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A STUDY OF ANCHORAGE CAPACITIES OF
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by

Jose L. G. Marques

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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CHAPTER 1

INTRODUCTION

1.1. Object and Scope

The object of this study was to examine some of the factors which influence the anchorage capacities of reinforcing bars in beam-column joints of reinforced concrete structures.

The study consisted of testing a series of eighteen specimens simulating typical beam-column joints in a structure in order to evaluate the capacity of the anchored beam reinforcement subjected to varying degrees of confinement at the joint.

The four types of confinement considered included column axial load, vertical column reinforcement, side concrete cover and lateral reinforcement (ties) throughout the joint.

The anchored beam bars were either #7 bars or #11 bars and they were bent to different geometric configurations.

All data obtained by Marques (16) was reanalyzed and compared with the data obtained in the present study in order to obtain more general conclusions.
1.2. Definition of the Problem

In a general way the problem can be illustrated through a typical exterior beam-column joint (Figure 1). The tensile force which is part of the beam moment at the face of the column must be developed by bond on some length of the embedded reinforcing bars. In many cases, the column is not wide enough to permit the use of a straight bar embedded in the joint. To develop sufficient anchorage capacity, the bar must be bent. The anchored bar then includes either a 180° bend or a 90° bend with a straight portion extending either up or down into the column. Considering these conditions, the anchorage capacity of the bar is a function of a certain number of parameters, including the diameter of the bar, the type of lugs and surface of the bar, the strength of the steel, the geometry of the bend, the strength of the concrete, the axial load on the column, the amount of cover over the anchored bar, the restraint or confinement offered by the column reinforcement (longitudinal bars and ties) and the type of loading to which the bar will be subjected.

The lack of information regarding anchorage capacities of reinforcing bars in joints has become increasingly clear with the need for reinforced concrete structures designed to absorb energy and sustain large rotations in the members and joints. Typical of these refinements are structures subjected to blast or seismic loadings. A great deal of work has been done on the evaluation of the behavior...
of structural members, but the joints between members have not received as much attention. The problem is complicated by certain constructional limitations which may arise. For instance, in a joint where many bars converge, increasing the anchorage length of some of the bars, in order to increase the forces which the bars resist, will probably reduce congestion which makes construction difficult, if not impossible. Ties in the joints may also increase anchorage capacities; however, ties add to congestion in the joint with the attendant problem of bar placement and concrete placement. Therefore, the factors which affect anchorage capacity of bent reinforcing bars must be determined to permit better analysis and more realistic and safe design of joints.

1.3. Previous Studies

There is very little information available concerning anchorage capacities of hooked or bent bars which can be used as a background for the present study. In most cases, bond stresses on straight bars were evaluated and bent-bar anchorages were of secondary importance.

The following brief summary of previous research will illustrate the lack of information of anchorage strenght of hooked deformed bars in joints.

Pullout Tests. Abrams (3) and Mylrea (4) conducted
tests on plain bar hooks. The results were generally inconclusive and now inapplicable because only plain bars were studied. Fishburn (5) conducted studies of bent bar anchorages in lightweight aggregate and reported that "for like slips and loads below the yield point of the steel, the loads carried by the bent bar anchorages of deformed bars were slightly greater than those carried by the straight embeddings of the same kind of the bar. In general, the superiority of the bent-bar anchorages over the straight embeddings tended to decrease with the increase in slip and with the increase in embedded length. At maximum load, and for like slips at load above the yield point, there was no appreciable difference in the loads supported by either the bent- or straight-bar anchorages of the same length and kind of deformed bar." Menzel (6) tested a few specimens with hooked deformed bars and reported that "the load-slip characteristics are influenced more by the factor of settlement than by variations in the shape (straight or hooked), embedded length or of the cross-section of the bar (round or square)." Hribar and Vasko (8) conducted tests of end anchorage of high strength bars and reported the existence of three zones which are: 1) - Linear pullout region where "when equivalent hook lengths are compared for a given bar size, a decrease in the moduli of anchorage results as the hook angle increases. This implies that as far as hook type is concerned, straight bars are more efficient at lower
loads when equivalent embeddings are compared. Specimen geometry serves to either spread the load through a larger volume of concrete (the case for a straight bar) or confines the load to a smaller volume (the case for a 180° hook). The higher stresses in the latter causes larger strains in the vicinity of the hook for a given applied load and results in a smaller slope in the initial linear region of the stress versus displacement curve." 2) - Region between cessation of linearity and proportional limit of the steel where "with increase in hook angle, a consistent decrease in the pullout resistance in the early slip region (prior to yielding of the steel) is observed when equivalent lengths of bar are compared." 3) - Ultimate failure where "for an equivalent length of embedment, the resistance to pullout increases as the hook angle increases."

The modulus of anchorage is defined as the ratio of stress to displacement at loaded end displacement of 0.001 in.

**Beam Tests.** A number of tests have been conducted on beams containing bent bar anchorages to determine the influence of anchorage characteristics on the shear and moment capacity of the beam. Taub and Neville (7) concluded that "when web reinforcement of a reinforced concrete beam exhibiting shear-tension failure is inadequate, hooks at the ends of main steel, consisting of plain bars, can materially
increase the load-carrying capacity of the beam. This increase is greater the less sufficient the web reinforcement of the beam. At the collapse of a beam in shear-tension the hooks disrupt the beam ends but this distinctive action of the hooks can be lessened by an increase in the strength of the concrete by provision of spiral reinforcements in the end of the beam, by increase of cover to the hooks, or by increase of the area over which the hooks act." Ferguson (14) reported the effect of end anchorage or hooks on the development length of high strength deformed bars in beams. In his report it is written that "a total of 13 specimens with end anchorages were tested, eight with straight and end anchorages and five with end hooks. During the testing period, the behavior of these beams, especially those with straight bar anchorage, appeared quite favorable." Menzel (6) concluded that: 1) - "Beams made with hooked deformed bars developed higher load-carrying capacity than corresponding beams with hooked plain bars." 2) - "There was little difference between the rate of deflection and the number and width of cracks in the mid-portions of beams made with straight and hooked bars, either plain or deformed. There appears to be less tendency for the development of diagonal tension cracks in beams with hooked bars than with bars without hooks but a greater tendency for disruption of concrete at the ends of the beams under high loads. Hence adequate web reinforcement is needed in the anchorage zones whether straight or hooked bars are used." 3) - "Hooks provide a
positive means for off-setting the loss of anchorage which normally results when unit bond resistances are lowered by settlement of concrete under bars with straight ends."

Splitting Tests. The resistance of the concrete around the bar to splitting, as well as the influence of lateral confinement on bond strength has also been examined by a few investigators. Leonhardt (10) discussed qualitatively the effect of lateral stresses. Robinson (9) observed that data was needed on anchorage strength of hooked bars and states in his concluding remarks that "transverse reinforcement placed in the compression zone and in the concrete surrounding tension reinforcement increases the shear strength of beams with web reinforcement. It also improves the strength of the anchorages of tension reinforcement by preventing longitudinal splitting of the concrete cover."

Untrauer and Henry (11) concluded that "the bond strength increases with increased normal pressure. The increase is approximately in proportion to the product of the square root of the normal pressure and the square root of the compressive strength of the concrete."

Joints. Bertero and McClure (12) have shown the importance of anchorage capacity at joints in a series of frame tests. Several models of a simple one story, single bay rigid frame were subjected to repeated alternating overloads. They reported that the bond strength at critical
sections was reduced due to longitudinal splitting of the concrete. However, the ultimate strength was not affected because special bearing plates were used, which provided anchorage at the joints. They state that "the above results are very important because they indicate that in cases where it should not be possible to provide mechanical anchorage for the steel it would be necessary to investigate another way to avoid the bond failure of the steel beyond the critical sections. It seems that it is not only a question of providing length of anchorage greater than 20 diameters, as is called for the 1956 ACI Code, but also a necessity to prevent the splitting of the concrete. From a practical viewpoint, the possibility of having considerable reduction in bond resistance has to be carefully considered in detailing the reinforcement whenever anchorage plates can not be provided." Full scale tests of the seismic resistance of beam-column joints at the PCA (21) provide some information on anchorage capacity of bars in a typical joint. For the types of anchorage used (hooked bars) it is reported that "although the steel yield zone penetrated into the columns in some of the tests, the bars did not show a serious loss in bond." The effect of axial loads on concrete cover in the anchorage capacity of the bars was not evaluated. In the analysis of the hooks only the shearing forces acting in the joint are taken into account.

The results of the various studies summarized above do not permit the establishment of exact guides for
determining anchorage capacities. However, the reported studies do provide information regarding the important parameters to be considered in an investigation of hooked bar anchorages.

There are two studies on the subject which will be described with slightly more detail since they are directly related with the present study. The first is Minor's (13) study, where the geometric factors which influence the anchorage capacities of bent deformed bars were studied. A series of eighty specimens containing reinforcing bars bent to different geometric configurations were tested to determine the effect of bond length, bend angle, and bar diameter on the response and strength of hooks. A diagram with dimensions of the test specimens used is shown in Figure 2.

The variables included bar diameters, 5/8, 7/8 and 1-1/8 inch, bond length to diameter ratios (l/d) ranging from 2.4 to 9.6 and bend angles ranging from 0° to 180° with inside radius to bar diameter ratio (r/d) varying from 1.6 to 4.6.

The experimental program consisted primarily of measuring slips between the reinforcing bar and concrete of each specimen at 36 load stages before yield of the bar. The slips were measured at three points, located at the loaded end, unloaded end and an intermediate point. The influence of geometric parameters on the anchorage behavior including initial stiffness, slip at a given stress and ultimate strength were discussed and the following conclusions were
reached:

1. For equal bond length to bar diameter ratios, bent-bar anchorages slip more at a given bar stress than straight bars.

2. For hooks of equal bond length to bar diameter ratios, hooks with larger angles of bend tend to slip more at a given bar stress.

3. For hooks of equal bond length to bar diameter ratios, hooks with smaller inside radius to bar diameter ratios tend to slip more at a given bar stress.

4. Where a bent bar anchorage consists of straight and curved sections, most of the slip is developed in the curved section.

5. Hooks have little effect on ultimate strength except for very short bond lengths. The second is Marques' (16) study where some of the factors which influence the anchorage capacities of bend deformed reinforcing bars in joints of reinforced concrete structures were examined. Ten full scale specimens each simulating an exterior beam-column joint in a structure, were tested in order to evaluate the anchorage capacity of the beam bars subjected to varying degrees of confinement and bent to different geometric configurations.

High strength deformed bars (ASTM A-615 Grade 60) with diameter of 7/8 inch were used in the tests. The hooks that were used, both 90° and 180°, conformed to ACI
specifications and recommendations. The following types of confinement were studied:

a) longitudinal column bars
b) column ties through the joint ranging from none to #3 bars @ 2.5 in.
c) concrete cover

Axial forces ranged from 270 kips to 550 kips.

The experimental program consisted primarily of measuring the slip between the bent deformed reinforcing bar and concrete at four points and the strains on the reinforcing bars at three points, at all the load stages until the failure of the specimen.

The influence of the different variables on the anchorage capacity of the bent deformed reinforcing bars were considered and the following conclusions were reached:

1. It appears that the axial load on the column does not influence the slip or the stress at failure significantly, and it seems that when steel confinement is present, the axial load seems to be detrimental to the slip behavior of the anchored bars.

2. The variation of the thickness of the side concrete cover does not seem to affect the slope of the bar stress-slip curve but it does affect the stress at failure. The smaller the thickness of the concrete cover the less the bar stresses were at failure and the more brittle the failure.
3. The effect of the confinement provided by the column bars depended upon their position with respect to the beam-bars. When the column bars were placed outside the beam-bars, the connection was more ductile (greater slips at failure) than when placed inside the beam-bars.

4. The presence of ties through the joint did not seem to increase the stress at failure in the beam bars. It appears that the ductility was slightly increased, but only up to a certain point. There seems to be a minimum spacing beyond which ties will not increase ductility.

5. The lead embedment length is the most important of all variables considered in the study. Even a very small lead embedment reduced the slip at given stresses and improved strength and ductility of the connection.

6. In general the 180° hook appeared to be less stiff than the 90° and also tended to result in a more brittle failure.

The cross sections and reinforcement of the column used in the above reported work, as well as the load system, instrumentation and material properties were exactly the same as the ones used in the present study and are described in Chapter II.

In Table 1 a summary of hook anchorage tests performed by Marques (16) is presented.

This chapter will be closed quoting a paragraph from the report of ACI Committee 408 (15), where it is stated that: "since bond stress distribution cannot be established on the
basis of the present knowledge of strains, experimental studies for each particular bond situation are necessary to predict numerical values accompanying failure. Only a few cases have been evaluated to date, and some of these have been more by sampling than by systematic exploration. Knowledge of the actual splitting forces developed by deformed bars and the resistance of members to splitting are needed for a more fundamental study."
CHAPTER 2

EXPERIMENTAL PROGRAM

2.1. Introduction

In order to evaluate the anchorage capacity of the hooked bars under realistic conditions, the following criteria had to be satisfied:

1. Specimens had to be full-scale models of beam-column joints in order to eliminate scale effects and to permit the use of fairly large diameter bars.

2. The specimen had to duplicate the constraints and loadings which would be expected on the region of the joint in which the bar was anchored.

3. Most of the hooked bars had to conform to current ACI standards for hook geometry.

2.2. Test Specimen

2.2.1. General Considerations

A diagram of the specimen selected to simulate a typical exterior beam-column joint is shown in Figures 3 and 4. In the specimens tested, the column cross section was
either 12 x 15 in. or 12 x 12 in. The assumed beam was 12 in. wide and 20 in. deep. The dimensions of the beam and the column were chosen so that the specimen would be a realistic full-scale simulation of a beam-column joint. In addition, it was desirable to select a specimen that was small enough to be easily handled and inexpensive to fabricate. The length of the column in all the tests was 50 in. The length was selected after taking into account the need for a small specimen and in addition the requirement that the column be long enough to permit embedment of the hooked bars, which in some cases extended into the column past the assumed beam. In addition, it was necessary to extend the column above and below the assumed beam in order to eliminate lateral constraints at the joint region produced by the axial loading heads.

To facilitate fabrications, the beams were not cast with the columns. The anchored bars extended past the face of the column and were connected to threaded rods which were loaded with hydraulic rams. The compression zone of the beam was simulated with a steel bearing plate against the face of the column over an area which approximated that of the compression zone of the assumed beam.

The column longitudinal reinforcement consisted of six #8 bars (ASTM A-615 Grade 60) in the 12 x 15 in. columns and four #8 bars (ASTM A-615 Grade 60) in the 12 x 12 in. columns. The transverse reinforcement consisted, in either
case, of #3 closed ties at 5 in. on center outside the joint. In some specimens the beam bars were placed outside the column bars in order to obtain information on the effect of confinement provided by the column bars. In other specimens ties were continued throughout the joint to provide information regarding the confining influence of lateral ties. In one specimen, the ties throughout the joint were provided with a cross tie. The clear cover over the ties was 1-1/2 in.

2.2.2. Variables

Table 2 summarizes the properties of the eighteen specimens tested in this study. Figure 5 shows plan and side views of the columns at the point where the beam-bars were anchored in the joint. The variables considered are discussed below.

1) Size of Anchored Bars. The tests were conducted with either #7 or #11 reinforcing beam-bars. The #7 bar represents a fairly realistic anchorage situation. The same cannot be said about a #11 reinforcing beam-bar anchored either in a 12 x 15 in. or 12 x 12 in. column though, to determine the limits of anchorage strength, it was felt that increasing the beam reinforcement to #11 bars would give an indication of the anchorage capacities of a fairly small joint. In Figure 5 are shown the details of the specimens
reinforced with #7 and #11 beam-bars, respectively (the specimens tested in Marques' (16) study are included).

2) **Hook Geometry.** Twelve tests were conducted using either 90° or 180° hooks conforming to the ACI 318-71 specifications and recommendations. For the 90° hook the extension beyond the bend, which will hereafter be referred to as the tail extension, was 12 bar diameters and for a 180° bend the tail portion was 4 bar diameters.

Examining the bar stress-slip curves (Figures 20 to 29) in Marques' (16) study, it can be seen that the bar did not slip at the end of the tail extension until failure was imminent. So, to have an assessment of the effect of the length of the tail extension on hook anchorage capacity, six tests were conducted with 90° hooks having tail extensions with half of the standard length, that is, reduced to 6 bar diameters. Three of the six specimens had #7 reinforcing beam-bars and the remaining three had #11 reinforcing beam-bars.

3) **Confinement.** Three types of confinement were considered.

(a) **Influence of the Longitudinal Column Bars on the Anchorage Strength.** To determine the influence of the column bars, tests were run with the column bars placed
outside the anchored beam-bars and comparison tests were conducted with the column bars placed inside the beam bars. In either case 2-7/8 in. concrete cover over the anchored bars was used. The specimens with the column bars outside the anchored bars represented what would normally be done in practice.

(b) **Influence of Column Ties Through the Joint.** In order to isolate the effect of the ties, the same column steel was retained, the column bars were placed inside the beam-bars and ties were placed throughout the joint. In this case the confinement consisted of #3 ties at either 5 in. or 2-1/2 in. spacing throughout the joint and a concrete cover of 2-7/8 in.

(c) **Influence of Concrete Cover.** To determine the effect of the concrete cover, the beam-bars were placed outside the column bars and the concrete cover over the beam-bars was reduced to 1-1/2 in.

4) **Lead Embedment Length.** By varying the size of the column, the lead embedment length before the hooked portion of the anchored bar was varied from 9.5 in. to 6.5 in. for the #7 bars and from 6 in. to 3 in. for the #11 bars.

No test was run with a #11 bar having a 180° bend in a 12 x 12 in. column, owing to the fact that the tail extension was so long that it did not fit within the column cross
section.

5) **Column Axial Load.** There is a general belief that column axial loads would increase the anchorage capacity of bars in joints. As has already been reported in Chapter 1, based upon the results of pullout tests, Untrauer and Henry (11) found an increase in bond strength with the increase of the normal pressure. Marques (16), using levels of axial load varying from 270 to 550 kips, reports that axial loads does not seem to affect the behavior of the anchored beam bars in joints and, in some cases, the axial load appears to be even detrimental to the slip behavior of the anchored bars. Thus, in order to determine the effect of normal stresses on the anchorage capacity, it was decided, in the present study, to keep the higher load at the level of 550 kips and reduce the lower load from 270 kips to 140 kips, which was expected to make more clear the effect of the column axial load on the behavior of the anchored bars. A load of 420 kips was used in the 12 x 12 in. column, in order to obtain the same level of stresses in the concrete and in the steel produced by the 550 kip load acting on the 12 x 15 in. columns. Those stresses are about 2500 psi in the concrete and 20 ksi in the steel.

Each specimen is designated by a group of letters and numbers, defining each one of the variables involved. For instance, specimen J11-90/1-15-1-L:
J11 - joint test with #11 reinforcing bars
90/1-90° hook with standard tail extension. If, instead of a standard tail extension, a half-tail extension was used, the hook was designated by 90/.5.
15 - column thickness in in.
1 - Type of confinement
L - level of the axial column load.

The different types of confinement are indicated in Tables 1 and 2 and are as follows:

1 - column bars plus 2-7/8 in. concrete cover
2 - only 2-7/8 in. concrete cover
3 - 2-7/8 in. concrete cover plus #3 ties at 5 in.
   throughout the joint
3a - 2-7/8 in. concrete cover plus #3 ties at 2-1/2
    in. throughout the joint
4  - Only 1-1/2 in. concrete cover
5  - 2-7/8 in. concrete cover plus #3 ties with a
    cross tie at 2-1/2 in. throughout the joint.

The different levels of axial load are indicated as follows:

H for 550 kips in 12 x 15 in. columns and
420 kips in 12 x 12 in. columns.
M for 270 kips and L for 140 kips.

As the results of the tests run in this study were
compared with the ones obtained by Marques (16), when referring to the latter tests, the same designations were used.

It should be noted that the geometry of the test specimens was selected so that a systematic evaluation of the variables affecting anchorage capacity could be carried out. The specimen J11-90/1-15-1-H was selected as a standard specimen. With the exception of the very small length of the lead embedment which was not sufficient, along with the hook, to develop the full strength of the bar by the provisions of ACI 318-71, the details of that specimen could be considered typical of practice in much of the United States. The column bars are outside of the anchored beam-bars, ties extend to the top and bottom of the beams, and the anchored bar includes a 90° hook with a tail extension conforming to ACI standards (17). In Marques' (16) study, the standard specimen was J7-90/1-15-1-H. All the other tests made possible the study of the effect of the variables considered, by comparison of their results with the results obtained in the standard tests.

2.2.3. Material Properties

Reinforcement. The deformations on the bars used throughout the study were the same and can be seen in Figure 6-b. The anchored bars, the longitudinal bars and ties conform with ASTM A-615 Grade 60.
Typical stress-strain diagrams for the #7 and #11 beam-bars (hooked bars) are shown in Figure 6-a.

**Concrete.** One basic mix was used throughout the study. The mix proportions were selected to give an average compressive strength, $f'_c$, of 4500 psi $\pm$ 10% at 14 to 21 days. All the specimens and control cylinders were cured at room conditions. Each specimen required casting two batches of concrete. The first batch completed the column to a level just below the horizontal portion of the anchored bars. The concrete strengths for both batches are listed in Table 2. As can be seen, the concrete strengths ranged, generally, from about 4.4 to 4.8 ksi. Few specimens varied from the nominal 4.5 ksi by values slightly greater than $\pm$ 10%.

2.3. **Loading System**

A load frame shown in Figure 7 and 8 was constructed to apply axial load to the column and tensile and compressive beam forces in the joint.

To load the column, four 100-ton center hole hydraulic rams (A) were used. The rams were placed below the bottom support platform and 2-1/8 in. alloy steel rods (B), were passed through the rams and bottom platform along the side faces of the column to the top head. The alloy steel rods were instrumented with strain gages and served as load cells to monitor the axial load. The rams were run with an
electric hydraulic pump (C). The top head and the bottom platform were fabricated using a series of channels to provide a rigid loading surface and distribute the load uniformly over the column section.

The anchored beam-bars protruded from the column about 12 in. Threaded rods were attached to the reinforcing bars using a Cadweld* splice. Two center hole hydraulic rams (D), with 60 tons capacity, were placed on the threaded rods and reacted against a column made up of two 18 in. channels. The reaction column transferred a compressive load to the test column through a large base section (E), to simulate the compressive zone of the beam. To equalize the overturning moment on the reaction column, threaded rods connected the bottom of the reaction column to the large channels supporting the bottom platform. In addition, it was necessary to provide a horizontal reaction (F) at the top of the test column to balance the moment imposed by the simulated beam. With this system the compressive beam force was about 20% greater than the tensile beam force, however, this was not considered to be detrimental to the behavior of the test specimen and was felt to be warranted considering the resulting simplicity of the test specimen.

*Registered Trademark, Erico Products, Inc.
2.4. **Instrumentation**

2.4.1. **Slip**

In order to measure the slip of the bar relative to the concrete, the following procedure was used. A 0.059 inch diameter piano wire was attached to the bar by making a short 90° bend at the end of the wire and inserting it into a drilled hole of equal diameter at the points where slip was to be determined. The wire was long enough to reach outside the form or surface of the specimen. The initial portion of the wire was maintained parallel to the displacement which was to be measured. In Figure 9 the points where the slips were measured and the expected directions of movement are shown. In all tests slip was measured at four points along the length of the anchored bars.

A neoprene tube was placed over the entire length of the piano wire to prevent bond. A small amount of permagum (sealer) was placed at the connection between the wire and the anchored bar to seal the tube and allow movement of the wire in the direction it was expected to travel. The sealer was extremely important to prevent interlock between the protruding wire and the concrete. The amount of permagum used was small, roughly 1/8 of a square inch of bar surface area, and the reduction of bond area was considered negligible. Figure 10 shows the wires and neoprene tubes in place on a specimen in the form prior to casting.
To reduce the wobble of the wire within the tube, the piano wire was placed in tension using a spring between the concrete surface and a small brass block fastened to the wire with a set screw. The installation is shown in Figure 11. This technique was developed by Minor (13) for his studies of hooked bars.

To measure the movement of the wires, dial gages with an accuracy of 0.0001 inch were mounted on a wood block which was fastened to the surface of the column with Eastman 910 adhesive, as shown in Figure 12. The dial gages rested against the brass block which was attached to the end of the wires.

The choice of the back surface as a reference surface instead of an externally supported reference was mainly based upon the following facts:

1) In most of the tests the back surface of the column remained virtually uncracked up to the moment of failure.

2) The readings were not affected by the deflections of the column which were appreciable in some of the tests run with #11 bars.

2.4.2. Bar Stresses

To measure the stresses in the anchored reinforcing bars, three electrical strain gages were mounted on each bar. Two were mounted on the bars and waterproofed prior to
casting and the third gage was mounted just prior to testing. Figure 9 shows the points where the strain gages were mounted. The gages on the protruding bars gave an estimate of the stress transfer from the bar to the concrete along the length of the bar.

2.5. **Test Procedure**

**Placement of the Specimen.** The specimens were seated in the test frame with a gypsum compound (Hydrocal) to reduce stress concentration between the specimen and the support platform and top loading head. After the column was seated in the test frame, the box section which transferred compressive beam forces to the column was seated with Hydrocal against the column face.

**Loading Sequence.** Before any load was applied to the anchored bars, the column was first loaded to the full axial load specified, which was held constant throughout the test. This could be considered to duplicate a common situation in a structure in which the column load remains nearly constant while the beam moment increases.

The reinforcing bars of the assumed beam were loaded in increments of roughly 2000 psi which provided nearly 30 load stages before yield. The stress increment was equivalent to a load of about 1200 and 3100 pounds for #7 and #11 bars, respectively.
The load increments were applied continuously at two-minute intervals. The load pattern could be considered equivalent to roughly 8000 psi per eight minute period and it was continued until termination of the test.

Crack patterns were marked at all load stages and photographs taken at critical states. The test was terminated when one of the anchored bars ceased to hold the load. In general, failure was sudden and brittle and resulted in the entire side face of the column spalling off.
CHAPTER 3

EVALUATION OF MEASURED RESULTS

3.1. **Introduction**

Measured data collected from all tests run in the present study, as well as in Marques' (16) study, is presented graphically in Appendix A. Three types of curves are plotted:

(a) Lead bar stress versus slip at points shown in Figure 9, are shown in Figures A-1 through A-28.
(b) Stress at start of hook versus slip at start of hook, shown in Figures A-29 through A-37.
(c) Stress before (BEF) and after (AFT) the hook versus lead bar stress, shown in Figures A-38 through A-47.

For each specimen and each kind of curve, curves are plotted for both bars. Although a more general and deep discussion of the results is presented in the Appendix A, the more salient trends shown in Figures A-1 through A-47 will subsequently be described:

1. In agreement with trends noted in Minor's (13) study, most of the slip takes place over the lead bar
straight embedment and curved portion of the hooked bar. In all cases, the lead slip (point 1H) was the greatest recorded. If the lead embedment was very small, like in the specimens with #11 beam-bars, the values of the recorded slip at point 2H was, in some cases, almost as large as the recorded lead slip. For most of the specimens whose beam-bars had hooks conform to ACI specifications and recommendations, the slip at points 3H, 3V and 4H or 4V was very small, being at points 4H or 4V, not measurable in most of the test until just prior to failure, or, in a few tests, not measurable at all. The specimens having hooked bars with half tail extension, mainly the ones with #11 hooked bars and low axial load on the column, showed a fairly large slip at point 4V. Though, the recorded slip does not represent the true vertical slip of the unloaded end but rather the circular displacement of point 4V with a radius close to the length of the half tail and center somewhere near to points 3H and 3V. This can be understood looking, for instance, at Figure A-28, where it can be seen that the movement at point 4V starts at much lower stresses than at points 3H and 3V and the last recorded slip at 4V is ten times greater than the last one recorded at 3V. Furthermore it can be seen, looking at the picture of specimen J11-90/.5-15-1-L included in Figure 34, that the specimen failed with the half tail extension kicking back.

2. Taking as positive displacement of 4V the one
shown at Figure 9, the measured slip indicates that in three
tests out of four runs with 180° hooks, the hook pulled
toward the front face of the specimen rather than around the
bend, as it is shown in Figures A4, A5, A18 and A19.

3. The initial stiffness (stress/slip) up to levels
about 30 kis of #7 bars was nearly three times greater than
that of #11 bars. At failure the lead slip of the #7 bars
was two to three times greater than that of #11 bars, the
greatest slip at failure of the #7 bars (J7-90/1-15-3-H)
was nearly four and one-half times greater than the greatest
slip at failure of the #11 bars (J11-90/1-15-3-L).

4. The difference between the lead stresses and the
stresses before the hook, i.e., the stress transfer along
the lead embedment, varied about 20 to 40 Ksi for the #7
bars and was practically negligible for the #11 bars, as is
shown in Figures A-38 through A-47 with the curves design-
nated with BEF.

5. As shown in Figures A-38 through A-47 with the
curves designated with AFT, the stress transferred to the
tail extension was generally small, normally less than 20
ksi until near failure. When failure was imminent, the
stresses transferred to the tail extension increased at very
fast rates, and the curves show a sudden and sharp increase
in slope.

In Table 3 the measured slip behavior of all speci-
mens is summarized and the following quantities are listed:
the applied lead stresses at lead slips of 0.005, 0.016 and 0.05 in., the slip at points 1H and 2H under applied bar stresses at 0.6 of the computed strength of the anchorage, the approximate slip at failure and the stress at failure.

The stress levels at lead slip of 0.005 in. indicate that in the J7 series the stresses are nearly the same except the two specimens with a 12 x 12 in. column and three with a 12 x 15 in. column, one of these having a 180° hook and the two remaining ones having a 90° hook with half tail extension (J7-90/.5-15-1-H and J7-90/.5-15-1-L). For these five specimens the stresses were about 60 to 70 percent of the measured stresses in the other specimens of the J7 series, which were between 30 and 40 ksi. At 0.016 in lead slip the measured stresses were between 50 and 60 ksi with the exception of the two specimens with the 12 x 12 inch column and two with a 12 x 15 inch column, one of these having a 180° hook and the other having a 90° hook with half tail extension (J7-90/.5-15-1-L). Again, for these four specimens, the stresses were about 60 to 70 percent of the measured stresses in the other specimens. At 0.05 in lead slip the measured stresses were between 70 and 80 ksi for all the specimens but the two with 12 x 12 inch columns, which had stresses about 60 ksi.

The stresses at failure for the #7 bars were nearly double those measured for the #11 bars. For #7 bars the lead slip at failure ranged from 0.052 to 0.418 inch, with
most of the specimens failing at lead slips beyond 0.140 inch. The #11 bars failed at lower lead slip values, which ranged from 0.028 to 0.091 inch.

In Table 3, the computed values of the strength of the anchorage, considering both hook and lead embedment, were obtained using the provisions given in Chapter 12, Sections 12.5 and 12.8 of ACI 318-71 (17). The value of the slips at 1H and 2H at stress levels of 0.6 times the computed strength of the anchorage will give a measure of the serviceability of the hooked bar, according to the suggested simplification made in Chapter 10, Section 10.63 of ACI 318-71 (17). The stress levels at 0.016 inch of lead slip were also listed in Table 3 in order to give a measure of the serviceability of the hooked bar, according to Chapter 10, Section 10.6.A of ACI Committee 318 Report (18), if it is assumed that the crack width in the beam column joint is nearly equal to the slip of the anchored beam-bar.

In order to evaluate the influence of each variable considered in this study, average values for the two bars of a specimen are compared in each case. The lead bar stress-slip curves used for the comparisons are the curves for the lead slip only.

3.2. Effect of Axial Load

In Figure 14 seven specimens are compared being three
of the J7 series and the remaining four of the J11 series. All seven specimens had the same lateral confinement, column bars outside the beam-bars and 2-7/8 inches concrete cover. The axial load varied from 140 to 550 kips. In both series, for all specimens with a 90° hook, the stress-slip curves are virtually identical. The specimens with a 180° hook show an appreciable difference, which is not very easy to explain owing to the fact that out of the 18 tests run in this study and 10 in Marques' (16) study, only four specimens with a 180° hook were tested. However, as in all specimens the plan of the hook was oriented in the direction of the axial load, in the case of the 180° hook there is an area of concrete enclosed by the hook while in the case of a 90° hook there is no concrete enclosed, as can be seen in the following sketch:

Thus, in the case of the 180° hook, the higher the axial forces the higher will be the stress concentration at the interface between the enclosed concrete and the hook, which
will eventually cause the hook to slip at earlier load stages. In other words, it seems that the 180° hook, by the reasons pointed out above, is more sensitive to the different levels of axial loads than the 90° hook.

In Figure 15 four specimens with the same lateral confinement are compared. In all four specimens the confinement consisted of only 2-7/8 inches of concrete cover and the beam-bars had a 90° hook. Once more, the stress-slip curves are virtually the same.

Figure 16 shows, for the J7 series, the curves obtained by plotting the stresses after and before the hook versus the lead bar stresses. The top curves are for the specimens with confinement consisting of column bars plus 2-7/8 inches of concrete cover and the bottom curves for the specimens with confinement consisting only of 2-7/8 inches of concrete cover. With the exception of J7-90/1-15-1-M all the curves are, for the two types of confinement and different levels of axial load, nearly equal. In Figure 17 the specimens of the J11 series are compared and it can be seen that the curves are essentially identical.

Based on the stresses and slip measurements for the tests in which axial loads were varied, the influence of column axial load appears to be negligible, seeming to be even detrimental when the anchored beam-bars are placed inside the longitudinal reinforcement of the column. The longitudinal column bars, mainly the ones that are closer to
the front face of the column, seem to be forced outward by lateral forces developed as the beam bars are pulled out of the joint. This is illustrated in Figures 33 through 35. Thus, when the axial load is increased, the stresses in the reinforcing column bars and the lateral strains in the column will increase and tend to increase the proclivity of the column bars to deform outward. Consequently, under larger axial loads the anchored beam-bars will slip more and the specimen eventually will fail with the beam-bars at lower stresses than for low axial loads. As can be seen in Table 3, all the specimens with the anchored beam-bars placed inside the reinforcing column bars and with low level of axial load, failed with the beam-bars at higher stresses than in the case of the corresponding specimens with high level of axial loads (J7-90/1-15-1-H and J7-90/1-15-1-M; J11-90/1-15-1-H and J11-90/1-15-1-L; J11-180-15-1-H and J11-180-15-1-L). The same trend has been previously observed in Marques' (16) study.

3.3. **Effect of Bend Angle and Lead Embedment**

Figure 18 shows the effect of bend angle and lead embedment on the slip at point 1H for hooked bars. In the nine tests where slip behavior is compared in Figure 18, the confinement and axial force remained constant, the variables being the bend geometry and the length of the lead embedment (the embedded straight portion of the beam-bars before the
hook) which was achieved by varying the thickness of the column.

The curves do not fully confirm the trend observed by Minor (13), that 90° hooks tend to be stiffer (slip less at given stresses) than 180° hooks, though, as can be seen in Table 3, all specimens with reinforcing beam-bars bent to 180° failed at lower stresses than the corresponding specimens with 90° hooks. Furthermore, all the specimens whose beam-bars were bent to 180° had a more brittle failure than the ones with the beam bars bent to 90°. It also must be pointed out that after failure, as can be seen in Figures 33 through 35, in all specimens with 180° hooks, the beam-bar that failed was with the hook uncovered, that is, completely loose, while most of the specimens with 90° standard hooks had, after failure, at least the tail extension still firmly bonded in the concrete.

For the J7 series Figure 19 shows the influence of bend angle on stresses after and before the hook. The curves representing the variation of the stresses before the hook with the variation of the lead stresses are practically the same. The curves representing the variation of the stresses after the hook are contradictory.

For the J11 series the same curves are shown at the top of Figure 21 and in both cases the two curves, after and before, are very much alike.

As far as the lead embedment is concerned, for the J7
series it is clear (Figures 18 and 19) that the length of
the lead embedment influenced significantly the strength and
stiffness of the hook anchorages. The beam-bars anchored in
12 x 15 inch columns reached stresses, at failure, greater
than 90 ksi. Those in 12 x 12 inch columns reached stresses,
at failure, just over 60 ksi. Figure 20 shows the influence
of the lead embedment on the stresses for the J7 series. The
stresses, in both cases (90° and 180° hooks) with the short
lead embedment, increased at a very high rate.

For the J11 series the lead embedment to bar diameter
ratio was very small and in either case, 12 x 12 inch or 12
x 15 inch columns, only a very small amount of stress was
transferred to the concrete, as shown in Figures A-45 through
A-47. As shown in Figure 18, the lead slip was always
greater for the same lead bar stress in the J11-90/1-12-1-H
than in the J11-90/1-15-1-H, and the latter specimen failed
with the beam-bars at higher stress. At the bottom of
Figure 21 is shown the influence of the lead embedment on
stresses for the J11 series. The curves representing the
stresses before the hook are practically the same in both
cases but for the stresses after the hook, the curve repre-
senting the specimen whose column was 12 x 12 inches, shows
a very rapid increase of stresses. It must be pointed out
that the difference in lead embedment provides not only ad-
ditional length for stress transfer before the hook but
also, and more important, when large bars are used, it pro-
vides increased area of concrete which improves the lateral
restraint against splitting as the bar is loaded.

3.4. **Effect of Lateral Confinement**

Figure 22 shows the effect of confinement on lead slip for the J7 series. On all five specimens the axial load was kept constant, and the confinement was varied. All the five curves are within a very narrow band, which indicates that there was a very small difference in slip behavior for all the specimens but J7-90/1-15-5-H, whose confinement was only 1-1/2 inches of concrete cover. Yet, it should be noted that such a severe reduction of the concrete cover did not change the shape of the curve but did reduce the stress and slip at failure. Stress at failure was about 75 percent and slip around 1/4 of that in the other four tests.

As has already been said before, the other four specimens compared did not show a great deal of difference in slip behavior. There was a slight increase in stresses at a given slip when the beam-bars were placed inside the column bars. Ties throughout the joint with the beam-bars placed inside the column bars produced a slight increase of stresses at a given slip. Yet, it seems that neither the location of the beam-bars nor the placement of ties throughout the joint appreciably influenced the stress-slip behavior. It might be worth pointing out that the behavior of specimen J7-90/1-15-3a-H would probably have been different if the concrete strength had not been so low for this specimen. As listed in Table 1, the value of $f'_c$ for J7-90/1-15-3a-H was about 3.8 ksi while for specimen
J7-90/1-15-3-H $f_c'$ was about 4.7 ksi. For the J7 series the effect of confinement on stresses is shown at the top of Figure 24. The curves confirm the trends indicated by the lead stress - lead slip curves.

The response of #11 bars to lateral confinement is shown in Figure 23. It is apparent that the position of the beam-bars outside or inside of the column bars did not appreciably change the lead bar stress-lead slip curves, either with high level of axial load or with low level of axial load on the column. It seems that the location of the column bars offers very little, if any, lateral restraint. The fact might be due to the very low stiffness of #8 bars unsupported by ties over a 2 foot length. It might be possible that the combination of column bars outside the hooked bars and ties throughout the joint would have improved further the slip behavior, owing to the fact that ties supporting the longitudinal bars would improve the stiffness of those bars. Yet, this combination was not considered in the present study.

Once again, a severe reduction of concrete cover from 2-7/8 inches to 1-1/2 inches did not change the shape of the lead bar stress - lead slip curve but did reduce the stress and slip at failure. Even though the confinement in this case was only 1-1/2 inches of concrete cover (J11-90/1-15-4-L) and the ratio of thickness of concrete cover to bar diameter was practically equal to one, the value of the measured anchorage strength was nearly 35 percent greater
than the value obtained using the ACI 318-71 (17) provision.

Ties throughout the joint did improve appreciably the strength of the joint. All the specimens where ties were present through the joint failed after the beam-bar reached yield, whereas all the others without ties through the joint failed at stresses of about 50 ksi, at most. The curves for specimens J11-90/1-15-3a-L and J11-90/1-15-5-L are practically the same, and the stresses at failure are quite close. The only difference is that the slip at failure for the latter specimen was about 30 percent greater than the slip at failure for the former specimen. It seems that the cross tie provided a little more ductility (larger slips at failure). The effect of confinement on stresses is shown in Figure 24 at the bottom, where the same trends are indicated. At given stresses, ties throughout the joint reduced the stresses after as well as before the hook. The results show that for the #11 hooked bars the ductility as well as the strength did really improve with the placement of the ties throughout the joint either without or with cross ties.

3.5. **Effect of the Reduction of the Tail Extension**

Figure 25 shows the curves of the slip behavior for the specimens reinforced with beam-bars having 90° hooks with half tail extension. It can be seen that, in this case, the placement of the beam-bars inside the column bars and a high level of axial load improved appreciably the slip and stress at failure, in both cases, that is, for the J7 series and the J11 series. With low axial load the specimens that
showed less ductility were, for the J7 series as well as for
the J11 series, the ones with the beam-bars inside the col-
umn bars and a low level of axial load on the column.
Figure 26 shows the effect of the reduction of the tail ex-
tension on stress. The trends are the same.

In Figures 27, 29 and 31 the slip behavior of the
specimens reinforced with beam bars having hooks with half
tail extension is compared with the corresponding specimens
with standard hooks. As in Marques' (16) study there was
no J7-90/1-15-2-L, the specimen J7-90/.5-15-2-L has been com-
pared with J7-90/1-15-2-M, which is not unreasonable due to
the fact that, as has already been said, the differences in
axial load did not seem, in the tests run with specimens
reinforced with beam-bars having standard hooks, to alter
very much the slip behavior. In Figures 28, 30 and 32 where
the stresses after and before the hook are compared for the
corresponding specimens, specimens where all variables but
one were kept constant, confirm the trends observed by com-
paring the lead bar stress - lead slip curves. From the
test results it seems that, when the length of the tail ex-
tension is reduced, part of the hook action is lost, that is,
when the beam-bars are being pulled, the unloaded end of the
tail starts moving on a circle whose radius is nearly equal
to its length and with the center of rotation somewhere close
to points 3V, 3H (Figure 9). Thus, the hook starts its move-
ment outward, as the tail kicks back due to the increase of
its stiffness, and will rather move sliding through the
concrete sleeve than crushing the concrete underneath the bend, as it did when standard hooks were used. Figures A-12, A-13, A-14, A-26, A-27 and A-28 show the lead bar stress-slip for all five points at the hook (Figure 9). For both series, J7 and J11, when the tests were run with low levels of axial load and with the beam-bars either inside or outside of the column bars, the respective figures show that the movement of point 4V started at lower lead bar stresses than the movement at points 3H and 3V and at failure the slip of 4V is greater than 3V, which can only be explained by the rotation of the tail extension. The fact is much more salient in the J11 series, due most probably to the much larger value of the stiffness for the tail extension of #11 bars. In order to clarify the above explanation, a sketch is shown:

![Diagram](image_url)

The specimen which had the beam-bars inside the column bars and a high level of axial load perform as well as the corresponding specimens with standard hooks, which might
possible be explained by the proximity of the tail extension of the hook to the compressed column bars. The tail section of the hooked bars was anchored immediately adjacent to a #8 compressed column bar and stress may have been transferred directly from the column bar to the tail extension, which, as a result, performed more efficiently. In the case of the #11 bar, the tail section was also anchored next to the column bar. Yet, as the #11 bar is much larger than the #8 column bar, the stress transferred from the column bar to the tail extension was not as significant as in the J7 series. Though, as shown in Figure 27, the lead bar stress - lead slip curves, even for the J11 series are very much alike.

Summarizing, it can be said that the performance of the specimens with hooked bars having half tail extensions was not too poor. In the J7 series all the specimens reached stresses, at failure, far beyond the yield, the specimens with low levels of axial load on the column failing at stresses about 20 percent less than the corresponding specimens with standard hooks. In the J11 series, the specimens with low level axial load in the column also failed at stress about 20 percent less than the corresponding specimens with standard hooks. In both series, the specimen that showed less ductility was the one with the beam-bars inside the column bars and a low level of axial load. Those specimens, as has already been said before, failed at stresses about 20 percent less than the corresponding ones with the standard
hooks but the slips at failure were about one-half of the slips at failure of the corresponding specimens with standard hooks.

In Table 3 the stresses at given slips as well as the stress and slip at failure are also listed for the specimens with the hooks having half tail extensions. These specimens have not been included in Table 4, because there are no provisions, in the Building Code Requirements for Reinforced Concrete (17), for hooks with the tail extension reduced. Table 5 summarizes the effect of the reduction of tail extension on lead slip behavior.

3.6. Stress-Slip Curve for Hooks

Figures A-29 through A-37 show the curves which were obtained by plotting the measured stresses at the end of the straight lead embedment versus the measured slip at point 2H (at the start of the hook). With those curves, the stress-slip behavior of the hooks used in this study can approximately be estimated. The curves generally duplicate the trends observed for the lead-slip. Comparison curves are not presented because they would not make the present work more clear. It should be pointed out that the measured stresses before and after the hook were influenced by bending and therefore can not be considered accurate over the whole range of loading.

For the J7 series the following can be seen:
(a) Figure A-30--the performance of hooks in the 12 x 12 inch column is poor when compared with the hooks in 12 x 15 inch columns.

(b) Figures A-31 and A-33--the performance of standard hooks and hooks with the tail extension reduced to half of the standard length are very much alike.

(c) Figure A-32--the large slip measured for J7-90/1-15-3a-H might be due either to deficient placement of the concrete in the joint owing to the congestion produced by the close spaced ties or a reflection of the lower concrete strength of that specimen, mainly if compared with the measured slip behavior for specimen J7-90/1-15-3-H.

For the J11 series, as has already been seen using the lead bar stress - lead slip curves, the most salient result is the appreciable reduction in slip at given stress levels for the specimens where closely spaced ties throughout the joint were used.

3.7. Mode of Failure

In most of the tests the sequence of cracking and subsequent failure followed a similar pattern. Cracking always began on the front face of the column, just below and above the protruding bars. These cracks always progressed towards the corners of the column, forming a V, as described in
Bond Stress - The State of the Art (15) for a very wide beam. The first crack appeared at early load stages. At intermediate load stages, cracks parallel to the vertical corners of the column appeared on the side face of the column about 2 inches from the corner. The vertical cracks on the front face extended to the compression zone of the beam. At high load stages cracks began to appear on all four faces. In many tests a horizontal crack appeared in a plane through the column at the level of the beam-bars. Figure 13 shows typical crack patterns prior to failure.

At high stress levels, when a load increment was applied, slip continued to increase while lead bar stresses tended to reduce slightly. In some cases failure occurred while the load was being held or was dropping with slip increasing steadily.

The test was terminated when one of the anchored bars could not hold the load anymore, which was normally followed by complete and sudden failure of the specimen, with the entire side cover spalling away to the level of the anchored bars and the value of the load dropping suddenly from its previous level.

Generally the failure patterns did not differ too much in specimens with the hooked beam-bars inside or outside the column bars. In most tests a very large area of the side face of the column was destroyed, as can be seen in the photographs included in Figures 33 through 35 which
provide an idea of the type of failure observed. Of all the specimens tested in both series, specimens J7-90/1-15-4-H and J11-90/1-15-4-H which had, as confinement, only 1-1/2 inches of concrete cover over the anchored beam-bars, were the ones that had the smallest area at side cover spalled off.

In Figures 36 and 37 a few close-ups of the zone around the hook are shown. It can be clearly seen that there is a gap between the outer surface of the hook and the concrete which was crushed at the inner surface. That can be explained by the tendency of the bent portions, when the hook is being loaded, to straighten, pulling the bar towards the center of the bend. This produces high lateral compressive stresses at the bend, traveling towards the unloaded end with the increase of the bar stresses due to the applied load increments, which in effect "punch out" the side cover at the bend forcing the entire side to spall away.

Specimens J11-90/1-15-3a-L and J11-90/1-15-5-L did not fail by loss of cover. These tests had been stopped because the flexural cracks became very wide and column distortion very large.

It was felt, prior to testing, that shear probably would influence the failure in the specimen. Even though shear cracks did appear in the specimens of the J11 series with a low level of axial loads on the column, in none of the tests was shear considered to be a contributing factor
in producing failure.

Specimens reinforced with beam bars having 180° hooks or 90° hooks having tail extensions reduced to a half of the standard length had a more brittle failure than all the other specimens.

The mode of failure was different for the two types of specimens. The specimens with standard hooks failed when the side face of the column spalled off, with the exception of one test specimen which failed like the specimens with the tail extension reduced to half of the standard length.

The specimens reinforced with beam-bars having hooks with half tail extensions all failed with the tail extensions kicking back and destroying the back of the column over its full length at the same time the side face spalled off. These specimens showed a much more brittle failure and the beam-bar that failed was usually totally uncovered at failure.

3.8. Failure Hypothesis for Hooked Bars in Beam-Column Joints

After observing the modes of failure and analyzing the data, a reasonable explanation of the failure phenomena of a hooked bar in joints can be made. In all but two specimens tested it was clear that failure was governed by loss of cover rather than by pulling out of the bar. The failure was sudden with the hooked bars suddenly and almost
instantaneously losing their load carrying capacity. As has already been said, specimens J11-90/1-15-5a-H and J11-90/1-15-5-H were showing deformations so large and cracks so wide due to flexure that the tests had to be stopped in order to protect the testing material.

In the J7 series, where the length of the lead embedment was appreciable, the increase of lead slip decreased the stress transfer capacity along the straight embedment. At the same time, the increase in lead slip produced splitting of the side cover at the lead end of the anchorage, which progressed backward towards the bent part of the anchored beam-bars. In the J11 series the effect of the lead embedment was negligible. When the bend started to be loaded with appreciable loads transferred by the lead embedment, very large compressive stresses at the inner surface of the hook were produced which also tended to split the cover. Near failure the hook acted as a wedge making the concrete cover spall off. This action produced either an outward deformation of the ties, when they were present, or an outward deformation of the unsupported longitudinal column bars, when the beam-bars were placed inside the column bars, increasing the instability of the bars, mainly those close to the front face of the column, under axial loads.

Now that a failure hypothesis has been established, based upon the test results, the effect of each one of the
variables, considered in this study, on the anchorage capacity of hooked bars in joints can be better understood.

1. Effect of Axial Force: while appearing beneficial, the axial load produces lateral strains in the same direction of the strains produced by the hooked bars. Furthermore, the action of the axial load on the concrete cover, just prior to failure, will have the same effect of inplane forces applied to opposite edges of a slab loaded with a transverse load distributed in a small area (that is similar to the punching out effect of the hook). In this case, the axial load will make the deflection increase very rapidly and will eventually make the slab fail by buckling. The specimens tested with high level of axial loads had, after failure, the concrete cover deflected outward at the level of the hooks.

2. Effect of Unrestrained Column Bars: column bars not restrained by ties through the joint are not stiff enough to withstand the very large strains which arise in the joint. The situation seems to be aggravated by high levels of axial load, which is reasonable owing to the fact that high levels of axial load tend to increase the proclivity of the column bars to deform outward.

3. Effect of Ties Carried Throughout the Joint: ties throughout the joint did improve the slip behavior, mainly in the J11 series. The closer the ties the more
efficient they were. The test results indicated that ties throughout the joint are most effective and consequently beneficial if the spacing is about equal to or less than the radius of the hook. In the J7 series, #3 ties spaced at 5 inches did improve slightly the performance of the anchorage and #3 ties spaced at 2-1/2 inches also improved it a little bit more. However, in both cases, the improvement was not considerable. In the J11 series #3 ties at 5 inches increased appreciably the strength and ductility of the anchorage and when the spacing was reduced to 2-1/2 inches the performance was substantially improved. It should be pointed out that in the J7 series the lead stresses at failure were high, beyond the yield point of the steel bars in all cases and it might have been possible to obtain more meaningful and definitive results with shorter straight lead embedments.

4. Effect of Concrete Cover: the reduction of concrete cover to 1-1/2 inches did decrease the strength and the ductility of the anchorage. In both series, the smallest slip at failure was recorded for this case. It is impossible to determine any limit value for the optimum thickness of concrete cover based on the very limited test results. Though it seems that 1-1/2 inches of cover, ties throughout the joint with a spacing about equal to or less than the radius of the hook and hooked bars placed inside of the column bars will be enough to prevent a localized failure of the side cover.
5. Effect of the Length of the Straight Lead Embedment: the analysis of the data shows that the performance of the hooked bars in 12 x 15 inch columns was very much improved compared to the performance of the anchorages in 12 x 12 columns. This can be attributed, in the J7 series, mainly to the added straight lead embedment. Yet, in the J11 series, the added straight lead embedment was negligible when compared with the bar diameter. Thus, for larger bar diameters, the increased thickness of the column provides more concrete area and, therefore, the total tensile force developed before the side cover spalls has obviously to be larger and consequently the performance of the anchorage is improved.
CHAPTER 4

SIGNIFICANCE OF RESULTS

4.1. Introduction

The design provision for hooked bar anchorages in ACI 318-71 (7) are mainly based on provisions appearing in previous codes. The Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-71) (18), states:

12.8. Standard hooks

Hook anchorage tests performed at Carnegie Mellon University (8) have reinforced the belief, reflected in the 1956 and 1963 ACI Codes, that the anchorage capacity of a hook in mass concrete is typically about the same as that of a straight bar with the same embedment length. Hooks in structural members are often placed relatively close to a free surface where splitting forces roughly proportional to the total bar pull may determine the hook capacity. Additional restraints, therefore, were imposed in updating the anchorage values of hooks from those in the 1963 Code. When $f'_c = 3000$ psi is used, (1) the tensile stress was limited to 0.5 $f_y$, which is similar to that allowed for Grade 40 steel in the 1963 Code, (2) the tensile force was limited in any bar to 50,000 lb, and (3) the average bond stress over the equivalent embedment length $a_e$ was limited to the allowable bond stress in the 1963 Code. Table 12.1 shows the maximum tensile force which is considered developed by a standard hook with $f'_c = 3000$ psi using the constants for standard hooks, $\xi$, of the code. The footnotes indicate the limits.

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TABLE 12-1
MAXIMUM TENSILE FORCE, POUNDS, DEVELOPED IN STANDARD HOOKS FOR $f'_c = 3000$ psi

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>$f_y = 60$ ksi</th>
<th></th>
<th>$f_y = 40$ ksi</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top Bar</td>
<td>Other Bars</td>
<td>Top Bars</td>
<td>Other Bars</td>
</tr>
<tr>
<td># 3</td>
<td>3,250*</td>
<td>3,250*</td>
<td>2,170*</td>
<td>2,170*</td>
</tr>
<tr>
<td># 4</td>
<td>5,920*</td>
<td>5,920*</td>
<td>3,940*</td>
<td>3,940*</td>
</tr>
<tr>
<td># 5</td>
<td>9,170**</td>
<td>9,170*</td>
<td>6,110*</td>
<td>6,110*</td>
</tr>
<tr>
<td># 6</td>
<td>10,850</td>
<td>13,020*</td>
<td>8,680*</td>
<td>8,680*</td>
</tr>
<tr>
<td># 7</td>
<td>11,830</td>
<td>17,750*</td>
<td>11,830*</td>
<td>11,830*</td>
</tr>
<tr>
<td># 8</td>
<td>15,580</td>
<td>23,370*</td>
<td>15,580*</td>
<td>15,580*</td>
</tr>
<tr>
<td># 9</td>
<td>19,720</td>
<td>29,580*</td>
<td>19,720*</td>
<td>19,720*</td>
</tr>
<tr>
<td>#10</td>
<td>25,040</td>
<td>33,390*</td>
<td>25,040*</td>
<td>25,040*</td>
</tr>
<tr>
<td>#11</td>
<td>30,760*</td>
<td>35,880</td>
<td>30,760**</td>
<td>30,760*</td>
</tr>
<tr>
<td>#14</td>
<td>40,660*</td>
<td>40,660</td>
<td>40,660*</td>
<td>40,660</td>
</tr>
<tr>
<td>#18</td>
<td>48,200++</td>
<td>48,200++</td>
<td>48,200++</td>
<td>48,200++</td>
</tr>
</tbody>
</table>

*Tensile stress slightly less than 0.5 $f_y$ for $f'_c = 3000$ psi.

*Bond stress based on equivalent development length, $L_e$, about equal to or less than specified in Section 1801 of ACI 318-63 for $f'_c = 3000$ psi.

which influenced the selection of $\xi$ values. The Code no longer includes the clause authorizing hooks to be evaluated as bar extensions.

As can be read above, only previous codes are mentioned and the recent pull out tests of hooked bars embedded in massive slabs (8), which add some more information. However, there is not too much information available regarding hooked bar strength in typical joints of reinforced concrete structures.
4.2. **ACI 318-71 Provisions Related with the Present Study**

The sections, or part of them, pertinent to the present study are quoted directly, as well as the corresponding sections, or parts of them, of the Commentary on Building Code Requirements for Reinforce Concrete (ACI 318-71).

Chapter 10. Flexure and Axial Loads

**Code**

10.6.3. When the design yield strength \( f_y \) for tension reinforcement exceeds 40,000 psi, the cross sections of maximum positive and negative moment shall be so proportioned that the quantity \( z \) given by

\[
a = f_s \sqrt[3]{d_c A}
\]

(10-2)

does not exceed the value given by Section 10.6.4. The calculated flexural stress in the reinforcement at service loads \( f_s \), in kips per square inch, shall be computed as the bending moment divided by the product of the steel area and the internal moment arm. In lieu of such computations, \( f_s \) may be taken as 60 percent of the specified strength \( f_y \).

10.6.4. The quantity \( z \) shall not exceed 175 kips per inch for interior exposure and 145 kips per inch for exterior exposure. Equation (10-2) does not apply to structures subjected to very aggressive exposure or designed to be watertight; special precautions are required and must be investigated for such cases.

The symbols used stand for:

\( A \) = effective tension area of concrete surrounding the main tension reinforcing bars and having the same centroid as that reinforcement, divided by the number of bars, square inch. When the main reinforcement consists of several bar sizes the number of bars shall be computed as the total steel area divided by the area of the largest bar used.
\( d_c \) = thickness of concrete cover measured from the extreme tension fiber to the center of the bar located closest thereto.

\( f_s \) = calculated stress in reinforcement at service loads, kis.

**Commentary**

10.6.4. The numerical limitations of \( z = 175 \) and 145 kips per inch for interior and exterior exposure, respectively, correspond to limiting crack widths of 0.016 and 0.013 inch.

**Chapter 12. Development of Reinforcement**

**Code**

12.5. Development length of deformed bars and deformed wire in tension.

The development length \( \lambda_d \), in inches, of deformed bars and deformed wire in tension shall be computed as the product of the basic development length of (a) and the applicable modification factor or factors of (b), (c) and (d), but \( \lambda_d \) shall not be less than 12 inches.

(a) The basic development length shall be: For #11 or smaller bars...0.04 \( A_{bfy} / \sqrt{f'_c} \) but not less than...0.004 \( d_{bfy} \).

(b) The basic development length shall be multiplied by the applicable factor or factors for:

Top reinforcement..........1.4.

The factors may be multiplied by an applicable factor or factors for:

Reinforcement being developed in the length under consideration and spaced laterally at least 6 inches on center and at least 3 inches from the side face of the member....0.8.

12.8. Standard hooks

12.8.1. Standard hooks shall be considered to develop a tensile stress in bar reinforcement \( f_h = \xi \sqrt{f'_c} \) where \( \xi \) is not greater than the values in Table 12.8.1. The value of \( \xi \) may be increased 30 percent where enclosure is provided perpendicular to the plane of the hook.
### Table 12.8.1

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>$f_y=60$ ksi</th>
<th>$f_y=40$ ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top Bars</td>
<td>Other Bars</td>
</tr>
<tr>
<td>#3 to #5</td>
<td>540</td>
<td>540</td>
</tr>
<tr>
<td>#6</td>
<td>450</td>
<td>540</td>
</tr>
<tr>
<td>#7 to #9</td>
<td>360</td>
<td>540</td>
</tr>
<tr>
<td>#10</td>
<td>360</td>
<td>480</td>
</tr>
<tr>
<td>#11</td>
<td>360</td>
<td>420</td>
</tr>
<tr>
<td>#14</td>
<td>330</td>
<td>330</td>
</tr>
<tr>
<td>#18</td>
<td>220</td>
<td>220</td>
</tr>
</tbody>
</table>

#### 12.8.2
An equivalent embedment length $\lambda_e$ shall be computed using the provisions of section 12.5 (a) by substituting $f_h$ for $f_y$ and $\lambda_e$ for $\lambda_d$.

#### 12.9. Combination development length

Development length $\lambda_d$ may consist of a combination of the equivalent embedment length of a hook or mechanical anchorage plus additional embedment length of the reinforcement.

The symbols used stand for:

- $A_p$ = area of an individual bar, square inch.
- $d_b$ = nominal diameter of bar, wire, or prestressing stand, inch.
- $\sqrt{f_{c}}$ = square root of specified compressive strength of concrete, psi.
- $f_h$ = tensile stress developed by a standard hook, psi.
- $\lambda_d$ = development length, inch.
- $\lambda_e$ = equivalent embedment length, inch.

#### Commentary

12.8. Standard hooks

Already quoted in 4.1.

Hereafter, the Building Code Requirements for Reinforced Concrete (ACI 318-71) (17) will be referred to as ACI Code and the Commentary on Building Code Requirements
for Reinforced Concrete (ACI 318-71) (18) as Commentary.

4.3. Measured and Computed Strength

The strength of the hooked bars was evaluated, based on data obtained from the tested specimens and also computed using the provisions of the ACI Code, which have been quoted in 4.2. The tensile stress developed by the hook was calculated and the contribution of the straight lead embedment was obtained manipulating the expression that gives the development length, which has been solved for $f_{x}$ in terms of a known anchorage length, becoming

$$f_{x} = \xi \sqrt{f_{c}} / 0.04A_{b}$$

The strength of the anchored hooked bars was obtained summing $f_{h}$ and $f_{x}$ and is listed in Tables 3 and 4. The values listed were obtained using for $\xi$ the numbers listed under "Other bars" (see 4.2), because, as the hook extends vertically, it is not too clear which value of $\xi$ should be used.

It must be pointed out that the recorded stresses at the start of the hook should only be considered as estimates of stresses, owing to the bending stresses in the bars as slip progressed toward the hook.

As listed in Table 4 the ratio $f_{h} \text{ meas.} / f_{h} \text{ comp.}$ varied from 1.59 to 1.93 in the J7 Series and from 1.58 to
2.32 in the J11 series and the ratio $f_{\text{meas.}}/f_{\text{comp.}}$ varied from 1.20 to 1.60 in the J7 series and from 1.28 to 1.89 in the J11 series. Thus, the results indicate that the ACI Code provisions are quite conservative, mainly when estimating the hook strength, at least when applied to beam-column joints like the ones whose test results are presented in this study. Also based on these results, it is difficult to accept the value of $f_y/2$ as a limit for $f_h$, as noted in the Commentary for section 12.8. The ratio $f_{\text{meas.}}/f_{\text{comp.}}$ was always lower, for the same specimen, than the ratio $f_h_{\text{meas.}}/f_h_{\text{comp.}}$. Yet, in no case was the stress underestimated. As the ratio of measured to computed hook stress $f_h$ is always, for the same specimen, less than the ratio of measured to computed anchorage stress $(f_h+f_s)$ it might be concluded that the values of $f_y$, computed using the ACI Code expression, for obtaining the development length, tend to be unconservative. Such conclusion would be erroneous. The equation for development length was not intended for bar lengths shorter than those required to develop yield stress, therefore it is not realistic to compute the anchorage on that basis. The fact that the ACI Code provisions seem unrealistically low for specifying hook strength in many cases, had already been shown in Marques' (16) study, where it was verified that specimens with approximately 45 percent and even 30 percent of the required (by the ACI Code) lead embedment, reached stresses at failure well beyond the yield point
of the steel.

As has already been shown in previous investigations, the worse stress transfer condition is near the loaded end of the bar and, of course, it travels toward the unloaded end with the increase in applied load. The very same phenomenon has been observed in the tests run in this study. A very short straight embedment is not efficient in transferring stresses to the concrete.

In the J7 series the increase in lead embedment length did increase the strength of the anchorage but, in both cases, that is, with either 12 x 15 inch columns or 12 x 12 inch columns, the hook stress at failure was at or exceeding yield in all cases. In the J11 series the increase in lead embedment length did not increase substantially the strength of the anchorage. Therefore, the importance of short straight lead embedment should probably be minimized in making design calculations. Based on test results, a short straight lead embedment may be defined as a straight lead embedment having 30 percent, or less, of the length which will be obtained using the ACI Code provisions.

The comparison of computed and measured results does not indicate, on the basis of strength and slip, any salient difference between 90° and 180° standard hooks. However, the 180° hook tends to result in a more brittle failure and typically came completely loose at failure, with the whole length of the hook totally uncovered. The hooked bars with
90° bends, with few exceptions, were firmly bonded to the concrete, at least over the length of the tail extension, after failure. The 180° hook seems to be more sensitive to the effect of the axial load than the 90° bend, which might lead to a faster deterioration of bond in the case of cyclic loads. Thus, it seems preferable to use the 90° bend.

4.4. Strength of Hooks in Mass Concrete

As reported by Hribar and Vasko (8) the standard hooks in mass concrete reach stresses well beyond yield. In Figure 38 stress-slip curves for #7 and #11 hooks in mass concrete, taken from reference (8), are compared with the stress-slip curves of the hooks obtained in this study. As it must be expected, the initial stiffness of the hook in a typical beam-column joint was smaller than in mass concrete. For the #7 bars anchored in joints there was practically no change in slope up to a slip of about 0.02 inches while the #7 bars embedded in mass concrete show a reduction at a slip of less than 0.01 inch. For the #11 bars anchored in the joint a reduced stiffness is apparent throughout the load range.

The improved behavior of the #7 bar in joints is not very easy to explain, though it is not unreasonable to accept the same hypothesis used to explain the improved behavior of the specimens reinforced with beam bars having 90° hooks with half tail extension, when high level of axial loads were
applied to the column. Hence, a possible explanation might be on the proximity of the tail section of the hook to the compressed longitudinal column bars. The tail section of the #7 hooked beam-bar was anchored immediately adjacent to a #8 column bar in compression and stress may have been transferred directly from the column bar to the tail section which, as a result, made the anchorage perform more efficiently. For the #11 bars, the tail section was also anchored by the side of the #8 compressed column bars. Yet, due to the fact that the #11 bars are so much larger than the #8 bars, the stress transferred from the column bars to the tail section of the hooked bars was not as significant.

4.5. Development Length for Reinforcement of Beam-Column Joints

In the Commentary Section 12.3, negative moment reinforcement, contains the following statement:

The anchorage value of a hook, plus an extension beyond the hook, should not be computed as greater than that of a hook unless a larger than minimum radius bend is used, or the hook is in confined concrete.

Confined region is defined in the ACI Code in Section A.2, Definitions:

Confined region--Region with transverse reinforcement conforming to the required area and spacing of Section A.5.9 and A.5.10 or Section A.6.4 and regions within beam-column connections conforming to Section A.7.
Sections A.5, A.6 and A.7 are:

A.5. Flexural members of special ductile frames.
A.6. Special ductile frame columns subjected to axial loads and bending
A.7. Beam-column connections in special ductile frames.

Section A.5.5 states:

A flexural member framing into a column where there is no flexural member on the opposite side shall have top and bottom reinforcement extended to the far face of the confined region and anchored to develop the specified yield strength. Development length shall be computed beginning at the near face of the column. Every bar shall terminate with a standard 90° hook or with such a hook and additional bar extension where needed to provide the required development length.

As can be seen, reading the above quoted sections of the ACI Code and Commentary dealing with the problem of the anchorage at hooked beam-bars, they are not very clear being even, in few cases, either contradictory or misleading.

Thus, for instance, in Section 12.3 of the Commentary no guidance is given as to the amount of increase in stress to be allowed if the radius is increased. What is stated in the same 12.3 does not agree with what is said in Section A.5.5. Figure 12.4 of the Commentary can lead to the idea that the total length of embedment from the critical section, including the hook, may be considered as development length, particularly in view of the recommendations contained in Section A.5.5.

From the test results, it can be concluded that, at
least for static loads, any additional length added to the
tail extension cannot be considered as adequate for provid-
ing development length. The failure was in no case produced
by pullout of the hooked bar, but by side splitting of the
joint. Therefore, it does not seem reasonable to consider
tail extensions as being effective in increasing anchorage
strength. Tail extensions beyond those required in a stan-
dard hook may provide a margin against bond deterioration
under cyclic loads, such as would be expected in earthquakes.
But even for this purpose, the value of extensions would
seem to be very dubious.
CHAPTER 5

FINAL CONSIDERATIONS

5.1. Proposed Design Recommendations

Based on the data obtained in Marques’ (16) study and on the data obtained from nine tests run in the present study, a report for the Reinforced Concrete Research Council has been written by Jirsa and Marques (19). In Chapter 5 of that report proposed design recommendations are presented which will subsequently be quoted directly.

5. Proposed Design Recommendations

5.1. Introduction

The design recommendations proposed for computing the anchorage capacities of hooked bars reflect the trends observed in the experimental program and are summarized below.

(1) Strength is increases as restraint against side splitting is increased. Standard hooks embedded in mass concrete exhibit strengths well in excess of yield (8).

(2) Much higher strengths can be permitted than in current specifications. For example, there is no reason a standard hooked bar with 60 ksi yield strength should be permitted to develop 30 ksi, while an identical hook of 40 ksi yield strength should be permitted to develop only 20 ksi. For small bar sizes these values are unrealistically prohibitive and as a result are probably frequently violated by designers. However, to allow higher stresses, minimum straight embedments before the hook are required, especially for larger bar sizes.

(3) No distinction is made between the strength of 90° and 180° standard hooks.
(4) Additional tail extensions over those required for a standard hook are not effective in increasing strength.
(5) Strength of #14 and #18 bars is not changed from ACI 318-71, because no valid test results are available for large hooks and such hooks would likely not commonly be used in frame structures.

To take these factors into account in developing design recommendations, the strength of hooked bar anchorage will be divided into three classes with the distinction between classes based on the lateral restraint to splitting which is provided.

5.2. Design Recommendations

Standard hooks shall be considered to develop tensile stresses in bar reinforcement equal to \( f_h = \xi \sqrt{f_{c'}} \) but not greater than \( f_y \), where \( \xi \) is based on the confining conditions of the hooked anchorage.

Class 1. For all bar sizes \( f_h \) shall be computed using \( \xi \) not less than 700 (1-0.3 \( d_b \)).

Class 2. For bar sizes #11 or smaller \( f_h \) may be computed using \( \xi = 1000 \) (1-0.3 \( d_b \)), provided the lead straight embedment between the standard hook and the critical section is not less than 4 bar diameters or 4 inches whichever is greater, side concrete cover normal to the plane of the hooked bar is not less than 2.5 inches, and cover on the tail extension is not less than 2 inches.

Class 3. For bar sizes #11 or smaller, \( f_h \) may be computed using \( \xi = 1250(1-0.3 \ d_b) \), if in addition to meeting the requirements for Class 2 hooks the bar is enclosed by ties spaced apart not further than 3 \( d_b \). Standard hooks embedded in mass concrete where side splitting is of no concern may be considered as Class 3.

No increase in \( f_h \) shall be permitted for tail extensions or bend radii greater than required for a standard hook. To meet equivalent embedment length \( l_e \) may be computed using Section 12.5 (a) by substituting \( f_h \) for \( f_y \) and \( l_e \) for \( l_d \). Only straight lead embedments greater than 4\( d_b \) or 4 inches, whichever is greater, shall be considered effective in computing development length requirements. For better control of deflections and cracking, 90° hooks are preferable.

5.3. Comparison of Proposed Recommendations with ACI 318-71 and with Test Results

The proposed design recommendations are summarized in
Figures 39 and 40. Both figures also show the computed strengths which could be obtained using current ACI 318-71 provisions. Figure 39 shows the $\xi$ values for bar sizes from #3 to #18. For hooks which do not meet the lateral restraint requirements for Class 2 and Class 3 joints, the values of $\xi$ are very similar to those given in 318-71. For bars which meet Class 2 and Class 3 cover requirements and tie requirements through the joints, the values of $\xi$ are increased roughly 40 percent for Class 2 and about 80 percent for Class 3 hooks. Furthermore, no distinction is made for anchorage capacities of bars with different yield stresses.

Figure 40 shows the computed hook strength for standard hooks based on the proposed design recommendations and on ACI 318-71 provisions. The stresses assigned to standard hook anchorages are considerably increased if the confinement conditions for Class 2 and Class 3 hooks are met. For example, under present Code restrictions the maximum stress on a #8 hook is 38 ksi (50 ksi if the 30 percent increase is permitted for "enclosure" in Section 12.8.1); however, with proper attention to cover and tie requirements, that same hook would now be rated at 60 ksi, if the concrete strength is 5000 psi. It should be noted that the type of confinement needed to increase the hook capacity for Class 2 and 3 is explicitly stated in the proposed recommendations, whereas the Code is vague about the type of enclosure needed.

Computed strengths using these recommendations for the bars tested in this study are listed in Table 6. Ratios of measured-to-computed strength vary from a minimum of about 1.15 to 1.7. Table 6 also lists the stress corresponding to 0.6 of the computed strength of the hooked bar anchorage and the measured lead slip for the test specimens at 0.6 of the computed strength. The rationale behind these comparisons is to give an indication of the possible crack width at the beam-column joint due to slip of the anchorage at service loads. The figures indicate that in none of the specimens was the slip greater than 0.016 at the assumed service stress. The limit for crack width suggested in the Commentary (Section 10.6) is 0.016 inch. The crack width at the face of the column is assumed to be about equal to the lead slip of the anchored bar. In general, for the #7 bars, measured slip values at "service loads" were in the range of 0.005 to 0.008. The only specimens which exceeded these values were those with 180° hooks and the specimens with 12 inch column thickness. In the case of the #11 bars, the slip at service loads tended to be higher but still less than the 0.016 inch crack width
suggested in the Commentary.

In evaluating the proposed design recommendations or those contained in ACI 318-71 it is apparent that the main "gap" in the experimental evidence is the suggested computation for additional straight lead embedment to achieve desired stresses at critical sections. While the stress capacity of the hook can be reasonably estimated, the reliability of the equation for development length in computing embedment lengths for stress values less than $f_y$, say 10 or 20 ksi, cannot be verified from the available test results.

Based on observations, it would appear that design recommendations can be adjusted to realistically reflect the actual strength of hooks and to satisfy required serviceability criteria.

5.2. Additional Research Needs

Most of the tests whose results have been used in this study were reinforced with #7 and #11 beam-bars having standard hooks. As said, the loss of anchorage capacity was due to a loss of concrete cover, mainly due to the destructive action of the hook. Based on test results it can be said that the decrease of concrete cover decreases the anchorage strength, and ties throughout the joint, closely spaced, do increase the anchorage strength. Increasing the thickness of the side cover, beyond a certain value, in order to lessen the destructive action of the hook, is not economical and, unless a few precautions are taken, such procedure will increase the possibility of shrinkage on temperature cracks developing in the concrete surface. As is known, the destructive action of the hook can also be reduced by an
increase in the area over which the hook acts. This can be achieved in a number of ways. Two of the simplest ways are the following:

1. Increasing the radius of the hook.
2. Increasing the area over which the hook acts using a transverse bar on the inside of the hook.

The first solution is vaguely referred in the ACI Code. Joint tests are needed to establish the relationship between the radius of the hook and the anchorage capacity of the hooked beam-bar in joints. A few joint tests are also necessary in order to evaluate the effectiveness of the second solution, that is, the use of a transverse bar on the inside of the hook, which not only will reduce the pressure on the concrete but also will act as tension reinforcement decreasing, therefore, the probability of splitting. The effect of confinement provided by column bars supported by ties throughout the joint and the effect of spandrel beams on the anchorage capacity of hooked beam-bars in column-beam joints need to be studied. The effect of using hooked beam-bars with the plane of the hook tilted toward the axis of the column needs also to be studied. The contribution of small straight lead embedments to the anchorage capacity of hooked bars must be studied, and the length below which the contribution is negligible must be defined.
CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1. **Summary**

The object of this study was to examine some of the factors which influence the anchorage capacities of reinforcing bars in beam-column joints of reinforced concrete structures.

Eighteen full scale specimens simulating a beam-column joint in a structure were tested. All the data obtained in Marques' (16) study was reanalyzed, that is, the data used in the present study corresponds to 28 tests. Fourteen specimens had #7 beam-bars (10 specimens from Marques' (16) work) and the remaining 14 were reinforced with #11 beam-bars. The anchored bars were subjected to varying degrees of confinement and bent to different geometric configurations. Six of the tests, 3 with #7 and 3 with #11 bars, had the tail extensions of the 90° standard hooks reduced to half of the length required by the ACI Code, that is, equal to six bar diameters.

In Chapter 1 studies of anchorage and bond problems are briefly discussed. In Chapter 2 the experimental program including discussion of specimen size, loading system,
test procedure and instrumentation used were described. The measured results were discussed in Chapter 3. In Chapter 4 the significance of the results was discussed, primarily in view of the ACI Code (318-71) provisions applicable to the present study. In Chapter 5 proposed design recommendations are presented and some of the points that still need to be cleared with further research are listed.

6.2. Conclusions

Based on test results:

1. The level of axial load did not significantly influence the behavior of the hooked bar anchorages, even regarding the slip or stresses at failure. When steel confinement was present, the axial load seemed to be detrimental to the slip behavior of the anchored bars.

2. The variation of the thickness of the side concrete cover did not affect the slope of the bar stress-slip curve but did affect the stress at failure, as well as the type of failure. The smaller the thickness of the concrete cover the less the bar stresses were at failure and the more brittle the failure.

3. The confinement provided by the column bars did not appreciably influence the behavior of the hooked bar anchorage.

4. Ties through the joint did improve the anchorage
capacity but only when the tie spacing was small compared to the diameter of the bend of the anchored beam-bar.

5. The lead embedment length was extremely important in determining the anchorage capacity of the hooked beam-bars.

6. Very small straight lead embedments were not effective in transferring stresses to the concrete.

7. At least for static loads, additional bar extension beyond that required for a standard 90° hook will not improve the anchorage capacity of hooked beam-bars.

8. The reduction of the tail extension, in 90° standard hooks, to half of the standard length was detrimental to the slip behavior of the hooked beam-bars. There was an appreciable decrease of slip at failure and the failure was very brittle. Stresses were generally not very much reduced at failure.

9. It appeared that the 180° hook was more sensitive to the effect of the axial load than the 90° and also tended to result in a more brittle failure.

10. There does not seem to be any justification to use values equal to \( f_y / 2 \) or less as a limit for the strength of a standard hook.
REFERENCES


17. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71), American Concrete Institute, Detroit, 1971.

18. ACI Committee 318, "Commentary on Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Detroit, 1971.


**TABLE 1**

**SUMMARY OF HOOK ANCHORAGE TESTS - MARQUES' (16)**

<table>
<thead>
<tr>
<th>SPECIMEN</th>
<th>Col. Size</th>
<th>Hook Angle</th>
<th>Tail Extension</th>
<th>Lead Embedment</th>
<th>Lateral Confinement (+)</th>
<th>P,Col. Load Kips</th>
<th>Conc.Str.f'c,ksi</th>
<th>Bar Failed in Marques' Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>J7-90/1-15-1-H</td>
<td>12x15</td>
<td>90</td>
<td>12d_b</td>
<td>9.5</td>
<td>1</td>
<td>550</td>
<td>4.6</td>
<td>LEFT J7-I</td>
</tr>
<tr>
<td>J7-90/1-15-1-M</td>
<td>12x15</td>
<td>90</td>
<td>12d_b</td>
<td>9.5</td>
<td>1</td>
<td>270</td>
<td>5.0</td>
<td>RIGHT J7-Ia</td>
</tr>
<tr>
<td>J7-90/1-12-1-H</td>
<td>12x12</td>
<td>90</td>
<td>12d_b</td>
<td>6.5</td>
<td>1</td>
<td>420</td>
<td>4.4</td>
<td>LEFT J7-II</td>
</tr>
<tr>
<td>J7-180-15-1-H</td>
<td>12x15</td>
<td>180</td>
<td>4d_b</td>
<td>9.5</td>
<td>1</td>
<td>550</td>
<td>3.9</td>
<td>RIGHT J7-IV</td>
</tr>
<tr>
<td>J7-180-12-1-H</td>
<td>12x12</td>
<td>180</td>
<td>4d_b</td>
<td>6.5</td>
<td>1</td>
<td>420</td>
<td>4.0</td>
<td>LEFT J7-V</td>
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<tr>
<td>J7-90/1-15-2-H</td>
<td>12x15</td>
<td>90</td>
<td>6d_b</td>
<td>9.5</td>
<td>2</td>
<td>550</td>
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<tr>
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<td>12x15</td>
<td>90</td>
<td>6d_b</td>
<td>9.5</td>
<td>2</td>
<td>270</td>
<td>4.7</td>
<td>RIGHT J7-VIa</td>
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<tr>
<td>J7-90/1-15-3-H</td>
<td>12x15</td>
<td>90</td>
<td>6d_b</td>
<td>9.5</td>
<td>3</td>
<td>550</td>
<td>4.5</td>
<td>RIGHT J7-VIb</td>
</tr>
<tr>
<td>J7-90/1-15-3a-H</td>
<td>12x15</td>
<td>90</td>
<td>6d_b</td>
<td>9.5</td>
<td>3a</td>
<td>550</td>
<td>3.9</td>
<td>RIGHT J7-VIc</td>
</tr>
<tr>
<td>J7-90/1-15-4-H</td>
<td>12x15</td>
<td>90</td>
<td>6d_b</td>
<td>9.5</td>
<td>4</td>
<td>550</td>
<td>4.4</td>
<td>RIGHT J7-VII</td>
</tr>
</tbody>
</table>

 (+) LATERAL CONFINEMENT
1 Column Bar + 2-7/8 in. cover
2 Only 2-7/8 in. cover
3 2-7/8 in. cover + #3 ties @ 5 in. throughout the joint
3a 2-7/8 in. cover + #3 ties @ 2-1/2 in. throughout the joint
4 Only 1-1/2 in. cover

(*) As viewed from back of column.
TABLE 2
SUMMARY OF HOOK ANCHORAGE TESTS

<table>
<thead>
<tr>
<th>SPECIMEN</th>
<th>Column Size In.</th>
<th>Hook Angle Degrees</th>
<th>Tail Extension</th>
<th>Lead Embedment In.</th>
<th>Lateral Confinement (+)</th>
<th>P,Col. Load Kips</th>
<th>Conc.Strength, f_c ksi</th>
<th>Bottom Batch</th>
<th>Top Batch</th>
<th>Bar Failed (*)</th>
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</thead>
<tbody>
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<td>J7-90/1-15-L</td>
<td>12x15</td>
<td>90</td>
<td>12d_b</td>
<td>9.5</td>
<td>1</td>
<td>140</td>
<td>4.8</td>
<td>4.8</td>
<td></td>
<td>RIGHT</td>
</tr>
<tr>
<td>J7-90/.5-15-1-H</td>
<td>12x15</td>
<td>90</td>
<td>6d_b</td>
<td>9.5</td>
<td>1</td>
<td>550</td>
<td>4.3</td>
<td>4.3</td>
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<td>RIGHT</td>
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<tr>
<td>J7-90/.5-15-1-L</td>
<td>12x15</td>
<td>90</td>
<td>6d_b</td>
<td>9.5</td>
<td>1</td>
<td>140</td>
<td>4.4</td>
<td>4.3</td>
<td></td>
<td>RIGHT</td>
</tr>
<tr>
<td>J7-90/.5-15-2-L</td>
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<td>90</td>
<td>6d_b</td>
<td>9.5</td>
<td>2</td>
<td>140</td>
<td>4.6</td>
<td>4.4</td>
<td></td>
<td>LEFT</td>
</tr>
<tr>
<td>J11-90/1-15-1-H</td>
<td>12x15</td>
<td>90</td>
<td>12d_b</td>
<td>6.0</td>
<td>1</td>
<td>550</td>
<td>4.8</td>
<td>5.0</td>
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<td>LEFT</td>
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<tr>
<td>J11-90/1-15-1-L</td>
<td>12x15</td>
<td>90</td>
<td>12d_b</td>
<td>6.0</td>
<td>1</td>
<td>140</td>
<td>4.8</td>
<td>4.7</td>
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<tr>
<td>J11-180-15-1-H</td>
<td>12x15</td>
<td>180</td>
<td>4d_b</td>
<td>6.0</td>
<td>1</td>
<td>420</td>
<td>4.5</td>
<td>4.7</td>
<td></td>
<td>RIGHT</td>
</tr>
<tr>
<td>J11-180-15-1-L</td>
<td>12x15</td>
<td>180</td>
<td>4d_b</td>
<td>6.0</td>
<td>1</td>
<td>140</td>
<td>4.3</td>
<td>4.4</td>
<td></td>
<td>LEFT</td>
</tr>
<tr>
<td>J11-90/1-15-2-H</td>
<td>12x15</td>
<td>90</td>
<td>12d_b</td>
<td>6.0</td>
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<td>550</td>
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<td>4.9</td>
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<tr>
<td>J11-90/1-15-2-L</td>
<td>12x15</td>
<td>90</td>
<td>12d_b</td>
<td>6.0</td>
<td>2</td>
<td>140</td>
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<td>4.4</td>
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<tr>
<td>J11-90/1-15-3-L</td>
<td>12x15</td>
<td>90</td>
<td>12d_b</td>
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<td>3</td>
<td>140</td>
<td>4.9</td>
<td>4.8</td>
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<td>J11-90/1-15-4-L</td>
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<td>90</td>
<td>12d_b</td>
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<td>4</td>
<td>140</td>
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<td>4.1</td>
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<td>RIGHT</td>
</tr>
<tr>
<td>J11-90/1-15-5-L</td>
<td>12x15</td>
<td>90</td>
<td>12d_b</td>
<td>6.0</td>
<td>5</td>
<td>140</td>
<td>5.1</td>
<td>4.9</td>
<td></td>
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<tr>
<td>J11-90/.5-15-1-H</td>
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<td>90</td>
<td>6d_b</td>
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<td>1</td>
<td>550</td>
<td>4.3</td>
<td>4.4</td>
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<tr>
<td>J11-90/.5-15-1-L</td>
<td>12x15</td>
<td>90</td>
<td>6d_b</td>
<td>6.0</td>
<td>1</td>
<td>140</td>
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<td>4.4</td>
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<td>LEFT</td>
</tr>
<tr>
<td>J11-90/.5-15-2-L</td>
<td>12x15</td>
<td>90</td>
<td>6d_b</td>
<td>6.0</td>
<td>2</td>
<td>140</td>
<td>4.9</td>
<td>5.0</td>
<td></td>
<td>RIGHT</td>
</tr>
</tbody>
</table>

(+) LATERAL CONFINEMENT

1. Column Bars + 2-7/8 in. cover
2. Only 2-7/8 in. cover
3. 2-7/8 in. cover + #3 ties @ 5 in. throughout the joint

3a. 2-7/8 in. cover + #3 ties @ 2-1/2 in. throughout the joint
4. only 1-1/2 in. cover
5. 2-7/8 in. cover + #3 ties with a cross tie, @ 2-1/2 in. throughout the joint

(*) As viewed from back of column.
TABLE 3

SUMMARY OF MEASURED SLIP BEHAVIOR

<table>
<thead>
<tr>
<th>SPECIMEN</th>
<th>Stress, ksi, at Lead Slip of 0.005 in 0.016 in 0.05 in</th>
<th>Computed $f_{h}+f_{b}$ ksi (*)</th>
<th>Lead Slip, in. at $0.6(f_{h}+f_{b})$ Lead Stresses</th>
<th>Slip at 2H, in. at $0.6(f_{h}+f_{b})$ Lead Stresses</th>
<th>Approximate Slip at Failure in.</th>
<th>Stress at Failure ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>J7-90/1-15-1-H</td>
<td>33 55 77</td>
<td>36.6+26.8</td>
<td>0.006</td>
<td>0.002</td>
<td>0.154</td>
<td>91</td>
</tr>
<tr>
<td>J7-90/.5=15-1-H</td>
<td>24 53 80</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.185</td>
<td>99</td>
</tr>
<tr>
<td>J7-90/1-15-1-M</td>
<td>35 61 78</td>
<td>38.4+28.2</td>
<td>0.006</td>
<td>0.001</td>
<td>0.276(+)</td>
<td>100</td>
</tr>
<tr>
<td>J7-90/1-15-1-L</td>
<td>30 52 78</td>
<td>37.4+27.4</td>
<td>0.008</td>
<td>0.001</td>
<td>0.212</td>
<td>100</td>
</tr>
<tr>
<td>J7-90/.5=15-1-L</td>
<td>23 39 78</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.104</td>
<td>84</td>
</tr>
<tr>
<td>J7-90/1-12-1-H</td>
<td>20 31 59</td>
<td>35.0+17.6</td>
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<td>0.082</td>
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<tr>
<td>J7-180/15-1-H</td>
<td>24 45 72</td>
<td>34.2+25.0</td>
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<td>0.003</td>
<td>0.149</td>
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<tr>
<td>J7-180/12-1-H</td>
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<td>35.6+17.9</td>
<td>0.011</td>
<td>0.007</td>
<td>0.073</td>
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<tr>
<td>J7-90/1-15-2-H</td>
<td>33 52 75</td>
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<td>0.008</td>
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<td>J7-90/1-15-2-M</td>
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<td>J7-90/.5=15-2-L</td>
<td>38 56 72</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.221(+)</td>
<td>91</td>
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<tr>
<td>J7-90/1-15-3-H</td>
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<td>36.2+26.6</td>
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<td>0.002</td>
<td>0.052</td>
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(*) ACI 318-71 - other bars values

(+) Values obtained using expressions in Tables B1 (#7) and B2 (#11).
TABLE 3 (CONTINUED)

<table>
<thead>
<tr>
<th>SPECIMEN</th>
<th>Stress, ksi, at Lead Slip of 0.005 in 0.016 in 0.05 in</th>
<th>Computed ( f_{h} + f_{h} ), ksi (^+)</th>
<th>Lead Slip, in. at 0.6 ( f_{h} + f_{h} ) Lead Stresses</th>
<th>Slip at 2H, in. at 0.6 ( f_{h} + f_{h} ) Lead Stresses</th>
<th>Approximate Slip at Failure in.</th>
<th>Stress at Failure ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>J11-90/1-15-2-H</td>
<td>22 35 48</td>
<td>29.7+6.8</td>
<td>0.005</td>
<td>0.001</td>
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<td>J11-90/1-15-2-L</td>
<td>18 28 50</td>
<td>28.2+6.4</td>
<td>0.008</td>
<td>0.003</td>
<td>0.057</td>
<td>53</td>
</tr>
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<td>J11-90/.5-15-2-L</td>
<td>18 29 Failed</td>
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<td>--</td>
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</tr>
<tr>
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<td>16 28 53</td>
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<td>0.005</td>
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</tr>
<tr>
<td>J11-90/1-15-3a-L</td>
<td>24 42 66</td>
<td>29.7+6.8</td>
<td>0.004</td>
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<td>0.081</td>
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\(^{(*)}\) ACI 318-71 - other bars values

\(^{(+)}\) Values obtained using expressions in Tables B1 (#7) and B2 (#11).
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<th>f_h, comp., ksi(*)</th>
<th>f_h, Meas. ksi</th>
<th>f_h, meas. (**) f_h, comp. ksi</th>
<th>f_l, comp. (**)</th>
<th>f = f_h + f_l comp. (**)</th>
<th>Meas. f at Failure</th>
<th>f Meas. f, comp. (**)</th>
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(+) ACI 318-71

(*) Using values for other bars.
### Table 5

**Summary of the Effect of the Reduction of Tail Extension on Lead Slip Behavior**

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<thead>
<tr>
<th>Specimen</th>
<th>LEAD STRESSES, KSI</th>
<th>Lead Slip, In</th>
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<tr>
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<td>----------------------</td>
<td>------------------------</td>
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FIGURE 1. END ANCHORAGE IN EXTERIOR BEAM-COLUMN JOINT
FIGURE 2. TEST SPECIMEN AND DIMENSIONS

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<th>Bar Size</th>
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<th>e</th>
<th>g(min)</th>
<th>l</th>
<th>( \theta )</th>
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<td>6 in</td>
<td>2-7/8 in</td>
<td>VAR</td>
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<tr>
<td>#9</td>
<td>16 in</td>
<td>12 in</td>
<td>7-1/2 in</td>
<td>2-7/8 in</td>
<td>VAR</td>
<td>VAR</td>
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FIGURE 3. JOINT ANCHORAGE TEST SPECIMEN
#3 TIES AT 5" OUTSIDE JOINT - ALL SPECIMENS

#8 LONGITUDINAL COLUMN BARS

HEIGHT OF COL = 50 IN.
ASSUMED BEAM DEPTH = 20 IN.
CLEAR COVER = 1-1/2"

FIGURE 4. DETAILS OF TEST SPECIMENS
FIGURE 5. DETAILS OF JOINTS
FIGURE 6. TYPICAL STRESS-STRAIN CURVES FOR #7 AND #11 BARS (ASTM A-615 GRADE 60)
FIGURE 7. LOAD FRAME
FIGURE 8. LOAD FRAME
### SLIP MEASUREMENT POINTS

### STRAIN GAGE LOCATIONS

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<th>C</th>
<th>D</th>
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<td>8 1/4</td>
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<td>3 1/4</td>
<td>16 1/2</td>
<td>9</td>
<td>3 1/4</td>
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</table>

**FIGURE 9. INSTRUMENTATION ON THE BARS**
FIGURE 10. SET UP FOR SLIP

FIGURE 11. SPRING AND PIECE OF BRASS
FIGURE 12. DIAL GAGES SET UP

FIGURE 13. CRACK PATTERNS PRIOR TO FAILURE
FIGURE 14. INFLUENCE OF COLUMN AXIAL LOAD ON SLIP
FIGURE 15. INFLUENCE OF COLUMN AXIAL LOAD ON SLIP
FIGURE 16. INFLUENCE OF COLUMN AXIAL LOAD ON STRESSES
FIGURE 17. INFLUENCE OF AXIAL LOAD ON STRESSES
FIGURE 18. INFLUENCE OF BEND ANGLE AND LEAD EMBEDMENT ON SLIP
FIGURE 19. INFLUENCE OF BEND ANGLE ON STRESSES
FIGURE 20. INFLUENCE OF LEAD EMBEDMENT ON STRESSES
FIGURE 21. INFLUENCE OF BEND ANGLE AND LEAD EMBEDMENT ON STRESSES
FIGURE 24. INFLUENCE OF CONFINEMENT ON STRESSES
FIGURE 25. LEAD BAR STRESS-LEAD SLIP CURVES FOR SPECIMENS WITH HALF TAIL HOOKS
FIGURE 26. BAR STRESS-LEAD BAR STRESS CURVES FOR SPECIMENS WITH HALF TAIL
FIGURE 28. INFLUENCE OF THE LENGTH OF TAIL EXTENSION ON STRESSES
FIGURE 29. INFLUENCE OF THE LENGTH OF TAIL EXTENSION ON SLIP
FIGURE 30. INFLUENCE OF THE LENGTH OF TAIL EXTENSION ON STRESSES
FIGURE 31. INFLUENCE OF THE LENGTH OF TAIL EXTENSION ON SLIP
FIGURE 32. INFLUENCE OF THE LENGTH OF TAIL EXTENSION ON STRESSES
FIGURE 33. APPEARANCE OF SPECIMENS AFTER FAILURE
FIGURE 34. APPEARANCE OF SPECIMENS AFTER FAILURE
FIGURE 35. APPEARANCE OF SPECIMENS AFTER FAILURE
FIGURE 37. CRUSHING AT THE REGION AROUND THE HOOK
FIGURE 38. COMPARISON OF STRESS-SLIP CURVES WITH HOOKS IN MASS CONCRETE
Figure 39. Proposed and Current $\xi$ Values for Strength of Standard Hooks (Ref. 19)
FIGURE 40. PROPOSED AND CURRENT VALUES FOR STRENGTH OF STANDARD HOOKS (REF. 19)
APPENDIX A

TEST RESULTS

A.1. Introduction

To determine the behavior of the anchorage of hooked beam-bars in joints, stress and slip were measured at several points along the bars. All the data obtained from the tests run in this study, as well as the data collected in Marques' (16) study, is presented graphically in this appendix for both bars in each specimen. The designation of left and right bars are as viewed from the back surface of the column, that is, the face opposite to that on which the bars protrude from the column.

Three different types of curves are presented, which will subsequently be discussed.

A.2. Lead Bar Stress-Slip Curves

The lead stress versus the slip measured at each one of the points shown in Figure 9 (1H, 2H, 3V, 3H and 4V or 4H) was plotted. The reference surface, for measuring the slip at the five points of each bar, was the back surface of the column. The lead stress or load on the bar was the value on the bar protruding from the column on which load was applied.
As mentioned previously, strains were measured using strain gages at three points along each embedded bar (see Figure 9). However, due to bending effects, the value obtained using the strain gages, as the ones placed on the projecting part of the embedded bar, tended to be slightly inconsistent. The inconsistency was due to bending stresses on the bars produced during application of load. Because it was nearly impossible to align the Cadweld splice and the bar so that all components were symmetrical, the values given by the strain gages mounted on the bars outside the concrete column did not always compare exactly with the bar forces obtained using hydraulic pressures. As a result, the stresses used to plot all the curves were those obtained using a carefully calibrated pressure gage attached to the hydraulic line from the pump to the 60 ton jacks. The bar forces or stresses given by the strain gages were only used to check the values given by the pressure gage. The differences between these values were pronounced in the J7 series.

The lead bar stress-slip curves are presented in Figures A.1 to A.28. In a few cases, the loading of one of the bars was not satisfactory. When this happened, only the curves corresponding to the properly loaded bar were plotted.

A.3. Stress-Slip Curves for Hooks

Figures A.29 through A.37 show the curves obtained
plotting the measured stresses at the end of the straight lead embedment (start to the bend) versus the slip at point 2H. With these curves the stress-slip behavior of the hooks in joints can approximately be estimated. It should be noticed that as the bar pulled out bending stresses were induced and therefore, the measured stresses before and after the hook can not be considered accurate over the whole loading range. At large slips the measured stresses can only be considered an estimate, mainly the stresses measured after the bend.

A.3. Stress Transfer Along the Bar

The location of the strain gages as shown in Figure 9 provided an assessment of the stress transferred to the concrete over the region between the strain gages, that is, over the straight embedment before the hook and the 90° bend portion between horizontal and vertical tangents to the hook. It should be noted that the stresses obtained can only be considered as estimates of stresses, because bending stresses were induced as the bars pulled out and mainly at large slips, the measured stresses may not be accurate. In the J11 series this discrepancy was particularly evident.

Plotting the lead bar stress versus the stress after the hook (curve designated by AFT) and versus the stress before the hook (curve designated by BEF), the curves shown in
Figures A.38 through A.47 were obtained.

Observing the curves it is apparent that the maximum stress transferred over the lead embedment varied between about 20 ksi and 40 ksi for the J7 series and can practically be disregarded for the J11 series, where in most of the tests the lead stress and the stress at the start of the hook were the same at lead stress levels of about 30 ksi.

In the J7 series the stresses after the hook remained very low, less than 20 ksi, until lead stresses between 60 ksi and 80 ksi were reached. When those stresses were reached, the stress after the hook increased sharply and in some cases nearly reached yield. In the J11 series, most of the specimens showed stresses after the hook generally at a level less than half the lead bar stress. Yet, specimen J11-90/1-12-1-H had, prior to failure, practically the same stresses after and before the hook and J11-90/.5-15-1-H as well as J11-90/1-15-5-L specimens show, prior to failure, a small difference in stresses after and before the hook.
FIGURE A.1. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.2. MEASURED LEAD-STRESS-SLIP RELATIONSHIPS
FIGURE A.3. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.5. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.6. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.10. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.11. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.12. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
Figure A.13. Measured Lead Stress-Slip Relationships

SLIP

LEAD BAR STRESS, KSI

0.002 in.

RIGHT — LEFT

1H

2H

3H

3V

4V

J7-9.0/5.15-1-L
FIGURE A.14. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.15. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.16. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.17. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.18. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.19. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.20. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.21. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
Figure A.23. Measured Lead Stress-Slip Relationships

- Right
- Left

JII-90/H15-3a-L

SLIP

0.02 in.

LEAD BAR STRESS, KSI

100  80  60  40  20  0

2H

3H

4V
FIGURE A.24. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
Figure A.26. Measured Lead Stress-Slip Relationships
FIGURE A.27. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.28. MEASURED LEAD STRESS-SLIP RELATIONSHIPS
FIGURE A.29. STRESS-SLIP CURVES FOR HOOKS
FIGURE A.30. STRESS-SLIP CURVES FOR HOOKS
FIGURE A.31. STRESS-SLIP CURVES FOR HOOKS
FIGURE A.32. STRESS-SLIP CURVES FOR HOOKS
FIGURE A.33. STRESS-SLIP CURVES FOR HOOKS
FIGURE A.34. STRESS-SLIP CURVES FOR HOOKS
FIGURE A.35. STRESS-SLIP CURVES FOR HOOKS
FIGURE A.36. STRESS-SLIP CURVES FOR HOOKS
FIGURE A.37. STRESS-SLIP CURVES FOR HOOKS
FIGURE A.38. MEASURED STRESSES
FIGURE A.39. MEASURED STRESSES
FIGURE A.40. MEASURED STRESSES
FIGURE A.41. MEASURED STRESSES
FIGURE A.42. MEASURED STRESSES
FIGURE A.43. MEASURED STRESSES
FIGURE A.44. MEASURED STRESSES
Figure A.46: Measured Stresses

- BEE
- AFT

Stress (ksi) vs. Lead Bar Stress (ksi)

J90/I5.3-L
J90/I5.5-L
J90/I5.2-L
J90/I5.3-L

Right - Left

20 KSI

169
FIGURE A.47. MEASURED STRESSES
APPENDIX B

TENTATIVE APPROACH TO DETERMINE A GENERAL
ANALYTICAL EXPRESSION FOR THE LEAD
BAR STRESS - LEAD SLIP CURVES

B.1. Introduction

For trying to determine the general analytical expression for the lead bar stress - lead slip curves, the method of plotting found most convenient was $f_s$, measured lead bar stress, versus log S1 on a semilog paper, being

$$S1=10^4x(\text{measured lead slip}).$$

The value $10^4$ as multiplier of the measured slip was used as a scale factor which made it possible to obtain fairly clear graphs, that is, graphs with the points neither too close nor too far apart from each other.

B.2. Lead Bar Stress - $\log (10^4x\text{Lead Bar Slip})$ Graphs

Figures B.1 through B.8 show the graphs obtained. As shown for all specimens but two, with #7 beam-bars, the graphs are composed of five straight lines whose slopes $(df_s/d[\log(10^4 \text{ slip})]$ ) keep increasing until the last
straight line is reached, where suddenly, the slope decreases. The graphs corresponding to specimens J7-90/1-15-3-H and J7-90/1-15-4-L are composed of six straight lines instead of five. However, the same general trend can be observed. For all the specimens but three, reinforced with #11 beam-bars, the graphs are composed of four straight lines whose slopes keep increasing until the last line is reached, where, suddenly, the slope decreases. The graph corresponding to specimens J11-180-15-1-H, J11-90/1-14-1-H and J11-90/.5-15-2-L are composed of five straight lines instead of four. Yet, the same general trend can be observed.

In five of the tests, in order to avoid instrument damage, the dial gages, mainly the ones monitoring points 1H and 2H, were taken off before the failure of the specimen. The five tests were those of specimens J7-90/1-15-1-M, J7-90/.5-15-2-L, J7-90/1-15-3-H, J11-180-15-1-L and J11-90/1-15-4-L. Thus, for these specimens there were no recorded slips at failure. Looking at Figures B.1 through B.8, it is clear that, mainly in J7 series with 12x15 inch column, there is a straight line going from about 65 to 70 ksi up to the stress at failure in all the specimens where the slips were recorded up to failure. This last straight line is not so well defined in the specimens of the J7 series with 12x12 inch columns as well as in almost all the specimens reinforced with #11 bars.

For the specimens mentioned above, the slip at failure
was obtained by extending the last straight line, in the corresponding lead bar stress - log(10^4 x lead slip) graph, from the last point recorded until the stress at failure.

For obtaining the graphs, an average value of the lead slip has been used.

B.3. **Analytical Expression for the Lead Bar Stress - Lead Slip Curves**

Using the graphs discussed in B.2, analytical expressions of the form slip = f(f_s) and f_s = f(slip) have been obtained and listed in Tables B1 and B2, as well as the limits within which each expression can be used. The general form of the expression is:

a) for lead slip \[ S = A \times 10^{-4} \times \text{EXP}(B f_s) \]*

b) for lead bar stress \[ f_s = C + D \log S \]

being \[ C = \frac{1}{B \log e} (4 - \log A) \]

where 4 is the exponent of the scale factor used \( (10^4) \),

and \[ D = \frac{1}{B \log e} \]

*For the sake of simplicity in typing, \text{EXP}(BX) will always be used instead of e^{BX}.**
B.4. **Significance of the Parameters A, B, C and D**

It is impossible, with the limited number of tests available, to determine which variables have the most influence on the variation of the parameters A, B, C and D, which ones make them increase and which ones make them decrease, or in other words, what are the relationships between the parameters A, B, C, D and the variables whose effect on the anchorage strength is considered to be most important. It is believed that the task of determining those relationships, even though extremely difficult, is not impossible. It is also believed that it would be necessary, for achieving any significant result, to separate the effect of the bend, as well as the effect of the straight lead embedment of the hook, on the parameters A, B, C and D.

Even though, as has already been said, no conclusions can be drawn, some interesting trends will subsequently be presented.

Looking at the expressions representing the curves $f_s = f(s)$ near failure, it can be seen that:

1. The parameters C and D increased slightly when the axial load decreased for specimens J7-90/1-15-1-H,M,L and for specimens J11-90/1-15-1-H,L parameter C increases about 50 percent and D about 200 percent.

2. The parameters C and D decreased drastically when the straight lead embedment was reduced in the J7 series,
J7-90/1-15-1-H and J7-90/1-12-1-H, and remained practically
the same in the J11 series, J11-90/1-15-1-H and J11-90/1-12-
1-H.

3. The parameter C remains practically the same and
D increases nearly 30 percent, in the J7 series, when a 180°
bend is compared with a 90° bend, J7-90/1-15-1-H and J7-180-
15-1-H. In the J11 series the parameter C increases nearly
20 percent and D nearly 100 percent when a 180° bend is com-

B.5. **Conclusion**

All the material contained in this appendix was in-
cluded in the present study, because it is believed to be
useful for further research in the same field within the
same line of thought.
### Table B1

**Expressions for Lead Stress-Lead Slip Curves**

<table>
<thead>
<tr>
<th>Material</th>
<th>Lead Slip (fs)</th>
<th>Lead Stress (fL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J7-90/1-15-1-H</td>
<td>55 fL = 0.60 x 10^-4 x s + 0.200 fS</td>
<td>13 fL = 2.34 x 10^-4 x s + 0.099 fS</td>
</tr>
<tr>
<td>J7-90/1-15-1-H</td>
<td>28 fL = 3.71 x 10^-4 x s + 0.056 fS</td>
<td>53 fL = 8.38 x 10^-4 x s + 0.037 fS</td>
</tr>
<tr>
<td>J7-90/1-15-1-H</td>
<td>69 fL = 2.95 x 10^-4 x s + 0.081 fS</td>
<td></td>
</tr>
<tr>
<td>J7-90/1-15-1-L</td>
<td>10 fL = 0.25 x 10^-4 x s + 0.209 fS</td>
<td>19 fL = 1.69 x 10^-4 x s + 0.109 fS</td>
</tr>
<tr>
<td>J7-90/1-15-1-L</td>
<td>28 fL = 5.08 x 10^-4 x s + 0.048 fS</td>
<td>45 fL = 8.50 x 10^-4 x s + 0.032 fS</td>
</tr>
<tr>
<td>J7-90/1-15-1-L</td>
<td>60 fL = 1.57 x 10^-4 x s + 0.042 fS</td>
<td></td>
</tr>
<tr>
<td>J7-90/1-12-1-H</td>
<td>6 x 10^-4 x s + 0.198 fS</td>
<td>15 x 10^-4 x s + 0.065 fS</td>
</tr>
<tr>
<td>J7-90/1-12-1-H</td>
<td>28 x 10^-4 x s + 0.048 fS</td>
<td>53 x 10^-4 x s + 0.037 fS</td>
</tr>
<tr>
<td>J7-180/15-1-H</td>
<td>7 x 110^-4 x s + 0.187 fS</td>
<td>15 x 10^-4 x s + 0.123 fS</td>
</tr>
<tr>
<td>J7-180/15-1-H</td>
<td>28 x 10^-4 x s + 0.038 fS</td>
<td>53 x 10^-4 x s + 0.060 fS</td>
</tr>
<tr>
<td>J7-180/15-1-H</td>
<td>61 x 10^-4 x s + 0.20 fS</td>
<td></td>
</tr>
<tr>
<td>J7-180/12-1-H</td>
<td>4 x 10^-4 x s + 0.33 fS</td>
<td>7 x 10^-4 x s + 0.173 fS</td>
</tr>
<tr>
<td>J7-180/12-1-H</td>
<td>26 x 10^-4 x s + 0.048 fS</td>
<td>49 x 10^-4 x s + 0.037 fS</td>
</tr>
<tr>
<td>J7-180/12-1-H</td>
<td>47 x 10^-4 x s + 0.079 fS</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- $f_S$ = ksi
- $s$ = in.
<table>
<thead>
<tr>
<th>TABLE B1 (CONTINUED)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J7-90/1-15-2-H</td>
</tr>
<tr>
<td>$7^2f_s=13$</td>
</tr>
<tr>
<td>$s=0.5x10^{-4}xe0.208fs$</td>
</tr>
<tr>
<td>$0.00022\leq s\leq0.00073$</td>
</tr>
<tr>
<td>$f_s=47.82+110.7x10^3g s$</td>
</tr>
<tr>
<td>13$^2f_s=29$</td>
</tr>
<tr>
<td>$s=1.5x10^{-4}xe0.108fs$</td>
</tr>
<tr>
<td>$0.00075\leq s\leq0.0039$</td>
</tr>
<tr>
<td>$f_s=80.16+213.2x10^3g s$</td>
</tr>
<tr>
<td>29$^2f_s=50$</td>
</tr>
<tr>
<td>$s=1.0x10^{-4}xe0.084fs$</td>
</tr>
<tr>
<td>$0.00039\leq s\leq0.0095$</td>
</tr>
<tr>
<td>$f_s=69.54+27.4x10^3g s$</td>
</tr>
<tr>
<td>40$^2f_s=71$</td>
</tr>
<tr>
<td>$s=2.0x10^{-4}xe0.039fs$</td>
</tr>
<tr>
<td>$0.0005\leq s\leq0.032$</td>
</tr>
<tr>
<td>$f_s=160.59+59.0x10^3g s$</td>
</tr>
<tr>
<td>71$^2f_s=99$</td>
</tr>
<tr>
<td>$s=1.2x10^{-4}xe0.078fs$</td>
</tr>
<tr>
<td>$0.032\leq s\leq0.29$</td>
</tr>
<tr>
<td>$f_s=114.54+29.5x10^3g s$</td>
</tr>
<tr>
<td>J7-90/1-15-2-M</td>
</tr>
<tr>
<td>$15f_s=25$</td>
</tr>
<tr>
<td>$s=0.27x10^{-4}xe0.169fs$</td>
</tr>
<tr>
<td>$0.00034\leq s\leq0.0020$</td>
</tr>
<tr>
<td>$f_s=62.13+15.63x10^3g s$</td>
</tr>
<tr>
<td>25$^2f_s=38$</td>
</tr>
<tr>
<td>$s=0.98x10^{-4}xe0.124fs$</td>
</tr>
<tr>
<td>$0.0020\leq s\leq0.010$</td>
</tr>
<tr>
<td>$f_s=74.28+18.57x10^3g s$</td>
</tr>
<tr>
<td>38$^2f_s=56$</td>
</tr>
<tr>
<td>$s=0.37x10^{-4}xe0.060fs$</td>
</tr>
<tr>
<td>$0.010\leq s\leq0.028$</td>
</tr>
<tr>
<td>$f_s=116.67+38.38x10^3g s$</td>
</tr>
<tr>
<td>56$^2f_s=69$</td>
</tr>
<tr>
<td>$s=0.41x10^{-4}xe0.055fs$</td>
</tr>
<tr>
<td>$0.028\leq s\leq0.046$</td>
</tr>
<tr>
<td>$f_s=157.89+65.79x10^3g s$</td>
</tr>
<tr>
<td>69$^2f_s=95$</td>
</tr>
<tr>
<td>$s=0.718x10^{-4}xe0.060fs$</td>
</tr>
<tr>
<td>$0.046\leq s\leq0.22$</td>
</tr>
<tr>
<td>$f_s=121.27+38.38x10^3g s$</td>
</tr>
<tr>
<td>J7-90/1-15-3-H</td>
</tr>
<tr>
<td>$16f_s=21$</td>
</tr>
<tr>
<td>$s=0.08x10^{-4}xe0.228fs$</td>
</tr>
<tr>
<td>$0.00031\leq s\leq0.00094$</td>
</tr>
<tr>
<td>$f_s=51.31+10.1x10^3g s$</td>
</tr>
<tr>
<td>21$^2f_s=32$</td>
</tr>
<tr>
<td>$s=0.92x10^{-4}xe0.110fs$</td>
</tr>
<tr>
<td>$0.00094\leq s\leq0.0051$</td>
</tr>
<tr>
<td>$f_s=84.57+20.93x10^3g s$</td>
</tr>
<tr>
<td>32$^2f_s=42$</td>
</tr>
<tr>
<td>$s=0.27x10^{-4}xe0.075fs$</td>
</tr>
<tr>
<td>$0.0003\leq s\leq0.0065$</td>
</tr>
<tr>
<td>$f_s=109.50+30.7x10^3g s$</td>
</tr>
<tr>
<td>42$^2f_s=58$</td>
</tr>
<tr>
<td>$s=0.10x10^{-4}xe0.043fs$</td>
</tr>
<tr>
<td>$0.0065\leq s\leq0.013$</td>
</tr>
<tr>
<td>$f_s=158.50+53.5x10^3g s$</td>
</tr>
<tr>
<td>58$^2f_s=70$</td>
</tr>
<tr>
<td>$s=0.36x10^{-4}xe0.050fs$</td>
</tr>
<tr>
<td>$0.013\leq s\leq0.019$</td>
</tr>
<tr>
<td>$f_s=218.32+85.28x10^3g s$</td>
</tr>
<tr>
<td>70$^2f_s=104$</td>
</tr>
<tr>
<td>$s=0.56x10^{-4}xe0.090fs$</td>
</tr>
<tr>
<td>$0.019\leq s\leq0.42$</td>
</tr>
<tr>
<td>$f_s=113.60+25.59x10^3g s$</td>
</tr>
<tr>
<td>J7-90/1-15-3a-H</td>
</tr>
<tr>
<td>$12f_s=20$</td>
</tr>
<tr>
<td>$s=0.05x10^{-4}xe0.271fs$</td>
</tr>
<tr>
<td>$0.00013\leq s\leq0.0011$</td>
</tr>
<tr>
<td>$f_s=45.21+8.5x10^3g s$</td>
</tr>
<tr>
<td>20$^2f_s=26$</td>
</tr>
<tr>
<td>$s=0.56x10^{-4}xe0.144fs$</td>
</tr>
<tr>
<td>$0.00013\leq s\leq0.0024$</td>
</tr>
<tr>
<td>$f_s=67.80+15.99x10^3g s$</td>
</tr>
<tr>
<td>26$^2f_s=44$</td>
</tr>
<tr>
<td>$s=0.88x10^{-4}xe0.061fs$</td>
</tr>
<tr>
<td>$0.0024\leq s\leq0.0072$</td>
</tr>
<tr>
<td>$f_s=125.32+37.75x10^3g s$</td>
</tr>
<tr>
<td>44$^2f_s=73$</td>
</tr>
<tr>
<td>$s=0.89x10^{-4}xe0.064fs$</td>
</tr>
<tr>
<td>$0.0072\leq s\leq0.028$</td>
</tr>
<tr>
<td>$f_s=147.75+49.9x10^3g s$</td>
</tr>
<tr>
<td>73$^2f_s=98.5$</td>
</tr>
<tr>
<td>$s=0.53x10^{-4}xe0.085fs$</td>
</tr>
<tr>
<td>$0.028\leq s\leq0.23$</td>
</tr>
<tr>
<td>$f_s=115.95+27.09x10^3g s$</td>
</tr>
<tr>
<td>J7-90/1-15-4-H</td>
</tr>
<tr>
<td>$8f_s=15$</td>
</tr>
<tr>
<td>$s=1.49x10^{-4}xe0.171fs$</td>
</tr>
<tr>
<td>$0.00059\leq s\leq0.0014$</td>
</tr>
<tr>
<td>$f_s=51.76+13.47x10^3g s$</td>
</tr>
<tr>
<td>15$^2f_s=32$</td>
</tr>
<tr>
<td>$s=5.21x10^{-4}xe0.076fs$</td>
</tr>
<tr>
<td>$5.0014\leq s\leq0.0059$</td>
</tr>
<tr>
<td>$f_s=99.38+30.3x10^3g s$</td>
</tr>
<tr>
<td>32$^2f_s=43$</td>
</tr>
<tr>
<td>$s=9.85x10^{-4}xe0.055fs$</td>
</tr>
<tr>
<td>$0.0059\leq s\leq0.111$</td>
</tr>
<tr>
<td>$f_s=125.35+41.12x10^3g s$</td>
</tr>
<tr>
<td>43$^2f_s=64$</td>
</tr>
<tr>
<td>$s=1.94x10^{-4}xe0.040fs$</td>
</tr>
<tr>
<td>$0.011\leq s\leq0.025$</td>
</tr>
<tr>
<td>$f_s=156.58+57.5x10^3g s$</td>
</tr>
<tr>
<td>64$^2f_s=72$</td>
</tr>
<tr>
<td>$s=5.43x10^{-4}xe0.060fs$</td>
</tr>
<tr>
<td>$0.025\leq s\leq0.41$</td>
</tr>
<tr>
<td>$f_s=91.02+13.63x10^3g s$</td>
</tr>
<tr>
<td>J7-90/1-15-4-H</td>
</tr>
<tr>
<td>$4f_s=15$</td>
</tr>
<tr>
<td>$s=2x10^{-4}xe0.166fs$</td>
</tr>
<tr>
<td>$0.00039\leq s\leq0.0025$</td>
</tr>
<tr>
<td>$f_s=51.60+13.87x10^3g s$</td>
</tr>
<tr>
<td>15$^2f_s=28$</td>
</tr>
<tr>
<td>$s=8.88x10^{-4}xe0.069fs$</td>
</tr>
<tr>
<td>$0.0025\leq s\leq0.0061$</td>
</tr>
<tr>
<td>$f_s=101.45+33.37x10^3g s$</td>
</tr>
<tr>
<td>28$^2f_s=56$</td>
</tr>
<tr>
<td>$s=20.4x10^{-4}xe0.039fs$</td>
</tr>
<tr>
<td>$0.0061\leq s\leq0.018$</td>
</tr>
<tr>
<td>$f_s=158.23+59.04x10^3g s$</td>
</tr>
<tr>
<td>56$^2f_s=73$</td>
</tr>
<tr>
<td>$s=31.1x10^{-4}xe0.031fs$</td>
</tr>
<tr>
<td>$0.018\leq s\leq0.029$</td>
</tr>
<tr>
<td>$f_s=187.18+74.28x10^3g s$</td>
</tr>
<tr>
<td>73$^2f_s=99.2$</td>
</tr>
<tr>
<td>$s=1.88x10^{-4}xe0.069fs$</td>
</tr>
<tr>
<td>$0.029\leq s\leq0.18$</td>
</tr>
<tr>
<td>$f_s=124.14+33.37x10^3g s$</td>
</tr>
</tbody>
</table>

$f_s = \text{ksi}, \ s = \text{in.}$
<table>
<thead>
<tr>
<th>J7-90/.5-15-1-L</th>
<th>$7 \leq f_g &lt; 11$</th>
<th>$11 \leq f_g &lt; 20$</th>
<th>$20 \leq f_g &lt; 38$</th>
<th>$38 \leq f_g &lt; 71$</th>
<th>$71 \leq f_g &lt; 84$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = 4.11 \times 10^{-4} x_e$</td>
<td>$s = 5.15 \times 10^{-4} x_e$</td>
<td>$s = 8.23 \times 10^{-4} x_e$</td>
<td>$s = 6.51 \times 10^{-4} x_e$</td>
<td>$s = 5.50 \times 10^{-4} x_e$</td>
<td></td>
</tr>
<tr>
<td>$f_g = 0.0066 &lt; s &lt; 0.0016$</td>
<td>$f_g = 0.016 &lt; s &lt; 0.040$</td>
<td>$f_g = 0.0040 &lt; s &lt; 0.047$</td>
<td>$f_g = 0.040 &lt; s &lt; 0.069$</td>
<td>$f_g = 0.029 &lt; s &lt; 0.063$</td>
<td></td>
</tr>
<tr>
<td>$e = 40.20 + 8.47 x \log s$</td>
<td>$e = 74.05 + 22.85 x \log s$</td>
<td>$e = 89.77 + 19.15 x \log s$</td>
<td>$e = 177.86 + 79.40 x \log s$</td>
<td>$e = 120.29 + 36.68 x \log s$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>J7-90/.5-15-2-L</th>
<th>$17 \leq f_g &lt; 24$</th>
<th>$24 \leq f_g &lt; 37$</th>
<th>$37 \leq f_g &lt; 43$</th>
<th>$43 \leq f_g &lt; 61$</th>
<th>$61 \leq f_g &lt; 90.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = 0.0095 \times 10^{-4} x_e$</td>
<td>$s = 0.046 x e$</td>
<td>$s = 0.067 x e$</td>
<td>$s = 0.113 x e$</td>
<td>$s = 1.01 x e$</td>
<td></td>
</tr>
<tr>
<td>$f_g = 0.00012 &lt; s &lt; 0.00096$</td>
<td>$f_g = 0.00096 &lt; s &lt; 0.0053$</td>
<td>$f_g = 0.0053 &lt; s &lt; 0.0085$</td>
<td>$f_g = 0.0085 &lt; s &lt; 0.019$</td>
<td>$f_g = 0.019 &lt; s &lt; 0.22$</td>
<td></td>
</tr>
<tr>
<td>$e = 48.28 + 7.94 x \log s$</td>
<td>$e = 77.11 + 17.85 x \log s$</td>
<td>$e = 102.54 + 28.45 x \log s$</td>
<td>$e = 145.02 + 48.99 x \log s$</td>
<td>$e = 108.36 + 27.09 x \log s$</td>
<td></td>
</tr>
</tbody>
</table>

$f_g \rightarrow \text{ksi}, \ s \rightarrow \text{in.}$
<table>
<thead>
<tr>
<th>J11-90/1-15-1-H</th>
<th>8&lt;fs&lt;16  s=0.97x10^{-4} 0.211fs  0.00053&lt;s&lt;0.0028  ( f_s=43.65+10.91x\log s )</th>
<th>16&lt;fs&lt;25  s=4.03x10^{-4} 0.122fs  0.0028&lt;s&lt;0.0085  ( f_s=64.18+18.88x\log s )</th>
<th>25&lt;fs&lt;46  s=13.24x10^{-4} 0.074fs  0.0085&lt;s&lt;0.040  ( f_s=89.62+31.12x\log s )</th>
<th>46&lt;fs&lt;48.5  s=0.24x10^{-4} 0.161fs  0.040&lt;s&lt;0.059  ( f_s=65.79+14.30x\log s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>J11-90/1-15-1-L</td>
<td>8&lt;fs&lt;14  s=0.1x10^{-4} 0.398fs  0.0024&lt;s&lt;0.0025  ( f_s=28.93+5.79x\log s )</td>
<td>14&lt;fs&lt;24  s=2.52x10^{-4} 0.160fs  0.0025&lt;s&lt;0.012  ( f_s=51.51+14.39x\log s )</td>
<td>24&lt;fs&lt;50  s=27.22x10^{-4} 0.061fs  0.012&lt;s&lt;0.058  ( f_s=96.65+37.75x\log s )</td>
<td>50&lt;fs&lt;51.8  s=36.85x10^{-4} 0.055fs  0.058&lt;s&lt;0.064  ( f_s=102.15+41.87x\log s )</td>
</tr>
<tr>
<td>J11-90/1-12-1-H</td>
<td>9&lt;fs&lt;17  s=0.3x10^{-4} 0.293fs  0.004&lt;s&lt;0.002  ( f_s=35.53+7.86x\log s )</td>
<td>17&lt;fs&lt;23  s=4.75x10^{-4} 0.125fs  0.004&lt;s&lt;0.0085  ( f_s=61.15+18.42x\log s )</td>
<td>23&lt;fs&lt;40  s=11.54x10^{-4} 0.087fs  0.0085&lt;s&lt;0.038  ( f_s=78.34+26.47x\log s )</td>
<td>40&lt;fs&lt;41.7  s=0.57x10^{-4} 0.163fs  0.038&lt;s&lt;0.051  ( f_s=59.90+14.15x\log s )</td>
</tr>
<tr>
<td>J11-180/1-15-1-H</td>
<td>3&lt;fs&lt;9  s=1.54x10^{-4} 0.312fs  0.0039&lt;s&lt;0.0025  ( f_s=28.04+7.39x\log s )</td>
<td>9&lt;fs&lt;17  s=6.27x10^{-4} 0.147fs  0.0025&lt;s&lt;0.0076  ( f_s=50.15+15.07x\log s )</td>
<td>17&lt;fs&lt;24  s=15.61x10^{-4} 0.093fs  0.0076&lt;s&lt;0.015  ( f_s=69.33+24.76x\log s )</td>
<td>24&lt;fs&lt;40  s=36.40x10^{-4} 0.058fs  0.015&lt;s&lt;0.037  ( f_s=96.87+39.70x\log s )</td>
</tr>
<tr>
<td>J11-180/1-15-1-L</td>
<td>17&lt;fs&lt;22  s=0.0006x10^{-4} 0.596fs  0.0015&lt;s&lt;0.003  ( f_s=31.85+3.87x\log s )</td>
<td>22&lt;fs&lt;29  s=1.56x10^{-4} 0.134fs  0.003&lt;s&lt;0.0077  ( f_s=65.50+17.19x\log s )</td>
<td>29&lt;fs&lt;42  s=6.02x10^{-4} 0.086fs  0.0077&lt;s&lt;0.024  ( f_s=84.78+26.17x\log s )</td>
<td>42&lt;fs&lt;49.7  s=3.35x10^{-4} 0.102fs  0.024&lt;s&lt;0.053  ( f_s=78.56+22.58x\log s )</td>
</tr>
<tr>
<td>J11-90/1-15-2-H</td>
<td>6&lt;fs&lt;10  s=0.55x10^{-4} 0.305fs  0.0034&lt;s&lt;0.0011  ( f_s=32.31+7.55x\log s )</td>
<td>10&lt;fs&lt;15  s=2.37x10^{-4} 0.152fs  0.0011&lt;s&lt;0.0023  ( f_s=55.15+15.15x\log s )</td>
<td>15&lt;fs&lt;27  s=4.31x10^{-4} 0.112fs  0.0023&lt;s&lt;0.0089  ( f_s=69.08+20.56x\log s )</td>
<td>27&lt;fs&lt;44  s=11.36x10^{-4} 0.076fs  0.0089&lt;s&lt;0.032  ( f_s=89.68+30.30x\log s )</td>
</tr>
</tbody>
</table>

\( f_s \) ksi, \( s \) in.
| J11-90/1-15-2-L | 8cf_s<11  
| s=0.62x10^-4 xe0.271f_s  
| 0.00054<e<0.0012  
| f_s=35.69+8.50xlog s | 11cf_s<22  
| s=1.78x10^-4 xe0.177f_s  
| 0.0012<e<0.00086  
| f_s=48.92+13.01xlog s | 22cf_s<28  
| s=8.94x10^-4 xe0.102f_s  
| 0.0086<e<0.016  
| f_s=68.63+22.58xlog s | 28cf_s<52.8  
| s=36.55x10^-4 xe0.052f_s  
| 0.016<e<0.057  
| f_s=108.04+44.28xlog s |
| J11-90/1-15-3-L | 10cf_s<18  
| s=0.62x10^-4 xe0.255f_s  
| 0.00079<e<0.0063  
| f_s=37.93+9.03xlog s | 18cf_s<32  
| s=1.58x10^-4 xe0.079f_s  
| 0.0065<e<0.020  
| f_s=61.61+29.15xlog s | 32cf_s<60  
| s=48.14x10^-4 xe0.044f_s  
| 0.020<e<0.069  
| f_s=121.41+52.33xlog s | 60cf_s<62  
| s=0.55x10^-4 xe0.120f_s  
| 0.069<e<0.090  
| f_s=82.12+19.19xlog s |
| J11-90/1-15-3a-L | 14cf_s<19  
| s=0.045x10^-4 xe0.350f_s  
| 0.00058<e<0.0034  
| f_s=55.27+6.58xlog s | 19cf_s<35  
| s=8.05x10^-4 xe0.077f_s  
| 0.0034<e<0.012  
| f_s=93.30+29.91xlog s | 35cf_s<57  
| s=25.81x10^-4 xe0.044f_s  
| 0.012<e<0.032  
| f_s=156.07+52.33xlog s | 57cf_s<68.9  
| s=15.23x10^-4 xe0.053f_s  
| 0.032<e<0.059  
| f_s=121.65+43.45xlog s |
| J11-90/1-15-4-L | 8cf_s<12  
| s=0.14x10^-4 xe0.313f_s  
| 0.00017<e<0.00059  
| f_s=55.91+7.36xlog s | 12cf_s<26  
| s=0.8x10^-4 xe0.164f_s  
| 0.00059<e<0.0056  
| f_s=57.28+14.04xlog s | 26cf_s<59  
| s=6.51x10^-4 xe0.082f_s  
| 0.0056<e<0.016  
| f_s=89.86+28.08xlog s | 59cf_s<44.1  
| s=1.94x10^-4 xe0.113f_s  
| 0.016<e<0.028  
| f_s=75.80+20.38xlog s |
| J11-90/1-15-5-L | 10cf_s<21  
| s=0.06x10^-4 xe0.320f_s  
| 0.00015<e<0.0048  
| f_s=37.70+7.20xlog s | 21cf_s<54  
| s=9.34x10^-4 xe0.076f_s  
| 0.0048<e<0.013  
| f_s=92.10+30.50xlog s | 54cf_s<59  
| s=24.50x10^-4 xe0.048f_s  
| 0.013<e<0.042  
| f_s=124.72+47.97xlog s | 59cf_s<65.4  
| s=2.79x10^-4 xe0.085f_s  
| 0.042<e<0.072  
| f_s=96.44+27.09xlog s |
| J11-90/1-15-1-H | 9cf_s<14  
| s=1.78x10^-4 xe0.208f_s  
| 0.0012<e<0.0032  
| f_s=41.62+11.07xlog s | 14cf_s<27  
| s=5.76x10^-4 xe0.122f_s  
| 0.0032<e<0.016  
| f_s=61.16+18.88xlog s | 27cf_s<43  
| s=31.25x10^-4 xe0.060f_s  
| 0.016<e<0.041  
| f_s=96.71+38.38+log s | 43f_s<47.6  
| s=5.76x10^-4 xe0.099f_s  
| 0.041<e<0.064  
| f_s=75.35+23.26xlog s |

f_s = ksi, s = in.
<table>
<thead>
<tr>
<th>J11-90/.5-15-1-L</th>
<th>14 ≤ f_g ≤ 17</th>
<th>17 ≤ f_g ≤ 25</th>
<th>25 ≤ f_g ≤ 39</th>
<th>39 ≤ f_g ≤ 42.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>s = 0.0025x10^{-4}xe 0.491f_g</td>
<td>s = 0.32x10^{-4}xe 0.210f_g</td>
<td>s = 9.69x10^{-4}xe 0.076f_g</td>
<td>s = 0.56x10^{-4}xe 0.149f_g</td>
<td></td>
</tr>
<tr>
<td>0.00024 ≤ s ≤ 0.0011</td>
<td>0.0011 ≤ s ≤ 0.0063</td>
<td>0.0065 ≤ s ≤ 0.019</td>
<td>0.019 ≤ s ≤ 0.032</td>
<td></td>
</tr>
<tr>
<td>f_g = 30.95 + 4.69x10^1log s</td>
<td>f_g = 49.56 + 10.97x10^1log s</td>
<td>f_g = 90.89 + 30.30x10^1log s</td>
<td>f_g = 65.53 + 15.46x10^1log s</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>J11-90/.5-15-2-L</th>
<th>11 ≤ f_g ≤ 15</th>
<th>15 ≤ f_g ≤ 19</th>
<th>19 ≤ f_g ≤ 27</th>
<th>27 ≤ f_g ≤ 43</th>
</tr>
</thead>
<tbody>
<tr>
<td>s = 0.0008x10^{-4}xe 0.682f_g</td>
<td>s = 0.34x10^{-4}xe 0.273f_g</td>
<td>s = 7.41x10^{-4}xe 0.108f_g</td>
<td>s = 32.40x10^{-4}xe 0.054f_g</td>
<td></td>
</tr>
<tr>
<td>0.00015 ≤ s ≤ 0.0021</td>
<td>0.0021 ≤ s ≤ 0.0059</td>
<td>0.0059 ≤ s ≤ 0.014</td>
<td>0.014 ≤ s ≤ 0.034</td>
<td></td>
</tr>
<tr>
<td>f_g = 23.91 + 3.38x10^1log s</td>
<td>f_g = 37.79 + 8.44x10^1log s</td>
<td>f_g = 66.52 + 21.32x10^1log s</td>
<td>f_g = 105.75 + 42.64x10^1log s</td>
<td></td>
</tr>
</tbody>
</table>

f_g = ksi, s = in.

\[ 181 \]
FIGURE B.1. PLOT OF LEAD BAR STRESS-LOG($10^4 \times$LEAD SLIP)

(x) - from this point on stopped recording to avoid instrument damage
FIGURE B.4. PLOT OF LEAD BAR STRESS-\text{LOG}(10^4 \times \text{LEAD SLIP})
Figure B.5. Plot of Lead Bar Stress Log(10^4 x Lead Slip) vs Log(Slip)
Figure B.6. Plot of lead bar stress-log(10⁴ x lead slip) vs. log(10⁴ x slip).

(x) - From this point on, stopped recording to avoid instrument damage.
FIGURE B.7. PLOT OF LEAD BAR STRESS-LOG(10^4 x LEAD SLIP)
FIGURE B.8. PLOT OF LEAD BAR STRESS-\(\log(10^4 \times \text{LEAD SLIP})\)
APPENDIX C

THEORETICAL APPROACH TO HOOK ACTION

In order to have a better understanding of the behavior of a hook when loaded, a theoretical solution based on principles of strength of materials will be presented.

The problem is to find the distribution of axial load, moment and radial deflection of a steel reinforcing bar bent to 180° and embedded in concrete, which constitutes the elastic foundation.

Starting with the solved problem of an arch on an elastic foundation, which can be found in Beams on Elastic Foundations (20), the solution has to include the addition of tangential forces, bond and frictional forces, acting along the bar (arch) and the application of the loads at an end instead of distributed across the arch.

The following assumptions were made:

1. The bar acts as an arch of flexural rigidity EI, the neutral axis of the undeflected shape being a circular arc of radius R.
2. The reaction forces are normal to the axis of the bar at every point and proportional to the radial deflection.
3. The foundation modulus is numerically equal to the modulus of elasticity of the concrete which constitutes the elastic foundation.

4. The tangential force per unit length, $P$, is sufficient to hold the bar in equilibrium and is equal to $P_x + \mu P_r$.

Taking the differential element in Figure B.2, using the differential equation of a circular arch of radius $R$ and flexural rigidity $EI$, $M = -EI(d^2Y/dx^2 + Y/R^2)$, and considering the conditions of static equilibrium, the following differential equation is obtained.

$$Y'' + 2Y''' + \left( \frac{KR^4}{EI} + 1 \right) Y' - \mu \frac{KR^4}{EI} Y = P_x \frac{R^4}{EI}$$

where $K = \phi FM$

- $FM$ = foundation modulus
- $\phi$ = diameter of the bar
- $\mu$ = friction coefficient for concrete placed against the steel bar
- $R$ = radius of hook or arch to center line
- $P = P_x + \mu P_r$
- $P_x = KA \pi \phi = U \theta \phi$ (linear variation)
- $U = \frac{N}{\pi \phi L} = \frac{N}{(\pi \phi) \pi \phi RN} = \frac{N}{(\pi \phi)^2 RN} = \text{bond stress}$
- $RN = \frac{R}{\phi}$
- $P_r = KY = \text{radial stresses on concrete}$
\[ Y = \text{radial deflection of the bar} \]
\[ EI = \text{flexural rigidity of the bar} \]
\[ Y' = \frac{dY}{d\theta} \]
\[ Y'' = \frac{d^2Y}{d\theta^2} \]
\[ Y''' = \frac{d^3Y}{d\theta^3} \]
\[ Y'''' = \frac{d^4Y}{d\theta^4} \]
\[ Y''''' = \frac{d^5Y}{d\theta^5} \]

To solve the characteristic equation, a computer program has been written and the subroutine

\[ ZPOLYR(A,N,Z,Z,IER), \]

from the IBM Scientific package, has been used. The roots were of the form:

\[ \lambda_1 = C, \quad \lambda_{2,3} = -AA\pm Bj, \quad \lambda_{4,5} = D\pm Ej. \]

Thus the general solution of the differential equation is:

\[ Y = C_1 \exp(C\theta) + \exp(-A\theta)F_1 + \exp(D\theta)F_2 + Y_p \]

where

\[ F_1 = C_2 \cos(B\theta) + C_3 \sin(B\theta) \]
\[ F_2 = C_4 \cos(E\theta) + C_5 \sin(E\theta) \]
\[ Y_p = -\frac{U_{\phi}}{\mu K} \left( \theta + \frac{1}{\mu} + \frac{EI}{\mu KR^4} \right) \]

\[ \dagger \text{For the sake of simplicity in typing } \exp(f(x)) \text{ substitutes for } e^{f(x)}. \]
and the solutions for the moment, shear and axial load are:

\[ M = -\frac{EI}{R^2} \left( Y^{II} + Y \right) \]

\[ Q = -\frac{EI}{R^3} \left( Y^{III} + Y^{I} \right) \]

\[ N = KR + \frac{EI}{R^3} \left( Y^{IV} + Y^{II} \right) \]

For obtaining the value of the constants \( C_1, C_2, C_3, C_4 \) and \( C_5 \) the following boundary conditions were used:

\[ \theta = 0, \ M = N = Q = 0 \]

\[ \theta = \pi, \ N = T, \ Y = 0 \]

With these five boundary conditions, five simultaneous linear equations were obtained, which are:

\[ \theta = 0, \ M = 0 \quad \text{1st equation} \quad Y^{II}_O + Y_O = 0 \]

\[ \theta = 0, \ N = 0 \quad \text{2nd equation} \quad \frac{KR^4}{EI} Y_O + Y^{IV}_O + Y^{II}_O = 0 \]

\[ \theta = \pi, \ Y = 0 \quad \text{3rd equation} \quad Y_\pi = 0 \]

\[ \theta = 0, \ Q = 0 \quad \text{4th equation} \quad Y^{III}_O + Y^I_O = 0 \]

\[ \theta = \pi, \ N = T \quad \text{5th equation} \quad Y^{IV}_\pi + Y^{II}_\pi = \frac{TR^3}{EI} \]

For obtaining the values of moments, displacements
and stresses in the concrete along the bar, a computer pro-
gram was written. This program as well as the one used to
obtain the roots of the characteristic equation were written
in Fortran IV and are presented, as well as their flow
charts, at the end of this Appendix. The subroutine SLVE
used to solve the simultaneous linear equations is a stan-
dard Gauss subroutine, which solves \( AX = B \) by triangular de-
composition with interchanges. The original \( A \) and \( B \) matrices
are destroyed and the results appear in \( B \). If the subrou-
inte is reentered with \( ISW \neq 0 \), it reuses the old decomposi-
tion of \( A \) and processes a new \( B \) matrix. It uses an auxiliary
index array \( M(N) \). If the \( A \) matrix is singular the computa-
tions are stopped.

The symbols used in the computer programs stand for:

(a) Computer program used for solving the fifth order
characteristic equation

\[
\begin{align*}
N & \quad \text{Degree of polynomial} \\
NOP & \quad \text{Number of problems to be solved} \\
\pi & = \pi; \text{ ES = modulus of elasticity of steel;} \\
\mu & = \mu; \text{ RN = R/\phi; FPC = concrete strength;} \\
\phi & = \phi; \text{ FM = modulus of elasticity of concrete;} \\
\text{GAMA} & = KR^4/EI; \text{ A(I) are the coefficients of the} \\
\text{polynomial, being in this case} \\
A(1)X^5 + A(3)X^3 + A(5)X - A(6) & = 0.
\end{align*}
\]
(b) Computer program used to obtain the value of the
five constants, $C_1$, $C_2$, $C_3$, $C_4$ and $C_5$

$$\text{RN} = \frac{R}{\phi}; \text{ FPC } = \text{ concrete strength; PHI } = \phi;$$
$$\text{FCOF } = \mu; \text{ ES } = \text{ modulus of elasticity of steel;}$$
$$\text{PI } = \pi; \text{ FM } = \text{ modulus of elasticity of concrete;}$$

$$A_1 = \frac{KR^4}{EI}; \quad DM = \frac{EI}{R^2}; \quad AA + jB \text{ and } D + jE \text{ and } C$$

are the roots of the characteristic equation; $T =$
force applied at the loaded end of the bent bar;

$$D_1 = \frac{TR^3}{EI}; \quad U = \frac{XT}{(\pi \phi)^2 \text{RN}},$$

$X$ being the coefficient which will permit any
given value to the bond stress; $U_1 = 7.5\sqrt{f_c}$,
that is the maximum value which $U$ can have;

$$B3 = \frac{U\phi}{\mu K}; \quad B5 = \frac{1}{\mu}; \quad B6 = \frac{1}{\mu A_1}; \quad Z(J) = 0; \quad Y(I) =$$
radial displacements; $YM(I) = \text{ moments along the}$
bent bar; $W(I) = \text{ concrete stresses underneath the}$
bent steel bar.

A few problems have been worked, with $X = 1$, and the
results are plotted.

In Figure C.4 the numerical values for the radial
deflection for a #3 bar are plotted for six different cases,
which are:

1. Hook radius equal to 3.5\(\phi\) and embedded in three different types of concrete.

2. Hook radius equal to 4.5\(\phi\) and embedded in three different types of concrete.

As should be expected and can be observed in Figure C.4:

(a) for the same hook radius, the maximum radial deflection decreases when the concrete strength increases.

(b) for the same type of concrete, the maximum radial deflection decreases when the hook radius increases.

In Figure C.5 the numerical values for the radial deflection for a #11 bar are plotted for six different cases, which are listed on the figure. As can be seen, the trends are the same for both bars. It can also be seen in both figures that the maximum value of the radial deflection has a tendency to shift toward the unloaded end, with the increase of either the concrete strength or the hook radius.

In Figure C.6 the radial deflections for a #3 bar with a standard hook radius are compared with the deflections for a #11 bar, also with a standard hook radius. Three different types of concrete were used. It can be seen that the maximum radial deflection for the #3 bar is:
(a) smaller than the maximum radial deflection for the #11 bar for \( f'_c = 1000 \text{ psi} \);
(b) about the same for \( f'_c = 3000 \text{ psi} \);
(c) larger for \( f'_c = 5000 \text{ psi} \).

However, the more important fact which can be observed in Figure C.6 is that the radial displacements for the #3 bar are negligible at about 2 inches, measured along the hook (arch), from the loaded end while for the #11 bar they are negligible at about 12 inches from the loaded end, also measured along the hook. That is, the hook whose bar has the smaller diameter may produce higher stresses on concrete. However, as for the hook with the smaller bar diameter the stresses are distributed along a very small length, the problem of splitting will increase with the use of the hook with the larger bar diameter.

In Figure C.7 moments are plotted for the cases listed on the figure. As can be seen, the value of the moment decreases very rapidly and is practically equal to zero at about 50 to 60 degrees from the loaded end.

A few problems have been solved with \( X = 0 \), that is, with zero bond stress. The results are not present because they were too unrealistic. Yet, even using \( X = 0 \), the value of the moment was practically equal to zero at about 50 to 60 degrees from the loaded end, as in the case with bond stresses.
In Table 1C the values of maximum moment at the loaded end are listed for a few cases. The axial force applied in all cases developed, in the bar, a stress equal to $f_y$, which was assumed to be equal to 60 ksi. In Figure C.3 an interaction diagram for a circular cross section is presented. Assuming that splitting was prevented, it can be seen that in all cases listed in Table 1C the anchorage would fail by overstress in the steel bar.

In Table 2C are listed the results obtained solving the same problems applying an axial force which in all cases developed, in the bar, a stress equal to 0.75$f_y$, $f_y$ again being taken equal to 60 ksi. At the interaction diagram it can be seen that for $N = 0.75 N_p$ the value at $M$ is 0.27 $M_p$, that is, $M \approx 0.30 M_p$. A few values of the ratio of maximum computed moment to plastic moment listed in Table 2C are very close to 0.30, that is, if splitting was prevented, the anchorage in those cases would resist to the applied axial forces.

It should be noted that the solution of the problem of a hook in elastic media, as treated here, will only be able to give reasonable qualitative answer, that is, will give a fairly good insight into the action of a hook embedded in concrete.

The values of the radial deflections observed in the tests are different from the values given by this solution. Some of the reasons for the discrepancies can be listed as
follows:

1. The fact that the concrete behaves non-linearly, becoming gradually stiffer for greater deflections was neglected.

2. It has been assumed that the foundation, the concrete, behaves without continuity, that is, the deflection at any point was assumed to be caused by the load acting only on that point and was totally independent of the other loads nearby.

3. Some of the boundary conditions may have been violated.

4. The assumed bond stress distribution, even though not unreasonable, is rather arbitrary.

5. The possible changes in bond stress distribution during loading were neglected.

6. The relative movement between the bar and the concrete was not taken into account.

As has already been said, the present solution was worked only for giving some insight into the problem of the action of a hook embedded in concrete. Thus, no further steps were taken in order to eliminate or even correct the discrepancies listed above.
### TABLE 1C

**SUMMARY OF HOOK BEHAVIOR**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>5.5</td>
<td>233486</td>
<td></td>
<td>240000</td>
<td>115131</td>
<td>2.03</td>
</tr>
<tr>
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<td></td>
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REAL A(I)
CAMEX Z(I)
READ 60,*,NOP
60 FORMAT(21E)
READ 10,(A(I),I=1,4)
10 FORMAT(4F10.6)
FI=3.141593
ES=2900000.
FCOF=1.
DO 20 J=1,NOP
READ 30,FN,FPC,PHI
30 FORMAT(3F10.4)
PRINT 40,FN,FPC,PHI
40 FORMAT(4X,*RN=HCKX FADII/EAR DIAMETER=*,F10.4,/4X,*FFC=
*CONCRETE STRENGTH=*,F10.4,4X,*PHI=BAR DIAMETER=*,F10.4,/)
FM=(145.*1.5)*33.*SQR(FPC)
GAMA=(64.*FM*PHI*(RN*4))/(ES*PI)
ETA=GAMA*1.
A(5)=ETA
A(6)=-BETA
PRINT 90,(A(I),I=1,6)
90 FORMAT(4X,*COEF. CF PCLY, ARE=*,6F15.6)
CALL ZPCLYR(A,N,Z,Z,IER)
PRINT 50,(Z(K),K=1,N)
50 FORMAT(4X,*THE ROOTS ARE=*,F15.6)
CC 70 K=1,N
70 Z(K)=Z(K)**5+2.*Z(K)**3+A(5)*Z(K)*A(6)
20 PRINT 80,(Z(K),K=1,N)
80 FORMAT(4X,*THE REST IS=*,2F15.6)
STOP
END
IMPLICIT REAL*4 (A-H,O-Z)
DIMENSION A(10),CE(10),Z(25),Y(25),W(25),M(10),YU(25)
300 READ RN,IP,R,PHI
IF(RN.EQ.0.0)GO TO 400
10 FORMAT(3F10.5)
PRINT 20,RN,FPC,PHI
20 FORMAT(4x,RN=HOOK RAD,/*BAR DIAM=*F10.5,/,4x,FPC=CONCRETE
*STRENGTH=*F10.5,4x,PHI=BAR DIAM=*F10.5,/,4x,*)
FPC=1.
FS=29C0000.
PI=3.141593
FM=(145.4+1.5)*33.5/DSQRT(FPC)
PRINT 30,FM
30 FORMAT(4x,*THE FOUNDATION MOD. IS=*F15.5,/) 
AI=(64.*FM/(RN**4)*PHI)/(ES*PI)
DM=-(ES*PHI*PHI*PI)/(64.*RN**4)
READ 40,A,B,D,E,C
40 FORMAT(5F10.5)
PRINT 50,A,B,D,E,C
50 FORMAT(4x,*THE ROOTS ARE:*4X,F10.5,2X,F10.5,5X,F10.5,
*2X,F10.5,5X,F10.5,/) 
G=DEXP(C*PI)
H=DEXP(-A*PI)
D=DEXP(D*PI)
PP=DSIN(B*PI)
QQ=DCOS(B*PI)
RR=DSIN(F*PI)
SS=DCOS(F*PI)
P1=2.*D*E*(2.*E*E-2.*D*D-1.)
V1=D*E*A-A*D*D*E**2+D-D-E*C
A(1,1)=1.+C*C
A(1,3)=-2.*A*A*A
A(1,4)=D*D-E+1.
A(1,5)=2.*D*E
A(2,1)=A1+C*C*(1.+C*C)
A(2,2)=A1+V
A(2,3)=-P
A(2,4)=A1+V1
A(2,5)=-P1
A(3,1)=G
A(3,2)=H*Q0
A(3,3)=H*PP
A(3,4)=0.*SS
A(3,5)=0.*PR
A(4,1)=C*(1.+C*C)
A(4,4)=D*(D*D-3.*E+F+1.)
A(4,5)=E*(3.+D*D-E+F+1.)
A(5,1)=C*(C**4+C*C)
A(5,2)=(P*P+Q0*V)
A(5,3)=(P*P+Q0*P)
A(5,4)=0.*(RR+V1-SS*V1)
A(5,5)=0.*(RR+V1-SS*P1)
16=1
200 READ 6C,T
   IF(T.EQ.0.0)GO TO 300
60 FORMAT(F10.5)
   PRINT 70,T
70 FORMAT(4X,'AXIAL FORCE IS=',F15.5,/) 
   D1=(T*64.*(RN*3))/((4*PI*PHI)
500 READ 80,X
   I6=I6-1
   IF(X.LT.0.0)GO TO 200
30 FORMAT(F10.5)
   U=(X*T)/(PI*PHI*PHI*RN)
   U1=7.5+0.5SQR(T(FP))
   IF(U-U1)165,165,155
155 U=U1
105 PRINT 255,U
255 FORMAT(4X,'BUND STRESS=',F10.5,/) 
   33=U/(FCOF*FM)
   BS=1./FCOF
   B6=1./(FCOF*A1)
   BI=03.*(PI+BS+.6)
   B2=BS*(B5+B6)
   CE(1)=H2
   CE(2)=A1*B2
   CE(3)=B1
   CE(4)=B3
   CE(5)=D1
   N=S
   ISW=16
   CALL SLVE(A,CE,N,ISW)
   IF(A(5,5).EQ.0.0)GO TO 400
   PRINT 100,(CE(I),I=1,N)
100 FORMAT(4X,THE COEFF. OF DISPL. FUNCT. ARE/,4X,C1=',F12.5, 
   Z(I)=0.0
   DC 110 J=1,18
110 Z(J+1)=Z(J)+F1/18.
   DC 910 I=1,19 
   Y(I)=CE(1)*DFXP(CZ(I))+DFXP(-AA*Z(I))+(CE(2)*DCOS(3*Z(I))) 
   **+CE(3)*DSIN(BZ(I))+DFXP(DZ(I)+(CE(4)*DCOS(EZ(I))) 
   *CE(5)*DSIN(EZ(I)))+B3*(Z(I)+B5+B6)
   YM(I)=D yn*(CE(1)*DFXP(CZ(I)) *(1.+C*C)+DFXP(-AA*Z(I))+(DSIN(3* 
   *Z(I))+(CE(2))*AA*B*(CE(3)+AA*AA-3*B+1.))-DCOS(H*Z(I))+(CE(2)* 
   *(B*B*AA-AA-1.)+CE(3)*AA*B)+DFXP(DZ(I))*DSIN(E*Z(I))*(CE(4) 
   **+(-2.*D*E)+CE(5)*(D*D-E^2+1.))+DCOS(EZ(I))*(CE(4)*(D*D-E^2+1.) 
   *CE(5)*2.*D*F)))-B3*(Z(I)+B5+B6)
910 W(I)=Y(I)*FM
   PRINT 120,(W(I),I=1,19)
   PRINT 130,(Y(I),I=1,19)
   PRINT 140,(YM(I),I=1,19)
120 FORMAT(4X,'CONCRETE STRESS AT 1C DEG. INT',4X,5E16.7,/) 
   *5E16.7,4X,5E16.7,4X,4E16.7,/) 
130 FORMAT(4X,'DISPL. AT 10 DEG. INT',4X,5E16.7,4X,5E16.7,4X 
   *5E16.7,4X,4E16.7,/) 
140 FORMAT(4X,'MOMENT AT 10 DEG. INT',4X,5E16.7,4X,5E16.7, 
   *5E16.7,4X,4E16.7,/) 
GO TO 500
400 STOP
SUBROUTINE SLVE(A,BH,N,ISW)
IMPLICIT REAL*(A-H,Z)
DIMENSION A(10,10),BH(10),M(10)
IF(ISW.NE.0)GO TO 1
DO 1000 K=1,N
IF(K.EQ.1)GO TO 1001
K1=K-1
DO 2000 J=1,K1
I=M(J)
S=A(I,K)
A(I,K)=A(J,K)
A(J,K)=S
J1=J+1
DO 3000 I=J1,N
3000 A(I,K)=A(I,K)-A(I,J)*S
2000 CONTINUE
1001 IF(K.EQ.N)GO TO 1190
I=K
S=A(K,K)
K2=K+1
DO 4000 J=K2,N
IF(ABS(A(J,K))+S.GT.ABS(S))GO TO 400C
I=J
S=A(J,K)
400C CONTINUE:
IF(S.EQ.0.0)GO TO 9991
M(K)=I
A(I,K)=A(K,K)
A(K,K)=S
K3=K+1
DO 7000 I=K3,N
7000 A(I,K)=A(I,K)/S
1000 CONTINUE:
1190 IF(A(N,N).EQ.0.0)GO TO 9991
1 N1=N-1
DO 7000 J=1,N1
I=M(J)
S=BH(I)
BR(I)=BH(J)
BH(J)=S
J2=J+1
DO 8000 I=J2,N
8000 BR(I)=BR(I)-A(I,J)*S
7000 CONTINUE
DO 9000 I1=1,N
J=N-(I1-1)
BH(J)=BH(J)/A(J,J)
IF(J.EQ.1)GO TO 900C
J3=J-1
DO 9009 I=1,J3
9009 BH(I)=BH(I)-A(I,J)*BH(J)
9000 CONTINUE
GO TO 9999
9991 A(N,N)=0.0
9999 RETURN
END
FIGURE C.3. INTERACTION DIAGRAM FOR CIRCULAR CROSS SECTION
# 3 BAR

HOOK RADI. $\phi < 3.5$

$\phi < 4.5$

$f_c = 1000 \text{ psi}$

$f_c = 3000 \text{ psi}$

$f_c = 5000 \text{ psi}$

FIGURE C.4. RADIAL DEFLECTIONS
FIGURE C.5. RADIAL DEFLECTIONS
FIGURE C.6. RADIAL DEFLECTIONS
Figure C.7. Moment