minimum are shown in figure B2.2.
Figure B2.4. POLARIMETER ANALYZING POWER

The energy dependence of the analyzing power for the coincidence polarimeter is shown. The analyzing power has been averaged over the angular acceptance of the system.
APPENDIX B3  CROSS SECTION FORMULAE

The number of particles scattered into a given detector is given by

\[ N = n \; \mathcal{I} \; \Omega \; \varepsilon \; \sigma \]

\( N \) = total number of counts  
\( n \) = number of target nuclei per unit volume  
\( \mathcal{I} \) = beam integration  
\( \Omega \) = effective solid angle of detector  
\( \varepsilon \) = detector efficiency  
\( \sigma \) = unpolarized cross section

If the beam is polarized, the number of particles scattered to the left and right differ by the laboratory asymmetry \( \alpha \) for transverse spin orientations up (↑) and down (↓):

\[ L(↑) = n \; \mathcal{I}(↑) \; \Omega(L) \; \varepsilon(L) \; \sigma \; (1+\alpha) \]
\[ R(↑) = n \; \mathcal{I}(↑) \; \Omega(R) \; \varepsilon(R) \; \sigma \; (1-\alpha) \]
\[ L(↓) = n \; \mathcal{I}(↓) \; \Omega(L) \; \varepsilon(L) \; \sigma \; (1-\alpha) \]
\[ R(↓) = n \; \mathcal{I}(↓) \; \Omega(R) \; \varepsilon(R) \; \sigma \; (1+\alpha) \]

When a polarimeter monitors the beam polarization continuously, it is possible to extract the unpolarized
laboratory cross section $\sigma_3(\psi_3)$ if the detector efficiencies and laboratory monitor cross section $\sigma_m(\psi_m)$ are known. With the subscripts (superscripts) $m$ denoting the monitor, the beam integrations in the scattering cell and in the monitor are simply related:

$$n_m I_m = h_o I_n$$

The beam integration ratio is a constant:

$$r = \frac{I(\uparrow)}{I(\downarrow)} = \frac{I_m(\uparrow)}{I_m(\downarrow)}$$

as are the solid angle ratios:

$$\frac{\Omega}{\Omega_L} = \frac{\Omega R}{\Omega_L} \frac{\epsilon_R}{\epsilon_L} \quad \frac{\Omega_m}{\Omega_m} = \frac{\Omega R}{\Omega_m} \frac{\epsilon_R}{\epsilon_m}$$

It is easily shown that

$$\frac{\sigma_3(\psi_3)}{\sigma_m(\psi_m)} = \left( \frac{N_o}{m(\psi)} \right) h_o \left[ \frac{\Omega R}{\Omega L} \frac{\epsilon_R}{\epsilon_L} \right] \left[ \frac{(1-a_m) + (1+a_m) \Delta_m}{(1-a) + \Delta m + (1+a) \Delta o} \right]$$

This ratio can be transformed to the center of mass

$$\frac{\sigma(\theta_3)}{\sigma_m(\psi_m)} = \left[ \frac{\sigma_3(\psi_3)}{\sigma_m(\psi_m)} \right] \left[ \begin{array}{c} \sigma(\theta_3) \\ \sigma(\psi_3) \end{array} \right]$$

or, if the 4He is detected,

$$\frac{\sigma(\theta_4)}{\sigma_m(\psi_m)} = \left[ \frac{\sigma(\psi_4)}{\sigma_m(\psi_m)} \right] \left[ \begin{array}{c} \sigma(\theta_4) \\ \sigma(\psi_4) \end{array} \right]$$
where
\[
\frac{\sigma(\theta_3)}{\sigma(\psi_3)} = \left( \frac{m_4}{m_3} \right) \frac{c}{(\cos\psi_3 + c)^2}
\]
with
\[
C = \left[ \left( \frac{m_4}{m_3} \right)^2 - \sin^2\psi_3 \right]^{1/2}
\]
and
\[
\frac{\sigma(\theta_4)}{\sigma(\psi_4)} = \left( 4 \cos\psi_4 \right)^{-1}
\]
The errors are given in quadrature:
\[
(\delta f)^2 = \sum \left( \frac{\partial f}{\partial \chi_i} \right)^2 (\delta \chi_i)^2
\]

\[
f(\chi_i) \equiv \frac{\sigma}{\sigma_m} \equiv f(N_0, M_\psi, a_m, \Omega_m, \tilde{\Omega}, a, \Omega)
\]
The following relations are the various derivatives:
\[
\begin{align*}
\frac{2f}{2N_0} &= \frac{1}{N_0} \left( \frac{\sigma}{\sigma_m} \right) \\
\frac{2f}{2a} &= \left[ \frac{(1 + \alpha \Omega) - (\Omega + \alpha)}{D} \right] \left( \frac{\sigma}{\sigma_m} \right) \\
\frac{2f}{2M_\psi} &= -\frac{1}{M_\psi} \left( \frac{\sigma}{\sigma_m} \right) \\
\frac{2f}{2\Omega} &= \left[ \frac{(1 - \alpha \Omega) + (\Omega - \alpha)}{D} \right] \left( \frac{\sigma}{\sigma_m} \right) \\
\frac{2f}{2a_m} &= \left( \frac{a_m - 1}{N} \right) \left( \frac{\sigma}{\sigma_m} \right) \\
\frac{2f}{2\tilde{\Omega}} &= \left[ \frac{(1 - a) \Omega + (\Omega + a)}{D} \right] \left( \frac{\sigma}{\sigma_m} \right) \\
\frac{2f}{2\Omega_m} &= \left( \frac{1 + a_m}{N} \right) \left( \frac{\sigma}{\sigma_m} \right) \\
N &= (1 - a_m) + (1 + a_m) \Omega_m
\end{align*}
\]
\[ D = (1-a)(1+\Omega) + (1+a)(\bar{\Omega} + \Omega) \]

We can write all the quantities in terms of counts in the detectors:

\[
\begin{align*}
N_0 &= L^\uparrow + L^\downarrow + R^\uparrow + R^\downarrow \\
M^\downarrow &= L_m^\downarrow + R_m^\downarrow \\
\bar{\chi} &= \frac{L_m^\uparrow + R_m^\uparrow}{L_m^\downarrow + R_m^\downarrow}
\end{align*}
\]

\[
\Omega^2 = \frac{R^\uparrow R^\downarrow}{L^\uparrow L^\downarrow}, \quad \Omega_m^2 = \frac{R_m^\uparrow R_m^\downarrow}{L_m^\uparrow L_m^\downarrow}
\]

\[
\rho^2 = \frac{L^\uparrow R^\downarrow}{L^\downarrow R^\uparrow}, \quad \rho_m^2 = \frac{L_m^\uparrow R_m^\downarrow}{L_m^\downarrow R_m^\uparrow}
\]

\[
\alpha = \frac{\rho^{-1}}{\rho + 1}, \quad \alpha_m = \frac{\rho_m^{-1}}{\rho_m + 1}
\]

The errors are statistical

\[
\begin{align*}
\delta N_0 &= \sqrt{N_0} \\
\delta M^\downarrow &= \sqrt{M^\downarrow} \\
\delta \bar{\chi} &= \frac{\bar{\chi}}{M^\downarrow} (1+\bar{\chi}) \\
\delta \alpha &= \left( \frac{\delta \rho^2}{\rho^2} \right) \frac{\rho}{(\rho + 1)^2}
\end{align*}
\]

\[
\left( \frac{\delta \Omega}{\Omega} \right)^2 = \left( \frac{\delta \rho^2}{\rho^2} \right) = \frac{1}{L^\uparrow} + \frac{1}{L^\downarrow} + \frac{1}{R^\uparrow} + \frac{1}{R^\downarrow}
\]
### APPENDIX B4

#### 7Be PHASE SHIFTS

<table>
<thead>
<tr>
<th>Energy</th>
<th>22.0000</th>
<th>24.0000</th>
<th>30.0000</th>
<th>32.0000</th>
<th>37.9300</th>
<th>45.2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi</td>
<td>92.8000</td>
<td>127.1000</td>
<td>205.2000</td>
<td>118.9000</td>
<td>5.7700</td>
<td>2.6300</td>
</tr>
</tbody>
</table>

#### REAL PHASE SHIFTS

<table>
<thead>
<tr>
<th>1/2+</th>
<th>253.90</th>
<th>-110.45</th>
<th>-128.13</th>
<th>-130.80</th>
<th>-133.06</th>
<th>-142.27</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2-</td>
<td>259.72</td>
<td>-102.62</td>
<td>-132.27</td>
<td>-129.56</td>
<td>-128.64</td>
<td>-143.10</td>
</tr>
<tr>
<td>3/2-</td>
<td>258.58</td>
<td>-102.01</td>
<td>-117.51</td>
<td>-120.59</td>
<td>-128.48</td>
<td>-139.27</td>
</tr>
<tr>
<td>5/2+</td>
<td>176.38</td>
<td>-8.47</td>
<td>-9.45</td>
<td>-10.69</td>
<td>-11.06</td>
<td>-14.71</td>
</tr>
<tr>
<td>5/2-</td>
<td>145.40</td>
<td>144.68</td>
<td>141.62</td>
<td>147.94</td>
<td>151.72</td>
<td>146.38</td>
</tr>
<tr>
<td>7/2-</td>
<td>160.93</td>
<td>157.60</td>
<td>150.95</td>
<td>156.80</td>
<td>160.19</td>
<td>163.24</td>
</tr>
<tr>
<td>7/2+</td>
<td>4.08</td>
<td>4.45</td>
<td>9.87</td>
<td>11.93</td>
<td>20.69</td>
<td>33.25</td>
</tr>
<tr>
<td>9/2+</td>
<td>4.79</td>
<td>5.10</td>
<td>11.87</td>
<td>9.77</td>
<td>22.29</td>
<td>46.67</td>
</tr>
<tr>
<td>9/2-</td>
<td>8.78</td>
<td>11.04</td>
<td>19.87</td>
<td>21.00</td>
<td>31.97</td>
<td>41.83</td>
</tr>
<tr>
<td>11/-</td>
<td>8.89</td>
<td>11.26</td>
<td>22.64</td>
<td>24.82</td>
<td>31.91</td>
<td>43.16</td>
</tr>
<tr>
<td>11/+</td>
<td>-0.39</td>
<td>-0.43</td>
<td>-0.27</td>
<td>-0.03</td>
<td>0.65</td>
<td>2.23</td>
</tr>
<tr>
<td>13/+</td>
<td>-0.39</td>
<td>-0.43</td>
<td>-0.27</td>
<td>-0.03</td>
<td>0.65</td>
<td>2.23</td>
</tr>
<tr>
<td>13/-</td>
<td>0.49</td>
<td>0.63</td>
<td>1.63</td>
<td>2.19</td>
<td>3.35</td>
<td>5.29</td>
</tr>
<tr>
<td>15/-</td>
<td>0.49</td>
<td>0.63</td>
<td>1.63</td>
<td>2.19</td>
<td>3.35</td>
<td>5.29</td>
</tr>
</tbody>
</table>

#### IMAGINARY PART

<table>
<thead>
<tr>
<th>1/2+</th>
<th>11.86</th>
<th>19.26</th>
<th>10.56</th>
<th>13.50</th>
<th>11.48</th>
<th>13.56</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2-</td>
<td>7.09</td>
<td>15.61</td>
<td>40.28</td>
<td>48.96</td>
<td>15.03</td>
<td>19.62</td>
</tr>
<tr>
<td>3/2-</td>
<td>9.54</td>
<td>15.55</td>
<td>26.15</td>
<td>50.95</td>
<td>16.81</td>
<td>20.02</td>
</tr>
<tr>
<td>3/2+</td>
<td>11.81</td>
<td>7.55</td>
<td>9.30</td>
<td>4.21</td>
<td>14.85</td>
<td>17.52</td>
</tr>
<tr>
<td>5/2+</td>
<td>11.21</td>
<td>8.98</td>
<td>9.51</td>
<td>6.51</td>
<td>13.04</td>
<td>13.49</td>
</tr>
<tr>
<td>5/2-</td>
<td>22.72</td>
<td>16.73</td>
<td>13.84</td>
<td>3.43</td>
<td>20.71</td>
<td>23.65</td>
</tr>
<tr>
<td>7/2-</td>
<td>14.94</td>
<td>12.11</td>
<td>7.91</td>
<td>3.10</td>
<td>16.65</td>
<td>11.41</td>
</tr>
<tr>
<td>7/2+</td>
<td>4.61</td>
<td>5.87</td>
<td>9.17</td>
<td>17.15</td>
<td>11.62</td>
<td>14.86</td>
</tr>
<tr>
<td>9/2+</td>
<td>3.65</td>
<td>5.82</td>
<td>11.13</td>
<td>16.27</td>
<td>13.92</td>
<td>20.60</td>
</tr>
<tr>
<td>9/2-</td>
<td>1.27</td>
<td>1.27</td>
<td>9.33</td>
<td>21.04</td>
<td>16.69</td>
<td>27.80</td>
</tr>
<tr>
<td>11/-</td>
<td>0.48</td>
<td>1.55</td>
<td>11.94</td>
<td>25.06</td>
<td>19.38</td>
<td>26.93</td>
</tr>
<tr>
<td>11/+</td>
<td>0.00</td>
<td>0.03</td>
<td>0.06</td>
<td>0.18</td>
<td>0.67</td>
<td>1.25</td>
</tr>
<tr>
<td>13/+</td>
<td>0.00</td>
<td>0.03</td>
<td>0.06</td>
<td>0.18</td>
<td>0.67</td>
<td>1.23</td>
</tr>
<tr>
<td>13/-</td>
<td>0.00</td>
<td>0.04</td>
<td>0.22</td>
<td>0.36</td>
<td>0.69</td>
<td>1.63</td>
</tr>
<tr>
<td>15/-</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY

Ba51  S. Bashkin et.al. Phys Rev 84(1951)1124.
Ba64  A.C.L. Barnard et.al. Nucl Phys 50(1964)629.
Be63  H. Beaumovieille et.al. Compte Rendu 256(1963)1494.
Be64  H. Beaumovieille CEA report R.2624.
Be68  F. Bertrand et.al. CEA report R.3428.
Be69  P.R. Bevington DATA REDUCTION AND ERROR ANALYSIS FOR THE PHYSICAL SCIENCES 1969.
Bi76  E.K. Biegert M.A. Thesis Rice Univ. 1976.
Bl52  J.M. Blatt et.al THEORETICAL NUCLEAR PHYSICS, 1952.
Bo60  R. Bouchez et.al. Journal de Phys 21(1960)356.
Ca75  P.A. Carruthers  Los Alamos report LA-6193-PR.
Do75  D.C. Dodder  Proceedings 4th Int Symp on
       Polarization Phenomena in Nuclear Reactions 1975.
Do76  D.C. Dodder et.al  Phys Rev C to be published.
Do77  D.C. Dodder  private communication.
Fe73  W. Fetscher et.al.  Nucl Phys A216(1973)47.
Go64  F.S. Goulding et.al.  Nucl Inst and Meth 31(1964)1.
Ha72  D.M. Hardy et.al.  Nucl Phys A195(1972)250.
Ha76  G.M. Hale  Private communication.
Hi69  B. Hind et.al.  Nucl Inst and Meth 71(1969)231.
Ja51  J.L. Jackson  Phys Rev 83(1951)301.
Ka53  N.G. VAN Kampen  Rev Mex Fis 2(1953)233.
Ka76  O. Karban  Univ of Birmingham Report 76-05.
Ka77  O. Karban et.al.  Nucl Inst and Meth 141(1977)387.
Ko77  J.A. Koepke et.al.  Phys Rev C to be published.
La58  A.M. Lane et.al.  Rev Mod Phys 30(1958)257.
Ne64   J.A. Nelder et.al. Computer Journ 7(1964)308.
Ox53   C.L. Oxley et.al. Phys Rev 91(1953)419.
Ro71   D.J. Rose Science 172(1971)797.
Ro75   S. Roman Proceedings 4th Int Symp on Polarization
       Phenomena in Nuclear Reactions 1975.
Sa53   R.G. Sachs NUCLEAR THEORY.
Si59   E.A. Silverstein Nucl Inst and Meth 4(1959)53.
Sn77   Pat von Behren, Private communication.
See also Nucl Phys A196(72)634.

Te52  Teichmann and Wigner  Phys Rev 87(1952)123.
Va76  V. Valkovic  Private communication.
Wa75  R.H. Ware et.al.  Univ. Minnesota Report COO-535-710.