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INTEREST RATES AND INFLATION IN CANADA

BY

JAMES F. MCCOLLUM

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

Thesis Director's signature:

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May, 1973
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PREFACE

Beginning in the late 1960s both economists and public policy-makers have laid stress on the role of price-expectations in influencing interest rates. This study was undertaken to clarify and test some of the theories relating real rates of interest, nominal rates of interest, price-expectations and inflation. While this study is primarily empirical, several theoretical issues are examined. In contrast to earlier work in this area this study concentrates on the issue of how an inflationary premium may become embedded in nominal interest rates. Canadian data has been used.

This study was begun during the academic year 1969-1970 at Rice University. Most of the computational work was carried out while the author was on staff at the Prices and Incomes Commission, Ottawa, Canada, during 1970-1971. A number of revisions were incorporated while the author was on staff at the Economic Council of Canada.

I would like to thank the members of my thesis committee, Stanley Besen and Joseph Burns for comments and criticisms which were instrumental in improving and completing my dissertation. A number of useful comments were also given by Ronald Soligo. Fred Nold, a former staff member of the Prices and Incomes Commission, made a number of useful comments on the statistical work in the study. I would also like to thank M. C. McCracken, former director of the Candide project at the Economic Council of Canada, for assistance in implementing the non-linear regression procedure that was used in chapter IV.
CHAPTER I
INTRODUCTION

This study is an investigation into the influence of inflation in Canada on Canadian interest rates. It focuses on the impact of inflation on nominal rates of interest via the route of price-expectations. At the present time no consensus among economists concerning the impact of inflation on interest rates has emerged in the literature. This lack of concord reflects disagreements over theory and over the nature of the existing empirical evidence. ¹/

Recent statements by public authorities in Canada suggest that the influence of price-expectations upon interest rates is accepted knowledge. "The best way to reduce high interest (in Canada) is to reduce inflation, (Canadian)." ²/

Thus stated Edgar Benson, the Minister of Finance, before the House of Commons Committee on Interest Rates, on October 30, 1969. Other testimonies given before that Committee suggested


that the recent inflation in Canada was directly responsible for the high levels reached by interest rates at that time.\footnote{See in particular the statement made by Professor E. Neufeld, \textit{ibid.}, pp. 75-90.}

More recently, the Chairman of the Economic Council of Canada has argued that, "During the sixties, interest rates have climbed to new highs mainly in response to upward revision in price-expectations. High interest rates incorporated ... and still do ... an inflation premium."\footnote{A. Raynauld, \textit{Inflation and Unemployment.} Speech before the Canadian Chamber of Commerce (September 20 1972).}

Despite the wide recognition by official policy-makers in Canada of the influence of price-expectations on interest rates and assertions that nominal rates of interest currently carry a substantial inflationary premium, no empirical evidence of the influence of Canadian inflation on rates of interest in Canada currently exists. Since policy is being formulated in the belief that inflation via inflationary-expectations play an important role in determining the level of nominal interest rates in Canada, it behooves us to attempt to quantify and assess this effect.

It is commonly argued that inflationary premiums become embedded in nominal rates of interest through the mutual actions of borrowers and lenders.\footnote{See for example, I. Fisher, \textit{The Theory of Interest} (New York: Macmillan 1930) and M. Friedman, \textit{op. cit.}} Milton Friedman has recently suggested a framework whereby changes in the money supply result in a temporary depressive effect on
nominal interest rates which is ultimately reversed as inflationary-expectations resulting from the rise in prices consequent upon the change in the money supply take hold.\(^1\)

Friedman's suggestion incorporates elements of Wicksell's and Fisher's analysis of the determinants of interest rates.\(^2\)

Nevertheless, a formal model is not presented by Friedman. In chapter II, we attempt to formalize Friedman's synthesis of Wicksell's and Fisher's views concerning the relationship among money, interest rates, and prices. The results obtained from our analysis appear to be at variance with Friedman's in several regards. Within the framework of the model developed in chapter II, it demonstrated that under certain conditions the monetary authorities can have a permanent effect on the real rate of interest. The condition required in the model to produce a steady rate of inflation also results in a permanent reduction in the real rate of interest.

In addition to the permanent effect of an increased rate of expansion of the money supply on the real interest rate, a temporary effect also may occur. It is this temporary effect which is focused on in Friedman's synthesis.\(^3\) The temporary effect occurs not because of inflation per se, nor

---

\(^1\) Ibid., pp. 5-11.

\(^2\) It is generally believed that Wicksell completely ignored the influence of price-expectations in his analysis. This impression is incorrect. See K. Wicksell, Interest and Prices (New York: Augustus M. Kelly 1965):1-3. It is reasonable to argue, however, that Wicksell did not build price-expectations into his analysis in a formal way. An early work anticipating in part both Wicksell's and Fisher's analysis is Henry Thornton, The Nature of Paper Credit (London 1802).

\(^3\) Ibid., pp. 7-11.
even because inflation is not fully anticipated, but rather is due to asymmetric expectations of inflation on the part of capital market participants. If participants on both sides of the market form expectations in the same manner, then the temporary effect disappears. This chapter also serves to clarify the relationship of the real term structure of interest rates to the nominal term structure and develop the "differential effects according to term to maturity" hypothesis.

The naive Fisherian model is a special case of the more general model developed in chapter II. Empirical testing of the naive Fisherian model forms the basis of chapter III. The results obtained suggest a number of serious shortcomings in past and contemporaneous work by other authors on the effect of inflation on nominal rates of interest. The "differential effects according to term to maturity" hypothesis developed in chapter II and based on the naive Fisherian model is also subjected to testing in chapter III. A comparison of my own results to those of other authors, and a critique of earlier empirical work on the naive Fisherian model is contained in the chapter. The chapter also contains a criticism of the real rate of interest series published by the Federal Reserve Bank of St. Louis.

The results obtained in chapter III suggest that there are severe limitations to the use of the naive Fisherian model as a framework for assessing the impact of inflation on interest rates. Chapter IV contains tests of the hypothesis developed in chapter II, that it is the
reactions of primary borrowers and lenders which are responsible for the rise in nominal rates of interest during inflationary episodes. An attempt is made to test the "asymmetric-expectations" hypothesis which also was developed in chapter II. As it turned out, severe data problems prelude thorough testing of the issues raised in this chapter. The work contained in this chapter should be regarded as a first step in assessing the reactions of capital market participants to inflation. Chapter V serves to draw the results obtained in the three previous chapters together.
CHAPTER II

PRICE-EXPECTATIONS AND CREDIT MARKETS

Introduction

It is commonly argued that the reason why interest rates rise during an inflationary episode is that:

Borrowers will then be willing to pay, and lenders will then demand higher interest rates -- as Irving Fisher pointed out decades ago.1/

It is frequently hypothesized, as in the above quotation, that inflationary premiums become embedded in nominal rates of interest through the mutual reactions of lenders and borrowers. This chapter, accordingly, focuses on the behaviour of lenders and borrowers in adjusting to inflation.

There are two recognized bodies of economic theory in which the relationships between inflation and interest rates are of central concern. These are the Fisherian2/ price-expectations theory and the Wicksellian3/ market-natural rate theory. In Fisher's analysis, inflation leads to higher nominal interest rates via price-expectations, while


in Wicksell's analysis a fall in the market rate of interest below the natural rate leads to inflation. Whereas Fisher tended to disregard the initial impact of money supply changes on interest rates, Wicksell tended to disregard the impact of price-expectations on interest rates. It would appear natural then to combine the Fisherian and Wicksellian models.

Recently Milton Friedman\(^1\) has attempted to integrate the Fisherian and Wicksellian relationships among money supply changes, interest rates, and inflation. The analysis presented in this chapter builds on the previous work of Fisher, Wicksell, and Friedman and attempts to formalize Friedman's synthesis.

The analysis begins by considering a highly simplified Fisherian model of the credit market. The behaviour of credit market participants under varying assumptions of adaption to the rate of inflation is considered. The model suggests that under certain conditions the monetary authorities can have a temporary effect on the real rate of interest, which disappears once capital market participants have fully adjusted to a change in the rate of inflation. It is suggested that it is this temporary effect which is focused on in Friedman's synthesis.\(^2\) Friedman appears to have incorrectly attributed the temporary effect to Wicksell.\(^3\) In a later part of the

\(^1\) M. Friedman, op. cit.

\(^2\) Ibid., pp. 5-11.

\(^3\) Ibid., pp. 7-8.
chapter it is suggested that the Wicksell effect is conceptually distinct from the temporary effect considered by Friedman. It is argued that Friedman's temporary effect depends critically on the assumption that borrowers adapt more quickly to inflation than do lenders. The hypothesis that lenders and borrowers adjust to inflation at different rates may be termed the asymmetric-expectations hypothesis. This hypothesis is examined empirically in chapter IV.

The Wicksellian relationships between changes in the money supply, the market rate of interest, and the natural rate of interest are then introduced into the model. The resulting model is consistent with both Fisher's views on the role of price-expectations in influencing interest rates and Wicksell's views on the effect of money supply changes on interest rates. In addition, this model would appear to be in the spirit of Friedman.1/ It does not appear possible to integrate the Fisherian and Wicksellian models of the credit market in such a way as to produce Friedman's results. As pointed out earlier, however, Friedman-type results can be obtained from a variant of the pure Fisherian model. There appears to be no need to add Wicksellian wrinkles to Fisher.

A Simplified Model of the Credit Market

For present illustrative purposes a highly simplified model will be used. This model, however, is

1/ Ibid., pp. 7-11.
adequate to capture both Fisher's and Wicksell's views concerning the relationships between interest rates and inflation.

It is assumed that there is a single homogeneous market for credit, in which a single rate of interest is determined. It is assumed that all finance is direct, and is not indirectly routed from primary savers through financial intermediaries to ultimate borrowers.\(^1\) It is assumed that credit contracts are made for one period, or, if not, the terms are recontracted each period. This assumption permits us to evade a number of stock-flow difficulties. Changes in the price-level are assumed to arise from disequilibrium in the money market, while changes in interest rates are assumed to arise from disequilibrium in the credit market.

The No-Inflation Case

Assuming an absence of inflation, the rate of interest (in both real and nominal terms) is determined along with the volume of credit by the interaction of supply and demand in the credit market. The following simplified model of the credit market is utilized.

\[
\begin{align*}
(A1) \quad d_t &= a + b_i_t \\
(A2) \quad s_t &= h + k_i_t \\
(A3) \quad d_t &= s_t
\end{align*}
\]

---

where equations (1), (2), and (3) are the demand for credit equation, the supply of credit equation, and the market-clearing assumption, respectively, and \( i \) is the nominal (and real) rate of interest. Instantaneous clearing of the market is assumed.

The equilibrium rate of interest is:

\[
(A4) \quad \hat{i}_t = \frac{h - a}{b - k}, \quad t = 1, \ldots, \infty,
\]

while the equilibrium real quantity of credit extended is:

\[
(A5) \quad \hat{\varepsilon}_t = \frac{bh - ak}{b - k}, \quad t = 1, \ldots, \infty. \quad 1/
\]

**Perfectly-Anticipated Inflation**

Assume that the economy moves from one steady state rate of inflation (for example, zero) to a new permanent higher level (\( \hat{\pi} \)). 2/ Assume that capital market participants are concerned with the real cost of borrowing and the real return on lending, and that they correctly perceive the new rate of inflation. It is also assumed that the rate of inflation has no effect on any other of the determinants of the demand and supply functions of credit. The aggregate demand and supply functions for credit are functions of the

---

1/ Symbols with an asterisk represent steady state values of the corresponding variable.

2/ In this and the remaining sections of this chapter we deal only with the case of an increase in the rate of inflation. For convenience it is assumed that the initial rate of inflation is zero. Symmetric results can be obtained for the case of deflation.
perceived real cost (return) on credit transactions. The model, thus, alters to:

\[(B1) \quad d_t = a + b(i - \bar{\pi})\]

\[(B2) \quad s_t = h + k(i - \bar{\pi})\]

\[(B3) \quad d_t = s_t.\]

In equilibrium we have:

\[(B4) \quad i_t^* = \frac{h - a}{b - k} + \bar{\pi}.\]

Equation (B4) demonstrates that under the assumed conditions of our model, the nominal rate of interest has risen by the rate of inflation.

From equation (B4) it is also evident that the real rate has remained unchanged. That is:

\[(B5) \quad i_t^* - \bar{\pi} = \frac{h - a}{b - k}\]

which was the result obtained under the condition of no-inflation.

The volume of real credit in equilibrium is also unaltered at:

\[(B6) \quad z_t^* = \frac{bh - ak}{b - k}\]

which is the same as the result obtained in the no-inflation case.
Under the assumptions made in this model, introduction of inflation, where it is perfectly anticipated by all credit market participants, produces results equivalent to the no-inflation case for all of the real variables in the model. In particular, neither the value for real interest rate, nor the real volume of credit is affected.

**Imperfectly Foreseen Inflation With Symmetric Expectations**

The assumption that inflation is perfectly anticipated by all credit market participants is dropped at this point. Instead, it is assumed that borrowers and lenders react gradually to a change in the rate of inflation. As the evidence accumulates that there has been a change in the rate of inflation, the inflationary expectations of credit market participants converges towards the new rate and eventually reach it. It is assumed that credit market participants react only to the past history of prices in forming price-expectations. It is assumed that the redistribution of wealth, which occurs during the adjustment period when the inflation is not fully anticipated, has only a negligible impact on the behaviour of lenders and borrowers. The model now becomes:

\[
\begin{align*}
(C1) \quad d_t &= a + b(i_t - \pi_t^e) \\
(C2) \quad s_t &= h + k(i_t - \pi_t^e) \\
(C3) \quad d_t &= s_t \\
(C4) \quad \pi_t^e &= \pi_{t-1}^e + \lambda(\pi_t^a - \pi_{t-1}^e)
\end{align*}
\]
or \[ \pi_t^e = \sum_{i=0}^{\infty} (1 - \lambda)^i \lambda \pi_{t-i}^a \]

\[(C5) \quad r_t^e = i_t - \pi_t^e \]

\[(C6) \quad r_t^a = i_t - \pi_t^a \]

\[(C7) \quad \pi_t^e = \pi_t^a \quad \text{(in steady state equilibrium)} \]

where \(r_t^e\) is the real ex-ante rate of interest at time \(t\), \(r_t^a\) is the ex-post real rate of interest, and \(\pi_t^e\) and \(\pi_t^a\) are the expected and actual rates of inflation. Equation (C4) describes the adaptive expectations mechanism. If the coefficient of adaptive expectations "\(\lambda\)" is equal to unity then this period's rate of inflation is perfectly-anticipated. On the other hand, if \(\lambda = 0\), then the actual rate of inflation has no effect on this period's expected rate of inflation. The closer \(\lambda\) is to zero the longer the time horizon of market participants.

Assume that the rate of inflation increases from zero to a new steady state rate \(\bar{\pi}\). \((\pi_t^a = \bar{\pi} \text{ for } t = 1 \ldots N)\). The full equilibrium values for the nominal rate of interest, the volume of borrowing, and the real rates ex-ante and ex-post can be derived by substituting \(\bar{\pi}\) for \(\pi_t^e\) into equations (C1) and (C2). The steady state results are identical to those obtained under perfectly-foreseen inflation. The real rate of interest ex-post in equilibrium appears as:

---

1/ This assumes that the distribution effects generated on the path to equilibrium did not in fact alter the final equilibrium position.
\( i_t^* - \bar{\pi} = \frac{h - a}{b - k} \),

while the real rate ex-ante is:

\( i_t^* - \pi^e = i_t^* - \bar{\pi} = \frac{h - a}{b - k} \).

In full equilibrium, the real rates ex-ante and ex-post, are unchanged from the no-inflation case. In this model, as it stands, the authorities cannot change either the real rate of interest or the real volume of credit by inflation in the long run. We now consider the process of adjustment on the way to equilibrium when \( \pi^e \neq \bar{\pi} \), since it is commonly alleged that it is unanticipated inflation rather than inflation per se which has real effects.

At any time \( t > 0 \), the expected rate of inflation is:

\( \pi_t^e = \sum_{i=0}^{N} (1 - \lambda)^i \lambda \pi_t^{e} \).

Substituting this result into equations (C1) and (C2) and solving for the temporary equilibrium value for the nominal rate, we have:

\( i_t^e = \left[ \frac{h - a}{b - k} \right] + \pi_t^e = \left[ \frac{h - a}{b - k} \right] + \sum_{i=0}^{N} (1 - \lambda)^i \lambda \pi_t^{e} \),

where \( i_t^e \) is the market clearing nominal rate of interest at time \( t \), and \( \lim_{t \to \infty} i_t^e = i^* \). Thus, at any time, \( t \), the nominal rate of interest exceeds the no-inflation real rate by the expected rate of inflation. It follows, therefore, that the real ex-ante rate has not altered, since:

\( r_t^e = i_t^e - \pi_t^e = \frac{h - a}{b - k} \).
Similarly, the real volume of credit extended remains at:

\[ z_t^* = \frac{bh - ak}{b - k} \]  \hspace{1cm} (13)

Equations (C12) and (C13) demonstrate that the perceived market-clearing results are not altered by unanticipated inflation. That is, there is neither an increase in the amount of real credit extended nor a temporary lowering in the real ex-ante rate of interest, during the process of adjustment from one rate of inflation to another.

The real rate of interest ex-post, however, does alter. The ex-post real rate at any time \( t \) is:

\[ \left( C14 \right) \quad r_t^a = \left( \frac{h - a}{b - k} \right) - \pi_t^a + \sum_{i=0}^{N} \lambda (1 - \lambda)^i \pi_{t-i}^a \]

or alternately:

\[ \left( C15 \right) \quad r_t^a = \left[ \frac{h - a}{b - k} \right] - \left[ \pi_t^a - \pi_t^e \right] \]

Figure 1 illustrates the impact on the real ex-ante and real ex-post rates of interest over time of a rise in the rate of inflation.

Short of full equilibrium,\(^{1/}\) however, with an increase in the rate of inflation, borrowers gain at lenders' expense.\(^{2/}\) Consider the temporary equilibrium that occurs immediately after the rise to a new higher rate of price increase.

---

\(^{1/}\) The word "equilibrium" is being used in two senses here. Full equilibrium refers to the notion that there is no inherent reason for values of variables to change. Temporary equilibrium refers to the notion that the market clears.

\(^{2/}\) With a fall in the rate of inflation, a transfer in the reverse direction occurs.
Ultimate savers extended $\lambda^*$ of loans in terms of current purchasing power, expecting to receive $\lambda^*(1 + r^e)$ in return. In fact, they received $\lambda^*(1 + r^a)$ or $\lambda^*(\pi^a - \pi^e)$ less than they anticipated. $(\pi^a - \pi^e)$ is a measure of the unanticipated inflation. Borrowers anticipate paying a real rate of interest of $r^e_t = (i_t - \lambda \pi^a_t) = (i_t - \pi^e_t)$. They actually paid an actual ex-post rate of $r^a_t = (i_t - \pi^a_t)$.

Retrospectively, the system was in disequilibrium. If lenders could have foreseen correctly the extent of the inflation, at the real rate of interest, $r^a$, they only would have been willing to extend a smaller amount of credit. In a similar fashion, had borrowers foreseen correctly the real ex-post rate of interest, they would have acquired more loans, for any given nominal rate.
There is a lump-sum transfer of $x(\pi_t^a - \pi_t^e)$ from primary lenders to primary borrowers. This transfer is a distribution effect and should cancel out for the economy as a whole. It is assumed in our analysis that wealth transfers of this type have no effect on the behaviour of lenders or borrowers.\footnote{Should ultimate lenders attempt to recoup their transferred savings, or the behaviour of borrowers be affected by their gain, the demand and supply curves would shift. This phenomenon is generally known as Historisis. (The path to equilibrium affects the ultimate equilibrium achieved). Historisis is generally ignored in economics. And for good reason, for should trading occur at what is ultimately a non-equilibrium price on the way to equilibrium, the whole analysis of demand and supply appears to break down. See John R. Hicks, Value and Capital (2nd ed.; London: Oxford University Press, 1946): 128.} The net transfer from lenders to borrowers in any particular period may be represented by:

\[(C15) \quad T_t = \left[\frac{bh - ak}{b - k}\right] \left[\pi_t^a - \pi_t^e\right]\]

where $\lim_{t \to \infty} T_t = 0$. That is to say, in steady state equilibrium, the transfer effect is zero.

If it is possible to identify lenders and borrowers with specific groups (e.g., once a lender, always a lender), then the transfer effect is cumulative and is at any particular time, "t",

\[(C16) \quad CT_t = \int_0^T(N)dN\]

or in discrete time,

\[CT_t = \sum_{i=0}^{N} T_{t-i}\]
If, as is likely the case, lenders are not forever lenders, and borrowers are not forever borrowers, $CT$, the cumulative transfer, has no precise meaning.

Equations (C14) and (C15) demonstrate that the effect of unanticipated inflation, where both sides of the market are equally poor forecasters, causes a temporary fall in the real ex-post rate of interest, and a transfer from lenders to borrowers. Ex-ante magnitudes are not affected. Insofar as it is legitimate to neglect distribution effects, no real effects occur. It should be noted that if either borrowers or lenders fail to fully adjust to inflation, permanent effects do occur.

The important point here is that not only does perfectly-anticipated inflation not have any real effects, but unanticipated inflation does not affect behaviour either, so long as we assume that the distributional effects cancel out.

In the analysis, there is neither a temporary nor a permanent trade-off between inflation and real effects.¹/ Capital market decisions, the real ex-ante rate of interest and the real value of loan contracts remain unchanged throughout. The most that occurs is a distribution effect which

¹/ These results would seem to run counter Friedman's argument that: "The monetary authority can make the market rate less than the natural rate only by inflation. It can make the market rate higher than the natural rate only by deflation," ibid., p. 8, and "the temporary trade-off comes not from inflation per se but from unanticipated inflation," ibid., p. 11. In a later section of this chapter, it is argued that the market rate in Friedman's analysis is not the same thing as Wicksell's market rate.
results from the transfer of wealth from lenders to borrowers (if $\lambda < 1$). This transfer effect occurs because loan contracts are written in nominal money terms. At the same time, this distributional effect is customarily neglected in economics. In order to have a real impact (aside from wealth transfers) something else must be true other than the fact that inflation is imperfectly foreseen.

Fisher's hypothesis that the level of nominal interest rates adjusts to fully incorporate the expected rate of inflation can be represented using equation (C11) above. If the real rate of interest is not restricted to be constant and the nominal interest rate is not required to fully reflect inflationary expectations, equation (C11) can be rewritten as:

\[(C17) \quad i_t = r_t + b\pi_e^t,\]

where $i$ is the nominal rate of interest, and $r_t$ is the real rate of interest. Fisher's price-expectations hypothesis in the context of equation (C17) is represented by the hypothesis that $b$, the coefficient on the price-expectations variable is equal to unity. If $b$ is not equal to unity, then the nominal rate of interest does not rise by the expected rate of inflation. The less specific hypothesis that price-expectations exert a positive influence on the nominal rate of interest is represented by the hypothesis $b > 0$.

Price-Expectations and the Term Structure of Interest Rates

It is convenient, at this point, to extend the results
obtained in the last section to the term structure of interest rates. Little attention has been paid to the role of price-expectations in the term structure of interest rates as David Meiselman has pointed out.

I should think that in the huge amount of research on term structure, perhaps the greatest single deficiency has been the lack of a clear distinction between the real and nominal rates. One of the areas for fruitful research in the future is to make that distinction clearer by bringing prices and price-expectations into the analysis explicitly.1/

The purpose of this section is to explore the relationship between the real term structure of interest rates, the nominal term structure of interest rates, and price-expectations. In particular, a distinction is drawn between the hypothesis that the steady state or Fisher effect of price-expectations on nominal rates of interest varies across the term structure with the hypothesis that the time horizon over which price-expectations are formed varies with the term to maturity of financial instruments.

The most frequently suggested hypothesis concerning price-expectations and the term structure of interest rates is concerned with the time horizon over which expectations

are formed.\textsuperscript{1} This hypothesis has no commonly accepted name in the literature, but it seems appropriate to refer to it as the "differential effects according to term to maturity" hypothesis, or simply the "differential effects" hypothesis. The differential effects hypothesis which concerns the process of adjustment, has no immediate connection with the Fisher price-expectations hypothesis which is concerned with the steady state or equilibrium impact of inflation on nominal interest rates. As a preliminary matter, however, we will consider the circumstances under which the term structure of interest rates will be invariant in equilibrium to the rate of inflation.\textsuperscript{2}

A theory of the term structure of interest rates is contained within the Fisherian theory of the determination of interest rates. This term structure theory\textsuperscript{3} follows


almost directly from the two-period case analysed in depth by Fisher. In the "n" period case, Fisher's model permits the derivation of a "... separate rate of interest for each period." 1/

There single-period rates of interest can be represented by the symbols: \( r_1, r_2, \ldots r_n \). Fisher further points out that:

Since the element of risk is supposed to be absent it does not matter whether we consider these second year rates of interest \((r_2)\) and time preference as the ones which are expected, or those which will actually obtain for, under our assumed addition of no risk, there is no discrepancy between expectations and realizations. 2/

Consequently, the single-period rates of interest (i.e., the current-period rate, and the future single-period rates), can be represented by the expected rates: \( \bar{r}_1, \bar{r}_2, \bar{r}_3, \ldots \bar{r}_n \).

Under the usual optimum conditions, 3/ there will be no incentive for intertemporal arbitrage. Thus, for an interval greater than one period in length, say two periods

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1/ Ibid., p. 313. The Fisherian model for the determination of multi-period interest rates is a general equilibrium model. It is mathematically equivalent to the general equilibrium model of price-theory, and international trade, which did not arise until the 1930's and 1940's, with J. R. Hicks, Value and Capital (2nd ed.; Oxford: Clarendon Press, 1946) and Abba Lerner's, Economics of Control (New York; Macmillan, 1944). The first complete presentation of Fisher's multi-period model can be found in I. Fisher, The Rate of Interest (New York: Macmillan, 1907).


long, the same rate of return may be obtained by engaging in a two-period contract as is anticipated on the engagement of two successive one-period contracts. This provides a theory of the relationship among interest rates over the term structure.

Since in practice, no loan contracts are made in advance so that there are no market quotations for a rate of interest connecting, for example, one year in the future with two years in the future, we never encounter such separate year to year rates. We do, however, have such rate implicitly in long-term loans. The rate of interest on a long-term loan is virtually an average of the separate rates for the separate years constituting that long term.\(^1\)

Denoting longer-term interest rates by capital letters we have for the two-period case:

\[(1 + R_2)^2 = (1 + r_1)(1 + r_2)\]

where the preceding subscript refers to the length of the loan and the following subscript refers to the period in which the contract begins.\(^2\) If the expected real rate for the future one-year contract is substituted into equation (1) the "Expectations Theory" of the term structure of interest rates emerges.

\[R_1 = \sqrt{(1 + r_2)(1 + r_2)} - 1\]

---


\(^2\) It is assumed that all interest accrues at the end of the period of the loan. Alternatively, if interest is paid during the life of the contract, this formulation assume that it can be reinvested at the rate \(R_1\).
In the "n" period case we have:

\[ nR_1 = \frac{n\sqrt{(1 + r_1)(1 + r_2) \ldots (1 + r_n)} - 1}{n} \]

The term structure of interest rates so derived by Fisher is a term structure of real rates since, as Fisher argued, individuals are interested in intertemporal trading in real terms. It gives the rates at which real purchasing power over current goods can be traded for purchasing power over future goods at various times in the future.

If the Fisher price-expectations hypothesis holds for the individual rates of interest in the term structure of interest rates, then the nominal rate of interest for each maturity will be raised by the average rate of inflation expected over that period. Assuming that the individual real rates of interest are invariant to the rate of inflation,\(^1\) then a shift in the rate of inflation to a higher level, will result in an equal increment in all nominal rates of interest once expectations have adjusted to the higher rate of inflation.\(^2\)

The Hicks-Lutz\(^3\) "Expectations Theory" of the term

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1/ This assumption is implicit in Fisher's work. See I. Fisher, *op. cit.*, Chapter XIX.

2/ This statement ignores the interaction terms between the real rate and the rate of inflation. Consideration of the interaction term would require a slight modification in the above statement.

structure of interest rates, however, is presented in nominal rather than real terms. Let $i_1$ represent the nominal one-period rate in the current period, let $i_j$ represent the one-period rate of interest expected to prevail in period $j$, and $i_{j+1}$ the nominal long-term rate of interest on a loan contract of $j$ years. By parity of reasoning, with the case of the real term structure, the nominal term structure of interest rate relationships appear as:

\[
2i_1 = \sqrt{(1 + i_1)(1 + i_2) - 1} \\
\vdots \\
ni_1 = \sqrt[2n]{(1 + i_1)(1 + i_2) \cdots (1 + i_n) - 1}
\]

If the Fisherian "price-expectations" hypothesis is correct for each period, 1 then:

\[
i_1 = r_1 + B_1 \pi_1^e \\
2i_1 = 2r_1 + B_2 2\pi_1^e \\
\vdots \\
ni_1 = nr_1 + B_n n\pi_1^e
\]

and $B_1 = B_2 \ldots B_n = 1$.

1/ The interaction term has been ignored here. It should be noted that the real rates are not assumed to be constant; all that is assumed is that inflation does not affect these real rates. $\pi_1^e$ refers to the average rate of inflation expected to prevail over the next $n$ periods.
Assuming that the Fisherian theory of the real term structure is correct and that the constituent real rates of interest are not affected by the rate of inflation, what conditions must be placed on the "B" coefficients above if the nominal term structure is to conform to the "expectations hypothesis" of the term structure?

With a steady state of inflation where $\pi_t^e = \bar{\pi}$ for all $t$, the condition required is that $B_i = B_j$ for all $i, j$. Suppose in our empirical work we find that $B_i \neq B_j$ for $i \neq j$, then inflation alters the nominal term structure of interest rates in equilibrium. The observed nominal term structure cannot be consistent with the "expectations hypothesis," when it is assumed that this hypothesis is obeyed by the underlying real term structure. This proposition can readily be demonstrated. According to the Fisherian theory of the real term structure, we have:

$$(1 + 2R_1)^2 = (1 + r_1)(1 + r_2).$$

Assume that a steady state rate of inflation is experienced so that $\pi_t^e = \bar{\pi}$. The relationship between price-expectations, real rates and nominal rates for the two-interval case appears as:

$$i_1 = r_1 + b_1\pi_1^e$$
$$i_2 = r_2 + b_2\pi_2^e$$
$$2I_1 = 2R_1 + B_2\pi_1^e.$$

Since we assumed a steady state rate of inflation, we have
\[ \pi_1^e = \pi_2^e = 2\pi_1^e \]. Furthermore, \( b_1 = b_2 \), if inflation is to have the same effect continuously on the one-period rate. If \( B_2 \neq b_1 = b_2 \), then:

\[(7) \quad (1 + 2I_1)^2 \neq (1 + i)(1 + \tilde{i}_2) \.

Furthermore, if \( B_2 = b_1 = b_2 \), then \( (1 + 2I_1)^2 = (1 + i_1)(1 + \tilde{i}_2) \). \(^1/\)

Thus, on the premise that the term structure of real rates of interest obeys the Hicks-Lutz conditions, in a steady state inflation, a necessary and sufficient condition for the nominal term structure to obey the Hicks-Lutz conditions is that \( B_i = B_j \) for all \( i \) and \( j \).

The Fisher price-expectations hypothesis (i.e., \( B_i = B_j = 1 \) for all \( i, j \)), is a sufficient condition for the proposition to hold but it is not necessary.

The notion that the "B" coefficients may not be equal for all interest rates should not be confused with the "differential effects according to term to maturity" hypothesis discussed below. The "differential effects" hypothesis is actually an hypothesis about the formation of price-expectations per se; it is not an hypothesis about the impact of price-expectations on interest rates as such.

The "differential effects" hypothesis is based on the notion that, as the time horizon about which expectations are formed increases, the period of history upon which expectations are based increases in length. Applied to the capital market,

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\(^1/\) The proof can be extended to the \( n \) period case by induction.
it suggests that the past time horizon relevant to the formation of price-expectations varies directly with the security's term to maturity.

As a purely theoretical matter, one would expect that it would take longer for longer rates than for short rates. When you are buying a security with a short life, you are really interested in extrapolating price movements over a shorter future period of time than when you are buying a very long-term security. It seems not unreasonable that if you are extrapolating for a short period, you will look back for a shorter period than when you are extrapolating for a longer period.1/

The "differential effects" hypothesis is an hypothesis about the formation of price-expectations rather than about the effect of price-expectations on interest rates. There is a tendency in the literature to confuse this notion with the notion that the estimated coefficients of the price-expectational variable should vary according to the term to maturity of the security.2/

Lacking observed data on price-expectations for various time horizons, the "differential effects" hypothesis cannot be tested directly. A weak indirect test can be made using the "naive Fisherian" model. If the "differential effects" hypothesis is correct, the mean lag (or the length of the lag), in the formation of price-expectations for short-term rates of interest should be significantly less than the mean lag


for long-term rates of interest. Friedman notes that:

I regard it as very strong empirical confirmation . . .
that the period it takes to get to full adjustment tends
to be much longer for long rates than it does for short
rates. . . . The mean period of price anticipation turns
out to be something like 10 years for short rates and
20 years for long rates. Since these are average
periods, they imply that people take an even longer
period of past history into account. The results are
wholly consistent with Fisher's.1/

If differential effects are present, the nominal term
structure of interest rates will not reflect the underlying
real term structure during the period under which expectations
are adjusting, even though the Fisher price-expectation
hypothesis may hold for all rates of interest. Empirical
results on the differential effects hypothesis can be found
in chapter III. The evidence suggests that "differential
effects" are not important.

**Inflation and Asymmetric Expectations**

In the previous two sections of this chapter, the
assumption that lenders and borrowers formed inflationary
expectations in an identical manner was made. This assumption
is relaxed in this section, otherwise, the model remains the
same as before. Unless explicitly mentioned to the contrary, it
is assumed that borrowers adapt to inflation more quickly than
do lenders.

1/ Milton Friedman, op. cit., p. 21. It should be pointed out
that Fisher did not explicitly consider the "differential
effects" hypothesis. However, Fisher did use both short
and long rates of interest in his empirical work. It is
interesting to note that the relative effects found by
Fisher on short- and long-term rates of interest were the
opposite to those effects mentioned by Friedman. See
I. Fisher, op. cit., Chapter XIX.
The model now alters to:

\[(D1)\quad d_t = a + b(i_t - d_{\pi t}^e)\]

\[(D2)\quad s_t = h + k(i_t - s_{\pi t}^e)\]

\[(D3)\quad d_t = s_t\]

\[(D4)\quad d_{\pi t}^e = d_{\pi t-1}^e = \gamma(\pi_t^a - d_{\pi t-1}^e)\]

or

\[d_{\pi t}^e = \sum_{i=0}^{\infty} (1 - \gamma)^i \gamma \pi_{t-i}^a\]  
(formation of expectations by borrowers)

\[(D5)\quad s_{\pi t}^e = s_{\pi t-1}^e + \delta(\pi_t^a - s_{\pi t-1}^e)\]

\[s_{\pi t}^e = \sum_{i=0}^{\infty} (1 - \delta)^i \delta \pi_{t-i}^a\]  
(formation of expectations by lenders)

\[(D6)\quad d_t^e = i_t - d_{\pi t}^e\]  
(real ex-ante rate for borrowers)

\[(D7)\quad s_t^e = i_t - s_{\pi t}^e\]  
(real ex-ante rate for lenders)

\[(D8)\quad r_t^a = i_t^ - \pi_t^a\]  
(ex-post real rate of interest)

\[(D9)\quad d_{\pi t}^e = s_{\pi t}^e = \pi^a = \pi, \text{ in steady state equilibrium.}\]

In the previous model, it was assumed that \(\delta = \gamma\), otherwise nothing has been altered. The steady state equilibrium results are as before.\(^1\) That is to say, the steady equilibrium nominal rate is given by:

\[(D10)\quad i_t^* = \frac{h - a}{b - k} + \pi_t + \pi_t^e.

\(^1\) An asterisk indicates a steady state or full equilibrium solution value for an endogenous variable.
The real volume of credit contracts remains at:

(D11) \[ \xi_t^* = \frac{bh - ak}{b - k} \]

The real rate ex-ante to borrowers appears as

(D12) \[ d^*_{re_t} = i_t^* - d^*_e = i_t^* - \bar{\pi} = \left[ \frac{h - a}{b - k} \right] \]

while for lenders we have:

(D13) \[ s^*_{re_t} = i_t^* - s^*_e = \left[ \frac{h - a}{b - k} \right] \]

In steady state equilibrium the real ex-post rate of interest is unchanged at:

(D14) \[ r^*_{a_t} = \frac{h - a}{b - k} \]

The path to equilibrium, however, is altered considerably. It is no longer true that the real volume of loan contracts is unaffected by unanticipated inflation, nor is it the case that the real ex-ante rate of interest remains unchanged.\(^1\)

The market-clearing nominal rate \(i_t^*\) in any period \(t\), can be found by solving equations (D1) and (D2) for \(i_t^*\), to obtain:

(D15) \[ i_t^* = \frac{h - a}{b - k} + \frac{b d^*_e - k s^*_e}{b - k} \]

Alternatively, we have:

(D16) \[ i_t^* = \frac{h - a}{b - k} + (b - k)^{-1} \sum_{i=0}^{\infty} (b(1-\gamma)^i \gamma - k(1-\delta)^i \delta) \cdot \tau_{a_t-1} \]

\(^1\) See p. 18.
where \( \lim_{t \to \infty} \frac{h - a}{b - k} \).

Suppose that the real factors in determining the rate of interest remain unchanged. Equation (D16) shows that the nominal rate at any time \( t \) may not be a simple addition of the real rate and the expected rate of inflation.\(^1\)

Equation (D16) also demonstrates one of the reasons why regressions of nominal rates of interest on past rates of inflation yield highly misleading results. If equation (D16) represents the true reduced form expression for the nominal rate of interest, then the estimated coefficients of lagged inflation terms represent a complex mixture of the slopes of the two demand and supply functions and expectations on either side of the market.

At any time, \( t \), the real ex-ante rate of interest to borrowers is lower than it initially was, until full

\(^1\) Previous empirical work on the influence of price expectations in determining the nominal rate of interest has been based on the assumption that the nominal rate of interest can be represented as a simple addition of the expected rate of inflation and the real rate of interest. See, for example, I. Fisher, *The Theory of Interest* (New York: Macmillan, 1930), Chapter XIX.


equilibrium is reached.\(^1\) (See Figure 2). The path of the real ex-ante rate of interest to borrowers is given by:

\[
(D17) \quad d_{r_t} e = i_t - d_{n_t} e .
\]

Alternatively

\[
(D18) \quad d_{r_t} e = \frac{h - a}{b - k} + (b - k)^{-1} \left\{ \varphi \left( b(1-\gamma)^i \gamma_k - (1-\delta)^i \delta \pi_{t-i}^a \right) \right\}_i + \sum_{i=0}^{\infty} (1-\gamma)^i \gamma \pi_{t-i}^a .
\]

We also have:

\[
(D19) \quad \lim_{t \to \infty} d_{r_t} e = \frac{h - a}{b - k} .
\]

Under the hypothesis that \( \gamma > \delta \),\(^2\) equation (D18)

---

\(^1\) This assumes, of course, that we are dealing with an increase in the rate of inflation and that borrowers adapt more quickly to inflation than do lenders.

\(^2\) This is the Fisher-Friedman Asymmetric Expectations hypothesis. A rationalization for this asymmetry in the formation of price expectations may be that borrowers are frequently business firms which have fewer prices to predict than do ultimate savers. Fisher argued as follows:

"... in general, borrowers foresee better than lenders. The great borrowers of today are not, as is often supposed, the ignorant poor, but the alert and well informed rich. It is the function of these people to look ahead, and the consequence is that they foresee a rise or fall of prices more quickly than the lenders or bondholders, who are only silent partners in the business... The consequence, therefore, is an inflation of loans stimulated from both sides of the market." Irving Fisher, Elementary Principles of Economics (New York: Macmillan, 1911):362-363.
Figure 2
TIME PATH OF REAL MARKET RATE
FACING BORROWERS

Figure 3
TIME PATH OF REAL MARKET RATE
FACING LENDERS
demonstrates that there is a temporary trade-off between inflation and the real ex-ante rate of interest to borrowers. If the authorities are interested in the real cost of credit to borrowers, then \( d_{\text{re}} \) is the relevant rate for consideration.

The ex-ante rate of interest to lenders rises and is represented by:

\[
(D20) \quad s_{\text{re}}^e_t = i_t^e - s_{\pi_t}^e, \quad \text{or}
\]

\[
(D21) \quad s_{\text{re}}^e_r = \frac{h - a}{b - k} + (b - k)^{-1} \left( \Sigma b(1 - \gamma)^i \gamma - k(1 - \delta)\delta \pi_{t-1}^a \right)
\]

\[
- \sum_{i=0}^{\infty} (1 - \delta) \delta \pi_{t-i}^a,
\]

where,

\[
(D22) \quad \lim_{t \to \infty} s_{\text{re}}^e_t = \frac{b - a}{h - k}.
\]

It is this temporary effect which appears to be at the heart of Friedman's analysis as well, when he suggests:

"To state this conclusion differently, there is always a temporary trade-off between inflation and unemployment (real rate to borrowers); there is no permanent trade-off. The temporary trade-off comes not from inflation per se, but from unanticipated inflation." M. Friedman, "The Role of Monetary Policy," The American Economic Review 58(March 1968):11. See also ibid., pp. 7-8. The phrase in parenthesis has been added by the author.

Friedman's views on the temporary nature of the trade-off between unemployment and inflation appear to have been anticipated by Irving Fisher. See I. Fisher, "A Statistical Relation Between Unemployment and Price Changes," International Labour Review (June 1926):785-792.
Short of steady state equilibrium, the real ex-ante rate of interest to lenders is above its equilibrium value. (See Figure 3). Equations (D18) and (D21) reveal that, in the short run, the real ex-ante rate of interest perceived by lenders rises, and the real ex-ante rate of interest perceived by borrowers falls. Consequently, the real volume of loan contracts in the short run is higher than its equilibrium value. It is also clear that the term "real rate of interest" does not have an unambiguous meaning short of steady state equilibrium.

The real volume of loan contracts rises, in the short run, above its long-run equilibrium value. (See Figure 5). This short-run effect is due to asymmetric expectations and may be termed "bilkage". It is not the same thing as the transfer effect discussed previously. The transfer effect occurs because of disequilibrium in retrospect, while bilkage results from a change in the temporary equilibrium position induced by asymmetric expectations. Bilkage leads to a change in the real ex-ante equilibrium position, and so has real effects (aside from distribution effects).

The real volume of credit extended in any period can be obtained by substituting the value for \( i^*_t \) into equation (D1) or (D2) to obtain:

\[
(D23) \quad \kappa^*_t = (b - k)^{-1} \{bh - ak + bk \cdot (d_{\pi^e} - s_{\pi^e})\}.
\]

1/ Bilkage effects have been commonly ignored in the previous literature. Hence there is no accepted term for this concept.
Figure 4

SHORT AND LONG RUN TRADE-OFF
BETWEEN REDUCTIONS IN THE REAL RATE
TO BORROWERS AND INFLATION

Figure 5

SHORT RUN TRADE-OFF
BETWEEN BILKED FUNDS AND INFLATION
In the long-run, the real volume of credit extended is unaffected, since:

\[(D24) \lim_{t \to \infty} \ell_t = \frac{bh - ak}{b - k}.\]

The amount of bilkage occurring each period is given by:

\[(D25) \beta_t = \ell_t^* - \ell_t,\] or \[(D26) \beta_t = (b - k)^{-1} \sum (1 - \gamma)^i \gamma(1 - \delta)^i \delta^i t = 1 \sum a_t.\]

From (D26) it is seen that in the long-run, bilkage is zero for:

\[(D27) \lim_{t \to \infty} \beta_t = 0.\]

From equation (D26), it is also evident that the amount of bilkage occurring each period is affected not only by the coefficients of expectations but also by the elasticity of demand for and supply of credit with respect to the rate of interest. The cumulative total increase in the amount of bilked credit over a given period of length (a, b) is given by the expression:

\[(D28) DB_t = \int_a^b B(z) \, dz.\]

The conclusion that inflation causes a temporary reduction in the real rate of interest perceived by borrowers and, "... an inflation of loans stimulated from both sides of the market," is crucially dependent on the assumption

that borrowers adapt to inflation at a faster pace than do lenders, \( (\gamma > \delta) \).\(^1\) If \( \delta > \gamma \), the effect of inflation in the short-run is to withdraw credit from the market and raise the ex-ante real rate to borrowers. If \( \delta = \gamma \), then neither the real volume of credit contracted nor the real rate ex-ante to borrowers or lenders is affected in the short-run.\(^2\)

The ex-post real rate of interest is readily derived using equation (D8). The path of the ex-post real rate of interest is influenced by the asymmetry in expectations, since the market-clearing nominal rate is affected. In addition, the amount and nature of the ex-post transfer effect is altered.

There are a number of reasons to regard the application of the above model to the real world with suspicion. One of the more important reasons is that we have ignored bilkage effects on the real determinants of the system. If the amount of bilkage is sizable, then one would expect that


\(^2\) The bilkage model is formerly analogous to the Friedmanian explanation of the Phillips curve. (See M. Friedman, op. cit.). The conditions required to generate a short-run Phillips curve are similar to those required for bilkage to occur. The analysis is critically dependent upon the assumption that factor suppliers are less perceptive about the rate of inflation or adapt less quickly than do factor purchasers. In neither case is there a long-run trade-off between inflation and real effects. Both the bilkage model and Friedman's analysis of the Phillips curve have the interesting property that, should factor suppliers adapt to inflation more quickly than factor purchasers, the real effects are opposite in direction. Under this set of conditions, Friedman's short-run Phillips curve would be positively inclined, and bilkage would be negative.
the size of the capital stock would be altered from what it otherwise might be. This, in turn, will have an effect on the real rate of interest. Should this occur, inflation would produce real effects even in the long-run. Nevertheless, we disregard effects of this type.

A Fisher-Wicksell-Ohlin Model of Money, Credit Interest Rates and Inflation

In the previous section of this chapter it was argued that a change in the rate of inflation may have a temporary impact on ex-ante real rates of interest in the absence of perfect foresight, if expectations are asymmetric. In the introductory section of this chapter it was suggested that Milton Friedman appears to have incorrectly attributed this temporary effect on real rates of interest to Wicksell.\(^1\)

In this section it is argued that the Wicksell effect is conceptually distinct from the temporary asymmetric expectations effect.

The Fisherian theory of interest rates, which served as a basis for our previous analysis, is a non-monetary theory. That is to say, money does not enter as one of the basic determinants of real interest rates. Fisher, for example, argued

\(^1\) See page 7.
that:

In other words, interest changes with absolutely no relation to the quantity of money in circulation.1/

The role of money, in Fisher's analysis, is to determine the price level or its rate of change. Introducing money, in the Fisherian manner, into the previously discussed model does not alter the conclusions of the analysis. The Fisherian system is bloc-recursive. The rate of change of the money supply determines the rate of increase of the price level. The rate of increase in the price level generates inflationary expectations, which in turn, affect the nominal rate of interest through their impact on the demand and supply of credit. In this setting, the monetary authorities can have a temporary effect on real variables such as the ex-ante real rate of interest if and only if there is some asymmetry in the manner in which price-expectations are formed. The real effects are limited to short-run situations and disappear once inflation has been fully adjusted to.

The main difference between the Fisherian model analysed above and the Fisher-Wicksell-Ohlin model considered below is that the latter is not bloc-recursive. Changes in

the money supply directly affect the volume of real credit extended. Changes in the aggregate price level are a consequence of disequilibrium in the money market, while changes in the interest rate arise from disequilibrium in the capital market.

In Wicksell's analysis, however, disequilibrium in the commodity market (excess aggregate demand or supply) arises from the effect of changes in the money supply on the credit market. In particular, increasing the supply of money has the effect of augmenting the supply of loanable funds and so permits excess aggregate demand to occur.

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A simple version of the Fisher-Wicksell-Ohlin model can be formulated as follows:

\[(E1) \quad d_t = a + b(i_t - \frac{d}{\pi_t})\]

\[(E2) \quad s_t = h + k(i_t - s_{\pi_t}) + (\frac{\Delta M}{p})_t\]

\[(E3) \quad d_t = s_t\]

\[(E4) \quad d_{\pi_t} = d_{\pi_{t-1}} + \gamma(\pi^a_t - d_{\pi_{t-1}})\]

\[(E5) \quad s_{\pi_t} = s_{\pi_{t-1}} + \delta(\pi^a_t - s_{\pi_{t-1}})\]

\[(E6) \quad d_{r^e_t} = i_t - d_{\pi_t}\]

\[(E7) \quad s_{r^e_t} = i_t - s_{\pi_t}\]

\[(E8) \quad r^a_t = i_t - \pi^a_t\]

\[(E9) \quad d_{\pi_t} = s_{\pi_t} = \pi^a_t = \pi \quad \text{(in a steady state equilibrium)}\]

\[(E10) \quad \frac{\pi^a_t}{\Delta M} = f(\frac{\Delta M}{p})_t\] \quad 1/

The essential modifications made in this model are the endogenous nature of inflation, and the direct effect of additions to the money supply on the credit market.

It is convenient to begin the analysis by considering the version of the model commonly attributed to

1/ Assume that \( v \) and \( y \) are constant, then \( M_1 y = p_1 y \), \( M_2 y = p_2 y \) and

\[
\left(\frac{p_2 - p_1}{p_1}\right) = \left(\frac{v}{y}\right) \cdot \left(\frac{M_2 - M_1}{p_1}\right) . \text{ Thus, } \pi^a_t = f(\frac{\Delta M}{p})_t .
\]
In most of Wicksell's writings, there is evidence of a tendency to neglect the role price expectations in affecting the behavior of capital market participants. Thus we initially assume that: \( \lambda = \delta = 0 \). It is assumed that inflation is initially absent, so that \( d_{t+1} \) and \( s_{t+1} \) are zero. Initially \( \frac{\Delta M}{P} \) and \( \pi^a_t \) are zero, and the Wicksellian market and natural rates are identical and equal to:

\[
(E11) \quad i^m_t = i^n_t = \frac{h - a}{b - k}.
\]

Consider the result of increasing the money supply by the same nominal amount period after period. The money supply function which is added to the system of nine equations above is:

\[
(E12) \quad \Delta M^S_t = \Delta M \text{ for } t = 0, \ldots, \infty.
\]

As the price level rises, the net addition to the supply of credit each period falls. This in turn puts less pressure on the price level, so that:

\[
(E13) \quad \lim_{t \to \infty} \left( \frac{\Delta M}{P} \right)_t = 0, \text{ and}
\]

\[
(E14) \quad \lim_{t \to \infty} \pi^a_t = 0.
\]

The natural rate of interest appears as:

---


2/ For an exception, see ibid., pp. 3-4.
\[(E15) \quad i^n_t = \frac{h - a}{b - k},\]

while the Wicksellian market rate is:

\[(E16) \quad i^m_t = (b - k)^{-1}(h - a) - (\frac{\Delta M}{p})_t, \quad \text{where}\]

\[(E17) \quad \lim_{t \to \infty} i^m_t = i^n_t.\]

The increase in the supply of money leads to forced savings of the amount \((\frac{\Delta M}{p})_t\) each period. The amount of forced savings falls with time,\(^1\) and approaches zero in the limit. In the limit the market rate converges to the natural rate.\(^2\)

The notion of forced savings is conceptually distinct from the transfer effect and the bilkage effect which were discussed earlier. Forced savings arise through the ability of the authorities to increase the supply of credit by printing money. Bilkage occurs because of different speeds of reaction on the part of capital market participants to inflation. Transfer effects arise because of differences in ex-ante and ex-post equilibrium positions.

Consider, now, the situation where the monetary authorities increase the money supply at a constant percentage rate so that \((\frac{\Delta M}{p})_t\) is constant and equal to \(c\) for all \(t\).

\(^1\) Under the assumption that the nominal money supply increases by the same nominal amount each period.

\(^2\) These results are equivalent to those obtained in: K. Wicksell, Interest and Prices (New York: Augustus M. Kelly, 1965); see also the Introduction to Interest and Prices, pp. 19-20, by B. Ohlin.
The money supply function is altered to:  

$$ (E18) \quad \frac{M^s}{M_t} = \pi. $$

By equation (E10), the rate of inflation will increase from zero to a constant rate $\bar{\pi}$. In such a situation the movement of prices is cumulative (but does not accelerate). There is a permanent divergence between the natural rate of interest which remains at $(b - k)^{-1} (h - a)$, and the market rate which is equal to:

$$ (E19) \quad i^m_t = (b - k)^{-1} (h - c - a). $$

The ex-post real natural and market rates are given by:

$$ (E20) \quad r^{na}_t = (b - k)^{-1} (h - a) - \pi, \text{ and } $$

$$ (E21) \quad r^{ma}_t = (b - k)^{-1} (h - c - a) - \pi. $$

There is no tendency for the real market rate ex-post to converge towards the real natural rate ex-post.

A distinction must be drawn between ex-ante nominal and real, market and natural rates of interest once the assumption that $\delta = \lambda = 0$ is relaxed. If expectations are formed in a symmetrical fashion, but there is not perfect foresight, then equations (E4) and (E5) can be

---

1/ In the remainder of this chapter, the assumption that the nominal money supply is increasing at a constant percentage rate will be retained.

replaced by: 1/

\[(E22) \quad \pi^e_t = \pi^e_{t-1} + \lambda(\pi_t^a - \pi^e_{t-1}) , \text{ where } 0 < \lambda < 1.\]

At any time, \(t\), the nominal market rate is given by:

\[(E23) \quad i^m_t = (b - k)^{-1} (h - c - a) + \sum_{i=0}^{n} (1-\lambda)^i \lambda^i \pi^a_{t-i} .\]

The nominal natural is given by:

\[(E24) \quad i^n_t = (b - k)^{-1} (h - a) + \sum_{i=0}^{n} (1-\lambda)^i \lambda^i \pi^a_{t-i} .\]

There is no tendency for the nominal market rate to converge to the nominal natural rate, since \(\lim_{t \to \infty} i^m_t \neq \lim_{t \to \infty} i^n_t \). 2/

On the path to equilibrium, the nominal market and nominal natural rates of interest are less than their full equilibrium values. The real ex-ante natural rate of interest is not affected, with a change in the rate of inflation, while the real ex-ante market rate of interest remains equal to \((b - k)^{-1} (h - c - a)\) throughout the adjustment process. 3/

---

1/ Equations (E1), (E2), (E3), (E10), and (E22), at first sight, appear to reflect the verbal model analysed in: M. Friedman, "The Role of Monetary Policy," The American Economic Review 47(March 1968):1-17. This system of equations contains the Wicksellian distinction between the market and natural rates of interest, the Fisherian distinction between the real and nominal rates of interest, and the notion that lenders and borrowers gradually adjust their price-expectations to a change in the rate of inflation.

2/ Friedman suggests that inflation would have to occur at an accelerating pace to create a permanent divergence between the nominal market and nominal natural rates. This result cannot be obtained in the above model. Nor is it the case that the market rate will tend to converge to the natural rate once price-expectations take hold. See ibid., pp. 7-8.

3/ See Figure 6.
Under the set of assumptions considered here the monetary authorities cannot have a temporary effect on the expected real market rate of interest which disappears once capital market participants have fully adjusted to inflation.¹/

¹/ This suggests that Wicksell's market rate of interest is conceptually distinct from the market rate of interest discussed by Friedman. In Wicksell's analysis, the monetary authorities can depress the market rate below the natural rate, through their ability to print money and augment the supply of loanable funds. In Friedman's analysis, the monetary authorities can create a temporary fall in the market rate of interest, if borrowers adapt to inflation at a faster pace than do lenders.
It only appears to be possible to generate temporary effects on real variables, such as the real market rate of interest which disappears once price-expectations have adjusted to a change in the rate of inflation, if asymmetry in the formation of price-expectations is present.

Introducing asymmetric-expectations into the above model modifies the results in a straightforward way. The full equilibrium values for the nominal market and nominal natural rates of interest are unchanged from the previous case and are given by:

(E25) \[ i^m_\text{t} = (b - k)^{-1} (h - c - a), \text{ and} \]

(E26) \[ i^n_\text{t} = (b - k)^{-1} (h - a) \text{.} \]

The nominal market-clearing rate, \( i^m \), in any period, \( t \), can be found by solving equations (E1), (E2), (E4), (E5), (E10), and (E18) to obtain:

(E27) \[ i^m_\text{t} = (b - k)^{-1} (h - c - a) \]

\[ + (b - k)^{-1} \sum_{i=0}^{\infty} \{b(1-\gamma)^i\gamma - k(1-\delta)^i\delta\} \pi^a_{t-1} \text{.} \]

The nominal natural rate and the nominal market rate are not equal on the path to, or in, equilibrium.

Two ex-ante real market rates of interest exist in this model, the real rate facing borrowers and the real rate facing

---

1/ Drop equation (E22) and reintroduce equations (E4) and (E5) so that the model now consists of equations (E1), (E2), (E3), (E4), (E5), (E6), (E7), (E8), (E10), and (E18).
lenders. These rates are given by:

\[(E28) \quad d_{rme} = i_t^m - d_e^p, \text{ and} \]

\[(E29) \quad s_{rme} = i_t^m - d_e^p, \text{ and are} \]

equal only in full equilibrium. A temporary departure of the real ex-ante market rates of interest from their new steady state value, can occur only in the presence of asymmetric-expectations.

**Summary**

In this chapter, an attempt has been made to construct a relatively simple model of the credit market which would contain the essential elements of both Wicksell's distinction between natural and market rates of interest, and Fisher's distinction between real and nominal rates of interest. We were unable to produce results consistent with those suggested in M. Friedman's well-known article, "The Role of Monetary Policy,"\(^1\) using this model. Results which closely resemble Friedman's, however, can be obtained from the pure Fisherian model under the assumption that price-expectations are asymmetric, and that borrowers anticipate inflation more quickly than do lenders. Friedman's results appear to depend sensitively on this assumption. If, for example, expectations are symmetric, but foresight is not perfect, the effects suggested by Friedman are not observed. If, on the other hand, lenders adapt to inflation at a faster pace than do borrowers, effects opposite to those suggested by Friedman are obtained.

\(^{1}\) *Ibid.* pp. 5-11.
There would appear to be no need to add Wicksellian "wrinkles" to Fisher in order to produce the results suggested by Friedman. Indeed, to do so appears to make it impossible to produce Friedman's results.

In the Fisher-Wicksell-Ohlin model developed in the latter part of this chapter we were able to distinguish between three conceptually distinct effects of changes in the money supply and the resulting change in the price level on interest rates, which are often confused in the literature.\(^1\) The effects dealt with include: transfer effects, bilkage effects, and forced savings.

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CHAPTER III

THE NAIVE FISHERIAN MODEL AND RELATED HYPOTHESES:
EMPIRICAL RESULTS

Introduction

This chapter serves four purposes. In the first section of the chapter the "naive Fisherian" model\(^1\) is developed and tested using Canadian data over the period 1952-70. A comparison of my own results with other earlier work is also contained in this section. The second section of this chapter contains a discussion of the problems of interpreting several real rate series which have recently been published.\(^2\) This is followed by a section devoted to the, "differential effects according to term to maturity" hypothesis. The chapter concludes with a discussion of the limitations on the use of the Fisher

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1/ By the "naive Fisherian" model, I am referring to that model of nominal interest rate determination in which it is assumed that the real rate of interest is constant, and that price-expectations are formed on the basis of a weighted average of past rates of inflation. The "naive Fisherian" model employs the same basic set of assumptions used by Irving Fisher in his own empirical work. See I. Fisher, The Theory of Interest (New York: Macmillan, 1930):399-451.

equation.

The Naive Fisherian Model

Fisher's basic equation relating the real rate of interest, the nominal rate of interest, and inflation is usually presented using the following formula:¹

\[(1) \quad (1 + i) = (1 + r)(1 + \pi) \]

where \(i\) refers to the nominal rate of interest, \(r\) represents the real rate of interest, and \(\pi\) represents the rate of inflation. It is customary to ignore the term \(\pi r\) which is usually quite small. Thus equation (1) usually appears as:

\[(2) \quad i = r + \pi \]

Equation (2) is subject to a variety of interpretations, only one of which is representative of Fisher's proposition that during periods of inflation the nominal rate of interest adjusts to incorporate the expected rate of inflation. Equation (2) can be variously interpreted as:

a) An identity relating the nominal rate of interest the actual rate of inflation, and the ex-post real rate of interest

b) A definition of the expected real rate of interest

c) A definition of the expected rate of inflation

d) An equilibrium condition

e) An hypothesis which suggests that the level of nominal interest rates adjusts to incorporate the expected rate of inflation.

It is possible for any of interpretations (a) through (d) to be true without (e) being true.\(^1\) It can be argued that failure to distinguish among these possibilities has led several

economists into a number of inconsistencies in their empirical work. In particular it is incorrect to argue that since (a), (b), or (c) happen to be true, (as a matter of definition), nominal interest rates reflect the expected rate of inflation. It is, of course, possible that interpretation (d) could hold but Fisher's hypothesis did not. This would be the case if inflationary expectations had an impact on the real rate.\(^1\)

In order to test the Fisher hypothesis using equation (2) as a basis, we require a representation of the real rate of interest, and the expected rate of inflation. In general we may write:

\[
i_t = g(x)_t + \pi^e_t + u_t,
\]

where \(i\) is the nominal rate of interest, \(g(x)\) is a function which determines the real rate of interest, \(\pi^e_t\) is the expected rate of inflation, and \(u_t\) is an error term. Equation (C11) in chapter II, can be interpreted as a particular version of equation (3) above.\(^2\) Equation (3) forms the basis for all

---


\(^2\) Equation (3) is more general than equation (C11) in two senses. In equation (3) the real rate of interest need not be assumed constant. Equation (3) is also consistent with a variety of other models that have been used to study the Fisher effect. See, for example, T. J. Sargent, "Commodity Price-expectations and the Interest Rate," *The Quarterly Journal of Economics* 83(February 1969): 127-140, and M. Feldstein and O. Eckstein, "The Fundamental Determinants of the Interest Rate," *Review of Economics and Statistics* 52(October 1970):363-375.
previous or contemporaneous empirical work on the Fisher effect.\footnote{1} Implicit in the use of equation (3) alone as a test of the Fisher effect is the assumption that the real rate of interest is unaffected by the rate of inflation.\footnote{2}

The Fisher price-expectations hypothesis in the context of equation (3), above, is represented by the hypothesis that the coefficient \(b\) is equal to unity. If \(b\) is not equal to unity then the nominal rate of interest does not rise by the expected rate of inflation contrary to hypothesis. The less specific hypothesis that price-expectations exert a positive influence on the nominal rate of interest is represented by the hypothesis, \(b > 0\).


\footnote{2}{This is not to say that price-expectations may not affect the real rate of interest in actual practice. Nevertheless, if equation (2) or (3) alone is to form the basis of testing for the Fisher price-expectations hypothesis, the assumption that the real rate of interest is unaffected by price-expectations is required. The reason for this assumption is straight-forward, for if the rate of inflation were to impact real rate\(\text{we could not place a prior on the coefficient of the expectation variable in equation (3).}}
It should be stressed that equation (3) always provides a conditional test of the Fisher price-expectations hypothesis. The test is conditional upon a maintained hypothesis on the formation of price-expectations, and a maintained hypothesis on the derivation of the real rate of interest.\(^1\)

Insofar as there are countless combinations of hypotheses on the real rate of interest and the formation of price-expectations, the Fisher hypothesis can never be generally rejected using an operational variant of equation (3).\(^2\) Clearly, if the data does not support the conditional test of the Fisher hypothesis, one cannot thereby claim that one or both of the maintained hypotheses can be rejected. If one or both of the maintained hypotheses were known to be false, the estimated equation would not provide a test of the Fisher price-expectations hypothesis.

Equation (3) must be specified in greater detail if it is to be used to test the Fisher hypothesis. The "naive Fisherian" model is based on the assumptions that the real rate of interest is constant over time, and that price-expectations are a linear function of past rates of change of prices.\(^3\)

---

1/ The conditional nature of the test of the Fisher price-expectations hypothesis which equation (3) allows was apparently not recognized by, Yohe and Karnosky, op. cit., Gibson, op. cit., or Feldstein and Eckstein, op. cit. Consequently, as is later argued in this chapter, these authors appear to have incorrectly interpreted their results.

2/ The strongest statistical result one can obtain using equation (3) is a rejection of a conditional version of the Fisher hypothesis.

3/ This assumption is made despite the warning by Samuel Taylor Coleridge that: "To most men, experience is like the stern lights of a ship, which illumines only the track it has passed."
Other variables such as government policy instruments, and unemployment which might influence the formation of price-expectations, are ignored. It is also assumed that expectations are single valued.\textsuperscript{1} Since there are no reasonable a priori grounds for preferring one method of representing the expectation variable over another method comparatively simple ways of representing distributed lags will be used.

These assumptions provide us with the following estimating equation:

\[(4) \quad i_t = a + \sum_{i=0}^{N} w_i \left( \frac{\Delta P}{P} \right)_{t-i} + u_t, \]

where \( \left( \frac{\Delta P}{P} \right)_t \) represents the actual rate of change of the price level at time \( t \).\textsuperscript{2} The coefficient "\( b \)" on the price-expectations variable can be retrieved from equation (4) by summing the weights on the inflation variable.\textsuperscript{3}

\textsuperscript{1} Alternatively it may be assumed that individuals form a subjective probability distribution about the future course of inflation, but react only to the mean of that function.

\textsuperscript{2} Equation (4) is the same estimating equation employed by Gibson, op. cit., and W. P. Yohe and D. S. Karnosky, op. cit., in their empirical work.

\textsuperscript{3} This is only true under certain assumptions on the formation of price-expectations. If it is assumed that the expected rate of inflation is a linear function of past rates of inflation we have:

\[(5) \quad \pi^e_t = \Sigma a_i \left( \frac{\Delta P}{P} \right)_{t-i}. \]

Each "\( w_i \)" coefficient in equation (4) is the product of the coefficient "\( b \)" and the corresponding "\( a_i \)." An estimate of "\( b \)" can be obtained if it is assumed that \( \Sigma a_i = 1. \)
Monthly Data

Estimates of equation (4) using monthly data over the period January 1952 to December 1970 appear in Table I-A. The rate of price change is compounded and expressed as an annual percentage rate. The Canadian Consumer Price Index was used. Yields to maturity on a slate of Government of Canada securities ranging from 90-day treasury bills to long-term bonds were used as dependent variables. An equation was also estimated for the McLeod, Young, Weir, industrial bond yield index.

The t value for each variable appears in parentheses below the corresponding estimated regression coefficient. The summary statistics appearing below each regression are: D.W. -- Durbin-Watson Statistic, S.E.E. -- the standard error of the estimate, $R^2$ -- the adjusted coefficient of determination, A.L. -- the average lag, S -- the sum of the coefficients on the price terms, $1^/$ S.E.S. -- the standard error of S, and $St_1$ -- the "t" statistic for S under the hypothesis that $S = 1$.

The average lag is not reported if negative coefficients appear in the regression. Under this circumstance there is no generally meaningful formula for computing the average lag. $2^/$

---

$1^/$ S is our estimate of $b$ in equation (3) or (4).

The Fisher hypothesis, interpreted as suggesting that the sum of coefficients is equal to unity, can be rejected for all maturities at the .01 level of significance. The results do not appear to be inconsistent with the weaker version of the Fisher hypothesis ($\beta > 0$), since the hypothesis that the sum of the weights is equal to zero can also be rejected for all maturities at the .01 level of significance.  

Mean lags are not calculable for the three shortest term rates, since negative coefficients enter into the lag distribution. There is no apparent difference among the mean lags of the remaining rates. The adjusted coefficients of determination decline with the term to maturity on Government securities, the highest being .42 for the Treasury Bills, and the lowest being .32 for long-term issues.

The pattern of intercept terms suggests that the real rate of interest increases with term to maturity. The relatively high real rate for (4.61) the McLeod, Young, Weir industrial bond index suggests that there is a risk premium attached to industrial securities in comparison with Government securities. The pattern of real rates is in accord with standard liquidity preference notions.

In Table I-B, results using 18 unconstrained lags on

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1/ In Table I-A "t" values for the hypothesis $\beta = 0$ do not appear. They can be easily calculated from the information in the table, however. For example, the relevant $t$ value in the case of the three-month treasury bill rate is 12.722.
<table>
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<tr>
<th>T.B.</th>
<th>Intercept</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
<th>$b_9$</th>
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<td>(2.27)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M.Y.W.</td>
<td>4.61</td>
<td>0.073</td>
<td>0.069</td>
<td>0.053</td>
<td>0.062</td>
<td>0.053</td>
<td>0.055</td>
<td>0.052</td>
<td>0.028</td>
<td>0.044</td>
<td>0.058</td>
<td>0.020</td>
<td>-0.0121</td>
</tr>
<tr>
<td></td>
<td>(37.77)</td>
<td>(3.49)</td>
<td>(3.14)</td>
<td>(2.40)</td>
<td>(2.82)</td>
<td>(2.45)</td>
<td>(2.55)</td>
<td>(2.37)</td>
<td>(1.34)</td>
<td>(2.06)</td>
<td>(1.91)</td>
<td>(1.00)</td>
<td>(-0.62)</td>
</tr>
<tr>
<td></td>
<td>D.W. = 0.077</td>
<td>S.E.E. = 1.142</td>
<td>$R^2$ = 0.39</td>
<td>S.E. = 0.5671</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average Lag = 2.473 Months</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table I-A**

**RATES OF PRICE CHANGE AND INTEREST RATES**

JANUARY 1952 - DECEMBER 1970, MONTHLY OBSERVATIONS (228)

O.L.S. 12 UNCONSTRAINED LAGS
<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Intercept</th>
<th>Sum of Weights</th>
<th>D.W.</th>
<th>S.E.E.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.B.</td>
<td>2.32</td>
<td>.667</td>
<td>.191</td>
<td>1.334</td>
<td>.41</td>
</tr>
<tr>
<td></td>
<td>(15.46)</td>
<td>(- 5.431)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>3.12</td>
<td>.597</td>
<td>.193</td>
<td>1.126</td>
<td>.41</td>
</tr>
<tr>
<td></td>
<td>(24.59)</td>
<td>(- 8.530)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-5</td>
<td>3.50</td>
<td>.569</td>
<td>.166</td>
<td>1.081</td>
<td>.40</td>
</tr>
<tr>
<td></td>
<td>(28.78)</td>
<td>(- 9.497)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-10</td>
<td>3.70</td>
<td>.555</td>
<td>.205</td>
<td>1.134</td>
<td>.36</td>
</tr>
<tr>
<td></td>
<td>(29.98)</td>
<td>(- 9.344)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10+</td>
<td>3.95</td>
<td>.504</td>
<td>.095</td>
<td>1.120</td>
<td>.31</td>
</tr>
<tr>
<td></td>
<td>(31.34)</td>
<td>(-10.551)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M.Y.W.</td>
<td>4.54</td>
<td>.596</td>
<td>.110</td>
<td>1.147</td>
<td>.39</td>
</tr>
<tr>
<td></td>
<td>(35.14)</td>
<td>(- 8.182)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
monthly data over the period January 1952 to December 1970 are found. Only the sum of the coefficients is presented. The "t" statistic appearing under the sum of the weights has been calculated for the null hypothesis that the sum is equal to unity. None of the six additional lag terms were significant and most were negative. The summary statistics remained essentially unchanged. Similar results are obtained if the length of the lag is extended to 24 months.

In summary, while these results suggest that price expectations may affect the level of nominal interest rates, the evidence is not clear cut. The Fisher hypothesis is uniformly rejected by the data. The weaker version of the Fisher hypothesis, however, is not. A number of statistical problems are inherent in the result reported in Tables I-A and I-B.

There is a relatively serious multicollinearity problem. In many cases, the correlation matrix of regressors is close to being singular. This, in itself, would tend to raise the standard errors of the coefficients. The degree of serial correlation is also quite severe. This means that our estimators are not efficient. At the same time the variances of the obtained estimators will be biased downward, tending to raise the t statistics. This combination of severe serial correlation combined with multicollinearity means that it is difficult to place much meaning on the tests of significance.

Autocorrelation by itself still leaves us with unbiased estimators. There are reasonable grounds for suspecting, however, that the coefficients on the lagged
price terms are biased upwards. The serial correlation suggests that important variables may have been omitted. If the omitted variables depart substantially from orthogonality with the included regressors, the estimated coefficients will be biased. Thus, it appears that the equations as they stand suffer from specification error. The low proportion of the variance explained is also suggestive of omitted variables. In the following section a variety of efforts are made to circumvent the statistical difficulties encountered in estimating equation (4).

One way of evading the multicollinearity problem in a regression equation such as equation (4), involving direct estimation of a long distributed lag, is through the use of the Almon lag technique.\(^1\) Results using a 3rd degree polynomial, and 12, 24 and 36 lags are recorded in Tables II-A and II-B. These results are substantially unchanged from our earlier results using unconstrained lags. Serial correlation remains a distinct difficulty.

Representative results using two standard techniques for reducing the serial correlation problem are reported in Table III. Results for two interest rates, the 90-day Treasury Bill rate, and the long-term Government of Canada bond yield, are recorded.\(^2\)

---


\(^2\) The results were not particularly sensitive to the maturity of the bond chosen. Since broadly similar results were obtained, only results for a representative short and long-term rate of interest are reported.
### Table II-A

**RATES OF INTEREST AND RATES OF PRICE CHANGE**  
**JANUARY 1952 - DECEMBER 1970, MONTHLY OBSERVATIONS**  
**3RD DEGREE POLYNOMIAL, 12 LAGS**

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Intercept</th>
<th>Sum of Weights</th>
<th>D.W.</th>
<th>S.E.E.</th>
<th>$R^2$</th>
<th>Average Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.B.</td>
<td>2.40 (16.88)</td>
<td>.618 (-7.351)</td>
<td>.10</td>
<td>1.352</td>
<td>.40</td>
<td>4.018</td>
</tr>
<tr>
<td>1-3</td>
<td>3.24 (27.15)</td>
<td>.534 (-10.669)</td>
<td>.10</td>
<td>1.135</td>
<td>.40</td>
<td>4.310</td>
</tr>
<tr>
<td>3-5</td>
<td>3.62 (31.79)</td>
<td>.510 (-11.761)</td>
<td>.09</td>
<td>1.082</td>
<td>.40</td>
<td>4.418</td>
</tr>
<tr>
<td>5-10</td>
<td>3.82 (32.22)</td>
<td>.496 (-11.608)</td>
<td>.13</td>
<td>1.128</td>
<td>.36</td>
<td>4.644</td>
</tr>
<tr>
<td>10+</td>
<td>4.06 (34.86)</td>
<td>.451 (-12.880)</td>
<td>.04</td>
<td>1.107</td>
<td>.33</td>
<td>4.599</td>
</tr>
<tr>
<td>M.Y.W.</td>
<td>4.67 (38.91)</td>
<td>.535 (-10.609)</td>
<td>.04</td>
<td>1.141</td>
<td>.39</td>
<td>4.646</td>
</tr>
</tbody>
</table>

### Table II-B

**JANUARY 1952 - DECEMBER 1970, MONTHLY OBSERVATIONS**  
**3RD DEGREE POLYNOMIAL, REPRESENTATIVE RESULTS**

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Intercept</th>
<th>Sum of Weights</th>
<th>Number of Lags</th>
<th>D.W.</th>
<th>S.E.E.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.B.</td>
<td>2.39 (14.65)</td>
<td>.616 (-6.344)</td>
<td>24</td>
<td>.123</td>
<td>1.370</td>
<td>.38</td>
</tr>
<tr>
<td>10+</td>
<td>3.66 (28.74)</td>
<td>.625 (-7.652)</td>
<td>24</td>
<td>.067</td>
<td>1.030</td>
<td>.42</td>
</tr>
<tr>
<td>T.B.</td>
<td>2.54 (12.74)</td>
<td>.523 (-6.417)</td>
<td>36</td>
<td>.067</td>
<td>1.520</td>
<td>.23</td>
</tr>
<tr>
<td>10+</td>
<td>3.29 (25.11)</td>
<td>.804 (-3.520)</td>
<td>36</td>
<td>.035</td>
<td>.96</td>
<td>.50</td>
</tr>
</tbody>
</table>
Using first-differences, the serial correlation is considerably reduced. The regressions, however, do not explain any of the variation in the rate of interest. Using an F test—the regressions as a whole would be rejected. The sum of the coefficients is .0004 for the short-term rate and -.007 for the long-term rate.\footnote{The hypothesized value is unity in each case.} Similar results are obtained using the Hildreth-Lu procedure. The estimated auto-regressive parameter is .99 (i.e., it is at the extremity of the range) suggesting that the serial correlation may be of a higher order than the first. The adjusted coefficients of determination are effectively zero. In all cases the coefficient on the price expectational variable is significantly different from one, at the .01 level, and insignificantly different from zero.

None of the individual coefficients in any of the regressions is significantly different from zero. At the same time, several coefficients are negative in sign. At face value, these results suggest that our earlier regressions of interest rates on lagged rates of price change may have been picking up bogus lags induced by the existence of serial correlation.

Table IV contains results using the Durbin Two-Step estimation procedure. This procedure allows us to
<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Intercept</th>
<th>Sum of Weights</th>
<th>D.W.</th>
<th>S.E.R.</th>
<th>$R^2$</th>
<th>$\rho$</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.B.</td>
<td>2.98</td>
<td>.076</td>
<td>1.449</td>
<td>.33</td>
<td>.00</td>
<td>.99</td>
<td>Hildreth-Lu</td>
</tr>
<tr>
<td></td>
<td>(1.84)</td>
<td>(-15.584)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T.B.</td>
<td>.02</td>
<td>.0005</td>
<td>1.545</td>
<td>.34</td>
<td>.00</td>
<td>1</td>
<td>First Difference</td>
</tr>
<tr>
<td></td>
<td>(.94)</td>
<td>(-162.5270)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10+</td>
<td>5.02</td>
<td>.028</td>
<td>1.337</td>
<td>.13</td>
<td>.01</td>
<td>.99</td>
<td>Hildreth-Lu</td>
</tr>
<tr>
<td></td>
<td>(7.77)</td>
<td>(-41.589)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10+</td>
<td>.03</td>
<td>-.007</td>
<td>1.242</td>
<td>.26</td>
<td>.04</td>
<td>1</td>
<td>First Difference</td>
</tr>
<tr>
<td></td>
<td>(1.58)</td>
<td>(-214.257)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
obtain estimates which are asymptotically efficient.\textsuperscript{1}

It was assumed that the residuals followed a first order autoregressive scheme. The estimated autoregressive parameter appears as $\theta$, among the summary statistics reported for each regression. The Durbin Two-Step procedure tended to yield estimates of the autoregressive parameter which were lower than those obtained using the Hildreth-Lu procedure. Autoregressive parameters in the neighbourhood of .9 were usually obtained. The results are, however, in accord with our earlier efforts using first-differences and the Hildreth-Lu procedure. The sum of the coefficients drops substantially in comparison with our initial estimates. For example, in the case of 3-month Treasury Bills the sum of the coefficients drops from .67 (in Table I-A) to .173 (in Table III). In the case of the long-term Government bond yield the sum of the coefficients falls from .48 to .143. The Fisher hypothesis can be rejected for all maturities. The adjusted coefficients of determination are either zero or a negligible distance from zero.

The results of our experimentation with the naive Fisherian model and monthly data may be summarized as follows:

Table IV

INTEREST RATES AND RATES OF PRICE CHANGE
DURBIN TWO-STEP PROCEDURE
JANUARY 1952 - DECEMBER 1970
12 UNCONSTRAINED LAGS, MONTHLY OBSERVATIONS

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Intercept</th>
<th>Sum of Weights</th>
<th>Auto Regressive Coefficient</th>
<th>D.W.</th>
<th>S.E.E.</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.B.</td>
<td>.22</td>
<td>.173</td>
<td>.940</td>
<td>1.471</td>
<td>.33</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>(9.84)</td>
<td>(-13.193)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>.42</td>
<td>.254</td>
<td>.098</td>
<td>1.301</td>
<td>.33</td>
<td>.05</td>
</tr>
<tr>
<td></td>
<td>(17.22)</td>
<td>(-12.046)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-5</td>
<td>.50</td>
<td>.282</td>
<td>.886</td>
<td>1.184</td>
<td>.31</td>
<td>.08</td>
</tr>
<tr>
<td></td>
<td>(21.10)</td>
<td>(-12.480)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-10</td>
<td>.28</td>
<td>.087</td>
<td>.943</td>
<td>2.437</td>
<td>.35</td>
<td>.00</td>
</tr>
<tr>
<td></td>
<td>(11.97)</td>
<td>(-13.745)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10+</td>
<td>.47</td>
<td>.143</td>
<td>.908</td>
<td>1.013</td>
<td>.26</td>
<td>.05</td>
</tr>
<tr>
<td></td>
<td>(24.77)</td>
<td>(-17.449)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M.Y.W.</td>
<td>.62</td>
<td>.233</td>
<td>.890</td>
<td>.903</td>
<td>.30</td>
<td>.09</td>
</tr>
<tr>
<td></td>
<td>(27.61)</td>
<td>(-13.729)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Using unconstrained lags the Fisherian hypothesis was rejected by the data. The weaker hypothesis that price-expectations exert a positive influence on nominal interest rates was not immediately rejected. Two statistical problems were evident, multicollinearity and serial correlation. An attempt was made to circumvent the multicollinearity problem by reestimating the equations using the Almon lag technique. This has little impact on the estimated coefficients. Attempts to treat serial correlation, however, resulted in the effective disappearance of the relationship between inflation and interest rates. No support for the Fisherian price-expectations hypothesis emerges from confronting the naive Fisherian model with monthly data. The severe degree of serial correlation suggests that the naive Fisherian model may not be a good approximation to reality for monthly observations and that there are other factors which may have a systematic impact on the rate of interest.

Comparison With Other Studies Using Monthly Data

It is useful to compare our results with those which were recently obtained by W. P. Yohe and D. S. Karnosky\(^1\) using the naive Fisherian model. Yohe and Karnosky claim to have found support for the Fisher hypothesis and concluded that:

---

price level changes since 1952 have evidently
come to have a prompt and substantial effect on price
expectations and nominal interest rates.1/

Yohe and Karnosky's work appears to have received widespread
approval and is frequently referred to in the literature.2/
There are a number of serious short-comings in the evidence
presented by Yohe and Karnosky. Interestingly Yohe and
Karnosky did not explicitly test either the weak or strict
version of Fisher's price-expectations hypothesis.

Table V contains comparable results from the Yohe
and Karnosky study and the present study. Yohe and Karnosky
typically used longer lags than those contained in this study.
Our results, however, indicated that there was little value
in extending the length of the lag much beyond twelve months.
The evidence presented earlier in this chapter indicates that
serial correlation is a problem when the naive Fisherian model
is confronted with monthly data. Nevertheless autocorrelation

1/ Ibid., p. 36.

2/ The following comments by various authors are suggestive.
"W. P. Yohe and D. S. Karnosky in a comprehensive article
... provide a succinct statement of the Fisher theory and a
careful discussion of the theoretical aspect of the Gibson
Paradox. They derive alternative estimates of the real rate,
and relate this analysis to explain interest rate movements
in recent years." D. I. Fand, "Monetarism and Fiscalism,"
"More recently, the Federal Reserve Bank of St. Louis has been
estimating the 'real rate', and their estimates are remarkably
stable despite very large changes in nominal rates." M.
Friedman, "A Monetary Theory of Nominal Incomes," Journal
a detailed study of interest rates and the Fisher effect, see
W. P. Yohe and D. S. Karnosky ... " L. C. Anderson and
The Federal Reserve Bank of St. Louis Review 52(April 1970):
10.
<table>
<thead>
<tr>
<th>Estimation Technique</th>
<th>Length of Lag</th>
<th>SHORT-TERM RATE&lt;sup&gt;1&lt;/sup&gt;</th>
<th>LONG-TERM RATE&lt;sup&gt;1&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fisher Effect</td>
<td>Adjusted Coefficient of Determination</td>
</tr>
<tr>
<td>Yohe &amp; Karnosky</td>
<td>O.L.S.</td>
<td>24</td>
<td>.76</td>
</tr>
<tr>
<td>McCollum</td>
<td>O.L.S.</td>
<td>12</td>
<td>.67</td>
</tr>
<tr>
<td>Yohe &amp; Karnosky</td>
<td>O.L.S.</td>
<td>24</td>
<td>.67</td>
</tr>
<tr>
<td>McCollum</td>
<td>O.L.S. 6th degree Almon</td>
<td>12</td>
<td>.62</td>
</tr>
<tr>
<td>McCollum</td>
<td>O.L.S. 3rd degree Almon</td>
<td>24</td>
<td>.62</td>
</tr>
<tr>
<td>McCollum</td>
<td>Hildreth-Lu</td>
<td>12</td>
<td>.08</td>
</tr>
<tr>
<td>McCollum</td>
<td>Durbin Two-Step</td>
<td>12</td>
<td>.17</td>
</tr>
</tbody>
</table>

1 The U.S. short-term rate is the four to six-month commercial paper rate, while the Canadian short-term rate is the 90-day treasury bill yield. A sufficiently long time series for the Canadian commercial paper rate does not exist. The U.S. long-term rate, is the yield to maturity on corporate Aaa bonds, while for Canada the long-term Government of Canada bond yield was used. The sample periods are almost identical, the U.S. results were obtained from 1952-69 data, while the Canadian results were obtained using the period 1952-70. Price changes were calculated using the consumer price index in each country.
difficulties are never mentioned by the authors and no Durbin-Watson statistics were reported. Our results suggest that attempts to reduce the serial-correlation problem result in the disappearance of the relationship between past rates of price change and interest rates, at least in the case of monthly data. Consequently the use of monthly data and the naive Fisherian model does not produce evidence in support of Fisher's price-expectations hypothesis. It should also be pointed out that a number of the conclusions which were drawn by Yohe and Karnosky violate the assumptions under which they estimated the model.\(^1\)

Quarterly Data

Results for the naive Fisherian model, using Canadian quarterly data over the period 1955-1970 are recorded in Table \(^2\) only the sum of the coefficients appears. As it turned out, however, the coefficients on lags after the second or third quarter were not significantly different from zero at the .05 level. Furthermore most of the response tends to occur within the first three quarters. For example, in the case of the Treasury Bill rate the sum of the coefficients is

\(^1\) For example, the real market rate calculations on page 24, and the experimental real rate series 2, on page 34 are inconsistent with the use of the naive Fisherian model to test for price expectations. Two different and inconsistent definitions of the real market rate are given on pages 24 and 38.

\(^2\) The sample period 1955-1970 was chosen rather than 1952-1970 in order to avoid a discontinuity in the price series.
.90, while the sum of the first three coefficients is .80. A similar statement can be made for all maturities. To the extent that these results can be construed as evidence that price expectations influence nominal interest rates, they suggest that the response of interest rates to a new rate of inflation occurs rapidly, with most of the effect occurring within the first year. In this respect (rapid adjustment) our results are similar to those claimed by W. P. Yohe and D. S. Karnosky\(^1\) and at odds with those of Irving Fisher,\(^2\) Thomas Sargent,\(^3\) and Milton Friedman\(^4\) all of whom report long lags in the adjustment of nominal interest rates to inflation.

The adjusted coefficients of determination range from .38 for the Government of Canada long-term bond yield to .54 for the Government of Canada 1 to 3 year bond yield. The initial difference between these results and those previously obtained using monthly data is that the sum of the coefficients is closer to unity. In three cases out of six we are unable to reject the Fisher hypothesis. (using a one-tailed test)\(^5\)

---


5/ The reason for using a one-tailed test here is that the most reasonable alternative, a priori, is $S = 0$. The possibility that $S > 1$ was thought most unlikely. A one-tailed test provides a more powerful test under these conditions. See, H. D. Brunk, Mathematical Statistics (Waltham Mass.: Blaisdell, 1965): 254-258.
The same econometric difficulties plague the quarterly results as was the case with the results using monthly data. Using a Almon distributed lag to overcome the multicollinearity problem led to results which were consistent with the Fisher hypothesis. These are recorded in Table V-B.

In contrast to the monthly results, the quarterly results using the Durbin Two-Step procedure are broadly consistent with the Fisher hypothesis (see Table VI). The hypothesis that the sum of the coefficients is equal to unity can be rejected only for the industrial bond yield and the long-term Government bond yield. On the other hand the hypothesis that the sum of the coefficients is equal to zero can be rejected for all maturities.\(^1\) Since our results using quarterly data and the naive Fisherian model are broadly consistent with the Fisher's price-expectations effect hypothesis, in all further work quarterly data is used.\(^2\)

---

\(^1\) The t statistics for the null hypothesis that the sum of the coefficients is equal to zero do not appear in Table V. They are easily calculated from the information available in that table, however. For the 3 month treasury bill rate the relevant t value is 8.837 while for the long-term Government bond yield it is equal to 4.319.

\(^2\) There is another pragmatic reason as well. There are relatively few Canadian rime series available on a monthly basis, consequently it is difficult to include other variables in an expanded model of interest rate determination.
### Table VII

**RATES OF PRICE CHANGE AND INTEREST RATES**  
1955-1970, QUARTERLY OBSERVATIONS  
O.L.S. 12 UNCONSTRAINED LAGS

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Intercept</th>
<th>Sum of Weights</th>
<th>D.W.</th>
<th>S.E.E.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.B.</td>
<td>1.83</td>
<td>.890</td>
<td>.551</td>
<td>1.273</td>
<td>.46</td>
</tr>
<tr>
<td></td>
<td>(5.92)</td>
<td>(-.020)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>2.50</td>
<td>.880</td>
<td>.562</td>
<td>.992</td>
<td>.54</td>
</tr>
<tr>
<td></td>
<td>(10.39)</td>
<td>(-1.245)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-5</td>
<td>2.95</td>
<td>- .821</td>
<td>.457</td>
<td>.984</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td>(12.35)</td>
<td>(-1.667)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-10</td>
<td>3.13</td>
<td>.816</td>
<td>.390</td>
<td>1.030</td>
<td>.46</td>
</tr>
<tr>
<td></td>
<td>(12.51)</td>
<td>(-1.038)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10+</td>
<td>3.45</td>
<td>.735</td>
<td>.247</td>
<td>1.067</td>
<td>.38</td>
</tr>
<tr>
<td></td>
<td>(13.30)</td>
<td>(-2.556)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M.Y.W.</td>
<td>3.93</td>
<td>.875</td>
<td>.334</td>
<td>1.056</td>
<td>.48</td>
</tr>
<tr>
<td></td>
<td>(15.30)</td>
<td>(-1.215)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent Variable</td>
<td>Intercept</td>
<td>Sum of Weights</td>
<td>D.W.</td>
<td>S.E.E.</td>
<td>$R^2$</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------</td>
<td>----------------</td>
<td>------</td>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>T.B.</td>
<td>1.92 (8.64)</td>
<td>1.034 (.6029)</td>
<td>.757</td>
<td>.853</td>
<td>.695</td>
</tr>
<tr>
<td>1-3</td>
<td>2.50 (15.12)</td>
<td>.993 (-.096)</td>
<td>.853</td>
<td>.655</td>
<td>.773</td>
</tr>
<tr>
<td>3-5</td>
<td>2.99 (18.45)</td>
<td>-.953 (.722)</td>
<td>.723</td>
<td>.623</td>
<td>.775</td>
</tr>
<tr>
<td>5-10</td>
<td>3.14 (19.01)</td>
<td>-.976 (.357)</td>
<td>.640</td>
<td>.634</td>
<td>.778</td>
</tr>
<tr>
<td>10+</td>
<td>3.42 (21.24)</td>
<td>-.923 (-1.186)</td>
<td>.347</td>
<td>.619</td>
<td>.767</td>
</tr>
<tr>
<td>M.Y.W.</td>
<td>3.78 (24.56)</td>
<td>1.092 (1.482)</td>
<td>.306</td>
<td>.592</td>
<td>.830</td>
</tr>
</tbody>
</table>
### Table VI

**INTEREST RATES AND RATES OF PRICE CHANGE**  
**DURBIN TWO-STEP PROCEDURE**  
**1955-1970, QUARTERLY OBSERVATIONS**  
**12 UNCONSTRAINED LAGS**

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Intercept</th>
<th>Sum of Weights</th>
<th>Auto Regressive Coefficient</th>
<th>D.W.</th>
<th>S.E.E.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.B.</td>
<td>.68 (.427)</td>
<td>.981 (-.099)</td>
<td>.665</td>
<td>1.108</td>
<td>.60</td>
<td>.28</td>
</tr>
<tr>
<td>1-3</td>
<td>.99 (.01)</td>
<td>.961 (-.303)</td>
<td>.622</td>
<td>1.347</td>
<td>.47</td>
<td>.46</td>
</tr>
<tr>
<td>3-5</td>
<td>.99 (8.54)</td>
<td>.895 (-.747)</td>
<td>.705</td>
<td>1.389</td>
<td>.40</td>
<td>.39</td>
</tr>
<tr>
<td>5-10</td>
<td>.86 (8.70)</td>
<td>.872 (-.873)</td>
<td>.741</td>
<td>1.421</td>
<td>.38</td>
<td>.35</td>
</tr>
<tr>
<td>10+</td>
<td>.54 (8.70)</td>
<td>.622 (-2.629)</td>
<td>.870</td>
<td>1.107</td>
<td>.23</td>
<td>.19</td>
</tr>
<tr>
<td>M.Y.W.</td>
<td>.44 (9.48)</td>
<td>.561 (-2.710)</td>
<td>.919</td>
<td>.883</td>
<td>.21</td>
<td>.17</td>
</tr>
</tbody>
</table>
Comparison With Other Studies Using Quarterly Data

Irving Fisher's\(^1\) original investigation into the relationship between inflation and interest rates was implicitly based on the naive Fisherian model. Quarterly U. S. data over the period 1890-1927 was used in a portion of his empirical work. The procedure adopted by Fisher was to correlate the U. S. commercial paper rate with a preconstructed weighted average of past rates of price change.\(^2\) The lag structure followed a normalized arithmetic progression, and summed to unity.\(^3\) The length of the lag was varied, and the weighted average of past rates of price change which was most highly correlated with the interest rate was chosen as being indicative of price-expectations. Fisher estimated that the effect of inflation on the short-term rate was spread over thirty years with an average lag of ten years.

The procedure used by Fisher does not permit a test of the Fisher hypothesis, since a measure of the impact of price-expectations on interest rates is not produced by this procedure.

In Table VII results from two other studies which were based on the naive Fisherian model and used quarterly U. S. data are compared with results obtained in this chapter. Yohe and Karnosky's quarterly results are broadly consistent


\(^2\) The U. S. wholesale price index was used.

\(^3\) The weights were obtained by taking the terms of an arithmetic progression and dividing by the sum of the weights.
with the results obtained here, and suggest that the Fisher hypothesis \( b = 1 \) probably cannot be rejected for the short-term rate, but can for the long-term rate.\(^1\) In addition some support is in evidence for the weak version of the Fisher hypothesis \( b > 0 \) since the null hypothesis that \( b = 0 \) can probably be rejected for both the short and the long-term rate. Only two results using quarterly data were presented by Yohe and Karnosky. The authors concentrated on monthly data where the evidence in favour of the Fisher hypothesis appears to be much more suspect.

The results reported by Gibson,\(^2\) which are most closely comparable to ours, conflict with both the strict Fisher hypothesis, and the weak version of that hypothesis. The sum of the coefficients is actually negative in the two results reported here. Or the basis of his results Gibson argues:

The evidence presented documents the effects hypothesized by Irving Fisher. There is indeed a positive relationship between nominal interest rates and expected rates of price change detectable in U. S. data.\(^3\)

This conclusion, however, is unjustified on the basis of the

---

\(^1\) An explicit test of the hypothesis in question cannot be made on the basis of the results reported by Yohe and Karnosky since a standard error for the sum of the weights on the price-terms is not presented.

\(^2\) W. E. Gibson, "Price Expectations Effects on Interest Rates," *The Journal of Finance* 25(March 1970):19-34. Gibson made no attempt to test the hypothesis that the sum of weights is equal to unity or zero.

<table>
<thead>
<tr>
<th>Estimation Technique</th>
<th>Length of Lag</th>
<th>SHORT-TERM RATE</th>
<th>LONG-TERM RATE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fisher Effect</td>
<td>Adjusted Coefficient of Determination</td>
</tr>
<tr>
<td>Gibson¹</td>
<td>O.L.S.</td>
<td>12</td>
<td>-.101</td>
</tr>
<tr>
<td>Yohe &amp; Karnosky²</td>
<td>O.L.S.</td>
<td>16</td>
<td>.84</td>
</tr>
<tr>
<td></td>
<td>6th degree Almon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>McCollum³</td>
<td>O.L.S.</td>
<td>12</td>
<td>.90</td>
</tr>
<tr>
<td>McCollum</td>
<td>O.L.S.</td>
<td>12</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>3rd degree Almon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>McCollum</td>
<td>Durbin Two-Step</td>
<td>12</td>
<td>.98</td>
</tr>
</tbody>
</table>


³ The Canadian 90-day treasury bill yield and the Government of Canada long-term bond yield were used. Quarterly Canadian data over the period 1955-70 were used.
results presented by Gibson.

The procedure used by Gibson was to regress various nominal rates of interest on current and lagged values of the rate of change of prices. In addition rates of change of interest rates were regressed on rates of change of the rate of inflation. The rates of change regressions, however, contradict the naive Fisherian model and implicitly assume that the real rate is changing over time. Consequently the rate of change regressions are not strictly comparable to the level regressions since they involve a different maintained hypothesis. Gibson used unconstrained lags, and both quarterly and annual data in his work.

Gibson's level regressions\(^1\) using annual data are best interpreted as evidence against the Fisher hypothesis. The coefficients were generally negative (sign conflicts with a priori notions), and occasionally negative and significant. Three of the four regressions presented\(^2\) displayed sums of coefficients which were negative. The level regressions on quarterly data,\(^3\) yield similar results. The sum of the coefficients on the inflation terms is negative for both regressions run. None of the individual coefficients is significant. The rate of change regressions on quarterly data also fall flat.\(^4\) The coefficients of determination

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\(^1\) Table II, *ibid.*, p. 25.


\(^3\) These are found in Table VII on the previous page and are originally contained in Gibson's Table III, *ibid.*, p. 34.

\(^4\) *Ibid.*, p. 34.
are low (.050 to .161) and none of the individual coefficients is significantly different from zero at the .05 level.\footnote{Thus the observation in \textit{ibid.}, p. 23, that the coefficients are, "positive as hypothesized, frequently significantly so," is rather unusual.}

The rate of change regressions on annual data,\footnote{\textit{Ibid.}, p. 24.} are the only results reported by Gibson in which there is any indication that price-expectations may influence nominal rates of interest. The evidence presented there is decidedly weak. Only the regressions for the call rate and the commercial paper rate produce any coefficients which are positive and significantly different from zero. The sum of the weights in these two regressions are .328 and .342 respectively.

Since a standard error for the sum of the weights was not calculated it is not possible to make an explicit test of the Fisher hypothesis. It would appear, however, that the sum of the coefficients, for both the call rate and the commercial paper rate are significantly less than unity, but significantly greater than zero.\footnote{At say the .05 level of significance.} Thus these results can be viewed as not being inconsistent with the weak version of the Fisher hypothesis, \(b > 0\).

The regressions for long-term rates of interest reported by Gibson using the rate of change specification, do not support the Fisher hypothesis in either its strict
(b = 1) or its weak (b > 0) form. None of the individual coefficients is significantly different from zero.

In contrast to the results presented in this chapter using quarterly data, the results reported by Gibson suggest that there is almost no evidence of price-expectations effects on nominal interest rates. Gibson's conclusion that there is ample evidence of the importance of price-expectations as a determinant of nominal interest rates would appear to fly directly in the face of his results.

It is difficult to compare studies other than those mentioned above to our own results, since they are not based on the same maintained hypotheses. Recent studies which have examined the impact of price expectations on interest rates include studies by M. Feldstein and O. Eckstein,¹/ T. J. Sargent,²/ and M. J. Hamburger and W. Silber.³/ A summary of the principal results obtained in these studies, as well as, in the original study by Irving Fisher are contained in Table VIII. These other studies differ primarily in the assumptions that are made about the formation of the real rate or g(x) in equation (3). No study attempts to study "Fisher's" hypothesis in terms of behavioural relationships, all rely on


<table>
<thead>
<tr>
<th>Estimation Technique</th>
<th>Length of Lag</th>
<th>Lag Structure</th>
<th>Model Used</th>
<th>Fisher Effect</th>
<th>Special Features</th>
<th>Observation Period and Data Used</th>
<th>Fisher Effect</th>
<th>Special Features</th>
<th>Observation Period and Data Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>M. J. Hamburger and W. Silber (2)</td>
<td>OLS</td>
<td>Contemporaneous rate of change. See &quot;length of lag&quot;.</td>
<td>Semi-reduced form, derived from the monetary sector of the FBI-SP model</td>
<td>So effect.</td>
<td>Inflation variable is redundant. Impossible to place a prior on the coefficient of the rate of price change variable.</td>
<td>Quarterly data 19511 to 196511.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T. J. Sargent (3)</td>
<td>OLS</td>
<td>Long-term rate: infinite lag; average lag of 33 years. Short-term rate: infinite lag; average lag of 44 years.</td>
<td>Geometric decay, estimated using the Hildreth-Liu procedure</td>
<td>Reduced form derived from a loanable funds model.</td>
<td>.064</td>
<td>Strict Fisher hypothesis is strongly rejected. Results suggest that price expectations are of little importance in determining nominal rates of interest. The model deteriorates with an extended sample period.</td>
<td>Annual data 1902-1940. Durand's ten-year basic yield. U.S. commodity price index.</td>
<td>.038</td>
<td>Strict Fisher hypothesis is strongly rejected. Results suggest that price expectations are of little importance. The model deteriorates with extended sample period 1902-1954.</td>
</tr>
</tbody>
</table>

the reduced form technique.

The study by Hamburger and Silber\(^1\) was not undertaken with a view to examining the Fisher hypothesis in any detail. The procedure adopted by these authors was to estimate a semi-reduced form, based on the monetary sector of the FRB-MIT\(^2\) model to determine the short-term rate of interest. Hamburger and Silber discovered that the contemporaneous rate of change of the price level did not enter their equation as a significant determinant of the interest rate. One of the difficulties with the approach adopted by Hamburger and Silber is that the monetary sector of the FRB-MIT model is designed to determine the nominal rather than the real rate, consequently the inflation term is redundant, and one should not expect it to appear significant in their equation. Given the form of the equation they were estimating it is impossible to place a prior on the coefficient of the inflation term. Redundancy is a problem that also affects the study by M. Feldstein and O. Eckstein.\(^3\) Feldstein's and Eckstein's model is based on a synthesis of the liquidity preference theory of interest rate determination and the Fisher price-expectations hypothesis. Since the nominal not

---


the real rate of interest is the opportunity cost of holding real money balances\(^1\) it is inappropriate to construct a model of real interest rate determination in terms of strict liquidity preference theory. It is impossible to place a prior on the size of the coefficient on the price-expectation variable in Feldstein's and Eckstein's equation since the demand and supply of money should determine the nominal not the real rate of interest. In particular there is no a priori reason why it be equal to unity. Feldstein and Eckstein claim that:

The data thus confirm the two basic Fisherian hypotheses: (1) in the long-run, the real rate of interest is (approximately) unaffected by the rate of inflation, but (2) in the short-run, the real rate of interest falls as the rate of inflation increases.\(^2\)

Actually the results reported by Feldstein and Eckstein support neither of these conclusions. Conclusion (1) is an assumption of the model while conclusion (2) violates the assumptions of the model.\(^3\)

T. J. Sargent\(^4\) has recently examined the impact of price-expectations on nominal rates of interest in the U. S. using a reduced form equation which was derived from a loanable funds model of interest rate determination. Sargent claims


\(^2\) Ibid., p. 366.

\(^3\) Conclusions (1) and (2) above would follow from Feldstein's and Eckstein's results if they are referring to the ex-post real rate at this point. If that is the case, however, conclusions (1) and (2) amount to essentially the same thing.

to have found evidence in favour of the Fisher theory. A
closer examination of Sargent's evidence, however, indicates
that his results would lead to a strong rejection of the strict
Fisher hypothesis.\(^1\) While the weak version of the Fisher
hypothesis receives some support in that the estimated
coefficient on the expected rate of inflation term is
significantly different from zero. Sargent's evidence
suggests that price-expectations have an extremely small and
essentially negligible impact on interest rates.

There are a number of other difficulties inherent
in the model used by Sargent. To explain the deviation of the
real market rate from the natural rate Sargent uses the rate
of change of the money supply.\(^2\) However, this variable is
not commensurable with the savings and investment flows which
determine the natural rate in his model. Since not all
variables prove significant in his reduced form it is difficult
to interpret the results.\(^3\) The model deteriorates substantially
when the sample period is extended,\(^4\) suggesting that his

---

\(^1\) Ibid., pp. 135-136.

\(^2\) Ibid., p. 131.

\(^3\) Sargent's results indicate that he has not correctly isolated
the determinants of the real rate of interest since not all
of the determinants of the real rate in the model enter his
equations significantly. The change in output variable,
for example, is never significant. Ibid., p. 133, 135, 136,
137, 139.

\(^4\) Ibid., p. 133. The monetary variable and the change in output
variable both prove to be insignificant. The only significant
variables are real gross national product and the distributed
lag on past rates of price change. The coefficient of
determination falls somewhat and the Durbin-Watson statistic
indicates a high degree of positive autocorrelation.
results are specific to the observation period chosen.

To summarize, the Fisher price-expectation hypothesis has been tested using models other than the naive Fisherian model. These results are not strictly comparable to ours since they involve testing a different conditional hypothesis. The other studies differ from the work contained in this chapter primarily in terms of the assumption made about the determination of the real rate of interest.

All of the other studies exhibit anomalous characteristics when examined closely, however. In the Feldstein and Eckstein study the real rate is determined by the demand and supply of money, whereas economic theory suggests that the demand for money is a function of the nominal and not the real rate of interest.\(^1\) The Sargent study is perhaps more solidly based on economic theory than the Feldstein and Eckstein study but even there a number of difficult problems of interpretation arise. Consequently it can be argued that there is no clearly superior existing alternative to the naive Fisherian model in examining Fisher's hypothesis if the single-equation reduced form approach is going to be adhered to.

**Price-Expectations and the St. Louis Real Rate Series**

Beginning in 1966 the Federal Reserve Bank of St. Louis has published a series for the real rate of interest.\(^2\)

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\(^1\) See R. A. Mundell, *op. cit.*, pp. 280-281.

\(^2\) The Federal Reserve Bank of St. Louis Review 50(December 1968):5.


Since at least one\(^1\) of the St. Louis real rate series is based on Yohe and Karnosky's empirical work using the naive Fisherian model, it seems appropriate to examine the basis for these rates in this study.

In order to test the hypothesis that price-expectations influence nominal rates of interest Yohe and Karnosky implicitly assumed that the real rate is constant over time. In constructing a real rate series Yohe and Karnosky make the following calculation using the estimated weights from equation (4):

\[
(5) \quad r_{mt} = i_t - \sum_{i=0}^{n} \hat{w}_i \left( \frac{\Delta P}{P} \right)_{t-i}
\]

where \(r_{mt}\) is the estimated real rate. This procedure, however, does not produce an interesting series since equation (4) in conjunction with equation (5) imply that:

\[
(6) \quad r_{mt} = \hat{a}_0 + \hat{u}_t
\]

That is to say Yohe and Karnosky's real rate is a random variable if the estimated model has random disturbances. It follows therefore that the time-series for the real rate derived by Yohe and Karnosky is without meaning and movements in this series are of little interest.

\(^1\) This is Yohe and D. S. Karnosky's "Real Rate 2," ibid., p. 34.
Yohe and Karnosky calculate another real rate which is inconsistent with the real rate discussed above.\(^1\)

Assuming an equilibrium position with expected price changes equal to zero, then \( r_{n_t} = r_{m_t} \). If price expectations increase by one per cent per year, after four years the nominal interest rate will rise by 69 basis points, thus:

\[
\begin{align*}
(1) & \quad r_{n_t+48} - r_{m_t+48} = 1.00 \\
(2) & \quad r_{n_t+48} - r_{n_t} = .69 \\
& \quad r_{n_t} = r_{m_t} \text{ equations (1) and (2) reduce to} \\
(3) & \quad r_{m_t+48} - r_{m_t} = -.31.
\end{align*}
\]

Thus, the market rate falls by 31 basis points following the increase in price expectations.\(^2\)

It is clear that \( r_{m_t} \) above is not the same \( r_{m_t} \) obtained from equation (6). To construct the real market rate \( r_{m} \) above, it is assumed that the Fisher equation always holds true. Consequently, price-expectations can depress the real rate of interest, and the nominal rate of interest need not rise by the full amount of the expected rate of change of prices. If this is the case, however, we could never reject the Fisher hypothesis using equation (4).

The inference that the real rate must have fallen if the sum of the coefficients on the lagged price-terms is less than unity is also drawn by W. E. Gibson.

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\(^{1}\) The notation used by Yohe and Karnosky suggests that these two real rates are equivalent. See, \textit{ibid.}, p. 24 and 38.

Nominal rates rise in response to positive price expectations but apparently not by the full amount of the percentage point increase in the expected rate of price change. Since the difference between the new levels of nominal and real rates equals the rate of price change expected, real rates appear to fall.1/

The real market rate discussed above will equal the real market rate of equation (6) only by chance. In this interpretation the purpose of estimating equation (4) is not to test the Fisher hypothesis, but simply to estimate the depressive effects of inflation on the real rate of interest, assuming that the Fisher equation holds true. Since the real market rate obtained in this fashion is inconsistent with the approach used in obtaining the price-expectation variable, it may be concluded that the calculations in the quotation are without meaning.

The contrast between these various rates is shown in Figure I. The estimated constant real rate is shown as a straight line. Assume that the rate of price change is one percent per year. The estimated intercept plus the computed residual for any point ($\hat{a}_o + \hat{u}_t$) is the real rate of interest presented by Yohe and Karnosky. The curve labelled $rm$ is Yohe and Karnosky's second estimate of the real market rate. In general, $rm_t \neq \hat{a}_o - \hat{u}_t$. This real market rate has absolutely nothing to do with Wicksell, contrary to the assertions by the authors, and is constructed on the basis that the Fisher

1/ W. E. Gibson, op. cit., p. 33.
Figure 1

ST. LOUIS REAL RATES

Nominal Rate

\[ \pi = 100 \]

Computed Real Rate \((\hat{A} + \hat{G})\)

Estimated Constant Real Rate

Time
equation always holds true.\textsuperscript{1/}

It is essentially impossible to interpret Yohe and Karnosky's "Real Rate 3" since they did not document the complete results for the equation which forms the foundation of their real rate series.\textsuperscript{2/}

In summary, efforts to construct a real rate series on the basis of estimates of the naive Fisherian model are destined to failure. The resultant series is either a random variable, or violates the assumptions made to estimate the initial model.

The Differential Effects According to Term to Maturity Hypothesis

A number of authors have suggested that the length of the lag in the formation of price expectations tends to vary with the term of the interest rate.\textsuperscript{3/} For example, it is argued that a longer history of past price performance is

\textsuperscript{1/} "Real Rate 1," on page 34 of the article is the original "real" rate series constructed at the St. Louis Federal Reserve Bank. This real rate series was constructed on the basis that price-expectations are known. A similar series for Canada may be found in T. J. Courchene, "Recent Canadian Monetary Policy," \textit{Journal of Money, Credit and Banking}\textsuperscript{3}(February 1971):39. Calculations of this sort have nothing to say about whether price-expectations affect nominal rate of interest; that is assumed to be true in the construction of the series.

\textsuperscript{2/} See: \textit{ibid.}, p. 38. Neither t statistics nor the usual schedule of summary statistics were presented.

relevant to the formation of price-expectations which influence the long-term rate of interest as opposed to the short-term rate. As was pointed out in chapter II, however, this involves an hypothesis on the formation of price-expectations per se, rather than a special modification of the Fisher hypothesis, which is concerned with the steady state impact of inflation on interest rates. At this date, no attempt to test the differential effects hypothesis has appeared in the literature.

The differential effects hypothesis is usually stated very loosely in the literature. When this hypothesis is placed on a more rigorous basis, several possible interpretations emerge.

Let \( L\pi^e \) and \( S\pi^e \) refer to price-expectations relevant to the long-term rate of interest and the short-term rate of interest respectively. Suppose that:

\[
L\pi^e = \sum_{i=0}^{n} w_i \pi_{t-i}^a, \text{ and}
\]

\[
S\pi^e = \sum_{i=0}^{n} v_i \pi_{t-i}^a, \text{ where}
\]

\( \pi^a \) refers to the actual rate of inflation. One possible interpretation of the differential effects hypothesis is that \( n > m. \) This version of the "differential effects" hypothesis does not imply that short-term rates of interest respond faster than long-term rates of interest to changes in the rate of inflation, since the speed of response is also conditioned by the shape of the lag distribution.

---

\(^1\) See M. Friedman, op. cit., p. 21.
The regression results previously reported in this chapter offer no support for the hypothesis that \( n > m \). Further evidence on the differential effects hypothesis is contained in Table IX.\(^1\)

The procedure used to obtain the results reported in Table IX was stepwise orthogonal regression. The lagged price terms were introduced one at a time in stepwise fashion. If the differential effects hypothesis as specified above is correct, one would expect to find more significant lagged price terms entering the regression for long-term rates than for short-term rates. Stepwise regression is a convenient procedure for determining the point at which to truncate the lag. Extending the lag further than the last lag reported in Table IX, did not raise the explained variation of interest rates. This procedure does not determine the interval over which expectations are formed. Rather it indicates the point at which there is no evidence in the data that the lag is any longer.

There is little support for the differential effects hypothesis contained in Table IX. In the case of Government securities the lag is truncated at 12 quarters for the Treasury Bill rate and 14 quarters for the long-term rate. The McCleod, Young and Weir industrial bond yield is somewhat of an anomaly with lag length of 20 quarters. These results suggest that there is little difference in the speed of response of long-term and short-term rates of interest to inflation. The

\(^1\) Since the naive Fisherian model could not be rejected using quarterly data the differential effects hypothesis is examined using quarterly data rather than monthly data.
<table>
<thead>
<tr>
<th>Table IX</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ORTHOGONAL REGRESSION</strong></td>
</tr>
<tr>
<td><strong>QUARTERLY OBSERVATIONS (64)</strong></td>
</tr>
<tr>
<td><strong>1955-1970</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b_1)</th>
<th>(b_2)</th>
<th>(b_3)</th>
<th>(b_4)</th>
<th>(b_5)</th>
<th>(b_6)</th>
<th>(b_7)</th>
<th>(b_8)</th>
<th>(b_9)</th>
<th>(b_{10})</th>
<th>(b_{11})</th>
<th>(b_{12})</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.R.</td>
<td>32.91</td>
<td>.0559</td>
<td>.0482</td>
<td>.0488</td>
<td>.0295</td>
<td>.0145</td>
<td>.0117</td>
<td>.093</td>
<td>.0186</td>
<td>.0197</td>
<td>.0241</td>
<td>.0281</td>
<td>.0280</td>
</tr>
<tr>
<td></td>
<td>(60.57)</td>
<td>(6.41)</td>
<td>(5.20)</td>
<td>(5.49)</td>
<td>(3.55)</td>
<td>(1.77)</td>
<td>(1.44)</td>
<td>(1.13)</td>
<td>(2.27)</td>
<td>(2.43)</td>
<td>(3.55)</td>
<td>(3.19)</td>
<td>(3.43)</td>
</tr>
<tr>
<td></td>
<td>D.W. = .653</td>
<td>S.E.E. = .82</td>
<td>(R^2 = .72)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(S_{1} = 329)</td>
<td>(S_{1} = 15.004)</td>
<td>Average Lag = 4.455 quarters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 1-3      | 37.32 | .0518  | .0449  | .0431  | .0305  | .0142  | .0117  | .075   | .0175  | .0180  | .0203  | .0207  | .0238  |
|          | (62.79) | (8.72) | (7.55) | (7.26) | (5.13) | (2.40) | (1.97) | (1.27) | (2.95) | (3.03) | (3.42) | (3.48) | (4.01) |
|          | \(S_{1} = 329\) | S.E.E. = .59 | \(R^2 = .81\) |
|          | D.W. = .727 | \(S_{1} = -21.908\) | Average Lag = 4.690 quarters |

| 3-5      | 39.87 | .0456  | .0437  | .0405  | .0296  | .0163  | .0132  | .086   | .0186  | .0156  | .0180  | .0219  | .0223  |
|          | (75.13) | (8.59) | (8.26) | (7.83) | (5.57) | (3.08) | (2.46) | (1.61) | (3.51) | (3.54) | (3.40) | (4.12) | (4.20) |
|          | \(S_{1} = 321\) | S.E.E. = .53 | \(R^2 = .84\) |
|          | D.W. = .708 | \(S_{1} = -23.591\) | Average Lag = 4.796 quarters |

| 5-10     | 41.20 | .0417  | .0421  | .0401  | .0320  | .0205  | .0158  | .089   | .0193  | .0195  | .0227  | .0220  | .0243  |
|          | (78.64) | (7.95) | (8.04) | (7.65) | (6.30) | (3.92) | (3.02) | (1.69) | (3.68) | (3.72) | (4.34) | (4.20) | (4.64) |
|          | \(S_{1} = 312\) | S.E.E. = .52 | \(R^2 = .85\) |
|          | D.W. = .546 | \(S_{1} = -21.551\) | Average Lag = 5.373 quarters |

| 10+      | 43.38 | .0342  | .0371  | .0325  | .0205  | .0241  | .0203  | .0156  | .0193  | .0212  | .0212  | .0205  | .0231  |
|          | (78.12) | (6.30) | (5.84) | (5.63) | (4.43) | (3.70) | (2.67) | (1.57) | (3.30) | (3.32) | (3.79) | (4.25) |
|          | \(S_{1} = 344\) | S.E.E. = .54 | \(R^2 = .82\) |
|          | D.W. = .214 | \(S_{1} = -23.321\) | Average Lag = 5.402 quarters |

| M.Y.W.   | 68.61 | .0438  | .0474  | .0445  | .0275  | .0229  | .0234  | .0186  | .0186  | .0201  | .0175  | .0165  | .0197  |
|          | (133.80) | (12.10) | (12.07) | (11.34) | (9.85) | (7.37) | (6.30) | (4.73) | (4.74) | (5.12) | (4.47) | (4.20) | (5.02) |
|          | \(S_{1} = 339\) | S.E.E. = .30 | \(R^2 = .92\) |
|          | D.W. = .441 | \(S_{1} = -28.531\) | Average Lag = 5.602 quarters |
results, however do not clearly contradict the differential effects hypothesis in the sense that there is no evidence that short-term rates respond more slowly than long-term rates to a change in the rate of inflation.\textsuperscript{1/}

Another version of the differential effects hypothesis can be specified in terms of the average lag. If the short-term rate of interest responds to inflation more rapidly than does the long-term rate, then the average lag will be smaller for the short-term rate. The average lag may be computed using the following formula:

\begin{equation}
A.L. = \sum_{i} w_i \frac{1}{\sum_{i} w_i}
\end{equation}

the $w_i$ are the estimated coefficients. Average lags cannot be computed for most of the regressions reported in earlier in this chapter due to the presence of negative coefficients in the lag structure. The regression results reported in Tables VI and IX, however, are amenable to average lag

\textsuperscript{1/} Contrary to popular belief, Irving Fisher did not hypothesize that the time-horizon over which price expectations are formed is related to the term to maturity of the security. Also contrary to popular belief Irving Fisher did not find evidence in support of the differential effects hypothesis. Indeed Fisher's empirical results suggest that the long-term rate of interest adjusts to a change in the rate of inflation, more rapidly than the short-term rate. Fisher's empirical results are recorded in; Irving Fisher, The Theory of Interest (New York: Macmillan, 1930):416-429. Also see Table VIII on page 85. There is no visible support for the differential effects hypothesis visible in the evidence provided by W. P. Yohe and D. S. Karnosky contrary to the claims of the authors. See W. P. Yohe and D. S. Karnosky, op. cit., pp. 25-26. The evidence that Milton Friedman cites on differential effects in "Factors Affecting the Level of Interest Rates," op. cit., p. 21 are inconsistent rather than consistent with Irving Fisher's results.
calculations, which are recorded in Table X.

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>Based on Table IV Quarterly Data</th>
<th>Based on Table IX Quarterly Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-month Treasury Bill</td>
<td>8.2 quarters</td>
<td>4.5 quarters</td>
</tr>
<tr>
<td>1-3 years</td>
<td>N. A.*</td>
<td>4.7</td>
</tr>
<tr>
<td>3-5</td>
<td>8.5</td>
<td>4.8</td>
</tr>
<tr>
<td>5-10</td>
<td>8.4</td>
<td>5.4</td>
</tr>
<tr>
<td>10+</td>
<td>8.5</td>
<td>5.6</td>
</tr>
</tbody>
</table>

*N. A. means that the average lag calculation is not applicable.

The average lags recorded in Table X which are based on results reported in Table VI, provide no support for the differential effects hypothesis. The average lags based on Table IX results using orthogonal stepwise regression provide extremely weak support for the differential effects hypothesis. The average lag for the long-term (10+) Government of Canada bond yield is, for example, only one quarter greater than the average lag for the 3 month Treasury Bill rate.

Another way to approach the differential effects hypothesis is by examining the percentage of the total response (say 50 per cent) occurs. Table XI contains computations based
<table>
<thead>
<tr>
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<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
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<tbody>
<tr>
<td></td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
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<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
</tr>
<tr>
<td>T.B.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1-3</td>
<td>16.94</td>
<td>31.55</td>
<td>45.13</td>
<td>53.92</td>
<td>58.32</td>
<td>61.87</td>
<td>64.69</td>
<td>70.32</td>
<td>76.29</td>
<td>83.60</td>
<td>91.51</td>
<td></td>
</tr>
<tr>
<td>3-5</td>
<td>16.13</td>
<td>30.11</td>
<td>43.54</td>
<td>53.04</td>
<td>57.46</td>
<td>61.10</td>
<td>63.47</td>
<td>68.92</td>
<td>74.52</td>
<td>80.85</td>
<td>87.29</td>
<td>94.71</td>
</tr>
<tr>
<td>5-10</td>
<td>14.61</td>
<td>28.60</td>
<td>41.58</td>
<td>51.06</td>
<td>55.22</td>
<td>60.51</td>
<td>63.26</td>
<td>69.22</td>
<td>74.21</td>
<td>79.99</td>
<td>86.99</td>
<td>94.14</td>
</tr>
<tr>
<td>10+</td>
<td>12.09</td>
<td>24.29</td>
<td>35.91</td>
<td>45.46</td>
<td>51.42</td>
<td>56.00</td>
<td>58.58</td>
<td>64.17</td>
<td>69.83</td>
<td>76.41</td>
<td>82.78</td>
<td>89.83</td>
</tr>
<tr>
<td>X.Y.N.</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
<td>96.03</td>
</tr>
</tbody>
</table>
on the results reported in Table IX.1/

The evidence presented in Table XI suggests that the difference between the rate of response of short and long-term rates of interest to a change in the rate of inflation is negligible.

The time pattern of response of most interest rates to a change in the rate of price change is remarkably similar. On balance it seems reasonable to conclude that using a variety of techniques, the hypothesis of differential effects does not receive any strong support from the data.

It was pointed out in chapter II, that there is a tendency to confuse the hypothesis that the speed of adjustment of expectations to inflation varies according to the bond maturity, with the hypothesis that the Fisher effect varies across bond maturities.2/ This latter hypothesis can be formulated as follows, using the naive Fisherian model. Suppose we perform two regressions, where L and S refer to long and short respectively:

\[
(10) \quad i_L = a_L + b_L \pi_L^e \quad \text{and,}
\]

\[
(11) \quad i_S = a_S + b_S \pi_S^e .
\]

1/ Since there was no evidence of differential effects in Table VI, only the computations based on Table IX are recorded here.

Table XII
DIFFERENCE BETWEEN TOTAL EFFECTS
ACCORDING TO TERM TO MATURITY*

<table>
<thead>
<tr>
<th></th>
<th>TB</th>
<th>1-3</th>
<th>35</th>
<th>5-10</th>
<th>10+</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB</td>
<td></td>
<td>0.964</td>
<td>1.423</td>
<td>1.691</td>
<td>2.942</td>
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<tr>
<td>1-3</td>
<td></td>
<td>0.479</td>
<td>0.800</td>
<td>2.125</td>
<td></td>
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<tr>
<td>3-5</td>
<td></td>
<td>0.347</td>
<td>1.612</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-10</td>
<td></td>
<td></td>
<td>1.265</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10+</td>
<td></td>
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<td></td>
<td></td>
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</tbody>
</table>

*Based on Table VI using quarterly data 1955-1970.

The hypothesis that the Fisher effect varies with the term to maturity of the security is the hypothesis that $b_L \neq b_S$. The implication of this result is, if the real term structure of interest rates obeys the Hicks-Lutz expectations theory, the nominal term structure will not, even after all adjustments to inflation have taken place. There is no a priori reasons to suppose that $b_L > b_S$ or that $b_S > b_L$ consequently a two-tailed test is used in the test results reported below.

Some tentative tests of this hypothesis are summarized in Table XII. These results are based on the regression results recorded in Table VI. The purpose of the test is to see if the equilibrium impact of inflation varies across the term structure. The entries in the table are the t values for the null hypothesis $(b_i - b_j) = 0$, against the alternative hypothesis $b_i \neq b_j$. Using the .01 level of
significance the critical t values are ± 2.655. The null hypothesis of no difference in the Fisher effect can only be rejected for the case of the Treasury Bill yield and the long-term Government of Canada bond yield.

Our results on the differential effects hypothesis using quarterly Canadian data over the period 1955-70 may be summarized as follows. There is no strong evidence that differential effects play an important role in influencing the term structure of nominal interest rates, while expectations are adjusting to changes in the rate of inflation. Evidence that the Fisher Effect varies across the term structure is rather weak. In only one case were we able to reject the hypothesis that the Fisher Effect does not vary along the term structure.

Summary of Single Equation Results

In this chapter the Fisherian price-expectations hypothesis and related hypotheses were tested using post-war Canadian data. The following conclusions may be drawn from the results presented in this chapter.

1) Estimates based on the naive Fisherian model and monthly data lead to a rejection of the strict Fisherian model. The weak version of that hypothesis, however, cannot be initially rejected.

2) The residuals of the monthly equations exhibited a high degree of auto-correlation. Transforming the data to correct for the serial correlation resulted in the effective disappearance of the relationship.
This suggested that our earlier results (as well as those of Yohe and Karnosky) were based on bogus lags induced by the serial correlation.

3) Estimates based on the naive Fisherian model and quarterly data do not lead to a rejection of Fisher's hypothesis. Transforming the equations to account for auto-correlation did not result in the relationship vanishing.

4) The "differential effects" hypothesis was not supported by the data.

5) The foundations for the Federal Reserve Bank of St. Louis real rate series were demonstrated to be extremely weak. The early St. Louis real rate series was shown to be based on an asserted measure of price-expectations and has no theoretical or empirical basis. It was argued that the real rate series suggested by Yohe and Karnosky was based on a misinterpretation of their work.

6) A number of authors have concluded that if the empirical evidence does not support the strict version of the Fisherian hypothesis the real rate must have altered. It was argued that this is not a legitimate inference and contradicts the assumptions of the model which was used to measure the effect of price-expectations.\(^1\)

---

In summary, it appears that an equation such as equation (3) is severely limited in the light that it can throw on the issue of the impact of price-expectations on the nominal rate of interest. Results using equation (3) are frequently ambiguous and a clear cut interpretation is not always possible. If price-expectations do affect nominal rates of interest, the behaviour of capital market participants must be altered in the presence of inflation. There would seem to be a strong a priori case then, for examining behavioural equations directly rather than relying solely on reduced form equations, like equation (3), for evidence of the role of inflation in influencing interest rates. An additional reason for examining structural equations directly is that it is impossible to test some of the hypotheses developed in chapter II using the reduced form approach.
CHAPTER IV

EMPIRICAL TESTS OF THE REACTIONS OF
BORROWERS AND LENDERS TO INFLATION

Introduction

Why do interest rates rise during an inflationary period? It is commonly alleged that lenders and borrowers react to inflation in such a way as to force up nominal interest rates.

The basic reason why interest rates have risen so high and bond prices have fallen so low is the belief of investors (lenders) and borrowers that the value of money will continue to decline significantly.\(^1\)

Previous empirical evidence on the influence of price-expectations on nominal interest rates has relied exclusively on the use of reduced forms.\(^2\) The procedure followed in previous work was to relate nominal interest rates to past rates of inflation and occasionally to other variables. In the concluding section of chapter III, it was argued that while some important insights can be gained through the use of the reduced form approach, there are some compelling reasons why one may not wish to rely solely on that method.

Since alternative specifications of structural systems can give to the same reduced form, the

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\(^2\) See chapter III, p. 55.
interpretation of the reduced form coefficients is often ambiguous.\(^1\) In this chapter behavioural equations for primary lenders and primary borrowers are specified and estimated directly. This has two advantages over the reduced form approach which was used in chapter III of this study. It can potentially provide more detailed evidence on the hypothesis that the reaction of borrowers and lenders to an increase in the rate of inflation leads to higher observed interest rates than it is possible to obtain using the reduced form method. It also permits an examination of the "Fisher-Friedman asymmetric-expectations" hypothesis which was introduced in chapter II.\(^2\) An examination of the Fisher-Friedman hypothesis is precluded if reduced forms only are estimated with no attempt to derive structural coefficients.

A thorough examination of the reactions of lenders and borrowers to inflation would be most satisfactorily conducted within the framework of a structural model of the capital market, where interest rates are determined by demand and

---

\(^1\) The finding of a series of positive coefficients in a regression equation such as equation (4) in chapter III is consistent with a number of competing theories. Thus even if the estimated coefficients in equation (4) are positive and significantly different from zero; the evidence for the Fisher hypothesis is weak. The most that equation (4) can do is produce evidence which is either consistent or inconsistent with the hypothesis that the reactions of borrowers and lenders to inflation influences nominal interest rates.

\(^2\) See chapter II, pp. 29-40.
supply forces in the corresponding markets.\textsuperscript{1/} A relatively straightforward examination of the hypotheses under consideration could then be conducted in terms of certain structural coefficients.\textsuperscript{2/} It is not possible, however, to specify and estimate a multi-market general equilibrium model of Canadian capital markets in which interest rates are determined by the direct interaction of demand and supply. Such an effort, which would be an enormous and difficult task,\textsuperscript{3/} is preluded by an absence of relevant balance sheet and interest rate information.


P. H. Hendershott, A Flow-of-Funds Model of Interest Rate Determination: Theoretical and Institutional Underpinnings, Krannert Graduate School of Industrial Administration Institute Paper No. 259 (North Carolina: Purdue University, October 1969).


\textsuperscript{2/} Ideally one would like to work in terms of a general equilibrium model of the financial claims matrix of the economy. See, for example, J. Tobin, Financial Intermediaries and the Effectiveness of Monetary Controls Cowles Commission Discussion Paper No. 63 (New Haven: Cowles Commission, 1958).


\textsuperscript{3/} There is little accepted econometric work, for example, on the nature of borrowing activity. See C. Christ, "A Model of Monetary and Fiscal Policy Effects on the Money Stock Price Level, and Real Output," Journal of Money, Credit and Banking 1 (November 1969): 690 and J. Tinbergen, op. cit., pp. 691-692.
What may seem to be an apparent route out of the data difficulties due to a lack of sufficient balance sheet information is to specify a model of financial markets in terms of financial-flows for which data does exist.\(^1\) This solution, however, is more apparent than real, for Canadian financial-flow data is extremely weak, particularly for final lenders and borrowers. Moreover the interest rate series which are available typically do not correspond to the flow categories.

Some light can be shed on the hypothesis that the reactions of borrowers and lenders to inflation result in higher nominal rates of interest, and the asymmetric-expectations hypothesis of Fisher and Friedman, without specifying and estimating a complete model of the financial sector.\(^2\) The procedure adopted in this study is to estimate individual demand and supply equations describing borrowing and lending activity in individual security markets. Data difficulties, however, do not permit a consideration of balance sheet constraints and the symmetry conditions of consumer demand theory.\(^3\) Nevertheless our procedure does

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\(^1\) This approach to financial model building is taken by Patric Hendershott. See P. H. Hendershott, A Flow-of-Funds Model of Interest Rate Determination: Theoretical and Institutional Underpinnings, Kraner Graduate School of Industrial Administration Institute Paper No. 259 (North Carolina: Purdue University, October 1969), and P. H. Hendershott, "A Flow-of-Funds Model," Journal of Money, Credit and Banking 3(November 1971):815-832.

\(^2\) Which is not possible in the face of current data difficulties in any case.

\(^3\) It is not possible, with the existing data base, to incorporate portfolio consistency constraints in our demand and supply functions. This means that we are forced to neglect a number of the recommendations made by J. Tobin and W. C. Brainard in, "Pitfalls in Financial Model Building," Papers and Proceedings of the American Economic Association 57(May 1968):99-122.
permit in principle an examination of both the behavioural implications of Fisher's price-expectations hypothesis and the Friedman-Fisher asymmetric hypothesis.

**Model Specification**

The approach followed in this chapter is to focus exclusively on the behaviour of ultimate borrowers and lenders. In the present context this appears reasonable since the upward pressure on interest rates, which is initiated by inflation is hypothesized to result primarily from the reactions of primary lenders and borrowers rather than financial intermediaries.\(^1\)

In the case of a financial institution, both sides of its balance sheet are in nominal terms and thus, to a first order of approximation we can assume that they react in a passive fashion to a change in the rate of inflation.

The model used to describe lending and borrowing activity is a combination of the partial adjustment model and the adaptive expectations model.\(^2\) In this model the desired stocks of financial assets and liabilities of lenders and borrowers depend on a number of variables including expected real rates of interest. The procedure whereby lenders and borrowers adjust their actual stocks of financial securities

---


toward the desired stocks is described by a partial adjustment process.

In the case of primary lenders desired real financial asset holdings are hypothesized to depend upon income, the expected real rate of interest on the security in question and the expected real rate of interest on substitute securities.\(^1/\)
The long-run demand function for a financial asset may be written in the following manner:

\[
A_t^* = a_0 + a_1 y_t + a_2 \pi_t^e + a_3 o_{r_t}^e
\]

where,

- \(A_t^*\) is the desired long-run real stock of a financial asset,
- \(y_t\) is real income,
- \(\pi_t^e\) is the own expected real rate of interest, and
- \(o_{r_t}^e\) is the expected real rate of interest on an alternative asset.

A priori we expect \(a_1 > 0, a_2 > 0\) and \(a_3 < 0\). Equation (1) can be written as:

\[
A_t^* = a_0 + a_1 y_t + a_2 (\pi_t^e - c_{\pi_t^e}) + a_3 (o_{r_t} - c_{o_{r_t}})
\]

and where,

- \(\pi_t^e\) is the expected rate of inflation,
- \(\pi_t\) is the own nominal rate of interest,
- \(o_{r_t}\) is the nominal rate of interest on an alternative asset, and

(3) \[ r_t^e = a_t^i - c\pi_t^e \] and

(4) \[ o_t^e = o_t^i - c\pi_t^e \]

Fisher's hypothesis that credit market participants react to a change in the rate of interest in such a way as to cause a compensating change in the nominal rate of interest, can be represented by the hypothesis that "c", the coefficient on the price expectational variable, is equal to unity.\(^1\)

Neither the expected rate of price change nor the desired real stocks for the financial assets are directly observable. These variables can, however, under certain assumptions be related to observable variables. The expected rate of price change can be related to past rates of price change with the use of the adaptive expectations model.\(^2\) Thus:

(5) \[ \pi_t^e = \pi_{t-1}^e + \lambda(\pi_t^a - \pi_{t-1}^e) \]

where \( \pi_t^a \) is the actual rate of inflation and \( \lambda \) is the coefficient of adaptive expectations. The expected rate of price change can be viewed as either the average rate of inflation expected in

---

\(^1\) See chapter II pages 15-19. The Fisher hypothesis is represented in terms of a long-run or steady state parameter in our model. This is to be distinguished from the Fisher-Friedman asymmetric expectations hypothesis which is represented in terms of the coefficient of adaptive expectations, and is an hypothesis about the adjustment process.

the long-run, or as an optimal forecast of next period's rate of inflation.\(^1\)

The stock adjustment model which has been used in a wide variety of other studies is employed here. Current demand for real asset holdings are related to long-run desired holdings by:

\[
A_t = A_{t-1} + \gamma(A_t^* - A_{t-1})
\]

where \(A_t\) is the actual stock of the security held and \(\gamma\) is the adjustment coefficient.

Equations (2), (5) and (6) can be combined to obtain:

\[
A_t = \lambda y_0 + \{(1 - \gamma)(1 - \lambda)\}A_{t-1} - \{(1 - \gamma)(1 - \lambda)\}A_{t-2} \\
+ \gamma a_1 y_t - (1 - \lambda)\gamma a_1 y_{t-1} + \gamma a_2^a i_t - (1 - \lambda)\gamma a_2^a i_{t-1} \\
+ \gamma a_3^o i_t - (1 - \lambda)\gamma a_3^o i_{t-1} - \{\gamma\lambda c(a_2 + a_3)\} \pi_t^a + u_t.
\]

Equation (7) contains only observable variables and may be estimated directly.\(^2\) Since equation (7) frequently encountered multicollinearity difficulties, two simpler model specifications were also appealed to. The first simpler expectation, is obtained by setting the coefficient of adjustment equal to unity to obtain:


\(2/\) A discussion of various aspects of estimating equation (7) can be found on pages 116-120.
\( A_t = \lambda a_o + (1 - \lambda) A_{t-1} + a_1 y_t - (1 - \lambda) a_1 v_t + a_2 a^*_t \)

\[- (1 - \lambda) a_2 a^*_t - 1 + a_3 o_i t - (1 - \lambda) a_3 o_i t_{-1} \]

\[- \{\lambda c(a_2 - a_3)\} \pi^a_t + u_t . \]

Equations (7) and (8) contain contemporaneous and lagged values for both the own rate of interest and the yield on an alternative asset. The second simpler specification involves the omission of the alternative interest rate, to obtain:

\( A_t = \lambda a_o + (1 - \lambda) A_{t-1} + a_1 v_t - (1 - \lambda) a_1 v_{t-1} \)

\[+ a_2 i_t - (1 - \lambda) a_2 i_{t-1} - \lambda c a^2 \pi^a_t + u_t . \]

All three specifications permit the retrieval of an estimate of "c" the coefficient on the expected inflation variable in equation (2), and \( \lambda \) the coefficient of adaptive expectations in equation (5). The strategy followed was to begin with specification (7), however, if this equation encountered estimation difficulties then specification (8) or (9) was used.

The model used to describe the behaviour of issuers of financial claims is similar in form to the model used on the asset demand side. The desired stock of a financial claim outstanding is hypothesized to be positively related to the stock of physical assets, negatively related to the expected real own rate of interest, and positively related
to the expected real rate on other sources of finance.\(^1\) The complete model describing borrowing behaviour appears as follows:

\[
L_t^* = b_0 + b_1 k_t + b_2 l^e_t + b_3 o^e_t \\
(11) \quad l^e_t = (l^i_t - c\pi^e_t) \\
(12) \quad o^e_t = (o^i_t - c\pi^e_t) \\
(13) \quad L_t - L_{t-1} = \psi(L_t^* - L_{t-1}) \\
(14) \quad \pi^e_t = \pi^e_{t-1} + \delta(\pi^a_t - \pi^e_{t-1}).
\]

On a priori grounds the following pattern of coefficient signs is anticipated: \(b_1 > 0\), \(b_2 < 0\), \(b_3 > 0\), \(0 \leq \psi \leq 1\), \(0 \leq \delta \leq 1\). Fisher's hypothesis suggests that \(c\) is equal to unity, while the Fisher-Friedman asymmetric expectations hypothesis suggests that \(\delta > \lambda\). Combining equations (10) to (14) and applying the Koyck transformation we obtain:

\[
L_t = \delta \psi b_0 + ((1 - \psi)(1 - \delta))L_{t-1} - ((1 - \psi)(1 - \delta))L_{t-2} \\
+ \psi b_1 k_t - (1 - \delta)\psi b_1 k_{t-1} - \psi b_2 l^i_t - (1 - \lambda)\psi b_2 l^i_{t-1} \\
+ \psi b_3 o^i_t - (1 - \lambda)\psi b_3 o^i_{t-1} - \psi\delta c(b_2 + b_3)\pi^a_t + u_t.
\]

Since problems of multicollinearity were generally encountered when equation (15) was estimated, two simpler

---

specifications which parallel the simpler asset demand equations were also employed.

In all cases the central parameters of interest, are "c" which appears in both the lender and borrower equations, \( \lambda \) which appears in the asset demand equations, and \( \delta \) which appears in the liability supply equations. Although the model specification used results in estimates of a number of other structural parameters, these estimates are not directly relevant for our purposes. Fisher's hypothesis suggests that "c" is equal to unity for both borrowers and lenders. It measures the long-run or steady-state reaction of borrowers and lenders to a change in the rate of inflation. The Fisher-Friedman asymmetric-expectations hypothesis, however, is concerned with the process adjustment from one rate of inflation to another. This hypothesis states that borrowers adapt to a change in the rate of price change at a faster pace than do lenders. Under this hypothesis we would typically expect to find \( \delta > \lambda \).

**Aspects of Estimation**

**The Identification Problem**

In implementing equations (7) and (15) empirically three distinguishable types of identification problems arise. These may be termed, theory identification, the classical identification problem, and within equation identification.

The first identification problem arises because different theoretical approaches can be used to explain the
same phenomena. For example, there are competing theories of inflation, interest rate determination and so on. In specifying structural equations it is already assumed that we have taken a position as to how certain economic processes work. Econometric theory per se does not provide a guideline as to the theoretical framework to be used.

If we were to specify a complete model of the economy which described behaviour in both the real and financial sectors, it is clear that equations (7) and (15) would be over-identified in the classical sense. At the same time it is not necessary to specify a complete model of the economy in order to estimate equations (7) and (15), since equations such as (7) and (15) are typically estimated directly using two-stage least squares in any event and the complete model would only provide a guideline to the slate of first round regressors to be used. The practice commonly followed in dealing with this type of situation is to use variables which may reasonably be taken to be exogenous to the money sector, at least in the short-run, as instrumental variables in the first round regressions of the two-stage procedure. Given the


2/ See, for example; E. L. Feige, "Expectations and Adjustments in the Monetary Sector," Papers and Proceedings of the American Economic Association 56 (May 1967): 468, and D. R. Starleaf, "The Specification of Money Demand-Supply Models which Involve the Use of Distributed Lags," The Journal of Finance 25 (September 1970): 756. The additional instrumental variables used in the first stage of the two-stage estimation procedure include: Bank Rate (the rediscount rate at the Bank of Canada), exports of goods and services in real terms, government expenditures on goods and services in real terms, the monetary base, population, and the average term to maturity of the Federal debt.
specification of our model application of ordinary least squares would lead to simultaneous equation bias since the error term is not independent of all of the regressors, consequently a two-stage procedure has been used.

The classical identification problem pertains to the composite coefficients in equations (7) and (15). The structural parameters; such as, $\lambda$, $\gamma$, $\psi$ and $\delta$ are overidentified in another sense, however. In equation (7); for example, there are ten estimated coefficients, yet the behavioural model underlying equation (7) contains seven structural parameters. If unconstrained two-stage least squares is applied to equation (7), multiple estimates of several parameters will be obtained. To circumvent this difficulty a constrained non-linear regression procedure was used in the estimation.\footnote{See D. W. Marquardt, "An Algorithm for Least-Squares Estimation of non-linear Parameters," Journal of the Society for Industrial and Applied Mathematics 1(June 1963).} This permits us to obtain unique estimates for the structural parameters.

The Data\footnote{Data sources are described in the Appendix.}.

The existing state of financial data in Canada does not permit the estimation of a detailed portfolio balance model for primary savers and borrowers. Financial data in Canada is primarily collected for the purpose of monitoring the activities of various financial institutions, consequently data on the extremities of the intermediation process tends to be incomplete. Since our purpose is to provide illustrative examples of attempts to test the behavioural implications of
the Fisher hypothesis and the Fisher-Friedman asymmetric-expectations hypothesis, construction of a closed and detailed model of the financial sector is not necessary.

The types of borrowing activity considered in this chapter include, borrowing by the household sector in the form of consumer credit and borrowing by the corporate sector in the form of bonds, and short-term loans. The consumer credit equation calls for a stock of consumer durables as one of the principal regressors. A series was constructed using the following formula:

$$STK = \sum_{i=0}^{n} (1 - \gamma)^i DUR_{t-1}$$

where DUR represents consumer expenditures on durable goods, $\gamma$ is an assumed depreciation rate, and STK is the calculated stock of consumer durables. The rate of depreciation was set equal to 0.25 and $n$ was set to 40.1/1

In the case of corporate borrowing, data from Statistics-Canada's Industrial Corporations Quarterly Financial Statistics (S. C. Cat. No. 61-003) was used. Coverage of this data is reasonably complete. Excluded are Crown corporations, provincial and local government enterprises, and financial institutions, all of which are desireable to exclude for the purposes of our study in any case.

The primary saver equations considered in this chapter include the demand for: personal saving deposits

1/ This rate can be justified on the basis of recent empirical evidence on the demand for durable goods in Canada. Using the Houthakker-Taylor model, T. T. Schweitzer has estimated a rate of depreciation for household appliances of 0.21 and for automobiles of 0.27. See T. T. Schweitzer, Personal Consumer Expenditures in Canada, 1926-1973 Part 2 (Ottawa: Economic Council of Canada, 1970):27 and 39.
at Chartered Banks and Canada savings bonds.

Occasionally a proxy interest rate series had to be used when the appropriate own interest rate series was unavailable. Since a number of the series used were not collected prior to 1962, quarterly data over the period 1962 to 1971 (IV) was used.

**Empirical Results**

A list of the variables used appears in Table I.\(^1\)

All of the equations were estimated in real terms, using quarterly data over the period 1962-1971.

Separate liability supply equations have been estimated for: industrial corporate bonds, short-term loans by industrial corporations, and consumer credit. In the case of the supply of financial claims we were forced to rely on the pure-expectation model in which only the own rate of interest was included.\(^2\)

\(^1\) Data Sources are described in the Appendix.

\(^2\) The supply function counterpart to equation (9) was used. A number of compromises have to be made in order to estimate supply functions for financial claims. In the case of consumer credit; for example, a proxy rate (the 90 day finance company rate) has to be used. In this case inclusion of an alternative rate would have made little sense. In the case of corporate bond borrowing and short-term corporate loans, alternative rates do exist. Estimation of the expanded model (equation 15), for corporate borrowers uniformly produced unsatisfactory results. In most cases the non-linear regression programme failed to converge on stable parameter values.
**Table I**

**DEFINITION OF MNEMONICS EMPLOYED**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCB</td>
<td>Industrial corporation liabilities in the form of bonds</td>
</tr>
<tr>
<td>LSL</td>
<td>Short-term loans of industrial corporations</td>
</tr>
<tr>
<td>CCR</td>
<td>Consumer credit</td>
</tr>
<tr>
<td>MSAVR</td>
<td>Saving deposits at Chartered Banks</td>
</tr>
<tr>
<td>CSBR</td>
<td>Canada Savings bonds</td>
</tr>
<tr>
<td>STK</td>
<td>Consumer durable stock</td>
</tr>
<tr>
<td>CPE</td>
<td>Plant and equipment of industrial corporations</td>
</tr>
<tr>
<td>Y</td>
<td>Gross National Product</td>
</tr>
<tr>
<td>RMYW</td>
<td>McLeod-Yound-Weir industrial bond rate</td>
</tr>
<tr>
<td>RTB6</td>
<td>6 month Treasury Bill rate</td>
</tr>
<tr>
<td>RDAY</td>
<td>Rate on day-to-day loans at banks</td>
</tr>
<tr>
<td>RSAV</td>
<td>Rate on Savings Deposits</td>
</tr>
<tr>
<td>RFIN</td>
<td>90 day finance company rate</td>
</tr>
<tr>
<td>RGOVLB</td>
<td>Rate on long-term Federal Government bonds</td>
</tr>
<tr>
<td>PYE</td>
<td>Rate of inflation, measures by the rate of change of the Consumer Price Index</td>
</tr>
</tbody>
</table>
Corporate bond borrowing was hypothesized to be positively related to the real value of plant and equipment, and negatively related to the yield on industrial bonds. The following estimating equation was employed:

$$L_{CBt} = d_0 + d_1 L_{CBt-1} + d_2 CPE_t + d_3 CPE_{t-1} + d_4 RMYW_t$$
$$\quad + d_5 RMYW_{t-1} + d_6 PYE_t + u_t$$

These coefficients are related to the structural parameters in the following way:

$$d_0 = \delta b_0 \quad d_4 = b_2$$
$$d_1 = (1 - \delta) \quad d_5 = - (1 - \delta)b_2$$
$$d_2 = b_1 \quad d_6 = \delta b_2 c$$
$$d_3 = -(1 - \delta)b_1$$

The demand for short-term loans on the part of industrial corporations was hypothesized to be positively related to the real value of plant and equipment, and negatively related to the day-to-day loan bank-loan rate.\(^1\)

The estimating equation employed is similar in form to the corporate bond equation and appears as:

$$L_{SLt} = d_0 + d_1 L_{SLt-1} + d_2 CPE_t + d_3 CPE_{t-1} + d_4 RDAY_t$$
$$\quad + d_5 RDAY_{t-1} + d_6 PYE_t + u_t$$

\(^1\) It is possible to use Chartered Bank prime rate here. However, Chartered Bank prime rate typically yields poor results. This is to be expected for two reasons. Over most of the period in question, the Chartered Banks were prevented by law from raising prime rate above six-percent, thus the series exhibits little variation. At the same time, when prime rate was at its ceiling level; it was generally unrepresentative of the cost of borrowing.
Results for corporate borrowing in these two forms are recorded in Table II.

Table II

TWO-STAGE NON-LINEAR LEAST SQUARES ESTIMATES OF THE STRUCTURAL PARAMETERS: PRIMARY BORROWERS, INDUSTRIAL CORPORATIONS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CORPORATE BONDS</th>
<th>SHORT TERM LOANS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Standard Error</td>
</tr>
<tr>
<td>(B_0)</td>
<td>-.0293</td>
<td>.0080</td>
</tr>
<tr>
<td>(B_1)</td>
<td>.4799</td>
<td>.0463</td>
</tr>
<tr>
<td>(B_2)</td>
<td>-2.601</td>
<td>.8922</td>
</tr>
<tr>
<td>(c)</td>
<td>.1568</td>
<td>.1535</td>
</tr>
<tr>
<td>(\delta)</td>
<td>.3635</td>
<td>.1585</td>
</tr>
<tr>
<td>(R^2)</td>
<td>.984</td>
<td></td>
</tr>
<tr>
<td>D. W.</td>
<td>1.637</td>
<td></td>
</tr>
</tbody>
</table>

The signs of all of the coefficients in the two liability supply functions are in agreement with a priori theoretical notions. In the case of corporate bonds "c", the critical parameter from the point of view of Fisher's hypothesis is significantly different from unity\(^1\) (contrary to the Fisher hypothesis) and

\(^1\) Using a one-tailed test at the .05 level of significance the appropriate critical t value is 1.693. The hypothesis that c is equal to unity can be strongly rejected, and produces a t statistic of 5.493.
insignificantly different from zero. In the case of corporate short-term loans the situation is not as clear cut. The hypothesis that \( c \) is equal to unity can be strongly rejected at the .05 level of significance.\(^1\) On the other hand the hypothesis that \( c \) is equal to zero can also be rejected at the .05 level of significance, and a "\( t \)" value of 2.538 is produced under the null hypothesis that \( c \) is equal to zero. It is also interesting to note that the point estimates of the Fisher effect are insignificantly different from each other in these two equations.\(^2\) The evidence produced suggests that inflation may have a small upward effect on the demand for loans, but that the full Fisher effect does not occur.

The coefficient of adaptive expectations lies between zero and unity in both the case of the corporate bond equation and the short-term loan equation. The point estimates of the decay parameter are extremely close, and the hypothesis that they are equal cannot be rejected.

In addition to the two corporate liability supply equations an equation for the supply of consumer credit liabilities\(^3\) was estimated. Consumer credit demanded was

---

\(^1\) The "\( t \)" statistic for the hypothesis that \( c \) is equal to unity is 4.364.

\(^2\) Using a two-tailed test at the .05 level of significance neither the hypothesis that \( c \) in the corporate bond equation is equal to \( c \) in the short-term loan equation or vice versa can be rejected.

\(^3\) It is more commonplace to view this as a demand for consumer credit.
hypothesized to a positively related to the real stock of consumer durables, and negatively related to the cost of borrowing. Since an interest rate series representative of the cost of consumer borrowing is not available, the yield on short-term finance company paper was employed. The estimating equation for consumer credit appears as:

$$ CCR_t = d_0 + d_1 CCR_{t-1} + d_2 STK_t + d_3 STK_{t-1} + d_4 RFIN_t + d_5 RFIN_{t-1} + d_6 PYE_t + u_t $$

In Table III, results for household borrowing in the form of consumer credit are recorded.

**Table III**

**TWO-STAGE NON-LINEAR LEAST SQUARES ESTIMATES OF THE STRUCTURAL PARAMETERS: PRIMARY BORROWERS, CONSUMER CREDIT**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_0$</td>
<td>-.0320</td>
<td>.0086</td>
</tr>
<tr>
<td>$B_1$</td>
<td>.1019</td>
<td>.0190</td>
</tr>
<tr>
<td>$B_2$</td>
<td>-.7448</td>
<td>.2346</td>
</tr>
<tr>
<td>$c$</td>
<td>-1.164</td>
<td>1.3803</td>
</tr>
<tr>
<td>$\delta$</td>
<td>.1988</td>
<td>.0582</td>
</tr>
</tbody>
</table>

$R^2$ .991

D. W. 2.350
With the exception of the estimate of "c", all of the point estimates of the structural parameters are correctly signed in accordance with a priori theoretical notions. Although the Fisher effect coefficient is negatively signed, the confidence interval around c is relatively broad.\(^1\) It is not possible; for example, to reject the hypothesis that c is equal to zero, or the hypothesis that c is equal to unity. Nor is it possible to reject the hypothesis that c in the consumer credit equation is equal to either of the point estimates of c obtained in the corporate borrowing equation.

The coefficient of adaptive expectations is estimated to be .1988. This value is significantly different from both zero and unity. It is interesting to note that this value is significantly less than the values obtained for corporate borrowing.

The empirical results for ultimate borrowers may be summarized as follows: The evidence in favour of the strong version of the Fisher hypothesis (c = 1) is extremely weak. In two of the three experimental borrowing equations examined the hypothesis that c is equal to unity can be rejected. In the one case (consumer credit) where the strong Fisher hypothesis cannot be rejected, the range of acceptable hypotheses is relatively wide and includes both unity and the origin. In two cases out of three the hypothesis that c is equal to zero cannot be rejected. On balance, the evidence suggests

\(^1\) Using a 95 percent confidence interval, hypothesized values of c between 1.649 and -3.977 are acceptable.
the presence of a weak positive Fisher effect which is closer to zero than it is to unity. There is also some evidence that corporate borrowers adjust to inflation at a marginally more rapid pace than household borrowers.

Two equations representative of primary saver behaviour were also estimated. Asset demand equations were estimated for personal saving deposits at Chartered Banks and Canada savings bonds.\(^1\)

The demand for personal saving deposits at Chartered Banks is hypothesized to be positively related to income, positively related to the rate on savings deposits, and negatively related to the rate on substitute assets. The rate on six month treasury bills is used as a measure of an alternative rate. The estimating equation for personal saving deposits appears as:

\[
MSAVR_t = d_0 + d_1 MSAVR_t + d_2 MSAVR_{t-1} + d_3 Y_t + d_4 Y_{t-1} \\
+ d_5 RSAV_t + d_6 RSAV_{t-1} + d_7 RTB6_t + d_8 RTB6_{t-1} \\
+ d_9 PYE_t + u_t .
\]

The "d" coefficients are related to the structural parameters in the following manner:

\(^1\) An attempt was made to estimate a demand function for term deposits at Trust and Mortgage loan companies. Our attempts in this direction were not particularly successful. The own interest rate variable typically assumed an incorrect sign in the estimated equations.
\begin{align*}
    d_0 &= \gamma \lambda a_0 \\
    d_1 &= (1 - \gamma) + (1 - \lambda) \\
    d_2 &= -((1 - \gamma)(1 - \lambda)) \\
    d_3 &= \gamma a_1 \\
    d_4 &= -(1 - \lambda) \gamma a_1 \\
    d_5 &= \gamma a_2 \\
    d_6 &= -(1 - \lambda) \gamma a_2 \\
    d_7 &= \gamma a_3 \\
    d_8 &= -(1 - \lambda) \gamma a_3 \\
    d_9 &= \lambda \gamma c(a_2 + a_3)
\end{align*}

Estimates of the structural parameters of the demand for personal saving deposits appear in Table IV. The signs of most of the coefficients are consistent with a priori notions. A notable exception is the stock adjustment parameter, which assumes a negative sign.

The Fisher effect coefficient \textquotedbl{}c\textquotedbl{}, is significantly different from zero and unity, suggesting the presence of a small positive Fisher effect. The coefficient of adaptive expectations is considerably higher than expected and exceeds unity.

Estimates of the structural parameters of the demand for Canada savings bonds are also included in Table IV. Since a yield to maturity series for Canada savings bonds is not available, a proxy series had to be used. The series used is the yield on Trust company investment certificates.\textsuperscript{1} As an alternative rate the yield on long-term Government securities was used. The estimating equation used is similar in form to that used for savings deposits.

The results for the savings bond demand function

\textsuperscript{1} Guaranteed investment certificates at Trust and Mortgage loan companies possess many of the attributes of Canada savings bonds.
appears in Table IV. The results parallel those obtained for demand for savings deposits in several ways. The Fisher effect coefficient $c$ is significantly different from unity, but insignificantly different from zero. The coefficient of adaptive expectations ($\lambda$) is on the high side. The point estimate exceeds unity, but it is not significantly greater than unity.

In the case of ultimate savers our results may be summarized in the following way: There is some evidence of a weak Fisher effect visible in our results. In both cases examined, however, the Fisher coefficient was significantly less than unity, thus the strong version of the Fisher hypothesis finds no support. At the same time the Fisher coefficient was insignificantly different from zero in one of the two demand functions. The coefficient of adaptive expectations is greater than anticipated in both equations. This result combined with the low value of the adjustment coefficient suggests that these two equations may suffer from specification bias.

While any conclusions drawn on the basis of these equations are extremely tentative, it is interesting to compare the results for the primary saver equations to the results for the primary borrower equations. In general both sets of equations yield similar information with regard to the Fisher hypothesis. The strong version of the Fisher hypothesis is rejected in all but one case. While the weak version of the Fisher hypothesis ($1 > c > 0$) is consistent with all of the equations examined. The hypothesis that $c$ is equal to zero
Table IV
TWO-STAGE NON-LINEAR LEAST SQUARES ESTIMATES OF THE STRUCTURAL PARAMETERS: PRIMARY LENDERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Personal Savings Deposits</th>
<th>Canada Savings Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Standard Error</td>
</tr>
<tr>
<td>a₀</td>
<td>4.294</td>
<td>1.7139</td>
</tr>
<tr>
<td>a₁</td>
<td>0.3077</td>
<td>0.1592</td>
</tr>
<tr>
<td>a₂</td>
<td>1407.00</td>
<td>449.95</td>
</tr>
<tr>
<td>a₃</td>
<td>-401.90</td>
<td>205.652</td>
</tr>
<tr>
<td>γ</td>
<td>-0.0855</td>
<td>0.0981</td>
</tr>
<tr>
<td>c</td>
<td>0.1125</td>
<td>0.0475</td>
</tr>
<tr>
<td>λ</td>
<td>1.776</td>
<td>0.171</td>
</tr>
<tr>
<td>R²</td>
<td>.987</td>
<td></td>
</tr>
<tr>
<td>D. W.</td>
<td>2.055</td>
<td></td>
</tr>
</tbody>
</table>
can be rejected by one of the primary saver equations and by one of the primary lender equations.

No support for the asymmetric-expectations hypothesis of Fisher and Friedman, \((\delta > \lambda)\) is visible in the data. To the extent that it is possible to make comparisons, the results are in conflict with the Fisher-Friedman hypothesis since all of the estimated values for \(\delta\) are significantly less than all of the estimated values for \(\lambda\).

**Summary**

In this chapter, an attempt was made to test the behavioural implications of the Fisher hypothesis, and the Fisher-Friedman asymmetric expectations hypothesis. While the results recorded in this chapter can only be viewed as a first attempt to explore these hypotheses, they do exhibit several interesting features.

The strong version of the Fisher hypothesis, \((c = 1)\) was generally rejected by the data. In the one case where the hypothesis that \(c\) was equal to unity could not be rejected, the corresponding standard error was relatively large. The hypothesis that the Fisher coefficient is equal to zero can be rejected in two cases out of five. There was no apparent difference in the steady state impact of inflation on borrower as opposed to lender behaviour, on balance. The evidence is consistent with the presence of a small positive Fisher effect.

The asymmetric expectations hypothesis \((\delta > \lambda)\) is not supported by our results. The evidence is consistent with
the hypothesis that lenders adapt to inflation at a faster pace than do borrowers, and conflicts with the Fisher-Friedman hypothesis. This result must be regarded as highly tentative, however, since it is possible that our equations are subject to specification error.
CHAPTER V

SUMMARY AND CONCLUSIONS

The notion that price-expectations play an important role in determining the nominal rate of interest has gained wide acceptance since Irving Fisher first wrote on the subject over seventy years ago.\(^1\) Nevertheless, empirical work on the influence of price-expectations on interest rates has been limited to a few recent studies.

As an aid to assessing the relationship between inflation, price-expectations, and interest rates a relatively simple model of the capital market was constructed in chapter II. Using this model a number of interesting results can be generated. For example, the model suggests that the authorities can affect the real rate of interest through two routes. They can have a permanent effect through their ability to augment the supply of loanable funds, and a short-run effect through asymmetric expectations. It was suggested that it is the short-run effect which is focused on by Milton Friedman in his well known article "The Role of Monetary Policy".\(^2\) It was also suggested that Friedman's argument hinges sensitively on the assumption that borrowers adapt more quickly to inflation than do lenders. Modification of this assumption changes the results in a substantive manner.

\(^1\) I. Fisher, Appreciation and Interest (New York: Macmillan, 1898).

In chapter III and IV, various implications of the theoretical work contained in chapter II were the subject of empirical investigation. Chapter III concentrated on the use of reduced-forms, while in chapter IV a structural approach was adopted.

In chapter III, Irving Fisher's hypothesis that nominal rates of interest adjust in a compensating manner to a change in the rate of inflation was examined with the aid of the naive Fisherian model. Using monthly data, we were able to reject the hypothesis that the Fisher price-expectations theory is true given our assumptions on the formation of the real rate of interest and the formation of price-expectations. The quarterly results were, in general, consistent with Fisher's hypothesis. The quarterly results did not, however, support the "differential effects" hypothesis.

Empirical work on the role of price-expectations and nominal rates of interest appears to have been frequently misinterpreted in the past. In chapter III, it was suggested that the inferences drawn by a number of other authors violated the assumptions under which they estimated their models, or that their conclusions were in conflict with their empirical results.

While the reduced form method does throw some light on the nature of the relationship between inflation, price-expectations and interest rates, it suffers from a number of important limitations. In order to overcome some of these limitations in chapter IV a structural approach was adopted.
This approach has the added advantage that it permits an investigation of the asymmetric-expectations hypothesis, which cannot be tested using the reduced form approach.

In chapter IV, behavioural equations for ultimate borrowers and lenders were specified and estimated using a two-stage non-linear regression technique. Attention was focused on the coefficient on the price-expectational variable and the price-expectation adjustment coefficient. A priori the coefficient on the price-expectational variable was hypothesized to be equal unity (Fisher), while the price-expectation adjustment coefficient for borrowers was hypothesized to be greater than the corresponding coefficient for ultimate lenders (Friedman).

The empirical results for the structural equations must be regarded as provisional estimates. These results, however, are consistent with the existence of a small positive Fisher effect. The hypothesis that the coefficient on the price-expectational variable is equal to unity does not appear to be supported by the data. The asymmetric expectations hypothesis of Fisher and Friedman was not supported by our results. In all of the equations examined the coefficient of adaptive expectations of primary savers exceeded the corresponding coefficient on the part of borrowers. These results suggest that a rise in the rate of inflation does not lead to a temporary fall in the real rate of interest facing borrowers due to the presence of asymmetric expectations.
APPENDIX

DATA SOURCES

Interest Rates

Interest rates on financial assets are in the form of yields to maturity. Most of the Canadian interest rates used can be found in The Bank of Canada Review or its predecessor The Bank of Canada Statistical Summary and Supplement. A number of unpublished interest rate time series were obtained from the Research Department of the Bank of Canada.

Financial Assets and Liabilities

Data on corporate financial liabilities was obtained from Industrial Corporations Financial Statistics (S. C. Cat. No. 61-003). Data on term deposits and guaranteed certificates of deposits appear in Financial Institutions (S. C. Cat. No. 61-006). Consumer credit data can be found in The Bank of Canada Review, as can data on term deposits at Chartered Banks.

Price Indexes

Time series for the Canadian Consumer Price Index (C. P. I.), and implicit deflators were obtained from Prices and Price Indexes (S. C. Cat. No. 62-112).

Other Data

Data on various macroeconomic aggregates (such as G. N. P. etcetera) were obtained from The National Income and
Expenditure Accounts (S. C. Cat. No. 531). Data from the second revised version of the accounts were used. Capital stock data can be obtained from Fixed Capital Flows and Stocks (S. C. Cat. No. 13-522). Since this data is in annual form an interpolation procedure must be used to generate quarterly estimates. Early results obtained using this method were not entirely successful. An alternative procedure of deflating the book-value of plant and equipment, which is available on a quarterly basis, by the implicit deflator for investment goods was used.
REFERENCES


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