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LIGHTNING CHANNEL STRUCTURE INSIDE AN ARIZONA THUNDERSTORM

by

Thomas L. Teer

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

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1. INTRODUCTION

1.1 The Typical Thunderstorm

Individual thunderstorms do not always fit a prescribed model. Theoretical models cannot predict winds and temperatures inside thunderstorms, and all experimentally determined data fluctuate widely. However, most convective thunderstorms do possess a morphology that is fairly consistent, and the Thunderstorm Project of 1946 and 1947 gathered a large amount of data from which storm morphology was derived (Byers and Braham, 1949). Byers' report is still the major reference on thunderstorm dynamics.

Thunderstorms consist of several regions of convection called cells, each of which is characterized by vertical winds and temperatures substantially different than the ambient air. Before one cell has gone through its entire life cycle (about 1 hour) others may form. If successive cells form close together, it becomes difficult to separate their effects on the ground. This resulting decay and regeneration of cells leads to a thunderstorm of a long lifetime (2 - 3 hours) with relatively constant electrical activity.

Following Byers and Braham (1949), the life cycle of a thunderstorm cell follows three stages: (1) the cumulus stage, (2) the mature stage and (3) the dissipating stage.
1.1.1 Cumulus Stage

The Cumulus stage features the existence of an updraft region in and below a cumulus cloud. The motion is a function of time and location within the cloud. Figure 12 is a vertical cross section of a thunderstorm in the cumulus stage. Temperatures in the cloud are generally higher than the ambient temperatures. Their typical magnitudes are seen by following the dashed lines at each level in Figure 12. The winds near the surface form a weak horizontal convergence area under the updraft section, and at all levels entrainment begins, i.e., air flows into the cell from all sides to conserve mass.

1.1.2 Mature Stage

The Mature stage, Figure 13, is defined by the onset of rainfall and is the most violent stage of the thunderstorm. As more and more water vapor cools and changes phase, the mass loading of water reaches the point where it cannot be supported by the updraft and rain begins.

The onset of rain triggers the onset of the downdraft which partially fills the volume previously filled by the updraft, and a true convection cell is generated. The wind vectors on Figure 12 and Figure 13 are schematic in that they do not represent true uniform wind fields; the draft regions are actually very gusty with an average vertical mass transport.

The downdraft is converted to horizontal motion at the surface, with the surface outflow marked by a rather large horizontal divergence. This change of wind from updraft to downdraft is accompanied by a sharp
decrease of temperature at the surface. The warmest regions in the cloud are still those of maximum updraft and the coldest regions occur where the downdraft is strongest.

1.1.3 Dissipating Stage

The mature stage usually lasts from 15 to 30 minutes in which the downdrafts take over all regions of the cell. During the dissipating stage, the downdraft decreases in magnitude until little or no vertical air motion persists. Heavy rains diminishes to showers and finally cease.

The temperatures initially quickly decrease within the entire cell to below ambient conditions as the downdraft spreads over the entire cell volume. As downdraft speeds diminish, the cell temperature approaches the ambient temperature. Figure 14 is a cross section of a thunderstorm in the dissipating stage when the vertical winds are still strong enough to measure. The duration of this stage is about 30 minutes.

1.1.4 Thunderstorm Electrification

The thunderstorm derives its name from the fact that thunder is heard and lightning is seen. It is obvious that out of the turmoil of wind, water, and temperature fluctuations, strong electric fields are generated within the storm which when large enough cause lightning to occur. Yet the exact nature of the processes involved in the generation of these electric fields and the emergence of charged regions in the thunderstorm is not yet fully understood.
It is generally believed that the upper part of the thunderstorm cloud is predominately positively charged and that the lower part is negatively charged with a small positively charged region near the cloud base. This positive dipole view was first postulated by Wilson (1916), (1920) and has been essentially verified by others up to the present time, e.g., Ogawa and Broor (1969).

It is commonly accepted that from 20 to 50 coulombs of charge are separated prior to lightning flashes. No direct measurements and only a little physical insight is available to explain the distribution of this charge or its generation, separation and destruction mechanisms (Taylor, 1972).

However, observations are available that link the electric charge dynamics to the storm's convection and precipitation processes, c.f., (Coroniti, Chaps. III and IV, 1965). The freezing level, -10°C, seems to be a fundamental altitude for separation-of-charge mechanisms (Taylor, 1972).

Lightning usually begins to occur in a thunderstorm when it enters the mature stage and usually lasts until fairly late in the dissipating stage when the cloud environment is settling down and charge generation and separation processes cease.
1.2 Section Synopsis

The electrical structure of a thunderstorm is complex and not easily experimentally studied. Traditionally, ground-based electric field and field change measurements are made in the vicinity of thunderstorms and attempts are made to define physically regions of electric charge and mechanisms responsible for the occurrence of lightning.

While these techniques have been successful in determining the gross nature of the electric field structure of thunderstorms, the vertically-oriented, positive-dipole model first suggested by Wilson (1920) cannot explain thunderstorm electrification in fine detail. Measurements derived from single electric field monitoring stations do not usually agree from experiment to experiment, and available theories, invoking the vertical dipole concept, do not generally account for these differences.

We feel that, while necessary, electric field measurements are not sufficient to successfully determine the fine structure of the origin and development of thunderstorm electricity. We have developed a technique which allows us to reconstruct the lightning channel source geometry both inside and outside the thunderstorm cell. Knowing the lightning channel structure allows us to apply electric field measurements to known source regions. With this closed set of measurements, we may experimentally probe the thunderstorm cell and derive unambiguous results that can be accurately compared to the macrophysical structure of the cloud. The inability heretofore to be able to define accurately the location of charge centers within the cloud, the source regions of a sequence of lightning flashes, the time required for the cloud to resupply charge
to specific regions, and the relationship between lightning source regions such as freezing levels, wind drafts, and precipitation zones has impeded progress toward understanding thundercloud electrification.

In this paper we lay the foundation for future work according to the above thesis. We first go through the thunder channel mapping technique to justify it theoretically (Section 2) and establish it as a viable technique. We explain how this system operates in the field (Section 3) and present the results of its use in a comprehensive study of the lightning channel structure inside a thunderstorm recorded on August 3, 1970 in Tucson, Arizona (Sections 4 and 5).

We present a discussion of the statistics of our acoustic reconstructions in Sections 6 and 7 and postulate reasons for the existence of several characteristic lightning structures observed. We also tie our observations to present theory and the few existing intra-cloud observations in Section 7.

While the lightning structure of thunderstorms is of profound interest, our thesis is at all times to elucidate the channel mapping technique. Our problem is very similar to the geophysical problem of acoustically mapping subsurface structure, except that we acoustically map the structure of the acoustic sources, the lightning channel.

We believe, as do Ogawa and Brook (1969), that the time has arrived when we must cast thunderstorm electricity studies against observable thunderstorm dynamics. There is little insight into the electrical structure of storms to be gained without relating charge structure to storm variables such as wind drafts, temperature, and precipitation distributions. As a result of these beliefs, we chose to
study the evolution of the dissipating stage of a nature thunderstorm. We have mapped every lightning flash (17 CG events and 20 IC events) that occurred in the 30 minute time interval preceding the end of all observed lightning flashes. Results of this study are also presented in Sections 6 and 7.

Our results establish the fact that both cloud-to-ground lightning flashes and intracloud flashes have extensive horizontal structure inside the cloud. This horizontal structure might be modelled as being contained in a rather thick, flat ellipsoid whose long axis is parallel to the earth's surface. A typical ratio of long horizontal axis to short horizontal axis to vertical axis is 3:2:1.

The most striking feature of the IC events and the intracloud portions of the CG events is their marked tendency to align themselves along the same direction, in a line running about 16° east of north. This alignment is normal to the direction of storm motion and parallel to the earth's geomagnetic field.

The cloud-to-ground channels all show a tendency to align themselves tilted from bottom to top in the direction of storm motion. Every event but one aligned itself in this fashion.

The IC events fell into two natural groups, those occurring just above the cloud's freezing level (-10°C) at 4.5 km and those occurring late in the dissipating stage at 7 km. The cloud-to-ground horizontal structure also seemed to be associated with the 0°C to -10°C levels in the storm.

Both CG events and IC events occurred 30% of the time with a dual nature to their structure. Each event had two layers of horizontal
structure separated by either a poor vertical channel connection or no connection at all (Section 6). A charge distribution model and an induction effect are postulated as possible source mechanisms for these events in Section 7.

Limitations of the technique are discussed in Section 7, and a complete discussion of errors is found in Section 8. The primary limitations of thunder sound ranging are those introduced by a lack of knowledge of the wind and temperature structure aloft and by range limitations due to atmospheric absorption.

Meteorological parameters in this study were derived from rawinsonde measurements made at Tucson 2 minutes after recording ceased. Deviations from this atmosphere produce no propagation effects that deter from quoted results.

All reported lengths should be interpreted as minimum lengths owing to atmospheric losses, but length ratios should be accurate. We mention (Section 8) in passing that the field deployment of multiple arrays several kilometers apart will provide us with less range limitation in future operations.

Appendices I and II contain the acoustic reconstructions derived for this study. We suggest that before reading further, you turn now to these sections and familiarize yourself with them, for, to use the one cliche' in this thesis, seeing is believing.
2. ACOUSTIC PROPAGATION PROBLEM

The acoustic reconstruction of lightning channels is a classical acoustic propagation problem. Our success in lightning channel mapping depends solely on our understanding of lightning as an acoustic radiator, the propagation of thunder through the atmosphere and the reception of the thunder at an array of microphones.

While our understanding of lightning as an acoustic radiator is incomplete, we are able to show that a simple approach is sufficiently correct.

Wave propagation theory is well understood and may be used exactly to predict the behavior of a thunder pulse in propagating through the atmosphere.

The array reception problem is only a special case of wave propagation theory and need be discussed only as it pertains to our particular location and array geometry.

In the next three sections we discuss these problems from the point of view of their application to lightning channel mapping. Experimental technique is resolved with popular theories of acoustic radiation and propagation.

2.1 Lightning as an Acoustic Source

A commonly accepted number for the energy input per unit length to a lightning stroke is $10^5$ joules/m. This energy is dissipated through (1) ionization, dissociation, and excitation of the lightning channel particles, (2) kinetic energy of the channel particles, (3) electromagnetic
radiation and (4) energy of expansion of the channel (Uman, 1969). Uman (1969) calculates that the energy input to atomic processes and kinetic energy is small, and observes through the work of Krider et al. (1968) and Horner (1964) that only a fraction of the available energy is lost through radiation processes.

Most workers believe that the available energy goes into the creation of a strong mechanical shock with overpressure on the order of 100 atm. (Uman, 1969). The shock reaches this pressure in 5 - 10 μ sec, leaving behind a hot plasma of T ≥ 30,000°k. The expanding strong shock decays rapidly over a distance of several meters to a weak shock with overpressures 0.1 - 10 atm. and subsequently this weak shock decays to and propagates as an acoustic wave with an overpressure at a range of 1 km of about 0.1 mb. (Few et al., 1970).

Details of the weak shock decay, while only qualitatively understood, have been experimentally studied (Uman et al., 1970; Few, 1969). These results seem consistent with the view that the weak shock decays to an acoustic wave over a range of tens of meters. We proceed with the hypothesis that the lightning channel may be modelled as a series of short (~10 m) cylindrical line sources. These line sources are oriented in a highly tortuous manner (Hill, 1968; Orville, 1969; Few 1969). Mesotortuosity, of a scale size < 50 meters (Few, 1969; Uman, 1969), and macrotortuosity, of a scale size > 50 meters, are responsible for the characteristic sounds of thunder. If the lightning channel were modelled as a semi-infinite line source, the shock wave would decay and propagate following cylindrical shock theory, yielding overpressures at a range of 1 km in excess of those observed (Few et al., 1970).
In any case, whether the shock is strong or weak, we are justified in our hypothesis that the shock decays to acoustic speeds after traveling a few meters from the discharge channel, and beyond several hundred meters the waves may be treated as having originated from acoustic point sources. Each channel irregularity acts as a sound source and, if the irregularities are mesotortuous, the sound will propagate beyond radii of several meters as spherically diverging waves. There are many such irregularities in a lightning channel, and the acoustic wave many meters from the channel represents the sum of the spherically diverging waves from each short cylindrical line source. It is obvious that the sound of thunder generated by means of this model will be strongly influenced by the lightning channel spatial geometry.

In Figure 1, a characteristic thunder record, we see the clap representing thunder onset followed by the rumbles and claps of shorter duration arriving from the complex spatial structure of the source. Nature has been kind to us, for it is this highly variable, non-stationary structure of the thunder signature that allows us to reconstruct the lightning channel geometry on the macrotortuous scale.

2.2 Acoustic Wave Propagation Applied to Thunder

Although a non-linear theory of sound propagation is needed in the weak shock-to-acoustic wave region, we believe that over the largest part of the propagation path, the linearized approach is sufficient. A brief review of this theory provides the framework in which we may discuss our technique and pertinent approximations.
Acoustic energy propagates through the atmosphere as a longitudinal wave whose pressure variations are small with respect to the ambient pressure. If the oscillation period is short compared to the time it takes for heat to be exchanged with the surrounding air, the process is adiabatic and the sound speed is a function of temperature. Assuming the equation of state for an ideal gas and the effects of the earth's gravity field are negligible,

\[ \alpha^2 = \frac{\gamma kT}{M} \]  

(1)

where \( \gamma \) = dry adiabatic gas constant = 1.402
\( k \) = Boltzmann constant = 1.38 x 10^{-23} \text{ J} \text{K}^{-1}
\( M \) = mean harmonic mass in kg
\( T \) = Temperature in °K

For dry air at 20°C, \( c = 344 \text{ m/sec} \).

Waves traveling in a homogenous, continuous medium are not refracted or scattered. However if variations exist in the refractive index of the medium, refraction and scattering will occur.

The most pronounced variation in the refractive index is the variation with altitude, a natural consequence of the constraints introduced by the earth's gravitational field. Consequently, we describe only the propagation of waves with physical parameters which are functions of altitude, the Z coordinate, along.

The principal deviation from this concept of horizontal stratification occurs when we introduce winds into the problem. Normally winds may be considered to have no vertical component, but in the disturbed atmosphere
typical of that inside thunderstorms, vertical winds may be quite strong. We confine this study to the dissipating stage of a thunderstorm during which Byers and Braham (1949) observe that downdraft velocities typically decrease to zero from 1 - 2 m/sec. We show in Section 8 that errors introduced in the channel coordinates by our neglect of vertical winds are on the same order as those arising from our lack of knowledge of the X and Y wind components. Therefore in the remainder of Section 2, we confine our discussion to propagation in horizontally stratified media.

2.2.1 Dispersion

A distinction must be made between the two concepts of phase velocity and propagation velocity (or ray velocity). In an isotropic, homogeneous medium, the adiabatic sound velocity, \( c \), is equal to the phase velocity, \( c \), and also equal to the group velocity \( v \) (or propagation velocity \( v \)). The wave vector \( k \) is parallel to \( c \) and \( v \). The frequency of the wave is given by \( \omega = Ck \) where \( C \) is not a function of \( \omega \).

In a density stratified or windy medium where the propagation is anisotropic, the phase velocity is a function of \( \omega \) and the directions and magnitudes of the phase velocity and the group velocity are different. The medium is said to be dispersive.

In the case of a density stratified atmosphere, neglecting gravity,

\[
\omega^2 = \omega_o^2 + C^2 k^2 \tag{2}
\]

\[
v = \frac{\omega}{k} = \frac{C}{\sqrt{1 - \omega_o^2/\omega^2}} \tag{3}
\]

\[
U = \frac{\partial \omega}{\partial k} = \frac{C}{\sqrt{1 - \omega_o^2/\omega^2}} \tag{4}
\]
where \( v \) = phase velocity

\( U \) = group velocity

\( C \) = adiabatic sound speed

and \( \omega_0 = \pi C/H_0 \), the low frequency cut-off for acoustic waves.

\( H_0 \) = scale height of the atmosphere

This dispersive effect is negligible for the lower atmosphere (< 100 km). Typical values of \( C \) (300 m/sec) and \( \omega_0 \) (2 \( \times \) 10^{-2} rad sec^{-1}) yield dispersion for one minute wave periods on the order of 2%. Significant dispersive effects exist only for frequencies \( \leq 0.02 \) Hz (Tolstoy, 1973), and Few (1969) has shown that typical thunder frequencies are two to three orders of magnitude higher than \( \omega_0 \).

\[ 2.2.2 \quad \text{Theory of Curved Paths} \]

The propagation paths may be approximated by considering the medium to be divided into a large number of layers bounded by plane interfaces parallel to the earth's surface. Each layer is of small thickness and in each layer the propagation velocity (or ray velocity) and velocity gradients are to be considered constant and equal to average values between this maxima and minima within that layer.

A process of continuous refraction takes places at each interface, and in the limit as the number of layers approaches infinity while the maximum layer thickness approaches zero, the ray path is described as an integral:

\[ \gamma(t) = \int v(t') dt' \]  

(5)
where \( \mathbf{r}(t) \) is the position vector of an element of the wave front, \( V(t) \) is the propagation velocity, at least piecewise continuous and \( t \) is time.

The significant effect of stratification is to continuously change the direction of \( \mathbf{k} \), the wave vector normal to the wavefront, always obeying Snell's law of refraction.

\[
ksin\theta = \text{constant} \quad (6)
\]

This changing wave vector causes the ray path to be curved as well, the particular curvature being calculated from equation (5).

2.2.3 Wind

Snell's law, equation (6), and the dispersion relation, equation (2), are valid only for sound waves propagated in a medium not moving with respect to an observer. In the coordinate system defined by the microphone array, which is fixed with respect to a moving homogeneous medium the frequency of a monochromatic wave becomes

\[
\omega = Ck + \mathbf{V}_w \cdot \mathbf{k} \quad (7)
\]

The group velocity, \( \mathbf{V}_g \), or the velocity with which a point on the wavefront propagates becomes

\[
V = \frac{\partial \omega}{\partial \mathbf{k}} = \frac{Ck}{k} + \mathbf{V}_w \quad (8)
\]

and the phase velocity, \( \mathbf{V}_p \), becomes
\[ \sim = \frac{\omega}{k} = Ck/k + (\omega_w \cdot \frac{k}{k}) \frac{k}{k} \]  \hspace{1cm} (9)

where \(\omega_w\) is the wind velocity and \(\k\) is the wave vector, always normal to the wavefront (Landau and Lifshitz, 1959).

The phase speed in a moving medium is simply the sum of the adiabatic sound speed in the direction of \(k\) and the component of wind parallel to \(k\). The normal component of wind serves only to alter the direction in which energy propagates, and, in general, the group velocity vector is not normal to the wavefront.

If we define \(\hat{n}\) as a unit vector normal to the wavefront, \(\hat{n} = k/k\), then

\[ \sim = CN + \omega_w \]  \hspace{1cm} (10)

\[ \sim = (CN + \omega_w \cdot \hat{n}) \hat{n} \]  \hspace{1cm} (11)

2.2.4 Ray Tracing

The form of Snell's law in equation (6) must be modified when winds are present in the medium. This modification may be seen in figure (2) where we visualize an acoustic interface between two fluid media, each having constant temperature and wind parameters. The necessary boundary condition that must be maintained is that the displacement of the two fluids normal to the interface must be continuous. This condition may only be satisfied if the X- and Y- components of the wave number \(k\) are maintained across the interface (figure 2).
The conservation of $k_x$ across the interface may be expressed from equation (9) as

$$k_x/w = \cos\alpha_i/c_i + \nabla_{w_i} \cdot n_i = \cos\alpha_T/c_T + \nabla_{w_T} \cdot n_T \quad (12)$$

Equation (12) may be solved for $\cos\alpha_T$, leading to a modified form of Snell's law for refraction in a windy atmosphere (no vertical winds):

$$\cos\alpha_T = \frac{c_T}{c_i/cos\alpha_i - (\nabla_{w_T} - \nabla_{w_i})} \quad (13)$$

where

$$\nabla_{w_i} = v_{w_i} \cos\alpha_i$$

$$\nabla_{w_T} = v_{w_T} \cos\alpha_T$$

The following development for the tracing of ray paths through a horizontally stratified atmosphere is a condensation of the treatment by A. A. Few (1968). A modified form of his program RATRAC, is currently being used to propagate thunder from the lightning channel to the microphone array.

By using rectangular coordinates in which $X, Y, Z$ are directed east, north, and up, the path of a ray is derived in component form as

$$\frac{dX}{dz} = \frac{cos\alpha}{cos\gamma} + \frac{u}{c} \frac{1}{cos\gamma} \quad (14)$$

$$\frac{dY}{dz} = \frac{cos\beta}{cos\gamma} + \frac{v}{c} \frac{1}{cos\gamma} \quad (15)$$
\[
(\frac{dS}{dz})^2 = 1 + (\frac{dx}{dz})^2 + (\frac{dy}{dz})^2
\]

where \((a, b, \gamma)\) are the direction cosines of \(n = \frac{k}{k}\); \(dS\) is an element of arc length along the ray; and \((u, v)\) are the \((X, Y)\) components of the wind. \(\cos\beta\) is determined exactly as is \(\cos\alpha\) and

\[
\cos^2 \gamma = 1 - \cos^2 \alpha - \cos^2 \beta
\]

Constant gradients within layers are introduced into equation (13):

\[
\cos \alpha_T = \cos \alpha_i \left[ \frac{1 + \Gamma_{cT} (Z_i - Z_T)}{1 - \Gamma_{wT} (Z_i - Z_T)} \right]
\]

where

\[
\Gamma_{cT} = \frac{1}{C_i} \frac{\partial C_T}{\partial z} = \text{constant}
\]

\[
\Gamma_{wT} = \frac{\cos \alpha_i}{C_i} \frac{\partial u_T}{\partial z} = \text{constant}
\]

A similar argument holds for \(\cos \alpha_T\). The acoustic ray path is now completely determined through equations (14 - 20).

2.2.5 Absorption

Although absolute amplitude and frequency measurements of thunder were not made during the 1970 Tucson experiment, the effects of absorption are certainly evident. The attenuation of acoustic waves as they propagate a distance \(d\) through the atmosphere may, to first order, be given by
\[ I = I_0 \exp\left(-Kl\right) \]  
(21)

where \( I \) and \( I_0 \) are sound intensities and \( K \) is the attenuation coefficient. \( K \) is a sum of three terms, \( K_c, K_s, \) and \( K_m \), or classical, molecular, and scattering attenuation.

Classical attenuation, \( K_c \), arises as a consequence of (1) wave energy absorption by the air owing to a finite air viscosity, (2) radiation, and (3) heat conduction from the wave peaks (Hall, 1972). Beranek (1954) gives \( K_c \) as

\[ K_c = 4.24 \nu^2 \times 10^{-11} \text{ m}^{-1} \]  
(22)

where \( \nu \) is the wave frequency. This type of attenuation is easily neglected in propagating thunder frequencies over ranges of a few kilometers.

Molecular attenuation in the atmosphere is a strong function of the amount of water vapor present. Energy is absorbed from the acoustic wave by transferring energy to water vapor molecules from the collisionally excited oxygen molecules. This energy is subsequently radiated away in the IR (Hall, 1972). This process may be described as a relaxation process with relaxation time \( T \). When molecular dissipation is introduced into a homogeneous atmosphere with \( \omega = c k \), the phase velocity (Tolstoy, 1973) for \( \omega T \ll 1 \), becomes

\[ \nu = c(1 + \omega^2 T^2 + \ldots) \]  
(23)
The medium is dispersive although, in cases of interest, in the lower atmosphere the correction is negligible. Even though the dispersive effect may be neglected, range damping might still be important. Tolstoy (1973) gives a first order calculation for the molecular attenuation coefficient as

$$K_m \approx \frac{w}{2c} \left( \frac{wT}{1 + w^2T^2} \right) \text{ m}^{-1}$$

(24)

We see that while $wT$ is a second order effect on the phase velocity, it is a first order effect on $K_m$. To study the effect of molecular dissipation on wave propagation, we examine Figure 3, a plot of molecular attenuation in db/100m versus temperature at 10% relative humidity (R.H.) for various frequencies (Harris, 1966). The value of $K_m$, the molecular attenuation coefficient in m$^{-1}$, is found by dividing the abscissa by 434.3.

A monochromatic wave of 125 Hz will be attenuated by 1 db after traveling 1 km in a medium at 5°C with 10% R.H., while a 1 kHz wave will be attenuated by 20 db after traveling 1 km in this same medium.

10% R.H. was chosen to illustrate the maximum absorption case. Higher relative humidities cause less absorption, about 1 order of magnitude less at 70% R.H., although wave frequencies near 1000 Hz will be damped ten times more strongly than wave frequencies near 100 Hz.

Molecular attenuation plays the most significant role in the propagation of thunder frequencies over ranges of a few kilometers.

Little is known about $K_s$, the scattering attenuation coefficient, other than to say that if an acoustic beam shines through a turbulent or
dusty medium, energy will certainly be scattered out of the beam. Rayleigh scattering and scattering from turbulent eddies both produce attenuation coefficients proportional to $u^2$ (Tolstoy, 1973), although the proportionality constants may only be derived experimentally. Some tests (Peran et al., 1970; Dneprovskaya, 1963) indicate that $K_s$ may be as important as molecular attenuation. More work on the importance of scattering to acoustic propagation is indicated.

2.3 Array Reception

Ideally the acoustic wave from a distant source may be treated as a plane wave at the array location as long as $d/R << 1$, where $d$ is the characteristic array dimension and $R$ is the range to the source. The wave front character should appear the same at each microphone, but displaced in time. In actual practice, random noise generated by man-made vibrations, wind and turbulent eddies add to the thunder being recorded. This random component reduces the coherence of the wave over the array, and in some cases, renders the data unusable.

Most of the above-mentioned noise sources are low-frequency phenomena, having peak frequencies in the range (0 - 5 Hz). Effects of noise are reduced with time-domain digital filtering techniques.

A common method used to increase signal coherence is to reduce the distance between microphones, although one must make a trade-off between time-delay resolution and signal coherence. The microphones used during the 1970 Tucson experiment were separated by 100 meters, although the signal coherence was poorer than in earlier data taken with 30 meter arrays. The array configuration is presented in Figure 4. It is shaped
like an equilateral triangle, with microphones 2, \( M_2 \), and 3, \( M_3 \), lying in a north-south line and microphone 1, \( M_1 \), lying to the east of \( M_2 \) and \( M_3 \).

We measure the time delays of the wave in traveling from \( M_i \) to \( M_j \) and use these delays to calculate the arrival angles, azimuth, \( \delta \), and declination, \( \theta \), of the phase velocity vector as follows.

From Figure 4 we see that

\[
\tau_{12} + \tau_{23} = \tau_{13}
\]  

(25)

where \( \tau_{ij} \) are position vectors from \( M_i \) to \( M_j \). \( \cos\alpha_{ij} \) are the direction cosines of \( \tau_{ij} \) with respect to \( \vec{v} \). They are given by

\[
\cos\alpha_{ij} = |\vec{v}|(\tau_{ij}/D)
\]  

(26)

where \( D \) is the distance from \( M_i \) to \( M_j \) and \( \tau_{ij} \) are the time lags of the acoustic wave in traveling from \( M_i \) to \( M_j \).

The azimuth angle, \( \delta \), (Figure 4) of all incident wave vectors is defined with respect to true north and is read as a compass bearing. The declination angle, \( \theta \), is the polar angle, referenced to the local zenith.

The phase velocity may be resolved into horizontal and vertical components;

\[
\vec{v} = v_h \hat{h} + v_z \hat{z}
\]  

(27)

and, writing the components of \( v_h \) along \( \tau_{ij} \):

\[
v_{ij} = v_h \cos(\text{angle between } \hat{h} \text{ and } \tau_{ij})
\]  

(28)
Combining equations (25), (26), (27), and (28) with some manipulation, we derive the arrival angles of the incident wave vector:

\[ \theta = \sin^{-1} \left[ \sqrt{\frac{N}{D}} \right] \]  
(29a)

\[ \delta = 270 - \tan^{-1} \left[ \sqrt{3} \frac{\tau_{13} - \tau_{12}}{\tau_{13} + \tau_{12}} \right] \]  
(30a)

where

\[ N = (\tau_{13} - \tau_{12})^2 + \frac{1}{3} (\tau_{13} + \tau_{12})^2 \]  
(31)

The propagation of a wave from source to observer in a windy atmosphere is equivalent to propagating a wave backwards in time from observer to source if all wind vectors are reflected through 180°. Consequently, we reverse the direction of the incoming wave and the direction of winds; i.e.,

\[ \psi_{R} = \psi \text{ (reversed)} = - \psi \text{ (incoming)} \]  
(32)

and the initial arrival angles, (θ, δ), become the initial angles for ray propagation (θ_R, δ_R).

\[ \theta_R = \sin^{-1} \left[ \sqrt{\frac{N}{D}} \right] \]  
(29b)

\[ \delta_R = 90 - \tan^{-1} \left[ \sqrt{3} \frac{\tau_{13} - \tau_{12}}{\tau_{13} + \tau_{12}} \right] \]  
(30b)

where the 'R' subscript refers to the direction of the reversed phase velocity vector.
A two-fold redundancy of equations (29), (30) exists. From (23) and (24) we see that

\[ \tau_{13} - \tau_{12} = \tau_{23} \]  \hfill (33)

If (33) is substituted into (30) and (31), we generate two other sets of \((\theta, \delta)\) which are functions of \((\tau_{12}, \tau_{23})\) and \((\tau_{13}, \tau_{23})\). As long as we may experimentally determine any two \(\tau_{ij}\), we may solve for \(\theta\) and \(\delta\) by substituting equation (33) for the appropriate \(\tau_{ij}\) in equations (29) and (30).

In equation (29b) \(v_R\) is the magnitude of the reversed phase velocity in the array vicinity, and under the influence of horizontal winds, this velocity is, from equation (9)

\[ v_R = C_o + V_{wo} \cdot \frac{\hat{n}_R}{\sqrt{N}} \]  \hfill (34)

\[ v_R = C_o + V_{wo} \sin \theta_R \cos(\delta_R - \delta_{WR}) \]  \hfill (35)

and, substituting (35) into (30) above, we derive the declination of the reversed phase velocity vector

\[ \sin \theta_R = \frac{\sqrt{N} C_o}{1 - \frac{\sqrt{N}}{V_{wo}} \cos(\delta_R - \delta_{WR})} \]  \hfill (36)

Equations (28) and (33) specify the orientation of \(n_R\), the reversed unit vector parallel to \(\hat{k}/k\) and normal to the wavefront which
propagates across the array. The ray-tracing program must have initial conditions specified by the direction of energy propagation, the ray velocity direction. As noted previously, this direction does not coincide with the phase velocity vector orientation. However, if we know the phase velocity, as we do from equations (31), (35), and (36), we may easily derive the energy propagation vector from equation (36).

\[
\vec{u}_R = C_o \hat{R}_R + \vec{V}_R^W
\]  

(37)

where \( C_o \) is the adiabatic sound speed in the array vicinity and \( \vec{V}_R^W \), the reversed wind velocity vector in the array vicinity, has magnitude \( V_R^W \) and lies at angle \( \delta_{WR} \) from the north. The vertical wind component is 0. The eastward and northward components are \( u_R \) and \( v_R \).

The ensuing result for \( \vec{u}_R \) is

\[
\vec{u}_R = \left[ C_o \sin \theta_R \cos \delta_R + u_R \right] \hat{i} + \left[ C_o \sin \theta_R \sin \delta_R + v_R \right] \hat{j} + C_o \cos \theta_R \hat{k}
\]  

(38)

and the azimuth, \( \delta'_R \), is

\[
\tan^{-1} \left[ \frac{U_{RY}}{U_{RX}} \right] = \tan^{-1} \left[ \frac{C_o \sin \theta_R \sin \delta_R + u_R}{C_o \sin \theta_R \cos \delta_R + u_R} \right]
\]  

(39)

\[
\tan \delta'_R = \frac{\sqrt{U_{RX}^2 + U_{RY}^2}}{U_{RZ}} = \frac{\sqrt{C_o^2 \sin \theta_R + \vec{V}_R^2}}{C_o \cos \theta_R} + \frac{2C_o \vec{V}_R^W \sin \theta_R \cos (\delta_R - \delta_W)}{C_o \cos \theta_R}
\]  

(40)

where \( \hat{i}, \hat{j} \) and \( \hat{k} \) are unit vectors pointing east, north and up, respectively.
Equations (39) and (40) are sufficient to define the ray input to the ray-tracing program to be traced back through the atmosphere to the source, a small section of the lightning channel.

As previously stated, the assumption implicit in this derivation is that of plane waves arriving at the array. If the characteristic array dimension is not trivially small with respect to the range of the source, the wavefronts may better be approximated as spherical in the array vicinity and a simple geometric correction can be made to the time delays $\tau_{ij}$.

Figure 5 illustrates the geometry used for the spherical correction. The source is located at 0 and a wave propagates across the microphone array defined by locations 1 and 2. The wavefront arrives at microphone 1 with phase velocity $V_p$ and propagates across the array to microphone 2 with time delay $\tau_{12}$. $R_1$ is the distance from source to microphone 1, and, to first order is given by

$$R_1 = V_p T$$

(41)

where $T$ is the propagation time from the source to the array. The distance covered by the wavefront in time $\tau_{12}$ is approximately given by

$$d = V_p \tau_{12}$$

(42)

and the distance, $R_2$, from the source to microphone 2 becomes

$$R_2 = R_1 + d,$$

(43)
fixing the angle $c$ as

$$\cos c = \frac{R_1^2 + R_2^2 - D^2}{2 R_1 R_2}$$  \hspace{1cm} (44)$$

Solving for $\Delta r$, the difference between the distance traversed by an assumed plane wave and the actual wavefront (Figure 5), we find

$$\Delta r + R_1 \left( \frac{1}{\cos c} - 1 \right)$$  \hspace{1cm} (45)$$

which yields the correction to the time delay, $\Delta \tau_{12}$

$$\Delta \tau_{12} = \frac{\Delta r}{V_p}$$ which, when added to $\tau_{12}$, yields the corrected time delay $\tau'_{12} = \tau_{12} + \Delta \tau_{12}$ \hspace{1cm} (46)$$

The new values of $\tau_{ij}$ are used to re-calculate the arrival angles. For ranges on the order of 2 - 5 kilometers the corrections to the time delays vary from 0 - 5 ms. While small, the correction may be on the same order of, or even exceed the uncorrected time delay, but when the entire set of $\tau_{ij}$ is examined, we see the net effect is a small change in lag time. These small lag time changes, however, may yield rather large changes in arrival angles. This effect is noticed when waves arrive at the array with steep declinations ($\theta \sim 90^\circ$). In these cases, the corrections to the time delays yield angular corrections up to 4 - 5 degrees for absolute ranges of several kilometers.
3. EXPERIMENTAL TECHNIQUE

3.1 In the Field

3.1.1 Field Layout and Instrumentation

The microphone array consisted of three microphones deployed at the vertices of an equilateral triangle 100 meters on a side as illustrated in Figure 6. Microphone 1, 30 meters from the truck, was far enough away for it not to be sensitive to people walking near the recording truck. Each microphone was mounted on three tripod legs with the pressure sensitive element 7 cm from the surface of the ground. An acoustic wave arriving at a microphone location is directly picked up by the microphone as is the first reflected wave. Thunder frequencies are low enough to insure little destructive interference from these two waves, although signal coherence may be slightly affected owing to different arrival angles at each microphone if the incoming waves are curved, not planar.

The instrumentation has been adequately discussed elsewhere (Few, 1968; Teer, 1972). We included the salient details here. The instruments deployed in the field were (1) three capacitor microphones (Globe model 100B), (2) one 'slow' electric field change meter, and (3) two 7.5 Hz seismometers.

The 3 db cutoffs of the Globe capacitor microphones were 0.1 Hz and 450 Hz with a flat frequency response over this range. The dynamic range of the microphones was 74 db with a sensitivity of 0.045 volt/dyne/cm². A pressure range of from 0.08 microbars to 400 microbars was required in order that the output voltage response be linear and within the limit ±10 volts.
To provide electric field change and timing information, the response from an electric field change meter (EFM) characterized by a 'slow antenna' (Kitagawa and Brook, 1960), with a 2.22 sec RC time constant was input to the tape recorder/oscillograph system.

The two seismometers, although not designed for this purpose, acted as rainfall indicators. A seismometer detects the impulse of a raindrop impinging on the outer casing. The arrival of each drop is recorded as a spike on an oscillograph trace. This oscillograph output later proved to be a useful diagnostic tool in interpreting the data.

Because of the limited dynamic range of the analog tape system, 36 db, and because of the dynamic range of a 'loud' thunder clap with respect to distant rumbles, it was necessary to shape the acoustic signal before it was recorded on tape. This was accomplished with a shaping preamplifier whose response characteristic is presented in Figure 7. The quality of the thunder data becomes, because of this shaping requirement, a strong function of the operator's judgement in selecting the proper amplifier gains. In general, when gains of less than five were used, and when the sources of the thunder were greater than a kilometer away (sound levels less than 120 db), the data were usable.

The tape recorder used was a Consolidated Electrodynamics VR3300 tape recorder. It has five FM channels, two direct recording channels, and one voice comments edge track on the 1/4-inch magnetic tape. At the field recording speed of 3-3/4 IPS, the center frequency of the FM carrier is 6750 Hz with the +40% deviation limits set at 9450 Hz and 4050 Hz. All frequencies are set to better than 1% precision.
3.1.2 Photographic Support

Mr. Leon Salanave, in charge of the University of Arizona Lightning Laboratory at that time, took all the lightning photographs during the 1970 experiment. The T-ll aerial surveying camera was used for the lightning photography. The T-ll has a calibrated focal length of 154.41 mm. The image is cast on a 9 inch by 9 inch format KODAK Aerographic PLUS-X film (ASA 125) from which negatives and contact prints were made. The camera is calibrated so a line extending from the lens center through the focal point is normal to the plane of the film. Therefore, cross hairs in the center of the photograph define the look direction of the camera. The horizontal and vertical field of view is about 70°.

The T-ll camera was located in the observation room at the lightning observatory. This room, about 3 m by 3 m by 3 m, could be rotated through 360°. The camera was rigidly mounted to the room and rotated with the room. It looked out of a window with a large overhanging window flap. The window flap limited the vertical field of view to about 30°. The flap was necessary to keep rain from being blown into the camera system during thunderstorms.

3.2 In the Laboratory

3.2.1 Photo Projection

To match a photograph of a lightning channel to an acoustic reconstruction, it is necessary to either project the photograph into the fixed frame of the array or cast the reconstructed coordinates of the
lightning channel into a rectangular coordinate system whose z axis is defined by the look direction of the camera. We have chosen the latter method.

If \((A, D)\) are the azimuth and declination of the camera look direction and \((\alpha_0, \delta_0)\) are the azimuth and declination of the survey point, a known point visible on the photo, then for a given azimuth and declination of a reconstructed source point, \((\alpha, \delta)\), we calculate the coordinates \((\zeta, \eta)\) to plot the source point in the frame of the camera look direction as follows (Teer, 1972)

\[
\eta = \eta' \cdot \frac{\sin \delta \cos D - \cos \delta \sin D \cos (\alpha - A)}{\sin \delta \sin D + \cos \delta \cos D \cos (\alpha - A)}
\]

\[
\zeta = \zeta' \cdot \frac{\cos \sin (\alpha - A)}{\sin \delta \sin D + \cos \delta \cos D \cos (\alpha - A)}
\]

where \((\zeta', \eta')\) are the \((x, y)\) coordinates of a point on the photograph and \(r'\) is the focal length of the T-11 camera.

\(A\) and \(D\) are first determined from a knowledge of \(\alpha_0\) and \(\delta_0\) and the \((x, y)\) coordinates of the survey point taken from the photograph \((\zeta_0, \eta_0)\), as follows (Teer, 1972)

\[
\sin (\alpha_0 - A) = \frac{\zeta_0 \cos \alpha_0}{\cos \delta_0}
\]

\[
\tan D = \frac{\sin \delta_0 - \eta_0 \cos \delta_0 \cos (\alpha_0 - A)}{\eta_0 \sin \delta_0 + \cos \delta_0 \cos (\alpha_0 - A)}
\]

Note that the camera may be pointed in any direction and the equations yield the projection onto the movable camera frame of a lightning event.
defined in the fixed array frame. After the acoustic map is appropriately rotated and projected so the reconstruction overlays the photograph and a model atmosphere is derived, the photograph is no longer needed, and the problem may be scaled and cast into any convenient frame.

3.2.2 Digital Data Processing

Selected data from the analog tapes were digitized on an SDS-92 computer at a sample rate of 2 Kc. The analog-to-digital converter produced an inverted 2's complement format with 10 bit words. The 2 Kc sample rate, corresponding to a Nyquist frequency of 1 Kc, was adequate to insure that no aliasing occurred. At the analog playback speed of 1-7/8 IPS, the signal passband extended from 0 Hz to 625 Hz with output voltages between ±1.0 volt.

3.2.3 Cross Correlation Analysis

It is evident from Figure 8 that the pressure variations caused by thunder form a time history that is random and non-stationary. These differences in the statistical structure of thunder are responsible for the observed time delays and characteristic sounds of thunder as the acoustic waves propagate across the microphone array.

An examination of Figure 8 illustrates several typical features of a thunder signal. Notice that records (b) and (c) have a low frequency present (about 2 Hz). If this low frequency component were due to thunder, it should propagate across the array as an acoustic wave, with phase velocity equal to the local sound speed, neglecting winds. If it were
due to local turbulence or wind gustiness, then we should (1) not be able to locate this component at each microphone owing to a very short lifetime on scale size larger than the array dimension, or (2) locate this component at each microphone and calculate the velocity of the small turbulent cell as it passes the array.

On the other hand, and more important for this experiment, if the high-frequency components in records (b) and (c) remain coherent over the array, and are truly caused by thunder, then, owing to their variable structure, we are able to calculate the lag times of arrival for these small wave packets as they cross the array. The detailed nature of these wave packets will be discussed later; it is sufficient at this time to note that the appearance of a typical thunder record fits nicely the "string of pearls" concept of thunder generation proposed by Few et al. (1967). Each mesotortuous section of the lightning channel acts as an independent source of sound, and because of the extended geometry of the channel, these sound pulses arrive at an observer's location with different amplitudes and from different directions.

We have shown in the previous sections that the fundamental experimental quantities measured are time delays of a signal across an array. The success of the experiment depends on an ability to calculate those time delays with sufficient precision. Sometimes these time delays may be measured by hand if the time series is presented at a large enough scale, but more often the time structure of the signal is too complex to handle this way and we turn to cross-correlation analysis.

The cross correlation function of two signals is extremely sensitive to the amplitude and frequency structure of each of the signals,
as well as the similarity between the two signals. That is, the average product of two related signals will always be a maximum when the time displacement between them is zero.

The time delay between two signals may be calculated by referencing one signal in time and combing the other signal through the fixed one. The location of the maximum correlation in time is defined as the delay time between the two signals. In Figure 9 (an example of a good thunder cross-correlation), this peak occurs 160 ms to the left of the zero lag position. By convention we say, "The signal from M3 lags the signal from M1 by -160 ms."

There are many references on cross-correlation techniques (Blackman and Tukey, 1959; Bevington, 1969; Bendat and Piersol, 1971). A review of their applicability to thunder propagation may be found in Few (1970) and Teer (1972).

The best estimate for the sample cross correlation function of two time sequences at lag positions \( r = 0, 1, 2, \ldots N \) is given by Bendat and Piersol (1971) as

\[
R_{xy}(r\Delta t) = \frac{1}{N-r} \sum_{n=1}^{N-r} X_n Y_{n+r}
\]  

(52)

where \( N \) is the number of samples in each sequence, \( n \) is the running sample index and \( \Delta t \) is the sample time. \( X_n \) and \( Y_n \) must have zero mean values.

The simplest way to assure the zero mean condition is to calculate \( \bar{n}_x \) and \( \bar{n}_y \), the sample means of \( X_n \) and \( Y_n \), subtract them from \( X_n \) and \( Y_n \) and input the result to (52).
Owing to our imposed constraint of fixed point arithmetic, however, we wanted \( X_n \) and \( Y_n \) to have strong positive bias to assure the occurrence of no negative values.

The effect of introducing positive bias and later removing it is equivalent to using (52). This may be shown as follows:

\[
X'_n = X_n - \eta_x \tag{53}
\]
\[
Y'_n = Y_n - \eta_y \tag{54}
\]

where \( X_n, Y_n \) are time sequences with non-zero mean values and \( \eta_x, \eta_y \) are sample means

\[
\eta_x = \frac{1}{N} \sum_{n=1}^{N} X_n \tag{55}
\]

Substitute (53), (54) into equation (52) and expand:

\[
R_{xy}(r\Delta t) = \frac{1}{N-r} \sum_{n=1}^{N-r} (X_n - \eta_x)(Y_n - \eta_y) \tag{56}
\]

\[
R_{xy}(r\Delta t) = \frac{1}{N-r} \sum_{n=1}^{N-r} X_n Y_{n+r} + \eta_x \eta_y - \frac{1}{N-r} \left[ \eta_x \sum_{n=1}^{N-r} Y_{n+r} + \eta_y \sum_{n=1}^{N-r} X_n \right] \tag{57}
\]

where the first term on the right is identical to equation (52) except \( X_n \) and \( Y_n \) have non-zero mean values. The following terms are necessary to extract the mean values and the weighted means from the resulting cross correlation coefficients.
If \( n \gg r \), (57) reduces to

\[
R_{xy}(r\Delta t) = \frac{1}{N-r} \sum_{n=1}^{N-r} X_n Y_{n+r} - \bar{X}\bar{Y}
\]  

(58)

This is the form commonly given in most signal analysis texts. (c.f. Bendat and Piersol, 1971)

At this point we stress the fact that the above formalism is strictly valid only for stationary data, whose moments are unchanged with the passage of time. If the data is non-stationary, as is thunder, the cross correlation of \( X \) and \( Y \) is represented as

\[
R_{xy}(t, \tau = t - \tau') = \frac{1}{T} \int_{0}^{T} X(t) Y(t + \tau) dt
\]  

(59)

where we have chosen to represent \( X \) and \( Y \) as continuous functions in order to illustrate the dependence of \( R \) on \( t_1 \) and \( t_2 \), very particular locations in \( X \) and \( Y \). The equation predicts the possibility of having a different \( R_{xy} \) each time \( t_1 \) is different. This is correct, in general, for the time series representing the entire thunder event caused by one lightning flash. The signal may be easily sixty seconds long, and allowing the cross-covariances to be functions of the observation time in that 60 seconds is an expression of the non-stationary structure of the thunder signal.

We are interested, however, in dividing the signal into very small time segments, not necessarily contiguous, of approximately 250 ms per segment. Then, as long as we choose similar 250 ms segments
from the output of another microphone (displaced 100 m from the first), the cross-correlogram becomes only a function of the time lag \( \tau \), the time difference between the two signals.

All we have done by our judicious choice of the windows is make order out of portions of the non-stationary data. Since we have constrained the three time windows with the requirement that the data be similar in each window, we have extracted the absolute time dependence from equation (59) and are now free to use equation (57) to calculate cross-correlograms. The resulting correlograms, \((M1 \times M2)\), \((M1 \times M3)\) and \((M2 \times M3)\), yielding \( \tau_{12} \), \( \tau_{13} \), and \( \tau_{23} \), will all be similar to each other if the similarity constraint has been carefully observed.

There is no a priori way to judge whether the cross-correlation technique will be successful when analyzing non-stationary thunder data. For instance, if a change in a window position of 1 ms causes a change of 1 ms in the resulting placement of correlation peaks, we might conclude that the analysis is so sensitive to our mathematical analysis technique that any physical significance is lost. In the example below, we have shown experimentally that this is not the case. Thunder is a physical phenomenon caused by a physical shource, and we have shown that the resulting correlograms are strongly related to the position of this source.

In Figure 10 we show two time series from M1 and M2 with correlation time lengths of 512 ms indicated on the traces. We also show the cross-correlograms of these correlation windows. The data from microphone 2 lags the data from microphone 1 by 273 - 274 ms. Correlation A indicates a time delay of 273 ms and correlation B yields 274 ms. The
actual correlation peak is relatively insensitive to the placement
of the time windows. The correlation peaks are time shifted by 1 ms
with respect to each other. This is about the most serious time shift
of this nature seen, and we conclude that our assumptions of treating
the correlation windows as corresponding to source points on the lightning
channel are justified.

An example of the response of our cross-correlation algorithm
to calibration signals is shown in Figure 11. The frequencies 20 Hz
and 50 Hz are representative of thunder frequencies. Various sinusoids
were input to all three analog-microphone channels simultaneously. The
analog data were processed through the system in identical fashion to
our thunder tapes. The peak in the auto-correlation occurs at the
zero lag position. The functions show no statistical instability near
maximum lags and the function envelopes have constant amplitude.

The cross-correlations are normalized to lie in the range

\[-1 \leq R'_{xy} \leq +1\]  \hspace{1cm} (60)

This is accomplished by dividing each cross-correlation coefficient by
the square root of the product of the zero-lag auto-correlation coeffi-
cients of \(X_n\) and \(Y_n\).

\[R'_{xy} = \frac{R_{xy}}{\sqrt{R_x(0)R_y(0)}}\]  \hspace{1cm} (61)
This normalization is useful in interpreting the degree of similarity in $X$ and $Y$. If the two signals are identical, then $R_x(o) = R_y(o)$ and $R_{xy}'$ is identically equal to 1. If $X$ and $Y$ are dissimilar, then $R_x(o)$ and $R_y(o)$ are not equal and $R_{xy}'$ will be less than 1. While we have observed cross-correlation peaks derived from thunder data to lie over the entire range of definition, peaks with correlation coefficients greater than 0.4 are usable.
4. THE 1970 TUCSON EXPERIMENT

4.1 Description

The Rice University Mobile Recording Laboratory was taken to Tucson, Arizona in August 1970 to conduct acoustic mapping experiments. These experiments were carried out in coordination with the University of Arizona Institute of Atmospheric Physics at their lightning observation laboratory located on the grounds of the Tucson Magnetic Observatory eight miles east of Tucson (Figure 15). Tucson was chosen as the 1970 summer site because of the lightning photography capability of the University of Arizona and also because of the type of thunderstorms that occur in the neighborhood. August is the peak thunderstorm month for the Tucson area. Usually every day or two, in the evening or during the night, small, localized, and intense convective thunderstorms are generated in this area of the Southwest.

The topography around Tucson was ideal for our studies. Tucson is located on an alluvial plain near the Catalina mountain range in a semi-arid desert area. The nearest mountains are 15 km to the north and they rise abruptly from the desert floor. The nearest buildings in the outskirts of Tucson are 6 - 7 km to the West. The microphone array was located in a sandy area with no ground cover, but with clumps of chaparral or cactus spaced about every five meters. Thunderstorms usually arrived from the East and swung along the southern edge of the Colorado National Forest, north of Tucson.
Conditions for recording were excellent. Owing to the isolation of the array site from city noise, automobiles, and machinery, a very low baseline noise level was recorded. Occasionally aircraft from nearby Monahan Air Force Base and from Tucson International Airport was recorded, but this noise was minimized during thunderstorm occurrences. Both the airport and the Air Force Base were located too far away for any surface activity to be detected.

4.2 Atmospheric Parameters on 03 Aug 70

The National Weather Service at Tucson International Airport reported the sighting of thunderstorms on August 3, 1970 between the hours of 0156 and 0526 MST. Steady winds from the east and east-southeast at 10 - 16 mph were reported, with maximum gusts from the east at 21 mph. The thunderstorms passed overhead at the array site, about 15 miles NNE of the airport; consequently, reported wind speeds at the airport were very likely less than those at the site.

A rawinsonde was launched from the airport at 0422 MST on August 3 and the measured winds and temperatures aloft are plotted in Figure 16. The observed lapse rate to an altitude of 10,700 meters of -6.5°K/km is in good agreement with the NACA Standard Atmosphere (Haltiner and Martin, 1957). A small temperature inversion at 500 meters, 1.5°C above the surface temperature 22°C, is present. This is typical of early morning conditions, although the inversion is usually more strongly defined in the Tucson area. For example, the inversion on 01 Aug 73 was 3.5°C above the surface temperature of 23°C at 500 meters.
The O°C level occurs at an altitude of 3.1 km for a lapse rate of 6.5°C/km with surface temperature 22°C. The O°C level measured by the rawinsonde occurred at 4.1 km and the model atmosphere used (see Section 5) placed O°C at 4.5 km.

It is likely that these atmospheric parameters are similar to those at the array site during the time interval 0246-0420 MST in which we recorded data from several cells over their lifetimes as they moved over the array from east to west. Meteorological measurements were not made at the array site, but the rawinsonde data is representative of average, post-storm, prevailing conditions. Our last recorded event occurred at 0418 MST, about 4 minutes earlier than the rawinsonde ascent.

Typical temperature fluctuations in thunderstorms have been given by Byers and Braham (1949). Figure 17 represents the difference in thunderstorm temperatures and environmental air. Curves $t_1$ and $t_3$ represent maximum temperature differences between environmental air and draft air. These deviations represent those encountered at a given level as a function of time, and those at various levels at a given time. The deviations are not serious with respect to wave propagation. In Section 8 we will refer to this figure to examine the effects of temperature fluctuation on tracing a ray through an atmosphere defined by the environmental conditions.

Byers and Braham (1949) calculate various entrainment models of a thunderstorm to produce Figure 18. They show that these models fit nicely the measured environmental temperature lapse rate and no data sets lie close to a calculated dry adiabatic lapse rate or a saturated
lapse rate. The rawinsonde atmosphere in Figure 16 with the measured lapse rate plotted as well as the computed wet and dry adiabatic lapse rates is similar to Figure 18. The fact that current models predict temperature behavior near that of the environmental temperatures with no greater than ±4°C deviation allows us to put a high reliability on using Figure 16 as good temperature data for wave propagation and channel modelling on August 3, 1970.

Winds are closely related to the temperature structure observed inside thunderstorms, and the wind field is highly variable. With no wind data available during storm times, it is of relatively little value to determine a wind model significantly different than the rawinsonde winds at 0422 MST.

Errors in channel coordinates introduced by vertical winds are of the same order as errors introduced by a lack of knowledge of horizontal winds. For instance, if a ray is propagated parallel to the earth's surface in an atmosphere characterized by a horizontal gradient in the vertical wind component, \( \frac{dw}{dx} \), of 0.03 m/sec-m, the deviation of the ray from a straight line after 3.5 sec is 60 meters. Section 8 presents a complete discussion of errors introduced by horizontal winds. This discussion may refer to vertical wind errors as well if the diagrams in Section 8 are rotated by 90°.

We are confident that the rawinsonde atmosphere sufficiently defines our horizontal wind field. Figure 14 predicts very small (~1 - 2 m/sec) vertical winds in the dissipating stage of a storm, and as it is exactly this stage being studied, we conclude that there
have been no serious errors introduced in lightning channel coordinates by winds. We believe that each point is located to within ±500 m of its actual value. (See Section 8 for details.)

4.3 Storm Summary

Figure 19 represents a potpourri of system operations and observed thunderstorm parameters such as rain rate, thunder levels, electric field event counting rates and counting rates of photographed lightning flashes.

The electric field meter (EFM) was turned on at 0252 MST and began to fail at 0332 MST, just as rain began to fall on the parallel plate EFM antenna. The EFM operated intermittently until total failure at 0355 MST when the rain rate approached a maximum. After 0332 MST electric field data was supplemented by the long wire response to a changing electric field from the seismometer cable and the WWV antenna.

Throughout the first hour of recording, the environmental noise was high; the most quiet data occurred in the time interval 0324-0415 MST, when the thunderstorm was less than five miles distant. Even when unity-gain amplifiers were used the thunder levels were severely shaped, as discussed in the text.

Throughout the entire 2-1/2 hours of recording there was one two-minute time interval when thunder was not heard.

The lower half of Figure 20 is a scatter plot of the azimuth angle of photographed lightning events versus time of occurrence. Two trends are apparent. As time increases, the azimuth angle distribution spreads to 130° at 0355 MST. As time increases the relative intensity
of the lightning channels on the photographs increases. Both these
trends support the conclusion that the storm, or storms, are moving
toward the array, initially from the east. As the storm approaches
overhead, the luminosity of the photographed lightning channels increases
and the azimuth spread naturally broadens out.

The symmetry is misleading, however, because our camera only
had a 70° horizontal field of view and the operator pointed it in the
direction of maximum activity. Many lightning events fell out of his
field of view. Field reports indicate that as time increased, he
steadily turned the camera from pointing due east at 0230 MST to 10°
west of north at 0330, indicating storm movement slightly to the north
of our location, moving from east to west, ±20°.

Evidence of this movement is more clearly seen in the top frame
of Figure 20, a scatter plot of acoustically reconstructed azimuth angles
of CG events versus time-of-occurrence. The same general features exist,
as they should, but the density of points in the time span covered by
the photographs (see lower part of Figure 20) exceeds the density of
points in that time span of acoustically reconstructed azimuth. This is
evidence of a phenomenon we encounter quite often. The acoustic signature
of a lightning event ranges in time from a few seconds to a minute or
more. Thunder events are masked by the occurrence of a nearby flash
which produces loud, continuing thunder.

Figure 21 illustrates this overlap. The period of acoustic
event occurrence peaks throughout the storm time at about 15 - 20 seconds
until 0340 MST, when the period increases and approaches infinity at
at the end of all activity. We may expect acoustic overlap until the
time between successive acoustic events becomes greater than the time
length of each event.

Refraction and absorption effects, as previously discussed,
serve to reduce the number of recoverable acoustic events. This loss
of data may alter results concerning the electrical evolution of a thunder-
storm. As a result, we chose a time region in our data characterized
by a minimum of thunder event overlap, that period 0350-0420 MST. This
time period, as we show in the next section, encompasses most of the
dissipating stage of one thunderstorm cell and results concerning this
stage are not distorted by unrecoverable events.

Figure 22 and Figure 23 are polar histograms of number of
acoustically reconstructed cloud to ground events per azimuth interval
versus azimuth.

Figure 22 covers the entire recording time span and indicates
the existence of two regions of electrical activity. We believe these
regions of activity belong to different thunderstorms. This belief is
supported by Figure 23, where we present similar histograms as a function
of time. Time is split into four contiguous time intervals, Figure 23
(a), (b), (c), and (d), starting at 0242 MST, the onset of recording time
when the storm activity occurred at ranges from 4 - 10 miles.

The principal azimuth during the first time interval was 70°-80°.
This azimuth shifts to 50°-60° during time interval 2, and we notice the
appearance of events at almost all azimuths, indicating the presence
of activity in the array vicinity.
The next time interval, Figure 23(c), only lasts for 11 minutes, compared to the other intervals of 38, 39, and 27 minutes, while the number of events is still quite high. The principal azimuth shifts to 10°-20° and still only a few events occur in the immediate array vicinity.

The Fourth time interval of 27 minutes beginning at 0350 MST covers the time span defined as the dissipation stage time period. Some events are overhead, but most occur due north of the array or roughly 40° west of north.

When we sum up all time spans, Figure 23(a), (b), (c), and (d), we produce Figure 22. The azimuth peak at 10°-20° in Figure 22 is primarily due to the sum of the last two time intervals in Figure 23, and because of the measured ranges (see Appendix I), we conclude that the activity around due north is from a different cell than that activity near 60°-70°. This assumption is strengthened when we note the shape of the azimuth envelope near 60°. It is characterized by a sharp onset at 100°, a peak at 75° and a gradual decay in amplitude at 30°. This is what we expect for the recorded electrical activity over the lifetime of a thunderstorm cell. The sharp onset is produced because we started recording when this cell was already electrically active. The decay results from a possible combination of the ceasing of electrical activity and its moving away from the array.
5. ACOUSTIC RECONSTRUCTIONS

Our goal in the next two sections is to present the lightning channel reconstructions of the storm's dissipating stage previously discussed. We have reconstructed 40 lightning events, every one that occurred in the time interval 0352 MST to 0420 MST. The entire set of these events in time sequence is presented in Appendix I and we will refer to these figures periodically by their time of occurrence.

5.1 Photographic Verification

In order to experimentally verify our reconstructions, we cast the acoustic reconstructions, whenever possible, against a photograph of the visible portion of the lightning channel and examine in detail the resulting match. A mismatch of the reconstruction to the photograph is caused by the choice of an incorrect set of atmospheric parameters. In general, the mismatch is small and we do not need to be extremely careful of our atmosphere. A more complete discussion of the expected range of errors is found in Section 8.

We have no photographs of lightning channels during the dissipating phase. While we believe the rawinsonde atmosphere is valid, without a check as to its validity, confidence in its use is diminished. Consequently we used the rawinsonde atmosphere to acoustically reconstruct the lightning channel occurring at 0334:55 MST (Appendix I), about 16 minutes earlier than the first event analyzed as belonging to the dissipating phase. An analysis of the storm's history indicates that this event occurred in the mature phase when the atmosphere was very
turbulent.

The time from lightning initiation to thunder first arrival was 7.5 sec and the duration of thunder was 17 sec. The thunder on all three microphones and the corresponding electric field change is presented in Figure 1.

The acoustic profile in Figure 25(a) is generated by propagation through a 'first look' atmosphere. This atmosphere is isothermal and homogeneous (no winds), and the ray from observer to source is a straight line. The orientation of this ray in space is determined from the initial condition equations (30) and (31) with v set equal to \( C_0 \). We have used \( C_0 = 340 \) m/sec corresponding to a temperature of 14.6°C as our initial sound speed.

A model atmosphere was derived from the rawinsonde measurements in Figure 16 and was used to derive the acoustic reconstruction in Figure 25(b). The model, Figure 24, maintains the features of the measurements in allowing a small inversion to exist at the surface and following a 6.5°F/km lapse rate to about 17 km. Winds in the model do not change with height as in the measurements, but the major wind shears and gradients are kept.

The match of the rawinsonde atmosphere, which was measured an hour later than the occurrence of event 02-0086, (0334:55), to the photograph is good except in the vicinity of a few hundred meters of the surface.

Vertical wind shears near the earth's surface are among the strongest wind gradients encountered in the atmosphere. Acoustic
waves propagating nearly parallel to the earth's surface spend enough time in these gradient fields to be strongly refracted either upwards away from the array or downward into the ground where they may be reflected upwards, scattered or absorbed due to surface roughness effects. Again, we postpone a discussion of errors to Section 8, but the mismatch in Figure 25(b) near the surface is accounted for by the above effects. A slightly different model atmosphere has been developed that yields a nearly perfect overlay of photo to this acoustic reconstruction, Teer (1972).

We conclude, therefore, that the rawinsonde atmosphere will yield sufficiently accurate acoustic profiles to derive meaningful statistics of the lightning channel geometry inside the storm clouds during the dissipating stage.

5.2 Representative Cloud-to-Ground Event Analysis

In order to make clear the acoustic reconstruction procedure, we will go through it step-by-step. Figure 26 represents a cloud-to-ground lightning event occurring at 0414:01 MST and the resulting thunder on the three microphone channels. The time delay from lightning initiation to thunder onset is 11 sec and the duration of detectable thunder is 31 sec.

Several features in this cloud-to-ground thunder signature are worth noting.

(1) The signature is characterized by a very sharp onset of high amplitude thunder, always noticed in a CG event and never from an
IC event. (Note: In the remainder of this paper we represent an intra-
cloud event as an IC event and a cloud-to-ground event as a CG event.)

(2) As time increases the signature loses most of its high
frequency structure and the low frequencies are enhanced. This is indica-
tive of acoustic absorption losses and we deduce that the visible portion
of the channel is nearer to the array than the intracloud portion.

(3) Low amplitude, high frequency thunder precedes the sudden
onset. We term this the signature precursor. It is indicative of either
an intracloud channel segment nearer to us than the vertical portion
of the channel, or a branch off the main channel nearer to us than the
main channel. In this case, the precursor seems to belong to a branch
off the main channel.

(4) The sudden onset portion of the signature has constant ampli-
tude. This is not characteristic of any true signature. The constant
amplitudes are present because of improper gain settings on the system
amplifiers, and the signal has been routed through the shaping pre-
amplifiers as discussed earlier. The data in this portion is rendered
useless for the generation of power spectra because true amplitudes are
not represented.

(5) Sections of the signature are identifiable on all three
microphones, but with different arrival times. An analysis of these time
delays allows us to reconstruct the lightning channel.
The 31 seconds of thunder were partitioned into 248 overlapping time windows of 0.256 sec each. Three 512 point cross-correlation functions from which the time delays were measured, were generated for each time window from the data in each microphone channel falling within that window.

Six representative cross-correlation sets, together with the time windows of data that went into the generation of the correlograms, are presented in Figure 27. While their interpretation has been previously discussed, we point out that the widths of the maximum cross-correlation peaks increase from 6 ms in 27(a) to 75 ms in 27(f). The correlation peak width is one-half the period of the most prevalent frequency in the data window, therefore the peak frequency has shifted from 83 Hz to 6.7 Hz in 31 seconds.

The time delays for each window, the propagation time from lightning onset to each window, the initial angles generated from these time delays and the final angles after propagating rays back in time to the source are presented in Table I together with the atmospheric conditions in the surface 150 meter layer.

Absolute range data is available directly from the acoustic reconstructions in Figures 28, 29, and 30.

Forty-one sets of time delays were generated from the 248 cross-correlation functions. We were unable to use 35% of the correlations owing to destructive interference effects, and turbulence-generated incoherence. The signal to noise ratio for 20% of the data windows was much less than 1 and rendered correlations useless. The overlapping of
windows provides a redundant measurement of the time lag present in any particular pulse or burst, consequently we use only one set of time delays for each identifiable portion of signal.

As a consequence of propagating through the atmosphere in Figure 16, the initial azimuth angles, $\delta_i$, have been rotated less than $1^\circ$ westward and the initial declinations, $\theta_i$, have been changed by less than $1^\circ$, typically a few tenths of a degree. Propagation in this atmosphere over distances less than 20 - 30 km is not much different than propagation in a 'first look' homogeneous atmosphere.

The cloud-to-ground channel reconstruction in three orthogonal projections, east vs. altitude, north vs. altitude, and east vs. west, is presented in Figures 28, 29, and 30. A smaller projection of this channel is in Appendix I at 0414:01 MST.

Orientation errors, discussed in a later section, are strong functions of all angular variables, but the size of the plotting symbols roughly determines the average error per plotted point.

The main channel rises to 3.6 km. Branches are evident from 2.2 km to 3.6 km. The structure above 3.6 km is horizontal, with a well defined channel extending 8.5 km in range and traversing a vertical thickness of about 1 km, except a section to the northwest that rises vertically from 4.5 to 6.2 km. Further discussion on the significance of these results is delayed to Section 7.
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Temperature in array vicinity = 22°C
Adiabatic sound speed in array vicinity = 345.2 m/sec
Wind speed in array vicinity = 6.6 m/sec
Wind velocity azimuth = 241°
5.3 **Representative Intraccloud Event Analysis**

Figure 31 presents an IC event occurring at 0359:19 MST, and the resulting thunder signatures on the three microphone channels. The thunder occurring near the EFM response belongs to the preceding CG event at 0358:54 MST. The time from lightning initiation to thunder first arrival is 11.5 sec and the duration of thunder is 15 sec.

The same features discussed with respect to CG events occur here with two major exceptions. (1) There is no sudden onset of high amplitude thunder. This is the major difference between acoustic signatures of CG and IC events. (a) The amplitudes in this event appear to be smaller than those in the previous CG event. This is true in general (Holmes et al., 1971). However intraccloud thunder amplitudes are highly variable. Many are not audible at all, and many generate as much acoustic energy as a typical CG event.

The signature of an IC event is much more complex than that of a CG event. We do not usually obtain as many good points for acoustic reconstructions as for the CG events. This is indicative of the much more complex structure of the IC event, as evidenced in Appendix I at 0359:19 MST, the acoustic reconstruction.

There is no well defined channel for the IC event as for the CG event, and the structure of this event may best be described as volume distributed. The one lone point separated from the others in the reconstruction was derived from one isolated low frequency pulse which occurred at 0359:46 MST, 25 sec after the end of the intraccloud thunder signature. There is no indication of an intervening electric field change in our EFM or the long wire antennas.
It has not been determined that lightning is the only existing source of thunder (Wilson, 1920; Dessler, 1973), consequently, we are not certain that the isolated point belongs to this channel. The appearance of isolated low frequency pulses throughout the storm history causes us to conclude that the point is not a source point on the lightning channel, and it is not used in the subsequent analyses.

The mean altitude of the IC event is 5 km and the average vertical thickness is 1 km. The long horizontal axis is oriented on a north-south line and extends 6 km. The short horizontal axis extends 2.6 km making the volume involved $6 \times 2.6 \times 1 \text{ km}^3$. Thirty points were used in generating the reconstruction, or roughly one point per spherical volume of radius 400 m.
6. **LIGHTNING CHANNEL GEOMETRY**

6.1 **Variance Analysis Technique**

Most of the IC events and the intracloud portion of the CG events seem to be volume distributed rather than lying on a very thin line. It is quite impossible to "connect the dots" making up these events because not enough channel points have been generated. Obvious intracloud channels pervade a volume of space, i.e., a long, straight channel might be modeled as a cigar shaped volume oriented arbitrarily in a fixed coordinate frame.

A system of presenting the statistics of the reconstructed channels is necessary. The best system, in our opinion, treats the distribution of acoustic source points as a collection of mass loaded points arbitrarily oriented in space. While the mass loading might be used as a statistical weighting factor, we have chosen to assign each point a unit mass. Resulting statistics reflect the actual orientation of the points and not that of statistically weighted points.

The distribution of points may be represented as a randomly oriented rigid body in space. We first calculate the centroid of the object

\[
<X_i> = \frac{1}{N} \sum_{j=1}^{N} X_{ij}
\]  

(62)
where \( X_i, i = 1, 2, 3 \) are \( x, y, z \) components of the centroid referenced to our fixed array coordinate system and \( j \) is the summation index running over \( N \) points. \( <> \) denote spatial averaging.

We translate our coordinate system to the centroid and calculate the principal axes of the rigid body by the variance ellipsoid technique described in Sonnerup and Cahill (1967).

The largest principal axis of the mass distribution of points is obtained by maximizing

\[
\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} \left( R_i \cdot \hat{n} - < R > \cdot \hat{n} \right)^2
\]

(63)

where \( R_i \) is the position vector in the fixed array coordinate frame to the \( i^{th} \) mass point in the distribution, \( < R > \) is the position vector to the centroid and \( \hat{n} \) is a unit vector along the semimajor axis of the variance ellipsoid.

The maximization of \( \sigma^2 \) is equivalent to finding the maximum eigenvalue of the variance matrix

\[
\sigma^2 = \begin{bmatrix}
< R_\alpha R_\beta > & - < R_\alpha > & < R_\beta > \\
- < R_\alpha > & < R_\beta > & \cdot \\
< R_\beta > & \cdot & \cdot
\end{bmatrix}
\]

(64)

where \( R_\alpha, R_\beta \) are the cartesian components of \( R \) in the fixed frame (Trumpler and Weaver, 1953).

The largest eigenvalue represents the variance in the direction of the corresponding eigenvector which gives the direction of the semimajor axis of the variance ellipsoid. The smallest eigenvalue repre-
sents the variance in the direction of the semiminor axis of the ellipsoid and the other eigenvalue is the variance of the third orthogonal axis of the variance ellipsoid.

Consequently nine parameters are generated in the analysis.

(i) The centroid coordinates $X_0$, $Y_0$, $Z_0$.

(ii) The three eigenvectors, orientations of the semiprincipal axes.

(iii) The three eigenvalues, variances along the semiprincipal axes.

The square root of each eigenvalue gives the standard deviation of the distribution from the centroid in the direction of its corresponding eigenvector. To generate the entire standard deviation ellipse, we reflect each eigenvector ($\mathbf{n} \rightarrow -\mathbf{n}$) and apply the corresponding standard deviation in this new direction. The reflection symmetry is possible because the variance matrix $\sigma^2_{ab}$ is symmetric, i.e.,

$$\sigma^2_{ab} = \sigma^2_{ba}$$

(65)

The resulting ellipsoid is oriented with its origin at the centroid of the mass distribution and principal axes along the maximum, median and minimum variances directions. For example, the ellipsoid corresponding to a simple vertical cloud-to-ground stroke would look like a thin cigar oriented normal to the earth's surface. If the points are normally distributed, two standard deviations, or one standard deviation on each side of the centroid, should contain at least 0.68 of all the points along that direction.
6.2 Time History

This study includes 40 lightning events, 20 IC channels and 20 CG channels.

Three of the CG events (0334:16, 0334:55, the last two figures in Appendix I) were not included in the variance analysis during the storm's dissipating stage. They were used to check our model atmosphere since we had their photographs, and they were used as checks against our statistics derived from the 37 events in the time interval 0350 to 0420 MST.

Figure 32 is a plot of the areal distribution of CG events as a function of the time-of-occurrence of the event. Each point, representing the bottom of the vertical part of a CG event, seems to have been randomly generated in time, although points 6 - 11 occurred in a common region, at 10° east of north at a range of 6 km.

The intracloud centroid locations in time sequence (Figure 32) are randomly generated. The striking feature visible in the intracloud plot is the ordered occurrence along a line 30° east of north. We return to this point in Section 7.

The centroids in plan view of all 37 events are shown in Figure 33. There is no preferred area for CG events versus IC events and the areal extent of all the events indicates the presence of one thunderstorm cell.
6.3 Horizontal Distribution

6.3.1 Cloud-to-Ground Events

We have plotted the length and orientation of the maximum variance (2 σ) axis for each event in plan view, the horizontal plane, at the centroid location of each event in Figure 34(a). The projections in plan view of the variance axis which yields the largest projected spread normal to the maximum variance axis is in Figure 34(b).

Since points along the vertical portion of the CG channels are included in the variance analysis of Figure 34, the X, Y-coordinates of the centroids are weighted toward the point at which the channel struck the ground. The lengths of the variance axes are slightly smaller than they would be if only the horizontal portion of the channel were analyzed.

To derive a more complete picture of the statistics of the CG events, we performed the variance analysis on the sets of CG points after having extracted points belonging to the vertical channels. Similar plots as above were constructed and are presented in Figure 35(a), (b).

The end position of the small perpendicular tick on each maximum variance axis in Figure 35(a) represents the location of the centroid from Figure 34 that closely corresponds to the point where the main channel went to ground. The numbers on Figures 34 and 35 are the time-sequence numbers and provide a means of locating identical events. An axis from Figure 35(a) and the corresponding axis from 35(b) are the tangent-plane projections of the major and minor axes of the variance ellipse calculated after having extracted the points lying on the main channel to ground.
These plots illustrate 4 points about the horizontal structure of cloud-to-ground events.

1. The eccentricities are all very large with $\bar{e} = 0.9$ and standard deviation $\sigma_e = 0.1$.

2. The variance ellipses are oriented in similar directions. The mean azimuth, $\bar{\phi}$, is 8.6° with standard deviation 24.1°. There seem to be two directions involved. Choosing the 10 events with positive slopes (azimuth angles east of north) yields $\bar{\phi} (+) = 26° \pm 15°$ and the average azimuth of the 7 events with negative slopes (azimuth angles west of north) yields $\bar{\phi} (-) = -16° \pm 15°$.

3. The channels all have considerable horizontal extent. The average horizontal range is 6229 meters with standard deviation 2277 meters. The longest event is 11.55 km and the shortest event extended 2.55 km horizontally. These distances represent standard deviation distances, which, if gaussian distributed, should be multiplied by 1.58.

4. It is striking that the location of the vertically weighted centroids all lie to the right side of the variance axes plotted in Figure 35(a). This indicates that the main channels are slightly tilted away from vertical, the direction normal to that of its maximum variance axis.

6.3.2 **Intracloud Events**

Similar plots of the horizontal extent of intracloud events are presented in Figures 36 and 37. The similarity in the directional alignments of all the events is quite impressive. The average azimuth, $\bar{\phi}$, is $15.9° \pm 15.5°$. 
The horizontal projections of the variance ellipsoids are all highly eccentric ellipses with average eccentricity, \( \bar{e} = 0.854 \pm 0.185 \). Since the average eccentricity for the IC events is less than that for cloud-to-ground events and the spread is larger, we see that, in general, the IC events cover a larger horizontal area.

The average length of the major axes is \( 6.466 \text{ km} \pm 2.523 \text{ km} \); the average length of the minor axes is \( 2.843 \text{ km} \pm 1.673 \text{ km} \), making the average horizontal area covered by an intracloud flash \( 18.4 \text{ km}^2 \). The length of the maximum horizontal event recorded was \( 12.3 \text{ km} \) with 673 meters lateral extent. The smallest horizontal event traversed was \( 1.8 \text{ km} \) with 804 meters lateral extent.

### 6.4 Vertical Distribution

The vertical structure of CG and IC events is presented in Figures 38 through 41.

Figure 38 presents the height of the intracloud centroids versus time of event occurrence together with the height to the top of the main channel versus time of the CG events.

Several features are worth noting.

1. Although data is sparse, we definitely observe the CG events progressively shifting from 2.5 km to higher levels in the cloud from 0352 MST to 0400 MST. They finally level off at about 4.5 km.

2. The activity of the CG events is high at 0350 MST and dwindles away until the end of the storm at 0425 MST. During the time interval in which there was much high level IC activity only 2 CG events occurred.
The IC events fall into two natural groups, those occurring at an average height of 4.5 km and those at an average height of 7.0 km. The lower group is active from 0352 MST until 0410 MST. The upper group begins at 0404 MST and lasts until 0415 MST. Although there is overlap of these times, we may interpret this behavior as indicative of the storm dynamics. Whatever cloud process responsible for the low level IC events either ended or shifted to a higher level in the thunderstorm cell.

The highest IC event occurs at an altitude of 9.3 km. The high level IC activity then seems to drop to the lower region near 7 km. Figure 39 was drawn to observe whether there was any horizontal shift of the charge regions as the low level IC activity shifted to higher regions in the cloud. We observe no such lateral shift. The activity seems to have shifted upwards over the entire cell.

The vertical extent of the IC events is presented superimposed at each event's centroid altitude in Figure 40. The average "thickness" of the region traversed vertically is 3.073 km for the high level IC events and is 1.864 km for the low level IC events. While this is more or less "thick" compared to a very thin layer, the average ratio of horizontal extent to vertical thickness, \( L/T \), is 2.9 ± 0.9, or between 2 and 4. We see that the horizontal structure of the IC events exceeds that of the vertical structure by two to four times.

The low level IC events seem to be dynamically associated with the freezing zone (-10°C to 0°C) of the storm between 4.1 km and 6 km as measured by the rawinsonde at 0422 MST. They are not associated at all with the cloud base at 1.8 km.
The high level IC events pervade a much greater volume, typically, than do the low level events. We observe one high level IC event 12.33 km long and the longest reported low level IC event was 8.79 km long.

Numbers by each event give the length of the maximum variance vector times two, i.e., in the principal axis coordinate system, the maximum variance axis is representative of the greatest extent of the event regardless of orientation in the fixed frame. A rough idea of the entire extent of the event may be easily visualized. For completeness, the small black triangles on the time scale mark the locations of the CG events.

Figure 41 is a similar plot of the CG events with one exception. Variances are plotted symmetrically about the centroid that was calculated by omitting points lying on the main vertical channel. The lined circles near the bottom of the variances represent the maximum height of the main channel after it leaves the horizontal structure of the CG events. These heights were taken directly from the acoustic reconstructions in Appendix I.

There are important features to be seen in Figure 41.

(1) The average centroid height is 4.577 km and the average vertical variance is 2.533 km (neglecting main channel points).

(2) The average ratio of horizontal extent to vertical thickness is $2.73 \pm 1.58$. The CG events are horizontally extended from 1.15 to 4.31 times their vertical thickness. We are neglecting the main channel height; otherwise this ratio would be meaningless.

(3) It is difficult to say whether the CG events are dynamically connected to the cloud's freezing level because of the problem in locating
this level. However, 76% of the events occur within or at the edge of the altitude zone marking the extreme of its probable location. Most of the centroids lie nearer the 4.5 km upper 0°C boundary. The rawinsonde measurement at 0422 MST placed the 0°C level at 4.1 km and the -10°C level at 6 km.

6.5 Dual Events

Perhaps the most surprising structure form observed in the study was the occurrence of many events, 30% of the IC events and 35% of the CG events, which appear to have two major unconnected channel portions.

After careful analysis, we are firmly convinced that only one electric field event is present which could have accounted for the thunder records yielding these events.

There are three possible explanations for the existence of these dual events.

1. The uppermost portion in each case resembles a reflection of the lower portion. Consequently, if a good acoustic interface existed nearby, then we might expect these events to be simply explained. There is no physical structure of which we are aware within 15 km that could be responsible for this phenomenon.

Remillard (1960) postulated that the presence of graupel layers in the cloud could cause a thunder reflection.

However, our own observations indicate that no reflecting interface is present in the cloud. Figure 42, the thunder record for event 88-926, Appendix I, illustrates the feature common in all dual events.
The amplitudes late in the record are as high as amplitudes early in the record. If the uppermost portion were truly reflected energy, then we would expect sharply reduced amplitudes for the thunder from the uppermost portion. This is not observed. Thunder amplitudes from the uppermost portion and the lower portion in the dual events are quite similar.

(2) The dynamics of the storm and the distribution of charge in the thunderstorm cell may be responsible for these dual events. We postulate that this is indeed the case and delay further comment for Section 7.

(3) The existence of dual events may have arisen from inadequate sampling of the thunder signal. If an entire section of the thunder record yielded unusable correlation functions, a priori we cannot discount the possibility that this missing data would provide the linking channel connection.

Initially we proceeded under assumption (3) and it became obvious that two features of the thunder record led to the reconstruction of dual events.

(1) There were no neglected portions of the thunder record. The re-confirmed cross-correlation functions simply yielded time delay sets which led to the reconstruction of a dual event.

(2) There were areas in the data which yielded unusable or no cross-correlations at all. We have re-examined each of the thunder records illustrating this problem and are able to show in all cases that (a) the thunder amplitudes in the unusable sections were simply too small to be able to discern any signal coherence and (b) when signal amplitudes were high, but the waves were incoherent, the absolute propagation times
were not correct for a signal of any initial angle to fall within the region between the two separated channel segments.

Appendix II contains plots of seven dual events with our explanation for the gaps between channel segments. Our results concerning the possible connection of the region is presented in Table II. In each case low amplitude data sections yielded incoherent cross-correlation functions. There are three cases in which no incoherence occurred throughout the entire thunder record, yet the dual nature of the reconstruction still emerges. We do not conclude that there is absolutely no gap connection for these 3 cases, because it is possible that the high amplitude, coherent thunder simply masks the low amplitude, incoherent structure responsible for the gaps in the other cases.

The incoherence observed in nine of the dual events may have arisen from destructive interference of the sound emitted by a vertical channel with the sound emitted by the upper and lower horizontally extended parts of the discharge.

We conclude that these are dual events possibly connected by a vertical channel. This vertical channel linking the two regions must be confined to a small region of space; it cannot appear in the acoustic reconstruction to be volume distributed. Possible generating mechanisms for this type of event are discussed in Section 7.
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7. DISCUSSION

7.1 Limitations

Our measurements are subject to several limitations and therefore must not be over-interpreted. These limitations include (1) the lack of detailed electric field and electric field change information, (2) the inability to discuss true amplitudes and, therefore energy release considerations, (3) the lack of synoptic atmospheric parameter measurement, and (4) a physical limitation regarding thunder propagation.

Absorption of acoustic energy by the atmosphere limits the absolute length of channel we are able to reconstruct. The destructive interference of waves arriving at our array site from several directions leaves gaps in the reconstruction and could lead to misinterpretation of the discharge process.

The acoustic reconstructions in this study are excellent maps, however, of the major portion of lightning discharge paths. As a consequence of the absorption process, they must represent minimum lengths, and it should be understood that all numbers quoted represent minimum values.

These limitations are rather simple functions of experimental technique. Since the 1970 experiment many of these limitations have been overcome. Future work will involve the use of calibrated, ground-based, hemispherical electric-field change meters and balloon-borne electric field instruments.
The dynamic range of the acoustic data acquisition system has been doubled to 74 dB (acoustic) through the use of digital recording techniques.

We have shown that the lack of adequate atmospheric parameter definition does not seriously affect this study, but it could seriously affect results concerning the mature stage of thunderstorms. We will overcome this difficulty in the future by (1) operating our own balloon-borne weather measurement package and (2) coordinating our activities with other groups who will provide us with atmospheric data.

Absorption and interference effects are minimized by using several arrays of microphones separated from each other by a few kilometers. The feasibility of this type of deployment was demonstrated in our Colorado 1972 experiment and will become standard procedure in the future.

The acoustic reconstruction of a lightning channel represents the cumulative effect of all process involved in the lightning event. We have been careful to label our reconstructions as belonging to events because each event represents the sum of many sound-generating mechanisms, all occurring within the time period of one second. We cannot, with the acoustic record alone, resolve the portions of the signal belonging to these internal processes.

The majority of knowledge concerning lightning and intracloud electrical processes has been experimentally gathered through the use of fast response, high time resolution, electric field measuring instruments (Uman, 1969).
In the remainder of this section we attempt, where possible, to differentiate our insight into the structure of the lightning event and to cast the resulting implications against measurements made at the higher time resolution. This is possible because the aim of past research has been to build an integrated picture of the entire lightning event through measurement and explanation of each internal process whose cumulative sum represents the lightning discharge.

7.2 The Cloud-to-Ground Discharge

The essential features of a cloud-to-ground discharge are depicted in Figure 43b(1) through (8). When the potential difference between the earth and the cloud grows large enough, electrical breakdown begins near the cloud base and the negatively charged stepped leader begins to propagate earthward (Figure 43b1). As one of the branches of the stepped leader nears the ground, commonly a positively charged leader propagates upward from the earth and makes contact with the negatively charged branch (Figure 43b2). These two steps typically take 20 ms for a 3 km cloud base height. A return stroke ionizing wavefront propagates back up the channel causing a very high current (about 2 x 10^4 amps) to flow for about 40μ sec (Figure 43b3). Subsequently currents of 100 amps may flow for a few milliseconds. The return stroke is responsible for the lightning channel luminosity commonly observed.

After the negative charges have moved to ground and because strong electric fields still exist, the channel is left positively charged. Current momentarily ceases to flow as the channel becomes non-conductive.
because of recombination and diffusion of ions (Figures 43b4 and 43b5). The lightning event is now over unless more negative charge is made available to the top of the channel.

The theory to this point is straightforward and has been experimentally confirmed (Schonland, 1938), Malan and Schonland, 1947). Considerable controversy still exists, however, concerning charge movement models and initiation mechanisms, although the observed features are well known.

The most accepted theory for subsequent strokes (Uman, 1969) is that due to Malan and Schonland (1951) (Figure 43a). The progression of steps beyond Figure 43b4 and in 43a are identical for the visible portion of the channel, but are markedly different in the intracloud portions.

Following Malan and Schonland (1951), the entire region of negative charge is not neutralized by the first return stroke because of the low conductivity inside the storm cloud (Kasemir, 1960). Consequently a strong potential difference exists between the top of the positively charged return stroke and the negatively charged region nearby. The ground connection is broken because of the recombination effects discussed above. As a result of this potential difference, J-, or junction-streamers penetrate into the previously untapped region (t in Figure 43a) and initiates a dart leader process which returns negative charge to earth (t in Figure 43a). Once the dart leader reaches earth a new return stroke occurs, leaving the channel at positive potential with respect to the higher negative region in the cloud (t in Figure 43a). The sequence of events starts over with the initiation
of a new J-process (t in Figure 43a). Typically three or four strokes per flash occur, but as many as 26 strokes in one flash have been reported (Uman, 1969).

The J-process of Malan and Schonland (1951) sequentially taps negative charge at higher and higher altitudes in one homogeneous region of negative charge. They observe from their electric field measurements that the CG lightning flash is therefore almost entirely vertical, with no horizontal extent.

On the other hand, Bruce and Golde (1941) postulate that rather than the J-process, leader strokes originate at the top of the channel and move towards new regions of negative charge (Figure 43b, 5 and 6). The resulting dart leader and return stroke process, (Figure 43b, 7 and 8), thereby removes negative charge from adjacent negative charge centers. Bruce and Golde (1941) predict that CG lightning flashes should have intensive horizontal structure and present one photograph as verification.

Results of this study cannot differentiate between these two concepts, but after comparing our results to those reported in the literature, we are firmly convinced that any theory of cloud-to-ground discharges must account for their large horizontal extent.

Table III is a compendium of these measurements and our own. The last column in this table indicates each investigator's interpretation of his data as favorable for CG events to be vertically extended or horizontally extended.

It is surprising to us that with the published evidence overwhelmingly in favor of horizontally extended cloud-to-ground discharges, many papers published in 1973, c.f. Uman et al., (1973a, b), Vonnegut (1973),
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<tr>
<th>Investigator</th>
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***M&S theory is that due to Malan and Schonland (1941).

**H indicates large horizontal extent.
V indicates large vertical extent.
still are using theory and concepts pertinent to vertically extended discharges. Note: Uman (1973a, b) and M. Brook usually include a discussion on the effects of horizontally extended channels on their results. Dessler (1973) bases a model of infrasonic thunder on the existence of a horizontally extended charge distribution.

7.2.1 Dual Cloud-to-Ground Discharges

In Section 6 we discussed dual events, those events with one vertical main channel but two intracloud, horizontally branched regions with either no connecting link or a very weak connecting link.

We postulate that these events may be explained by a merging of the Bruce and Golde theory with the Malan and Scholand theory. Both of these theories require J-streamers to propagate to adjacent regions of negative charge.

We imagine that if the Malan and Schonland theory is operable, the J-streamers will permeate the negatively charged region in a highly complex and non-homogeneous fashion. This behavior near the top of the channel after the first return stroke has been reported in a very good study of lightning discharges with VHF radio direction finding (Proctor, 1971). Brook and Kitagawa (1964) report that intense microwave emission occurs during the dart leader stage. We observe strong horizontal, complex branching to occur at the height which most investigators believe is typical of the negative charged region center.

If the Bruce and Golde theory is correct, and the leader must bridge a relatively poorly conducting gap to a new negatively charged region, the channel connection will probably be well defined and considerable
branching will not occur. Proctor (1971) reports many instances of dual events with a "waist", or a region between 3.8 and 4.5 km free of branches, whereas the regions near 3.8 and 4.5 km were strongly branched with considerable horizontal structure (> 5 km in horizontal extent). This structure is verified by our study through an examination of our dual CG events. (See Section 6 and Appendix II.)

Malan and Schonland (1951) initially provided us with the idea. In searching for a physical reason for the existence of dual events we closely examined Figure 43b. If we overlay the outline of the lightning channel at each time interval, \( t_1 \) through \( t_5 \), in Figure 43b, we produce the channel on the extreme right. It, surprisingly, has most of the characteristics of our dual events and fits Proctor's observations except for the horizontally extended negative charge region.

Therefore, our J-process model invokes the existence of several thin layers of negative charge separated by regions of uncharged, low conductive atmosphere. A layer is thin if its vertical thickness is much less than its horizontal extent. We invoke the Bruce and Golde process for the vertical development of the cloud-to-ground discharge and the Malan and Schonland process for the horizontal development.

7.2.2 More on Cloud-to-Ground Events

The positive dipole model of a thunderstorm proposed by Wilson (1916, 1920) is not refuted at all. If we examine the ratio of the vertical extent of all our cloud-to-ground discharges to their horizontal extent, we find \( \frac{H_V}{H_H} = 1.04 \pm 3 \). That is, the CG event is usually as vertically
extended as it is horizontally, and the vertical component of the current is still considerable. Electric field and electric field change meters will still see the thunderstorm as bipolar, but the investigator must now include the horizontal structure in his analysis.

Another possibility for the existence of dual lightning events is that of the induced secondary. If we assume that the entire cloud is in a state of electrostatic stress owing to a rather complex charge distribution, then it is not unfeasible to assume that a lightning discharge, resulting in the readjustment of the field configuration in the entire cloud, will induce a secondary discharge at a different altitude with no channel connection existing between the two regions. We have no conclusive data to confirm this model, other than the acoustic reconstructions, and this model has not been tested theoretically.

7.3 *Intracloud Discharges*

Eventhough intracloud discharges are the most common type of lightning discharges, their investigations are limited, and results of these investigations vary widely.

The commonly accepted model for the intracloud discharge is that of the vertical positive dipole (Reynolds and Neil, 1955), (Smith, 1957), and (Ogawa and Brook, 1964).

Reynolds and Neil (1955) found from their analysis of 35 IC discharges that the vertical separation of charge was 0.6 km and that the horizontal separation of charge averaged 0.5 km. Their electric field measurements indicated a positive charge center above the negative.
The IC discharges occurred at a mean altitude of 5.5 km.

Smith (1957) found that 80% of his data (54 discharges) fit a vertical dipole model.

Ogawa and Brook (1964) have shown that their electric field data from IC discharges fit the vertical dipole model.

Workman et al., (1942) calculated the location of charge centers for 100 IC discharges and found that they were horizontally extended 5 times more than vertically. They measured an average vertical separation of 0.6 km and an average horizontal separation of 3 km. The maximum range observed was 10 km and minimum range was 1 km. The average height of the dipole center was 8 km.

Takeuti (1965) found that his data (3 discharges) fit a positive vertical dipole model even though the horizontal extent of the channels ranged from 2 km to 8 km. The discharge occurred at heights between 6 km and 12 km.

Ligda (1956) reports from radar observations the existence of extremely long, horizontal, intracloud discharges. He observed one discharge approaching 150 km in length.

Our measurements of 20 IC discharges, from Section 6, indicate the average horizontal extent is 7 km ± 3 km. The length of the maximum horizontal event recorded was 12.4 km. The average thickness, or vertical extent, of the IC discharge is 3.07 km for the upper level events with mean altitude of occurrence at 7 km and is 1.86 km for the lower level IC discharges occurring at a mean height of 4.5 km. The average ratio of horizontal extent to vertical thickness is $2.9 \pm 0.9$, much larger than previously reported.
Again, we observe that most measurements indicate extensive horizontal extent to the IC discharges, but most investigators report that the typical IC discharge involves the net vertical movement of charge (Kasemir, 1960; Vonnegut, 1973). However, we believe that this trend is slowly changing (Ogawa and Brook, 1969).

We find that most of the IC discharges in this study fall into two groups. (1) The point distribution is homogeneous and no apparent channel occurs. (2) There is a well defined IC channel observed.

This structure may be indicative of the charge distribution which existed prior to the discharge. We would expect very complex branching without one well defined channel to occur in highly homogeneous regions of charge.

That the structure of an IC discharge is complex is not in doubt. Brook and Kitagawa (1964) observe strong microwave emission associated with the initial section of the IC discharge. Kitagawa and Brook (1960), in a widely accepted paper, report that the late stage of an IC discharge is similar to the J-streamer process occurring between strokes in a CG discharge. This common feature which produces strong branching in a CG discharge probably produces strong branching in the IC discharge.

We observe this complex structure in many of our IC events, and Kitagawa and Brook (1960) infer from their electric field data that the IC discharge is a result of a "very complex and non-homogeneous charge distribution".
7.4 Storm Dynamics and Lightning Channel Structure

7.4.1 Convection and Temperature

The production and separation of charge in a thunderstorm is certainly associated with thunderstorm dynamics. We observe 65% of the lightning activity in the storm’s dissipating stage to occur near or a few hundred meters above the freezing level.

As turbulence and vertical convection in the dissipating stage ceases, we observe a decrease and final ceasing of lightning activity.

The IC activity occurs from successively higher altitudes in the cloud as the storm nears completion, and, in the mature stage when we observe much wind noise and turbulence, the lightning activity is at a maximum (Figure 19).

We are unable to relate our observations more closely to convection and temperature because of our lack of atmospheric data.

7.4.2 Storm Motion

The most surprising result of this study, and one which we intend to confirm, indicates that both the CG discharges and the IC discharges are aligned, horizontally, in the same direction defined by a narrow cone of azimuth angles.

There is an obvious ordered structure to the data (Figures 34, 35, 36, and 37), and we infer that one physical storm variable, perhaps direction of storm motion, is responsible for this striking alignment. The storm was traveling from east to west, ± 20°, as reported by the
National Weather Service and as deduced from Figures 20 and 23. The horizontal portions of all the channels are aligned perpendicular to the direction of storm motion.

This observation does not agree with Ogawa and Brook (1969) who report that their observations indicate the tendency of the horizontal inner cloud channel structures to align with the direction of storm motion. They also re-studied the IC discharge data of Workman et al., (1942) and report the alignment of IC discharges with the direction of storm motion.

It seems that the alignment of lightning channels in a common direction is well confirmed, although the reason for this alignment must still be explained.

7.4.3 Geomagnetic Field Alignment?

We observe from Figures 35 and 36 that the majority of intra-cloud lightning channels are roughly parallel to the geomagnetic field (D = 15° in Tucson, Arizona). This alignment is most probably accidental, but an examination of Ogawa and Brook's (1969) data from six different storms yields the average channel orientation in an azimuth cone of 80° spread, centered at an azimuth of 23°.

Ogawa and Brook (1969) also report on the data of Workman et al., (1942) which includes 105 IC discharges from two storms in 1940. We believe that these were in the southwestern United States. They show that all the IC discharges were remarkably well aligned along the direction 41° east of north with a 30° spread.
These measurements, in similar environments, but all in different years, from 1940 to 1970, all show similar alignment. That this is coincidental seems unlikely. It may be argued that since all the measurements were taken in similar locations, and since the wind directions reported for each of the three cases are always east to west or west to east, the alignment is occurring with respect to the direction of storm motion. Ogawa and Brook's (1969) report of alignment along storm motion is not verified by looking at their published data, but a tendency toward this alignment is evident.

On the other hand, our data shows misalignment with the direction of storm motion. We conclude that other storm parameters or possibly the geomagnetic field must be responsible for this remarkable repeatability of azimuthal alignment in the data.

Appendix III presents a plausibility argument for the existence of an anisotropic conductivity in the region of lightning channel formation which might account for this alignment.

7.5 Amplitude Considerations

We saw in Section 6 (Figure 27) that selective damping of high acoustic frequencies occurs as the range from the lightning channel source increases. The frequency content of the thunder signature is certainly representative of the energy per unit length in the lightning channel source.

Figures 44 and 45 are acoustic reconstructions of event 89-2004 (Appendix I) where we have identified each point in 44 with an amplitude
scale and a thunder type scale and each point in 45 with a peak frequency scale.

We observe that even though absolute range increases as we go higher in altitude, the amplitude of recorded thunder is still appreciable.

As altitude increases we observe a marked decrease in peak frequency for the inner cloud portions of the channel, indicative of the selective absorption process.

However, we observe that the main channel portion of the event (Figure 45) is composed of points whose peak frequencies vary widely. Many points have peak frequencies between 50 Hz and 100 Hz, while others peak between 12.5 Hz and 25 Hz. According to Few (1969), the energy per unit length is related to the observed peak frequency, implying a variable energy per unit length along the main channel. This is contrary to the constant energy per unit length which most investigators assume.

The energy per unit length delivered to the lightning channel at sea level is (Few, 1969; Holmes et al., 1971)

\[ E = 4.65 \times 10^4 \cdot \left( \frac{c^2}{f^2} \right) \exp \left( -\frac{z}{H} \right) \]  

(66)

where \( C \) = local sound speed, m/sec

\( Z \) = altitude in km

\( H \) = scale height = 8 km

\( f \) = frequency in Hz

and \( E \) = energy per unit length, joules/m.

We find that over the main channel, the energy per unit length changes by about one order of magnitude for frequencies taken from Figure 45.
7.6 Reflections

The possibility of an extremely efficient, acoustic interface existing inside a thunderstorm is remote. Remillard (1960) has shown that, for typical thunder frequencies, cloud particles and humidity changes cause the reflected signal intensity to be reduced by 60 db from the incident signal. This is well beneath the noise level of our recording system.

We have re-examined Remillard's conclusions concerning the reflections one might expect from rain and graupel particles in the thunderstorm environment. In both cases we consider Rayleigh scattering to be responsible for whatever reflection might occur because the average distance between graupel particles or rain particles is about 100 to 200 times the particle diameter. Consequently, we expect negligible interaction between particles, and the scattering is weak.

Maximum coefficients may be approximated by assuming that all the energy scattered out of an incident beam is backscattered. The Rayleigh scattering cross section is

\begin{equation}
\sigma_R = \frac{256\pi^5}{9} \left( \frac{f}{c_o} \right)^4 \alpha^6 m^2
\end{equation}

where \( f \) is the incident wave frequency in Hz, \( c_o \) is the local sound speed in m/sec, and \( \alpha \) is the radius of the scattering particle in meters.

The assumption of hard, immovable scattering is implicit here, that is, the boundary condition at a rigid surface is that the surface pressure equals twice the pressure of the incident wave.
If we assume that the spheres move with the medium as the acoustic wave passes by, we must impose the boundary condition that the pressure of the surface of the sphere equals the pressure of the incoming wave. This means that equation (67) would be reduced by $\frac{1}{2}$.

Writing (67) in terms of the geometric cross section $\sigma = \pi a^2$ and allowing the beam to penetrate a distance $L$ into the scattering medium, we calculate the reduction in intensity in traveling $L$ meters as

$$\frac{\delta I}{I} = \frac{256\pi^4}{9} \left( \frac{f}{c_0} \right)^4 a^4 (\eta \sigma)L$$

(68)

where $\eta$ is the number of scatters per unit volume.

Writing the reduction in wave amplitude for a path length $L$ as

$$\left( \frac{\delta A}{A_0} \right) = \left( \frac{\delta I}{I} \right)^{1/2}$$

(69)

we see

$$\frac{\delta A}{A} = \frac{16}{9} \pi^2 \left( \frac{f}{c_0} \right)^2 s^2 (\eta \sigma L)^{1/2}$$

(70)

The only change necessary to convert to the case for scatterers oscillating with the medium is to divide (70) by $2^{-1/2}$. This implies a 3db reduction in transmitted amplitude.

The ratio $\left( \frac{\delta A}{A_0} \right) = R$ is calculated for typical radii and concentrations of graupel particles and rain drops (Remillard, 1960), and the results are presented in Table IV. We have assumed $c_0 = 335 \text{ m/sec}$, $f = 50 \text{ Hz}$ and a 1 km path length.
In both cases we see that reflections from rain sheets or graupel layers are totally negligible with respect to the typical thunder signature in this study.

Little (1972) states that the scatter of acoustic waves by hydrometeors may be studied with the acoustic echo sounder out to ranges of 300 meters. The acoustic echo sounding system is several orders of magnitude more sensitive than is our system, and it is clear that we are able to distinguish no backscattered acoustic radiation from hydrometeors.

**TABLE IV: Acoustic Reflectivities**

<table>
<thead>
<tr>
<th></th>
<th>a, cm</th>
<th>n, cm⁻³</th>
<th>R</th>
<th>db</th>
</tr>
</thead>
<tbody>
<tr>
<td>graupel</td>
<td>2.5 x 10⁻²</td>
<td>6 x 10⁻⁴</td>
<td>1.2 x 10⁻⁷</td>
<td>-70 db</td>
</tr>
<tr>
<td>rain</td>
<td>10⁻¹</td>
<td>1.5 x 10⁻⁴</td>
<td>3.8 x 10⁻⁶</td>
<td>-54 db</td>
</tr>
</tbody>
</table>
8. **ERROR ANALYSIS**

8.1 *Propagation in the Array Vicinity*

It can be shown that the per cent error in the polar angle, \( \theta \), is approximately given by

\[
E_\theta = \frac{\sin \theta}{\theta} \left[ E_v + E_d + \frac{4}{3} \left( 2\tau_{13} - \tau_{12} \right) \Delta \tau x 10^2 \right]
\]

(71)

where \( E_v = \% \) error in the phase velocity \( \sim 3\% \)

\( E_d = \% \) error in the array dimension \( \sim 0.1\% \)

\( N = (\tau_{13} - \tau_{12})^2 + \frac{1}{3} (\tau_{13} + \tau_{12})^2 \)

\( \Delta \tau = \) time resolution in \( \tau_{ij} \sim 1 \) msec

For the first tabulated values in Table I, we find \( \theta = 50.2^\circ \) and \( E_\theta = 3.02\% \). This corresponds to a vertical displacement of \( \pm 300 \) meters at a horizontal range of \( 5 \) km.

The error in the azimuth of a reconstructed subsource point after propagation through an isothermal, homogeneous atmosphere is given by

\[
\Delta \delta = \Delta \gamma + \sin \delta \left[ \frac{1}{\sqrt{N}} + \frac{\cos \delta}{\tau_{13} + \tau_{12}} \right] \Delta \tau
\]

(72)

where \( N \) is defined in equation 32 and \( \Delta \gamma \) is the error in the magnetic declination. For the first tabulated values in Table I,

\[
\delta_1 = 333.2^\circ
\]
and with $\Delta \gamma = 0.03^\circ$, $\Delta \tau = 1$ msec, the error in the calculated azimuth becomes $\Delta \delta = 0.3^\circ$, corresponding to a horizontal displacement error of $\pm 26$ meters at a range of 5 km.

8.2 Ray Propagation

The errors in propagating a ray to its source through a real, turbulent, spatially varying atmosphere may at first sight seem insurmountable. However, several simplifying assumptions regarding ray propagation have been made earlier in this paper, and, in fact, the ray analysis is quite accurate for the absolute tolerances we are willing to accept.

8.2.1 Propagation Time

We have seen that we may neglect the errors in total propagation time owing to shock propagation, because the shock decays to a wave traveling at the local sound speed in a few meters. If the shock front were traveling at an average speed of 2 km/sec and decayed to an acoustic wave in 300 meters, the absolute range error would be 150 meters. We expect it is much smaller than this.

Typically 3 - 4 strokes occur in a lightning flash, spanning an absolute time not exceeding 0.3 sec. An error in propagation time of 0.3 sec yields a range error of 100 meters at a sound speed of 330 m/sec.

8.2.2 Atmospheric Fluctuations

Small scale fluctuations in temperature and wind velocity range
between 0.1°K to 1°K and 0.1 m/sec to 1 m/sec in the atmosphere. These
random fluctuations insure that a wave will have significant energy
scattered forward into a cone of about 1 radian after traveling 100 to
1000 wavelengths, although the θ = 0, forward direction will still be
preferred.

The result of these fluctuations is to distort a traveling wave-
front, and, as it crosses the array, the coherence of the wave will be
reduced. This is one justification for using cross-correlation techniques
to extract the arrival angles of the wave as it crosses the array.
The cross-correlation function may be distorted by atmospheric inhomoi-
geneties, but it remains sensitive to the direction of arrival of maximum
energy, i.e., the forward direction to an incoming wave, and will produce
a correlation peak at the time delay corresponding to this direction.

In many instances, there is very poor correlation between two
microphone receivers, resulting in uninterpretable correlation functions.
Appreciable incoherence does not usually arise from inhomogeneities in
the array vicinity because the distance between microphones, 100 m, does
not greatly exceed characteristic thunder wavelengths (1 m - 300 m).
We have seen previously that the observed incoherence is usually due to
destructive interference of waves arriving simultaneously at the array
from different source locations.

Consequently, reception problems cause us to delete points from
our analysis but do not cause us to assign large errors to the points
we keep.
8.3 Wavefront Analysis

8.3.1 Test Atmospheres

The source of possible range errors of greatest concern to us regards our lack of knowledge of the large scale temperature and wind velocity fluctuations in the atmosphere. By large scale, we mean those fluctuations whose scale size greatly exceeds our maximum wavelength. As long as this condition holds, we are in the domain of geometric optics, and ray tracing techniques are valid. (Implicit in this section is that all phenomena fluctuate on a time scale greater than our propagation time.)

While ray tracing is permissible, we do not usually know wind profiles better than \( \pm 100\% \) and temperature (in °C) profiles better than \( \pm 50\% \). We investigate the effects of this inadequacy in the remainder of this section.

Cumulative error in propagating a ray through a relatively unknown atmosphere is a complicated function of altitude, range and initial angles. We have chosen to examine the maximum errors expected after propagating a ray through several simple, although realistic, atmospheres. We end the section by illustrating the differences between straight-ray propagation and curved ray propagation through the atmosphere used in deriving the 1970 Tucson acoustic reconstructions.

The following eight figures, 46 through 53, illustrate the effects of propagating a wave through several test atmospheres. We assume a point source at the origin of an infinite halfspace and allow a wavefront
to expand outward through the atmosphere and follow its progression in time.

We have chosen to view the wavefront and its projections after 20 sec of propagation time, typical for most thunder events.

The spheroidal wavefront is cut by an east-west meridional plane, a north-south meridional plane and an orthogonal local tangent plane. The intersection of the wavefront and each of these planes defines a curve representing the actual location of the wavefront after 20 sec propagation. The position of the wavefront viewed in each of these planes may be read directly from the horizontal or vertical scales of all the following figures.

In some cases, for illustrative purposes, we have drawn projections of the spheroidal wavefront into the meridonal planes. Since these interior points represent projections, the actual deviations visible are projected deviations; consequently, another orthogonal plot is needed to calculate true deviations.

The wavefront from a point source after 20 sec propagation through an isothermal, homogeneous atmosphere is a sphere of radius

$$r = 40.1/\sqrt{T(\degree K)}$$
meters, and each orthogonal cut will produce a circle of radius \(r\) centered at the origin.

In order to examine the effects of variable atmospheres on the wavefront, we cast our results often against this reference sphere.

The most common atmospheric variation is the observed temperature lapse rate in the atmosphere of 6.5\(\degree\)K/km. We present the east-west meridonal cut in Figure 46. The curved, heavy lines represent the wave-
front, and the straight parallel lines are representative projections of the wavefront into this meridional plane.

At large declination angles near the local horizon the wavefronts propagate with almost the same speed, and the horizontal range difference between propagation in the isothermal atmosphere and the lapse rate atmosphere tends to zero while the vertical deviation is about 200 meters. As declination is decreased toward the local zenith, the horizontal difference increases and the vertical deviation decreases, passes through a minimum at $\theta = 45^\circ$ (an altitude of 4.8 km) and increases to 200 m at $\theta = 0^\circ$. We stress the fact that for a propagation time of 20 sec there is no detectable vertical difference between curved ray propagation and straight ray propagation for rays inclined 45$^\circ$ from the zenith at an altitude of 4.8 km.

Sources that originate at altitudes between 3.5 and 6 km and that subtend an angle between 30$^\circ$ and 60$^\circ$ with the local zenith do not suffer large vertical deviations from straight ray propagation if they propagate in an atmosphere of typical lapse rate.

We observed from the Tucson 1970 data that 90% of the initial angles generated fell into this polar angle range.

The effects of propagating a wave through an atmosphere characterized by a 6.5$^\circ$K/km lapse rate but with different surface temperatures are presented in Figure 47. Unless the surface temperatures are quite different, the range difference after 20 sec propagation is negligible.

Many times we have surface wind and temperature information available, but we lack data concerning upper levels. Figure 48 illustrates the range deviations one might encounter if he assumed a 6.5$^\circ$K/km lapse
rate, when the lapse rate was actually adiabatic, about 9°K/km. Although range deviations are present they are small. We notice again the shift in sign of the vertical deviation as altitude increases. Over most of the polar angle range and at altitudes between 2 km and 6 km the vertical deviations are negligible.

The previous three figures were all symmetric about the origin as we expect for changes only in the adiabatic sound speed. In Section 2 we saw that the presence of winds in the propagation medium introduces anisotropy into wavefront propagation because the direction of energy propagation is no longer normal to the wave vector and the wave velocity is not equal to the ray velocity.

The simplest wind configuration is a constant, horizontal wind, and the effects of propagating a wave through this atmosphere (chosen to be isothermal) are shown in Figure 49 where we compare to the no wind, isothermal case.

The effects of wind are immediately seen to be larger than the effects of temperature alone. A change in temperature, T, changes sound speed as T^{1/2}, while a change in wind adds vectorially to the ray velocity, producing a large change in C. We interpret Figure 49 in the following way. If we assume a first-look, isothermal, no-wind atmosphere to propagate rays, but allow a constant 10 m/sec wind, directed to the East, we find range errors on the order of ± 200 m in an east-west direction after 20 sec. As we look more normally to the wind direction, this horizontal range error decreases to zero. The entire wavefront remains a sphere whose origin has been shifted in the direction of the wind and has a radius equal to that of the no-wind, isothermal wavefront.
Our next complexity closely approaches the model of a real atmosphere. We introduce wind gradients in a logarithmic fashion by setting the wind according to Table V.

**TABLE V: Test atmosphere with wind gradients.**

<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>Wind Speed (East) m/sec</th>
<th>Wind Speed (North) m/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>10.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>20.0</td>
<td>0.0</td>
</tr>
<tr>
<td>20.0</td>
<td>20.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

There is a vertical wind speed gradient of 1 sec\(^{-1}\) to an altitude of 100 meters, a gradient of .01 sec\(^{-1}\) between 100 m and k km and a constant wind beyond 1 km to 20 km. These gradients are typical with the very strong gradient near the earth's surface and lesser gradients at higher altitudes.

The choice of no y-wind component allows us to examine the maximum effect of the gradient structure because the east-west meridional plane is the plane illustrating the maximum ray curvature and wavefront displacement.

The sharp gradient near the earth's surface causes a ray to be sharply curved into the ground or away from the ground according to the direction of the ray with respect to the wind direction. Maximum wavefront displacement between propagation in an isothermal atmosphere and propagation
in an isothermal-test wind atmosphere is about 1.5 km for rays inclined at polar angles between 85° and 90° (Figure 50). The vertical deviation quickly falls off as polar angle decreases and we enter the region of very small gradients (θ ≤ 80°). Rays that are not inclined much from the zenith and are oriented anti-parallel with the wind do not spend much time in the region of the strong gradients. They are quickly curved upwards and leave the layer. Once they leave the strong-gradient region, gradients are small and ray paths are consequently not so sharply curved. In this region most wavefront deviation is due to the presence of the wind magnitudes as illustrated in Figure 49.

Figure 51 is a plan view of the wavefront displacement presented in Figure 50. We present this figure to illustrate clearly the effect of propagating rays in a half-space, as is the earth-atmosphere system. The top dashed curve ends abruptly at θ = 30°. This curve terminates because the wavefront strikes the ground at this point, as evidenced in Figure 50. Reflection occurs, but we have chosen not to depict reflections in this study. We also see that we may expect horizontal deviations of about 1 km for rays propagated near the surface.

In order to examine the maximum effects of wind errors, we chose to plot the effects of propagation in a test atmosphere whose winds are twice the magnitude of those in Table V. These winds are unrealistically high (40 m/sec = 89 mph) and represent the maximum deviations likely to occur. As expected (Figure 52) the maximum deviations occur at very large polar angles and may be as large as 1 km, when compared to half-magnitude winds. The absolute deviations from an isotropic, homogeneous atmosphere are considerably greater, about 2.5 km near θ = 90°.
Figure 53 represents expanding wavefronts and their positions at the end of 20 sec and 40 sec propagation. The dashed line is the wavefront resulting from propagating through the wind system in Table V in a temperature environment defined by a 6.5°K/km lapse rate. The maximum displacement at 40 sec for low polar angles is on the order of 1 km. The larger propagation time, for a given azimuth and declination, allows the wavefront time to penetrate into portions of the atmosphere previously untapped by the wavefront at 20 sec.

A study of these plots yields the following conclusions:

1. The largest errors always occur for polar angles approaching 90°. This is due to the strong wind shears near the earth's surface.

2. Wind errors of ± 100% in each layer yield absolute range propagation errors of ~ ± 1 km after 20 sec propagation.

3. Temperature deviations from a 6.5°K/km lapse rate yield much smaller errors in absolute range, typically ± 200 m.

4. The relative error in locating points whose initial angles are similar is very small, ~ 50 meters typically.

Regarding the acoustic reconstruction of lightning channels, we believe that the preceding set of figures adequately demonstrates the need for some absolute measurements of winds and temperatures aloft. These measurements need not be extensive because the relative error between points is small, and once the gross nature of the atmosphere is defined, the accuracy of our reconstructions increases by about 50%.

The need for photographing lightning channels is demonstrated. We may use the known sources provided by the photographs to calibrate our array and measure the wind and temperature field between the lightning
channel and array. Propagation errors are then due only to our lack of knowledge of the intracloud atmospheric parameters.

8.3.2 Tucson 1970 Atmosphere

Figure 54 presents a ray plot through the atmosphere used to generate the reconstructions in this study. Wind magnitudes are usually small (see Figure 24), and the temperature structure is represented by a 6.5°K/km lapse rate with a minor inversion near the surface.

The striking feature concerning this atmosphere is the lack of ray curvature. At θ = 45° there is almost no difference between straight ray and curved-ray propagation. Because we believe this is a valid model atmosphere (Section 5), we believe that deviations in wind and temperature structure (Figure 17) probably occur to within ± 30% of their actual values, making the acoustic reconstruction accurate to within ± 500 meters when all error affects are accounted for.

8.4 Other Channel Mapping Techniques

In terms of absolute accuracy in the determination of lightning channel coordinates, we feel the acoustic technique offers higher resolution than techniques involving multi-station arrays of electric field meters. Electric field measurements are used to locate charge distribution centers, not coordinates of the lightning channel path. These measurements fundamentally depend on the assumption of a spherical charge distribution or a charge distribution whose largest dimension is much smaller than the height of the charge center and the distance between detectors.
Ogawa and Brook (1969) deployed two electric field stations, and, in order to describe the structure of the volume distribution of charge, they made three further assumptions:

(1) The altitude of the charge center for every first return stroke is assumed to be 3 km.

(2) The main channel to ground is assumed to be vertical. This assumption yields ~ 1 km error in altitude for a tilt of 20°.

(3) The extension in length of the channel between return strokes is constant and equal to 0.5 km and 1.7 km for long continuing-current strokes.

Ogawa and Brook (1969) give no data concerning errors in the charge center coordinates, but because of these assumptions we feel their data may be used primarily to deduce probable trends in the structure of the volume distribution of charge.

The electric field measurements are able to resolve field changes due to each component of a lightning flash and are necessary to complement the acoustic mapping of lightning channels for determining the distribution and subsequent readjustment of charge.

Proctor (1971) deployed one array of five VHF receiver stations, and, by taking time differences in VHF pulse arrivals at different stations, he was able to map the portions of the lightning channel which emitted coherent VHF radiation. His technique compares with the acoustic technique quite well, yielding horizontal range errors ~ 25 m and height errors from 200 m to 1300 m. As in our technique, the largest height errors occur at altitudes less than 2 km and are near 200 m for altitudes exceeding 2 km. He is able to resolve and map each component of the
lightning event, although regions of a stroke yielding no VHF radiation or incoherent radiation are missing.
APPENDIX I

SET OF ACOUSTIC RECONSTRUCTIONS

The following set of 37 figures are the acoustic reconstructions of all the lightning events which occurred in the dissipating stage of a thunderstorm cell near Tucson, Arizona on August 3, 1970 between the hours of 0352 and 0421, Mountain Standard Time. The figures are in consecutive time sequence and the time of lightning occurrence is printed in large type in the lower right corner of each figure.

Each figure presents three orthogonal views of the lightning event, the altitude versus east range, altitude versus north range and north range versus east range projections. The north versus east projection is often referred to as the plan view of the lightning event.

The event number is a bookkeeping number and is of no physical significance. All ranges are in kilometers.

Following the set of 37 figures are two figures representing the acoustic reconstruction of events 86-811 (0334:55 MST) and 86-658 (0334:16 MST). 86-658 is a reconstruction of two separately occurring but acoustically superimposed CG events. These events occurred in the storm cell's mature phase and have been included for calibration and cross-checking purposes as explained in the text.
TUCSON 1970 EXPT.
03AUG70
EVENT  88 -- 358
0354:04 MST
TUCSON 1970 EXPT.
03AUG70
EVENT 89 -- 1320
041107 MST
APPENDIX II

DUAL EVENTS

We present seven examples of dual events and illustrate on their reconstructions the probable link up region if one exists. Lightly shaded regions on five of the plots indicate (1) thunder existed at the indicated range but was low amplitude and incoherent, yielding no cross-correlation derived time delays, and (2) if a link up exists it must pass from the lower channel segment to the upper one through the shaded region.

Dashed lines with their associated question marks indicate that there was no data gap in the thunder signature due to signal incoherence. All cross-correlation factions yielded usable time delays and no points were generated to cross the gap. As explained earlier, this does not mean that a weak link does not exist; the high amplitude coherent thunder may have masked any thunder generated by a weak channel segment linking the two regions.

Propagation time and absolute range conditions yielded the shaded regions. To show exactly how each region was derived, we need three orthogonal plots, but in each case we present the projection which most clearly illustrates the existence of the dual events. The three orthogonal projections of each event may be found in Appendix I.

We present two projections of 88-1052, the altitude versus north range, and the plan view, north range versus east range, to illustrate the most complex event. These are two IC regions which may be linked owing to the presence of low amplitude incoherent thunder. The CG portion
and one of the IC portions do not appear to be linked because there is no data missing which could account for the link up. It appears that this event was made up of two CG channels and has an extensive IC portion. The existence of two CG channels is not too unusual. Four of the 17 CG events in this study appear to have two CG channels.
APPENDIX III

GEOMAGNETIC FIELD-ALIGNMENT OF LIGHTNING CHANNELS

The concept of the geomagnetic field alignment of lightning channels is surprising, and, at first sight, against physical intuition. However, with the strong suggestion of alignment present in the data, we have re-examined the interaction of the geomagnetic field with charged particles in the troposphere. A plausible explanation for a magnetic field effect is the existence of an anisotropic conductivity in the vicinity of a lightning discharge.

The presence of electrons and ions in the atmosphere allows current to flow and makes the atmosphere slightly conductive. It has been commonly supposed that, because of the presence of $\sim 10^{25}$ neutral particles/m$^3$ in the troposphere, collisional effects so predominate the behavior of charged particles that no geomagnetic field effects are apparent, c.f. Coroniti, P. 535, 1965.

This is indeed the case for the undisturbed, neutral atmosphere, but this has not been confirmed for conditions occurring inside a thunderstorm's highly charged and electrically stressed regions.

These regions typically consist of a mixture of electrons, ions and neutral particles. Three types of collisions occur: those between electrons and neutrals; those between ions and neutrals; and those between electrons and ions. The electron-ion collision frequencies are typically $< 2000$ sec$^{-1}$. Collision frequencies among these components
under conditions of local thermodynamic equilibrium (LTE) are

\[ \nu_{en} = 2.34 \times 10^{-15} \sqrt{\frac{T}{\eta_n}} \text{ sec}^{-1} \]

\[ \nu_{in} = 9.5 \times 10^{-16} \left( \frac{M_n}{M_i + M_n} \right) \eta_n \text{ sec}^{-1} \]

where \( T \) is the electron temperature in \( ^\circ \text{K} \), \( M_n \) and \( M_i \) are the neutral and ion masses, and \( \eta_n \) is the neutral number density in \( m^{-3} \). The ion-neutral collision frequency is based on a collision cross section inversely proportional to the relative velocity such that \( \sigma_{in} |v_i - v_n| \approx 9.5 \times 10^{-16} \text{ m}^3/\text{sec} \).

The collision frequencies must be compared to the gyro-frequency of a charged particle in a magnetic field

\[ \omega_j = \frac{e_j B}{M_j} \text{ rad/sec} \]

where \( e_j \) is the charge in coulombs, \( B \) is the magnetic field in \( \text{web/m}^2 \) (\( 1 \text{ web/m}^2 = 10^4 \) gauss), and \( M_j \) is the mass of the charged particle in kg.

The electron cyclotron-frequency in a magnetic field of 0.45 gauss is \( 7.92 \times 10^6 \) rad/sec and the nitrogen ion, NII, cyclotron-frequency is \( 154 \) rad/sec.

For room temperatures and ambient neutral densities of \( 10^{25} \text{ m}^{-3} \) the electron-neutral collision frequency is \( \sim 10^{11} \text{ sec}^{-1} \), exceeding \( \omega_e \) by 5 orders of magnitude. The ion-neutral collision frequency is \( 6 \times 10^9 \text{ sec}^{-1} \), exceeding \( \omega_i \) by 7 orders of magnitude.

If, however, one envisions a short time interval immediately after the initiation of a J-process if the neutral number density is low,
about \(10^{20} \text{ m}^{-3}\) or \(10^{21} \text{ m}^{-3}\), the collision frequencies of electrons and ions will be about the same as the cyclotron frequencies. In this case magnetic field effects will be noticed.

Three specific conductivities are defined as follows:

\[
\sigma_0 = \frac{\eta_0 e^2}{m_e v_{en}} \text{ m}^{-1} \text{ m}^{-1}
\]

\[
\sigma_1 = \eta_0 e^2 \left[ \frac{Z v_{in}}{m_i (v_{in}^2 + \omega_i^2)} + \frac{v_{en}}{m_e (v_{en}^2 + \omega_e^2)} \right] \text{ m}^{-1} \text{ m}^{-1}
\]

\[
\sigma_2 = \eta_0 e^2 \left[ \frac{Z \omega_i}{m_i (v_{in}^2 + \omega_i^2)} + \frac{\omega_e}{m_e (v_{en}^2 + \omega_e^2)} \right] \text{ m}^{-1} \text{ m}^{-1}
\]

where we have included only electrons, neutrals and NII, singly ionized nitrogen. \(Z_e\) is the ionic charge, \(\sigma_0\) is the conductivity parallel to the impressed magnetic field, or that which occurs in the absence of a magnetic field. \(\sigma_1\), the Pederson conductivity, is perpendicular to the magnetic field, and \(\sigma_2\), the Hall conductivity, is perpendicular to both the electric field and the magnetic field.

For an electron-neutral collision frequency of \(10^6 \text{ sec}^{-1}\) and an ion-neutral collision frequency of \(10^3 \text{ sec}^{-1}\), with \(\eta_e = \eta_i = 10^{15} \text{ m}^{-3}\), we find

\[
\sigma_0 \approx 30 \text{ m}^{-1} \text{ m}^{-1}
\]

\[
\sigma_1 \approx 1.5 \text{ m}^{-1} \text{ m}^{-1}
\]

\[
\sigma_2 \approx -0.3 \text{ m}^{-1} \text{ m}^{-1}
\]
Electric currents flowing in this medium of anisotropic conductivity may be expressed in terms of the components of \( \mathbf{E} \) parallel to and perpendicular to the magnetic field. Thus:

\[
\mathbf{j} = \sigma_0 \mathbf{E}
\]

\[
\mathbf{j}_\perp = \left(\sigma_1^2 + \sigma_2^2\right)^{1/2} \mathbf{E}_\perp
\]

If \( \mathbf{E} \) lies \( 45^\circ \) to \( \mathbf{B} \) such that \( \mathbf{E} = \mathbf{E}_\perp \), we may solve for \( \mathbf{j}_\perp \) in terms of \( \mathbf{j} \):

\[
\mathbf{j}_\perp = \left[\left(\sigma_1^2 + \sigma_2^2\right)^{1/2} / \sigma_0\right] \mathbf{j} \ll \mathbf{j}
\]

The current normal to \( \mathbf{B} \) is strongly damped. This impedance to current flow normal to \( \mathbf{B} \) arises as a consequence of the particles tendency to perform cyclotron motion about \( \mathbf{B} \), while the particles do not feel the influence of \( \mathbf{B} \) when moving parallel to \( \mathbf{B} \).

When lightning streamers are initiated, perhaps through the J-process, in a negatively charged region with a radially symmetric electric field, currents tend to flow along \( \mathbf{E} \) in all directions. However, in the presence of the geomagnetic field the only preferred direction of current flow is parallel to the field. Once streamers oriented along the field are developed, all subsequent lightning processes will follow these streamers.

The existence of an anisotropic conductivity depends strongly on the neutral number density while the electron density effects only
the magnitudes of $j_0$, $\sigma_1$, and $\sigma_2$. The value of $10^{15}$ m$^{-3}$ for $n_e$ and $n_i$ was chosen as a good median value in the range quoted by Uman, P. 231, 1969, in which he concludes that electron densities in dart leader channels probably lie between $10^{13}$ - $10^{19}$ m$^{-3}$. Neutral densities as low as $10^{20}$ - $10^{21}$ m$^{-3}$ are extrapolated from Brode's (1956) results concerning the time variation of density behind a strong shock from a spherical blast wave.

We postulate that number densities low enough ($n_n = 10^{22}$ m$^{-3}$) to allow anisotropy in the conductivity arise as a result of the sweeping away of particles in the channel vicinity by the shock.

This mechanism is meant, at the moment, to be a plausibility argument for the occurrence of lightning discharges which are aligned with the geomagnetic field. Further work will include the gathering of more experimental data on the orientation of lightning channels inside clouds and the derivation of a theory capable of explaining this alignment. We believe the plausibility argument herein is a good beginning.
ACKNOWLEDGEMENTS

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I want to posthumously thank my good friend Charles Conly of the Rice University Dept. of Electrical Engineering for his assistance in the computer programming for this project. He was killed in an auto accident while acting as our field engineer for the 1972 Colorado experiment.

Finally I would like to thank my wife, Reggie, who has worked as hard as I have for this project's completion. Her loyalty and encouragement have helped both of us to endure, and occasionally to enjoy, five years of graduate life.

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REFERENCES


Figure 1. A typical thunder event. The top three traces represent the output from the three microphones in our array. The bottom trace is the output from our parallel plate electric field change meter (EFM). EFM event A is responsible for the thunder in the top three traces. EFM events B, C, D are field changes not associated with thunder arrivals at the array. They probably represent distant intra-cloud discharges. The thunder is from lightning event 2.
Figure 2. Refraction and reflection at an acoustic interface.

The boundary condition at the interface between two fluids, continuity of the normal component of fluid displacement, leads to the conservation of the horizontal component of $\tilde{k}$ across the interface. $\tilde{k}$ is the wave vector. Subscripts $i, T, r$ refer to the incident ray, the transmitted ray and the reflected ray. $k_x$ is the horizontal component of $k$. $\theta$ is the incidence angle.
Figure 3. Molecular attenuation coefficients in dB/100 m for acoustic waves as a function of temperature for various frequencies, at 10% R.H. To convert to $K_m$ in m$^{-1}$, divide abscissa by 43.43 (after Harris, 1966).
Figure 4. The Tucson 1970 experimental array geometry. Microphone locations are represented by 1, 2, and 3. Indicated directions are with respect to magnetic north. In order to orient our coordinate systems with respect to true north, we add the magnetic declination at Tucson, 13°, to all azimuth angles.
\[ V_{Ph} = V_{h} + V_{z} \]

DIRECTIONS ARE WITH RESPECT TO MAGNETIC NORTH
Figure 5. Spherical correction geometry. The wavefront passes from microphone 1 to microphone 2, and the resulting $\tau_{12}$ will be that for a spherical wavefront. However, the theory interprets the time delay as that arising from a plane wave propagating from microphone 1 to microphone 2.
Figure 6. The Array Configuration. Notice that the IAP Lightning Lab is oriented with respect to true north, while the array is oriented with respect to magnetic north.
Figure 7. Shaping preamplifier response. Signal shaping is necessary due to the limited dynamic range of the analog recording system.
Preamplifier Shaping Characteristics

04 JAN 1971
Figure 8. The Non-Stationary Structure of Thunder.

(a) Time-varying mean square value.  (b) Time-varying mean value.

(c) Time-varying mean and mean-square value.
Figure 9. Example of a representative cross-correlogram. Peaks occurring to the left of the zero-lag position are defined as negative lag peaks. In this case, the signal from M3 lags the signal from M1 by 160 ms. The correlograms are normalized such that the amplitude of the peak value lies between -1 and +1. The bias at each end of the correlogram is a visible representative of the normalization of the correlogram to a zero mean value.
Figure 10. The Identification of Sub-Source Points.

The data from M1, Window A is correlated with the corresponding window on Microphone trace 2, and we see that Channel 2 lags Channel 1 by 273 ms. Channel 2 lags Channel 1 by 274 ms when the B windows are correlated. This difference of 1 ms is equal to the sampling error and is not significant, so the two correlations yield the same time delays.
Figure 11. Response of cross-correlation algorithm to calibration signals. The top function on the right side represents the cross-correlation between a 20 Hz sine wave (left-side, top) and an identical sine wave (not shown) which has been shifted later in time by 15 msec. The resulting correlation is also a 20 Hz sine wave.

The bottom function on the right is the auto-correlation of the time signal shown on the bottom left side. This calibration signal was created by recording a uniform 20 Hz signal and quickly sweeping to 50 Hz. The correlation window was chosen to bracket the transition from 20 Hz to 50 Hz. Since this is an auto-correlation, the maximum correlation coefficient (0.997) occurs at zero log. The peak width, 15 msec, corresponds to a frequency of 33 Hz and represents the time and amplitude weighted average of the frequency content of the original signal.
Figure 12. Cumulus Stage of Thunderstorm Development. The symmetry in the cross-section is for illustrative purposes only, and the numbers represent common occurrences - not limiting cases. (After Byers and Braham, 1949.)
Figure 15. Mature stage of Thunderstorm Development.

Visible symmetries represent idealized structure for illustrative purposes. Numbers represent commonly occurring values but are not limiting cases. (After Byers and Braham, 1949.)
Figure 14. Dissipating Stage of Thunderstorm Development. Visible symmetries represent idealized structure for illustrative purposes. Numbers represent commonly occurring values but are not limiting cases. (After Byers and Braham, 1949.)
Figure 15. Location of the Tucson 1970 Acoustic Profiling Experiment. The array location is marked with a cross and the arrows indicate some of the larger peaks visible on photographs of lightning flashes. Scale is 1 ft. to 250,000 ft.
Figure 16. Rawinsonde measurements of temperature and winds aloft at 0422 local time on August 3, 1970. The rawinsonde was launched at the Tucson International Airport, about 15 km WSW of the array site. Temperature in °K is plotted as a function of pressure and altitude, referenced to ground level. For comparison, the dry adiabatic lapse rate and the saturated adiabatic lapse rate are shown. Wind magnitude and direction as measured by the rawinsonde are shown in the middle figure. Wind directions indicate the direction to which the wind was blowing. The last two plots are of the wind components, allowing a close examination of the wind shear and relative magnitudes.
Figure 17. An illustrative drawing of temperature deviations commonly observed between the air inside a thunderstorm cell and the environmental air outside the cell. Figures $t_1$ and $t_2$ refer to the conditions in the cumulus stage and dissipating stage, while $t_3$ refers to conditions at a later time in a given stage at successively higher levels. These same temperature fluctuations will be observed at a given level as a function of time. (After Byers and Braham, 1949.)

Figure 18. Thermodynamic diagram showing calculated temperature lapse rates for various amounts of entrained air. The environmental lapse rate is a measured rate. All models of the lapse rate inside the thunderstorm cell lie closely to the environmental lapse rate, and the set of these curves lies roughly midway between the dry adiabatic lapse rate and the saturated adiabatic lapse rate. The environmental relative humidity scale is on the right. (After Byers and Braham, 1949.)
Figure 19. Storm summary, a plot of system operations and storm parameters on August 3, 1970. The extraneous noise level is derived by our subjective interpretation of the oscillogram record which was continuous from 0230 to 0420 local time. The portion of the storm reported on in this thesis is from 0355 to 0420 MST. Small vertical tick marks on the thunder level plot show that part of the acoustic data in the indicated time windows is overmodulated and absolute levels cannot be given.
Figure 20. Scatter plot of azimuth angles of observed cloud-to-ground discharges versus time of occurrence. The bottom figure was derived from photographs of CG events and the dot size is roughly correlated with brightness. The azimuth angles of all CG events derived from acoustic reconstructions is shown in the top figure as a function of time. The lower figure should overlay the upper figure in the portion below the first dashed line. The time section marked by the vertical arrow represents the dissipating stage of one thunderstorm cell reported in this thesis.
Figure 21. Histograms of time interval between thunder occurrence versus time. An event is defined as the time difference between two adjacent thunder events. Data presented does not distinguish between CG and IC events. 10 minute intervals between histograms cover entire recording range. The vertical axes represent the number of events in the nth 5 sec interval, and time is plotted on horizontal axes.
Figure 22. Polar histogram of no. of CG events per $10^\circ$ azimuth angle increment versus azimuth angle. The data spans the entire recording time, 0242 - 0420 MST, consequently spatial and time dependence is mixed. All CG data are derived from acoustic reconstructions. Scale is 2 events per inch for the shaded region and 10 events per inch for the dark region. The dark region is a similar plot for data taken from available photographs.
Figure 23. Polar histograms of no. of acoustically reconstructed CG events per n\textsuperscript{th} 10° azimuth angle increment versus azimuth angle for selected storm-time intervals. Scale is 2 events per inch. The sum of all histograms yields the polar histogram in Figure 22.
Figure 24. Model atmosphere taken from rawinsonde data in Figure 16.
TUCSON 1970 EXPT.

MEASURED SOUND SPEED PROFILE FROM ESSA RAWINSONDE ASCENT
0422 MST. 03AUG70
TUCSON INTERNATIONAL AIRPORT

HEIGHT (KM)
24.00
20.00
16.00
12.00
8.00
4.00
0.00
280.00
300.00
320.00
340.00
360.00
SPEED OF SOUND (M/S)

TUCSON 1970 EXPT.

MEASURED WIND VELOCITY PROFILE FROM ESSA RAWINSONDE ASCENT
0422 MST. 03AUG70.
TUCSON INTERNATIONAL AIRPORT
NORTH COMP. EAST COMP.

HEIGHT (KM)
24.00
20.00
16.00
12.00
8.00
4.00
0.00
-4.00
0.00
4.00
12.00
WIND COMPONENTS (M/S)
Figure 25. (a) This profile was generated by propagating thunder through an isothermal, homogeneous atmosphere. Dashed lines represent the photograph (not shown) of lightning event 2.

(b) This profile was generated by propagating thunder through an atmosphere characterized by vertical wind and temperature gradients (Figure 24).
ACOUSTIC PROFILES OF LIGHTNING CHANNEL 2
Figure 26. Thunder signature on 3 microphone channels of a
typical cloud-to-ground lightning discharge marked by the heavy, vertical
arrow at 0414:01 MST. Sections marked a, b, c, d, e, and f refer to
correlation windows in Figure 27.
Figure 27. Cross-correlation functions of selected data windows from thunder signature in Figure 26. The time windows from the 3 microphone channels are shown to the left of each set of cross-correlations. In each case, $\tau_{12}$ is determined from the bottom correlogram, $\tau_{13}$ from the middle correlogram and $\tau_{23}$ from the top correlogram. Ideally, $\tau_{13} - \tau_{12} = \tau_{23}$. The microphone channels are numbered from bottom to top, 1, 2, 3, respectively. Each time window represents .256 sec. The time delay, in msec, of the central peak is found to the right of each correlogram, and the cross-correlation coefficient of the central peak is found below the time delay.
Figure 28. The east-west versus altitude projection of a typical CG lightning discharge. This event occurred at 0441:01 MST. The time delay from lightning initiation to thunder onset was 11 sec, and the duration of detectable thunder was 31 sec.
Figure 29. The north-south versus altitude projection of the CG event described in Figure 28.
Figure 30. The plan view, east versus north, projection of the CG event described in Figure 28.
Figure 31. Thunder signatures on the 3 microphone channels of a typical intracloud lightning discharge. The lightning event is marked by the heavy vertical arrow at 0355:19 MST.
Figure 32. The plan view, east range versus north range, of both CG events (upper) and IC events (lower). Numbers represent time sequence of occurrence. Plotted points represent the centroids of the acoustic reconstructions.
Figure 33. Plan view, east range versus north range, of both CG events and IC events. All points represent the centroids of the acoustic reconstructions.
Figure 34. Cloud-to-ground events in plan view. (a) Each line is drawn symmetrically about its centroid and represents the horizontal projection of the axis of maximum variance (2 σ). These variance axes represent the direction and magnitude of the greatest horizontal extent of the lightning discharge. (b) Projections in plan view of the variance (2 σ) axis which yields the largest spread normal to its corresponding maximum variance axis in (a). Numbers represent time sequence of occurrence.
Figure 35. Horizontal variance of cloud-to-ground events.

(a) The lines represent the horizontal projection of the maximum variance of CG events. Centroid locations and variances were calculated without including points lying on the main channel-to-ground. The tips of indicated tick marks normal to the axes represent the location of the centroid weighted by vertical main channel points (Figure 34). Numbers represent time sequence of occurrence. (b) Projection in plan view of the variance axes which yield largest spread normal to the maximum variance axes in (a). Vertical channel points are not used.
Figure 36. Intracloud events. Maximum variance projection onto north-east plane. Dots denote IC centroid.
Figure 37. Intraclass events. Variance about axis which yields maximum spread normal to maximum variance projection in Figure 36. Dots denote IC centroids.
Figure 38. Time-of-occurrence versus altitude for all lightning discharges. Altitudes of the IC events are defined by the Z-coordinate of IC centroids. Altitudes for the CG events are defined by the height to the beginning of significant horizontal structure.
Figure 39. Location of the IC centroids. Lower level IC events and upper level IC events refer to observations taken from Figure 38.
Figure 40. Vertical variance of IC events versus time-of-occurrence. The variance axis (2σ) which yields maximum vertical projection is plotted symmetrically about each IC centroid. Black triangles at bottom indicate the time position of CG events. Numbers give the 2σ length of the maximum variance axis in the principal axis coordinate system.
Figure 41. Vertical variance of CG events versus time-of-occurrence. The variance axis (2σ) which yields maximum vertical projection is plotted symmetrically about each CG event. CG centroids were calculated after omitting points lying on the vertical, main channel. θ represent altitudes at which the vertical, main channel entered significant horizontal structure.
Figure 42. Thunder signature characteristic of dual event (88-926, Appendix I). Electric field change corresponding to this lightning discharge is not shown. The electric field event shown is that for event 88-1052, Appendix I.
Figure 43. (a) Illustration of the several stages in a cloud-to-ground discharge. Steps 1 - 4 are after Schonland (1938). Steps 5 - 8 are after Bruce and Golde (1941). (b) Malan and Schonland (1951) theory of the J-process occurring between strokes in a cloud-to-ground discharge. \( t_1 \rightarrow t_5 \) represent 5 time steps in the evolution of a multistroke discharge. The sum of all time steps equals the channel shape at the extreme right.

The acoustic reconstruction technique is only sensitive to the cumulative sum of the five processes. The region above cloud base in each case is assumed to be a homogeneous region of negative charge.
Figure 44. Acoustic reconstruction of cloud-to-ground event 89-2004 (Appendix I). East range versus altitude. "First arrival" is the first arrival of thunder yielding a usable point. "Precursor" indicates that the acoustic point was generated in the thunder occurring prior to the large amplitude clap.
Figure 45. Acoustic reconstruction of cloud-to-ground event 89-2004 (Appendix I). East range versus altitude. "First arrival" is the first arrival of thunder yielding a usable point. "Precursor" indicates that the acoustic point was generated in the thunder occurring prior to the large amplitude clap.
Figure 46. Heavy outlines represent the wavefront in the east-west meridonal plane after 20 sec propagation. Interior lines are projections of the wavefront into this plane. Unbroken lines refer to propagation in an isothermal (T = 15°C), homogeneous atmosphere, and dashed lines refer to propagation through an atmosphere characterized by a 6.5°C/km lapse rate with no winds.
Figure 47. Heavy outlines represent the wavefronts in the east-west meridional plane after 20 sec propagation. The curves represent propagation through an atmosphere characterized by a 6.5°C/km lapse rate but with different initial temperatures. $T_o = 20°C$, unbroken line. $T_o = 15°C$, dashed line. No winds are present.
Figure 48. Heavy outlines represent the wavefronts in the east-west meridional plane after 20 sec propagation. Interior lines are projections of the wavefront into this plane. Both curves represent propagation through atmospheres characterized by the same initial temperature, $T_o = 15^\circ C$, but with different lapse rates. $\Gamma = 9^\circ C$, unbroken line. $\Gamma = 6.5^\circ C$, dashed line. No winds are present.
Figure 49. Heavy outlines represent the wavefronts in a local tangent plane after 20 sec propagation. Interior lines are projections of the wavefront into this plane. The unbroken curves represent propagation in an isothermal \( (T_o = 20^\circ C) \), homogeneous atmosphere, and the dashed lines represent propagation in an isothermal \( (T_o = 20^\circ C) \) atmosphere in which a constant 10 m/sec wind is blowing from west to east.
Figure 50. Heavy outlines represent the wavefronts in the east-west meridional plane after 20 sec propagation. Interior lines are projections of the wavefronts into this plane. The unbroken curves represent propagation in an isothermal \(T_0 = 13.3^\circ C\), homogeneous atmosphere, and the dashed lines represent propagation through an isothermal \(T_0 = 13.3^\circ C\) atmosphere with varying winds and wind gradients. (See Table V.)
Figure 51. A local, tangent plane view of the wavefronts defined in Figure 50. This plane is normal to the east-west meridonal plane of Figure 50.
Figure 52. Heavy outlines represent the wavefronts in the east-west meridonal plane after 20 sec propagation. Interior lines are projections of the wavefront into this plane. The solid lines represent propagation through an isothermal \((T_o = 20^\circ C)\), homogeneous atmosphere. The dashed lines and the dash-dot-dash lines represent propagation through an atmosphere characterized by surface temperature \(T_o = 20^\circ C\) with a lapse rate of \(6.5^\circ C/\text{km}\). The dashed line atmosphere is also characterized by the test winds of Table V. The dash-dot-dash line atmosphere is characterized by the test winds of Table V where all wind magnitudes are doubled.
Figure 53. Outlines represent the wavefronts in the east-west meridonal plane. The interior lines represent positions of the wavefronts after 20 sec propagation, and the outer lines represent the wavefront positions after 40 sec propagation. Solid lines represent propagation through an isothermal \((T_o = 15^\circ C)\), homogeneous atmosphere, and dashed lines represent propagation in an atmosphere characterized by test winds (Table V), initial temperature \((T_o = 15^\circ C)\) and a lapse rate of \(6.5^\circ C/km\).
Figure 54. Acoustic ray plots in the east-west meridonal plane.

Propagation of rays through the 1970 Tucson rawinsonde atmosphere modeled in Figure 24. All rays started from initial azimuth angle of 90° (due East) in the right half plane, and all rays started from initial angle of 180° (due West) in the left half plane. The initial polar angles (from local zenith) are indicated at each ray path. No absolute propagation times are implied.