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Neutron-Proton Final State Interactions in
Proton-Proton Pion Production at 800 MeV

by

Richard Douglas Felder

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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I. INTRODUCTION

A. N-N Pion Production

Pion production in nucleon-nucleon (N-N) interactions has been the subject of considerable interest since the first pions were artificially produced in 1948. The relation between this inelastic process and ordinary N-N scattering provides a means of studying the fundamental principles of the meson theory of nuclear forces. This theory, originally conceived by Yukawa (Yu35), assumes that strongly interacting mesons are responsible for the coupling between nucleons, via a virtual meson field. In the collision of two nucleons, a virtual meson can be materialized as a free and observable particle if the available energy in the center-of-momentum (c.m.) system is greater than the meson mass. Experimental measurements of N-N pion production are thus useful in learning more about the nuclear force, as well as understanding the mechanisms of pion production.

The pion is the lightest ($m_\pi \approx 140$ MeV) of the known strongly interacting particles and, thus in terms of the Yukawa theory, is responsible for the long range components of the nuclear force. The pion exists in three charge states ($\pi^+, \pi^0, \pi^-$) and is classified as a pseudoscalar boson. This means the pion is a spin zero particle with negative intrinsic parity and obeys Bose-Einstein
statistics. At lab energies below 300 MeV the pion-nucleon (π-N) interaction is dominated by a very strong, broad resonance known as the Δ(1232), or Δ(3/2, 3/2), with total spin and isospin quantum numbers of 3/2, 3/2. The Δ resonance plays a significant role in all π-N scattering and N-N pion production reactions at intermediate energies.

At incident energies above ∼290 MeV, N-N pion production becomes kinematically possible for the following reactions:

\begin{align*}
\text{p + p} & \rightarrow \text{d} + \pi^+ & (I-1a) \\
\rightarrow & \text{p} + \text{n} + \pi^+ & (I-1b) \\
\rightarrow & \text{p} + \text{p} + \pi^0 & (I-1c) \\
\text{p + n} & \rightarrow \text{d} + \pi^0 & (I-2a) \\
\rightarrow & \text{n} + \text{n} + \pi^+ & (I-2b) \\
\rightarrow & \text{p} + \text{n} + \pi^0 & (I-2c) \\
\rightarrow & \text{p} + \text{p} + \pi^- & (I-2d) \\
\text{n + n} & \rightarrow \text{d} + \pi^- & (I-3a) \\
\rightarrow & \text{p} + \text{n} + \pi^- & (I-3b) \\
\rightarrow & \text{n} + \text{n} + \pi^0 & (I-3c)
\end{align*}

Because of the obvious difficulties in measuring reactions (I-2) and (I-3), N-N pion production at intermediate energies (\(T_p \lesssim 1 \text{ GeV}\)) has been studied most extensively for reactions (I-1). Historically, these reactions have been investigated in inclusive counter
experiments (Ba70, Co72) measuring only the pion angle and energy or in experiments employing visual techniques (Ba62, Gu64, Bu64), such as bubble chambers. The visual experiments have the advantage of identifying particular reactions over a large geometry, but often suffer from low statistics. The inclusive experiments in general do not completely determine the reaction kinematics, making the investigation of specific reaction mechanisms extremely difficult. Thus, few three-body pion production measurements have been made which allow careful study of individual production mechanisms.

The most prominent mechanism for reaction (I-1b) is that in which the reaction proceeds through the formation of an intermediate Δ resonance; i.e., \( pp \to \Delta^{++} n - p\pi^+ n \) (see Figure II-2f). This mechanism has been the subject of numerous studies (Ma57, Bu64, Mo75) one of which (Hu76) is still in progress (Fe76a) with hopes of providing very complete results. A second mechanism which is worthy of consideration involves a nucleon-nucleon final state interaction (FSI). For this mechanism, reaction (I-1b) proceeds as \( pp - d^*\pi^+ - p\pi^+ \) (see Figure II-2e). Here \( d^* \) refers to an unbound neutron-proton pair interacting with small relative energy in the \( ^3S_1 \) and \( ^1S_0 \) spin states. No previous detailed measurements have been made of the role of nucleon-nucleon FSI in N-N pion production.
B. N-N Final State Interactions

Nucleons produced in a reaction often interact strongly with each other before getting outside the range of their mutual forces. Such interactions are termed "final states interactions," and they may greatly influence the distributions of the reaction cross sections. The FSI nucleons interact in relative S-states with the strength of the interaction increasing with decreasing relative energy. The Pauli principle allows n-p pairs to interact in the \(^3S_1\) spin, \(T = 0\) isospin state or the \(^1S_0\), \(T = 1\) state. However, p-p pairs and n-n pairs are restricted to the \(^1S_0\), \(T = 1\) state.

Nucleon-nucleon FSI has been experimentally studied over a wide range of energies (Pl76, Br70, Fu73) for a variety of reactions (Br70, Wa75). Since N-N FSI effects are more dramatic at lower energies, most experiments have been performed at incident nucleon energies below 100 MeV, commonly considering the p-d breakup reaction, pd \(\rightarrow\) pnn. Measurements of n-p FSI in this reaction have recently been made for incident proton energies as high as 600 MeV (Fu73, Wi75) and 800 MeV (Fe75b, Fe76c). The observation of prominent FSI effects in these experiments is evidence that FSI may play a significant role in N-N pion production at these energies.

The subject of N-N FSI in pion production has been addressed frequently in the literature (Wa52, Ge54, Am67)
but never studied explicitly experimentally. Although the effects of FSI are known (Ge54, Ba62) to be important in the energy range where reaction (I-1a) dominates pp pion production (see Figure II-1), they are generally thought (Fe63) to be insignificant at higher energies (≥ 1 GeV).

In a recent inclusive experiment (Hu75), n-p FSI was observed to play a definite role in reaction (I-1b) at 800 MeV. Data from this experiment are given in Figure I-1, where the inclusive cross section for pp-dπ⁺ and pp-d*π⁺ is shown. The contribution from these processes is deduced from a characteristic, narrow peak in π⁺ momentum spectra at maximum pion momentum. Also shown in Figure I-1 is the cross section for just the pp-dπ⁺ reaction (Ri70). Since differences between these two cross sections are attributable to the d* mechanism, strong evidence exists for an n-p FSI contribution.

C. Motivation for the Present Experiment

In order to fully understand pion production in N-N collisions, it is necessary to study particular reaction mechanisms in isolation by performing kinematically complete experiments. The scarcity of previous experiments of this nature at intermediate energies is the primary motivation for the present experiment. The role of the n-p FSI in pp pion production is of special interest since evidence for this often-mentioned, but seldom-measured, mechanism has been observed recently at 800 MeV. Thus,
Figure I-1

\[ pp + d \pi^+ \text{ PLUS } pp + d^* \pi^+ \text{ CROSS SECTIONS FROM} \]

INCLUSIVE EXPERIMENT AT 800 MeV

The pp+\(d\pi^+\) data of reference Ri70 are shown for comparison. The solid lines are guides to the eye.
the purpose of the present investigation is to measure with good statistics the angular distribution of the cross sections for reactions (I-la) and (I-lb) at 800 MeV, with kinematical conditions specifically chosen to observe the n-p FSI.

This measurement contributes to the knowledge of pp pion production in several meaningful ways. The cross section for the pp-d*π⁺ mechanism has not been measured accurately in the past, and its size relative to the pp-dπ⁺ cross section is of basic importance. One reason for this is that the pp-dπ⁺ reaction is often studied by determining the energy spectra of the π⁺ without detecting other particles in coincidence; e.g., reference Ba70. The reaction pp-d*π⁺ is an inevitable background in such experiments and must be subtracted so that precise measurements of its cross section improves the knowledge of pp-dπ⁺. On the basis of this measurement, it may be possible to draw general conclusions on the ratio of these two cross sections at other energies. The adequacy of simple FSI theories to describe the experimental data can also be ascertained, and the possible separation of the n-p FSI into the 3S₁ and 1S₀ states can furnish information on the angular momentum transitions (see Section II.A) involved. Finally, this experiment provides useful input and tests for both the isobar and one-pion exchange models of pion production (see Section II.B), both of which are applicable in the energy range near 800 MeV.
II. THEORETICAL CONCEPTS

A. General

Following the early studies of N-N pion production near threshold, Watson and Brueckner (Wa51) were able to interpret the results using the conservation of angular momentum, parity, and isotopic spin. By assuming charge symmetry and charge independence, Van Hove et al. (Va52) expressed all the possible N-N reactions in terms of only three independent amplitudes, which represent transitions from one isospin state of the nucleons to another. The total cross sections may be written in terms of these transitions as (Ro54)

\[
\begin{align*}
\sigma(pp-d\pi^+) &= \sigma_{10}(d) \quad \text{(II-1a)} \\
\sigma(pp-pn\pi^+) &= \sigma_{10} + \sigma_{11} \quad \text{(II-1b)} \\
\sigma(pp-pp\pi^0) &= \sigma_{11} \quad \text{(II-1c)} \\
\sigma(np-d\pi^0) &= 1/2 \sigma_{10}(d) \quad \text{(II-1d)} \\
\sigma(np-n^-) &= 1/2 (\sigma_{11} + \sigma_{01}) \quad \text{(II-1e)} \\
\sigma(np-pn\pi^0) &= 1/2 (\sigma_{10} + \sigma_{01}) \quad \text{(II-1f)}
\end{align*}
\]

where the first index represents the isotopic spin state of the initial nucleons, and the second that of the final nucleons. The cross section \(\sigma_{10}(d)\) represents the case where there is a deuteron in the final state, whereas \(\sigma_{10}\) denotes the cross section for an unbound neutron-proton pair in the final state. The general energy dependence of the isospin cross sections contributing in pp reactions is
given (Lo70) in Figure II-1. The pp pion production cross section is seen to rise rapidly above threshold; by 800 MeV, the total reaction cross section exceeds the elastic scattering cross section.

For energies below 700 MeV, pion production is often analyzed in terms of transitions between a small number of angular momentum states. This partial wave analysis becomes less meaningful at higher energies, where many transitions contribute. The conservation laws for these transitions require that the two-nucleon initial state must have the same isotopic spin, angular momentum, and parity as the final state. If it is assumed that the final state nucleons are in a relative S state and that the pion is in a s or p state with respect to the two-nucleon system, then only the transitions given (Ro54) in Table II-1 are allowed for \( \pi^+ \) production in p-p collisions. The notation in Table II-1 is that of Rosenfeld (Ro54), where capital letters refer to the orbital angular momentum of the two-nucleon system and the small letters to the orbital angular momentum of the pion with respect to the two-nucleon system. The spin and angular momentum of the two-nucleon system are given in the standard notation \( \frac{1}{2}L_{sJ} \), and the last subscript of the final state symbol indicates the overall angular momentum.

Also given in Table II-1 are the pion angular distributions which are expected in the absence of inter-
approximate energy dependence of $\sigma_{10}$, $\sigma_{10}(d)$, and $\sigma_{11}$ (Lo70)
<table>
<thead>
<tr>
<th>Isospin Reaction</th>
<th>Class</th>
<th>Initial State</th>
<th>Final State</th>
<th>Angular Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>ς_{10} (d)</td>
<td>Ss</td>
<td>^3P_1</td>
<td>^3S_1s_1</td>
<td>Isotropic</td>
</tr>
<tr>
<td></td>
<td>Sp</td>
<td>^1S_0</td>
<td>^3S_1p_0</td>
<td>Isotropic</td>
</tr>
<tr>
<td></td>
<td>Sp</td>
<td>^1D_2</td>
<td>^3S_1p_2</td>
<td>1/3+cos^2θ</td>
</tr>
<tr>
<td>ς_{10}</td>
<td>Ss</td>
<td>^3P_1</td>
<td>^3S_1s_1</td>
<td>Isotropic</td>
</tr>
<tr>
<td></td>
<td>Sp</td>
<td>^1S_0</td>
<td>^3S_1p_0</td>
<td>Isotropic</td>
</tr>
<tr>
<td></td>
<td>Sp</td>
<td>^1D_2</td>
<td>^3S_1p_2</td>
<td>1/3+cos^2θ</td>
</tr>
<tr>
<td>ς_{11}</td>
<td>Ss</td>
<td>^3P_0</td>
<td>^1S_0s_0</td>
<td>Isotropic</td>
</tr>
</tbody>
</table>
ferences. If the total angular momentum is zero or the pion is produced in an s state, the distribution will be isotropic; the only anisotropic angular distribution shown is for the $^1D_2^3S_1^1P_2$ transition. The $^1D_2^3S_1^1P_2$ transition is also the only reaction of class Ss or Sp for which the $\Delta(3/2,3/2)$ resonance can occur. For higher partial waves, the $\Delta$ can also occur in the Ps and Pp classes of reactions, which often are included for $\sigma_{11}$.

B. Pion Production Models

An early phenomenological theory for pion production near threshold was developed by Watson and Brueckner (Wa51) using a partial wave analysis. Rosenfeld (Ro54) and others (Ge54) compared the results of this theory with low energy ($\approx 500$ MeV) experiments and found most of the data could be fit with a small number of parameters. A significant feature of their results is that pion production takes place predominantly in angular momentum states which allow the pion and one of the final state nucleons to resonate in the $\Delta(3/2,3/2)$ state.

Mandelstam (Ma58) modified the Watson-Brueckner approach to form an isobar model for pion production. The assumptions of this model are that the outgoing pion is in a resonant $\Delta (3/2,3/2)$ state with one of the nucleons and that production takes place via transitions between only a few low angular momentum states. It is also assumed that the matrix element for each particular transition is
constant except for factors due to the final state pion-nucleon and nucleon-nucleon interactions. Using this theory, satisfactory three-parameter fits were made (Ma58, Vo66) to the pp pion production data up to 700 MeV, for both the total and differential cross sections.

Above 700 MeV the Mandelstam model begins to fail to adequately describe pion production, since more partial waves are involved and the constancy of the reaction matrix amplitudes becomes questionable. However, in the 800 MeV to 3 GeV energy range, the one-pion-exchange (OPE) model with form factors (Fe61, Fe62) may be used to describe the data. This model assumes that the pion is produced in a peripheral collision, or, in other words, that the interaction is transmitted by the least massive strongly interacting particle, which is the pion. The formation of the Δ isobar is included in the OPE model (Fe63, Am67), and at energies above 1.5 GeV other contributing isobars are often considered (Ch62). Final state interactions leading to deuteron formation have also been described in terms of the OPE model (Tu63, Ch63, Ya64), but such effects are seldom included in general applications (Am67).

Feynman diagrams illustrating OPE for π⁺ production in p-p interactions are shown in Figure II-2. The diagrams for the pp-Δπ⁺ reaction (a,b) and for pp-pnπ⁺ (c,d) are similar except for the final state interaction which forms the deuteron in the former case. The n-p FSI can
FEYNMAN DIAGRAMS FOR $\pi^+$ PRODUCTION IN PP INTERACTIONS

(a) $\pi^+$ exchange

(b) $\pi^0$ exchange

(c) $\pi^+$ exchange

(d) $\pi^0$ exchange

(e) n-p FSI ($d^*$)

(f) Isobar formation

(g) One neutron exchange
also occur leaving the n-p pair unbound (d*) as shown in diagram (e). Isobar formation is illustrated in diagram (f), where a peripheral collision excites one nucleon to an isobar, which then decays to a pion-nucleon pair. Intermediate isobar formation contributes in the pp-d\pi^+ and pp-d^*\pi^+ reactions as well as in pp-pn\pi^+ (Ya64). A simple one-neutron exchange diagram (g) has also been considered for the pp-d\pi^+ reaction (Pe63), but is generally thought to be unimportant at energies below 2 GeV (Ya64). Nevertheless, evidence exists for the OPE model breaking down in certain kinematical reactions at higher energies (Ch62, He68).

C. Final State Interaction Theory

Neutron-proton FSI data are commonly analyzed using the Goldberger-Watson (Go64) formalism for final state interactions. This theory assumes a two-step reaction mechanism in which the squared three-particle reaction matrix element \(|T_{fi}|^2\) can be written as

\[|T_{fi}|^2 = F(k) |T_{fi}^0|^2,\] (II-2)

where \(F\) is the enhancement factor for the n-p FSI and \(T_{fi}^0\) is the matrix element for the production of the n-p pair. According to Watson (Wa52), \(T_{fi}^0\) should depend only very weakly on the n-p relative momentum \(k\), and hence the variation of \(T_{fi}^0\) with \(k\) is neglected. The enhancement factor \(F\) is proportional to the square of the n-p
relative wave function at zero separation. When properly normalized, this factor reduces to unity in the case of no FSI, and thus includes the contribution from diagrams with no n-p rescattering. The enhancement factor may be expressed as

$$F(k) = \left| \frac{(k^2 + \alpha^2)r/2}{-1/a+(r/2)k^2-ik} \right|^2, \quad (II-3)$$

where \(a\) and \(r\) are the scattering length and effective range, respectively, and

$$\alpha = (1 + \sqrt{1-2r/a})/r. \quad (II-4)$$

Here \(k\) is given nonrelativistically as

$$k = \left( \frac{4m_p m_n T_{np}}{m_p + m_n} \right)^{1/2}, \quad k_{np} = \hbar k, \quad (II-5)$$

where \(m_n\) and \(m_p\) are the neutron and proton masses and \(T_{np}\) is the total kinetic energy in the n-p c.m. system. By using the appropriate scattering lengths and effective ranges, \(F\) may be determined for both the spin singlet \((^1S_0)\) and triplet \((^3S_1)\) interactions.

Three body FSI cross sections are often expressed as an incoherent sum of singlet and triplet terms given by

$$\frac{d^5\sigma(k)}{dp_1 dp_2 d\Omega_1 d\Omega_2} = I\left[x^S F^S(k) + x^T F^T(k)\right], \quad (II-6)$$

where \(I\) is a phase space factor described in Section VI.B. The FSI factor \(F\) contains the dominant dependence upon the
relative momentum \( k \), and the factor \( X \) is essentially independent of \( k \) for \( n-p \) relative energies which are small compared to the bombarding energy (Wa52). Equation (II-6) can be used to determine the singlet and triplet contributions in \( n-p \) FSI by allowing coefficients \( X^s \) and \( X^t \) to vary in least-square fits to the experimental cross sections (Br70, Wi76b). In studies of FSI in \( p-d \) breakup at energies > 500 MeV (Wi75, Fe76c), it was found useful to include a constant term with the \( X F(k) \) terms in describing the data. The constant term was due mainly to contributions from significant four-body reactions. With this addition the resulting fits described the shape of the measured cross sections quite well.

Final state interaction theory based upon Equation (II-6) produces a simple result for the ratio of scattering into the \( ^3S_1 \) continuum state \((\sigma^t)\) to scattering into the deuteron bound state \((\sigma^d)\). Provided the factors \( X \) are independent of small \( k \) so that the amplitudes can be factored as in Equation (II-6), the \( \sigma^t \) and \( \sigma^d \) cross sections are proportional to the square of their respective wave functions evaluated at the origin (Du76, Fe76c, Br70); that is,

\[
\frac{\sigma^t}{\sigma^d} = \frac{\int \frac{d^3k}{(2\pi)^3} \left| \psi_L(r=0) \right|^2}{\left| \psi_d(r=0) \right|^2} ,
\]

(II-7)
where the integration over the final state relative momentum \( k \) extends up to the maximum energy \( T_{np} \) included in the continuum channel. The wave functions are normalized to

\[
\int \left| \psi_d(r) \right|^2 d^3r = 1 \quad \text{(II-8)}
\]

and

\[
\psi_t(r) = e^{i \vec{k} \cdot \vec{r}} + f(k) \frac{e^{ikr}}{r} \quad \text{(II-9)}
\]

for \( r \to \infty \).

The deuteron wave function may be evaluated by using one of several common functions, such as that due to Hulthén (Hu42), Yamaguchi (Ya54), or the Moravcsik (Mo58) form of the Gartenhaus (Ga50) wave function. These wave functions normally include small percentages of D-state. The Hulthén S-state wave function is given by

\[
\psi_d(r) = \left[ \frac{\alpha \beta (\alpha + \beta)}{2\pi} \right]^{\frac{1}{2}} e^{-\alpha r} e^{-\beta r} \frac{e^{i kr}}{(\beta-\alpha)r} \quad \text{(II-10)}
\]

where \( \alpha = 0.232 \ \text{fm}^{-1} \) and \( \beta = 1.20 \ \text{fm}^{-1} \). The triplet continuum wave function can be approximated (Du76, Fr54) as

\[
\psi_t(r) = \sin \frac{kr}{kr} + f(k) \frac{e^{ikr} - e^{-\beta r}}{r} \quad \text{(II-11)}
\]

with the scattering amplitude \( f(k) \) given by

\[
f(k) = \frac{1}{k \cot \delta - ik} \quad \text{(II-12)}
\]

where

\[
k \cot \delta = -\frac{1}{\alpha} + \frac{x}{2} k^2 \quad \text{(II-13)}
\]
via the effective range approximation. Other forms of the
scattering amplitude $f(k)$ may be used, such as that pro-
vided by the Yamaguchi separable potential model (Ya54).
In addition, hard-core effects may be incorporated in
the evaluation of Equation (II-7) by considering a finite
interaction range (Du76). This procedure requires
averaging the wave functions over an interaction volume
and usually decreases $\sigma^t/\sigma^d$ (Fe76c).
III. DESIGN CONSIDERATIONS

A. Geometry and Kinematics

This experiment was designed to measure pion production from the reactions

\[ pp \rightarrow d\pi^+ \]  \hspace{1cm} (III-1)

and

\[ pp \rightarrow p\pi^+n \]  \hspace{1cm} (III-2)

in a kinematically-complete experiment. To determine the kinematics of three-body reactions completely, it is necessary to measure five of the nine independent kinematic variables which describe the final state. This experiment measured the spherical scattering angles \( \theta_1, \phi_1, \theta_2, \phi_2 \) of the two charged particles in the final state and the momentum \( p_1 \) of one of them. Using conservation of energy and momentum, all remaining kinematic quantities of interest were calculated.

The laboratory arrangement used in the experiment is shown in Figure III-1. This arrangement was a variation of that employed in a previous experiment which studied p-d breakup mechanisms (Fe76b, Fe76c). The proton beam was incident on a liquid hydrogen target, \( T \), and the scattered particles were detected in coincidence in a magnetic spectrometer arm and a time-of-flight (TOF) arm. Scintillation counters, S1-S4, provided the fast coincidence signals which electronically enabled six multiwire proportional counters (MWPC), P1-P6. Counters P1 and P2 measured the scattering angles \( \theta_2, \phi_2 \) while P3-P6 determined the
Figure III-1

EXPERIMENTAL ARRANGEMENT

T is the liquid hydrogen target.
S1-S4 are scintillation counters.
P1-P6 are multiwire proportional counters.
M is the spectrometer magnet.
LION and RION are ionization chambers.
M1-M4 are monitor scintillation counters.
PM is the multiwire position monitor.
FCUP is the Faraday cup.
angles $\theta_1, \phi_1$ and measured the momentum $p_1$ by the angle of bend through the spectrometer magnet, $M$. The TOF of the charged particles in each arm was measured along with the pulse heights in scintillators $S1$ and $S3$.

The location of the MWPC detectors in the experimental arrangement were determined by maximizing the scattering angle and magnet bend angle resolutions, while providing a sufficient flight path in the TOF arm and maintaining a reasonable solid angle. A sufficient flight path was that necessary to separate background reactions from the reaction of interest on the basis of the TOF measurement. A reasonable solid angle was one which produced counting rates compatible with the time limitations of the experiment and with accidental rates. The resulting MWPC positions were determined by compromising among the desired objectives.

In a kinematically-complete experiment, particular regions of phase space can be investigated by choosing the appropriate kinematic conditions. For this study the central arm angles and $p_1$ momentum range were chosen to measure the role of neutron-proton final state interactions in reaction (III-2). Since the deuteron is bound by only 2 MeV the best angles for observing the $n$-$p$ FSI mechanism, denoted as

$$pp-d^*\pi^+\to(pn)\pi^+,$$  \hspace{1cm} (III-3)

are the two-body angles of reaction (III-1), for which the neutron and proton must have a minimum relative energy.
To distinguish between reactions (III-1) and (III-2) the momentum of the deuterons and d* protons must be measured in the spectrometer arm. Since the d* denotes an unbound n-p pair, only the charged proton is bent in the magnet and detected by the system. Although the deuteron and d* proton have essentially the same velocity, the deuteron momentum will be approximately twice that of the d* proton. In practice the momentum acceptance of the magnet was not large enough to detect both deuterons and protons from reactions (III-1) and (III-3) simultaneously. These two reactions thus required two different magnetic field strengths for detection and were cleanly separable.

The laboratory scattering angles dictated by the relativistic kinematics of reaction (III-1) are summarized in Table III-1. The deuteron angle $\theta_d$ is a double-valued function of the pion angle $\theta_\pi$, corresponding to two physically allowed solutions in the kinematics. The maximum $\theta_d$ is approximately $14.67^\circ$ for which $\theta_\pi = 41.7^\circ$.

The range of kinematic conditions over which reaction (III-1) was measurable with the experimental apparatus was limited by several physical constraints. In a previous arrangement (Fe76b) of the same experimental apparatus, the central angle of the spectrometer arm, $\Theta_1$, was restricted to $60^\circ \geq \Theta_1 \geq 22^\circ$. The maximum angle was limited by the traverse of the magnet stand, while the minimum angle was constrained by shadowing from the target
TABLE III-1

Spectrum of Laboratory Scattering Angles for
$pp-d\pi^+$ at $T_p = 800$ MeV ; $\varphi_d = 0^\circ$, $\varphi_\pi = 180^\circ$

<table>
<thead>
<tr>
<th>$\theta_d$</th>
<th>$P_d$</th>
<th>$\theta_\pi$</th>
<th>$P_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.47°</td>
<td>897</td>
<td>10°</td>
<td>581</td>
</tr>
<tr>
<td>11.23°</td>
<td>965</td>
<td>20°</td>
<td>550</td>
</tr>
<tr>
<td>13.82°</td>
<td>1057</td>
<td>30°</td>
<td>505</td>
</tr>
<tr>
<td>14.65°</td>
<td>1153</td>
<td>40°</td>
<td>454</td>
</tr>
<tr>
<td>14.37°</td>
<td>1243</td>
<td>50°</td>
<td>403</td>
</tr>
<tr>
<td>13.47°</td>
<td>1322</td>
<td>60°</td>
<td>356</td>
</tr>
<tr>
<td>12.27°</td>
<td>1387</td>
<td>70°</td>
<td>314</td>
</tr>
<tr>
<td>10.95°</td>
<td>1441</td>
<td>80°</td>
<td>278</td>
</tr>
<tr>
<td>9.63°</td>
<td>1484</td>
<td>90°</td>
<td>248</td>
</tr>
<tr>
<td>8.33°</td>
<td>1518</td>
<td>100°</td>
<td>223</td>
</tr>
</tbody>
</table>
chamber flanges and the interference of the spectrometer magnet with the beam line. To allow smaller $\theta_1$ angles, the target chamber was rotated 90° with respect to the beam direction, permitting the beam to pass through the scattering windows. This eliminated chamber restrictions at small $\theta_1$ but imposed the additional condition that $\theta_1, \theta_2 \leq 60^\circ$. The spectrometer magnet was reconfigured to form a modified "C" magnet, which, given the magnet position, allowed central angles of $60^\circ \geq \theta_1 \geq 13.8^\circ$. The lateral position of the magnet relative to the central line of the spectrometer was chosen to maximize the momentum acceptance as determined by a Monte Carlo calculation. This calculation also predicted the FWHM $\theta_1$ acceptance as $-3.0^\circ$ to $+2.6^\circ$ of the central angle, allowing detection of particles with angles as small as $\theta_1 \approx 10.8^\circ$. The smallest $\theta_2$ was limited to $20.5^\circ$ by scintillator S2 hitting the beam pipe.

As a result of the modifications to the basic experimental apparatus, reaction (III-1) was able to be measured at five angle pairs with the pion angles ranging from $22^\circ$ to $50^\circ$ (see Table III-2). For the same angles, reaction (III-3) was also measurable since the lower limit on the detectable proton momentum was less than $\sim 300$ MeV/c.

B. Spectrometer Magnet

The spectrometer magnet is a picture-frame bending magnet with a 6" pole face gap height which is 18" wide
TABLE III-2
Kinematic Conditions for Measuring Reactions \(pp \rightarrow d\pi^+\) and \(pp \rightarrow d^*\pi^+\)

<table>
<thead>
<tr>
<th>(\theta^d, d^*_{\text{lab}}) (deg.)</th>
<th>(p^d_c) (MeV/c)</th>
<th>(p^c_{\text{lab}}) (MeV/c)</th>
<th>(\theta^\pi_{\text{lab}}) (deg.)</th>
<th>(p^\pi, \sim) (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.9</td>
<td>970</td>
<td>485</td>
<td>22</td>
<td>550</td>
</tr>
<tr>
<td>12.8</td>
<td>1010</td>
<td>505</td>
<td>25</td>
<td>525</td>
</tr>
<tr>
<td>13.8</td>
<td>1055</td>
<td>530</td>
<td>30</td>
<td>505</td>
</tr>
<tr>
<td>14.7</td>
<td>1160</td>
<td>580</td>
<td>40.6</td>
<td>445</td>
</tr>
<tr>
<td>14.4</td>
<td>1240</td>
<td>620</td>
<td>50</td>
<td>405</td>
</tr>
</tbody>
</table>

\(p_c\) refers to the central momentum as established by the spectrometer magnet; \(\theta^d, d^*_{\text{lab}} = 0^\circ\); \(\theta^\pi_{\text{lab}} = 180^\circ\).
and 36" deep. The central field of the magnet was measured as a function of the voltage drop across the power supply shunt by using a Hall probe and NMR gaussmeter.

The reconfiguration of the spectrometer magnet not only altered the physical magnet shape, but also changed the field characteristics. In particular, the maximum field was reduced, the fringe field was increased, and the shape of the field was slightly affected. The maximum magnetic field placed an upper limit on the momentum which could be accepted in the spectrometer given the 30° central bend angle required for less than 2% dp/p resolution. This maximum momentum was ≈1500 MeV/c at half-maximum acceptance. The increased magnetic fringe field near the beam line required special measures to be taken to prevent significant deflection of the proton beam. Steel tubing was used to shield the beam line from the magnetic field, supports were constructed to secure the beam pipe rigidly, and additional steel plates were added to the magnet to confine the fringe field.

To measure any changes in the magnetic field due to reconfiguration, the magnet was mapped in two horizontal planes at three different field strengths. The results of the magnet map revealed the magnetic field to be very uniform, permitting the particle momentum to be calculated using the uniform field approximation, which has been used previously with success (Fe76b). The details of this momentum calculation are given in Appendix A.
C. Background Reactions

To determine the ability of the experimental system to separate reactions (III-1) and (III-2) from other interactions, a careful study of the kinematics of background reactions was undertaken. At the kinematical conditions for measuring (III-2), the only reactions capable of producing significant background are

$$pp \rightarrow \pi^+ pn$$  \hspace{1cm} (III-4)

and

$$pp \rightarrow pp\pi^0,$$  \hspace{1cm} (III-5)

where the first two final state particles, as written, are those detected in the spectrometer and TOF arms, respectively. Reactions (III-2) and (III-4) produce background in measuring (III-1), but this can easily be removed since the system overdetermined the kinematics for two-body scattering. Reaction (III-4) can also be easily separated from the data of interest by using the measured TOF in either arm. The most prominent background is due to reaction (III-5), but this too is separable from the data by using the TOF and scintillator pulse height as measured in the TOF arm. Figure III-2 is a plot of the loci for reactions (III-2), (III-3), and (III-4) at the $\theta_1/\theta_2$ angle pair, $12.8^\circ/25^\circ$. In this figure the spectrometer momentum is plotted versus the measured TOF in the TOF arm. One can see that the background
Figure III-2

LOCI FOR COMPETING PION PRODUCTION REACTIONS AT 800 MeV

The spectrometer momentum is given as a function of the time-of-flight measured in the right arm for the angles $\theta_1 = 12.8^\circ$, $\theta_2 = 25^\circ$. The location of the $d^*$ and $\Delta$ resonances are given for $pp\rightarrow p\pi^+n$, which is the reaction of interest.
\[ \begin{align*}
\text{+} \quad P+P &= P+P^+ + N \\
\times \quad P+P &= P+P+P^0 \\
\ldots \quad P+P &= P^+ + P + N
\end{align*} \]
reactions can be separated from the reaction of interest on the basis of this TOF measurement alone, provided the TOF resolution is \( \leq 1.5 \) nsec.
IV. EXPERIMENTAL TECHNIQUE

A. Basic Features

This experiment was performed on the 800 MeV external proton beam of the Los Alamos Meson Physics Facility (LAMPF) linear accelerator. The arrangement of the experimental apparatus has been described in Chapter III. The time structure of the LAMPF proton beam is described by a macrostructure of 120 pulses per second, each pulse being 450 μsec long. Each macropulse, in turn, has a microstructure consisting of a .25 nsec burst every 5 nsec. The proton beam was completely contained within ±3.5 MeV of the 800 MeV energy peak. The beam profile at the target had a Gaussian shape with a FWHM of approximately .5 cm in both the horizontal and vertical planes. The average beam current during the experiment was 5 to 10 picoamperes.

The liquid hydrogen target was contained in a thin-walled (.0127 cm), cylindrical Kapton flask which was 6.54 cm long and 2.54 cm in diameter. A cryogenic system (Cr70) was provided by LAMPF to liquify the hydrogen and maintain the target at a temperature of 20°K. The target was housed in a specially designed cryogenic target chamber (Ma74) which permitted the cryogenic target, a polyethylene calibration target, and a remote-controlled phosphor screen to be easily moved in and out of the beam. The target chamber was maintained at a pressure near
$2 \times 10^{-5}$ mm Hg and provided .025 cm-thick mylar windows for the scattered particles to escape. To reduce multiple scattering within the system, helium-filled polyethylene bags were placed between each MWPC detector.

The experimental system was calibrated using p-p elastic scattering, for which the kinematic variables are overdetermined by the system. The measured central momentum of the magnet was used to calibrate the magnet shunt voltage, and the momentum resolution $\Delta p/p$ was found to be 1.5%. This resolution was the result of the magnet bend angle resolution, which is limited by multiple scattering and the MWPC wire spacing. The angular resolution was measured to be approximately .25° FWHM for each scattering angle.

The angle and momentum acceptances of the system were determined by Monte Carlo calculations, which are described in Section VI.B. The $\theta_1$ acceptance was typically 5.6° FWHM while the $\theta_2$ FWHM acceptance was about 5°. The FWHM momentum acceptance of the spectrometer was typically from 75% to 145% of the central magnet momentum.

B. Detectors

The charged-particle detectors employed in the experiment consisted of scintillation counters and MWPC detectors. The scintillation counters provided the fast timing signals which established the coincidence conditions and measured the TOF of particles in both arms. They also
determined the relative energy loss of the scattered particles by means of pulse height, which assisted in particle identification. The position-sensitive MWPC detectors accurately determined the trajectories of the scattered particles in the system.

All scintillation counters consisted of NE-102 plastic scintillator material optically coupled to adiabatic UVT Plexiglas light guides. The light was detected by RCA 8575 photomultiplier tubes which were powered by Ortec 265, or equivalent, phototube bases. All photomultiplier tubes were equipped with magnetic shielding. Further description of the scintillators is given in reference Fe75a.

The construction and operation of the MWPC detectors developed at Rice University have been described in detail by Buchanan et al. (Bu72a). Each MWPC consists of an x,y grid of signal wires capable of determining the point at which a charged particle passed through the detector plane. These detectors have a very high detection efficiency and are able to count at rates greater than $10^5$ Hz per wire. The wire planes also have large sensitive areas, a low mass per unit area ($\sim 20$ mg/cm²), and a spatial resolution of less than 2 mm. The counters operate in the proportional region at 5-5.5 kV in a gas mixture of 30% argon and 70% dimethyl-propane at atmospheric pressure. In the experimental system the wire counters determine the scattered particle trajec-
tories which are used to calculate the scattering angles, the momentum $p_1$, and numerous geometrical quantities that assist in background elimination. The dimensions and spacing of the wire planes are summarized in Table IV-1.

The MWPC electronic readout system used in this experiment has been described by Buchanan (Bu75). This system is implemented entirely in CAMAC electronics and provides several event selection mechanisms. In addition, the system has the capability to process up to sixteen multiple readouts on each $x, y$ wire plane coordinate. A concise description of the readout process is given in reference Fe75a.

C. Beam Monitors and Normalization

The beam monitoring system consisted of a profile monitor (PM), a double-arm scintillator telescope system (M1-M4), two ionization chambers (RION,LION), and a Faraday cup (FCUP). These monitors are shown in Figure III-1 with the designations given above.

The beam profile monitor is a 8.1 cm x 8.1 cm multi-wire counter which is similar to a MWPC but employs an integrating capacitive readout (Bu76). This allows the counter to operate in high particle fluxes, as encountered in the proton beam. The outputs of the $x$ and $y$ coordinates were displayed on an oscilloscope, producing the beam profile with a .25 cm spatial resolution. This display served as a continuous monitor of any changes in the beam
TABLE IV-1

Dimensions and Spacing of Wire Counters Used in the Experiment

The notation is that of Figure III-1; $M_c$ refers to the center of the magnet.

<table>
<thead>
<tr>
<th>Counter</th>
<th>$x$ Dimension (cm)</th>
<th>$y$ Dimension (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>12.19</td>
<td>12.19</td>
</tr>
<tr>
<td>P2</td>
<td>28.48</td>
<td>28.45</td>
</tr>
<tr>
<td>P3</td>
<td>12.19</td>
<td>7.62</td>
</tr>
<tr>
<td>P4</td>
<td>20.32</td>
<td>12.19</td>
</tr>
<tr>
<td>P5</td>
<td>77.22</td>
<td>28.45</td>
</tr>
<tr>
<td>P6</td>
<td>77.22</td>
<td>28.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spacing</th>
<th>Distance (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-P1</td>
<td>101.0</td>
</tr>
<tr>
<td>P1-P2</td>
<td>179.1</td>
</tr>
<tr>
<td>T-P3</td>
<td>65.7</td>
</tr>
<tr>
<td>P3-P4</td>
<td>109.2</td>
</tr>
<tr>
<td>P4-$M_c$</td>
<td>112.1</td>
</tr>
<tr>
<td>$M_c$-P5</td>
<td>113.7</td>
</tr>
<tr>
<td>P5-P6</td>
<td>101.8</td>
</tr>
</tbody>
</table>
shape, position, and intensity. A television image of the beam striking a remote-controlled phosphor screen in the target chamber was also used to observe the beam size and position.

The scintillator monitor consisted of two scintillator telescope arms each fixed at 40° with respect to the beam. These telescopes measured p-p elastic scattering from a 7.62 cm x 7.65 cm x .5 cm-thick CH₂ target which was housed in a helium-filled scattering chamber located ~10 m downstream of the main target. The four-fold coincidence signals of the scintillators were scaled and used as a relative monitor of the beam intensity. These signals were also gated by the system "busy" signal and used to determine the dead time of the electronics system.

Two ionization chambers, one constructed by Rice University (RION) and the other by LAMPF (LION), were used as absolute monitors of the beam intensity. The Rice ion chamber consisted of an aluminum collecting foil sandwiched between two high voltage foils which were separated by 1.905 cm of argon gas at atmospheric pressure. The chamber was operated in the ionization plateau region at 1500 V, and the current was integrated and digitized by a Brookhaven Instruments Corporation Model 1000 Current Integrator. The construction and characteristics of LION and the Faraday cup are described in reference Ba75. The Faraday cup was nonoperative during the majority of the experiment and consequently was used only as a relative intensity monitor.
The gain of RION can be calculated using the relation

\[ G_{\text{RION}} = CP \frac{\left[ \frac{tA}{2.24 \times 10^{-4}} \frac{P}{760} \frac{273.3}{T} \right] \frac{dE}{dx}}{W_{\text{Ar}}} \]  

(IV-1)

where \( t \) is the ion chamber thickness of 1.905 cm; \( A \) is the gram atomic weight of argon (39.948); \( P \) and \( T \) are the operating temperature and pressure in mm Hg and °K, respectively; \( \frac{dE}{dx} \) \( \text{Ar} \) is the energy loss per path length of 800 MeV protons in argon (1.697 \times 10^6 \text{ eV/gm/cm}^2 \) (Ja66)); \( W_{\text{Ar}} \) is the average energy per ion pair in eV (26.4 eV/ion pair (Ba51)); and \( CP \) is the calibration factor obtained by comparison with a calibrated current monitor. Assuming LION to be an accurate current monitor (Ba75), RION was calibrated against LION over a wide range of intensities, and \( G_{\text{RION}} \) was measured to be 159.6 ± 8 at 12.8° C, .767 atm., compared to the calculated value of 160.0; hence, \( CP = 1.00 ± .05 \). The activation reaction, \( ^{12}\text{C}(p,pn)^{11}\text{C} \), (Cu63), was measured during the calibration, and the cross section was found to agree within 8% with the previously interpolated value (Ho76b).

The Rice ion chamber subsequently served as the absolute beam intensity monitor during the experiment, using 160 ± 8 for \( G_{\text{RION}} \). LION could not be used directly at the low beam currents at which the experiment operated due to a slowly-varying, low-level background in the digitized signal. The temperature of RION and the atmospheric
pressure were recorded throughout the experiment, but fluctuations produced less than 1% changes in $G_{\text{RION}}$.

Several scalar ratios were continuously observed during the experiment to insure consistency among the monitors. In addition, pp elastic scattering was periodically measured, and the resulting cross section (see Section VII.A) was found to agree very closely with previous measurements (Wi76a, Fe76c). Thus, the absolute uncertainty of the beam normalization is estimated to be ±5%.

D. Electronic Logic

The purpose of the fast scintillator logic is to reduce accidental events and thus improve the data quality by requiring stricter time coincidences of the data than are available from the MWPC electronics alone. The wire plane strobe logic enables (strobes) the MWPC electronics to accept particle tracks whenever particles pass through the spectrometer and TOF arms in coincidence. If a second, more stringent scintillator coincidence condition (tag condition) is met, the MWPC information is processed as an event.

A block diagram of the electronic logic which enabled the wire planes is shown in Figure IV-1. This logic system was set up in the experimental area, or cave, in order to minimize the time required for the strobe signal to be transmitted to the wire planes. The coincidence condition which was necessary to generate a strobe can be
Figure IV-1

BLOCK DIAGRAM OF ELECTRONICS SYSTEM USED TO STROBE

THE MWPC DETECTORS

AND - EGG C144 majority coincidence
DISC - LRS 621 discriminator
FI(FO) - LRS 428 linear fan-in/fan-out
G & D - LRS 222 gate and delay generator
OR - LRS 429 logic fan-in/fan-out
STRB GEN - Strobe signal generator
written symbolically as S2*S3*S4, where S2, etc. refer to detectors indicated in Figure III-1. This enable pulse was set 200 nsec wide and occurred approximately 150 nsec after the passage of the particles. A veto input into the strobe coincidence module prevented the formation of a strobe while the beam gate (BG) was off, during the processing of an event, or during an adjustable time delay following the previous strobe. This delay time was set at 2 to 3 µsec, depending upon the beam intensity.

After an enable signal was generated, a readout of the wire plane coordinates was initiated if the logic condition Tag 8 = (S1*S2*S3*S4)* PCOIN was met, where PCOIN refers to the wire plane coincidence requirements, which will be discussed later (see Section V.B).

The electronics system which produced the tag condition is shown in Figure IV-2. The MWPC electronics allow simultaneous data accumulation under four different tag conditions, but only one tag was used in this experiment. The formation of the tag was vetoed by the computer busy signal (BUSY) or the presence of a "pile-up" (PU). A "pile-up" was produced whenever S2*S3*S4 occurred during the 200 nsec strobe interval; this condition occurred for less than 2% of the strobes throughout the experiment.

The timing for the scintillator coincidence conditions was made "loose" enough to easily encompass all reactions of interest but "tight" enough to eliminate
Figure IV-2

BLOCK DIAGRAM OF ELECTRONICS ARRANGEMENT PRODUCING
THE TAG CONDITION AND MEASURING THE TOF

ADC       - LRS 2249A analogue-to-digital
           converter
AND       - LRS 365 coincidence or equivalent
FO(FI)    - LRS 429 logic fan-in/fan-out or
           equivalent
G & D     - LRS 222 gate and delay generator
LEV CONV  - LRS 688 level adapter
PG        - EGG GP100 pile-up gate
TDC       - LRS 2226 time-to-digital converter
accidental events as much as possible. In general, the plane strobe electronics, located in the cave, were timed "looser" than the tag electronics, which were located in the Rice University data-acquisition trailer.

The electronics arrangement effecting the TOF measurements is also shown in Figure IV-2. The common start signal for the time-to-digital converter (TDC) came from scintillator S3, while S2 and S3 provided the stop signals. The TDC start was inhibited if a strobe signal was not present. An analogue-to-digital converter (ADC), also gated by the strobe, was used to measure the pulse height in scintillators S1 and S3.

The electronics configuration used to monitor the proton beam is shown in Figure IV-3. The ion chamber (RION) and Faraday cup signals were fed into digital current integrators and the output pulses were scaled along with the digitized LION signal provided by LAMPF. The output of the profile monitor was visually displayed on an oscilloscope. Coincidences between the monitor scintillators M1-M4 were formed in gated and ungated modes and scaled, along with accidental M1-M4 coincidences. In addition, several other scintillator and logic signals were scaled as measures of the relative beam intensity. Among these were S1, S2, S3, S4, STROBE, BG, PU, and the logic signals S1*S2, S3*S4, and S1*S2*S3*S4. These numbers were useful in normalizing the data and checking the operation of the detectors.
Figure IV-3

ELECTRONICS ARRANGEMENT USED TO MONITOR THE BEAM
AND MEASURE SCINTILLATOR EFFICIENCIES

AND - EGG 203 coincidence or equivalent
CI  - Brookhaven Instru. Corp. Model 1000
current integrator or Ortec 439
digital current integrator
DISC - LRS 621 discriminator
FO  - LRS 428 linear fan-in/fan-out
OR  - EGG OR102 dual OR/NOR module
SCALERS - L2550 12 channel scaler
SCOPE - Tektronix RM 564 storage oscilloscope
Also shown in Figure IV-3 is the electronic logic which measured the detection efficiency of scintillators S1-S4. This measurement employs additional scintillators, EF1 and EF2, which formed crossed paddles behind S2, and scintillators EF3 and EF4 which did likewise behind S4. The efficiencies were determined by taking ratios between the S1-S4 scintillator signals and the appropriate logic signal, EF1*EF2 or EF3*EF4. These efficiencies were observed during the experiment to insure proper operation of the scintillators.

The scintillator coincidence logic provided the event selection criteria for the MWPC electronics. Using the Rice Micro-programmed Input-Output Processor (Bu72b), the MWPC readout system was in turn interfaced to a DEC PDP-11/45 computer. This computer controlled the data acquisition which grouped events into data buffers and recorded these buffers on magnetic tape. As written on tape, an event consisted of the twelve MWPC readouts, the TOF and pulse height measurements, and information on the logic tag and wire plane coincidence conditions. Also included were multiple readout registers and the second readout of each coordinate, if such was present. In addition, the contents of all the scalars were written to tape every 120 seconds as a scalar buffer. The computer also performed an on-line analysis of the experimental data, which provided the primary monitor of the data quality.
V. DATA ANALYSIS

A. Event Identification and Background Reduction

The analysis of the data consisted of identifying the events resulting from the reaction of interest and displaying these events as functions of useful kinematic variables. Background events included data from other reactions and events which possessed improper scattering geometries. Such background events were removed from the events of interest, or "good" events, by a set of rejection criteria, or cuts, which were applied to the data. The logic and ordering of the cuts were found to be important in correctly identifying the good events. The resulting good events were histogrammed as functions of kinematic variables which displayed the dynamics of the reaction mechanisms under study.

The particle trajectories determined from the wire plane data provided several means of removing background events on the basis of geometry. The position of closest approach between the trajectories in the two arms was calculated and required to originate in the beam-target overlap volume. This position was described by histograms XTARG, YTARG, and ZTARG, which refer to a right-handed coordinate system with the origin at the target center, positive z downstream, and positive y upward. Events were also histogrammed and cut as a function of DVTARG, the
distance between the two trajectories at the point of closest approach. Typical target geometry distributions are shown in Figure V-1, along with the cuts employed. Since the target was housed in an evacuated chamber, few events were observed to originate outside the target volume. Those events which failed the target geometry cuts included accidental events, pion-decay events, and events originating from the points at which the proton beam passed through the scattering chamber windows.

Geometry restrictions were also included in the spectrometer arm in order to eliminate particles which scattered from the magnet pole faces. Geometric quantities given by the input and output magnet trajectories were compared with the same quantities calculated using the input trajectories and the modified uniform field model. For example, DXMAG was the difference between the measured and calculated horizontal position of the trajectory intersection with the magnet output face; DYMAG was the measured minus calculated difference in the vertical position at P5; and DSY was the difference between the measured and calculated vertical slope between P5 and P6. Typical magnet geometry distributions and cuts are shown in Figure V-2. The magnet geometry cuts were generally made quite "loose" due to the dispersion present in the calculated quantities.

The major criteria for removing events due to background reactions were the measured TOF values in each arm.
Fig. V-1

TARGET GEOMETRY DISTRIBUTIONS (--- Cuts)

ZTARG

COUNTS

YTARG

COUNTS

XTARG

COUNTS

MEAS.-CALC. (.05"")
Figure V-2

MAGNET GEOMETRY DISTRIBUTIONS

(--- CUTS)

COUNTS

DSY

4000

2000

-10 -5 0 5 10 15 20

COUNTS

DYMAG

4000

2000

-10 -5 0 5 10 15 20

COUNTS

DXMAG

4000

2000

-10 -5 0 5 10 15 20

MEAS.-CALC. (.05")
By assuming the reaction of interest in the kinematics and using the measured momentum $p_1$, the difference (DTOF) between the measured and calculated TOF in each arm was computed. Placing restrictions on these TOF differences was equivalent to requiring the particle mass in each arm be that specified by the reaction used in the kinematics. Hence, only events from the reaction of interest were accepted by the DTOF cuts. Shown in Figure V-3 is the TOF (right) arm DTOF spectrum for only protons being accepted in the spectrometer (left) arm. The peak centered at zero is due to pions from reaction (III-2) while the peak at larger DTOFR is due to protons from the background reaction (III-5) (see Figure III-2). The ability to separate the background from the good events by using the DTOF spectra was quite good even for this, one of the worst angle pairs for interference from background reactions.

As a check of the above procedure, the mass of the particle in the spectrometer arm was calculated directly from the measured momentum and TOF. Such a mass spectrum is shown in Figure V-4 for the $\theta_1/\theta_2$ angle pair, 12.8°/25°. For three-body reactions, by requiring a proton in the left arm, the mass of the particle in the right arm was also calculated (Ho76a). Cuts on these mass spectra provided a redundant means of removing events from background reactions.
Figure V-3

TIME-OF-FLIGHT SPECTRUM IN THE RIGHT ARM FOR
PROTONS DETECTED IN THE SPECTROMETER ARM

--- CUTS 12.8° / 25°

\[ \pi^+ (pp \rightarrow p\pi^+ n) \]

\[ p (pp \rightarrow pp\pi^0) \]

COUNTS

DTOFR (nsec)
Figure V-4

MASS SPECTRUM OF PROTONS DETECTED IN THE SPECTROMETER
The pulse heights measured in scintillators S1 and S3 provided additional information for particle discrimination. For the right arm, in which the particle type was a question, the protons were slower than the pions and gave a larger pulse height in S1. The pion pulse height peak was not cleanly separable from proton peak, however, since due to Landau straggling, a fraction of the pions had a pulse height comparable to the proton pulse height. The pulse height spectrum was more useful in forming a two-dimensional, S1 pulse height versus DTOFR histogram, in which the particle distinction was more apparent. It was then possible to make a two-dimensional cut on this histogram to separate particles, although the particle discrimination was seldom improved over that obtainable by using the DTOFR spectrum alone.

After applying all the rejection criteria to the data, the resulting good events were displayed as functions of numerous kinematic variables. Among these variables were measured quantities, such as the momentum $p_1$ and the scattering angles, and also calculated quantities such as the other particle momenta and the relative energy between each pair of particles in three-particle final states. For the n-p relative energy ($T_{np}$) of interest in studying mechanism (III-3), the relative momentum $k_{np}$ was also calculated.

A small background was found to remain in the DTOFR spectra after making cuts on the target geometry and TOF in
the spectrometer arm. This contribution is present in Figure V-3 and is seen most clearly at negative DTOFR values. Since events with TOF values less than that of the pions cannot correspond to true coincidences, these events are accidentals, which produce a white background in the DTOFR spectra. The relative amount of accidental background within the accepted DTOFR pion peaks varied from \( \sim 7.7\% \) at \( \theta_\pi = 22^\circ \) to \( \sim 3.6\% \) at \( \theta_\pi = 50^\circ \). In order to remove the contribution from accidentals, a second set of cuts was placed on the DTOFR spectra. These cuts were located at negative DTOFR values and enclosed the same width as the primary cuts. The events within the secondary cuts were analyzed as normal events and after complete analysis were subtracted from the various good event spectra.

The possibility of a significant contribution to the measured cross sections from reactions in the target cell was also considered in the analysis. In passing through the target cell, the beam traversed .0254 cm of Kapton, which has an atomic composition of \( C_{22}H_{10}N_2O_4 \) and a density of \( \sim 1.1 \text{ gm/cm}^3 \). When both scattering arms were placed at small angles, the acceptance of the system extended over a wide range of the z coordinate of the interaction position. Thus, scattering from the target chamber windows, located \( \sim 17 \text{ cm} \) upstream and downstream of the target center, was accepted by the system at small angles. The contribution from each window, which was .0254 cm of mylar, composition
C_{10}H_{8}O_{4}, density \sim 1.39 \text{ gm/cm}^3, was found to be \sim 0.7\% of the contribution from the target for both reactions of interest. This number may be considered as a rough estimate of the contribution from the target cell.

An attempt was made to determine explicitly the target cell contribution to the measured cross section by using the ZTARG distribution. Scattering from the target cell was present only at the edges of this distribution, but the resolution in ZTARG and the magnitude of the contribution were not sufficient to "see" any effect. The target cell contribution was calculated, however, by determining the difference between the cross section found in using the complete ZTARG distribution and that found by using only a central portion of this distribution. The thickness of the target was, of course, adjusted in the latter cross section calculation. The resulting target cell contribution was found to be 2\% \pm 3\% of the measured cross section for all reactions of interest in this experiment. Since this contribution was small compared to the statistical and systematic errors of the data, the effect was considered negligible. However, in future experiments it is suggested that target-empty runs be taken routinely in order to determine precisely the amount of scattering from the target cell.
B. Detector Efficiencies

The efficiency of the detector system constituted a major correction to the data. Based on previous experience and measurements described in Section IV.D, the scintillator detection efficiencies were generally assumed to be 100%. The MWPC total data efficiencies, however, were substantially lower. The operating characteristics of the wire planes are a function of the gas mixture, high voltage, bias voltage, and count rate (Bu72). As a result, the wire counters often have time varying efficiencies, which must be calculated for each individual data run. Two different types of wire plane efficiencies must be determined: the zero efficiency and the multiple readout efficiency.

The zero efficiency (ZEF) is the probability of a particle passing through all the wire counters without giving a zero readout in any coordinate. A zero readout results when either the coordinate does not detect the particle or some type of electronic malfunction occurs. For each run data were accumulated under three different wire plane coincidence conditions, which were cycled by computer during the course of the run; these conditions were

\[
\text{PCOIN} = P_1 \times P_3 \times P_5,
\]

\[
\text{PCOIN} = P_2 \times P_4 \times P_6,
\]

and

\[
\text{PCOIN} = P_1 \times P_2 \times P_3 \times P_4 \times P_5 \times P_6,
\]

where PCOIN was required in forming the tag condition (see Section IV.D). The data taken under the first two conditions
were used to measure the zero efficiencies of plane coordinates not in coincidence. That is, the zero efficiencies of the even planes were measured when the odd planes were required in coincidence, and vice versa.

To measure the zero efficiency of a particular wire plane, only the events detected within a restricted region of an adjacent, "shadow," plane were considered, where all events were assumed to originate in the target. The restrictions, or cuts, placed on the shadow plane coordinates were chosen well within the observed penumbral regions to ensure a true measure of the zero efficiency for each coordinate. Cuts were also placed on the TOF of the events considered in the ZEF calculation in order to make the results typical of the reactions of interest. The resulting zero efficiency of the MWPC system was just the product of the individual coordinate efficiencies,

$$ZEF = \prod_{i=1}^{12} ZEF_i.$$  \hspace{1cm} (V-1)

The value of ZEF for detecting the pion production reactions was typically 90%. Although the pulse height in the wire counters varies as $\sim \bar{\theta}^2$, no strong $\bar{\theta}$ dependence was observed in the measured ZEF, and hence, no such dependence was assumed in correcting the measured cross sections by ZEF.

The multiple readout efficiency is the probability of the particles of an event passing through all the wire counters without producing a multiple readout in any
coordinate. A readout is a group of adjacent activated wires in a MWPC coordinate. Each readout is characterized by the central position of the group of wires and the number of adjacent activated wires, or wire width. A multiple readout is an additional, separate group of activated wires in a given MWPC coordinate.

Multiple readouts are caused by electronic feed-throughs, delta rays, accidentals, or actual multi-prong events. Electronic feedthroughs are the result of crosstalk between adjacent amplifiers on the MWPC amplifier cards, described in reference Bu72. Normally amplifier crosstalk merely increases the wire width of a readout by one wire and changes the position by a half-wire. However, when adjacent amplifiers do not correspond to adjacent wires, as for the case of amplifiers one and nine, crosstalk produces a multiple readout. This phenomenon has been previously documented (Ga73, Ho76a) and was observed to account for approximately 30-40% of the multiple readout events in this experiment. Accidentals occur whenever particles from separate events traverse a plane coordinate within the \( \lesssim 700 \) nsec resolving time of the MWPC system. Accidentals are identical in appearance to actual multi-prong events, in which the separate particles belong to the same event. The production of delta rays is considered a probable cause of second readouts which occur in the near vicinity of first readouts.
In this experiment only events with no multiple readouts in the entire MWPC system were considered analyzable events. All multiple readout events were rejected, and the measured cross section was then corrected for the multiple readout efficiency (MREF). In determining MREF, cuts were placed on the TOF measured in each arm, so that MREF was calculated for only the reaction of interest. The multiple readout efficiency was found to increase with increasing $\beta$ of the proton detected in the spectrometer and thus, was calculated as a function of the momentum $p_1$ (see Figure V-5). The $\beta$ dependence of MREF is due to the fact that the MWPC pulse heights vary as $\sim \beta^{-2}$ and electronic crosstalk increases with pulse height. Thus, smaller $\beta$ implies larger pulse heights, more electronic feedthroughs, more multiple readouts, and hence, a smaller MREF.

In order to apply MREF as a correction factor to the cross section, it was necessary to prove that the rejected multiple readout events differed from the accepted events only in that they possessed additional readouts. To this end single multiple readout (1MR) events, which have only one additional readout in only one MWPC coordinate, were also analyzed. The 1MR events were analyzed with the same set of rejection criteria normally used to establish the fraction of zero multiple readout (OMR) events that are good events. The 1MR events were first analyzed using
Figure V-5

TYPICAL VARIATION IN MULTIPLE READOUT EFFICIENCY

(MREF) WITH PROTON MOMENTUM
the first readout, and if this produced a cut failure, the second readout was tried. By trying both readouts, if necessary, it was found that within statistics

\[
\frac{\text{good LMR}}{\text{LMR}} \sim \frac{\text{good OMR}}{\text{OMR}}, \quad (V-2)
\]

when identical cuts were applied in each case. Thus, the OMR and LMR events were found to belong to the same class of events.

The OMR and LMR data accounted for 60-70\% of all non-zero events. The remaining events possessed one or more multiple readouts in more than one MWPC coordinate, making the analysis of such events extremely difficult, if not impossible. However, if single additional readouts are considered to occur randomly (Ho76a), there exists to first order a fraction of events

\[
f_{\text{MR}} \approx P_1 + P_1^2 + P_1^3 + \ldots \approx \frac{P_1}{1-P_1} \quad (V-3)
\]

which are equivalent to OMR events, where \( P_1 \) is the probability of an event having one multiple readout. The fraction of total events which are equivalent to OMR events is then

\[
f = f_0 + f_{\text{MR}} \quad (V-4)
\]

where \( f_0 \) is the fraction of OMR events. Values of \( f \) were typically .85-.90, meaning that the MREF correction was accurate to approximately the 90\% level. Previous data
(Ho76a) have shown that a significant fraction of the remaining \((1-f)\) multiple readout events are due to the detection of multiple particles. In this experiment the additional particle is very likely an accidental, and thus, these events are also expected to be equivalent to the OMR events.

C. Error Analysis

Several sources of statistical and systematic errors are common to all cross sections measured in this experiment. The statistical errors arise from the number \((N)\) of data counts, the multiple readout efficiency \((\sim 1\%)\), and the calculated solid angle \((\sim 1\%)\). The statistics of the solid angle distributions were improved at each point by smoothing the calculated points by eye. The statistical errors are given by

\[
\frac{\Delta \sigma}{\sigma}_N = \frac{1}{\sqrt{N}} \text{ etc.}, \tag{V-5}
\]

and the total statistical error is calculated from (Be69)

\[
\frac{\Delta \sigma}{\sigma}_{\text{stat}} \approx \sqrt{\left(\frac{\Delta \sigma}{\sigma}_N\right)^2 + \left(\frac{\Delta \sigma}{\sigma}_{\text{MREF}}\right)^2 + \left(\frac{\Delta \sigma}{\sigma}_{\Omega}\right)^2} \tag{V-6}
\]

since the individual errors are essentially uncorrelated. Systematic errors in the experiment are due mainly to beam normalization \((\sim 5\%)\), the solid angle calculations \((\sim 2\%)\), the uncertainty in the areal target density \((\sim 2\%)\), and the correction for detector efficiencies \((\sim 2\%)\). Thus, by
quadratic addition as in Equation (V-6), the total systematic error is estimated to be ±6%. Unless otherwise noted, only statistical errors are included in the error bars for the cross sections presented in Chapter VII.
VI. CROSS SECTION CALCULATIONS

A. Three Body Cross Section

Kinematically-complete measurements of three body reactions can be used to determine fifth order differential cross sections. The three body cross section measured in this experiment is given by

\[
\frac{d^5 \sigma}{dp_1 d\Omega_1 d\Omega_2} = \frac{N/(MREF \times ZEF \times DT)}{N_0 n_t \Delta \Omega_1 \Delta \Omega_2 \Delta p_1}, \tag{VI-1}
\]

where \(N\) is the number of good events in the momentum interval \(p_1\) to \(p_1 + \Delta p_1\); \(MREF\) is the multiple readout efficiency; \(ZEF\) is the zero efficiency; \(DT\) is the electronic dead time correction; \(N_0\) is the number of incident protons; \(n_t\) is the number of target protons per \(cm^2\); and \(\Delta \Omega_1 \Delta \Omega_2\) is the average solid angle of the experimental system over the momentum interval \(p_1\) to \(p_1 + \Delta p_1\). The determination of \(N\), \(MREF\), and \(ZEF\) has been discussed in the preceding chapter. The solid angle is determined by a Monte Carlo calculation, which will be discussed in Section VI.B. The determination of the remaining parameters requires further explanation.

The incident beam current was measured by the ion chamber RION as described in Section IV.C. Thus, the number of incident protons was given by

\[
N_0 = \frac{\text{RION}}{G_{\text{RION}} \times 1.602 \times 10^{-19} \text{ coul.}}, \tag{VI-2}
\]
where here RION refers to the digitized output in units of 2 \times 10^{-11} \text{ coulombs}. The number of target protons was simply

\[ n_t = \frac{\mathcal{L} \rho N_a}{A}, \]  

(VI-3)

where \( \mathcal{L} \) is the target length (6.55 cm), \( \rho \) is the LH\(_2\) density (0.0708 gm/cm\(^3\)), \( N_a \) is Avagadro's number (6.022x10\(^{23}\)), and \( A \) is the gram atomic weight of hydrogen (1.008).

The dead time correction was determined by the gated and ungated M1-M4 coincidence signals, corrected for accidental coincidences. An approximate correction for strobe pile-ups was also included. In the notation of Figures IV-2 and IV-3,

\[ DT = \frac{\text{MONG} - \text{MGA}}{\text{MONF} - \text{MFA}} \times \frac{\text{STROBE}}{\text{STROBE} + \text{PU}}. \]  

(VI-4)

This equation assumes pile-ups are a small fraction (\( \approx 2\% \)) of the strobes, as was the case in this experiment. Typical values of \( DT \) for the event rates which existed in the experiment were 90-95%.

The cross section given in Equation (VI-1) is an average of the true cross section, or interaction probability, over the kinematically-allowed portion of the experiment acceptance. Although this cross section is usually specified at the point \( p_1, \theta_1, \phi_1, \theta_2, \phi_2 \), it can be identified by other independent kinematical variables, one of which may be the relative momentum between particular final state particles.
B. Monte Carlo Calculations

Calculation of cross sections as in Equation (VI-1) requires knowledge of the experimental solid angle $\Delta \Omega_1 \Delta \Omega_2$ subtended by the detectors. Since several detectors were employed in this experiment and the target was of finite size, an accurate calculation of the solid angle in closed form is extremely difficult. Moreover, the presence of the magnetic spectrometer in the detection system makes the solid angle a complex function of the scattering angle and momentum of the spectrometer particle. Physical effects such as energy loss, particle decay, multiple scattering, and kinematics also influence the solid angle. Therefore, the only convenient means of calculating the solid angle is by using Monte Carlo techniques.

The Monte Carlo programs used in the analysis are essentially computer simulations of the physical experiment. A random interaction position is chosen within the beam-target overlap volume, and isotropic scattering angles ($\phi$, $\cos \theta$) are randomly selected for particle masses corresponding to the reaction of interest. A random momentum $p_1$ is then selected within a physically realistic momentum range to complete the set of kinematical variables describing a three-body event. This event is propagated through the experimental system to see if it intersects all detectors and the magnet gap. In this propagation the effects of magnetic fields, energy loss, multiple scattering,
and particle decay are considered. If the event meets all the requirements of the experimental geometry and is kinematically allowed, it is binned as a function of $p_1$ and other kinematically calculated quantities.

For the analysis of three-body final state reactions, it is also useful to produce histograms in which each good Monte Carlo event is renormalized by a three-body Lorentz invariant phase space factor, $I$, given by

$$I = \frac{p_1^2 p_2^2}{8E_1[p_2 E_3 + E_2 p_2 - E_2 p_0 \cos \theta_2 - E_2 p_1 \cos (\theta_1 - \theta_2)]}, \quad (VI-5)$$

where the subscripts 0, 1, 2, and 3 refer to the beam particle, the momentum-analyzed particle, the other detected particle, and the undetected particle, respectively. Details of the calculation of this phase space factor are given in Appendix B.

The effect of including energy loss in the Monte Carlo calculation of solid angle for reaction (III-2) was quite significant for proton momenta below 400 MeV/c. The energy loss of the proton in the magnet arm resulted in a lower $p_1$ momentum with a different momentum acceptance and hence a different solid angle. This shifted the acceptance toward higher momenta, reducing the acceptance for 400 MeV/c protons by 15-20%. Due to energy loss, the momentum measured in the spectrometer was smaller than the true event momentum. Shown in Figure VI-1 is the calculated difference between the true and measured momentum as a
Figure VI-1

DIFFERENCE BETWEEN TRUE AND MEASURED PROTON MOMENTUM RESULTING FROM ENERGY LOSS IN THE SYSTEM

\[
(p_t^i - p_t^m) \quad \text{(MeV/c)}
\]

\[
p_t^m \quad \text{(MeV/c)}
\]
function of the measured momentum. This distribution applied for all pp-$d^*\pi^+$ angle pairs and was used in the analysis program to transform the measured momentum into the true event momentum. Such a correction was necessary in order to associate the proper momentum with the measured angles before calculating other kinematical variables.

For the range of angles over which reactions (III-1) and (III-2) were measured in this experiment from $\sim 9\%$ to $\sim 12\%$ of the pions decayed before completely traversing the TOF arm. Many of the muons resulting from these decays were detected in the TOF arm; however, changes in the measured scattering angles were large enough to cause most of the muon events to be rejected by target geometry cuts. The Monte Carlo calculation revealed that typically 90\% of the pion decay events were rejected by the geometry restrictions imposed by the physical apparatus or cuts in the data analysis. The result of including multiple scattering in the solid angle calculation was found to be negligible due to the very small scattering angles involved.

The specific Monte Carlo programs which were employed in the cross section calculations are described in Appendix C. The basic Monte Carlo technique was checked by comparing the calculated solid angle distributions with those measured using pp elastic scattering. The agreement between the measured and calculated acceptances was very good for elastic scattering. The results of the Monte Carlo calcu-
lations for three body reactions were compared with results obtained from independent calculations (Ho76a, Wi75) which also utilized Monte Carlo techniques. In all cases there was excellent agreement between the independent calculations.

C. Singly Differential Cross Sections

The two body reactions measured in this experiment are presented as singly differential cross sections given by

\[ \frac{d\sigma}{d\Omega_2} = \frac{N/(\text{MREF*ZEF*DT})}{N_0 n_t \Delta\Omega_2}, \]  

(VI-6)

where all quantities are defined as in Equation (VI-1).

The term "singly differential" is being used loosely here since \( d\Omega_2 = d\cos\theta_2 d\phi_2 \). Implicit in the calculation of the average solid angle \( \Delta\Omega_2 \), was the fact that the coincident detection of the second particle was required in the spectrometer arm.

In order to compare explicitly the cross sections for the pp-\( \pi^+ \) and pp-\( d^*\pi^+ \) reactions, a singly differential cross section was also calculated for the latter reaction. Using the Monte Carlo routine FSIEF, described in Appendix C, the average solid angle \( \Delta\Omega_2 \) was determined as a function of the n-p final state relative momentum, \( k_{np} \). This solid angle was greatly reduced at large values of \( k_{np} \) due to the rapidly falling detection efficiency for the \( d^* \) proton in the spectrometer arm. Typical results for this calculated efficiency are shown as a function of \( k_{np} \) in Figure
Figure VI-2

TYPICAL $d^*$ DETECTION EFFICIENCY VERSUS $k_{np}$

$P^*(k_{np})$

$14.7^\circ / 40.6^\circ$

$k_{np}$ (MeV/c)
VI-2. The calculation assumed that the d* system was produced in a pure S-state, and all results were found to agree very well with an independent Monte Carlo calculation (Wi76b). Using the number of good events, N, which were also binned as a function of $k_{np}$, the singly differential cross section was determined by integrating over $k_{np}$ from zero to small maximum values. Thus, the resulting $d\sigma/d\Omega_2$ represented the cross section for the pseudo-two-body reaction $pp \rightarrow d^*\pi^+$, where $d^*$ was defined only for $k_{np} \leq k_{np}^{max}$. 
VII. MEASURED CROSS SECTIONS

A. pp-pp

The pp elastic scattering cross section was measured in order to calibrate the experimental system, determine the resolution of the apparatus, and check the normalization and solid angle calculation. The measured differential cross section at 800 MeV is presented in Figure VII-1 and Table VII-1 as a function of the c.m. scattering angle. For comparison, the data of Willard et al. (Wi76a) are shown in Figure VII-1, with error bars which represent the overall uncertainty. Since the two measurements agree within the statistical errors given for the present data, the estimate of 5% uncertainty in the beam normalization is supported. Additional pp elastic scattering data exist at 800 MeV (Fe76b) which are also in good agreement with the cross sections presented here.

B. pp-dπ⁺

The pair of reactions

\[ pp - \pi^+ d \]  \hspace{1cm} (VII-1)

and

\[ \pi^+ d - pp \]  \hspace{1cm} (VII-2)

comprise one of the rare examples in particle physics where a reaction and its inverse can both be measured conveniently. These reactions have been used to test the
Figure VII-1

DIFFERENTIAL C.M. CROSS SECTION FOR P-P ELASTIC
SCATTERING AT 800 MeV

The data of the present experiment are compared to
that of Willard et al. (Wi76a) at the same energy;
the data points at 90° overlap completely. The
solid line is a guide to the eye.
$pp \rightarrow pp$

- Present work
- Wi76a
### TABLE VII-1

Proton-Proton Elastic Scattering at 800 MeV

<table>
<thead>
<tr>
<th>$\bar{\theta}_{\text{lab}}$ (deg)</th>
<th>$\frac{d\sigma}{d\Omega}_{\text{lab}}$ (mb/sr)</th>
<th>$\bar{\theta}_{\text{cm}}$ (deg)</th>
<th>$\frac{d\sigma}{d\Omega}_{\text{cm}}$ (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.8</td>
<td>13.9 ± .6</td>
<td>53.4</td>
<td>2.99 ± .12</td>
</tr>
<tr>
<td>26.0</td>
<td>10.1 ± .4</td>
<td>60.4</td>
<td>2.31 ± .10</td>
</tr>
<tr>
<td>30.1</td>
<td>6.29 ± .28</td>
<td>69.4</td>
<td>1.56 ± .07</td>
</tr>
<tr>
<td>35.0</td>
<td>4.10 ± .14</td>
<td>79.8</td>
<td>1.14 ± .04</td>
</tr>
<tr>
<td>40.0</td>
<td>3.16 ± .12</td>
<td>89.9</td>
<td>1.00 ± .04</td>
</tr>
</tbody>
</table>
principle of detailed balance, by which a process can be simply related to its inverse process (Ro67). Measurements of these reactions have verified this principle to within 5% (Ri70), although the same has been accomplished at the .3% level in low energy nuclear interactions (Wi68).

Assuming detailed balance valid, either of the above reactions may be used to gain knowledge of its inverse. Thus, both reactions (VII-1) and (VII-2) are employed in the measurement and analysis of pion production via pp→π⁺d.

The c.m. differential cross section for the pp→π⁺ reaction measured at 800 MeV is presented in Figure VII-2 as a function of the cosine of the c.m. pion angle (also see Table VIII-2). The data of Richard-Serre et al. (Ri70) are shown for comparison; these data, which correspond to an equivalent incident proton energy of 810 MeV, were measured for the inverse reaction and transformed using detailed balance. The agreement between the two measurements is very reasonable, where in both cases the error bars are due to statistical errors, which vary with angle.

The measured cross section was fitted to the parameterization,

\[ \frac{d\sigma}{d\Omega} \bigg|_{\text{c.m.}} = K(A + \cos^2 \theta_{\text{c.m.}} - B \cos^4 \theta_{\text{c.m.}}) \]

(VII-3)

the results of which are presented in Figure VII-2 and Table VII-2. The first two terms of this parameterization
Figure VII-2

DIFFERENTIAL C.M. CROSS SECTION FOR pp→π⁺ at 800 MeV

Also shown are the data of Richard-Serre et al. (Ri70) at an equivalent proton energy of 310 MeV.
The solid line is a fit to the data of the present experiment using the parameterization of Eq. (VII-3).
\[ \frac{d\sigma}{d\Omega} = 171.(281 + \cos^2 \theta - 2.33 \cos^4 \theta) \]

- Present experiment
- Reference RI70 (810 MeV)
TABLE VII-2

Parameters Describing the Differential Cross Section

for $pp \rightarrow d\pi^+$ Near 800 MeV

<table>
<thead>
<tr>
<th>$T_p$ (lab) (MeV)</th>
<th>A</th>
<th>B</th>
<th>$K$ (mb/sr)</th>
<th>$\sigma_{tot}$ (mb)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>.322±.020</td>
<td>0</td>
<td>.151±.006</td>
<td>1.25±.09</td>
<td>This work</td>
</tr>
<tr>
<td>800</td>
<td>.28 ± .04</td>
<td>.23±.23</td>
<td>.171±.023</td>
<td>1.22±.19</td>
<td>This work</td>
</tr>
<tr>
<td>(810)</td>
<td>.28 ± .08</td>
<td>.57±.17</td>
<td>.212±.018</td>
<td>1.33±.12</td>
<td>Ne 58</td>
</tr>
<tr>
<td>(810)</td>
<td>.275±.023</td>
<td>.43±.06</td>
<td>.177±.007</td>
<td>1.16±.05</td>
<td>Ri 70</td>
</tr>
</tbody>
</table>
account for the production of s- and p-wave pions (see Table II-1). The $\cos^4 \theta$ term is included to provide for the production of d- and f-wave pions. Evidence for including the $\cos^4 \theta$ term at this energy has been established in very complete angular distribution measurements from 600-800 MeV (Ri70). For the limited angular range measured here, the $\cos^4 \theta$ term is not required for a reasonable least-squares fit to the cross section; nevertheless, fits were made both with and without the $\cos^4 \theta$ term for the sake of completeness and for comparison with other work.

In terms of the parameters in Equation (VII-3), the total cross section for reaction (VII-1) at 800 MeV was calculated using

$$\sigma_{tot} = 4\pi K \left( A + \frac{1}{3} - \frac{B}{5} \right), \quad (VII-4)$$

and is given in Table VII-2, along with the $\sigma_{tot}$ and fitted parameters from two previous experiments which studied the inverse reaction. The quoted errors in the total cross section include all statistical and systematic errors, added quadratically. The agreement between the present and previous experiments is quite good; however, the errors in the present work are rather large due to the small number of measured points.

It is significant to note that the approximate $\cos^2 \theta$ dependence of the angular distribution strongly suggests a
dominant $^1D_2 \rightarrow ^3S_1P_2$ transition (see Table II-1), as previously established in numerous works (Ma58, Ri70). This observation is not surprising since the $^1D_2$ transition is favored statistically ($2J+1 = 5$) and, more importantly, is strongly enhanced by the formation of the intermediate $\Delta$ isobar. The fact that a $\cos^4\theta$ contribution appears in the angular distribution has been explained as due to $f$-wave interference with the dominant $^1D_2$ transition (Sc68).

C. pp-$p^+n$

The fifth order differential cross sections for the pp-$p^+n$ reaction were calculated from the data by using Equation (VI-1). These cross sections were measured for the angles given in Table III-2 and represent averages of the true cross sections over the angular acceptance of the system. The range of proton momenta, $p_1$, over which the cross sections were measured was determined by the central momentum and experimental momentum acceptance for each angle pair.

The three body differential cross sections are presented in Figure VII-3 as a function of $p_1$. The dashed curve which appears in each spectrum is a plot of $p_1$ versus the n-p relative momentum ($k_{np}$) as given by the scale on the right. The pronounced enhancement observed in the cross section near $k_{np} = 0$ is clear evidence of this reaction proceeding via the neutron-proton final state interaction. A broad, second peak occurring near the upper limit of the
Figure VII-3

DIFFERENTIAL CROSS SECTIONS FOR $pp+p^+n$ AT 800 MeV

$$\frac{d^5\sigma}{dp_1d\Omega_1d\Omega_2}$$ versus $p_1$

a) $\theta_p = 11.9^0$; $\theta_\pi = 22.0^0$

b) $\theta_p = 12.8^0$; $\theta_\pi = 25.0^0$

c) $\theta_p = 13.8^0$; $\theta_\pi = 30.0^0$

d) $\theta_p = 14.7^0$; $\theta_\pi = 40.6^0$

e) $\theta_p = 14.4^0$; $\theta_\pi = 50.0^0$

The n-p relative momentum ($k_{np}$) is given as a function of $p_1$ by the dashed line and the scale on the right.
$P + P = P + \pi^0 + N$  11-9/22  FIG VII-3A
P+P = P+P+I+N 128/25. FIG VII-3B

CROSS SEC (µb/ST^2*MEV/C)

P1 (MEV/C)

KNP (MEV/C)
$P + P = P + P + N$ 13.8/30.°  FIG VII-3C
P+P = P+π+π 147/406 FIG VII-3D

CROSS SEC ($\mu B/\text{sr}^2/\text{MeV/c}$)

P1 (MeV/c)

KNP (MeV/c)
momentum acceptance in each distribution is an enhancement due to the formation of the \( \Delta^{++}(p\pi^+) \) and \( \Delta^+(n\pi^+) \) isobars in the pion production process. The shape of this latter peak is distorted by averaging over the large angular acceptance of the apparatus and possibly by interference effects between the two isobars.

In order to examine more carefully the n-p FSI enhancement and minimize the effect of averaging over the angular openings, the data are presented also as a function of the n-p relative momentum and given in Figure VII-4. To produce these spectra, the data were binned as a function of the \( k_{np} \) calculated for each event and divided by the Monte Carlo calculated \( k_{np} \) distribution which included the phase space factor, \( I \) (see Section VI.B). The resulting cross section is the same as that in Figure VII-3, with the exception that it is divided by phase space and projected on the n-p relative momentum axis. The resolution in \( k_{np} \) was found to be \( \lesssim 5 \text{ MeV/c} \) for small relative momenta.

The rapid increase in the cross sections in Figure VII-4 as \( k_{np} \) approaches zero exhibits the general behavior expected for n-p FSI. Other processes also contribute to these spectra, however, especially at large \( (k_{np} > 80 \text{ MeV/c}) \) momenta. In particular, the broad \( \Delta \) peak seen in Figure VII-3 extends into the region of the \( d^* \) peak in some cases. This produces a contribution in the spectra of Figure VII-4 which can be considered a non-FSI, \( \Delta \) background. In
Figure VII-4

DIFFERENTIAL CROSS SECTIONS FOR $pp \rightarrow p\pi^+ n$ PROJECTED
ON THE $n-p$ RELATIVE MOMENTUM AXIS

$$\frac{d^5 \sigma \left( k_{np} \right)}{dp_1 d\Omega_1 d\Omega_2} \quad \text{versus} \quad k_{np}$$

a) $\theta_p = 11.9^{\circ}; \quad \theta_\pi = 22.0^{\circ}$
b) $\theta_p = 12.8^{\circ}; \quad \theta_\pi = 25.0^{\circ}$
c) $\theta_p = 13.8^{\circ}; \quad \theta_\pi = 30.0^{\circ}$
d) $\theta_p = 14.7^{\circ}; \quad \theta_\pi = 40.6^{\circ}$
e) $\theta_p = 14.4^{\circ}; \quad \theta_\pi = 50.0^{\circ}$

The lines are the result of fits to the data described in section VIII.A.
$P + P = P + P + N$  

\[ \mu B / Sr^2 \text{ vs. MeV}^2 \]

- **DATA**
- $X^s F^s + X^t F^t + C$
- $X^t F^t + C$
- $C$

KNP (MeV/C)
\( P + P = P + \pi^0 + N \) 12.8/25.

(b) \( + \text{DATA} \)

\(- X^s F^s + X^t F^t + C \)

\(-- X^t F^t + C \)

\( \cdots C \)

CROSS SEC / PHSP

(\( \mu B / \text{SR}^2 \times \text{MEV}^2 \))

KNP (MEV/C)
$P+P = P+PI+N \quad 14\cdot7/40\cdot6$

\begin{align*}
+ & \text{ DATA} \\
- & X^sF^s + X^tF^t + C \\
-- & X^tF^t + C \\
---- & C
\end{align*}

CROSS SEC / PHSP

(\mu B/\text{SR}^2 \text{ * MEV}^2)

KNP (MEV/C)
\[ P + P = P + \pi^0 + N \quad 14.4/50. \]

**Graph:**

- **DATA**
- \( X^s F^s + X^t F^t + C \)
- \( X^t F^t + C \)
- \( C \)

**Axes:**
- **CROSS SEC / PHSP (\( \mu B / sr^2 * \text{MeV}^2 \))**
- **KNP (MEV/C)**
order to estimate the magnitude and extent of this background, the $p_\perp$ spectra in Figure VII-3 were divided by phase space and the low-$p_\perp$ half of the $d^*$ peak was assumed to have negligible background. The Gaussian shape of the low-momentum half of the $d^*$ peak then was assumed to describe also (symmetrically in $k_{np}$) the high-momentum half of this peak, and differences between the cross section and this shape were ascribed to a $\Delta$ background. For the $k_{np}$ spectra with $\theta_\pi \geq 30^\circ$, such a background was found to be essentially negligible in the region, $k_{np} \leq 75$ MeV/c. At smaller pion angles, this background was very small for $k_{np} \lesssim 60$ MeV/c.
VIII. DISCUSSION

A. FSI Fits

The cross sections in Figure VII-4 were compared to Goldberger-Watson FSI theory by performing least-square fit calculations to the expression

\[
\frac{d^5\sigma}{dp_1 d\Omega_1 d\Omega_2} \frac{1}{I} = x^S F^S(k) + x^T F^T(k) + C, \quad (VIII-1)
\]

which is similar to Equation (II-6). Here, C is a constant as a function of k, and \(F^S, F^T\) are the Goldberger-Watson (GW) enhancement factors given in Equation (II-3). These enhancement factors were evaluated with the following set of singlet-triplet scattering length and effective range parameters (Br70):

\[
\begin{align*}
    r^T &= 1.75 \pm .015 \text{ fm}, \\
    r^S &= 2.67 \pm .02 \text{ fm}, \\
    a^S &= -23.68 \pm .03 \text{ fm}, \\
    a^T &= 5.41 \pm .01 \text{ fm}.
\end{align*}
\]

In the fitting process, the experimental momentum and angular resolutions were folded into the enhancement factors by using the Monte Carlo routine TSMER, described in Appendix C. The constant term was included in the fits since such a term has been found useful in describing n-p FSI data at high energies (Wi75, Fe76c). Thus three factors, \(x^S, x^T\), and C, were allowed to vary in the fitting procedure.
The results of the fits to the GW theory are given in Figure VII-4 and Table VIII-1a. Shown in Figure VII-4 are the contributions from all terms (solid line), from the triplet plus constant terms (dashed line), and from the constant term alone (dotted line). The data at each angle pair were fit only over the region where no obvious $\Delta$ background was present, and this region is indicated by the extent of the lines in Figure VII-4. As seen in this figure, the GW theory with a constant term gives an excellent description of the shape of the n-p relative momentum spectra at all angles.

Several observations can be made by considering the fitting parameters given in Table VIII-1a. The most significant of these is that the n-p FSI is due in large part to scattering in the $^3S_1$ (triplet) state. This is indicated by the ratio $X^+/X^S$, which is much larger here than the 3:1 predicted by simple spin statistics. Although the $^3S_1$ contribution is small, it is important at very small $k_{np}$, as indicated in Figure VII-4. The predominance of triplet scattering in the n-p FSI of pp-d*π$^+$ can be understood by recalling the results for pp-dπ$^+$. In the latter reaction the transition $^1D_2$-$^3S_1P_2$ was found to be dominant, mainly because it was favored by the formation of the intermediate $\Delta$ isobar. This transition is expected to be dominant for pp-d*π$^+$ also, since in the OPE model (see Figure II-2) the two reactions differ only in the
### TABLE VIII-1a

Results of Fitting the Data with the Goldberger-Watson FSI Theory with a Constant Term

<table>
<thead>
<tr>
<th>$\theta^{\pi}_{\text{lab}}$</th>
<th>$\theta^{\pi}_{\text{cm}}$</th>
<th>$x^s$</th>
<th>$x^t$</th>
<th>$x^{sF^s}_{np=0}$</th>
<th>$x^{tF^t}_{np=0}$</th>
<th>$c$</th>
<th>$x^t/x^s$</th>
<th>$\chi^2_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(degrees)</td>
<td>(nb/sr$^2$MeV$^2$)</td>
<td>(nb/sr$^2$MeV$^2$)</td>
<td>(nb/sr$^2$MeV$^2$)</td>
<td>(nb/sr$^2$MeV$^2$)</td>
<td>(nb/sr$^2$MeV$^2$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.</td>
<td>40.8</td>
<td>3.14±.77</td>
<td>67.4±9.6</td>
<td>1201.±301.</td>
<td>1040.±148.</td>
<td>170.±79.</td>
<td>21.5</td>
<td>.60</td>
</tr>
<tr>
<td>25.</td>
<td>46.1</td>
<td>2.79±.59</td>
<td>58.9±6.4</td>
<td>1081.±217.</td>
<td>909.±98.</td>
<td>119.±49.</td>
<td>21.1</td>
<td>.62</td>
</tr>
<tr>
<td>30.</td>
<td>54.5</td>
<td>2.37±.44</td>
<td>51.4±3.9</td>
<td>919.±170.</td>
<td>793.±60.</td>
<td>57.±27.</td>
<td>21.7</td>
<td>.68</td>
</tr>
<tr>
<td>40.6</td>
<td>71.1</td>
<td>1.29±.40</td>
<td>34.6±3.6</td>
<td>500.±155.</td>
<td>534.±52.</td>
<td>36.±23.</td>
<td>26.8 1.06</td>
<td>.87</td>
</tr>
<tr>
<td>50.</td>
<td>84.3</td>
<td>.78±.30</td>
<td>27.0±2.5</td>
<td>300.±116.</td>
<td>418.±39.</td>
<td>47.±17.</td>
<td>34.9</td>
<td>.87</td>
</tr>
</tbody>
</table>

$\chi^2_R$ refers to the reduced chi-square of the fits.
n-p FSI, which may or may not leave the nucleons in a bound state.

As seen also in Table VIII-la, the ratio $X^t/X^s$ tends to decrease with decreasing $\theta^\Pi$ while the constant term $C$ increases. The decrease in $X^t/X^s$ suggests that $1_S^0$ scattering may become more important relative to $3_S^1$ scattering at forward pion angles. In order to determine the effect of including the constant term in Equation (VIII-1), the data were fit also with $C = 0$, the results of which are given in Table VIII-1b. These fits are inferior to those with non-zero C terms, and show a slightly different trend in $X^t/X^s$ with angle. The exclusion of the constant term resulted in $X^t$ coefficients which were typically 25% larger and $X^s$ values which were typically 35% smaller; however, the shapes of the distributions of $X^t$ and $X^s$ remained largely unchanged.

The constant term in Equation (VIII-1) describes a contribution to the cross section which is independent of the n-p relative momentum. The size and probable shape (phase space) of this term are shown in Figure VII-3 as a function of momentum $p_\perp$ for the $13.8^\circ/30^\circ$ angle pair. Such a constant contribution is expected for the physical case in which no n-p rescattering occurs after the production of the n-p pair, but in principle the factor $F(k)$ accounts for this case (see Section II.C). However, the fact that a second, much stronger enhancement ($\Delta$) exists
TABLE VIII-1b

Results of Fitting the Data to the Goldberger-Watson Theory without a Constant Term

<table>
<thead>
<tr>
<th>$\Theta_{\text{cm}}^\pi$</th>
<th>$X^s$</th>
<th>$X^t$</th>
<th>$X^t/X^s$</th>
<th>$\chi^2_{\text{R}}$</th>
<th>$d\sigma/d\Omega^s_{0-40}$</th>
<th>$d\sigma/d\Omega^t_{0-40}$</th>
<th>$d\sigma/d\Omega^t_{0-65}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(deg)</td>
<td>(nb/sr$^2$MeV$^2$)</td>
<td>(nb/sr$^2$MeV$^2$)</td>
<td></td>
<td></td>
<td>(\mu b/sr)</td>
<td>(\mu b/sr)</td>
<td>(\mu b/sr)</td>
</tr>
<tr>
<td>40.8</td>
<td>1.97\pm.54</td>
<td>87.8\pm1.9</td>
<td>44.6</td>
<td>1.11</td>
<td>.62\pm.17</td>
<td>8.92\pm.40</td>
<td>29.6\pm1.4</td>
</tr>
<tr>
<td>46.1</td>
<td>1.83\pm.44</td>
<td>73.9\pm1.6</td>
<td>40.4</td>
<td>1.15</td>
<td>.60\pm.14</td>
<td>7.78\pm.35</td>
<td>25.5\pm1.3</td>
</tr>
<tr>
<td>54.5</td>
<td>1.78\pm.34</td>
<td>59.2\pm1.2</td>
<td>33.3</td>
<td>1.00</td>
<td>.57\pm.10</td>
<td>6.04\pm.27</td>
<td>19.9\pm1.0</td>
</tr>
<tr>
<td>71.1</td>
<td>.90\pm.31</td>
<td>39.7\pm1.0</td>
<td>44.1</td>
<td>1.18</td>
<td>.29\pm.10</td>
<td>4.26\pm.24</td>
<td>13.5\pm.8</td>
</tr>
<tr>
<td>84.3</td>
<td>.25\pm.23</td>
<td>33.7\pm.8</td>
<td>134.</td>
<td>1.41</td>
<td>.09\pm.08</td>
<td>3.54\pm.16</td>
<td>10.7\pm.6</td>
</tr>
</tbody>
</table>

$\chi^2_{\text{R}}$ refers to the reduced chi-square of the fits.
in a nearby region of phase space may alter this situation. For example, a contribution may exist from an interference between the \(d^*\) and \(\Delta\) mechanisms. Also the \(\Delta\) enhancement is not entirely absent in the \(n-p\) FSI region, although it is greatly diminished. Thus the constant term may be attributed in part to data from the \(\Delta\) tail, even though these data show some \(k_{np}\)-dependence in the fitting region.

Other possible sources of the constant term include accidental events and background reactions. The analysis of accidentals in the DTOFR spectra (see Section V.A) revealed that these events contributed solely to the constant term and that their removal reduced this term by typically 30%. Thus any incomplete removal of the accidentals will strengthen this term. Additional background sources are the reaction \(pp\rightarrow ppn^0\) and the four-body (double pion production) reactions. In the event of incomplete proton rejection in the TOF arm, \(pp\rightarrow ppn^0\) can produce a \(k_{np}\)-independent background; however, this effect should be small. The cross sections for double pion production at 800 MeV are only a few percent of that for \(pp\rightarrow pn^+n\) (Lo70, Co72). These four-body reactions can produce a small number of events, mainly at the more forward pion angles, in which protons are scattered into the spectrometer arm and \(\pi^\pm\) are detected in the TOF arm. Since such events have the signature of \(pp\rightarrow pn^+n\) and show little dependence on the calculated \(k_{np}\), they may also contribute
slightly to the constant term. In any case, the constant term is generally small, but its inclusion in Equation (VIII-1) greatly improves the fits to the measured cross sections.

B. Singly Differential d* Cross Sections

The singly differential d* cross sections were calculated using the procedure outlined in Section VI.C. The data counts \( N(k_{np}) \) were corrected for the solid angle \( \Omega_2(k_{np}) \) which included the d* detection efficiency of the spectrometer arm. As a function of \( k_{np} \), the fractions of the data due to singlet scattering and to triplet scattering were determined from the GW fits of Table VIII-1a and applied to the corrected data counts. Then by summing over \( k_{np} \), the singly differential cross sections were found for the singlet (s), triplet (t) and total (d*) contributions to the scattering.

The singlet cross section was calculated only for \( k_{np} \leq 40 \text{ MeV/c} \) (\( T_{np} \leq 1.7 \text{ Mev} \)) since its contribution is very small at higher \( k_{np} \). The triplet cross section was determined for \( k_{np} \leq 40 \text{ MeV/c} \) and for \( k_{np} \leq 65 \text{ MeV/c} \) (\( T_{np} \leq 4.5 \text{ Mev} \)). At higher relative momenta, background begins to contribute strongly to the cross section, especially at the more forward pion angles. Thus the integration limit was chosen large enough to utilize most of the d* data but small enough to prevent the cross section from being strongly influenced by less statistically significant data, which contained a larger background.
The singly differential c.m. cross sections for \( pp-d^*\pi^+ \) are presented in Figure VIII-1 and Table VIII-2. The contributions from the n-p singlet, triplet, and total (including C term) FSI are given along with the cross section for \( pp-d\pi^+ \). Equivalent cross sections calculated from the C = 0 fits are given in Table VIII-1b. The angular shapes of the triplet and total \( d^* \) cross sections closely approximate that of \( pp-d\pi^+ \), which is strong evidence for a dominant \( ^1D_2-^3S_1P_2 \) transition in \( pp-d^*\pi^+ \). The large uncertainty in the singlet contribution permits few observations other than the fact that the cross section is very small and the angular distribution is not isotropic. Anisotropy implies that transitions of higher order than class Ss (given in Table II-1) are important in this \( \sigma_{11} \) isospin reaction. The Ps and Pp classes are known to contribute significantly to \( \sigma_{11} \) in this energy range (Lo70), but such transitions do not leave the final state nucleons in a relative S-state. Thus the next highest transition class which is expected to contribute to the \( ^1S_0 \) nucleon final state is Sd. The \( pp-d^*\pi^+ \) cross section in Figure VIII-1 is smaller than the \( pp-d\pi^+ \) cross section, but, of course, the relative magnitude is a function of the \( k_{np} \) integration limit.

C. The \( \sigma^+/\sigma^d \) Ratio

As discussed in Section II.C, n-p FSI theory can be used to predict the ratio of scattering into the triplet
Figure VIII-1

SINGLY DIFFERENTIAL pp→d*π+ CROSS SECTIONS AT 800 MeV

The individual contributions of the singlet ($^1S_0$) and triplet ($^3S_1$) cross sections are shown integrated over $k_{np}$ from 0-40 MeV/c or 0-65 MeV/c. Also shown are the cross section for the total d* contribution from 0-40 MeV/c and the pp→dπ+ cross section. The solid lines are guides to the eye.
<table>
<thead>
<tr>
<th>θ^{d*}/θ^{π^+})_{lab}</th>
<th>θ^{π}_cm (deg)</th>
<th>dσ/dΩ^S_{0-40} (μb/sr)</th>
<th>dσ/dΩ^{t}_{0-40} (μb/sr)</th>
<th>dσ/dΩ^{t}_{0-65} (μb/sr)</th>
<th>dσ/dΩ^{d*}_{0-40} (μb/sr)</th>
<th>dσ/dΩ^{d} (μb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.9/22.</td>
<td>40.8</td>
<td>.99 ± .26</td>
<td>7.0 ± 1.0</td>
<td>22.2 ± 3.4</td>
<td>9.51 ± .46</td>
<td>133.±5.</td>
</tr>
<tr>
<td>12.8/25.</td>
<td>46.1</td>
<td>.93 ± .21</td>
<td>6.31 ± .74</td>
<td>19.8 ± 2.3</td>
<td>8.34 ± .35</td>
<td>120.±4.</td>
</tr>
<tr>
<td>13.8/30.</td>
<td>54.5</td>
<td>.77 ± .14</td>
<td>5.34 ± .42</td>
<td>17.2 ± 1.4</td>
<td>6.60 ± .34</td>
<td>102.±3.</td>
</tr>
<tr>
<td>14.7/40.6</td>
<td>71.1</td>
<td>.43 ± .13</td>
<td>3.74 ± .38</td>
<td>11.7 ± 1.2</td>
<td>4.52 ± .21</td>
<td>65.±2.</td>
</tr>
<tr>
<td>14.4/50.</td>
<td>84.3</td>
<td>.26 ± .11</td>
<td>2.90 ± .28</td>
<td>8.49± .83</td>
<td>3.61 ± .22</td>
<td>49.8±1.5</td>
</tr>
</tbody>
</table>
continuum state to scattering into the deuteron bound state. This ratio is given in terms of the squares of the continuum and deuteron wave functions, as in Equation (II-7). Using the Hulthén form of the deuteron wave function given in Equation (II-10) and the triplet continuum wave function given in Equation (II-11), this ratio was calculated over the relative momentum range of interest (Du76). The Yamaguchi form of the scattering amplitude \( f(k) \) given by (Ya54)

\[
f(k) = \frac{\lambda}{(k-i\beta)^2 (k-i\alpha)(k+i\gamma)}
\]  

(VIII-2)

was used in Equation (II-11), since it is more appropriate for this form of the triplet wave function than the effective range amplitude of Equation (II-13). Here \( \gamma = 3.12 \text{ fm}^{-1} \), \( \lambda = 8.1 \text{ fm}^{-3} \), and \( \alpha, \beta \) are defined as in Equation (II-10).

The calculated \( \sigma^t/\sigma^d \) ratio is given in Figure VIII-2 as a function of the upper integration limit, \( k_{np} \), in the continuum channel. If the effective range approximation for \( f(k) \) is used instead, the ratios are typically 3-4% smaller. In Table VIII-3, the experimentally observed \( \sigma^t/\sigma^d \) ratios are compared with the calculated values for two sets of \( k_{np} \) integration limits, 0-40 MeV/c and 0-65 MeV/c. Excellent agreement is observed between the measured and theoretical ratios for both integration limits at all angles.
Figure VIII-2

CALCULATED $\sigma^t/\sigma^d$ RATIO USING THE YAMAGUCHI FORM OF THE SCATTERING AMPLITUDE

\[
\frac{\sigma^t}{\sigma^d} (\text{continuum}) / \sigma^d
\]

\[\text{Maximum } k_{np} \quad (\text{MeV}/c)\]
<table>
<thead>
<tr>
<th>$\theta^d, d^*/\theta^\pi$</th>
<th>$\theta^\pi_{c.m.}$</th>
<th>$\frac{\sigma_t}{\sigma^d}$ (0-40)</th>
<th>$\frac{\sigma_t}{\sigma^d}$ (0-40)</th>
<th>$\frac{\sigma_t}{\sigma^d}$ (0-65)</th>
<th>$\frac{\sigma_t}{\sigma^d}$ (0-65)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(deg)</td>
<td>(deg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.9/22.</td>
<td>40.8</td>
<td>$.053 \pm .008$</td>
<td>$.056$</td>
<td>$.167 \pm .026$</td>
<td>$.168$</td>
</tr>
<tr>
<td>12.5/25.</td>
<td>46.1</td>
<td>$.053 \pm .006$</td>
<td>$.056$</td>
<td>$.165 \pm .020$</td>
<td>$.168$</td>
</tr>
<tr>
<td>13.8/30.</td>
<td>54.5</td>
<td>$.052 \pm .004$</td>
<td>$.056$</td>
<td>$.169 \pm .014$</td>
<td>$.168$</td>
</tr>
<tr>
<td>14.7/40.6</td>
<td>71.1</td>
<td>$.058 \pm .006$</td>
<td>$.056$</td>
<td>$.180 \pm .019$</td>
<td>$.168$</td>
</tr>
<tr>
<td>14.4/50.</td>
<td>84.3</td>
<td>$.058 \pm .006$</td>
<td>$.056$</td>
<td>$.170 \pm .017$</td>
<td>$.168$</td>
</tr>
</tbody>
</table>
The agreement between the measured and calculated ratios was found to remain essentially unchanged for lower \( k_{np} \) integration limits. This is evidence that very little \( k_{np} \)-dependent background is present in data below \( k_{np} = 65 \text{ MeV/c} \). For an upper integration limit of \( k_{np} = 75 \text{ MeV/c} \), the agreement was also unchanged except at \( \theta_{\pi} = 22^\circ \), where the measured ratio showed a slight increase relative to the calculated value. This is not surprising since the \( \Delta \) background is expected to contribute above \( k_{np} = 60 \text{ MeV} \) at this angle. It was for this reason that the data beyond \( k_{np} = 60 \text{ MeV/c} \) were not incorporated in the GW fit at \( \theta_{\pi} = 22^\circ \). In determining \( \sigma^t \) at \( k_{np} \) beyond the fit region, it was necessary to extrapolate the fit in order to obtain the triplet fraction.

The excellent agreement obtained between the measured and calculated \( \sigma^t/\sigma^d \) ratio at this energy suggests this ratio may be valid also at other energies. In such a case, knowledge of the \( pp-d\pi^+ \) cross section at other energies can be used to predict the \( pp-d*\pi^+ \) cross section, or at least its triplet part. Thus in the region of \( 1^D_2-^3S_1p_2 \) dominance, the major portion of the \( pp-d*\pi^+ \) reaction can be predicted. In general, the singlet \( d* \) interaction cannot be predicted from the \( pp-d\pi^+ \) reaction. This is because the singlet state is not comparable to the triplet bound state (deuteron) due to spin dependence of the nuclear forces.
The calculation of $\sigma_t/\sigma_d$ is not a new idea (Ge54, Ro54, Br70), but this marks the first good comparison with pion production data measured under the proper kinematic conditions. It should be noted that the calculation of $\sigma_t/\sigma_d$ given in Figure VIII-2 provides the best agreement with the experimental data. Other ways of calculating this ratio, such as using the effective range approximation for $f(k)$, incorporating hard core effects, or including the D-state in the deuteron and triplet continuum wave functions, generally result in smaller $\sigma_t/\sigma_d$ values; however, within the experimental errors the agreement between measurement and calculation is usually preserved.

D. Future Studies

It would be useful to complement the results of the present experiment by measurements of $pp-d\pi^+$ and $pp-d\pi^+$ at other energies. Such measurements can be used to determine the general applicability of the results presented here, particularly that involving the calculated $\sigma_t/\sigma_d$ ratio. These reactions are also useful in understanding nucleon-nucleus pion production, since this process is normally the result of a single nucleon-nucleon collision within the nucleus. Similarly, knowledge of the inverse reactions, $\pi^+d\rightarrow pp$ and $\pi^+d^*\rightarrow pp$, is necessary in understanding the absorption of pions by quasi-deuterons in a nucleus, which is known to occur in light nuclei.
(No68). Finally, these reactions help determine the inelastic part of the \( 1D_2 \) phase shift in N-N scattering, which has been the subject of recent interest (Ka76, Wi76a).

The study of the n-p singlet FSI should be pursued since uncertainties in the present experiment did not permit definite conclusions regarding the \( 1S_0 \) cross section. To this end, the reaction,

\[
pp + ^2He \rightarrow (pp)\pi^0,
\]

(VIII-3)

has been measured recently (Fe76a); in this reaction the pp FSI must occur in the \( 1S_0 \) state since the Pauli principle excludes the \( 3S_1 \) interaction. Thus by assuming isospin invariance, reaction (VIII-3) can be used in principle to determine the n-p singlet FSI contribution in pp-d*\( \pi^+ \). It is not yet clear, however, whether the analysis of the recent data for reaction (VIII-3) will improve upon the results obtained here. Inherent difficulties exist in determining an angular distribution and in obtaining an absolute cross section. Nevertheless, it is hoped that the continuing study of \( pp + p\pi^0 \) in both the pp FSI and \( \Delta^+ \) kinematic regions will ultimately further the present results.
IX. APPENDICES

APPENDIX A

Calculation of Particle Momentum

The spectrometer momentum is calculated from the measured incoming and outgoing phase space of the particle passing through the magnet. The central momentum, \( P_c \), is the momentum required for a particle to pass through the magnet with a bend angle equal to the 30° bend of the spectrometer arm. The central momentum in MeV/c is given by

\[
P_c = 0.3001 \, B_Y^O \, R_c \quad \text{(A-1)}
\]

or,

\[
P_c = \frac{0.3001 \, B_Y^O \, \text{Leff}}{\sin \theta_1 + \sin \theta_2} \quad \text{(A-2)}
\]

where \( B_Y^O \) is the central magnetic field in kilogauss, \( R_c \) is the central radius of curvature in cm, \( \text{Leff} \) is the effective length of the magnet in cm (\( B_Y^O \, \text{Leff} = \int B_y \, dz \)), and \( \theta_1 \), \( \theta_2 \) are the input and output angles of the spectrometer arm (\( \theta_1 = \theta_2 = 15° \) here). In the uniform field approximation (Se67) the relationship between the input and output angles is independent of \( B_Y \) and depends only on \( \int B_y \, dz = B_Y^O \, \text{Leff} \). Thus, the momentum can be calculated from

\[
P = \frac{0.3001 \, B_Y^O \, \text{Leff}}{\sin \theta_1^i + \sin \theta_2^i}, \quad \text{(A-3)}
\]
where $\theta_1$ and $\theta_2$ are the input and output angles of the particle trajectory through the magnet. In terms of the input slope (SXI) of the particle relative to the central trajectory, $\sin \theta_1'$ is given by

$$\sin \theta_1' = \sin[\theta_1 - \tan^{-1}(\text{SXI})] \quad (A-4)$$

or

$$\sin \theta_1' = \frac{1}{\sqrt{1+(\text{SXI})^2}} \{\sin \theta_1 - \text{SXI} \cos \theta_1\} \quad (A-5)$$

Similarly $\sin \theta_2'$ is given in terms of the relative output slope (SXO) as

$$\sin \theta_2' = \frac{1}{\sqrt{1+(\text{SXO})^2}} \{\sin \theta_2 + \text{SXO} \cos \theta_2\} \quad (A-6)$$

In using Equation (A-3) to calculate the momentum, a minor (<.6%) variation in Leff across the pole face gap was taken into account, which slightly improved the momentum resolution.
APPENDIX B

Three-Body Lorentz Invariant Phase Space

The Lorentz invariant phase space available for a three body reaction is given by (Ni68)

\[ R_3 = \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \frac{d^3p_3}{2E_3} \delta^3(p_1 + p_2 + p_3 - p_0) \delta(E_1 + E_2 + E_3 - E), \]  
\[ (B-1) \]

where \( E \) is the total initial energy and subscripts 0, 1, 2, 3 refer to the incident particle and three final state particles, respectively. Integrating over \( d^3p_3 \), the delta function gives

\[ \vec{p}_3 = \vec{p}_0 - \vec{p}_1 - \vec{p}_2 \]  
\[ (B-2) \]

and hence,

\[ E_3(\vec{p}_3) = E_3(\vec{p}_0 - \vec{p}_1 - \vec{p}_2); \]  
\[ (B-3) \]

thus

\[ R_3 = \int \frac{d^3p_1}{8E_1} \frac{d^3p_2}{8E_2} \frac{1}{E_3} \delta(E_1 + E_2 + E_3 - E). \]  
\[ (B-4) \]

Since \( d^3p_1 = p_1^2dp_1d\Omega_1 \), etc., this may be written as

\[ d^5R_3 = \frac{p_1^2dp_1d\Omega_1}{8E_1} \int \frac{p_2^2dp_2}{8E_2} \frac{1}{E_3} \delta(E_1 + E_2 + E_3 - E), \]  
\[ (B-5) \]

where \( E_2 \) and \( E_3 \) remain functions of \( p_2 \). In order to integrate over the unobserved \( p_2 \), a useful property of the Dirac \( \delta \)-function may be applied:
\[ f(x) \delta(g(x)) dx = \left[ f(x) \left( \frac{\partial}{\partial x} g(x) \right) \right]^{-1} \bigg|_{x=x_0} \]  \hspace{1cm} (B-6)

where \( x_0 \) is determined by \( g(x_0) = 0 \). Let

\[ g(p_2) = E_1 + E_2 + E_3 \left( \vec{p}_o - \vec{p}_1 - \vec{p}_2 \right) - E \]  \hspace{1cm} (B-7)

or,

\[ g(p_2) = E_1 + \sqrt{2p_2^2 + m_2^2} + \sqrt{\left| \vec{p}_o - \vec{p}_1 - \vec{p}_2 \right|^2 + m_3^2} - E \]  \hspace{1cm} (B-8)

where by expanding \( \left| \vec{p}_o - \vec{p}_1 - \vec{p}_2 \right|^2 \), \( E_3 \) may be expressed as

\[ E_3 = \left[ p_2^2 + p_1^2 + p_2^2 - 2p_0 \cdot \vec{p}_1 - 2p_0 \cdot \vec{p}_2 + 2p_1 \cdot \vec{p}_2 + m_3^2 \right]^{1/2} \]  \hspace{1cm} (B-9)

Then

\[ \frac{\partial}{\partial p_2} \left[ g(p_2) \right] = \frac{p_2}{\sqrt{p_2^2 + m_2^2}} + \frac{2p_2 - 2p_0 \cos \theta_{02} + 2p_1 \cos \theta_{12}}{2 \sqrt{\left| \vec{p}_o - \vec{p}_1 - \vec{p}_2 \right|^2 + m_3^2}} \]

\[ = \frac{p_2}{E_2} + \frac{p_2 - p_0 \cos \theta_{02} + p_1 \cos \theta_{12}}{E_3} \]  \hspace{1cm} (B-10)

By projecting along the \( p_2 \) axis, conservation of momentum gives

\[ p_2 - p_o \cos \theta_{02} + p_1 \cos \theta_{12} = - p_3 \cos \theta_{23} \]  \hspace{1cm} (B-11)

and thus,

\[ \frac{\partial}{\partial p_2} \left[ g(p_2) \right] = \frac{p_2 E_3 - E_2 p_3 \cos \theta_{23}}{E_2 E_3} \]  \hspace{1cm} (B-12)

Now with
\[ f(p_2) = \frac{p_2^2}{E_2(p_2)} \times \frac{1}{E_3(p_0 - p_1 - p_2)} \]  

(B-13)

and with \( g(p_2) = 0 \) when \( p_2 \) is on the energy shell (i.e., when \( E_1 + E_2 + E_3 - E = 0 \)), Equation (B-6) may be used to obtain

\[ d^5R_3 = \frac{p_1^2 dp_1 d\Omega_1 d\Omega_2}{8E_1} \left[ \frac{p_2^2}{E_2E_3} \times \frac{E_2E_3}{p_2E_3 - p_3E_2 \cos \theta_{23}} \right]. \]  

(B-14)

The phase space factor \( I \) is now defined as

\[ I = \frac{d^5R_3}{dp_1 d\Omega_1 d\Omega_2} = \frac{p_1^2 p_2^2}{8E_1 (p_2E_3 - E_2 p_3 \cos \theta_{23})}, \]  

(B-15)

since this differential phase space is related to the set of observables and calculated cross sections in this experiment. By employing Equation (B-11), the factor \( I \) may also be written as

\[ I = \frac{p_1^2 p_2^2}{8E_1 [E_3 p_2 + E_2 (p_2 - p_0 \cos \theta_{02} + p_1 \cos \theta_{12})]} \]  

(B-16)

which has dimensions of \( \text{MeV} \) for the \( c = 1 \) system of units employed here.
APPENDIX C

Monte Carlo Programs

The Monte Carlo programs employed in the data analysis are computer simulations of the scattering and detection processes occurring in the physical experiment. Interaction positions are randomly chosen within the beam-target overlap volume, and scattering angles are randomly selected according to the laws of isotropic emission. The scattered particles are propagated through the system to test whether they meet the requirements of the experimental geometry. In this propagation, the effects of magnetic fields, energy loss, particle decay, and multiple scattering are considered. Particle decay is limited to pions in the TOF arm, and the spectrometer magnet is described in terms of the uniform field model. Four separate Monte Carlo programs were written for use in the data analysis and are designated: MC2B, MONTY, FSIEF, TSMER.

The routine MC2B calculates the average solid angle \( \Delta \Omega_2 \) for two-body reactions. Since \( d\theta_2/d\theta_1 \gg 1 \) for the reaction \( pp-d\pi^+ \), the scattering angles \( \theta_2, \phi_2 \) are chosen over the interval \( \Delta \Omega_2 \), and two-body kinematics are used to calculate \( \theta_1, \phi_1, \) and \( p_1 \). The events are propagated through the system, and the good events are histogrammed as functions of several kinematic and geometrical variables. The average solid angle is determined by the ratio of good
events to tries, multiplied by the trial interval \( \Delta \Omega_2 \).
The calculations of MC2B were found to agree with experimental measurements of the momentum and angular acceptance using pp elastic scattering.

The program MONTY calculates the solid angle for three-body interactions by performing the following sequence of computations:

1) Choose scattering angles \( \theta_1, \phi_1, \theta_2, \phi_2 \) over the trial intervals \( \Delta \Omega_1 \) and \( \Delta \Omega_2 \) which completely encompass the solid angles of the detectors.

2) Select a random momentum \( p_1 \) from within the \( p_1 \) momentum range of interest.

3) If the randomly selected \( p_1, \Omega_1, \Omega_2 \) are not kinematically allowed, begin again at step 1.

4) Propagate the kinematically allowed events through the experimental system to determine the good events, which meet all the geometry requirements.

5) Histogram the good events as functions of various geometric and kinematic quantities, including \( p_1 \) and \( k_{np} \); for each good event increment the appropriate histogram bin by unity or by the phase space factor, I.

6) Divide the data counts by the properly normalized and identically binned Monte Carlo counts to obtain the fifth-order differential cross section of Equation (VI-1) as a function of \( p_1 \) or \( k_{np} \).
The results calculated with MONTY were found to agree very well with an independent Monte Carlo calculation by Hogstrom (Ho76a).

The program FSIEF was used to determine the singly differential cross section for the reaction pp-d+π+. This program is similar to the two-body routine MC2B, but also calculates the d* detection efficiency, \( P^*(k_{np}) \), as a function of the n-p relative momentum. FSIEF performs the following sequence of steps:

1) Pick random \( \cos\theta_2, \phi_2 \) within the trial interval \( \Delta\Omega^r_2 \) which encompasses the right (pion) detector arm.

2) Check the acceptance in the right arm to determine the right solid angle \( \Delta\Omega^r_2 \) from the ratio of good events to tries.

3) For good events in the right arm, choose a random \( k_{np} \) (e.g., between 0-100 MeV/c) which corresponds to an effective mass

\[
m_L = m_p + m_n + k_{np}^2 \frac{(m_p + m_n)}{2m_p m_n}
\]

in the left arm. Histogram the number of tries versus \( k_{np} \).

4) Run two-body kinematics with \( \theta_2, \phi_2, m_2, \) and \( m_L \) to calculate \( \theta_L, \phi_L \) and \( p_L \) for the d* pseudo-particle.

5) Select random, isotropic c.m. "decay" angles \( \cos\theta_{np} \) and \( \phi_{np} \) which, given \( k_{np}/2 \) and \( p_L/2 \) determine the (proton) variables \( \theta_1, \phi_1, p_1 \) via relativistic transformations.
6) Propagate particle 1 through the spectrometer arm to determine the good events.

7) Histogram the number of good events versus $k_{np}$ and divide bin-by-bin by the number of tries versus $k_{np}$ to obtain the $d^*$ detection efficiency $P^*(k_{np})$.

8) Calculate the average solid angle, $\Delta \Omega_2(k_{np}) = P^*(k_{np}) \Delta \Omega'_2$, for use in determining the singly differential pp-$d^*\pi^+$ cross section via Equation (VI-6).

The results obtained from FSIEF were found to be in good agreement with those of an independent calculation by Witten (Wi76b).

The Monte Carlo routine TSMER was used to "smear" the Goldberger-Watson enhancement factors, $F(k)$, over the experimental resolution. These smeared factors were those used in fitting the data with Equation (VIII-1). The sequence of steps necessary to calculate the smearing functions is as follows:

1) Perform steps 1-4 of program MONTY.

2) Weight the good events by $I^*F^S$ and by $I^*F^t$ and histogram versus $k_{np}$.

3) Using a Gaussian-weighted random number generator, smear $p_1$, $\theta_1$, $\phi_1$, $\theta_2$, and $\phi_2$ according to their uncertainties in measurement. Recalculate $k_{np}$ using the smeared variables, and histogram the good events again weighted by $I^*F^S$ and $I^*F^t$ versus the new $k_{np}$.
4) Divide bin-by-bin the "smeared" $k_{np}$ spectra by the "normal" spectra to obtain the "smearing functions" for $F^S$ and $F^t$ versus $k_{np}$.

The smearing functions calculated for $F^S$ and $F^t$ were applied to these enhancement factors before fitting to the experimental data. The calculated function for the slowly-varying $F^t$ was essentially unity for all $k_{np}$. The result of smearing $F^S$ reduced this factor by as much as 15% for the smallest $k_{np}$ bins and increased $F^S$ slightly (< 2%) at larger $k_{np}$.
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