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ACCELERATION AND INJECTION OF
SOLAR-WIND PARTICLES AT THE MAGNETOPAUSE

by

Thomas W. Hill

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

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ABSTRACT

ACCELERATION AND INJECTION OF
SOLAR-WIND PARTICLES AT THE MAGNETOPAUSE

by

Thomas W. Hill

The earth's magnetopause is modeled as a tangential boundary between two regions of plasma having either parallel or antiparallel magnetic fields. The boundary has the geometry of a tangential discontinuity, but has a finite dimension in the direction of current flow. In a steady state, the loss of current-carrying particles from the boundary requires a flow of plasma into the boundary. The properties of this plasma flow are derived from the conditions of mass, momentum, and energy conservation. When the fields on either side are parallel, plasma flows through the boundary toward the region having the smaller ratio of (plasma concentration)/(field strength). When the fields are antiparallel, plasma flows toward the boundary from both sides. As applied to the magnetopause, the tangential-boundary model provides a mechanism for acceleration and injection of solar wind particles into the magnetosphere to populate the plasma sheet when the interplanetary magnetic field is northward. A mechanism is also provided for the depletion of the plasma sheet when the interplanetary magnetic field is southward. The results of the model are consistent with available observations of the characteristics of the plasma sheet. The model also offers an interpretation of the role played by the north-south component of the interplanetary magnetic field in the solar-wind/magnetosphere interaction.
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1. INTRODUCTION

Two of the fundamental unanswered questions in magnetospheric physics are (1) the origin of the hot plasma in the outer magnetosphere known as the plasma sheet, and (2) the control exerted by the north-south component of interplanetary magnetic field, over the dynamics of the plasma sheet and the associated geomagnetic phenomena. This thesis confronts these two questions by means of a dynamic model of the magnetopause, in which the magnetopause is viewed as a finite, tangential boundary between two regions of magnetized plasma, following the suggestion of Alfvén (1960).

Before developing the model, we briefly review the present knowledge of that portion of magnetospheric physics that is relevant to the above two questions.

1.1 The Magnetosphere

Currents flowing within the earth produce an intrinsic geomagnetic field that is approximately dipolar above the surface. Aside from small secular and periodic changes, the surface geomagnetic field often exhibits characteristic, nonperiodic perturbations known as geomagnetic storms. (For detailed descriptions of the geomagnetic field and its variations, see for example Chapman and Bartels, 1940; Dessler, 1965.) The study
of these transient variations, and of the associated auroral phenomena, have led many researchers to postulate the interaction of the geomagnetic field at large distances with streams of solar plasma expanding into interplanetary space. (See the review by Dessler, 1967, and references therein.) Parker (1958) argued, on the basis of a hydrodynamic flow model, that the expansion of the solar corona should result in a continuous, supersonic flow of plasma through interplanetary space (the "solar wind").

When the sustained existence of the solar wind was verified by deep-space-probe experiments (Gringauz et al., 1960; Neugebauer and Snyder, 1962), it became clear that the geomagnetic field should, at some distance, become permanently distorted by interaction with the solar-wind plasma. It has now been firmly established by numerous spacecraft measurements that the geomagnetic field is confined to a cavity in the solar-wind flow; this cavity is called the magnetosphere after the suggestion of Gold (1959). The magnetosphere exists because the surface geomagnetic field $B_0$ has a pressure $B_0^2/2\mu_0$ that exceeds the dynamic pressure $\rho v^2$ of the solar wind at the earth's orbit. A rough measure of the characteristic dimension of the magnetosphere is obtained by equating the (measured) solar-wind dynamic pressure to the geomagnetic dipole-field pressure $(B_0^2/2\mu_0)(R_E/r)^6$, where $r$ is geocentric distance in the equatorial plane, and $R_E$ is the radius
of the earth. Taking a typical solar-wind number density 
$n \sim 5/\text{cm}^3$ and flow speed $v \sim 500 \text{ km/sec}$ (see, for example, 
Brandt, 1970), the characteristic dimension is $r \approx 10 \text{ R}_E$. 
This dimension is nearly constant over a wide range of 
solar-wind parameters because of the $1/r^6$ dependence of 
the magnetic pressure of the geomagnetic dipole field.

The condition of pressure balance between the 
dynamic pressure of the solar wind (assumed to be un-
magnetized), and the magnetic pressure of the distorted 
geomagnetic field (assumed to be in vacuum), gives rise to 
a free-boundary problem, where the shape of the plasma/field 
boundary determines the field strength inside the boundary, 
and vice-versa. Magnetopause models developed with these 
assumptions are known as Chapman-Ferraro models. The 
shape and size of the magnetosphere, calculated numeri-
cally on the basis of this type of model, are in satisfac-
tory agreement with the true size and shape of the 
magnetosphere as determined experimentally (see, for example, 
the review by Spriter and Alkane, 1969). The distortion 
has a high degree of day-night asymmetry because of the 
highly-supersonic nature of the solar-wind flow. The 
boundary between the magnetosphere and the surrounding 
interplanetary medium is called the magnetopause (see 
figure 1).

Another consequence of the supersonic nature of the 
solar wind is the necessary existence of a detached bow
Sketch illustrating the geometry of the magnetosphere in the noon-midnight meridian cross-section. Labels indicate the principal regions of magnetospheric plasma that are of importance in the solar-wind/magnetosphere interaction. Note, in particular, that the plasma-sheet region forms a spatial link between the solar wind in the magnetosheath, and the auroral-zone ionosphere.
shock wave upstream of the magnetosphere, to divert the solar-wind flow around the cavity (see figure 1). Since the solar wind is essentially a collisionless plasma, the shock must be a collisionless shock; that is, the randomizing of particle velocities in the shock must be accomplished by particle-electromagnetic field interaction rather than by particle-particle interaction.

While neither the magnetopause nor the bow shock are thoroughly understood as regards their internal structure and dynamics, their shapes and positions are nonetheless accurately predicted by simple considerations of fluid dynamics (see, for example, reviews by Spriter and Alksne, 1969; Willis, 1971). The region of thermalized solar-wind flow between the shock and the magnetopause is called the magnetosheath. The geometry of the magnetosphere, magnetosheath, and bow shock are illustrated in figure 1.

Also shown in figure 1 is the "cusp" region, marking the separation between the shell of "closed" field lines surrounding the earth, and the bundles of "open" field lines extending into the geomagnetic tail. Figure 1 also shows a pair of attached shocks, proposed by Walters (1966) as a means of diverting the magnetosheath flow around the magnetospheric bulge just downstream from the cusp region.
1.2 Solar-wind/Magnetosphere Interaction at the Magnetopause

While the Chapman-Ferraro model (a perfect boundary between unmagnetized plasma on one side and vacuum magnetic field on the other) successfully predicts the shape and location of the magnetopause, it does not allow for any transfer of plasma or magnetic flux across the magnetopause. It is now known that the solar wind contains a magnetic field that is compressed (enhanced) in the magnetosheath, so that the magnetic field outside the magnetopause may be non-negligible with respect to the field inside. It is also now known that the outer magnetosphere contains plasma whose energy density may be non-negligible with respect to that outside. Thus the magnetopause should be viewed, not as a plasma-vacuum field interface, but rather as a boundary between two regions of different plasma/magnetic-field characteristics. It is to be expected that this more general approach may lead to an understanding of certain important features of the solar-wind/magnetosphere interaction that are not contained in the Chapman-Ferraro idealization. These features include the formation of the geomagnetic tail, the transfer and storage of solar-wind energy in the tail, and the transfer of magnetosheath plasma across the magnetopause. These features, which are almost certainly interrelated, will be discussed in turn.
If the shape of the magnetosphere were determined entirely by pressure balance between the geomagnetic field and the magnetosheath plasma, then the cavity should close downstream from the earth at an angle given roughly by the arctangent of the reciprocal of the sonic mach number of the magnetosheath flow, as illustrated in figure 2 (see Lees, 1964). Satellite observations indicate, to the contrary, that the magnetosphere has an elongated tail stretching far behind the earth, as indicated in figure 1.

The tension in the distended magnetic field of the tail must be balanced either by an additional internal pressure or by a transfer across the magnetopause of the tangential component of solar-wind momentum, or a combination of the two. Magnetohydrodynamic wave pressure and solar plasma pressure (Dessler, 1964), and anisotropic interior plasma pressure (Rich et al., 1972), have been suggested as possible internal mechanisms for balancing the magnetic tension. Proposed mechanisms for the transfer of solar-wind momentum across the magnetopause include magnetic field-line connection or merging between interplanetary and geomagnetic fields (discussed in section 2.2.1 below), turbulent mixing of magnetosheath and magnetosphere plasma by micro-instabilities in the boundary (Eviatar and Wolf, 1968), and an unspecified form of viscous drag (Axford and Hines, 1961). The model
Teardrop model of the magnetosphere, showing the expected shape of the tail in the absence of internal plasma pressure or boundary stresses. Since the geomagnetic dipole field pressure decreases as $r^{-6}$, the solar-wind cavity would close downstream almost as if expanding into a vacuum. The solar wind would then fill the void with its speed of sound $v_s$ while moving downstream with its flow speed $v$. The angle at which the tail boundary would converge would be given approximately by $\arctan(v_s/v)$, which is of the order of $10^0$ for the magnetosheath flow.
developed in this thesis offers a possible mechanism for a viscous type transfer of solar-wind momentum across the magnetopause without turbulence.

The argument that the formation of the tail is due at least in part to the transfer of solar-wind momentum across the magnetopause, is supported by a large body of experimental evidence indicating that both the total magnetic energy and the total plasma energy of the tail depend strongly on solar-wind conditions, in particular, on the direction of the solar-wind magnetic field (section 2.1.4 below, and references therein).

There is also a considerable amount of experimental evidence indicating that solar-wind particles, as well as solar-wind momentum and energy, are transported across the magnetopause into the magnetosphere. This evidence is also discussed in section 2.1.5 below. Thus it appears that the solar wind is the ultimate source of the particles making up the plasma sheet, although the mechanism whereby the solar plasma enters the magnetosphere to form the plasma sheet is not well-understood. The model developed in this thesis offers a possible mechanism for the population of the plasma sheet.

The point to be made here is that the magnetopause is not an impervious boundary between solar-wind plasma on one side and the geomagnetic field on the other, as envisioned in the Chapman-Ferraro idealization. While that
idealization allows a good quantitative understanding of the size and shape of the magnetopause, it does not include the interesting and important phenomena of plasma, momentum, and energy transport from the solar wind to the magnetosphere. These phenomena can only be understood on the basis of a model that neglects neither the plasma nor the magnetic field on either side of the magnetopause.

1.3 Thesis Objectives

The Chapman-Ferraro type boundary discussed above is a special case of a tangential discontinuity, which may be defined as any planar boundary between two regions of magnetized plasma, where the magnetic field on both sides is parallel to the plane of the boundary. It is easily shown (section 3.1 below) that in the magnetohydrodynamic approximation, the component of plasma flow velocity normal to such a boundary must be zero. Following the suggestion of Alfvén (1968), we have considered in this thesis a tangential-type boundary that is part of a finite system in space, that is not a closed system with respect to conservation of particles. Owing to a loss of particles from the boundary at its edges, the normal component of plasma velocity is no longer zero, thus allowing for the possibility of plasma, momentum, and energy flux into the boundary. Boundary conditions are derived from the conditions
of mass, momentum, and energy conservation; these boundary conditions are then applied to the magnetopause in order to investigate the injection and acceleration of solar-wind particles at the magnetopause.

The model developed below offers a mechanism for the origin of the plasma sheet, as well as a possible description of the plasma-sheet time variations associated with geomagnetic activity. The model also offers a theoretical interpretation of the observed dependence of geomagnetic activity on the north-south component of the interplanetary magnetic field.
2. PLASMA IN THE OUTER MAGNETOSPHERE

The following is a brief review of presently-available observations and theories of the plasma environment of the outer magnetosphere, insofar as these observations and theories relate to the model developed in the following chapter. (The term "outer magnetosphere" is here applied to that portion of space beyond the plasmapause and up to and including the magnetopause. The plasmapause is the outer boundary, located at a geocentric distance of about $3-5 R_E$, of the plasmasphere, the domain of cold ($\sim 1$ eV) plasma of ionospheric origin.) For a more detailed review of these topics, the reader is referred to Axford (1969) and Vasyliunas (1972).

2.1 Survey of Relevant Observations

The term "magnetospheric plasma" is applied here to the population of charged particles in the outer magnetosphere having energies in the range $10^2$-$10^4$ eV. This definition is intended to include those plasma regions usually referred to as the plasma sheet (Bame et al., 1967), the extraterrestrial ring current (Frank, 1967), and the polar cusp (Frank, 1971b). The classification of outer magnetospheric plasma into these three regions is largely a matter of convention, and is not meant to imply that there is any natural division between the three regions; there is
reason to believe that the three regions are merely observationally-defined subsets of a single plasma population (see, for example, Frank, 1971a,b; Hill, 1971).

The energetic ($\geq 10^4$ eV) particles of the trapped radiation zones, and the cold ($\leq 10$ eV) particles of the plasmasphere, are excluded from the above definition of magnetospheric plasma, although they occupy overlapping regions of space (see, for example, Vasyliunas, 1972).

Since the first in situ observations of outer-magnetospheric plasma (Gringauz et al., 1960; Freeman, 1964; Bame et al., 1966, 1967), it has become apparent that plasma in the energy range $10^2-10^4$ eV accounts for virtually all of the particle energy content of the outer magnetosphere; the plasma energy density is often comparable to, and in some cases exceeds, the local magnetic-field energy density. For example, as shown in figure 3, the magnetospheric tail may be divided into three regions of distinct plasma/magnetic-field character. These are (1) the high-latitude tail, in which magnetic-field pressure dominates particle pressure; (2) the plasma sheet, in which the two pressures are comparable; and (3) the narrow field-reversal region imbedded within the plasma sheet, where the particle pressure dominates.

Observations in the magnetosphere are usually described in the geocentric solar-magnetospheric coordinate system (Ness, 1965). In this system, the x axis
FIGURE 3

Illustration of the magnetosphere in the noon-midnight meridian cross-section, showing the asymmetric distortion of the geomagnetic field by the solar wind. The standing bow shock diverts the supersonic solar-wind flow around the magnetospheric cavity. The magnetosheath is the region of post-shock solar-wind flow past the magnetosphere. The magnetopause is the boundary between the magnetosheath plasma and the geomagnetic field. The geomagnetic tail can be divided into three regions of distinct plasma/magnetic-field character. In the high-latitude tail, magnetic-field pressure dominates plasma pressure; in the plasma sheet, the two pressures are comparable; in the neutral sheet, where the field reverses direction, the plasma pressure dominates.
points toward the sun, the z axis points northward in the plane defined by the earth-sun line and the geomagnetic dipole axis, and the y axis completes the right-handed orthogonal system, pointing dawn-to-dusk.

2.1.1 Time-averaged Characteristics of Magnetospheric Plasma

The plasma sheet in the tail is defined observationally as a persistent region of enhanced flux of low-energy \((10^2-10^4)\) eV electrons and protons, extending typically about 3-6 \(R_E\) on both sides of the tail field reversal, and across the tail from the dawn to the dusk magnetopause. The plasma-sheet fluxes have been observed extensively by the Vela satellites in the range \(-15 R_E < x_{SM} < -20 R_E\) (Bame et al., 1966, 1967; Hones, 1968a,b, 1969, 1970; Hones et al., 1967, 1968, 1970), and by the Imp 3 satellite out to a geocentric distance of 40 \(R_E\) (Meng and Anderson, 1971). Plasma-sheet fluxes terminate abruptly at an inner boundary which lies typically at a geocentric distance of about 10 \(R_E\) behind the earth in the dusk-to-midnight quadrant, according to OGO satellite measurements (Vasyl'liunas, 1968). This inner boundary is called an Alfvén layer (see Schield et al., 1969; Wolf, 1970; Hill, 1971).

The Alfvén layer is not symmetric in longitude, but the plasma sheet apparently encircles the earth at a distance of 5-12 \(R_E\) in the equatorial plane (see figure 4). The high-latitude boundary or plasma-sheet "envelope"
FIGURE 4

Illustration of average position and geometry of the plasma sheet. The three projections are (a) noon-midnight meridian cross section (solar-magnetospheric xz plane); (b) equatorial cross section (solar-magnetospheric xy plane); and (c) tail cross section in a plane $x_{SM} = \text{constant}$ $\sim -20 R_E$. The dawn-dusk asymmetry of the Alfvén layer is shown schematically in (b), and of the outer envelope, in (c).
(see figure 4) is likewise sharply defined in terms of observed particle flux, and lies roughly parallel to the xy plane in the tail. The envelope and the Alfvén layer converge at the earthward end to form a pair of "horns" extending down to the ionosphere in the neighborhood of the auroral oval (see figure 4). This description of the average plasma-sheet geometry is based principally upon electron observations in the earthward end of the tail by the Vela satellites (Bame et al., 1967), observations near the Alfvén layer by the OGO 1 (Vasyliunas, 1968) and OGO 3 (Frank, 1971a) satellites, and observations in the "horns" by the Mars I (Gringauz et al., 1964) and Electron 2 (Gringauz et al., 1966) satellites.

The distance to which the plasma sheet extends down the tail is not well-established. Observations by the lunar-orbiting Explorer 35 satellite (Prakash, 1972; Nishida and Lyon, 1972) and the lunar-surface based ALSEP SIDE experiment (Garrett et al., 1971) indicate the appearance of plasma-sheet fluxes at lunar distance ($\approx 60 R_E$) in the tail, principally during geomagnetically disturbed periods. Observations by the lunar-surface based ALSEP CPLEE detector (Burke and Reasoner, 1972) indicate the absence of plasma-sheet fluxes during geomagnetically quiet periods. Therefore we tentatively assign an order-of-magnitude length $\sim 50 R_E$ to the plasma sheet, although there is still considerable controversy on this point.
Plasma-sheet electrons are characterized by a quasi-thermal energy spectrum, with mean energy in the range 0.2-1 keV and with spectral intensity in the range $10^7 - 10^9$ electrons/(cm$^2$·second·steradian·keV) near the peak energy. Plasma-sheet protons also appear quasi-thermal, but with a mean energy in the range 1-5 keV, and intensity roughly two orders of magnitude below that of the electrons. These flux measurements give a number density for electrons and protons typically in the range $n_e = n_p = 0.1-1$/cm$^3$, and electron and ion "temperatures" typically $T_e \sim 5 \times 10^6$ oK and $T_p \sim 2 \times 10^7$ oK. Thus the plasma energy density is of the order of $10^{-9}$ erg/cm$^3$, which is comparable to the magnetic-field energy density in the tail.

The diamagnetic effect of this plasma pressure has been detected in the form of a region of depressed field strength coincident with the plasma sheet (Lazarus et al., 1968; Behannon, 1968, 1970; Nihalov et al., 1970).

The velocity distribution of plasma-sheet electrons often appears isotropic, but has also been observed to be anisotropic by factors of two or more, with peak velocity components aligned with the local magnetic field (Hones et al., 1970).

Satellite spin-modulated proton fluxes (Freeman et al., 1968; Hones et al., 1972b) have indicated the existence of a layer about $10^3$ km thick in which the plasma-sheet plasma flows along the magnetopause in the direction of...
the adjacent magnetosheath flow. This flow resembles the viscous boundary-layer flow postulated by Axford and Hines (1961).

The principal dawn-dusk asymmetry of the near-earth plasma sheet is in the structure of the inner edge. The electron plasma-sheet flux is most intense in the dawn and predawn hours (local time), and intense plasma-sheet electron fluxes may extend inward as far as the plasmapause at geocentric distance ~ 5 R_E. The proton flux varies in the opposite manner; the most intense fluxes and those closest to the earth are observed on the evening side (Vasyliunas, 1972). The dawn-dusk asymmetry is also observed farther down the tail, where the electron component of the plasma sheet appears to be hotter and thicker toward the dawn side, and just the opposite for the proton component (Bame et al., 1967; see figure 4).

The region separating the shell of closed, dipole-like field lines on the dayside, from the open, polar-cap field lines extending into the magnetospheric tail, is commonly called the polar cusp. Observations by the Injun 5 satellite (Frank and Ackerson, 1971), the IMP 5 satellite (Frank, 1971b), and the ISIS 1 satellite (Heikkila and Winningham, 1971) have established that magnetosheath plasma penetrates into the magnetosphere through the polar cusps, forming a sheet of magnetosheath plasma extending from the magnetopause down to the ionosphere along geomagnetic field lines, with a latitudinal
width of a few degrees, extending at least four hours of local time on either side of noon (see figure 1). The dayside plasma sheet apparently lies on geomagnetic field lines just equatorward of the cusp latitude of about 80°. This fact has led to the suggestion that magnetosheath plasma in the cusp is the source of plasma-sheet plasma (see section 2.2.2 below).

The extraterrestrial ring current (Frank, 1967) is a permanent population of electrons and protons with energy ~10 keV, circling the earth at a distance of 5-10 RE in the equatorial plane. These particles form a ring current because their motion is dominated by curvature and gradient drifts, which are westward for protons and eastward for electrons, giving rise to a roughly circular loop of westward current. (The additional ring current due to the cold plasmasphere plasma and the high-energy trapped radiation is negligible by comparison, because of the much smaller energy density in these two particle populations.) The ring current is probably just the earthward extension of the tail plasma sheet.

The effect of this ring current is to decrease the geomagnetic field inside, and enhance that field outside the ring current. During geomagnetic storms, the ring current is asymmetrically enhanced to form the stormtime ring current, with the chief enhancement occurring in the evening sector. This enhancement can be understood on the basis of enhanced convection of plasma-sheet particles
from the tail, since plasma-sheet protons have about five times the energy density of plasma-sheet electrons, the ring-current enhancement should be about five times as great in the evening sector, where protons are injected, than in the morning sector, where electrons are injected.

It is probably significant that the proton-to-electron temperature ratio (or mean-energy ratio) has roughly the same value (5:1) in the magnetosheath, the plasma sheet, and in the ring current, while the individual temperatures may vary by two orders of magnitude among the three regions. This fact can be interpreted as supporting the view that the three plasma regions have a common source (i.e., the solar wind).

2.1.2 Time Variations of Magnetospheric Plasma

The plasma sheet exhibits systematic time variations in association with geomagnetic activity; in particular, both the Alfvén layer and the outer envelope have characteristic motions associated with magnetospheric substorms. (The term "magnetospheric substorm" refers to the non-equilibrium state of the magnetosphere, lasting about one hour, when the magnetospheric tail releases energy into the nightside magnetosphere at a suddenly-enhanced rate of the order of 1 terrawatt (TW) = 10^{12} watts (see, for example, Hultqvist, 1969). This energy release results in the enhancement of the ring current that occurs during the magnetic-storm main phase, and a variety of terrestrial phenomena including enhanced high-latitude
particle precipitation, intensification and breakup of
auroral arcs (the auroral substorm), and magnetic bays on
surface magnetograms (the polar magnetic substorm).
For descriptions of these phenomena, the reader is
referred to Akasofu, 1968; Feldstein, 1969; and Hultqvist,
1969.)

The Alfvén layer has been observed to approach the
earth, in the evening quadrant, from a quiet-time distance
of about 10 $R_E$ to a distance of around 5-8 $R_E$ during
magnetospheric substorms (Freeman and Maguire, 1967;
Vasyliunas, 1968; 1972; Schield and Frank, 1970; see
figure 5). This earthward motion appears to begin shortly
after the onset of the polar magnetic substorm as determined
from surface magnetograms (Vasyliunas, 1968), and may be
understood in terms of enhanced convection under the
influence of an enhanced dawn-dusk electric field in the
tail. Plasma-sheet convection velocities of the order of
100 km/sec have been inferred from satellite-spin modu-
lation of proton fluxes at the Vela orbital distance
(Hones et al., 1972b).

The substorm-associated motions of the high-latitude
plasma-sheet envelope have been studied extensively by
Vela satellites at distances of 15-20 $R_E$ in the tail
(Hones, 1968a,b, 1969, 1970; Hones et al., 1967, 1968,
one-half to one hour before the sudden onset of the polar
FIGURE 5

Illustration of the apparent motions of the plasma-sheet boundaries during a magnetospheric substorm. The quiet-time or pre-substorm configuration is shown in (a). As shown in (b), the plasma sheet appears to thin just before the substorm onset, with the envelope approaching the neutral sheet from both sides, followed by earthward motion of the Alfvén layer. Following the substorm onset, the plasma sheet expands (c), sometimes exceeding its pre-substorm dimensions, and the Alfvén layer recedes away from the earth. As discussed in the text, the motions of the envelope are only apparent, and are probably caused by an escape of plasma-sheet particles followed by injection of new particles, rather than compression and expansion of the same particle population.
(a.) QUIET-TIME

(b.) PRE-ONSET "DROP OUT"

(c.) POST-ONSET "RECOVERY"
magnetic substorm as determined from surface magnetograms, and usually begin to recover, sometimes abruptly, at or following the peak of the surface magnetic bay. This phenomenon has been termed "thinning" or "contraction" of the plasma sheet followed by "expansion" (Hones et al., 1967). However, simultaneous measurements by pairs of Vela satellites have shown that the plasma is not heated during this process, but in fact is slightly cooler in the thin remanent plasma sheet than in the pre-"thinning" plasma sheet. Thus the disappearance and reappearance of the plasma indicates not a compression and subsequent expansion of a permanent population of particles, but rather the escape of particles from the plasma sheet followed by the appearance of newly-injected particles (Hones et al., 1970, 1971).

The direction in which the plasma-sheet particles escape during the pre-onset "dropout" has not been firmly established. Garrett et al. (1971) have interpreted ALSEP SIDE measurements of ion bursts at the lunar surface in the tail, as indicating a flow of plasma-sheet ions down the tail, away from the earth, preceding substorm onset. Hones et al. (1972a) have reported the observation of plasma-sheet ions escaping into the magnetosheath at the VELA orbital distance during the pre-onset dropout. Burko et al. (1972) have reported a similar observation for plasma-sheet electrons at the lunar orbital distance.
The post-onset recovery of the plasma sheet appears to be caused by a flow of newly-injected hot plasma downstream into the tail from a source earthward of the Vela orbit, moving tailward with speeds \( \geq 40 \text{ km/sec} \) (Hones et al., 1967, 1968, 1970). The recovering plasma sheet is, in many instances, both hotter and thicker than the pre-onset plasma sheet.

Figure 5 illustrates the apparent motions of the inner and high-latitude boundaries of the plasma sheet during substorms.

2.1.3 Measurements of Magnetospheric Electric Field

Another set of experiments of interest here are those that measure, either directly or indirectly, the steady electric-field configuration of the magnetosphere. Within the plasmasphere, the convection electric field has been measured extensively by an indirect technique in which the motion of plasma in the equatorial plane is inferred from observed surface motions of whistler ducts (see Carpenter et al., 1972, and references therein). At geosynchronous orbit (McIlwain, 1971) and at the Vela orbital distance in the tail (Hones et al., 1972b), the convection electric field \( \mathbf{E} = -v \times \mathbf{B} \) has been deduced from the direct measurement of plasma convection velocity \( v \). Optical tracking of artificially-injected Barium ion clouds in the ionosphere (Heppner et al., 1971) and
near geosynchronous orbit (Rieger et al., 1972) has also been used to infer the convection electric field from the relation \( \mathbf{E} = -\mathbf{v} \times \mathbf{B} \). Other indirect methods of deducing magnetospheric electric fields include the optical tracking of auroral forms (Axford and Hines, 1961; Davis, 1971), the measurement of ionospheric drifts by incoherent radar backscatter techniques (Douplik et al., 1972), and the observation of asymmetries in the trapped-radiation flux (McDiarmid et al., 1969; Roederer and Hones, 1969). Finally, direct measurement of electric fields has been accomplished with double electrostatic probes, either balloon-borne (Mozer and Manka, 1971), rocket-borne (Kelley et al., 1971), or satellite-borne (Cauffman and Gurnett, 1971).

Measurements obtained by these diverse techniques have generally agreed on the following gross aspects of the large-scale electric-field configuration in the outer magnetosphere. The electric field is dominated in the outer magnetosphere by a dawn-to-dusk component, with a magnitude typically in the range \( 10^{-5} \) – \( 10^{-3} \) V/m. This generally dawn-to-dusk electric field is consistent with the idea of sunward plasma convection in the closed portion of the magnetosphere. The dawn-to-dusk field tends to be enhanced during magnetospheric substorms, especially in the tail.

Observations in the auroral-zone ionosphere typically show a reversal of the electric field near the latitude
of the limit of closed field lines, with generally sunward convection below that latitude and antisolar convection at higher latitudes. There is some evidence that the transition from sunward to antisolar convection occurs below the latitude of the magnetopause, consistent with the idea of a boundary flow layer in which the plasma-sheet particles flow in the same direction as the adjacent magnetosheath flow. Finally, the whistler observations in particular indicate that the dawn-dusk component (sunward convection) may be reversed (antisolar convection) during the recovery of magnetospheric substorms, indicating a return flow of plasma towards the magnetospheric tail.

2.1.4 Correlation of Interplanetary Parameters with Geomagnetic Activity

In studying the effects of interplanetary parameters on geomagnetic activity, a distinction must be made between those geomagnetic phenomena that are global in character (e.g., the sudden storm commencement (SSC) and the sudden impulse (SI)), and those that are localized in the nightside auroral zone (the magnetospheric substorm).

The global events are associated with a large-scale compression or decompression of the entire magnetospheric cavity in response to sudden changes in the dynamic pressure of the incident solar wind. The expected association between solar-wind pressure discontinuities and
SI events (see, for example, Dessler and Parker, 1959) has been confirmed by satellite measurements (Gosling et al., 1967; Burlaga and Ogilvie, 1969).

The relationship between interplanetary parameters and substorm phenomena is more complex, and is still being actively investigated. While the global events can be explained by sudden changes in the solar-wind energy flux impinging the magnetosphere, the occurrence of magnetospheric substorms is not clearly related to the available flux of solar-wind energy, but rather is controlled by the efficiency with which that energy is injected into the magnetosphere, and the rate at which the energy thus stored in the geomagnetic tail is released and dissipated in the inner magnetosphere. The mechanism of energy transfer at the magnetopause is still an unsolved problem (see section 1.2 above), and the efficiency of that mechanism is sensitive to variations in interplanetary parameters in a way that is not thoroughly understood.

The most significant interplanetary parameter, in terms of correlation with magnetospheric substorms, has proved to be the component of the interplanetary magnetic field perpendicular to the solar-magnetospheric equatorial plane (Fairfield and Cahill, 1966; Rostoker and Fälthammer, 1967; Schatten and Wilcox, 1967; Wilcox et al., 1967; Hirshberg and Colburn, 1969; Arnoldy, 1971). These studies have demonstrated that a negative (southward) zSM
component of the interplanetary magnetic field is a favorable (but not a necessary) criterion for the occurrence of substorms. Rostoker and Fälthammare (1967) have pointed out that this correlation probably reflects the importance of the $y_{SM}$ component of the electric field in the magneto-sheath, in the transfer of solar-wind energy across the magnetopause. This correlation is consistent with the reconnection model of Dungey (1961), and also with the theoretical picture of Alfvén and Fälthammare (1971) that emphasizes the importance of electrostatic particle acceleration in the magnetopause current layer as a mechanism for solar-wind energy transfer into the magnetosphere.

A detailed correlation analysis by Arnoldy (1971) indicates that, of several interplanetary field and plasma parameters, the one that is most significantly correlated with substorm activity (as measured by the AE index; see Davis and Sugiura, 1966) is the time-integral of the southward component of interplanetary magnetic field over the one-hour interval preceding the substorm activity. The time delay ~1 hr between the arrival of the southward interplanetary field and the associated substorm activity indicates that the southward interplanetary magnetic field does not merely act as a trigger mechanism for the release of energy already stored in the magnetosphere, but rather is related to the rate at which solar-wind energy is transferred into the magnetosphere. This conclusion is
supported by the observation (Fairfield and Ness, 1970) that both the total magnetic flux in the tail, and the tail field strength outside the plasma sheet, increase by as much as 10% during the substorm "growth phase", i.e. the time between the southward-turning of the interplanetary magnetic field and the sudden onset of the substorm energy release.

Other interplanetary parameters that appear to correlate with substorm activity are the interplanetary magnetic field strength (Wilcox et al., 1967; Schatten and Wilcox, 1967), the level of disturbance of the interplanetary magnetic field (Bellaf et al., 1967), the sector polarity of the interplanetary magnetic field (Schatten and Wilcox, 1967), and the solar-wind velocity (Snyder et al., 1963). Since the various interplanetary parameters are not mutually independent, it is difficult to isolate the geomagnetic effects of a single interplanetary parameter. However, the analysis of Arnoldy (1971) strongly indicates that the prolonged (~1 hr) existence of a southward magnetic-field component in the solar wind is the most important interplanetary parameter in regulating the efficiency with which solar-wind energy is transferred to the magnetosphere.
2.1.5 Observations Related to the Origin of Magnetospheric Plasma

There are two important classes of observations relating to the question of the source of plasma in the outer magnetosphere. The first of these, discussed in section 2.1.1 above, is the observation of magnetosheath plasma penetrating into the polar cusps. Frank (1971b) presents comparisons of energy spectra measured in the magnetosheath, the polar cusp, and the plasma sheet, supporting the view that plasma-sheet particles originate in the magnetosheath.

The second class of observations involves the measurement of the ionized Helium-to-Hydrogen abundance ratios ($\text{He}^+ : \text{H}^+$) and ($\text{He}^{++} : \text{H}^+$) in precipitating beams of auroral primary particles. The available data indicate that the $\text{He}^{++} : \text{H}^+$ ratio in auroral particles significantly exceeds the $\text{He}^+ : \text{H}^+$ ratio (Whalen et al., 1971; Reasoner, 1973). This result supports the hypothesis that the auroral particles originate in the solar wind (in which the Helium concentration is dominated by $\text{He}^{++}$) rather than in the ionosphere (where the ionized Helium component is almost entirely $\text{He}^+$). Since the auroral precipitation occurs, at least in part, on field lines that pass through the plasma sheet (Schield et al., 1969; Vasyliunas, 1970), this observation also supports the idea that plasma-sheet particles have their origin in the solar wind.
2.2 Theoretical Considerations of Magnetospheric Plasma

The plasma sheet plays a fundamental role in the transfer of solar-wind energy into the magnetosphere, and the dissipation of that energy in the form of magnetospheric substorms (see, for example, Frank, 1971b; Hill and Dessler, 1971; Dessler, 1972). As seen in section 2.1.5 above, the solar wind is the most likely source of plasma-sheet particles. The mechanism by which solar-wind particles are injected into the magnetosphere is an important missing link in our understanding of the solar-wind/magnetosphere interaction. Three injection mechanisms have been proposed, corresponding to three different magnetosphere models. These three models, all highly qualitative at present, will be briefly outlined below.

2.2.1 The Reconnection Model

The reconnection model, proposed by Dungey (1961) and developed by Levy et al. (1964) and Axford et al. (1965), is illustrated in figure 6. In this model it is proposed that geomagnetic and interplanetary magnetic fields will merge and become topologically connected wherever their directions are antiparallel. Figure 6 illustrates the reconnection for the case of a predominantly southward interplanetary field. In this case the geomagnetic and interplanetary field lines are connected at the dayside magnetopause, swept across the polar cap by the solar
FIGURE 6

The reconnection model of the magnetosphere, after Axford (1969). Geomagnetic field lines in this model are severed at a neutral point on the dayside magnetopause (position 1 in the figure), and connected with interplanetary field lines. The connected geomagnetic-interplanetary field lines are swept over the polar caps by the solar wind (positions 2-5), and disconnected at another neutral point in the tail (position 6). The reconnected geomagnetic field lines are then convected back towards and around the earth (positions 7-10). The plasma sheet in this model represents solar-wind plasma that is trapped within the reconnected field lines at position 6 and convected toward the earth. Note that the plasma-sheet envelope in this model is defined by the highest-latitude closed field lines on the night side.
wind, and finally disconnected downstream from the earth at the neutral sheet, with the reconnected geomagnetic field lines returning to the nightside magnetosphere. As shown in figure 6, the plasma sheet in this model is composed of solar-wind plasma that has been trapped on reconnected geomagnetic field lines in the tail, and subsequently convected toward the earth. The order-of-magnitude increase in the plasma-sheet temperature as compared to the magnetosheath temperature is attributed to compression of the plasma as it is convected toward the earth, in the direction of increasing magnetic field (see Axford, 1969).

There is, at present, no generally-accepted theory of magnetic merging or field-line reconnection in a collisionless plasma. In the absence of such a theory, it is difficult to assess the importance of field-line connection in the solar-wind/magnetosphere interaction. However, we note the following two difficulties encountered in the reconnection model: (1) The permanent existence of the geomagnetic tail, irrespective of interplanetary-magnetic-field orientation, indicates that some process other than reconnection is responsible for the momentum transfer necessary for tail-formation; (2) It is difficult to see how a compression of trapped magnetosheath plasma to form the plasma sheet could result in both an order-of-magnitude increase in temperature and at the same time an order-of-magnitude decrease in density.
2.2.2 Models Involving Polar-cusp Entry of Magnetosheath Plasma

In these models, the plasma sheet is formed of magnetosheath plasma that enters the magnetosphere through the dayside polar cusps, and drifts around to the nightside to form the plasma sheet. The model of Frank (1971b) is a modified reconnection model in that the interplanetary magnetic field is assumed to be connected to the geomagnetic field in the region of the polar cusps, and convected along with the plasma into the nightside plasma sheet. The model of Hill and Dessler (1971) also proposes the polar-cusp entry of magnetosheath plasma, but in that model the plasma is assumed to become detached from the interplanetary magnetic field and to drift longitudinally in quasi-trapped orbits within the magnetosphere. In either case, the magnetosheath plasma is not compressed in forming the plasma sheet, so that the increase in temperature and simultaneous decrease in number density can be consistently interpreted in terms of an energy-dependent injection into the nightside magnetosphere.

2.2.3 Models Involving Electrostatic Acceleration and Injection at the Dayside Magnetopause

In the model of Alfvén (1958) and Alfvén and Fälthammer (1971), the dayside magnetopause is represented by an equivalent planar current sheet. For the case of
southward interplanetary magnetic field, the current sheet resembles the neutral sheet model discussed by Alfvén (1968) and illustrated in figure 7. In this model it is assumed that current-carrying particles are lost from the neutral-sheet system at its edges, and replaced by a flow of plasma into the sheet from both sides. The current-carrying particles are accelerated by the electric field \( \mathbf{E} = -\mathbf{v} \times \mathbf{B} \), where \( \mathbf{v} \) is the velocity of plasma flow into the boundary. The condition of particle-conservation leads to the self-consistency requirement \( E = B^2/(\mu_0 d n e) \), where \( B \) is magnetic-field strength, \( d \) is the neutral-sheet dimension in the direction of current flow, \( n \) is plasma concentration, and \( e \) is the electronic charge. This expression gives a potential drop of the order of a few kilovolts across the dayside magnetopause. It is proposed that particles are accelerated in the magnetopause and then injected into pseudo-trapped orbits in the magnetosphere to form the plasma sheet.

The model developed in the next chapter is a generalization of the above neutral-sheet model, for a tangential discontinuity of finite dimensions, between two regions of arbitrary plasma parameters, and either parallel or antiparallel magnetic fields of arbitrary strength. Before developing this model, we consider, in the next section, the magnetohydrodynamic limit of the model, and show that the usual magnetohydrodynamic approach may have
FIGURE 7.

Illustration of the neutral-sheet model of Alfvén (1968). The sheet current $J$ separates two regions of oppositely-magnetized plasma. Loss of current-carrying particles at the edges of the current sheet results in a flow of plasma toward the boundary from both sides, and a resultant electric field in the direction of current flow.
to be modified in considering the dynamics of the solar-
wind/magnetosphere interaction at the magnetopause.

2.2.4 Limitations to the Magnetohydrodynamic Approach

Consider the configuration shown in figure 8, where
a magnetic field $\mathbf{B} = B\hat{z}$, varying only in the $x$ direction,
is imbedded in a collisionless plasma. The plasma is
two-dimensional in that all quantities are independent
of the $z$ coordinate. Consider the time-independent state.

The condition of $z$-independence implies that the
electric field $\mathbf{E}$ must lie in the $xy$ plane, so that $\mathbf{E} \cdot \mathbf{B} = 0$.
We assume that the scale length of the magnetic field
$(B/\nabla B)$ is much larger than the ion cyclotron radius, so
that the first-order guiding-center drift approximation
applies. If the electric field $\mathbf{E} = 0$, then the Maxwell
equation $\mu_0 \mathbf{j} = \nabla \times \mathbf{B}$, together with the guiding-center
expressions for magnetization and gradient-drift current,
is equivalent to the condition of static two-dimensional
pressure balance

$$\frac{B^2}{2\mu_0} + n k T_\perp = \text{Constant}$$

2.1

where $n$ is plasma concentration and $T_\perp$ is plasma temper-
ature perpendicular to $\mathbf{B}$. The guiding centers drift along
contours of constant $B$, so that the invariance of $I = k T_\perp / B$
implies that if 2.1 is satisfied initially, it will be
satisfied at any later time.

If we now add an electric field $\mathbf{E} = E\hat{x}$, the plasma
will drift with the velocity $\mathbf{v}_E = E \times \mathbf{B} / B^2$. Both the plasma
FIGURE 8.
Illustration of the two-dimensional magnetized plasma configuration discussed in the text.
drift velocity \( \mathbf{v}_E \) and the particle gradient-drift velocities \( \mathbf{v}_G = kT_\perp \mathbf{B} \times \nabla \mathbf{B} / qB^3 \) are in the +y direction (q is the charge including sign), so that the guiding centers again drift on contours of constant B. Thus by the invariance of I (or, equivalently, by the fact that \( \mathbf{j} \cdot \mathbf{E} = 0 \)), there is no acceleration associated with these drifts. The component of \( \mathbf{E} \) in the direction of \( \nabla B \) does not change the results obtained for the case \( \mathbf{E} = 0 \).

We now consider the effect of an electric field \( \mathbf{E} = E\hat{y} \neq 0 \). In this case the plasma drift \( \mathbf{v}_E \) is in the direction of + \( \nabla B \), and the gradient-drift velocity \( \mathbf{v}_G \) is in the direction of + \( \mathbf{E} \), so that the particles are accelerated in the course of their drifts. An observer moving with the plasma sees no electric field, but sees a changing magnetic field; in this frame of reference, the acceleration is viewed as betatron acceleration. The invariance of the first adiabatic invariant I yields

\[
\frac{d}{dt} (kT_\perp) = (kT_\perp / B) \frac{dB}{dt} = kT_\perp E \frac{\partial B}{\partial x} / B^2 \tag{2.2}
\]

An observer at rest with respect to the field gradient would view the acceleration as electrostatic acceleration due to particles' gradient drift through the electric field:

\[
\frac{d}{dt} (kT_\perp) = \mathbf{v}_G \cdot (q\mathbf{E}) = kT_\perp E \frac{\partial B}{\partial x} / B^2
\]

in agreement with 2.2.

As a result of this particle acceleration, the usual magnetohydrodynamic approximation is not consistent with
the two-dimensional pressure balance 2.1. That is, for
the case $\tilde{E} \times \nabla B \neq 0$, it is impossible to satisfy simulta-
neously the following three requirements:

(1) static two-dimensional pressure balance,
equation 2.1

(2) conservation of first adiabatic invariant,
$$ \nabla \cdot (kT_{\perp}/B) = 0 \quad 2.3 $$

(3) condition of frozen-in-flux,
$$ \nabla \cdot (n\tilde{E}B/B^2) = \varepsilon y \frac{\partial}{\partial x} (n/B) = 0 \quad 2.4 $$

For example, if the first adiabatic invariant is conserved
and frozen-in-flux is satisfied, then $\beta = 2\mu_0 nkT_{\perp}/B^2$ is
constant, and it is impossible to satisfy the pressure
balance condition 2.1.

There are several known phenomena in space having
$\tilde{E} \times \nabla B \neq 0$, but where the static two-dimensional pressure
balance 2.1 need not apply. For example, in the extra-
terrestrial ring current, the tension in the (three-
dimensional) magnetic field balances the radial gradient
in field plus plasma pressure. In the solar wind, for
another example, small relative changes in the dynamic
(flow) pressure can easily balance any gradient in static
plasma plus field pressure. In these cases, the pressure
balance condition 2.1 does not apply even approximately,
and the first adiabatic invariant and the frozen-in-flux
condition probably remain excellent approximations.
However, it is commonly assumed that the static pressure is constant across the magnetopause boundary, although the possibility exists that $\tilde{E} \times \nabla \theta \neq 0$ there. (Indeed, the idea of magnetic merging or field-line reconnection requires that $\tilde{E} \times \nabla \theta \neq 0$.) In this case the magnetohydrodynamic approximation must be violated, by violation of either the first adiabatic invariant, or the frozen-in-flux condition, or both. Violation of the first invariant requires that the magnetic field have significant (spatial/temporal) changes on a scale comparable to the cyclotron (radius/period) of the particles; violation of frozen-in-flux requires dissipation of magnetic-field energy by some mechanism analogous to Joule heating.

It is likely that both processes occur at the magnetopause, although it is not obvious which is quantitatively more significant. Table 1 lists some typical values of parameters measured in the plasma sheet and in the magnetosheath. If the plasma-sheet plasma comes from the magnetosheath, then the relative changes in the two ratios $(n/b)$ and $(kT_{\perp}/b)$ from the magnetosheath to the plasma sheet, should indicate, respectively, the extent of violation of frozen-in-flux, and of the first invariant, in the injection process. As seen in Table 1, both of these ratios change significantly between the magnetosheath and the plasma sheet, indicating that neither the frozen-in-
<table>
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<th></th>
<th>n (cm$^3$)</th>
<th>$kT_\perp$ (eV)</th>
<th>B (Y)</th>
<th>n/B</th>
<th>$kT_\perp$/B</th>
<th>β</th>
</tr>
</thead>
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<tr>
<td>Magnetosheath</td>
<td>10</td>
<td>50/500</td>
<td>12</td>
<td>.83</td>
<td>4.17/41.7</td>
<td>16</td>
</tr>
<tr>
<td>Near-Earth Plasma Sheet</td>
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<td>500/5000</td>
<td>35</td>
<td>.014</td>
<td>14.3/143</td>
<td>.9</td>
</tr>
</tbody>
</table>

Relative Change = $\frac{[(\text{Pl. Sheet}) - (\text{M'.sheath})]}{[(\text{Pl. Sheet}) + (\text{M'.sheath})]}$ = -97% +55%

**TABLE 1.** Typical values for plasma and magnetic-field parameters in the magnetosheath and in the near-earth plasma sheet, and the relative change between the two regions in the ratios (n/B) and ($kT_\perp$/B).
flux concept nor the first adiabatic invariant are useful approximations in considering the injection process at the magnetopause.
3. DYNAMIC MODEL OF A TANGENTIAL BOUNDARY
WITH FINITE DIMENSIONS

We define here a "tangential boundary" as a planar boundary between two regions of magnetized plasma, where the magnetic field on either side of the boundary is uniform, and parallel to the plane of the boundary. This definition includes the static magneto-hydrodynamic tangential discontinuity and the perpendicular magneto-hydrodynamic shock wave, as well as the dynamic, non-magneto-hydrodynamic boundary discussed below. This definition, for example, describes the interplanetary magnetic-field sector boundaries in the solar wind (Siscoe et al., 1968; Siscoe and Coleman, 1969), and applies, at least approximately, to the magnetopause (Cahill and Patel, 1967; Cummings and Coleman, 1968; Willis, 1971). Such boundaries are likely to exist in other astrophysical systems as well (see, for example, Michel, 1968, 1969).

3.1 The Magneto-hydrodynamic Approximation

In the magneto-hydrodynamic approximation, a tangential discontinuity is a static configuration where the boundary is at rest with respect to the plasma on either side, and is taken to be infinite in the direction of current flow. Let \( \hat{x} \) be the normal to such a boundary (see figure 9), and let \( \mathbf{B} = B_\hat{z} \) on one side of the boundary. If the boundary is at rest with respect to the plasma on
FIGURE 9.

Illustration of a tangential discontinuity. The discontinuity lies in the yz plane, and the fields $B_1$ ($x < 0$) and $B_2$ ($x > 0$) are parallel to this plane. The current $J$ flows in the yz plane, and is related to $B_1$ and $B_2$ by $\mu_0 J = (B_{1z} - B_{2z}) \hat{y} + (B_{2y} - B_{1y}) \hat{z}$. 
either side, then $E_y = 0$, as required by $\nabla \times B = E_y/B_z = 0$. Suppose, to the contrary, that $E_y \neq 0$. Then, since all parameters are independent of $y$ and $z$, Maxwell's equation can be written
\[ -\frac{\partial B}{\partial t} = \nabla \times E = \frac{\partial E_y}{\partial x} \hat{z} = 0 \]
so that $E$ must be the same on both sides of the boundary. Then if $B = B\hat{z}$ on one side of the boundary, the condition $E \cdot B = 0$, required by the steady-state assumption, requires that $B = B\hat{z}$ on the other side as well, that is, the magnetic fields on either side must lie in the same plane. Then the condition of conservation of particles is
\[ \nabla \cdot (n\vec{E}) = 0 = \nabla \cdot (n\nabla \times B^2) = \frac{\partial}{\partial x}(nE_y/B_z) = E_y \frac{\partial}{\partial x}(n/B_z) \]
Thus for $E_y \neq 0$, we have $n/B = \text{constant}$ across the boundary (the two-dimensional frozen-in-flux condition). But if $n/B = \text{constant}$, it can be shown (see, for example, Longmire, 1963), that the conservation of momentum and energy require velocities exceeding the magnetoacoustic speed, so that the boundary may be identified as a shock wave propagating perpendicular to $B$.

Thus we conclude that, in the magnetohydrodynamic approximation, a tangential boundary must either be at rest with respect to the plasma (tangential discontinuity), or must propagate through the plasma as a perpendicular shock, with a speed exceeding the magnetoacoustic speed. In this one-dimensional picture, the only boundary conditions are that (a) the total pressure (dynamic plus
thermal plus magnetic) must be the same on both sides of
the boundary, and (b) the current in the boundary must
satisfy Maxwell's equation \( \mathbf{\mu} \mathbf{j} = \nabla \times \mathbf{B} \).

From the considerations discussed in chapter 2, it is
clear that neither the static tangential discontinuity
nor the perpendicular shock provide a realistic descrip-
tion of the magnetopause. For this reason, we adopt
here a modified version of the magnetohydrodynamic
approach, following the suggestion of Alfvén (1968), that
includes the effect of the finite dimensions of the system
in the direction of current flow.

3.2 Boundary Conditions Imposed by Finite Dimensions

Consider the system illustrated in figure 10, where
the plane \( x=0 \) separates two regions of plasma having
either parallel or antiparallel magnetic fields. Subscripts
1 and 2 denote region 1 (\( x < 0 \)) and region 2 (\( x > 0 \)) re-
spectively. We define the following quantities:

\[ \mathbf{v} = \text{plasma flow velocity} = v_x \hat{x} \]
\[ \mathbf{E} = \text{electric field} = E_y \hat{y} \]
\[ \mathbf{B} = \text{magnetic field} = B_z \hat{z} \]
\[ n = \text{number density of ions (and of electrons) in} \]
\[ \text{the plasma} \]
\[ m = \text{ion mass} \]
\[ e = \text{absolute value of electronic charge} \]
\[ d = \text{dimension of system in the y direction} \]
\[ \mathbf{J} = \text{current density} = j_y \hat{y} \]
\[ j_z = \text{sheet current density} = \int_{-\infty}^{\infty} j_y \, dx = J_y \hat{y} \]
\[ p = \text{plasma pressure} \]
\[ W_{\text{e,1}} = \text{mean thermal energy per electron/ion} \]
Figure 10.

Tangential discontinuity with either parallel or antiparallel fields on either side of the boundary.

The discontinuity lies in the yz plane and has dimension d in the y direction.
\[ \beta = 2\mu_0 P/B^2 = \text{(plasma pressure)/(magnetic pressure)} \]
\[ M = \sqrt{\mu_0 n m} v/B = \text{Alfvén mach number} \]

We assume for simplicity that the plasma pressure on either side of the boundary is isotropic, with the scalar pressure given by
\[ P = (\gamma - 1)n(\omega_i + \omega_e) \tag{3.1} \]

We further assume that the ions are singly charged, and we neglect the electron mass with respect to the ion mass. Neither the magnetic nor the electric field has a component normal to the boundary. Observations (see, for example, Willis, 1971) indicate that the field configuration illustrated in figure 10 is a realistic (though simplified) description of the magnetopause.

3.2.1 Potential Drop Across the Current Sheet

From Maxwell's equations it follows that
\[ \mu_0 J_y = B_{1z} - B_{2z} \tag{3.2} \]
and that, in a steady state,
\[ E_{1y} - E_{2y} = 0 \tag{3.3} \]
Since the electric field vanishes in the plasma rest frame, it follows from equation 3.3 that
\[ E_{1y} = E_{2y} = v_{1x}B_{1z} = v_{2x}B_{2z} \tag{3.4} \]
From equation 3.4, we see immediately that if the magnetic field is in the same direction on both sides of the
boundary, then the plasma flow is in the same direction on both sides; i.e., the plasma will flow through the boundary. On the other hand, if the fields are antiparallel, then the plasma flow will be in opposite directions on either side of the boundary.

Equation 3.4 provides one equation in the two unknowns $v_{1x}$ and $v_{2x}$. The second equation is provided by the continuity equation expressing the conservation of particles. The flux of ions into the boundary, per unit yz area, per unit time, is given by

$$ F = n_1 v_{1x} - n_2 v_{2x} $$

with an equal influx of electrons. This influx of particles is balanced, in a steady state, by a loss of current-carrying particles from the edges of the current sheet. Let $N$ be the number of particles (of each sign) escaping the current sheet at its edges, per unit time, per unit distance in the $z$ direction. Then the continuity equation reads

$$ Fd = N $$

The loss rate $N$ is related to the sheet-current density $J$. If we define the following integrals,

$$ N_{id} = \left[ \int_{-\infty}^{\infty} n_{i} v_{i} y \, dx \right]_{y=d} \quad N_{i0} = \left[ \int_{-\infty}^{\infty} n_{i} v_{i} y \, dx \right]_{y=0} $$

$$ N_{ed} = \left[ \int_{-\infty}^{\infty} n_{e} v_{e} y \, dx \right]_{y=d} \quad N_{e0} = \left[ \int_{-\infty}^{\infty} n_{e} v_{e} y \, dx \right]_{y=0} $$

then the loss rate $N$ and the current density $J$ can be defined in terms of these integrals:
\[ N = N_{10} - N_{10} = N_{ed} - N_{e0} \]

\[ J_y = e(N_{10} - N_{e0}) = e(N_{1d} - N_{ed}) \]  \hspace{1cm} (3.7)

The condition that particles are lost from the system at its edges can be expressed by

\[ N_{1d} \geq 0, \quad N_{ed} \geq 0, \quad N_{10} \leq 0, \quad N_{e0} \leq 0 \]  \hspace{1cm} (3.8)

Using the definitions 3.7 and inequalities 3.8, it is easily shown that

\[ N \geq J/e \]  \hspace{1cm} (3.9)

where \( J \) is the absolute value of \( J_y \). The inequality 3.9 means that the current \( J \) in the boundary requires a minimum loss rate that is necessary to support that current; the equality holds if ions escape only in the direction of \( J \) and electrons escape only in the direction of \( -J \).

We denote by \( r \) the ratio of the total flow of charge from the system \( N_e \), to the directed flow of charge from the system \( J \).

\[ r = N_e/J \geq 1 \]  \hspace{1cm} (3.10)

With this definition, equations 3.5 and 3.6 give

\[ n_1v_{1x} - n_2v_{2x} = RJ/e \]  \hspace{1cm} (3.11)

Simultaneous solution of 3.4 and 3.11 gives the velocities

\[ \begin{bmatrix} v_{1x} \\ v_{2x} \end{bmatrix} = \begin{bmatrix} B_{2z} \\ B_{1z} \end{bmatrix} \frac{RJ}{[de(n_1B_{2z} - n_2B_{1z})]} \]  \hspace{1cm} (3.12)

Equation 3.12 indicates that for the case of parallel fields, the plasma will flow through the boundary from the
region of greater n/B toward the region of lesser n/B, and that for the case of antiparallel fields the plasma will flow toward the boundary from both sides.

The potential drop across the region is obtained by combination of 3.4 and 3.12:

$$\Phi = E_d = r B_{1z} B_{2z} (B_{1z} - B_{2z}) / [\mu_0 e (n_1 B_{2z} - n_2 B_{1z})]$$  \hspace{1cm} 3.13

where the sign of the potential drop indicates the sign of \( J \cdot E \). For the special case of a neutral sheet, where \( B_{1z} = -B_{2z} = B \) and \( n_1 = n_2 = n \), the potential drop 3.13 reduces to the result of Alfvén (1968) for \( r = 1 \):

$$\Phi_{ns} = B^2 / (\mu_0 n e)$$

Note also that the potential drop is zero in the Chapman-Ferraro limit (i.e., for \( B_{1z} = 0 \)).

3.2.2 Particle Acceleration in the Current Sheet

As a result of the potential drop given by equation 3.13, the particles that carry the current in the boundary will undergo electrostatic acceleration. By requiring that the total energy in the boundary be conserved, we can place limits on the extent of this acceleration, regardless of the details of the boundary-layer structure and the plasma distribution function.

The conservation of energy for a steady state can be written

$$\nabla \cdot [ \nabla (n m v^2 / 2 + n w_i + n w_e + P) + E \times B / \mu_0 ] = 0$$  \hspace{1cm} 3.14
where \( \nu \) is the mass flow velocity

\[
\nu = (n_i m_i v_{i i} + n_e m_e v_{e e}) / (n_i m_i + n_e m_e) \approx \nu_i
\]

In the plasma outside the boundary, \( \nu = E \times B / B^2 \), but within the boundary there is a \( y \)-component: \( \nu = E \times B / B^2 + v_{iy} \hat{y} \).

Integrating 3.14 over a volume containing the boundary layer, we have

\[
\left[ n_2 \nu_{2x} (m v^2 / 2 + 2w_{i2} + 2w_{e2}) - n_1 \nu_{1x} (m v^2 / 2 + 2w_{i1} + 2w_{e1}) \right. \\
- J \cdot \mathbf{E} \right] d + I = 0
\]

3.15

where

\[
I = \left[ \int_1^n \nu_{iy} (m v^2 / 2 + 2w_i + 2w_e) \ dx \right]_{y=0}^{y=d}
\]

3.16

In writing 3.15 and 3.16, we have used equation 3.1 with \( \gamma = 2 \), since all acceleration is in the \( xy \) plane. In terms of the integrals 3.7, we rewrite 3.16:

\[
I = N_{id} (m v^2 / 2 + 2w_i + 2w_e)_{y=d} \\
- N_{i0} (m v^2 / 2 + 2w_i + 2w_e)_{y=0}
\]

\[
= N (m v_f^2 / 2 + 2w_{if} + 2w_{ef}) + (2J_y / \varepsilon) (w_e)_{y=d} - (w_e)_{y=0}
\]

3.17

where we have defined the average final energies with which particles escape the current sheet:

\[
m v_f^2 / 2 + 2w_{if} = N_{id} (m v^2 / 2 + 2w_i)_{y=d} - N_{i0} (m v^2 / 2 + 2w_i)_{y=0}
\]

\[
w_{ef} = N_{ed} (w_e)_{y=d} - N_{e0} (w_e)_{y=0}
\]

3.18

Combining 3.15 and 3.17, we have
\[
\begin{align*}
&\left[ n_1 v_{1x} (m v_1^2 / 2 + 2 w_{11} + 2 w_{e1}) \\
&\quad - n_2 v_{2x} (m v_2^2 / 2 + 2 w_{12} + 2 w_{e2}) + J \cdot \mathbf{E} \right] d \\
&\quad = N (m v_f^2 / 2 + 2 w_{if} + 2 w_{ef}) - (2 j_y / \varepsilon) \left[ w_e \right] y=d \quad (2 j_y / \varepsilon) \left[ w_e \right] y=0 = 0
\end{align*}
\]

3.19

3.2.2.1 Antiparallel Fields

Consider first the case where the fields on either side of the boundary are antiparallel. Then the average value of the quantity \((m v^2 / 2 + 2 w_i)\) for ions entering the boundary from both sides is

\[
\langle m v^2 / 2 + 2 w_i \rangle = \frac{n_1 v_{1x} (m v_1^2 / 2 + 2 w_{11}) - n_2 v_{2x} (m v_2^2 / 2 + 2 w_{12})}{(n_1 v_{1x} - n_2 v_{2x})} \tag{3.20a}
\]

and the average energy of electrons entering the boundary from both sides is

\[
\langle w_e \rangle = \frac{n_1 v_{1x} w_{e1} - n_2 v_{2x} w_{e2}}{(n_1 v_{1x} - n_2 v_{2x})} \tag{3.20b}
\]

We define \(a_i, a_e\) as the average change in ion and electron energies due to electrostatic acceleration, in units of the total potential-energy difference \(e \Phi\). Thus by definition

\[
\begin{align*}
&\quad m v_f^2 / 2 + 2 w_{if} = \langle m v^2 / 2 + 2 w_i \rangle + 2 a_i e \Phi \\
&\quad w_{ef} = \langle w_e \rangle + a_e e \Phi \tag{3.21}
\end{align*}
\]
(a\textsubscript{i} is the average change in the quantity \( W_i + \frac{mv^2}{2} \); this is approximately equal to the change in ion energy \( W_i + \frac{mv^2}{2} \), provided the flow energy \( \frac{mv^2}{2} \) is small compared with the thermal energy \( W_i \).

Combining 3.19, 3.20, and 3.21, we have, for anti-parallel fields,

\[ 2N(a\textsubscript{i} + a\textsubscript{e})e\Phi = \frac{dJ}{gL} - (2J_y/e) \begin{bmatrix} \mathcal{W}_i \end{bmatrix}\bigg|_{y=0}^{y=d} \]

\[ \text{3.22} \]

The quantity \( \begin{bmatrix} \mathcal{W}_e \end{bmatrix}\bigg|_{y=0}^{y=d} \) represents the difference in the electron temperatures measured at \( y=d \) and at \( y=0 \).

To evaluate this quantity, we consider the conservation of momentum in the \( y \) direction, expressed by

\[ \frac{\partial}{\partial y} \left( nvm^2 + n_i W_i + n_e W_e + B^2/2\mu_0 \right) = 0 \]

\[ \text{3.23} \]

The magnetic-field pressure \( B^2/2\mu_0 \) is independent of \( y \).

We assume that the dynamic pressure \( nvm^2 \) is negligible with respect to the static pressure \( n_i W_i + n_e W_e \). We further assume that the current sheet is charge-neutral:

\[ (n_i - n_e)/(n_i + n_e) \ll 1 \]

(this is required by our previous assumption of uniform \( \mathcal{E}_x \)).

Then 3.22 reduces to

\[ \frac{\partial}{\partial y} (W_i + W_e) = 0 \]

\[ \text{3.24a} \]

or

\[ \begin{bmatrix} \mathcal{W}_i \end{bmatrix}\bigg|_{y=0}^{y=d} = - \begin{bmatrix} \mathcal{W}_e \end{bmatrix}\bigg|_{y=0}^{y=d} \]

\[ \text{3.24b} \]

Equation 3.24 says that the sum of ion and electron temperatures is independent of \( y \). Using this condition,
and equations 3.7, it is easily shown that

\[
N(w_i + w_e) = N\langle w_i \rangle + N\langle w_e \rangle + N(a_i + a_e)e \Phi \\
= N(w_i + w_e)_{y=0} - (J_y/e) \left[ w_e \right]_{y=0} = d 3.25
\]

Using 3.25 to eliminate \( \left[ w_e \right]_{y=0} \) from 3.21, we have

\[
2r \left[ (w_i + w_e)_{y=0} - (\langle w_i \rangle + \langle w_e \rangle) \right] = e \Phi 3.26
\]

The quantity \( (w_i + w_e) \) is independent of \( y \), and represents, for a given value of \( y \), the sum of ion and electron temperatures averaged across the boundary. In writing 3.26, we have used the fact that \( J \cdot E > 0 \) for the case of antiparallel fields.

It is shown in the Appendix that, for this case, the sum of the average temperatures of escaping particles, defined by 3.18, satisfies the inequality

\[
(w_i + w_e)_{y} \leq w_{if} + w_{ef} \leq 2(w_i + w_e)_{y} 3.27
\]

Using this inequality, equation 3.26 implies that

\[
\frac{1}{2} \leq r(a_i + a_e) \leq 1 + \frac{1}{2}(B_1\beta_1 + B_2\beta_2)/(B_1 + B_2) 3.28
\]

Using the expression 3.13 for the potential drop \( \Phi \), we find that the average ion-electron pair escaping the current sheet has been accelerated by an amount

\[
e\Phi_a \leq (a_i + a_e)e \Phi \leq 2e\Phi_a[1 + \frac{1}{2}(B_1\beta_1 + B_2\beta_2)/(B_1 + B_2)] 3.29
\]

where

\[
e\Phi_a = B_1B_2(B_1 + B_2)/2\mu_0(n_1B_2 + n_2B_1) 3.30
\]
3.2.2.2 Parallel Fields

If the fields $B_1$ and $B_2$ are parallel, then plasma flows into the boundary from only one side; some of the particles escape the current sheet at its edges, and the remainder pass through the boundary to supply the plasma on the other side. We define the coordinate system of figure 10 such that $B_{1z}$ and $B_{2z}$ are positive, and region 1 has the greater ratio ($n/B$); then the plasma flow is in the $+x$ direction. In this case, the average energies of ions and electrons entering the sheet are simply

$$\langle mv^2/2 + 2w_i \rangle = mv_1^2/2 + 2w_{i1}$$
$$\langle w_e \rangle = w_{e1}$$

We retain the definitions 3.21, so that $\alpha_i$ and $\alpha_e$ represent the average energy changes for ions and electrons escaping the current sheet at its edges. We also define $\delta_i$ and $\delta_e$ as the corresponding energy changes for ions and electrons passing through the boundary into region 2; i.e.,

$$mv_2^2/2 + 2w_{i2} = mv_1^2/2 + 2w_{i1} + 2\delta_i e\Phi$$
$$w_{e2} = w_{e1} + \delta_e e\Phi$$

With these definitions, the energy-conservation equation 3.19 becomes

$$2 \left[ N(\alpha_i + \alpha_e) + n_2 v_{2x} d(\delta_i + \delta_e) \right] e\Phi =$$
$$J \times E_d = (2J_y/e) \left[ w_e \right]_{y=d} -$$

We eliminate $[w_e]_{y=d}$ using equation 3.25, to obtain
\[ N[(w_i + w_e) - (w_i + w_e)] + n_2 v_2 c(\delta_i + \delta_e) \Phi = \]
\[ \frac{1}{2} \bar{J} \cdot \bar{E}_d \]  
\[ 3.34 \]

It is shown in the Appendix that, if \[ J \cdot \bar{E} > 0 \], then the inequality 3.27 is satisfied, and that if \[ J \cdot \bar{E} < 0 \], then the inequality

\[ 0 < w_{if} + w_{ef} < (w_i + w_e) \]
\[ 3.35 \]

is satisfied. We combine equation 3.34 with inequalities 3.27 and 3.35 for the respective cases \[ J \cdot \bar{E} > 0 \] and \[ J \cdot \bar{E} < 0 \], to obtain

\[ \frac{1}{2} \leq r[(a_i + a_e) + n_2 B_1(\delta_i + \delta_e)/(n_1 B_2 - n_2 B_1)] \]
\[ \leq 1 + \frac{1}{2}(B_1 \beta_1 - B_2 \beta_2)/|B_1 - B_2| \]
\[ \text{for } J \cdot \bar{E} > 0 \]  
\[ 3.36a \]

\[ -\frac{1}{2} \geq r[(a_i + a_e) + n_2 B_1(\delta_i + \delta_e)/(n_1 B_2 - n_2 B_1)] \]
\[ \geq -B_1 \beta _1 /2 |B_1 - B_2| \]
\[ \text{for } J \cdot \bar{E} < 0 \]  
\[ 3.36b \]

Using the expression 3.13 for the potential drop, we find that the average ion-electron pair in the boundary is accelerated by an amount

\[ \Phi_p \leq \left[ (1 - \frac{n_2 B_1}{n_1 B_2}) (a_i + a_e) + \frac{n_2 B_1}{n_1 B_2} (\delta_i + \delta_e) \right] \Phi \]
\[ \leq 2 \Phi_p [1 + \frac{1}{2}(B_1 \beta_1 - B_2 \beta_2)/(B_1 - B_2)] \]
\[ \text{for } J \cdot \bar{E} > 0 \]  
\[ 3.37a \]

\[ -\Phi_p \geq \left[ (1 - \frac{n_2 B_1}{n_1 B_2}) (a_i + a_e) + \frac{n_2 B_1}{n_1 B_2} (\delta_i + \delta_e) \right] \Phi \]
\[ \geq -\Phi_p B_1 \beta_1 /|B_1 - B_2| = -(w_i + w_e) \]
\[ \text{for } J \cdot \bar{E} < 0 \]  
\[ 3.37b \]

where

\[ \Phi_p = B_1 |B_1 - B_2| / (2 \mu_0 n_1) \]
(The last inequality is the obvious statement that an
ion-electron pair cannot lose more than its initial energy.)

3.2.3 Pressure Balance at the Boundary

The upper limits derived above (3.29 and 3.37) for
particle acceleration in the boundary, are expressed in
terms of the two pressure ratios \( \beta_1 \) and \( \beta_2 \). By imposing
the requirement of static pressure balance at the boundary,
we can eliminate one of these two ratios, and show that
in the parallel-field case, \( \mathcal{J} \cdot \mathcal{E} \) must be negative.

With the assumptions made above, the law of conser-
vation of momentum can be written

\[
\nabla (n m v^2 + p + B^2/2 \mu_0) = 0
\]

3.38

Integrating the \( x \) component of 3.38 across the
boundary gives

\[
B_1^2(1 + \beta_1 + 2m_1^2) = B_2^2(1 + \beta_2 + 2m_2^2)
\]

3.39

We now make the further assumption that the flow into
the boundary is much slower than the Alfvén speed:

\[
m_1^2 \ll 1, \quad m_2^2 \ll 1
\]

3.40

(The validity of this assumption as applied to the mag-
netopause is discussed in the next chapter.) Then 3.39
reduces to

\[
B_1^2(1 + \beta_1) \approx B_2^2(1 + \beta_2)
\]

3.41

Using 3.41 we can rewrite the quantity in brackets on
the right-hand side of 3.29;
\[ 1 + \frac{1}{2}(B_1\beta_1 + B_2\beta_2)/(B_1+B_2) = \frac{1}{2}[1 + B_2(1+\beta_2)/B_1] \]
\[ = \frac{1}{2}[1 + B_1(1+\beta_1)/B_2] \] 3.42

Then 3.29 becomes

\[ e\Phi_a \leq (\alpha_i+a_e)e\Phi \leq e\Phi_a[1 + B_2(1+\beta_2)/B_1] \]
\[ = e\Phi_a[1 + B_1(1+\beta_1)/B_2] \] 3.43

for antiparallel fields.

Using 3.41 to rewrite the quantity in brackets on the right-hand side of 3.37a, we have

\[ 1 + \frac{1}{2}(B_1\beta_1 - B_2\beta_2)/(B_1-B_2) = \frac{1}{2}[1 - B_2(1+\beta_2)/B_1] \]
\[ = \frac{1}{2}[1 - B_1(1+\beta_1)/B_2] \]
\[ < 0 \] 3.44

(The inequality follows from the fact that \( \beta_1 \) and \( \beta_2 \) are non-negative by definition, and \( B_1 \neq B_2 \).) Using the result 3.44 in 3.37a, we find that 3.37a is self-contradictory. Thus the conservation of energy and the condition (3.41) of static pressure balance require that

\[ \gamma e < 0 \] for the parallel-field case. This means that a self-consistent, steady-state flow solution, for the case of parallel fields, requires that the side having the greater ratio (\( n/B \)) also have the smaller value of \( B \). That is, for steady-state flow from region 1 to region 2, we must have

\[ B_1/B_2 < \text{smaller of } (n_1/n_2 , 1) \] 3.45

This requirement is illustrated in figure 11.
FIGURE 11.

Illustration of the parameter domains in which the different flow solutions apply. In domain I, the fields are antiparallel on either side of the boundary, and the flow is toward the boundary from both sides. In domain II, the flow is through the boundary from region 1 to region 2. In domain III, the flow is through the boundary from region 2 to region 1. In the shaded domains, no steady-state flow solutions exist, as parameter sets in these domains do not satisfy the condition 3.45.
The limits given by 3.43 for antiparallel fields, and by 3.37b for parallel fields, are illustrated in figure 12 for several sets of parameters \( \left( \frac{n_1}{n_2}, \frac{B_1}{B_2}, \beta_1 \right) \).
FIGURE 12.

Illustration of the limits of particle acceleration in the boundary, given by expressions 3.43 and 3.37b. The shaded area in each case represents, as a function of $B_1z/B_2$, the allowed range of values for the average change in kinetic energy in the boundary per ion-electron pair. The unit of energy is $e\phi_0 = B_2^2/2\mu_0n_1$, which is of the order of 2 keV for the magnetoopause.

(a) $\beta_1 = 0.1$, $n_1/n_2 = 1$
(b) $\beta_1 = 0.1$, $n_1/n_2 = 2$
(c) $\beta_1 = 0.1$, $n_1/n_2 = 10$
(d) $\beta_1 = 1.0$, $n_1/n_2 = 1$
(e) $\beta_1 = 1.0$, $n_1/n_2 = 2$
(f) $\beta_1 = 1.0$, $n_1/n_2 = 10$
(g) $\beta_1 = 10$, $n_1/n_2 = 1$
(h) $\beta_1 = 10$, $n_1/n_2 = 2$
(i) $\beta_1 = 10$, $n_1/n_2 = 10$
4. APPLICATION TO THE MAGNETOPAUSE

The model developed in the last chapter is a generalization of the model proposed by Alfvén (1968) to explain how the solar wind generates a potential drop across the dayside magnetopause. This model will be referred to hereinafter as the tangential-boundary model.

It is quite obvious that some of the assumptions made above are not strictly valid at the magnetopause, most notably the assumptions of planar geometry and of time-independence. However, the qualitative nature of the solutions is not expected to be highly geometry-dependent, and the steady-state solutions should be approximately valid over sufficiently long time scales. Specifically, as long as the plasma density and field strength on either side of the magnetopause remain reasonably constant over a time scale longer than the period of magnetopause oscillations (~10-100 sec), then the results of the time-independent analysis should provide a valid description of the relationships among the various parameters averaged over the oscillations.

With these reservations in mind, we consider the consequences of applying the tangential-boundary model to the magnetopause. The principal conclusions that result are:

1. The solar wind can generate, by this mechanism, a potential drop of a few kilovolts across the dayside
magnetopause, the electric field is directed dawn-to-dusk when the magnetosheath magnetic field has a southward component, and dusk-to-dawn when the magnetosheath magnetic field has a northward component. The potential drop is smaller when the fields are parallel on either side of the magnetopause, but is non-zero as long as there is a change in magnetic field across the magnetopause.

(2) Charged particles carrying the current in the magnetopause will undergo electrostatic acceleration up to energies of a few keV.

(3) The model provides a mechanism for the source and loss of outer-magnetosphere plasma, when the magnetosheath magnetic field has, respectively, a northward and a southward component.

4.1 Potential Drop Across the Dayside Magnetopause

In the above derivation, we identify region 2 as the magnetosphere, just inside the dayside magnetopause; then $B_2$ can be considered (roughly) as a fixed parameter having a value $B_2 \approx 60 \gamma$. Then region 1 is identified as the magnetosheath, just outside the magnetopause, and $n_1$ can be considered (roughly) as a fixed parameter having a value $n_1 \approx 10/\text{cm}^3$. The potential drop across the dayside magnetopause, given by equation 3.13, is of the order of $
abla \Phi = B_2^2 / \mu_0 n_1 \varepsilon \approx 2 \text{ kV}$. If $B_{1z}$ is the $z_{SM}$ component of
the magnetosheath magnetic field (positive when northward, negative when southward), then the potential drop across the dayside magnetopause, from 3.13, is given roughly by

$$\Phi \geq \Phi_0 (B_1/B_2)(1 - B_{1z}/B_2)/(1 - n_2B_{1z}/n_1B_2)$$  \hspace{1cm} 4.1

where the inequality follows from the requirement that $r \geq 1$ in equation 3.13. Figure 13 illustrates the lower limit $\Phi(B_{1z})$ given by 4.1, for several values of the ratio $n_2/n_1$. As indicated by figure 13, the potential drop is expected to be greater when the interplanetary magnetic field has a southward component, and smaller when the interplanetary field has a northward component; the difference between the two increases with increasing interplanetary field strength, and may be as great as an order of magnitude if the magnetosheath field strength is comparable to the geomagnetic field strength just inside the magnetopause. This is in agreement with the conclusion of Alfvén and Fälthammar (1971) based on the neutral-sheet model of Alfvén (1968).

This dependence of the cross-magnetosphere potential drop on the interplanetary-field direction, is qualitatively the same as that deduced from a simple reconnection model (see, for example, Dungey, 1961). Indeed, the tangential-boundary model, for the case of antiparallel fields, is similar to a reconnection model, in that it involves charged-particle acceleration by dissipation of magnetic-field energy. However, two important distinctions
The lower limit, given by equation 4.1, to the potential drop across the dayside magnetosphere as deduced from the tangential-boundary model. $B_{1z} = z_{SM}$ component of magnetosheath magnetic field; $B_2$ = geomagnetic-field strength just inside magnetopause; $n_{1,2} = \text{number density of plasma just outside and inside the magnetopause, respectively}; \text{unit of potential} = \phi_0 = B_2^2/\mu_0 n_1 e \sim 2 \text{ kV}$. A negative potential drop means that $J \cdot E$ is negative.
can be made between the tangential-boundary model and the reconnection model: (1) The reconnection model requires a magnetic-field component normal to the boundary, and the corresponding flow of plasma on reconnected field lines away from the merging region in the +z direction of figure 10. (2) In the reconnection model, the potential drop is zero when the fields are parallel on either side of the boundary. By contrast, the flow-through solution of the tangential-boundary model, for the parallel-field case, involves a negative dissipation of electromagnetic energy ($\mathbf{J} \cdot \mathbf{E} < 0$), so that plasma energy is converted into magnetic-field energy in the boundary; this can be viewed as a "reverse merging" process.

The potential drop derived here will exist in the absence of any field-line reconnection; any reconnection that occurs will increase the magnitude of the potential drop (for the case of a southward interplanetary magnetic-field component), but reconnection is not required to produce the qualitative increase in the potential drop as the interplanetary field turns southward.

4.2 Acceleration and Injection of Magnetosheath Plasma

According to the tangential-boundary model, when the magnetosheath magnetic field has a component parallel to the geomagnetic field inside the magnetopause, the flow of magnetosheath plasma across the magnetopause provides a source of plasma in the outer magnetosphere adjacent
to the magnetopause, and thus a mechanism for populating the plasma sheet. The particle-trajectory calculations of Roederer (1969) have shown that the region just inside the dayside magnetopause, and the nightside plasma-sheet region, are connected by magnetic-field drift orbits of quasi-trapped particles. Similar calculations by the present author (unpublished) have included the effect of \( \mathbf{E} \times \mathbf{B} \) drift, for several reasonable models of the magnetospheric convection electric field; in this case the two regions are still connected by drift orbits for particles having energies \( \gtrsim 1 \) keV. Thus the problem of populating the plasma sheet reduces to the problem of injecting and accelerating magnetosheath plasma at the front and sides of the magnetopause.

By considering the acceleration process that occurs in the boundary, we can deduce both the number density and the temperature of the resultant magnetospheric plasma in terms of the corresponding quantities in the magnetosheath.

Consider first the limiting case in which the particle cyclotron radius is small compared with the thickness of the boundary layer, i.e., compared with the scale length of the magnetic-field gradient \( \mathbf{B}/(\partial \mathbf{B}/\partial x) \). This approximation should apply, for example, to the electrons from the magnetosheath. A particle's speed along the boundary is then given approximately by its gradient-drift speed.
\[ v_y \approx v_G = w \frac{\partial B}{\partial x} / qB^2 \]  

where \( W \) represents the particle energy perpendicular to the magnetic field. In this case, the particle acceleration rate is just

\[ \frac{dW}{dt} = qv \cdot E = WE \frac{\partial B}{\partial x} / B^2 \]  

In the boundary rest frame, the time derivative can be written

\[ \frac{d}{dt} = v_x \frac{\partial}{\partial x} \]

Thus, using \( v_x = E/B \), equation 4.3 gives

\[ \frac{dW}{dx} = \frac{w}{B} \frac{\partial B}{\partial x} \]

Equation 4.4 implies that

\[ \frac{w}{B} = \text{constant across boundary} \]

i.e., the conservation of the first adiabatic invariant. Thus, in crossing the boundary, the particle is accelerated such that

\[ w_2 \approx B_2 w_1 / B_1 \]

with the parallel energy remaining unchanged.

Consider now the opposite limiting case, where the cyclotron radius is comparable to the boundary thickness, i.e., \( a_C = mv/eB \sim B/(\partial B/\partial x) \). (Here \( v \) is the component of velocity perpendicular to \( B \).) This approximation is probably applicable to ions in the magnetopause current sheet. Figure 14 illustrates schematically an ion trajectory at such a boundary. To estimate the drift
Sample ion trajectory in a magnetic-field gradient whose dimension is comparable to the ion-cyclotron radius. The real ion trajectory (a) is approximated by the simplified trajectory (b) made up of arcs of circles having alternate radii $a_{c1}$ and $a_{c2}$ corresponding to cyclotron radii in fields $B_1$ and $B_2$.

**FIGURE 14.**
speed in the y direction, we replace the true trajectory of figure 14a by the simplified trajectory of figure 14b, consisting of arcs of circles having alternating radii \( a_{c1} \) and \( a_{c2} \) corresponding to fields \( B_1 \) and \( B_2 \) on alternate sides of the plane \( x=0 \). If the arcs intersect the plane \( x=0 \) at an angle \( \theta \) for a given trajectory, then the ion executes a net y displacement \( \delta y = 2(a_{c1}-a_{c2})\sin\theta \) in a time \( \delta t = 2\theta/\omega_{c1} + 2\theta/\omega_{c2} \). The drift speed in the y direction is then

\[
v_y \approx \sin\theta (a_{c1}-a_{c2})/((\theta/\omega_{c1} + \theta/\omega_{c2})
= v \sin\theta (B_2-B_1)/[\theta(B_1+B_2)] \tag{4.7}
\]

Averaging 4.7 over all possible values of the angle \( \theta \), we have that the average drift velocity, for a given thermal velocity \( v \), is

\[
v_y \approx 2v(B_2-B_1)/[\pi(B_1+B_2)] \tag{4.8}
\]

For self-consistency, the average field gradient in the boundary layer must be given approximately by

\[
\frac{\partial B}{\partial x} \approx (B_2-B_1)/2(a_{c1}+a_{c2})
= qB_1B_2(B_2-B_1)/2mv(B_1+B_2) \tag{4.9}
\]

Combining 4.8 and 4.9 gives the drift speed in terms of the field gradient:

\[
v_y \approx \frac{B}{\pi} \frac{1}{w} \frac{\partial B}{\partial x} /qB^2 \tag{4.10}
\]

where we have used the geometric mean of \( B_1 \) and \( B_2 \) for the value of \( B \) averaged over the trajectory. Comparing 4.10 with 4.2, we conclude that the y-drift speed will be
proportional to the quantity $w \frac{\partial B}{\partial x}/qB^2$, with the constant of proportionality being of order unity. Thus we write

$$v_y = a w \frac{\partial B}{\partial x}/qB^2, \quad 1 \leq a \leq 3 \quad 4.11$$

with $a=1$ corresponding to the limiting case of small cyclotron radius, and a value $a \approx 2$ being more representative of particles whose cyclotron radius is comparable to the current-sheet thickness.

The same argument that leads from 4.2 to 4.5, can be applied to 4.11, giving

$$w/B^a = \text{constant across boundary}, \quad 1 \leq a \leq 3 \quad 4.12$$

so that

$$w_2 = w_1 (B_2/B_1)^a \quad 4.13$$

Equation 4.13 gives the expected relationship between the temperature in the outer-magnetospheric plasma and the temperature in the magnetosheath.

Given the temperature ratio 4.13, the density ratio $n_2/n_1$ is then determined by the condition of static pressure balance (equation 3.41):

$$B_1^2/2\mu_0 + n_1w_1 = B_2^2/2\mu_0 + n_2w_2 \quad 4.14$$

Combining 4.13 and 4.14, we can write

$$n_2/n_1 = (B_1/B_2)^a \beta_2/ \left[ 1 + \beta_2 - (B_1/B_2)^2 \right] \quad 4.15$$

The pressure ratio $\beta_2$ can be eliminated from 4.15 by considering that the quantity $2\mu_0 n_1w_1/B_2^2$ is roughly a constant that is fixed by the boundary conditions of the problem. This quantity is the ratio of magnetosheath plasma pressure to geomagnetic field pressure just inside
the boundary. This ratio may not be strictly constant, of course, but its value is determined by the gas dynamics of the magnetosheath flow rather than by the boundary interaction discussed here, and for the sake of illustration, we assign to this ratio the constant value unity. This means that we are provisionally assuming an approximate pressure balance between the magnetosheath plasma pressure and the geomagnetic field pressure, as is done in the Chapman-Ferraro models; then any additional external pressure provided by the magnetosheath magnetic field must be balanced by an additional internal pressure of magnetosheath plasma.

With this assumption, equation 4.15 assumes the simple form

$$n_2/n_1 \approx (\beta_1/\beta_2)^{\alpha+2}$$

Equations 4.13 and 4.16 give the plasma-sheet parameters \((n_2, W_2)\) in terms of the corresponding parameters in the magnetosheath \((n_1, W_1)\), for a given ratio of outside-to-inside field strength \((\beta_1/\beta_2)\), that would be expected if the plasma sheet is populated by the flow of magnetosheath plasma across the magnetopause as in the tangential-boundary model.

If the original velocity distribution in the magnetosheath, and the resultant velocity distribution in the plasma sheet, are approximated by Maxwellians with densities and temperatures \((n_1, W_1)\) and \((n_2, W_2)\) respectively, then the energy spectra in the two regions can be compared
as shown in figure 15. The figure presents comparisons of spectra based on two possible values of the power-law exponent \( a = 1, 2 \), and for a range of possible field ratios \((B_1/B_2)\). Because of the uncertainty in some of the assumptions made in the above derivation, the comparisons presented in figure 15 should be viewed as illustrative rather than unique. However, there are two qualitative features illustrated by figure 15 that are not sensitive to the assumptions made as to boundary-layer structure; they are (1) a decrease in number density, and (2) an increase in temperature, in the plasma-sheet plasma as compared to the source magnetosheath plasma.

4.3 Rate of Population and Depletion of the Plasma Sheet

The tangential-boundary model also allows an order-of-magnitude estimate of the source rate for population of the plasma sheet during periods of northward-directed magnetosheath magnetic field, and the loss rate for depletion of the plasma sheet during periods of southward-directed magnetosheath magnetic field. The rate at which plasma enters or leaves the outer magnetosphere is, in this model,

\[
R = n_2 v_2 x d\Delta z \tag{4.17}
\]

where \( \Delta z \) is the effective magnetopause dimension in the direction perpendicular to the current flow. Taking \( \Delta z \sim d \), and \( n_2 \) from equation 4.16, equation 4.17 becomes
FIGURE 15.

Comparisons of particle differential energy spectra for the source (magnetosheath) plasma and the resultant (plasma sheet) plasma, as deduced from the tangential-boundary model, using a variety of parameters $B_1/B_2$ and the two values $\alpha = 1, 2$ for the power-law exponent in equations 4.13 and 4.16. The case $\alpha = 1$ corresponds to conservation of the first adiabatic invariant, and applies to particles whose cyclotron radii are much smaller than the current-sheet thickness. The case $\alpha = 2$ is more nearly applicable to particles whose cyclotron radii are comparable to the current-sheet thickness. The magnetosheath spectra in each case are represented by Maxwellian distributions with $n = 10/\text{cm}^3$, and mean energies $W_e = 50 \text{ eV}$, $W_i = 500 \text{ eV}$ for electrons and ions respectively. The plasma sheet spectra are represented by Maxwellian distributions with $n_2$ given by equation 4.16 and $W_{e2}$ and $W_{i2}$ given by equation 4.13. In each case, curve 1 is the magnetosheath spectrum, curve 2 is the plasma-sheet spectrum for $\alpha = 1$, and curve 3 is the plasma-sheet spectrum for $\alpha = 2$. 
\( \frac{B_1}{B_2} = 0.25 \)
\[ R \approx \left( n_1 \Phi_d / B_2 \right) (B_1 / B_2)^{a+2} \]  \[ 4.18 \]

Substituting \( \Phi \) from 3.13 gives

\[ R \approx \pm \left( B_2 d / \mu_0 e \right) (B_1 / B_2)^{a+3} \left( 1 \mp B_1 / B_2 \right) \left[ 1 \mp \left( B_1 / B_2 \right)^{a+3} \right] \]  \[ 4.19 \]

with (upper/lower) sign applying to (northward/southward) magnetosheath magnetic field. The expected source/loss rate as a function of \( (B_1 / B_2) \) is shown in figure 16, for \( B_2 = 60Y, d = 20 R_E \), and the values \( a=1,2 \). As indicated in the figure, the source/loss rate is of the order of \( 10^{25} \) ion-electron pairs/sec when the magnetosheath magnetic field is comparable in magnitude to the geomagnetic field inside the magnetopause. This is comparable to the rate at which magnetosheath plasma is expected to penetrate the polar cusps (see, for example, Willis, 1969; Hill and Dessler, 1971).

In order to estimate the characteristic time for population or depletion of the plasma sheet, we need an estimate of the total plasma-sheet particle content. From the observations described in section 2.1.1 above, the particle content can be estimated (very crudely) as \( (0.5/cm^3)(6 R_E)(40 R_E)(50 R_E) \sim 10^{30} \) pairs. Then the characteristic time for population or depletion of the entire plasma sheet by the tangential-boundary model would be of the order of \( 10^5 \) sec (\( \sim \) one day).
The dependence on $B_1/B_2$ of the rate of source or loss of magnetospheric plasma expected on the basis of the tangential-boundary model. The number $a$, defined by equation 4.12, depends on boundary-layer structure, but is of order unity.
4.4 Comparison with Observations

As discussed in section 2.1.4 above, it has been demonstrated observationally that a southward component of the interplanetary magnetic field is a favorable criterion for the occurrence of magnetospheric substorms. It is currently thought (see, for example, the review by Axford, 1969) that substorms are caused by an increase in the magnetospheric dawn-dusk electric field. Thus the tangential-boundary model is consistent with the observations in that an increased dawn-dusk electric field is expected in response to increasing southward interplanetary magnetic field.

As discussed in section 4.1 above, the reconnection model predicts the same correlation (for essentially the same reason), so that it is impossible to distinguish the relative importance of the two processes from the correlation observations alone. Observations of the component of magnetic field normal to the magnetopause would be helpful in distinguishing the dominant interaction, but such observations are very difficult to obtain and to interpret, and the available results are inconclusive (see Willis, 1972). However, the tangential-boundary model, for the case of northward magnetosheath magnetic field, contains observable features that have no counterpart in the reconnection model.

The electrostatic acceleration that occurs in the current sheet of the tangential-boundary model should
result in a layer of energetic particles at the magnetopause; such a layer is in fact consistently observed (Meng and Anderson, 1970). The electric field generated in the dayside magnetosphere by the tangential-boundary model is of the order of 0.01 mV/m; this is consistent with the values measured indirectly for geomagnetically quiet times, as discussed in section 2.1.3 above. However, the cross-tail electric field of the order of 0.3 mV/m that is thought to exist during magnetospheric substorms, requires an additional source, for example magnetic merging across the neutral sheet in the tail (Dessler, 1971). The dusk-to-dawn electric field that arises in the tangential-boundary model for northward-directed interplanetary magnetic field does not imply a net dusk-to-dawn electric field across the magnetosphere, but rather a decrease in the dawn-to-dusk component arising from other sources.

The plasma flow velocities corresponding to this electric field are of the order of a few hundred m/sec inside the magnetopause, and a few km/sec in the magnetosheath. Since the Alfvén speed is of the order of $10^3$ km/sec just inside the magnetopause, and $10^2$ km/sec in the magnetosheath, we are certainly justified in neglecting the dynamic pressure in the above considerations of pressure balance at the magnetopause.

Since the magnetospheric plasma flow is toward the boundary for antiparallel fields, and away from the boundary
for parallel fields, we conclude that, in the magnetosphere rest frame, the magnetopause should move inward when the magnetosheath magnetic field is southward, and should move outward when the magnetosheath magnetic field is northward. The speed of this magnetopause motion according to the tangential-boundary model (equation 3.12) should be of the order of

\[ v_2 = \frac{r B_1 (B_2 - B_1)}{[\mu_0 d e (n_1 B_2 - n_2 B_1)]} \]

\[ \sim \frac{B_1}{(\mu_0 d n_1 e)} \]

Using the parameters adopted above, this expression gives a speed \( \sim 0.2 \, R_E/\text{hr} \) for the magnetopause motion if the magnetic-field strengths are comparable on either side of the magnetopause. This result may account in part for the observed "erosion" of the dayside magnetopause during the growth phase of substorms, when the magnetosheath magnetic field is southward (see, for example, Aubry et al., 1970).

We note that the transport of magnetosheath plasma across the magnetopause, when the magnetosheath magnetic field has a northward component, implies a transport of the parallel component of momentum across the boundary. This transport of parallel momentum across the magnetopause may be partially responsible for the "viscous" boundary-layer flow postulated by Axford and Hines (1961) and observed by Freeman et al. (1968) and by Hones et al. (1972b). This transport of magnetosheath plasma and
momentum would be additional to that caused by turbulent mixing of magnetosheath and magnetospheric plasma (see, for example, Eviatar and Wolf, 1968).

We have estimated the source/loss rate for plasma-sheet particles, on the basis of the tangential-boundary model, to have a value of the order of $10^{25}$ electron-ion pairs/sec. This rate gives a time constant of the order of one day for the depletion of the entire plasma sheet during periods of southward interplanetary magnetic field, or for the repopulation of the plasma sheet during periods of northward interplanetary magnetic field. This is slower, by one to two orders of magnitude, than the time constant for the observed disappearance of the plasma sheet during the growth phase of magnetospheric substorms (see section 2.1.2 above). Thus we conclude that either (a) only 5-10% of the total plasma-sheet particle content is lost from the magnetosphere during a given substorm "dropout" as observed at the WELA orbit, or (b) there is another loss mechanism more efficient than the one discussed above. The loss mechanism proposed here is supported by the WELA observations of Hones et al. (1972a), which they interpret as indicating the escape of plasma-sheet ions into the magnetosheath during the growth-phase dropout of the tail plasma sheet.

The tangential-boundary model for the source of plasma-sheet particles offers a simple explanation of the simultaneous decrease in density and increase in
temperature that is observed in the plasma sheet as compared to the magnetosheath. Figure 17 compares typical energy spectra for particles measured in the magnetosheath and in the plasma sheet, with Maxwellian spectra deduced from the present model, taken from figure 15. As noted above, the deduced spectra are by no means uniquely determined by the model; the comparison of figure 17 is intended to show only that the model is consistent with observations, not necessarily verified by observations. However, the qualitative decrease in number density and increase in temperature are natural consequences of the model.

Finally, we note that, according to the tangential-boundary model, the transport across the magnetopause of magnetosheath plasma and momentum should become more efficient as the fields on either side of the magnetopause become more nearly parallel. This result is in agreement with the electric-field measurements reported recently by Heppner (1972). Heppner shows clear evidence that the anti-solar convection over the polar caps is enhanced on the side (dawn or dusk) where the geomagnetic field just inside the magnetopause has a component parallel to the interplanetary magnetic-field direction, and that the convection is slower on the side where the two fields have an antiparallel component.
Comparison of observed and theoretical energy spectra for particles in the magnetosheath and in the plasma sheet. The theoretical (Maxwellian) spectra are taken from figure 15b (protons) and figure 15c (electrons), corresponding to $B_1/B_2 = 0.5$, with $\alpha = 2$ for the proton plasma-sheet spectrum, and $\alpha = 1$ for the electron plasma sheet spectrum. The number density of the Maxwellian spectra has been increased by a factor of three from those of figure 15, in order to fit the theoretical magnetosheath spectra to the observed magnetosheath spectra, but the relationship between the magnetosheath and plasma-sheet theoretical spectra is not altered by this transformation. As noted in the text, the theoretical spectra are not uniquely determined by the model; the comparison is intended only to illustrate that the model is consistent with the observed energy spectra, with reasonable choices of parameters.

(a) proton spectra
(b) electron spectra
5. SUMMARY AND CONCLUSIONS

A dynamic model has been constructed for a tangential boundary with finite dimensions, to include the effect of loss of current-carrying particles from the edges of the boundary. The model amounts to a generalization of the neutral-sheet model of Alfvén (1956) for arbitrary plasma/field parameters and for either parallel or antiparallel magnetic fields on either side of the boundary. The particles lost from the edges of the current sheet are replaced, in a steady state, by a flow of plasma into the boundary; this flow of plasma normal to the boundary does not arise in the magnetohydrodynamic description of a tangential discontinuity.

The plasma flow is directed toward the boundary from both sides when the fields are oppositely-directed on either side; the flow is directed through the boundary when the fields on either side are parallel, toward the region that has the smaller ratio of plasma density to field strength. The change in the ratio \( n/B \) is determined by the loss rate of particles from the edges of the current sheet, and is a measure of the extent to which the condition of frozen-in-flux is violated in the boundary.

The plasma flow normal to the boundary plane is associated with an electric field, in the boundary rest frame, in the direction of current flow. For the case of
antiparallel fields, \( J \cdot E > 0 \) in the boundary, and magnetic-field energy is "annihilated", i.e., converted into particle energy. For the case of parallel fields, \( J \cdot E < 0 \) in the boundary, and plasma energy is converted into magnetic-field energy. The resulting potential drop across the system is generally greater for the case of antiparallel fields than for the case of parallel fields.

The model is applied to the magnetopause in an attempt to describe the dynamic interaction whereby solar-wind plasma, momentum, and energy are transported into the magnetosphere. Numerous observations have indicated the existence of such an interaction, and the important role of the north-south component of interplanetary magnetic field in regulating this interaction. Such an interaction is not included in the classical Chapman-Ferraro model of the magnetopause, in which the external magnetic field and the internal plasma pressure are neglected. The tangential-boundary model allows a description of the dynamic interaction, while retaining the assumptions of a steady state and a balance of static pressure across the boundary.

The principal conclusions of the model as applied to the magnetopause are as follows:

(1) The potential drop across the dayside magnetosphere should be greater when the interplanetary magnetic field has a southward component, and smaller when the interplanetary magnetic field has a northward component.
The potential drop should also increase with increasing magnitude of the southward component of interplanetary magnetic field.

(2) A southward-directed interplanetary magnetic field should result in a loss of plasma from the outer magnetosphere. The entire observed particle content of the plasma sheet could be removed from the magnetosphere by this mechanism in a time of the order of one day. Thus we conclude that, unless some more efficient loss mechanism is operating, the observed disappearance of the plasma sheet at the VELA orbit during the substorm growth phase, involves the loss from the magnetosphere of only 5-10% of the total particle content of the plasma sheet. It is plausible that the remainder of the particles "lost" from the region of the VELA orbit convect earthward to form the stormtime ring current, and ultimately populate the radiation belts.

(3) A northward-directed interplanetary magnetic field should result in a net transfer of solar-wind plasma and momentum across the magnetopause into the magnetosphere. This plasma and momentum transfer may contribute significantly to the quiet-time source of plasma-sheet particles and the formation of a "viscous" boundary layer just inside the magnetopause where the magnetospheric plasma flows along the boundary in the direction of the magnetosheath flow. This source mechanism
can supply the entire observed plasma-sheet particle content in a time of the order of one day.

(4) The injection mechanism results in a plasma-sheet energy distribution that has both a smaller number density and a higher temperature than the energy distribution of the source particles in the magnetosheath.

The conclusions of the tangential-boundary model are found to be in essential agreement with the available observations; however, the model is still largely qualitative, and the assumed geometry is still an extreme idealization to the true magnetopause geometry. In particular, we note the following limitations of the model as developed above:

(1) For simplicity, we have restricted the model to the case $B_1 \times B_2 = 0$; i.e., we have considered only the cases of parallel or antiparallel fields. It would be of interest to extend the above analysis to see if similar steady-state flow solutions exist for arbitrary angle between the two fields.

(2) It has been implicitly assumed that the plasma in the boundary will provide the necessary current without violating the condition of charge neutrality. Since ions and electrons enter the boundary at the same rate (by means of $E \times B$ drift), they must also escape the boundary at its edges at the same rate, unless strong electric fields are built up normal to the boundary.
This requirement can probably be met if the boundary has a thickness comparable to the ion-cyclotron radius; then the ions will move on non-adiabatic trajectories as shown in figure 14, while the electrons, because of their smaller cyclotron radii, will move on trajectories more nearly characterized as adiabatic guiding-center drift. Then the ions will move along the boundary with a significant fraction of their thermal speed, while the electrons will move along the boundary with a drift speed that is much smaller than their own thermal speed but comparable to the ion thermal speed.

(3) Even if the boundary, as a whole, remains uncharged, the differential drift of ions and electrons in the boundary results in a tendency for a build-up of positive charge at one end of the current sheet (the end to which the vector $J$ points), and a corresponding build-up of negative charge at the other end. Like any charge-separation effect, this tendency is self-limiting in that a small charge separation generates the electric field required to impede further charge separation. The resultant charge-separation electric field may seriously distort the pattern of $\mathbf{E} \times \mathbf{B}$ drift into the boundary. The distortions of the flow pattern have been considered in detail by Cowley (1972) for the neutral-sheet model of Alfvén (1968), which is a special case of the tangential-boundary model developed above. While the potential drop is redistributed along the current sheet, the total
potential drop across the current sheet, as a function of external boundary conditions, is not significantly affected by the charge-separation effects. Thus we do not expect the charge-separation effects to alter the qualitative conclusions reached above.

(4) A complete dynamic model of the magnetopause would require, in addition to the boundary conditions derived above, a detailed self-consistent model of the boundary layer, where the magnetic-field structure of the boundary layer determines the particle trajectories, which in turn determine the current density, which in turn determines the magnetic-field structure of the boundary layer.

Obviously, the formulation of a self-consistent boundary-layer model for the magnetopause is a problem whose complete solution, even for the simple geometry assumed here, will require iterative numerical techniques for a large number of specific parameter sets \((n_1, B_1, P_1, n_2, B_2, P_2)\). The above formulation is intended to establish the limits imposed on any such solution, by the general laws of particle, momentum, and energy conservation, as functions of the external boundary conditions. We believe that this approach yields some insight, that is not available from either the closed (Chapman-Ferraro) model or the open (reconnection) model, into the dynamic
solar-wind/magnetosphere interaction occurring at the magnetopause, as well as offering a mechanism for the source of plasma in the outer magnetosphere.
APPENDIX

To justify the inequalities 3.27 and 3.35, we first note that the first half of 3.35 is true by definition (kinetic energy is defined to be positive), and that the second half of 3.27 follows from the definitions 3.7 and 3.8:

\[
N(w_{if} + w_{ef}) = N_{id}w_{id} - N_{io}w_{i0} \\
+ N_{ed}w_{ed} - N_{eo}w_{e0} \quad \text{(By definition)} \\
\geq N(w_{id} + w_{i0} + w_{ed} + w_{e0}) \quad \text{(By 3.8)} \\
= N(w_{i+d} + (w_{i+e})_0 + N(w_{i+e})_d \\
= 2N(w_{i} + w_{e})_y \quad \text{(By 3.24a)}
\]

Thus we have, independent of the sign of \(J \cdot \vec{E}\),

\[0 \leq w_{if} + w_{ef} \leq 2(w_{i}+w_{e})_y \quad \text{A1}\]

To justify the other half of each inequality, we note that

\[
Nw_{id} = (N_{id} - N_{io})w_{id} \\
= N_{id}w_{id} - N_{io}w_{i0} + N_{io}(w_{i0} - w_{id}) \\
= Nw_{if} - N_{io} \left[ w_{i} \right]_{y=0}^{y=d} \quad \text{A2}
\]

Similarly, for electrons we have

\[
Nw_{ed} = Nw_{ef} - N_{eo} \left[ w_{e} \right]_{y=0}^{y=d} \quad \text{A3}
\]

Combining A2 and A3, we have
\[ N(\omega_{id} + \omega_{ed}) = N(\omega_i + \omega_e)_y \]
\[ = N(\omega_i + \omega_e)^d - N_{i0} \left[ \omega_i \right]^d_0 - N_{e0} \left[ \omega_e \right]^d_0 \]
\[ = N(\omega_i + \omega_e)^d - (N_{i0} - N_{e0}) \left[ \omega_i \right]^d_0 \]
(BY 3.24a)
\[ = N(\omega_i + \omega_e)^d - (J_y/e) \left[ \omega_i \right]^d_0 \quad \text{A4} \]
(BY 3.7)

The difference \[ \left[ \omega_i \right]^d_0 = \omega_{id} - \omega_{i0} \] must have the same sign as \( E_y \); that is, the effect of electrostatic acceleration must be to increase the ion temperature in the direction of \( \vec{E} \) (or to decrease the ion temperature in the direction of \(-\vec{E}\)); thus the second term on the right-hand side of A4 has the same sign as \( J \cdot \vec{E} \). Thus we have
\[ \omega_i + \omega_e = (\omega_i + \omega_e)_y + kJ \cdot \vec{E} \quad \text{with } k > 0 \quad \text{A5} \]

Combining A5 with A1 gives the desired result:
\[ 0 \leq \omega_i + \omega_e \leq (\omega_i + \omega_e)_y \quad \text{if } J \cdot \vec{E} < 0 \]
\[ (\omega_i + \omega_e)_y \leq \omega_i + \omega_e \leq 2(\omega_i + \omega_e)_y \quad \text{if } J \cdot \vec{E} > 0 \]
REFERENCES


Feldstein, Y. I., Polar auroras, polar substorms, and their relationships with the dynamics of the magnetosphere, Rev. Geophys., 7, 179, 1969.


Freeman, J. W., Jr., The morphology of the electron distribution in the outer radiation zone and near the magnetospheric boundary as observed by Explorer 12, *J. Geophys. Res.*, 69, 1691, 1964.


McIlwain, C. E., Plasma convection in the vicinity of the geosynchronous orbit, paper presented at the Summer Institute on Earth's Particles and Fields, Cortina, Italy, 1971.


