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ANALYSIS OF A CIRCULAR WAVEGUIDE CAVITY LOADED
WITH THICK MAGNETIZED FERRITE DISKS

BY

JOSEPH MILES HELLUMS

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

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PREFACE

In microwave ferrite devices there are several ferrite/dielectric interface problems and this thesis is concerned with the analysis of one of them. The E-plane circulator has some advantages over the H-plane circulator but has not received such widespread attention due to its inherent complexity. One of the important problems in this device is the ferrite/dielectric interface formed by the flat surface of the disk. In order to concentrate on this problem, a ferrite-disk-loaded cavity has been studied and experiments performed to check the theory.

After a brief introduction in Chapter 1, Chapter 2 analyzes the dielectric filled circular waveguide using a scaling required in the next Chapter. In Chapter 3, the analysis developed by Suhl and Walker is extended to include evanescent behaviour and mode plots are developed for a specific set of parameters. These plots contain a great deal of information, and are a guide to understanding the behaviour of this complicated structure. They are required for the analysis of the cavity problem in Chapter 6. In Chapter 4 the preliminary experimental work is described along with measurements to determine the dielectric constant of the isotropic material and the ferrite used. The analysis and behaviour of the cavity loaded with unmagnetized disks is described in Chapter 5. This proved to be very valuable in understanding the behaviour of the magnetized cavity. The possibility of either propagating or evanescent fields in the dielectric region was included and the facility to move the cavity with respect to the coupling hole helped in mode identification.

In Chapter 6 the analyses of Chapters 2 and 3 are combined and a single basis mode approximation is used to predict resonances of the magnetized cavity.
The results of Chapter 5 are used in identifying certain observed resonances and associating them with the predicted ones, and these resonance frequency pairs are then compared. The results and their implications for future work are discussed in Chapter 7.
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CHAPTER 1

INTRODUCTION

The three-port E-plane waveguide junction circulator may be smaller than the H-plane configuration, but as shown in Figure 4.3, it has the disadvantage that the junction is loaded with axially magnetized ferrite disks placed against the narrow walls, while in the H-plane junction the axially magnetized ferrite may extend the entire distance between the broad walls. Therefore, in the analysis of the H-plane junction it can be assumed that there is no r.f. field variation in the direction of the bias field. However, this simplifying assumption cannot be applied to the E-plane junction. It has been recognized that an important consideration in understanding operation of the latter configuration would be the ferrite/air interfaces at the flat disk surfaces. In order to concentrate on this particular problem it was decided to consider the end-loaded cavity also shown in Figure 4.3, in which the other boundary conditions are more easily defined.
CHAPTER 2

MODES IN DIELECTRIC - FILLED CIRCULAR GUIDE

2.1 Introduction

Expressions for the familiar TE and TM modes of dielectric-filled circular guide will be derived in a less familiar form. This form is important here because it is later used in describing the field in the center section of the magnetized end-loaded cavity. Incorporated into the expressions will be the same scaling of magnetic field and of length as will be used in the theory of the ferrite-filled guide. This development facilitates the handling of the continuity condition required at the interfaces between cavity regions. The orthogonality property of the modes is noted and is extended to orthonormality by adjusting arbitrary coefficients.

2.2 Scaling of Magnetic Field and Length

The permittivities of the ferrite and dielectric must be made distinguishable in the notation, because the ferrite permittivity as well as that of the dielectric appears in the theory of the dielectric modes. The notation used is as follows:

\[
\varepsilon_{p} = \text{ferrite permittivity} \quad (2 - 1)
\]

\[
\varepsilon_{d} = \text{dielectric permittivity} \quad (2 - 2)
\]

\[
\varepsilon_{fr} = \frac{\varepsilon_{p}}{\varepsilon_{o}} = \text{ferrite relative permittivity} \quad (2 - 3)
\]

\[
\varepsilon_{dr} = \frac{\varepsilon_{d}}{\varepsilon_{o}} = \text{dielectric relative permittivity} \quad (2 - 4)
\]

\[
\varepsilon_{r} = \frac{\varepsilon_{d}}{\varepsilon_{p}} = \text{ratio of dielectric permittivity to ferrite permittivity} \quad (2 - 5)
\]
Before the dielectric modes are derived, the required scaling is applied to Maxwell's Equations. Initially, these have the form:

\[
\nabla \times \mathbf{E} = -j \omega \mu_0 \mathbf{H} \quad (2 - 6)
\]
\[
\nabla \times \mathbf{H} = j \omega \varepsilon_0 \mathbf{E} . \quad (2 - 7)
\]

These equations are then multiplied by the factor \((\mu_0 / \varepsilon_F)^{1/2}\), which has the units of impedance, and \(\mathbf{H}\) is renamed as the product of this factor and the actual magnetic field, thus giving the new magnetic field the units of electric field:

\[
\nabla \times \mathbf{E} = -j \beta_0 \mathbf{H} \quad (2 - 8)
\]
\[
\nabla \times \mathbf{H} = j \omega \varepsilon_0 (\mu_0 / \varepsilon_F)^{1/2} \mathbf{E} . \quad (2 - 9)
\]

Here, \(\beta_0 = 2 \pi / \lambda_0\), and \(\lambda_0\) is the wavelength of a plane wave in an infinite region of unmagnetized ferrite. Now, the two equations are multiplied by the factor \(\sqrt{\beta_0}\), and the independent variables initially having units of length becomes unitless. In a cylindrical coordinate system these are \(r\) and \(\phi\).

The resulting form of Maxwell's Equations is

\[
\nabla \times \mathbf{E} = -j \mathbf{H} \quad (2 - 10)
\]
\[
\nabla \times \mathbf{H} = j \varepsilon_R \mathbf{E} . \quad (2 - 11)
\]

2.3 The TE Modes

To obtain modes with electric field transverse to the \(\phi\) direction this field is assumed to derive from a vector potential according to the relationship

\[
\mathbf{E} = -j \nabla \times \Pi_b , \quad (2 - 12)
\]

where the vector potential satisfies the condition

\[
\Pi_b = \Pi_h \hat{z} . \quad (2 - 13)
\]
Substituting (2-12) into (2-10), one finds the r.f. magnetic field can also be given in terms of the vector potential, according to the relationship

\[ \mathbf{H} = \nabla \times \nabla \times \mathbf{H}_h. \]  (2-14)

Using equation (2-11) and (2-12) one obtains

\[ \nabla \times (\mathbf{H} - \epsilon_R \mathbf{H}_h) = 0, \]  (2-15)

which indicates that the magnetic field can also be obtained from the vector potential \( \mathbf{H}_h \) and a second potential, which is a scalar, as shown by

\[ \mathbf{H} = \epsilon_R \mathbf{H}_h + \nabla \Phi. \]  (2-16)

Using (2-14) and (2-16) one can write

\[ \nabla \times \nabla \times \mathbf{H}_h = \epsilon_R \mathbf{H}_h + \nabla \Phi. \]  (2-17)

Then, making use of the vector identity,

\[ \nabla \times \nabla \times \mathbf{A} = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}, \]  (2-18)

equation (2-17) becomes

\[ \nabla (\nabla \cdot \mathbf{H}_h) - \nabla^2 \mathbf{H}_h = \epsilon_R \mathbf{H}_h + \nabla \Phi. \]  (2-19)

If \( \mathbf{H}_h \) and \( \Phi \) are assumed to be related by \( \nabla \cdot \mathbf{H}_h = \Phi \), equation (2-19) becomes

\[ \nabla^2 \mathbf{H}_h + \epsilon_R \mathbf{H}_h = 0. \]  (2-20)

From this equation one can determine the vector potential. The spatial dependence of \( \mathbf{H}_h \) in a cylindrical coordinate system is assumed to take the form

\[ \mathbf{H}_h = \mathcal{T}(r, \phi) \mathcal{Z}(z). \]  (2-21)

When this is substituted into (2-20) and the operator \( \nabla^2 \) is written as

\[ \nabla^2 + \frac{\lambda^2}{\lambda^2 z^2} \],

one obtains

\[ \frac{1}{\lambda} \nabla^2 \mathcal{T} + \frac{\lambda''}{\lambda} + \epsilon_R = 0. \]  (2-22)

Since the coordinate \( z \) is independent of the transverse coordinates, the terms \( \frac{1}{\lambda} \nabla^2 \mathcal{T} \) and \( \frac{\lambda''}{\lambda} \) must be independent and therefore constant, and if, in the
case of the former, this requirement is expressed in the form

\[ \frac{1}{T} \nabla^2_T T = -k_c^2, \quad (2-23) \]

and this is taken into account in equation (2-22), separate differential equations for \( T \) and \( Z \) are obtained

\[ Z'' + (\varepsilon_R - k_c^2) Z = 0 \quad (2-24) \]
\[ \nabla^2_T T + k_c^2 T = 0. \quad (2-25) \]

The corresponding solutions for \( Z \) and \( T \) are

\[ Z = A e^{-j\beta z} + B e^{j\beta z} \quad (2-26) \]
\[ T = C \text{J}_n(k_c r) e^{jn\phi} \quad (2-27) \]

where \( \beta \) is given by

\[ \beta^2 = \varepsilon_R - k_c^2 \quad (2-28) \]

and \( k_c \) will be determined by boundary conditions on the field components.

Following Suhl and Walker\(^{(1)}\), only the cases \( n = \pm 1 \) will be considered.

The electric and magnetic fields for TE modes have already been expressed entirely in terms of the vector potential \( \Pi_h \), as is recalled from equations (2-12) and (2-14). Using again the identity (2-18), the right hand side of (2-14) can be rewritten, so that \( H \) is given by

\[ H = \nabla (\nabla \cdot \Pi_h) - \nabla^2 \Pi_h. \quad (2-29) \]

The second term on the right can be replaced according to (2-20), so that, repeating (2-12), the electric and magnetic fields are given in terms of \( \Pi_h \) by

\[ E = -j \nabla \times \Pi_h \quad (2-30) \]
\[ H = \nabla (\nabla \cdot \Pi_h) + \varepsilon_R \Pi_h. \quad (2-30) \]

If one now refers to the general form taken by \( \Pi_h \), as revealed by (2-13) and
(2-21) it is seen that
\[ \nabla \cdot \Pi_h = T Z^\prime \]  \hspace{1cm} (2-31)

and that
\[ \nabla (\nabla \cdot \Pi_h) = Z^\prime \nabla_t T + T Z'' \hat{z} \]  \hspace{1cm} (2-32)

Using (2-24), this can be further written as
\[ \nabla (\nabla \cdot \Pi_h) = Z^\prime \nabla_t T + k_c^2 T Z \hat{z} - \epsilon_R T Z \hat{z} \]  \hspace{1cm} (2-33)

Finally, substituting this expression for \( \nabla (\nabla \cdot \Pi_h) \) into (2-30) one obtains for the fields the relationships
\[ E = -j Z \nabla_t T \times \hat{z} \]  \hspace{1cm} (2-34)
\[ H = Z^\prime \nabla_t T + k_c^2 T Z \hat{z} \]  \hspace{1cm} (2-35)

The electric field and the transverse portion of the magnetic field can be written as
\[ E = \nabla e \]  \hspace{1cm} (2-36)
\[ H_t = \nabla h \]  \hspace{1cm} (2-37)

where the vectors \( e \) and \( h \), called the "mode vectors", contain the transverse spatial dependence, and the functions \( \nabla \) and \( \nabla_t \) depend only on the coordinate \( \epsilon \).

Based on these two equations and on the equation pair (2-34) and (2-35), it is seen that the mode vector can be written as
\[ e = \hat{z} \times \nabla_t T \]  \hspace{1cm} (2-38)
\[ h = D \nabla_t T, \]  \hspace{1cm} (2-39)

where \( D \) is to be arbitrarily chosen. Now, referring to (2-34), this determines that
\[ \nabla = j Z \]  \hspace{1cm} (2-40)
and this in turn determines, along with (2-26) that $V$ can be written as

$$V = V^+ e^{-j \beta z} + V^- e^{j \beta z}.$$  \hfill (2-41)

Substituting the assumed form of $h_r$ equation (2-39), into equation (2-37), and comparing the resulting expression for $H_\perp$ to the transverse component of equation (2-35), one sees that

$$I = \frac{I}{D} Z' ,$$  \hfill (2-42)

and, using the relationships (2-40) and (2-41), that

$$I = \left(-\beta/D\right)\left(V^+ e^{-j \beta z} - V^- e^{j \beta z}\right).$$  \hfill (2-43)

Thus, if $V$ and $I$ are considered to be voltage and current on an equivalent transmission line, the characteristic impedance on that line may be adjusted by choosing various values of $D$. Setting $D$ to $-1$ and using equation (2-27) for $T$ in the resulting equations (2-38) and (2-39) leads to the following expressions for the mode vector components:

$$e_r = - j \frac{n}{\tau} C J_1 (k_c r) e^{jn \phi}$$  \hfill (2-44)

$$e_\phi = k_c C J_1'(k_c r) e^{jn \phi}$$  \hfill (2-45)

$$h_r = - k_c C J_1'(k_c r) e^{jn \phi}$$  \hfill (2-46)

$$h_\phi = - j \frac{n}{\tau} C J_1 (k_c r) e^{jn \phi} .$$  \hfill (2-47)

The expressions for $V$ and $I$ are

$$V = V^+ e^{-j \beta z} + V^- e^{j \beta z}$$  \hfill (2-48)

$$I = \gamma \left( V^+ e^{-j \beta z} - V^- e^{j \beta z} \right)$$  \hfill (2-49)

$$\gamma = \beta .$$  \hfill (2-50)

At the wall of the circular guide, $r = r_0$, the condition $E_\phi = 0$ must be satisfied.
According to (2-45), one must therefore have
\[ J_i'(k_c r_0) = 0, \tag{2 - 51} \]
which results in
\[ k_c r_0 = j_m' \tag{2 - 52} \]
where \( j_m' \) is the \( m \)th zero of \( J_i'(u) \). The resulting mode vectors are
\[ \mathbf{e}_m \quad \text{and} \quad \mathbf{h}_m \quad \text{, where the first subscript is due to the restriction } |m| = 1. \]
The corresponding modes are denoted as \( \text{TE}_{m}^{0} \) modes.

It can be shown that the mode vectors obey an orthogonality condition, such that the relationships
\[ \int_0^{\pi} \int_0^{r_0} \mathbf{e}^*_i \cdot \mathbf{e}_j r \, dr \, d\phi = 0 \quad \text{for} \quad i \neq j \tag{2 - 53} \]
\[ \int_0^{\pi} \int_0^{r_0} \mathbf{h}^*_i \cdot \mathbf{h}_j r \, dr \, d\phi = 0 \quad \text{for} \quad i \neq j \tag{2 - 54} \]
\[ \int_0^{\pi} \int_0^{r_0} \mathbf{h}^*_i \cdot \mathbf{e}_j r \, dr \, d\phi = 0 \quad \text{for} \quad i \neq j \tag{2 - 55} \]
\[ \int_0^{\pi} \int_0^{r_0} \mathbf{e}^*_i \cdot \mathbf{e}_j r \, dr \, d\phi = 0 \quad \text{for} \quad i \neq j \tag{2 - 56} \]
are satisfied. By a proper choice of the arbitrary coefficient \( C \) appearing in
the mode vectors, i.e.
\[ C = \left( \frac{2}{\pi^{1/2}} \right) \left\{ k_c r_0^{2} \left( J_0'^2(k_c r_0) + J_1'^2(k_c r_0) \right) \right\}^{1/2} \tag{2 - 57} \]
\[ + \left( k_c r_0^{2} - (n-1)^2 \right) J_0'^2(k_c r_0) + \left( k_c r_0^{2} - (n+1)^2 \right) J_1'^2(k_c r_0) \right\}^{1/2}, \]
the further conditions
\[ \int_0^{\pi} \int_0^{r_0} \mathbf{e}^*_i \cdot \mathbf{e}_i r \, dr \, d\phi = 1 \tag{2 - 58} \]
\[
\int_{0}^{2\pi} \int_{0}^{r_{0}} h_{iL}^{*} \cdot h_{iL} \, r \, dr \, d\phi = 1 \quad (2-59)
\]

can be established.

2.4 The TM Modes

To generate dielectric modes TM to \( \varepsilon \) the magnetic field is given in terms of a vector potential as follows:

\[
\mathbf{H} = j \varepsilon_{r} \nabla \times \Pi_{e} \quad (2-60)
\]

A development similar to that given for the TE modes leads to the mode vector components

\[
\begin{align*}
e_{r} &= k_{c} C \mathbf{J}_{1}'(k_{c} r) e^{j n \phi} \quad (2-61) \\
e_{\phi} &= j \frac{n}{r} C \mathbf{J}_{1}(k_{c} r) e^{j n \phi} \quad (2-62) \\
h_{r} &= -j \frac{n}{r} C \mathbf{J}_{1}(k_{c} r) e^{j n \phi} \quad (2-63) \\
h_{\phi} &= k_{c} C \mathbf{J}_{1}'(k_{c} r) e^{j n \phi} \quad (2-64)
\end{align*}
\]

where the constant \( k_{c} \) must satisfy

\[
k_{c} r_{0} = j \alpha m \quad (2-65)
\]

and \( j \alpha m \) is the \( m \)th zero of \( \mathbf{J}_{1}(\alpha) \). In order to establish the orthogonality property for the TM mode vectors, the coefficient \( C \) is chosen according to

\[
C = (2 \pi^{-1/2}) \left[ k_{c}^{-2} \mathbf{r}^{2} \left( \mathbf{J}_{0}''(k_{c} r_{0}) + \mathbf{J}_{2}''(k_{c} r_{0}) \right) + \left( k_{c}^{2} \mathbf{r}^{2} - (n - j)^{2} \right) \mathbf{J}_{0}''(k_{c} r_{0}) + \left( k_{c}^{2} \mathbf{r}^{2} - (n + j)^{2} \right) \mathbf{J}_{2}''(k_{c} r_{0}) \right]^{-1/2} \quad (2-66)
\]

The transverse electric and magnetic fields themselves become

\[
\begin{align*}
(\mathbf{E}_{t})_{im} &= \mathcal{V}_{im}(\alpha) \mathbf{E}_{i} m \\
(\mathbf{H})_{im} &= \mathcal{I}_{im}(\alpha) \mathbf{h}_{i} m
\end{align*} \quad (2-67, 2-68)
the equivalent transmission line voltage and current are

\[ V_{lm}(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z} \quad (2-69) \]

\[ I_{lm}(z) = \gamma (V^+ e^{-j\beta z} - V^- e^{j\beta z}) \] \quad (2-70)

the admittance is

\[ \gamma = \frac{\varepsilon_R}{\beta} \quad , \quad (2-71) \]

and the propagation constant is given by

\[ \beta^2 = \varepsilon_R - k_c^2 \] \quad (2-72)
CHAPTER 3

MODES IN CIRCULAR WAVEGUIDE

FILLED WITH AXIALLY MAGNETIZED FERRITE

3.1 Introduction

The theoretical investigation of the circular waveguide cavity end-loaded with axially magnetized ferrite is strongly based on the treatment of modes in ferrite-filled circular guide. A theory of these modes is presented in this chapter, with the approach relying closely on the treatment by Suhl and Walker. The special properties of the magnetized ferrite are taken into account by introducing a tensor permeability into Maxwell's Equations for the medium. Dependence of the elements of this tensor on the quantities characterizing the medium is stated. From Maxwell's Equations general solutions for the field components in the direction of magnetization are obtained. The remaining field components are expressed in terms of the longitudinal ones. Applying these general expressions to a ferrite-filled circular guide with magnetization along the axis and requiring that the implied boundary conditions be satisfied yields a characteristic equation for propagation in the guide. A graphical approach to treatment of this equation is then discussed. The analysis is extended beyond that by Suhl and Walker to include the situation for evanescent modes. The approach is then applied to the specific case of interest in the present cavity problem, and solution plots are constructed which will be used later in the prediction of cavity resonance frequencies.
3.2 The Tensor Permeability

If the coordinate system is chosen so that the z-axis lies along the d.c. magnetic field, Maxwell's Equations in the magnetized ferrite can be written as

$$\nabla \times \mathbf{E} = -j\omega \mu \mathbf{H} \quad (3 - 1)$$

$$\nabla \times \mathbf{H} = j\omega \varepsilon \mathbf{E} \quad (3 - 2)$$

where the permeability is given by

$$\mathbf{H} = \begin{pmatrix} H & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{pmatrix}. \quad (3 - 3)$$

If the definitions of the unitless quantities $\sigma$ and $\rho$ are

$$\sigma \equiv \tau H_{dc}/\omega \quad (3 - 4)$$

$$\rho \equiv \tau M_{dc}/\mu_0 \omega, \quad (3 - 5)$$

the elements of the permeability tensor can be written as

$$\mu/\mu_0 = 1 - \rho \sigma/(1 - \sigma^2) \quad (3 - 6)$$

$$\kappa/\mu_0 = \rho/(1 - \sigma^2). \quad (3 - 7)$$

3.3 Solution of Maxwell's Equations

3.3.1 Introduction of a useful operator

Before proceeding with the solution of Maxwell's Equations it will be useful to establish a special vector notation and to define an associated operator.

A vector $\mathbf{A}_t$, which is transverse to the z direction, may be operated on by an
operator $P$ to form a new vector $\mathbf{A}_t$, according to the following definitions:

\begin{align}
\mathbf{A}_t &= A_x \hat{x} + A_y \hat{y} \\
\mathbf{A}_t^+ &= A_y \hat{x} - A_x \hat{y} \\
PA_t &= \mathbf{A}_t^+ .
\end{align}

Examining these three equations, one sees that the operator $P$ simply rotates $\mathbf{A}_t$ by $90^\circ$ in the clockwise sense. It is noted that

\begin{equation}
P^2 = -1 ,
\end{equation}

since

\begin{equation}
P^2 \mathbf{A}_t = P(\mathbf{A}_t^+ ) = PA_t^+ = -\mathbf{A} .
\end{equation}

$P$ and $\mathbf{A}_t^+$ will retain their meanings when $\mathbf{A}_t$ is replaced by

\begin{equation}
\mathbf{V}_t = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} .
\end{equation}

3.3.2 Coupled differential equation for $E_z$ and $H_z$

It is assumed that the $z$-dependence of $E$ and $H$, which are complex time independent vectors, takes the form of the factor $e^{-j\beta z}$. Under these conditions, equation (3-2) can be written as

\begin{align}
\nabla_t \times H_t &= j \omega \varepsilon E_z \hat{z} \\
-\hat{z} \times \nabla_t H_z - j\beta \hat{z} \times H_t &= j \omega \mu E_t .
\end{align}

A similar procedure can be carried out for equation (3-1), resulting in

\begin{align}
\nabla_t \times E_t &= -j \omega \mu_0 H_z \hat{z} \\
-\hat{z} \times \nabla_t E_z - j\beta \hat{z} \times E_t &= (-j \omega \mu \varepsilon H_t )_z .
\end{align}

Now, making use of the definition of $\mathbf{A}_t^+$, these four equations can be rewritten again:
\[ \nabla_t \cdot H_t^\dagger = j \omega e E_z \]  
(3 - 18)

\[ \nabla_t H_z + j \beta H_t^\dagger = j \omega e E_t \]  
(3 - 19)

\[ \nabla_t \cdot E_t^\dagger = -j \omega \mu H_z \]  
(3 - 20)

\[ \nabla_t E_z + j \beta E_t^\dagger = -j \omega \mu H_t - \omega \chi H_t^\dagger . \]  
(3 - 21)

Finally, the scaling of r.f. magnetic field and length as applied to Maxwell's

Equations in Chapter 2 is carried out for this set of equations. If, in addition, the definitions

\[ \gamma \equiv \mu / \mu_0 \]  
(3 - 22)

\[ \rho \equiv \kappa / \kappa \]  
(3 - 23)

are made, equations (3-18) through (3-21) become

\[ \nabla_t^\dagger H_z + j \beta H_t^\dagger = j E_t \]  
(3 - 24)

\[ \nabla_t \cdot H_t^\dagger = j E_z \]  
(3 - 25)

\[ \nabla_t^\dagger E_z + j \beta E_t^\dagger = -\gamma (j H_t + \rho H_t^\dagger) \]  
(3 - 26)

\[ \nabla_t \cdot E_t^\dagger = -j H_z . \]  
(3 - 27)

It is recalled that \( \beta \) in these equations is the original propagation constant divided by \( \beta_0 \), where \( \beta_0 = 2\pi / \lambda_0 \) and \( \lambda_0 \) is the wavelength of a plane wave in an infinite region of unmagnetized ferrite.

If one operates on (3-24) and (3-26) with \( \nabla_t^\dagger \) and uses (3-25) and (3-27) one obtains

\[ j \beta \nabla_t \cdot H_t^\dagger = j \nabla_t \cdot E_t = -\beta E_z \]  
(3 - 28)

\[ j \beta \nabla_t \cdot E_t^\dagger = -\gamma (j \nabla_t \cdot H_t + j \rho E_z) = \beta H_z . \]  
(3 - 29)

Operating on (3-24) and (3-26) with \( \nabla_t^\dagger \) and using (3-25) and (3-27) one obtains
\[ \nabla_t^2 H_z + j \beta \nabla_t \cdot H_t = -H_z \quad (3-30) \]
\[ \nabla_t^2 E_z + j \beta \nabla_t \cdot E_t = -\gamma (E_z + \rho \nabla_t \cdot H_t) . \quad (3-31) \]

Elimination of \( \nabla_t \cdot H_t \) and \( \nabla_t \cdot E_t \) gives
\[ \nabla_t^2 H_z + (1 - \beta^2/\gamma) H_z = j \beta \rho E_z \quad (3-32) \]
\[ \nabla_t^2 E_z + \gamma (1 - \beta^2 - \beta^3/\gamma) E_z = -j \beta \rho H_z . \quad (3-33) \]

These are coupled differential equations in \( E_z \) and \( H_z \). They show that, because \( \rho \) is non-zero in general, \( E_z \) and \( H_z \) will both be non-zero in general. Later, it is shown that the transverse field components can be expressed in terms of the two longitudinal ones, but first the problem of solving for \( E_z \) and \( H_z \) is considered.

### 3.3.3 General solutions for \( E_z \) and \( H_z \)

A linear combination of \( E_z \) and \( H_z \) is defined as
\[ \Psi = E_z + j \Lambda H_z \quad (3-34) \]

and the possibility of a value of \( \Lambda \) for which (3-32) and (3-33) can be used to obtain a differential equation for \( \Psi \) is investigated. If one multiplies equation (3-32) by \( j \Lambda \) and adds the resulting equation to (3-33), the result is
\[ \nabla_t^2 (E_z + j \Lambda H_z) + \gamma (1 - \beta^2 - \beta^3/\gamma) E_z + j \Lambda (1 - \beta^2/\gamma) H_z \]
\[ = -j \beta \rho H_z - \beta \rho \Lambda E_z. \quad (3-35) \]

This equation is rewritten as
\[ \nabla_t^2 \Psi + \gamma (1 - \beta^2 - \beta^3/\gamma) \rho \Lambda \nabla E_z \]
\[ + \gamma (1 - \beta^2/\gamma) + \beta \rho / \Lambda \nabla j \Lambda H_z = 0 \quad (3-36) \]

If this is to be a differential equation in the unknown \( \Psi \), one must have
\[ \gamma (1 - \beta^2 - \beta^3/\gamma) + \beta \rho / \Lambda = 1 - \beta^2/\gamma + \beta \rho / \Lambda = \gamma. \quad (3-37) \]
The two values of $\Lambda$ for which this condition is satisfied are the roots of the equation

$$\Lambda^2 + \frac{1}{\beta\rho} [\nu (1 - \rho^2 - \beta^2/\nu) - 1 + \beta^2/\nu] \Lambda - 1 = 0. \quad (3.38)$$

They are given by

$$\Lambda_{1,2} = - \left[ \nu (1 - \rho^2 - \beta^2/\nu) - 1 + \beta^2/\nu \right] / 2 \beta\rho \quad (3.39)$$

$$+ \left\{ \left[ \nu (1 - \rho^2 - \beta^2/\nu) - 1 + \beta^2/\nu \right]^2 / 4 \beta^2 \rho^2 + 1 \right\}^{1/2}.$$

The corresponding values of $\chi^2$ are $\chi^2_1$ and $\chi^2_2$:

$$\chi^2_{1,2} = \nu (1 - \rho^2 - \beta^2/\nu) + \beta\rho \Lambda_{1,2} \quad (3.40)$$

or

$$\chi^2_{1,2} = 1 - \beta^2/\nu - \beta\rho \Lambda_{1,2} \quad (3.41)$$

where the identity

$$\Lambda_1 \Lambda_2 = -1 \quad (3.42)$$

has been used. We then have

$$\Psi_1 = E_z + j \Lambda_1 H_z \quad (3.43)$$

$$\Psi_2 = E_z + j \Lambda_2 H_z \quad (3.44)$$

and

$$\nabla^2 \Psi_1 + \chi^2_1 \Psi_1 = 0 \quad (3.45)$$

$$\nabla^2 \Psi_2 + \chi^2_2 \Psi_2 = 0. \quad (3.46)$$
The general solutions for $\Psi_1$ and $\Psi_2$ become

$$\Psi_1 = A_1 J_n (\chi_1 r) e^{j n \phi}$$  

$$\Psi_2 = A_2 J_n (\chi_2 r) e^{j n \phi}.$$  

(3-47)  

(3-48)

3.3.4 The transverse components in terms of $E_\perp$ and $H_\perp$

If one recalls the definition of the operator $P$ as given in (3-8) through (3-10), equations (3-24) and (3-26) can be written as

$$jP E_t + j\beta H_t = -\nabla_t H_\perp$$  

(3-49)

$$\nabla (\rho - jP) H_t + j\beta E_t = -\nabla_t E_\perp.$$  

(3-50)

First, one eliminates $H_t$ between these. Equation (3-49) is multiplied by the operator $\nabla (\rho - jP)$ while (3-50) is multiplied by the operator $j\beta$.

The latter result is then subtracted from the first:

$$(\nu - \beta^2 - j\nu P) E_t = -\nabla (\nu P - \rho) \nabla_t H_\perp - j\beta \nabla_t E_\perp.$$  

(3-51)

One then returns to equations (3-49) and (3-50) and eliminates $E_t$. Equation (3-49) is multiplied by $j\beta$, equation (3-50) is multiplied by $jP$, and the two are subtracted:

$$(\nu - \beta^2 - j\nu P) H_t = j\beta \nabla_t H_\perp + jP \nabla_t E_\perp.$$  

(3-52)

In order to remove the operator $P$ from the expressions in (3-51) and (3-52), both equations are operated upon by $(\nu - \beta^2 + j\nu P)/\nu$ and the results are

$$((\nu - \beta^2)/\nu - \nu \beta^2) E_t = j(\nu \beta^2 - (\nu - \beta^2)) \nabla_t H_\perp$$

$$+ (\rho(\nu - \beta^2) - \nu P) \nabla_t H_\perp.$$
\[-j\beta \left( 1 - \beta^2/\nu \right) \nabla_t E_z + \beta \rho \nabla_t^\dagger E_z \]
\[((\nu - \beta^2)/\nu - \nu \rho^2)H_z = j\beta \left( 1 - \beta^2/\nu \right) \nabla_t H_x \]
\[-\beta \rho \nabla_t H_z + j(1 - \beta^2/\nu) \nabla_t^\dagger E_z + \rho \nabla_t^\dagger E_z \]. \quad (3-54)

These equations express the transverse portions of the electric and magnetic field in terms of the longitudinal components.

3.4 The Waveguide Characteristic Equation

3.4.1 The region within a circular waveguide

In this section, a characteristic equation is derived from which allowed values of the propagation constant $\beta$ can be found. Previously in this development no boundary conditions have been impressed upon the solutions to Maxwell's Equations, the only conditions having been that the d.c. magnetic field and magnetization are in the $z$ direction, and that field dependence on $z$ takes the form of a factor $\exp(-j\beta z)$. It will now be required that the solution of Maxwell's Equations applies to the region within a ferrite-filled circular guide with the magnetization lying along the guide axis. The result is that the fields must satisfy certain further conditions at the guide walls. Eventually it will be seen that only certain values of $\beta$ will allow these conditions to be fully satisfied.
3.4.2 The boundary conditions on $E_z$

Equations (3-45) and (3-46), which define the functions $\Psi_1$ and $\Psi_2$, can be solved simultaneously for the longitudinal electric and magnetic fields. The latter are thus given by

$$E_z = \frac{\lambda_2 \Psi_1 - \lambda_1 \Psi_2}{\lambda_2 - \lambda_1} \quad (3-55)$$

$$H_z = j \frac{\Psi_1 - \Psi_2}{\lambda_2 - \lambda_1} \quad (3-56)$$

Substituting (3-47) and (3-48) into (3-55), one obtains

$$E_z = \frac{\lambda_2 A_1 J_n(\chi_1 r) - \lambda_1 A_2 J_n(\chi_2 r)}{\lambda_2 - \lambda_1} e^{in\phi} \quad (3-57)$$

If it is now required that $E_z$ be zero at the waveguide wall, $r = r_0$, the coefficients $A_1$ and $A_2$ must satisfy the condition

$$\lambda_2 A_1 J_n(\chi_1 r_0) = \lambda_1 A_2 J_n(\chi_2 r_0) \quad (3-58)$$

This leads to

$$A_1 = A \frac{J_n(\chi_2 r_0)}{\lambda_2} \quad (3-59)$$

$$A_2 = A \frac{J_n(\chi_1 r_0)}{\lambda_1} \quad (3-60)$$

where $A$ is an arbitrary coefficient. The longitudinal field components become

$$E_z = A \frac{J_n(\chi_2 r_0) J_n(\chi_1 r) - J_n(\chi_1 r_0) J_n(\chi_2 r)}{\lambda_2 - \lambda_1} e^{in\phi} \quad (3-61)$$

$$H_z = j \frac{A}{\lambda_1 A_2} \frac{J_n(\chi_2 r_0) J_n(\chi_1 r) - J_n(\chi_1 r_0) J_n(\chi_2 r)}{\lambda_2 - \lambda_1} e^{in\phi} \quad (3-62)$$
3.4.3 Solutions for the transverse components

If the expressions for $E_z$ and $H_z$ given in equation (3-61) and (3-62) are substituted into (3-53) and (3-54), i.e. the expressions giving the transverse field components in terms of the longitudinal ones, explicit expressions for the former components will be obtained. In doing this, one makes the definitions

$$\lambda_1 \equiv \beta \Lambda_2 \quad (3-63)$$

$$\lambda_2 \equiv \beta \Lambda_1 \quad (3-64)$$

$$\Omega \equiv (\Lambda_2 - \Lambda_1) \nu ((1 - \beta^2/v^2)^2 - \rho^2), \quad (3-65)$$

uses the identity

$$\Lambda_1 \Lambda_2 \equiv -1, \quad (3-66)$$

and applies equations (3-40) and (3-41) giving $\chi_1^z$ and $\chi_2^z$ in terms of $\Lambda_1$ and $\Lambda_2$. The expressions which result from this are given in equations (3-67) through (3-70), these having been multiplied by $\exp(-\imath \beta a)$ for completeness.

(These are displayed on the next page.)

3.4.4 The ferrite modes

To this point, the values which the propagation constant can take on have not been restricted. If $\beta$ is real and positive, the expressions (3-67) through (3-70) explicitly allow only for propagation in the $+z$ direction, and if $\beta$ is negative imaginary they imply decay. Field expressions will now be written which represent propagation and evanescence in both directions. Equations (3-67)
\[ E_r = j \beta \left\{ n \frac{X_z^2}{\lambda_z r} J_n(x_z r) J_n(x_r) - n \frac{X_z^2}{\lambda_z r} J_n(x_z r) J_n(x_r) - \chi_z^2 \chi_z J_n(x_z r) J_n(x_r) + \chi_z^2 \chi_z J_n(x_z r) J_n(x_r) \right\} \frac{A}{\omega} e^{j\phi} e^{-j\beta z} \] (3.67)

\[ E_\phi = \beta \left\{ n \frac{X_z^2 l}{\lambda_z} J_n(x_z r) J_n(x_r) - n \frac{X_z^2 l}{\lambda_z} J_n(x_z r) J_n(x_r) - \frac{X_z^2}{\lambda_z} \chi_z J_n(x_z r) J_n(x_r) + \frac{X_z^2}{\lambda_z} \chi_z J_n(x_z r) J_n(x_r) \right\} \frac{A}{\omega} e^{j\phi} e^{-j\beta z} \] (3.68)

\[ H_r = \left\{ -n \frac{X_z^2}{l} J_n(x_z r) J_n(x_r) + n \frac{X_z^2}{l} J_n(x_z r) J_n(x_r) - \left[ \lambda_z (1 - \beta^2 / \nu) - \rho \right] \chi_z J_n(x_z r) J_n(x_r) + \left[ \lambda_z (1 - \beta^2 / \nu) - \rho \right] \chi_z J_n(x_z r) J_n(x_r) \right\} \frac{A}{\omega} e^{j\phi} e^{-j\beta z} \] (3.69)

\[ H_\phi = j \left\{ -n \left[ \lambda_z (1 - \beta^2 / \nu) - \rho \right] \frac{1}{l} J_n(x_z r) J_n(x_r) + n \left[ \lambda_z (1 - \beta^2 / \nu) - \rho \right] \frac{1}{l} J_n(x_z r) J_n(x_r) - \chi_z^2 \chi_z J_n(x_z r) J_n(x_r) + \chi_z^2 \chi_z J_n(x_z r) J_n(x_r) \right\} \frac{A}{\omega} e^{j\phi} e^{-j\beta z} \] (3.70)
through (3-70) can be rewritten as

\[
E_r = j F_1 B e^{j\phi} e^{-j\beta z} \quad (3 - 71)
\]

\[
E_\phi = F_2 B e^{j\phi} e^{-j\beta z} \quad (3 - 72)
\]

\[
H_r = F_3 B e^{j\phi} e^{-j\beta z} \quad (3 - 73)
\]

\[
H_\phi = j F_4 B e^{j\phi} e^{-j\beta z}, \quad (3 - 74)
\]

where a new arbitrary coefficient has been defined according to

\[
B \equiv A/\Omega. \quad (3 - 75)
\]

The definition of the F's can be seen by comparing (3-71) through (3-74) with (3-67) through (3-70). Complex values of \( \beta \) will not be considered, since these would correspond to propagation losses which could not be explained by the model presently being used for the ferrite. The definition

\[
\beta^- \equiv -\beta \quad (3 - 76)
\]

is made. Referring to (3-39), (3-40), (3-63) and (3-64), it can be shown that a sign change in the propagation constant has the following effects:

\[
\exp \left( -j\beta^- z \right) \rightarrow \exp \left( -j\beta z \right) = \exp \left( j\beta z \right) \quad (3 - 77)
\]

\[
\Lambda_{1,2} (\beta) \rightarrow \Lambda_{1,2} (\beta^-) = -\Lambda_{2,1} (\beta) \quad (3 - 78)
\]

\[
\lambda_{1,2} (\beta) \rightarrow \lambda_{1,2} (\beta^-) = \lambda_{2,1} (\beta) \quad (3 - 79)
\]

\[
\gamma_{1,2}^z (\beta) \rightarrow \gamma_{1,2}^z (\beta^-) = \gamma_{2,1}^z (\beta). \quad (3 - 80)
\]

Noting the implied definitions of \( F_1, F_2, F_3 \) and \( F_4 \) it can be shown that
The transverse components can now be written for the case

\[ \beta \rightarrow \beta^- = -\beta : \]

\[ E_r = j F_1 B^- e^{j\phi} e^{j\beta z} \]
\[ E_\phi = F_2 B^- e^{j\phi} e^{j\beta z} \]
\[ H_r = - F_3 B^- e^{j\phi} e^{j\beta z} \]
\[ H_\phi = - j F_4 B^- e^{j\phi} e^{j\beta z}. \]

Combining these expressions for a propagation constant \(-\beta\) with the expressions in (3-71) through (3-74), the transverse fields for the ferrite modes become

\[ E_r = j F_1 e^{j\phi} \left( B^+ e^{-j\beta z} + B^- e^{j\beta z} \right) \]
\[ E_\phi = F_2 e^{j\phi} \left( B^+ e^{-j\beta z} + B^- e^{j\beta z} \right) \]
\[ H_r = F_3 e^{j\phi} \left( B^+ e^{-j\beta z} - B^- e^{j\beta z} \right) \]
\[ H_\phi = j F_4 e^{j\phi} \left( B^+ e^{-j\beta z} - B^- e^{j\beta z} \right). \]

If the definitions

\[ e = \left( j F_1 \hat{r} + F_2 \hat{\phi} \right) e^{j\phi} \]
\[ h = \left( F_3 \hat{r} + j F_4 \hat{\phi} \right) e^{j\phi} \]

are made, the ferrite modes can be rewritten as

\[ E_t = \left( V^+ e^{-j\beta z} + V^- e^{j\beta z} \right) e = V(e) e \]
\[ H_z = ( V^+ e^{-j\beta z} - V^- e^{j\beta z} ) h = I(z) h \]

(3.96)

where \( e \) and \( h \) can be called the mode vectors. When both forms of \( z \)-dependence are included, definition (3-75) is taken into account, and identity (3-66) is used, the expressions for the longitudinal field components become

\[ E_z = \frac{\alpha}{\Lambda_2 - \Lambda_1} ( J_n(\chi z r_o) J_n(\chi z r) - J_n(\chi z r) J_n(\chi z r_o) ) e^{j\phi} ( V^+ e^{j\beta z} - V^- e^{j\beta z} ) \]

(3.97)

\[ H_z = \frac{\alpha}{\Lambda_2 - \Lambda_1} j ( \Lambda_2 J_n(\chi z r_o) J_n(\chi z r) - \Lambda_1 J_n(\chi z r) J_n(\chi z r_o) ) e^{j\phi} ( V^+ e^{j\beta z} + V^- e^{j\beta z} ) \].

(3.98)

3.4.5 The boundary condition on \( E_\phi \) and the characteristic equation

Noting again that the modes in ferrite-filled circular waveguide must satisfy certain boundary conditions at the guide wall, the condition on \( E_\phi \) is

\[ E_\phi |_{r=r_o} = 0 \]

(3.99)

Restricting the analysis to \( n=\pm 1 \) and referring to (3-68) and (3-90) this results in the condition \( F_\phi |_{r=r_o} = 0 \) or

\[ n \chi_z \frac{1}{r_o} J_1(\chi z r_o) J_1(\chi z r) - \frac{\chi_z}{\Lambda_1} \chi_1 J_1(\chi z r_o) J_1(\chi z r) \]

\[ - n \chi_z \frac{1}{r_o} J_1(\chi z r) J_1(\chi z r_o) + \frac{\chi_z}{\Lambda_2} \chi_2 J_1(\chi z r) J_1(\chi z r_o) = 0 \].

(3.100)

This is rearranged to give

\[ \frac{1}{\chi_1} \left( \frac{\chi z}{r_o} J_1(\chi z r_o) J_1(\chi z r) - n \right) = \frac{1}{\chi_2} \left( \frac{\chi z}{r_o} J_1(\chi z r_o) J_1(\chi z r) - n \right) \]

(3.101)

Referring to definitions (3-22) and (3-23) for \( \nu \) and \( \rho \) and to equations (3-6) and (3-7) which give \( \mu \) and \( \kappa \) in terms of \( \sigma \) and \( p \), \( \nu \)
and \( \rho \) can be expressed as
\[
\nu = \left( 1 - \sigma^2 + p \sigma \right) / \left( 1 - \sigma^2 \right) \tag{3-102}
\]
\[
\rho = p / \left( 1 - \sigma^2 + p \sigma \right). \tag{3-103}
\]

Using these, the resulting expressions for \( \Lambda_1 \) and \( \Lambda_2 \), and the definitions of \( \lambda_1 \) and \( \lambda_2 \), it can be shown that
\[
\lambda_{1,2} = -\frac{\sigma}{2} \beta^2 + \frac{1}{2} (\sigma + p) \pm \beta \left[ -\frac{\sigma}{2} \beta + \frac{1}{2} \frac{\sigma + p}{\rho} \right]^2 + 1 \right]^{1/2}. \tag{3-104}
\]

From these, two more relationships can be derived:
\[
\lambda_1 \lambda_2 = -\beta^2 \tag{3-105}
\]
\[
\lambda_1 + \lambda_2 = -\sigma \beta^2 + \sigma + p = \sigma \lambda_1 \lambda_2 + \sigma + p. \tag{3-106}
\]

If \( \nu \) and \( \rho \) in the expression
\[
\chi_{1,2} = 1 - \beta^2 / \nu - \rho \lambda_{1,2} \tag{3-107}
\]
are also written in terms of \( \sigma \) and \( p \), \( \beta^2 \) replaced by \( -\lambda_1 \lambda_2 \), and \( \lambda_2 \) or \( \lambda_1 \) eliminated by using equation (3-106) one can show that
\[
\chi_{1,2} = (1 - \lambda_{1,2}^2) / (1 - \sigma \lambda_{1,2}^2). \tag{3-108}
\]

The waveguide characteristic equation can now be written as
\[
G \left( \sigma, \lambda_1, \eta_0 \right) = G \left( \sigma, \lambda_2, \eta_0 \right) \tag{3-109}
\]
or
\[ G_{1}^{\pm} = G^{\pm}, \]  \hspace{1cm} (3-109)

where the definition
\[ G^{\pm}(\sigma, \lambda, r_{0}) = \frac{l^{-\sigma \lambda}}{1-\lambda^{2}} \left[ \frac{J_{1}(r_{0})}{1-\lambda^{2}} \right] \left[ \frac{J_{1}(r_{0}) J_{1}(r_{0})}{J_{1}(r_{0})} \right] (\pm 1) \]  \hspace{1cm} (3-110)

is used, both the \( n = +1 \) and the \( n = -1 \) cases being included.

### 3.4.6 Treatment of the \( n = \pm 1 \) cases

In equation (3-110), the magnetization, the frequency, the ferrite relative permittivity, the gyromagnetic ratio and the guide radius are considered fixed, thus fixing \( p \) and \( r_{0} \). Then, the behaviour of the ferrite mode propagation constants as the bias magnetic field is varied is determined by investigating the behaviour (as \( \sigma \) is varied) of the values of the propagation constant \( \beta \) that cause the characteristic equation to be satisfied. It must be recalled that \( \lambda_{1} \) and \( \lambda_{2} \) are determined by \( \sigma \), \( p \) and \( \beta \). Thus if \( p \) and \( r_{0} \) are considered fixed, and if a choice of \( \sigma \) and \( \beta \) is made, both \( G_{1}^{+} \) and \( G_{2}^{+} \) are determined.

For \( n = -1 \), one has
\[ G^{-(\sigma, \lambda, r_{0})} = \frac{l^{-\sigma \lambda}}{1-\lambda^{2}} \left[ \frac{J_{1}(r_{0})}{1-\lambda^{2}} \right] \left[ \frac{J_{1}(r_{0}) J_{1}(r_{0})}{J_{1}(r_{0})} \right] + 1 \]  \hspace{1cm} (3-111)

It has been assumed so far that \( \sigma \) and \( p \) are positive. Making the definitions
σ' ≡ -σ \quad (3-112)

t' ≡ -t \quad (3-113)

and if \( \lambda'_1 \) and \( \lambda'_2 \) denote \( \lambda_1 \) and \( \lambda_2 \) evaluated at the same \( \beta \) for \( \sigma' \) and \( t' \), one finds that

\[
\begin{align*}
\lambda'_1 &= -\lambda_2 \\
\lambda'_2 &= -\lambda_1.
\end{align*}
\quad (3-114) (3-115)

One can then show that

\[
\begin{align*}
G^+ (\sigma', \lambda'_1, r_0) &= -G^- (\sigma, \lambda_2, r_0) \\
G^+(\sigma', \lambda'_2, r_0) &= -G^-(\sigma, \lambda_1, r_0).
\end{align*}
\quad (3-116) (3-117)

If one assumes that a value of \( \beta \) has been found such that

\[
G^-(\sigma, \lambda_1, r_0) = G^-(\sigma, \lambda_2, r_0),
\quad (3-118)
\]

it is implied that

\[
G^+(\sigma', \lambda'_1, r_0) = G^+(\sigma', \lambda'_2, r_0),
\quad (3-119)
\]

and similarly, if \( \beta \) has been found such that

\[
G^+(\sigma', \lambda'_1, r_0) = G^+(\sigma', \lambda'_2, r_0)
\quad (3-120)
\]

is satisfied, it can be seen that

\[
G^- (\sigma, \lambda_1, r_0) = G^- (\sigma, \lambda_2, r_0).
\quad (3-121)
\]

Equations (3-118) through (3-121) demonstrate that the set of \( \beta \)'s satisfying the \( n = +1 \) form of the characteristic equation with negative \( \sigma \) and \( t \) is identical to the set that causes the \( n = -1 \) form with \( \sigma \) and \( t \) positive to be satisfied.

This shows that the characteristic equation for \( n = -1 \) can be investigated
by examining the same equation with \( n = +1 \) and with the signs of \( \sigma \) and \( p \) reversed. This development leads to the conclusion that the solution values of \( \beta \) with \( n = -1 \) and magnetization in the \( +z \) direction are the same as those with \( n = +1 \) and the magnetization in the \(-z\) direction. Such a statement can also be made with \( n = +1 \) and \( n = -1 \) interchanged. Therefore, it is the sense of field pattern rotation with respect to the direction of magnetization that effects the solution values of \( \beta \), not the sense of rotation with respect to the \( +z \) direction.

If one is interested in a treatment of the characteristic equation for both senses of rotation relative to the \( +z \) direction and for \( \sigma \) and \( p \) actually positive, the \( n = -1 \) case can be investigated by treating the \( n = +1 \) characteristic equation with the sign of \( \sigma \) and \( p \) reversed.

Since only \( G^+ \) will be used, the superscript ++ will be omitted. It is now necessary to investigate the solutions of \( G_1 = G_2 \) for positive and negative \( \sigma \), \( p \). In this investigation \( p \) and \( r_o \) can be considered as parameters which are determined by frequency, ferrite parameters and guide radius. The sign of \( p \) must be the same as that of \( \sigma \), the magnitude of \( p \) being fixed.

3.5 The \( \lambda , \sigma^- \) Plane

3.5.1 Approach to a graphical solution

It is clear that the dependence of \( G(\sigma, \lambda_1, r_o) \) and \( G(\sigma, \lambda_2, r_o) \) on \( \sigma, p, \beta \) and \( r_o \) is complicated, and the treatment will therefore be numerical and graphical. One searches for \( \lambda_1, \lambda_2 \) pairs that correspond to a given \( \beta \) and lead to \( G_1 = G_2 \).
Consider a $\lambda, \sigma$ plane above which it can be imagined that a surface of $G(\sigma, \lambda, r_0)$ is constructed. Note that this surface does not depend explicitly on the parameter $p$, but $r_0$ must be known. This $\lambda, \sigma$ plane is then divided into $\lambda_1, \sigma$ regions and $\lambda_2, \sigma$ regions. These are regions into which $(\lambda_1, \sigma^{-})$ and $(\lambda_2, \sigma^{+})$ points respectively must fall.

Portions of the surface lying above the $\lambda_1, \sigma^{-}$ regions will be the $G_1$ surface. Portions lying above the $\lambda_2, \sigma^{-}$ regions will be the $G_2$ surface. For a given $\sigma^{-}$, it will always be possible to find several $\lambda_1, \lambda_2$ pairs for which $G_1 = G_2$ but these do not necessarily correspond to the same values of $p$. This difficulty is overcome by noting that either $\lambda_1$ or $\lambda_2$ can always be written as a function of the other and of $\sigma^{-}$ and $p$. For positive values of $\sigma^{-}$, the $G_2$ surface is then thought of as being transformed to lie above the $\lambda_1, \sigma^{-}$ regions as does $G_1$ while for negative values of $\sigma^{-}$ the $G_1$ surface is transformed to lie above the $\lambda_2, \sigma^{-}$ regions as does $G_2$. This can be viewed as follows: For $\sigma > 0$ the height of the $G_1$ surface at a point $(\lambda_1, \sigma^{-})$ is given by the value of $G_1$ corresponding to that pair. The height of (transformed) $G_2$ at that point is given by the value of $G_2$ at the $\lambda_2, \sigma^{-}$ pair corresponding to $\lambda_1, \sigma^{-}$. Intersections of the $G_1$ and transformed $G_2$ surfaces now correspond to actual solutions of the characteristic equation, $G_1 = G_2$. For positive values of $\sigma^{-}$ these intersections lie above the $\lambda_1, \sigma^{-}$ regions, and correspond to the $n = +1$ case with magnetization in the $+z$ direction, while for negative values of $\sigma^{-}$ they lie above the $\lambda_2, \sigma^{-}$ regions and correspond to the $n = -1$ case.

The $\lambda_1, \sigma^{-}$ or $\lambda_2, \sigma^{-}$ pair corresponding to an intersection of $G_1$
and \( G_2 \) can be used with the appropriate \( p \) to find the related value of \( \beta \) by finding \( \lambda_2 \) or \( \lambda_1 \) respectively and by utilizing the relationship \( \beta^2 = -\lambda_1 \lambda_2 \).

Projections of intersections of \( G_1 \) and \( G_2 \) onto the \( \lambda, \sigma \) plane are called solution curves.

### 3.5.2 Propagating ferrite modes

The positive \( \sigma \) portion of the \( \lambda, \sigma \) plane is considered first. Recall that for \( \sigma^+ \) positive \( p \) is positive. One must find \( \lambda_1 \) and \( \lambda_2 \) as the propagation constant \( \beta \) is varied. The constant \( \beta \) is allowed to be positive real, negative imaginary, or zero.

The positive real case is considered first. The equations for \( \lambda_1 \) and \( \lambda_2 \)

\[
\lambda_{1,2} = -\frac{\sigma}{2} \beta^2 + \frac{1}{2} (\sigma^+-p) \pm \left\{ \left[-\frac{\sigma}{2} \beta^2 + \frac{1}{2} (\sigma^+-p) \right]^2 + \beta^2 \right\}^{1/2}, \tag{3-122}
\]

show that \( \lambda_1 \) and \( \lambda_2 \) are real, since

\[
\left[-\frac{\sigma}{2} \beta^2 + \frac{1}{2} (\sigma^-+p) \right]^2 + \beta^2
\]

is always positive. For \( \beta = 0 \) it can be shown that

\[
\lambda_1 = \sigma + p \tag{3-123}
\]

\[
\lambda_2 = 0. \tag{3-124}
\]

As \( \beta \to \infty \) it can be seen that

\[
\lambda_1 \to 1/\sigma \tag{3-125}
\]

\[
\lambda_2 \to -\infty. \tag{3-126}
\]
Further investigation of the expressions for \( \lambda_1 \) and \( \lambda_2 \) when \( \sigma, p \) are positive and \( \beta \) is positive real shows that the region of the positive-half plane into which all \((\lambda_1, \sigma)\) points fall is that located between the line \( \sigma = \lambda + p \) and the curve \( \sigma = 1/\lambda \), the latter not being included. The \( \lambda_2, \sigma \) region is the second quadrant. These regions are illustrated by shading in Figure 3.1, for the case \( p = .8 \).

### 3.5.3 Evanescent ferrite modes

Assume \( \beta \) takes the form \(-j\alpha\) where \( \alpha \) is positive real. For \( \exp(-j\beta z) \) this leads to \( \exp(-\alpha z) \). It is important not only to find the \( \lambda_1, \sigma \) and \( \lambda_2, \sigma \) regions corresponding to this class of \( \beta \), but to learn if such values of \( \beta \) lead to complex \( \lambda_1 \) and \( \lambda_2 \) which would not be possible to treat by the method proposed. The expressions for \( \lambda_1 \) and \( \lambda_2 \) can now be written as

\[
\lambda_{1,2} = \frac{\alpha^2}{2} \sigma^2 + \frac{1}{2} \sigma (\sigma + p) \pm j \alpha \left\{ -\left[ \frac{\sigma^2 \alpha^2 + 1 \sigma + p}{\alpha^2} \right]^2 + 1 \right\}^{1/2}
\]

or

\[
\lambda_{1,2} = \frac{\sigma}{2} \alpha^2 + \frac{1}{2} \sigma (\sigma + p) \pm \left\{ \left[ \frac{\sigma^2 \alpha^2 + 1 \sigma + p}{\alpha^2} \right]^2 - \alpha^2 \right\}^{1/2}.
\]

The condition for real \( \lambda_1 \) and \( \lambda_2 \) (\( \lambda_1, \lambda_2 \) are either both real or both complex) is

\[
\left[ \frac{\sigma^2}{2} \alpha^2 + \frac{1}{2} \sigma (\sigma + p) \right]^2 - \alpha^2 \geq 0.
\]

This in turn leads to the condition

\[
\alpha^2 - \frac{1}{\sigma^2} \alpha + \left( 1 + \frac{p}{\sigma} \right) \geq 0,
\]
where the restriction that \( \sigma \) and \( p \) are positive has been used. The definition

\[
Q(\infty) \equiv \infty^2 - \frac{p}{\sigma} \infty + (1 + \frac{p}{\sigma})
\]  \hspace{1cm} (3-151)

is made. For non-negative \( Q(\infty) \), \( \lambda_1 \) and \( \lambda_2 \) are real. If, for a particular \( \sigma \) and \( p \) there is a positive \( \infty \) which leads to \( Q(\infty) < 0 \), the corresponding \( \lambda_1 \) and \( \lambda_2 \) are complex. It is known furthermore, that such an \( \infty \) will indeed exist if there are two distinct real roots of \( Q(\infty) = 0 \). Such distinct roots exist if the condition \( \sigma < \frac{p}{\lambda^2} + \left(\frac{p^2}{4\lambda^2} + 1\right)^{1/2} \) is satisfied.

The value of \( \sigma \) given by \( \sigma = \frac{p}{\lambda^2} + \left(\frac{p^2}{4\lambda^2} + 1\right)^{1/2} \) will be called \( \sigma_0 \) and corresponds to the intersection of \( \sigma = \lambda + p \) and \( \sigma = 1/\lambda \) in the first quadrant.

For \( \sigma < \sigma_0 \) values of \( \infty \) in the range

\[
\frac{1}{\sigma} - \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma^2}\right]^{1/2} \leq \infty \leq \frac{1}{\sigma} + \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma^2}\right]^{1/2}
\]  \hspace{1cm} (3-132)

produce complex \( \lambda_1 \) and \( \lambda_2 \). For \( \sigma = \sigma_0 \), \( \lambda_1 \) and \( \lambda_2 \) become equal when

\[
\infty = \frac{1}{\sigma} \pm \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma^2}\right]^{1/2}.
\]  \hspace{1cm} (3-153)

For \( \sigma > \sigma_0 \) it can be shown that as \( \infty \) increases from zero, \( \lambda_1 \) increases from \( \sigma = \lambda + p \) while \( \lambda_2 \) increases from \( \lambda = 0 \) and approaches \( \sigma = 1/\lambda \) as \( \infty \) approaches \( \infty \).

For \( \sigma = \sigma_0 \) it can be demonstrated that \( \lambda_1 \) remains constant at \( i/\sigma \) as \( \infty \) increases from zero, until \( \infty = 1/\sigma \), at which point \( \lambda_2 \) remains constant and \( \lambda_1 \) increases to infinity.
Last is the case $\sigma < \sigma_0$. Using the expressions for $\lambda_1$ and $\lambda_2$ with $\beta = -j\omega$, $\omega \geq 0$, it can be shown that as $\omega$ increases from zero to $\frac{1}{\sigma} - \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma}\right]^{1/2}$, $\lambda_1$ decreases from $\lambda = \sigma + p$ to $\lambda = \frac{1}{\sigma} - \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma}\right]^{1/2}$ and then becomes complex. As $\omega$ increases from $\frac{1}{\sigma} + \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma}\right]^{1/2}$ to infinity, $\lambda_1$ is again real, and increases indefinitely from $\frac{1}{\sigma} + \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma}\right]^{1/2}$.

In this same process, $\lambda_2$ increases from zero until $\lambda_1 = \lambda_2$ at $\lambda = \frac{1}{\sigma} - \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma}\right]^{1/2}$, after which $\lambda_2$ becomes complex. At $\omega = \frac{1}{\sigma} + \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma}\right]^{1/2}$, $\lambda_2$ becomes real again, $\lambda_2 = \lambda_1 = \frac{1}{\sigma} - \left[\frac{1}{\sigma^2} - 1 - \frac{p}{\sigma}\right]^{1/2}$, and $\lambda_2$ decreases to $\lambda = 1/\sigma$ as $\omega$ increases indefinitely.

An investigation of the characteristic equation has been made for values of $\omega$ which lead to complex $\lambda_1$ and $\lambda_2$ (when $\lambda_1$ and $\lambda_2$ are complex, $\lambda_1 = \lambda_2^*$) and it is believed that none causes the characteristic equation to be satisfied.

The regions of the $\lambda$, $\sigma$ plane corresponding to imaginary $\beta$'s are illustrated by shading in Figure 3.2, which has been constructed for $p = .8$.

3.5.4 The negative-$\sigma$ half-plane

The above considerations for $\sigma > 0$ and for both real and imaginary values of $\beta$ determine the $\lambda_1$, $\sigma$ and $\lambda_2$, $\sigma$ regions in the first two quadrants of the $\lambda$, $\sigma$ plane. These have been seen in Figure 3.1 and 3.2 to occupy the entire half-plane. Further, there is no overlapping of regions. One can locate $\lambda_1$, $\sigma$ and $\lambda_2$, $\sigma$ regions in the $\sigma < 0$ half-plane as well.

Consider $\lambda_1$ and $\lambda_2$ evaluated for $\sigma' = -\sigma$ and $p' = -p$, where $\sigma, p > 0$.
**FIGURE 3.2**

IMAGINARY $\beta$ REGIONS

OF THE $\lambda, \sigma$ PLANE

\[ \sigma^* = \lambda + \rho \]
\[ \sigma^* = 1/\lambda \]
\[ \lambda^2, \sigma \]
\[ \lambda = \frac{1}{\rho} \left[ \frac{1}{\rho} - 1 - \frac{2}{\rho} \right]^{1/2} \]
Substituting $-\sigma$ and $-p$ for $\sigma'$ and $p'$ in the resulting expression one finds that

$$
\lambda_{1,2}' = \frac{\sigma}{\beta} \beta^2 - \frac{1}{\beta} (\sigma+p) \pm \left\{ \left[ -\frac{\sigma}{\beta} \beta^2 + \frac{1}{\beta} (\sigma+p) \right]^2 + \beta^2 \right\}^{1/2} = -\lambda_{2,1}.
$$

Here, the prime denotes evaluation at $\sigma'$ and $p'$. These equalities show that the $\lambda_1$, $\sigma$ regions in the $\sigma < 0$ half-plane are just the reflection through the origin of the $\lambda_2$, $\sigma$ regions for $\sigma > 0$. Similarly, the $\lambda_2$, $\sigma$ regions for $\sigma < 0$ are the reflections through the origin of the $\lambda_1$, $\sigma$ regions for $\sigma > 0$.

### 3.6 The Solution Curves

#### 3.6.1 An approximate method

In the two-dimensional graphical treatment the $G_1$ and $G_2$ surfaces cannot actually be represented. Instead, infinity and zero curves of $G_1$ and $G_2$ are located in the appropriate regions, and are used in roughly locating the solution curves. Infinity and zero curves of $G_1$ are denoted by $I$ and $O$ respectively, while $I'$ and $O'$ are used for $G_2$.

#### 3.6.2 The infinity curves

First, infinity curves of $G$ in the $\lambda$, $\sigma$ plane are considered. Curves of

$$
J_i \left( r \frac{\lambda}{1 - \sigma \lambda} \right)^{1/2} = 0
$$

correspond to infinite $G$. These occur for

$$
r_0 \left( \frac{(1-\lambda^2)/(1-\sigma \lambda)}{J_i(x)} \right)^{1/2} = j_n
$$

where the $j_n$ are the zeros of $J_i(x)$.
A portion of such a curve lying in a \( \lambda_1, \sigma^- \) region will be denoted by \( \mathbf{I}_n \), a portion lying in a \( \lambda_2, \sigma^- \) region by \( \mathbf{I}'_n \). Each infinity curve consists of two branches, one being the reflection through the origin of the other. There are two other infinity curves, these corresponding to \( \lambda = -1 \) and \( \lambda = 0 \), and being called \( \mathbf{I}_b \) or \( \mathbf{I}'_b \) and \( \mathbf{I}_a \) or \( \mathbf{I}'_a \), respectively. After the \( \mathbf{I} \) and \( \mathbf{I}' \) curves are located, transformations are carried out so that the \( \mathbf{I}' \) curves for \( \sigma > 0 \) are transformed to lie in the \( \lambda_1, \sigma^- \) regions while for \( \sigma < 0 \) the \( \mathbf{I} \) curves are transformed to lie in the \( \lambda_2, \sigma^- \) regions.

For \( \sigma < 0 \), the \( \mathbf{I}'_n \) curves are reflections through the origin of the \( \mathbf{I}_n \) curves for \( \sigma > 0 \) while the transformed \( \mathbf{I}_n \) curves for \( \sigma < 0 \) are reflections through the origin of the transformed \( \mathbf{I}'_n \) curves for \( \sigma > 0 \).

### 3.6.3 The zero curves

Recall the expression for \( G \)

\[
G = \frac{1}{\chi^2} \left[ \frac{1}{\lambda} F (\nu \chi) - 1 \right]
\]

(3-135)

where the definitions

\[
\chi^2 = \frac{1 - \lambda^2}{1 - \sigma \lambda}
\]

(3-136)

and

\[
F = \frac{\nu \chi J'_1(\nu \chi)}{J_1(\nu \chi)}
\]

(3-137)

have been used. Note that since all real \( \lambda, \sigma^- \) pairs are allowed, all real values of \( \chi^2 \), positive and negative, will occur. When \( \chi^2 \) is positive, \( \chi \), the square root of \( \chi^2 \), is positive. When \( \chi^2 \) is negative, \( \chi \) is positive imaginary. All positive real and imaginary values of \( \chi \) will occur in
the \( \lambda, \sigma \) plane. It is seen, however, that \( F \), and therefore \( G \), is always real. A plot of \( F(x) \) for real and imaginary \( x \) is given in Figure 3.3.

The branches of the curve \( \sigma = 1/\lambda \) are zero curves of \( G \) when approached from the side on which \( \chi \) is negative. Regions of \( \chi < 0 \) are shown in Figure 3.4. As \( \sigma \) approaches \( 1/\lambda \) along the vertical from a direction such that \( \chi \) is positive, \( \chi \) is real and grows indefinitely. Referring to Figure 3.3 it can be seen in this situation that infinities and zeros of \( F \) occur more and more rapidly as \( 1/\lambda \) is neared. Thus \( G \) does not approach a limit.

If, however, \( \sigma \) approaches \( 1/\lambda \) from a direction such that \( \chi \) is negative, \( \chi \) will be imaginary, and it can be seen that \( F(\rho \chi) \to \rho \chi \to 1/\lambda \) as \( |\chi| \to \infty \), so that \( \frac{1}{\chi} \left[ \frac{1}{\lambda} F(\rho \chi) - 1 \right] \to \frac{\rho |\chi|}{\lambda \chi} \to 0 \).

The branch of the curve \( \sigma = 1/\lambda \) in the first quadrant of the \( \lambda, \sigma \) plane becomes the \( O_C \) curve, while the branch in the third quadrant becomes the \( O'_C \) curve. It is also seen that \( G \to 0 \) as \( \lambda \to \pm \infty \).

As seen from equation (3-135), the other kind of zero for \( G \) occurs along curves corresponding to \( F(\rho \chi) = \lambda \). The plot of \( F(x) \) versus \( x \) exhibits a branch structure. For each branch, \( F \) takes on all real values. The choice of a value for \( \chi \rho^2 = \rho^2 \left( \frac{1-\lambda}{1-\sigma \lambda} \right) \) determines a pair of curves in the \( \lambda, \sigma \) plane. Along these curves \( F(\rho \chi) \) takes on the constant value which is indicated by the plot of \( F(x) \) to correspond to the chosen \( \chi \rho^2 \) value. These curves of constant \( F \) in the \( \lambda, \sigma \) plane exist for any \( \lambda \) (except \( \lambda = 0 \)). At a point along such a curve where the condition \( \lambda = F \) is satisfied, one has
FIGURE 3.3

$F(x)$ for real and imaginary $x$
$\chi^2 < 0$

$\lambda = \frac{1}{\sigma} \pm \left[ \frac{1}{\sigma^2} - 1 - \frac{p'}{\sigma} \right]^{1/2}$

$\sigma = \lambda + p$

$\lambda = 1$

$\sigma = 1/\lambda$

$\sigma = 1/\lambda$

$\sigma = 1/\lambda$

$\lambda = \frac{1}{\sigma} \pm \left[ \frac{1}{\sigma^2} - 1 - \frac{p}{\sigma} \right]^{1/2}$

**Figure 3.4**

Negative $\chi^2$ Regions

Of the $\lambda, \sigma$ Plane
\( G = 0 \). This is a point on a \( O_n \) or \( O_n' \) curve, depending on the location of the point and the branch to which \( \nu \) corresponds. For \( \sigma > 0 \) \( O_n' \) curves are transformed to lie in the \( \lambda_1, \sigma \) region, while for \( \sigma < 0 \) \( O_n \) curves are transformed to lie in the \( \lambda_2, \sigma \) region. In contrast to the infinity curves, zero curves in the \( \lambda_2, \sigma \) region of the third quadrant in the \( \lambda, \sigma \) plane can not be obtained by reflecting the zero curves of the first quadrant through the origin. This will be seen in Figures 3.7 through 3.12.

### 3.6.4 The \( \sigma = \sigma_0 \) line

The intersection of the \( \sigma = 1/\lambda \) and \( \sigma = \lambda + p \) curves in the first quadrant of the \( \lambda, \sigma \) plane occurs at \( \sigma = \sigma_0 \) and \( \lambda = \lambda_0 \) (Figure 3.5), where \( \sigma_0 \) and \( \lambda_0 \) are given by

\[
\begin{align*}
\sigma_0 &= \frac{p}{2} + \left( \frac{p^2}{4} - 1 \right)^{1/2} \\
\lambda_0 &= \frac{p}{2} + \left( \frac{p^2}{4} - 1 \right)^{1/2}.
\end{align*}
\]

For all real values of \( \beta \), it can be shown that for \( \sigma = \sigma_0 \), \( \lambda_2 \) varies from \(-\infty \) to 0 while \( \lambda_1 = \lambda_0 \). As \( \beta \) becomes imaginary, \( \lambda_1 \) remains constant at \( \lambda_0 \) while \( \lambda_2 \) continues to increase. At a certain value of \( \beta \), \( \lambda_2 = \lambda_1 \) and as \( \beta \) becomes more imaginary, \( \lambda_2 \) remains constant at \( \lambda_0 \) and \( \lambda_1 \) increases instead. Thus, for \( \sigma = \sigma_0 \), all \( \lambda_1 's > \lambda_0 \) correspond to \( \lambda_2 = \lambda_0 \).

It is important to understand the behavior of \( G_\infty \) as the line \( \sigma = \sigma_0 \), \( \lambda_1 > \lambda_0 \) is approached from above. If one is initially at a point \( (\lambda_1, \sigma) \) above the horizontal line,
\[ \sigma = \lambda + p \]
\[ \sigma = \frac{1}{\lambda} \]
\[ \lambda = \frac{1}{2} \left( \frac{1}{\sigma} - \frac{1}{\lambda} \right) \]

**Figure 3.5**

Location of the point \((\lambda_0, \sigma_0)\)
and to the right of the line $\sigma = \lambda + p$ the point $(\lambda_2, \sigma)$ will be in the region to the left of $\sigma = 1/\lambda$ and to the right of $\lambda = 0$. As one approaches $\sigma = \sigma_0$ with $(\lambda_1, \sigma)$, $(\lambda_2, \sigma)$ moves downward and to the right, approaching $\sigma = \sigma_0$, $\lambda = \lambda_0$. At the same time $G_2 \to 0$.

If the point $(\lambda_1, \sigma)$ approaches the line $\sigma = \sigma_0$, $\lambda > \lambda_0$ from below, the point $(\lambda_2, \sigma)$ approaches the point $(\lambda_0, \sigma_0)$ from below and to the right. Thus, the $\sigma = 1/\lambda$ line is approached from a region of positive $\lambda_0^2$ and $G_2$ has no limit. For negative $\sigma$, the point $(\lambda_1, \sigma)$ approaches the point $(-\lambda_0, -\sigma_0)$ from below and to the right as $(\lambda_2, \sigma)$ approaches $\sigma = -\sigma_0$ from below, and $G_1$ therefore approaches zero. As $(\lambda_2, \sigma)$ nears the $\sigma = -\sigma_0$ line for $\lambda_2 < -\lambda_0$, the point $(\lambda_1, \sigma)$ approaches $(-\lambda_0, -\sigma_0)$ from above and to the left, and $G_1$ has no limit. These results lead to the conclusion that the line $\sigma = \sigma_0$, $\lambda_1 > \lambda_0$ is a zero-$G_2$ curve when approached from above, while the $\sigma = -\sigma_0$, $\lambda_2 < -\lambda_0$ line is a zero-$G_1$ curve when approached from below.

3.6.5 Location of the solution curves

Solution curves occur only in regions of the $\lambda, \sigma$ plane which correspond to $G_1$ and $G_2$ of like sign, since an intersection of the $G_1$ and $G_2$ surfaces can occur only in such regions. The $I, I', O$ and $O'$ curves generally divide the allowed regions of the $\lambda, \sigma$ plane into smaller regions of like or unlike signed $G_1$ and $G_2$. Once it is known that the signs of $G_1$ and $G_2$ are alike in a given region, it is easy to determine sign information about $G_1$ and $G_2$. 


in other regions. A solution curve may still fail to occur in such a region, however, and one may have to visualize the surfaces over the region before the question is settled. Finally, various points on the solution curve can be located by noting intersections of I and I', and O and O' curves. At these intersections G₁ and G₂ are equal, and a solution curve must pass through each one of them.

3.7 A Specific Case

3.7.1 The ferrite cavity section

The graphical method for analyzing the modes in circular waveguide filled with axially magnetized ferrite will now be applied in order to acquire an understanding of the ferrite modes which will be involved in solving the magnetized cavity problem. For this problem, the investigation has been restricted to a single guide radius, .587 cm, and a single ferrite type, TT-2-118. In order to search for cavity resonance frequencies in the range 8-12.4 GHz, the ferrite modes must be considered for that entire band. Once the guide radius and the ferrite parameters are chosen, each solution plot pertains to a single frequency only, and since the plots are quite complicated, only a small number of frequencies can be examined graphically. In this case, the frequencies chosen are 8, 10 and 12 GHz. Each plot consists of only one quadrant of the $\lambda, \sigma$ plane, the first quadrant being for $n = +1$ and the third for $n = -1$. An $n = \pm 1$ pair of plots is constructed for each of the three frequencies. As will be seen later, these plots provide a
general understanding of the mode behaviour, not only for the specific frequencies for which they are drawn but for those between, and this makes possible the quantitative analysis required to identify cavity resonances. In addition to the six solution plots, an illustration of the various regions of the $\lambda, \sigma$ plane for a frequency of 8 GHz has been included.

The infinity and zero curves which are the basis for the solution plots shown here have been located according to data generated by computer programs written specifically for this purpose. These programs must convert input data into unitless forms such as $r_o$ and $p$, and a discussion of the determination of these quantities is given below, with emphasis on the required change in systems of units. Two programs are used, one for infinity curves and one for zero curves, and each is discussed.

3.7.2 Calculation of parameters

If a mode plot is to be constructed for a given ferrite, guide radius and frequency, one must find $p$ and $r_o$. The calculation of $r_o$ is relatively simple, since

$$r_o = (\text{actual } r_o) \cdot \beta_o,$$

where

$$\beta_o = \frac{\omega}{\sqrt{\mu_o \epsilon_p}}$$

and $\epsilon_p$ is the permittivity of the ferrite. Finding $p$ may be somewhat more difficult, since the rationalized MKS system of units has been used in the
development of the ferrite-filled guide theory, while the parameters of ferrites are often tabulated in Gaussian units. In the following development, quantities given in Gaussian units are subscripted with G while those in MKS units are not subscripted.

The definition of $p$ is given as

$$p = \frac{5 M_{\text{sat}}}{\mu_0 \omega} ,$$  \hspace{1cm} (3-142)

where the quantities are given in MKS units. In MKS units, the magnetic moment is given by

$$m = \mu_0 \pi a^2 i ,$$ \hspace{1cm} (3-143)

or

$$\frac{m}{\mu_0} = \pi a^2 i ,$$ \hspace{1cm} (3-144)

while in Gaussian Units one has

$$m_G = \pi a_G^2 i_G / c_G ,$$ \hspace{1cm} (3-145)

where $c_G$ is the speed of light in cgs units. One can then write the following relationships between MKS quantities and the corresponding Gaussian quantities:

$$a^2 = 10^{-4} a_G^2$$ \hspace{1cm} (3-146)

$$i = i_G / 3 \times 10^3 ,$$ \hspace{1cm} (3-147)

This leads to
\[
\frac{m}{\mu_0} = \frac{1}{3 \times 10^{15}} \pi d_{\text{e}}^2 i_{\text{e}} = \frac{1}{3 \times 10^{15}} c_\text{e} m_\text{e} = 10^{-3} m_\text{e}, \tag{3-148}
\]

Noting, then the definition of magnetization, one has

\[
\frac{M}{\mu_0} = 10^{-3} M_\text{e}. \tag{3-149}
\]

The gyromagnetic ratio \( \gamma \) is the ratio of magnetic moment \( m \) to angular momentum \( p^* \):

\[
\gamma = \frac{m}{p}. \tag{3-150}
\]

Referring to equation (3-142), one must then find the relationship between \( p \) and \( P_\text{e} \):

\[
p = \text{mass} \times \text{length} \times \text{velocity} = (10^{-3} \text{ mass}_\text{G}) \times \]
\[
(10^{-2} \text{ distance}_\text{G}) \times (10^{-2} \text{ velocity}_\text{G}) = 10^{-7} P_\text{e}. \tag{3-151}
\]

This leads to

\[
\gamma = \frac{m}{p} = \frac{\mu_0 10^{-3} m_\text{e}}{10^{-7} P_\text{e}} = 10^4 \frac{\mu_0}{\mu_0} \gamma_\text{e}, \tag{3-152}
\]

Combining now the result for \( \gamma \) and \( \frac{M}{\mu_0} \), one can arrive at

\[
\frac{\gamma M_{\text{sat}}}{\mu_0} = \mu_0 10^4 \times 10^5 \gamma_\text{e} (M_{\text{sat}})_\text{e} = 4 \pi \gamma_\text{e} (M_{\text{sat}})_\text{e}, \tag{3-153}
\]

since

\[
\mu_0 = 4 \pi \times 10^{-7} \text{ henrys/meter}. \tag{3-154}
\]

* This \( p \) should not be confused with the term used to normalize the saturation magnetization used earlier in this chapter.
[Note that $4\pi (M_{\text{sat}})_e$ is tabulated in Lax and Button (3) for several materials.] Applying equation (3-153) to equation (3-142), one obtains

$$p = \frac{\gamma_e \cdot 4\pi (M_{\text{sat}})_e}{\omega}.$$ 

One must also find $\gamma_e$. From Lax and Button (3)

$$\gamma_e = \left( \frac{q_{\text{eff}} e}{2 m c} \right)_e,$$  \hspace{1cm} (3-156)

where $e$ is the magnitude of the electron charge and $m$ is the mass of the electron.

For $q_{\text{eff}} = 2$, this becomes

$$\gamma_e = 1.76 \times 10^7.$$  \hspace{1cm} (3-157)

Then,

$$p = \frac{(1.76 \times 10^7) 4\pi (M_{\text{sat}})_e}{\omega}$$ \hspace{1cm} (3-158)

for $q_{\text{eff}} = 2$. Expressing $\omega$ as $2\pi f$, where $f$ is in MHz, this becomes

$$p = (2.8)(4\pi M_{\text{sat}})_e/f.$$ \hspace{1cm} (3-159)

For the case of $q_{\text{eff}} \neq 2$, one can write

$$p = 1.4 q_{\text{eff}} (4\pi M_{\text{sat}})_e / f.$$ \hspace{1cm} (3-160)

Similarly, it can be shown that

$$\sigma = i \frac{q_{\text{eff}} (h_{\text{INT,DC}})_e}{f}.$$ \hspace{1cm} (3-161)

The relationship (3-161) for $\sigma$ is not needed in constructing the solution plots.
but is required in their use. For the nickel ferrite, Trans Tech \# TT-2-118 (3) to which the plots correspond, one obtains

\[
p = 1.4 \cdot 2.55 \cdot 1800 / f \\
\sigma = 1.4 \cdot 2.55 \cdot (H_{INT,DC})_0 / f.
\] (3-162) (3-163)

3.7.3 The infinity curve computer program

(a) Description and discussion

This computer program makes use of the property that infinity and transformed infinity curves lying in the allowed \( \lambda_2, \sigma \) region of the third quadrant can be obtained by reflection through the origin of the corresponding curves in the allowed region of the first quadrant. Curves corresponding to roots of \( J_i (\kappa) \) up to the tenth can be generated. \( L_n \) curves are given after their transformation from the \( \lambda_2, \sigma \) region to the \( \lambda_1, \sigma \) region. Input data are frequency in GHz, \( 4 \pi M_{sat} \) in gauss, guide radius in meters, relative permittivity, the gyromagnetic ratio and the effective g-factor. Values for the first ten roots of \( J_i (\kappa) \) are contained within the program itself. A minimum and maximum value of \( \lambda \) are given as input. For the chosen infinite - \( G \) curves, the range of \( \lambda \) values is scanned according to a specified increment, and for each \( \lambda \) the corresponding point on the infinite - \( G \) curve is located by calculating \( \sigma \). If this point lies in the positive \( \sigma \) plane, but is smaller than an established maximum, the program determines whether the point lies in a region corresponding to \( G_1 \) or \( G_2 \). If it falls in a \( G_2 \) region, a transformation to the \( G_1 \) region is carried out, and the
point is associated with the appropriate $I_n^\prime$ curve. After the entire range of $\lambda$'s has been scanned, and $\lambda, \sigma$ coordinates defining the $I_n$ and $I_n^\prime$ curves have been stored, the scan of $\lambda$ is then repeated, with points on the next infinite-G curve being located and transformed when appropriate.

3.7.4 The zero-curve computer program

(a) The computer program for locating curves of zero-$G_1$ and zero-$G_2$ is considerably more lengthy and complex than that for the infinity curves. The behaviour of the curves of zero-$G$ in the $\lambda, \sigma$ plane is more complicated, their trajectories being much more difficult to determine, and the arrangement of $O_n$ and $O_n^\prime$ curves in the allowed third quadrant region cannot be determined by reflection through the origin of those in the first quadrant. Zero-$G$ curves occur in the $\lambda, \sigma$ plane when the equation

$$F(x) = \lambda$$

is satisfied, where $F$ is defined according to

$$F(x) \equiv x \frac{J_i'(x)}{J_i(x)}$$

and $x$ is given by

$$x = r_0 \left\{ (1 - \lambda^2) / (1 - \sigma \lambda) \right\}^{1/\kappa}.$$  

If all parts of the $\lambda, \sigma$ plane are considered, $x$ may be either real or imaginary. The behavior of $F$ for all such values has been given earlier in Figure 3.3, which shows that $F(x)$ is always real. When $x$ is imaginary, the
function \( J_i(x) \) becomes the real function \( I_i(|x|) \). In the zero \( G_1 \) and \( G_2 \) curve computer program, required values of \( F(x) \) are based on values generated for \( J_1(z) \) with \( z \) complex. In generating coordinate tables to be used in plotting zero-\( G_1 \) and zero-\( G_2 \) curves, the program utilizes a mechanism by which the distances between points to be plotted are automatically kept within a prede-
termined range. Referring to Figure 3.3, \( F(x) \) is seen to have branches, and each such branch becomes associated with a \( O_n \) curve of \( G \) in the \( \lambda, \sigma \) plane, the first branch resulting in the \( O_n \) curve. The asymptote locations having been given in the program, determination of the \( O_n \) curve is based on consecutive scans of the branches of \( F(x) \) by scanning ranges of \( x \) between corresponding pairs of asymptotes.

Once a value of \( x^2 \) is chosen, its sign is known and whether \( x \) is real or imaginary, and the corresponding value of \( F \) is determined. As shown by equation (3-166), the expression for \( x^2 \) in terms of \( \lambda \) and \( \sigma \) implies that the chosen value of \( x^2 \) corresponds to a curve in the \( \lambda, \sigma \) plane, and \( F(x) \) has a fixed value everywhere on this curve. It is then necessary to find the point on this curve for which the corresponding value of \( \lambda \) is equal to the constant value of \( F(x) \). At this point, \( F(x) = \lambda \) and the condition \( G = 0 \) is satisfied. It remains to determine the region of the plane in which this point lies. This determines whether the particular point corresponds to a zero of \( G_1 \) or \( G_2 \) and whether it lies in the positive \( \sigma \) or negative \( \sigma \) half-plane. If the point is in the positive-\( \sigma \) half-plane and is determined to lie on a zero-\( G_2 \) curve, it will be transformed to the \( \lambda_1, \sigma \) region of the first quadrant. If the point is located
in the negative $\sigma^-$ part of the plane and falls in the $G_1$ region, its transformed location in the $\lambda_2, \sigma^-$ region is found.

3.7.5 The graphical solution

(a) Discussion of the plots

Pairs of solution plots ($n = \pm 1$) for 8, 10 and 12 GHz are given in Figures 3.7 through 3.12. However, before discussing these it will be desirable to consider Figure 3.6, in which the various regions of the $\lambda, \sigma$ plane are illustrated for the frequency of 8 GHz. This plot and the solution plots correspond to a circular waveguide radius of .587 cm., a saturation magnetization $4\pi M_{sat}$ of 1800 gauss, a $g_{eff}$ of 2.55 and a ferrite permittivity 9.5. For each solution plot, the quantities $r_0$ and $p$ have been calculated according to Section 3.7.2, and the resulting values of both parameters are indicated in each solution plot. The value of $r_0$ does not appear in Figure 3.6, since it does not affect the boundaries in the $\lambda, \sigma$ plane.

Referring to Figure 3.6, the shaded portions of the first quadrant correspond to allowed regions for $(\lambda_1, \sigma)$ points when $\sigma^-$ is positive. The $G_1$ surface is considered to be constructed originally above this portion of the half-plane, while the $G_2$ surface must be transformed to lie above this region. For negative values of $\sigma^-$, the shaded regions are those in which all $(\lambda_2, \sigma)$ points fall, and the $G_1$ surface in this half-plane must be transformed from the unshaded regions. The more heavily shaded regions correspond to real values for the propagation constant $\beta$, while the more lightly shaded areas correspond to
FREQUENCY = 8 GHz.
ACTUAL \( p = 0.8033 \)
\( M_{\text{sat}} = 1800 \text{ Gauss} \)
\( g_{\text{eff}} = 2.55 \)
\( r_0 = 3.031 \)
(ACTUAL RADIUS = 0.587 CM.)

\[ \lambda = \frac{\sigma}{\rho} \pm \left[ \frac{1}{\sigma^2} - 1 - \frac{\rho^2}{\sigma^2} \right]^{1/2} \]

\[ \sigma = \lambda + p \]

\[ \sigma = \frac{1}{\lambda} \]

\[ \lambda = \frac{\sigma}{\rho} \pm \left[ \frac{1}{\sigma^2} - 1 - \frac{\rho^2}{\sigma^2} \right]^{1/2} \]

\[ p > 0 \]

\[ p < 0 \]

**FIGURE 5.6**
REGIONS OF THE \( \lambda, \sigma \) PLANE
extensions of the Suhl and Walker treatment to imaginary values of $\beta$.

Inclusion of the latter regions in the solution plots permits the ferrite modes to be considered in their evanescent forms.

An illustration of the manner in which solution curves are approximately located is given by considering in Figure 3.7 the curve which extends to the top of the 8 GHz, $n = +1$ plot between the $I'_1$ and $O'_0$ lines. It can be shown that $G_1$ and $G_2$ are both positive in the region bounded by the curves $I_1$, $O_1$, and $I'_1$. Choosing a value for $G_1$, a corresponding constant $G_1$ contour can be visualized between $I_1$ and $O_1$. Moving along the segment of this contour extending between the intersection of $I_1$ and $O_1$ and the curve $I'_1$, $G_2$ will pass through all values from 0 to $\infty$, and in particular, will somewhere take on the fixed value of $G_1$. The point at which this occurs must lie on a solution curve, so that such a curve is known to exist in the region being considered. It can further be reasoned that this curve passes through the intersection of $I_1$ and $I'_1$, and through the point at which $O_1$ intersects $O'_0$. Except for a very small number of cases in which the $G_1$ and $G_2$ surfaces must be more closely examined, the solution curves, denoted by dashed lines, can be approximately located by using the reasoning just indicated.

The solution curve just referred to can be associated with the TE$_{11}$ mode in circular waveguide of the same radius and filled with an isotropic material having the ferrite permittivity. This is demonstrated by obtaining an expression for the propagation constant in the limiting case of large $\sigma$ and showing that it approaches that for the TE$_{11}$ mode in isotropically filled guide. This is reasonable since for an infinite d.c. field, the ferrite becomes isotropic, so that one of the
ferrite modes must behave this way in the limit. Further examination of Figure 3.7 reveals that no other ferrite mode remains in the propagating state as $\sigma$ grows indefinitely. This conclusion is in agreement with the fact that the $\text{TE}_{11}$ circular guide mode is the only $n = \pm 1$ TE or TM mode that will propagate in a guide of the present radius filled with a dielectric with $\varepsilon = \varepsilon_F$. The solution curve between $O'_o$ and $I'_B$ for $\sigma < \sigma_o$ is also associated with the $\text{TE}_{11}$ mode in isotropically filled guide.

All of the solution curves originating at $\lambda_1 = \sigma = 1$ except the first eventually pass through an $I_r - I_A$ intersection and then extend into the region of imaginary propagation constants. Only the first few of these are shown. Those which intersect the $O'_b$ line terminate and then resume further to the right. Another array of solution curves exists below the $O'_b$ line.

Proceeding to the 10 GHz, $n = +1$ plot, Figure 3.8, various changes due to the frequency increase are pointed out and the resulting modifications in the solution curves are noted. The parameter $p$ has been decreased from .8033 to .6246, causing the $\sqrt{r} = \lambda + p$ line to shift to the left. The pattern of $I_r$ and $O_r$ curves originating at $(\lambda_1, \sigma) = (1, 1)$ has rotated in a counterclockwise sense. An examination of the first quadrant branch of the infinite $G$ curve associated with the condition

$$\frac{1 - \frac{\lambda^2}{1 - \sigma \lambda}}{1 - \sigma \lambda} = \left( \frac{\mu}{\varepsilon_o} \right)^2,$$  \hfill (5-147)

where $j_{11}$ is the first zero of $J_1(\lambda)$, will reveal that after this curve passes
downward through \((\lambda_1, \sigma) = (1,1)\), it extends well below \(\sigma = \sigma_0\) in the \(\lambda_2, \sigma\) portion of the first quadrant and then turns upward again, quickly growing quite close to the positive \(\sigma\) axis. The transformation into the \(\lambda_1, \sigma\) region of the portion of this curve lying in the \(\lambda_2, \sigma\) region takes on an interesting form, as evidenced by the loop in the lower left part of the solution plot. A further consequence is that \(I_A'\) and \(I_1'\) nearly coincide for values of \(\sigma\) greater than \(\sigma_0\). This results in the apparent anomaly in the path of the solution curve crossing the \(I_A', I_1'\) line at \(I_2\).

In Figure 3.9, the solution plot for 12 GHz, \(n = +1\), the shift of \(I_n\) and \(O_n\) curves has been sufficient to allow two of the solution curves to remain in the propagating region as \(\sigma\) increases indefinitely. The second of these corresponds to the TM mode in isotropically filled guide. Again, the most significant change due to the increase in frequency is due to the change in infinity curves located according to equation (3-167). The branch occurring in the \(\lambda > 0\) half-plane does not turn upwards after passing through the point \((\lambda, \sigma) = (1,1)\), but continues downward into the fourth quadrant. Similarly, the \(\lambda < 0\) branch extends upward into the second quadrant. This portion of the \(\lambda < 0\) branch then transforms into the \(\lambda_1, \sigma\) region of the first quadrant, becoming the \(I_1'\) curve passing through the point \((\lambda_0, \sigma_0)\) and lying between \(O_0'\) and \(I_A'\). A portion of the \(\lambda > 0\) branch now extends into the \(\lambda_1, \sigma\) region below \(\sigma = \sigma_0\), as is indicated in the lower left part of Figure 3.9. A new solution curve has appeared in this part of the plot, which corresponds to evanescent modes. The remaining part of the \(\lambda > 0\) branch lying in the first quadrant must be transformed and becomes the \(I_1'\) curve moving through \((\lambda_0, \sigma_0)\) and sharply
off to the right.

As suggested earlier while discussing the zero curves of $G_1$ and $G_2$, the third quadrant solution plots which apply to $n = -1$ are significantly different from those in the first quadrant for $n = +1$. This is reasonable, since it is expected that the propagation constant behavior for the two senses of rotation will be different.

Referring now to the plot for 8 GHz, $n = -1$, which is Figure 3.10, and comparing this with Figure 3.7 for $n = +1$ at the same frequency, one sees several very apparent differences in the arrangement of infinity and zero curves. There is no analogy in the $n = +1$ case to the $O_0$ curve which lies near to the $O_c'$, $O_\infty$ curve for $n = -1$. A solution curve occurs between these two curves in the region for $\sigma > -\sigma_0$ but is not shown in the plot. There is also no analogy for $n = +1$ to the $I_B'$ curve and the related transformed $I_B'$ curve for $n = -1$, nor is there in the $n = -1$ case a curve corresponding to the $I_B'$ curve for $n = +1$. Of course, these differences are due to the fact that all $I_B$ and $I_B'$ curves originate from the $\lambda = -i$ line, which is asymmetrically located. All $O_n$ and $O_n'$ curves are different for the two signs of $n$, especially for the $O_0$ and $O_0'$ curves, again because the original zero curves of $G$ are not symmetrically placed about the origin of the plane.

Solution curves in the first and third quadrants are different as well, especially in the case of the curve between $O_0$ and $O_c'$ for $n = -1$ and in the case of the other solution curve for $\sigma > -\sigma_0$ and $n = -1$. However, since very large $\sigma$ corresponds to the case of isotropic loading, and since the propagation constants for $n = +1$ and $n = -1$ are the same for the isotropic case, each $n = -1$ plot should indicate the same number of propagating modes when $|\sigma|^2$ becomes very large as does the corresponding
n = +1 plot. In the cases of n = -1 for 10 and 12 GHz, it can be seen that the lower left portions of the plots, Figures 3-11 and 3-12, are more complicated than for the n = +1 plots.
CHAPTER 4

THE EXPERIMENTAL METHOD

4.1 Introduction

The final goal of the experimental portion of the present investigation is evaluation of the predictions made for the magnetized end-loaded cavity (Figure 6.1). The experimental arrangement consists of the cavity assembly (Section 4.2) and measurement arrangement (Section 4.3). The arrangement of equipment used in performing measurements on the cavity is described, with a block diagram included, and the procedure by which the measurements are carried out is discussed. The electromagnet used in magnetizing the ferrite disks is considered in Chapter 6. Development of the cavity assembly in its final form and the series of preliminary experiments involved in this process are reviewed. Finally, investigations are carried out for three relatively simple configurations involving the cavity assembly. These help to check the experimental arrangement and procedure, and establish ferrite and dielectric permittivity values that will be used in later predictions.

4.2 Discussion of the Cavity Assembly

The cavity assembly consists of brass, ferrite and dielectric pieces which are used in realizing the theoretical cavity and in providing an interface between this cavity and the measurement arrangement. The assembly components are shown in Figures 4.1 and 4.2. The cavity fixture is brass and consists of the cavity body, two cavity end-plates and two cavity end-plugs. These are illustrated in
Figure 4.1. The loading pieces, made of dielectric and ferrite, are described in Figure 4.2.

The cavity body is a cylinder with a circular hole extending axially through its center. This hole serves as the length of circular waveguide from which the waveguide cavity is formed. The radius of this guide is .587 cm as determined by the geometry of the E-plane waveguide junction circulator as shown in Figure 4.3. A section has been removed from the cavity body in order that it can be secured to a rectangular, waveguide flange. When the body is in place, the section of circular guide is centered with respect to the rectangular guide cross-section and aligned in the H-plane. The depth of the removed section is such that the circular guide wall comes to within $\sim 1$ mm of the rectangular flange. A coupling hole of .476 cm. diameter is located in the circular guide wall so that it will be centered in the rectangular guide opening when the fixture is in place.

To form cavity ends, two end plates have been fabricated as shown in Figure 4.1. These have posts that fit snugly and symmetrically into the circular guide when the end plates are in place to form the required 2.286 cm. (.9") long cavity. Cavity ends can also be formed by using two brass plugs, also shown in Figure 4.1. These allow the cavity length to be adjusted and the position of the cavity to be shifted relative to the coupling hole. The ferrite and dielectric pieces are shown grouped in Figure 4.2 according to the three end-loaded configurations to be examined. Each set consists of two ferrite disks and one disk of dielectric, corresponding to a total length of 2.286 cm. The three ferrite disk thicknesses are .4, .6 and .8 cm., and the corresponding dielectric rod
FIGURE 4.3
THE E-PLANE WAVEGUIDE JUNCTION
CIRCULATOR AND THE END-LOADED CAVITY
lengths are 1.486, 1.086 and .686 cm. In the preliminary experimental work described in Section 4.4 these loading pieces are also used in combinations other than those of Figure 4.2, and a fourth dielectric piece 2.286 cm. in length, as illustrated in Figure 4.2, is also used.

4.3 The Measurement Arrangement and Procedure

A block diagram of the measurement arrangement is shown in Figure 4.4. The purpose of this arrangement is to detect and observe energy absorptions due to cavity resonances in the 8 - 12.4 GHz range. It will be necessary to determine the frequencies at which these occur and their relative strengths, and to record this information.

A swept frequency generator is used as the source of microwave energy in the experimental work. Two directional couplers are utilized, one to sample power incident on the cavity assembly and provide feedback for the generator's external leveling function, and the other to sample reflected power. This power is applied through the frequency meter to a crystal detector, whose output is then coupled to the Y-input of an X - Y plotter or an oscilloscope. When the X - Y plotter is used, the generator is operated at a low sweep rate and in the single sweep mode. For a given cavity configuration, several sweeps are made. One sweep is obtained with the rectangular waveguide section terminated in a short circuit plate. This is followed by a series of sweeps with the cavity in place and for different coupling hole positions or different applied d.c. magnetic fields. The first sweep is a reference sweep corresponding to a total reflection of the incident
power and is compared with the other sweeps to determine the presence of absorption due to cavity resonances. The X-Y plotter is calibrated so that the relationship between horizontal deflection and frequency is known and the frequency of an absorption can be read directly from the plot.

The X-Y plotter can be replaced with an oscilloscope if an absorption is very weak. In this case, the generator is used in a multiple sweep mode at a higher sweep rate, and the oscilloscope display is observed while the cavity coupling hole position is being varied. The resulting time-varying absorption level for each cavity resonance makes the absorption more apparent and its frequency easier to determine.

4.4 Preliminary Experimental Work

4.4.1 Testing of the cavity assembly

Several early versions of the cavity assembly were used in first attempts to observe resonances. Initially, a coupling hole diameter of .318 cm. and an air-filled center section were tried, with no provision being made for changing the effective position of the coupling hole. The ferrite disk thicknesses were chosen to result in cavity resonances in the 8-12.4GHz frequency range. When no resonances were observed for the three end-loaded, air-filled center section configurations, a ferrite-filled configuration was examined, but still without success. Failure also resulted when the cavity was filled with dielectric. First
observation of a resonance absorption occurred with the dielectric-filled cavity when the coupling hole was enlarged to a diameter of .476 cm. This resonance occurred at 9.70 GHz, and when associated with the \( \text{TE}_{111} \) cavity mode, led to an experimentally determined dielectric relative permittivity of 2.85. Brass end-plugs were fabricated so that any odd-symmetry modes possible in the dielectric filled cavity could be observed, and the \( \text{TE}_{112} \) cavity mode was located.

Attempts to detect a resonance in the end-loaded cavity with an air-filled center section continued to meet with failure, even with the enlarged coupling hole. The brass plugs were not utilized, due to the resulting difficulty in maintaining the proper spacing of disks, so that only the centered hole position was considered. Assuming that the absence of absorptions was related to the evanescent condition prevailing in the center section, three dielectric cylinders were fabricated for the purpose of filling this section in all three disk thickness cases. Absorptions were then observed. Furthermore, the dielectric cylinders provided the proper spacing between the ferrite disks, thus allowing for more convenient use of the brass plugs for shifting the coupling hole position.

### 4.4.2 Infinite guide loaded with dielectric cylinders

Since the dominant mode in circular waveguide of radius .587 cm does not propagate at frequencies below 15 GHz, the case of a dielectric cylinder centered in the cavity body with the cavity ends removed can be analyzed for the 8 - 12.4 GHz range by assuming that the circular guide is infinitely long. This will result in predicted resonance frequencies which can be checked by experiment. Such an investigation has been carried out for six different dielectric cylinder lengths,
some of them being formed by combinations of the three fabricated cylinders. A relative permittivity of 2.85 is assumed. Only TE_{11} resonances occur, since TM modes are not significantly excited (Section 5) and the remaining TE modes do not propagate in the dielectric-filled portion of the guide within the 8 - 12.4 GHz range.

Theory for the cylinder-loaded infinite guide is given in Appendix 3 where a characteristic quantity whose zeros correspond to resonance frequencies is derived. These zeros are found by a computer implemented search. The predicted and observed results are compared in Table 4.1. The predicted resonance frequency lies in the 100 MHz range just below the frequency tabulated. The comparison indicated by the table is good, with a maximum difference between predicted and observed frequency of \( \sim 2\% \).

4.4.3 Infinite guide loaded with ferrite cylinders

The analysis applied in Section 4.4.2 and described in Appendix 3 has also been applied to the case of the cavity fixture body center-loaded with unmagnetized ferrite cylinders. This configuration is more complicated than for the dielectric, since the permittivity of the ferrite is much greater. This means that several waveguide modes will lead to resonances and that the number of absorptions will be greater. Cylinder lengths of .4, .6, .8, 1.0 and 1.2 cm are considered, again using combinations of existing disks. Resonance frequencies have been predicted for a range of assumed ferrite relative permittivities, with the value 8 seeming to produce the best correspondence between theory and
experiment. A comparison of measured frequencies with frequencies predicted on the basis of this permittivity is given in Table 4.2. Again, predicted resonance frequencies fall in the 100 MHz interval immediately below the tabulated frequency. When making the observations, the ferrite load has been moved from side to side in order to observe the dependence of absorption on coupling hole position and to determine the number of maxima occurring. This number has been noted for each measurement in the table, odd numbers corresponding to even modes and even numbers to odd modes. Similarly, a note concerning the expected number of absorption maxima for each predicted frequency is given. These help substantiate the matching of measured frequencies with predicted ones. It is seen for this situation that observed frequencies tend to be high for predicted resonance frequencies at the low end of the band and low for predictions at the high end. However, the discrepancies are less than 5%.

4.4.4 Ferrite-filled cavity

Using the brass plugs as cavity ends, several cavity lengths have been investigated for the case of unmagnetized ferrite filling. Eight lengths ranging from .4 cm to 1.8 cm have been considered by using combinations of the six available disks. A relative permittivity of 8 has been assumed, and the $TE_{11}$, $TE_{21}$, $TE_{01}$ and $TE_{31}$ circular waveguide modes have been taken into account as possible sources of cavity resonances. Predicted and observed resonance frequencies have been paired when possible and are given in Table 4.3. Again, both theoretical and observed information has been obtained concerning the number of
absorption maxima occurring as the coupling hole is shifted, and this has been noted for each pair in the table. As seen for the infinite guide partially loaded with ferrite, observed resonance frequencies for predictions at the low end of the band tend to be high, while the reverse is true for predicted frequencies at the high end.
<table>
<thead>
<tr>
<th>DIELECTRIC CYLINDER LENGTH (CM)</th>
<th>PREDICTED RESONANCE FREQUENCY (GHz)</th>
<th>OBSERVED RESONANCE FREQUENCY (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.683</td>
<td>11.0</td>
<td>10.82</td>
</tr>
<tr>
<td>1.090</td>
<td>10.2</td>
<td>10.07</td>
</tr>
<tr>
<td>1.486</td>
<td>9.8 [12.1]</td>
<td>9.70 [12.09]</td>
</tr>
<tr>
<td>1.773</td>
<td>9.6 [11.5]</td>
<td>9.54 [11.46]</td>
</tr>
<tr>
<td>2.169</td>
<td>9.4 [10.8]</td>
<td>9.38 [10.83]</td>
</tr>
<tr>
<td>2.576</td>
<td>9.3 [10.4] [12.1]</td>
<td>9.28 [10.40] [12.14]</td>
</tr>
</tbody>
</table>

**Table 4.1**

Comparison of predicted and observed TE resonance frequencies for infinitely long circular waveguide loaded with a dielectric cylinder.
<table>
<thead>
<tr>
<th>Ferrite Cylinder Length (cm)</th>
<th>Predicted Resonance Frequency (GHz)</th>
<th>Observed Resonance Frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.8 (1) 11.3 (1)</td>
<td>8.20 (1) 11.02 (1)</td>
</tr>
<tr>
<td>0.4</td>
<td>10.4 (1) 11.8 (2) 12.5 (1)</td>
<td>10.44 (1) 11.75 (2) 12.20 (1)</td>
</tr>
<tr>
<td>0.6</td>
<td>9.9 (1) 10.0 (2) 12.1 (1) 13.0 (2)</td>
<td>10.07 (2) 10.19 (1) 11.85 (1) 12.65 (2)</td>
</tr>
<tr>
<td>0.8</td>
<td>8.9 (2) 9.6 (1) 11.8 (1) 11.9 (3) 12.7 (3) 12.8 (1)</td>
<td>9.30 (3) 9.53 (1) 11.62 (3) 11.69 (1) 12.50 (3) 12.70 (1)</td>
</tr>
<tr>
<td>1.0</td>
<td>8.2 (2) 9.4 (1) 11.2 (3) 11.2 (3) 11.6 (1) 12.6 (1)</td>
<td>8.78 (3) 9.88 (1) 11.14 (2) 11.14 (3) 11.57 (1) 12.65 (1)</td>
</tr>
</tbody>
</table>

**Table 4.2**

Comparison of Predicted and Observed TE Resonance Frequencies for Infinitely Long Circular Waveguide Loaded with an Unmagnetized Ferrite Cylinder.
<table>
<thead>
<tr>
<th>CAVITY LENGTH (CM.)</th>
<th>ORIGIN OF DATA</th>
<th>RESONANCE FREQUENCY (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PR.</td>
<td>OB.</td>
</tr>
<tr>
<td>.4</td>
<td>10.3</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>10.6</td>
<td>12.3</td>
</tr>
<tr>
<td>.6</td>
<td>8.5</td>
<td>11.0</td>
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<tr>
<td></td>
<td>9.3</td>
<td>11.1</td>
</tr>
<tr>
<td>.8</td>
<td>7.5</td>
<td>10.5</td>
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<tr>
<td></td>
<td>8.7</td>
<td>10.6</td>
</tr>
<tr>
<td>1.0</td>
<td>6.9</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>8.3</td>
<td>10.2</td>
</tr>
<tr>
<td>1.2</td>
<td>9.2</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>9.8</td>
<td>10.0</td>
</tr>
<tr>
<td>1.4</td>
<td>8.5</td>
<td>9.4</td>
</tr>
<tr>
<td></td>
<td>9.3</td>
<td>9.9</td>
</tr>
<tr>
<td>1.6</td>
<td>7.9</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>8.9</td>
<td>9.9</td>
</tr>
<tr>
<td>1.8</td>
<td>7.9</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>8.9</td>
<td>9.9</td>
</tr>
</tbody>
</table>

**Table 4.5**

Comparison of predicted and observed TE resonance frequencies for a circular waveguide cavity filled with unmagnetized ferrite
CHAPTER 5
THE END-LOADED CAVITY
WITH UNMAGNETIZED FERRITE

5.1 Introduction

5.1.1 The cavity fixture

In this chapter an investigation of the end-loaded cavity configuration without an applied bias field is carried out. The cavity loading arrangements considered in this investigation are identical to those which will be used when a bias field is applied. The cavity fixture and the associated parts have already been described in Chapter 4. The cavity end-plates are not used in the experimental work, the end plugs being used instead in order to permit observation of absorption for coupling hole positions over the entire cavity length.

5.1.2 Experimental arrangement and procedure

The experimental arrangement and procedure are as described in Chapter 4, where a block diagram has been given. The cavity fixture is attached to an X-band waveguide flange. A cavity resonance will result in absorption through the coupling hole of incident microwave energy. This absorption will be seen as a decrease in the reflected power, which is monitored by a directional coupler and a crystal detector. Experimental results are recorded by plotting the crystal detector output while frequency is swept from 8 GHz to 12.4 GHz. Presence of an absorption is determined by comparing this plot with
one obtained with the cavity fixture replaced by a short-circuit plate.

5.1.3 Purpose of the investigation

An understanding of the unmagnetized cavity is a desirable preliminary step to the study of the magnetized cavity. This is true for several reasons. First, only with this approach can the magnetized case be investigated with proper weight given to the question of how the bias field alters the cavity behavior. Second, though the two situations are different, both are end-loaded, and experience from treatment of the simpler one will be of help in dealing with the more complicated one. Insight into the measurement problem is acquired, and feasibility of the intended experimental approach is determined. Finally, some discrepancies between theory and experiment in the two situations may be related. An understanding of discrepancies studied in the unmagnetized case, which is less complicated, can be applied in the investigation of related disagreements in the case with bias field applied.

5.1.4 Description of the approach

The location of the coupling hole and orientation of the cavity fixture are expected to permit only the generation of TE cavity modes. Analysis and prediction are carried out only for those modes expected to occur within the 7 - 13 GHz range, which contains the 8 - 12.4 GHz experimental range. First, therefore, the modes of interest are identified. When the analysis is carried out, a half-cavity is assumed and a characteristic equation for cavity solutions is developed. Using this equation, resonance frequencies for the cavity are predicted
by numerical methods, and these frequencies are tabulated according to ferrite disk thickness and circular waveguide mode classification. Then, using the form of the characteristic equation already developed, a graphical representation of the solution for resonance frequencies is presented and discussed, thus providing further insight into the cavity problem and providing a check on the numerical results. As a final step in the analysis, a series of plots is generated, each one representing the variation of the amplitude of the longitudinal magnetic fields along the cavity length for one of the predicted resonances. These are very useful in characterizing the modes and in making the comparison between theory and experiment.

5.2 Analysis and Predictions

5.2.1 Expected waveguide modes

Only resonances due to TE circular waveguide modes will be observed in the experimental work with the unmagnetized end-loaded cavity. This is expected because of the orientation of the cavity with respect to the rectangular waveguide. Since the coupling hole is small, it is assumed that fields near the end of the rectangular waveguide will be much like those occurring if the cavity fixture were replaced by a short circuit plate. In this case, there would be no electric field, but a transverse magnetic field would exist. Thus, when the cavity fixture is in place, coupling to the cavity can be considered mainly in terms of the transverse magnetic field of the rectangular waveguide TE_{10} mode. This field component is thought of as coupling into the axial magnetic field in the cavity at the
wall. Cavity TM modes are thus unlikely, since they possess no axial magnetic field. It is then concluded that the only cavity modes to be considered (all cavity modes are derived directly from either the TE or TM circular waveguide modes) are those arising from TE circular waveguide modes.

One is then left with the need to determine which TE modes can lead to observed resonances in the experimental frequency range. Although only the 8 - 12.4 GHz interval is examined experimentally, the entire 7 - 13 GHz range will be considered when making predictions. Determination of allowed TE modes is based on the requirement that any cavity field solution must derive from a waveguide mode which is propagating in the ferrite region. This results from the fact that one must have propagation in at least one of the regions, and that if the mode is not propagating in the ferrite, it will not be propagating in the dielectric. This is because of the larger relative permittivity of the ferrite (≈ 8 as compared to ≈ 3 for the plastic). Therefore, a mode spectrum for the ferrite region has been represented in Figure 5.1. Only the five lowest order TE circular guide modes are considered. The same kind of representation is given in Figure 5.2 for the dielectric-filled circular guide. These representations show that only the four lowest order TE modes have cutoff frequencies below 13 GHz in the ferrite-filled guide, while only one mode propagates below 13 GHz in the dielectric-filled guide.

The two figures are based on the information tabulated in Table 5.1, which is based on the theory for circular waveguide modes. The portions of this theory referred to below do not utilize the scaling of magnetic field and length which was applied in discussing the ferrite-filled guide. For TE modes in isotropically
**Figure 5.1** Modes Propagating in the End Regions
Figure 5.2 Modes Propagating in the Center Region
loaded waveguide, the characteristic equation for the propagation constant \( \beta \) can be expressed as

\[
\beta = \left\{ \varepsilon_r (2\pi / \lambda)^2 f^2 - k_c^2 \right\}^{1/2}.
\]

(5 - 1)

Here, \( \varepsilon_r \) is the relative permittivity of the dielectric filling the guide, \( f \) is the frequency in GHz, and \( k_c \) is a quantity depending on the mode being considered.

The quantity \( k_c \) is given by the expression

\[
k_c = x_{nm}' / r_0,
\]

(5 - 2)

where \( x_{nm}' \) represents the \( m \)th zero of \( J_n'(u) \) and \( r_0 \) is the radius of the circular guide in meters, .00587 in this case. Using the expression for the guide propagation constant, the expression for cutoff frequency, corresponding to \( \beta = 0 \), can be written as

\[
f_c = \left( \frac{\pi k_c / 2\pi}{\varepsilon_r} \right)^{1/2}.
\]

(5 - 3)

| TE MODE | \( x' \) | \( k_c \) | \( f_c, \text{GHz} \) | \( \varepsilon_r = 2.85 \) | \( \varepsilon_r = 8 \)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1.841</td>
<td>313.6</td>
<td>8.87</td>
<td>5.29</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>3.054</td>
<td>520.3</td>
<td>14.71</td>
<td>8.78</td>
<td></td>
</tr>
<tr>
<td>01</td>
<td>3.832</td>
<td>652.8</td>
<td>18.46</td>
<td>11.02</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>4.201</td>
<td>715.7</td>
<td>20.24</td>
<td>12.08</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>5.317</td>
<td>900.5</td>
<td>25.47</td>
<td>15.29</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 5.1

This makes apparent the fact that smaller values of \( k_c \) correspond to lower cutoff frequencies – that is, to modes which begin to propagate at lower frequencies.
In turn, from the expression for $k_c$ it is seen that smaller values of $x_{nm}$ correspond to lower cutoff frequencies or to lower order modes. Based on an examination of the set of functions $J_n(x)$, the five smallest zeros of $J_n(x)$ have been entered in Table 5.1. These are $x_{11}$, $x_{21}$, $x_{01}$, $x_{31}$, and $x_{41}$ and correspond to the modes $TE_{11}$, $TE_{21}$, $TE_{01}$, $TE_{31}$, and $TE_{41}$ - the five lowest order TE circular waveguide modes. Their cutoff frequencies are tabulated for both ferrite and dielectric filling. The relative permittivities, as shown, are 2.85 for dielectric and 8 for ferrite. Only the $TE_{11}$, $TE_{21}$ and $TE_{01}$ modes are considered in the following treatment since the $TE_{31}$ mode is not expected to lead to detectable resonances.

5.2.2 The cavity characteristic equation

Frequencies at which non-zero cavity fields can exist are called cavity resonance frequencies. Each such frequency corresponds to an allowed field solution or cavity mode. A characteristic equation can be found such that frequencies at which the equation is satisfied are the resonance frequencies. This equation is obtained by writing general expressions for the cavity fields and then requiring that these satisfy certain boundary conditions. For the end-loaded cavity, the problem is simplified somewhat by noting the symmetry of the configuration about the transverse plane through the cavity center. Because of this symmetry, it is assumed that each cavity field solution will possess either even or odd symmetry about this plane. The terms 'even' and 'odd' refer to spatial dependence of the transverse electric field. Thus, the cavity fields are fully described if they are known only to one side of the center plane. The original problem can therefore be replaced by a simpler one - the half-cavity with either an electric short circuit
or a magnetic short circuit at the center plane.

Both the end-loaded cavity and the half-cavity are shown schematically in Appendix 4, where a complete derivation of the characteristic equation is given. The half-cavity has only two regions. Region 1 is filled with unmagnetized ferrite which is assumed to behave electrically like a lossless, isotropic dielectric. Region 2 is filled with dielectric. The length of region 1 is equal to the ferrite disk thickness, while the length of region 2 is half that of the original cavity center section.

Each half-cavity region can be treated as a length of circular waveguide filled with the appropriate dielectric. Therefore, for any cavity mode, the fields in either region correspond to a superposition of circular waveguide modes. The radial dependence of any such mode does not depend on the dielectric loading, so that boundary conditions at the ferrite-dielectric interface can be satisfied by using the same, single mode in both regions. Then, each cavity solution will be associated with a single circular waveguide mode.

General expressions for the field components with arbitrary constant coefficients are written in each region. There are two cases, corresponding to whether or not the waveguide mode is propagating in region 2. The expressions depend parametrically on the waveguide mode being considered. The number of arbitrary coefficients is then reduced to one-half by applying the boundary conditions at the end wall in each region. This separates each existing case into two cases, corresponding to even and odd symmetry. Finally, continuity of the transverse field components at the interface must be established. The longitudinal component of the magnetic field is of special interest due to the part played by this
component in the coupling process. Therefore, the continuity conditions on the transverse electric and magnetic field components are expressed instead in terms of the longitudinal component of magnetic field and its derivative with respect to $z$, where the $z$-axis lies along the axis of the cavity. The fields are forced to conform to the conditions at the ferrite-dielectric interface in two steps. First, the magnetic field $z$-component is required to be continuous. This condition can be satisfied by adjusting the arbitrary coefficient. Finally, the $z$-derivative of this component of the magnetic field must be continuous. This condition, which can be expressed independently of the remaining arbitrary constant, will be satisfied only at the resonance frequencies. It is given below for the four cases:

$$\frac{1}{u} \tan u \lambda_{ri} = \begin{cases} -\frac{1}{v} \coth v \lambda_{rz} & \text{below} \\ \frac{1}{v} \cot v \lambda_{rz} & \text{above} \end{cases} \quad (5-4)$$

$$\begin{cases} \frac{1}{v} \tanh v \lambda_{rz} & \text{below} \\ \frac{1}{v} \tan v \lambda_{rz} & \text{above} \end{cases} \quad (5-5)$$

where

$$u = \beta_l \quad (5-8)$$

$$v = \begin{cases} \alpha_{z \ell} & \text{below cutoff in 2} \\ \beta_z \ell & \text{above cutoff in 2} \end{cases} \quad (5-9, 5-10)$$

$$\lambda_{ri} = \ell_i / \ell \quad (5-11)$$

$$\lambda_{rz} = \ell_{z \ell} / \ell \quad (5-12)$$
\( L_1 = \) length of region 1 \hfill (5-13)

\( L_2 = \) length of region 2 \hfill (5-14)

\( L = \) length of half cavity \hfill (5-15)

\( \beta_1 = \left\{ \varepsilon_{r_1} \left( \frac{2\pi}{3} \right)^2 f^2 - k_c^2 \right\}^{1/2} \) \hfill (5-16)

\( \alpha_2 = \left\{ k_c^2 - \varepsilon_{r_2} \left( \frac{2\pi}{3} \right)^2 f^2 \right\}^{1/2} \) \hfill (5-17)

\( \beta_2 = \left\{ \varepsilon_{r_2} \left( \frac{2\pi}{3} \right)^2 f^2 - k_c^2 \right\}^{1/2} \) \hfill (5-18)

\( f = \) frequency in GHz \hfill (5-19)

\( \varepsilon_{r_1}, \varepsilon_{r_2} = \) relative permittivities of regions 1 and 2 \hfill (5-20)

\( k_c = \frac{j_{nm}}{r_0} \) \hfill (5-21)

\( j_{nm} = \) \( m^{th} \) zero of \( J_n'(x) \) \hfill (5-22)

\( r_0 = \) radius of circular waveguide \hfill (5-23)
5.2.3 The numerical method

Two methods have been mentioned for treating the cavity characteristic equation problem. One involves a purely numerical solution carried out by computer, while the other involves a graphical treatment. The former approach is more practical because it is much less time consuming, but in isolated cases the second approach is a useful supplement to the first. The purely numerical approach is discussed in this section, the graphical approach in the next.

The first type of solution consists simply of a numerical search for zeros. The characteristic equation for each of the various cases is re-expressed so that resonance frequencies correspond to zeros of a characteristic quantity. Beginning with the lower limit of the frequency range which is to be considered, the frequency is increased in arbitrarily small increments until the upper frequency limit is reached. At each frequency, the characteristic quantity for the appropriate waveguide mode and the particular case is evaluated. When two consecutive evaluations result in different algebraic signs for the quantity, it is generally assumed that a zero has occurred between the two frequencies, and thus that a resonance frequency falls within the interval. The characteristic equation, and the characteristic quantity contain frequency dependent terms which are first evaluated and then substituted into the final expression.

In this work the numerical solution of the characteristic equation was carried out with the aid of a small computer. Several problems arise in such a treatment, especially when the behavior of the characteristic quantity is not already well known. Some of the functions appearing in the characteristic equation possess singularities within the frequency range scanned. The associated
sign change would have to be distinguished from one in which the characteristic quantity has passed through a zero. To avoid this difficulty, the rewriting of the characteristic equation to obtain a characteristic quantity is carried out in such a way that the singularities do not appear. In the present case, this causes sine and cosine functions to appear in the characteristic quantity rather than the tangent and cotangent. The resulting expression should be continuous and well-behaved. A difficulty which still remains is the possibility that two closely located zeros of the characteristic quantity be overlooked due to the use of insufficiently small frequency increments. That this has not occurred in the present predictions is verified by the graphical treatment to be considered later.

The characteristic equation has been given earlier in (5-4) through (5-23). For the purpose of carrying out the numerical solution, the singularities are removed by multiplying all four equations by \( u \cos \varphi_k \), and multiplying the first by \( u \sinh \varphi_{r2} \), the second by \( u \sin \varphi_{r2} \), the third by \( u \cosh \varphi_{r2} \), and the fourth by \( u \cos \varphi_{r2} \). Then, all four equations are rearranged in such a way that the right-hand sides are zero.

\[
\begin{align*}
\left[ u \sinh \varphi_{r2} \sin \varphi_{r1} + u \cosh \varphi_{r2} \cos \varphi_{r1} \right] &= 0 \\
\left[ u \sin \varphi_{r2} \sin \varphi_{r1} - u \cos \varphi_{r2} \cos \varphi_{r1} \right] &= 0 \\
\left[ u \cosh \varphi_{r2} \sin \varphi_{r1} + u \sinh \varphi_{r2} \cos \varphi_{r1} \right] &= 0 \\
\left[ u \cos \varphi_{r2} \sin \varphi_{r1} + u \sin \varphi_{r2} \cos \varphi_{r1} \right] &= 0
\end{align*}
\]

(5-25) (5-26) (5-27) (5-28)

The left-hand sides of these equations become the characteristic quantities. Before a search for resonance frequencies is carried out, the quantities \( \varphi_{r1} \) and \( \varphi_{r2} \) must
be evaluated according to the disk thickness being considered. The assumed mode symmetry determines whether the upper or lower pair of equations is to be used. For each circular waveguide mode being considered with its associated value of $k_c$, the cutoff frequency must be calculated according to equation (5-3). This frequency is compared with that at which the characteristic quantity is to be evaluated and a final choice of the equation to be used is made.

Evaluation of the characteristic quantity is now begun by first evaluating $u$ and $v$, making use of the fact that the definition of $v$ depends on whether or not region 2 is above or below cutoff. As well as depending on frequency, $u$ and $v$ depend on the waveguide mode, the waveguide radius, the half cavity length, and the permittivities. Finally, the values of $u$ and $v$ are substituted into the expressions in the characteristic quantity and the quantity is evaluated. The algebraic sign of this value is compared with that for the previous frequency to determine if a resonance has occurred between the two frequencies.

A brief flow chart describing the manner in which the numerical solution is carried out is given in Figure 5.3.
Set Fixed Parameters:
Permittivities
Cavity radius
Cavity length

Establish lower, upper limits of frequency range and frequency increment

Set variable parameters:
Mode constant
Disk thickness

Find normalized lengths of regions 1 and 2

Calculate cutoff frequencies in regions 1 and 2, set frequency to cutoff frequency in region 1

Is frequency below lower limit of frequency range?

Yes

No

Calculate μ
Display frequency - resonance occurs between this frequency and the last one - even mode

Does the odd mode characteristic quantity have the same sign as for the previous frequency? Yes

No

Display frequency - resonance occurs between this frequency and the last one - odd mode

Save newly calculated values of even and odd mode characteristic quantities for comparison with values at next freq.

Increment frequency

Is frequency above upper limit of frequency range? Yes

No

END

FIGURE 5.3  THE FLOW CHART
5.2.4 The graphical method

The derivation of a characteristic equation for resonance frequencies of the end-loaded cavity has been discussed above along with a method for numerically solving this equation. It is revealing to supplement this numerical treatment with a graphical solution. Generating such a solution is quite time consuming, but worthwhile because of the clarification it offers. Its main advantage here is that it verifies the frequencies predicted numerically. In particular, it will show that the method utilized in the numerical approach has not resulted in the omission of any resonance. Furthermore, the plots provide a means of identifying the solutions for different disk thicknesses and relating them to each other. Finally, these plots allow a deeper understanding of the present problem by providing insight into the cavity behavior outside the frequency range considered or for different values of various parameters.

As will be seen in the next section, a solution plot is generated for each circular waveguide mode and each disk thickness. Each plot extends from 7 GHz to 13 GHz. When cutoff frequency in the dielectric for a given mode falls in the 7 - 13 GHz range, both the below and above cutoff situations are included in the same plot. Even and odd cases are also included in the same plot. The three waveguide modes to be considered are TE_{11}, TE_{21} and TE_{01}. For each of these, even and odd cavity modes are considered, and for each symmetry, the mode may in general be either below or above cutoff in the dielectric section, region 2. It has been shown that, in fact, only the TE_{11} mode reaches the propagating state in region 2, within the plotted frequency range. Thus, the plots for TE_{21} and TE_{01} solutions only need account for the below-cutoff conditions in that region.
TABLE 5.2 The Characteristic Equation

To facilitate the discussion of the construction of the solution plots, the important expressions are given in Table 5.2. The quantities involved in these expressions have been defined in equation (5-8) through (5-23), and values of $k_c$ for the three waveguide modes have been given in Table 5.1. In general, for each cavity solution plot, there will be three curves (Figure 5-4). The first of these $+$ is a plot of $\frac{1}{u} \tan u \cdot \ell_{pi}$. The second curve is for even modes ($x$), and will be a plot of $-\frac{1}{u} \coth u \cdot \ell_{pi}$ below cutoff and $\frac{1}{u} \cot u \cdot \ell_{pi}$ above cutoff. Intersections of this curve with the first correspond to resonance frequencies associated with even cavity modes. The third and final curve is for modes with odd symmetry ($\omega$), and will be a plot of $-\frac{1}{u} \tanh u \cdot \ell_{pi}$ below cutoff $-\frac{1}{u} \tan u \cdot \ell_{pi}$ above cutoff. Intersections of this curve with the first one correspond to resonances associated with odd cavity modes.
<table>
<thead>
<tr>
<th>Mode Designation</th>
<th>Disk Thickness (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.4</td>
</tr>
<tr>
<td></td>
<td>.6</td>
</tr>
<tr>
<td></td>
<td>.8</td>
</tr>
<tr>
<td>$TE_{11}$</td>
<td>Fig. 1</td>
</tr>
<tr>
<td>$TE_{21}$</td>
<td>Fig. 2</td>
</tr>
<tr>
<td>$TE_{01}$</td>
<td>Fig. 3</td>
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<tr>
<td></td>
<td>Fig. 4</td>
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<tr>
<td></td>
<td>Fig. 5</td>
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<td></td>
<td>Fig. 6</td>
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<td></td>
<td>Fig. 7</td>
</tr>
<tr>
<td></td>
<td>Fig. 8</td>
</tr>
<tr>
<td></td>
<td>Fig. 9</td>
</tr>
</tbody>
</table>

**TABLE 5.3.**

The solution plots are given in Figure 5.4 through 5.12, and a key to these is given in Table 5.3. The basic horizontal scale used in the plots is for frequency, which is labeled along this axis in GHz. The frequency axis is the same for all the plots. Though none of the quantities plotted depends on $f$ explicitly, this is the independent variable of primary interest, and the frequency scale allows resonance frequencies to be read directly according to the location of intersections. Four additional horizontal axes are included. These involve variables which depend on frequency and are the independent variables appearing explicitly in the functions plotted. The first two of these are $u$ and $u\ell_n$. It is recalled that $u$ and $u\ell_n$ appear only in the expression $\frac{1}{u} \tan u\ell_n$, which is associated with region 1 and with both even and odd solutions. The other two horizontal scales are for $v^2$ and $v\ell_n$. These variables are involved in curves for both even and odd modes, above and below cutoff. If cutoff for a given waveguide mode in one of the cavity regions occurs in the 7 - 13 GHz range, it is indicated by a vertical dashed line. This condition occurs at $u = 0$ for region 1.
and \( v = 0 \) for region 2. In the plots for TE\(_{21} \) (Figures 5.7 - 5.9) and TE\(_{01} \) (Figures 5.10 - 5.12) modes the cutoff in region 1 occurs in the 7 - 13 GHz range, while cutoff in region 2 occurs above this range. In the plots for the TE\(_{11} \) waveguide mode (Figures 5.4 - 5.6), cutoff for region 1 occurs below the frequency range included, while in region 2, cutoff occurs within the range. On the \( u_{\lambda_{11}} \) axis, which is labeled only for frequencies above cutoff in region 1, integral multiples of \( \pi/4 \) are marked. The function \( \frac{1}{u} \tan u_{\lambda_{11}} \) has zeros at \( n\pi \) and takes on infinite values at \( (2n+1)\pi/2 \) where \( n = 0, 1, 2, \ldots \). A similar procedure is followed for \( u_{\lambda_{12}} \) above cutoff, since the functions plotted involve \( \tan u_{\lambda_{12}} \) or \( \cot u_{\lambda_{12}} \), and these functions also have special behavior at certain integral multiples of \( \pi/4 \).

In locating values along the \( u, u_{\lambda_{11}}, u_{\lambda_{12}}, v \) and \( u_{\lambda_{12}} \) axes, the expressions for the variables are solved for frequency, \( f \). The desired value of the given variable is then substituted into the resulting expression to determine the corresponding frequency. This allows the particular value of the variable to be correctly located relative to the frequency axis. Although the curves are not actually plotted with respect to the \( u, v, u_{\lambda_{11}} \) and \( u_{\lambda_{12}} \) axes, but with respect to the frequency axis, the presence of these axes is useful because the dependence of the expressions involved in the characteristic equation on the associated variables is familiar. A further advantage is that these additional axes facilitate an understanding of effects on the curves (and hence on resonance frequencies) due to parameter changes.
FIGURE 5.4  SOLUTION PLOT, $TE_{11}$ WAVEGUIDE MODE, .4 CM DISK THICKNESS
Figure 5.8 Solution plot, TE$_{21}$ waveguide mode, .6 cm disk thickness
Figure 5.9 Solution plot, TE₂₁ waveguide mode, 0.8 cm disk thickness
FIGURE 5.11  SOLUTION PLOT, $T_{E_{01}}$ WAVEGUIDE MODE, .6 CM DISK THICKNESS
5.2.5 Discussion of predictions

The results of the numerical solution of the characteristic equation are given in Table 5.4. The predicted resonance frequencies are separated into three main classifications according to circular waveguide mode type. These three types \( TE_{11}, TE_{21}, TE_{01} \) are indicated along the left side of the chart. The three ferrite disk thicknesses are indicated across the top of the table. Also included in the chart is a designation of mode symmetry for each resonance. The frequency entered in the table for a given resonance is actually the upper limit of a 50 MHz interval in which the resonance frequency is predicted to lie.

The nine plots in Figures 5.4 through 5.12 comprise the graphical treatment of the characteristic equation. The resonance frequencies indicated in the graphical representation agree with those shown in Table 5.4. The two solutions thus substantiate each other, demonstrating that the algorithm used in the purely numerical method has led to location of all the resonances.

In at least one respect, the solution plots for the end-loaded unmagnetized cavity have contributed information to the table of predicted resonance frequencies. Cavity solutions occurring for different ferrite disk thicknesses but corresponding to a given curve intersection which has simply shifted in frequency with changing thickness can be thought of as the same mode. This approach is based on the fact that although the fields for the mode will change as the disk thickness changes, other characteristic properties of the solution will not be altered. This thinking has been reflected in the construction of Table 5.4, where predicted frequencies for different disk thicknesses have been grouped horizontally and associated with single cavity modes. These cavity modes are designated according to a numbered
<table>
<thead>
<tr>
<th>Waveguide Mode Classification</th>
<th>Cavity Mode Sequence</th>
<th>Mode Symmetry</th>
<th>Disk Thickness (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>Even</td>
<td>.4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Odd</td>
<td>.6</td>
</tr>
<tr>
<td>TE₁₁</td>
<td>3</td>
<td>Even</td>
<td>.8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Odd</td>
<td></td>
</tr>
<tr>
<td>TE₂₁</td>
<td>1</td>
<td>Even</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Odd</td>
<td></td>
</tr>
<tr>
<td>TE₀₁</td>
<td>1</td>
<td>Even</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Odd</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.4. Predicted Resonance Frequencies in GHz**
sequence for each waveguide mode. In the purely numerical treatment this association between resonances for different disk thicknesses is not apparent, although it might be inferred by assuming that the frequency of a cavity mode will tend to decrease as the ferrite section becomes thicker and by noting that related modes must have similar symmetry.

Referring to Figure 5.4, the $\text{TE}_{11}$ plot for 0.4 cm, three solutions are noted. One of these, corresponding to the first $\text{TE}_{11}$ mode in Table 5.4, occurs outside the vertical range of the plot. The intersection is at 8.70 GHz and corresponds to an even mode. If the 0.6 cm plot for the same waveguide mode is examined (Figure 5.5), it is seen that the corresponding intersection has shifted to 7.60 GHz, while in the final $\text{TE}_{11}$ plot (Figure 5.6) the intersection occurs outside the range of the plot to the left. The second $\text{TE}_{11}$ cavity mode, indicated in Table 5.4, corresponds to the intersection at 9.30 GHz on the 0.4 cm. plot, shifts to 8.00 GHz in the second plot, and finally to 7.40 GHz in the 0.8 cm. plot. The third $\text{TE}_{11}$ cavity mode appears at 11.55 GHz in the 0.4 cm. plot and shifts downward as indicated in the remaining two plots. The last $\text{TE}_{11}$ mode does not appear until the third solution plot, since the corresponding intersection occurs above the frequency range in the first two plots.

Cavity modes can also be followed from thickness to thickness for the remaining waveguide modes. For the $\text{TE}_{21}$ mode with a 0.4 cm. disk thickness two solutions occur very near to each other at 12.40 and 12.45 GHz, as seen in Figure 5.7. These two modes can be followed in Figure 5.8 and 5.9 as they shift lower in frequency while disk thickness increases. For the $\text{TE}_{01}$ waveguide mode, no resonances occur at the 0.4 cm. disk thickness (Figure 5.10), but two
modes appear at 13.00 GHz for the .6 cm. thickness (Figure 5.11) and move to approximately 12.30 GHz when the thickness is increased to .8 cm. (Figure 5.12).

The solution plots clearly reveal whether or not a cavity mode is above or below cutoff in the dielectric section. For the TE_{11} waveguide mode (Figures 5.4 - 5.6) cavity mode 1 is below cutoff for all three disk thicknesses. Cavity mode 2 is above cutoff at the .4 cm. thickness but has shifted below cutoff for the two greater thicknesses. The remaining two cavity modes remain above cutoff. In the TE_{21} and TE_{01} plots all the cavity modes occur below cutoff in region 2.

The effect of changing disk thickness has been mentioned above in terms of specific resonance frequencies and illustrated by comparing different plots for a given waveguide mode. The effect of varying a parameter is thought of in terms of the resulting changes in the $u$, $v$, $u_{r_1}$ and $v_{r_2}$ axes. While the frequency axis remains unchanged, values on the other horizontal axes may shift relative to frequency. Since, effectively, the curves are plotted relative to the $u$, $v$, $u_{r_1}$ and $v_{r_2}$ axes, they also are changed.

When the disk thickness is changed without altering the overall cavity length, $u$ and $v$ are not affected, because neither depends on $l_{r_1}$ or $l_{r_2}$, and there will be no change in the $u$ and $v$ axes. Values of $u_{r_1}$ and $v_{r_2}$, however, are shifted along the respective axes, except for the points $u_{r_1} = 0$ and $v_{r_2} = 0$, which remain fixed. From the definitions of $l_{r_1}$ and $l_{r_2}$ as given in equations (5-11) through (5-17), it is seen that an increase in one must mean a decrease in the other. If $l_{r_1}$ is increased, points
on the $u_{\ell_{r_1}}$ axis will shift nearer to the origin, which is at $u_{\ell_{r_1}} = 0$ and corresponds to cutoff in the ferrite region. Points corresponding to different values of $u_{\ell_{r_1}}$ shift different distances, with larger values shifting more. At the same time, the parameter $l_{r_2}$ will decrease, and points on the $u_{\ell_{r_2}}$ axis will shift outward from the origin in both directions, again with larger values of $u_{\ell_{r_2}}$ shifting further. If $l_{r_1}$ is decreased instead, so that $l_{r_2}$ increases, the shifts will reverse. Thus, the $u_{\ell_{r_1}}$ and $u_{\ell_{r_2}}$ axes contract and expand, respectively, about their origins when $l_{r_1}$ is increased, while the effects are reversed if $l_{r_1}$ is decreased.

The effect on the curves when $l_{r_1}$ and $l_{r_2}$ are changed is even more complex. Because $u$ and $\nu$ appear in the functions being plotted as factors in the products $u_{\ell_{r_1}}$ and $u_{\ell_{r_2}}$, changes in each curve can first be thought of as the result of points on the curve being shifted to the right or left in step with the movement of corresponding points on the $u_{\ell_{r_1}}$ and $u_{\ell_{r_2}}$ axes. However, $u$ and $\nu$ also appear in the factors $1/u$ and $1/\nu$, so that while each point on a curve is shifted horizontally in such a way that the quantity $u_{\ell_{r_1}}$ and $u_{\ell_{r_2}}$ does not change, the factor $1/u$ or $1/\nu$ will increase or decrease and points on the curves must also shift in the vertical direction. If points on a curve shift horizontally toward the origin of the $u_{\ell_{r_1}}$ or $u_{\ell_{r_2}}$ axis, they will shift vertically away from the frequency axis, whereas if the horizontal shift is away from the origin, the vertical shift will be toward the frequency axis.

Finally, this complicated effect on the curves due to changes in disk thickness leads to changes in resonance frequencies which are difficult to predict,
especially for small changes in thickness. In the range between cutoff in the
dielectric section and cutoff in the ferrite section, the horizontal movement of
the different solution curves is in the same direction, while to the right of cutoff
in the dielectric those plotted with respect to $u$ and $uL_{r1}$ move
oppositely to those plotted as functions of $\nu$ and $uL_{r2}$. Because
of this behavior and because intersecting curves always have slopes of opposite
sign, the direction of intersection movement would ordinarily be certain in the
first region while impossible to determine by inspection in the second. But because
the curves shift vertically as well as horizontally, even the former case cannot be
determined by inspection.

However, if the changes in $L_{r1}$ and $L_{r2}$ are not restricted
to being small, the situation is quite different, and some conclusions about the
eventual effect on resonance frequencies can be made by examination of a given
plot. This can best be done by describing changes in the curves in terms of the
movement of asymptotes, these being the single most descriptive characteristic for a
given curve. An example of this is the intersection at 11.50 GHz in Figure 5.4,
the TE$_{11}$ plot for .4 cm thickness. This predicted resonance frequency must
eventually decrease if $L_{r1}$ is increased sufficiently, although the
direction of its movement due to small changes in $L_{r1}$ is not obvious.
As $L_{r1}$ is increased, the asymptote of the $\frac{1}{u}\tan uL_{r1}$ curve at
$uL_{r1} = \frac{3\pi}{2}$ will shift to the left, approaching $u = 4.7$ as $L_{r1}$
approaches unity. This is slightly to the left of the $\nu = 0$ asymptote of the
$\frac{1}{\nu}\cot \nu L_{r2}$ curve. The solution corresponding to the 11.50 GHz
resonance will remain to the left of the \( u \cdot l_{r1} = \frac{3 \pi}{2} \) asymptote while continuing to occur between the same two curve branches until the point

\[ u \cdot l_{r1} = \frac{3 \pi}{2} \]

has moved to the left of \( \nu = 0 \). Then, the original pair of branches no longer intersects. Since the \( u \cdot l_{r1} = \frac{3 \pi}{2} \) asymptote can shift to the left of 11.50 GHz but still be to the right of \( \nu = 0 \) at 8.87 GHz, the resonance at 11.5 GHz must shift to the left. Eventually, this frequency will shift past 8.87 GHz, as the shifting asymptote passes the fixed one, and the corresponding intersection will occur between different branches of the two curves.

More generally speaking, if \( l_{r1} \) approaches unity and \( l_{r2} \) approaches zero, the \(- \frac{1}{\nu} \text{tan} \, \nu \cdot l_{r2} / - \frac{1}{\nu} \text{tan} \, \nu \cdot l_{r2} \) curve approaches the frequency axis whereas the \(- \frac{1}{\nu} \text{coth} \, \nu \cdot l_{r2} / \frac{1}{\nu} \text{cot} \, \nu \cdot l_{r2} \) curve approaches infinity everywhere. The result of this is that the zeros and infinities of the \( \frac{1}{\nu} \text{tan} \, u \cdot l_{r1} \) curve become the odd and even solutions of the characteristic equation, respectively, and correspond to the resonance frequencies. The locations of these points are relatively easy to determine.

A similar but less complex situation results when \( l_{r1} \) and \( l_{r2} \) remain constant, and \( \nu \) is changed. Here, the \( u \), \( \nu \), \( u \cdot l_{r1} \) and \( \nu \cdot l_{r2} \) axes are all affected. The origins do not shift relative to frequency, but other values on the axes shift toward the origins for increasing \( \nu \) and away for decreasing \( \nu \), with larger values undergoing greater shifts. These changes in the axes can be thought of as contractions or expansions centered on the origins. Because \( l_{r1} \) and \( l_{r2} \) are constant, the ratios of \( u \cdot l_{r1} \) to \( u \) and \( \nu \cdot l_{r2} \) to \( \nu \) remain unchanged as \( \nu \) varies, so that corresponding points on the \( u \) and \( u \cdot l_{r1} \) axes and corresponding points on the \( \nu \) and \( \nu \cdot l_{r2} \)
axes shift identically.

Changes in the plotted curves can be described in a manner similar to that for changing \( l_{r_1} \) and \( l_{r_2} \) — points on each curve shift with the corresponding points on the u, v, u\( l_{r_1} \) and v\( l_{r_2} \) axes. Thus, the curves contract horizontally toward their respective origins when \( l \) is increased and expand away from the origins when \( l \) is decreased. They shift in the same direction in the region above cutoff in the dielectric and shift oppositely in the region below that cutoff. Vertical distortion of the curves is absent due to the fixed relationships between u and u\( l_{r_1} \) and between v and v\( l_{r_2} \).

As in the case of changing \( l_{r_1} \) and \( l_{r_2} \), the effect on predicted resonance frequencies as \( l \) is changed is not generally foreseeable.

In the region between the two cutoffs, where curve shifts are in opposite directions, the direction in which an intersection will shift from its initial frequency can be determined by inspection of the given plot only if a sufficiently large change in \( l \) is allowed. But for arbitrarily small changes in \( l \), the direction cannot be predicted.

To demonstrate the predictability when the change in \( l \) is allowed to become large, an example is considered. For the \( \frac{1}{u} \tan u l_{r_1} \) curve an increase in \( l \) causes the asymptotes to shift lower in frequency, approaching \( u = 0 \) as \( l \) grows larger. Thus, any given asymptote can be shifted to the left of cutoff in region 2 if \( l \) is sufficiently increased. For curves dependent on v and v\( l_{r_2} \), the asymptotes shift toward v = 0 from both sides. Referring to Figure 5.4, the intersection at 11.50 GHz will shift constantly to the left as \( l \) is increased. This trend must continue until the asymptote at
\[ U_{\theta 1} = \frac{3\pi}{2} \] reaches the \( \nu = 0 \) line, at which condition the two branches involved no longer intersect.

When \( U_{\theta 1} = \frac{3\pi}{2} \) shifts to the left past \( \nu = 0 \), the intersection between the branch of \( \frac{1}{U} \cot \nu \epsilon_{r z} \) lying between \( \nu \epsilon_{r z} = 0 \) and \( \nu \epsilon_{r z} = \pi \) and the branch of \( \frac{1}{U} \tan U_{\theta 1} \) lying between \( U_{\theta 1} = \pi/2 \) and \( U_{\theta 1} = 3\pi/2 \) no longer exists. It is replaced by an intersection between \( -\frac{1}{U} \coth \nu \epsilon_{r z} \) and the branch of \( \frac{1}{U} \tan U_{\theta 1} \) between \( U_{\theta 1} = 3\pi/2 \) and \( U_{\theta 1} = 5\pi/2 \). Although it is quite possible that this intersection, which corresponds to the original resonance at 11.50 GHz, will move uniformly to the left as \( \epsilon \) increases, this cannot be demonstrated, since the intersecting curves are shifting oppositely. By continuing to increase \( \epsilon \), however, the intersection can always be brought arbitrarily close to the \( \mu = 0 \) line.

Similarly, referring again to Figure 5.4, the intersection initially at 9.30 GHz will move uniformly to the left until it reaches the \( \nu = 0 \) line. For an increase in \( \epsilon \) once again it can not be shown that the intersection always moves to the left, but as above, it can be seen that it can still be brought arbitrarily near to \( \mu = 0 \). Finally, a similar statement is made for the intersection at 8.70 GHz, which already lies in the region of oppositely shifting curves. Again, the direction of its shift at any given value of \( \epsilon \) as \( \epsilon \) is increased cannot be seen by inspection, but the resonance frequency can still be brought arbitrarily close to the region 1 cutoff frequency.

Changes in the permittivities are like those in \( \epsilon \) in that corresponding points on the \( \mu \) and \( U_{\theta 1} \) axes or on the \( \nu \) and \( \nu \epsilon_{r z} \) axes are shifted
identically. Although all four axes are influenced by $\varepsilon$, only $\mu$ and $\mu \varepsilon_{\text{r}1}$ are affected by changes in $\varepsilon_{\text{r}1}$, and only $\nu$ and $\nu \varepsilon_{\text{r}2}$ are affected by changes in $\varepsilon_{\text{r}2}$. The expressions for $\mu$ and $\nu$ reveal that an increase in $\varepsilon_{\text{r}1}$ or $\varepsilon_{\text{r}2}$ causes all points on the respective pair of axes to shift to lower frequencies, while a decrease in the permittivity causes a shift to higher frequencies. It can also be shown that these changes in the axes are not simple shifts, since the movement is greater for points at larger initial frequencies.

As was true for changes in $\varepsilon$, the curves can be considered to undergo only horizontal shifts, with no distortion in the vertical direction. As a permittivity in increased, the associated curves and related intersections will move to lower frequencies. If the permittivity is decreased, the shift of curves and intersections reverses its direction. If both permittivities are increased or both decreased, it is clear that all predicted resonance frequencies will decrease or increase, respectively. If one permittivity is increased and the other decreased, the corresponding curves will shift oppositely, and directions of change in different resonance frequencies may in general be different and are no longer apparent by inspection.

If these opposite curve shifts are restricted to being very small and are known, along with the slopes of the two intersecting curves, the corresponding resonance frequency change can actually be determined. Unfortunately, such small changes in permittivity are generally of no practical interest in understanding the effect on predictions of errors in the permittivities. The case of opposite
changes in the permittivities is potentially of special interest from the standpoint of explaining experimental discrepancies, since it offers the possibility of simultaneous high and low predictions. But such an explanation would be impossible to substantiate for errors great enough to cause discrepancies of significant size.

The last parameter to be discussed is \( r_o \), the waveguide radius. When this parameter is changed, the \( \mu, \mu_{\ell_1}, \nu \) and \( \nu_{\ell_2} \) axes are all affected, with corresponding points on the \( \mu \) and \( \mu_{\ell_1} \) axes and corresponding points on the \( \nu \) and \( \nu_{\ell_2} \) axes again shifting identically. Examination of the manner in which \( r_o \) enters into the expressions for \( \mu \) and \( \nu \) reveals that increase in \( r_o \) leads to a shifting of \( \mu \) and \( \nu \) values to the left, with this shift being different for different initial values of frequency. A decrease in \( r_o \) causes a shift to higher frequencies. Again, the resulting effect on plotted curves is not a simple one, but all curves will move in the same direction, so that the direction for shifts in resonance frequencies can be established.

5.2.6 The field amplitude plots

For each predicted cavity mode, a plot has been constructed to describe the variation along the cavity length of the longitudinal, time-varying magnetic field. These plots are given in Figures 5.13 through 5.31. The method by which they are generated is discussed in detail in Appendix 5. Coupling of microwave energy from the rectangular waveguide into the experimental cavity through
the circular hole in the curved wall is believed to be closely related to the longitudinal magnetic cavity field at this hole. Since the cavity field is a pure standing wave, the longitudinal magnetic field component is completely described by its amplitude. While searching for cavity resonances, the dependence of energy absorption on coupling hole position is observed, and is expected to bear a close resemblance to the variation along the cavity length of magnetic field amplitude. If this occurs, the plots will help in establishing the desired one-to-one correspondence between predicted modes and observed resonances. Otherwise, this correspondence is difficult to establish because many resonances may occur and because predicted and observed frequencies for these may be substantially different.

Construction of the plots is based on the complex expressions for the cavity fields. The \( z \)-component of the time-varying magnetic field is obtained according to the usual convention - the corresponding complex component is multiplied by the factor \( \exp(j \omega t) \) and the real part of the product is taken. The form taken by the expression for the field component depends on the mode symmetry, or the cavity region, and on whether or not the cavity mode is propagating in that region. The numerical evaluation of the expression at a specific \( z \) will depend on the waveguide mode involved, on the actual resonance frequency, and on the various parameters of the problem. Only the half-cavity is included in the amplitude plots, since each mode is either even or odd, and no new information will be contributed by a plot for the entire cavity length.

The plots are normalized, and a detailed description of how this normalization is carried out is included in Appendix 5. It is convenient to determine
analytically the overall maximum taken on by the amplitude for a given resonance, since this eliminates the need for a point by point numerical search. This maximum will depend on the expressions for the field component, and can itself be written in terms of the arbitrary constant and various other quantities appearing in those expressions. A maximum value of unity is achieved by properly adjusting the arbitrary constant. The four main cases corresponding to even and odd symmetry and above or below cutoff in region 2 must be considered separately. Each of these must in general be further separated according to the values of quantities derived from the field component expressions, as shown in the appendix.

One of the most noticeable things revealed by these plots is that some even symmetry modes lead to amplitudes, and therefore to absorption dependence, which resemble those for an odd mode. These situations are especially prominent when region 2 is substantially below cutoff, as seen in the following cases:

the $\text{TE}_{21}$ resonance predicted at 12.40 GHz for the .4 cm disk thickness (Figure 5.16),
the $\text{TE}_{21}$ resonance predicted at 10.90 GHz for the .6 cm disk thickness (Figure 5.20),
the $\text{TE}_{01}$ resonance at 13.00 GHz for the .6 cm disk thickness (Figure 5.23),
the $\text{TE}_{21}$ resonance at 10.15 GHz for the .8 cm thickness (Figure 5.26)
and the $\text{TE}_{01}$ resonance at 12.25 GHz for the .8 cm thickness (Figure 5.30).
Figure 5.13 Amplitude Plot. Resonance No. 1
Figure 5.14 Amplitude Plot. Resonance No. 2
Figure 5.18 Amplitude Plot. Resonance No. 6.
Figure 5.21 Amplitude Plot. Resonance No 9
Figure 5.27 Amplitude Plot. Resonance No 15
Figure 5.29 Amplitude Plot, Resonance No. 17

Region 1

Region 2
Figure 5.30 Amplitude Plot. Resonance No 18
5.3 Measurements and Comparison with Predictions

5.3.1 The experimental absorption plots

The theory developed for the unmagnetized end-loaded cavity will be evaluated mainly through the use of experimentally obtained plots describing energy absorption by the cavity. These plots are given in Figures 5.32 through 5.34. The measurement arrangement and the cavity assembly used in obtaining the plots have been described in Chapter 4. A sequence of curves was obtained for each ferrite disk thickness. Each curve in a sequence corresponds to a different position of the coupling hole relative to the cavity, and is obtained by slowly sweeping the frequency of the incident power from 8 GHz to 12.4 GHz while plotting the voltage output of the detector. For comparison, a plot is also made with a short-circuit plate substituted for the cavity fixture. The coupling hole positions given with the plots are determined approximately. Frequency is marked horizontally at the bottom of each set of plots, and a key is given in Table 5.5.

5.3.2 The comparison table

The purpose of Table 5.6(a-c) is to present a summary of the theoretical and experimental results for the unmagnetized cavity and a comparison of these results. The table is separated into three parts according to ferrite disk thickness. In a given part, each row corresponds to a predicted cavity mode. All predicted resonance frequencies in the 7-13 GHz range are included. These are ordered according to increasing frequency, as in the case of the theoretical amplitude plots.
The sequence numbers included in the table have also been noted in the table of predicted resonance frequencies, Table 5.4. The grouping according to disk thickness corresponds to the grouping of curves in the experimental absorption plots (Figures 5.32 - 5.34), while the ordering according to increasing frequency corresponds to the order in which resonances occur in absorption plot curves. For each predicted resonance, the corresponding circular waveguide mode is noted (column 2). The predicted and observed resonance frequencies are then given (columns 3 and 4). Column 5 shows the mode symmetry. A comment on the strength of the observed absorption is included (column 6).

Columns 7 and 8 reveal the agreement between the theoretical amplitude plots and the information contained in the absorption plots. In addition to stating a symmetry type, this data describes the expected and observed number and locations of absorption maxima. An attempt is made to include information about both location and strength of absorption maxima. Information in column 7 is based on the theoretical amplitude plots. The distance of a given amplitude maximum from the cavity center at ɛ = L is given in centimeters. It is noted that if a predicted mode is even, an amplitude maximum will generally occur at the cavity center, except in the case of modes which are cut off in the center cavity region. The experimental case is somewhat more complicated. As absorption plots are being recorded, the coupling hole position must be varied in steps. This means that an absorption maximum may effectively occur between two hole positions, and the actual location can only be estimated. A similar statement is true for the absorption strength data. Location and relative strength of an absorption maximum become difficult to determine when the absorption is very weak. In fact, theoretical
data often help in locating such resonances.

A detailed study has not been made of the relationship between the r.f. magnetic field and absorption. It is reasonable to assume that maxima in amplitude and absorption will occur at the same location, but it is less apparent that a given percentage difference in magnetic field amplitude will correspond to a similar percentage difference in absorption strength. Therefore, care must be used when drawing conclusions from a comparison of this aspect in theory and measurement. Because the state of the cavity mode in the center region - whether or not the mode is propagating or below cutoff there - has a great deal to do with the characteristics of the mode, both predicted and observed, and may well have much bearing on the comparison between theory and experiment, this information as determined theoretically is included in the table (column 9). The comments included in the table will be discussed in 5.3.3.

It is in the construction of Table 5.6 that the value of the theoretical amplitude plots becomes most evident. The arrangement of the table assumes a one-to-one correspondence between predicted and observed resonances. In practice such a correspondence is extremely difficult to achieve. First, predicted and observed resonance frequencies are sometimes substantially different. Therefore, even when it is known whether the predicted resonance is that of an even or odd mode and the symmetry of the actual resonances can be observed, conclusions about the correspondence between predicted and observed resonances may be uncertain. Second, some observed resonances lead to such weak absorption that it is difficult to locate the resonance on a plot. The additional information provided by the amplitude plots makes the matching of predicted and observed resonances surer,
and in the worst cases helps substantiate the presence of a weak resonance.

5.3.3 Comparison of predictions and measurements

The discussion of the agreement between theory and experiment is based on the predicted resonance frequencies, the theoretical amplitude plots, and the experimental absorption plots. Since the comparison table, Table 5.6, summarizes this information, it will serve as the center of this discussion. A comparison of resonance frequencies cannot be carried out at all unless each observed resonance is identified with a predicted one, an operation largely made possible by the amplitude plots. Furthermore, the agreement between the observed effect of coupling hole position on absorption and the effect indicated by the theoretical amplitude plots is of interest in its own right.

An examination of the absorption plots reveals an ambiguity due to the fact that the observed resonance frequency depends on the position of the coupling hole. This dependence is systematic, with the frequency moving lower as the absorption strengthens. Frequencies have been entered in the table according to the convention that the frequency at which the greatest absorption appears is the resonance frequency.

In completing the comparison table, the experimental absorption plots have been supplemented with observations made by oscilloscope. Such observations are important when plotted evidence of a resonance is too weak. While the oscilloscope display was monitored, the coupling hole was shifted back and forth between cavity ends. The changing display, due to the appearance and disappearance of resonances, made faint absorptions more noticeable.
The fact that all resonances found experimentally and theoretically have been paired not only indicates general agreement between theory and experiment, but also supports the view that TM cavity modes are not excited to a significant degree in the present experimental arrangement, and lends further credibility to the assumption concerning the relationship between coupling and magnetic field at the hole.

A feeling for the agreement between the amplitude plots and observed variation of absorption can only be gained by examining the plots, but the extent of this agreement is certainly evident in the table. In all cases, the number of absorption maxima observed as the coupling hole is moved is the same as that expected, and in many cases the location of these maxima is very close to that expected from theory. This is particularly evident for resonances with the .4 cm disk thickness, and is quite good for those .6 cm resonances which were within the range of the measurement plots. Most impressive are those cases which have the largest number of absorption maxima. Examples of these are resonances # 3, # 10, # 16 and # 17. The absorption due to resonance # 3 is strong. The predicted and observed locations of absorption maxima agree well for # 3, there being only a 5.5% error for the off-center maximum. The observed off-center absorption is very distinct, further emphasizing the experimental agreement.

Resonance # 10 corresponds to the same cavity mode as resonance # 3 with the disk thickness increased to .6 cm. Theory predicts very little change in resonance frequency and this is borne out by experiment, though there is a slight increase as opposed to the expected decrease. The observed absorption for a centered hole is again strong, but the off-center absorption, expected to be weaker, is actually barely discernible. Location of this absorption is expected to
change very little, and the observations are in agreement with this. The same mode appears again as resonance #16 when the disk thickness is increased to .8 cm. In this case the absorption becomes weak, making the off-center maximum almost impossible to find. Again, predicted location has changed very little, and observation is in fairly good agreement with this. Resonance #17, which corresponds to a mode not appearing for the other disk thicknesses, is especially interesting because of its odd symmetry with two maxima occurring on either side of cavity center and because, in agreement with theory, all maxima have the same strength.

Certain distinctive even/odd resonance pairs have been predicted for every one of the disk thicknesses. Out of nineteen resonances predicted for the 7 - 13 GHz range, ten belong to such pairs. In each case, the predicted frequencies are quite closely located, and both modes are below cutoff in the cavity center-section. Resonances #4 and #5 constitute such a pair, as do resonances #8 and #9, #14 and #15, #18 and #19. Another pair, #11 and #12, is predicted but occurs above the plotted range. In the experimental location of these resonances the theoretical amplitude plots have again been important. First, the frequencies for a given pair are predicted to be so similar that they are expected to be experimentally superimposed. Since one mode of each pair is an even mode, a superposition of the two in an absorption plot might be expected to exhibit the behavior of an even mode, with an absorption maximum at the center of the cavity. Such a characteristic was not found, however, and it was thought initially that the even mode did not occur. The amplitude plots show, however, that an even mode which is below cutoff in region 2 exhibits a behavior in that
region much different from that of an even mode which is propagating in region 2, and instead of a maximum at the cavity center, has an amplitude minimum there. This is not an amplitude zero, but in some cases the amplitude is relatively small and would suggest an absorption behavior like that of an odd mode. So it is revealed by the theoretical amplitude plots that the even mode of each such pair does not necessarily fail to occur, but may indeed be superimposed on the odd mode, the combination of the two appearing to be a single odd mode. Evidence that this is a valid interpretation comes when closer examination of each of the absorptions reveals a faint, non-zero level when the coupling hole is centered.

All of the even/odd resonance pairs exhibit very pronounced off-center absorption in the experimental plots. All have two absorption maxima, one to either side of center-cavity. In each case, the observed absorption maximum is closer to the center than indicated by theory, with the error ranging from a 3.5% for the # 4/# 5 pair to 17.5% for the # 14/# 15 pair. More important is the fact that the trend suggested by prediction of shift in the absorption peak location as disk thickness is increased is found to occur in experiment.

Resonance # 1 is unique in the set, since the theoretical amplitude plot for this mode predicts a nearly uniform level of observed absorption for coupling hole positions over the entire length of the dielectric center section.

By referring to the absorption plot for .4 cm ferrite disk thickness, Figure 5.32, one can find the weak absorption at 9.15 GHz and clearly see that the behavior expected for the absorption is obeyed.

Referring to column 6 of the comparison chart, it is seen that absorptions observed in the lower part of the band tend to be weak. Noticeable examples
are the resonances $\# 1$, $\# 2$, $\# 7$ and $\# 13$. The behavior may be attributed to variations in effective coupling hole size with frequency.

The discrepancies in predicted resonance frequency, $f_p$, and observed resonance frequency, $f_o$, are summarized in Figure 5.35 and $|(f_o-f_p)/f_p|$ is less than 10% and typically $\sim 3\%$. It is noticeable that the error tends to be positive at the lower resonance frequencies for a given disk size and negative at the higher ones. However, it is also seen that the errors associated with $\text{TE}_{11}$ modes tend to be positive while those for $\text{TE}_{21}$ and $\text{TE}_{01}$ tend to be negative (with two exceptions). This behavior may be due to the presence of the coupling hole since the effect of the hole on cavity operation depends on both frequency and cavity mode. Errors in assumed permittivity may also be a contributing factor but, as discussed in Section 5.2.5, the behavior of the inhomogeneous cavity with variations in permittivity is complex and cannot easily be characterized.
Distance of Coupling Hole from Center
of Cavity in 1/32's of one inch

<table>
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<tr>
<th>Sweep #</th>
<th>.4</th>
<th>.6</th>
<th>.8</th>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td>6</td>
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<td>7</td>
<td>6</td>
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<td>12</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>14</td>
<td>13</td>
</tr>
</tbody>
</table>

KEY TO EXPERIMENTAL ABSORPTION PLOTS

TABLE 5.5
FIG. 5.32  ABSORPTION PLOT, UNMAGNETIZED CAVITY, .4 CM DISKS
FIG. 5.33 ABSORPTION PLOT, UNMAGNETIZED CAVITY, .6 CM DISKS
FIG. 5.34  ABSORPTION PLOT, UNMAGNETIZED CAVITY, .8 CM DISKS
<table>
<thead>
<tr>
<th>Resonance</th>
<th>Waveguide Mode</th>
<th>Predicted Frequency (GHz)</th>
<th>Observed Frequency (GHz)</th>
<th>Symmetry</th>
<th>Absorption Strength</th>
<th>Position in cm of absorption maximum Relative amplitude of maximum</th>
<th>Below Cutoff in Plastic?</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TE_{11}</td>
<td>8.70</td>
<td>9.15</td>
<td>Even</td>
<td>Weak</td>
<td>.75/1.00</td>
<td>.79/1.00</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>TE_{11}</td>
<td>9.30</td>
<td>9.66</td>
<td>Odd</td>
<td>Weak</td>
<td>.80/1.00</td>
<td>.79/1.00</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>TE_{11}</td>
<td>11.55</td>
<td>11.55</td>
<td>Even</td>
<td>Strong</td>
<td>.00/1.00</td>
<td>.00/1.00</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>TE_{21}</td>
<td>12.40</td>
<td>12.14</td>
<td>Even</td>
<td>Very Strong</td>
<td>.86/1.00</td>
<td>.83/1.00</td>
<td>Yes</td>
</tr>
<tr>
<td>5</td>
<td>TE_{21}</td>
<td>12.45</td>
<td>12.14</td>
<td>Odd</td>
<td>Very Strong</td>
<td>.86/1.00</td>
<td>.83/1.00</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**TABLE 5.6 (a) Comparison table. Disk thickness 0.4 cm**
<table>
<thead>
<tr>
<th>Resonance</th>
<th>Waveguide Mode</th>
<th>Predicted Frequency (GHz)</th>
<th>Observed Frequency (GHz)</th>
<th>Symmetry</th>
<th>Absorption Strength</th>
<th>Position in cm of absorption maximum / relative amplitude of maximum</th>
<th>Below cutoff in plastic?</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>TE_{11}</td>
<td>7.60</td>
<td>-</td>
<td>Even</td>
<td>-</td>
<td>.65/1.00 / -/ -</td>
<td>Yes</td>
<td>Not found experimentally - probably occurs below frequency range of observations</td>
</tr>
<tr>
<td>7</td>
<td>TE_{11}</td>
<td>8.00</td>
<td>8.35</td>
<td>Odd</td>
<td>Very Weak</td>
<td>.65/1.00 / .71/1.00</td>
<td>Yes</td>
<td>Observations of max location is difficult due to very low absorption</td>
</tr>
<tr>
<td>8</td>
<td>TE_{21}</td>
<td>10.90</td>
<td>10.90</td>
<td>Even</td>
<td>Strong</td>
<td>.74/1.00 / .71/1.00</td>
<td>Yes</td>
<td>See note for # 4.</td>
</tr>
<tr>
<td>9</td>
<td>TE_{21}</td>
<td>11.00</td>
<td>10.90</td>
<td>Odd</td>
<td>Strong</td>
<td>.74/1.00 / .71/1.00</td>
<td>Yes</td>
<td>Absorption coincides with # 8. Result appears very nearly odd except for faint non-zero absorption at center-cavity. See note for # 5.</td>
</tr>
<tr>
<td>10</td>
<td>TE_{11}</td>
<td>11.45</td>
<td>11.60</td>
<td>Even</td>
<td>Strong</td>
<td>.88/ .40 / .87/ -</td>
<td>No</td>
<td>Smaller absorption max is barely discernible in experimental plot</td>
</tr>
<tr>
<td>11</td>
<td>TE_{01}</td>
<td>13.00</td>
<td>12.65</td>
<td>Even</td>
<td>Medium</td>
<td>.74/1.00 / -/ -</td>
<td>Yes</td>
<td>Observed only by oscilloscope-resonance is outside frequency range of experimental plot</td>
</tr>
<tr>
<td>12</td>
<td>TE_{01}</td>
<td>13.00</td>
<td>12.65</td>
<td>Odd</td>
<td>Medium</td>
<td>.74/1.00 / -/ -</td>
<td>Yes</td>
<td>Observed only by oscilloscope-coincides with # 11 — combined absorption expected to resemble that for odd mode</td>
</tr>
</tbody>
</table>

TABLE 5.6 (b) Comparison Table. Disk Thickness 0.6 cm
<table>
<thead>
<tr>
<th>Resonance</th>
<th>Waveguide Mode</th>
<th>Predicted Frequency (GHz)</th>
<th>Observed Frequency (GHz)</th>
<th>Symmetry</th>
<th>Absorption Strength</th>
<th>Position in cm. of absorption relative maximum amplitude of maximum</th>
<th>Cutoff in Plastic?</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>TE_{11}</td>
<td>7.35</td>
<td>8.00</td>
<td>Odd</td>
<td>Very Weak</td>
<td>.63/1.00 —/—</td>
<td>Yes</td>
<td>Observed only by oscilloscope—resonance is outside frequency range of experimental plot.</td>
</tr>
<tr>
<td>14</td>
<td>TE_{21}</td>
<td>10.15</td>
<td>10.40</td>
<td>Even</td>
<td>Strong</td>
<td>.63/1.00 .52/1.00</td>
<td>Yes</td>
<td>See comment for # 4, # 5 — absorption like for odd mode but does not reduce to zero at center — this is borne out by observation</td>
</tr>
<tr>
<td>15</td>
<td>TE_{21}</td>
<td>10.30</td>
<td>10.40</td>
<td>Odd</td>
<td>Strong</td>
<td>.63/1.00 .52/1.00</td>
<td>Yes</td>
<td>Observed absorption superimposed on that for # 14 has small non-zero absorption at center due to # 14.</td>
</tr>
<tr>
<td>16</td>
<td>TE_{11}</td>
<td>10.70</td>
<td>11.00</td>
<td>Even</td>
<td>Weak</td>
<td>.00/1.00 .00/1.00</td>
<td>No</td>
<td>Smaller maximum is nearly indiscernible in experimental plot — distance of max from center is very difficult to determine</td>
</tr>
<tr>
<td>17</td>
<td>TE_{11}</td>
<td>11.85</td>
<td>11.90</td>
<td>Odd</td>
<td>Weak</td>
<td>.40/1.00 .40/1.00</td>
<td>No</td>
<td>Experimental frequency is difficult to determine — situation is complicated by strong nearby absorption</td>
</tr>
<tr>
<td>18</td>
<td>TE_{01}</td>
<td>12.25</td>
<td>12.10</td>
<td>Even</td>
<td>Strong</td>
<td>.63/1.00 .56/1.00</td>
<td>Yes</td>
<td>See notes for # 4, # 14 — in this case, small non-zero absorption at cavity-center is not seen in experiment.</td>
</tr>
<tr>
<td>19</td>
<td>TE_{01}</td>
<td>12.30</td>
<td>12.10</td>
<td>Odd</td>
<td>Strong</td>
<td>.63/1.00 .56/1.00</td>
<td>Yes</td>
<td>See notes for # 5, # 15.</td>
</tr>
</tbody>
</table>

**TABLE 5.6 (c) Comparison table. Disk Thickness 0.8 cm**
Figure 8.35

Resonance frequency differences

.4 cm. disk thickness

.6 cm. disk thickness

.8 cm. disk thickness

$\nu_0 - \nu_p$, GHz
CHAPTER 6

THE END-LOADED CAVITY

WITH MAGNETIZED FERRITE

6.1 Introduction

The theories of dielectric-filled circular waveguide reviewed in Chapter 2 and guide filled with magnetized ferrite discussed in Chapter 3 are combined in a theoretical treatment of the circular waveguide cavity end-loaded with axially magnetized ferrite disks. The cavity configurations considered are identical to those of Chapter 5. Emphasis is on obtaining predicted resonance frequencies which are then compared with experiment.

6.2 Theory of the Cavity

6.2.1 Formulation of the problem

The cavity analyzed here is of length 2.286 cm (.9 inch) and radius .587 cm (.231 inch). It is symmetrically end-loaded with three different disk thicknesses in turn: .4 cm (.158 inch), .6 cm (.236 inch), and .8 cm (.315 inch). The ferrite type is the same as that of Chapters 3, 4, and 5. It is characterized by a $4\pi M_{\text{SAT}}$ of 1800 gauss and an effective g-factor of 2.55. Since a relative permittivity value of 9.5 was used in the ferrite-filled waveguide study of Chapter 3, this value is also used in the predictions of this chapter. The ferrite is assumed to be uniformly magnetized along the cavity axis. The cavity center
section is filled with dielectric of relative permittivity 2.85, as determined by experiments performed in Chapter 4 and as used in the treatment of Chapter 5. Resonance frequencies of this configuration are predicted.

6.2.2 Mode expansions

In the cavity shown in Figure 6.1, a field for which the transverse electric component exhibits either even or odd axial symmetry is assumed. This means effectively that a magnetic or electric short circuit wall can be inserted across the cavity at \( z = \ell/2 \), and the fields found only on one side of the wall.

This leads to a new cavity problem, as shown in Figure 6.2.

The transverse field in the dielectric section (with the same scaling as in Chapter 2) is expanded in the dielectric-filled circular guide modes and takes the form

\[
\begin{align*}
E^b_x &= \sum_i V_i^b \, E_i^b \quad (6 - 1) \\
H^b_z &= \sum_i I_i^b \, H_i^b .\quad (6 - 2)
\end{align*}
\]

This notation makes no distinction between TE and TM modes. The dielectric mode vectors, \( E_i^b \) and \( H_i^b \) are given as

\[
\begin{align*}
E_i^b &= -j \frac{n}{r} \, C_i \, J_i(\kappa_i \, r) \, e^{j n \phi \hat{r}} \\
&\quad + \kappa_i \, C_i \, J_i'(\kappa_i \, r) \, e^{j n \phi \hat{\rho}} \quad (6 - 3) \\
H_i^b &= -\kappa_i \, C_i \, J_i'(\kappa_i \, r) \, e^{j n \phi \hat{r}} \\
&\quad - j \frac{n}{r} \, C_i \, J_i(\kappa_i \, r) \, e^{j n \phi \hat{\rho}} \quad (6 - 4)
\end{align*}
\]
FIGURE 6.1
THE END-LOADED CAVITY WITH MAGNETIZED FERRITE
Figure 6.2

The Half-Cavity
\[ E_i^p = + k_{ci} C_i J_i'(k_{ci} r) e^{jn\phi} \hat{r} + j \frac{n}{r} C_i J_i(k_{ci} r) e^{jn\phi} \hat{\phi} \quad (6 - 5) \]
\[ h_i^b = - j \frac{n}{r} C_i J_i'(k_{ci} r) e^{jn\phi} \hat{r} + k_{ci} C_i J_i'(k_{ci} r) e^{jn\phi} \hat{\phi} \quad (6 - 6) \]

where the value of \( n \) is restricted to +1 or -1 to correspond to the theory of ferrite-filled guide in Chapter 3. \( V_i^b \) and \( I_i^b \) take the general forms
\[ V_i^b(z) = V_i^{b+} e^{-j\beta_{bi}z} + V_i^{b-} e^{j\beta_{bi}z} \quad (6 - 7) \]
\[ I_i^b(z) = Y_i V_i^{b+} e^{-j\beta_{bi}z} - Y_i V_i^{b-} e^{j\beta_{bi}z} \quad (6 - 8) \]

The characteristic admittance \( Y_i \), the propagation constant \( \beta_{bi} \), the coefficient \( C_i \) and the mode number \( k_{ci} \) are given in Chapter 2.

It is assumed that the transverse field in the ferrite section is also expandable in a set of basis modes, and that the ferrite modes of Chapter 3 will serve as such a set. The form of this expansion is
\[ E_t^F = \sum_k V_k^F E_k \quad (6 - 9) \]
\[ H_t^F = \sum_k I_k^F H_k \quad (6 - 10) \]

where the ferrite mode vectors are given by
\[ E_k^F = j F_{1k} \hat{r} + F_{2k} \hat{\phi} \quad (6 - 11) \]
\[ H_k^F = F_{3k} \hat{r} + j F_{4k} \hat{\phi} \quad (6 - 12) \]

and the \( z \)-dependent quantities \( V_k^F \) and \( I_k^F \) have the general form
\[ V_k^F(z) = V_k^{F+} e^{-j\beta_{fk}z} + V_k^{F-} e^{j\beta_{fk}z} \quad (6 - 13) \]
\[ I_k^F(z) = V_k^{F+} e^{-j\beta_{fk}z} - V_k^{F-} e^{j\beta_{fk}z} \quad (6 - 14) \]
\( F_{ik}, \quad F_{xk}, \quad F_{sk}, \quad \text{and } F_{zk} \) are defined in Chapter 3 by equations (3-67) through (3-74), and \( \beta_{Fk} \) is the ferrite propagation constant.

For each dielectric basis mode, either the transverse electric field or the transverse magnetic field is set to zero at \( z = L/2 \). This requirement allows one of the two arbitrary constants \( V_{i}^{B+} \) and \( V_{i}^{B-} \) to be determined in terms of the other. The resulting forms for \( V_{i}^{B}(z) \) and \( I_{i}^{B}(z) \) depend not only on the choice of axial symmetry but also on whether or not the dielectric mode is propagating:

**Odd Solution, Propagating**

\[
V_{i}^{B}(z) = -jz e^{-j\beta_{Di}z/L} \sin \beta_{Di} (z - L/2) \quad V_{i}^{B+} \tag{6-15}
\]

\[
I_{i}^{B}(z) = +z e^{-j\beta_{Di}z/L} \cos \beta_{Di} (z - L/2) \quad V_{i}^{B+} \tag{6-16}
\]

**Even Solution, Propagating**

\[
V_{i}^{B}(z) = +z e^{-j\beta_{Di}z/L} \cos \beta_{Di} (z - L/2) \quad V_{i}^{B+} \tag{6-17}
\]

\[
I_{i}^{B}(z) = -jz e^{-j\beta_{Di}z/L} \sin \beta_{Di} (z - L/2) \quad V_{i}^{B+} \tag{6-18}
\]

**Odd Solution, Evanescent**

\[
V_{i}^{B}(z) = -z e^{-\alpha_{Di}z/L} \sinh \alpha_{Di} (z - L/2) \quad V_{i}^{B+} \tag{6-19}
\]

\[
I_{i}^{B}(z) = +z e^{-\alpha_{Di}z/L} \cosh \alpha_{Di} (z - L/2) \quad V_{i}^{B+} \tag{6-20}
\]

**Even Solution, Evanescent**

\[
V_{i}^{B}(z) = +z e^{-\alpha_{Di}z/L} \cosh \alpha_{Di} (z - L/2) \quad V_{i}^{B+} \tag{6-21}
\]

\[
I_{i}^{B}(z) = -z e^{-\alpha_{Di}z/L} \sinh \alpha_{Di} (z - L/2) \quad V_{i}^{B+} \tag{6-22}
\]
In the evanescent cases, one has \( Y_i = -j \alpha_{b_i} \) for TE modes
and \( Y_i = j \varepsilon_{e} / \alpha_{b_i} \) for TM modes.

For each ferrite basis mode it is required that \( E_t \) be zero at \( z = 0 \),
and equations (6-13) and (6-14) become

**Propagating in the Ferrite**

\[
\begin{align*}
V_k^F(\xi) &= -j 2 \sin \beta_{f_k} \varepsilon_{f_k} V_k^{F*} \\
I_k^F(\xi) &= + 2 \cos \beta_{f_k} \varepsilon_{f_k} V_k^{F*}
\end{align*}
\]  \( (6-23) \)

**Evanescent in the Ferrite**

\[
\begin{align*}
V_k^F(\xi) &= - 2 \sin \alpha_{f_k} \varepsilon_{f_k} V_k^{F*} \\
I_k^F(\xi) &= + 2 \cos \alpha_{f_k} \varepsilon_{f_k} V_k^{F*}
\end{align*}
\]  \( (6-25) \)

6.2.3 The ferrite-dielectric interface

For any cavity field solution, the transverse parts of the electric and magnetic fields must be continuous at the interface \( z = t \) between magnetized ferrite and dielectric. This condition can be expressed as

\[
\begin{align*}
E_t^D(\xi = t) &= E_t^F(\xi = t) \\
H_t^D(\xi = t) &= H_t^F(\xi = t)
\end{align*}
\]  \( (6-27) \)

When this continuity condition is written in terms of the field expansion in each region, it becomes

\[
\begin{align*}
\sum_{l} V_l^b(t) e_{l_i}^b &= \sum_{k} V_k^F(t) e_{k_i}^F \\
\sum_{l} I_l^b(t) h_{l_i}^b &= \sum_{k} I_k^F(t) h_{k_i}^F
\end{align*}
\]  \( (6-29) \)
Examination of the expressions for the dielectric mode vectors, equations (6-3) through (6-6), and for the ferrite mode vectors, equations (3-67) through (3-74), reveals that the functional dependences on \( r \) for the two cases are quite different. If an infinite number of terms is included in the mode expansion for each region, it is assumed that for certain frequencies there are sets of mode coefficients \( V^b_\lambda^+ \) and \( V^f_\lambda^+ \) that cause conditions (6-29) and (6-30) to be satisfied everywhere on the interface. It is further assumed that the continuity conditions on transverse \( E \) and \( H \) can be satisfied to any degree of accuracy by retaining a sufficiently large but finite number of these terms.

Making use of the dielectric mode vector orthonormality property, as defined for \( n = \pm 1 \) in equations (2-53) through (2-59), and operating on both sides of (6-29) and (6-30), one obtains

\[
V_i^b(t) = \sum_k V_k^b(t) \int \mathbf{e}_i^b \cdot \mathbf{e}_k^b \, ds \quad (6-31)
\]

\[
I_i^b(t) = \sum_k I_k^b(t) \int \mathbf{h}_i^b \cdot \mathbf{h}_k^b \, ds \quad (6-32)
\]

where the integrals are over the circular waveguide cross-section. These equations are then rewritten in such a way that the mode coefficients appear explicitly:

\[
A_i V_i^{b+} + \sum_k C_{ik} V_k^{f+} = 0 \quad (6-33)
\]

\[
B_i V_i^{b+} + \sum_k C_{ik} V_k^{f+} = 0 \quad (6-34)
\]

where the \( A, B, C \) and \( D \) coefficients are given by
Propagating in the ferrite
\[
C_{ik} = + j \sin \beta_{Fk} t \int \mathbf{E}_i^* \cdot \mathbf{E}_k^F \, ds \tag{6-35}
\]
\[
D_{ik} = - \cos \beta_{Fk} t \int \mathbf{H}_i^* \cdot \mathbf{H}_k^F \, ds \tag{6-36}
\]

Evanescent in the ferrite
\[
C_{ik} = + \sinh \alpha_{Fk} t \int \mathbf{E}_i^* \cdot \mathbf{E}_k^F \, ds \tag{6-37}
\]
\[
D_{ik} = - \cosh \alpha_{Fk} t \int \mathbf{H}_i^* \cdot \mathbf{H}_k^F \, ds \tag{6-38}
\]

Even mode, propagating in the dielectric
\[
A_i = e^{-j\beta_{di} \ell z} \cos \beta_{di} (\ell z - t) \tag{6-39}
\]
\[
B_i = j e^{-j\beta_{di} \ell z} \sin \beta_{di} (\ell z - t) \tag{6-40}
\]

Even mode, evanescent in the dielectric
\[
A_i = e^{-\alpha_{di} \ell z} \cosh \alpha_{di} (\ell z - t) \tag{6-41}
\]
\[
B_i = e^{-\alpha_{di} \ell z} \sinh \alpha_{di} (\ell z - t) \tag{6-42}
\]

Odd mode, propagating in the dielectric
\[
A_i = j e^{-j\beta_{di} \ell z} \sin \beta_{di} (\ell z - t) \tag{6-43}
\]
\[
B_i = e^{-j\beta_{di} \ell z} \cos \beta_{di} (\ell z - t) \tag{6-44}
\]

Odd mode, evanescent in the dielectric
\[
A_i = e^{-\alpha_{di} \ell z} \sinh \alpha_{di} (\ell z - t) \tag{6-45}
\]
\[
B_i = e^{-\alpha_{di} \ell z} \cosh \alpha_{di} (\ell z - t) \tag{6-46}
\]
A pair of equations like (6-33) and (6-34) can be written for each dielectric basis mode. The number of equations is equal to the number of unknown mode coefficients if the finite number of basis modes in each region is the same, and the result will be a set of linear, simultaneous equations for these unknowns. This set of equations is homogeneous, so that its solution for the mode coefficients will be the null one unless the A, B, C and D coefficients satisfy a special condition called the cavity characteristic equation. This condition is obtained by setting the determinant of the equation set to zero. Frequencies at which non-zero solutions for the mode coefficients occur are the cavity resonance frequencies.

It is recalled that the number of basis modes retained in the expansions is assumed sufficient to allow accurate satisfaction of the interface continuity requirement. For each resonance frequency found as a solution to the characteristic equation, there is a corresponding set of dielectric and ferrite mode coefficients. To find the field associated with the corresponding cavity mode, these are substituted into the field expansions (equations 6-1) and (6-2).

In the limiting case of very large d.c. magnetic fields the ferrite becomes isotropic, its permeability taking on the form corresponding to zero saturation magnetization. In agreement with this it can be shown that in the same limiting case the ferrite basis modes become the usual TE and TM modes of circular waveguide filled with a dielectric having the ferrite permittivity. This implies that predicted modes and resonance frequencies for the magnetized cavity reduce to those for the unmagnetized cavity as the d.c. magnetic field grows sufficiently large. Since each mode for the unmagnetized cavity is associated with only one dielectric waveguide mode, and the same is true for each ferrite basis mode, only one mode
coefficient for each region will remain non-zero for a given cavity resonance in the limiting situation.

In order to reduce the scale of the problem, it is desirable to retain as few basis modes as possible. Predictions for the unmagnetized cavity arrived at by retaining only a small number of basis modes in the field expansions are not generally expected to be accurate for any of the cavity modes, but in the infinite d.c. field limit some of the unmagnetized cavity resonances will be accurately predicted. These resonance modes are associated with dielectric-filled guide modes which correspond to the ferrite basis modes retained. If a given unmagnetized cavity resonance is predicted both by using the accurate field expansions and by using much shorter expansions, agreement between the two approaches should deteriorate smoothly as the d.c. magnetic field is reduced, implying that the approximate predictions are expected to compare well with the accurate expansions over some range of d.c. field and for some cavity modes. These magnetized cavity modes would be those which reduce to the unmagnetized cavity modes predicted in the limit by the less accurate approach.

6.3 Predicted Resonance Frequencies

6.3.1 The single basis mode approximation

Resonance frequency predictions will be made for the case of only one basis mode in the field expansion for each region. The ferrite basis mode chosen is the TE_{11} - limit mode, i.e. that which becomes the TE_{11} circular guide mode in
the limit of infinite d.c. magnetic field. It is expected that internal bias fields in the experimental investigation will be of the order of 400 oersted, which corresponds to \(|\sigma| < \sigma_0\) for the ferrite of Chapter 3 and for frequencies in the 8 - 12 GHz range. The theoretical bias field is restricted in such a way that the solution curve for the ferrite basis mode remains isolated on the solution plot for the entire frequency range being investigated (see Section 6.3.2) and always corresponds to a propagating mode. The basis mode chosen for the dielectric section is the TE_{11} circular waveguide mode. This mode does not propagate over the entire 8 - 12 GHz range in the dielectric section, and predictions have been restricted to the part of the band above the cutoff frequency, which is 8.87 GHz.

Based on the discussion in Section 6.2.3, it is seen that the single basis mode expansions would lead in the infinite d.c. field limit to accurate predictions for unmagnetized cavity resonances corresponding to TE_{11} circular waveguide modes. From this it is concluded that the predicted resonance frequencies should be compared with those for observed resonances known to be associated with TE_{11} cavity modes for the unmagnetized ferrite.

Both even and odd symmetry cavity modes are considered. The equations for the mode coefficients (6-33) and (6-34) are rewritten for the case of one basis mode in each region:

\[
A V^{D^+} + C V^{F^+} = 0 \quad (6-47)
\]

\[
B V^{D^+} + D V^{F^+} = 0 \quad (6-48)
\]

Here the subscripts of A, B, C, D and the mode coefficients have been omitted, since there is no need to distinguish between basis modes in a given region. The characteristic equation for the special case becomes
\[ A \cdot D - B \cdot C = 0, \quad (6-49) \]

where

\[ C = + j \sin \beta_0 t \int e^{b} \cdot e^F ds \quad (6-50) \]
\[ D = - \cos \beta_0 t \int h^b \cdot h^F ds, \quad (6-51) \]

and for

**Even cavity modes**

\[ A = e^{-j\beta_0 l/z} \cos \beta_0 (l/z - t) \quad (6-52) \]
\[ B = j \beta_0 e^{-j\beta_0 l/z} \sin \beta_0 (l/z - t) \quad (6-53) \]

**Odd cavity modes**

\[ A = j e^{-j\beta_0 l/z} \sin \beta_0 (l/z - t) \quad (6-54) \]
\[ B = \beta_0 e^{-j\beta_0 l/z} \cos \beta_0 (l/z - t) \cdot (6-55) \]

Written in terms of (6-50) through (6-55), equation (6-49) becomes

**Even cavity modes**

\[ - \cos \beta_0 (l/z - t) \cos \beta_0 t \int h^b \cdot h^F ds \]
\[ + \beta_0 \sin \beta_0 (l/z - t) \sin \beta_0 t \int e^{b} \cdot e^F ds = 0 \quad (6-56) \]

**Odd cavity modes**

\[ - \sin \beta_0 (l/z - t) \cos \beta_0 t \int h^b \cdot h^F ds \]
\[ - \beta_0 \cos \beta_0 (l/z - t) \sin \beta_0 t \int e^{b} \cdot e^F ds = 0 \cdot (6-57) \]

In the two forms of the characteristic equation, (6-56) and (6-57), the integrands do not depend on the variable \( \phi \), and the factors \( 2\pi \) resulting from the integration over \( \phi \) can be removed. The integrands do not depend on assumed cavity mode symmetry, and the following definitions holding for both
symmetries are made:

\[
I_1 = \frac{1}{C} \int_0^{R_c} e^b_r \cdot e^F_r \, r \, dr \quad (6 - 58)
\]

\[
I_2 = \frac{1}{C} \int_0^{R_c} h^b_r \cdot h^F_r \, r \, dr \quad (6 - 59)
\]

The two forms of the characteristic equation become

**Even cavity modes**

\[
- \cos \beta_p (\ell z - t) \cos \beta_p t \, I_2
+ \beta_p \sin \beta_p (\ell z - t) \sin \beta_p t \, I_1 = 0 \quad (6 - 60)
\]

**Odd cavity modes**

\[
- \sin \beta_p (\ell z - t) \cos \beta_p t \, I_2
- \beta_p \cos \beta_p (\ell z - t) \sin \beta_p t \, I_1 = 0 \quad (6 - 61)
\]

where the factor \( C \) has been divided out of the equations. The dielectric basis mode vectors are given according to Chapter 2 as

\[
e^b_r = -j \frac{n}{r} C J_1(k_c r) \quad (6 - 62)
\]

\[
e^b_\phi = +k_c C J_1'(k_c r) \quad (6 - 63)
\]

\[
h^b_r = - \frac{n}{r} C J_1'(k_c r) \quad (6 - 64)
\]

\[
h^b_\phi = -j k_c C J_1(k_c r) \quad (6 - 65)
\]

while the ferrite basis mode vectors are given according to Chapter 3 as

\[
e^F_r = j F_1 \quad (6 - 66)
\]

\[
e^F_\phi = F_2 \quad (6 - 67)
\]

\[
h^F_r = F_3 \quad (6 - 68)
\]

\[
h^F_\phi = j F_4 \quad (6 - 69)
\]
where the factor \( \exp(-j\eta \phi) \) has been omitted in both sets of components.

Using equations (6-62) through (6-69), the integrals in (6-58) and (6-59) become

\[
I_1 = \int_0^\infty \left[ -\frac{n}{r} J_1(k_r r) F_1 + k_c J_1'(k_r r) F_2 \right] rdr \quad (6-70)
\]

\[
I_2 = \int_0^\infty \left[ -k_c J_1'(k_r r) F_3 - \frac{n}{r} J_1(k_r r) F_4 \right] rdr \quad (6-71)
\]

6.3.2 The computer treatment

The characteristic equations (6-60) and (6-61) for even and odd cavity mode symmetries must be solved for the predicted cavity resonance frequencies. This solution has been implemented by computer for the cases \( n = +1 \) and \( n = -1 \).

The left-hand sides of (6-60) and (6-61) are evaluated at successive frequencies over the desired range, with zeros being located by noting the occurrence of sign changes. Such evaluations at a given frequency can be separated into the following steps:

a) evaluate \( \beta_n \) for \( n = +1 \) and \( n = -1 \).

b) evaluate \( I_1 \) and \( I_2 \)

c) evaluate the characteristic quantities.

a) Evaluation of \( \beta_n \)

The particular method by which the ferrite propagation constants are found for the two values of \( n \) is based on various characteristics of the special case. These characteristics are best summarized by referring to the six solution plots which were originally given in Chapter 3 and are repeated in Figures 6.3 through 6.8. In these figures the solution curve for the ferrite basis mode is in the lower left part of the
Figure 6.3: The Ferrite Basis Mode and Restricted σ-range—From Figure 3.7
THE FERRITE BASIS MODE AND RESTRICTED $\sigma$-RANGE — FROM FIGURE 3.8
FIGURE 6.8: THE FERRITE BASIS MODE AND RESTRICTED \( \sigma \)-RANGE — FROM FIGURE 3.10
plot and has been designated by an arrow. The range of \( \sigma \) for which the special requirements on the ferrite mode are satisfied is also indicated. Predictions are made for internal d.c. magnetic fields of 345, 390 and 435 oersted, all of which correspond to values of \( \sigma \) within the allowed range.

To begin the determination of \( \beta_+ \) for \( n = +1 \) (\( \beta_+ \)) and \( n = -1 \) (\( \beta_- \)), the parameters \( \beta_0 \), \( r_0 \), \( \sigma \), and \( p \) must be evaluated for the present frequency. They are given by

\[
\beta_0 = \omega (\epsilon F \mu_0)^{1/2} \tag{6 - 72}
\]

\[
r_0 = (0.00587 \text{ meters}) x \beta_0 \tag{6 - 75}
\]

\[
\sigma = 1.4 x 2.55 x H_{bc,\text{int}}(\text{oer}) / (f(\text{GHz}) \times 1000) \tag{6 - 74}
\]

\[
p = 1.4 x 2.55 x 1800 / (f(\text{GHz}) \times 1000) \tag{6 - 75}
\]

The propagation constants are found by searching along the appropriate constant-\( \sigma \) line for the desired solution curve. For \( n = +1 \) this search takes place in the first quadrant of the \( \lambda, \sigma \) plane and is carried out by scanning through values of \( \lambda_1 \). As seen in Figures 6.3, 6.4 and 6.5, the desired solution curve lies between the \( I_B' \) and \( C_0' \) lines for the entire 3 - 12 GHz frequency range. The scan of \( \lambda_1 \) will begin at the \( I_B' \) line, which is defined by \( \lambda_1 = (\sigma + p + i) / (1 + i) \), and will continue until the solution curve has been passed. At each value of \( \lambda_1 \), the functions

\[
S_1 = \frac{F(\chi_1 r_0) / \lambda_1 - i}{\chi_1^2} \tag{6 - 76}
\]

\[
S_2 = \frac{F(\chi_2 r_0) / \lambda_2 - i}{\chi_2^2} \tag{6 - 77}
\]

are evaluated, where \( r_0, \sigma \) and \( p \) have been given in equations (6-72) through (6-75), and
\[ \lambda_2 = \frac{\sigma + p - \lambda_1}{1 - \sigma \lambda_1} \]  \hspace{.5cm} (6 - 78)

\[ F(x) = x \frac{J_1'(x)}{J_1(x)} \text{ for real } x \]  \hspace{.5cm} (6 - 79)

\[ F(x) = |x| \frac{I_1'(|x|)}{I_1(|x|)} \text{ for imaginary } x \]  \hspace{.5cm} (6 - 79)

\[ \chi_{1, z} = \left[ \frac{1 - \lambda^2_{1, z}}{1 - \sigma \lambda_{1, z}} \right]^{1/2}. \]  \hspace{.5cm} (6 - 80)

The difference \( \Delta G = G_1 - G_2 \) is then determined. When the sign of \( \Delta G \) for two consecutive values of \( \lambda_1 \) is different, it is known that the condition \( G_1 = G_2 \) has occurred between the two values, and the intersection with the solution curve has been located. An approximate value of \( \lambda_1 \) for this intersection, denoted by \( \lambda_1^+ \), is known and the corresponding value of \( \lambda_2 \) denoted by \( \lambda_2^+ \) and determined by the relationship (6-78), can be used to find the propagation constant according to

\[ (\beta^+_{p})^2 = -\lambda_1^+ \lambda_2^+. \]  \hspace{.5cm} (6 - 81)

Determination of \( \beta^-_p \) is accomplished by a constant \( -\sigma \) search in the third quadrant of the \( \lambda, \sigma \) plane. The following conventions are adopted:

\[ \sigma \quad \rightarrow \quad \sigma^+ = -\sigma \]  \hspace{.5cm} (6 - 82)

\[ p \quad \rightarrow \quad p^- = -p \]  \hspace{.5cm} (6 - 83)

Examination of Figures 6.6, 6.7 and 6.8 reveals that the solution curve of interest will intersect a constant \( \frac{\sigma}{\sigma^+} \) line directly to the left of either the \( \Im_\alpha \) line or the \( \Re \) line, depending on which of these lines intersects the constant \( \frac{\sigma}{\sigma^+} \) line farthest to the right. There is a frequency below which the \( \Im \) curve does not exist in the region of interest and the scan of \( \lambda_2 \) simply begins at the \( \Im_\delta \) line
and proceeds to the left. This frequency is given by the condition \( r_0 = x_{ii} \), where \( x_{ii} \) is the first zero of \( J_i(x) \). For greater frequencies, the intersection of \( I_i \) and the constant-\( \lambda \) line is determined by substituting the value of \( \lambda_1 \) given by

\[
\hat{\lambda}_1 = \frac{1}{2} \hat{\sigma} \left( \frac{x_{ii}}{r_0} \right)^2 + \frac{1}{2} \left[ \hat{\sigma}^2 \left( \frac{x_{ii}}{r_0} \right)^4 + 4 \left( \frac{x_{ii}}{r_0} \right)^2 \right]^{1/2} \tag{6-84}
\]

into the equation

\[
\hat{\lambda}_2 = \left( \hat{\sigma} + \hat{p} - \hat{\lambda}_1 \right) / (1 - \hat{\sigma} \hat{\lambda}_1), \tag{6-85}
\]

which gives \( \hat{\lambda}_2 \) in terms of \( \hat{\lambda}_1 \). The intersection of the \( I_i' \) line with the constant-\( \sigma \) line always occurs at \( \hat{\lambda}_2 = -1 \). The scan of \( \hat{\lambda}_2 \) values will then begin at the most negative of these and proceed to the left in the direction of less negative values. Again, the functions \( \hat{G}_i \) and \( \hat{G}_2 \) given by

\[
\hat{G}_i = \left( F(\hat{\lambda}_i r_0)/\hat{\lambda}_i - 1 \right) / \hat{\lambda}_i^2 \tag{6-86}
\]

\[
\hat{G}_2 = \left( F(\hat{\lambda}_2 r_0)/\hat{\lambda}_2 - 1 \right) / \hat{\lambda}_2^2 \tag{6-87}
\]

where

\[
\hat{\lambda}_i = \left( \hat{\sigma} + \hat{p} - \hat{\lambda}_2 \right) / (1 - \hat{\sigma} \hat{\lambda}_2) \tag{6-88}
\]

\[
F(x) = \left\{ \begin{array}{ll}
J_i(\hat{\lambda}_i x) / J_i(x) & \text{for real } x \\
|x| J_i'(|x|) / I_i(|x|) & \text{for imaginary } x
\end{array} \right. \tag{6-89}
\]

\[
\hat{\lambda}_{i,2} = \left[ (1 - \hat{\lambda}_{i,2}) / (1 - \hat{\sigma} \hat{\lambda}_{i,2}) \right]^{1/2}, \tag{6-90}
\]

are evaluated at successive values of \( \hat{\lambda}_2 \) until a zero of the function \( \Delta \hat{G} = \hat{G}_i - \hat{G}_2 \) has occurred. An approximate value of \( \hat{\lambda}_2 \) for the solution curve intersection is then known, as well as the corresponding value of \( \hat{\lambda}_i \). The corresponding \( \lambda_i \) and \( \lambda_2 \) for \( \sigma \) and \( p \) are given by
\[ \lambda^{-}_{1} = - \hat{\lambda}^{-}_{k} \]  
\[ \lambda^{-}_{2} = - \hat{\lambda}^{-}_{1} \]  
\hspace{1cm} (C - 92) \hspace{1cm} (C - 93)

according to equations (3-114) and (3-115), and the value for the \( n = -1 \)

propagation constant is given by

\[ (\beta^{-}_{F})^2 = (\lambda^{-}_{1} \lambda^{-}_{2}) \]  \hspace{1cm} (C - 94)

b) Evaluation of \( I_{1} \) and \( I_{2} \)

\( I_{1} \) and \( I_{2} \) are found by numerical integration. The interval \([0, r_{0}]\), where \( r_{0} \) is frequency dependent, is divided into \( N \) intervals defined by the intermediate points

\[ r_{1} = 0, \ r_{2} = r_{0}/N, \ \ldots, \ r_{N} = (N-1)r_{0}/N, \ r_{N+1} = r_{0}. \]  \hspace{1cm} (C - 95)

The integrands of \( I_{1} \) and \( I_{2} \),

\[ S_{1} (r) \equiv -n \ J_{1} (k_{c} r) \ F_{1} + k_{c} r \ J_{1} ' (k_{c} r) \ F_{2} \]  \hspace{1cm} (C - 96)

\[ S_{2} (r) \equiv -k_{c} r \ J_{1} ' (k_{c} r) \ F_{3} - n \ J_{1} (k_{c} r) \ F_{4} \]  \hspace{1cm} (C - 97)

are evaluated at each \( r_{n} \), the corresponding values becoming \( S_{1} (r_{n}) \) and \( S_{2} (r_{n}) \).

The integrals are then approximated by the summations

\[ I_{1} \approx \Delta r \ \sum_{n=1}^{N} \left[ \frac{S_{1} (r_{n}) + S_{1} (r_{n-1})}{2} \right] \]  \hspace{1cm} (C - 98)

\[ I_{2} \approx \Delta r \ \sum_{n=1}^{N} \left[ \frac{S_{2} (r_{n}) + S_{2} (r_{n-1})}{2} \right] \]  \hspace{1cm} (C - 99)
The evaluations of $S_1$ and $S_2$ must be carried out for both signs of $n$. Referring to equations (3-67) through (3-74), the expressions for $F_1$, $F_2$, $F_3$ and $F_4$ are seen to involve explicitly the quantities $n$, $\lambda_1$, $\lambda_2$, $\chi_1$, $\chi_2$, $r_o$ and $r$. The scaled quantities $r$ and $r_o$ do not depend on the sign of $n$. The remaining quantities do, however, and their values for the two cases have already been found. Thus, for $n = +1$, the values are $\lambda_1 = \lambda_1^+$, $\lambda_2 = \lambda_2^+$, $\chi_1 = \chi_1^+$, $\chi_2 = \chi_2^+$, while at $n = -1$, they are $\lambda_1 = \lambda_1^-$, $\lambda_2 = \lambda_2^-$, $\chi_1 = \chi_1^-$, and $\chi_2 = \chi_2^-$. The Bessel functions appearing in the $F$'s are generated for specific arguments by algorithms in the computer program. The resulting values for the integrals become $I_1^+, I_2^+$ and $I_1^-, I_2^-$. 

c) Evaluation of the characteristic quantity

The characteristic quantities for odd and even symmetry are given in equations (6-60) and (6-61). Each of these must be evaluated at the present frequency for $n = +1$ and $n = -1$. The quantities $t$, $z$, and $\beta_o$ are first found according to

$$L = .02286 \text{ meters } \times \beta_o \quad (G - 100)$$

$$t = (\text{actual } t \text{ in meters}) \times \beta_o \quad (G - 101)$$

$$\beta_o = (\varepsilon_R - k_e^2)^{1/2} \quad (G - 102)$$

$$\varepsilon_R = \varepsilon_R/\varepsilon_0 \quad (G - 103)$$

The ferrite propagation constants for $n = +1$ and $n = -1$ have already been determined and are denoted by $\beta_p^+$ and $\beta_p^-$. Similarly, $I_1$ and $I_2$ have been found and are given by $I_1^+$, $I_2^+$ for $n = 1$ and $I_1^-$, $I_2^-$ for $n = -1$. The cavity characteristic quantity for the various cases can then be evaluated.

The listing for a computer program used in carrying out the procedure
just described for predicting resonance frequencies for the special case is given in Appendix 6.
6.3.3 Discussion of predictions

Resonance frequencies for the magnetized end-loaded cavity predicted with the aid of the computer program listed in Appendix 6 are given in Table 6.1. As described in the previous sections, these predictions are based on the single-basis-mode assumption and are affected by various restrictions, such as the requirement that the dielectric mode be propagating. Since cutoff frequency for the TE_{11} circular waveguide mode in the dielectric filled guide in 8.87 GHz, frequencies examined by the computer program are restricted to the range 8.87 to 12.00 GHz. Although only disk thicknesses of .4, .6 and .8 cm. will be examined experimentally, the entire range of thicknesses from .1 cm to 1.0 cm in steps of .1 cm has been included in the predictions. The table is separated into halves according to the two signs for n. Each of these portions is broken down according to mode symmetry, and finally according to internal d.c. magnetic field.

The predictions will be affected by several choices which are made in carrying out the computer algorithm. In scanning over the frequencies at which the cavity characteristic quantity is evaluated, a frequency increment of 200 MHz was used, and the values entered in the table are the center frequencies of intervals in which zeros of the characteristic quantity occurred. Results are also affected by the number of terms used in the numerical integration and by the accuracy to which the various Bessel functions are evaluated by subprograms.

Figures 6.9 and 6.10 summarize the information in Table 6.1. Here, the information has first been separated according to mode symmetry, Figure 6.9 containing the even mode predictions, Figure 6.10 the odd. Examination of the table of predicted resonance frequencies reveals that d.c. magnetic field has little
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**Table 6.1** Predicted Resonance Frequencies, GHz.
Figure 6.9 Predicted Resonance Frequencies, Even Modes
effect on predicted resonance frequency for the range of field considered. Therefore, in constructing the plots only data for 390 oersted has been represented. The frequency range is indicated to be 8 - 12.4 GHz, with the area to the left representing the restriction due to $\text{TE}_{11}$ cutoff and that on the right reflecting the fact that the resonance frequency search was not extended above the frequency of the final solution plot. The 200 MHz frequency intervals in which resonances were predicted are marked "+" or "-" according to the sign of $n$.

Examination of the table and plots indicates a general downward trend in resonance frequency as disk thickness is increased, with cavity modes passing out of the allowed frequency range at the low end and others appearing at the high end. It is noted that Figures 6.9 and 6.10 emphasize this dependence on disk thickness, while the measured plots discussed later emphasize changes with d.c. field. As ferrite disk thickness approaches zero, the cavity resonances should become just those of a circular waveguide cavity filled with isotropic dielectric. This is seen to occur in Figure 6.9, when the frequencies of two predicted cavity modes for $n = +1$ and $n = -1$ at the left of the plot are apparently nearing 9.7 GHz, the resonance frequency for the lowest order $\text{TE}_{11}$ mode for the dielectric-filled cavity.

6.4 Measurement of Resonance Frequencies

6.4.1 The experimental approach

In the experimental work carried out here as a check on the magnetized cavity theory, the arrangement and procedure are identical to that described in
Chapter 4 except for the application of the d.c. magnetic field. This is accomplished by placing the cavity fixture between the pole pieces of an electromagnet while resonances are being sought. The fixture must be aligned with its axis along the applied magnetic field. The magnet is used in two configurations. Observations are first made with the cavity end plates in place, so that the coupling hole is centered and its position fixed. This leads to observations of even modes only. This situation uses the electromagnet in a standard configuration, for which a calibration is carried out. In order to consider magnetized cavity modes having odd symmetry it is necessary to make use of the end plugs, but when in place these do not allow the fixture to be placed between the magnet pole pieces. The magnet was then modified to allow proper positioning of the cavity and to allow effective shifting of the coupling hole with the cavity in place. A second magnet calibration was carried out. For the centered cavity coupling hole and the first magnet configuration experimental absorption plots were obtained, while for the odd cavity modes only oscilloscope observations were made.

6.4.2 The internal d.c. magnetic field

The d.c. magnetic field appearing in the theory of ferrite-filled guide and the theory of the magnetized end-loaded cavity is the total d.c. field existing within the ferrite. This field, called the internal field, and the ferrite magnetization are assumed to be uniform. The electromagnet calibration, however, relates the winding current to the bias field without the ferrite in place. Therefore, a relationship between this field, which will be called the applied field, and the
internal field must be found. The internal field is due to the combined effect of the applied d.c. field and the demagnetizing field. For the present disk-shaped samples the demagnetizing field will not be uniform, even if the magnetization in the sample is assumed to be so.

It will be assumed for the sake of simplification that the magnetization is uniform. Furthermore, in order to arrive at a uniform demagnetizing field from which a uniform internal field can be derived, each disk shape will be approximated by an ellipsoid. For an ellipsoid, uniform magnetization leads to a uniform demagnetizing field, and if the magnetization lies along a principal axis, the demagnetizing field lies along the same axis and is given by the MKS relationship

\[ H_{dem} = -N \frac{M_{SAT}}{\mu_0}, \quad (C -104) \]

where \( N \) is the demagnetizing factor for the given axis. Since the magnetization and demagnetizing field are to be given in Gaussian units, this relationship must be transformed. Making use of the identities

\[ \frac{M_{SAT}}{\mu_0} = 10^3 \times \left( \frac{M_{SAT}}{\mu_0} \right)_G \quad (C -105) \]

\[ (H_{dem})_G = 4\pi \times 10^{-3} x H_{dem}, \quad (C -106) \]

one obtains

\[ (H_{dem})_G = -N \left( 4\pi \left( \frac{M_{SAT}}{\mu_0} \right)_G \right). \quad (C -107) \]

If each ferrite disk is now associated with an ellipsoidal shape, where the minor axis lies along the axis of the disk, and the remaining two principal axes are equal in length, the ratios of minor axis to major axis for the three thicknesses are
\[ t = 0.4 \text{ cm} \rightarrow 0.341 \quad (G-108) \]
\[ t = 0.6 \text{ cm} \rightarrow 0.511 \quad (G-109) \]
\[ t = 0.8 \text{ cm} \rightarrow 0.681 \quad (G-110) \]

The corresponding values of \( N \) are
\[ t = 0.4 \text{ cm} : N = 0.63 \quad (G-111) \]
\[ t = 0.6 \text{ cm} : N = 0.54 \quad (G-112) \]
\[ t = 0.8 \text{ cm} : N = 0.41 \quad (G-113) \]

Based on equation (6-107), with \( 4\pi (M_{\text{sat}}) G = 1800 \), these correspond to demagnetizing fields of
\[ t = 0.4 \text{ cm} : (H_{\text{dem}}) G = -1134 \text{ oe.} \quad (G-114) \]
\[ t = 0.6 \text{ cm} : (H_{\text{dem}}) G = -972 \text{ oe.} \quad (G-115) \]
\[ t = 0.8 \text{ cm} : (H_{\text{dem}}) G = -738 \text{ oe.} \quad (G-116) \]

The internal d.c. magnetic field for each disk thickness can now be found according to
\[ (H_{\text{int, d.c.}}) G = (H_{\text{app, d.c.}}) G - (H_{\text{dem}}) G \quad (G-117) \]

Calibration plots for the two electromagnet configurations are given in Figure 6.11. These were obtained by placing a Hall probe at the approximate location of one of the ferrite disks and measuring the d.c. field for various winding currents. These plots, then, give the applied d.c. magnetic field at the ferrite disk for a given winding current. For each disk thickness this field can be converted to internal d.c. field by using the appropriate one of equations (6-114) through (6-116) with (6-117).
Figure 6.11 Electromagnet Calibration Curves
6.4.3 Results of the measurements

Experimental absorption plots for the magnetized end-loaded cavity are given for the three disk thicknesses in Figures 6.12, 6.13 and 6.14. All of these plots were obtained with the coupling hole centered and with the electromagnet in its first configuration. Each sweep extends from 8 to 12 GHz, successive sweeps corresponding to larger values of applied d.c. magnetic field. The plots are keyed according to Tables 6.2, 6.3 and 6.4. Included in each table are the winding current, the resulting d.c. field, and the corresponding internal d.c. field which depends on disk thickness according to the relationships in (6-114) through (6-117). Many plotted absorptions are extremely difficult to locate, so that the observed resonance frequencies have been included in the tables (with each column corresponding to a separate mode) and can be used while examining the plots to aid in identifying associated absorptions. Other weaker absorptions have not been included in the tables. For applied d.c. fields less than the demagnetizing field, no internal field is entered.

6.5 Comparison of Experiment and Theory

Figures 6.15 and 6.16 are versions of Figures 6.9 and 6.10 with additional information. It is recalled that Figures 6.9 and 6.10 are plots of predicted resonance frequencies for the magnetized end-loaded cavity, the first plot being for even cavity modes, the second for odd modes. These predictions make use of a mode expansion in each cavity region which includes only one basis mode. This is the TE_{11} wave-guide mode in the dielectric section while the ferrite basis mode retained is the one
Figure 6.12 Absorption plot, magnetized cavity, .4 cm diers
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**Table 6.2**

Absorption Plot Key: .4 cm. Disk Thickness
Figure 6.15 Absorption Plot, Magnetized Cavity, .6 cm Disks
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**Table 6.3**

*Absorption Plot Key: 0.6 cm disk thickness*
Figure 6.14: Absorption plot, magnetized cavity, .8 cm disks.
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**Table 6.4**

Absorption Plot Key, .8 cm Disk Thickness
which becomes the $\text{TE}_{11}$ mode in the limit of very large d.c. field. It is concluded, then, that in the large field limit, the approximate expansion will lead to accurate predictions for all $\text{TE}_{11}$ cavity modes. Since the ferrite becomes isotropic in this limiting situation, these are identical to the $\text{TE}_{11}$ modes for the experimental cavity with the ferrite unmagnetized. Based on an understanding of the accurate magnetized cavity theory in which a sufficiently large number of basis modes is used, every cavity mode becomes an unmagnetized cavity mode in the large field limit. Some in particular will evolve into the same $\text{TE}_{11}$ modes accurately predicted in the limit by using the approximate expansions. This implies that, as the d.c. field is reduced and the cavity resonances are again affected by the ferrite anisotropy, the approximate method will continue to provide satisfactory predictions for these resonances down to some lower limit of d.c. field.

The theory of Section 6.2.3 excludes many resonances which one expects to see experimentally. For example, magnetized cavity modes associated with isotropically-filled waveguide modes of higher order azimuthal dependence are not included by the present theory, while such modes are indeed observed experimentally. This situation means that one is especially pressed to determine which observed magnetized-cavity resonances are to be compared with predictions. It then becomes desirable to relate various observed resonances and modes under magnetized conditions to observed $\text{TE}_{11}$ cavity modes occurring in the isotropic case. And if the predicted magnetized cavity resonances can be paired with the predicted $\text{TE}_{11}$ isotropic ferrite cavity resonances, predicted and observed modes associated with like $\text{TE}_{11}$ modes can be related and their resonance frequencies compared.
The comparison plots, Figures 6.15 and 6.16, are concerned with the task of associating predicted modes for the magnetized cavity with $\text{TE}_{11}$ modes for the unmagnetized case. The predictions do not allow the evaluation of a given mode to be observed as d.c. field grows large, so that this approach cannot be used in finding the related $\text{TE}_{11}$ mode. The predictions do allow, however, for examination of the resonance frequencies as the ferrite disk thickness nears zero. The limiting modes in this case will be $\text{TE}_{11}$ modes of the dielectric-filled cavity. It will be assumed that the correspondence between magnetized cavity modes and $\text{TE}_{11}$ modes in the thin disk limit is identical to that found in the large d.c. magnetic field limit.

To achieve the identification with $\text{TE}_{11}$ unmagnetized cavity modes, data derived from predictions and measurements for the unmagnetized cavity, as well as for the dielectric-filled cavity, have been added to the plots. The first four dielectric-filled cavity resonances corresponding to $\text{TE}_{11}$ modes occur at 9.7 GHz, 11.8 GHz, 14.7 GHz and 17.9 GHz. These are designated as the $\text{TE}_{111}$, $\text{TE}_{112}$, $\text{TE}_{113}$ and $\text{TE}_{114}$ modes respectively, and each has its counterpart as the ferrite disk thickness becomes non-zero with the ferrite unmagnetized. Unmagnetized end-loaded cavity modes corresponding to these have been discussed at length in Chapter 5 for disk thicknesses of .4, .6 and .8 cm, and have been identified in Table 5.4 as $\text{TE}_{11}$ cavity modes 1, 2, 3 and 4 (cavity mode sequence column). $\text{TE}_{111}$ and $\text{TE}_{112}$ resonance frequencies for the dielectric-filled cavity are indicated in Figures 6.15 and 6.16 on the frequency axis, which corresponds to zero disk thickness. Locations of the remaining two resonance frequencies are to the right of the 8 - 12.4 GHz range and must be visualized. Also added to both plots are the
$TE_{11}$ resonance frequencies predicted in Chapter 5 for the unmagnetized end-loaded cavity with .4, .6 and .8 cm disks. For the .4 cm case (see Table 5.6) these are the $TE_{111}$ mode predicted at 8.70 GHz (even, # 1), the $TE_{112}$ mode predicted at 9.30 GHz (odd, # 2) and the $TE_{113}$ mode predicted at 11.55 GHz (even, # 3). For the .6 cm disk the $TE_{111}$ mode is predicted to occur at 7.60 GHz (even, # 6), the $TE_{112}$ mode at 8.00 GHz (odd, # 7), and the $TE_{113}$ mode at 11.45 GHz (even, # 10). Finally, for the .8 cm disk thickness, the $TE_{111}$ mode occurs below the range of predictions, the $TE_{112}$ mode is predicted to occur at 7.35 GHz (odd, # 13), the $TE_{113}$ mode at 10.70 GHz (even, # 16) and the $TE_{114}$ mode at 11.85 GHz (odd, # 17). The $TE_{111}$ and $TE_{112}$ resonance frequencies occur below the frequency range of the plots for the .8 cm disk thickness, while the $TE_{114}$ resonance frequencies are above the range except for the largest thickness. Also included in Figures 6.15 and 6.16 for each case possible are the corresponding observed resonance frequencies. Comparing the predicted data for the magnetized case with the data just mentioned for the unmagnetized case, all the resonance frequencies associated with the former have been assigned to modes of the magnetized cavity and these groups of data points have then been labelled according to their association with the various modes of the unmagnetized case.

A similar process has been carried out in the case of the experimental data for the magnetized cavity. In this case, the plots in Figure 6.12 through 6.14 have been examined and the resonances associated with $TE_{11}$ modes in the isotropic end-loaded case have been determined. This is not accomplished by observing very large applied d.c. magnetic field, but by noting the frequency
shift of absorptions and their final locations as the actual magnetization is reduced to zero by reducing the applied field. The appropriate observed modes can then be identified with the predicted ones. Then, for the applied d.c. field corresponding to an internal field of \(~ 390\) oe., the resonance frequency for each of these chosen modes is entered in the comparison plots. In the case of Figure 6.16, which is for odd modes, all experimental information has been based on observations made with the aid of the oscilloscope, since no plots for odd symmetry modes were made. In the \(0.6\) cm thickness case there was experimental evidence of odd mode resonances in the frequency range \(9 - 10\) GHz. Although encouraging, identification with the isotropic \(TE_{112}\) mode was inconclusive and these data have not been included in the plot.

Some points of particular interest in the resulting comparison should be mentioned. The two \(TE_{114}\) modes and the two \(TE_{115}\) modes in the magnetized \(0.4\) cm disk case have been experimentally confirmed. The higher \(TE_{115}\) resonance occurs slightly above the band, having been seen for lower d.c. fields, and its location is approximated in Figure 6.15 for even modes. Comparison between theory and experiment, especially as far as trends are concerned, is good for this disk thickness and for even mode symmetry. Fairly good agreement is seen elsewhere on the plot. Of special interest for the odd mode plot is the appearance of the \(TE_{114}\) mode, which exhibits four absorption maxima along the cavity length and is therefore very distinctive. In the treatment of the unmagnetized cavity for a \(0.6\) cm disk thickness, this mode is neither predicted nor observed. Predictions for the magnetized case, however, suggest appearance of the related mode at the upper part of the band and experiment has borne out this expectation, as indicated in Figure 6.16.
Figure 6.16 Comparison Plot, Odd Modes
CHAPTER 7

CONCLUSIONS AND COMMENTS

7.1 Conclusions and Discussion

This work has shown that Suhl and Walker's analysis of the ferrite-filled waveguide can be extended to include evanescent behaviour. The resulting mode plots, although involved, provide useful insight into the complicated behaviour of the loaded waveguide system. In Chapter 6, where the inhomogeneous cavity is treated, the decision to limit the analysis to a single basis mode is shown to be reasonable, and the experimental evidence confirms the validity of this approach. It is likely that the use of additional terms in the field analysis would yield even better results. It is concluded, therefore, that this method has general applicability to the dielectric/ferrite waveguide interface problem.

The computer program of Chapter 6, which searches for resonance frequencies, is based on a study of the mode plots of Chapter 3, which in turn were generated for specific parameter values. The program was also written for a single basis mode, and the values of normalized internal magnetic field and r.f. frequency were restricted. The internal magnetic field range insured that the basis mode was propagating in the ferrite and was isolated from others, and the frequency range guaranteed that the dielectric mode would also be propagating. The specialization of the approach in this way, in order to generate solutions in a reasonable timescale that could be checked in the laboratory, is felt to have been an appropriate course of action.
7.2 Comments and Suggestions for Future Work

The full effect of the numerical approximation used in Chapter 6, e.g. the number of iterations in the Bessel function subroutines and terms in the numerical integration, has not been studied in depth and may have contributed to minor inaccuracies.

The mode plots contain a great deal of information, and more detailed study of the evanescent regions may provide further useful insight which is not available in the Suhl and Walker treatment.

Inclusion of a larger number of basis modes in the cavity analysis may provide more precise agreement with experiment. However, some of these ferrite modes will be associated with TM dielectric modes and therefore, in order to provide insight and data to aid evaluation of the resulting predictions, it would be necessary to extend the procedure of Chapter 5 to TM modes. This would require another cavity with a different coupling arrangement. It would be helpful to be able to take into account a small annular air gap by extending the Chapter 3 treatment to the case of a coaxial ferrite rod and allowing the rod radius to approach that of the guide. This suggests another possibility, the case of letting the guide radius approach infinity, which may lead to a ferrite-rod-waveguide configuration. Other useful extensions of the ferrite-filled guide analysis would be: cases for \(|n| > 1\); other cavity loading configurations; and various ferrite-dielectric interface problems in infinite or semi-infinite circular guide which occur in devices.
REFERENCES


APPENDIX 1

C

I(N) CURVES OF G1 AND G2 FOR POSITIVE SIGMA

C

DIMENSION OUT1(1200), OUT2(1200), RINT1(10)

101 FORMAT (E20,8)
102 FORMAT (120)
103 FORMAT ('//40X*40HI(N) CURVES OF G1 AND G2* POSITIVE SIGMA//')
104 FORMAT ('//2X*12HFREQUENCY = *F4,1*4H GH7')
105 FORMAT ('//2X*12H*PI*NSAT = *F6,1*6H GA155')
106 FORMAT ('//2X*16HACTUAL RADIUS = *F7,5*4H CM.')
107 FORMAT ('//2X*2AHRELATIVE PERMITTIVITY = *F4,1')
108 FORMAT ('//2X*14HEFFECTIVE G = *F4,2')
109 FORMAT ('//2X*20HNORMALIZED RADIUS = *F7,5')
110 FORMAT ('//2X*4HP = *F7,5')
111 FORMAT ('//56X*1H1*11,AH CURVE/')
112 FORMAT ('//53X*1H1*11,AH=YNHM CURVE/')
113 FORMAT ('//2X*15HNO CURVE FXISTS')
114 FORMAT ('//2X*6E20,8')
115 FORMAT ('//13X*6MLANDA1,14X*5HSIGMA2(15X*6MLANDA1,14X*5HSIGMA2)')
116 FORMAT ('//2X*32HSCANNING OF BRANCHES IS COMPLETE')
READ(5,101) FRQGHZ
READ(5,101) RMSAT
READ(5,101) RADUIS
READ(5,101) RELPER
READ(5,101) GEFF
READ(5,101) SIGMAX
READ(5,101) RLMIN
READ(5,101) RLM1
READ(5,101) RLAMF
READ(5,102) NRORI
READ(5,102) NRDHF
READ(5,101) DELLAM
ROOTJ1(1) = 3.83171
ROOTJ1(2) = 7.01559
ROOTJ1(3) = 10.17347
ROOTJ1(4) = 13.32369
ROOTJ1(5) = 16.47063
ROOTJ1(6) = 19.61586
ROOTJ1(7) = 22.76008
ROOTJ1(8) = 25.90367
ROOTJ1(9) = 29.04683
ROOTJ1(10) = 32.18968
PI = 3.1415926536
RNORM = RADIUS**2.*PI*FRQGHZ*SQRTRELPER)/29.9776
P = 1.4*GEFF*RMSAT/(FRQGHZ*1000.)
WRITE(6,103)
WRITE(6,104) FRQGHZ
WRITE(6,105) RMSAT
WRITE(6,106) RADIUS
WRITE(6,107) RELPER
WRITE(6,108) GEFF
WRITE(6,109) RNORM
WRITE(6,110) P
N = NRDHF
NCOUNT = 1
w1 = 1
w2 = 1
\[ \text{RLAMDA} = \text{RLAM1} \]

GO TO 220

208 \[ \text{SIGMA} = 1_*/\text{RLAMDA} = (\text{RNDNRM}/\text{ROOTJI}(N))^{**2}(1_*/\text{RLAMDA}**2)/\text{RLAMDA} \]

RLMDAK = RLAMDA

IF((SIGMA \leq 0) OR (\text{ABS}(\text{SIGMA}) > \text{SIGMAX})) GO TO 204

IF(\text{RLAMDA} > 0) GO TO 202

203 IF((1_*/\text{RLAMDA}) \leq 0) GO TO 206

RLAMDA = (\text{SIGMA} + P - \text{RLAMDA})/(1_*/\text{SIGMA} * \text{RLAMDA})

GO TO 211

202 IF(((\text{SIGMA} > 0) OR (\text{RLAMDA} > 0) AND (\text{SIGMA} > \text{RLAMDA} - P)) OR ((\text{SIGMA} \leq 0) OR (\text{RLAMDA} \leq 0) AND (\text{SIGMA} + P - \text{RLAMDA} > 0))) GO TO 204

GO TO 203

204 OUT1(M1) = RLAMDA

OUT1(M1+1) = SIGMA

M1 = M1 + 2

GO TO 206

211 OUT2(M2) = RLAMDA

OUT2(M2+1) = SIGMA

M2 = M2 + 2

206 RLAMDA = RLMDAK + DELLAM

220 IF(\text{ARMS}(\text{RLAMDA}) > \text{RLAMIN}) GO TO 207

RLAMDA = RLAMIN

207 IF(\text{RLAMDA} \leq \text{RLAMF}) GO TO 208

WRITE(6, 111) N

IF(M1 > 1) GO TO 214

218 WRITE(6, 113)

IF(NCOUNT \leq 2) GO TO 213

GO TO 217

214 WRITE(6, 115)

NROWS = (M1 - 1)/8,
NSTART = 1
IF(NROWS .GT. 0) GO TO 215
GO TO 216
215 WRITE (6,114) (OUT1(J), J = NSTART, NSTART+5)
NSTART = NSTART + 6
IF(NSTART .LT. NROWS*A) GO TO 215
IF(NSTART .EQ. M1) GO TO 212
216 WRITE (6,114) (OUT1(J), J = NSTART, M1-1)
212 IF(NCOUNT .EQ. 2) GO TO 213
217 NCOUNT = 2
WRITE(6,112) N
IF(M2 .EQ. 1) GO TO 218
M1 = M2
219 DO 219 J = 1, M1-1
219 OUT1(J) = OUT2(J)
GO TO 214
213 IF(N .GE. NMOD) GO TO 209
N = N + 1
GO TO 210
209 CONTINUE
WRITE (6,116)
STOP
END
**APPENDIX 2**

C
C
C


de=0 G1 AND G2 CURVES
C

**DIMENSION U(3), V(3), UAPPRX(3), VAPPRX(3), RR1(3), RR2(3)**

**DIMENSION XSLOAD(11), YSWHI(11), NOUT(4), OUTPUT(4,300)**

100 FORMAT (E20.8)
101 FORMAT (I20)
102 FORMAT (/58X*21HZERO G1 AND G2 CURVES///)
103 FORMAT (/3X*16HACTUAL RADIUS = *F7.5*4H CM///)
104 FORMAT (3X,12HA*P1*MSAT = *F6*1*6H GAUSS///)
105 FORMAT (3X,24HRELATIVE PERMITTIVITY = *F5.2///)
106 FORMAT (3X,21HEFFECTIVE G FACTOR = *F4.2///)
107 FORMAT (3X,12HFREQUENCY = *F9*1*5H GHZ///)
108 FORMAT (3X,20HNORMALIZED RADIUS = *F7.5///)
109 FORMAT (3X,4HP = *F7.5///)
110 FORMAT (/5X,6E20.8)
111 FORMAT (1X,3(15X,5HLAWDA*15X,5XSIGMA///)
112 FORMAT (/3X*27HNEGATIVE REAL ARGUMENT OF F)
113 FORMAT (/3X*34HATTEMPT TO EVALUATE F WHEN J1 = .0)
116 FORMAT (/3X*4HSEARCH FOR STARTING X=SQUARED HAS JUMPED BRANCH
117 FORMAT (/3X*4HBRANCH J1///)
118 FORMAT (/64X*2H0(11*7H CURVE///)
119 FORMAT (/39X,3H(0(11,16H)=PRIMEDT CURVE///)

120 FORMAT (/60X*2H0(11*14H)=PRIMED CURVE///)
121 FORMAT (/62X,*3H(0(11,9H))T CURVE///)
FORMAT (6IX,12HNG PUXI'TS FOUND,)
FORMAT (////5IX,32HSCANNING OF BRANCHES IS COMPLETE)
READ (5,100) F0WGMZ
READ (5,100) MAUINS
READ (5,100) RMSAT
READ (5,100) HELPER
READ (5,100) GEFF
READ (5,101) IHNMB
READ (5,101) IHNNE
READ (5,100) FMAX
READ (5,100) FMIN
READ (5,100) ABSIGM
READ (5,100) AHLMX
READ (5,100) DISTMX
READ (5,100) DIJSHN
C
C RMSAT IS 4*PI*(SATURATION MAGNETIZATION IN GAUSS). NO
C UNTRANSFORMED VALUES OF LAMDA GREATER THAN FMAX OR LESS THAN FMIN
C ARE CONSIDERED. THE DISTANCE BETWEEN TWO PLOTTED POINTS ON A
C GIVEN CURVE IS NOT ADJUSTED WHEN, FOR BOTH POINTS, THE MAGNITUDE
C OF SIGMA IS GREATER THAN ABSIGM OR THE MAGNITUDE OF LAMDA IS
C GREATER THAN AHLMX, WHEN THE RESOLUTION IS ADJUSTED, THE UPPER
C AND LOWER LIMITS FOR SEPARATION ARE, RESPECTIVELY, DISTMX AND
C DIJSHN. THESE DEPEND ON THE SCALE USED FOR EACH AXIS. SCANNING
C BEGINS WITH BRANCH IHNMB AND ENDS WITH BRANCH IHNNE.
C
WRITE (6,102)
WRITE (6,103) RADIUS
WRITE (6,104) RMSAT
WRITE (6,105) HELPER
WRITE (6,106) GEFF
WRITE (6, 107) FHGFHZ
PI = 3.1415926536
RNORM = RADIUS**2 * PI * FHGFHZ * SQRT(HELPER) / 29.9776
P = 1.4 * GEFF * RMSAT / (FHGFHZ * 100)**
WRITE (6, 108) RNORM
WRITE (6, 109) P

THE FOLLOWING PAIRS OF CONSTANTS CONFINES THE FIRST ELEVEN BRANCHES

XSQLO(1) = -1.0, XSQH1(1) = 3.431**2
IF (FMAX * GT * 1.0) XSQLO(1) = *FMAX**2 = FMAX = .25
XSQLO(2) = 3.832**2, XSQH1(2) = 7.015**2
XSQLO(3) = 7.016**2, XSQH1(3) = 10.173**2
XSQLO(4) = 10.174**2, XSQH1(4) = 13.323**2
XSQLO(5) = 13.324**2, XSQH1(5) = 16.470**2
XSQLO(6) = 16.471**2, XSQH1(6) = 19.615**2
XSQLO(7) = 19.616**2, XSQH1(7) = 22.759**2
XSQLO(8) = 22.761**2, XSQH1(8) = 25.903**2
XSQLO(9) = 25.904**2, XSQH1(9) = 29.046**2
XSQLO(10) = 29.047**2, XSQH1(10) = 32.189**2
XSQLO(11) = 32.190**2, XSQH1(11) = 35.332**2

THIS PROGRAM WILL NOT SCAN PAST BRANCH 10.

IF (IBRNE * GT * 10) IBRNE = 10
IBRN = IBRN8

THE BRANCH NUMBER IS SET. THE RANGE OF XSQ OVER WHICH THE
STARTING SEARCH IS ALLOWED TO TAKE PLACE IS DEFINED BY THE LOWER
LIMIT XSQLO AND THE UPPER LIMIT XSQH1. THE STARTING INCREMENT IN
XSQ USED FOR THE SEARCH DEPENDS ON THE BRANCH CONCERNED, AND IS
DETERMINED IMMEDIATELY BELIEVE.

DXSQS = 0.5 * (1.0 + IARN)

XSQ = XSQLO(IARN + 1)

NRETWL = 1

XR = 0.0  J XI = 0.0  I RMINA = 1.

IF (XSQ .LT. 0.0) XI = SQRT(-XSQ)

IF (XSQ .GT. 0.0) XR = SQRT(XSQ)

IF (XSQ .EQ. 0.0) GO TO 201

CALL FCMLPL(XH, XI, RLMDA, FI, U, V, VAPPRK, VAPPRK, RH1, RH2,

C IERHOR)

IF (IERHOR .EQ. 0) GO TO 201

IF (IERHOR .EQ. 1) WRITE (6,112)

IF (IERHOR .EQ. 2) WRITE (6,111)

GO TO 254

GU TO (204,205,209,217), NRETWL

DELF = RLMDA = FM0

IF (ABS(DELF) .LT. .1) GO TO 211

IF (DELF .LT. 0.0) DXSQS = -DXSQS

WE MAKE USE OF THE FACT THAT F IS A MONOTONE DECREASING FUNCTION.

XSQ1 = XSQ J XSQ = XSQ + DXSQS J DELF1 = DELF

NRETWL = 2

IF (XSQ .LT. XSQHIC(IARN + 1)) NO TO 202

WRITE (6,116) J GO TO 254

ONLY IN BRANCH ZERO COULD THE FIRST INCREMENT OF XSQ BE NEGATIVE.

THIS POSES NO PROBLEM, SINCE THAT BRANCH EXTENDS INDEFINITELY TO

THE LEFT.
C

205 DELF = HLMDA - FMAX
IF (ABS(DELF) .LT. 1) GO TO 211
IF (DELF*DELF1 .LT. 0) GO TO 206
XSQ = XSQ1 - XSQ = XSQ + DXSQS / DELF1 = DELF
IF (XSQ .LT. XSW) (XSW1 = 1)) :00 TO 202
WRITE (6,116) J GO TO 254

C

A COARSE LOCATION OF THE STARTING XSQ HAS BEEN OBTAINED; WE MUST
NOW IMPROVE THE ACCURACY OF THIS LOCATION UNTIL ABS(DELF) IS LESS
THAN .1 .

C

206 IF (XSQ .LT. XSW1) GO TO 208
DELF2 = DELF 1 XSQ2 = XSQ J GO TO 207
208 DELF2 = DELF1 1 DELF1 = DELF J XSQ2 = XSQ1 1 XSW1 = XSQ
207 XSQ = (XSQ1 + XSQ2)/2.
NRETL = 3 J GO TO 209
209 DELF = HLMDA - FMAX
IF (ABS(DELF) .LT. 1) GO TO 211
IF (DELF*DELF1 .LT. 0) GO TO 210
DELF1 = DELF 1 XSQ1 = XSQ J XSQ = (XSQ1 + XSQ2)/2, J GO TO 202
210 DELF2 = DELF 1 XSQ2 = XSQ J XSQ = (XSQ1 + XSQ2)/2, J GO TO 202

C

A STARTING XSW HAS BEEN DETERMINED FOR THE PRESENT BRANCH.

C

211 DXSQ = .5 J NRETL = 4
NOUT(1) = 1 J NOUT(2) = 1 J NOUT(3) = 1 J NOUT(4) = 1
NFWFAK = 1
212 IF (RLMDA .LT. FMIN) GO TO 244
IF (((XSQ .NE. 0) .AND. ((RLMDA .NE. 0) .AND. (RLMDA*2 .NE. 1, C .))))) GO TO 213
NGWFAM = 1 J XSQ = XSQ + DXSQ

IF (XSQ LT XSWHICBN + 1) GO TO 202

GO TO 244

SIGMA = 1/RLMDA = (R0RM**2/XSQ)*(1.0 - RLMDA**2)/RLMDA

IF (SIGMA NE 0 GO TO 214

NGWFAM = 1 J XSQ = XSQ + DXSQ

IF (XSQ LT XSWHICBN + 1) GO TO 202

GO TO 244

C

A POINT OF ZERO-G HAS BEEN LOCATED IN THE SIGMA-LAMDA PLANE.

ON WHICH CURVE THE POINT LIES MUST NOW BE DETERMINED.

C

IF (SIGMA LT 0 GO TO 236

P = ABS(P)

IF (RLMDA GT 0 GO TO 237

RLMDA = (SIGMA + P - RLMDA)/(1.0 - SIGMA*RLMDA)

NFAM = 2 J GO TO 239

IF (((SIGMA LT 1/RLMDA) AND (SIGMA GT (2*RLMDA - P)/ (RLMDA**2 + 1.0))) OR (((SIGMA GT 1/RLMDA) AND (SIGMA LT (2*RLMDA - P)/ (RLMDA**2 + 1.0))) GO TO 238

IF (((SIGMA GT 1/RLMDA) AND (SIGMA GT (2*RLMDA - P)/ (RLMDA**2 + 1.0))) OR (((SIGMA LT 1/RLMDA) AND (SIGMA LT (2*RLMDA - P)/ (RLMDA**2 + 1.0))) GO TO 240

NGWFAM = 1 J XSQ = XSQ + DXSQ

IF (XSQ LT XSWHICBN + 1) GO TO 202

GO TO 244

NFAM = 1 J GO TO 239

P = ABS(P)

IF (RLMDA LT 0) GO TO 241

RLMDA = (SIGMA + P - RLMDA)/(1.0 - SIGMA*RLMDA)

NFAM = 4 J GO TO 239
IF (((SIGMA, GT, 1, /RLM), AND, (SIGMA, LT, (?,*RLMDA = P)/C
(RLMDA**2 + 1,)))) OR (((SIGMA, LT, 1, /RLMDA), AND, (SIGMA, GT,
C (2,*RLMDA = P)/(RLMDA**2 + 1,)))) GO TO 242
IF (((SIGMA, LT, 1, /RLMDA), AND, (SIGMA, GT, (?,*RLMDA = P)/C
(RLMDA**2 + 1,)))) OR (((SIGMA, GT, 1, /RLMDA), AND, (SIGMA, LT,
C (2,*RLMDA = P)/(RLMDA**2 + 1,)))) GO TO 243
NUMFAM = 1 J XSQ = XSQ + DXSQ
IF (XSQ, LT, XSQ1(1,0N + 1)) = 0 TO 202
GO TO 244

243 NFAM = 3
239 GO TO (215*216*224*229*233), NUMFAM
215 OUTPUT(NFAM, NOUT(NFAM)) = RLMDA
OUTPUT(NFAM, NOUT(NFAM) + 1) = SIGMA
NOUT(NFAM) = NOUT(NFAM) + 2

C
C DEETERMINATION OF THE NEXT VALUE OF XSQ = 
C THIS DETERMINATION IS USED ONLY IF NO RESOLUTION ADJUSTMENT IS
C BEING MADE. IT IS BASED ON A CRITERION OF UNIFORM X=INCREMENT.
C THE PRESENT XSQ AND THE PREVIOUS DXSQ MUST BE KNOWN.
C
216 XSQB = XSQ = UXSQ
SYNX28 = 1, ) SYNX2 = 1,
IF (XSQ, LT, 00) SYNX28 = -1,
IF (XSQ, LT, 00) SYNX2 = -1,
UX = SYNX2*SYN(ABS(XSQ)) J UXN = SYNX28*SYN(ABS(ABS(XSQ)))
XSQB = XSQ
UXN = 2*UX = UX
SYNUXN = 1.
IF (UXN, LT, 00) SYNUXN = "10
XSQ = SYNUXN*UXN**2
NFAHB = NFAM J SIGMA8 = SIGMA ; RLMDAB = RLMDA ; DXSQ = XSQ = XSQB
IF (XSW *GE* XSW1) GO TO 244
NGWFAM = 2 J GO TO 202

C IF THERE IS A PREVIOUS POINT BELONGING TO THE SAME BRANCH, IT MAY
C BE NECESSARY TO CHECK THE RESOLUTION AND MAKE AN ADJUSTMENT. IF
C THE PRESENT POINT, FOR WHICH NO ADJUSTMENT HAS BEEN MADE, AND THE
C PREVIOUS POINT ARE NOT ON THE SAME ZERO-CURVE, THE APPARENT
C DISTANCE BETWEEN THE TWO IS NOT CHECKED.

218 IF (NFAM *GW* NFAMR) TO 219
GO TO 215

C IF THE PRESENT AND PREVIOUS POINTS ARE BOTH OUTSIDE THE REGION OF
C INTEREST, THE APPARENT SEPARATION IS NOT CALCULATED.

219 IF (((ABS(SIGMA) *LE* ABSIGN) AND (ABSRLMDA) *LE* ABLAMX)) OR
C (((ABS(SIGMAW) *LE* ABSIGN) AND (ARSLMDA) *LE* ABLAMX))
C GO TO 220
GO TO 215

C IT HAS BEEN FOUND THAT THE APPARENT SEPARATION OF THE TWO POINTS
C MUST BE CHECKED. THE LAMDA=AXIS SCALE IS USED IN MEASURING
C DISTANCE. IF THE SEPARATION IS WITHIN PREVIOUSLY SET LIMITS,
C CALCULATION PROCEEDS TO THE NEXT XSW.

220 DIST = SQRT(16.*(SIGMA - SIGMA2)**2 + (KLM1A - RLMDAH)**2)
IF ((DIST <LT* DISTMN) OR (DIST >GT* DISTMX)) GO TO 221
GO TO 215

C THE CASES OF EXCESS SEPARATION AND INSUFFICIENT SEPARATION
C ARE TREATED INDEPENDENTLY.
C

221 IF (DIST .GT. DISTMX) GO TO 227
NGWFAM = 4 J GO TO 227

C

229 IF (NFM .NE. NFMFA) GO TO 215
DIST = SQRT(16.*((SIGMA - SIGMA2)**2 + (KLMDA - KLMDAH)**2))
IF ((DIST .LT. DISTMN) .OR. (DIST .GT. DISTMX)) GO TO 230
DXSQ = XSQ - XSQF J GO TO 215

230 IF (DIST .GT. DISTMX) GO TO 231

227 XSQ = XSQ + DXSQ
IF (XSQ .GE. XSUM(18QN + 1)) GO TO 244
GO TO 202

231 XSQL = XSQ - DXSQ J XSQR = XSQ J NGWFAM = 5 J GO TO 232

233 DIST = SQRT(16.*((SIGMA - SIGMA2)**2 + (KLMDA - KLMDAH)**2))
IF (DIST .GT. DISTMX) GO TO 234
IF (DIST .LT. DISTMN) GO TO 239
DXSQ = XSQ - XSQF J GO TO 215

234 XSQR = XSQ J GO TO 237

C

235 XSQL = XSQ

232 XSQ = (XSQL + XSQR)/2. J GO TO 202

222 XSQL = XSQF J XSQR = XSQ J NGWFAM = 3 J GO TO 223

C

224 DIST = SQRT(16.*((SIGMA - SIGMA2)**2 + (RLMCA - RLMDA)**2))
IF ((DIST < LT, DISTMN) OR (DIST < UT, DISTMX)) GO TO 225

DXSQ = XSQ = XSWH I GO TO 215

225 IF (DIST < LT, DISTMN) GO TO 224

XSR = XSQ J GO TO 227

226 XSQL = XSQ

223 XSQ = (XSQL + XSR)/2. J GO TO 202

C

C OUTPUT CYCLE = = =

C

244 WRITE (6,117) IHBAN

DO 245 NFAM = 1,4+1

245 GO TO (246,247,248,249), NFAM

246 WRITE (6,118) IHBAN J GO TO 250

247 WRITE (6,119) IHBAN J GO TO 250

248 WRITE (6,120) IHBAN J GO TO 250

249 WRITE (6,121) IHBAN

250 IF (NOUT(NFAM) * GT, 11) GO TO 251

WRITE (6,122) J GO TO 245

251 WRITE (6,111)

JOUT = 1

252 KOUT = JOUT + 5

IF (KOUT * GT, NOUT(NFAM) = 1) KOUT = NOUT(NFAM) = 1

WRITE (6,110) (OUTPUT(NFAM, IO:1)), IOUT = JOUT, KOUT)

253 JOUT = JOUT + 6

IF (JOUT * LT, NOUT(NFAM) = 1) GO TO 252

245 CONTINUE

C

C OUTPUT FOR THE PRESENT BRANCH HAS BEEN COMPLETED. WE NOW PROCEED

C TO THE NEXT BRANCH, IF ANY REMAIN.

C

IHBAN = IHBAN + 1
IF (IBRN .GT. IBRNE) go to 253
GO TO 255

253 CONTINUE

C

C SCANNING OF BRANCHES IS COMPLETE.

C

WRITE (6,121)

254 CONTINUE

STOP

END
SUBROUTINE FCMLX (X*Y*FR,FI,U,V,UAPPRX,VAPPRX,RHI,RH2,IEROR)

DIMENSION U(3), V(3)* UAPPRX(3), VAPPRX(3), RR1(3), RH2(3)

IEROR = 0
ND = 5
NMAX = 2

IF ((X .LE. 0) .AND. (Y .EQ. 0)) GO TO 1100

EPSLON = .5/10.**ND

DO 100 N = 1, NMAX+1, 1

UAPPRX(N) = 0

100 VAPPRX(N) = 0

Y1 = ABS(Y)

RO2 = X**2 + Y**2

RO = SQRT(RO2)

IF (X .EQ. 0) PHI = 1.5707963268

IF (X .GT. 0) PHI = ATAN(Y1/X)

IF (X .LT. 0) PHI = 1.1415926536 + ATAN(Y1/X)

C = EXP(Y1)

SUM1 = C*COS(X)

SUM2 = C*SIN(X)

D1 = 2.3026+NU + 1.3843

ARGT = S=D1/NMAX

CALL SUBT (T, ARGT)

R = 0

IF (NMAX .GE. 0) R = NMAX*T

ARGT = 7.3576*(D1*Y1)/RO

CALL SUBT (T, ARGT)

S = 1.3591*RO

IF (Y1 .LT. D1) S = 1.3591*RO*T

NU = 1 + INT(S)

IF (R .GT. S) NU = 1 + INT(R)
N = 0  \; BESL = 1, \; J \; C1 = 1, \; J \; C2 = 1

N = N + 1

\begin{align*}
BESL & = BESL \times N / (N+1) \\
C & = C1 \; J \; C1 = C2 \; J \; C2 = C \\
\text{IF} \; (N \lt N) & \; \text{GO TO 300} \\
R1 & = 0 \; J \; R2 = 0 \; J \; S1 = 0 \; J \; S2 = 0
\end{align*}

\begin{align*}
C & = (2, * N = X \times R1 + Y1 \times R2) + (X \times R2 + Y1 \times C) \\
R1 & = (2, * N \times X = R02 \times R11) / C \\
R2 & = (2, * N \times Y1 = R02 \times R11) / C \\
BESL & = BESL \times (N+1) / N \\
C & = 2, * N \times BESL \\
\text{RLMDA1} & = C \times C1 \\
\text{RLMDA2} & = C \times C2 \\
C & = C1 \; J \; C1 = C2 \; J \; C2 = C \\
S & = R1 \times (\text{RLMDA1} + S1) - R2 \times (\text{RLMDA2} + S2) \\
S2 & = R1 \times (\text{RLMDA2} + S2) - R2 \times (\text{RLMDA1} + S1) \\
S1 & = S \\
\text{IF} \; (N \ge N) & \; \text{GO TO 500} \\
\text{RH1}(N) & = R1 \\
\text{RR2}(N) & = R2
\end{align*}

N = N + 1

\begin{align*}
\text{IF} \; (N \ge 1) & \; \text{GO TO 400} \\
C & = (1, + S1) \times 2 + S2 \times 2 \\
U(1) & = (SUM1 \times (1, + S1) + SUM2 \times S2) / C \\
V(1) & = (SUM2 \times (1, + S1) + SUM1 \times S2) / C \\
\text{DO 600} \; N = 0 \times N \text{MAX} / 1 + 1 \\
U(N+2) & = \text{RH1}(N+1) \times U(N+1) - \text{RR2}(N+1) \times V(N+1) \\
V(N+2) & = \text{RH1}(N+1) \times V(N+1) + \text{RR2}(N+1) \times U(N+1) \\
\text{IF} \; (Y \ge 0) & \; \text{GO TO 600} \\
\text{DU 700} \; N = 0 \times N \text{MAX} / 1 \\
V(N+1) & = -V(N+1)
\end{align*}
800  DO 1000  N = 0, NMAX+1
     IF(SQRT(((U(N+1)-UAPPRX(N+1))**2 + (V(N+1)-VAPPRX(N+1))**2)**0.5) < EPSLON) GO TO 1000
     D U 900  M = U, NMAX+1
     UAPPRX(M+1) = U(M+1)
900  VAPPRX(M+1) = V(M+1)
     NU = NU + 5
     GO TO 200
1000  CONTINUE
     IF (U(2)**2 + V(2)**2 < EPSLON) GO TO 1200
     IERROR = 2
     RETURN
1200  UD = (U(1) - U(3))/2,
     VD = (V(1) - V(3))/2,
     FR = (UD*(U(2)*X+V(2)*Y) - VD*(U(2)*Y+V(2)*X))/(U(2)**2+V(2)**2)
     FI = (VD*(U(2)*X+V(2)*Y) + UD*(U(2)*Y+V(2)*X))/(U(2)**2+V(2)**2)
     RETURN
1100  IERROR = 1
     RETURN
END
SUBROUTINE SUB1 (Y)
    IF (Y .GT. 10.) GO TO 100
    P = 5.7941E-05 * Y - 1.76148E-03
    P = Y * P + 2.08645E-02
    P = Y * P + 1.29013E-01
    P = Y * P + 8.5777E-01
    T = Y * P - 1.0125
    RETURN

100  Z = ALOG(Y) - .775
    P = (6.775 - ALOG(Z)) / (1.0 + Z)
    P = 1.0 / (1.0 + P)
    T = Y * P / Z
    RETURN
END
APPENDIX 3

INFINITELY LONG CIRCULAR WAVEGUIDE
LOADED WITH A DIELECTRIC CYLINDER

In an infinitely long circular waveguide partially loaded with dielectric as shown in Figure A.3.1, waveguide modes which are cutoff in the air-filled regions (2 and 3), on either side of the dielectric, can lead to resonances. A characteristic equation for the resonance frequencies is derived for TE modes. The transverse electric field will possess either even or odd symmetry about the plane at \( z = 0 \), resulting in the equivalent problem depicted in Figure A.3.2. The waveguide mode must always be propagating in region 1. General expressions for the transverse components of TE fields are given by

\[
E_t = V(z) \ e^{i(r, \phi)} \tag{A-3-1}
\]

\[
H_t = I(z) \ h(z, \phi) \tag{A-3-2}
\]

where

\[
V(z) = V_1^+ e^{-jz} + V_1^- e^{jz} \equiv V_1(z) \tag{A-3-3}
\]

\[
I(z) = \beta \left( V_1^+ e^{-jz} - V_1^- e^{jz} \right) \equiv I_1(z) \tag{A-3-4}
\]

in the dielectric section (region 1),

\[
V(z) = V_2^+ e^{-\alpha z} \equiv V_2(z) \tag{A-3-5}
\]

\[
I(z) = -j \alpha V_2^+ e^{-\alpha z} \equiv I_2(z) \tag{A-3-6}
\]

in the air section (region 2), and \( \epsilon \) and \( \mu \), the mode vectors, are transverse vectors which are the same for both regions. The propagation constants are given by
\[ \beta = \left\{ + \varepsilon_r \left( \frac{2 \pi}{\lambda} \right)^2 f^2 - k_c^2 \right\}^{1/2} \]  
\[ \alpha = \left\{ - \left( \frac{2 \pi}{\lambda} \right)^2 f^2 + k_c^2 \right\}^{1/2}, \]  
where

\[ f = \text{frequency in GHz} \]  
\[ \varepsilon_r = \text{relative permittivity of the dielectric} \]  
\[ k_c = \chi_{nm}'/r_0 \]  
\[ \chi_{nm}' = \text{m}^{th} \text{ zero of } J_n(x) \]  
\[ r_0 = \text{radius of circular waveguide} \]  

Taking the boundary condition at \( z = 0 \) into account,

\[ V_1(z) \rightarrow -j 2 \begin{array}{c} V_1^+ \sin \beta z \\
I_1(z) \rightarrow + 2 \beta V_1^+ \cos \beta z
\end{array} \]  
\[ (A-3-15) \]  
\[ (A-3-14) \]

for odd symmetry, and

\[ V_1(z) \rightarrow + 2 \begin{array}{c} V_1^+ \cos \beta z \\
I_1(z) \rightarrow -j 2 \beta V_1^+ \sin \beta z
\end{array} \]  
\[ (A-3-15) \]  
\[ (A-3-16) \]

for even symmetry. Requiring now that the transverse field be continuous at the
dielectric-air interface, \( z = t/2 \), one obtains

\[ -j 2 \begin{array}{c} V_1^+ \sin \beta t/2 = + \end{array} \begin{array}{c} V_2^+ e^{-\alpha t/2} \\
+ \begin{array}{c} 2 \beta \end{array} V_1^+ \cos \beta t/2 = -j \alpha \begin{array}{c} V_2^+ e^{-\alpha t/2}
\end{array}
\end{array} \]  
\[ (A-3-17) \]  
\[ (A-3-18) \]

for the odd case and

\[ + 2 \begin{array}{c} V_1^+ \cos \beta t/2 = + \end{array} \begin{array}{c} V_2^+ e^{-\alpha t/2} \\
- j 2 \beta \begin{array}{c} V_1^+ \sin \beta t/2 = -j \alpha \begin{array}{c} V_2^+ e^{-\alpha t/2}
\end{array}
\end{array}
\end{array} \]  
\[ (A-3-19) \]  
\[ (A-3-20) \]

for the even case. Finally, for non-trivial solutions to be possible, the condition
\[ \beta \cos \beta t/2 + \alpha \sin \beta t/2 = 0 \]  \hspace{1cm} (A-5-21)

must hold for odd symmetry, and

\[ \beta \sin \beta t/2 - \alpha \cos \beta t/2 = 0 \]  \hspace{1cm} (A-5-22)

must hold for even symmetry. These are the characteristic equations for the two cases, and the frequencies at which they are satisfied are the resonance frequencies.
FIGURE A.3.1
INFINITE CIRCULAR GUIDE
PARTIALLY LOADED WITH DIELECTRIC
FIGURE A.3.2
THE EQUIVALENT PROBLEM
APPENDIX 4

RESONANCE FREQUENCIES FOR AN END-LOADED
CIRCULAR WAVEGUIDE CAVITY

Consider the circular waveguide cavity represented cross-sectionally in Figure A.4.1. Each of the three regions shown is filled with lossless, isotropic dielectric. The lengths of regions 1 and 3 are equal, as are the permittivities, with the permittivity of the center region being less than that of the end regions. A method for predicting the resonance frequencies for a cavity of this type is discussed. Because the configuration is symmetrical about the transverse plane \( z = L \), it is assumed that all resonances will correspond to field solutions which possess even or odd symmetry about this plane. Thus, one can alternatively consider a new, simpler configuration, the half-cavity shown in Figure A.4.2.

The discussion of the cavity will reflect an assumption that resonance frequencies are measured by observing absorption of power through a coupling hole in the curved wall of the cavity. Dependence of the dissipated power on longitudinal positioning of this coupling hole can be observed and considered along with predicted field dependence to aid in identifying observed resonances with predicted ones. In describing the cavity fields, the coordinate system is chosen so that its \( z \)-axis lies along the axis of the cavity.

Only cavity resonances corresponding to transverse-electric (TE) circular waveguide modes will be considered. In either region, the field expressions for such modes take the general form
\[ E_r = \frac{n}{r} J_n(k_r r) e^{j n \phi} (A e^{-S z} + B e^{S z}) \quad (A-4-1) \]
\[ E_\phi = j k_c J'_n(k_r r) e^{j n \phi} (A e^{-S z} + B e^{S z}) \quad (A-4-2) \]
\[ H_r = -j \delta k_c J'_n(k_r r) e^{j n \phi} (A e^{-S z} - B e^{S z}) \quad (A-4-3) \]
\[ H_\phi = -j \delta \frac{n}{r} J_n(k_r r) e^{j n \phi} (A e^{-S z} - B e^{S z}) \quad (A-4-4) \]
\[ H_z = k_c^2 J_n(k_r r) e^{j n \phi} (A e^{-S z} + B e^{S z}) \quad (A-4-5) \]

where \[ S = \left\{ k_c^2 - \varepsilon_r \left( \frac{2\pi}{\lambda} \right)^2 - f \right\}^{1/2} \]

Any cavity field solution must have continuous transverse electric and magnetic fields at the interface between the regions. From the general expressions it is seen that continuity of the transverse electric field is equivalent to continuity of the longitudinal magnetic field, while continuity of the transverse magnetic field is equivalent to continuity of the \( z \)-derivation of the longitudinal magnetic field. Thus, the continuity conditions at \( z = t \) can be expressed in terms of the longitudinal magnetic field and its \( z \)-derivative. It will be advantageous to make use of this fact in deriving the cavity characteristic equation, since it is useful to specify this magnetic field component for each mode (Appendix 5). When searching experimentally for resonance frequencies, coupling to the cavity is believed to be closely related to the longitudinal magnetic field at the coupling hole, with observed coupling expected to vary with hole location according to the amplitude of the field component. Thus, if predicted cavity modes can be characterized according to \( z \)-dependence of the longitudinal magnetic field, in addition to resonance frequency, a much better basis for identification of observed resonances with predicted ones is provided.
More specific cavity field expressions are obtained by taking into account the boundary conditions at the ends of the half-cavity. These also can be given in terms of the longitudinal magnetic field and its \( \varepsilon \)-derivative. The arbitrary coefficients \( A \), \( B \) and \( S \) are subscripted to indicate the appropriate region. There are two cases:

**Even Cavity Solution**

\[
E_{\perp}(z = 0) = 0 \Rightarrow H_{\perp}(z = 0) = 0 \Rightarrow B_1 = -A_1 \quad (A-4-6)
\]

\[
H_{\parallel}(z = l) = 0 \Rightarrow \frac{\partial H_{\perp}}{\partial z}(z = l) = 0 \Rightarrow B_2 = A_2 e^{-2\varepsilon l} \quad (A-4-7)
\]

**Odd Cavity Solution**

\[
E_{\perp}(z = 0) = 0 \Rightarrow H_{\perp}(z = 0) = 0 \Rightarrow B_1 = -A_1 \quad (A-4-8)
\]

\[
E_{\perp}(z = l) = 0 \Rightarrow H_{\perp}(z = l) = 0 \Rightarrow B_2 = -A_2 e^{-2\varepsilon l} \quad (A-4-9)
\]

Every cavity solution must correspond to propagation in region 1, since at least one of the regions must be propagating and region 1 has the greater permittivity. Thus, \( S_1 \) becomes \( j\beta z_1 \), while \( S_2 \) becomes either \( z_2 \) or \( j\beta_2 z_2 \).

Having taken into account the conditions at \( z = 0 \) and \( z = l \), and making the appropriate substitutions for \( S_1 \) and \( S_2 \), the expressions for \( H_{\parallel} \) and \( \frac{\partial H_{\perp}}{\partial z} \) are:

**Region 1**

\[
H_{\parallel} = -j \beta z_1 k z_1 J_n(k r) e^{j n \phi} \sin \beta_1 z_1 \quad (A-4-10)
\]

\[
\frac{\partial H_{\perp}}{\partial z} = -j \beta_1 A_1 k z_1 J_n(k r) e^{j n \phi} \cos \beta_1 z_1 \quad (A-4-11)
\]
Region 2

Even, below cutoff:

\[ H_z = \sum A_2 k_c^2 J_n(k_c r) e^{in\phi} e^{-\alpha_2 l \cosh \alpha_2 (l-z)} \quad (A-4-12) \]
\[ \frac{\partial H_z}{\partial z} = -2\alpha_2 A_2 k_c^2 J_n(k_c r) e^{in\phi} e^{-\alpha_2 l \sinh \alpha_2 (l-z)} \quad (A-4-13) \]

Even, above cutoff:

\[ H_z = \sum A_2 k_c^2 J_n(k_c r) e^{in\phi} e^{-\beta_2 l \cos \beta_2 (l-z)} \quad (A-4-14) \]
\[ \frac{\partial H_z}{\partial z} = 2\beta_2 A_2 k_c^2 J_n(k_c r) e^{in\phi} e^{-\beta_2 l \sin \beta_2 (l-z)} \quad (A-4-15) \]

Odd, below cutoff:

\[ H_z = \sum A_2 k_c^2 J_n(k_c r) e^{in\phi} e^{-\alpha_2 l \sinh \alpha_2 (l-z)} \quad (A-4-16) \]
\[ \frac{\partial H_z}{\partial z} = -2\alpha_2 A_2 k_c^2 J_n(k_c r) e^{in\phi} e^{-\alpha_2 l \cosh \alpha_2 (l-z)} \quad (A-4-17) \]

Odd, above cutoff:

\[ H_z = j \sum A_2 k_c^2 J_n(k_c r) e^{in\phi} e^{-\beta_2 l \sin \beta_2 (l-z)} \quad (A-4-18) \]
\[ \frac{\partial H_z}{\partial z} = -j \beta_2 A_2 k_c^2 J_n(k_c r) e^{in\phi} e^{-\beta_2 l \cos \beta_2 (l-z)} \quad (A-4-19) \]

The \( r \) and \( \phi \) spatial dependence is omitted in the remaining expressions. Further generality is removed from the field expressions by requiring continuity of \( H_z \) at \( z = t \), where \( A_1 \) and \( A_2 \) are newly defined arbitrary constants:

Even, below cutoff:

\[ A_1 \sin \beta_1 t = A_2 \cosh \alpha_2 (l-t) \]
\[ \Rightarrow A_2 = \frac{A_1 \sin \beta_1 t}{\cosh \alpha_2 (l-t)} \quad (A-4-20) \]
Even, above cutoff:

\[ A_1 \sin \beta_1 t = A_2 \cos \beta_2 (l-t) \]

\[ \Rightarrow A_2 = A_1 \sin \beta_1 t / \cos \beta_2 (l-t) \] \hspace{1cm} (A-4-21)

Odd, below cutoff:

\[ A_1 \sin \beta_1 t = A_2 \sinh \alpha_2 (l-t) \]

\[ \Rightarrow A_2 = A_1 \sin \beta_1 t / \sinh \alpha_2 (l-t) \] \hspace{1cm} (A-4-22)

Odd, above cutoff:

\[ A_1 \sin \beta_1 t = A_2 \sin \beta_2 (l-t) \]

\[ \Rightarrow A_2 = A_1 \sin \beta_1 t / \sin \beta_2 (l-t) \] \hspace{1cm} (A-4-23)

This results in the following expressions:

Region 1

\[ H_z = A_1 \sin \beta_1 z \] \hspace{1cm} (A-4-24)

\[ \frac{\partial H_\phi}{\partial z} = \beta_1 A_1 \cos \beta_1 z \] \hspace{1cm} (A-4-25)

Region 2

Even, below cutoff:

\[ H_z = A_1 \frac{\sin \beta_1 t}{\cosh \alpha_2 (l-t)} \cos \alpha_2 (l-z) \] \hspace{1cm} (A-4-26)

\[ \frac{\partial H_\phi}{\partial z} = -\alpha_2 A_1 \frac{\sin \beta_1 t}{\cosh \alpha_2 (l-t)} \sinh \alpha_2 (l-z) \] \hspace{1cm} (A-4-27)

Even, above cutoff:

\[ H_z = A_1 \frac{\sin \beta_1 t}{\cos \beta_2 (l-t)} \cos \beta_2 (l-z) \] \hspace{1cm} (A-4-28)

\[ \frac{\partial H_\phi}{\partial z} = \beta_2 A_1 \frac{\sin \beta_1 t}{\cos \beta_2 (l-t)} \sin \beta_2 (l-z) \] \hspace{1cm} (A-4-29)
Odd, below cutoff

\[ H_z = A_1 \frac{\sin \beta_1 t}{\sinh \alpha_2 (l-z)} \sinh \alpha_2 (l-z) \]  
(A-4-30)

\[ \frac{\partial H_z}{\partial z} = -\alpha_2 A_1 \frac{\sin \beta_1 t}{\sinh \alpha_2 (l-t)} \cosh \alpha_2 (l-z) \]  
(A-4-31)

Odd, above cutoff

\[ H_z = A_1 \frac{\sin \beta_2 t}{\sin \beta_2 (l-z)} \sin \beta_2 (l-z) \]  
(A-4-32)

\[ \frac{\partial H_z}{\partial z} = -\beta_2 A_1 \frac{\sin \beta_1 t}{\sin \beta_2 (l-t)} \cos \beta_2 (l-z) \]  
(A-4-33)

It is seen that \( H_z \) is identically continuous at \( z = t \), but that this is not the case for \( \frac{\partial H_z}{\partial z} \). Frequencies at which both quantities are continuous correspond to cavity resonances.

One first expresses the continuity of \( \frac{\partial H_z}{\partial z} \) at \( t \) algebraically:

Even, below cutoff:

\[ -\beta_1 \cos \beta_1 t = \alpha_2 \frac{\sin \beta_1 t}{\cosh \alpha_2 (l-t)} \sinh \alpha_2 (l-t) \]  
(A-4-34)

Even, above cutoff:

\[ -\beta_1 \cos \beta_1 t = -\beta_2 \frac{\sin \beta_1 t}{\cos \beta_2 (l-t)} \sin \beta_2 (l-t) \]  
(A-4-35)

Odd, below cutoff:

\[ -\beta_1 \cos \beta_1 t = \alpha_2 \frac{\sin \beta_1 t}{\sinh \alpha_2 (l-t)} \cosh \alpha_2 (l-t) \]  
(A-4-36)
Odd, above cutoff:

$$-\beta_1 \cos \beta_1 t = \beta_2 \frac{\sin \beta_2 t}{\sin \beta_2 (l-t)} \cos \beta_2 (l-t) \quad (A-4-37)$$

Making the definitions \(l_1 = t\) and \(l_2 = l - t\), as shown in Figure A.4.2, these conditions can be rewritten to obtain a cavity characteristic equation for each of the various cases. Frequencies at which this equation is satisfied are resonance frequencies for the end-loaded cavity.

**Even Cavity Solution**:

$$\frac{1}{\beta_1} \tan \beta_1 l_1 = \begin{bmatrix} -\frac{1}{\alpha_2} \coth \alpha_2 l_2 \\ \frac{1}{\alpha_2} \cot \alpha_2 l_2 \end{bmatrix}$$

below cutoff in 2

$$\frac{1}{\beta_2} \tan \beta_2 l_2 = \begin{bmatrix} -\frac{1}{\alpha_2} \tanh \alpha_2 l_2 \\ \frac{1}{\alpha_2} \tan \alpha_2 l_2 \end{bmatrix}$$

above cutoff in 2

(A-4-38) (A-4-39)

**Odd Cavity Solution**:

$$\frac{1}{\beta_1} \tan \beta_1 l_1 = \begin{bmatrix} -\frac{1}{\alpha_2} \tanh \alpha_2 l_2 \\ -\frac{1}{\alpha_2} \tan \alpha_2 l_2 \end{bmatrix}$$

below cutoff in 2

(A-4-40) (A-4-41)

where

$$\beta_1^2 = \varepsilon_1 \left(\frac{2 \pi}{\lambda_1^2}\right)^2 f^2 - k_c^2 \quad , \quad f \text{ in GHz}$$

(A-4-42)

and

$$\alpha_2^2 = -\left[\varepsilon_2 \left(\frac{2 \pi}{\lambda_2^2}\right)^2 f^2 - k_c^2 \right]$$

below cutoff in region 2

(A-4-43)

or

$$\beta_2^2 = \left[\varepsilon_2 \left(\frac{2 \pi}{\lambda_2^2}\right)^2 f^2 - k_c^2 \right]$$

above cutoff in region 2

(A-4-44)

The constant \(k_c\) depends on the circular guide mode being considered and is given by \(k_c = x_{nm}'\), where \(x_{nm}'\) is the \(m^{th}\) zero of \(J_n'(x)\).
Defining four new quantities,

\[ l_{r1} = \frac{l_1}{l} \quad (A-4-45) \]
\[ l_{r2} = \frac{l_2}{l} \quad (A-4-46) \]
\[ \alpha = \beta_1 \cdot l \quad (A-4-47) \]
\[ \nu = \begin{bmatrix} \alpha_2 \cdot l \\ \beta_2 \cdot l \end{bmatrix} \quad \begin{array}{c} \text{below cutoff} \\ \text{above cutoff} \end{array} \quad (A-4-48) \]

the cavity characteristic equation can be rewritten a final time:

**Even Cavity Modes**:

\[ \frac{1}{u} \tan u_l r_l = \begin{bmatrix} \frac{1}{u} \coth \nu \cdot l_{r2} \\ \frac{1}{u} \cot \nu \cdot l_{r2} \end{bmatrix} \quad \begin{array}{c} \text{below cutoff} \\ \text{above cutoff} \end{array} \quad (A-4-50) \]

**Odd Cavity Modes**:

\[ \frac{1}{u} \tan u_l r_l = \begin{bmatrix} \frac{1}{u} \tanh \nu \cdot l_{r2} \\ \frac{1}{u} \tan \nu \cdot l_{r2} \end{bmatrix} \quad \begin{array}{c} \text{below cutoff} \\ \text{above cutoff} \end{array} \quad (A-4-52) \]
\[ \varepsilon_{r1} > \varepsilon_{r2} \]

**Figure A.4.1**

The End-Loaded Cavity

\[ \varepsilon_{r1} > \varepsilon_{r2} \]

**Figure A.4.2**

The Half-Cavity
APPENDIX 5

FIELD AMPLITUDE PLOTS FOR AN END-LOADED CIRCULAR WAVEGUIDE CAVITY

As mentioned in Appendix 4, coupling into an end-loaded circular waveguide cavity by means of a hole in the curved wall is believed to be closely related to the longitudinal magnetic field at the hole. A strong correspondence is expected between the dependence on hole position of energy absorbed by the cavity and the axial variation of the longitudinal magnetic field. Therefore, it is useful to carry out the derivations necessary for constructing plots which give for each predicted mode the \( z \)-dependence of this magnetic field component. These plots can then be studied in conjunction with observed behavior of energy absorption as hole position is varied to aid in the identification of resonances and the evaluation of predictions.

The \( z \)-component of the time-varying magnetic field, with \( r \) and \( \phi \) dependence omitted, can be derived from the corresponding complex field component by multiplying by \( \exp(j\omega t) \), assuming \( A_1 \) to be real, and taking the real part of the product.

**Region 1**

\[
H_z = A_1 \sin \frac{\nu l}{\ell} \cos \omega t \quad (A-5.1)
\]

**Region 2**

Even, below cutoff

\[
H_z = A_1 \frac{\sin \frac{\nu l r_1}{\ell}}{\cosh \nu \frac{(z-\ell)}{\ell} \cos \omega t} \quad (A-5.2)
\]
Even, above cutoff

\[ \mathcal{H}_z = A_1 \frac{\sin \frac{u \ell}{r_1}}{\cos \frac{u \ell}{r_z}} \cos \frac{u (L-z)}{L} \cos \omega t \quad (A-5-3) \]

Odd, below cutoff

\[ \mathcal{H}_z = A_1 \frac{\sin \frac{u \ell}{r_1}}{\sinh \frac{u \ell}{r_z}} \sinh \frac{u (L-z)}{2} \cos \omega t \quad (A-5-4) \]

Odd, above cutoff

\[ \mathcal{H}_z = A_1 \frac{\sin \frac{u \ell}{r_1}}{\sin \frac{u \ell}{r_z}} \sin \frac{u (L-z)}{L} \cos \omega t \quad (A-5-5) \]

It is seen that the time dependence of the field component appears as a factor independent of \( z \). Thus, the longitudinal variation of \( \mathcal{H}_z \) is fully described by a plot of the time-independent factor.

To reduce the amount of space required, only the magnitude of the \( z \)-dependent factor need be plotted, with sign symbolically denoted by '+' or '-'. The plot extends only over the half-cavity. It is understood that the longitudinal magnetic field possesses either even or odd symmetry about the actual cavity center plane, according to the cavity mode designation.

The quantity whose magnitude is to be plotted will be denoted by 'h\(_z\)', and is given by:

**Region 1**

\[ h_z = A_1 \sin \frac{u \pi}{L} \quad (A-5-6) \]

**Region 2**

Even, below cutoff

\[ h_z = A_1 \frac{\sin \frac{u \ell}{r_1}}{\cosh \frac{u \ell}{r_z}} \cosh \frac{u (L-z)}{2} \quad (A-5-7) \]
Even, above cutoff

\[ h_z = A_1 \frac{\sin \frac{ul_1}{\cos \frac{ul_2}{}}}{\cos \frac{ul_1}{}} \cos \frac{\nu (l-z)}{l} \quad (A-5-8) \]

Odd, below cutoff

\[ h_z = A_1 \frac{\sin \frac{ul_1}{\sinh \frac{ul_2}{}}}{\sinh \frac{ul_2}{}} \sinh \frac{\nu (l-z)}{l} \quad (A-5-9) \]

Odd, above cutoff

\[ h_z = A_1 \frac{\sin \frac{ul_1}{\sin \frac{ul_2}{}}}{\sin \frac{ul_2}{}} \sin \frac{\nu (l-z)}{l} \quad (A-5-10) \]

The constant \( A_1 \) is arbitrary and will be chosen so that the maximum value attained in the plot of \( |h_z| \) is unity. One must now determine a consistent method by which this normalization can be carried out. The task is separated into four general cases, according to mode symmetry and whether or not the solution is propagating in region 2. Then each of these situations is broken down further, if necessary. The subscript has been omitted in the notation for the constant \( A_1 \).

Even solution, region 2 below cutoff

Inspection of the expression for \( h_z \) in region 2 reveals that the sign of the quantity does not change and that its magnitude is always decreasing as \( z \) increases. Because the derivative of \( h_z \) must be continuous for a cavity solution, and in particular, is continuous at \( z = l \), the maximum value of \( |h_z| \) must occur in region 1. Since \( |h_z| \) must be decreasing at \( z = l \), it is known that \( ul_1/l = ul_2 > \pi/l \). Thus, \( |\sin ul_2/l| \) reaches a value of unity in region 1 and the maximum value of \( |h_z| \) occurring in region 1 is \( A \). This is therefore the overall maximum of
\[ |h_{\bar{z}}| \leq 1. \text{ A is thus set to 1.} \]

**Even solution, region 2 above cutoff**

The expression for \( h_{\bar{z}} \) in region 2 takes on its maximum magnitude at \( \bar{z} = l \). This magnitude is given by \( A |\sin \frac{u.l_{1}}{\cos v.l_{2}}| \). Since \( |h_{\bar{z}}| \) can under no conditions take on a greater value than \( A \) in region 1, the overall maximum of \( |h_{\bar{z}}| \) will occur in region 2 if \( |\sin \frac{u.l_{1}}{\cos v.l_{2}}| > 1 \), this maximum will be given by \( A |\sin \frac{u.l_{1}}{\cos v.l_{2}}| \) and normalization is obtained by requiring \( A = |\cos v.l_{2}/\sin u.l_{1}| \). If \( |\sin \frac{u.l_{1}}{\cos v.l_{2}}| < 1 \), the maximum magnitude of \( h_{\bar{z}} \) may occur in either region. In the case that \( u.l_{1} \geq \pi/2 \) a maximum value of \( A \) is attained by \( |h_{\bar{z}}| \) in region 1, this becomes the overall maximum and the normalization becomes \( A = 1 \). For \( |\sin \frac{u.l_{1}}{\cos v.l_{2}}| < 1 \), the case remains. For this condition, \( h_{\bar{z}} \) increases monotonically over the range \( \bar{z} = 0 \) to \( \bar{z} = t \), reaches its greatest value of region 1 at \( \bar{z} = t \), and has a positive derivative there. Since \( \frac{dh_{\bar{z}}}{d\bar{z}} \) is continuous, \( h_{\bar{z}} \) must attain a greater value in region 2 than it does at \( \bar{z} = t \). Thus the overall maximum of \( h_{\bar{z}} \) occurs in region 2, and is given by \( A |\sin \frac{u.l_{1}}{\cos v.l_{2}}| \), thus leading to the normalization condition \( A = |\cos v.l_{2}/\sin u.l_{1}| \). Only the case \( |\sin \frac{u.l_{1}}{\cos v.l_{2}}| = 1 \) has not been specifically considered and it is easily shown that this leads to \( A = 1 \).

**Odd solution, region 2 below cutoff**

As in the first main case, the expression for \( h_{\bar{z}} \) in region 2 is always decreasing in magnitude as \( \bar{z} \) increases from \( t \) to \( l \). Again because of the continuity of the
derivative of $h_z$, the magnitude of $h_z$ must reach its maximum in the first region and the condition $u \cdot \ell_{r_1} > \pi/2$ must be satisfied. This implies that $\left| \sin \theta z / \ell \right|$ attains a maximum value of unity in region 1 and that the overall maximum of $\left| h_z \right|$ is A. The proper normalization is thus A = 1.

Odd solution, region 2 above cutoff

Here is the most complicated case of the four. The slope of $h_z$ is not single-signed in either of the two regions and in neither region is it certain that the expression for $\left| h_z \right|$ reaches its maximum possible value. That is, it is not immediately obvious that $\sin \theta z / \ell$ reaches a value of 1 in region 1 or that $\sin \theta (l - z) / \ell$ reaches a value of 1 in region 2 — although at least one of these conditions must occur. This case will be divided into five possible situations based on the values of $u \cdot \ell_{r_1}$ and $v \cdot \ell_{r_2}$. The situation $u \cdot \ell_{r_1}, v \cdot \ell_{r_2} < \pi/2$ cannot occur because of the requirement that $\frac{d h_z}{d z}$ be continuous. The situation $u \cdot \ell_{r_1}, v \cdot \ell_{r_2} > \pi/2$ can occur, however, and must itself be separated into two cases. Both $\sin \theta z / \ell$ and $\sin \theta (l - z) / \ell$ reach the maximum possible value of 1 in the respective regions. Thus, the overall maximum of $\left| h_z \right|$ is either A or $A \left| \sin u \cdot \ell_{r_1} / \sin v \cdot \ell_{r_2} \right|$, depending on the value taken by $\left| \sin u \cdot \ell_{r_1} / \sin v \cdot \ell_{r_2} \right|$. For $\left| \sin u \cdot \ell_{r_1} / \sin v \cdot \ell_{r_2} \right| < 1$ the normalization becomes A = 1, while for $\left| \sin u \cdot \ell_{r_1} / \sin v \cdot \ell_{r_2} \right| > 1$ the proper normalization is $A = \left| \sin v \cdot \ell_{r_2} / \sin u \cdot \ell_{r_1} \right|$. The first normalization, A = 1, also holds if $\left| \sin u \cdot \ell_{r_1} / \sin v \cdot \ell_{r_2} \right| = 1$. The conditions $u \cdot \ell_{r_1} < \pi/2, v \cdot \ell_{r_2} > \pi/2$ and $u \cdot \ell_{r_1} > \pi/2, v \cdot \ell_{r_2} < \pi/2$ are
somewhat simpler. Based on knowledge of the function \( \sin \frac{u \pi}{2}, u \leq \frac{\pi}{2} \) implies that \( h \) is monotonically increasing over the interval between \( z = 0 \) and \( z = t \), that \( h \) reaches its greatest region 1 value at \( z = t \), and that the slope of \( h \), given by \( \frac{dh}{dz} \), is positive at \( z = t \). Because \( \frac{dh}{dz} \) is continuous everywhere, a value larger than that at \( z = t \) must be attained by \( h \) in region 2. Thus, the overall maximum of \( |h| \) occurs in the second region of the half-cavity and is given by \( A \left| \frac{\sin u l_{r1}}{\sin v l_{r2}} \right| \). The required normalization is therefore \( A \left| \frac{\sin v l_{r2}}{\sin u l_{r1}} \right| \). By a very similar argument it can be shown that for \( u l_{r1} > \frac{\pi}{2}, v l_{r2} < \frac{\pi}{2} \) one must require that \( A = 1 \). The final case is for \( u l_{r1} = \frac{\pi}{2} \) or \( v l_{r2} = \frac{\pi}{2} \). Based on knowledge of the functions involved and taking into account the required continuity of \( \frac{dh}{dz} \), this condition implies that \( h \) has the same maximum value in both regions, that this value is \( A = A \left| \frac{\sin u l_{r1}}{\sin v l_{r2}} \right| \), and therefore that \( \left| \frac{\sin u l_{r1}}{\sin v l_{r2}} \right| = 1 \) and \( A \) should be set to 1. The normalizations are now summarized according to the cases for which they apply.

**Even solution, region 2 below cutoff**

\[
A = 1 \quad (A \cdot 5 \cdot 11)
\]

**Even solution, region 2 above cutoff**

\[
\left| \frac{\sin u l_{r1}}{\cos v l_{r2}} \right| \geq 1 \Rightarrow A \left| \frac{\cos v l_{r2}}{\sin u l_{r1}} \right| \quad (A \cdot 5 \cdot 12)
\]

\[
\left| \frac{\sin u l_{r1}}{\cos v l_{r2}} \right| < 1, u l_{r1} \geq \frac{\pi}{2} \Rightarrow A = 1 \quad (A \cdot 5 \cdot 13)
\]

\[
\left| \frac{\sin u l_{r1}}{\cos v l_{r2}} \right| < 1, u l_{r1} < \frac{\pi}{2} \Rightarrow A = \left| \frac{\cos v l_{r2}}{\sin u l_{r1}} \right| \quad (A \cdot 5 \cdot 14)
\]
Odd solution, region 2 below cutoff

\[ A = 1. \]  \hfill (A-5-15)

Odd solution, region 2 above cutoff

\[ u - l_{r1} , v - l_{rz} < \pi / 2 \quad \text{cannot occur} \]  \hfill (A-5-16)

\[ u - l_{r1} , v - l_{rz} > \pi / 2 \]

\[ |\sin u - l_{r1} / \sin v - l_{rz}| \leq 1 \Rightarrow A = 1 \]  \hfill (A-5-17)

\[ |\sin u - l_{r1} / \sin v - l_{rz}| > 1 \Rightarrow A = |\sin v - l_{rz} / \sin u - l_{r1}| \]  \hfill (A-5-18)

\[ u - l_{r1} < \pi / 2 , v - l_{rz} > \pi / 2 \Rightarrow A = |\sin v - l_{rz} / \sin u - l_{r1}| \]  \hfill (A-5-19)

\[ u - l_{r1} > \pi / 2 , v - l_{rz} < \pi / 2 \Rightarrow A = 1 \]  \hfill (A-5-20)

\[ u - l_{r1} = \pi / 2 \text{ or } v - l_{rz} = \pi / 2 \Rightarrow A = 1 \]  \hfill (A-5-21)
RESONANT FREQUENCIES OF FERRITE-END-LOADED
CIRCULAR WAVEGUIDE CAVITY

THIS PROGRAM PREDICTS X-BAND RESONANT FREQUENCIES OF A
CIRCULAR WAVEGUIDE CAVITY END-LOADED WITH THICK FERRITE
DISKS. THESE PREDICTIONS ARE MADE UNDER SEVERAL RESTRICTIONS.
ONLY AZIMUTHAL DEPENDENCE CORRESPONDING TO N = ±1 OR
N=±1 IS CONSIDERED. WE ASSUME A CAVITY SOLUTION THAT HAS
EVEN AXIAL SYMMETRY. ONLY ONE BASIS MODE IS RETAINED IN
EACH CAVITY RESONANCE. ONLY FREQUENCIES FOR WHICH THE DIELETRIC BASIS MODE PROPAGATES ARE INVESTIGATED. THE DC
MAGNETIC FIELD IS CHOSEN SO THAT A SINGLE PROPAGATING
FERRITE MODE SEEKS MOST IMPORTANT THROUGHOUT THE X-BAND.
THE CHOICE OF THIS FIELD INFLUENCES THE PROCEDURE BY
WHICH THE FERRITE PROPAGATION CONSTANTS ARE FOUND. THE
RESONANT FREQUENCY INFORMATION IS PRESENTED IN THE FORM OF
A TABLE GIVING THE CHARACTERISTIC QUANTITY FOR N=1 AND
N=±1 AS A FUNCTION OF FREQUENCY. THE RADIUS OF THE CA
VITY IS .507 CM., THE LENGTH IS 2.246 CM.

INPUT PARAMETERS
DIMENSION HJ(3), H1(3), RNRNM(20), RNF(2), ALMRA1(2),
          ALMRA2(2), NCASE(2), SF1(20), SF2(20), COP(2),
          CWJ(2)

REAL KC, NU, LAM1A1, LAM1A2, K11S0, K12SQ, LAM2P, LAM2P2,
          LM31, LM32, K11S0P, K12SQP, K11S0N, K12SQN, 11, 12

100 FORMAT (E20.4)
101 FORMAT (I20)
102 FORMAT (/36x,61MCIRCULAR WAVEGUIDE CAVITY END-LOADED WITH FERRITE DISKS, 37x,60MIREVERSAL OF RESONANT FREQUENCIES FOR A SPECIAL CASE, 90x,91MONLY GIVING THE N = +1 AND N = -1 CHARACTERISTIC QUANTITIES AS FUNCTIONS OF FREQUENCY/)
103 FORMAT (2X,2AHCAVITY LENGTH = E2.286 CM./)
104 FORMAT (2X,2AHCAVITY RADIUS = E2.287 CM./)
105 FORMAT (2X,3AHTHICKNESS OF FERRITE DISKS = E2.53,4H CM./)
106 FORMAT (2X,32HPI+MSAT FOR THE FERRITE = E2.54,6H GAUSS/)
107 FORMAT (2X,45HRELATIVE PERMITTIVITY OF THE FERRITE = E2.31/)

C INITIAL FREQUENCY IN GHZ = E3.401
C FINAL FREQUENCY IN GHZ = E3.401
C FREQUENCY INCREMENT IN GHZ = E3.190
C RELATIVE PERMITTIVITY OF THE DIELECTRIC = E3.190
C DISK THICKNESS IN CENTIETERS = E3.190
C UNIFORM INTERNAL MAGNETIC BIAS FIELD = E3.190
C NUMBER OF INTERVALS USED IN NUMERICAL INTEGRATIONS = E3.190
C REAL PART OF CHARACTERISTIC QUANTITY = E3.190
C IMAGINARY PART OF THE CHARACTERISTIC QUANTITY = E3.190
108 FORMAT (2x,25H EFFECTIVE G FACTOR = $F4.7/)
109 FORMAT (2x,25H INTERNAL bias FIELD = $F4.0/)
110 FORMAT (2x,35H ELECTRIC RELATIVE PERMITTIVITY = $F3.1/)
111 FORMAT (2x,30H FREQUENCY RANGE SCANNED = $F4.1, $1H GHz, TO $F4.1, $5H GHz/)

112 FORMAT (2x,25H FREQUENCY INCREMENT = $F4.7, $5H GHz/)
113 FORMAT (2x,61H NUMBER OF INTERVALS USED IN NUMERICAL INTEGRATION = $14/)
114 FORMAT (/$2x,94MIN A SEARCH FOR THE +1 FERRITE PROPAGATION$)
   C CONSTANT K1150 HAS BECOME NON-NEGATIVE, /2x, 17H EXECUTION HALTED
   CED)
115 FORMAT (/$2x,94MIN A SEARCH FOR THE +1 FERRITE PROPAGATION$)
   C CONSTANT K1254 HAS BECOME NON-POSITIVE, /2x, 17H EXECUTION HALTED
   CED)
116 FORMAT (/$2x,42HERMUT IN J BESSEL FUNCTION GENERATION/2x$
   C 17H EXECUTION HALTED)
117 FORMAT (/$2x,42HERMUT IN J BESSEL FUNCTION GENERATION/2x$
   C 17H EXECUTION HALTED)
120 FORMAT (/$2x,94MIN, NO FERRITE SOLUTION CURVE IN A +1 SEARCH/2x, 17H EXECUTION HALTED)
119 FORMAT (/$2x,76MIN, THE SEARCH FOR THE N = +1 SOLUTION CURVE$)
   C K1250 HAS BECOME NEGATIVE, /2x, 17H EXECUTION HALTED)
120 FORMAT (/$2x,94MIN, NO-POSITIVE K1150 HAS BEEN GENERATED IN$)
   C THE SEARCH FOR THE N = +1 SOLUTION CURVE/2x, 18H EXECUTION HALTED)
121 FORMAT (2x,34H FREQUENCY(GHz), $1H MCG, 4H MCG, 4H MCG/)
122 FORMAT (2x,23, $8, $4E25,A/)
123 FORMAT (/$2x,39H SCANNING OF FREQUENCIES IS COMPLETE$)
C

C

READ (5,100) FM
READ (5,100) IR
READ (5,100) NM
READ (5,100) RPM
READ (5,100) T
READ (5,100) NINTS

C

C THE FERRITE DISK THICKNESS IS CONVERSION FROM CENTIMETERS
C TO METERS.

T = 0.1 * T

C

C THE NUMBER OF VALUES OF NORMALIZED RADIUS USED IN THE
C NUMERICAL INTEGRATIONS IS GIVEN BY

NORM = NINTS + 1

C

WRITE (6,102)
WRITE (6,103)
WRITE (6,104)
WRITE (6,105) T

C

C PARAMETERS FOR THE TI-P-118 FERRITE ARE AS FOLLOWS

FMSAT = 1800

RPERMF = 9.5

GEFF = 2.55

C

WRITE (6,106) FMSAT
WRITE (6,107) RPERMF
WRITE (6,108) GEFF
WRITE (*,107) MIOM1
WRITE (*,110) NPEMND
WRITE (*,111) FRQI, FROF
WRITE (*,112) MI, M0
WRITE (*,113) NINTS
WRITE (*,121)
C
PI = 3.1415927
FROGHZ = FRQI
C
C CALCULATION OF THE NORMALIZED QUANTITIES
C
200 HETAOI = 2.*PI*FROGHZ*SQRT(NPEMND)/3
RONRM = 0.05987*HETAOI
LONRM = 0.02246*HETAOI
THNRM = TNFSS*HETAOI
KC = 1.84118/RONRM
C
C THE CALCULATION OF THE CAVITY CHARACTERISTIC EQUATION IS
C NOT CARRIED OUT AT A GIVEN FREQUENCY UNLESS THE DIELEC=
C TRIC SECTION IS PROPAGATING.
C
IF ((KPEFM/3NPEFM - KC**2) .LE. .0) GO TO 550
C
RETANU = SQRT(KPFRM**2/KPFM**2)
SIGMA = 1.6*GELF*HOCINT/(FROGHZ**1000.*)
P = 1.6*GELF*FPHSAT/(FROGHZ**1000.*)
MU = (1. - SIGMA**2 - P*SIGMA)/(1. - SIGMA**2)
RNM = P/(1. - SIGMA**2 - P*SIGMA)
C
DETERMINATION OF THE NORMALIZED FERITYPE PROPAGATION CONSTANT
FOR THE N=1 AND N=1 CASES

THE N=1 CASE

K1ST = 0

THE STARTING LAMDA1 IS CALCULATED

LAMDA1 = (SIGMA + P + 1.)/(1. + SIGMA)

LAMDA1 IS RECHENEMTED

LAMDA1 = LAMDA1 = .001

K11SQ = (1. - LAMDA1**2)/(1. - SIGMA*LAMDA1)

LAMDA2 = (SIGMA + P - LAMDA1)/(1. - SIGMA*LAMDA1)

K12SQ = (1. - LAMDA2**2)/(1. - SIGMA*LAMDA2)

AS LONG AS THE POINT (LAMDA1, SIGMA) REMAINS IN THE
REGION BETWEEN THE (1-R)PRIME AND (2-R=0)PRINT
CURVES; NEITHER K11SQ NOR K12SQ CAN BE ZERO. IF
THE LATTER OF THESE CURVES IS CHANGED BEFORE A SO-
LUTION IS FOUND, EXECUTION IS Halted, WE EXPECT K11SQ
TO BE NEGATIVE AND K12SQ TO BE POSITIVE. THESE CONDI-
TIONS ARE CHECKED.

IF (K11SQ .LT. 0) GO TO 220

WRITE (6,114) I, GO TO 560

220 IF (K12SQ .GT. 0) GO TO 230

WRITE (6,115) I, GO TO 560
230 X = RUNNRM*SORT(K1750) 1 Y = RUNRNM*SORT(K1150)

270 DD 270 NORDER = 0,2,1

270 CALL HESJ (X, NORDER, R, IEN)

270 IF (IEN .EQ. 0) GO TO 250

280 WRITE (6,116) 1 GO TO 560

250 BJ(NORDER + 1) = H

250 CALL HESJ (Y, NORDER, R, IEN)

250 IF (IEN .EQ. 0) GO TO 270

260 WRITE (6,117) 1 GO TO 560

270 BJ(NORDER + 1) = H

C

C WE EXPLECT AJ(2) AND B1(2) TO BE NON-ZERO.

C

F1 = X*(BI(1) + BI(3))/2*BI(2)

F2 = X*(BJ(1) - BJ(3))/2*BJ(2)

G1 = (F1/LAMDA1 - 1.)/K1159

G2 = (F2/LAMDA2 - 1.)/K1750

DG = G2 - G1

IF (DG .EQ. 0) GO TO 300

IF (NTEST .EQ. 1) GO TO 280

NTEST = 1; DG1AST = DG; G2LAST = G2; GO TO 210

C

C WE MUST VERIFY THAT THE SEARCH FOR THE SOLUTION CURVE

C HAS NOT FAILED. WE ASSUME THAT THE REGION ADJACENT TO

C THAT IN WHICH THE SOLUTION CURVE LIES HAS NOT AFFECTED

C

C

260 IF (G2LAST .GE. G1, .0) GO TO 290

260 WRITE (6,118) 1 GO TO 560

290 IF (DG1AST .GE. G1, .0) GO TO 300

290 DGLAST = DG; G2LAST = G2; GO TO 210
300 CONTINUE

C A SOLUTION OF THE FERRITE FILLED GUIDE CHARACTERISTIC
C EQUATION FOR N=1 HAS BEEN FOUND.

LM11P = LAMDA1 ; LM11P = LAMDA2
K11SOP = PI150 ; K12SOP = K1250

C FNFP IS THE NORMALIZED FERRITE PROPAGATION CONSTANT FOR
C THE N = +1 CASE.
FNFP = SQRT((LM11P*LM11P)

C
C SOLUTION FOR THE FERRITE PROPAGATION CONSTANT FOR N = -1
C
SIGMA = 4SIGMA ; PN = -P ; NTEST = 0

C A SEARCH FOR THE APPROPRIATE SOLUTION CURVE IS CARRIED
C OUT IN THE THIRD QUADRANT OF THE LANDA, SIGMA PLANE.
C FIRST, A STARTING LAMDA2 IS FOUND.

IF (ROMNORM .GT. 3.932) GO TO 310

LAMDA2 = 3.932 AND GO TO 320

310 CONTINUE

C
C AN II CURVE IS KNOWN TO EXIST IN THE NEGATIVE LAMDA
C SIDE OF THE IA CURVE FOR SIGMA GREATER THAN -1. THUS,
C WE MUST DECIDE WHETHER THE SCAN WILL BEGIN AT THE II-
C PRIME CURVE OR THE II CURVE.
C
LAMDA1 = SIGMA = (1.832/ROMNORM)**2 +
C \[ Q = 0.5 \cdot \text{SQRT}(\text{SIGN} + 2 \cdot (3.83/\text{WONORM})^2 + 4) \cdot \]
\[ Q^2 \cdot (3.83/\text{WONORM})^2 \]
\[ \text{LAMDA} = (\text{SIGN} + \text{PN} - \text{LAMDA})/(1 - \text{SIGN} \cdot \text{LAMDA}) \]
\[ \text{LAMDA}^2 = \text{LAMDA}(1 - \text{LAMDA}) \]

C Since it is difficult to determine \text{LAMDA} at which the search for the solution curve must be discontinued, if
C assume the search will be successful,
C
LAMDA2 is incremented

320 \text{LAMDA} = \text{LAMDA} + 0.001

C \text{KI2SW} = (1 - \text{LAMDA}^2)/(1 - \text{SIGN} \cdot \text{LAMDA})

C \text{KI2SW} is expected to be non-zero, if this condition is
C not met, an appropriate message is generated, and execution
C is deleted.

C
IF (\text{KI2SW} .NE. 0) GO TO 330
WRITE (6, 119) 1 GU TO 560

330 \text{LAMDA}1 = (\text{SIGN} + \text{PN} - \text{LAMDA})/(1 - \text{SIGN} \cdot \text{LAMDA})
\text{KI15Q} = (1 - \text{LAMDA}^2)/(1 - \text{SIGN} \cdot \text{LAMDA})

C

C We expect that this \text{KI15Q} will be positive, if not, an
C error message will be generated and the execution deleted.

C
IF (\text{KI15Q} .GT. 0) GO TO 340
WRITE (6, 120) 1 GU TO 560

340 \text{X = WONORM} \cdot \text{SQRT} (\text{KI15Q})

DO 350 NORDER = 0, 2, 1
CALL BESJ (X, NORDER, R, IER) IF (IER .GT. 0) GO TO 240
350 RJ(NODDER + 1) = H
  F1 = R*(RJ(1) - HJ(3))/(3*RJ(3))
  G1 = (F1/THETA - 1.0)/THETA
  IF (THETA .LT. 0.0) GO TO 370
  ARG = R*SQRT(RJ(3))
  DO 360 NODDER = 0,2,1
       CALL BFSJ (ARG, NODDER, R, IER) IF (IER .GT. 0) GO TO 240
  360 RJ(NODDER + 1) = R
       F2 = ARG*(RJ(1) - HJ(3))/(2*RJ(3))
  GO TO 390
  370 ARG = R*SQRT(-THETA)
  DO 380 NODDER = 0,2,1
       CALL HFSJ (ARG, NODDER, R, IER) IF (IER .GT. 0) GO TO 260
  380 RJ(NODDER + 1) = H
       F2 = ARG*(RJ(1) + HJ(3))/(2*RJ(3))
  390 G2 = (F2/THETA - 1.0)/THETA
       DG = G2 - G1
       IF (DG .EQ. 0.0) GO TO 410
       IF (NIEST .EQ. 1) GO TO 400
       NIEST = 1.1 DGLAST = DG J GO TO 320
C
C  FOR THE N = -1 CASE, NO CHECK HAS BEEN ARRANGED FOR
C  FAILURE TO LOCATE THE SOLUTION CURVE.

C

  400 IF (DGLAST*DG .LT. 0.0) GO TO 410
       DGLAST = DG J GO TO 320

  410 CONTINUE
C
C  A SOLUTION OF THE FILLED-GUIDE CHARACTERISTIC EQUATION FOR
C  N = -1 HAS BEEN FOUND.
C
THE N = +1 AND N = -1 FERRITE PROPAGATION CONSTANTS HAVE BEEN DETERMINED. THEY ARE NOT USEFUL IN FINDING THE SPECTIVE VALUES OF THE CAVITY CHARACTERISTIC QUANTITY. THIS QUANTITY IS OUTPUT AS A COMPLEX NUMBER, THOUGH IT WILL BE EITHER REAL OR IMAGINARY, AND WE WILL FIND COR AND COI FOR N = +1 AND N = -1 CASES.

DRNUM = K0/NORM/KIN15

DO 420 LNNORM = 1, NORM + 1
420 RNUM(RNORM) = DRNUM*(IRNUM = 1)

BNF(1) = BNF ; HNF(2) = RNFIN

ALMDA1(1) = LMDA1P J ALMDA1(2) = LMDA1N
ALMDA2(1) = LMDA2P J ALMDA2(2) = LMDA2N
NCASE(1) = 1 ; NCASE(2) = -1

DO 540 ICASE = 1, 2 + 1
LMDA1 = ALMDA1(ICASE) J LMDA2 = ALMDA2(ICASE)
N = NCASE(ICASE)
BNFER = BNF(ICASE)

KI15Q = (1. - LMDA1**2)/(1. - SIGMA*LMDA1)
KI25Q = (1. - LMDA2**2)/(1. - SIGMA*LMDA2)

HERE, WE USE THE FACT THAT KI25Q IS POSITIVE.

ARG = RNUM*SQRT(KI25Q)

CALL BESJ(ARG, 1, MKIPRO, IER) ; IF (IER .GT. 0) GO TO 240
C IF THE ARGUMENT OF THE J BESSEL FUNCTION IS IMAGINARY, 
C THE VALUE OF J IS REPLACED BY THE CORRESPONDING VALUE OF 
C THE I BESSEL FUNCTION.

IF (K1150 .LT. 0) GO TO 430
   ARG = RNDRM*SQRT(K1150)
   CALL BFSJ (ARG, 1, BK1140, IER) J IF (IER .GT. 0) GO TO 240
GO TO 440.

430 ARG = RNDRM*SQRT(-K1150)
   CALL BFSJ (ARG, 1, BK1140, IER) J IF (IER .GT. 0) GO TO 240

440 DD 510 IRNUMR = 2, RNORDM, 1
   RNNRM = RNDRM(INNRM)
   ARG = RNNRM*SQRT(K1250)
   DO 450 NORDER = 0, 2, 1
      CALL BFSJ (ARG, NORDER, P, IER) J IF (IER .GT. 0) GO TO 240
   450 BJ(NORDER + 1) = H

   B(KJ2) = BJ(2)
   RPK12 = .5*(HJ(1) - HJ(3))/BJ(2)
   ARG = WC*KNURM
   DO 460 NORDER = 0, 2, 1
      CALL BFSJ (ARG, NORDER, R, IER) J IF (IER .GT. 0) GO TO 240
   460 BJ(NORDER + 1) = B

   BKCI = BJ(2)
   RPKCI = .5*(BJ(1) - BJ(3))/BJ(2)
   IF (K1150 .LT. 0) GO TO 480
   ARG = RNNRM*SQRT(K1150)
   DO 470 NORDER = 0, 2, 1
      CALL BFSJ (ARG, NORDER, K, IER) J IF (IER .GT. 0) GO TO 240
   470 BJ(NORDER + 1) = H

   BK111 = BJ(2)
   RPK11 = .5*(BJ(1) - HJ(3))/BJ(2)
GO TO 590

590 ARG = RNORM1*SQR1(K115Q)
DO 490 NUMBER = 0,2,1
CALL BFS1(ARG,NORFR, N, IER) J IF (IER .GT. 0) GO TO 590

490 BI(NORDER + 1) = N
BK111 = H1(2)
RPK111 = 5*(H1(1) + A1(3))/A1(2)

500 A1 = LAMDA1*(1. - RNFER**2/NU) - RH0
A2 = LAMDA2*(1. - RNFER**2/NU) - RH0

C

F11 = RNFER*(K125Q/LAMDA1)*((1./RNORM1)*HK12RO*RAK111
C = RNFER*K125Q*SQT(ABS(K115Q))*HK12RO*RPK111
C = RNFER*(K115Q/LAMDA2)*((1./RNORM1)*HK11RO*RAK121
C + RNFLH*K115Q*SQT(K125Q)*BK1110*RPK121

C

F21 = RNFER*K125Q*(1./RNORM1)*HK12RO*HK111
C = RNFER*(K125Q/LAMDA1)*SQT(ABS(K115Q))*BK12RO*RPK111
C = RNFER*K115Q*(1./RNORM1)*BK11RO*BK121
C + RNFER*(K115Q/LAMDA2)*SQT(K125Q)*AK11RO*RPK121

C

F31 = RH*K125Q*(1./RNORM1)*HK12RO*RAK111
C = A2*SQT(ABS(K115Q))*AK12RO*RPK111
C + K115Q*(1./RNORM1)*BK11RO*BK121
C + A1*SQT(K125Q)*AK11RO*RPK121

C

F41 = RH*A2*(1./RNORM1)*AK12RO*BK111
C = K125Q*SQT(ABS(K115Q))*BK12RO*RPK111
C + RH*A1*(1./RNORM1)*AK11RO*AK121
C + K115Q*SQT(K125Q)*AK11RO*RPK121

C

SF1(IRNUHM) = N*BKCI*F11 + KC*RNORM1*RPF*CI*F21
.510 SF2(IRNORM) = -KC*KNORM1*KPC1*131 = N*KC1*F41

   SF1(1) = .0 , SF2(1) = .0

   11 = .0 , 12 = .0

   DO 520 IRNORM = 2 , IRNORM+1

      11 = 11 + IRNORM*(SF1(IRNORM) + SF1(IRNORM = 1))/2.

      520 I2 = 12 + IRNORM*(SF2(IRNORM) + SF2(IRNORM = 1))/2.

   C

   C   CO = COS(2*PI+T) = (LNORM/2 - TNORM)*COS(HYPER*TNORM)*12

   C   = HYPER*SIN(2*PI+T) = (LNORM/2 - TNORM)*SIN(HYPER*TNORM)*12

   C

   C   COP IS EITHER THE REAL OR IMAGINARY PART OF CO.  THE

   C   CHARACTERISTIC QUANTITY.  RECALL THAT K1250 IS POSITIVE.

   C

   C   IF (K1250 .GT. 0) GO TO 530

   C   C0R(CASE) = .0 , C0T(CASE) = COP , GO TO 560

   C   530 C0R(CASE) = COP , C0T(CASE) = .0

   C   CONTINUE

   C   WRITE (6,122) FREQHZ, COR(1), CO(1), COR(?), CO(?)

   C   550 FREQHZ = FREQHZ + DFREQ

   C   IF (FREQHZ .LE. FREQ) GO TO 200

   C   WRITE (6,123)

   C   CONTINUE

   C

   C   STOP

   C

   C
SUBROUTINE BESJ (X, N, RJ, IER)
   D  =  .001
   BJ  =  .0
   IF (N) 10*20*20
   10  IER  =  1
   RETURN
   20  IF (X) 30*30*31
   30  IER  =  2
   RETURN
   31  IF (X<15.) 32,32,34
   32  NTEST  =  20. + 10.*X  =  X**2/3
   33  GO TO 36
   34  NTEST  =  .90. + X/2.
   35  IF (N=NTTEST) 40,35,36
   36  IER  =  4
   RETURN
   40  IER  =  0
   N1  =  N  +  1
   BPREV  =  .0
   IF (X<5.) 50,60,60
   50  MA  =  X  +  6.
   GO TO 70
   60  MA  =  1.4*X  +  60./X
   70  MB  =  N  +  IFIX(X)/8  +  2
   MZERO  =  MAXU(MA,MB)
   MMXX  =  NTEST
   100  DO 190 K  =  MZERO,MMXX,3
   190  EM1  =  1.E0-2K
   FH  =  .0
   ALPHA  =  .0
IF (M*(M/2)+2) > 120, 110, 120

110 JT = -1
     GO TO 130
120 JT = 1

130 M2 = M-2
     DN 160 K = 1, M2
     MK = M - K
        BMK = 2*FLUAT(MK)*F41/X = FM
        FM = F41
        F41 = RMK

        IF (MK-N1) > 150, 140, 150

140 BJ = BMK
150 JT = -JT
       S = 1 + JT

160 ALPHA = ALPHA + BMK*S
        RMK = 2*F41/X = FM

        IF (N) > 190, 170, 180

170 BJ = BMK
180 ALPHA = ALPHA + BMK
         BJ = BJ/ALPHA

        IF (ABS(BJ-HPREV) = ARS(Of+HJ)) > 200, 200, 190

190 BPREV = BJ
        IEB = 3

200 RETURN
      END
SUBROUTINE BESI(x, n, RI, IFK)
    IER = 0
    A1 = 1.0

    IF (N) 150, 15, 10
    10 IF (X) 160, 20, 20
    15 IF (X) 140, 17, 20

    17 RETURN

    20 TOL = 1.E-6
    IF (X=12.) 40, 40, 30

    30 IF (X=FINAT(N)) 40, 30, 110

    40 XX = X/2.
    50 TERM = 1.0

    55 IF (N) 70, 70, 55

    60 DO 60 I = 1,N
        FI = 1
        IF (ABS(TERM)=1.E-45) 56, 60, 60
        56 IER = 3
        BI = 0.0

    65 RETURN

    60 TERM = IFK*X X/I
    70 BI = TERM

    75 XX = XX+XX

    80 DO 90 I = 1,1000
        IF (ABS(TERM)=ABS(HI+TL)) 100, 100, 90

    90 FW = K*(I + K)
    TERM = TERM*XX/IFK
    95 BI = BI + TERM

100 RETURN

110 FN = FINAT
    IF (X=12.0) 115, 111, 111
111  IE4 = 1
    RETURN
115  XX = 1,/(a.*X)
    TERM = 1,
    RI = 1,
   DO 130 K = 1,30
    IF (ABS(TFRM)-ABS(1.0LRI)) .GT.140,140,120
   120  F = (2*K-1)**2
    TERM = TERM*XX*(FK-FN)/FLOAT(K)
    RI = RI*TERM
   GO TO 130
140  PI = 3.1415927
    RI = RI*EXP(X)/SQR(2.*PI*X)
   GO TO 160
150  IER = 1
   GO TO 100
160  IER = 2
   GO TO 100
END