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THE FISCAL FEASIBILITY OF LAND VALUE TAXATION
IN URBAN METROPOLITAN COMMUNITIES

by

Gerald W. Stone, Jr.

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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ABSTRACT

THE FISCAL FEASIBILITY OF LAND VALUE TAXATION
IN URBAN METROPOLITAN COMMUNITIES

by

Gerald W. Stone, Jr.

Site value taxation is a levy solely on the value of land and is by no means a new idea. The movement toward increased use of land as a tax base is often cited as desirable because of efficiency aspects. Unfortunately, the idea is all too often dismissed for alleged lack of adequacy. This study focuses on the revenue adequacy aspects of site value taxation.

Local government revenue and expenditure functions are introduced into both Ricardian and three factor neo-classical models of economic growth. Tax revenues are assumed to be derived solely from land rents, while public expenditures are related to per capita income and population. Through a neo-classical model we show that the elasticities of factor substitution, the expenditure elasticities of income and population, and factor shares are important parameters to consider when attempting to answer questions regarding revenue sufficiency of land value taxation.
This study shows we can expect that land rents will grow relatively more rapidly than will local expenditure needs. We conclude that site value taxation is potentially a very fertile source of tax revenues, particularly in urban communities.
CHAPTER I

INTRODUCTION

Property taxes are the primary source of local government revenues in the United States. The present property tax distorts allocative decisions because improvements are included in the tax base. Land value taxes are taxes levied exclusively on the value of land. Thus, the general defects of the present property tax, particularly as it relates to the taxation of improvements are absent in land value taxation.

High absolute levels of urban property tax rates discourage investment in urban real property relative to other forms of capital, discourage restoration and rebuilding in central cities, and other things equal, provide an incentive to locate investment in suburban areas where tax rates are generally lower. The property tax tends to discourage consumption in housing relative to consumption in other goods.¹ This departure from neutrality has become

¹However, federal income taxes encourage the consumption of housing services as imputed rent is excluded from federal income taxation, and mortgage interest and property taxes are permitted as deductions. When comparing
more important as effective tax rates have increased over the years.

Site value taxation, on the other hand, taxes location rents which are independent of the improvements on a particular site. Location rents constitute a surplus; taxing this surplus will not result in a reduction of sites offered. The tax is neutral with respect to operating decisions since no response by the landowner can increase his return.

Site value taxation is neutral with regard to intensity of land use since the tax is a lump sum charge. The land component in the present property tax also possesses this neutrality. However, the tax on improvements discourages development, and tends to encourage less intensive land usage. Thus exempting improvements from the present property tax and increasing the tax rate would increase the holding costs of land and tend to foster more intensive

the effects of the offsetting unneutralities, Netzer [67] concludes:

On balance, therefore, the combined property tax/income tax system discourages expenditure for owner-occupied housing at the lower end of the scale and encourages it for better off families. Moreover, it surely discourages consumption of rental housing, which has no income tax advantage, relative to owner-occupied housing and to other forms of consumption. Since tenants tend to be lower income people, the overall system is decidedly unneutral with respect to housing consumption in the lower income groups.
development.

Moreover, the present property tax method of financing education is under attack in the courts. Recent court cases in California, Minnesota, Texas and New Jersey have ruled that financing education services with property taxes is unconstitutional because "it makes the quality of a child's education a function of the wealth of his parents and neighbors."\(^2\) If the United States Supreme Court upholds these decisions,\(^3\) substantial intergovernmental reform will

\(^2\)See Herber [42] for an extended discussion of these cases. The same problem exists between cities and their suburbs. Generally middle and high income families cluster in the suburbs while low income groups seek housing where they can afford it—namely, in the core city. Furthermore business has tended to follow the high income population shift and has migrated from central cities in increasing numbers over the past decade. The result of these industrial and residential shifts has been to leave cities with a dwindling tax base while at the same time imposing the dual responsibility of providing for low income populations and serving the needs of commuting populations. For additional evidence see Advisory Commission on Intergovernmental Relations [4], Neenan [65], Bowen and Davis [16], and Kee [48]. Kee argues that wide discrepancies exist between core cities and their suburbs in their fiscal capabilities of raising adequate revenues on the one hand, and their populations' needs for public services on the other. For a different approach to a similar problem see Murray [61], who examines the level and composition of central cities' expenditures as one possible set of determinants of the number of city residents migrating to suburban or fringe areas.

\(^3\)The Rodriguez v. San Antonio Independent School District decision is currently being appealed to the Supreme Court of the United States by the Texas Board of Education.
be necessary.\textsuperscript{4} Since it appears that the property tax structure may be reformed considerably, the incremental economic as well as "administrative" trauma associated with conversion to site value taxation will be at a minimum during this period of transition.

In light of these problems, site value taxation has much to offer local governments. First, site value taxation discourages vacant land speculation and tends to encourage more efficient use of land within communities. Second, since population growth and community improvements financed by local government enhance the value of land, communities have the right to tax these "unearned increments" to land value.\textsuperscript{5} Finally, as this study demonstrates in the chapters which follow, land values in urban communities are rising rapidly enough to permit tax revenues to keep pace with

\textsuperscript{4}Some have suggested that local governments continue to finance education, but substitute alternative revenue sources (income and sales taxes) for the present property tax. Clearly, this also will be rejected by the courts as unequal capacities of school districts will continue to yield unequal educational expenditures per pupil. See Herber [42] for a more complete discussion of the alternatives.

\textsuperscript{5}The notion of land value taxation goes back to Adam Smith and David Ricardo. It was developed and popularized by Henry George [35], who argued for the elimination of all taxes except those on land. Thus, his proposal was labeled the "single tax." In this study we do not argue that all public expenditures should be supported by a site levy.
rising urban fiscal needs.

Previous research on the revenue adequacy of site value taxation has been concentrated on determining the tax rate necessary for a site levy to generate the yield obtained from present property taxes. This algebraic approach to the problem offers little to explain the dynamic properties of the tax. The question of whether the site levy remains adequate in the future is left unanswered. Furthermore, if the tax is inadequate at present, this static approach gives little insight into the question of whether or not the tax will become adequate at some point in the future. The answers to these questions depend upon current levels of expenditures and revenues, and their respective rates of growth. We assume that a given proportion of total expenditures can be currently financed by a site levy, and then determine what characteristics are required of the economy for a site levy, with a constant tax rate, to provide sufficient revenue over time.

The plan of the study is as follows: Chapter II examines site value taxation from the viewpoints of historical background, allocation, equity, administrative feasibility, and revenue adequacy. The revenue adequacy section presents a summary of previous research, all of which attempted to answer questions dealing with concurrent revenue replacement.
In Chapter III revenue and expenditure functions are generated. Tax revenues are assumed to be derived solely from the value of land, while public sector expenditures are related to income and population.

The Ricardian and Neo-Classical models developed in Chapters IV and V respectively permit analysis of the revenue adequacy question over time. The latter model places particular emphasis on the relationships between revenue adequacy and (1) the elasticities of substitution between factors and, (2) the elasticities of expenditures with respect to per capita income and population.

In Chapter VI an empirical model is developed based on the theoretical models presented in Chapters IV and V. Empirical evidence on the national economy for the years 1929-1970 is presented. The evidence suggests that site levies would prove adequate for many urban communities. In addition, we show that the question of revenue sufficiency at the national level is not closed.
CHAPTER II

SITE VALUE TAXATION

INTRODUCTION

Site value taxation is a levy solely on the value of land and is by no means a new idea. Proponents since Henry George have tended to make extreme claims for their proposals; generally they have been ignored by the economic profession. However, today revenue needs at local levels are severe enough that alternative sources are receiving considerable attention. The movement toward increased use of land as a tax base is often cited as desirable because of efficiency aspects. Unfortunately, the idea is all too often dismissed for alleged lack of adequacy.¹ In this section we examine the historical, allocational, administrative, equity, and revenue adequacy aspects of site value taxation. Each of these aspects has been discussed in some detail elsewhere, but each deserves mention and analysis in this study.

¹See Netzer [67], Heilbrun [40] and Pechman [75]. Both Netzer and Heilbrun discuss site value taxation at some length theoretically, but their empirical conclusions regarding revenue adequacy are based on aggregate tests of rather poor data.
HISTORICAL BACKGROUND

The concept of economic rent has its roots in classical economic theory. Many writers in the early nineteenth century singled out land as possessing special characteristics. As early as Adam Smith, land rent was promoted as a desirable tax base. Smith wrote:

Both ground rents [urban land rent], and the ordinary rent of land [agricultural land rent], are a species of revenue, which the owner in many cases enjoys, without any care or attention of his own. Though a part of this revenue should be taken from him, in order to defray the expenses of the State, no discouragement will thereby be given to any sort of industry. The annual produce of the land and the labor of the society, the real wealth and revenue of the great body of the people, might be the same after the tax as before. Ground rents, and the ordinary rent of land are, therefore, perhaps, the species of revenue, which can best bear to have a peculiar tax imposed upon them.²

In Ricardian theory, rent was the result of two factors. First, if land is homogeneous, supply limitations create scarcity rents. Second, if land is heterogeneous, the relative supplies of the various types of land give rise to differential rents. Thus, land rent, being a return to a nonreproducible natural factor, was especially suitable for tax purposes because such taxes could not be

²Shoup [86], p. 80.
shifted and would not discourage production.³

Henry George [35] applied the Ricardian analysis to urban land. Urban land which is fixed in supply at favorable locations, earns an economic (location) rent which can be taxed with no resulting distortion of economic activity. George proposed to tax all land rents. He was convinced that this would abolish poverty and economic crises, the latter being the result of speculation in land values. He proposed the land tax as a "single tax" because he thought the revenue collected would be sufficient to finance the entire government sector at that time. To a considerable extent, George's position was often misinterpreted as advocacy of nationalization of land. In fact, he simply proposed a tax on pure site rents.⁴

³See Shoup [86], especially Chapter V, and Blaug [14]. As Shoup points out, Ricardo might be expected to support a tax on land rent as the tax has no tendency to discourage production. However, for Ricardo, an equity issue was of primary importance: "Rent often belongs to those who, after many years of toil, have realized their gains, and expended their fortunes in the purchase of land or houses, and it certainly would be an infringement of that principle which should ever be held sacred, the security of property, to subject it to unequal taxation."

⁴George [35], p. 328 concludes: "This, then, is the remedy for the unjust and unequal distribution of wealth apparent in modern civilization, and for all the evils which flow from it: WE MUST MAKE LAND COMMON PROPERTY." This to George was the remedy that would abolish poverty and extreme business cycles. An application of the remedy required: "Now, inasmuch as the taxation of rent, or land
Substantial effort was made to introduce the single-tax notion into the public sector in the 1890's. This effort was generally unsuccessful. However, site value taxation is not a dead issue. As we show below, site values are a substantial portion of the real property tax base and are potentially more revenue productive than casual analysis of present property tax structures would suggest. That is, we show that land rent as a tax base is growing more rapidly than is the present property tax base.

ALLOCATION ASPECTS

In this section we briefly examine the case for site value taxation. We do not intend to provide an exhaustive theoretical treatment of these arguments. We give a brief summary to point out the benefits of such a tax.

A site value levy is neutral with respect to operating decisions because the value of any particular site is independent of the nature of improvements 5 existing on that property. In contrast, the value of improvements is highly dependent on the nature of the land on which they are placed. This property tax problem is illustrated by the following two quotations from the works of Adam Smith and Adam Smith's father, Thomas Smith.

---

5 Unless otherwise indicated the terms "improvements" and "buildings" will be used interchangeably. However, "improvements" is a more inclusive term than "buildings."
site. As Netzer [67] has noted:

Location rents constitute a surplus and taxing them will not reduce the supply of sites offered; instead, the site value tax will be entirely neutral with regard to landowners decisions, since no possible response to the tax can improve the situation, assuming that landowners have been making maximum use of their sites prior to imposition of the tax. [p. 205]

The site value levy is neutral with regard to intensity of land use. The tax is a lump sum charge that is unrelated to the value of the particular improvements on that site. The market and assessed values of a site are based on the advantages and disadvantages of the site's location. Converting to heavy taxation of land will increase the holding costs of land which will encourage a more intensive utilization of existing landholdings.

The present property tax distorts allocation decisions because improvements are included in the tax base. High absolute levels of urban property tax rates tend to discourage investment in urban real property relative to investment in other forms of capital.6

The present property tax is unneutral among inputs. It encourages the substitution of other inputs (e.g., labor) for real property. To the extent that firms and

6See Netzer [67] for an extended discussion of this phenomenon.
industries are unable to affect such substitutions without suffering cost disadvantages relative to their competition, the tax will tend to shift resources out of these particular firms and industries. Land value taxation is not afflicted with this substitution effect. Finally, through an income effect, both property and land value taxation tend to discourage consumption of real estate relative to all other goods and services, unless real estate is an inferior good.

Consequently, a change from the present property tax would have two primary effects. First, the much higher tax rate on land would encourage landowners in metropolitan areas to develop their land (extensive development); this would reduce pure land speculation as the holding costs of land rise. Second, lowering the relative tax rate on buildings and improvements would stimulate building development and renewal (intensive development).

Increased land taxes will reduce the attractiveness of investing in land vis-a-vis other assets, but will not destroy the land itself. Therefore taxes on site values will be born by the owners of the sites when the tax is initially levied. The tax cannot be shifted because the supply of land is for all practical purposes perfectly
inelastic.  

Land speculation keeps land out of use, or unused, disrupting the regular growth of cities. Relatively low property tax rates, together with capital gains taxation, encourages non-development of land within the metropolis. The result is urban sprawl which requires public services to be extended further into the countryside than would normally be necessary. Several studies have indicated that per capita expenditures for particular services are negatively related to population density. Therefore, increases in efficiency in the public sector can be expected as a result of shifting to heavier taxation of land. As redevelopment and a more intensive utilization of land occur, various public services, e.g., sewer, water, and transportation services can achieve greater economies of scale. Thus, exempting improvements from taxation could substantially reduce speculation and the urban sprawling effects associated with the present property tax.

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7 See Netzer [67], p. 33. For additional elaboration on the incidence of the present property tax see Simon [88], and Mieszkowski [59].

8 See Chapter III for a discussion of these studies.

9 See Committee on Governmental Operations [25]. However, urban sprawl and speculation may make private redevelopment more likely since neighborhoods will vary in age and will not deteriorate simultaneously. This phenomenon gives rise to a classic case of the prisoner's
ADMINISTRATIVE FEASIBILITY

If the problems associated with administering a tax on rent or site values are insurmountable then the proposal has little value even if it is to be preferred for other reasons. Netzer [67] has discussed the administrative feasibility of site value taxation at great length and concludes that it should be ruled out on administrative grounds only if the problems are far greater than those associated with administration of the current property tax.\textsuperscript{10} Some economists feel that it would be much more difficult to assess unimproved land values than it is to assess land and improvement values combined. This belief is generally fostered by the lack of sufficient numbers of sales upon which to base valuations.\textsuperscript{11} However, many
dilemma. Heterogeneous aging of the community may mean that at any particular moment, only a portion of the area is blighted, and therefore, renewal may occur privately.

\textsuperscript{10}Netzer [67], p. 202.

\textsuperscript{11}An example of this is Ursula K. Hicks, Public Finance, (London, Cambridge University Press, 1947), cited in Heilbrun [40], p. 134. However, the same argument can be made as to the sales sample size available for a particular type of real property. As buildings vary in age and composition, additional comparisons are required to evaluate both land and improvements. When buildings are to the point at which redevelopment occurs, prices will reflect the value of the site less destruction costs. Substantially the same problems arise when evaluating site and improvements or when only the site is to be valued.
states currently publish separate values of land and improvements. 12

In order to overcome the general problems associated with property assessment, "self-assessment" schemes have been proposed. The essential nature of the proposal for general property taxes has been explained succinctly by Harberger [39] as follows:

...allow each property owner to declare the value of his own property, make these declared values a matter of public record, and require that an owner sell his property to any bidder who is willing to pay say, 20 percent more than the declared value. This simple scheme is self-enforcing, allows no scope for corruption, has negligible cost of administration, and creates incentives, in addition to those already present in the market, for each property to be put to use in which it has the highest economic productivity. [p. 119]

Requiring the owner to sell his underassessed property is a harsh penalty and amendments have been offered to permit the owner to reassess his property at a value equal to the bid.

This proposal faces several severe problems of implementation for land value taxation. First, some object strongly to the requirement of forced sale against the will of the property owner. Modification of the plan to permit reassessment frustrates potential bidders and would probably render the proposal administratively costly. Second, land

12 See Kieper [49].
values, like other prices, may be subject to considerable inflation. Inflation poses a problem for "self-assessment" because of the added amount of bidding incurred due to inflation alone. This problem could be reduced by annually correcting each assessment for inflation and permitting the owner to request a reduced valuation. Third, uncertainty of tax payments is introduced. Under government assessment tax liability is known. However, "self-assessment" introduces a whole range of possible valuations. In addition, for land taxation "self-assessment" would only be workable for vacant land, or land which could be costlessly vacated. Otherwise, the new owner of the land could inflict severe losses on the owner of the improvements in the short run by charging a rent equal to the difference between total revenue and total variable costs.

Finally, the use of encumbrances on the property could reduce the effectiveness of the proposal. Encumbrances on land can reduce the value of its potential use to any bidder, as can the existence of leaseholds, rights of entry, life estates, etc. The property can be pledged as security for debt, and a potential bidder cannot annul these interests by purchasing the underlying estate.13

13See Holland and Vaughn [45] for a more extensive review of "self-assessment."
Although "self-assessment" appears unworkable at present, valuation of land separate from improvements is clearly feasible. Professional assessors in many states having been doing this for many years. Thus we do not consider the administration of a land value levy insurmountable. The existence of land value taxation in various parts of the world is at least presumptive evidence of the view that it is administratively feasible.

EQUITY ASPECTS

Land rent is derived from a simultaneous interaction of several factors: Natural features, public spending, spillover benefits of private spending by individuals other than the landowner, and demand for land as a spacial

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14 California has been separately assessing land, improvements, and personal property since 1935. Netzer [67], after considerable analysis concludes that "there is no fundamental conceptual problem in valuing sites for taxation, but there would be formidable difficulties in practice, related to problems such as encumbrances."

15 Netzer [67], p. 202. In addition, see Tait [91] for a discussion of the 1947 "Compensation and Betterment Act" in the United Kingdom. This act was designed to insure that profits on land converted to urban or industrial use from farming, accrued to the state and not individual land owners. The principle problem with this type of increment levy is that the taxpayer can decide when to incur the tax liability by timing the development or sale of land. The tax became a political issue and was abolished soon after its inception.
resource. The equity argument for site value taxation asserts that a substantial share of the value of land is a consequence of population growth and public actions which are independent of the private actions of landowners.

Since much of land value is due to community development and growth, it is argued that the community has the right to recapture this unearned increment.\textsuperscript{16} However, a shift today to a land value tax with high rates would involve some difficulties. Present owners of land have paid anywhere up to full value for their investments, where previous owners may have enjoyed unearned increments. Taxing present owners will result in an expropriation of that part of their investment that does not fall into the unearned increment category.\textsuperscript{17}

The weight of this argument is reduced by the fact that a shift from the present property tax to a land value tax would result in removing the tax from buildings and placing it on land. If every parcel of land has the same building value to land value ratio, then no party is worse off (his tax bill would remain the same). If this condition did not prevail then those with ratios below the aggregate would find their tax bills increased, and those with above

\textsuperscript{16}Ibid., p. 209.
\textsuperscript{17}Heilbrun [40], p. 164, and Netzer [67], p. 290-310.
average ratios would find their tax bills decreased.\textsuperscript{18}

The extent to which a shift to land value taxation would cause inequities depends on the increase in the tax rates. If a shift required substantial expropriation of rents, or very high tax rates on land value, then substantial inequities would result. It has been argued that the problem of equity for present owners might be mitigated by taxing only increments to land value. By taxing only increments, taxation or expropriation of past increases in land values can be reduced.\textsuperscript{19}

**REVENUE ADEQUACY**

Existing studies, both theoretical and empirical, have used only static analysis. The question asked has been: Are land values sufficiently high that a site value tax at this time will reproduce revenues from the current real estate tax? This section will not attempt to prove or disprove the revenue adequacy of a site levy; instead we briefly summarize the sketchy evidence from other sources as to the possible revenue that would be generated by a

\textsuperscript{18}Heibrun [40], p. 164.

\textsuperscript{19}One drawback to this proposal is that if the value of land is equal to the discounted value of expected rents, then taxing incremental increases in land value will reduce the present value of land.
land value levy. In the final section we discuss problems
with existing studies, and introduce an approach used to
answer revenue adequacy questions in the succeeding chap-
ters.

Heilbrun [40] provides a mathematical solution to
the general question of adequacy. The rent on land will
exceed the yield of the present property tax if the ratio
of the interest rate for capitalizing land rents to the
average effective property tax rate is greater than the
ratio of the capital value of buildings to the capital
value of land.\footnote{Heilbrun [40], p. 151.} Gaffney [33] extended this analysis and

\[ N_L = \text{rent of land gross of property tax on land} \]
\[ R = \text{the yield of the present property tax} \]
\[ r_a = \text{the rate of the present property tax} \]
\[ C_L = \text{Capital value of land after capitalization of present land tax} \]
\[ C_B = \text{Capital value of buildings after capitalization of present tax} \]
\[ i_L = \text{rate of capitalization of the rent of land.} \]

Now:
\[ C_L = \frac{N_L}{i_L + r_a} \]

thus,
\[ N_L = C_L (i_L + r_a) \]

and
\[ R = r_a (C_L + C_B) \].

Therefore \( N_L > R \), if

\[ C_L (i_L + r_a) > r_a (C_L + C_B) \]
or
\[ i_L C_L + r_a C_L > r_a C_L + r_a C_B \]

which reduces to \( i_L C_L > r_a C_B \) or \( \frac{i_L}{r_a} > \frac{C_B}{C_L} \).
reached the not unexpected result that maintenance of the same revenues from a land tax requires that the tax rate on the value of land be equal to the old tax rate on all property times the ratio of the value of both land and buildings to the value of land.\footnote{Gaffney [33], p. 189-190.}

Goldsmith [36] estimated the ratio of the value of buildings to land to be 4.4:1, with an average property tax rate of 1.4 percent. Using the Heilbrun relationship, unless the interest rate for capitalizing land is

\[ r'_{a} = \text{new tax rate necessary to maintain the same levy} \]
\[ C'_L = \text{new land value when buildings are untaxed.} \]

From footnote (20) above,

(i) \[ C_L = N_L/(i_L + r_a), \]

and thus if the current tax is replaced by a tax on land values

(ii) \[ C'_L = (N_L + r_a C_B)/(i_L + r_a'). \]

Clearing denominators, and subtracting equation (ii) from equation (i) yields

\[ i_L [C_L - C'_L] + r_a C_L - r'_a C'_L = -r_a C_B. \]

In order to maintain the existing revenues,

\[ r'_a C'_L = r_a [C_L + C_B]. \]

Substituting, \[ C_L = C'_L \] gives the result that

(iii) \[ r_a' = r_a [(C_L + C_B)/C_L]. \]
substantially in excess of 6 percent, the yield from property taxes is equivalent to nearly all the rent on land.\textsuperscript{22} Netzer cited results using data from a California study which indicated that the ratio of improvement value to land value was equal to 1.35:1.\textsuperscript{23} This is substantially different from Goldsmith's estimate. Using his figures an interest rate on land of only 2.2 percent would be required to replace the yield of the existing tax.

Using a mapping technique and assessed values to arrive at real estate values Gaffney [33] estimates that the ratio of the value of land and buildings to land values in Milwaukee in 1965 was 1.4:1, or equivalently, the improvement to land value ratio was 0.4:1. This implies that an increase of 40 percent in the tax rate, with improvements exempted could replace the property tax in Milwaukee.\textsuperscript{24} Results of this type of analysis for California are set forth in Table II-1. The values presented are the percent increase in the tax rate required to maintain the same revenue when improvements are exempted

\textsuperscript{22}Netzer [67], p. 211, \( i_L/.014 = 4.4 \), this implies that \( i_L = 6.16 \) percent.

\textsuperscript{23}Study by Ronald Welch reported in a letter to Netzer [67], p. 211.

\textsuperscript{24}Gaffney [33], the calculation is based on equation (iii) in footnote (20) above.
### Table II-1

**Required Percentage Increase in Property Tax Rate and the Required Land Tax Rate Per $100 Market Value, 1970**

*If improvements are excluded from the present property tax*

<table>
<thead>
<tr>
<th>County</th>
<th>% Increase in Tax Rate</th>
<th>Tax Rate/$100 Total Land Value</th>
<th>County</th>
<th>% Increase in Tax Rate</th>
<th>Tax Rate/$100 Total Land Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alameda*</td>
<td>230</td>
<td>$6.53</td>
<td>Orange*</td>
<td>236</td>
<td>$4.90</td>
</tr>
<tr>
<td>Alpine</td>
<td>203</td>
<td>2.43</td>
<td>Placer*</td>
<td>202</td>
<td>3.90</td>
</tr>
<tr>
<td>Amador</td>
<td>245</td>
<td>3.55</td>
<td>Plumas</td>
<td>339</td>
<td>4.78</td>
</tr>
<tr>
<td>Butte</td>
<td>219</td>
<td>4.38</td>
<td>Riverside*</td>
<td>214</td>
<td>4.43</td>
</tr>
<tr>
<td>Calaveras</td>
<td>159</td>
<td>2.67</td>
<td>Sacramento*</td>
<td>280</td>
<td>7.07</td>
</tr>
<tr>
<td>Colusa</td>
<td>154</td>
<td>2.25</td>
<td>San Benito</td>
<td>177</td>
<td>2.71</td>
</tr>
<tr>
<td>Contra Costa*</td>
<td>323</td>
<td>9.00</td>
<td>San Bernardino*</td>
<td>292</td>
<td>6.28</td>
</tr>
<tr>
<td>Del Norte</td>
<td>169</td>
<td>3.82</td>
<td>San Diego*</td>
<td>219</td>
<td>4.86</td>
</tr>
<tr>
<td>El Dorado</td>
<td>174</td>
<td>3.25</td>
<td>San Francisco*</td>
<td>276</td>
<td>7.41</td>
</tr>
<tr>
<td>Fresno*</td>
<td>234</td>
<td>4.78</td>
<td>San Jauquin*</td>
<td>272</td>
<td>6.70</td>
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<tr>
<td>Glenn</td>
<td>160</td>
<td>2.60</td>
<td>San Luis Obisop</td>
<td>222</td>
<td>4.23</td>
</tr>
<tr>
<td>Humboldt</td>
<td>235</td>
<td>5.53</td>
<td>San Mateo*</td>
<td>240</td>
<td>5.40</td>
</tr>
<tr>
<td>Imperial</td>
<td>162</td>
<td>3.12</td>
<td>Santa Barbara*</td>
<td>222</td>
<td>4.81</td>
</tr>
<tr>
<td>Inyo</td>
<td>153</td>
<td>2.56</td>
<td>Santa Clara*</td>
<td>249</td>
<td>6.13</td>
</tr>
<tr>
<td>Kern*</td>
<td>182</td>
<td>3.71</td>
<td>Santa Cruz</td>
<td>207</td>
<td>4.30</td>
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<tr>
<td>Kings</td>
<td>213</td>
<td>4.24</td>
<td>Shasta</td>
<td>277</td>
<td>5.13</td>
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<tr>
<td>Lake</td>
<td>163</td>
<td>2.73</td>
<td>Sierra</td>
<td>146</td>
<td>2.21</td>
</tr>
<tr>
<td>Lassen</td>
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<td>4.46</td>
<td>Siskiyou</td>
<td>192</td>
<td>3.42</td>
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<tr>
<td>Los Angeles*</td>
<td>224</td>
<td>5.34</td>
<td>Solano*</td>
<td>251</td>
<td>5.50</td>
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<tr>
<td>Madera</td>
<td>200</td>
<td>3.09</td>
<td>Sonoma</td>
<td>231</td>
<td>4.95</td>
</tr>
</tbody>
</table>

II-17
<table>
<thead>
<tr>
<th>County</th>
<th>% Increase in Tax Rate</th>
<th>Tax Rate/ $100 Total Land Value</th>
<th>County</th>
<th>% Increase in Tax Rate</th>
<th>Tax Rate/ $100 Total Land Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marin*</td>
<td>229</td>
<td>$5.35</td>
<td>Stanislaus</td>
<td>263</td>
<td>$5.41</td>
</tr>
<tr>
<td>Mariposa</td>
<td>146</td>
<td>1.55</td>
<td>Sutter</td>
<td>184</td>
<td>2.90</td>
</tr>
<tr>
<td>Mendocino</td>
<td>189</td>
<td>3.52</td>
<td>Tehama</td>
<td>183</td>
<td>3.21</td>
</tr>
<tr>
<td>Merced</td>
<td>198</td>
<td>1.75</td>
<td>Trinity</td>
<td>150</td>
<td>2.29</td>
</tr>
<tr>
<td>Modoc</td>
<td>159</td>
<td>2.57</td>
<td>Tulare</td>
<td>229</td>
<td>4.33</td>
</tr>
<tr>
<td>Mono</td>
<td>205</td>
<td>2.68</td>
<td>Tuolumne</td>
<td>183</td>
<td>3.74</td>
</tr>
<tr>
<td>Monterey</td>
<td>238</td>
<td>4.43</td>
<td>Ventura*</td>
<td>191</td>
<td>4.05</td>
</tr>
<tr>
<td>Napa*</td>
<td>225</td>
<td>4.23</td>
<td>Yolo*</td>
<td>199</td>
<td>4.31</td>
</tr>
<tr>
<td>Nevada</td>
<td>172</td>
<td>2.68</td>
<td>Yuba</td>
<td>230</td>
<td>4.57</td>
</tr>
</tbody>
</table>

**SMSA Counties**  
(*After Name)  
(Average)  
238  
$5.46

**Non-SMSA Counties**  
(Average)  
198  
3.46

**California**  
(Total)  
230  
5.30

Source: California State Board of Equalization [21].
from the property tax. In addition, the tax rate on land per $100 market value if improvements were excluded from the tax base is presented.

Table II-1 provides evidence of the fact that highly developed counties in California, with high intensity usage of land, high value improvements, and little farm land would find it slightly more difficult to reduce the taxation on improvements. This is evidenced by the somewhat larger required percentage increase in the tax rate for SMSA's (2.38) than for non-SMSA areas (1.98). Moreover, the tax rate on land would be higher in urban areas. On the other hand, jurisdictions with considerable under-developed, potentially valuable urban land, or a large proportion of valuable farmland, and with lower proportions of improvements to land value, may be able to eliminate the tax on improvements with somewhat less difficulty. Assume

25California currently taxes personal property in addition to the values of improvements and land. We assume that personal property would not continue to be taxed and the tax on improvements would be eliminated. The method of calculation is described in footnotes (19) and (20) above. The tax rate per $100 total land value was calculated by dividing the current property tax rate by the assessment ratio and multiplying by the percent increase in the tax rate when improvements are exempted.

26The increased tax rate could be more dependent on service requirements and cost structures rather than intensity of land use in metropolitan communities. The significance and possibility of this problem is considered below.
the interest rate for capitalization on land is 5 percent. Then the percent of annual rent that must be taxed in order to maintain existing revenues ranges from a low of 31.0 percent to a high of 180.0 percent. Clearly, the higher the discount rate, the lower the values in this range and vice versa.

This evidence and the method of analysis employed may yield some erroneous implications about the revenue adequacy of site value taxation. Rural counties can replace the yield from current property taxes more easily than can urban counties at present, but the ability of rural land to support rising expenditure needs without continually increasing the land tax rate seems doubtful. The question of whether or not land taxation will continually yield adequate revenues must be considered. The answer to this question depends upon the current levels of expenditures and revenues, and their respective rates of growth.

We do not argue that all local expenditures should be financed by a site levy. For this analysis a site levy is considered to be adequate whenever it will continually finance some given proportion of the public sector. An

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27 The study by Gaffney [33], suggests that the value of urban land relative to the value of improvements may be considerably understated in California.
upper limit for revenue adequacy discussions is the present yield of the real estate tax (land plus improvements). Consequently if present expenditures (those the land tax is expected to finance) equal or exceed present revenues from a site levy then adequacy in the future depends on the relative rates of growth of revenues and expenditures. If revenues are growing more rapidly than expenditures, adequacy conditions will be attained at some point in the future. This is a sufficient condition. The length of time required for fiscal balance depends on the size of the initial deficit, and the difference between the rates of growth of revenues and expenditures. On the other hand, if the growth rate of expenditures exceeds that of revenues then a site levy would never be adequate.

Similarly, if revenues from the land tax initially exceed expenditure requirements, and if the rate of growth of the tax base exceeds that of expenditures, adequacy is assured. However, if expenditure needs are growing more rapidly than site values, fiscal deficits will eventually materialize. The length of time before this occurs again depends on the size of the initial surplus, and on the

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28 Personal property is excluded in our discussion. Several states at present tax this type of property. However, throughout this study we assume that personal property would be exempt from taxation.
respective rates of growth of the two components.

This approach to answering the question of revenue adequacy forms the foundation of the theoretical and empirical analysis which follows. The next chapter develops revenue and expenditure relations needed to answer the adequacy question.

CONCLUSION

This chapter has shown that the site value levy is neutral with regard to the following decisions: (1) operating, (2) rate of replacement of structure, (3) renovation, and (4) new construction. Administratively the tax seems feasible. For the most part those who doubt the value of a shift to site value taxation do so because of the revenue adequacy and equity aspects of such a tax. Most economists believe that insufficient revenues would be generated without "excessive" tax rates. At the same time, they are concerned about the effect of the tax on the present value of real property. The import of this complaint is diminished by taxing only increments to land value.\(^9\)

Generally, the adequacy of site value taxation has

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\(^9\)Netzer [67] discusses this proposal at some length. He cites several American proposals to tax the increment only at the time of transfer. This proposal, while administratively simple, has the drawback of encouraging owners to postpone realization.
been analyzed with national or state boundaries in mind. Goldsmith, Heilbrun, and Netzer reject site value taxation on grounds of inadequacy through use of aggregate United States data. Gaffney presents data from one local community where substitution of a land tax, increasing tax rates by 40 percent and exempting improvements, would replace the current property tax yield. Gaffney's study suggests that a shift to land taxation is likely to be more successful in urban communities where population concentrations generate high demands for space and result in high land values. With the exception of Gaffney's [33] study of Milwaukee, existing studies suffer the defect of requiring site levies to replace revenues all across the nation. There may exist particular characteristics necessary for the levy to generate sufficient revenues; however, the nation as a whole does not appear to possess these characteristics.

All of these studies rely on static analysis. They determine the land tax rate necessary for a tax on site values to replace the revenues from the present real estate tax. We assume that some fraction of public output is to be financed by a site levy with a given tax rate. We then ask: If this level of expenditures exceeds (equals) tax revenues, what community characteristics are necessary for revenues to become (remain) adequate or for tax rates to
be reduced over time? The uniqueness of this study is not found in the idea of site value taxation, but in the approach used to answer questions of revenue sufficiency.
CHAPTER III

THE REVENUE AND EXPENDITURE FUNCTIONS

INTRODUCTION

In this chapter the revenue and expenditure functions are developed and described in considerable detail. For the revenue function, we assume that local taxes are derived entirely from the real rent on land.¹ We consider local expenditures to be related to income and population. It may appear that specific urban services are functionally related to any number of factors. We will argue, however, that total urban expenditures are related principally to income and population.²

THE EFFECTS OF POPULATION ON EXPENDITURES

Three population factors can be expected to exert

¹Throughout this study we assume that available land is fixed in supply. However, the implications of more or less land resources are discussed in various sections of the study.

²The discussion in this section is in per capita terms, while the formal relationship developed in the model is in total terms.
considerable influence over public expenditures for goods and services; (1) the number of people, or city size, (2) density, and (3) the rate of change of population. Since geographical area is assumed constant any change in population will be directly reflected in density. However, this fact does not eliminate the necessity of separately analyzing the effects of city size and density on expenditures.

City population growth may lead to lower per capita expenditures if economies of scale exist. Since there are overhead expenses in the provision of water and sewage facilities, and fire and police protection, which do not increase in proportion to the population served, economies of scale would be expected to exist. As overhead costs are spread over a larger number of people, per unit costs of these and general administrative services will decline. However, as overhead costs are spread more widely with population growth, the rate of fall in per capita cost declines. Consequently, the advantages resulting from population growth decline.

Alternatively, city size may be directly related to per capita expenditures for several reasons. First, city size--beyond a certain point--may lead to diseconomies of scale in the same manner that diseconomies are assumed to occur in business firms. Second, the community may face
capacity constraints in the short run. For example, in the case of water supply, the city has to seek supplies further afield so that water must be piped longer distances to the city. Finally, diseconomies of scale may exist in local government administration. Local administrations may become less efficient if graft, corruption, and a myriad of other problems are positively related to bureaucratic size.³

The second population factor, density, provides a measure of average population concentration. Per capita expenditures in the community may be directly related to population density as crowding makes the provision of various public services more difficult, and therefore more

³See Neutze [70], for a more extended theoretical discussion of these issues. Available empirical research is in conflict regarding the relationship between population and per capita expenditures. There are numerous instances in which population is positively related to per capita expenditures for a particular function. For example, Brazer [18], finds that in cities with population greater than 25,000, per capita police expenditures are positively associated with population, while Vieg [94] found per capita operating expenditures positively related with population in 303 California cities in 1957. For similar results see Wood [98], and Shapiro [84]. On the other hand, many studies have failed to find a significantly consistent relationship between the two variables, and the writers have concluded that neither economies nor diseconomies in the public sector exist at local levels. Gabler [34], finds that generally there is no consistent relationship between city size and functional categories analyzed. Further evidence of the same nature can be found in Scott and Feder [83], Hirsch [44], Schmandt and Stephens [82], Bollens [15], and Sacks and Hellmuth [81]. For a good review of this voluminous literature see Wilensky [97], and Barlow [9].
expensive. For example, a positive association between density and per capita fire expenditures would be expected due to: (1) increased fire hazards associated with higher population densities (greater congestion), (2) larger number of inaccessible buildings, and (3) fewer fire breaks. Similarly, greater density would be directly related to per capita police expenditures since there is a tendency for crowding to breed crime, due to the irritation and discomfort that emerge from forced interaction. Alternatively, concentration of people may permit a more intensive utilization of existing facilities thereby lowering per unit costs. Of course, this result is contingent upon service quality remaining constant.\(^4\)

The rate of population growth is expected to have its major effect on the decision making process of local government officials. If the fiscal planning of municipalities is inadequate, there will be inadequate governmental response to the public service needs of newcomers to the

\(^4\)For a very extensive theoretical discussion of the effects of density and area served on the cost functions of public services see Shoup [85], especially Chapter 5. Per capita fire expenditures were found to be positively and significantly related to density by Brazer [18], Wood [98], Fisher [30], and Sacks [80]. However, Shapiro [84] was unable to detect a significant relationship. Brazer [18] was able to detect a positive relationship between density and per capita police expenditures, while Wood [98], Shapiro [84], Fisher [30], and Sacks [80] could not.
community. If budgeting for increased demands for public services tends to lag these increases, then there will probably be a negative relationship between per capita expenditures and the rate of population change.  

THE EFFECT OF INCOME ON EXPENDITURES

As per capita income in the economy rises through time, there are several reasons why expenditures by the local public sector can be expected to rise. As per capita income rises the demands for most goods and services also rise. The extent to which the demand for state and local government services will grow relative to income is dependent on the income-elasticity of demand for public services.

Baumol [11], has argued that the majority of

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Brazer [18] found that the rate of growth of population was generally not significant in explaining municipal expenditure levels. However, in the more rapidly growing cities of California he found that the rate of population increase was significant; in all such cases the relationship was inverse. Studies by Scott and Feder [83] and Elsner and Sosnick [28], also found the relationship to be inverse. However, Spangler [90], using state and local data, found significant and positive relationships between the rate of population growth from 1950-1960 and per capita expenditures in several categories. His rationale for this relation emphasizes the disruptive effects of rapid population growth which prevent jurisdictions from operating at minimum points on long-run average cost curves. Given the results of other studies, Spangler's results may be a direct consequence of using aggregated data (state and local). The majority of other studies deal with city expenditures exclusively.
services provided by local governments are services for which increases in productivity are difficult to achieve. The production process is highly labor intensive so that productivity growth is lower than in the private sector. The net result is that unit costs in the public sector rise relative to those in the private sector. If demands for public services are relatively "tax price" inelastic or relatively insensitive to costs, then, ceteris paribus, we would expect that over time local public spending will increase relatively faster than does per capita income. Empirical confirmation of this hypothesis can be found in Bradford, Malt and Oates [17]. Consequently, the larger is the income-elasticity, the lower is the rate of growth of productivity in the public sector, and the less "tax price" elastic is the demand for government activity, the more likely it is that a rising share of national income will be devoted to the local public sector. The overall effect of per capita income on the level of expenditures is an empirical question.

All existing studies have been based on cross-section data, and have reported the pattern of public spending for

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6 This is more likely at the state and local level because of the large defense component in federal expenditures. See McLure [56] for an extended analysis of the implications of these and other issues for intergovernmental financing.
given points in time. In the past the choice has been limited by the general non-availability of data. Since income and other socioeconomic data have generally not been available on a regular basis, determinent studies of necessity have been cross-section in nature. Our study is directed toward the nature of public spending in given communities over time. Thus time series data will be utilized.

SPECIFICATION OF THE EXPENDITURE FUNCTION

The simple model presented below provides the theoretical foundations for the local aggregate expenditure function used throughout the remainder of this study. This model parallels a model introduced by Goodman and Bergstrom [37], which drew heavily from Baumol [11], and Litvack and Oates [52]. The model generates an aggregate local public expenditure relationship in which expenditure elasticities are constant with respect to income and population.

The composition of services provided by the public sector is assumed constant; only the scale may change. Each member of the community has identical per capita effective demand for this output. We assume that decision making is such that total output demanded is exactly supplied. 7

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7 This formulation is similar to that used by Rothenberg [79]; however, his resource expenditure functions differ considerably as he concentrates on the central city - suburb relationship.
This formulation distinguishes between the output of the public sector and the inputs necessary to produce this output. The distinction permits comparison between ultimately desired services and (1) those commodities which are the result of public production, and (2) the numerous inputs that enter the public "production" function. For example, consider police services. It is assumed that individuals in the community seek a particular level of security, rather than a given number of policemen, minimum response time to calls for help, suspect apprehension rates, etc.

If all public services provided by local communities were "pure public" goods, the consumption of the service by any consumer would not reduce its usefulness to others. If this were the case, the major effect of a change in population would be a reduction in per capita cost.\(^8\) A secondary effect of population size on the level of public services would be the tendency for residents to vote for more public services due to the lower per capita costs.

However, for locally supplied public services the level of utility derived by the individual depends not only on the level of output of the service, but also on the size of the group consuming the good. Impure public goods are

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\(^8\)This assumes a fixed geographical service area. The effect of different service areas on public expenditures is discussed extensively in Shoup [85].
usually characterized by costs of congestion; after some point the addition of yet another person to the group reduces the level of consumption of the good for the other members.

The utility function of the $i^{th}$ individual can be expressed as,

$$U_i[X_i,E^*], \text{ where } E^* = N^{-\gamma}E,$$  \hspace{1cm} (1)

and $X_i$ is the quantity of private goods he consumes. The usefulness of any public service ($E^*$) is related to ($N$) the number of people sharing the service and, ($E$) the quantity of the service supplied. For example, the level of protection ($E^*$) provided for each individual by a particular police force ($E$) depends on the number of persons ($N$) for whom surveillance and other police services must be provided. If $\gamma = 0$, the public service is a pure public good. When $\gamma > 0$, the individual's satisfaction from the public good is diminished as new residents move into the community.

Suppose the unit cost of the private good is $p_x$ and the unit cost of the public service is $p_e$. Assume further, that the total cost of all public facilities and total income is shared equally by all residents of the community. In this case, the demand function for $E$ for any individual is found by maximizing equation (1) subject to,
\[ p_x X_i + \frac{p_e}{N} E \leq \left( \frac{Y}{N} \right)_i \]  

(2)

where \((Y/N)_i\) is the income of the \(i^{th}\) individual or per capita income. Substituting \(N^Y E^*\) for \(E\) from equation (1), permits equation (2) to be written as

\[ p_x X_i + p_e^{NY-1}E^* \leq (Y/N)_i. \]  

(3)

Thus determination of the demand for \(E^*\) is formally equivalent to finding an ordinary demand function when the price is \(p_e^{NY-1}\).

Suppose there are constant per capita income and price elasticities \(\psi\) and \(\alpha\), respectively, for the commodity \(E^*\). Assume that the units of the public service are measured in such a way that the unit cost of \(E\), \(p_e\), initially equals 1. Further, assume that relative prices between public and private goods remain constant.\(^9\) Then the demand function for \(E^*\) would be of the form

---

\(^9\)This is a simplifying assumption. If \(p_e/p_x\) is not constant then equation (5) would include an additional term \(p_e^a\). If the demand for public services is relatively price inelastic, which to the author seems likely, this assumption does not represent a severe restriction on the model. Goodman and Bergstrom [37] estimate the tax price elasticity to be approximately \(-.20\) for 826 municipalities with 1960 populations between 10,000 and 150,000, located in 10 states.
\[ A[N^{\gamma-1}]^\alpha \left[ \frac{Y}{N} \right]^\psi. \]  

(4)

The quantity of \( E \) demanded is \( N^\gamma \) times the quantity of \( E^* \) demanded. Consequently, the demand for \( E \) is

\[ N^\gamma A[N^{\gamma-1}]^\alpha \left[ \frac{Y}{N} \right]^\psi = AN^\psi + \alpha(\gamma - 1) \left[ \frac{Y}{N} \right]^\psi. \]  

(5)

We can now specify the resource expenditure function for the community. Total public expenditures (\( E \)) are related to total population (\( N \)) and per capita income (\( Y/N \)) in the following manner,

\[ E = AN^\phi \left[ \frac{Y}{N} \right]^\psi, \]  

(6)

where \( A \) enters as a scale factor and \( \phi \) and \( \psi \) are population and per capita income elasticities respectively. Thus, the coefficient, \( \phi \), for the elasticity of expenditures with respect to population would be interpreted to be \( \gamma + \alpha(\gamma - 1) \). If the demand for public output is relatively insensitive to costs (small \( \alpha \)), then we would expect \( \phi \) to closely approximate the crowding parameter \( \gamma \).

THE REVENUE FUNCTION

A tax revenue scheme that taxes land values or incremental land values must ultimately tax the real rent
on land. Consequently, the tax function relates total tax revenue \( T \) to aggregate rent \( R \) times the tax rate \( g \). That is,

\[
T = gR, \quad 0 \leq g \leq 1.
\]  

(7)

At any point in time the amount of deficit (+) or surplus (-) a community experience is equal to

\[
D = E - T
\]  

(8)

and the budget is balanced when \( D = 0 \). We now turn to an analysis of revenue adequacy of a site value tax in a Ricardian economy.
CHAPTER IV

REVENUE ADEQUACY IN A
RICARDIAN ECONOMY

INTRODUCTION

In this chapter we analyze the revenue adequacy of a site value tax in a Ricardian model of economic growth. Existing analyses have attempted to answer the question as to whether or not site value taxation is capable of generating the same revenues as are received from real estate taxes at present. Our model allows analysis of revenue adequacy over time, by introducing revenue and expenditure relations developed in Chapter III into the Ricardian system of economic growth. Finally, results of the Ricardian model are summarized and limitations of the model are presented.

DESCRIPTION OF THE MODEL

In the Ricardian system economic growth primarily results from the activities of capitalists. Economic growth must eventually wither away as the process of
transforming profits into capital ends. Due to decreasing marginal returns of new capital (and labor) applied to the same quantity of land, or to less productive land, rent increases over time; consequently the rate of profit declines.\(^1\)

The Ricardian system can be stated formally in several equations. Assume output is a function of three factors; land, labor and capital. As land is fixed in supply we will exclude it from the explicit production function and output will be a function of labor and capital. For simplicity,

\[ Y = f(N); \quad (1) \]

with the following properties:

\[ f'(N) > 0 \quad (1a) \]

\[ f''(N) < 0 \quad (1b) \]

\[ f'''(N) \leq 0 \quad (1c) \]

where \( Y \) denotes output (income) and \( N \) is the number of doses of capital-and-labor. To avoid the necessity of handling two variables at one and the same time, capital and labor are assumed to be applied in equal doses. The

\(^1\)Pasinetti [74], p. 81.
average product curves in the graphs reflect the fact that labor and capital are increased in a given ratio; for every unit of labor applied to the land, capital of x units is applied. Changing this ratio would result in a shift of the average product curves. If the ratio were reduced (less capital per unit of labor) average produce would shift to the left and vice versa for an increase in the ratio. This presumes constant proportions of the two variable inputs, which inevitably places an additional restriction on the model.

Real values equal money values as all output is homogeneous. Marginal product is assumed positive [inequality (1a)]. The principle of diminishing marginal returns at a constant or increasing rate is assumed.

\(^2\)To avoid excessive notation we are using "N" to represent both doses of capital-and-labor and population. Labor is assumed to consist of the population and for every unit of labor applied to the land, capital of x units is applied. Consequently, population and the units of capital-and-labor are numerically the same. That is for every dose of labor (person) some constant multiple of capital is applied, leaving capital-and-labor doses and population equal numerically. Whenever an important distinction between population and capital-and-labor doses is necessary this distinction will be noted in the text.

\(^3\)By assuming one homogeneous output we exclude valuation considerations. Our approach is similar to that of Kaldor [47] and Barkai [8]. Pasinetti [74], presents a Ricardian model with two commodities in which he considers various valuation questions. For our purposes value does not introduce any important problems nor any relevant questions and is therefore excluded from the analysis.
The total wage bill ($W$), total rent ($R$), and total profits ($\pi$), are defined as follows:

\[
W = Nw
\]

(2)

\[
R = f(N) - Nf'(N)
\]

(3)

\[
\pi = N[f'(N) - w],
\]

(4)

where $w$ is the real wage rate. The magnitude of rent is dependent solely on the size of the gap between average product and marginal product. Total profits consist of a residual equal to total product-less-rent minus the wage bill.

Capital is considered to be circulating capital, and therefore consists entirely of the wage bill.\textsuperscript{5} Symbolically,

\[
K = W.
\]

(5)

Thus far, seven variables have appeared: $Y$, $X$, $W$, $R$, $\pi$, $K$, $w$. However, there are only five equations. Two additional

\textsuperscript{4}The assumption of marginal returns diminishing at an increasing rate characterizes typical production functions used throughout microeconomic theory. In his examples, Ricardo assumed that returns diminished at a constant rate.

\textsuperscript{5}See Pasinetti [74], p. 125.
equations are needed to make the system determinate. In the equilibrium which Ricardo considered natural the following two data would prevail:

\[ w = \bar{w} > 0 \]  \hspace{1cm} (6)

and

\[ K = \bar{K} \]  \hspace{1cm} (7)

where \( \bar{K} \) = given stock of capital at the beginning of the year,\(^6\) and \( \bar{w} \) is the natural real wage rate.\(^7\) The system is determinate. The solution to the system of equations (1) - (7) is the natural equilibrium of the Ricardian system.

The linear system is diagrammed at population \( N_1 \) in Figure IV-1. Area \( ON_1DC \) is total product, area \( BHDC \) is total rent on land and \( BC \) is the per capita rent on land. The Supply curve of labor is assumed to be infinitely elastic at the subsistence wage rate \( OW \) and profits are defined as the total product minus rent minus wages and they are equal to area \( WKHB \). In all graphs after Figure IV-1

\(^6\)See Pasinetti [74], p. 127. The production process requires exactly one year to complete, and capital (all circulating) takes one year to be re-integrated.

\(^7\)The natural real wage rate is defined as that wage-rate which keeps population constant.
Rents, Profits, Wages, and Expenditures ($\psi=0$, $\phi=1$) in The Ricardian System
subsistence wages will be considered to be the X axis such that point W in Figure IV-1 represents point 0 in subsequent figures (this simplifies the graph considerably).

We now turn to an analysis of revenue adequacy when expenditures are related to income and community size by introducing the expenditure equation developed in Chapter III. We will end this section with a discussion of limitations of this analysis, and develop the need for a more detailed specification of the economy.

EXPENDITURES A FUNCTION OF POPULATION

In this section we assume that expenditures are related solely to population, ($\psi = 0$), or

$$E = A[N]^\phi.$$  \hspace{1cm} (8)

Initially, assume that $\phi = 1$. Total expenditures are a linear function of population and per capita expenditures are a constant level A. At any point in time tax revenues will cover or exceed expenditures if

$$AN \leq gR$$ \hspace{1cm} (9)

or
A \leq \frac{gR}{N}.

(10)

If the tax rate \( g = 1 \), then the community can cover expenditures or run a surplus if per capita rent is greater than or equal to per capita expenditures demands. This is shown in Figure IV-1.

For exposition purposes the origin for expenditures has been raised to point B. Per capita expenditures \( A = BF \) and total expenditures \( E = (ON_1)(BF) \). This facilitates the comparison of aggregate rent BHDC and aggregate expenditures BHGF as well as permitting the respective per capita comparisons BC and BF. Thus if at any point in time \( BC > BF \) then the community generates a surplus and the budget can be balanced by setting \( g = BF/BC < 1 \), generating just enough revenue to cover expenditures.

However, consider the case where \( A > \frac{R}{N} \), \( (BF > BC) \); per capita expenditures exceed per capita rents. The result is clear in the situation depicted in Figure IV-2. With community size originally \( N_1 \), per capita rent is BC; per capita expenditures are BF and exceed per capita rent by CF. As the community grows, aggregate rent and rental share expand so that at population \( N_2 \), per capita rent is B'C' exceeding per capita expenditures B'F'(=BF), by F'C'.

However, consider the case depicted in Figure IV-2 when per capita expenditures are equal to OF'. Total
System in which both (1) rents are sufficient to eventually finance the public sector and, (2) expenditure deficits are inevitable.
expenditures exceed total rents even when profits are driven to their minimum. When the community expands as far as it can (population = ON), per capita expenditures (OF') still exceed per capita rents (OG). The following section will set out the conditions required for aggregate rent to finance aggregate public expenditures within the Ricardian framework.

We continue to assume that $\phi = 1$. With total expenditures linearly related to population and total revenue equal to some fraction, $g$, of aggregate rent. Aggregate public deficits can be stated as

$$D = AN - gR.$$  \hspace{2cm} (11)

Initially, if the public sector is operating at a deficit ($D > 0$), then a necessary condition for restoring a balanced budget is

$$\frac{dD}{dN} < 0,$$  \hspace{2cm} (12a)

which is

$$\frac{dD}{dN} = A + g f''(N) N < 0,$$  \hspace{2cm} (12b)

which reduces to,

$$A < -gf''(N) N.$$  \hspace{2cm} (13)
Per capita expenditures, A, are constant. Therefore, with a given tax rate (g), the budget will be more easily balanced when \(|f''(N)|\) is larger. That is, the more rapidly returns to the variable factor decline, the more likely a tax on aggregate rent will provide sufficient revenue.

Figure IV-3 incorporates the same expenditure relationship as Figure IV-2, but the production function is characterized by returns to the variable factor decreasing at an increasing rate. At the initial population \(N_0\), per capita expenditures OF exceed per capita rents BC. As population increases to \(N_1\), per capita rents increase and eventually exceed per capita spending (DG > OF). When population reaches \(N_1\) a tax rate of less than unity (OF/DG) can be used to balance the budget.

Thus far it has been assumed that \(\phi = 1\). Next consider briefly the problem when \(\phi \neq 1\). In this event equation (13) becomes,

\[
\phi AN^{\phi - 1} + g f''(N) N < 0,
\]

which reduces to

\[
A < - \frac{g}{\phi} f''(N) N^{2-\phi}.
\]

If \(\phi > 1\), then the right hand side of equation (15) is smaller than the right hand side of equation (13).
Expenditures and rents when returns to the variable factor decline at an increasing rate
Consequently, equation (15) is harder to satisfy in that a greater degree of diminishing returns to the variable factor is required. The result is that the expenditure line curves up and to the right, thus narrowing the population range in which a balanced budget or a surplus will be achieved. Alternatively, if $\phi < 1$, the opposite occurs because the right hand side of equation (15) is always larger than the right hand side of equation (13). As the community grows, per capita expenditures decline and the budget is more easily balanced.

EXPENDITURES A FUNCTION OF PER CAPITA INCOME

This section considers expenditures solely related to per capita income, ($\phi = 0$), or

$$E = A\left(\frac{Y}{N}\right)^\psi.$$  \hfill (16)

For this section of the analysis we will further assume that $\psi = 1$, so that expenditures are linearly related to per capita income. Revenues will equal or exceed expenditures whenever,

$$gR \geq A\left(\frac{Y}{N}\right).$$  \hfill (17)
Dividing equation (17) and equation (16) by $Y$, and assuming $g = 1$ yields

$$\frac{R}{Y} \geq \frac{A}{N} = \frac{E}{Y}. \quad (18)$$

Thus whenever the share of income going to land is greater than the share devoted to public output government can finance its services with a tax on rent. Aggregate public deficits can be written as,

$$D = A[\frac{Y}{N}] - gR. \quad (19)$$

Initially, if the public sector is operating at a deficit ($D > 0$), a necessary condition for the budget to eventually be balanced is

$$\frac{dD}{dN} < 0, \quad (20)$$

or

$$\frac{dD}{dN} = A[\frac{Nf'(N) - f(N)}{N^2}] + gf''(N) \quad N < 0. \quad (21)$$

As the community grows, $dN > 0$, the deficit will be smaller when,

$$A[\frac{Nf'(N) - f(N)}{N^2}] < -gf''(N) \quad N. \quad (22)$$
Rewriting equation (22) results in

$$\frac{A}{N^2} \left[ \frac{Nf'(N)}{f(N)} - 1 \right] < - \frac{gf''(N)N}{f(N)}. \quad (23)$$

The bracket term on the left side of equation (23) is always negative because labor's share of income is always less than one, while the right hand side is always positive because $f''(N) < 0$, by assumption. As the economy expands (population increases), due to diminishing marginal returns to labor and the fixed land resource, per capita income declines. Total expenditures, $E$, also decline because $A$ is a fixed parameter. At the same time aggregate rent, $R$, increases because

$$\frac{dR}{dN} = -f''(N)N > 0. \quad (24)$$

Consequently the aggregate deficit will fall.

Up to this point we have assumed that $\psi = 1$. Consider briefly the model when $\psi \neq 1$. Again we assume the public sector is operating at a deficit. Consider the necessary conditions for the budget to eventually be balanced. When $\psi \neq 1$, equation (23) becomes

$$\left\{ \psi \left[ \frac{f(N)}{N} \right]^{\psi - 1} \right\} \frac{A}{N^2} \left[ \frac{Nf'(N)}{f(N)} - 1 \right]$$

$$< - \frac{gf''(N)N}{f(N)}. \quad (25)$$
Equation (25) is exactly the same as equation (23) except for the term in the brackets, \{\}. When \(\psi > 1\) the bracketed term, \{\} > 1. Therefore for any population size, the larger is \(\psi\), the smaller will be the deficit as the community grows. Alternatively, if \(\psi < 1\), the bracketed term is less than one and the opposite result occurs. Again, this is the result of falling per capita income when population rises.

The fact that a larger per capita income elasticity is beneficial to revenue adequacy may seem perverse. This is due to two factors. First, capital is only permitted to grow as rapidly as does population (capital-and-labor are applied to land in fixed proportions). Because of a fixed land resource, per capita income falls as population rises. However, if capital were permitted to grow more rapidly than labor, per capita income could rise, and a larger per capita income elasticity would hinder the attainment of adequacy. Second, we have not introduced technical change into this model. Technical change would tend to increase per capita income as the economy grew and therefore, a larger per capita income elasticity would work against the attainment of a balanced budget.

Fixed capital-and-labor doses, and the absence of technical progress are restrictions to this model.
Therefore, our conclusions as to the effect of different per capita income elasticities on revenue adequacy may require revision. The incorporation of variable capital to labor ratios and the introduction of technological progress will tend to yield the more intuitively plausible result that a larger per capita income elasticity will hinder attainment and revenue adequacy.

EXPENDITURES A FUNCTION OF BOTH POPULATION AND INCOME

In this section assume that expenditures are related to both income and population ($\psi > 0$ and $\psi > 0$). We assume that at a given point in time some fraction of expenditures can be financed by a tax on rent or land value. For this rent levy to continue to support this proportion of expenditures the rate of growth of rents must equal or exceed the rate of growth of expenditures. The rate of growth of rents, $r$, is,

$$
 r = \frac{1}{R} \frac{dR}{dt} = \frac{f'(N) \frac{dN}{dt} - [Nf''(N)+f'(N)] \frac{dN}{dt}}{f(N)-Nf'(N)},
$$

or

$$
 r = \left[ \frac{-Nf''(N)}{f(N)} \right] n, \text{ where } n = \frac{1}{N} \frac{dN}{dt}.
$$

(26)  

(27)
The rate of growth of expenditures, \( \varepsilon \), is

\[
\varepsilon = \frac{1}{E} \frac{dE}{dt} = (\phi - \psi)n + \psi y,
\]

where \( y \) is the rate of growth of aggregate income;

\[
y = \frac{1}{Y} \frac{dY}{dt} = \left[ \frac{Nf'(N)}{f(N)} \right] n.
\]

Adequacy requires that

\[
r \geq \varepsilon.
\]

Substituting equations (27), (28), and (29) into equation (30) yields,

\[
\left\{ \frac{-Nf''(N)}{N} \right\} n \geq (\phi - \psi)n + \psi \left\{ \frac{Nf'(N)}{f(N)} \right\} n
\]

dividing both sides by \( n \) and rewriting results in

\[
\frac{-Nf''(N)}{N} \left[ 1 - \frac{Nf'(N)}{f(N)} \right] \geq \phi - \psi \left[ 1 - \frac{Nf'(N)}{f(N)} \right].
\]

The results of this section, as expected, are similar to those of the previous sections. Deficits will be smaller over time, ceteris paribus; (1) the faster returns to the variable factor decline, (2) the smaller is the population expenditure elasticity, \( \phi \), and (3) the
larger is the per capita income elasticity, \( \psi \). \(^8\) The effect of changes in relative factor shares on revenue adequacy can not be determined within the context of this model.

**SUMMARY AND LIMITATIONS OF THE RICARDIAN MODEL**

This section is devoted to a brief summary of the results of the Ricardian analysis and a discussion of the limitations of this model. The latter part develops the reasons for the neo-classical chapter which follows.

Use of the Ricardian model has shown that when revenue needs are primarily dependent upon the population and the fisc initially is operating at a deficit, for a tax on site rent to permit attainment of balance per capita rents must be increasing over time. The extent of this increase will determine the length of time or population required for revenues to equal needs. In addition, the production function must be subject to decreasing returns at a constant or increasing rate if the fisc is to be balanced eventually.

If the public sector is characterized by diseconomies of scale adequacy conditions are most difficult to achieve. The greater the share of public output devoted to pure public goods, the more easily the budget can be balanced. As the community grows, the total cost of public services are

\(^8\)See the previous section.
spread over more individuals.

When the economy's public service demand is primarily dependent upon income, deficits will not occur if rental share exceeds the share of income devoted to public output. Since not all income is allocated to fiscal output, total rent must eventually exceed local expenditures if returns to the variable factor decline at a constant or at an increasing rate. How large the community must grow before rents are sufficiently large depends on the proportion of output designated as public output and how rapidly returns to the variable factor decline.

The Ricardian model has two primary limitations. First, the rate at which returns to the variable factor decline is a crucial element to revenue adequacy (equation 32). This derivative is well defined mathematically and graphically; however, this concept has little to offer analytically or empirically. In this model, the more relevant elasticities of substitution between factors are not well defined. Second, the analysis suggests that rental share increases can occur only with population growth and rapidly diminishing returns to the variable factor. This is true for any given average product curve. However, by changing the capital-to-labor doses, we can shift the average product curve and consequently change
aggregate rent. Thus this model only permits growth in the variable factor to occur in fixed proportions.

The upshot of these limitations is that a specific three factor model is required so that the effects of factor substitutability and the rate of growth of capital on aggregate rents can be determined. This suggests that the appropriate question to ask is: What rate of growth of capital improvements (given that the rate of increase of the labor force and the degree of factor substitutability is known) will permit a tax on site rents to support fiscal activities?
CHAPTER V

REVENUE ADEQUACY IN A
NEO-CLASSICAL ECONOMY

INTRODUCTION

We begin with an extensive description of a three factor production function introduced by Meade [58]. The characteristics of the production function are briefly analyzed. We then adapt this function to analyze the revenue adequacy question when public output is related to income and population. The public sector is introduced through the expenditure equation developed in Chapter III.

DESCRIPTION OF THE PRODUCTION FUNCTION

Consider a closed economy in which the net output \((Y)\) produced depends upon the amount of land \((L)\), labor \((N)\), and capital improvements \((K)\) at any point in time.\(^1\) We can write this relationship as a constant-returns-to-scale production function

\[ Y = f(L, N, K) \]

\(^1\)This Descriptive Section summarizes Appendix I-1 from Meade [58]. In this chapter "N" will refer to labor or population only, not capital-and-labor doses.
\[ Y = F[K, L, N]. \quad (1) \]

Growth in the economy results from changes in capital \((dK)\), labor \((dN)\), and land \((dL)\). For convenience, in this section we will assume that land can vary such that \(dL > 0\), however, this assumption will be dropped in several subsequent sections. We will write the marginal products of the factors \(K, L,\) and \(N\), as \(F_K, F_L,\) and \(F_N\) respectively, and the partial derivatives of the marginal products as \(F_{KK}, F_{KL},\) etc. Further,

\[ F_K > 0, \quad F_L > 0, \quad F_N > 0, \quad \text{and} \]

\[ F_{KL} = F_{LK}, \quad F_{NK} = F_{KN}, \quad F_{LN} = F_{NL}. \quad (2) \]

Moreover, the proportions of total output which accrue to capital, land, and labor are \(U, Z,\) and \(Q\) respectively, where \(U = F_KK/Y, \ Z = F_LL/Y\) and \(Q = F_NN/Y\). Because constant returns to scale are assumed, \(U + Z + Q = 1\). In what follows we will (a) develop definitions for \(F_{KK}, F_{LL},\) and \(F_{NN}\) and, (b) develop definitions for \(F_{KL}, F_{NL},\) and \(F_{NK}\) in terms of three elasticities of substitution \(\sigma_{LN}, \sigma_{KN},\) and \(\sigma_{KL}.\)

Differentiating equation (1) yields

---

\(^2\)Following Young's theorem; we assume that all cross partial derivatives are continuous. For additional information see Chiang [23], and Allen [1].
\[ \text{d}Y = \text{F}_K \text{d}K + \text{F}_L \text{d}L + \text{F}_N \text{d}N, \text{ for all } \text{d}K, \text{d}L, \text{d}N. \]  
(3)

Because the production function is linearly homogeneous,

\[ Y = \text{F}_K K + \text{F}_L L + \text{F}_N N. \]  
(4)

Differentiating equation (4) gives

\[ \text{d}Y = K[\text{F}_{KK} \text{d}K + \text{F}_{KL} \text{d}L + \text{F}_{KN} \text{d}N] \]
\[ + L[\text{F}_{LK} \text{d}K + \text{F}_{LL} \text{d}L + \text{F}_{LN} \text{d}N] \]
\[ + N[\text{F}_{NK} \text{d}K + \text{F}_{NL} \text{d}L + \text{F}_{NN} \text{d}N] \]
\[ + \text{F}_K \text{d}K + \text{F}_L \text{d}L + \text{F}_N \text{d}N, \text{ for all } \text{d}K, \text{d}L, \text{d}N. \]  
(5)

Subtracting equation (3) from equation (5), and rearranging terms results in

\[ 0 = \text{d}K[\text{F}_{KK} + \text{L} \text{F}_{LK} + \text{N} \text{F}_{NK}] \]
\[ + \text{d}L[\text{F}_{LK} + \text{L} \text{F}_{LL} + \text{N} \text{F}_{NL}] \]
\[ + \text{d}N[\text{F}_{NK} + \text{L} \text{F}_{LN} + \text{N} \text{F}_{NN}], \text{ for all } \text{d}K, \text{d}L, \text{d}N. \]  
(6)

Since this relationship is true for all values of \text{d}K, \text{d}L, and \text{d}N then all terms in each bracket must sum to zero and we know that,
$$F_{KK} = -\frac{L}{K} F_{KL} - \frac{N}{K} F_{KN}, \tag{7a}$$

$$F_{LL} = -\frac{K}{L} F_{KL} - \frac{N}{L} F_{LN}, \tag{7b}$$

$$F_{NN} = -\frac{K}{N} F_{KN} - \frac{L}{N} F_{LN}. \tag{7c}$$

The signs of equations (7a)-(7c) will be discussed below.

In an economy with three factors of production, the effect of changes in the supplies of capital, labor, and land upon the distribution of output between profits, wages, and rents depends upon three elasticities of substitution: (a) the elasticity of substitution between land and labor \([\sigma_{LN}]\); (b) that between land and capital \([\sigma_{KL}]\); and (c) that between capital and labor \([\sigma_{KN}]\).

The elasticity of substitution between any two factors A and B is defined as the percentage increase in the ratio of B to A which would be necessary to cause a one percent decline in the ratio of the price per unit of B to the price per unit of A when (1) the quantity of the third factor, C, is held constant, (2) the increase in the amount of B is just sufficient to maintain output constant in the face of a reduction in the amount of C, and (3) factors are paid their marginal products. Thus a formal definition of the elasticity of substitution between capital and land is
\( \sigma_{KL} = \frac{(dK/K) - (dL/L)}{(dF_K/F_K) - (dF_L/F_L)} \),

(8)

when \( dK, dL, dF_K \) and \( dF_L \) have the values they would when \( dY = dN = 0 \). Similar equations define \( \sigma_{KN} \) and \( \sigma_{LN} \). From equation (3), when output and the amount of labor are constant \( (dY = dN = 0) \) then

\[
dL = - \frac{F_K}{F_L} dK .
\]

(9)

Differentiating \( F_L \) when \( dN = 0 \) yields

\[
dF_L = F_{KL} dK + F_{LL} dL .
\]

(10)

Substituting equation (7b) and equation (9) into equation (10) we obtain

\[
dF_L = F_{KL} dK + [- \frac{K}{L} F_{KL} - \frac{N}{L} F_{LN}][- \frac{F_K}{F_L} dK].
\]

(11)

Algebraic manipulation of (11) results in

\[
dF_L = dK\{ F_{KL}(1 + \frac{KF_K}{LF_L}) + F_{LN} \frac{NF_K}{LF_L} \}.
\]

(12)

Similarly, differentiating \( F_K \) when \( dN = 0 \) yields

\[
dF_K = F_{KK} dK + F_{KL} dL ,
\]

(13)
and substituting equation (7a) and equation (9) into equation (13) and rearranging terms yields,

\[ dF_K = - dK \left[ F_{KL} \left( \frac{L}{K} + \frac{F_L}{F_K} \right) + \frac{N}{K} F_{KN} \right]. \quad (14) \]

Substituting the value of \( dL \) from (9) and \( dF_L \) from (12) and \( dF_K \) from (14) into equation (8) and rearranging terms and writing \( F_{KL} = UY, F_{NL} = ZY, \) and \( F_{KN} = QY \) yields,

\[ \frac{1}{\sigma_{KL}} = \frac{KN}{(U+Z)Y} \{ F_{KL}(2 + \frac{U}{Z} + \frac{Z}{U}) + F_{NL} \frac{NU}{KZ} + F_{KN} \frac{NZ}{LU} \}. \quad (15a) \]

By similar reasoning we conclude that

\[ \frac{1}{\sigma_{KN}} = \frac{KN}{(U+Q)Y} \{ F_{KN}(2 + \frac{U}{Q} + \frac{Q}{U}) + F_{NL} \frac{LU}{KQ} + F_{KL} \frac{LQ}{NU} \}; \quad (15b) \]

\[ \frac{1}{\sigma_{LN}} = \frac{LN}{(Q+Z)Y} \{ F_{LN}(2 + \frac{Q}{Z} + \frac{Z}{Q}) + F_{NK} \frac{KZ}{LQ} + F_{KL} \frac{KQ}{NZ} \}. \quad (15c) \]

These three equations provide relationships for the three terms \( F_{KL}, F_{NL}, \) and \( F_{KN} \). Solving (15a), (15b), and (15c) for these three terms results in,

\[ F_{KL} = \frac{Y}{2KL} \left\{ \frac{(U+Z)[(1-Z)(1-U)+UZ]}{\sigma_{KL}} - \frac{Z(1-Z)(1-2U)}{\sigma_{NK}} \right. \]

\[ - \frac{U(1-U)(1-2Z)}{\sigma_{NL}} \} ; \quad (16a) \]
\[ F_{KN} = \frac{Y}{2KN} \left\{ \frac{(U+Q)[(1-Q)(1-U)+UQ]}{\sigma_{KN}} - \frac{Q(1-Q)(1-2U)}{\sigma_{KL}} \right\} \]

\[ - \frac{U(1-U)(1-2Q)}{\sigma_{NL}} \right\}; \quad (16b) \]

\[ F_{LN} = \frac{Y}{2NL} \left\{ \frac{(Z+Q)[(1-Q)(1-Z)+QZ]}{\sigma_{NL}} - \frac{Q(1-Q)(1-2Z)}{\sigma_{LK}} \right\} \]

\[ - \frac{A(1-Z)(1-2Q)}{\sigma_{KN}} \right\}. \quad (16c) \]

**PRODUCTION FUNCTION CHARACTERISTICS**

Since these equations are extremely complex, we will at various times throughout what follows assume that \( \sigma_{KN} = \sigma_{KL} = \sigma_{NL} = \sigma \). This special case simplifies the analysis considerably, but represents an additional restriction on the model. The assumption that all three elasticities of substitution are equal is unrealistic, especially when you consider spacial and fixity characteristics of land vis-a-vis labor and capital. Nevertheless, for our purposes this assumption will be utilized for clarification in several sections below.

We are now in a position to evaluate the signs of \( F_{KK}, F_{LL}, \text{ and } F_{NN} \). Substituting equations (16a)-(16c) into equations (7a)-(7c) where appropriate, and rearranging terms
results in,

\[
F_{KK} = - \frac{Y}{2K^2} \left\{ \frac{2U[U(Z-1)+(1-Z)]}{\sigma_{KN}} + \frac{2U[U(U-1)+(1-U)]}{\sigma_{KL}} \right. \\
- \frac{2U^2(1-U)}{\sigma_{NL}} \right\}; \\
\quad (17a)
\]

\[
F_{LL} = - \frac{Y}{2L^2} \left\{ \frac{2Z(Q(Z-1)+(1-Z))}{\sigma_{KL}} + \frac{2Z[U(Z-1)+(1-Z)]}{\sigma_{NL}} \right. \\
- \frac{2Z^2(1-Z)}{\sigma_{NK}} \right\}; \\
\quad (17b)
\]

\[
F_{NN} = - \frac{Y}{2N^2} \left\{ \frac{2Z[Q(Z-1)+(1-Z)]}{\sigma_{KN}} + \frac{2Z[U(Z-1)+(1-Z)]}{\sigma_{NL}} \right. \\
- \frac{2Z^2(1-Z)}{\sigma_{KL}} \right\}. \\
\quad (17c)
\]

The more difficult is substitution between any two factors, the greater will be the impact on output when additional units of a third factor are supplied. If labor and land are poor substitutes for each other, the third term in the brackets, {}, of equation (17a) becomes very large, making the bracketed term negative and \(F_{KK} > 0\). Thus when the elasticity of substitution between any two factors is small, marginal productivity of the third factor may be
increasing. The conditions necessary for the production function to exhibit increasing or decreasing returns to any factor are summarized in Table V-1.

**TABLE V-1**

**CONDITIONS FOR MARGINAL PRODUCTS TO EXHIBIT INCREASING OR DECREASING RETURNS**

<table>
<thead>
<tr>
<th>$F_{ii}$</th>
<th>$F_{ii} &gt; 0$</th>
<th>$F_{ii} &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{KK}$</td>
<td>$\sigma_{NL} \to 0$</td>
<td>$\sigma_{NK} \to 0$, or $\sigma_{LK} \to 0$, or $\sigma_{NL} \to \infty$</td>
</tr>
<tr>
<td>$F_{LL}$</td>
<td>$\sigma_{NK} \to 0$</td>
<td>$\sigma_{LK} \to 0$, or $\sigma_{LN} \to 0$, or $\sigma_{NK} \to \infty$</td>
</tr>
<tr>
<td>$F_{NN}$</td>
<td>$\sigma_{LK} \to 0$</td>
<td>$\sigma_{NK} \to 0$, or $\sigma_{NL} \to 0$, or $\sigma_{LK} \to \infty$</td>
</tr>
</tbody>
</table>

Note: The arrow indicates that a particular parameter approaches zero or infinity. For example, $\sigma_{NL} \to 0$ would read as follows: the elasticity of substitution of labor for land approaches zero, or becomes small.

Consider the special case of $\sigma_{KN} = \sigma_{LN} = \sigma_{KL} = \sigma$. Equations (17a)-(17c) reduce to,
\[ F_{KK} = -\frac{Y}{2K^2} \left[ \frac{2U(1-U)}{\sigma} \right] < 0, \text{ since } 0 < U < 1; \quad (18a) \]

\[ F_{LL} = -\frac{Y}{2L^2} \left[ \frac{2Z(1-Z)}{\sigma} \right] < 0, \text{ since } 0 < Z < 1; \quad (18b) \]

\[ F_{NN} = -\frac{Y}{2N^2} \left[ \frac{2Q(1-Q)}{\sigma} \right] < 0, \text{ since } 0 < Q < 1. \quad (18c) \]

Consequently, when the elasticities of substitution between all factors are equal, all marginal products are subject to diminishing returns. We now turn to an analysis of revenue adequacy when expenditures are related to population.

EXPENDITURES A FUNCTION OF POPULATION

From the discussion in Chapter IV if the budget is to be balanced per capita rent must equal or exceed per capita expenditures. Throughout this section we will assume that \( \phi = 1 \), and available land \( (L) \) to the community is fixed in supply. The implications of relaxing the latter assumption are noted at the end of this section. Then budget balance requires that

\[ \frac{R}{N} = \frac{F_L L}{N} \geq A. \quad (19) \]

Since per capita expenditures are constant over time, if the community is initially operating at a deficit, budget
balance will require per capita rent to be growing. Because \( A \) is a constant parameter, whether or not inequality (19) is satisfied will depend on the time path of per capita rents as the community grows. Specifically,

\[
\frac{\partial [R/N]}{\partial N} = \frac{\partial (F_L L/N)}{\partial N} = \frac{LF_{LN} N - F_L L}{N^2}. \tag{20}
\]

Substituting equation (16c) for \( F_{LN} \) in (20) yields,

\[
\frac{\partial [R/N]}{\partial N} = \frac{L}{N} \left\{ \frac{Y}{2NL} \left[ \frac{(Z+Q)((1-Q)(1-Z)+QZ)}{\sigma_{NL}} \right] \frac{Q(1-Q)(1-2Z)}{\sigma_{LK}} \right. \\
- \left. \frac{Z(1-Z)(1-2Q)}{\sigma_{KN}} \right\} - \frac{F_{LL}}{N^2}. \tag{21}
\]

For simplicity assume that \( \sigma_{NK} = \sigma_{NL} = \sigma_{KL} = \sigma \). Therefore the partial derivative reduces to,

\[
\frac{\partial [R/N]}{\partial N} = \frac{1}{N^2} \left\{ \frac{YQZ}{\sigma} - F_{LL} \right\}
= \frac{1}{N^2} \left\{ \frac{QF_{LL}}{\sigma} - F_{LL} \right\}. \tag{22}
\]

Thus \( \frac{\partial [R/N]}{\partial N} \geq 0 \) when \( \frac{QF_{LL}}{\sigma} \geq F_{LL} \)

or when

\[
\frac{Q}{\sigma} \geq 1. \tag{23}
\]
Three cases emerge

(a) \(3\) when \(Q < \sigma\) \(\frac{\partial (R/N)}{\partial N} < 0\)
(b) when \(Q = \sigma\) \(\frac{\partial (R/N)}{\partial N} = 0\)
(c) when \(Q > \sigma\) \(\frac{\partial (R/N)}{\partial N} > 0\).

Consider the first case. If the elasticity of substitution is greater than labor's share of output, per capita rents will decline as more labor is applied to the existing stock of land and capital. However, when \(\sigma\) is less than labor's share the opposite occurs, and when \(\sigma = Q\), per capita rents are constant over time. In addition, capital can be added to the existing land, so that the time path of per capita rents with respect to capital improvements must be considered. That is,

\[
\frac{\partial (R/N)}{\partial K} = \frac{\partial (FL/L/N)}{\partial K} = \frac{L}{N} F_{LK}.
\]  

(24)

Equation (24) reduces to

\[
\frac{\partial (R/N)}{\partial N} = \frac{F_K Z}{N\sigma} > 0,
\]

(25)

since all components of equation (25) are positive. Thus

---

\(^3\) Case (a) is the most likely case. In a Cobb-Douglas production function where the factor exponents sum to unity, \(Q\) is always less than \(\sigma\) because \(0 < Q < 1\) while \(\sigma = 1\). Cases (b) and (c) can be exhibited in a C.E.S. production function.
an increase in population alone is likely to lead to a reduction in per capita rents (if \( Q < \sigma \)) while an increase in improvements results in an increase in per capita rents.

These results are illustrated in Figure V-1. Per capita rent declines and approaches zero as population increases for a given capital stock (we have assumed \( Q < \sigma \)). If buildings in the community are maintained at a given level, \( K = K_1 \) for example, then per capita rents will support public expenditures up to a population of \( N_1 \), beyond that level deficits are incurred. If property owners could be induced to increase existing capital stock, shifting the curve out to say \( K = K_2 \) then fiscal imbalance could be forestalled until the population has grown to \( N_2 \). If improvements are made sufficiently rapidly, the fiscal crisis could be forestalled forever.

Consider the system when, for example, population is \( N_1 \) and capital stock equals \( K_1 \). If the tax rate, \( g \), is unity, the public budget is balanced (point H, Figure V-1). In order for this tax to continue to finance this level of total expenditures, per capita rents must rise at least as rapidly as does per capita expenditures. The same level can be continually financed, or, if per capita rents grow faster, the land tax rate can be reduced. The growth of per capita rents will exceed the rate of growth.
Growth of per capita rent as population changes for given levels of improvements on existing property
of per capita expenditures (from equation (19)) if,

\[ f_L + \ell - n \geq 0 \]  \hspace{1cm} (26)

where

\[ f_L = \frac{1}{F_L} \frac{dF_L}{dt} , \]

\[ \ell = \frac{1}{L} \frac{dL}{dt} = 0 , \]

\[ n = \frac{1}{N} \frac{dN}{dt} , \]

and, because available land is fixed (\( \ell = 0 \)),

\[ f_L - n \geq 0 . \]  \hspace{1cm} (27)

Now,

\[ f_L = \frac{1}{F_L} \frac{dF_L}{dt} = \frac{1}{F_L} \{ F_{LK} \frac{dK}{dt} + F_{NL} \frac{dN}{dt} \} \text{ since } \frac{dL}{dt} = 0 . \]  \hspace{1cm} (28)

Substituting (28) into (27) gives

\[ \frac{1}{F_L} [K F_{LK}^k + N F_{NL} n] - n \geq 0 , \text{ where } k = \frac{1}{K} \frac{dK}{dt} . \]  \hspace{1cm} (29)

Substituting equation (16a) for \( F_{LK} \) and equation (16c) for \( F_{NL} \) into equation (29), and rearranging terms yields,
\[ k \left[ \frac{(U+Z)[(1-Z)(1-U)+UZ]}{2Z\sigma_{KL}} - \frac{Z(1-Z)(1-2U)}{2Z\sigma_{NK}} ight] - \frac{U(1-U)(1-2Z)}{2Z\sigma_{NL}} \]

\[ + n \left[ \frac{(Z+Q)[(1-Q)(1-Z)+QZ]}{2Z\sigma_{KL}} - \frac{Q(1-Q)(1-2Z)}{2Z\sigma_{LK}} - \frac{Z(1-Z)(1-2Q)}{2Z\sigma_{KN}} \right] - n \geq 0 . \quad (30) \]

Transferring \( n \) and the term multiplied by \( n \) to the right hand side of the inequality, and solving for the rate of growth of capital improvements, \( k \), we obtain,\(^4\)

\[ k \geq n \left[ 1 - \left\{ \frac{(Z+Q)[(1-Q)(1-Z)+QZ]}{2Z\sigma_{NL}} - \frac{Q(1-Q)(1-2Z)}{2Z\sigma_{LK}} - \frac{Z(1-Z)(1-2Q)}{2Z\sigma_{KN}} \right\} \right] . \]

\[ \left\{ \frac{(U+Z)(1-Z)(1-U)+UZ}{2Z\sigma_{KL}} - \frac{Z(1-Z)(1-2U)}{2Z\sigma_{NK}} - \frac{U(1-U)(1-2Z)}{2Z\sigma_{NL}} \right\} . \]

(31)

Thus the rate at which capital must grow to permit a balanced public budget depends on the rate of growth of population, the share distribution of income and the elasticities of substitution.

\(^4\)It is necessary to assume that the denominator of the term in the brackets is positive in order to maintain the inequality greater than or equal to. Whenever this assumption poses a problem it will be noted in the text.
If the elasticity of substitution between land and capital is small an increase in the supply of capital will cause a large decline in the profits per unit of capital and a large increase in the rent per unit of land. This can be seen in equation (31). A smaller $\sigma_{KL}$ results in a larger absolute increase in the denominator than the resultant increase in the numerator.\(^5\) In this case a smaller $\sigma_{KL}$ leads to a slower rate of capital growth necessary for per capita rents to remain constant or grow. Of course, if substitution between capital and land is easier, then the opposite result holds and capital in the community must grow at a more rapid rate if site value taxation is to prove adequate.

A similar result emerges in the analysis of the

\(^5\) We need to show that the absolute increase in the denominator of equation (31) is larger than the increase in the numerator when $\sigma_{KL}$ becomes smaller. Consequently, we need to show that

\begin{align*}
\text{(i) } & (U+Z)[(1-Z)(1-U)+UZ] > Q(1-Q)(1-2Z).
\end{align*}

Remembering that $Q+U+Z = 1$, then (i) reduces to

\begin{align*}
\text{(ii) } & (1-Z)(1-U)+UZ > Q(1-2Z),
\end{align*}

which is

\begin{align*}
\text{(iii) } & 1-U-Z+2UZ > Q-2QZ.
\end{align*}

Again as $Q+U+Z=1$, then inequality (iii) reduces to

\begin{align*}
\text{(iv) } & 2UZ+2QZ > 0,
\end{align*}

and consequently inequality (i) holds.
elasticity of substitution between land and labor ($\sigma_{NL}$). If land and labor are poor substitutes then an increase in the labor force will force wages down considerably while raising rents rapidly. Capital need not grow as fast in order to maintain per capita rents at some constant level. In equation (31) the numerator declines more rapidly than the denominator as $\sigma_{NL}$ becomes smaller, which allows the required rate of capital growth to be lower.\(^6\)

When labor and capital are very good substitutes the required capital growth rate is primarily dependent on the other two elasticities of substitution. If labor and capital are poor substitutes for land then the wage rate and profits per unit of capital will fall and rent per unit of land will rise in order to employ the increased labor and capital on the existing stock of land. This is clear in equation (31). As $\sigma_{KN}$ becomes larger the two terms with $\sigma_{KN}$ in the denominator approach zero, and the remaining four terms determine the necessary rate of capital increase. When $\sigma_{KL}$ and $\sigma_{NL}$ are smaller the result is that the entire term in the brackets is smaller. Intuitively, this case

\(^6\)For reasons similar to Footnote (4), we need to show that


A proof similar to that in Footnote (4) results in $2QZ+2UZ > 0$ as the final inequality.
would seem to best represent the real world.

When labor and capital are poor substitutes for each other, the opposite result occurs. When $\sigma_{KN}$ is lower, the denominator of and numerator of the term in the brackets, fall and rise respectively with the result that the necessary rate of growth of capital is larger than previously. Consequently, the more difficult it is to substitute capital and/or labor for land the more viable is a tax on site rent.

Again consider the special case when $\sigma_{KL} = \sigma_{KN} = \sigma_{NL} = \sigma$. With this simplifying assumption equation (31) reduces to

$$k \geq n \left[ \frac{\sigma - Q}{U} \right].$$

Clearly the more difficult is substitution, the larger labor's share and the larger capital's share, the slower capital improvements must grow for a site value tax to remain adequate over time. Table V-2 presents various required rates of growth of capital improvements for various assumed parameters for the economy.\(^7\) If the economy is

---

\(^7\)The average annual rate of growth of population in the United States since 1900 has been 1.4 percent. The labor force tends to show different growth patterns depending on the time period. For example from 1900 to 1929 the labor force grew at a 1.9 percent rate while from 1929-1965
TABLE V-2

THE REQUIRED RATE OF GROWTH OF CAPITAL FOR VARIOUS PARAMETER VALUES WITH n = 1.5 PERCENT

<table>
<thead>
<tr>
<th>Q</th>
<th>.70</th>
<th>.70</th>
<th>.75</th>
<th>.75</th>
<th>.80</th>
<th>.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>U</td>
<td>.25</td>
<td>.20</td>
<td>.20</td>
<td>.15</td>
<td>.15</td>
</tr>
<tr>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>0.3</td>
<td></td>
<td>-2.40</td>
<td>-3.00</td>
<td>-3.375</td>
<td>-4.50</td>
<td>-5.00</td>
</tr>
<tr>
<td>0.4</td>
<td></td>
<td>-1.80</td>
<td>-2.25</td>
<td>-2.625</td>
<td>-3.50</td>
<td>-4.00</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>-1.20</td>
<td>-1.50</td>
<td>-1.875</td>
<td>-2.50</td>
<td>-3.50</td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>-0.375</td>
<td>-0.50</td>
<td>-1.00</td>
</tr>
<tr>
<td>0.7</td>
<td></td>
<td>0.60</td>
<td>0.75</td>
<td>.375</td>
<td>.50</td>
<td>0.00</td>
</tr>
<tr>
<td>0.8</td>
<td></td>
<td>1.20</td>
<td>1.50</td>
<td>1.125</td>
<td>1.50</td>
<td>1.00</td>
</tr>
<tr>
<td>0.9</td>
<td></td>
<td>1.80</td>
<td>2.25</td>
<td>1.875</td>
<td>2.50</td>
<td>2.00</td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td>2.40</td>
<td>3.00</td>
<td>2.625</td>
<td>3.50</td>
<td>3.00</td>
</tr>
<tr>
<td>1.1</td>
<td></td>
<td>3.00</td>
<td>3.75</td>
<td>3.375</td>
<td>4.50</td>
<td>4.00</td>
</tr>
<tr>
<td>1.2</td>
<td></td>
<td>3.60</td>
<td>4.50</td>
<td>4.125</td>
<td>5.50</td>
<td>5.00</td>
</tr>
</tbody>
</table>

characterized by a Cobb-Douglas production function then the line with σ = 1 is applicable and the rate of growth of capital must equal or exceed values ranging from it only grew 1.3 percent per year. During the period of 1900-1965 the labor force grew at an annual rate of 1.6 percent in contrast to the 1.4 percent for the population. Table 1 assumes that the rate of growth of labor in the economy is 1.5 percent.
1.8 percent to 3.0 percent annually.

The preceding section dealt with expenditures under the assumption that \( \phi = 1 \). We now consider the expenditure equation when \( \phi \) is unconstrained. Again for simplicity assume that all elasticities of substitution are equal to a common \( \sigma \). We can then write an equation analogous to (32) as,

\[
k \geq n \left[ \frac{(\phi)\sigma - Q}{U} \right].
\]

The only significant change in the introduction of the economies of scale parameter. The larger the fiscal diseconomies associated with community size, the more rapid must be the growth of property improvements relative to the growth of population. Clearly, extreme diseconomies in the production of public output will render any tax inadequate. Hence, the value of \( \phi \) is important for public policy formulation. Similarly the greater the economies achieved in any particular function of local government the easier it will be for a land tax to provide necessary revenues. If \( \phi \) is less than unity, then population growth can be larger, and the levy will remain adequate over time.

The introduction of only "pure" public expenditures\(^8\)

\(^8\)The term "pure public good" is used in a restricted sense. The quantity of public expenditures is fixed. New residents can be served a zero marginal cost, as we assume that service quality remains constant.
$(\phi = 0)$ into the model permits $k$ to be smaller. Increasing the community's size allows the pure public cost to be spread over more residents. Per capita expenditures from this component will be reduced as the community grows, thus reducing the required growth of per capita rents and consequently the necessary $k$. However, the magnitude of the effect declines as population expands, as the marginal change in per capita expenditures from this component diminishes rapidly.

Finally, we turn briefly to an analysis of the system when available land is not fixed ($\lambda \neq 0$). We continue to assume that $\phi = 1$. If the community is presently operating at either a deficit or a balanced budget, then adequacy requires that the rate of growth of per capita rents respectively exceed or equal the rate of growth of per capita expenditures. Analysis of this situation was completed above when $\lambda = 0$ (equations 26-31). The following parallels that analysis; however, available land is allowed to grow ($\lambda > 0$). In order to simplify the notation, let the following equation represent equation (31),

$$k \geq n \left[ \frac{1 - \{c^*\}}{\{a^*\}} \right]$$

(34)

where $\{c^*\}$ and $\{a^*\}$ represent the appropriate bracketed terms in equation (31). Remembering (from equations (26))
that revenue adequacy requires

\[ f_L + \lambda - n \geq 0 ; \] \hspace{1cm} (35)

we now permit \( \lambda > 0 \).

Solving the system in the same manner as in equations (26)-(31) yields the following condition for the rate of growth of capital,

\[ k \geq n[ \frac{1-c^*}{a^*} ] + \lambda \left[ 1 - \left( \frac{1-c^*}{a^*} \right) \right]. \] \hspace{1cm} (36)

Equation (36) is the same as equation (34) except for the term multiplied by \( \lambda \). The introduction of a positive rate of land growth in general hinders revenue adequacy. However, when capital and labor are extremely poor substitutes for land, revenue adequacy will be aided by \( \lambda > 0 \). The difficulty in substituting capital and labor for land leads to rapidly increasing returns to land as a factor (see Table V-1), and this causes rents to rise rapidly. In a community where capital and labor are poor substitutes, an increase in land area will work against land taxation. Thus, for the most part, communities with easy access to additional land will find the rate of growth of revenues from land taxation too low to meet their rising expenditure needs.
EXPENDITURES A FUNCTION OF PER CAPITA INCOME

This section analyzes revenue adequacy when expenditures are related solely to per capita income. Assume that $\phi = 0$ for this section of the chapter. The expenditure function becomes

$$E = A[\frac{Y}{N}]^\psi.$$  \hspace{1cm} (37)

Assume that at some point in time the budget is balanced ($E = R$ if $g = 1$). If the public sector is to avoid deficits while financing the budget with land taxes, the rate of growth of expenditures, $\varepsilon$, must equal or exceed the rate of growth of aggregate rents, $r$, or

$$f_L + \varepsilon \geq \psi(y-n).$$ \hspace{1cm} (38)

For convenience, assume that $\varepsilon = 0$, and $\sigma_{KL} = \sigma_{KN} = \sigma_{NL} = \sigma$. The implications of relaxing these two assumptions were discussed above and are similar in this case. Solving equation (38) for the required rate of growth of capital permitting a balanced budget yields

$$k[U(\frac{1}{\sigma} - \psi)] \geq n[\psi-Q(\frac{1}{\sigma} - \psi)].$$ \hspace{1cm} (39)

It is clear in equation (39) that whether or not $\frac{1}{\sigma} > \psi$ is crucial to the problem of adequacy. First, if $\frac{1}{\sigma} < \psi$, the
bracketed term on the left hand side of the inequality is negative while the bracketed term on the right hand side is positive. Therefore, no rate of capital formation will satisfy the inequality; rent cannot grow fast enough to keep pace with expenditures. On the other hand, if $\frac{1}{\sigma} > \psi$, we can rearrange equation (39) to obtain

$$K \geq n\left[ \frac{\psi - Q\left( \frac{1}{\sigma} - \psi \right)}{U\left( \frac{1}{\sigma} - \psi \right)} \right].$$

(40)

Consequently, if $\frac{1}{\sigma} > \psi$ then rents can grow fast enough if capital formation proceeds at a rapid enough pace. How rapidly capital must grow depends on (1) the difference between $\frac{1}{\sigma}$ and $\psi$, (2) the values of $Q$ and $U$, and (3) the rate of growth of population. Whether or not $\frac{1}{\sigma} > \psi$ is an empirical question; the values are estimated in Chapter VI. In addition, larger per capita income elasticities necessitate larger rates of capital growth and adequacy is more difficult to achieve.  

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The value of $\psi$ for state and local governments is estimated to be approximately 0.7. Thus for $\sigma < 1.43$, $1/\sigma$ is greater than $\psi$, and inequality (40) follows from inequality (39). Thus, at least some rate of capital growth will permit rents to grow fast enough to meet new expenditure needs.

These results may appear to conflict with those of the Ricardian model. A larger per capita income elasticity was beneficial to revenue adequacy in the Ricardian analysis. This was because capital was only permitted to
EXPENDITURES RELATED TO BOTH
POPULATION AND INCOME

As we have noted earlier, for a tax on site rent to be considered adequate, the growth rate of the tax base must equal or exceed the growth rate of the public sector. In the present case the growth rate of rent must exceed the growth rate of expenditures. Again we will assume that all \( \sigma \)'s are equal. The expenditure equation relates total expenditures to both income and population,

\[
E = AN^\phi \left[ \frac{Y}{N} \right] \psi .
\]  

Solving the system in the same manner as previously for the rate of growth of capital that will permit rents to keep pace with expenditures yields,

\[
k \geq n \left[ \frac{(\phi - \psi) - Q \left( \frac{1}{\psi} - \frac{1}{\sigma} \right)}{U \left( \frac{1}{\sigma} - \psi \right)} \right] .
\]  

Equation (42) differs from equation (40) only in that the grow as fast as did population (capital-and-labor applied to land in fixed proportions). Because of the fixed land resource and the nature of the production function, per capita income fell as population rose. However, if capital were allowed to grow faster than labor in the Ricardian model, per capita income would rise and the same implications as above would result.
population elasticity ($\phi$) enters the equation. The results of this section are summarized below:

[1] The lower the elasticity of factor substitution the easier adequacy is achieved.

[2] The larger the per capita income elasticity of expenditures the more difficult adequacy is achieved.

[3] The larger are the shares of income accruing to labor and capital the slower capital must grow for rents to keep pace with the public sector.

[4] Population diseconomies (large $\phi$) will preclude adequacy if they are sufficiently large.

CONCLUSION

The model presented above has shown that the elasticities of factor substitution, factor shares, and the expenditure elasticities of income and population are important parameters to consider when attempting to answer the question regarding revenue sufficiency of land value taxation. Further, the model has demonstrated that this question can be answered by comparing the actual rate of growth of capital improvements to that rate determined necessary in equation (42). This important result forms an integral
part of the empirical analysis due to limited land value data.

The more difficult is substitution of land for capital or labor, the easier adequacy conditions can be met. Conversely, if substitution of land for capital or labor is easier, then capital in the community must grow more rapidly if site value taxation is to provide adequate revenue in the future. The larger the share of income accruing to capital and/or labor; and consequently, the smaller the share received by land, the easier adequacy conditions can be fulfilled.\textsuperscript{11}

Communities that are providing more "pure" public goods and services rather than impure public goods will find the required rate of capital growth lower. Although per capita costs of "pure" public goods decline as the community grows, the extent of this effect diminishes rapidly. And finally, in general communities that are able to absorb additional land easily will find adequacy

\textsuperscript{11}We are using the word adequacy in a fairly restricted sense: the growth rate of tax revenues equals or exceeds the growth rate of expenditure needs. The smaller is the share of national income accruing to land the less significant the tax becomes. The absolute level of revenue raised becomes inconsequential. However, the data for California and Milwaukee presented in Chapter II indicate that land's share may equal or exceed capital's share. This is not an insignificant potential tax base.
conditions more difficult to achieve. However, if capital and labor are very difficult to substitute for land, required capital growth may be lower. We now turn to an examination and analysis of the available empirical data.
CHAPTER VI

EMPIRICAL EVIDENCE

INTRODUCTION

In this chapter we present an analysis of available data to determine if revenues from a site levy could be expected to keep pace with some given fraction of expenditure needs. The estimates are for the aggregate U.S. economy. Because of relatively greater rates of capital development and of growth in the demand for land as a spacial resource, revenues from a site value tax would be expected to grow relatively more rapidly in metropolitan areas.

Empirical work on the revenue adequacy of a site value tax is handicapped by the scarcity of available data. For the national economy data on the value of land are virtually nonexistent.¹ In the following sections we develop empirical models for testing revenue adequacy and present modifications of these models necessitated by insufficient data on land value.

¹With the exception of a few estimates of particular years by Goldsmith [36], Manvel [53], and Keiper [49].

VI-1
THE EMPIRICAL MODEL

The analysis in Chapter V shows that five parameters are important in determining the adequacy of a site value levy over time: (1) The population elasticity of expenditures, (2) the per capita income elasticity of expenditures, (3) the elasticity of factor substitution,\(^2\) (4) the share of national product allocated to labor, and (5) the share of national product accruing to capital. In this section we present and discuss estimates of these five parameters.

Production Parameters

Considerable empirical research effort has been devoted to estimating production functions for the aggregate economy, and for particular industries. In general researchers have attempted to estimate coefficients for Cobb-Douglas or Constant Elasticity of Substitution (CES) production functions.\(^3\)

\(^2\)In this section we assume that the three elasticities of substitution introduced in Chapter V are equal. Although \(\sigma_{LK} = \sigma_{LN} < \sigma_{KN}\) may be more realistic, this assumption would require very extensive calculation, without yielding much additional information. In addition, estimates of \(\sigma_{LK}\) and \(\sigma_{LN}\) are unavailable. Consequently, our estimates of the rate of capital growth required for land to be an adequate tax base are somewhat overstated.

\(^3\)The empirical evidence on the two factor Cobb-Douglas production function is summarized in Walters [95], Nerlove [66], summarizes evidence from time-series and
Theoretical and empirical evidence suggest that production parameters are sensitive to small changes in the data, measurement of variables, and method of estimation.\(^4\) Most of the time series and cross-section estimates of aggregate \(\sigma\) are less than unity; however, the cross-section estimates are generally higher than are the time-series estimates.\(^5\) Estimates of the elasticity of substitution of capital for labor for various industries differ

cross-section estimates of two factor CES production functions for the postwar period through 1965. The more recent literature (since 1965) is compiled in Nadiri [64]. As the Cobb-Douglas function is a special case of the CES, we will confine our discussion to the estimates of the CES production function. Evidence on three factor production functions in manufacturing is unreliable and sketchy due to poor data on the stock of land used in manufacturing. Some evidence is available for agriculture, but knowledge of this sector offers little insight into the current problem.

\(^4\)Fisher [29] has argued that considerable care must be exercised when using an aggregate production function to characterize an economy. Where capital is firm-specific or immobile, problems are introduced not only for capital aggregation but for labor and output aggregation as well. Because land is an immobile factor these aggregation problems exist in our model. We recognize that some degree of approximation is involved in representation of the economy by an aggregate production function. However, the solution to this problem is beyond the scope of this study. Partly because of this problem, large ranges of values for \(\sigma, z, Q,\) and \(U\) are considered in this chapter.

\(^5\)Nadiri [64], p. 1151 cites several recent studies in which various estimates of \(\sigma\) are distinctly less than unity.
substantially, but the majority of these estimates are unmistakably less than one.\textsuperscript{6} Estimated values for parameters Q and U vary considerably depending on the periods of time particular studies cover.\textsuperscript{7} Because we cannot place enough confidence in any particular estimate, the following ranges of values for each parameter are considered:

\begin{align*}
\sigma & : \text{0.3 to 1.2, } \\
Q & : \text{0.70 to 0.80, } \\
U & : \text{0.15 to 0.25, } \\
Z & : \text{0.05 to 0.10. }
\end{align*}

**Expenditure Parameters**

Revenue adequacy requires that tax revenues must finance not only today's needs, but tomorrow's needs as well. As a result, we begin by developing estimating equations for the community's expenditure requirements over time.

Numerous expenditure studies have been done on the relationship between local public expenditures and various

\textsuperscript{6}See Nerlove [66] and Nadiri [64].

\textsuperscript{7}See Walters [95]. Estimates of the share of national product accruing to land (Z) are normally omitted from these two factor studies.
socio-economic variables. These studies have generally been cross-sectional. More recently, effort has been directed towards time-series estimates in order to uncover the cyclical nature of public expending.\footnote{See McMahon [57].}

One major methodological difficulty characterizes expenditure studies. Expenditure studies in general do not provide separate estimates of "supply" and "demand" schedules. This identification problem arises in the analysis of public spending because those variables that systematically affect the demand for public services are also associated with variations in the supply of these services and in the determination of tax revenues. For example, personal income is a determinant of both the demand for and the supply of public services in as much as income affects both tax revenues and the factor costs of services such as education and police protection. That is, areas with higher per capita incomes will tend to have higher wages and salaries. Since the public sector employs considerable labor services, per unit costs will be higher, and total expenditures will tend to be higher in these areas.

In order to answer the question of the revenue sufficiency of a site levy, delineation of this supply and demand dicotomy is not necessary. It is the rate of growth
of, or the level of, the expenditures that is important, not the underlying reasons for this growth or level. What we are interested in is the equilibrium values, not the individual supply and demand curves that comprise the equilibrating scissors.\textsuperscript{9} We are interested primarily in how these equilibrium values vary over time in relation to values of income and population.\textsuperscript{10}

For this study estimates of state and local expenditure elasticities of population and per capita income are required. Furthermore, the elasticities should be independent of cyclical variations in public output. Consequently, the level of unemployment is introduced into our

\textsuperscript{9}This is not to say that estimating individual supply and demand curves is not interesting nor necessary in general. This is undoubtedly one of the most pressing empirical needs in the field of state and local finance. However, this estimation problem (identification problem) is not the interest of this study.

\textsuperscript{10}Every study of local public expenditures embodies the implicit assumption that local public spending can be isolated and studied in relation to selected explanatory variables. For our study the question of causality between state and local expenditures and per capita income is somewhat important. While we cannot argue that income is totally unaffected by state and local spending, clearly the influence of this component on Gross National Product or Personal Income will be minor when compared to the influence of federal stabilization activities. Thus we assume that per capita income, for all practical purposes, is independent of variations in state and local expenditures on goods and services.
empirical model.\textsuperscript{11} The level of unemployment is related to needs and demands for unemployment compensation as well as cyclical variations in public assistance needs of those unemployed, or those outside the labor force.\textsuperscript{12}

The expenditure equation developed in Chapter III is estimated for state and local governments in the United States. The available data were separated into two classes: (1) annual data for the period 1929 to 1970, and, (2) quarterly data from the first quarter of 1948 to the second quarter of 1971. Gross national product per capita and personal disposable income per capita were employed to test the stability of the elasticities. Personal income would be expected to exhibit less cyclical variation due to the nature of built-in stabilizers. The general estimating equation is

\begin{equation}
E_t = A[N]_t^e \left( \frac{Y}{N} \right)^{e2i} U_t^{e3} \epsilon_t, \quad i=1,2
\end{equation}

where

\begin{itemize}
    \item \(E\) = State and local purchases of goods and services (1958 dollars),
\end{itemize}

\textsuperscript{11}The theoretical model developed above implicitly assumes full employment. The introduction of unemployment into the empirical model is recognition of this limitation.

\textsuperscript{12}See McMahon [57].
\[ A = \text{Multiplicative constant,} \]
\[ N = \text{Civilian population}^{13} \]
\[ \left[ \frac{Y}{N} \right]_1 = \text{Gross national product per capita (1958 dollars)}, \]
\[ \left[ \frac{Y}{N} \right]_2 = \text{Disposable personal income per capita (1958 dollars)}, \]
\[ U = \text{Total unemployment as a percent of total labor force}, \]
\[ e_k = \text{Expenditure elasticity of the } k^{th} \text{ variable}, \]
\[ \varepsilon_t = \text{Error term}. \]

Assuming that the elasticities \( e_1, e_{2i}, \) and \( e_3 \) are constant through time, and dropping the time subscript for simplicity, equation (1) can be rewritten as,

\[
\ln E = \ln A + e_1 \ln N + e_{2i} \ln \left[ \frac{Y}{N} \right]_1 + e_3 \ln U \\
+ \ln \epsilon
\]

and ceteris paribus.

\[ ^{13} \text{Throughout the theoretical model we used "N" to represent both labor and population. We have chosen civilian population in this case because expenditure rather than production relationships are estimated.} \]
\[ e_1 = \frac{\ln E}{\ln N}, \quad e_2 = \frac{\ln E}{\ln \left[ \frac{Y}{N_i} \right]}, \quad e_3 = \frac{\ln E}{\ln U} \quad . \tag{2} \]

Equation (2) was estimated by ordinary least squares regression. Two distinct sets of data were analyzed: (1) quarterly data from 1948 to 1971, and (2) annual data from 1929 to 1970. The results are presented in Table VI-1.\(^{15}\)

ANALYSIS OF THE RESULTS

As expected, all elasticities are positive and significant at levels greater than 95 percent. The population elasticity estimates were roughly constant in all four equations, ranging in value from 2.30 to 2.78. The income elasticity estimates on the other hand, exhibit marked variation between the two sets of data (ranging from .30

\(^{14}\)In any analysis of national income data where aggregate variables have been continually rising over time, we expect considerable autocorrelation. Cochran and Orcutt [24], and Johnston [46] provide a method of estimating when autocorrelation is present. Estimates incorporating this technique are presented in Table VI-1. In all equations this technique removed a sufficient amount of the autocorrelation to permit acceptance of the hypothesis of no autocorrelation using the Durbin-Watson method.

\(^{15}\)Both income variables were highly correlated with unemployment over the 1929-1970 period covered by the annual data. Colinearity was not present in the quarterly estimates. Consequently, we have more confidence in the quarterly estimates.
## TABLE VI-1

EXPENDITURE ELASTICITY ESTIMATES FOR
AGGREGATE STATE AND LOCAL EXPENDITURES

<table>
<thead>
<tr>
<th>#</th>
<th>N</th>
<th>Y₁</th>
<th>Y₂</th>
<th>U</th>
<th>R²</th>
<th>D-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>2.53**</td>
<td>.603**</td>
<td>0.08**</td>
<td>.962**</td>
<td>1.68b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.212)</td>
<td>(0.144)</td>
<td>(0.02)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[11.94]</td>
<td>[4.19]</td>
<td>[3.90]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[2]</td>
<td>2.30**</td>
<td>.70**</td>
<td>0.05**</td>
<td>.981**</td>
<td>1.75c</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.152)</td>
<td>(0.095)</td>
<td>(0.014)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[15.18]</td>
<td>[7.34]</td>
<td>[3.59]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[3]</td>
<td>2.71**</td>
<td>.364*</td>
<td>0.173**</td>
<td>.921**</td>
<td>1.55a</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.284)</td>
<td>(0.182)</td>
<td>(0.035)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[9.56]</td>
<td>[2.00]</td>
<td>[4.92]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>2.78**</td>
<td>.288*</td>
<td>0.147**</td>
<td>.930**</td>
<td>1.50a</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.269)</td>
<td>(0.168)</td>
<td>(0.026)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[10.33]</td>
<td>[1.71]</td>
<td>[5.77]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Figures in parentheses, ( ), are standard errors of the coefficients. Figures in brackets [ ] are t values. The significance of the regression coefficients is tested using a one-tail t test, and the significance of the coefficients of multiple determination are tested using the F test.

*Indicates coefficient is significant at the 95 percent level.

**Indicates coefficient is significant at the 99 percent level.

The existence of serial correlation was tested with the Durbin-Watson statistic. The letters a, b, and c indicate the probability assigned to the left hand tail of the distribution, where a, b, and c equal .01, .025, and .05 respectively. In all equations, we can accept the hypothesis of no autocorrelation.
to .70). This difference seems to be explained by the unemployment variable. This can be attributed to the greater fluctuations in economic activity over the four decades covered in the latter equations. This period included both the Great Depression and World War II, making the accuracy of these estimates somewhat suspect. In all cases, the hypothesis of no autocorrelation can be accepted at the \( \alpha = .01 \) level of significance.

Chapter V provides a means by which the revenue adequacy question can be answered. Rewriting equation (42) from Chapter V results in,

\[
k \geq n \left[ \frac{(\phi - \psi) - Q(\frac{1}{\sigma} - \psi)}{U(\frac{1}{\sigma} - \psi)} \right].
\]

(4)

With this formulation we estimate the necessary rate growth of capital structures for the growth of land value to be sufficient to maintain adequate public revenues. Estimates incorporating the various ranges of production parameters, and the elasticities estimated in equations (1)-(4), are tabulated in Tables VI-2, VI-3, and VI-4. These tables are constructed to encompass the lowest, middle, and highest values we would anticipate for \( \phi \) and \( \psi \) respectively.

The required rates of capital growth vary from a low of -2.1 percent per year in Table VI-2 when \( \sigma = 0.3 \),
TABLE VI-2

REQUIRED ANNUAL PERCENTAGE RATE OF GROWTH OF CAPITAL FOR VARIOUS PRODUCTION PARAMETERS WHEN

\( n = 1.5\% \)
\( \phi = 2.3 \)
\( \psi = 0.3 \)

<table>
<thead>
<tr>
<th>Q</th>
<th>.70</th>
<th>.70</th>
<th>.75</th>
<th>.75</th>
<th>.80</th>
<th>.80</th>
</tr>
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<tr>
<td>U</td>
<td>.25</td>
<td>.20</td>
<td>.20</td>
<td>.15</td>
<td>.15</td>
<td>.10</td>
</tr>
<tr>
<td>( \sigma )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>-.24</td>
<td>-.30</td>
<td>-.56</td>
<td>-.68</td>
<td>-1.40</td>
<td>-2.10</td>
</tr>
<tr>
<td>0.4</td>
<td>1.25</td>
<td>1.57</td>
<td>1.20</td>
<td>1.59</td>
<td>1.09</td>
<td>1.64</td>
</tr>
<tr>
<td>0.5</td>
<td>2.86</td>
<td>3.57</td>
<td>3.20</td>
<td>4.26</td>
<td>3.76</td>
<td>5.65</td>
</tr>
<tr>
<td>0.6</td>
<td>4.59</td>
<td>5.69</td>
<td>5.25</td>
<td>6.99</td>
<td>6.55</td>
<td>9.84</td>
</tr>
<tr>
<td>0.7</td>
<td>6.41</td>
<td>8.03</td>
<td>7.63</td>
<td>10.15</td>
<td>9.71</td>
<td>14.60</td>
</tr>
<tr>
<td>0.8</td>
<td>8.43</td>
<td>10.54</td>
<td>10.16</td>
<td>13.55</td>
<td>13.05</td>
<td>19.60</td>
</tr>
<tr>
<td>0.9</td>
<td>10.59</td>
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<td>12.87</td>
<td>17.09</td>
<td>16.62</td>
<td>25.04</td>
</tr>
<tr>
<td>1.0</td>
<td>12.94</td>
<td>16.18</td>
<td>15.80</td>
<td>21.07</td>
<td>20.57</td>
<td>30.86</td>
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<td>15.49</td>
<td>19.30</td>
<td>18.93</td>
<td>25.26</td>
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<td>1.2</td>
<td>18.38</td>
<td>22.94</td>
<td>22.51</td>
<td>30.00</td>
<td>29.51</td>
<td>44.18</td>
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TABLE VI-3

REQUIRED ANNUAL PERCENTAGE RATE OF GROWTH OF CAPITAL FOR VARIOUS PRODUCTION PARAMETERS WHEN

\( n = 1.5\% \)
\( \phi = 2.3 \)
\( \psi = 0.7 \)

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( Q = 0.7 )</th>
<th>( 0.7 )</th>
<th>( 0.75 )</th>
<th>( 0.75 )</th>
<th>( 0.8 )</th>
<th>( 0.8 )</th>
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</thead>
<tbody>
<tr>
<td>( U )</td>
<td>0.25</td>
<td>0.20</td>
<td>0.20</td>
<td>0.15</td>
<td>0.15</td>
<td>0.10</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.55</td>
<td>0.68</td>
<td>1.06</td>
<td>1.40</td>
<td>1.89</td>
<td>2.85</td>
</tr>
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<td>0.4</td>
<td>1.113</td>
<td>1.42</td>
<td>1.04</td>
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<td>1.33</td>
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<td>0.5</td>
<td>3.18</td>
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<td>4.81</td>
<td>4.30</td>
<td>6.46</td>
</tr>
<tr>
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<td>7.25</td>
<td>6.87</td>
<td>9.16</td>
<td>8.66</td>
<td>13.00</td>
</tr>
<tr>
<td>0.7</td>
<td>8.95</td>
<td>11.19</td>
<td>10.81</td>
<td>14.41</td>
<td>13.91</td>
<td>20.87</td>
</tr>
<tr>
<td>0.8</td>
<td>12.94</td>
<td>16.18</td>
<td>15.80</td>
<td>21.07</td>
<td>20.53</td>
<td>30.80</td>
</tr>
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<td>19.21</td>
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<td>23.63</td>
<td>31.51</td>
<td>31.02</td>
<td>46.53</td>
</tr>
<tr>
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<td>45.33</td>
<td>68.00</td>
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<td>51.78</td>
<td>69.03</td>
<td>68.55</td>
<td>102.84</td>
</tr>
<tr>
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<td>84.98</td>
<td>84.58</td>
<td>112.50</td>
<td>112.05</td>
<td>168.50</td>
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</table>
TABLE VI-4

REQUIRED ANNUAL PERCENTAGE RATE OF GROWTH OF CAPITAL FOR VARIOUS PRODUCTION PARAMETERS WHEN

\( n = 1.5\% \)
\( \phi = 2.7 \)
\( \psi = 0.7 \)

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>U</th>
<th>.70</th>
<th>.70</th>
<th>.75</th>
<th>.75</th>
<th>.80</th>
<th>.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>.25</td>
<td>.36</td>
<td>.46</td>
<td>.09</td>
<td>.11</td>
<td>-.39</td>
<td>-.59</td>
</tr>
<tr>
<td>0.4</td>
<td>.20</td>
<td>2.47</td>
<td>3.08</td>
<td>2.70</td>
<td>3.61</td>
<td>3.11</td>
<td>4.67</td>
</tr>
<tr>
<td>0.5</td>
<td>.20</td>
<td>5.02</td>
<td>6.29</td>
<td>5.91</td>
<td>7.88</td>
<td>7.38</td>
<td>11.08</td>
</tr>
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<td>8.15</td>
<td>10.21</td>
<td>9.82</td>
<td>13.05</td>
<td>12.58</td>
<td>18.87</td>
</tr>
<tr>
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<td>.15</td>
<td>12.21</td>
<td>15.31</td>
<td>14.92</td>
<td>19.86</td>
<td>19.40</td>
<td>29.18</td>
</tr>
<tr>
<td>0.8</td>
<td>.10</td>
<td>17.61</td>
<td>22.09</td>
<td>21.64</td>
<td>28.91</td>
<td>28.35</td>
<td>42.54</td>
</tr>
<tr>
<td>0.9</td>
<td>.10</td>
<td>25.02</td>
<td>31.28</td>
<td>30.95</td>
<td>41.21</td>
<td>40.78</td>
<td>61.10</td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td>35.80</td>
<td>44.75</td>
<td>44.37</td>
<td>59.17</td>
<td>58.67</td>
<td>88.00</td>
</tr>
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<td>66.38</td>
<td>66.14</td>
<td>88.17</td>
<td>87.69</td>
<td>131.55</td>
</tr>
<tr>
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<td></td>
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<td>107.70</td>
<td>107.15</td>
<td>142.80</td>
<td>142.38</td>
<td>213.61</td>
</tr>
</tbody>
</table>
Q = .80, and U = .10, to a high of 213.61 percent in Table VI-4 when \( \sigma = 1.2, Q = .80, \) and \( U = .10. \) In general very high rates of capital growth are required when \( \sigma > 0.7. \) However, as we have argued above, the elasticity of substitution is probably located in the lower end of our range.

The elasticities estimated in equation [2] with quarterly data and disposable personal income are clearly superior to the other estimates. Multicollinearity of the independent variables was virtually nonexistent, the lack of serial correlation was assured, and the coefficient of determination \( (R^2) \) was the highest. For these reasons, we feel a population elasticity of 2.3 and a per capita income elasticity of .7 best characterizes aggregate state and local spending in the United States. The value of the elasticity of substitution, for the reasons cited above, probably ranges between 0.4 and 0.6.\(^{16}\) Consequently, we would expect that a rate of capital growth between 3.60% and 9.16% (assuming labor's share of national income is

\(^{16}\) Ideally, we would desire estimates of \( \sigma_{KL}, \sigma_{KN}, \) \( \sigma_{NL} \) individually. Estimates of individual \( \sigma \)'s are not available at present for the reasons noted above. We would expect that the value of \( \sigma_{KL} \) and \( \sigma_{NL} \) would be considerably less than the value of \( \sigma_{NK} \). The value of \( \sigma_{NK} \) has generally been estimated to be less than unity (see footnote 6). The value of \( \sigma \) assumed in Tables VI-2, VI-3, and VI-4 represents some average approximation of all three. Consequently, a value ranging between 0.4 and 0.6 does not seem unlikely.
approximately .75) would permit land value taxation to be an adequate revenue source.

Wasson, Musgrave and Harkins [96] have estimated that the average rate of growth of capital stocks for all industries since World War II has been between 5.5 percent and 6.0 percent.\(^\text{17}\) Rates of capital growth vary considerably depending on the type of equipment considered. These average rates range from a high of 20 percent per year for office computing and accounting machinery to a low of .5 percent per year for mining and oil field machinery.\(^\text{18}\) Because communities are generally characterized by the nature of the industries located therein, capital growth will vary considerably between urban communities.

Communities with higher than average rates of capital growth and with a relatively constrained land area will find substituting a site levy for the present property tax quite adequate. Moreover, substitution of land taxation for present property taxation may stimulate further building and redevelopment as taxes on improvements

\(^{17}\)The exact value depends upon the nature of the depreciation schedule assumed, and the method used to value used equipment. See Wasson, Musgrave, and Harkings [96] for a complete description of the computing methods.

\(^{18}\)This data was not reported by industry. However, some industry classifications could reasonably be inferred from reported data; for example, railroad equipment and service industry machinery.
are reduced or eliminated entirely. Consequently, our analysis suggests that the revenue adequacy of a site tax is not a closed question. Communities expecting rapid growth would do well to consider the alternative of a land value tax as a weapon in their fiscal arsenals.

Finally our results suggest that the crowding parameter, γ, generally has a value approaching three.\textsuperscript{19} This can be interpreted to mean that as city size increases, the advantages of sharing the costs of public services among more people are offset sharply by the losses incurred from sharing the services with more people. This suggests that there are considerable congestion costs associated with urban public service expenditures.

\textsuperscript{19}The crowding parameter γ = (φ+α)/(1+α), where α is the price (cost) elasticity of demand for public services. When public service demand is relatively insensitive to costs, the value of γ approaches the value of φ. However, because α < 0, the value of γ will always exceed the value of φ.
CHAPTER VII

CONCLUSION

One of the main contributions of the model developed in this study is that the nature of the community's needs is considered explicitly in the attempt to determine the revenue adequacy of a site value tax. Previously, adequacy of a site value tax has been defined as the ability of such a tax to reproduce existing yields from current real estate taxes. The problem of whether or not such a tax could be adequate through time has been ignored. The proposed site value levy may not be able to reproduce the revenues from a tax on both land and improvements presently, but some years into the future the tax may prove adequate. Furthermore, even if land value taxes could reproduce existing property tax revenues, if revenues did not grow rapidly enough to meet future needs then substitution of tax bases would not represent a viable alternative.

Previous empirical research effort has been directed towards determination of the effect that income and population have on public expenditures. Generally, this effort has been of a cross-sectional nature. For the purpose of VII-1
adequately answering questions of revenue sufficiency, time-series analysis is required and has been used in this study. Because we are interested in the behavior of land values and public sector needs over time, existing analyses of public expenditures are of little value in determining whether a land tax will generate sufficient revenue over time.

When the public fiscal economy is primarily dependent on the population of the community and the fisc is initially operating at a deficit, for a tax on site rent to provide a viable solution to elimination of this deficit, per capita rents must be rising over time. The larger the population elasticity of public output the greater the rise in per capita rents must be in order to eventually balance the budget. In the Ricardian framework this required the production function to be subject to decreasing returns at a constant or an increasing rate if the fisc was to eventually be balanced. The larger the population elasticity, the more rapidly returns to the variable factor must decline. The larger is the share of the public sector devoted to "pure" public goods, the less restrictive are the conditions on the production function. Thus, in communities where overhead needs represent a considerable fraction of public output, if site value taxation is not adequate
initially, it will eventually provide necessary revenues as population grows and per capita needs decline. On the other hand, communities with high, "impure" public demand, will find generating sufficient revenue from any tax difficult. When the community's public output is principally related to per capita income a site value levy will supply necessary revenues if rental share exceeds or is equal to the share of income devoted to the public sector.

The neo-classical model developed in Chapter V focused upon the effect of capital formation on aggregate rents. Generally, population growth alone is insufficient to cause per capita rents to rise as rapidly as is required in order to balance local budgets. Capital improvements must also be growing if rents are to rise rapidly enough. How fast capital improvements must be made to existing land depends on the elasticities of substitution between factors and relative factor shares. The more difficult it is to substitute labor and capital for land, or the larger the share of income accruing to capital and labor, the lower is the rate of growth of capital required to generate sufficient rents for budget balance. Communities which have severe land area constraints, and which experience rapid population and capital growth will simultaneously experience rapidly increasing land values. Communities which are able
to absorb surrounding land at relatively low prices usually will not experience nearly so rapid a rate of increase in aggregate rents or land values unless labor and/or capital are extremely poor substitutes for land. Consequently, communities with virtually unlimited land are more likely, in general, to find a site value levy inadequate.

The model as developed and tested assumes a closed economy. That is, there is no mention of factor mobility. This is similar to assuming equal land tax rates in many communities. Thus, if there was mobility of capital or labor before the tax was introduced, the mobility parameters would remain unchanged if all communities simultaneously substituted a land tax for a tax on both land and improvements.¹

However, if just one community or region decides to substitute a site levy for the current tax, the flow of capital into that region would increase, as the marginal return from capital would rise. The end result is that the required rate of capital growth from within the community is less and a site levy under these conditions is more apt to produce sufficient revenue over time.²

¹This assumes that all communities have the same initial property tax structures and rates.

²The magnitude of capital migration under these assumptions is an empirical question. A number of studies
Part of any community's public provided output includes capital improvements. Although the property itself is not subject to direct taxation, the externalities produced generally lead to increased property values surrounding the outlay. An obvious example of this is capital outlays for highways and streets. Nearness to highways or streets generally increases property values for obvious traffic and transportation reasons. Other examples include urban renewal, parks and sports amphitheaters, etc. As a result, the greater the proportion of total expenditures going into capital improvements, the smaller the necessary increase in capital from the private sector. In both the revenue and expenditure formulations, we have neglected to account for the effects of public capital outlays on revenue adequacy.

The most important result of this study is that it rebuts the argument that land value taxation should be

have been directed toward determining the effect of tax incentives on industrial location. See McLure [67], for a theoretical treatment of the issue and Due [68], for a summary of the empirical studies. There have not been any explicit studies dealing with the land value tax as an incentive for industrial location in the United States in the recent past. The general conclusion of most empirical studies is that taxation is of minor importance when the magnitudes of other differentials (primarily labor costs) are considered. However, depending on the rise in the tax rate, removing improvements from the tax rolls could create tremendous incentives for the relocation of very capital intensive industries.
dismissed for lack of revenue potential. Table II-1 demonstrates the fact that on the average aggregate land value in California is approximately equal to aggregate improvement value. If the share of national income received by labor is approximately 75 to 80 percent then the share accruing to land would be approximately 10 percent; roughly equal to that accruing to capital. This suggests very strongly that the absolute amount of revenue that could be raised via such a levy is large.

The evidence presented in Table VI-3 suggests that since expenditure needs can be expected to rise over time, a rate of capital growth between 1.4 percent and 9.2 percent will be needed to keep land rents rising sufficiently rapidly to meet these increased demands. The evidence on real capital growth by Wasson, Musgrave, and Harkins [96], suggests that a rate of capital growth within this range is not unlikely for the United States economy. Consequently, the capability of site value taxation to meet rising expenditure needs at the aggregate level seems assured. Particular urban communities with a greater rate of capital growth may even be able to reduce land tax rates while meeting their rising expenditure requirements over time.

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3We have assumed that Q=.75, U=.15, Z=.10, .4≤ρ≤.6, ψ=2.3, and ψ=0.7. Of the ranges of estimates available, these are the values in which we place the most confidence.
BIBLIOGRAPHY


