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PROTON–PROTON QUASI–FREE SCATTERING FROM DEUTERIUM
AT 800 MeV

by

Thomas M. Williams

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IN PARTIAL FULFILLMENT OF THE
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Doctor of Philosophy

Thesis Director's Signature: 

Houston, Texas
February 1977
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I. INTRODUCTION

In recent years there has been much interest in proton induced deuteron breakup. Most of this work has been at energies below 200 MeV,\textsuperscript{1-6}) although some work has been done at higher energies.\textsuperscript{7-11}) The three-body reaction with the highest cross section is in the area of quasi-free scattering. Quasi-free scattering (QFS) is the scattering of one of the target nucleons as if it were a free nucleon. The cross section falls off rapidly as the spectator momentum increases. The specific reaction covered in this thesis is

\[ p + d \rightarrow p + p + n, \]

where \( n \) is a spectator neutron.

At low energies the proton-deuteron system can be treated using the Faddeev equations\textsuperscript{12}) which provide an exact formalism for the three-body system. For higher energies it is difficult to obtain a numerical solution. The common approach used to describe this reaction at medium energies is the simple impulse approximation (SIA).\textsuperscript{13}) Since it is assumed that the neutron was a spectator to the reaction, its final momentum should be equal to its initial Fermi momentum. There are several assumptions underlying the impulse approximation. First is that the incident particle
never interacts with two target nucleons at the same time. This should be valid if the wavelength of the incident particle is less than the average separation of the target nucleons. Second that the amplitude of the incident wave falling on each nucleon is nearly the same as if that nucleon were alone, and third that the binding forces between the target nucleons are negligible during the decisive phase of the collision, when the incident particle interacts strongly with the system.

The impulse approximation or spectator model predicts that the cross section is directly proportional to the free p-p cross section multiplied by the square of the deuteron ground state wave function.

In general the impulse approximation is valid for small spectator momenta, however corrections which include the effects of multiple scattering, final state interactions, and off energy shell cross sections are often added to describe p-d scattering over a broader range of kinematic conditions.

Chapter 2 contains a description of the layout and equipment used in this experiment. The theory of QFS is discussed in chapter 3. This is followed by a discussion of how the data is analyzed in terms of efficiency, normalization, solid angle correction, etc. Chapter 4 also contains a section on possible causes of errors. The results of the analysis is given
in chapter 5. Conclusions and suggestions for future work are given in chapter 6.
II. EXPERIMENTAL DESIGN

A. PHYSICAL CONSIDERATIONS

The experiment was designed to measure the cross section for various reactions resulting from 800 MeV protons incident on deuterons. A diagram of the experimental layout is shown in figure 2-1. This experiment was performed on the External Proton Beam (EPB) at the Los Alamos Meson Physics Facility (LAMPF) in Los Alamos, New Mexico. Two protons were detected in coincidence by a spectrometer and a time of flight (TOF) arm so that the measurement was kinematically complete for the three-body reaction.

Angle pairs were selected in two different ways. First the angle pairs were selected so the neutron was at rest (p-p elastic angles). The second set of angle pairs were selected so the neutron was scattered parallel to the beam (symmetric angles) and thus the p-p scattering was 90° in the c.m. system. In none of these angles was the final state interaction contribution expected to be large.

B. TARGET

The deuteron target was 1.062 g/cm² of LD₂ centrally located in a 20 cm by 40 cm diameter target chamber. The cryogenic equipment used for liquefying the deuterium
Figure 2-1

The arrangement of detectors is shown here. P indicates wire planes. S indicates scintillators used in the logic of the experiment. M1-M4 are the monitor scintillators used in 1976. The dotted lines show the position of the monitor scintillation system used in 1974. The profile monitor (PM) and the Rice ion chamber (RION) were used only in 1976. The target used was liquid deuterium.
was furnished by LAMPF. This cryogenic system, manufactured by Cryodyne\textsuperscript{(R)}, consists of three principal components: the refrigerator unit, a compressor unit, and a control unit. Figure 2-2 is a diagrammatic flow chart of the cryogenic system.

In order to liquefy deuterium the following sequence of events occurs. First, the volume inside the high vacuum jacket is pumped down using the roughing pump and diffusion pump. Second, the purge valve and the target fill valve are opened allowing the deuterium supply line system and the refrigerator to be pumped down. Third, flushing of this system with deuterium and subsequent pumping is then repeated a few times to insure pure deuterium in the system. Fourth, the refrigerator begins cooling down the deuterium.

The refrigerator consists of two expansion chambers which expand ultra pure helium within their system. After expansion, the helium is recirculated, passing first through the compressor for compression and next through an air cooled heat exchanger for cooling. The refrigerator has a 10 watt cooling capacity at 20\degree K.\textsuperscript{14)} As the deuterium begins liquefying additional gas must be added to the system. Liquid begins dropping to the target where it evaporates and returns to the refrigerator until eventually the target is cooled and liquid builds up in the target. The temperature of the refrigerator
Figure 2-2

This shows a flow diagram of the hydrogen refrigerator and associated plumbing.

<table>
<thead>
<tr>
<th>SYMBOL</th>
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<tbody>
<tr>
<td>V</td>
<td>Remotely controlled valve</td>
</tr>
<tr>
<td></td>
<td>(pneumatic or electric)</td>
</tr>
<tr>
<td>Θ</td>
<td>Manual valve</td>
</tr>
<tr>
<td>EVV</td>
<td>Emergency Vent Valve</td>
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<tr>
<td>SV</td>
<td>Supply Valve</td>
</tr>
<tr>
<td>TFV</td>
<td>Target Fill Valve</td>
</tr>
<tr>
<td>TV</td>
<td>Target (Empty) Valve</td>
</tr>
<tr>
<td>HVV</td>
<td>High Vacuum Valve</td>
</tr>
<tr>
<td>FV</td>
<td>Fore Valve</td>
</tr>
<tr>
<td>PV</td>
<td>Purge Valve</td>
</tr>
<tr>
<td>RV</td>
<td>Roughing Valve</td>
</tr>
<tr>
<td>TC</td>
<td>Thermocouple Vacuum Gauge</td>
</tr>
<tr>
<td>IG</td>
<td>Ion Discharge Vacuum Gauge</td>
</tr>
<tr>
<td>DP</td>
<td>Diffusion Pump</td>
</tr>
<tr>
<td>FP</td>
<td>Fore Pump</td>
</tr>
<tr>
<td>RP</td>
<td>Roughing Pump</td>
</tr>
<tr>
<td>T</td>
<td>Target cell</td>
</tr>
<tr>
<td>Res.</td>
<td>LD₂ reservoir</td>
</tr>
</tbody>
</table>
is measured by a hydrogen vapor pressure sensitive thermometer. The vapor pressure of the liquid deuterium is maintained at 13.5 psia (23.3°K) by means of a heater. This pressure is about 2 psi above the Los Alamos atmospheric pressure. Thus any leak in the external plumbing will not leak air into the system. All pressure gauges, vacuum pumps and remotely controlled valves are operated from a master control panel external to the target location.

The actual target container is a Kapton cylinder with an active region 2.54 cm in diameter and 6.53 cm in length. The Kapton walls are 0.0127 cm in thickness and are radiation resistant for the levels of radiation encountered. Kapton is a polyamide film (H-film) which has the strength of mylar, a density 1.08-1.14 g/cm$^3$, and an atomic composition of C$_{22}$H$_{10}$N$_2$O$_4$.$^{15}$ The density of liquid deuterium is 0.163 g/cm$^3$ at 23.3°K,$^{16,17}$ which gives a target thickness of 1.062 g/cm$^2$ or 3.20 × 10$^{23}$ atoms/cm$^2$.

C. BEAM

The beam protons used in the experiment were accelerated to 800 MeV by the LAMPF linear accelerator. LAMPF is now doing simultaneous acceleration of H$^+$ and H$. The beams are separated in the switchyard and the H$^+$ is sent down line X. In line X the beam is split and
some protons are sent to area C (High Resolution Spectrometer) and some to line B. In line B, a stripper is used so part of the beam can be sent to EPB. The remainder goes to B Room.

Under normal operation, the beam bursts are \( \approx 400 \ \mu \text{sec} \) long at 120 Hz. The microstructure was such that each burst contained narrow (1/4 nsec) spikes of beam 5 nsec apart. There were periods in which the beam was "chopped." This was done by removing the first 50 to 100 \( \mu \text{sec} \) from each burst. For us, this just reduced our data rate and increased our accidental rate.

The available current ranged from 0.1 pA to 100 nA. Over the entire range of currents, the beam spot at the target remained approximately 0.4 cm in diameter. The divergence of the beam was very low, however multiple scattering in the target and various vacuum windows and ion chambers made the beam much larger down stream at the profile monitor and Faraday cup.

D. DETECTORS

All scintillators in the experiment consisted of NE102 scintillation plastic, optically epoxied to UVT Plexiglas light pipes. The light was detected by RCA 8575 photomultiplier tubes, which were powered by Ortec 265 or equivalent tube bases. All photomultiplier tubes near the magnet had magnetic shields. The
The beam at LAMPF is pulsed at 120 Hz. Each burst is 400 μsec long. (a) shows the macrostructure of the beam and its burst width compared to the time between bursts. (b) shows the microstructure in each burst.
scintillators were used for measuring time of flight, scaling, and fast logic. Pulse height information was acquired from some tubes.

The Rice developed Multi-Wire Proportional Counters (MWPC) have been described in previous work. Each MWPC consists of an x, y grid of wires, sandwiched between high voltage cathodes, capable of determining the point where a charged particle traversed the detector plane. The MWPC have a spatial resolution $\approx 0.25$ cm, a count rate capability $10^5$ Hz/wire, and a multiple readout capability. They operate in the proportional region at 5 to 5.5 kV with a gas mixture of 25% Argon and 75% dimethylpropane.

E. BEAM MONITORING

The system of monitors for the 1976 run differed from the system used in 1974. In 1974 a thin aluminum target was placed in the beam approximately three meters downstream from the LD$_2$ target (fig. 2-1). Two scintillator paddles were set up as a single arm to record scattering from this target. Still further downstream were the LAMPF ion chamber (LION) and the Faraday cup.

In 1976 we removed the aluminum target and instead of it, we used a CH$_2$ target placed between LION and the Faraday cup. At this target four scintillator paddles were arranged into two scintillator telescopes to measure p-p coincidences from the CH$_2$ target. Behind the CH$_2$
target was the profile monitor followed by the Rice ion chamber (RION). RION seemed to be the most reliable monitor during this set of runs.

In the LD$_2$ target chamber was a remotely controlled phosphor that had a + marked at the target center. By using a TV monitor it was possible to view the beam spot size and position immediately upstream of the target. The position at the target could also be calculated by projecting back to the target the trajectories of the detected particles. The profile monitor enabled us to monitor beam position and size at the CH$_2$ target. By centering the beam at both the targets, we could be sure the beam was coming straight down the center of the beam pipe. The profile monitor was identical to the wire planes but had an integrating readout system.

We noticed that LION had a noise problem that gave a background of 3 to 20 Hz. At low beam intensities the background might be as much or greater than the data. Because of the fluctuations in the background it was not a reliable monitor. The RION noise was very low, never measured to be more than 10 pulses/hr. The gain of RION was calibrated by using LION at high beam currents and found to be 160 ± 8.$^{15,19}$

To correct for dead time, the output of the monitor scintillators were sent to two modules: one module was free running and the second was gated by the beam gate
and computer busy. By taking the ratio of monitor free
to the gated monitor it was possible to determine the
dead time of the system (see fig. 2-6). Monitor accidentals were also measured. The accidentals were usually
a few percent of the total coincidences and the ratio
of gated to free accidentals was approximately the ratio
of gated to free total. This is as it should be if
the beam was of uniform intensity. It was observed
that the monitor accidentals increased with the beam
rate as expected.

F. ELECTRONIC LOGIC AND COMPUTER HARDWARE

The detector signals were processed using NIM and
CAMAC electronics before being sent to a PDP 11/45
computer. A block diagram of the electronics is shown
in the next three figures.

A MWPC readout would occur following a "strobe"
pulse. The requirement for a strobe was a three fold
coincidence between scintillators S2, S3, and S4. Before
the signals go to the computer, a TAG D signal requires
a four fold coincidence of S1, S2, S3, and S4. This
signal is sent to the MWPC logic so it will know which
coordinates are required to have an acceptable event.
These required coordinates are "in coincidence". Thus
each event requires at least some non zero MWPC readouts.
This greatly reduces the nonanalyzable data on tape.
It was necessary for all the electronics used for the MWPC fast logic to be in the experimental area. This figure shows that electronics and the signals sent to the trailer from those electronics.
The trailer electronics were used to process the signals that came from the modules in figure 2-4. These signals were processed and sent to the computer. The results of these signals and the wire plane signals were sent to tape if the necessary conditions were met.
Figure 2-6

The monitor scintillator signals (labeled AM) were sent to the trailer for processing and scaling.

The efficiency scintillators (EF) were used to check the efficiency of S1, S2, S3, and S4.

Other beam monitors also sent signals to the trailer so we could get an accurate absolute monitor of the beam.
If all the coordinates that are in coincidence have non-zero readouts, then the event goes to the computer to be written to tape. The coordinates that are in coincidence are changed approximately every 100 buffers.

The strobe is gated by the beam gate, computer busy and is self gated such that it is disabled for at least 2 µsec after a strobe. This 2 µsec will allow time for the computer to send a busy signal to gate off the strobe. If a second strobe occurs during the 200 nsec interval required to read out the MWPC coordinates, the event is rejected as a "pile-up" because of MWPC ambiguity. Normally the pile-up has less than 1/2 percent effect on the results. The computer busy and beam gate are also used for vetoing various scalers. This is necessary to make dead time corrections.

The timing requirements for the scintillator coincidences were made wide enough such that the reaction of interest was easily encompassed but tight enough to reduce background reactions as much as possible.

Each event included readout of scintillator pulse heights (ADC), time-of-flight (TDC), MWPC readouts, tag identification, and two MWPC multiplicity words. Events were buffered with 256 words per buffer and written on magnetic tape. A more detailed and technical explanation of the computer interfacing is discussed in previous publications.²⁰,²¹)
Signals from RION and the Faraday cup were sent through digital current integrators and the output was sent to the computer. These current integrators also had meters so beam current could be read directly. Both short and long term fluctuations in beam intensity were observed. These fluctuations were often as much as 100% of the desired intensity. That is one reason the wire plane efficiencies could vary so much between runs.

During the 1976 run, counts in each scintillator, current integrator, time, pile-ups, strobos, tags, etc. were scaled and dumped to tape every minute. These numbers were used in normalizing the data and made it possible to use part of a run in case of equipment failure during the run.
III. THEORY

The deuteron is a very loosely bound nucleus. It has a binding energy of 2.2 MeV with an observed radius of 4 fm.\textsuperscript{4)} This is larger than the range of nuclear forces, 1.7 fm, and thus the nucleons in the deuteron spend a large fraction of time outside each others influence. If the wavelength of the incoming beam (0.14 fm at 800 MeV) is less than the average separation of the nucleus, then it is quite likely that an interaction between the beam and one of the target nucleons will occur and the second target nucleon is likely to remain as a spectator. Under these conditions the impulse approximation is expected to be valid. If this is a valid assumption, the momentum of the spectator nucleon is expected to be the same as its Fermi momentum in the deuteron nucleus. Several papers\textsuperscript{7-9)} have been written indicating that the cross section as a function of spectator momentum agrees out to \( \sim 200 \) MeV/c. Beyond this momentum there are definite indications that double scattering or some other form of scattering occurs.

Under the impulse approximation, the undetected particle (in this case the neutron) had a final momentum of \( \overset{\rightarrow}{p_3} \). Since it is a spectator, it had an initial momentum of \( \overset{\rightarrow}{p_3} \) as well and the target proton had an initial momentum of \( -\overset{\rightarrow}{p_3} \) assuming the deuteron was at rest.
Regardless of the validity of the SIA, there are certain quantities which we can evaluate without any doubts. These are the kinematical relations between the particles. A system of three particles in the final state is described by nine parameters. The laws of conservation of momentum and energy impose four conditions on these parameters with the result that there are only five independent parameters. We measured five parameters in this experiment and thus it was kinematically complete. From these five measured parameters, the other four can be calculated. According to Kullander et al.\textsuperscript{22} the differential cross section is

\[
\frac{d^9\sigma}{dp_1dp_2dp_3} = \frac{4\pi^2}{\beta_0} |t_{fi}|^2 \delta[p_1 + p_2 + p_3 - p_0] \delta[E_1 + E_2 + E_3 - E_0 - m_d],
\]

where 0, 1, 2, 3 refer to the beam proton, spectrometer proton, TOF proton, and neutron respectively. \(t_{fi}\) is the transition matrix element defined by

\[
|t_{fi}|^2 = \frac{E_0^2}{\pi^2 E_0 E_1 E_2 E_3} \left(\frac{d\sigma}{d\Omega}\right)_{c.m.}^{p-p} |\phi(p_3)|^2.
\]

If we integrate over \(p_3\) and the magnitude of \(p_2\), the
result is:

\[
\frac{d^5\sigma}{d\Omega_1 d\Omega_2 dp_1} = \frac{4\pi^2 E_2 E_3 p_1^2 p_2^2}{\beta_0 [p_2 E_2 + p_3 E_3 + E_2 (p_1 \cos(\theta_1 - \theta_2) - p_0 \cos \theta_2) / \beta_0]} |t_{fi}|^2
\]

\[
= \frac{32E_0^2}{\beta_0 E_0} |\phi(p_3)|^2 \left( \frac{d\sigma}{d\Omega} \right)^{P-P}_{c.m.} I ,
\]

where all angles refer to the angle from beam line (\(\vec{p}_0\))
and I is the Lorentz invariant phase space for three body reactions,

\[
I = \frac{p_1^2 p_2^2}{8E_1 [E_2 p_2 + E_3 p_2 + E_2 (p_1 \cos(\theta_1 - \theta_2) - p_0 \cos \theta_2)]}.
\]

Our experimental results show good agreement with
the SIA out to \(p_3 \sim 200\) MeV/c when we use the Hulthen
deuteron wave function with 7\% Yamaguchi\(^{23}\) \(D\) state.
This wave function is expressed by:

\[
U = \frac{0.2771}{(\alpha^2 + p^2)(\beta^2 + p^2)},
\]

\[
W = \frac{0.7850p^2}{(\alpha^2 + p^2)(\gamma^2 + p^2)^2},
\]

and

\[
|\phi(p)|^2 = U^2 + W^2 ,
\]

with \(\alpha = 0.2316\) fm\(^{-1}\), \(\beta = 5.181\alpha\), and \(\gamma = 6.771\alpha\). The
assumption made above ignored any double scattering
or final state interaction.

The total Hamiltonian of the reaction can be expressed by \(^{24}\)

\[ H = H_0 + V \]

where

\[ H_0 = K_1 + K_2 + K_3 \]

and

\[ V = V_1 + V_2 + V_3. \]

\(K_j\) is the relativistic kinetic energy operator of particle \(j\) and \(V_j\) is the interaction of particle \(k\) and \(n\) with \(j, k, n\) a permutation of \(1, 2, 3\). \(E\) will denote the energy of the system.

The cross section for scattering from a state described asymptotically by the wave function \(\psi_i\) to the state \(\psi_f\) is proportional to the absolute value squared of the transition amplitude

\[ \langle \psi | T | i \rangle = \langle \psi_f | V | \psi_i \rangle \]

where \(\psi_i\) is the outgoing scattering state of the system which originates from the state \(\phi_i\). That is

\[ \psi_i = \lim_{\varepsilon \to 0^+} \frac{i\varepsilon}{E - H + i\varepsilon} \phi_i. \]

For the interaction under consideration, \(\phi_f = |\vec{k}_1\rangle |\vec{k}_2\rangle |\vec{k}_3\rangle |L\rangle\), the three particle plane-wave state. Here \(\vec{k}_i\) is the final state momentum of particle \(i\) and \(|L\rangle\) the three particle
spin state. \( L \) is an index ranging from 1 to 8 and 
\[ \phi_i = |p_0\rangle |s\rangle |P,N\rangle, \]
where \( |p_0\rangle |s\rangle \) is the plane-wave state of the projectile proton, which has momentum \( p_0 \) and spin projection \( s = \pm 1/2 \), and \( |P,N\rangle \) is the wave function for a deuteron with momentum \( P \) and spin projection \( N = -1, 0, 1 \). Following Faddeev\textsuperscript{12) we may write the scattering state in the form \( \psi_i = \psi^{(1)} + \psi^{(2)} + \psi^{(3)}. \)

The \( \psi^{(j)} \) satisfy the matrix equation

\[
\begin{pmatrix}
\psi^{(1)} \\
\psi^{(2)} \\
\psi^{(3)}
\end{pmatrix} =
\begin{pmatrix}
0 \\
0 \\
\phi_i
\end{pmatrix} + G
\begin{pmatrix}
0 & T_1 & T_1 \\
T_2 & 0 & T_2 \\
T_3 & T_3 & 0
\end{pmatrix}
\begin{pmatrix}
\psi^{(1)} \\
\psi^{(2)} \\
\psi^{(3)}
\end{pmatrix},
\]

where

\[ G = \frac{1}{(E - H_0 + i\epsilon)} \]

and

\[ T_j \equiv V_j + V_j G V_j \]

are the free-particle two body transition matrices.\textsuperscript{24)}

The \( T_j \) satisfy the Lippmann-Schwinger equation,

\[ T_j = V_j + V_j G T_j. \]

From the previous equations the following multiple scattering series is obtained:

\[
\langle f | T|i \rangle = \langle \phi_f | T_3 | \phi_i \rangle + \langle \phi_f | T_2 | \phi_i \rangle + \langle \phi_f | T_3 GT_2 | \phi_i \rangle \\
+ \langle \phi_f | T_2 GT_3 | \phi_i \rangle + \langle \phi_f | T_1 GT_3 | \phi_i \rangle + \langle \phi_f | T_1 GT_2 | \phi_i \rangle + O(G^2). \]

The first two terms on the right hand side of this equation
are single-scattering or impulse terms. The SIA consists in keeping only the first two terms in the above equation and in replacing them by their values for free scattering at the corresponding energy and angle. The terms containing two factors of $T_j$ are double-scattering contributions. The remaining terms describe successively higher-order scattering processes.

Because the successively higher-order terms become increasingly difficult to calculate, a simplification of the multiple scattering series is necessary for application. The Glauber approximation for the transition amplitude has been used successfully in describing the elastic process at intermediate through high energies.$^{25)}$

This approximation includes only the impulse terms and the energy conserving parts of the double-scattering terms which appear in the multiple-scattering series. The relation

$$\frac{1}{E - H_0 + i\varepsilon} = p\left[\frac{1}{E - H_0}\right] - i\pi\delta(E - H_0)$$

is used to split the double-scattering contributions into two parts. The principal-value part of double scattering exactly cancels the infinite sum of triple and higher-order scattering terms, which contain the complete eikonal propagator$^{26)}$. Now the elastic scattering and the deuteron breakup processes essentially differ
only in the final state of the scattering system. Hence, returning to the breakup process, we retain as a reasonable first correction to the impulse terms the energy conserving parts of double scattering:

\[
\langle f | T | i \rangle = \langle \phi_f | T_3 | \phi_i \rangle + \langle \phi_f | T_2 | \phi_i \rangle \\
- i \pi [ \langle \phi_f | T_3 \delta(E-H_0)T_2 | \phi_i \rangle + \langle \phi_f | T_2 \delta(E-H_0)T_3 | \phi_i \rangle \\
+ \langle \phi_f | T_1 \delta(E-H_0)T_3 | \phi_i \rangle + \langle \phi_f | T_1 \delta(E-H_0)T_2 | \phi_i \rangle ].
\]

This equation can be written explicitly as

\[
\langle f | T | i \rangle = (2\pi)^{3/2} (2M)^{1/2} \left[ \langle L | T_3 (p_0, -\vec{k}_3 + \vec{k}_1, \vec{k}_2) \phi_N(k_3) | s \rangle \\
+ \langle L | T_2 (p_0, -\vec{k}_2 + \vec{k}_1, \vec{k}_3) \phi_N(k_3) | s \rangle \right] \\
+ \frac{i}{\delta} \left( \frac{M}{\pi} \right)^{1/2} \left( \int_{(p_0)_L} d^2q \frac{\langle L | T_3 (\vec{k}_1 + \vec{k}_2 + \vec{q}, -\vec{q} + \vec{k}_1, \vec{k}_2) T_2 (p_0, \vec{q} + \vec{k}_1 + \vec{k}_2 + \vec{q}, \vec{k}_3) \phi_N(q) | s \rangle}{| (q_u + | \vec{k}_1 + \vec{k}_2 |) E(\vec{q}) + q_u E(\vec{k}_1 + \vec{k}_2 + \vec{q}) |} \right) \\
+ \int_{(k_1 + k_3)_L} d^2q \frac{\langle L | T_2 (\vec{k}_1 + \vec{k}_3 - \vec{q}, \vec{q} + \vec{k}_1, \vec{k}_3) T_3 (p_0, -\vec{q} + \vec{k}_1 + \vec{k}_3 - \vec{q}, \vec{k}_2) \phi_N(q) | s \rangle}{| (q_u + | \vec{k}_1 + \vec{k}_3 |) E(\vec{q}) + q_u E(\vec{k}_1 + \vec{k}_3 - \vec{q}) |} \\
+ \int_{(k_2 + k_3)_L} d^2q \frac{\langle L | T_1 (\vec{k}_2 + \vec{k}_3 - \vec{q}, \vec{q} + \vec{k}_2, \vec{k}_3) T_3 (p_0, -\vec{q} + \vec{k}_1 + \vec{k}_3 - \vec{q}, \vec{k}_2) \phi_N(q) | s \rangle}{| (q_u - | \vec{k}_2 + \vec{k}_3 |) E(\vec{q}) + q_u E(\vec{k}_2 + \vec{k}_3 - \vec{q}) |} \\
+ \int_{(k_2 + k_3)_L} d^2q \frac{\langle L | T_1 (-\vec{q}, \vec{k}_2 + \vec{k}_3 + \vec{q}, \vec{k}_2, \vec{k}_3) T_2 (p_0, \vec{q} + \vec{k}_1, \vec{k}_2 + \vec{k}_3 + \vec{q}) \phi_N(q) | s \rangle}{| (q_u + | \vec{k}_2 + \vec{k}_3 |) E(\vec{q}) + q_u E(\vec{k}_2 + \vec{k}_3 + \vec{q}) |}
\]

where \( \phi_N(q) \) is the deuteron internal wave function in momentum space and \( \int_{(k)_L} d^2q \) indicates a two-dimensional integration over the components of \( \vec{q} \) perpendicular to \( \vec{k} \).
The component of $\mathbf{q}$ parallel to $\mathbf{k}$, denoted by $q_n$, is determined at each integration point by the appropriate energy conserving $\delta$ function.

The first impulse term in the above equation describes a process in which the projectile proton experiences a single 90° c.m. collision with the target proton. Notice that the initial target proton momentum is $-\mathbf{k}_3$ in the impulse term and not 0. This is one of the corrections necessary to produce an accurate description of the scattering. The SIA normally uses the free cross section for a target proton at rest. The second impulse term describes a single collision of the projectile proton with the target neutron. We have neglected this term, as it is strongly suppressed with the present kinematics. The third term describes a Glauber double-scattering process in which the projectile is scattered first from the target proton and subsequently from the neutron. The fourth term describes a similar process where the projectile interacts first with the neutron. The fifth term describes a process where the projectile is scattered from the target proton which then nearly forward scatters from the neutron. This may be thought of as a single-scattering process followed by a final-state interaction of the target nucleons. Finally, the sixth term describes a similar process where the projectile collides first.
with the neutron which then collides with the target proton, transferring most of its momentum to the proton. The various single- and double-scattering processes are illustrated diagrammatically in figure 3-1.

Most theoretical work has been done at energies below 200 MeV. At these energies the only results that come close to fitting the data include the entire multiple scattering series. The curves that use a truncated series using single-, double- or even triple-scattering come nowhere near giving the correct results.\textsuperscript{1,2,32) Work done by Wallace \textsuperscript{24) at 600 MeV indicates that the multiple scattering effects become significant at high spectator momenta, but they cause only about a factor 2 difference where a factor of 10 is needed. Wallace indicates that the off-energy-shell behavior of $T_j$ is not well known in the 400 to 1000 MeV range and the use of on shell p-p amplitudes are not adequate.
Figure 3-1

(a) shows the impulse approximation for p-p QFS.
(b) shows the impulse approximation for p-n QFS.
(c) and (d) show Glauber double scattering.
(e) and (f) show FSI scattering. This does not mean that the proton and neutron have a low final relative energy, but that the two target nucleons interact with each other after the target proton strikes one of them.
IV. DATA ANALYSIS

A. INTRODUCTION

Analysis of kinematically complete three body data leads to the determination of a fifth order differential cross section. This cross section is an average of the actual cross section over the kinematically allowed portion of the experimental acceptance and is calculated by

\[ \frac{d^5 \sigma}{d\omega_1 d\omega_2 dp_1} = \frac{(N_S - N_{bak})}{N_0 n \Delta \omega_1 \Delta \omega_2 \delta p_1^m \left| \frac{dp_1}{dp_1^m} \right|} \cdot \left( E_{wp} E_{dt} \right), \]

where \( N_S \) is the number of events detected to have scattered into the experimental system in the spectrometer momentum interval \( p_1 \) to \( p_1 + \delta p_1 \). \( N_{bak} \) is the background or accidental events, \( E_{wp} \) is the wire plane efficiency, \( E_{dt} \) is the dead time efficiency, \( N_0 \) is the number of target deuterons/cm², \( \Delta \omega_1 \Delta \omega_2 \) is the average solid angle of the experimental system over \( p_1 \) to \( p_1 + \delta p_1 \), and \( \left| \frac{dp_1}{dp_1^m} \right| \) is the Jacobian which corrects the measured momentum interval, \( \delta p_1^m \), to the actual interval \( \delta p_1 \).

B. DETECTOR EFFICIENCY

Corrections were made to the data for detector efficiency. Based on previous experience and online measurements, scintillator efficiencies were assumed
to be 100 percent. The online checks of scintillator efficiencies were done by placing small scintillators (EF in fig. 2-6) behind S2 and S4. If one of these scintillators detected a particle, then it was assumed that the scintillator in front of it should have seen that particle. It was not true that all particles detected by the efficiency counters passed through S1 or S3, but the fraction of them that did should be constant for a given angle pair and magnetic field.

Unfortunately the Multi-Wire Proportional Counters were not so efficient. A MWPC can be inefficient in two ways. It can fail to detect a particle when one passes through it, or it can detect two or more particles when it should detect only one. The first case is called a "zero" and the second case a "high bit." The efficiency of the wire plane is a function of the velocity of the particle detected, the gas mixture, the high voltage, and the bias voltage. A too intense beam can also cause MWPC efficiency to be low. This leads to time varying efficiencies and thus they must be calculated run by run.

A zero may occur when the signal was not of sufficient gain to trigger the discriminator, when the particle passed outside the active region of that coordinate, or when there was an electronic failure. In order not to consider the particles that passed outside the active region of a plane as an inefficiency, the efficiency
of each MWPC was measured by making cuts on its shadow plane. The shadow plane is the adjacent plane on the same side of the magnet. By requiring a particle to pass near the center of the shadow plane, it was assumed the particle passed in the active region of the MWPC in which the efficiency was being calculated. Since all the particles I was interested in were protons and the reaction of interest was the dominant reaction, the planes could be easily adjusted to detect protons with a good efficiency. The zero efficiency for the twelve coordinates was usually on the order of 95 percent.

High bit efficiency is the probability of the event particles passing through the MWPC without giving any multiple readouts. A readout occurs when one or more adjacent wires fire (detect a signal). If two adjacent wires fire, then the particle is recorded as hitting the "half-wire" between those two wires. If three adjacent wires fire, it is assumed the particle passed nearest the center wire of the three. Multiple readouts occur when at least one wire between the readouts does not detect a particle. Multiple readouts may be caused by noise, accidentals, and electronic feedthrough. Electronic feedthrough may cause high bits because the first and ninth amplifiers are physically adjacent on an amplifier card. Normally feedthrough would not create a problem
because it just causes an adjacent wire to fire. Accidentalss occur whenever two separate particles traverse the MWPC within its resolving time.

An assumption was made that every high bit event also included a real event. A very few may include two real events. To test this assumption the same run was analyzed several times accepting different number of high bit coordinates each time. First events with no high bits were analyzed. For this particular run approximately 88 percent of the analyzed events were good. Then those events with exactly one high bit were analyzed. The fraction of those that were good was approximately 44 percent. Then events with two high bits were analyzed with 21 percent good. This is as expected. With one double readout there is a 50 percent chance that the readout chosen is the correct one. With two double readouts, there is only a 25 percent chance of choosing the correct readout in both coordinates with high bits. This analogy holds well except for four high bits. The most likely cause of four high bits is that two particles pass through the TOF arm. If this happens, it is very likely that the first readouts in Y (1Y and 2Y) correspond to one particle and the second readouts correspond to the other particle. To a limited extent this will also be true for the X readouts. Thus for four high bits, the ratio of good is expected to be
as much or more than the ratio for three high bits. This does nothing to invalidate the assumption that every high bit event contains a real event. The ratios of good to analyzed events can be found in table 4-1.

C. EVENT IDENTIFICATION

From the data, events were individually analyzed to determine which of these events were good events. For an event to be considered good it must satisfy several conditions. The rays of both protons were traced back to the target. The intersection of these rays (or point of closest approach) was required to be in the target. The maximum separation of the two rays was limited to about 0.9 cm. The entrance and exit rays of the magnet are projected onto the rear face of the magnet. Here the separation of the two rays was calculated and required to be small. The planes before the magnet were used to calculate the Y position at plane 5. This was then compared to the measured position at 5Y. The difference between the output slope and the calculated output slope was required to be small. By rejecting particles that do not meet the above criteria we eliminate those particles that scattered from things other than the deuterium in the target. A final requirement is to require the measured time of flight of each arm to be within 3 nsec of the
<table>
<thead>
<tr>
<th>Number of High Bits</th>
<th>Analyzed Events</th>
<th>Good Events</th>
<th>Percent Good</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>35421</td>
<td>31270</td>
<td>88.3</td>
</tr>
<tr>
<td>1</td>
<td>11259</td>
<td>4986</td>
<td>44.3</td>
</tr>
<tr>
<td>2</td>
<td>9381</td>
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<td>3157</td>
<td>376</td>
<td>11.9</td>
</tr>
<tr>
<td>4</td>
<td>2662</td>
<td>406</td>
<td>15.2</td>
</tr>
<tr>
<td>5</td>
<td>698</td>
<td>50</td>
<td>6.4</td>
</tr>
</tbody>
</table>
Figure 4-1

The position of the reaction in the target was calculated by projecting back to the target the proton in each arm. This figure shows the profile of the events as a function of x, y, and z. The units used are half-wires (1/20 inch or 0.13 cm). The counts shown are per halfwire for x and y and per 4 halfwires for z. The FWHM of the beam appears to be 3/16 inch (0.5 cm). The dotted lines show the placement of the cuts. Any event that fell outside the cuts was rejected as not coming from the target.
Figure 4-2

These three histograms show the difference between the particle positions calculated using the wire planes in front of the magnet vs. the positions calculated by using the planes behind the magnet. All units are in half wires (1/20 inch or 0.13 cm). DX is the difference between X at the rear magnet face. DSY is the difference between 5Y and 6Y compared to the calculated difference. DY is the difference between 5Y and 5Y calculated by using planes 3 and 4.
Figure 4-3

The measured time of flight of each particle was compared to the calculated time of flight. The difference between the results was required to be small.

The dash-dot lines indicate the cuts placed on the delta-TOF spectrum. The dashed part of the histograms have the counts/channel multiplied by 100 so the background can be seen. Most of this background is the result of accidental coincidences. L refers to to the left (spectrometer) arm and R refers to the right (TOF) arm.
calculated time of flight of that arm. Any deuteron or pion that got into the system could usually be eliminated by TOF cuts. Many accidentals could also be rejected.

D. SOLID ANGLE DETERMINATION

In order to calculate a cross section it is necessary to know the solid angle subtended by the detector system. If we had a point target and one detector in each arm the answer could be easily calculated by

\[ \Delta \Omega_1 \Delta \Omega_2 = \int \frac{dA}{r_2} \int \frac{dA}{r_1}, \]

where \( dA \) is the differential element of area and \( r \) is the location of that element with respect to the target. Since our target is not a point target and we have multiple detectors in each arm, the problem is much more difficult. The magnet increases the difficulty of the problem by making the solid angle a function of particle momentum in the spectrometer arm. Physical considerations such as energy loss, multiple scattering, and kinematics also need to be considered. Due to the difficulty of the problem the most common method of calculating the solid angle, \( \Delta \Omega_1 \Delta \Omega_2 \) and the Jacobian \( dp_1/dp_1^M \) is through the Monte Carlo technique. A program written by Tom Witten and revised by myself was used to calculate the solid angle of the system.
The events in the Monte Carlo program were randomly chosen within preselected limits. First \( X \), \( Y \), and \( Z \) of the target were selected within the intersection of the beam and the target. Second the \( X \) slope, the \( Y \) slope, and momentum of the particle in the spectrometer arm, \( p_1 \), were chosen. These slopes were so selected that an increase in the maximum allowed slope would not result in any more particles passing through the system. Choosing slopes instead of angles is an approximation but since \( \theta = \tan \theta \) for \( \theta \leq 70 \text{ mrad} \) (4°), it is a good approximation. If the particle successfully passes through all detectors before the magnet, then \( X \) slope and \( Y \) slope of particles passing through the TOF arm are selected. If this particle successfully passes through the TOF arm, the program returns to the spectrometer arm particle. The momentum of this particle is then corrected for energy loss. The particle is passed through the magnet assuming five sections of magnet field. The field is assumed uniform over each section. The central section (section 3) is 36 inches (91.4 cm) long and the field strength is calculated from the central momentum of the magnet and effective length using the uniform field approximation. Sections 2 and 4 are each 5 inches (12.7 cm) long and of 60 percent of central field strength. Sections 1 and 5 are each 7.5 inches (19.1 cm) long and of 20 percent central field strength. In bending power, this is equi-
valent to a field 45 inches long with the field of section 3. If the particle passes through the magnet without striking the magnet coils or pole tips, the particle is projected onto the detectors behind the magnet.

An event in which both particles pass through all detectors is then sent to a kinematics subroutine to determine if the event is kinematically possible. If the event is possible its Lorentz invariant phase space is calculated. The event is then histogrammed as a function of its \( p_1 \), \( Z \) target, \( \theta_L', \phi_L', \theta_R', \phi_R' \), and \( p_3 \). Also \( p_1 \) and \( p_3 \) are histogrammed after multiplying by phase space. The program then begins on a new event. If at any time a particle misses a detector the program immediately increments the number of tries scaler and starts over on a new event.

Some results of a typical Monte Carlo program are shown in figures 4-4 and 4-5. Notice the \( \phi \)'s have a sharper cut off than the \( \theta \)'s. This is caused by the target having a length of \( \approx 6 \) cm but the beam has a height of only \( \approx 0.3 \) cm. The acceptable \( \theta_L \) is also a function of momenta. That is the reason \( \theta_L \) is not flat across the center 4°. The system acceptance of particles as a function of momentum is shown in figure 4-5. Notice there is a peak approximately 150 MeV/c wide near the central momentum and then the acceptance
These are histograms of the acceptable angles using the Monte Carlo program. The acceptance in $\theta_L$ is a function of the particle momentum and the magnet field. $\phi_L$ is limited by the gap in the magnet. $\theta_R$ and $\phi_R$ are limited by the size of the detectors in the TOF arm.
Figure 4-5

This shows the particle acceptance as a function of momentum. The central momentum of the magnet (PC) is shown in the figure. The peak is approximately flat for about 25 percent of PC. The long tail on the low momentum side is cut off by the event being kinematically impossible. This kinematic cut off is shown by the dashed line.
begins to fall off. Perhaps this should be phrased as a peak having a width of 25 percent the central momentum. Beyond this range the acceptance begins to fall off. There is a long tail on the high momentum side. For the reaction I am interested in, this high momentum tail is usually cut off by the reaction being kinematically impossible instead of failure to pass through the system. The dotted line shows the kinematic cut off.

E. BEAM NORMALIZATION

To calculate a cross section, an absolute normalization of the beam was necessary. In 1974 we measured the beam three ways. First we had two scintillators forming a telescope aimed at a thin aluminum target, second was a LAMPF ion chamber, and third was the Faraday cup. The ratios of these three monitors should have been constant since they were measuring the same beam. Unfortunately the ratios were not constant, in fact they varied widely. There are several possible causes of this. The ion chamber has a background count that was not measured until the 1976 run. The movement of the beam could effect the solid angle accepted by the monitor scintillators. Leakage in the Faraday cup could reduce its output.

To determine which monitor was the most accurate, I compared their outputs to the Tag D scaler adjusted to
a free Tag D. Also the coincidences between S1 and S2 (S12) were used in this comparison. All runs used were
40°-40° p-p elastic runs with a CH₂ slab target and the magnet was set at 950 MeV/c central momentum. According
to the results (Table 4-2) the most reliable monitor was the Faraday cup with an average variation of 2.5 percent.
S12 had a smaller variation but it could not be used as a monitor because it changed with angle and target. It also
was directly tied to Tag D since Tag D is a four fold coincidence of S1-S4.

In 1976 we had four ways of measuring the beam intensity. We still had the ion chamber and Faraday cup used in 1974.
We replaced the single arm scintillator monitor by a two arm, four scintillator coincidence monitor whose scintilla-
tor telescopes detected protons from a slab of CH₂. Also added was a Rice ion chamber. During 1976 the Faraday cup
did not work properly. The LAMPF ion chamber had a background rate that was significant at low intensities. Because of
these problems and because we were able to keep the beam centered on the Rice ion chamber by using the profile monitor,
the Rice ion chamber was the best absolute beam monitor we had with an average variation of 1%. This chamber had a
gain of about 160 and had no background.

F. ERRORS

Errors in this experiment can come from several
<table>
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<tr>
<th>Tag D/S12</th>
<th>Tag D/FCup</th>
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<th>Tag D/LION</th>
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<tr>
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<tr>
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<td>327</td>
<td>4.78</td>
<td>2.60</td>
</tr>
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</table>

Average = 2.08      328      4.88      2.64

\[
\frac{|x - \overline{x}|}{\overline{x}} = 0.020\quad \frac{.025}{.020} = 0.030\quad \frac{.047}{.020}
\]
sources. They may be caused by detectors not working properly, detectors not being in the correct position, beam off center, scattering from material other than the target, accidental coincidences, and statistical errors in both data and Monte Carlo runs. In this section I will try to cover each source of error.

The problem with beam normalization has been discussed in the previous section. It was noted that the average variation between Tag D and Faraday cup was 2.5%. That does not mean the actual error in normalization was 2.5%. The output of the Faraday cup could be reduced by leakage, by multiple scattering causing some beam to miss the cup, and by the beam being improperly positioned. The error caused by improper normalization was estimated to be less than seven percent.

Monte Carlo runs assuming the beam off center by 1/4 inch (0.6 cm) or equipment misaligned by 0.1 inch (0.25 cm) indicated the change in solid angle was small, less than one percent. However by moving two detectors in the magnet arm it was possible to change the momentum acceptance of the system enough to cause effects up to 5% for $p_3 > 250$ MeV/c. For $p_3 < 250$ MeV/c the effect was less than 1 percent.

Scattering from things other than the LD$_2$ could also produce background. The things most likely to scatter the beam so it would be detected are the mylar vacuum windows
on the target chamber and the kapton windows on the target cell. The events from the vacuum windows can be easily rejected because they obviously did not come from the target. The events from the target cell ends cannot be easily rejected because the point of interaction cannot be accurately located. By using two target empty p-p elastic runs it was possible to isolate the contributions from the kapton. This contribution was calculated to be about 0.4\% of a target full run. For QFS, all the contribution from the kapton was in the first two p_{3} channels (<10 MeV/c).

Accidental coincidences will occur in any experiment of this type. If two particles from one event and one particle from a separate event are detected, the event will be rejected because of high hits in the wire planes. But if one particle from each of two separate events is detected, it is possible that this "event" could pass all cuts. A measure of the frequency of this happening can be determined by the background in the time-of-flight spectrum. For one typical run (34^{0}-34^{0}) the background in the TOF arm was 0.28\% and 0.16\% in the spectrometer arm. Some of this background will be eliminated by x, y, z, and dv at the target cuts. Thus the background will surely be less than 1/2\% and probably only about one half of that.

Detector efficiencies were calculated and corrections were made. This is covered in section IV-B. The uniform
field approximation for the magnet could produce errors. If so, they should appear in elastic runs where the kinematics are overdetermined. No such indications appeared.

Statistical errors occur in both the experimental data and the Monte Carlo results. These errors can be reduced by taking longer runs and by taking larger bins. To reduce these errors, the Monte Carlo results were smoothed, the experimental data for $150 < p_3 < 250$ MeV/c were binned in 10 MeV/c bins instead of 5 MeV/c bins, and the data for $p_3 > 250$ MeV/c were binned in 25 MeV/c steps. The error bars shown in figures 5-4 and 5-5 are only due to statistical errors in the experimental data.

The total error in the system must include the contribution from each possible source of error. By including all the above mentioned error sources, the error of each point should be less than 10% except for a few data points that have very poor statistics.
V. RESULTS

The data was analyzed and compared to the SIA cross section, \(^{22}\)

\[
\frac{d^5\sigma}{d\theta_1 d\theta_2 dp_1} = \frac{32\overline{E}}{\beta_0 E_0} |\phi(p_1)|^2 \frac{d\sigma}{d\Omega}_{\text{c.m.}}^{P-P} \frac{p_1 p_2^2}{8E_1(E_2p_2+E_3p_2+E_2[p_1\cos(\theta_1-\theta_2)-p_3\cos\theta_2])},
\]

where all angles refer to angle from beam line. Data was taken at two different types of angle pairs as shown in Table 5-1. The first type of angle pairs were those in which we varied the spectrometer and TOF arms to keep them at equal angles. This resulted in the spectator being scattered approximately parallel (or antiparallel) to the beam. This reaction always occurs at or near 90° c.m. but the impact energy is not always equal because the target proton has an average momenta of other than zero. The exact impact energy varies with the direction and momenta of the target proton. The equivalent laboratory collision energies in the two body system range from 565 to 1170 MeV. In determining these effective collision energies it is assumed that \(\hat{p}_3\) is essentially parallel or antiparallel to the incident beam proton. The elastic proton-proton c.m. cross section varies considerably over this energy range as can be seen in figure 5-1.

The second type of angle pairs were p-p elastic angles for 800 MeV. At these angles \(\hat{p}_3 = 0\) was kinematically
## TABLE 5-1

Kinematic Conditions Chosen for QFS Studies

<table>
<thead>
<tr>
<th>$\theta_1$ (deg)</th>
<th>$\theta_2$ (deg)</th>
<th>$\theta_3 \pm 10$ (deg)</th>
<th>$p_1$ (MeV/c)</th>
<th>$p_2$ (MeV/c)</th>
<th>$p_3$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-62.2</td>
<td>80</td>
<td>1306</td>
<td>507</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>25</td>
<td>-56.4</td>
<td>80</td>
<td>1230</td>
<td>625</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>30</td>
<td>-50.5</td>
<td>80</td>
<td>1144</td>
<td>740</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>35</td>
<td>-45.0</td>
<td>80</td>
<td>1050</td>
<td>850</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>40</td>
<td>-40.0</td>
<td>80</td>
<td>953</td>
<td>953</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>45</td>
<td>-35.0</td>
<td>80</td>
<td>850</td>
<td>1050</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>30</td>
<td>-30</td>
<td>180</td>
<td>942</td>
<td>942</td>
<td>169</td>
</tr>
<tr>
<td>34</td>
<td>-34</td>
<td>180</td>
<td>948</td>
<td>948</td>
<td>109</td>
</tr>
<tr>
<td>40</td>
<td>-40</td>
<td>80</td>
<td>953</td>
<td>953</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>44</td>
<td>-44</td>
<td>0</td>
<td>949</td>
<td>949</td>
<td>97</td>
</tr>
<tr>
<td>48</td>
<td>-48</td>
<td>0</td>
<td>936</td>
<td>936</td>
<td>210</td>
</tr>
</tbody>
</table>
Figure 5-1

The p-p elastic cross section at 90° c.m. varies with energy as shown here. The $T_{lab}$ is the kinetic energy of the beam proton assuming a stationary target proton.
allowed. The impact energy was assumed to be 800 MeV but
the c.m. scattering angle differed for different angle pairs.
The p-p elastic cross section variation with c.m. angle is
shown in figure 5-2.

Data was taken at five pairs of symmetric angles.
At 48° special program changes were necessary to eliminate
p-d elastic events. The large peak at 1080 MeV/c in figure
5-3b shows the large contamination from elastic events.
Deuterons in the magnet were easily eliminated but deuterons
in the TOF arm were more difficult. If an event had the
proper angles and momenta for an elastic event and was co-
planar, it was assumed that it was an elastic event. This
assumption seemed to be valid because other angle pairs had
very few coplanar events. Thus the cuts appeared to cut all
elastic events and few if any QFS events. However some
p + d \rightarrow p + d^* events appear to have been accepted. That
is the probable cause of the peak at 450 MeV/c in the p_3
spectrum for 48°. Kinematics show that the relative energy
between the neutron and one of the protons is less than
5 MeV/c for p_3 between 450 and 600 MeV/c. The results of the
symmetric data is given in Table 5-2 and figure 5-4.

The symmetric angles are perhaps not the best way to
study QFS because small p_3 is not allowed except near 40°.
The other set of angles taken were the p-p elastic angles
for 800 MeV. The 40° data is on the p-p elastic angles
Figure 5-2

The p-p elastic cross section at 800 MeV as a function of $\theta_{\text{c.m.}}$ is shown.
Figure 5-3

This figure shows the momentum of the particles through the magnet for a 48°-48° run. (a) is after cuts and (b) is before cuts. Nearly 90 percent of the events between 1050 and 1100 MeV/c failed the cuts. Most of these events are p-d elastic events. Only about 25 percent of the events around 950 MeV/c failed the cuts.
TABLE 5-2

Symmetric Angle QFS Results

<table>
<thead>
<tr>
<th>θ (deg)</th>
<th>T_{eff} (MeV)</th>
<th>(dσ/dΩ)^{P-P}_{c.m.} (mb/sr)</th>
<th>d^3σ_{exp} (μb/sr^2MeV/c)</th>
<th>d^3σ_{SIA} (μb/sr^2MeV/c)</th>
<th>exp/SIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>565</td>
<td>2.80 ± .15</td>
<td>22.6 ± 2.5</td>
<td>19.5 ± 1.1</td>
<td>1.16</td>
</tr>
<tr>
<td>34</td>
<td>643</td>
<td>1.85 ± .12</td>
<td>92.3 ± 9.5</td>
<td>94.7 ± 6.2</td>
<td>.97</td>
</tr>
<tr>
<td>40</td>
<td>800</td>
<td>.994 ± .03</td>
<td>3690 ± 400</td>
<td>3800 ± 190</td>
<td>.97</td>
</tr>
<tr>
<td>44</td>
<td>961</td>
<td>0.68 ± .09</td>
<td>83.7 ± 8.4</td>
<td>78.1 ± 10.4</td>
<td>1.07</td>
</tr>
<tr>
<td>48</td>
<td>1170</td>
<td>0.57 ± .08</td>
<td>4.17 ± .43</td>
<td>2.73 ± .38</td>
<td>1.53</td>
</tr>
</tbody>
</table>
Figure 5-4

The results of the symmetric angle data as a function of the spectator momentum are shown. The curves are the Hulthen wave function. No adjustment in the wave function has been made to improve the fit on any curve.
but it is also a symmetric angle so $\theta_{\text{c.m.}} = 90^\circ$. In the elastic angle data, $p_3 = 0$ was kinematically allowed. The variation in the two body interaction energy was much smaller for these angles than for the symmetric angles. The results of the elastic angle data are shown in figure 5-5. The Hulthen wave functions shown in figure 5-5 have been multiplied by an adjustment factor. This adjustment factor was determined by minimizing $\chi^2$ in a least-squares fit\textsuperscript{31}) to the 28 data points between 10 and 150 MeV/c. The only variable in the fit was the adjustment factor. This factor was expected to be less than one since part of the time the proton was shadowed by the neutron. In each case this adjustment factor was less than one, usually about 0.8, and the reduced $\chi^2$ was less than one in each case.

Some typical raw $p_3$ data is illustrated in figure 5-6. The top histogram (a) shows the counts as a function of neutron momenta. The center histogram shows the results of a Monte Carlo run for the same data. The bottom histogram shows the Monte Carlo results multiplied by phase space on an event-by-event basis. This will also give an indication of the statistical accuracy of the various data points.
Figure 5-5

The elastic angle data as a function of spectator momenta is shown. The smooth curves are the Hulthen deuteron wave function individually normalized to fit each curve.
<table>
<thead>
<tr>
<th>$\theta_1$ (deg)</th>
<th>$\theta_2$ (deg)</th>
<th>$\theta_{c.m.}$ (deg)</th>
<th>$(d\sigma/d\Omega)^{P-P}_{c.m.}$ (mb/sr)</th>
<th>Adjustment Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-62.2</td>
<td>47.0</td>
<td>4.02 ± .18</td>
<td>.82</td>
</tr>
<tr>
<td>25</td>
<td>-56.4</td>
<td>58.2</td>
<td>2.58 ± .06</td>
<td>.82</td>
</tr>
<tr>
<td>30</td>
<td>-50.5</td>
<td>69.2</td>
<td>1.55 ± .05</td>
<td>.73</td>
</tr>
<tr>
<td>35</td>
<td>-45.0</td>
<td>79.8</td>
<td>1.14 ± .04</td>
<td>.58</td>
</tr>
<tr>
<td>40</td>
<td>-40.0</td>
<td>90.0</td>
<td>.994 ± .03</td>
<td>.97</td>
</tr>
<tr>
<td>45</td>
<td>.35.0</td>
<td>100.2</td>
<td>1.14 ± .04</td>
<td>.76</td>
</tr>
</tbody>
</table>
Figure 5-6

The top histogram (a) shows the neutron momentum calculated from the detected protons. The counts are per 5 MeV/c. (b) shows the neutron momentum from the Monte Carlo run corresponding to (a). (c) is the same as (b) but each event was multiplied by a constant and phase space. To get the cross section as a function of $p_n$, (a) is divided by (c) and the results multiplied by a normalization constant.
VI. CONCLUSIONS

Quasi-free scattering has been observed under various kinematical conditions at 800 MeV. The data agree well with the simple impulse approximation out to a spectator momentum of about 200 MeV/c. Above this point, a significant departure from theory is observed.

Very little theoretical work has been done above 200 MeV. The work done by Wallace\(^{24}\) indicates that multiple scattering should become important at high spectator momenta, but the correction he calculates comes nowhere near being adequate to describe the data. Low energy (<200 MeV) results have shown that it is necessary to use a complete multiple scattering series to explain the data. At higher energies (600–1000 MeV) the higher order scattering terms should rapidly become negligible.

The results of Aladashvili et al.\(^{33,34}\) indicate that an intermediate isobar may be produced at high spectator momenta (>300 MeV/c). They observed a two proton effective mass distribution at 2170 ± 10 MeV, with a peak width of 50 MeV. This is the sum of the \(\Delta\) and nucleon mass.

It is obvious that further theoretical work needs to be done on this problem. It may also be useful to
redo part of this experiment using two spectrometers so there will be no error in the calculation of \( p_3 \), where the relative energy of particles two and three approaches zero.

If the deutron wave function really follows our data instead of the Hulthen wave function, it indicates that the nucleons in the deuteron nucleus spend much time close (<1/2 fm) together. Using the results of this experiment, it is possible to explain some of the results of Hogstrom.\textsuperscript{15}
VII REFERENCES


