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A Study of the Neutron-Proton Singlet
Final State Interaction in
Proton-Induced Deuteron Breakup

by

Ronald Eugene Plasek

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

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May, 1974
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It has been the author's experience that the educational program received at Rice has been very meaningful and rewarding and that the practical experimental techniques learned have proved very valuable in job-related applications. It has been time well-spent.
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I. Introduction

A. The Three-Body Problem

Nuclear reactions with three final state particles have been studied quite frequently both in low- and high-energy physics. Recently, considerable interest has been generated by the interpretation of several experimental studies as evidence for possible states in the mass three system. In addition, considerable progress along theoretical lines has come about largely due to 1) the exact solution of the three-body integral equations by Fadeev [Fa 62, Fa 65, Fa 70], the approximation of the dynamics with separable potentials, and the attainment of a high degree of calculational sophistication through the use of high-speed computer systems.

Although the Fadeev equations are exact and in principle have a unique solution, the difficulty in obtaining numerical solutions usually precludes their convenient use by experimentalists in analyzing data. In order to use the Faddev equations, effectively one must write down the potentials. In general, the numerical results obtained predict an overbinding of the triton by about 2 MeV. Also there is disagreement with n-d elastic scattering data in the forward direction, and only the general features of nucleon-deuteron breakup spectra are predicted. In addition, the computation of these results requires a considerable financial support level. In view
of the above, experimentalists and many theoreticians have tended to rely on specialized simpler theories, the numerical results of which are more readily and economically attainable. These theories have been divided primarily into two areas: final state interaction (FSI) theory and quasi-free scattering (QFS) theory.

Two FSI theories commonly used to describe spectra in which one or more final-state particle pairs possesses low relative momentum is that of Goldberger and Watson [Go 64] and Phillips, Griffy, and Biedenharn [Ph 60]. The QFS theory commonly used to describe spectra in which the energy of the undetected "spectator" particle is low is the simple impulse approximation (SIA). It is not the purpose here to describe mathematically the results of the above-mentioned theories or to point out the applications of each, but to emphasize why these theories are used so commonly to fit experimental spectra even though "exact" solutions of the three-body problem indeed do exist.
B. Mass Three States

1. An Astrophysical Significance

Nucleosynthesis is an area in which a mass-three system plays an important role. A star consisting of only hydrogen can release nuclear energy in essentially one way, the proton-proton (P-P) cycle,

\[ ^1H(p,\beta^+\nu)^2D, \]

\[ ^2D(p,\gamma)^3He, \] \hspace{1cm} (I-1)

\[ ^3He(^3He,2p)^4He. \]

If \(^{12}\text{C}\) is also present in the stellar core, hydrogen burning may proceed by the so-called carbon-nitrogen-oxygen cycle (CNO), with the end result, again, helium. Helium is burned in a process known as the 3\(\alpha\) reaction:

\[ ^4He(\alpha,\gamma)^8\text{Be} \hspace{1cm} -95\text{keV}, \] \hspace{1cm} (I-2)

\[ \text{Be}(\alpha,\gamma)^{12}\text{C} \hspace{1cm} +7.4\text{MeV}. \]

An interesting note here is that \(^{12}\text{C}\) is destroyed easily by the \((\alpha,\gamma)\) reaction to yield \(^{16}\text{O}\); however, this is inconsistent with the relative amount of \(^{12}\text{C}\) observed (terrestrial + solar + meteorite). In view of this abundance, Hoyle postulated that the 3\(\alpha\) process ought to be characterized by a resonance at 7.68 MeV. Subsequently, the resonance, the so-called "second excited state of
carbon," was found to lie at 7.644 MeV. Thus, the end result of helium burning is $^{12}\text{C}$ and $^{16}\text{O}$. At successively higher temperatures, heavier elements are formed. The point to be made is that not only nuclear ground states but also nuclear excited states can play integral roles in nucleosynthesis cycles.

Recall now the second stage of the P-P cycle, namely,

$$^2\text{D}(p,\gamma)^3\text{He}.$$  

Were there any states in $^3\text{He}$, what would be the probability of the reaction proceeding through one of them, similar to the $^{12}\text{C}(7.64)$ state in the $\text{Be}(\alpha,\gamma)^{12}\text{C}$ reaction? Does the observed relative abundance of $^3\text{He}$ either preclude or perhaps imply the existence of $^3\text{He}$ excited states at particular levels? In any case, experimental evidence for $^3\text{He}$ excited states would have to be consistent with the P-P cycle and the observed relative abundance of the elements.

2. Review of Experimental Searches

Considerable effort has been directed recently into experimental searches for states in the three-nucleon system. Listed below are several studies in which the experimental results have been interpreted by the authors as possibly having been evidence for such states.

1) the existence of a trinucleon bound by about 1 MeV in the reaction $^3\text{H}(n,p)3\text{n}$ [Aj 65].
2) three states in $^3$He with excitation energies of 8.2, 10.2, and 12.6 MeV with widths $= 0.9$ MeV in the $^3$He(p,p$'$) reaction at 30.2 MeV. [Ki 66].

3) two states in $^3$He with excitation energies of 10.2 and 12.6 MeV in the $^6$Li(p,$\alpha$) reaction at 30.2 MeV. [Fo 67]

4) a peak in the d* excitation function in the D(p,pn)p reaction at 12.4 MeV $^3$He excitation energy [Ni 70].

However, all the above experiments since have been repeated several times by independent experimental groups with negative results obtained in all the cases (Table I-1). The consistently negative results lead one to suspect some aspect of the experimental techniques or associated data analysis. Thereafter, possible mass three excited states were searched for via a variety of different reactions. A summary of this experimental evidence perhaps is presented best in tabular form (Table I-1), taken in part from Ref. [By 71].

There have been three previous experimental studies of interest regarding the (p,d*) mechanism in the D(p,pn)p reaction: Niiler et. al. [Ni 70] have reported structure (a "bump") in the d* excitation function in the D(p,pn)p reaction at about 10.5 MeV incident proton energy, corresponding to a possible resonance at 12.5 MeV excitation in $^3$He. The two final state protons were detected at angles chosen such that the d* was emitted at 30° in the laboratory for each incident proton energy in the range
Table I-1

Summary of experimental evidence for mass three excited states
<table>
<thead>
<tr>
<th>Reaction</th>
<th>Incident Energy (MeV)</th>
<th>Angular Range (deg.)</th>
<th>Cross Section (mb/sr)</th>
<th>Remarks</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^3\text{H}(n,p)3n )</td>
<td>14.4</td>
<td>5-20</td>
<td>( \frac{d\sigma}{d\Omega}(5^\circ) = 12.5 \pm 5 ) integrated from ( E_p = 2.7 ) to ( 6.1 ) MeV</td>
<td>Evidence for trineutron</td>
<td>Aj 65</td>
</tr>
<tr>
<td>( ^3\text{H}(n,p)3n )</td>
<td>14.1, 18.2, 21.5</td>
<td></td>
<td></td>
<td>No evidence for trineutron</td>
<td>De 67</td>
</tr>
<tr>
<td>( ^3\text{H}(n,p)3n )</td>
<td>15.2</td>
<td>0,15</td>
<td>( \frac{d\sigma}{d\Omega}(5^\circ) = 25 \pm 6 ), ( \frac{d\sigma}{d\Omega}(0^\circ) = 10 \pm 3 ), integrated from ( E = 3.5 ) to ( 6.0 ) MeV.</td>
<td>No evidence for trineutron</td>
<td>Fu 67</td>
</tr>
<tr>
<td>( ^3\text{H}(n,p)3n )</td>
<td>20.8</td>
<td>0</td>
<td>( \frac{d\sigma}{d\Omega}(0^\circ) = 3.5 \pm 2 ) integrated from ( E = 6.5 ) to ( 14.4 ) MeV</td>
<td>No evidence for trineutron</td>
<td>Th 66</td>
</tr>
<tr>
<td>( ^3\text{H}(t,^3\text{He})3n )</td>
<td>22.25</td>
<td>8-20</td>
<td></td>
<td>Weak evidence for trineutron unbound by ( 1 - 1.5 ) MeV</td>
<td>Oh 68</td>
</tr>
<tr>
<td>( ^3\text{He}(\pi^-,\pi^+)3n )</td>
<td>140</td>
<td>15-40</td>
<td></td>
<td>Evidence for a broad resonance in ( 3n ) system ((E_x = 2, \Gamma = 12 ) MeV)</td>
<td>Sp 70</td>
</tr>
<tr>
<td>( ^7\text{Li}(n,^3\text{n})^6\text{Li} )</td>
<td>14-19</td>
<td></td>
<td>( \sigma_{\text{tot}} &lt; 1 ) mb</td>
<td></td>
<td>Fu 68</td>
</tr>
<tr>
<td>( ^3\text{He}(n,p)nnp )</td>
<td>14.4</td>
<td>5</td>
<td>( \frac{d\sigma}{d\Omega}(5^\circ) = 2 \pm 1 ) integrated from ( E_p = 2.7 - 6.1 ) MeV</td>
<td>No evidence for three-nucleon final state interaction</td>
<td>An 66</td>
</tr>
</tbody>
</table>
\begin{table}
\begin{tabular}{|c|c|c|c|}
\hline
Process & \(Q^2\) & \(E_x\) & Remarks \\
\hline
\(^3\text{He}(t,^3\text{He}')^3\text{He}^*\) & 22.5 & 8-20 & No evidence for \(^3\text{He}^*\) Oh 68 \\
\(^6\text{Li}(\pi^-,t)^3\text{He}^*\) & \(\pi^-\) capture & & No evidence for \(^3\text{He}^*\) Mi 69 \\
\(^3\text{He}(p,p')^3\text{He}^*\) & 25.0 & 25, 35, 50 & Upper limit = 0.25 \(3 < E_x < 15\) MeV \\
\hline
\(^3\text{He}(p,p')^3\text{He}^*\) & 25.5 & 11 - 70 & Upper limit = 0.3+0.1 \(E_x < 12\) MeV \\
\hline
\(^3\text{He}(p,p')^3\text{He}^*\) & 30.2 & 10-40 & \(d\sigma/d\Omega(15^\circ) = 2\) \(E_x = 10.2\) MeV \\
& & & Evidence for \(^3\text{He}^*\), Ki 66 \\
& & & \(E_x = 8.2, 10.2\) \(\Gamma = 0.9\) MeV and 12.6 MeV \(\Gamma = 0.9\) MeV \\
\hline
\(^3\text{He}(p,p')^3\text{He}^*\) & 30.6 & 17,20,26 & Upper limit = 0.15 with \(\Gamma = 1\) MeV \\
& & & No evidence for \(^3\text{He}^*\) Ha 68 \\
\hline
\(^3\text{He}(p,p')^3\text{He}^*\) & 30.9 & 15,26 & Upper limit (26\(^\circ\)) = 0.25 with \(\Gamma = 1\) MeV \\
& & & No evidence for \(^3\text{He}^*\) Ma 67 \\
\hline
\(^3\text{He}(p,p')^3\text{He}^*\) & 34.2 & 17.5 & Upper limit = 0.6 with \(\Gamma = 1\) MeV \\
& & 25.0 & Upper limit = 0.3 with \(\Gamma = 1\) MeV \\
& & & No evidence for \(^3\text{He}^*\) Bar 69 \\
\hline
\(^3\text{He}(^3\text{He},^3\text{He}')^3\text{He}^*\) & 44,53 & 5-42 & Upper limit = 0.12 \(E_x < 30\) MeV \\
& & & No evidence for \(^3\text{He}^*\), Sl 67 \\
\hline
\(^3\text{He}(\alpha,\alpha')^3\text{He}^*\) & 42 & 17.5,20.0 & Upper limits 0.2,0.4, \(0.2,0.15\) with \(\Gamma = 1\) MeV \\
& & 22.0,25.0 & & No evidence for \(^3\text{He}^*\) Wa 67 \\
\hline
\(^3\text{He}(e,e')^3\text{He}^*\) & 200 & 60 & Upper limit = \(3 \times 10^{-33}\) cm\(^2\)/sr \\
& & & No evidence for \(^3\text{He}^*\), Fr 67 \\
\hline
\end{tabular}
\end{table}
<table>
<thead>
<tr>
<th>Reactions</th>
<th>Cross Section</th>
<th>Energy Range</th>
<th>Additional Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3\text{H}(p,n)^3\text{He}^*$</td>
<td>30.3</td>
<td>10-30</td>
<td>Evidence for broad resonances in $^3\text{He}$ system at 16 MeV ($\Gamma = 9$ MeV) and 9.6 MeV ($\Gamma = 5$ MeV)</td>
</tr>
<tr>
<td>$^6\text{Li}(p,\alpha)^3\text{He}^*$</td>
<td>20.0</td>
<td>15-85</td>
<td>Upper limit (20) = 0.3 with $\Gamma = 0.9$ MeV</td>
</tr>
<tr>
<td>$^6\text{Li}(p,\alpha)^3\text{He}^*$</td>
<td>30.2</td>
<td>10-40</td>
<td>Evidence for $^3\text{He}^*$, $E_x$ Fo 67 =10.2,12.6 MeV</td>
</tr>
<tr>
<td>$^6\text{Li}(p,\alpha)^3\text{He}^*$</td>
<td>45.0</td>
<td>15-90</td>
<td>Upper limit (25) =0.03 with $\Gamma = 0.5$ MeV</td>
</tr>
<tr>
<td>$^2\text{H}(p,p)^2\text{H}$</td>
<td>3-9</td>
<td></td>
<td>No evidence for $^3\text{He}^*$ By 71 (T = 1/2)</td>
</tr>
<tr>
<td>$^2\text{H}(p,d^*)p$</td>
<td>9-13</td>
<td>30-77</td>
<td>Excitation function for $d^*$ production</td>
</tr>
<tr>
<td>$^2\text{H}(p,d^*)p$</td>
<td>7-17</td>
<td>25</td>
<td>Excitation function for $d^*$ production</td>
</tr>
<tr>
<td>$^6\text{Li}(p,pd)\alpha$ all possible two-particle coincidences</td>
<td>9,10</td>
<td></td>
<td>No evidence for p-d Va 68 final-state interaction, $&lt;5.5&lt;E_x&lt;12.5$ MeV</td>
</tr>
<tr>
<td>$^6\text{Li}(p,ad)p$</td>
<td>45.0</td>
<td>$\theta_\alpha=50, 50&lt;\theta_d&lt;100$ $\theta_\alpha=30, \theta_d=80,100$</td>
<td>No evidence for p-d By 71 final-state interaction $5.5&lt;E_x&lt;20$ MeV</td>
</tr>
<tr>
<td>Reaction</td>
<td>Cross Section</td>
<td>Energy</td>
<td>Notes</td>
</tr>
<tr>
<td>----------</td>
<td>---------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>$^3\text{He}(p,n)3p$</td>
<td>13.1</td>
<td>20</td>
<td>$\frac{d\sigma}{d\Omega}(20^\circ)=0.005\pm0.018$, integrated from $E_n=1.7-4.1$ MeV</td>
</tr>
<tr>
<td>$^3\text{He}(p,n)3p$</td>
<td>14.1</td>
<td>3-90</td>
<td>Upper limit $\frac{d\sigma}{d\Omega}(3^\circ)=0.5\pm0.3$ integrated from $E_n=3$ to 5.85 MeV</td>
</tr>
<tr>
<td>$^3\text{He}(p,n)3p$</td>
<td>30.3,49.5</td>
<td>10-30 2-60</td>
<td>Evidence for a broad resonance in 3p system at 9 MeV, $\Gamma=10.5$ MeV.</td>
</tr>
<tr>
<td>$^3\text{He}(p,n)3p$</td>
<td>24.9</td>
<td>8</td>
<td>Deviation from four-body phase space prediction</td>
</tr>
<tr>
<td>$^3\text{He}(p,n)3p$</td>
<td>44</td>
<td>6-25</td>
<td></td>
</tr>
<tr>
<td>$^6\text{Li}(^3\text{He},^6\text{He})3p$</td>
<td>53.2</td>
<td>14.1</td>
<td></td>
</tr>
<tr>
<td>$^2\text{H}(d,dn)p$</td>
<td>925±13</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>$^2\text{H}(p,d^*)p$</td>
<td>7-14.5</td>
<td>60,90</td>
<td>Excitation function for $d^*$ production</td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>&amp;</td>
<td>No evidence for three Co 66 nucleon final-state interaction</td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>&amp;</td>
<td>No evidence for An 65 three-nucleon final state interaction</td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>&amp;</td>
<td>Evidence for a broad Wi 69 resonance in 3p system at 9 MeV, $\Gamma=10.5$ MeV.</td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>&amp;</td>
<td>Deviation from four-body phase space prediction</td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>&amp;</td>
<td>No evidence for To 68 three-nucleon final-state interaction</td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>&amp;</td>
<td>No evidence for three Ba-PC nucleon final-state interaction</td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>&amp;</td>
<td>No evidence for $d-n$ Va 72 final state interaction</td>
</tr>
<tr>
<td>&amp;</td>
<td></td>
<td>&amp;</td>
<td>No evidence for $^3\text{He}^*$. Br 71</td>
</tr>
</tbody>
</table>
9-13 MeV. On the other hand, Van der Weerd et. al. [We 71] reported that the excitation function showed little structure between 7 and 17 MeV. In this report, the proton and neutron from the d* were detected at 25° in the laboratory for each bombarding energy. Also d* angular distributions were taken at seven energies.

Braithwaite et. al. [Br 71] point out that a more appropriate detection geometry for observing ³He resonances would be varying laboratory angles which would correspond, for different bombarding energies, to fixed d* center of mass angles. Also, it is pointed out that the corresponding center of mass angles of Ref. [Ni 70] varied from 65° to 69.2° and the center of mass angles of Ref. [We 71] from 53.3° to 60.9° over the energy range of the excitation functions. Excitation functions in the proton energy range 7-14.5 MeV were measured by detecting the two final state protons such that a p-n system with zero relative energy was emitted at 90° and 60° in the center of mass system. [Given the detection angles enumerated in Fig. 1 of Ref. [Br 71], the 60° angle appears to have been reported erroneously as 30°]. No evidence was found for structure in either the excitation function for the differential cross section or in the excitation function for the primary interaction factor.
3. Motivations for Present Study

In the three above-mentioned studies, different coincidence detection geometries were used as well as different data analysis techniques, all of them being model-dependent. In certain cases, better statistics were needed as well as additional data collected over a wider range of bombarding energies and angles. In another case, no reference was made to the determination of the neutron counter absolute efficiency vs. neutron energy. The maintaining of fixed d* laboratory detection angles (corresponding to varying d* c.m. angles) has already been commented upon and cannot be disregarded in view of the known angular dependence of the (p,d*) cross section. There are, in addition, quite a few items relating to analysis techniques employed (e.g., fitting of spectral line shapes to FSI models, background subtraction, solid angle transformations, choice of integration limits, etc.) which, because of their specialized and detailed nature, have been discussed in the data analysis chapter. In light of the above discussion, it was deemed necessary to reinvestigate the d* mechanism in the D(p,pn)p reaction monitored by the following principles:

1) Record both p_1-p_2 and p_1-n coincidence spectra simultaneously. This implies that the same reaction mechanism would be studied via two independent detection schemes in different kinematic/geometric configurations.
The results of the two investigations should agree, not only qualitatively, but also quantitatively, thereby affording a higher degree of confidence in the results obtained.

2) Record the coincidence spectra over a wide range of bombarding energies and angles.

3) Vary detection angles so that the $d^*$ center of mass angle would remain constant for varying bombarding energies.

4) Subtract coincidences from mechanisms other than the n-p singlet interaction.

5) Determine accurately absolute neutron counter efficiency vs. neutron energy, so as to assure reliable absolute cross section values.

6) Pay special attention to energy and time-of-flight calibrations, so that the n-p relative energy could be determined accurately.

7) Record, for each neutron counter coincidence, a pulse shape signal, to be used in discriminating against random coincidences due to gamma-rays.

8) Obtain "reasonable" statistics while not jeopardizing principle #2.

9) Obtain, over a given n-p relative energy range, cross section values which are not final-state-interaction model-dependent - that is to say, take into account only geometric and kinematic factors. This principle avoids the use of factors which become infinite in the region of low n-p relative energy.
C. Overview of Present Experimental Studies

Excitation functions of both proton-proton and proton-neutron coincidences corresponding to d* production in the D(p, pn)p reaction were taken in the proton bombarding energy range 8-12 MeV for five different d* center of mass angles. The proton from the d* will be denoted by p₁, the first emitted proton (the "recoil" or "inelastically scattered" proton) by p₂, and the neutron by n. A kinematically complete experiment may be performed by determining the momenta of any two of the three final state particles. In the present study, parameters associated with coincidences between both p₁-p₂ and p₁-n were recorded simultaneously.

Referring to figure I-1, a deuterated polyethylene foil was bombarded by a proton beam provided by a tandem Van de Graaff accelerator. The production of each d* caused a proton and a neutron with low relative energy to be emitted in essentially the same direction. The detection geometry for the p-n system was a solid state detector and a neutron scintillator positioned coaxially at the same angle (same side of beam axis). The angle of another solid state detector, corresponding to the recoil proton p₂, was calculated from energy-momentum conservation. The kinetic energy of the protons was obtained by measuring the pulse height of their signals from commercial silicon surface barrier detectors. The
Figure I-1
Schematic arrangement of scattering chamber, target, and detectors
neutron detector was a NE-218 liquid organic scintillator, viewed by two phototubes. The neutron energy was obtained indirectly, using the associated particle time-of-flight technique, i.e., the time difference between the neutron detection and the associated proton detection was measured. Also the time of flight difference between the detection of the two protons was measured and used in off-line analysis as a criterion for eliminating coincidences other than those from the D(p,pn)p reaction.

NE-218 and other similar organic scintillators also respond to sources other than neutrons - in particular, gamma-rays. However, within the scintillator, the electrons (produced by gamma-rays) and the protons and alphas (produced by neutrons) generate light having significantly different decay constants. Therefore, it was possible to identify the nature of most of the scintillation events by performing pulse shape analysis on the signal from the 12th dynode of one of the phototubes.

Absolute cross section normalization was determined indirectly by monitoring the elastically scattered protons from deuterium with an additional solid state detector mounted at a fixed scattering angle. The various parameters associated with each p-p and p-n coincidence were written on magnetic tape and analyzed at the end of the experiment. Various one- and two-dimensional displays of the parameters were monitored throughout the course of the experiment.
The yields in the d* peaks were integrated and presented as excitation functions and angular distributions. Also taken into account in the subsequent extraction of the (p,d*) cross section were the d* geometric detection probability, phase space effects, d* solid angle transformations, and the subtraction of coincidences from "background" reactions.
II. Experimental Procedure

A. Physical Arrangement

The incident charged particle beam was obtained from a High Voltage Model EN tandem Van De Graaff accelerator located in the Bonner Nuclear Laboratories, Rice University, Houston, Texas. The kinetic energy of the beam was determined by measuring accurately the magnetic field of the 90° analyzing magnet. This was accomplished by measuring, using a nuclear magnetic resonance (NMR) technique, the Larmor frequency of a proton/lithium sample placed within the magnetic field. The (relativistic) expression relating the Larmor frequency \( f \) and the beam particle kinetic energy \( T \) (MeV) may be written as

\[
    f = \frac{1}{K_0} \left[ T(T+2M) \right],
\]

where \( Q \) is the number of electronic charges on the particle, \( m \) is the rest mass of the particle in MeV, and \( K \) is a constant. \( K \) may be determined by observing the NMR frequency corresponding to a resonance or threshold reaction of known particle kinetic energy. In the present work, the beam energies were determined using the value 6.08420134 MeV/MHz for \( K \).

The beam entering the chamber was defined by a pair of insulated tantalum disks, situated 60 and 30 cm. in front of the target holder. Each 1.5 mm diameter disk
was followed by an antiscattering disk. In order to minimize gamma and neutron background, the beam was transported through lead-lined beam pipe 15 meters beyond the chamber and deposited in a beam dump housing which was shielded heavily with concrete. Special care was taken to assure that the $p_1$ detector, for angles more forward than 30°, was shielded by the target frame from possible beam scattering from the entrance port of the chamber. The beam current registering on the tantalum disks and the beam dump was monitored to facilitate beam tuning for optimum transmission and minimum slit scattering. During the actual data collection, the beam registering on the slit system was generally less than 2% of the beam registering at the beam dump.

The targets used were foils of deuterated polyethylene having typical thicknesses of several hundred $\mu$g/cm$^2$. It was found that in order to avoid target overheating and subsequent deterioration, the beam current on target had to be kept less than about 50 nanoamperes.

Targets were prepared by suspending deuterated polyethylene powder in chemically pure trichloroethylene at the boiling point. The resulting suspension was poured (quickly) onto clean glass slides. After the trichloroethylene had evaporated, the foil was floated off the slide in water. Approximate target thicknesses were determined by weighing the aluminum target holders with and without their foils.
The chamber used was designed by C. Joseph and A. Niiler and has been used in numerous three body experiments. The chamber is described briefly in Ref. [Ni 69]. The chamber lid was rotatable and had provisions for the mounting of solid state detectors. Angular readings were scribed on the lid and could be read accurately to 0.1° divisions. Detectors also could be positioned on the chamber floor each 10°, or mounted on a special calibrated arm capable of rotating up to ±5° with respect to each fixed (10°) position. The target could be rotated (externally) a full 360°.

The chamber was constructed of aluminum and a neutron exit port was provided on one side of the chamber. Here the chamber wall was only 1/8" thick, the total vertical angular definition of the window being 9° (i.e. ±4.5°). Independent horizontal and vertical chamber motion was possible to facilitate alignment. There were also entrance- and exit-viewing quartz crystals. The entire assembly was attached rigidly to the concrete foundation. Special care (see Appendix B) was taken to assure that the reaction chamber, slit system, and beam dump were aligned optically when viewed through the 90° analyzing magnet.
B. Detectors, Associated Signals, and Electronics

1. Charged Particle Detectors and Signals

Charged particles were detected in commercial silicon surface barrier detectors (Ortec 1000 µ). It was of interest to derive two signals from the detectors: 1) a linear signal, the pulse height of which is proportional to the energy deposited in the detector and 2) a fast logic timing signal, for use in a fast coincidence requirement with other associated detectors.

Referring to figure II-1, connected to each detector was a charge-sensitive preamplifier (Tennelec Model TC 133). In order to obtain a linear signal, the preamplifier output was fed to a spectroscopy amplifier (Ortec Model 451), the unipolar output of which, after having been delayed suitably, was sent directly to an ADC. [In order to avoid noise pickup, special care should be taken to guarantee that "clean power" is supplied to the detectors, that the preamplifiers are properly grounded, and that the preamp coaxial connectors are not loose!]

The fast timing signals were obtained by double-delay-line shaping the preamplifier pulse and then detecting its zero-crossover in a timing single channel analyzer (TSCA). The advantage of this technique lies in the fact that zero-crossover timing, in contrast to leading-edge timing, is essentially independent of the pulse amplitude. Delay line amplifiers used were Ortec Models 410 and 460.
Figure II-1
Electronics block diagrams for charged particle-charged particle and charged particle-neutron coincidence experiments

PRE  = preamplifier
DHLA = double delay-line amplifier
TSCA = timing single channel analyzer
FO   = fanout (fast logic)
SA   = spectroscopy amplifier
GDG  = gate and delay generator
ADC  = analog to digital converter
TAC  = time to amplitude converter
TSCA's were Ortec Models 420A and 455. TSCA's were Ortec Models 420A and 455. Using a tail pulse generator and a fast oscilloscope, the timing walk on the TSCA's was adjusted for minimum walk over a 10:1 dynamic range.

2. Neutron Detector and Associated Signals

The neutron detector used was a liquid NE-218 (Nuclear Enterprises) organic scintillator, enclosed in a thin-walled aluminum cylindrical container 12.7 cm. in diameter and 7.62 cm. thick with a glass window on one face. Two photomultiplier tubes (Amperex Model 56 AVP) viewed the scintillator through the glass window. The assembly was mounted inside a large cylindrical shield containing graphite, lead, and paraffin to minimize background radiation from directions other than directly from the target. The effectiveness of this shield may be verified by removing it and noting at least a ten-fold increase in count rate. This system was hung from an overhead crane which was free to travel on a pivotable I-beam, the axle of which was supported directly over the center of the scattering chamber. The neutron counter was thus free to rotate from 0° to 180°. A most complete discussion of the entire system may be found in Ref. [Re 72].

It was of interest to derive two signals from this detector: 1) a fast logic timing signal and 2) a pulse shape signal, used to help identify the nature of the scintillation event. The phototube anode signals were
stub-clipped (about 15 cm of shorted 50 ohm coaxial cable), inverted (EGG IT100), and fed to fast discriminators (EGG Tl40/N) which triggered on the leading positive edge of the pulse at approximately 250 mv. The discriminator outputs were sent into fast triggers (EGG TR104), each of which had sufficient delay (dead time) to eliminate multiple firings of the discriminators, caused by ringing of the clipped anode signals. The trigger outputs were stub-clipped (about 1 meter of shorted 50 ohm cable) in order to produce fast logic pulses approximately 5 nanoseconds wide. These logic pulses were then fed to a fast overlap coincidence unit (EGG Cl04), the output of which was used as the neutron timing signal.

Unfortunately, organic scintillators (NE-213, NE-218, stilbene, etc.) respond not only to neutrons but also to gamma-rays. However, neutrons and gamma-rays produce light scintillations with significantly different decay characteristics. For the above-mentioned scintillators, the rise time of the integrated light output is about 10 nanoseconds for gamma-rays and about 130 nanoseconds for neutrons [Ku 68]. Therefore, if the integrated light signal were double-delay-line shaped, the resultant pulses would cross zero at different times, depending on the nature of the incident radiation. Zero crossover timing on a double-delay-line shaped signal is equivalent to 50% constant fraction timing [Or 70]. As pointed out in Ref [Ku 68], the optimum discrimination occurs for 70-80% constant fraction timing; however, such fractions were
not available on general-purpose "shelf" modules.

The output from the fast overlap coincidence unit, after having been delayed in a Gate and Delay Generator (Ortec Model 416A), was used as the start signal for a time to amplitude converter (TAC). The zero-crossover of the double-delay-line shaped dynode signal was used as the stop signal. Various delay-line amplifiers, shaping constants, and TSCA's were used. However, best results were obtained with the instruments and settings noted in figure II-2.

The ability of the above system to distinguish between neutrons and gamma-rays is dependent on the low level light threshold for pulse shape discrimination (PSD). Electronically, this corresponds to the variable lower level discriminator setting on the TSCA. In contrast, the low level light threshold of the neutron counter fast logic timing signal is determined by the (internal) trigger levels of the fast discriminators. The setting of the PSD threshold is really a compromise between the fraction of neutron counter events analyzed and the separation of the neutron and gamma-ray peaks. It was determined experimentally with a Pu-Be source that as the threshold was raised, the neutron-gamma separation did indeed become somewhat better. However, the difference was not sufficient to offset the rapid increase in the percentage of events which fell below the threshold and thus were not subject to PSD analysis. Accordingly, the PSD threshold was set as low as possible without the TSCA triggering on noise.
Figure II-2

Electronics block diagram for PSD

56 AVP = Amperex phototubes
I = EGG IT-100 inverting transformer
DDLA = = double delay-line amplifier
TSCA = = timing single channel analyzer
GDG = = gate and delay generator
TRIG = = trigger
FAST DISC = fast discriminator

DDLA = Ortec Model 460  Integ. = 0.1 usec
TSCA = Ortec Model 455  Bipolar mode
Walk adjust optimized for best resolution
C. Data Capture System

The data was captured using an IBM 1800 on-line computer with an eight parameter interface [Jo 69]. It is possible, utilizing these facilities, to measure up to eight different experiments simultaneously, and, for each experiment, to measure from one to eight parameters. Each parameter is recorded as an analog-to-digital converter (ADC) channel number. The ADC resolution for each parameter may be chosen to be 64, 128, 256, 512, 1024, 2048, 4096, or 8196 channels full scale.

Associated with each of the experiments is a gate (standard logic) pulse, which causes the computer to read and store the channel numbers of the chosen ADC's along with another number (Tag) which identifies the data as having been an event in experiment n (n=1,2,...8). Fig. II-3 enumerates the parameters recorded under each tag and the source of each gate pulse.

For example, consider the p-p coincidence experiment (Gate B). Whenever particles 1 and 2 are detected within the time resolution of the T12 TAC, a T12 signal will be generated. Therefore, a slow coincidence between an E1, E2, and T12 logic pulses was used as the gating pulse and the parameters E1, E2, and T12 were recorded whenever such a coincidence occurred. The triple coincidence facilitated the elimination of unwanted signals that might cause coincidences unrelated to the three-body breakup.
Figure II-3

Parameters recorded under each tag

"slow" coincidence = 0.5 usec overlap

Tag A not used
<table>
<thead>
<tr>
<th>ADC</th>
<th>PARAMETER</th>
<th>TAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E1</td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>E1</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>E2</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>T1N</td>
<td>X</td>
</tr>
<tr>
<td>6</td>
<td>T12</td>
<td>X</td>
</tr>
<tr>
<td>7</td>
<td>PSD</td>
<td>X</td>
</tr>
<tr>
<td>8</td>
<td>MON</td>
<td>X</td>
</tr>
</tbody>
</table>
coincidences (noise, elastically scattered particles, etc.). This was accomplished by setting discriminators and windows on the TSCA's located outside the target room area. The multiple coincidence unit also facilitated observation of free $E_1$, $E_2$, or TAC spectra without modifying the electronics set-up.

Therefore, for each gate signal, a four-parameter group of numbers was stored in the computer and eventually written on magnetic tape:

1. Some pre-determined hexadecimal code which identified the following data as having occurred under Gate B.

2. Channel number of $E_1$ ADC.

3. Channel number of $E_2$ ADC.

4. Channel number of $T_{12}$ ADC.

Proper timing between the gate pulses and the associated ADC (parameter) pulses was essential. The interface was designed such that a gate pulse must precede each of its associated ADC pulses by approximately 0.5 microseconds. This was accomplished by delaying each parameter pulse in a Delay Amplifier (Ortec Model 427A) and each logic pulse in a Gate and Delay Generator. Electronics block diagrams for both charged particle-charged particle and charged particle-neutron coincidence experiments have been presented in figure II-1.
It should be noted that the linear $E_1$ signal was split into two different ADC's: one for the $p-p$ coincidence and one for the $p-n$ coincidence. The two tags were operated in non-timesharing mode. This technique eliminated the possibility of losing a coincidence event, were both a $p-p$ and a $p-n$ coincidence to occur simultaneously. This is possible if all three particles are emitted into their respective solid angles. In such a situation, assuming a time-sharing mode, the higher numbered tag event would be ignored.

The linearity of the ADC's was checked with a precision tail pulse generator; several ADC's showed significant non-linear characteristics. These non-linear ADC's were not used to record any of the following parameters: $E_1$, $E_2$, $T_{12}$, $T_{1n}$. 
D. Calibrations and Normalizations

1. Energy and Time of Flight Calibrations

Charged particle energy calibrations were performed in the customary manner. Single particle spectra were taken for various angles and bombarding energies and the peaks identified. For example, protons on deuterated polyethylene yields proton and deuterium peaks from p-d elastic scattering as well as the elastic and first inelastic peaks from carbon. The energies of the various particle peaks were calculated and a linear relationship between energy E and ADC channel number C was assumed.

\[ C = A + BE \quad \text{II-2} \]

Coefficients A and B were found by a least squares fit.

The D(d,n)$^3$He reaction was used to calibrate the charged-particle-neutron relative time-of-flight axis. A linear relationship between channel number and relative time-of-flight (TOF) was assumed:

\[ C = A + B (T + D) \quad \text{II-3} \]

where A is the TAC channel corresponding to coincident start and stop pulses at the TAC input, B is the TAC calibration (channels per nanosecond), T is the relative time of flight of the two particles, and D is the relative
electronic delay of the start and stop channels. Since A, B, and D are all constants, the above equation is of the form

\[ C = F + BT \]

F being a constant. B was determined by adding precisely known amounts of delay (Ortec Model 425 Calibrated Delay Boxes) to the TAC stop pulse and observing over a 200 nsec range the shifts of the time peak from a precision tail pulse generator. T may be calculated as the difference in absolute times of flight between the two particles from the following equation

\[ \text{TOF} = 71.9917 \times \text{SQRT}(m/E) \times L \]

where m is the particle mass (amu), E is its kinetic energy (MeV), L is the target to detector distance (meters), and TOF is the absolute time-of-flight of the particle (nanoseconds). The energies also may be calculated from conservation of energy and momentum (two-body kinematics). C was observed as the channel number of the relative TOF peak. Thus, F may be calculated.

2. Neutron Detector Absolute Efficiency Determination

Neutron detector absolute efficiency is a function of the type scintillator used, the geometry of the scintillator and coupled phototube(s), the neutron energy, and the detector threshold, below which light from recoil particles is not detected.
The following experimental arrangement was employed for the efficiency measurements: the $^3$He particles from the D(d,n)$^3$He were detected in a $\Delta E$-$E$ counter telescope. Pulses from the telescope were displayed in the $\Delta E$ vs $E$ plane so that the $^3$He particles could be separated from all other charged particle products. The neutron counter was placed on the corresponding neutron recoil axis. The $^3$He particles both with and without coincidence with the associated neutrons were counted simultaneously. As explained previously, both $E$ and $\Delta E$ signals were split into two different ADC's and the two tags operated in non-timesharing mode, since roughly one-third of all $^3$He particles detected would have been detected simultaneously by both coincidence channels.

The difference between the live-times associated with the two spectra was found to be consistently less than 0.1%. At least 2500 $^3$He-n coincidences were obtained for each coincidence-gated spectrum. Assuming 100% efficiency for the solid state detector, the absolute neutron detection efficiency was determined as the ratio of the number of $^3$He particles recorded with and without coincidence requirements. Thereby, the efficiency was determined for numerous neutron energies by varying the $^3$He and neutron angles as well as the incident deuteron bombarding energy. It was not possible to use the D(d,n)$^3$He reaction below neutron energies of 2 MeV because of its positive Q-value.
The low energy bias or lower light threshold is defined as the light output value below which the associated "fast" electronics will not respond. Electronically, this corresponds to the lower level threshold on the anode pulse fast discriminators. This threshold is usually expressed in units of the light produced by a given energy electron. A dynode pulse height spectrum, gated by anode coincidences, was taken with a $^{137}$Cs source in order to determine the threshold. This spectrum is presented in fig. II-4. The energy of the Compton edge of the spectrum is given by

$$E = \frac{E_\gamma}{1 + \frac{.51}{(1-COS\theta)E_\gamma}}$$

where $E_\gamma$ is the energy of the gamma-ray. For $^{137}$Cs, $E_\gamma$ is 0.6616 MeV and the Compton edge energy is 0.477 MeV. According to Flynn [Fl 64], the half-height of the Compton edge lies 4% above the Compton edge electron energy. Thus, the mean threshold was calculated, assuming a linear relationship between channel number and equivalent electron energy light output. The threshold corresponded to 50 $\pm$3 keV equivalent electron energy. The results of the efficiency measurements along with theoretical efficiency predictions are presented in fig. II-5. The actual neutron counter efficiency values used in the data analysis were
Figure II-4

Cesium-137 dynode spectrum, anode coincidence gated
Figure II-5a

A comparison of experimental absolute neutron counter efficiency points with theoretical predictions. Programs DETEFF and O5S are described in Appendix A. The DETEFF curve is the sum of curves A, B, and C, which represent the efficiency contributions due to neutron scattering and rescattering from hydrogen and the \((n,\alpha)\) reaction on carbon, respectively.

Figure II-5b

Predictions of program DETEFF over the neutron energy range 0-45 MeV. The total efficiency is the sum of the contributions from the indicated interactions with hydrogen and carbon.
the values calculated by program DETEFF (for a 50 keV threshold and a 50% fractional resolution) reduced by the percent of neutrons excluded (about 10%) based on PSD gamma-ray rejection. Descriptions of efficiency programs DETEFF and O5S are presented in Appendix A.

3. Neutron Counter Angular Calibrations

The angular calibration was performed experimentally by counting the number of $^3\text{He}-\text{n}$ coincidences from the $\text{D}(d,n)^3\text{He}$ reaction for a fixed $^3\text{He}$ detection angle and a slightly varying neutron recoil angle. Given a $^3\text{He}$ detection angle, the corresponding neutron angle was determined uniquely by conservation of momentum and energy (two body kinematics). The neutron counter angle was varied in $1/2^\circ$ to $1^\circ$ steps about the overhead track angular reading which should have corresponded to the recoil neutron angle. The $^3\text{He}-\text{n}$ coincidences were counted and normalized to the d-d elastic yield, measured by another solid state detector fixed at $50^\circ$. In this way, an angular calibration was performed for several different neutron angles, typical results of which are shown in fig. II-6.

4. Beam Current Integration

In the determination of any absolute cross section it is necessary to know the product $N_pN_t$, where $N_p$ is the number of bombarding particles and $N_t$ is the number of target particles per unit area. $N_t$ is difficult to determine for several reasons: 1. the relative concentration of deuterium in the target is not known accurately, 2. the
Figure II-6

Neutron counter angular calibration points
target deteriorates from exposure to the beam, the target thickness thereby decreasing with time, and 3. $N_t$ may not be uniform over the entire target surface. The product $N_b N_t$ was determined indirectly by placing a solid state detector at a fixed angle and monitoring the elastically scattered protons from deuterium. The quantity $N_b N_t$ was found from the relation

$$N_b N_t = \frac{N}{\sigma_{el} \Delta \Omega},$$

where $N$ is the number of scattered protons, $\sigma_{el}$ is the two body cross section in barns, and $\Delta \Omega$ is the solid angle subtended by the monitor detector. The cross section values used were those determined by Wilson [Wi 69]. These values are enumerated in Table II-1.

5. Detector Solid Angles

The detector solid angles were determined by circular tantalum disks positioned in front of each detector at a known distance from the target. All solid state detector disks were positioned at 7.62 cm and the neutron scintillator at 1.62 m. The disk diameters were measured under a microscope fitted with a precision micrometer. The resulting solid angles subtended were 0.531, 2.111, 5.229, and 0.08185 msr, corresponding to the $p_1$, $p_2$, neutron, and monitor detectors, respectively.
<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>Cross Section (mb/sr)</th>
</tr>
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<tbody>
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<td>7.00</td>
<td>129.5</td>
</tr>
<tr>
<td>7.50</td>
<td>126.3</td>
</tr>
<tr>
<td>8.00</td>
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<tr>
<td>12.50*</td>
<td>86.0*</td>
</tr>
</tbody>
</table>

* denotes extrapolated cross section value
6. Missing Mass Calculations

As an example of the accuracies obtained using the above-described energy and time-of-flight techniques, it is possible to calculate for each event in the \( p_1 - p_2 \) or \( p_1 - n \) locus in the reaction \( D(p,pn)p \) a "missing mass," using kinetic energies determined from the energy and time-of-flight calibrations. Formulas used are included in the kinematics section. That is to say, the missing mass, which is quite sensitive to the energy and time-of-flight parameters, for the \( p_1 - p_2 \) (\( p_1 - n \)) locus should correspond to the mass of the neutron (proton). Examples of this missing mass calculation, using a randomly selected \( p_1 - p_2 \) and \( p_1 - n \) locus, are shown in fig. II-7. The missing mass resolution for the \( p-n \) case increases somewhat with longer neutron flight paths.
Figure II-7

Relative energy and missing mass plots from the D(p,pn)p reaction

(top): p-p coincidences - missing mass = neutron
(below): p-n coincidences - missing mass = proton
III. Theoretical Aspects

A. The Singlet Deuteron

In the 2.225 MeV bound state of the deuteron both particles, at least 97% or so of the time, are in the symmetric space-spin state: \( L = 0, S = 1 \). By the exclusion principle, two neutrons or two protons (identical particles) could not occupy this state; therefore \( T_Z \) (isospin projection) can be only zero, and thus \( T = 0 \), an isospin singlet. The total wave function is indeed antisymmetric when formulated as a triple product of a space, spin, and isospin function.

Similarly, two neutrons, two protons, or a proton and a neutron may form an antisymmetric space-spin state, for instance \(^1S_0\), in which the particles are in the same space state but opposite spin state (\( S = 0 \), "singlet"). Here \( T_Z = -1, 0, +1 \), the states being referred to as a dineutron, a virtual deuteron or singlet deuteron, and a diproton, respectively. The experimentally determined n-p singlet scattering length of \(-23.6 \text{ fm} \) implies, from effective range theory, that the singlet deuteron is just slightly unbound.

When viewed as taking part in a sequential three body breakup, the \( d^* \) behaves similarly to a short-lived particle. For example, Phillips, Griffy, and Biedenharn [Ph 60] have modified Wigner's formulation [Wi 55] to show that for a system of two particles within a radius
a, each partial wave will have a time delay in decaying
given approximately by

\[ \Delta T_1 = \hbar \frac{d}{dE} [\delta_1 + \phi_1(a)] \]

where \( \delta_1 \) is the \( l \) th partial wave phase shift, \( \phi_1 \) is
the hard sphere phase shift, and \( E \) is the relative energy
between the particles. It has been calculated [Ni 70]
that the time delay for an \( n-p \) system at a relative energy
of 60 keV (the most probable excitation energy) would be
approximately \( 3 \times 10^{-22} \) seconds, during which time the first
emitted proton would have traveled about 72 fm. Thus,
the first emitted proton would be well outside the range
of nuclear interaction when the pair broke up. This \( S = 0 \)
short-lived \( n-p \) system is referred to as a "singlet
deuteron" or "virtual deuteron," often abbreviated \( d^* \) or \( d \).

A matter of interest is the fact that a cluster
consisting of a ground state deuteron and some other
particle with isospin \( T \) can form states of only isospin
\( T \), but a \( d^* \) and some other particle with isospin \( T \) can
form states of isospin \( 1+T \), \( 1-T \), or \( T \).

The complexity of the detection techniques for
unstable particles varies from reaction to reaction. For
example, consider \( t(b, ^8\text{Be}) \) reactions in which the Be
nucleus has a lifetime of about \( 3 \times 10^{-16} \) seconds before
breaking up into two alpha particles with 90 keV energy.
release. This time corresponds, through the Uncertainty Principle, to a width of only about 2 eV, which is orders of magnitude less than typical energies of nuclear reaction products. Therefore, the two alpha particles would be strongly correlated both in energy and in direction, which would facilitate simple detection geometry.

On the other hand, a $3 \times 10^{-22}$ second lifetime for a d* would correspond through the Uncertainty Principle to a rather large energy width -- over 2 MeV. One method of describing this (continuum) distribution is the density of states function of Phillips, Griffy, and Biedenharn [Ph 60, Ph 67], an example of which is shown in figure III-1. Although the most probable value for the n-p singlet system relative energy is about 60 keV, the distribution is somewhat broad, in agreement with the Uncertainty Principle prediction. Thus, the d* breaks up in the laboratory into a cone, the maximum half angle of which is given by

$$\theta_{1/2} = \frac{V'}{V^*}, \quad \text{III-2}$$

where $V'$ is half the n-p relative velocity in the d* center of mass system, and $V^*$ is the velocity of the d* in the laboratory. Since the n-p system is in an $S$ state, the breakup would be isotropic in the d* center of mass system.
Figure III-1

PGB density of final states function for n-p singlet interaction

-23.7 fm = n-p scattering length

3.5 fm = effective range of n-p interaction

2.49 fm = renormalization radius ("hard sphere" radius of interaction)
The broad density of states distribution and the negative Q-value (-2.225 MeV for (p,d*) reactions) imply that, for typical Tandem bombarding energies, $\theta_{1/2}$ will have a rather broad probability distribution. Thus, not all the neutrons and protons from the d*'s will be strongly correlated in energy or in direction. This indeed compounds the d* detection problem in that if one chooses to perform a kinematically complete experiment by detecting the proton and neutron from the d*, not only must separate proton and neutron detectors be utilized, but also a certain fraction of the breakup protons and neutrons will miss their respective detectors. This fraction is a very sensitive function of the d* breakup energy, the breakup direction of the particles, the d* kinetic energy, and the solid angles of the respective proton and neutron detectors. This is essentially a solid angle transformation.

Clearly the most favorable geometry for n-p detection would be both detectors positioned coaxially. Alternatively, one might perform a kinematically complete experiment by detecting in coincidence the first emitted proton and the proton from the d*, but in either case, the probability of any two final state particles entering their respective detectors decreases rapidly with the d* breakup energy.
B. Features of a Kinematically Complete Measurement of a Three Body Final State Reaction

Let a three body final state reaction be represented in the form \( b(t,12)^3 \). Assuming that the masses of the three particles are known, a kinematically complete description of the experiment requires that the vector momenta \( \vec{p}_i \) of all three final state particles be determined. This is equivalent to determining nine scalar variables \( (p_i, \theta_i, \phi_i) \), \( i = 1,2,3 \). Conservation of kinetic energy and the three components of linear momentum reduces the number of independent variables to five. However, the solutions of the energy conservation equation are not generally unique, due to its quadratic nature, thereby requiring six parameters to be determined for a complete description. It is thus clear that determining the energies of any two of the three identifiable particles at fixed detector positions will determine uniquely the momentum vector of the third particle, and thereby describe completely the three particle final state.

For a given \( (\theta_i, \phi_i) \) and \( (\theta_j, \phi_j) \), the parameter pair \( (p_i, p_j) \) [or equivalently \( (E_i, E_j) \)] must lie along a definite quadratic locus in the \( E_i - E_j \) plane, \( i \) and \( j \) denoting any two of the three particles. Each point on the \( E_i - E_j \) locus corresponds to a particular energy of the third undetected particle and to a particular relative energy for each of the three possible particle pair permutations.
Although all events must lie along the locus, it is their distribution along the locus which provides clues to the possible reaction mechanisms. The coincidence events usually are projected onto one of the energy axes \( (E_i) \); this spectrum then is proportional to the differential or "locus" cross section \( \frac{d^3\sigma}{d\Omega_i d\Omega_j dE_i} \), the \( \Omega \)'s representing the detector solid angles. When related to the undetected third particle energy or to the various particle pair relative energies, the presence or absence of structure in the projected spectrum and its general shape often provide indications as to mechanisms which may or may not have taken place. Furthermore, it is often possible to obtain indications of the relative contributions of different mechanisms, whenever more than one is enhanced significantly.

In general it is not possible to describe completely an observed coincidence spectrum in terms of any one particular mechanism. For example, in the \( p + p + n \) final state, one may find contributions from the singlet interaction in the \( p-n \) systems \((1,3)\) and \((2,3)\) and in the \( p-p \) system \((1,2)\). Also the triplet interaction is possible in the \((1,3)\) and \((2,3)\) \( p-n \) systems. Also present is the quasi-free scattering process and simultaneous breakup. Higher order effects also could be possible. One can usually, however, find kinematic regions in which one particular mechanism is strongly enhanced and others become much less probable.
C. Kinematical Relationships

Results of calculations pertinent to the subjects listed below will be presented in this section: 1) three-body locus, 2) sequential decay and relative energies, 3) statistical phase space, 4) missing mass calculations, 5) coordinate system transformation Jacobians, and 6) \( d^* \) detection efficiency. For the first five subjects, the straightforward but tedious algebraic manipulations have been reported numerous times in the literature; therefore, only significant results will be summarized, with special attention paid to the designation of the coordinate systems and angular conventions employed.

1. Three-Body Locus: In the reaction \( o+t \rightarrow l+2+3 \), the objective is to express the momentum of particle 2 in terms of the momentum of particle 1 (or any other cyclic permutation of the indices). The laboratory momentum \( p_2 \) is a double-valued function of \( p_1 \) and is given by [Si 65]

\[
p_2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

where

\[
A = \frac{1}{2M_3} + \frac{1}{2M_2}
\]

\[
B = \frac{1}{M_3} \left[ p_1 \cos \theta_{12} - p_0 \cos \theta_2 \right]
\]
\[ C = P_0^2 \left( \frac{1}{2M_3} - \frac{1}{2M_0} \right) + P_1^2 \left( \frac{1}{2M_1} + \frac{1}{2M_3} \right) - \frac{1}{M_3} P_0 P_1 \cos \theta_1 - Q \] \[ \theta_{12} = \theta_1 + \theta_2 \]

The \( P_i \) represent the laboratory momenta \( [P_i = \sqrt{2M_i E_i}] \) of the \( i \)th particle. \( Q \) is the customary "Q-value" for the reaction, the masses are in amu and the energies are in MeV. The angle convention is, for \( \theta_1 \) and \( \theta_2 \) on opposite sides of the beam axis, both are positive. For both angles on the same side of the beam axis, \( \theta_1 \) is positive and \( \theta_2 \) is negative. This convention applies the constraint of coplanar geometry with respect to the azimuthal angles \( \phi_i \). The solutions for \( P_2 \) represent valid physical solutions only for \( B^2 - 4AC \) and \( P_2 \geq 0 \).

2. Sequential Decay and Relative Energies

The reaction will be considered to proceed by two distinct steps:

\[ b + t \rightarrow i + (j,k) \] \[ i + (j,k) \rightarrow i + j + k \]

allowing for the possible cyclic permutations of the indices. Then the kinetic energy of the center-of-mass of the \((j,k)\) system is given by [Si 65]
\[ T_{jk} = \frac{1}{2(m_j+m_k)} \left( P_0^2+P_1^2-2P_0P_1\cos\theta_1 \right) \]  

and the relative energy of the \((j,k)\) pair relative to the \((j,k)\) center of mass is given by

\[ E_{jk} = E_0 + Q - T_i - T_{jk} \]  

where \(T_{jk}\) is given above.

If particle 1 were the first emitted particle, then

\[ E_{23} = E_0 + Q - T_1 - \frac{1}{2(m_2+m_3)} \left( P_0^2+P_1^2-2P_0P_1\cos\theta_1 \right) \]  

Similarly, if particle 2 were the first emitted particle, then

\[ E_{13}^\pm = E_0 + Q - T_2^\pm - \frac{1}{2(m_1+m_3)} \left( P_0^2+P_1^2\pm2P_0P_2\cos\theta_2 \right) \]  

Similarly, if particle 3 were the first emitted particle, then

\[ E_{12}^\pm = E_0 + Q - \frac{P_0^2}{2M_3} + \frac{P_0}{M_3} \left( P_1\cos\theta_1 + P_2^\pm\cos\theta_2 \right) \]

\[-\frac{1}{2} \left( \frac{1}{m_3} + \frac{1}{m_2} \right) \left[ P_1^2+P_2^\pm^2+2P_1P_2^\pm\cos\theta_{12} \right] \]

where the notations and angular convention are the same as in the previous section.
3. Statistical Phase Space

Consider particle 1 emitted first and the coordinate system to be the center of mass of particle 1 and the (2,3) recoil system (C' system). The differential cross section in terms of particle 1 may be written as

\[
\frac{d^2\sigma}{d\Omega_{1-2,3} dE_{1-2,3}} = \frac{8\pi^2}{\hbar^2} \frac{\mu}{K} |m|^2 \rho_1 ,
\]

where \(\mu\) is the reduced mass of the \((m_o, m_t)\) system, and \(K\) is the relative momentum of the bombarding particle and the target particle. The quantity \(\rho_1\) is the "density of states." \(\Omega_{1-2,3}\) and \(E_{1-2,3}\) are the solid angle and energy as determined in the \(C'\) coordinate system. \(|m|^2\) is the square of the matrix element for the reaction.

If the matrix element were assumed to be constant, then the form of the resulting expression for the cross section is referred to as a "pure phase space" or "purely statistical" energy distribution. It may be shown by evaluating the Jacobian

\[
\frac{\partial (E_1, \Omega_1, \Omega_2)}{\partial (E_{1-2,3}, \Omega_{1-2,3}, \Omega_{2-3})} ,
\]

that [Oh 65]
\[ \rho_1(E_1)dE_1d\Omega_1d\Omega_2 = \]
\[ \frac{m_1m_3p_1p_2^2dE_1d\Omega_1d\Omega_2}{|p_2[(m_2+m_3)/m_2]+p_1\cos\theta_1-p_0\cos\theta_2|} \]

where \( p_1, \Omega_1 \) are the momenta and solid angles in the laboratory system. The notations and angular convention are the same as in the previous sections.

4. Missing Mass

For light bombarding particles and target particles, there is usually little ambiguity in identifying the third undetected particle in a three-body final state. However, for more complex bombarding particles and/or target particles, one or more of the resulting three particles may be a complex nucleus, which may have been excited into one of its nuclear states. Therefore, given any three-body locus, one should be able to identify uniquely a "missing mass" with it. This technique was useful in the \( D(p,pn)p \) reaction for checking energy and timing calibrations, since there was no ambiguity as to the missing masses.

The missing mass \( m_3 \) is given by

\[ m_3^2 = W_3^2 - p_3^2 \]

where
\[ p_3^2 = \overrightarrow{F}_3 \cdot \overrightarrow{F}_3 = p_0^2 + p_1^2 + p_2^2 - 2p_1p_0\cos\theta_1 - 2p_0p_2\cos\theta_2 - 2p_1p_2\cos\theta_{12}, \]

\[ W_3 = W_0 + W_t - W_1 - W_2, \]

\[ W_i = E_i + m_i, \]

\[ p_i = \sqrt{2m_iE_i}. \]

\( E_i \) is the laboratory kinetic energy of particle \( i \) (MeV) and \( m_i \), its rest mass (MeV).

5. Transformation Jacobians

Certain solid angle transformations are used commonly for transforming experimental spectra from one coordinate system to another. Take, for example, the \( p-p \) and \( p-n \) coincidence spectra from the \( D(p,pn)p \) reaction, to which one would like to compare a final-state interaction theory. Final-state interaction theories are expressed in terms of \( E_{ij} \), where \( E_{ij} \) is the relative energy of particles \( i \) and \( j \) in the \((i,j)\) recoil center of mass (RCM) system, whereas the data are taken in the laboratory and expressed in the form \( d^3\sigma/d\Omega_1d\Omega_2dE_1 \), the \( E \) and \( \Omega \)'s all being quantities measured in the laboratory.

The required quantity is the Jacobian that transforms the solid angle from the \((i,j)\) RCM to the LAB system.

This Jacobian has been evaluated by Bronson [Br 64] and Simpson [Si 65] as.
\[ J_{ij} = \left[ \frac{\sum_{i=1}^{\Sigma m_i}}{\Pi m_i} \right] \frac{P(E_i, \Omega_1, \Omega_2)}{P_k P_i'} \]

where \( P_k \) is the momentum of particle \( k \) in the SCM and \( P_i' \) is the momentum of particle \( i \) in the \((i,j)\) RCM system.

The SCM is the system such that the vector sum of all three final state momenta is identically zero. \( P(E_1, \Omega_1, \Omega_2) \) is the phase space factor derived in section III-C-3.

\( E_i \) is the axis upon which the three-body locus has been projected, and \( \Omega_1, \Omega_2 \) are the laboratory solid angles of the two detectors.

\( J_{ij} \) is thus the Jacobian which transforms the particle-pair recoil center of mass solid angle to the laboratory system. If \( E_{ij} \) is the relative energy in the \((i,j)\) RCM system, then

\[ P_i' = \sqrt{2\mu_{ij} E_{ij}} \]

where \( \mu_{ij} \) is the reduced mass of the \((i,j)\) pair.
The components of $P_i'$ are given for the SCM by

\[ P_{iz}' = \sqrt{2m_iE_i} \cos \theta_i - \sqrt{2m_i} a_i, \]

\[ P_{iy}' = \sqrt{2m_iE_i} \cos \theta_i \sin \phi_i, \]

\[ P_{ix}' = \sqrt{2m_iE_i} \cos \theta_i \cos \phi_i, \]

where

\[ P_i' = \sqrt{P_{ix}'^2 + P_{iy}'^2 + P_{iz}'^2}, \]

\[ a_i = \frac{\sqrt{2m_i} p_0}{(m_0 + m_t)} \sqrt{\frac{m_i}{2}}. \]

The angular prescription for the above system assumes $\phi_i_{\text{lab}} = 180^\circ$ for the first emitted particle and all $\theta_i$'s are non-negative.
6. d* Detection Efficiency

Part I - p-n coincidence geometry

A d* with kinetic energy T and direction \( \alpha \) breaks up at point \( o \) (fig III-2). The c.m. neutron velocity is of magnitude \( u \) and direction \( \theta \) and that of the proton is \( u \) and \( (\pi - \theta) \). The proton detector is distance \( L_p \) from the breakup point and its radius is \( R_p \). Similarly, \( L_n \) and \( R_n \) refer to the neutron detector. It is assumed that the breakup is isotropic in the d* recoil c.m. system, that \( R_p/L_p \) and \( R_n/L_n \ll 1 \), and that the flux of d*’s is uniform over the solid angles defined by the detectors.

For a given ratio \( \rho \) of the breakup energy \( E^* \) to \( T \), the d* kinetic energy, and in the case where the d* velocity is between \( \alpha \) and \( \alpha + d\alpha \) and the neutron c.m. velocity is between \( \theta \) and \( \theta + d\theta \), \( F(\rho, \alpha, \theta) d\alpha d\theta \) is defined as the fraction of events such that the breakup particles enter their respective detectors. Integrating over \( \theta \) gives the detection probability as a function of \( \rho \) and \( \alpha \).

\[
G(\rho, \alpha) = \frac{\int_{\alpha}^{\alpha_{mx}} F(\rho, \alpha, \theta) 2\pi \sin \theta d\theta}{\int_{0}^{\pi} 2\pi \sin \theta d\theta} = \frac{1}{2} \int_{0}^{\pi} F(\rho, \alpha, \theta) \sin \theta d\theta
\]

The integration over \( \alpha \) then gives the detection probability...
as a function of \( \rho \),

\[
P_{\text{det}}(\rho) = \frac{\alpha_{mx}}{\int_0^{\alpha_{mx}} G(\rho,\alpha) 2\pi \sin \alpha d\alpha} = \frac{\alpha_{mx}}{\int_0^{\alpha_{mx}} G(\rho,\alpha) \sin \alpha d\alpha} \frac{1 - \cos \alpha_{mx}}{\int_0^{\alpha_{mx}} 2\pi \sin \alpha d\alpha},
\]

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where \( \alpha_{mx} \) is the maximum of the angles \( \alpha_p = \arctan \left( \frac{R_p}{L_p} \right) \) and \( \alpha_n = \arctan \left( \frac{R_n}{L_n} \right) \).

On a reference plane which is parallel to the detector surfaces and a unit distance from the origin \( o \), circles of radius \( \approx \alpha_p, \alpha_n \) and \( \alpha \) will be defined by the cones of the two detectors and the direction of the \( d^* \). \( \theta_p \) and \( \theta_n \) are the laboratory angles describing the direction of the breakup particles with respect to the direction defined by \( \alpha \). If \( u \) is the c.m. velocity of the breakup particles and \( v \) is the laboratory velocity of the c.m., it can be seen from fig. III-2 that

\[
\tan \theta_n = \frac{u \sin \theta}{v + u \cos \theta},
\]

and

\[
\tan \theta_p = \frac{u \sin \theta}{v - u \cos \theta}.
\]

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Since \( T = \frac{1}{2} m v^2 \) and \( \frac{1}{4} E^* = \frac{1}{2} \left( \frac{1}{4} M \right) u^2 \),
\[ \rho = \frac{E^*}{T} = \left[ \frac{u}{v} \right]^2 \]  

Therefore

\[ \tan \theta_n = \frac{\sqrt{\rho} \sin \theta}{1 + \sqrt{\rho} \cos \theta} \]  

and

\[ \tan \theta_p = \frac{\sqrt{\rho} \sin \theta}{1 - \sqrt{\rho} \cos \theta} \]  

For given angles \( \alpha \) and \( \theta \), the various polar angles of the breakup particles will also define approximate circles on the reference plane of radii \( \approx \theta_n \) and \( \theta_p \).

The function \( F(\rho, \alpha, \theta) \) is determined by the portion of the \( \theta_n \) circle which lies inside the radius \( \alpha_n \) and which also lies symmetrically opposite the portion of the \( \theta_p \) circle enclosed by the radius \( \alpha_p \). Table III-1 shows the various cases to be considered and the corresponding values of \( F \) in terms of \( \phi_p \) and \( \phi_n \). \( \phi_p \) is defined as that fraction of the circumference of the circle of radius \( \theta_p \) which lies inside the circle of radius \( \alpha_p \), and likewise for \( \phi_n \). For the intersecting circles, the law of cosines can be used to calculate

\[ \phi_p = \frac{1}{\pi} \arccos \left[ \frac{\alpha^2 - \alpha_p^2 + \theta_p^2}{2 \alpha \theta_p} \right] \]  

and
| \( F \) | 1 | \( \theta_n \) | \( \theta_p \) | \( \theta_p + \theta_n - 1 \), if \( \geq 0 \) | 0 |

| conditions for \( \theta_p, \theta_n \) and \( \alpha \) in terms of \( \alpha_p \) and \( \alpha_n \) |
|---|---|---|---|---|---|
| \((\alpha + \theta_n) \leq \alpha_n\) | \((\alpha + \theta_n) > \alpha_n\) | \((\alpha + \theta_n) \leq \alpha_n\) | \((\alpha + \theta_n) > \alpha_n\) | \((\alpha + \theta_n) \leq \alpha_n\) | \((\alpha + \theta_n) > \alpha_n\) |
| \(|\alpha - \theta_n| \leq \alpha_n\) and \(|\alpha - \theta_p| \leq \alpha_n\) | \(|\alpha - \theta_n| \leq \alpha_n\) and \(|\alpha - \theta_p| > \alpha_n\) | \(|\alpha - \theta_n| > \alpha_n\) and \(|\alpha - \theta_p| \leq \alpha_n\) | \(|\alpha - \theta_n| > \alpha_n\) and \(|\alpha - \theta_p| > \alpha_n\) |
\[ \phi_n = \frac{1}{\pi} \arccos \left[ \frac{\alpha_n^2 + \theta_n^2}{2\alpha_\theta} \right] \]

Part II - p-p coincidence geometry

Consider now the situation for a given d* breakup energy in which the velocity of the recoil proton (p_1) is between \( \alpha_2 \) and \( \alpha_2 + \Delta \alpha_2 \). From two-body kinematics, a unique d* emission angle \( \theta'(E^*) \) is defined. One may thus calculate \( \alpha_1 = |\theta' - \theta_1|, \theta_1 \) being the angle at the center of the solid angle \( \Delta \Omega_1 \). Similar to the p-n case, the \( \theta \) integration yields

\[
G(E^*, \alpha_2) = \frac{\int_{0}^{\pi} F(E^*, \alpha_2, \theta) 2\pi \sin \theta d\theta}{\int_{0}^{\pi} 2\pi \sin \theta d\theta} = \frac{1}{2} \int_{0}^{\pi} F(E^*, \alpha_2, \theta) \sin \theta d\theta.
\]

The integration over \( \alpha_2 \) gives

\[
P_{\text{det}}(E^*) = \frac{\int_{0}^{\alpha_{2\text{mx}}} G(E^*, \alpha_2) 2\pi \sin \alpha_2 d\alpha_2}{\int_{0}^{\alpha_{2\text{mx}}} 2\pi \sin \alpha_2 d\alpha_2} = \frac{\int_{0}^{\alpha_{2\text{mx}}} G(E^*, \alpha_2) \sin \alpha_2 d\alpha_2}{\int_{0}^{\alpha_{2\text{mx}}} 1 - \cos \alpha_{2\text{mx}}}
\]

where \( \alpha_{2\text{mx}} \) is the half-angle subtended by the solid angle \( \Delta \Omega_2 \). Since the d* kinetic energy \( T \) is known \( p = E^*/T \) is defined for each \( E^* \) and therefore \( F(E^*, \alpha_2, \theta) \) is defined in part I.
IV. Presentation of Data

A. Multiparameter Data Analysis Program

Program ZROOM is a versatile program for the analysis of multi-parameter data. The data was written on tape in the following format: TAG number, ADC channel number of word 1, ADC channel number of word 2, ..., ADC channel number of word n, (maximum = 8). Zroom is divided into essentially three phases:

1. Event sorting based on one- and two-dimensional windows
2. Special calculations
3. Storage and output

1. Event sorting
   a. one-dimensional windows - valid events may be restricted to lie inside or outside a window (in channel numbers) of any two of the n valid words.
   b. two-dimensional windows - an "X,Y" window (64x64) may be defined, where X,Y are any two of the n valid words. The events may be restricted to lie inside or outside the window. Up to a total of four such windows may be applied. Of the four, one may be designated a "background window," such that events within this window are subtracted from events in another specified "real window."
   c. Energy loss addition - The energy loss of a particle in a thin transmission detector (one of the n valid words) is converted and added to the total energy signal (another of the n valid words).
d. TAC subtraction - A real window and up to two accidental windows may be defined for a relative time-of-flight spectrum between two charged particles. In this way, events random in time ("accidental" events) may be subtracted statistically from events correlated in time ("real" events).

2. Special Calculations

Given such parameters as masses, bombarding energy, angles, Q-value, energy and time-of-flight axis calibrations, the following special calculations may be performed directly from the data tape:

a. Calculate $E_1$ and $E_2$ (energy) from the event channel numbers.

b. Calculate relative energies $E_{12}$, $E_{13}$, $E_{23}$.

c. Given $E_1$ and $E_2$, calculate the "missing mass" $m_3$.
   Also, a missing mass window may be defined such that the events may be restricted to lie within the window.

d. For each neutron spectrum, the neutron counter absolute efficiency may be determined from a look-up table as a function of the neutron energy calculated in a.

3. Storage and output

a. Store event $X,Y$ in a 64x64 matrix, or

b. Store any two of the following six quantities $X, Y, E_{12}, E_{13}, E_{23}, m_3$ in 1024 channel real arrays, or
c. Store any four of the above six quantities in 1024 channel integer arrays.

d. Project events in a) onto either the X or Y axis.

e. Print the 64x64 matrix and/or 1024 arrays.

f. Punch on cards the 64x64 matrix, the X or Y projections of the matrix, or the 1024 channel arrays.
B. Processing of Data

As was mentioned previously, the raw data was collected and written on magnetic tape and analyzed at the conclusion of the experiment. The following discussion describes typically the numerous steps taken (by ZROOM) in the reduction of the raw data.

1. Proton-Proton Coincidences from the D(p,pn)p Reaction

Three words (ADC channel numbers) were written for each p-p coincidence: E_1, E_2, T, where the E's corresponded to the kinetic energies of the two protons, and T, the relative time of flight between the two particles. In the first reading of the tape, the p-p coincidence events were sorted into a 64x64 (16 channels compressed into 1 cell) matrix in the E_1-E_2 plane. In the second reading of the tape, the parameter T was sorted into a 1024 channel spectrum. The peak in this spectrum contained the TOF parameter of the locus of real events, whereas the small flat background represented the events caused by two (random in time) signals occurring within the time resolution of the TAC ("accidental" coincidences).

In order to eliminate any random coincidences which might exist in the locus region, two windows of equal width were defined: 1. a "real" window, centered about the peak, and 2. an "accidental" window, located anywhere in the flat region. In the third reading of the tape, the events (E_1, E_2, T) were processed as follows: For T
within the real window, a +1 was stored cumulatively at coordinates $E_1, E_2$. For $T$ within the accidental window, a -1 was stored cumulatively at coordinates $E_1, E_2$. Otherwise, the event was ignored. The subtraction in the region of the locus was usually quite small ($\leq 1\%$), most accidental events having been concentrated in "accidental bands" at the energies corresponding to the elastic and inelastic peaks from the deuterated polyethylene target. From the resulting matrix, a two-dimensional window was drawn about the locus in the $E_1 - E_2$ plane (64x64).

In the fourth reading of the tape, the events were restricted to lie inside the locus window, and the TAC subtraction was duplicated. Also included in the fourth reading were data cards defining the masses, bombarding energy, angles, Q-value, and both $E_1$ and $E_2$ axis calibrations (number of channels per MeV). The events were projected onto the $E_1$ axis (64 or 1024 channels) and onto a linear relative energy axis (64 or 1024 channels). These spectra not only were printed but also were punched on cards as a convenient input medium for subsequent plot and analysis programs. The relative energy plots were particularly useful since subsequent analyses required that the yield be summed over a given relative energy region.
2. Proton-Neutron Coincidences from the D(p,pn)p Reaction

Three words were written for each p-n coincidence: E, T, PSD, where E corresponded to the kinetic energy of the proton, T, the relative time of flight between the proton and the neutron, and PSD, a pulse shape discrimination pulse to aid in neutron-gamma-ray identification of the source of the neutron scintillator event. In the first reading of the tape, the p-n coincidence events were sorted into a 64x64 E-T matrix. In the second reading of the tape, the PSD parameter was sorted into a 1024 channel spectrum. From the PSD spectrum and previous spectra from a Pu-Be source, a one-dimensional neutron window was determined and, in the third reading of the tape, only the events with PSD parameters within the neutron window were sorted into another 64x64 E-T matrix. From this matrix, a two-dimensional window was drawn about the locus in the E-T plane. Also present was a (small) uniform background in the E-T plane. Therefore, a two-dimensional background window, identical in width and length to the locus window, was drawn in the uniform background region of the E-T plane.

In the fourth reading of the tape, data cards were read which defined the masses, bombarding energy, angles, Q-value, proton energy axis energy calibration, and neutron associated time of flight axis calibration. From this data, the neutron energy was calculated for each event,
and from an array of 100 neutron counter efficiency values calculated by program DETEFF in the energy range 0 to 10 MeV, the corresponding efficiency was calculated with Bessel's interpolation formula (third order interpolation). This efficiency \( \text{EFF} \) was calculated for each valid real and background event, and the numbers stored in the various arrays were actually \(+1/\text{EFF}\) and \(-1/\text{EFF}\). In this way, neutron counter absolute efficiencies could be folded automatically into the spectra.

The events with PSD parameter within the neutron window were analyzed as follows:

1. for \((E,T)\) within the real window, a positive number \(+1/\text{EFF}\) was stored cumulatively at coordinates \((E,T)\).
2. for \((E,T)\) within the background window, the event was transformed to its corresponding coordinate within the real locus window and a negative number \(-1/\text{EFF}\) was stored cumulatively. [This is the reason for having the real and background windows of equal length and width.]
3. otherwise, the event was ignored.

The events were projected onto the proton energy axis and onto a linear relative energy axis. The spectra were printed and also punched on cards.
C. Excitation Function Histograms

In fig. IV-1 a typical p-p and p-n locus are presented as density plots. In fig. IV-2 the TAC and PSD spectra from the loci of fig. IV-1 are presented. The loci of fig. IV-1 have been projected onto the $E_1$ axis and plotted in fig. IV-3 along with the various particle pair relative energy curves. Finally, the projected loci for both p-p and p-n coincidence spectra are presented in fig. IV-4 as a family of excitation function histograms.
Figure IV-1

Proton-proton and proton-neutron locii presented as density plots

The p-p locus is displayed in the $E_1$-$E_2$ plane.
The p-n locus is displayed in the $E_1$-TOF plane.

$E_0 = 11.0$ MeV and the d* c.m. angle = 75°.
Figure IV-2

A sample TAC and PSD spectrum; $E^\circ = 11.0$ MeV and the d* cm angle = 75°.

The FWHM of the TAC peak is 20 nanoseconds.

The neutron and gamma-ray peaks are separated by approximately 15 nanoseconds.
Figure IV-3

Coincidence spectra (from Fig. IV-2) projected onto the E1 axis

(top): p-p projected locus - particles 1,2,3 are the proton (from the d*), the recoil proton, and the neutron, respectively. Relative energy curves for all three possible particle-pair permutations are also shown.

(bottom): p-n projected locus - particles 1,2,3 are the proton (from the d*), the neutron, and the recoil proton, respectively.

The bombarding energy is 11.0 MeV and the d* center of mass angle is 75°.
Figure IV-4

Excitation functions for various d* center of mass angles presented as a family of histograms. The p-n spectra have been corrected for the absolute neutron counter efficiency.
V. Analysis, Results, and Conclusions

A. Integrated d* Peak Laboratory Yields

Before proceeding with a proper analysis of the coincidence spectra, it is illustrative simply to integrate the laboratory spectra in a region of low relative energy. In presenting the family of excitation function histograms (fig. IV-4) the following convention of units was employed.

\[
\frac{d^3 \sigma}{d \Omega_1 d \Omega_j dE_1} = \frac{N(E_1) F_N}{\Delta \Omega_i \Delta \Omega_j \Delta E_1} \quad (\text{mb/sr}^2\text{-MeV}) , \quad V-1
\]

where \(N(E_1)\) is the number of coincidences between \(E_1\) and \(E_1 + \Delta E_1, \Delta \Omega_i, \Delta \Omega_j\) are the solid angles subtended by the particular pair of coincidence detectors considered, and \(\Delta E_1\) is the energy cell considered. \(F_N\) converts \(N\) (counts) into cross section values (barns) based on the monitor spectrum. To obtain the integrated cross sections, the yields were summed over a region corresponding to low n-p relative energy, say less than 200 keV.

\[
\frac{d^2 \sigma}{d \Omega_1 d \Omega_j} = \int_{E_1^{\min}}^{E_1^{\max}} \left[ \frac{d^3 \sigma}{d \Omega_1 d \Omega_j dE_1} \right] dE_1 \approx \sum_{E_1^{\min}}^{E_1^{\max}} \int_{E_1^{\min}}^{E_1^{\max}} \frac{d^3 \sigma}{d \Omega_1 d \Omega_j dE_1} \Delta E_1 , \quad V-2
\]

where \(d^3 \sigma/d \Omega_1 d \Omega_j dE_1\) is given by eqn. V-1 above and \(E_1^{\min}\), \(E_1^{\max}\) define the \(E_1\) region such that the n-p relative energy is less than 200 keV. These yields are presented
Figure V-1

Integrated d* yield for n-p relative energy less than 200 keV plotted versus incident proton bombarding energy and d* cm detection angle.
P-n

10.5 MEV

11.0 MEV

11.5 MEV

12.0 MEV

\(\frac{\text{MB/} \text{SR}^2}{\text{MB/} \text{SR}^2}\)

\(\theta_{\text{CM}}^*\)
as excitation functions and angular distributions in fig. V-1.

That the integrated laboratory cross sections for the p-p and p-n coincidence geometry do not agree, especially when presented as angular distributions, is not surprising, since, depending upon the coincidence geometry (i.e., either p-p or p-n), marked differences exist in the following kinematic/geometric quantities, all of which directly influence the yield in the laboratory system:

1) the probability of either pair of particles entering their respective detectors versus the d* breakup energy.
2) three-body phase space available.
3) the "d*-defining" laboratory solid angles.

In order to compare quantitatively the results of the two observation geometries, the laboratory yields must be transformed to the system center of mass. These transformations also are different because of different d*-defining laboratory solid angles.

These effects will be taken into account in subsequent sections where it will be shown that the p-p and the p-n coincidence cross sections agree quite well over a wide range of bombarding energies and d* detection angles and that no anomalous behavior was observed in any of the excitation functions or angular distributions.
B. Understanding Spectral Line Shapes

In order to understand qualitatively the shapes of the projected spectra (fig. IV-4) it is illustrative to construct the spectra from final-state interaction (FSI) theory. A FSI theory which has been used quite successfully [Ni 69, Ni 70] in describing spectra in which one or more particle pairs possess low relative momentum is the density of final states (DOFS) theory of Phillips, Griffy, and Biedenharn (PGB) [Ph 60]. The DOFS function is given by

\[
\text{DOFS} = \frac{\mu}{\pi \hbar^2 k} \left[ \frac{d}{dk} \left[ \delta + \phi(a_0) \right] - \frac{1}{2k} \sin 2 \left[ \delta + \phi(a_0) \right] \right], \quad V-3
\]

where \( k \) is the wave number corresponding to the relative energy of the two interacting particles, \( \delta \) is the usual scattering phase shift, \( \phi \) is the hard sphere scattering phase shift, and \( a_0 \) is the radius of interaction of the two particles. Utilizing the effective range theory phase shift,

\[
\cot \delta = -\frac{1}{ka} + \frac{kr_0}{2} \quad \text{,} \quad V-4
\]

and \( \phi = ka_0 \) for the hard sphere phase shift yields

\[
\text{DOFS} = \frac{\mu}{\hbar^2 \pi k^2} \left[ ka_0 - \left( \frac{1}{ka} + \frac{kr_0}{2} \right) \sin^2 \delta - \frac{k}{2} \sin 2(ka_0 + \delta) \right]. \quad V-5
\]

The DOFS functions for the p-p singlet and p-n singlet and triplet FSI are shown in fig. V-2b.
Figure V-2

a. Singlet DOFS functions transformed from RCM to LAB for p-p coincidence geometry. Particles 1,2,3 correspond to the proton from the d*, the recoil proton, and the neutron, respectively. The superscript 's' denotes the singlet interaction. The Jacobians (J's) have been described previously (Chapter III).

b. Singlet and triplet DOFS functions plotted versus interacting particle-pair relative energy (in the RCM).

c. Singlet DOFS functions transformed from RCM to LAB for p-n coincidence geometry. Particles 1,2,3 correspond to the proton from the d*, the neutron, and the recoil proton, respectively. The Jacobians have been described previously (Chapter III).

In a and c, the bombarding energy is 11.0 MeV and the d* c.m. angle is 75°.
The DOFS function usually is transformed from the RCM (recoil center of mass of the two interacting particles) to the LAB system by the appropriate transformation Jacobians which have been discussed earlier. Results of such a transformation are shown in fig. V-2a and V-2c. Fig. V-2a depicts the p-p coincidence geometry and fig. V-2c, the p-n geometry for the D(p, pn)p reaction at an incident proton bombarding energy of 11 MeV and a d* c.m. angle of 75°.

In fig. V-2a only the p-p singlet and n-p singlet FSI contributions are shown. The triplet contributions have essentially the same shape as the phase space factor and are about an order of magnitude less in amplitude. The Jacobian $J_{13}$ which transforms the (1,3) system from the RCM to the LAB is plotted also. [Although not graphed as such, $J_{13}$ becomes infinite for zero relative energy.]

In fig. V-2c only the same three FSI contributions are shown. The triplet contributions, as well as the phase space factor, are more than an order of magnitude less than either the (1,3) or (2,3) singlet terms.

Note how the experimental spectra (fig. IV-4) are in agreement with the shapes predicted by the PGB theory. For example, the p-p coincidence spectra are characterized by a peak corresponding to the n-p singlet (d*) FSI and by a second peak corresponding to the overlap of the p-p singlet FSI with the n-p singlet FSI between the recoil proton and the neutron. The PGB theory predicts an improved separation of the two peaks (i.e., d* peak and
"background reactions" peak) as the range of kinematically possible $E_1$ values increases. This occurs for increasing bombarding energies and for forward d* angles. This may be observed in fig. IV-4 by comparing a p-p coincidence spectrum for say 11-12 MeV, $\theta^*=50^\circ$ or $60^\circ$ with one at 8-9 MeV for $75^\circ$ or $80^\circ$. The n-p coincidence experimental spectra also are in agreement with the PGB theory, which predicts one large d* peak superimposed upon a relatively small flat contribution from the other singlet FSI's.

Clearly some of the coincidences in the low n-p relative energy region are due to processes other than the n-p singlet FSI (d*). Based on calculations similar to those of fig. V-2, the PGB theory can be used to predict, for a fixed low n-p relative energy region, the "overlap" of the d* peak with the other "background" mechanisms for both p-p and p-n coincidence geometry. For the p-n coincidence geometry, the predicted background contributions were small and essentially constant for all experimental spectra taken. For the p-p spectra, the predicted background contributions were larger than for the p-n case; for a fixed d* detection angle, these predicted background contributions decreased smoothly with increasing bombarding energy, and for a constant bombarding energy, they increased slightly for the more backward detection angles. The use of this FSI theory in performing minimum Chi-square fits to experimental spectra will be covered in detail later in this chapter.
C. Detailed Analysis

1. Functional Form of Coincidence Spectra

The subsequent data analysis is based upon the following representation of the laboratory yield as a function of the n-p relative energy $E^*$:

$$N(E^*) = \left[ \frac{d^2\sigma}{d\Omega dE^*} \right] \ast T \ast P(E^*) \ast \Delta \Omega^*_L \ast PSF \ast \Delta E^* \ast F_N \ast \text{cm}$$

$N(E^*)$ is the number of coincidences (p-p or p-n) detected in the laboratory corresponding kinematically to the n-p relative energy ($E^*$) being between $E^*$ and $E^* + \Delta E^*$. For p-n coincidences, it is assumed that the absolute neutron counter efficiency has been taken into account. $d^2\sigma/d\Omega dE^*$ is the differential cross section for the $d^*$ mechanism in the center of mass for $E^*$ between $E^*$ and $E^* + \Delta E^*$. "$d\Omega^*$" is in the center of mass. PSF is the phase space factor, defined previously. $\Delta \Omega^*_L$ is the laboratory solid angle into which the $d^*$'s are emitted and is defined as the larger of the two coincidence detector solid angles. $T$ is the solid angle factor which transforms $d\Omega$ from the center of mass to the laboratory system. $P(E^*)$ is the probability, assuming that the appropriate particle was emitted into $\Delta \Omega^*_L$ of the "other" particle entering its detector, thereby resulting in a coincidence (geometrically). This factor is a sensitive function of $E^*$ and the relative
sizes of the finite detector solid angles. $\Delta E^*$ is the relative energy increment in the $E^*$ axis upon which the laboratory data have been projected. $F_N$ is the factor which converts cross section values (barns) to laboratory yields (counts) by normalization to a reaction of reasonably well-known cross section, as well as taking into account small dead-time corrections in the multi-parameter data acquisition interface. From eqn. V-6, it is clear that in order to compare validly the center of mass $d^*$ cross sections, the laboratory coincidences must be divided by the various geometric/kinematic factors indicated, and the results integrated over a suitable range of $n$-$p$ relative energy.

2. Projection of Spectra onto Relative Energy Axis

It should be noted that in eqn. V-6, the coincidence data is denoted as a function of $E^*$, but actually was projected as a function of $E_1$. One method of transforming this yield from a function of $E_1$ to a function of $E^*$ is to multiply the yield in each $E_1$ bin by the factor $\frac{\delta E_1}{\delta E^*} \approx \frac{\Delta E_1}{\Delta E^*}$. This technique must be considered carefully since $\Delta E^*$ becomes quite small in the region $E^*=0$. Furthermore, one must treat specially the $E_1$ bin in which $E^*=0$ is overlapped. A much simpler approach involves calculating, using general three-body kinematics, the $E^*$ corresponding to each data point in the projected $E_1$ spectrum and then storing the data in an array linear in $E^*$. This is illustrated in fig. V-3 in which the data
Figure V-3

Projection of p-p and p-n coincidence spectrum onto E* axis for E*<100 keV, E°=11 MeV, θ* = 75°. The neutron counter absolute detection efficiency has been taken into account.
in the region $E^* < 100$ keV as a function of $E_1$ has been
projected onto a linear $E^*$ axis in 1 keV steps. A similar
example of a spectrum in the region $E^* < 1$ MeV is presented
elsewhere in this chapter. The reason that some $E^*$ bins
are zero is due to the fact that in this particular
kinematic region, $E^*$ changes more rapidly than $E_1$.

In order for this representation to be valid, one
must be able to determine $E^*$ quite accurately, and since
$E_1$ is a double-valued function of $E^*$, the analysis should
be independent of this property. The accuracy of determining
$E^*$ in terms of the energy and time-of-flight calibrations
has already been commented on. In addition, in the region
of low relative energy, it was verified that although
$P(E^*)$ varies strongly with $E^*$, it is almost completely
insensitive to which value of $E_1$ is used to obtain the
value of $E^*$; that is to say, $P(E^*)$ is essentially the
same, irrespective of the proton from the d* being emitted
"forward" or "backward." Forward (backward) refers to
the branch of the locus in which $\frac{\delta E^*}{\delta E_1} > 0$ ($\frac{\delta E^*}{\delta E_1} < 0$).
Therefore, this representation, in light of its implied
usage (eqn. V-6), is justifiable.

3. d* Detection Efficiency

$P(E^*)$ represents the probability of both particles
entering their detectors, assuming a uniform flux of the
appropriate particle over $\Delta \Omega^*$, which is defined as the
larger of the two solid angles in question. This definition of $\Delta\Omega^*_L$ is more convenient for purposes of performing the numerical integrations required for the $P(E^*)$ calculation.

An example of $P(E^*)$ for both the p-p and p-n coincidence geometry is shown in fig. V-4. Note that for the p-p geometry, the probability is unity for $E^*=0$. That is to say, whenever $p_2$ is emitted into $\Delta\Omega_2 [=\Delta\Omega^*_L]$, $p_1$ is emitted into $\Delta\Omega_1$; however, this probability decreases rapidly as $E^*$ increases. For the p-n case, $\Delta\Omega_n [=\Delta\Omega^*_L]$ is almost ten times larger than $\Delta\Omega_1$; therefore, as a flux of d*'s of breakup energy $E^*=0$ is smeared across $\Delta\Omega_n$, coincidences will occur only for $p_1$ being emitted into $\Delta\Omega_1$. Hence, the maximum probability would be the ratio $\Delta\Omega_1/\Delta\Omega_n \approx 0.1$. Fig. V-4 illustrates the benefit of using so large a neutron solid angle, since the actual p-n experimental coincidence probability will be reduced by the neutron detector absolute efficiency, typically 30-40%. The prescriptions for calculating the detection probabilities have been presented previously (chapter III).

4. Solid Angle Transformation and Phase Space

The solid angle transformation factor T is required because $\Delta\Omega^*_L$ is defined by two different solid angles, depending on the coincidence geometry. In the p-p geometry, $\Delta\Omega^*_L$ is defined by $\Delta\Omega_2$ located at a back angle, whereas in the p-n geometry, $\Delta\Omega^*_L$ is defined by $\Delta\Omega_n$, located at a forward angle on the opposite side of the beam axis. In
Figure V-4

$P(E^*)$ vs. $E^*$ for both p-p and p-n coincidence geometry.

$E^0=11$ MeV, $\theta^*=60^\circ$. 
a real sense, one may view the reaction as a two-body problem, with the factor \( P(F^*) \) taking into account the detection efficiency due to the breakup of the \( d^* \).

For example, consider an elastic scattering experiment \( l(2,2)l \) in which detectors subtending equal solid angles are placed on the \( l,2 \) recoil axes. In general, the number of particle type "1" detected at \( \theta_1 \) will not be equal to the number of particle type "2" detected at \( \theta_2 \). They will be related by the expression \( \frac{\sin^2 \theta_1}{\sin^2 \theta_2} \), however, the cross section for the scattering in the c.m. will be the same no matter which particle type is counted. The transformation factor, \( T \) is given by

\[
T = \frac{\sin \theta_{cm} \sin \theta_{lab}}{\sin \theta_{cm} \cos (\theta_{cm} - \theta_{lab})}, \quad V-7
\]

where \( \theta_{cm}, \theta_{lab} \) are the angles of the particle in question in the system center-of-mass and laboratory, respectively.

The phase space factor is a purely kinematical quantity and is given by the expression [Si 65]

\[
PSF(E_1, \Omega_1, \Omega_2) = \frac{m_1 m_3 P_1 P_2^2}{|P_2 [(m_2 + m_3)/m_2] + P_1 \cos \theta_{12} - P_0 \cos \theta_2|}, \quad V-8
\]
where \((P_1, P_2)\) are the linear momenta of particles \((1, 2)\), and \(\theta_{12} = \theta_1 + \theta_2\). For the relative energy region restricted to a few hundred keV, the phase space factor is essentially constant, so that the value corresponding to \(E^* = 0\) was used for each spectrum.

A representative plot of both the solid angle transformation factor \(T\) and the phase space factor versus bombarding energy and \(d^*\) detection angle for \(p-p\) and \(p-n\) coincidence geometry is presented in fig. V-5.

5. Background Subtraction and Integration Limit

Due to the rapidly decreasing nature of \(P\) with \(E^*\), background subtraction techniques and the choice of an upper limit of relative energy (integration limit) must be considered carefully. Ideally, the limit should be chosen in a way so that 1) the cross section value will not be highly sensitive to errors in background determination and 2) the cross section integration will utilize all "statistically significant" data, consistent with the division by the factor \(P(E^*)\). With the limit chosen as such, the result would not be highly sensitive to errors in determining the channel number corresponding to the \(E^*\) upper limit nor to statistical fluctuations which, when divided by \(P(E^*)\), might become significant.

It was determined that these objectives may be achieved by restricting the integration limit to around 50 keV. For such a limit, the cross section error
Figure V-5

Typical dependence of phase space factor (PSF) and solid angle transformation (T) on bombarding energy and d* detection angle.
PHASE SPACE FACTOR

\[ \theta^{*}_{\text{CM}} = 75^\circ \]

SOLID ANGLE TRANSFORMATION

\[ T \]

\[ E_p (\text{MEV}) \]

8 9 10 11 12
associated with determining the channel corresponding to 50 keV is less than 3%; by contrast, the error is typically 8% at 25 keV. Also, the total effect of subtracting background versus not subtracting background is less than 5%. As is illustrated by re-examining fig. V-4, by 50 keV, both p-p and p-n detection probabilities have fallen by approximately two orders of magnitude, and, referring to fig. V-6, the vast majority of all d*-associated data is included in this region.

In order to estimate the extent of background reaction contributions, the coincidence spectra for E* <1 MeV were projected onto the E* axis and summed over 50 keV regions. A representative example is presented in fig. V-6. All spectra were the same shape: a pronounced peak in the low energy region due to almost solely the n-p singlet FSI, and a relatively flat region for E*>500-600 keV, due to almost entirely "background" reactions. The coincidences in the flat region were averaged and subtracted from the entire spectrum.

The results thereby obtained are in agreement with the predictions of the PGB theory cited earlier. This may be seen by expressing the background subtracted as a fraction of the total number of coincidences in a low relative energy region, say 300 keV. These fractions are graphed in fig. V-7; however, when the same fractions are calculated for the relative energy region 0-50 keV, all are very small, with the d* detection efficiency-
Figure V-6

Projection of p-p and p-n coincidence spectra onto $E^*$ axis in 50 keV steps. The neutron counter absolute detection efficiency has been taken into account. The regions attributed to background reactions are indicated. $E^o=11$ MeV, $\theta=75^o$. 
Figure V-7

Fraction background determined (for $E \leq 300$ keV) by subtracting average value in background region (fig. V-6) from coincidence spectra. The lines drawn through the points are not the predictions of any model; they are drawn solely to illustrate the monotonic variation of the background determined using this technique.
weighted (1/\theta) effects on the 50 keV cross section values being less than 5%. The weighted effects on cross section values determined with higher integration limits, however, increase rapidly, due to the greater percentage of total coincidences (vs. E*) due to background reactions (fig V-6) and to the division by \theta, which becomes progressively smaller and smaller (fig V-4).

6. Presentation of (p,d*) Cross Sections

The cross section values calculated from eqn. V-6 (after background subtraction) for both p-p and p-n coincidence spectra are presented in fig. V-8 as a family of excitation functions and angular distributions. The cross sections were integrated to two E* limits, 25 keV and 50 keV. The agreement of the extracted cross sections over the wide range of bombarding energies and angles reaffirms the validity of the description of the coincidence spectra by eqn. V-6. All excitation functions and angular distributions are smoothly varying and, in general, rather uninteresting. No anomalous behavior or evidence for "structure" of any kind was found.

The assignment of errors to the absolute cross sections derived from the p-p and p-n coincidence locii certainly involves more than the assessment of statistical uncertainties. There are sources of error both common to and unique to each detection geometry.

Sources of error common to both detection schemes include: 1) determining \Delta \Omega, 2) determining the number
of elastically scattered protons from deuterium (monitor spectrum), 3) uncertainties in the absolute p,d elastic cross section values, and 4) measurement of the monitor detector solid angle.

Although the relative errors of the cross section values used in this work [Wi 69] are typically 2-3%, the absolute errors are much larger due to primarily inconsistencies in the measured absolute values. For example, in Ref. [We 71], over-all uncertainties of about 15% are quoted due to primarily interpolations between values tabulated in the literature. The 15% is perhaps somewhat exaggerated.

The number of elastically scattered protons from deuterium is more difficult to determine for a deuterated polyethylene target than for a gas target. Since the polyethylene foils were several hundred \( \mu \)g/cm\(^2\) thick, the low energy "tail" from the elastically scattered protons from carbon extended under the p,d elastic peak. It is estimated that the number of elastically scattered protons from deuterium could be determined to 2 or 3%.

Also, there are additional errors which are peculiar to the coincidence geometry, with the p-n case having the larger share. For the p-p case, additional sources of error include 1) determination of \( \Omega \), 2) projection of coincidence spectra onto the \( E \) axis (kinematic broadening), 3) calculation of \( P(E^*) \) (finite number of integration
steps), 4) determining the exact channel corresponding to the E* integration limit, and 5) statistical uncertainties.

For the p-n case, in addition to the sources of error enumerated for the p-p case, are 1) determining absolute neutron counter efficiency, and 2) loss of neutron events due to PSD gamma-ray rejection. Due to the relatively short neutron flight path, the errors due to kinematic broadening in projecting the locus onto the E_1 axis were larger than for the p-p case.

The most significant contribution to the absolute uncertainties is inherent in the technique of dividing each increment in the integrated cross section by P(E*), which decreases rapidly with E*. For example, an error of only 1 keV in determining the total number of coincidences occurring for E* less than say 25 keV or 50 keV is insignificant; however, when weighted by the d* detection efficiency, the error increases to about 8% at 25 keV and about 3% at 50 keV.

When all the above sources of error were added, it was determined that realistic absolute errors of about 10% and 15% could be assigned to the cross sections derived from the p-p and p-n coincidences, respectively.

As a matter of interest, in fig. V-9, one of the integrated cross sections is plotted versus the integration limit employed.
Figure V-8

(p,d*) cross section values, calculated using eqn. V-6, presented in excitation functions and angular distributions. The cross sections were integrated to two $E^*$ limits, 25 keV and 50 keV, and are denoted as such. The p-p and p-n points have been displaced slightly to the left and right, respectively, of the exact plotting points, so that both results could be displayed in one figure without overlapping.
A representative \((p,d^*)\) cross section, plotted vs. the integration limit. Note the logarithmic scale. \(E^*=11\) MeV, \(\theta^*=75^\circ\).
D. Comments on Analysis Techniques

Three previous experimental studies [Ni 70, We 71, Br 71] of the \((p,d^*)\) mechanism in the \(D(p,pn)p\) reaction at low bombarding energies have been described briefly (chapter I). It is the opinion of this author that, in addition to the situation of not maintaining fixed \(d^*\) center-of-mass detection angles for varying bombarding energies, there exist serious shortcomings in certain aspects of the data analysis techniques employed in Ref. [Ni 70] and Ref. [We 71]. Before examining some of these in detail, a brief summary is presented:

Ref. [Ni 70]: 1) The use of Jacobians in transforming from the RCM to the LAB: The Jacobian, in general, does not describe accurately the \(d^*\) detection probability because it ignores the finite sizes of the detector solid angles. 2) The fitting of the entire spectrum with a series of the form \(\Sigma a_p J\), where \(\rho\) is the DOFS function and \(J\), the Jacobian, in order to subtract background: All the Jacobians become infinite in their regions of real interest, for example, \(J_{13}\) (\(d^*\) peak) at \(E^*=0\), and \(J_{12}, J_{23}\), at the "\(E_1\)-end" of the \(p-p\) locus (where the background mechanisms peak). Hence small errors in energy calibrations are grossly magnified. 3) Unreasonably large \(d^*\) cross section integration limits: Since for \(E^* = .3, .5,\) or \( .7\ MeV\), the \(d^*\) detection probability (also the Jacobian) has fallen
by over four orders of magnitude, statistical uncertainties and errors in background subtraction are magnified tremendously. 4) Results are FSI model-dependent.

Ref. [We 71]: 1) Incorrect DOFS function quoted (algebraic error). 2) No reference to determination of neutron counter absolute detection efficiency. 3) Background and cross section values FSI model-dependent (even if correct DOFS expression had been used). 4) Unreasonably large d* cross section integration limit: 1 MeV!

1. Transformation Jacobians

Consider the p-p detection geometry and assume E*=0. As p₂ is smeared across the solid angle ΔΩ₂, p₁ will be spread across some other solid angle, call it ΔΩ*. This ΔΩ* is defined uniquely by two-body kinematics. The critical situation occurs when the solid angle ΔΩ₁ is smaller than ΔΩ*; that is to say, a recoil proton p₂ is emitted into ΔΩ₂ but its corresponding proton p₁ is emitted outside ΔΩ₁. For ΔΩ₁ < ΔΩ* the actual detection probability is constant and equal to unity and for ΔΩ₁ > ΔΩ*, is somewhat less than unity. By contrast, the Jacobian has a constant value for a fixed value of E*, assuming, of course, fixed bombarding energy, detection angles, etc., regardless of the size of ΔΩ₁! The d* detection probability and hence the laboratory yields are dependent on the relative sizes of ΔΩ₂ and ΔΩ₁; the Jacobian is not.
Consider now the situation for which $\Delta \Omega_1 = \Delta \Omega^*$. Suppose $p_2$ were emitted such that it fell "just barely inside" its solid angle $\Delta \Omega_2$. For $E^*=0$, by definition of $\Delta \Omega^*$, $p_1$ also would fall "just barely inside" its solid angle $\Delta \Omega_1$. Now assume some non-zero value for $E^*$, as is usually the case. If the cm breakup of the d* were such that proton $p_i$ had a velocity component perpendicular to the velocity of the n-p cm but directed away from the center of solid angle $\Delta \Omega_1$, then $p_1$ would be deposited outside $\Delta \Omega_1$, even though $p_2$ were detected. There would be no coincidence. Since the breakup is essentially isotropic in the d* cm, this situation would occur 50% of the time for the special case described. Now suppose $\Delta \Omega_1$ were made larger than $\Delta \Omega^*$. For the same situation described above, the probability of $p_1$ entering $\Delta \Omega_1$ would be larger than it would have been for $\Delta \Omega_1 = \Delta \Omega^*$. Again, note that for this case, the Jacobian still would possess the same value, regardless of the size of $\Delta \Omega_1$.

In summary, the Jacobian does not describe accurately the d* detection probability for $\Delta \Omega_1 < \Delta \Omega^*$ or for $\Delta \Omega_1 > \Delta \Omega^*$. It describes only the very unique situation for which $\Delta \Omega_1 = \Delta \Omega^*$. Satisfying this constraint experimentally for all bombarding energies and detection angles is not feasible.
The situation is even more exaggerated in the p-n detection. Due to the coaxial detection geometry, the Jacobian describes the detection probability accurately only for $\Delta \Omega_1 = \Delta \Omega_n$. But usually $\Delta \Omega_n >\Delta \Omega_1$. This is due to the relatively low neutron counter efficiency, as explained in chapter V, sec. C3.

In general, for a fixed $d^*$ detection angle, the detection probability for some arbitrarily-picked value of $E^*$ increases with bombarding energy. Similarly, for fixed bombarding energy, the detection probability increases for the more forward angles. This is illustrated in fig. V-10 in which values of $P(E^*)$ are plotted vs. bombarding energy and detection angle for a fixed value of $E^*$.

It is not surprising, therefore, that when attempts were made to fit the $d^*$ peak with functions similar to those used by Niiler [Ni 70], 1) unreasonable background contributions were predicted and 2) the predicted n-p singlet FSI contributions did not agree between the p-p and p-n coincidence spectra.

The problem is pin-pointed in the following trivial example: One obtains a number of spectra, for fixed bombarding energy and detection angles, but changes the relative sizes of the coincidence detector solid angles for each spectrum. This will change the $d^*$ detection probability and hence, the laboratory yields will vary accordingly. If the $d^*$ detection probability were
Figure V-10

Variation of the d* detection efficiency factor P(E*) for fixed E*. The probabilities are expressed as the fraction of the maximum efficiency geometrically possible. These maximum efficiencies are unity for the p-p case and about 0.1 for the p-n case, as described previously. The reason that some value other than 1 keV was chosen for the p-n case is that the detection probability for E*=1 and 2 keV is maximum for most spectra. The choice of another value of E* better shows the variation of the detection efficiency.
calculated for each different solid angle combination and the cross section calculated per eqn. V-6, all results would be the same, as they ought to be. On the other hand, if one attempts to fit the varying laboratory spectra with a function of the form $\sigma^* \rho J$, $J$ will be the same for each spectrum, regardless of the varying detection solid angles. Therefore, the (absurd) conclusion is that a variety of values for $\sigma^*$ (the fitted n-p singlet FSI) will be obtained.

2. Minimum Chi-square Fits

In Ref. [Ni 70] the p-p coincidence spectra were fit with a function of the form $\Sigma a_i \rho i_j J_{ij}$, where $a$ is the parameter to be fitted, $\rho i_j$ is the DOFS function for particles $i,j$, and $J_{ij}$ is the Jacobian between the RCM and the LAB. The inadequacy, in general, of the Jacobian in describing the $d^*$ detection probability, or equivalently the solid angle transformation from the RCM to the LAB, was discussed in the previous section. However, even if a correct transformation had been used, there still exists a serious problem in that one must divide accurately the "upper branch" of the locus from the "lower branch," so that $E_2$ will remain a single-valued function of $E_1$. Near the maximum kinematically possible value of $E_1$, there are usually two real solutions of $E_2$ as a function of $E_1$.

The point of division is not simply determinable visually from the $E_1$-$E_2$ spectrum, due to kinematic
broadening. Alternatively, one could calculate (from kinematics) the division channel. But the point to be made is that, because the locus is so densely populated in this region, an error of a few channels in defining the division can scale the fitted background yield in this region by as much as 40%! Since this is the region in which the background reactions actually are being fitted, due to the dense population, this method of determining background is highly sensitive to precise energy calibrations and to an accurate division of the locus. As a result, this technique of determining the background under the d* peak is considered undesirable, since the background under the d* peak is considered to be the "tail" from the background peak.

In light of the above discussion, it was interesting to attempt to fit both p-p and p-n spectra with functions similar to those used in Ref. [Ni 70] but only in a region of low n-p relative energy, $<300$ keV. The p-p spectra were fit (minimum Chi-square) with the function

$$a_1 \rho_{13} J_{13} + a_2 \rho_{12} J_{12},$$

where the first term is the n-p singlet FSI and the second is the p-p singlet FSI. If more than one of the background contribution terms were included in the fitting function, one of the background terms became significantly negative, which is not physically acceptable, whereas the other
background term became unreasonably large and positive. This is indicative of an "overfit," i.e., too many fitting terms; therefore, only one background term could be included in the fitting function. This is a reasonable idea since, from fig. VI-a, the terms $\rho_{12} J_{12}$, $\rho_{23} J_{23}$, and $P$ all have essentially the same shape in the region of the $d^*$ peak. As a matter of fact, the ultimately desired quantity, $a_1$, was almost completely insensitive to the choice of $\rho_{12} J_{12}$, $\rho_{23} J_{23}$, or $P$ for the background term in the fitting function.

The p-n spectra were fit with the same function for $E^{*}\leq 300$ keV. For each fit, the percent background under the $d^*$ peak ($<300$ keV) was calculated based on the results of the fit, i.e.,

$$BKG(\%) = \frac{\Sigma a_2 \rho_B J_B}{\Sigma a_1 \rho J^* + \Sigma a_2 \rho_B J_B} \times 100 \quad V-10$$

where the superscript $*$ denotes the DOFS function and Jacobian corresponding to the $d^*$ contribution and the subscript $B$ denotes the DOFS function and Jacobian corresponding to the background contribution.

The results of such minimum Chi-square fits are presented in fig. VI-11, for both p-p and p-n coincidence spectra. On each page, the top graph is the fitted parameter $a$, which is labeled "FSI." The middle graph is Chi-square per degree of freedom and the bottom graph is the percent background calculated.
$80^\circ$

PP COINC

FSI

$X^2$

BKG

$E_P$

8 9 10 11 12
In view of the inadequacy of the Jacobian representation of describing the RCM to LAB solid angle transformation, it is not surprising that certain anomalies result. The most glaring one occurs at the forward angles, at which the fitted p-n FSI is some 50% larger than the fitted p-p FSI. Moreover, the fitted background is unreasonably large - about 40%-50% for the p-n case and 20%-30% for the p-p. That these background estimates are grossly in error may be seen at a glance by examining fig. IV-4 for $\theta=50^\circ$. In addition, all background predictions are excessive and are in disagreement with background estimates obtained directly from the experimental spectra (fig. V-7).

Another item of concern is that a reasonable Chi-square (near unity) does not necessarily guarantee realistic fits. Examples of this are most evident for the p-n case, $\theta=70^\circ$ and $75^\circ$. A smoothly varying background is not predicted and the fitted $d^*$ yields vary quite erratically.

Note that when the $(p,d^*)$ cross sections for the identical spectra were calculated using eqn. V-6, that the results (fig. V-8) agreed well over all energies and angles considered. No erratically varying excitation functions were observed.

In light of the above discussions, it is the opinion of this author that in the analysis of both p-p and p-n spectra
1. Detection probabilities (or, equivalently, RCM to LAB solid angle transformations) should be calculated geometrically, either by numerical integration or Monte Carlo technique.

2. Estimates of background contributions should be made directly from the shape of the experimental spectra.

The undesirable consequences of alternative approaches have been considered.

3. Choice of Integration Limit

As has been illustrated in fig. V-6, the vast majority of all n-p singlet FSI-related coincidence events occur for $E^* < 100$ keV. If one merely is summing the laboratory coincidences, then the background coincidences, in general, will not modify the result significantly for a few more hundred keV or so. On the other hand, the situation is completely different if one actually is calculating the cross section, which requires division of the coincidence spectrum by the appropriate solid angle transformation factor ($d^*$ detection efficiency). Using a Jacobian has the same qualitative effect. Since for both p-p and p-n geometry the detection efficiency decreases rapidly with n-p relative energy (fig. V-4), if the integration limit is too large, then the cross section result will depend more on the statistically poor data composed of a questionable amount of background coincidences than it will on the more statistically significant $d^*$-related data which is influenced much less by background.
It was determined that a realistic compromise between
1) choosing the limit large enough to utilize most d*-related data and 2) choosing the limit small enough so
that background effects are not significant can be achieved
by picking the integration limit at the n-p relative
energy such that the d* detection probability has fallen
by approximately two orders of magnitude from its value
at E*=0. Accordingly, 50 keV was considered a realistic
limit for the present study. Since the detection geometries
cited in Refs. [Ni 70] and [We 71] did not differ much
from those of the present study, it is the opinion of
this author that the results obtained in these two studies
by integrating the cross section to relative energies
between .3 and 1 MeV are highly questionable.

4. Miscellaneous

In Ref. [We 71] Appendix B, there is an algebraic
error in the statement of the PGB DOFS function. The
last term is stated in the form sin2(ka-δ), whereas it
should read sin2(ka+δ). It is not known whether this
represents the actual DOFS function used or whether it
is merely a typographical error.

In Ref. [Br 71] a more realistic approach was taken
toward the data collection and subsequent analysis. The
detector solid angles were changed for each bombarding
energy so that the d* angle remained fixed in the cm.
Only events with n-p relative energy less than 100 keV were considered, as determined from examination of the TAC spectrum. In transforming from the RCM to the LAB, a Monte Carlo-type sampling was performed over the detection solid angles. The square of the matrix element was calculated using Goldberger and Watson FSI theory.

The integrated laboratory cross sections presented agree quite well with those of the present work, when they are integrated over the same relative energy interval (100 keV). The resulting matrix elements, for θ*=60° and 90°, when plotted vs. incident proton energy, decreased monotonically over the region of investigation (7 - 14.5 MeV). For both excitation functions, the decrease in the matrix element amplitude (expressed in arbitrary units) was approximately 50% in the interval 8 - 12 MeV. The (p,d*) cross sections determined in the present work for E*≤50 keV for both p-p and p-n coincidences are in excellent agreement. They, too, decrease monotonically by approximately 50% in the interval 8 - 12 MeV.
E. Concluding Remarks

In the present work, all \((p,d^*)\) cross section excitation functions decreased monotonically between 8 and 12 MeV over the \(d^*\) cm angular range 50°-80°. The author considers this to be convincing negative evidence for any structure which might be due possibly to an excited state in \(^3\text{He}\), as suggested in Ref. [Ni 70]. The agreement between both \(p-p\) and \(p-n\) coincidence cross section values, as well as the "reasonableness" of the approaches taken in the data analysis should afford a relatively high degree of confidence in the results obtained.

Consider that, in contrast to all previous studies, the numerical results obtained here 1) were not dependent on the choice of detection geometry (either \(p-p\) or \(p-n\)), 2) were not based on any FSI model, 3) did not use any "fitting" process to determine background, 4) did not employ any quantities which became infinite in the regions of interest, and 5) were not sensitive to errors in background determination.

As such, it is hoped that this document will provide a useful guide for performing an accurate and realistic analysis of any multi-particle coincidence data, similar to that investigated here.
VI. Appendices
A. Computer Programs for Generating Absolute Neutron Counter Efficiencies

In a scintillator, neutrons are detected by the scintillation light produced by the charged products of interactions between the incident neutrons and the nuclei of the scintillator (hydrogen and carbon). For neutron energies below 10 MeV, the efficiency may be calculated reliably because only proton-neutron (p-n) interactions contribute. [Ku 64]. Above this energy, interactions with the carbon nuclei become significant and although considerable information is known for the p-n interactions, almost no systematics, other than the total cross section and elastic cross section are available for neutron-carbon (n-C) interactions.

1. Program "NEFF"

The absolute neutron efficiency of the NE-218 scintillator used in this work was calculated using the computer program NEFF. NEFF is a version of the program DETEFF [Th 71], modified only for input/output flexibility and for compatibility with the Bonner Nuclear Laboratory IBM 1800 computer. The actual calculations are those described by Kurz [Ku 64].

The calculations include first- and second-scattering contributions from interactions with both protons and
carbon, saturation effects in the production of scintillation light, and finite resolution of the detector threshold. The program calculates absolute efficiencies in the energy range 0.01 to 300 MeV for the following scintillation materials: NE-102, NE-211, NE-218, NE-213, NE-224, Pilot-B, stilbene, and anthracene. A cylindrical scintillator geometry is assumed with the neutrons incident on a flat face. For NE-218, the light output data of Masterson [Ma 70] was used.

NEFF calculates for a specified series of neutron energies, $E$, the total efficiency, $\varepsilon(E)$. This quantity is the result of a folding integration over the detection threshold, $T$, of the type

$$
\varepsilon(E) = C \int_0^\infty dT \, \exp \left[ -\left( \frac{T-T_0}{\tau T_0} \right)^2 \right] \varepsilon(E,T)
$$

$$
C^{-1} = \int_0^\infty dT \, \exp \left[ (-1/2)\left( \frac{T-T_0}{\tau T_0} \right)^2 \right]
$$

where $T_0$ is the mean threshold, $\tau$ is the fractional resolution, and $\varepsilon(E,T)$ is the efficiency at energy $E$ and threshold $T$.

The function $\varepsilon(E,T)$ is computed in two portions: $\varepsilon_H(E,T)$, the efficiency for an initial interaction with hydrogen, and $\varepsilon_C(E,T)$, the efficiency for an initial
interaction with a carbon nucleus. One subroutine, HYD(E,T) computes the single-scattering efficiency for n-p collisions. Another subroutine, CARB(E,T), computes the single-scattering efficiency for n-C collisions from the (n,α), (n,n'3α), and (n,p) reactions. Since the (n,n) and (n,n'γ) reactions contribute only through rescattering, they are treated separately.

Multiple-scattering effects are calculated by integrating over the neutrons in the final state of primary reactions that did not produce sufficient light to be detected. This integration involves the successive use of HYD(E',T') and CARB(E',T'), where E' is the scattered neutron energy and T' is the light threshold reduced by the light produced in the primary reaction.

2. Program "05S"

Program 05S is a Monte Carlo code for calculating pulse height distributions due to monoenergetic neutrons incident on organic scintillators, written by Textor and Verbinski [Te 67,Ve 68]. The Monte Carlo experiment follows the course of each neutron through the scintillator and obtains the energy-deposits of the ions produced by elastic scatterings and reactions. The light pulse produced by the neutron is obtained by summing up the contributions of the various ions with the use of appropriate light vs. ion-energy tables. The neutron energy range covered is 200 keV to 70 MeV.
B. Alignment Procedures

1. Reaction chamber, slit system, and beam dump

   The procedure followed in the alignment of the reaction chamber, slit system, and beam dump is presented here. Special attention to the small details has been included purposely with the forethought that someone performing similar tedious tasks in the future may find them particularly useful.

   An alignment telescope was mounted in back of the 90° analyzing magnet, and the beam pipe section between the 90° magnet and the switching magnet was removed. Target holder #1 was mounted from the side of the 90° magnet and a translucent target and a light source were inserted in the holder. The telescope was then leveled and adjusted for cross-hair alignment on the target, thereby defining the optical axis for subsequent alignments.

   The target and light source were removed and placed on the end of the short beam pipe section in back of the reaction chamber. The tensioning cables on that section were adjusted for cross-hair alignment, as viewed from the 90° magnet. Then the pre-chamber slits were inserted one at a time to check that they were on the optical axis.

   A silverized (reflective) target was placed on the front of the beam dump just before the bellows and the beam pipe set screws were adjusted for cross-hair alignment.
Proper alignment of the beam dump at the entrance to the neutron cave was assumed.

Proper positioning of the beam on target was further verified from examination of deteriorated (burned) regions on polyethylene target foils.

2. Alignment of neutron counter system

The neutron counter shield was aligned in both the vertical and horizontal planes with respect to the reaction chamber. Two aluminum rings were fitted over each end of the cylindrical aluminum container which enclosed the neutron counter and phototubes. These rings fit snugly inside the shield, thereby assuring that the neutron counter was positioned on the axis of the shield.

a. Horizontal plane alignment: A large steel square was hung over the lid of the reaction chamber such that the same edge lined up with both 90° scribe marks on the top of the chamber. A previously determined 90° scribe mark was located on the south target room wall. By a series of adjustments and readjustments, a transit was aligned between the 90° scribe mark and the left edge of the square. The transit was slightly higher than the "R1" beam line. The neutron counter shield then was leveled approximately in the vertical plane (by eyeball) and positioned at 90°, as indicated by the readings on the circular overhead track. Thin aluminum rings with both horizontal and vertical cross-hairs were inserted in the
front and back of the shield. By C-clamping the bases of the four overhead adjustable rods to the top of the shield structure at various places, the front and rear vertical cross-hairs were made to line up with the edge of the square, as viewed through the transit.

b. Vertical plane alignment: The neutron counter was set at about 40° and the transit leveled. Alignment through the transit was then made on the center of the milled neutron exit port on the side of the chamber. The front and rear horizontal cross-hairs on the neutron shield were thus aligned by adjusting only the four turnbuckles on the sides of the shield.
C. Kinematics Table

In this appendix, the angles for $d^*$ observation are tabulated for bombarding energies and $d^*$ cm angles used in the collection of data in the present work. The first angle in each pair is the $d^*$ angle; the second is that of the recoil proton (first emitted particle). These angles were calculated from two-body kinematics for the reaction $D(p,d^*)p$, using the sum of a proton and a neutron mass for the $d^*$ mass and a $Q$ value of $-2.225$ MeV.
**Angular Settings for d* Observation**

<table>
<thead>
<tr>
<th>$E^\circ$ (MeV)</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>75°</th>
<th>80°</th>
</tr>
</thead>
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<td>7.5</td>
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<td>29.2°,70.7°</td>
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<td>29.6°,71.6°</td>
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<td>33.6°,64.0°</td>
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<td>26.2°,81.5°</td>
<td>30.4°,73.0°</td>
<td>32.4°,69.0°</td>
<td>34.5°,65.1°</td>
</tr>
<tr>
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<td>26.4°,82.1°</td>
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<td>32.8°,69.5°</td>
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</tr>
<tr>
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<td>33.1°,69.9°</td>
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BIBLIOGRAPHY


