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NUCLEOSYNTHESIS OF LIGHT ELEMENTS
IN YOUNG SUPERNova REMNANTS
SURROUNDING PULSARS

by

ELIAHU DWEK

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

Thesis Director's Signature:

[Signature]

Houston, Texas
December, 1976
That Nova was a moderate star like our good sun; it stored
no doubt a little more than it spent
Of heat and energy until the increasing tension came to
the trigger-point
Of a new chemistry; then what was already flaming found a
new manner of flaming ten-thousandfold
More brightly for a brief time; what was a pin-point fleck
on a sensitive plate at the great telescope's
Eyepiece now shouts down the steep night to the naked eye,
a nine-day super-star.

---Robinson Jeffers
Acknowledgements

I wish to thank my advisor, Professor Donald D. Clayton, for introducing me to this problem, and for his faithful guidance throughout this work.

Special thanks are due to all the guys in Room 202 for the numerous discussions in 'real life' problems; and, finally, thanks to the organizers of the tennis tournament, which provided a continuous source of excitement—and frustrations.
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INTRODUCTION

In their classic review paper on the origin of the elements Burbidge et al. (1957: also referred to as $B^2FH$) proposed that stellar nucleosynthesis is the major source of the cosmological abundances of the elements from carbon through iron. This theory, however, cannot account for the synthesis of the light elements, lithium, beryllium and boron (Li Be B). In stellar interiors these elements are destroyed by nuclear reactions at temperatures that are cooler than those at which they can be created. For example, $^7\text{Li}$ is formed during the proton proton chain by the reaction $^3\text{He}(\alpha,\gamma)^7\text{Be}(\varepsilon^-\gamma)^7\text{Li}$ at temperatures around $2 \times 10^7 \, \text{°K}$, but is destroyed by $^7\text{Li}(\text{P},\alpha)^4\text{He}$ at $T \sim 2 \times 10^6 \, \text{°K}$. As a consequence, before a star evolves off the main sequence, all the light elements in the volume that contains the majority of the mass will be destroyed. It seems therefore, that to account for the synthesis of LiBeB, a non-equilibrium process occurring in a low-density, low-temperature environment in the universe is required. This process was appropriately coined by $B^2FH$ as the $x$-process.

Various different sites and mechanisms have been proposed for the $x$-process, which are reviewed in detail by Reeves (1974) in a paper on the origin of the light elements. These models for the synthesis of LiBeB can be classified into two categories:

1. Autogenic Models

These models assume that the observed stellar abun-
dances of the light elements are of local origin, and
that each star, therefore, generates its own LiBeB by
proton irradiation of the stellar atmosphere. This
model was first proposed by Bernas et al. (1967) and
was later reexamined by Ryter et al. (1970). The
latter found the main difficulty of this model to be
the high energy requirement for the production of the
observed amount of light elements in young stars (T
Tauri stars). This energy was found to be comparable
to, if not larger than, the total amount of gravita-
tional energy released by those stars since their
birth. Canal, Isern and Somahuja (1975), however,
considered the nucleosynthesis of LiBeB by low-
energy proton and α-fluxes and found the energy re-
quirements to be considerably lower than those pre-
viously estimated.

2. Galactogenic Models

These models assume that the abundance of light ele-
ments is of a universal nature and propose that LiBeB
are synthesized by the interaction of cosmic rays
with interstellar material (Reeves, Fowler, and
and Meneguzzi and Reeves (1975) calculated the amount
of LiBeB produced using a diffusion model of high-
energy cosmic rays in the galaxy. Their results are
shown in Table 1. It is clear from the table that
this theory can explain the abundances of ⁶Li, Be and
Boron within the uncertainties in the observations and in the theory itself, but that additional sources are required to account for the observed $^7$Li abundance. Several supplemental sources that produce $^7$Li but no appreciable amounts of other LiBeB elements have been suggested:

1. Synthesis during the "Big Bang" (Wagoner, 1973)
2. Synthesis of $^7$Be during helium shell flashes in the interior of red giants (Cameron and Fowler, 1971)
3. Production in exploding and in bouncing supermassive stars (Nørgaard and Arnould, 1975 and Nørgaard and Fricke, 1976, respectively)
4. Possible production in supernovae shock waves (Epstein, Arnett, and Schramm, 1975)
5. A possible production by a hypothetical low-energy component of the galactic cosmic rays either in interstellar space or trapped in supernova remnants (Meneguzzi and Reeves, 1975, and Bodansky, Jacobs, and Öberg, 1975)

Clayton and Dwek (1976) proposed specifically that $^7$Li is produced by $\alpha\alpha$ collisions in supernova remnants, assuming that the supernova explosion leaves a rotating neutron star within the expanding nebula of ejected matter. The newly-formed pulsar loses rotational energy in the form of magnetic dipole radiation that is capable of accelerating charged particles within the nebula. Presupernova models (Arnett, 1975) and observations of the Crab Nebula
(Woltjer, 1958; Davidson and Tucker, 1970) show a dominance of helium in the nebula. Therefore, the primary ions accelerated are α-particles and the nuclear collisions in the nebula will primarily be αα collisions with the production of lithium as a consequence.

Data given by Cameron (1973), based on Li observations in meteorites, imply a cosmic abundance of \( ^7\text{Li}/\text{H} \sim 1.5 \times 10^{-9} \); whereas observations of \( ^7\text{Li} \) abundances at the surface of young stars imply a slightly lower cosmic abundance of \( ^7\text{Li}/\text{H} = 9.2 \times 10^{-10} \) (Boesgaard, 1976). Assuming that the observed abundance of \( ^7\text{Li} \) in young stars is of a galactic nature, rather than a result of a local autogenic process within the stars themselves, and taking the mass of the interstellar medium to be \( \sim 10^{10} \, M_\odot \) of hydrogen implies 64-105 \( M_\odot \) of interstellar \( ^7\text{Li} \). If the lifetime of the medium against incorporation into stars is \( \sim 5 \times 10^9 \) yrs (Talbot, private communication), and if the galactic rate of pulsar-forming events is \((30 \, \text{yr})^{-1}\) (Gunn and Ostriker, 1970), then each event must, on the average, synthesize \( \sim 6 \times 10^4 - 10^5 \) \( ^7\text{Li} \) atoms.

In this work the possibility of synthesizing the above amount of \( ^7\text{Li} \) is reexamined. This work differs from that of Clayton and Dwek (1976) in the following:

(1) For simplicity we consider a homogeneous nebula expanding uniformly with a constant surface velocity \( v_{\text{SN}} \). The composition of the ejected nebula is that of a characteristic supernova ejecta, with carbon and oxygen
as the major other constituents.

(2) It is assumed that the injected particle spectrum can be written as a power law spectrum of injection energies. Then, following the injection, the evolution of the spectrum in the nebula is calculated, taking energy losses due to collisions and expansion into account. The effect of diffusion is considered as well.

(3) The production of other light elements by alpha collisions on carbon and oxygen, and the associated emission of gamma rays are also included in the calculations.

With these modifications of the earlier model it is found that the proposed scenario for the origin of \(^7\)Li seems more doubtful than previously thought.

More specifically it is found that:

(1) Collision losses affect the \(\alpha\)-particle spectrum in the nebula by preferentially depleting the low energy particles. Therefore in the early stages of the expansion the \(\alpha\)-particle spectrum is actually inverted, containing fewer particles at lower energies. As a result a \(^7\)Li/\(^6\)Li production ratio of only 10 can be achieved, requiring a special injection spectrum, sharply peaked between the \(\alpha\alpha\) production threshold of the two lithium isotopes.

(2) The energy requirement of the model is significantly higher than thought previously, requiring more than one-third of the pulsar's power to be converted into this special spectrum of \(\alpha\)-particles.
(3) Since $^7\text{Li}$ is the dominant nucleus produced in the nebula, gamma ray fluxes from the deexcitation of $^7\text{Li}^*$ (478 keV) and $^7\text{Be}^*(431 \text{ keV})$ exceed those from the deexcitation of $^{12}\text{C}^*(4.44 \text{ MeV})$ by a factor of three.

Gamma ray astronomy therefore provides the best hope of obtaining an observational clarification of the model.
CHAPTER 1
THE MODEL

§1 - The Mass and Chemical Composition of the Nebula

The suggestion that \(^7\)Li is synthesized in supernova remnants surrounding young pulsars raises the question about the characteristics of the event, that is, about the typical mass and composition of the ejected nebula in an 'average' supernova event. If each supernova event produces the same number of \(^7\)Li nuclei, as was assumed before, then clearly the characteristic supernova remnant will be related to an average star where the average is taken over the number of stars. However, if each supernova event produces an amount of \(^7\)Li proportional to the remnant's mass, then clearly the average supernova remnant will correspond to an average star, where the average is obtained in a mass-weighted calculation.

This same problem was encountered by Heinebach, Norman, and Schramm (1976) regarding the composition of cosmic rays accelerated in an 'average' supernova remnant. Using an initial mass function for the range of stellar masses which produce supernovae, they found that the average mass, \(M\), of the main sequence progenitor is about 13 to 19 \(M_\odot\) in a mass-weighted calculation, and around 10 to 13 \(M_\odot\), if there is no mass weighting.

After core hydrogen burning these main sequence progenitors develop a helium core surrounded by a hydrogen-rich envelope. This envelope has a negligible effect on
the evolution of the core (Arnett, 1972), so that the advanced evolution of massive stars may be investigated by evolving initially pure helium "stars". Such pure helium "stars" have been evolved up to a point prior to their collapse for several different masses (Arnett, 1973). The $M_c = 4$ and $8 M_\odot$ models evolved correspond to the cores of main sequence stars of 13 to 15 $M_\odot$ and 22 to 24 $M_\odot$, respectively (Arnett, 1975). The progenitor masses are close enough to the average masses described earlier, that in the following, the 8 and 4 $M_\odot$ evolved helium cores will be considered as the cores of the mass-weighted and number-weighted progenitors, prior to the supernova event.

The distribution of abundances in these cores prior to their collapse is given in Arnett (1975). In spite of some uncertainties in the evolution beyond this point, it will be assumed that the collapse is followed by an explosion, leaving a condensed object of $\sim 1.5 M_\odot$ and ejecting the rest of the mass. To characterize the corresponding mass and composition of the ejected material, two factors need be determined: (1) the exact location of the mass cut; and (2) The effect of explosive processing on the final composition of the ejecta.

The exact location of the mass cut in the presupernova models is not known at the present and depends primarily on the density distribution in the core, the energy released in the formation of the neutron star, and the coupling between the outgoing neutrinos and the infalling matter.
The effect of explosive burning on the abundances was estimated by Heinebach, Norman, and Schramm (1976). The result for the 4 \( M_\odot \) core is, that the net effect on the O-Ne-Mg and the He-C zones is to increase the abundance of \(^{16}\text{O}\) at the expense of the \(^4\text{He}\) and \(^{12}\text{C}\) abundances. Explosive processing of a C-O-Ne-Mg zone in the 8 \( M_\odot \) core has the effect of burning \(^{12}\text{C}\) to \(^{16}\text{O}\). These results depend critically on \( \theta_\alpha \), the reduced alpha width for the reaction \(^{12}\text{C}(\alpha,\gamma)^{16}\text{O}\), a number experimentally difficult to obtain (Dyer, 1973), and whose best current value known was used in the calculations. The main effect of explosive processing will therefore be to lower the amount of Li produced by \( \alpha\alpha \) collisions, and to lower the production of \(^{11}\text{B}\), since the cross section for its production by \( \alpha^{^{16}\text{O}} \) collisions is only one-third of that for its production by \( \alpha^{^{12}\text{C}} \) collisions (see Chapter 5).

With these problems in mind the following prescriptions were adopted:

1. The mass cut was arbitrarily taken to be such that all the silicon and nickel in the core is locked up in the remnant star. This corresponds to an ejected mass of 2.6 \( M_\odot \) for the 4 \( M_\odot \) core, and to an ejecta of 6.2 \( M_\odot \) for the 8 \( M_\odot \) core.

2. The composition of the nebula was taken to be the composition of the fraction of the core above the mass cut.

The two models chosen are summarized in Table 2. The just-
ification for this choice is that it is simple, that at this stage of knowledge any other choice will be equally hard to justify, and that the results of this work can be calibrated for different composition and masses of the nebula.

§2 - The Dynamics of the Nebula

In Ostriker and Gunn (1971) and Bodenheimer and Ostriker (1974) the pulsar formed after the supernova event plays an important dynamic role in the evolution of the supernova remnant. In their picture the formation of a neutron star in the center of the core leaves a cavity which fills with electromagnetic radiation emitted by the pulsar. The resulting radiation pressure in the cavity prevents any further collapse and drives the expansion of the envelope. However, the role of the pulsar in the ejection of the nebula is questionable. Pulsars lose their rotational energy progressively on a time scale of years, while high expansion velocities are observed only a few months after the explosion. A neutrino deposition mechanism (Colgate, 1975) may therefore be a more promising way of transporting the gravitational energy released in the collapse on a very short time scale to the outer layers of the imploding star, preventing further collapse and ejecting the envelope.

The details of the ejection mechanism are not relevant to this work, since the only assumption that will be made is, that as a result of the explosion the material is
ejected with a surface velocity ranging in values between $5 \times 10^8$ cms$^{-1}$ and $1.5 \times 10^9$ cms$^{-1}$, corresponding to observed velocities between these limits by Shlovskii (1960) and Kirshner et al. (1973).

Theoretical models of supernova explosions (Colgate and McKee, 1969) show that different layers of the ejecta expand with different velocities, the bulk ($\sim 90\%$) of the ejected mass expanding almost uniformly and the outer part of the ejecta speeding up with decreasing density. It will therefore be assumed that the nebula expands with a velocity distribution $V = \frac{r}{t}$ so that the density distribution remains homogeneous at all times. The density at time $t$ will therefore be given by

$$\rho(t) = \frac{M_{ej}}{\left(\frac{4\pi}{3} V_{SN}^2 t^3\right)} \quad (1-1)$$

where $V_{SN}$ is the constant velocity of the outer radius.

§3 - The Temperature of the Nebula

Initial temperatures in the nebula following the explosive event range from $\sim 10^8$ °K to $\sim 7 \times 10^3$ °K in the different zones (Trimble, 1973). Following the expansion, thermal energy is converted to kinetic energy, so that the effective temperatures at maximum light peak is $\sim 10^4$ °K falling to 5000°K in three weeks. To follow the evolution of the temperature in the nebula, one has to develop a full hydrodynamic code à la Chevalier (1974), taking the effect of the pulsar into account.
Fortunately, the nuclear reactions occurring in the nebula are of a non-thermal nature. The temperature dependence in this work enters indirectly in the expression for the rate of energy loss due to electronic collisions, \( \frac{dE}{dt}\text{coll} \), discussed in Chapter 3. The rate at which a non-relativistic particle loses energy by electronic collisions in a neutral gas is given by the Bethe formula (Bethe, 1933). When the gas is completely ionized, the process is slightly different, since the initial conditions of the electrons are not the same. Furthermore, as the electron temperature increases, the incident particle suffers smaller energy losses in each collision. Numerically one finds (Reeves, 1971)

\[
\frac{dE}{dt}\text{collision in } \text{ionized plasma} \propto 2 \frac{dE}{dt}\text{collision in neutral gas} \times S(x)
\]

where \( S(x) \) is a function of \( V(\text{incident particle})/V(\text{electron}) \) and is of order unity for most temperatures encountered in this work. The energy losses due to collisions are therefore taken to be

\[
\frac{dE}{dt}\text{coll} \propto 2 \frac{dE}{dt}\text{neutral gas}
\]

Although recombinations occur in the nebula during the expansion, a residual ionization is expected, so that the expression used overestimates the actual energy losses in
the nebula by less than a factor of two.

§4 - The Pulsar's Parameters

Throughout this work the pulsar will be regarded as a point source of energetic particles located at the center of the expanding nebula. The energy loss converted into energetic particles will be assumed proportional to the rate of energy loss in the form of electromagnetic radiation. This rate of energy loss is given by the magnetic dipole model of Ostriker and Gunn (1969):

\[ L(t) = L_0 (1 + t/\tau_0)^{-2} \]  \hspace{1cm} (1-2)

where \( L_0 \) is the initial pulsar luminosity and \( \tau_0 \) is the initial slowing down time, defined as the ratio:

\[ \tau_0 = \frac{1}{2} \frac{\Omega_0}{|\dot{\Omega}_0|} \]

where \( \dot{\Omega}_0 \) is the initial angular velocity of the pulsar. The total energy radiated over the lifetime of the pulsar is:

\[ E_{\text{pulsar}} = \int_0^\infty L(t) \, dt = L_0 \tau_0 \]

half of it being liberated during the first \( \tau_0 \) years. A conventional choice of parameters is \( L_0 = 10^{43} \) ergs sec\(^{-1}\) and \( \tau_0 = 3 \) yrs (Ostriker and Gunn, 1969), so that the total energy radiated by the pulsar over its lifetime amounts to \( \sim 10^{51} \) ergs. In this work the above choice
for $L_0$ and $\tau_0$ will be the conventional choice, unless explicitly stated otherwise.
CHAPTER 2

THE SPECTRUM AND CHEMICAL COMPOSITION OF THE SOURCE

Several models have been suggested for the acceleration, or ejection, of energetic particles by a pulsar. Some of them will be briefly reviewed, the main emphasis being on:

(a) The composition of the accelerated, or ejected, material.
(b) The characterization of the source spectrum.

In characterizing the spectrum of the source, the various acceleration mechanisms will be divided into two categories:

(1) Singular ejection mechanisms.
(2) Mechanisms that can provide a continuous source of energetic particles.

(1) Singular Injection

LeBlanc and Wilson (1970) conducted a numerical study of the collapse of a rotating magnetized star of 7 M_\odot.

They found that the combined effect of rotation and magnetic field produces an axial jet of material which forms shortly after the initial collapse ceases. The total emitted jet mass is estimated to be \( \sim 1.2 \times 10^{32} \) g (\( \sim 0.06 M_\odot \)) with a kinetic energy of \( \sim 1.2 \times 10^{51} \) ergs (\( \sim 7.5 \times 10^{56} \) MeV), corresponding to a kinetic energy of \( \sim 10 \) MeV per nucleon.

LeBlanc and Wilson (1970) did not specify the composition of the ejected material, which presumably consists of highly neutronized material. Nevertheless, we will consider the contribution of a hypothetical jet of material, consisting primarily of helium nuclei, to the nucleosynthesis of lithium in the nebula. Let \( N_0(E_0)dE_0 \) be the
number of $\alpha$-particles injected by the source in the energy range $E_0$ and $E_0 + dE_0$. For simplicity only, a power law injection with a spectral index $\gamma$ and low energy cutoff $E_c$ will be assumed, that is

$$N_0(E_0) = kE_0^{-\gamma} \quad \text{for} \quad E_0 \geq E_c$$

Requiring the total energy of the ejecta to be $7.5 \times 10^{56}$ MeV, that is,

$$E_{TOT} = \int_{E_c}^{\infty} E_0 N_0(E_0) dE_0 = 7.5 \times 10^{56} \text{ MeV}$$

The injection spectrum is then

$$N_0(E_0) = (\gamma-2) \frac{E_{TOT}}{E_c^2} \left(\frac{E_0}{E_c}\right)^{-\gamma}$$  \hspace{1cm} (2-1)

(2) **Continuous Injection**

Several mechanisms have been proposed that can provide a continuous source of energetic particles:

(a) **Acceleration of Plasma From the Surface of Pulsars**

The rotation of a neutral star through its own magnetic field generates an induced electric field that can extract charged particles from the surface of the star (Goldreich and Julian, 1969). Whether positive ions will escape from the surface depends critically on the work function for their extraction. No ions are expected to be pulled off the surface
if it is composed of iron nuclei which form molecular chains of high binding energy ($\sim 14$ keV) per ion (Ruderman and Sutherland, 1975). However, the induced electric field will have no problems of extracting ions from the surface if it is primarily composed of helium (Michel, 1975). Rosen and Cameron (1972) investigated the surface composition of magnetic neutron stars following the SN explosion. They found that the outermost layers of the envelope could be composed almost entirely of $^4\text{He}$, resulting from the photo-disintegration of the iron peak nuclei, amounting to approximately $1.3 \times 10^{21}$ gr. This total amount of helium was found compatible with the estimated time-integrated rate of mass loss from the surface of the pulsar (Michel, 1975).

(b) Particle Acceleration by the Pulsar's Electromagnetic Wave

The pulsar loses rotational energy in the form of gravitational-quadrupole and magnetic-dipole radiation (Ostriker and Gunn, 1969). These low-frequency electromagnetic waves are capable of accelerating particles to highly relativistic energies if they are injected in the vicinity of the neutron star (Gunn and Ostriker, 1969), and to moderately relativistic energies if they are injected from the expanding nebula (Kulsrud, Ostriker, and Gunn, 1972). The dominance of helium in the nebula (or in the ambient plasma surrounding the pulsar), will therefore be
reflected in the composition of the accelerated particles, which, for simplicity, will be taken as consisting only of \(\alpha\)-particles.

In calculating the injection spectrum in the continuous case the details of the injection mechanism are ignored and the only assumption made that the energy losses in energetic particles constitute a constant fraction of the total electromagnetic radiation loss of the pulsar.

Let \(N_s(E_0, t_0)\,dE_0\,dt_0\) be the number of \(\alpha\)-particles injected by the source in the energy range \(E_0\) and \(E_0 + dE_0\) and in the time interval \(t_0\) and \(t_0 + dt_0\); and let \(\alpha_{CR}\) be the fraction of radiation losses converted to energetic cosmic-ray \(\alpha\)-particles. The conservation of energy requires that \(\alpha_{CR} \times \) the energy lost by the pulsar between \(t_0\) and \(t_0 + dt_0\) must equal the total energy in \(\alpha\)-particles injected in that time interval, that is,

\[
\alpha_{CR}\,L(t_0)\,dt_0 = dt_0 \int E_0 N_s(E_0, t_0)\,dE_0 \tag{2-2}
\]

Again, for the sake of simplicity only, it is assumed that the injected spectrum has a power law form:

\[
N_s(E_0, t_0) = kE_0^{-\gamma} \quad \text{for} \quad E_0 > E_c \tag{2-3}
\]

where, as before, \(E_c\) is the low energy cutoff and \(\gamma\) is the spectral index. Using equations (1-2), (2-2), and (2-3), the injection spectrum in the continuous case is
then:
\[
N_s(E_o, t_o) = (\gamma-2) \frac{\alpha C R L_o}{E_c^2} \left(\frac{E_o}{E_c}\right)^{-\gamma} (1 + t_o/\tau_o)^{-2}
\] (2-4)

The total number of \(\alpha\)-particles injected into the nebula per unit time is given by the integral:
\[
\int_{E_c}^{\infty} N_s(E_o, t_o) dE_o
\] (2-5)

which yields
\[
\dot{N}_{TOT}(t_o) = \frac{\gamma-2}{\gamma-1} \frac{\alpha C R L_o}{E_c} (1 + t_o/\tau_o)^{-2}
\] (2-6)

which depends only weakly on \(\gamma\).
CHAPTER 3

THE ENERGY LOSSES IN THE NEBULA

Consider a group of $\alpha$-particles injected into the nebula with the same initial energy $E$. Following the injection these particles lose energy in the nebula. Collisions between the $\alpha$-particles themselves, that is, stochastic processes by which the $\alpha$-particles can gain or lose energy, are neglected. The primary modes of energy loss are therefore: (1) Collisions with the ambient plasma; (2) Energy losses due to the expansion of the nebula. The rate of energy loss can therefore be given by

$$\frac{dE}{dt} = -\left(\frac{dE}{dt}\right)_{\text{coll}} - \left(\frac{dE}{dt}\right)_{\text{exp}}$$  \hspace{1cm} (3-1)

$\frac{dE}{dt}$ defined by (3-1) is the velocity of a particle in the group in 'energy space', represented by a smooth and continuous function, where:

$\left(\frac{dE}{dt}\right)_{\text{coll}}$ is the rate of energy loss due to electronic collisions in the nebula, and

$\left(\frac{dE}{dt}\right)_{\text{exp}}$ is the rate of energy loss due to the expansion of the nebula.

For $\left(\frac{dE}{dt}\right)_{\text{coll}}$ an approximation is used for the energy losses in an ionized plasma given by Ryter et al. (1970)
\[
\left(\frac{dE}{dt}\right)_{\text{coll}} \approx 9 \times 10^{-11} \frac{n(t)}{E^{1/2} \text{ (MeV)}} \frac{\text{MeV}}{\text{sec}}^3
\] 

(3-1)

\(n(t)\) is the number density of the nebula, that decreases in time due to the expansion. For a constant expansion rate, \(n(t)\) can be written as:

\[n(t) = \tilde{\alpha} t^{-3}\]

Let \(M_{\text{ej}}\) be the total mass of the nebula and \(V_{\text{SN}}\) its expansion rate. Then \(\tilde{\alpha}\) is given by

\[\tilde{\alpha} \approx 6 \times 10^2 \left(\frac{M_{\text{ej}}}{M_{\odot}}\right) \left(\frac{10}{V_{\text{SN}}}\right)^3\] 

(3-3)

where the expression is scaled for a helium-dominated nebula of 1 \(M_{\odot}\) expanding at a constant rate of 10\(^3\) cm sec\(^{-1}\).

Considering the group of \(\alpha\)-particles to be an ideal non-relativistic gas, the rate of energy loss of the group due to the expansion of the nebula is simply \(\frac{2E_{\text{TOT}}}{t}\), where \(E_{\text{TOT}}\) is the total energy of the \(\alpha\)-particles in the group. Since stochastic losses are neglected this monoenergetic group of \(\alpha\)-particles remains monoenergetic at all times, so that the rate of energy loss of each individual \(\alpha\)-particle due to the expansion of the nebula can be written as:

\[\left(\frac{dE}{dt}\right)_{\text{exp}} = \frac{2E}{t}\]

The energy equation (3-1) can therefore be written as

\[\frac{dE}{dt} = -\frac{a}{t^3} E^{-1/2} - \frac{2E}{t}\] 

(3-4)
where  \[ a \equiv g \ 10^{-11} \ \text{a} \, . \]

Defining  \[ z \equiv E^{3/2} \ t^2 \, , \]
equation (3-4) can be written in an integrable form:

\[ \frac{dz}{dt} = -\frac{3}{2} \ a \]
yielding

\[ E_o^{3/2} \ t_o^3 + \frac{3}{2} \ a \ t_o = E^{3/2} \ t^3 + \frac{3}{2} \ a \ t \]  \hspace{1cm} (3-5)

The above equation describes the motion of a particle injected in the E-t plane at time \( t_o \) with an energy \( E_o \), as it loses energy in the nebula. Using equation (3-5) the initial energy or injection time can be expressed in terms of the other variables:

\[ E_o = E \left( \frac{t}{t_o} \right)^2 \ [1 + b \ \frac{t-t_o}{t}]^{2/3} \]  \hspace{1cm} (3-6)

where

\[ b \equiv b(E,t) = 3 \left( \frac{dE}{dt} \right)_{\text{coll}} / \left( \frac{dE}{dt} \right)_{\text{exp}} = \frac{3a}{2E^{3/2}t^2} \]  \hspace{1cm} (3-7)

and

\[ t_o = \left[ \frac{q}{2} + d^{1/2} \right]^{1/3} + \left[ \frac{q}{2} - d^{1/2} \right]^{1/3} \]  \hspace{1cm} (3-8)

where

\[ p \equiv 3a / (2E_o^{3/2}) \]
\[ q \equiv (E/E_o)^{3/2} \ t^3 + pt \]

and

\[ d \equiv q^2 / 4 + p^3 / 27 \]
Particles injected in the energy interval $E_0$ and $E_0 + dE_0$ will spread at time $t$ to energies between $E$ and $E + dE$.

The relation between these two intervals can be obtained by differentiating (3-5):

$$dE_0 = \left(\frac{E}{E_0}\right)^{1/2}\left(\frac{t}{t_0}\right)^3 dE$$

(3-9)
CHAPTER 4

THE EVOLUTION OF THE PARTICLE SPECTRUM IN THE NEBULA

In deriving an expression for the particle spectrum in the nebula, the following assumptions will be made:

1) The motion of a group of particles with energies between $E_0$ and $E_0 + dE_0$ injected by the source between $t_0$ and $t_0 + dt_0$ is described by their velocity in energy space given by equation (3-1). Fluctuations in the energy of the particles, or the possible existence of secondary acceleration mechanisms in the nebula are therefore neglected.

2) The number of particles in the group is a conserved quantity, that is, catastrophic losses, or collisions which remove or add particles to the group, are neglected.

The evolution of the particle spectrum in the nebula is examined for the following three cases:

Case 1: Following the Singular Injection Event

Let $N(E,t)$ be the number of particles in the nebula which at time $t$ have energies between $E$ and $E + dE$. These particles decayed from particles injected at time $t_0$ in the energy interval $E_0$ and $E_0 + dE_0$. The conservation of number of particles implies that

$$N(E,t)dE = N_0(E_0)dE_0$$

or

$$N(E,t)dE = (\gamma-2) \frac{E_{TOT}}{E_c^2} \left( \frac{E_0}{E_c} \right)^{-\gamma} dE_0 \quad (4-1)$$
Using equations (2-6) and (2-9) yields

\[
N(E,t) dE = (\gamma - 2) \frac{E_{\text{TOT}}}{E_c^2} \left( \frac{E}{E_c} \right)^{-\gamma} \left( \frac{t}{t_o} \right)^{-2\gamma} \times \\
\times \left[ 1 + b \frac{t-t_o}{t} \right]^{-\frac{2\gamma}{3}} \left( \frac{E}{E_o} \right)^{1/2} \left( \frac{t}{t_o} \right)^3 dE
\]

The spectrum of particles in the nebula will then be given as:

\[
N(E,t) = (\gamma - 2) \frac{E_{\text{TOT}}}{E_c^2} \left( \frac{E}{E_c} \right)^{-\gamma} \left( \frac{t}{t_o} \right)^{-\nu} \left[ 1 + b \frac{t-t_o}{t} \right]^{-\mu}
\]

with the low-energy cutoff:

\[
E_c(t) = E_c \left( \frac{t}{t_o} \right)^2 \left[ 1 - b_o \frac{t-t_o}{t} \right]^{2/3}
\]

where

\[
b_o \equiv b(E,t)|_E = E_c \\
t = t_o \\
= 3a \left[ 2E_c^{3/2} t_o^2 \right]^{-1}
\]

and

\[
\nu \equiv 2(\gamma - 1) \\
\mu \equiv \frac{2\gamma + 1}{3}
\]

**Case 2: The Particle Spectrum for the Continuous Injection Case**

Let \( dN(E,t,t_o) dE \) be the number of particles which, emitted between times \( t_o \) and \( t_o + dt_o \), have at time \( t \) energies in the range \( E \) and \( E + dE \). \( N_s(E_o,t_o) dE_o dt_o \) is the number of particles created by the source between
times \( t_0 \) and \( t_0 + dt_0 \) with energies between \( E_0 \) and \( E_0 + dE_0 \). The number of particles is conserved, implying that

\[
dN(E,t,t_0) dE = N_s(E_0,t_0) dE_0 dE_0 dt_0
\]  

(4-5)

Using (2-4), (3-6), and (3-9), equation (4-5) can be written as:

\[
dN(E,t,t_0) dE = (\gamma-2) \frac{\alpha_{CL_0} E_0^{-\gamma}}{E_c^2} \frac{dE_0 dt_0}{(1+t_0/\tau_0)^2}
\]

\[
= (\gamma-2) \frac{\alpha_{CL_0} E_0^{-\gamma} (t/t_0)^2}{E_c^2} \left[ 1 + \frac{b}{3} \right] \frac{dt}{(1+t_0/\tau_0)^2}
\]

(4-6)

\[
N(E,t), \text{ the total number of particles which at time } t \text{ have energies between } E \text{ and } E + dE, \text{ is obtained by integrating (4-6) over all the emission times which can contribute to } N(E,t). \text{ The lower limit of the integral corresponds to the time the source is switched on, that is, } t_0 = 0. \text{ The upper limit of the integral, } t_u, \text{ is determined by the lowest injection energy that can decay in the time interval } t - t_u \text{ to } E. \text{ If } E = E_c, \text{ then clearly, that energy is } E \text{ itself, and the upper limit of the integral in 'now', i.e., } t. \text{ However, if } E < E_c, \text{ then the lowest injection energy that can decay to } E \text{ is } E_c. \text{ } t_u \text{ can then be obtained by solving (3-5) for } E_0 = E_c. \text{ The last epoch that can contribute to } N(E,t) \text{ is thus given by:}
\]

\[
t_u = \begin{cases} 
E_c^{3/2} t_u^{3/2} + \frac{3}{2} a t_u = E^{3/2} t^{3/2} + \frac{3}{2} a t & \text{at } E < E_c \\
t & \text{at } E > E_c
\end{cases}
\]

(4-7)
and

\[ N(E,t) = (\gamma-2) \frac{a_{CR} L_0}{E_c^2} \left( \frac{E}{E_c} \right)^{-\gamma} \int_0^{t_u} \left[ 1 + b \frac{t-t_o}{E} \right]^{-\mu} \times \]

\[ \times \left( \frac{t_o}{t} \right)^{\nu} \frac{dt_o}{(1+t_o/t_o)^2} \]

(4-8)

where

\[ \mu \equiv \frac{2\gamma + 1}{3} \quad \text{and} \quad \nu \equiv (\gamma - 1) \]

(4-9)

The integral can be written in a dimensionless form:

\[ N(E,t) = (\gamma-2) \frac{a_{CR} L_0}{E_c^2} \left( \frac{E}{E_c} \right)^{-\gamma} \frac{U^{\nu+1}}{(1+b)^{\mu}} \int_0^1 \frac{X^{\nu}}{(1-VX)^{\mu}} \frac{dX}{(1+WX)^{2}} \]

(4-10)

where

\[ U \equiv \frac{t_u}{t} \quad \nu \equiv U \left( \frac{b}{1+b} \right) \quad \text{and} \quad W \equiv \frac{t_u}{t_o} \]

(4-11)

Figures 1(a) - 1(f) and 2(a) - 2(d) show the value of \( N(E,t) \) at different epochs of time for two values of the power law index, \( \gamma = 2.5 \) and \( \gamma = 8.5 \), respectively; and for four different values of the low energy cutoff, \( E_c \).

The general behavior of the particle spectrum as a function of time can be understood in the following way: At early epochs collisional losses dominate, i.e., the low energy particles are removed faster from the spectrum than those having higher energies. The shape of the spectrum is approximately given by \( N(E,t) \propto E^{1/2} \), which can be derived by taking the limit of \( N(E,t) \) given by
(4-10) for $b >> 1$. Figures 1(a) and 1(b) clearly show this behavior. In figure 1(c) the first appearance of a maximum in $N(E,t)$ is seen. This maximum is a result of the combined effects of collisional losses and power law injection, the latter replenishing the spectrum with low energy particles. As the nebula expands and collisional losses decrease, the maximum moves to lower energies until it occurs at the low energy cutoff. This effect is clearly seen in figure 1(d), where the maximum is at $E = 100$. When expansion losses dominate, the high energy particles are removed from the spectrum faster than those with lower energy. The maximum in $N(E,t)$ then shifts to energies increasingly lower than $E_c$, an effect readily seen in Figures 1(f), 2(c) and 2(d).

As can be seen from the injection spectrum, the total number of particles injected by the pulsar is greater for smaller, low energy cutoffs. However, since collisional losses dominate in the early stages of the expansion, spectra with lower values of $E_c$ contain fewer particles. As collisional losses decrease, these spectra build up and contain more particles than those with higher values of $E_c$. This transition can be seen in Figures 1(a) through 1(f), and 2(a) through 2(d).

Case 3: The Spectrum for Continuous Injection, Including Diffusion

When taking diffusion into account, it will be assumed that the diffusion coefficient is independent on position
but may be dependent on energy and time. The underlying assumption is that the density of the nebula and the magnetic field are spatially uniform, but evolve in time. The picture, therefore, is one of a point source, located at the center of the nebula, creating energetic particles which diffuse away from the origin in an isotropic and homogeneous medium. The magnetic field in the nebula is assumed to be chaotic, so that the diffusion of the particles across the field lines can be described as a random walk problem. The probability of finding at time \( t \) a particle, which at time \( t = 0 \) was located at \( r = 0 \), at a distance between \( r \) and \( r + dr \) from the origin is therefore given by Chandrasekhar (1943):

\[
P(r)dr = \frac{4\pi r^2 dr}{(4\pi Dt)^{3/2}} \times \exp\left(-\frac{r^2}{4Dt}\right) \quad (4-12)
\]

where \( D \) is a diffusion coefficient, which in our model is independent of position but may be a function of energy and time. Using (4-12), equation (4-5) can be generalized to give the number of particles which, emitted between times \( t_o \) and \( t_o + dt_o \) are at time \( t \) at a distance between \( r \) and \( r + dr \) from the origin with energies between \( E \) and \( E + dE \):

\[
dN(E,t,t_o,r)dEdr = N(E_o,t_o)dE_o dt_o \times
\]

\[
\times \frac{4\pi r^2 dr}{\left[4\pi \int_{t_o}^{t} D dt\right]^{3/2}} \times \exp\left[-\frac{r^2}{4\int_{t_o}^{t} D dt}\right] \quad (4-13)
\]
Let \( \hat{N}(E,t,r) \) be the integral of (4-13) over all emission times, with \( t_u \) given by (4-7). \( N(E,t,R(t)) \), the total number of particles which at time \( t \) are trapped in the nebula with energies between \( E \) and \( E + \Delta E \), is given by

\[
N(E,t,R) = \int_0^R \hat{N}(E,t,r) \, dr \quad (4-14)
\]

where \( R = R(t) \) is the radius of the nebula at time \( t \). \( \hat{N}(E,t,r) \) is an integral over emission times, and changing the order of integration in (4-14) yields

\[
N(E,t,R) = (\gamma - 2) \frac{\alpha_{C\text{R} \text{L}_0}}{E_c^2} \left( \frac{E}{E_c} \right)^{-\gamma} \frac{U^{\nu+1}}{(1+b)^{\mu}} \int_0^1 P_c(E,t,R,t_o) \frac{x^\nu}{(1-vx)^{\mu}} \frac{dx}{1+wx} \quad (4-15)
\]

where \( P_c(E,t,R,t_o) \) is the probability that a particle injected at time \( t_o \) and having at time \( t \) an energy \( E \), will be confined to the nebula at that time. \( P_c \) is given by (Appendix A)

\[
P_c(Z) = \text{erf} \, Z - \frac{2}{\sqrt{\pi}} \, Z \, e^{-Z^2} \quad (4-16)
\]

where

\[
Z^2 = (q+1)R^2/(4 D_o E^p t^{q+1} [1-(UX)^{q+1}]) \quad (4-17)
\]

and the diffusion coefficient \( D \) was taken to be dependent on energy and time only with the functional form:
\[ D = D_0 \exp^q t^q \]  

(4-18)

In Appendix B an expression for \( D \) of the general form of (4-18) is derived. Asymptotic values of \( P_c(Z) \) are:

\[
\lim_{Z \to 0} P_c(Z) = \begin{cases} 
0 & \text{when } Z \to 0 \\
1 & \text{when } Z \to 0 
\end{cases}
\]  

(4-19)

Therefore, since the value of \( Z^2 \) decreases as energy and time increase, one sees from equations (4-15) and (4-19) that as the nebula evolves, it becomes increasingly transparent to cosmic rays with increasingly lower energies.

The radial distribution of particles in the nebula is

\[
F(E,t,r)dr = \frac{\tilde{N}(E,t,r)dr}{N(E,t)} 
\]  

(4-20)

where \( N(E,t) \) is given by (4-10). This function is shown in Figure 3 for a power law index of \( \gamma = 5.5 \) for three different values of the diffusion coefficient \( D \). Only for \( D = 10^{26} \text{ cm}^2 \text{ sec}^{-1} \) a significant fraction (\( \sim 25\% \)) of the particles escape. For comparison, the value of the diffusion coefficient in the free interstellar space is \( 10^{28} \text{ cm}^2 \text{ sec}^{-1} \) (Cowsik and Wilson, 1975). Since the diffusion coefficient is of the general form \( D = D_0 \exp^p t^q \), the results of Figure 3 can be interpreted in two ways:

(1) For a given time the three curves show the increasing probability of particles with higher energy to escape the nebula.
(2) For a given energy the three curves show a time sequence in which the probability of a particle to escape the nebula increases in time. The nebula can therefore be pictured as a 'leaking sphere' from which, as time goes on, particles with increasingly lower energies can leak out.
CHAPTER 5

THE PRODUCTION OF LIGHT ELEMENTS IN THE NEBULA

§1 - Reaction Thresholds and Nuclear Cross Sections

The most abundant elements in the nebula are helium, carbon and oxygen; therefore the most important reactions for the production of the light elements will be those involving $\alpha$-particles incident on $^4\text{He}$, $^{12}\text{C}$, and $^{16}\text{O}$.

In Table 3 the thresholds for the production of LiBeB in $\alpha$-particle reactions with $^4\text{He}$, $^{12}\text{C}$, and $^{16}\text{O}$ are listed. The data represent the channels with the lowest threshold for the production of the given element (or its unstable parent), and are based on mass excesses tabulated by Wapstra and Gove (1971).

The $\alpha + \alpha$ Reaction

The $\alpha \alpha$ reactions $^4\text{He}(\alpha,\text{pn})^6\text{Li}$, $^4\text{He}(\alpha,\text{d})^6\text{Li}$, $^4\text{He}(\alpha,\text{p})^7\text{Li}$, and $^4\text{He}(\alpha,\text{n})^7\text{Be}$ are an important source of Li in the cosmic ray propagation model (Meneguzzi, Audouze, and Reeves, 1971; Mitler, 1972). The cross sections used for these reactions were obtained from the principle of detailed balance, and from data on the inverse reactions. Kozlovski and Ramaty (1974) pointed out that this method does not take into account the production of $^7\text{Li}$ and $^7\text{Be}$ in their first excited states at 478 and 431 keV, respectively.

Burcham et al. (1958) found that the cross section for producing $^7\text{Li}$ in its ground state is almost equal to the cross section for its production at its first excited
state. This similarity follows from the fact that the ground and first excited states of $^7$Li have the same spatial wave function, so that the only difference in the cross section is caused by the different penetrability factor of the outgoing protons, a difference that decreases as the incident energy is increased. Similar arguments apply for the reaction that produces $^7$Be and $^7$Be$^{*}$431keV, both states decaying by electron capture to $^7$Li. Direct measurements of the cross sections for $^7$Li and $^7$Be formation were made at $E_\alpha = 38.5$ MeV by Burcham et al. (1958) and at $E_\alpha$ between 38 and 140 MeV by King et al. (1975). The experimental results used in this work are shown in Figure 4.

Excited states make a negligible contribution to the formation cross section of $^6$Li (Bodansky, Jacobs, and Oberg, 1975). The first excited state at 2.18 MeV decays to $^d + ^4$He and the $T = 1$ second excited state at 3.56 MeV, which requires a change $\Delta T = 1$ of isotopic spin between target and product nuclei, is suppressed relative to reactions which produce $^6$Li in its ground state for which $\Delta T = 0$ (Bernas et al., 1967). Production of $^6$Li in excited states is therefore neglected in this work, and the values of the cross sections for $^6$Li production in its ground state are taken from Mitler (1972). These values are shown in Figure 5.

**The $\alpha + ^{12}$C Reaction**

The experimental cross sections for the formation of various light elements by alpha bombardment on $^{12}$C are
shown in Figure 6. The data for $^6\text{Li}$, $^7\text{Li} + ^7\text{Be}$ and $^9\text{Be}$ represent the total contribution to the yield of mass 6, 7, and 9, respectively. The contribution of the stable isotopes $^{10}\text{B}$ and $^{11}\text{B}$ to the mass-10 and mass-11 yield have not been measured. Monte-Carlo spallation calculations done by Ayres et al. (1973) at $E_\alpha = 920$ MeV suggest that the $^{10}\text{B}$ contribution to the mass-10 yield should be by about a factor of 3 larger than that of $^{10}\text{Be}$, while the $^{11}\text{B}$ contribution to the mass-11 yield approximately equals that of $^{11}\text{C}$.

There exists a general pattern in the cross section curves in Figure 3, which can be understood in terms of the "Serber two step model" for high-energy nuclear reactions (Serber, 1947). At high energies the mean free path of the incident particle in the target nucleus increases due to a decrease in the scattering cross sections between nucleons, and becomes comparable to the nuclear dimensions. The target nucleus is therefore partially 'transparent' to the incoming particle which interacts with only a few nucleons in the target.

The first step of the model consists of following the sequence of nuclear interactions which generate a cascade of collisions, best done by using a Monte-Carlo method (Schmitt et al., 1973). The incident and target nuclei are both clusters represented by discrete nucleons at fixed positions. As the incident, nucleus collides with the target, a nucleon of one cluster interacts with a nucleon of the other cluster. This interaction may result
in the scattering of the clusters; a capture of one cluster by another; or, most frequently, in the breakup of either or both clusters into smaller ones. For each collision the final momentum states of the scattered nucleons are calculated through ordinary kinematics; however, if a momentum state corresponds to an already occupied state, the collision is assumed not to have occurred (Pauli principle). After each collision, the scattered nucleon (or cluster of nucleons) can either undergo another collision, or, if it has enough energy, cross the potential barrier of the nucleus and escape. This cascade goes on until the energy of the nucleons is so low that their energy is shared with the nucleus as a whole. This whole step of 'intranuclear cascade' takes place during a very short time interval ($\sim 10^{-22}$ sec) at the end of which, a residual nucleus is left having a certain excitation energy $E^*$. 

The second step of the Serber model is the deexcitation of the residual nucleus. If the excitation energy is low, a statistical distribution of the excitation energy among the various degrees of freedom can be achieved (Weiskopf, 1937), after which, the nucleus deexcites by a slow evaporation process, 'boiling off' one particle after another. However, if the excitation energy is large compared to the total binding energy of the nucleus, the deexcitation is sudden and the various possible breakup modes, including the simultaneous emission of more than
one particle, occur according to their statistical weight (Fermi, 1950). From the preceding, the pattern of the cross section curves can be explained as follows: The rising part above threshold reflects the increase of statistical weight of the given reaction channel with energy and the increasing probability of the Coulomb-barrier penetration for the outgoing particles. The decreasing part in the $^{11}$C cross section is due to the opening of new channels. Since the total binding energy of the target nucleus is about 100 - 150 MeV, no new channels are expected to open up at higher energies. The individual cross sections are therefore expected to remain constant from that energy on, decreasing somewhat due to the increasing transparency of the target nucleus to the incident particle. The distribution of final nuclei is strongly dependent on $\Delta T_3$, the difference in the third component of the isospin ($T_3 = 1/2(N-Z)$) of the product and target nucleus, strongly favoring reactions for which $\Delta T_3 = 0$. This may be understood in terms of the Serber two step model in the following way:

The cascade — From purely statistical arguments the most probable reactions are those where the relative number of protons and neutrons ejected is closest to that of the target nucleus. For example, if the target is symmetric with $N = Z$, then ($\alpha, \alpha'pn$) reactions are expected to occur twice as frequently as ($\alpha, \alpha'pp$) or ($\alpha, \alpha'nn$) reactions. Therefore, the most probable events
are those having $\Delta T_3 = 0$. This simple argument is complicated by the difference in the cross section for the collisions between identical (p-p or n-n) and unlike (n-p) nucleons, and by the effect of the Coulomb barrier on proton emission at low energies.

**The breakup** — The important factor here is the statistical weight of the channel. This depends on the kinetic energy released by the breakup, given by $K = E^* - Q$, where $Q$ is the Q-value for the given breakup. The smaller $Q$ values are obtained for stable products, which have low $T_3$ values, so that again, reactions leading to low $\Delta T_3$ are more probable. In light of these arguments, one should expect large cross sections for the production of $^{10}$B and $^6$Li and a low cross section for the production of $^{10}$Be. As the number of ejected particles decreases, the above arguments, which are of statistical nature, become increasingly difficult to apply. For small $\Delta A$ (defined as $A_{\text{target}} - A_{\text{product}}$), extra contributions from 'peripheral' reactions become important.

**The $\alpha + ^{16}$O Reaction**

The experimental data on this reaction are very scarce; however, Monte-Carlo calculations of Ayres et al. (1973) suggest that the production cross sections from $^{16}$O are equal to those from $^{12}$C, except for $A = 11$, where the $^{16}$O cross section is approximately one-third as great,
§2 - Reaction Rates

The production rate of a light nucleus of type-\(i\) can be written as the following sum over target nuclei:

\[
\frac{dN_i}{dt} = \sum_j n_j(t) \int_{Q_{ij}}^{\infty} N(E,t) \sigma_{ij}(E)V(E)dE \quad (5-1)
\]

where \(N_i\) is the total number of nuclei of element \(i\) produced; \(n_j\) is the number density of the target nucleus \(j\), decreasing as \(t^{-3}\); \(N(E,t)\) is the number of \(\alpha\)-particles at time \(t\) in the nebula with kinetic energies between \(E\) and \(E + dE\); \(\sigma_{ij}(E)\) is the cross section for the reaction \(\alpha + i \rightarrow j\) at energy \(E\) (Lab frame), with a threshold \(Q_{ij}\); and \(V(E)\) is the velocity of the incident \(\alpha\)-particles with kinetic energy \(E\). In calculating the production rate, the effect of the expansion on the kinematics of the reaction is neglected, which introduces only minor errors as long as \(V_{SN} << V(E)\).

§3 - Light Element Production

Case 1: In the Singular Injection Event

Instead of calculating the contribution of this mechanism to the production of \(^7\)Li by use of the expressions derived for the particle spectrum, \(N(E,t)\), in equation (4-2), an order of magnitude estimate will be made in a simple, analytical way. The reaction rate in a slowing down phase can be written as:
\[ N(7\text{Li}) = n(4\text{He}) \int_{E_c}^{\infty} \int_{\max\{Q, E_c\}}^{\infty} \sigma(E)V(E) \left( \frac{dE}{dE_{\text{coll}}} \right)_{\text{coll}} dE dE' \]

where we will consider the production of \(^7\text{Li}\) due to the ambient helium in the nebula. Assuming optimum conditions for \(^7\text{Li}\) production, that is, a monoenergetic spectrum of \(\alpha\)-particles at \(E_\alpha = 45\) MeV (no \(^6\text{Li}\) will therefore be produced), equation (5-2) can be written as:

\[ N(7\text{Li}) = n(4\text{He}) N_\alpha \int_{Q}^{E_\alpha} V(E) \sigma(E) \left( \frac{dE}{dE_{\text{coll}}} \right)_{\text{coll}} dE \]

where \(N(E')\) was represented as a \(\delta\)-function at \(E' = E_\alpha\), normalized to give a total number, \(N_\alpha\), of injected particles. \(n_j\) is the number density of the target nucleus \(j\), and \(\left( \frac{dE}{dt} \right)_{\text{coll}}\) is the rate of energy loss due to collisions, given by (3-2). Since \(^7\text{Li}\) is produced only over the narrow energy interval \(E_\alpha - Q \approx 10\) MeV, \(\sigma(E)\) and \(V(E)\) can be approximated by an average value corresponding to an energy \(Q < E_\alpha < E_\alpha\) in that interval. Inserting the value of \(\left( \frac{dE}{dt} \right)_{\text{coll}}\) for that energy, one gets for the total number of \(^7\text{Li}\) produced:

\[ N(7\text{Li}) = \frac{n(4\text{He})}{n} N_\alpha \sigma(E_\alpha)V(E_\alpha) \frac{E_\alpha}{9 \times 10^{-11}} (E_\alpha - Q) \]

\(\frac{n(4\text{He})}{n}\) is the ratio of the density of target helium-nuclei to the total density in the deceleration region, which in the spirit of obtaining an upper value for \(N(7\text{Li})\) will
be taken to be of order unity. If the jet consists primarily of α-particles, then \( N_\alpha \sim 10^{35} \). Taking \( \sigma(E_\alpha) \sim 50 \text{ mb}, V(E_\alpha) \sim 4 \times 10^9 \text{ cm sec}^{-1} \) and \( E_\alpha^{1/2} \sim 6 \), one gets

\[
N(\text{\textsuperscript{7}Li}) \sim 10^{50} \text{ nuclei},
\]

which is equal to the total required amount of \textsuperscript{7}Li production. We see therefore, that a hypothetical jet of alpha particles with total mass and kinetic energy as suggested by the numerical studies of LeBlanc and Wilson (1970) is able to produce a significant amount of \textsuperscript{7}Li, when stopped in helium rich material. A certain amount of helium may be produced by the photodisintegration of the iron peak nuclei at the high temperatures encountered during the collapse. A production of \textsuperscript{7}Li can therefore not be ruled out entirely, and the underlying possibilities may justify a more detailed investigation of the problem.

**Case 2: In the Continuous Injection Case**

In the following, Model A and Model B will designate the two models of the nebula whose characteristics are listed in Table 2, with the choice of \( V_{SN} = 5 \times 10^8 \text{ cm sec}^{-1} \) as the velocity of the nebula and the conventional choice of pulsar parameters listed in Chapter 1. If other parameters are used instead of those above, they will be mentioned explicitly.

For a model to be a significant source of \textsuperscript{7}Li, the
following conditions should be met:

(1) The total number of $^7$Li nuclei produced should be between $6 \times 10^4$ and $10^5$ nuclei.

(2) No significant production of light elements other than $^7$Li should occur, and specifically, the relative yield of $^7$Li to $^6$Li should be $\approx 12$.

A computer model was developed to calculate the production of light elements in the nebula by integrating the reaction rate given by equation (5-1). In the beginning, the production rate is zero, rising rapidly as the total number of $\alpha$-particles injected into the nebula increases. At the same time, the density of the nebula decreases, so that a maximum production rate is reached at typical times of about two years, after which the production rate starts to decrease, so that no significant amount of light elements is produced after a period of about seven years.

Figures 8 and 9 show the resulting abundances of the light elements produced for Models A and B, respectively and for various different injection spectra. These spectra are characterized by their low-energy cutoff, $E_c$, and their spectral index, $\gamma$. In Figure 11(a) the relative production of $^7$Li and $^6$Li is shown for the two models. These graphs show that for spectra producing the required amount of $^7$Li, the highest $^7$Li/$^6$Li ratio achieved is about 8 for the number-averaged core (Model A), and only 3 for the mass-averaged core (Model B). This result could have been
predicted from the abundances of elements in the ejecta listed in Table 2. In Model B the abundance of carbon and oxygen is significantly enhanced, relative to the abundance of helium in the nebula. As a result, a significant amount of light elements other than $^7\text{Li}$ will be produced, primarily from $\alpha + ^{16}\text{O}$ reactions. This seems, therefore, to disqualify the mass-averaged supernova event as a typical site for $^7\text{Li}$ production.

A study of Figure 8 shows that the requirement for producing about $10^{50}$ $^7\text{Li}$ nuclei is increasingly difficult to meet for 'harder' injection spectra; whereas for a given low-energy cutoff, $E_c$, the requirement for a relative enhancement in the $^7\text{Li}$ production yield is more difficult to meet for 'flatter' injection spectra. This is due to the fact that a flat spectrum suffers less from collisional losses than a steep spectrum, since a larger fraction of its particles is injected at higher energies. These same particles also account for the enhancement in the production of other light elements by $\alpha + ^{12}\text{C}$ and $\alpha + ^{16}\text{O}$ collisions. For energies below 100 MeV, the main contribution to the production of lithium comes from the $\alpha\alpha$ reaction, whereas above that energy, the main contributors are the $\alpha + ^{12}\text{C}$ and $\alpha + ^{16}\text{O}$ collisions. This effect, which is shown in Figure 10, manifests itself in the production curves for model A by a 'levelling off' in the production of lithium for $\gamma = 2.5$ and an actual drop for harder spectra.
In Clayton and Dwek (1976) a ratio of $^{7}\text{Li}/^{6}\text{Li} \sim 12$ could be obtained for a steady state power law spectrum with $\gamma > 7$. As can be seen, this ratio is not obtainable in this model, even with a steep injection spectrum of $\gamma = 8.5$. The difficulty of this model to reach this ratio is a result of the more realistic treatment of the evolution of the spectrum in the nebula. Collision losses in the early stages of the expansion 'invert' the spectrum, which, as a result, goes as $\sim E^{1/2}$, so that the number of particles in the spectrum increases with energy. Therefore, although the threshold for $^7\text{Li}$ production by $\alpha \alpha$ reactions is 10 MeV lower than that for $^6\text{Li}$ production, no large $^{7}\text{Li}/^{6}\text{Li}$ ratio can be obtained.

To reduce the effect of collision losses, a computer run was made for Model A with an expansion velocity of $1.5 \times 10^3$ cm sec$^{-1}$. Increasing the expansion velocity by a factor of 3 has the immediate effect of decreasing the density of the nebula, and consequently, the production rate, by a factor of 27. The decrease in the density, however, increases the number of energetic particles in the nebula. This increase is larger for steeper spectra with lower values of $E_c$. The net effect of increasing the expansion velocity on the production yield of $^7\text{Li}$ is shown in Figure 13. For $E_c = 40$ and $\gamma = 8.5$, the average increase in the number of particles in the spectrum is by a factor of $\sim 100$, actually producing more ($\sim 2 \times 10^5$) $^7\text{Li}$ than the standard model. The $^7\text{Li}/^6\text{Li}$
ratio is accordingly increased (see Figure 11(b)), and reaches a value of \( \sim 10 \); and \( \frac{\frac{L_i}{\text{BeB}}}{\frac{L_i}{\text{BeB}}} \), the relative overproduction of lithium with respect to other light elements (shown in Figure 12), reaches a large value of \( \sim 100 \).

One can therefore conclude that to account for the observed amount of \(^7\text{Li}\), only \( \sim 30\% \) (\( \alpha_{\text{CR}} = 0.3 \)) of the pulsar's energy loss needs to be converted to medium-energy \( \alpha \)-particles. To achieve the observed \(^7\text{Li}/^6\text{Li} \) ratio an additional source of \(^7\text{Li} \) is required, producing about \( 1.2 \times 10^9 \) \(^7\text{Li} \) nuclei. This additional source may be a jet of material injected into the nebula at the early stages of the collapse. To supply the necessary amount of \(^7\text{Li} \) about \( \sim 12\% \) of the material in the jet has to be in the form of \( \alpha \)-particles which have to be slowed down in \( \sim 8 \times 10^{-2} \) gr cm\(^{-2} \) of helium-rich matter.

For sake of completeness, an additional computer run was made for Model A in which the number of target nuclei was retained but the total mass of the ejected nebula was increased to 20 M\(_\odot\). This model is also shown in Figure 13, so that from this figure the effects of different mass, composition, and velocity on the results of the standard Model A can be estimated.
CHAPTER 6

THE ASSOCIATED GAMMA-RAY FLUXES

Kozlovski and Ramaty (1974) first pointed out that in $\alpha\alpha$ reactions $^7\text{Li}$ and $^7\text{Be}$ are produced in their excited states at 0.478 MeV and 0.431 MeV, with about equal probability as in their ground state. These excited states of $^7\text{Li}$ and $^7\text{Be}$ decay to their ground state by photon emission, so that after allowing for electron capture, for each four $^7\text{Li}$ nuclei formed, about one photon of 0.478 MeV and one photon of 0.431 MeV are produced. Let $\frac{dN_{\gamma}(^7\text{Be}^*)}{dt}$ and $\frac{dN_{\gamma}(^7\text{Li}^*)}{dt}$ be the rate of photon emission of the specified type, then:

$$\frac{dN(\gamma)(^7\text{Be}^*)}{dt} = \frac{1}{2} \frac{dN(\gamma)(^7\text{Li})}{dt} \bigg|_{\alpha\alpha}$$

$$= \frac{1}{2} n(^4\text{He}) \int_{E}^{\infty} N(E,E) \sigma_{^7\text{Li}}(E) V(E) dE \frac{Q(7\text{Li})}{Q(7\text{Be})}$$

(6-1)

and

$$\frac{dN_{\gamma}(^7\text{Be}^*)}{dt} = \frac{1}{2} \frac{dN(\gamma)(^7\text{Be})}{dt} \bigg|_{\alpha\alpha}$$

$$= \frac{1}{2} n(^4\text{He}) \int_{E}^{\infty} N(E,E) \sigma_{^7\text{Be}}(E) V(E) dE \frac{Q(7\text{Be})}{Q(7\text{Li})}$$

(6-2)

where $\frac{dN(\gamma)(^7\text{Li})}{dt} \bigg|_{\alpha\alpha}$ and $\frac{dN(\gamma)(^7\text{Be})}{dt} \bigg|_{\alpha\alpha}$ are the rate of production of $^7\text{Li}$ and $^7\text{Be}$ in their ground and first excited state.
by $\alpha\alpha$ reactions.

Inelastic collisions of $\alpha$-particles with $^{12}\text{C}$ will produce $^{12}\text{C}$ in an excited state of 4.44 MeV, which decays by photon emission to its ground state. The differential cross section for this reaction is given by Meneguzzi and Reeves (1975) (see Figure 7); and following their assumption, the total cross section is calculated by assuming angular isotropy.

The production of lithium and other light elements in supernova remnants can therefore, in principle, be affirmed by observing the associated emission of gamma ray lines. To estimate the number of photons that emerge from the nebula without being absorbed or degraded in energy, the amount of absorption in the nebula must be taken into account. If $\frac{dn_\gamma}{dt}$ is the production rate of photons of energy $E_\gamma$ per unit volume in the nebula; and $\mu_\gamma$, the absorption coefficient of that photon per unit length; then, assuming that each unit volume radiates isotropically, the rate, $r_\gamma$, at which these photons are emitted from the nebula, will be given by

$$r_\gamma = \int_0^R 4\pi r^2 dr \frac{dn_\gamma(r,t)}{dt} \times \int_{\Omega} T_\gamma(r,\Omega) d\Omega$$

(6.3)

$T_\gamma(r,\Omega)d\Omega$ is the fraction of photons of energy $E_\gamma$, emitted in directions between $\Omega$ and $\Omega + d\Omega$ by a unit volume located at a distance $r$ from the center, that emerge from the nebula without being absorbed or degraded in energy.
\( T_\gamma \) is given by (see Appendix C)

\[
T_\gamma(r, \theta) = \frac{1}{2\pi} \exp[-\mu_\gamma r \left( \frac{R^2}{x^2} - \sin^2 \theta \right)^{1/2}] \cosh(\mu_\gamma r \cos \theta)
\]

(6-4)

Assuming a uniform distribution of sources in the nebula, that is,

\[
\frac{dn_\gamma(r,t)}{dt} = \left( \frac{4\pi}{3} R^3 \right)^{-1} \times \frac{dN_\gamma(t)}{dt}
\]

(6-5)

equation (6-3)

\[
r_\gamma = 6\pi \frac{dN_\gamma(t)}{dt} \int_0^1 \int_0^{\pi/2} T_\gamma(x, \theta) d\theta dx
\]

(6-6)

where \( \frac{dN_\gamma(t)}{dt} \) is the rate of production of photons of energy \( E_\gamma \), and \( x \) is the dimensionless parameter \( \frac{r}{R} \).

If the supernova event occurs at a distance \( D \), then the observed flux, \( F_\gamma \), will be given by

\[
F_\gamma = \frac{8.3 \times 10^{-45}}{D^2 (\text{Kpc})} \quad r_\gamma \left( \frac{\text{photons}}{\text{cm}^2 \text{ sec}} \right)
\]

(6-7)

where \( D \) is expressed in units of kiloparsecs (\( \sim 3 \times 10^{21} \text{ cm} \)).

Figure 14 shows the rate at which the 478 keV and 4.44 MeV photons are emitted from the nebula, where \( E_C = 40 \) MeV, \( \gamma = 8.5 \), and an expansion velocity of \( V_{\text{SN}} = 1.5 \times 10^9 \) cm sec\(^{-1} \) were used as parameters. The maximum emission rate is \( \sim 6 \times 10^{52} \) photons sec\(^{-1} \) for the 478 keV line and \( \sim 3 \times 10^{41} \) photons sec\(^{-1} \) for the 4.44 MeV line, corresponding to fluxes of \( \sim 5 \times 10^{-5} \) photons cm\(^{-2} \) sec\(^{-1} \) and \( \sim 2 \times 10^{-5} \) photons cm\(^{-2} \) sec\(^{-1} \), scaled for a supernova event at a distance of 10 kpc.
It is of interest to compare gamma ray fluxes predicted in this work with observed fluxes from the galactic center. The first two observations of gamma fluxes in this energy region (Johnson and Haymes, 1973) detected a flux of \((1.8 \pm 0.5) \times 10^{-3}\) photons cm\(^{-2}\) sec\(^{-1}\) at an average energy of \(476 \pm 24\) keV. This flux was originally interpreted by Fishman and Clayton (1972) as being the de-excitation of low-energy \(^7\)Li and \(^7\)Be cosmic rays inelastically scattering from the interstellar gas. A third observation (Haymes et al., 1975) detected a flux of \((8.0 \pm 2.3) \times 10^{-4}\) photons cm\(^{-2}\) sec\(^{-1}\) at an energy of \(0.53 \pm 0.01\) MeV, which was interpreted to be the 511 keV positron-annihilation line. Another line of interest to this work was detected at \(4.6 \pm 0.1\) MeV, most likely the 4.44 MeV deexcitation line of \(^{12}\)C\(^{+}\), with a flux of \((9.5 \pm 2.7) \times 10^{-4}\) photons cm\(^{-2}\) sec\(^{-1}\).

To explain the time behavior of the observed flux around 0.5 MeV as being due to \(\alpha\alpha\) collisions in a supernova remnant, a supernova event had to have occurred in their overlapping field of view at a mere distance of 2 kpc, about seven years ago. Clearly, an event of that magnitude and at that distance could hardly have occurred unnoticed. In Figure 14, the observed data is compared to the fluxes predicted in this work which are normalized to a source at a distance of \(\sim 2\) kpc to give the best fit to the observations.

This comparison with the observational data, does
point, to an interesting characteristic feature of the gamma ray fluxes predicted in this work. As can be seen in Figure 14, the predicted flux at 4.44 MeV is about a factor of three lower than the observed feature at that energy. This is a result of the requirement that \(^7\)Li be the main nuclei produced in the remnant which requires the dominance of \(\alpha\alpha\) reactions with the associated photon emission at around 0.5 MeV, over inelastic \(\alpha\)-collisions with \(^{12}\)C which produce 4.44 MeV photons. A necessary condition, therefore, for observed gamma fluxes to be associated with lithium production in supernova remnants, is the dominance of the 0.5 MeV feature over that at around 4.44 MeV by a factor of at least three. Thus, any future observation of these two lines from a single source with the relative intensity mentioned above, might be an indication of the dominance of \(\alpha\alpha\) collisions in that source, and the resulting production of \(^7\)Li; whereas the dominance of the 4.44 MeV line will definitely rule out this mechanism.
CHAPTER 7

THE PULSAR AS A COSMIC Ray SOURCE

A pulsar can be a cosmic ray source only if the energetic particles confined in the nebula can leak away before suffering severe energy losses. Cowssik and Wilson (1975) considered the efficiency at which cosmic rays trapped in a supernova remnant can be injected in the interstellar medium. Using a maximum diffusion coefficient (equal to that of test particles in the interstellar medium) and a continuous production of cosmic rays by the pulsar, they found that most of the cosmic ray energy is lost in the expansion, and only a small fraction ($\sim 10^{-3}$) is injected into the interstellar medium.

Similar conclusions are reached in this work, where the bulk of the cosmic ray particles is injected at energies around 40 MeV, so that most of their energy is lost by collisions. Furthermore, rough estimates of the diffusion coefficient places its value many orders of magnitude below that used in the work of Cowssik and Wilson.

Figure 13 shows the total energy put out by the pulsar as a function of time. The figure also includes the total energy in the form of energetic cosmic-ray particles in the nebula. This energy, denoted by $E_{CR}$, is obtained by performing the integral:
\[ E_{CR} = \int_{E_0}^{\infty} N(E,t)E \, dE \]  

where \( E_0 \) is some low energy cutoff, taken to be 5 MeV. The characteristics of the spectra are shown in the figure. We see that \( E_{CR} \) reaches a maximum value of \( \sim 5 \times 10^{49} \) ergs, a mere 5 percent of the total energy input in the form of \( \alpha \)-particles. Most of the energy is lost by collisions and is comparable in magnitude to the total energy output of the pulsar at the early stages of the expansion.

It may well be that this energy loss contributes to the early supernova light curve and to the heating and continuing acceleration of the expanding nebula.

It seems, therefore, that to account for the bulk of energy content in the form of cosmic rays in the galaxy, a secondary acceleration mechanism is needed, preferably delayed in time so that expansion losses will be negligible.

The pulsar may, however, still contribute a certain fraction of the total cosmic-ray energy in the galaxy. This contribution will probably be in the form of extremely high-energy cosmic rays accelerated from the pulsar's magnetosphere, or a small high-energy tail of the cosmic rays accelerated in the nebula.

The latter possibility is an exciting one, since the composition of these high-energy cosmic rays will reflect the composition of the material in the acceleration
region of the nebula; therefore, if light elements are indeed produced in the nebula, then an increase in their abundance in time will show up as a similar increase in their abundance in the escaping cosmic rays. The nebula in this model is a 'leaking sphere' which becomes increasingly transparent to lower energy cosmic rays. It is therefore interesting to speculate that the lower energy cosmic rays will contain a larger amount of light elements than the higher energy cosmic rays, which escaped the nebula at an earlier time. The ratio of light element abundance to that of another element whose abundance in the nebula does not change in time should therefore be an energy-dependent quantity, increasing for lower cosmic ray energies.

Such an enhancement for decreasing cosmic ray energies was indeed found in the Li+Be+B/C+O ratio by Webber et al. (1973). It may be premature to attribute this observation to the production of light elements in the cosmic ray sources, especially since other explanations have been offered to account for the variation in the Li+Be+B/C+O ratio (for a review article on this subject, see Cesarsky and Audouze, 1974). However, the light elements are usually assumed to be only secondary nuclei in the cosmic ray propagation model, their abundance being used to obtain the mean path length traveled by cosmic rays in the galaxy, or to deduce the mean cosmic
ray age in the galaxy. Their presence in the cosmic ray sources may therefore have significant consequences on the conventional interpretation of cosmic ray data.

The prospects, however, of this 'leaking sphere' picture to provide a test for \(^{7}\text{Li}\) production are very bleak. The requirement of the model to inject the \(\alpha\)-particles at a low energy cutoff of 40 MeV with a steep spectrum of \(\gamma = 8.5\) causes the amount of high-energy particles able to escape the nebula to be minimal. Nevertheless, this picture is introduced in this work, since it provides a 'natural' way of explaining the energy dependence of the Li+Be+B/C+O ratio in the case where the pulsar injects mostly highly energetic \(\alpha\)-particles into the nebula.
CONCLUSIONS

In this thesis a new model for the production of $^7$Li was presented, and tests by gamma ray astronomy were described. The site of production was an expanding nebula ejected from a $4 \, M_\odot$ core undergoing gravitational collapse. To account for the observed $^7$Li abundance, the newly-born pulsar needed to convert about 30% of its energy losses into medium-energy $\alpha$-particles, which, after injection into the nebula, had primarily undergone $\alpha\alpha$ collisions.

The requirement of injection energies around 40 MeV is the most difficult to justify, putting severe restrictions on the model; however, the other choices of parameters are conventional and need no special requirements of the model.

The possibility of synthesizing $^7$Li by a jet of material ejected during the collapse of the iron core was suggested. With this additional source, the $^7$Li/$^6$Li ratio can rise from the acquired 10 to the desired 12.

The gamma ray fluxes associated with the model were calculated. The source was found to be 'too close' to account for observed fluxes, and 'too weak' to be observed if its distance was 10 kpc. A characteristic feature of the emerging flux is the relative intensity of the various gamma ray lines. Although this feature cannot serve as a positive identification for the proposed scenario, its absence will rule out the production of $^7$Li in the source by the proposed model.
The evolution of the cosmic-ray $\alpha$-particles in the nebula was examined, taking their energy losses due to collisions and expansion into account. The diffusion of these cosmic-rays from their sources was also considered, and the picture of the nebula as a 'leaking sphere' was presented. Although this picture cannot provide a test for $^7\text{Li}$ production in the nebula, it was introduced as a possible explanation for the energy variation of the light-to-medium element ratio in high-energy cosmic-rays, in a situation where the cosmic-ray $\alpha$-particles are injected with high energies into the nebula.
APPENDIX A

THE CONFINEMENT PROBABILITY $P_c$

Equation (4-14) can be written explicitly:

$$N(E, t, R) = (\gamma - 2) \frac{\alpha CR_{LO}^2}{E_c^2} \left( \frac{E}{E_c} \right)^{-\gamma} \int_0^R 4\pi r^2 dr \times$$

$$\times \int_0^{t_u} \left[ 4\pi \int_0^t D(E, t) dt \right]^{-3/2} \times \exp\left[ -\frac{r^2}{4 \int_0^t D(E, t) dt} \right] \times$$

$$\times \left( \frac{t}{t_o} \right)^\mu \left[ 1 + b \frac{t - t_o}{t} \right]^{-1} \times \left( 1 + \frac{t_o}{t_o} \right)^{-2} dt_o \quad (A-1)$$

Defining: $\lambda^{-1} \equiv \lambda^{-1}(E, t, t_o) = 4 \int_{t_o}^t D(E, t) dt$

and performing the integral over $r$ first yields:

$$P_c(E, t, R, t_o) = \frac{4}{\sqrt{\pi}} \lambda^{3/2} \int_0^R r^2 \exp(-\lambda r^2) dr \quad (A-2)$$

Changing variables to

$$x = r^2$$

$$dr = \frac{1}{2} x^{-1/2} dx$$

one gets

$$P_c(E, t, R, t_o) = \frac{2}{\sqrt{\pi}} \lambda^{3/2} \int_0^{R^2} x^{1/2} \exp(-\lambda x) dx \quad (A-3)$$
The solution for the integral is given in Gradshteyn and Ryzhik (1965),

\[ P_c(E,t,R,t_o) = \frac{2}{\sqrt{\pi}} \Gamma \left( \frac{3}{2}, \lambda R^2 \right) \]  \hspace{1cm} \text{(A-4)}

where \( \gamma(a,x) \) is the incomplete gamma-function defined as:

\[ \gamma(a,x) = \int_0^x t^{a-1} e^{-t} \, dt \]

Using the recursion relation (Abramowitz and Stegun, 1972)

\[ \gamma \left( \frac{3}{2}, z^2 \right) = \frac{1}{2} \gamma \left( \frac{1}{2}, z^2 \right) - ze^{-z^2} \]

and the value:

\[ \gamma \left( \frac{1}{2}, z^2 \right) = \sqrt{\pi} \text{ erf} \, z \]

one gets

\[ P_c(E,t,R,t_o) = \text{ erf} \, z - \frac{2}{\sqrt{\pi}} ze^{-z^2} \]  \hspace{1cm} \text{(A-5)}

where

\[ z^2 = \lambda \langle E, t, t_o \rangle R^2 \] and \( \text{ erf } z \) is the error function

defined as

\[ \text{ erf } z = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} \, dt \]

Assuming that the diffusion coefficient is of the form

\[ D = D_o \, E^p \, t^q \]

one gets for \( \lambda \):
\[ \lambda^{-1} = 4 \int_{t_0}^{t} D_0 E^P t^q dt = \frac{4}{q+1} D_0 E^P t^{q+1} \left[ 1 - \left( \frac{t_0}{t} \right)^{q+1} \right] \]

so that

\[ z^2 = (q+1)R^2 \left\{ 4 D_0 E^P t^{q+1} \left[ 1 - \left( \frac{t_0}{t} \right)^{q+1} \right] \right\}^{-1} \]  \quad (A-6)
APPENDIX B

THE DIFFUSION COEFFICIENT

Following the injection, an energetic particle suffers a series of displacements in the nebula. These displacements will be treated as a series of steps of equal length, taken in a random direction with equal probability. Let \( \nu \) be the number of displacements suffered by the particle per unit time, and let \( \ell \) be the length of each step. The probability that a particle will find itself at distance between \( r \) and \( r + dr \) after a time \( t \) is then given by (Chandrasekhar, 1943)

\[
P(r,t)dr = \frac{4\pi r^2 dr}{(4\pi Dt)^{3/2}} \times \exp\left(-\frac{r^2}{4Dt}\right)
\]

(B-1)

where

\[
D = \frac{1}{2} \nu \ell^2
\]

(B-2)

This equation is quite general and the only restriction is that the size of the step must be much smaller than the dimensions of the problem, a restriction that is easily met for each of the following cases.

The Various Diffusion Coefficients

In the absence of external fields collisions with the ambient plasma impedes diffusion. \( \ell \), the length of each step, is just the collision path length and is given by

\[
\ell^2 \propto \langle V^2 \rangle \langle \tau_c^2 \rangle
\]
where $\tau_c = v_c^{-1}$ is the time step between collisions, $v_c$ the collision frequency, and $V$ the velocity of the particle. Using (B-2) one gets for the diffusion coefficient in a collision-dominated plasma:

$$D_c = \frac{1}{3} \frac{\langle v^2 \rangle}{v_c} \quad (B-3)$$

As can be seen, the diffusion coefficient decreases as the collision frequency increases.

In a magnetized plasma, collisions can cause diffusion. If $v_c \ll \omega_c$, where $\omega_c$ is the cyclotron frequency, the magnetic field strongly inhibits diffusion across its field lines, restricting the particles to gyrate, with a typical radius $a_c$ along the field. In this case collisions can move the particles a distance $a_c$ across the field lines, which are assumed to be tangled up so that a random walk appropriately describes the motion of the particles in the nebula. The diffusion coefficient in this case will be given as:

$$D_m = \frac{1}{a} a_c^2 v_c = \frac{1}{3} \frac{\langle v^2 \rangle v_c}{\omega_c^2} \quad (B-4)$$

Comparing this expression to (B-3), we see that the diffusion coefficient is reduced by a factor of $(\frac{v_c}{\omega_c})^2$.

Bohm suggested that collective processes in a plasma, like turbulences and instabilities, might lead to a different diffusion coefficient, and proposed

$$D_B = \frac{1}{48} \frac{\langle v^2 \rangle}{\omega_c} \quad (B-5)$$
The Bohm diffusion can be derived from (B-4) by maximizing the efficiency of collisions, i.e., by assuming \( \nu_c \propto \omega_c \).

**The Collision Frequency**

The collision frequency can be approximated by

\[
\nu_c = \frac{1}{\Delta E_c} \frac{(dE)}{dt}\text{coll}
\]

where \( \Delta E_c \) is the average energy loss in a collision. For a 1 \( M_\odot \) nebula expanding at \( V_{SN} = 10^9 \) cm sec\(^{-1}\), the rate of energy loss due to collisions can be written with the aid of equations (3-2) and (3-3) as

\[
\frac{(dE)}{dt}\text{coll} \propto 5 \times 10^{18} E^{-1/2} t^{-3}
\]

(B-6)

Since the average energy loss in one collision is approximately 40 eV, one gets for \( \nu_c \):

\[
\nu_c \propto 10^{23} E(\text{MeV})^{-1/2} t^{-3} \text{ sec}^{-1}
\]

(B-7)

**The Evolution of the Magnetic Field**

To evaluate the diffusion coefficient in a magnetized plasma, a model for the evolution of the magnetic field in the nebula has to be developed. Following Pacini and Salvati (1973) it was assumed that a fraction \( \alpha_B \) of the rotational energy of the pulsar is converted into magnetic energy in the nebula. The energy equation for the magnetic energy in the nebula is
\[
\frac{dE_B}{dt} = \frac{\alpha_B L_0}{(1+t/\tau_O)^2} - \frac{E_B}{t} \tag{B-8}
\]

where the last term takes the expansion losses into account.

If \( V \) is the volume of the nebula, then the average magnetic field in the nebula is related to \( E_B \) by the following equation:

\[
\frac{1}{8\pi} B^2 = \frac{E_B}{V} \tag{B-9}
\]

Integrating (B-8) and using (B-9) one gets

\[
B^2 = B_0^2 \left(\frac{R_O}{R}\right)^4 + \frac{6\alpha_B L_0 \tau_O}{R^3} = \frac{1}{x} \left(\ln(1+x) - \frac{x}{1+x}\right) \tag{B-10}
\]

where \( R \) is the radius of the nebula, and \( x \), the dimensionless parameter \( t/\tau_O \). The initial magnetic field \( B_0 \) decreases rapidly with the expansion and will therefore be neglected in the following. Inserting the values of the parameters used in this model, one gets

\[
B \text{ (gauss)} \approx 5 \frac{1}{x^2} \left[\ln(1+x) - \frac{x}{1+x}\right]^{1/2} \tag{B-11}
\]

**Cyclotron Frequency**

In practical units the cyclotron frequency for non-relativistic \( \alpha \)-particles is:

\[
\omega_c = 4.8 \times 10^3 B \text{ (gauss)} \tag{B-12}
\]
Using relations (B-7), (B-11), and (B-12), one gets for the various diffusion coefficients:

(a) \( D_c \approx D_c^0 E^{3/2} t^3 \) collision-dominated plasma

(b) \( D_m \approx D_m^0 E^{1/2} t \) strong magnetic field

(c) \( D_B \approx D_B^0 E t^2 \) Bohm diffusion

(B-13)

Expressed in units of cm\(^2\)/sec one gets for the various constants (with energies expressed in MeV and time in sec)

\[
\begin{align*}
D_c^0 & \approx 10^{-7} \\
D_m^0 & \approx 10^{-17} \\
D_B^0 & \approx 10^{-4}
\end{align*}
\]

(B-14)

In obtaining (B-13), terms slowly varying with time have been given an average constant value; therefore, the numerical values obtained in (B-14) should be regarded as an estimate only, and may be incorrect by an order of magnitude.

The transition from a collision-dominated plasma to a magnetized plasma can be done 'smoothly' if the expression for the Bohm diffusion coefficient is used instead of (B-13b). It has therefore been assumed that the Bohm coefficient is more appropriate in describing the diffusion in a magnetized plasma. For \( \alpha \)-particles with energies between \( 10^2 \) and \( 10^3 \) MeV, the condition \( v_c \ll \omega_c \) holds for \( t \approx 10^5 - 10^6 \) sec, so that the use of the Bohm diffusion coefficient is valid only after that time, prior to which (B-13a) has to be used.
The purpose of this appendix is not so much to provide an accurate value for the diffusion coefficient in the nebula, as to justify the general functional form

$$D = D_0 E^p t^q$$

In obtaining this expression for $D$, it was assumed that the nebula is homogeneous without density fluctuations and gradients, so that the diffusion coefficient is independent of spatial coordinates.
APPENDIX C

THE TRANSMISSION FUNCTION $T_\gamma$

The transmission function $T_\gamma$ is calculated for an expanding sphere with a homogeneous density distribution under the following assumptions:

(1) The distribution of sources in the nebula is isotropic, that is, the number density of sources is a function of $(r,t)$ only.

(2) The gamma rays are emitted isotropically from the source.

Consider a source located at distance $r$ from the center of a sphere, and photons emerging from opposite directions characterized by the polar angles $\theta$ and $\pi + \theta$ (see figure). Photons emitted in the direction $\theta$ travel a distance $y$ in the nebula, while photons emitted in the direction $\pi + \theta$ have to travel a distance $x$ to reach the surface. The fraction of photons emitted in those directions that reach the boundary $R$ without undergoing collisions is therefore:

$$T_\gamma(r,\Omega)d\Omega = \frac{1}{4\pi} \left[ e^{-\mu x(r,\theta)} + e^{-\mu y(r,\theta)} \right] \sin \theta d\theta d\phi$$  \hspace{1cm} (C-1)

where $\mu$ is the absorption coefficient per unit length.
Solving for \( x \) and \( y \) one gets:

\[
x = r\left[\cos\theta + \sqrt{a + \cos^2\theta}\right]
\]

\[
y = -r\left[\cos\theta - \sqrt{a + \cos^2\theta}\right]
\]

where

\[
a = \left(\frac{R}{l}\right)^2 - 1
\]

Inserting the expressions for \( x \) and \( y \), the transmission function becomes

\[
4\pi T_\gamma = \exp(-\mu rcos\theta - \mu r\sqrt{a + \cos^2\theta}) + \\
+ \exp(\mu r\cos\theta - \mu r\sqrt{a + \cos^2\theta})
\]

or

\[
T_\gamma = \frac{1}{2\pi} \exp[-\mu r \sqrt{a + \cos^2\theta}] \times \cosh(\mu R\cos\theta)
\]

(C-3)
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Ryter, C., Reeves, H., Gradsztajn, E., and Audouze, J.

Schmitt, W. F., Ayres, C. L., Marker, M., and Shen, B.S.P.
Figure 1(a) - (f)

$N(E,t)$ the spectrum of energetic particles in the nebula at different epochs of time, is shown for four different injection spectra. The injection spectra have the same spectral index, $\gamma = 2.5$, but different values $E_c$, the low-energy cutoff: $E_c = 500$ MeV (solid line); $E_c = 100$ MeV (dashed line); $E_c = 50$ MeV (dashed-dotted line); and $E_c = 10$ MeV (dotted line).
Model A

\( \nu = 2.5 \)

(a) \( t = 10^6 \text{ sec} \)

(b) \( t = 5 \times 10^6 \text{ sec} \)
(c) \( t = 10^7 \text{ sec} \)

Model A

\( \nu = 2.5 \)

(d) \( t = 5 \times 10^7 \text{ sec} \)
Figure 2(a) - (d)

Same as Figure 1, for injection spectra with $\gamma = 8.5$
Figure 3
Figure 3

$F(E,r,t)$ the probability density of finding a particle with energies between $E$ and $E + dE$ at a distance between $r$ and $r + dr$ at time $t = \tau_0$, is shown for three different values of the diffusion coefficient $D$. Only for a diffusion coefficient of $10^{26}$ cm$^2$ sec$^{-1}$ does a particle have a significant probability ($\sim 25\%$) to escape the nebula (hatched portion of the curve).
(a) $D=10^{24}\text{ cm}^2\text{ sec}^{-1}$
(b) $D=10^{25}\text{ cm}^2\text{ sec}^{-1}$
(c) $D=10^{26}\text{ cm}^2\text{ sec}^{-1}$
Figure 4
Figure 4

Reaction cross section for $^4\text{He}(\alpha,p)(^7\text{Li}_{gs} + ^7\text{Li}^*)$ (open circles) and for $^4\text{He}(\alpha,n)(^7\text{Be}_{gs} + ^7\text{Be}^*)$ (black circles). (From King et al., 1975)
Figure 5
Figure 5

Reaction cross section for $^4\text{He}(\alpha, x)^6\text{Li}$. The dashed curves represent estimates only (From Mitler, 1972)
Figure 6
Figure 6

Production yield of various nuclei by $\alpha + ^{12}\text{C}$ reactions. The contribution of $^{10}\text{B}$ and $^{11}\text{B}$ to the production curves of mass-10 and mass-11 was not measured.

(From Reeves, 1974)
Figure 7
Figure 7

The differential cross section for the reaction $^{12}\text{C}(\alpha,\alpha')^{12}\text{C}^* (4.44 \text{ MeV}) (\gamma)^{12}\text{C}$. (From Meneguzzi and Reeves, 1975)
$^{12}\text{C}(\alpha, \alpha')^{12}\text{C}^*(\gamma)^{12}\text{C}$

$E_\gamma = 4.44 \text{ MeV}$

- ZOBEL 68 ($\gamma$)
- MITCHELL 64 ($\gamma$)
- YAVIN
- MIKUMO 61
- BURDZIK 72
- SPECHT 71
- BARON 71
Figure 8
Figure 8(a) - (c)

The production yield of the light elements by various injection spectra. The mass and composition of the nebula is given by Model A (Table 2). The expansion velocity is $5 \times 10^8 \text{ cm sec}^{-1}$ and the parameters of the pulsar are given in Chapter 1.
(b) Model A
\[ \psi = 5.5 \]
Figure 9(a) - (c)

Same as Figure 8, for a nebula whose mass and composition are given by Model B (Table 1).
(b) Model B
\( \nu = 5.5 \)
\[ \log_{10} N_L \]

(c) Model B
\[ \nu = 8.5 \]
Figure 10(a) - (b)

(a) The contribution of the $\alpha\alpha$ and $\alpha + ^{12}\text{C}$ reactions to the production of $^7\text{Li}$ for spectra with $\gamma = 2.5$ and different values of $E_c$.

(b) Same as (a) for spectra with $\gamma = 8.5$. 
(a) Model A
\( \nu = 2.5 \)

(1) \( N(7\text{Li}) \) from \( \alpha \alpha \)
(2) \( N(7\text{Li}) \) from \( \alpha + 12\text{C} \)

\( E_c \) (MeV)

(b) Model A
\( \nu = 8.5 \)

(1)
(2)
Figure 11(a) - (b)

(a) The \((^7\text{Li} + ^7\text{Be})/^6\text{Li}\) ratio achieved by various injection spectra for Model A and Model B.

(b) The \((^7\text{Li} + ^7\text{Be})/^6\text{Li}\) ratio achieved by various injection spectra, where the expansion velocity of Model A was taken to be \(1.5 \times 10^9 \text{ cm sec}^{-1}\).
Figure 12(a) – (b)

(a) The overproduction of lithium with respect to other light elements by various injection spectra, for Model A.

(b) The same as (a), but for Model A, expanding at a higher velocity of $1.5 \times 10^9$ cm sec$^{-1}$. 
Figure 13
Figure 13(a) - (c)

The production yield of $^7\text{Li}$ by different injection spectra for the following models:

(a) Model A with standard parameters.

(b) Model A with an expansion velocity of $1.5 \times 10^3 \text{ cm sec}^{-1}$.

(c) Model A, in which the number of He, C, and O nuclei was kept the same, but the ejected mass was increased to $20 M_\odot$. 
$\log_{10}N(^{7}\text{Li})$

$E_e$(MeV)

$\gamma = 8.5$
Figure 14
Figure 14

The emission rate (read on the left ordinate scale) of various gamma-ray lines is shown as a function of time. The resulting fluxes (read on the right ordinate scale) assuming a source at a distance of \( \sim 2 \text{ kpc} \) are compared with observed features from the galactic center. The amount of self absorption in the nebula due to Compton scattering is very small at the stages of the expansion shown in the figure, and is therefore not shown.
$r_\gamma$ (photons sec$^{-1}$)

$F_\gamma$ (photons cm$^{-2}$ sec$^{-1}$)

(D = 2 kpc)

10$^{-3}$

10$^{-4}$

10$^{-5}$

$10^{40}$

$10^{41}$

478 keV

431 keV

4.44 MeV

Model A

$V_{SN} = 1.5 \times 10^9$ cm s$^{-1}$

$\nu = 8.5$

Theoretical fluxes predicted in this work

○ Obs. 476 keV feature
  (Johnson & Haymes, 1973)

● Obs. 0.5 MeV feature
  (Haymes et al., 1975)

△ Obs. 4.6 MeV feature
  (Haymes et al., 1975)
Figure 15
Figure 15

$E_{\text{CR}}$, the total energy in the form of cosmic-ray $\alpha$-particles in the nebula is compared to the total energy output of the pulsar, for an injection spectrum characterized by $E_C = 50$ MeV and $\gamma = 8.5$. The expansion velocity of the nebula was taken to be $V_{\text{SN}} = 1.5 \times 10^4$ kms$^{-1}$. 
TABLES
TABLE 1

COSMIC ABUNDANCES OF LIGHT ELEMENTS

<table>
<thead>
<tr>
<th>Observed Abundances*</th>
<th>(^{6}\text{Li}/\text{H})</th>
<th>(^{7}\text{Li}/\text{H})</th>
<th>\text{Be}/\text{H}</th>
<th>\text{B}/\text{H}</th>
<th>(^{7}\text{Li}/^{6}\text{Li})</th>
<th>(^{11}\text{B}/^{10}\text{B})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Boesgaard, 1976)†</td>
<td>7.6((-11))</td>
<td>9.2((-10))</td>
<td>1.3((-11))</td>
<td>1.0((-10))</td>
<td>12.1</td>
<td>4.0</td>
</tr>
<tr>
<td>(Cameron, 1973)‡‡</td>
<td>1.2((-10))</td>
<td>1.4((-9))</td>
<td>2.0((-11))</td>
<td>1.1((-8))</td>
<td>12.5</td>
<td>4.1</td>
</tr>
<tr>
<td>Abundances predicted by GCR spallation**</td>
<td>8((-11))</td>
<td>1.2((-10))</td>
<td>2.0((-11))</td>
<td>3.0((-10))</td>
<td>1.5</td>
<td>2.4</td>
</tr>
</tbody>
</table>

* Numbers in parentheses indicate powers of ten.
** Meneguzzi, Audouze and Reeves (1971).
† Based on observed stellar abundances.
‡‡ Based on abundances in carbonaceous chondrites.
\begin{table}
\centering
\caption{MODELS USED IN THIS WORK}
\begin{tabular}{lcc}
\hline
Composition & \textbf{MODEL A} & \textbf{MODEL B} \\
 & $M_{\text{CORE}} = 4 \ M_\odot$ & $M_{\text{CORE}} = 8 \ M_\odot$ \\
 & MASS (\(M_\odot\)) & MASS (\(M_\odot\)) \\
\hline
$^4\text{He}$ & 2.04 & 3.15 \\
C & 0.253 & 0.546 \\
O & 0.18 & 1.57 \\
Ne & 0.042 & 0.743 \\
Mg & 0.074 & 0.22 \\
TOTAL* & 2.59 & 6.23 \\
Si & 1.1 & 0.376 \\
Ni & 0.32 & 1.36 \\
TOTAL** & 1.42 & 1.736 \\
\hline
\end{tabular}
\end{table}

*Assumed ejected.
**Assumed locked up in remnant star.
# TABLE 3

REACTION THRESHOLDS FOR α-REACTIONS (MeV)

<table>
<thead>
<tr>
<th>Target Nuclei</th>
<th>$^6\text{Li}$</th>
<th>$^7\text{Li}$</th>
<th>$^7\text{Be}$</th>
<th>$^9\text{Be}$</th>
<th>$^{10}\text{Be}$</th>
<th>$^{10}\text{B}$</th>
<th>$^{11}\text{B}$</th>
<th>$^{11}\text{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4\text{He}$</td>
<td>44.8</td>
<td>34.7</td>
<td>38.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>31.6</td>
<td>32.8</td>
<td>32.9</td>
<td>32.9</td>
<td>36.3</td>
<td>31.6</td>
<td>21.3</td>
<td>25.0</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>24.1</td>
<td>25.5</td>
<td>26.5</td>
<td>30.4</td>
<td>38.3</td>
<td>33.0</td>
<td>28.9</td>
<td>30.4</td>
</tr>
</tbody>
</table>