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MEASUREMENT OF POLARIZATION IN THE $^{40}\text{Ca}(d,n)^{41}\text{Sc}$ REACTION BY RECOIL IMPLANTATION

by

Robert Worden Dougherty, Jr.

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Thesis Director's signature:

Houston, Texas
May, 1973
To

Mother
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CHAPTER I
Introduction

Magnetic moments of low-lying states of atomic nuclei furnish valuable information concerning nuclear structure, and therefore measurements of these quantities are of theoretical interest. Magnetic moments of the ground state of stable nuclei are, in general, well established. Consequently, current experimental interest concerns measurements of magnetic moments of very short-lived \(10^{-7}-10^{-12}\) sec excited states, and of ground states of radioactive nuclei whose lifetimes are less than a minute. In this context, the present work is directed toward the measurement of the magnetic moment of the ground state of \(^{41}\text{Sc}\), which decays by positron emission to \(^{41}\text{Ca}\) with a halflife of 0.6 sec and an endpoint energy of 5.5 MeV.

The ground state of the \(^{41}\text{Sc}\) nucleus is of particular interest because, in the shell model description, it consists of a doubly-magic \(^{40}\text{Ca}\) core plus a single valence proton in an \(f_{7/2}\) orbital. The simplest application of this model predicts that the magnetic moment of such a nucleus is the sum of the contributions of the orbital and spin magnetic moments of the proton in a single-particle configuration (Ma 55). The value of the scandium moment derived from this model (the Schmidt value) is 5.79 nuclear magnetons; however, such shell-model predictions
of magnetic moments of other single-particle nuclei differ significantly from experimental values (Su 66) indicating a departure from the pure single-particle state. Theoretical evidence for such a departure for the $^{41}$Sc nucleus is exhibited in a description by Gerace and Green (Ge 67) which indicates that, in addition to the single-particle configuration in the ground state wavefunction, other configurations corresponding to excitations in the core must be included. Thus a measurement of the magnetic moment of $^{41}$Sc would furnish a test of this description of the ground-state configuration.

The ultimate objective of the work reported herein is to measure the magnetic moment of $^{41}$Sc by the method of nuclear magnetic resonance (NMR). This method, in which transitions between nuclear substates are induced, has been widely used in determining magnetic moments (Fu 69). In application to short-lived beta-active nuclei, the resonant condition can be observed with a technique that uses the anisotropy in the beta decay of polarized nuclei (Co 59). If the nuclei can be polarized, an asymmetry in the beta yield will be observed with respect to the directions defined by the axis of polarization, provided the decay is via an allowed transition (Ja 57). Consequently, if nuclear transitions are induced by NMR thereby destroying the polarization, the beta asymmetry is relaxed. Thus the application of this method to detect nuclear
resonance depends upon the production of polarized beta-active nuclei. In the present case, this is to be accomplished with the $^{40}\text{Ca}(d,n)^{41}\text{Sc}$ stripping reaction. The beta-active scandium nuclei are ejected from the target by recoil and are implanted in an appropriate stopping medium or catcher foil (Su 66).

The basic experimental arrangement, depicted schematically in Figure 1, consists of a target and a suitable catcher foil, both of which are placed in a strong homogeneous magnetic field. The direction of the field coincides with the axis of polarization of the recoils, which is in the direction $\vec{k}_d \times \vec{k}_n$ (Ba 60). This field serves to decouple the nuclear spin from the atomic spin and to prevent the loss of polarization before the decay of the nucleus. As is standard in an NMR measurement, an oscillating magnetic field produced by a coil wound about the catcher foil is applied perpendicular to the axis of polarization. When the frequency of oscillation matches the Larmor precessional frequency of the nuclear spin, magnetic dipole transitions are induced, and the polarization is destroyed. Thus, if nuclear polarization is indicated by asymmetrical counting rates in beta counters located above and below the catcher foil, the resonant condition can be detected by the vanishing of the asymmetry. Finally, the Larmor frequency depends upon the magnetic moment of the nucleus through the relation (Se 64):
FIGURE 1

The physical arrangement of the experimental apparatus.
\[ \omega_L = \frac{uH}{I} \]

where \( u \) and \( I \) are the nuclear moment and spin respectively, and \( H \) is the strength of the homogeneous magnetic field.

This thesis concerns itself with the first phase of the effort to determine the magnetic moment of \( ^{41}\text{Sc} \); namely, the development of the apparatus and method for establishing the existence of nuclear polarization through an observation of an asymmetrical beta decay. The basic objective is then to produce polarized \( ^{41}\text{Sc} \) recoil nuclei and to detect the polarization by the method described above. Since depolarization mechanisms are operative both in the recoil process and on the stopper foil before decay, the experiment can only set a lower limit on the actual magnitude of the nuclear polarization as created by the nuclear reaction processes themselves.

The observation of the beta-decay anisotropy of polarized nuclei was first used by Chase and Igo (Ch 59) to establish the existence of polarization of recoil nuclei produced in nuclear stripping reactions. Examples of the use of this method and its successful application to the measurement of magnetic moments are listed in Table I. The existence of polarization in the \( ^{40}\text{Ca}(d,n)^{41}\text{Sc} \) reaction leading to the formation of \( ^{41}\text{Sc} \) in its ground state has been observed in the outgoing neutrons by Gedcke et al. (Ge 69). In order to predict the corresponding polarization
TABLE I
Summary of Properties of Nuclei in Recoil Polarization
and Magnetic Moment Experiments

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Decay</th>
<th>$^I\gamma \to ^{II}\gamma$</th>
<th>Type</th>
<th>Orientation Process</th>
<th>Atomic Configuration</th>
<th>Term</th>
<th>A</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\overline{\text{B}}$</td>
<td>$^{12}\text{C} + \text{B}^-$</td>
<td>$1^+ \to 1^+$</td>
<td>G. T.</td>
<td>$^{11}\text{B}(d,p)$</td>
<td>[He] $2s^22p$</td>
<td>$^2p_{\frac{3}{2}}$</td>
<td>-1</td>
<td>Su 67</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Wi 71</td>
</tr>
<tr>
<td>$^{12}\text{N}$</td>
<td>$^{12}\text{C} + \text{B}^+$</td>
<td>$1^+ \to 0^+$</td>
<td>G. T.</td>
<td>$^{10}\text{B}(^3\text{He},n)$</td>
<td>[He] $2s^22p^3$</td>
<td>$^4S_{3/2}$</td>
<td>1</td>
<td>Su 67</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{17}\text{F}$</td>
<td>$^{17}\text{O} + \text{B}^+$</td>
<td>$5/2^+ \to 5/2^+$</td>
<td>Fermi G. T.</td>
<td>$^{16}\text{O}(d,n)$</td>
<td>[He] $2s^22p^5$</td>
<td>$^3P_{3/2}$</td>
<td>0.99</td>
<td>Su 66</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{19}\text{Ne}$</td>
<td>$^{19}\text{F} + \text{B}^+$</td>
<td>$1/2^+ \to 1/2^+$</td>
<td>Fermi G. T.</td>
<td>Atomic Beam</td>
<td>[He] $2s^22p^6$</td>
<td>$^1S_0$</td>
<td></td>
<td>Be 67</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{23}\text{P}$</td>
<td>$^{23}\text{Si} + \text{B}^+$</td>
<td>$1/2^+ \to 1/2^+$</td>
<td>Fermi G. T.</td>
<td>$^{28}\text{Si}(d,n)$</td>
<td>[Ne] $3s^23p^3$</td>
<td>$^4S_{3/2}$</td>
<td>0.65</td>
<td>Su 71</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$^{35}\text{A}$</td>
<td>$^{35}\text{Cl} + \text{B}^+$</td>
<td>$3/2^+ \to 3/2^+$</td>
<td>Fermi G. T.</td>
<td>Atomic Beam</td>
<td>[Ne] $3s^23p^6$</td>
<td>$^1S_0$</td>
<td></td>
<td>Co 63</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{41}\text{Sc}$</td>
<td>$^{41}\text{Ca} + \text{B}^+$</td>
<td>$7/2^+ \to 7/2^+$</td>
<td>Fermi G. T.</td>
<td>$^{40}\text{Ca}(d,n)$</td>
<td>[A] $3d4s^2$</td>
<td>$^2D_{3/2}$</td>
<td>0.99</td>
<td></td>
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</table>
of the recoiling $^{41}$Sc nucleus, a description of the stripping reaction mechanism must be formulated.

The polarization process in stripping reactions has been explained in a semi-classical manner by Newns (Ne 53, Ne 58) whereby the incoming and outgoing particle wavefunctions undergo an absorption by the potentials describing their interaction with the target and residual nuclei respectively. In the case of $^{41}$Sc, this model leads to a prediction of the recoil polarization in the same sense to the outgoing neutron. A more complete model of the polarization process includes the role of the coulomb and spin-orbit potential in both the incoming and outgoing channels. At present an analysis with this model is not available, and hence the relative signs of the polarizations arising from these additional terms is not known. Therefore no precise prediction can be made of the magnitude and sign of the polarization of the recoiling nucleus. The theory does predict, however, that recoil polarization exists. Moreover, since the recoil polarization varies with bombarding energy (Pf 67), an energy can, in principle, be found where the polarization is different from zero.

The technique for establishing the polarization of recoil nuclei described herein utilizes the asymmetrical beta decay of aligned nuclei along the axis of polarization. For an allowed transition, the beta intensity distribution function is given by (Le 56, Ko 59):
\[ I(\theta) = I_o \left( 1 + \frac{\langle I_z \rangle}{I} A \frac{v_e}{c} \cos \theta \right) \]

\( I_o \) is the total decay intensity, and \( \langle I_z \rangle \) is the expectation value of the projection of the nuclear angular momentum \( I \) on the axis of polarization. \( \langle I_z \rangle / I \) is then the nuclear polarization \( P \), and \( \theta \) is the angle between the beta-ray velocity and the axis of polarization. \( v_e/c \) is the ratio of the beta velocity to that of light. For beta energies above 1 MeV, \( v_e/c \gtrsim 0.94 \), and therefore a negligible error is made in the case of the scandium decay by replacing this ratio by unity. The asymmetry factor \( A \) depends upon the relevant matrix elements of the decay, and is zero for a pure Fermi (\( \Delta I = 0 \)) transition. In a pure Gamow-Teller transition, \( A \) takes on the following values depending upon the angular momentum transfer (Sc 66):

\[
A = \begin{cases} 
1 & \Delta I = -1 \\
\frac{1}{I_1 + 1} & \Delta I = 0 \\
\frac{I_1}{I_1 + 1} & \Delta I = +1
\end{cases}
\]

where \( I_1 \) is the total angular momentum of the parent nucleus.

The \( \Delta I = 0 \) transition of the \( ^{41}\text{Sc} \) decay is a mixture of both Fermi and GT types, and \( A \) must be evaluated in terms of the matrix elements and strengths of each transition. This calculation is performed using single-particle wavefunctions to describe both the \( ^{41}\text{Sc} \) parent nucleus and the
daughter, its mirror $^{41}\text{Ca}$, with a result of $A = 0.99$ (Appendix G).

With the above simplifications, the beta-ray intensity distribution for the scandium decay becomes:

$$I(\theta) = I_0 (1 + P \cos \theta)$$

The survey of experimental results given in Table I indicates the types of beta-decay processes involved in each measurement, and gives the value of the asymmetry parameter $A$ calculated for each transition.

In addition to the nuclear processes governing the magnitude of the recoil polarization, there are other factors which lead to a depolarization of the scandium nucleus during the sequence of events which follow the nuclear reaction. The polarization must be maintained during recoil through the target lattice, during flight to the catcher foil, and during the stopping process in the implanting medium. Furthermore, the net polarization must persist until beta decay. The scandium is recoiled with a velocity on the order of $10^{+8}$ cm/sec, thus will spend about $10^{-13}$ sec in the target, $10^{-7}$ sec in flight, and about $10^{-12}$ sec in stopping. However, the implanted atom is at rest in the catcher for an average of 600 ms before decay, therefore methods to preserve polarization must be investigated.
Of primary consideration is the hyperfine coupling between the nuclear magnetic moment and the magnetic field $\vec{H}_J(0)$ at the nucleus produced by a given electron configuration. This interaction, which couples the nuclear angular momentum $\vec{I}$ to the electronic angular momentum $\vec{J}$, can destroy the polarization initially or after implantation in the catcher foil. Decoupling may be accomplished, however, by application of a strong external magnetic field $\vec{H}_e$ along the axis of polarization such that the electronic magnetic moment precesses about the external field thereby aligning $\vec{H}_J(0)$ along the direction of $\vec{H}_e$. Consequently, $\vec{I}$ and $\vec{J}$ both precess about the direction $\vec{H}_e$ instead of about their vector sum $\vec{F}$. The strength of the external field required for decoupling can be determined by considering the total interaction between the nuclear moment and the electrons and applied field (Ko 58):

\[
(I-1) \quad \mathcal{K} = \hbar a \vec{I} \cdot \vec{J} + \hbar b Q + g_J \mu_B \vec{J} \cdot \vec{H}_e + g_I \mu_N \vec{I} \cdot \vec{H}_e
\]

where: $\vec{I} \cdot \vec{J} = \frac{1}{2} \left[ I(I+1)-J(J+1)-I(I+1) \right] = \frac{c}{2}$

\[
Q = \frac{3(\vec{I} \cdot \vec{J}) + 3/2(\vec{I} \cdot \vec{J}) - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)}
\]

$h = $ Plank's constant,

$g_J \vec{J} = $ the magnetic dipole moment of the electrons in Bohr magnetons, and

$g_I \vec{I} = $ the magnetic dipole moment of the nucleus in nuclear magnetons.
The first term is the dipole-dipole interaction coupling the nuclear magnetic moment to the magnetic field produced by the electrons (or the electron magnetic moment). The coupling of the nuclear quadrupole moment to the electric field gradient produced at the nucleus by the electrons is represented by the second term. The last terms are the interaction of the magnetic moments with the applied field. Experimental measurements of the coupling constants $a$ and $b$ are summarized in Table II for various isotopes of neutral scandium in its atomic ground state ($^2D_{3/2}$) and first excited state ($^2D_{5/2}$). It is apparent that the values given are fairly independent of the nuclear mass value, thus can be expected to apply to $^{41}$Sc. Since the values of $a$ for the different isotopes listed differ only by the ratio of nuclear magnetic moments (or, for different atomic states of the same isotope, by the ratio of the values of $g_J$) (Se 64), the values for $^{41}$Sc can be readily predicted from any of the above measurements.

Evaluation of equation (I-1) indicates that the quadrupole interaction can be neglected. Thus, to decouple $\vec{I}$ and $\vec{J}$, the following relation must be satisfied:

$$|ha \vec{I} \cdot \vec{J}| \ll |g_J \mu_B \vec{J} \cdot \vec{H}_e|$$

or:

$$\frac{|hac|}{2} \ll |g_J \mu_B \vec{J} \cdot \vec{H}_e|$$

or:

$$He \gg \frac{|hac|}{2} \frac{1}{g_J \mu_B J}$$
TABLE II

Atomic Hyperfine Structure Constants for the Scandium Isotopes

<table>
<thead>
<tr>
<th>Isotope</th>
<th>I</th>
<th>Atomic State</th>
<th>a</th>
<th>b</th>
<th>( u_I )</th>
<th>Q</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{43}\text{Sc} )</td>
<td>7/2</td>
<td>( ^2D_{5/2} )</td>
<td>105.7</td>
<td>-44</td>
<td>4.61</td>
<td>-0.26</td>
<td>Co 66</td>
</tr>
<tr>
<td>( ^{45}\text{Sc} )</td>
<td>7/2</td>
<td>( ^2D_{3/2} )</td>
<td>269.56</td>
<td>-26.31</td>
<td>4.76</td>
<td>-0.22</td>
<td>Fr 59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( ^2D_{5/2} )</td>
<td>109.03</td>
<td>-37.31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( ^{47}\text{Sc} )</td>
<td>7/2</td>
<td>( ^2D_{5/2} )</td>
<td>122.2</td>
<td>-38</td>
<td>5.33</td>
<td>-0.22</td>
<td>Co 66</td>
</tr>
<tr>
<td>( ^{41}\text{Sc} )</td>
<td>7/2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.79*</td>
<td></td>
</tr>
</tbody>
</table>

*Schmidt value
This expression can be evaluated for the two electronic states of \(^{41}\text{Sc}\) using experimental values for the electronic g-factors (Mo 49).

\[^2D_{3/2} \quad H_e \gg 1.03 \cdot 10^3 \text{ gauss} \quad (g_J = 0.79)\]

\[^2D_{5/2} \quad H_3 \gg 1.14 \cdot 10^3 \text{ gauss} \quad (g_J = 1.20)\]

The above discussion concerns only those scandium ions recoiled in the neutral atomic state. This electron configuration, \([\text{A}\,3\,d\,4\,s^2]\), produces a field \(H(0)\) at the nucleus which can be attributed to the unpaired d electron and any polarization of the inner electron core. However, experimental measurements contained herein (Chapter IV) indicate that the more likely recoil charge state is the singly ionized configuration in which the scandium atom is left with an unpaired 4s electron. Because of the much greater probability density at the nucleus of an s-electron orbital, a much larger field \(H(0)\), and therefore a stronger hyperfine coupling between I and J, may be expected in this state.

No measurements of the hyperfine coupling constant \(a\) (which is proportional to \(H(0)\)) are available for singly ionized scandium, therefore an investigation must be made of other atoms which contain a single 4s electron for which such data are available. Listed in the table below is the value of the magnetic field at the nucleus for three elements
which contain a single 4s atomic electron, along with a varying number of 3d electrons. \( H(0) \) was calculated in each from experimentally determined values of a \( (\text{Fu 69}) \), averaged over all results when measurements were made on more than one isotope. Also shown is the magnitude of the externally applied magnetic field \( H_e \) required to decouple I and J for the electronic ground state \( (^3D_1) \) and first two excited states \( (^3D_2 \text{ and } ^3D_3) \) of singly ionized scandium.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Atomic Configuration</th>
<th>Magnetic Field ( H(0) ) (gauss)</th>
<th>Decoupling Field ( (10^3 \text{ gauss}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>[A] 4s</td>
<td>( 5.8 \cdot 10^5 )</td>
<td>( ^3D_1 ): ( H_e 2^&gt; 3.6 ) &lt;br&gt;( ^3D_2 ): 0.8 &lt;br&gt;( ^3D_3 ): 0.2</td>
</tr>
<tr>
<td>Cr</td>
<td>[A] 3d^5 4s</td>
<td>( 1.0 \cdot 10^6 )</td>
<td>( ^3D_1 ): ( H_e 2^&gt; 6.4 ) &lt;br&gt;( ^3D_2 ): 1.4 &lt;br&gt;( ^3D_3 ): 0.4</td>
</tr>
<tr>
<td>Cu</td>
<td>[A] 3d^10 4s</td>
<td>( 2.6 \cdot 10^6 )</td>
<td>( ^3D_1 ): ( H_e 2^&gt; 16.3 ) &lt;br&gt;( ^3D_2 ): 3.5 &lt;br&gt;( ^3D_3 ): 2.6</td>
</tr>
</tbody>
</table>

The values of \( H(0) \) in the above table indicate the increasing contribution of a greater number of d electrons in the heavier atoms. Since the scandium configuration contains only one d electron in its first ionized state, the value of \( H(0) \) can be expected to be near that of potassium.

The second ionized state of scandium, which can also be present in the recoil beam, has an electron configuration which, apart from the paired 4s electrons, is essentially the same as that of the neutral state; therefore approximately the same external field is required for decoupling in this state.
Proper choice of the recoil stopping medium is essential to the prevention of polarization relaxation after implantation. Previous work (see references in Table I) indicate that for a given stripping reaction, stopping foils with a face-centered cubic lattice give the greatest degree of beta-decay asymmetry. This has been attributed to the symmetric interaction of crystalline electric field gradients on the electric quadrupole moment of the recoil nucleus, provided the ion is trapped in a lattice site (Ab 53). In addition, materials of the highest purity must be used as contaminants present in the lattice can upset the crystalline symmetry (B1 49). Previous experience has also revealed deterioration of the stopping foils over periods of several hours because of radiation damage to host atoms.

When metallic crystals are used as catcher foils, an additional relaxation mechanism is operative due to magnetic fields produced at the nucleus by conduction electrons. However the noble metals, possessing fcc symmetry, have been used with success probably because of purity and lack of surface contaminants.

A final consideration is the relaxation due to thermal lattice vibrations in the host material. Korrenge (Ko 50) has shown that the product of the absolute temperature and spin-lattice relaxation time is a constant for a given catcher medium. This relation has been verified experi-
mentally for the case of $^{12}$B recoils in various metals by Williams and Madansky (Wi 70).

The ultimate test of the existence of scandium recoil polarization relies upon measurements of the beta-decay asymmetry using each of two catcher foils, one where the relaxation time is expected to be sufficiently long and one where the recoil polarization is destroyed immediately. Previous procedures use organic foils such as teflon or mylar as a null test, the polarization relaxation being rapid due to the lack of crystalline structure of the organic compound.

The present work has progressed to the point where a more systematic study of the catcher foils is necessary to identify those which minimize the effects of the relaxation mechanisms. The foils used in the present work—gold and aluminum—cover a range of atomic numbers, but were mainly chosen for convenience.

The preceding discussion illustrates the basic steps involved in the measurement of the scandium magnetic moment. This thesis focuses upon the nuclear physics aspects, namely to find the conditions under which the reaction will furnish sufficient yield of $^{41}$Sc recoils to render the measurement feasible. Related to this is the design and development of the apparatus suited for observation of the subsequent beta-decay anisotropy. The final extension of this work will consist of a detailed study of
the methods of preservation of nuclear polarization in the stopping medium such that the measurement of the ground state moment of the scandium nucleus can be accomplished.
CHAPTER II

The Experimental Problem;
The Feasibility of the Procedure

A brief description of the salient features entering into a measurement of the asymmetrical beta decay from polarized $^{41}$Sc recoil nuclei has been presented in Chapter I; now a more extensive examination of the experimental approach will follow. In addition to a summary of the essential elements involved in the method, the results of preliminary experiments will be contained in this discussion. Further details concerning these measurements are presented in Chapter IV.

A. Statement of the Experimental Problem

The anisotropy in the scandium beta decay was discussed in the preceding section in terms of the intensity distribution $I(\theta)$ of the positron emission (equation I-1). Experimentally, however, one measures the difference in count rates in the right and left (or upper and lower) counting assemblies. It is then convenient to define an experimentally observable asymmetry as:

$$A = \frac{S_r - S_l}{S_r + S_l} \pm \sqrt{S_r + S_l}$$  \hspace{1cm} \text{(II-1)}

where $S_r$ and $S_l$ are the number of counts detected by the right and left counting systems during a given run. This
expression is related to the nuclear polarization \( P \) through the quantities \( S_r \) and \( S_\perp \) which are integrals of the intensity distribution \( I(\theta) \) over the angular range of acceptance \( \alpha \) of the detectors. Furthermore, since the polarization \( P \) is a function of time due to the relaxation mechanisms in the catcher foil, the polarization \( P_0 \) at time \( t = 0 \) is related to the polarization \( P \) at any subsequent time \( t \) by the relation:

\[
P = P_0 e^{-t/\tau_r}
\]

where \( \tau_r \) is the relaxation time of the polarization in a particular medium. Incorporating the time dependence of \( P \) in \( S_r \) and \( S_\perp \), the expression for the experimental asymmetry \( A \) becomes (Appendix A):

\[
(II-2) \quad A = P_0 e^{-t/\tau_r} (1 + \cos \alpha)
\]

where \( t \) is the time between production of the recoil and detection of the subsequent beta decay.

In the present study, a value of \( \alpha \) of 26 degrees was chosen for the acceptance angle of the counting systems. Then for relaxation times long in comparison to the half-life of the scandium decay (\( \tau_{1/2} = 0.6 \text{ sec} \)), the most favorable circumstance for the measurement of polarization, the above expression leads to \( A \approx 1.9 P_0 \).

Information about the value of \( P_0 \) to be expected is available from measurements of the neutron polarization.
occurring in the $^{40}\text{Ca}(d,n)^{41}\text{Sc}$ ground state reaction. In these measurements, the average neutron polarization is about 8% at a deuteron energy of 6 MeV (Ge 69). If this value is taken to be an indication of the magnitude of the $^{41}\text{Sc}$ polarization, then it is seen that the resulting laboratory asymmetry is about 15%. A measurement of an asymmetry of this magnitude to an accuracy of $\pm$ 3% imposes only minimal requirements on counting statistics as is evident by the examination of the error term in equation (II-1). Indeed, the sum $S_r + S_\perp$ need be only about $10^3$ counts to furnish the necessary accuracy. In practice, however, it could not be assumed that the polarization could be so easily determined since there can be no guarantee that an implanting medium furnishing a relaxation time longer than the scandium half-life can be found.

In order to take into account the possibility that the polarization $P_0$ and relaxation time $\tau_r$ might be substantially smaller than in the foregoing estimates, it was deemed necessary to design the experiment to measure asymmetries as low as $2 \pm 1\%$. To achieve this degree of accuracy, it is necessary that the quantity $S_r + S_\perp$ be increased to $10^4$ counts.

The foregoing discussion assumes the measurement of the beta-decay asymmetry is made with an experimental arrangement free of instrumental asymmetries and interference from background radiations. In practice both sources of difficulty are present.
In the design of the experiment, every effort was made to minimize the inherent instrumental asymmetries; and, as discussed in Chapter V, it is believed that they are no greater than 1%. However, the experimental procedure must be extended to compensate for these asymmetries, thereby increasing the number of counts required for a given degree of accuracy in the measurement by a factor of four.

The effect of background radiations is to require an even greater accumulation of counts from the scandium decay to obtain a given degree of accuracy. In an extreme case where background yields equal that due to the scandium decay, an increase in the scandium yield by a factor of 2 or 3 is necessary.

The uncertainty \( \Delta A \) in the asymmetry \( A \) is shown in Appendix B to be:

\[
(II-3) \quad \Delta A = \left[ 1 + \frac{1}{2} R_b \right]^{\frac{1}{2}} \Delta A_{no}
\]

\( R_b \) is the relative background contribution and \( \Delta A_{no} \) is the experimental uncertainty if the background were not present. This expression has been evaluated as a function of relative background level to furnish the number of scandium counts per run required to yield a given degree of accuracy in the determination of \( A \). These results are shown in Figure 2.

To meet the accuracy requirement within running periods of acceptable length, it is necessary to optimize
FIGURE 2

Number of counts required per run to obtain various degrees of accuracy versus the relative background yield.
the yield of recoils implanted in the catcher foil and the efficiency of detection of the subsequent beta decay radiations in the presence of background. The experimental parameters that must be considered in order to achieve this objective will now be discussed.

B. Discussion of Recoil Yields

The yield of recoil ions implanted on the catcher foil is determined by the following expression:

\begin{equation}
Y_r = N_d N_t \sigma(\theta) \Omega
\end{equation}

where \(N_d\) is the number of incident deuterons, \(N_t\) is the number of target nuclei per unit area, \(\sigma(\theta)\) is the recoil differential cross section, and \(\Omega\) is the solid angle subtended at the catcher foil by the recoil collimator. These quantities will now be investigated in order to maximize the recoil yield \(Y_r\).

In a previous investigation (Do 70), the total reaction cross section for the \(^{40}\text{Ca}(d,n)^{41}\text{Sc}\) g.s. was measured over a deuteron energy range from threshold (1.20 MeV) to 4 MeV where a value of 18 mb was reached. A measurement of the angular differential cross section of the recoiling scandium ions was made at this energy, yielding a maximum of 10 mb/sr in the angular region about 30 degrees, as shown in Figure 3. This angle was then chosen as the recoil angle in the design of the polarization experiment.
FIGURE 3

The $^{40}$Ca(d,n)$^{41}$Sc ground state recoil differential cross section. The upper curve is the result of the predicted shape of the recoil angular distribution after multiple scattering theory was applied to an initial recoil angular distribution calculated from an experimental neutron distribution (Gr 70). The lower curve represents a scaling of this calculation to fit the data. These calculations were performed in previous work (Do 70).
Measurements of the recoil yield have been extended to $E_d = 6$ MeV (Chapter IV) where the cross section rises to about 20 mb/sr.

The maximum value of the target thickness term $N_t$ in the above expression, expressed in ug/cm$^2$, is limited by the need to have the scandium ions recoiled out of the target. For the beam energies and resulting recoil momenta appropriate to the present studies (see Chapter IV), the range of the recoils is approximately 100 - 150 ug/cm$^2$ which represents a practical upper limit on the target thickness. However, multiple-scattering effects in targets of this range of thickness are excessive and, in practice, a target of 30 ug/cm$^2$ was used. Even with a target this thin, it was estimated (Chapter IV) that only about 60% of the scandium ions produced actually recoil from the target. This quantity can be defined as the recoil efficiency $f_{re}$, characteristic of the thickness of the target used.

The physical dimensions of the recoil catcher system, which determines the recoil solid angle, are restricted by the width of the gap of the available electromagnet and the linear extent of the homogeneous portion of the magnetic field in that gap. As is described in Chapter III, the dimensions of these quantities limited the dimensions of the catcher foil to a $\frac{1}{2}$-inch diameter, and required that it be located no further than about 3 inches from the
target. The solid angle subtended by the catcher foil was $1.1 \cdot 10^{-2}$ steradians. However, the path of the recoiling ions is bent slightly by the magnetic field, and in order to restrict the area of implantation to the catcher foil only, the solid angle was actually made smaller than that allowed by the geometrical factors ($5.17 \cdot 10^{-3}$ steradians).

The expected yield of recoils in the catcher foil can now be estimated using equation (II-4), reexpressed in terms of experimental quantities (Appendix A):

(II-5) \[ Y_f = 9.47 \cdot 10^{-4} \sigma(\theta) Q \Omega_r t f_{re} \]

where $Q$ is the number of microcoulombs of charge collected, $\sigma(\theta)$ is the differential cross section in mb/sr, $t$ is the target thickness in mg/cm$^2$, $\Omega_r$ is the recoil solid angle in steradians, and $f_{re}$ is the recoil efficiency. For the values of these quantities given above, the yield of recoils will be 180 per microcoulomb of beam at $E_d = 6$ MeV.

C. The Beta Detection Efficiency

The beta decays emanating from the implanted nuclei were detected on the right and left sides of the catcher foil by two pair of solid-state counters, each pair operating in coincidence to aid in the suppression of background. The solid-state counters, chosen because of the spacial limitations already discussed, had a large surface
area so as to intercept as much of the beta-decay sphere as possible. In fact, the rear counter of each pair subtended a half-angle of 26 degrees with respect to the centroid of the catcher foil, leading to a total effective solid angle of 1.27 steradians which is 10% of the total decay sphere. In addition, to discriminate between betas from the decay of scandium nuclei and those from other sources with lower endpoint energies, aluminum absorbers were placed in front of each pair of detectors. While this method effectively reduced the background level, it also lowered the scandium beta yield by about 50%. Thus the final scandium beta detection efficiency is 0.1 times 0.5 or a factor of 0.05.

If the number of scandium recoils produced per microcoulomb of charge is multiplied by the beta detection efficiency, the resulting number of betas detected per microcoulomb is 9. If no background were present in the system, $4 \times 10^4$ counts per run would be required to obtain a 1% accuracy in the measurement. This can be accomplished in less than two hours with a beam current of $\frac{1}{2}$ microamp. (This beam intensity was the maximum that could be withstood by the thin calcium targets.) However, background is not only present, but is a major source of difficulty in the carrying out of the experiment. These difficulties will be discussed in the following section.
D. The Problem of Background Radiation

A major source of background in carrying out the measurement of the beta radiation as described above is the large flux of prompt gamma radiation produced by the deuteron when on target. This background can be eliminated --and the beta radiation still detected with only a small impairment of efficiency--by deflecting the beam off the target and counting the betas only during the subsequent interval. The alternative processes of beam deflection and beta counting were controlled by an electronic switching circuit operating on a total cycle time on the order of the scandium halflife. The switching sequence is illustrated in the diagram below.

![Diagram showing beam and counter states over time]

The yields estimated in the preceding section must, of course, be corrected for those beta radiations undetected during the bombarding periods. The factor by which the total beta yield per microcoulomb is reduced is given by the following expression (Appendix A):

\[
\frac{1}{\lambda T} \frac{1 - e^{-\lambda T}}{1 - e^{-\lambda \tau}} \left( e^{-\lambda t_1} - e^{-\lambda t_2} \right)
\]
where $\lambda$ is the decay constant of the $^{41}\text{Sc}$ nucleus. The values of these timing parameters adapted for reasons discussed in Appendix A are:

$$
\tau = 360 \text{ ms} \\
T = 180 \text{ ms} \\
t_1 = 7.5 \text{ ms} \\
t_2 = 172.5 \text{ ms}
$$

With these values the above correction factor is 0.46. Assuming that the steady-state beam intensity can be doubled so that the average is still one-half microamp, the expected scandium beta yield drops to about 4 counts per microcoulomb.

Another major source of background is the production of other beta-emitting nuclei whose decay products have endpoint energies approaching that of the $^{41}\text{Sc}$ decay. Their effects on the measurements are discussed in detail in Chapter IV. Briefly, their effects can be severely reduced, if not eliminated, by an aluminum absorber interposed between the catcher foil and the counters, but, at the same time, with a loss of about 50% in the efficiency of detection of the scandium betas as mentioned above.

There are two additional sources of background in the measurement not yet discussed, first delayed gamma rays associated with the beta decay of the various nuclides formed, and secondly the beta decay of the metastable
$^{38}$K·13 whose endpoint energy and half-life are close to that of $^{41}$Sc. The first of these sources of background could be reduced, but not eliminated, by careful shielding of the detectors as will be discussed in Chapter IV. The second source, that due to $^{38}$K*, was unavoidably present in the measurement, but its relative importance could be controlled by a choice of beam energy and was minimal at a deuteron energy of 4 MeV.

The combined effects of these residual backgrounds contributed from 26% of the total observed yield at 4 MeV to as much as 44% at 6 MeV. It was determined in Chapter IV, however, that even though the relative background level was higher at 6 MeV, the increased scandium beta yield per unit charge (or more appropriately, per unit time) slightly more than compensates for the reduction in accuracy that this increase in background gives. For this reason 6 MeV was chosen for the polarization measurement. Figure 4 displays the estimated effects of various levels of background upon the uncertainty in the experimental asymmetry $\Delta A$ in a determination of the accuracy achieved versus run time. These calculations are specifically correct for a deuteron energy of 6 MeV, but show typical behavior of the system over the range from 4 to 6 MeV.
FIGURE 4

The uncertainty in the asymmetry versus run time for various background levels added to the scandium yield.
CHAPTER III
The Experimental Apparatus

The measurements of this experiment were performed using the 12 MeV tandem Van de Graaff accelerator of the Bonner Nuclear Laboratories at Rice University. The deuterium beam was analyzed by the 90-degree magnet, and its energy was determined from the strength of the magnetic field measured with a nuclear magnetic resonance magnetometer. The magnetometer frequency was related to the beam energy by a calibration based on the location of the $^{40}$Ca$(p,p)^{40}$Ca resonance at 4.681 MeV.

The beam was directed into the target room by a smaller switching magnet approximately 30 feet from the target chamber. Located in the beam transport system were two parallel plates which could be charged with a high voltage supply to deflect the beam from the target. Approximately 7 feet from the chamber was the first set of beam-defining slits followed by diffusion pumps and two cold traps. The final beam collimation system consisted of three tantalum slits separated by four inches each and located eleven inches from the target.

After entering the scattering chamber, the beam passed through a thin calcium target and was stopped in a Faraday cup within the chamber. Solid state counters were used to detect the subsequent beta decay from the recoil nuclei
after implantation. The data were collected and analyzed with the use of an IBM 1800 on-line computer and data-acquisition system.

The following sections contain a discussion of the beam deflection system, the method of data acquisition, and a description of the design and construction of the polarimeter chamber.

A. The Beam Deflection and Beta Detection System

The basic functions of the electronic equipment, the arrangement of which is shown in Figure 5, were the detection of the flux of beta rays and amplification and storage of the associated pulses from the detection system. The electronics also provided the alternative switching of the deuteron beam and of the counting systems.

The basic beam switching system consisted of a set of deflector plates located about 8 feet from the chamber and connected to a 15 kilovolt power supply. A transistorized multivibrator circuit provided the control signal to switch the beam off the target and onto the first set of beam-defining slits. This system is described in detail by Cramer (Cr 61) and Youngblood (Yo 65). Simultaneously, another output signal from the control circuit temporarily stopped the operation of the scalars where the data were stored. The timing of the switching sequence was such that the counting system was operative only when the beam was
FIGURE 5

Schematic diagram of the beam deflection and beta detection system electronics.
off target. This relationship is diagrammed in Chapter II.

The total cycle time was adjustable from 120 to 360 milliseconds, while the bombarding time was variable from about 0.35 to 0.65 of the total cycle. The counting period could be extended from zero to within 7.5 ms of either side of the "beam-off" period. The values of the pulsing parameters chosen for operation are given in Appendix A.

Each of the detecting systems used in measuring the left-right beta-decay asymmetry consisted of two transmission-type solid state detectors operating in coincidence. These detectors (supplied by Princeton Gamma-Tech) were 1000 microns thick; the front detectors had a 200 mm$^2$ surface area while the rear detectors were 300 mm$^2$. Signals from the detectors were preamplified and amplified in the target room, then transmitted to the data acquisition area. At this point the signals were attenuated before entering double-delay line amplifiers. This extra stage of amplification before transmission was a necessary means of noise reduction because of pulse-pickup in the transmission lines. The double-delay line amplifiers were chosen for their timing stability.

After amplification, the signals were fed through timing single-channel analyzers to discriminate against electronic noise. At this stage, the timing was adjusted such that output pulses of each of a pair of counters arrived at a multiple coincidence unit within a $\frac{1}{2}$ us time
span; the resolving time was set at 2 us. The outputs of each of the single-channel analyzers and of the coincidence units were fed into scalars. Thus the singles rates of each of the four detectors could be monitored as well as the coincidence rates.

The deuteron beam was stopped in an insulated Faraday cup which was connected to a current integrator. The digital output of the integrator was fed into a counter located internally in the interface to the computer. When a predetermined integrated beam current was reached, an inhibit pulse was supplied from the computer interface to the gate of a master timer. This timer was used to terminate data storage in all scalars.

In order to determine the various levels of background radiation, it was necessary to measure the rate of decay of the total activity from the target over a period of seconds. To accomplish this measurement, an alternate method of data acquisition was devised, requiring a change in the electronics. The transistorized switching circuit was replaced by a slower mechanical timer with total cycle times available of 4, 10, 21.3, and 36 seconds. The remainder of the detection circuitry remained as previously described. Signals from the counting systems were sent to multiscalors which were an integral part of the on-line data acquisition system. The maximum allowable count rates were on the order of 1000 counts per second. The start
pulse beginning the multi-scalar sweep originated from the mechanical timer, and the time spent in each channel was adjusted such that one multiscalar sweep covered the entire cycle period of the timer.

The actual data-acquisition process was controlled by three signals from the mechanical timer which initiated the operations of beam deflection, multiscaling, and data recording. The sequence of these signals was the same as that in the foregoing mode of operation, but the timing involved was different. Here, the beam was on target during the first 10% of the cycle, and the multiscalar sweep covered the remainder. In a given run, many sweeps were made until enough counts were logged to define accurately the decay curve. The procedure by which the total activity is analyzed to give the constituent activities is described in detail in Appendix C.

The energy calibration of the counting system made use of the 662 keV radiation from a $^{137}$Cs gamma source. A sample pulse-height spectrum is shown in Figure 6. The gain of each amplifier was adjusted so that the Compton edge of the gamma ray fell in the same channel, then the baseline of the SCA's were raised just above the noise level. Each counter sampled an identical portion of the energy spectrum. The definition of the energy scale was completed by the location of the full energy peak at 662 keV. A $^{144}$Ce beta source with an endpoint energy of 3 MeV was then
Energy calibration spectra. The upper spectrum shows the Compton scattering of 0.662 MeV gamma rays from the decay of $^{137}$Cs. Calibration was based on the location of the Compton edge at 0.478 MeV and the full energy peak at 0.662 MeV. The baseline setting corresponds to an energy of $0.1725 \pm 0.0166$ MeV.

The lower figure represents the 3 MeV $B^{-}$ spectrum with a peak at $0.334 \pm 0.0166$ MeV.
used to set the timing of the coincidence circuitry.

The shape of the beta spectrum shown in the figure can be explained by the fact that the energy loss in silicon of all betas above about 1 MeV in energy is approximately 350 keV per 1000 microns, creating a peak in the spectrum at this energy. The long tail is the result of the extreme degree of multiple scattering of the electrons with a resulting increase in path length in the detector.

B. The Design of the Polarimeter Chamber

The scattering chamber used in the asymmetry measurement was designed to obtain a maximum counting rate from the decay of the recoil nuclei consistent with the spatial limitations imposed by the gap width and the pole-face diameter of the available electro-magnet. A cross-sectional view of the chamber is shown in Figure 7. Another view showing the recoil and counting geometry is presented in Figure 8.

The cylindrical brass body of the chamber was 4½ inches in diameter and 2 inches deep with walls ½ inch in thickness. At the center of the chamber was a brass target holder which was faced with lead on the side exposed to the beam. This holder was mounted on a base that could be rotated from outside the chamber. To prevent excessive heating of the thin targets due to energy loss by the beam, the target holder was mounted to insure good thermal contact with the chamber.
FIGURE 7

A cross-sectional view of the scattering chamber. The beam enters the chamber through port B, passes through the thin target at T, and is stopped on a gold disk G embedded in a lead Faraday cup L. The scandium recoil ions pass through the lead collimator C and are stopped on the catcher foil F. A counter M, used to monitor the elastically scattered deuterons from the target, is located 120 degrees to the beam direction. The slits for the monitor are located at R and S. The connector K provides a means of beam current integration. The window at W is used to observe the displacement of the beam by the magnetic field on a quartz placed at the target position. Shown with a dashed line is the position of the magnet pole face with respect to the chamber.
A cross-sectional view of a portion of the scattering chamber showing the recoil and counting geometry. The recoil ions leave the target T and travel through the lead collimator C until they are stopped on the catcher foil F. The positrons from the scandium decay travel through a thin brass window B into the counting telescopes. The two sets of solid state counters, A and B, are fitted into a holder H which is removable from the chamber. The front counters, AF and BF, have a 200 mm² surface area where the rear, AR and BR, have 300 mm² area. Also shown in front of and between the counters are the aluminum disks D used for energy discrimination. The mounting M for the target holder enables a rotation of the target from outside the chamber. Also shown is a glass window W and a portion of the Faraday cup Q.
Beam unscattered by the target was stopped by a piece of gold, 10 mil thick, embedded in a Faraday cup made of lead where the collected charge could be measured.

The ions recoiling from the target were collimated by a 3/16 inch aperture in a lead cylinder before implantation. This aperture was located 2.3 inches from the target, subtending an angle of 4.5 degrees and a solid angle of $5.17 \times 10^{-3}$ steradians. The recoils were stopped by a conically-shaped catcher foil subtending a slightly greater solid angle than the entrance aperture. The foil was glued to a brass mounting which was attached to the flange at the end of the recoil catcher tube and could be removed when it was necessary to change foils.

The beta particles from the decay of the recoil nuclei passed through both the catcher foil and a 2 mil brass window into the counting assemblies on an axis perpendicular to the scattering plane. The brass counter holder provided a mounting for the two sets of solid state detectors on either side of the recoil catcher tube as indicated in Figure 8. In addition, it also provided mounting shelves for the aluminum disks or absorbers used as a means of energy discrimination (Chapter IV). The holder could be removed from the chamber, then rotated by 180 degrees and reinserted into position.

Both ends of the cylindrical chamber could be removed to give access to its interior. One end contained the
mounting for the target and the Faraday cup; the other contained a 3-inch diameter glass window which allowed visual inspection of the target. Another window in the cylindrical wall of the chamber was used for observing the motion of the beam at the target position when the chamber was positioned in the magnetic field.

A solid state monitor counter was located at 120 degrees from the beam direction and about three inches from the target and served to determine target thicknesses and to monitor the buildup of contaminants. The cross section for the $^{40}$Ca(d,d)$^{40}$Ca elastic scattering at 120 degrees is 36.2 mb/sr (Gr 70) at 4 MeV, and this value was used to calculate target thicknesses from the measured yield of scattered deuterons. At 6 MeV the corresponding cross section is 20.8 mb/sr. For the solid angle of $6.80 \cdot 10^{-4}$ steradians used in these measurements, the relationship (Do 70) between the target thickness $t$ in mg/cm$^2$, the measured elastic yield $Y$, and the integrated beam current $Q$ in uC is:

<table>
<thead>
<tr>
<th>Beam Energy</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 MeV</td>
<td>$t = 4.32 \cdot 10^{-4} \frac{Y}{Q}$</td>
</tr>
<tr>
<td>6 MeV</td>
<td>$t = 7.53 \cdot 10^{-4} \frac{Y}{Q}$</td>
</tr>
</tbody>
</table>

A typical charged particle spectrum is shown in Figure 9.

The targets used in this experiment were produced by vacuum deposition of evaporated natural calcium metal upon
FIGURE 9

Typical monitor counter charged-particle spectrum.
a suitable substrate. Two types of targets were used. When a thick target was required (approximately 200-300 ug/cm$^2$), the calcium was self-supporting. Thinner targets (10-30 ug/cm$^2$) were evaporated onto 10 ug/cm$^2$ carbon foils mounted on tantalum frames. The thin targets were found to oxidize almost immediately, while the thick targets could be kept relatively oxygen-free for extended periods of time.

C. The Strong Magnetic Field

The strong magnetic field required for the polarization experiment was provided by a Varian Model V-4007 electro-magnet with 6-inch diameter pole faces and a gap adjustable from zero to several inches. To furnish a field with maximum strength and homogeneity, the gap was set as narrow (2$\frac{1}{2}$ inches) as allowed by the thickness of the target chamber. A more uniform magnetic field was obtained by using ring shim pole caps with a step of 0.080 inches. The magnet current was supplied by a Varian V-2200A regulated power supply capable of providing 1.4 amps. A calibration of magnet current versus magnetic field was measured using a NMR gaussmeter as shown in Figure 10. Also indicated are the results of the measurement of the field homogeneity, after the ring shims were properly adjusted. The magnet coils were water cooled with a closed circulatory system and chilled water heat exchanger.
FIGURE 10

Magnetic field calibration of the Varian electro-magnet. The insert shows the field homogeneity as a function of distance from the center of the pole faces.
The magnet assembly had to be mounted at a beam-line height of 69 inches; and for this purpose, a sturdy aluminum table was constructed to support the magnet. The elevation of the table was adjustable by means of jacks. The magnet assembly rolled on ball-bearing wheels in tracks on the supporting table. Movement of this assembly was necessary to give access to the chamber. Once in correct position, the magnet was securely fastened to the table.

The deuteron beam, upon entering the target chamber, had to pass through about 12 inches of the fringing field of the magnet. This field caused a slight deflection of the beam at the position of the defining slits. To reduce this effect, the beam tube was shielded with quarter-inch iron pipe. Because of the remaining 2½ inches of unshielded beam path immediately preceding the target, the beam was deflected about 1/8 inch at the target by the strong field itself. To nullify this deflection, a small permanent horseshoe magnet was mounted about 10 inches from the target with a field direction transverse to the beam path. The distance of this trimming magnet from the beam could be varied to provide correct compensation for the various values of the strong magnetic field.

D. System Alignment and Accuracy

In order to maximize the sensitivity in the detection of the asymmetry in the beta decay of the recoil nuclei, it
was necessary to minimize systematic asymmetries through the accurate design and construction of the chamber and counting systems. Care was taken to insure that the centroid of the recoil cone accurately intersected the center of the target, and that the polarimeter itself was aligned so that the chamber faces were parallel to the pole faces of the magnet. Typical tolerances during the construction of the apparatus were 0.001 - 0.002 of an inch.

It is of primary importance that the recoil ions be implanted symmetrically on the catcher foil with respect to the location of the detector assemblies. The sensitivity of the system to horizontal shifts in the centroid of the recoil cone is exemplified in that, based upon the dimensions of the catcher foil and counting geometry, every 0.004 of an inch movement of the recoil spot on the stopping foil results in a 1% change in the counting rates in the right and left counting systems (and consequently a 1% change in the experimental asymmetry A). Such a shift can result from a horizontal movement of the deuteron beam spot on the target of about 0.01 inch. Consequently, because of the need for accurate positioning, the beam was defined by a relatively narrow slit system (0.078 inch horizontally by 0.125 inch vertically) located eleven inches from the target. Finally, to prevent movements of the beam spot on target during measuring periods of 0.01 inch or greater, the horizontal movement of the beam at a distance of 89
inches from the chamber was limited to 0.125 inches.

Since both the deuteron and recoil beams consisted of charged particles, it was necessary to align the pole faces of the magnet exactly parallel to the plane defined by the centroid of the recoil cone and the beam axis, or, in practice, to the chamber faces to insure that no horizontal movement of either beam would be caused by the magnetic field.
CHAPTER IV

Performance Optimization of the Apparatus

The measurements discussed in this chapter were made for the purpose of optimizing the efficiency of the detection of the betas from the decay of scandium ions after recoil implantation. This was accomplished by maximizing the scandium beta yield in comparison to that of extraneous beta-active sources and other background activity. It was also necessary to study the scandium-recoil yield as a function of such factors as deuteron beam energy and target thickness, the latter quantity affecting the efficiency of the recoil process. The results of this work contributed to the design of the experimental apparatus for the asymmetry measurement. In addition, because of the effect of the strong magnetic field on the trajectories of the ions, a determination of the charge state of the recoils was also relevant to this design.

A. Methods of Discrimination Against Background Radiation

The production of positron-emitting nuclei with beta endpoint energies as high as 5 MeV could interfere with the detection of the beta decay of $^{41}\text{Sc}$ (endpoint energy = 5.5 MeV). These nuclei are not only recoiled along with the scandium onto the catcher foil, but are deposited in all portions of the polarimeter chamber. The most significant
of these, typically produced by the deuteron beam in an oxidized calcium target with a carbon backing, are listed in the table below.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Endpoint Energy</th>
<th>Halflife</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{40}\text{Ca}(d,\alpha)^{38}\text{K}$</td>
<td>2.68 MeV</td>
<td>7.67 min</td>
</tr>
<tr>
<td>$^{40}\text{Ca}(d,\alpha)^{38}\text{K}\cdot13$</td>
<td>5.03 &quot;</td>
<td>0.946 sec</td>
</tr>
<tr>
<td>$^{16}\text{O}(d,n)^{17}\text{F}$</td>
<td>1.74 &quot;</td>
<td>66 sec</td>
</tr>
<tr>
<td>$^{12}\text{C}(d,n)^{13}\text{N}$</td>
<td>1.2 &quot;</td>
<td>9.96 min</td>
</tr>
</tbody>
</table>

An additional source of background was due to Compton electrons produced by gamma radiation in the counter and catcher foil areas. This radiation emanated from three sources—-from the annihilation of positrons, from beta decays to excited states in various nuclei, and from the prompt decay of nuclei excited by the deuteron beam both in the Faraday cup and in the walls of the chamber.

The following sections will deal with the problems of reducing the gamma flux at the position of the counters and of retaining a sufficient scandium beta yield while lowering the efficiency of detection of background radiation.

**Reduction of gamma-ray background.** The most significant method of background reduction, as already mentioned in Chapter II, was the switching of the deuteron beam off target during measurement or counting periods. Without this procedure, the experiment would not have been feasible because of the large flux of prompt gamma radiation produced by the deuteron beam.
To further reduce the production of gamma radiation in the chamber, all portions likely to be exposed to the deuteron beam were constructed or lined with high-Z materials of high purity. Thus the beam was collimated with slits made of tantalum and was stopped in a gold absorber. Furthermore, to protect the counting systems from the gamma flux that is produced, the Faraday cup, enclosing the beam stop, was constructed of lead and shaped in such a manner as to form a shield between the chamber and counter areas. (See Figure 8.) In addition, the recoil collimator was constructed of lead.

The above efforts, while successful, were still not satisfactory; therefore, to further reduce the sensitivity to gamma-ray background, each of the two sets of counters were operated in coincidence. This brought about an additional factor of two reduction in background activity, thereby lowering the background to an acceptable level as to render sufficient sensitivity to beta detection. The absorption of competing beta rays was then studied with this arrangement.

Absorption of low energy beta rays. With the thin solid-state counters used in the present experiment, direct pulse-height discrimination was not possible; therefore, the energy loss and multiple scattering effects on positrons of thin sheets of aluminum placed in front of the counters were used to screen out the lower energy betas. Aluminum was
chosen over other low-Z materials because of its higher
density, availability and machinability.

Figure 11 shows the measured yields of the competing
high-energy components, the $^{41}\text{Sc}$ and $^{38}\text{K}\ast$ beta decays, as
a function of absorber thickness expressed in g/cm$^2$.
These data were accumulated by activation of a thick calcium
target with 4 MeV deuterons. After activation, the decay
rate of the total target activity was measured, and the
procedure outlined in Chapter III and Appendix C was used
to separate the scandium, potassium and background
contributions.

Since much of the background radiation can be attribu-
ted to the formation of $^{17}\text{F}$ from deuteron bombardment of
the oxygen layer formed on the calcium target, a lead oxide
target was investigated in a manner similar to that above.
It was found that the fluorine beta decay, with an endpoint
energy of 1.7 MeV, can be absorbed by 0.6 or 0.7 g/cm$^2$ of
aluminum as also indicated in Figure 11.

Under deuteron bombardment, the carbon backing on the
typical target led to the formation of $^{13}\text{N}$ ($\tau_{1/2} = 10$ min),
and $^{38}\text{K}$ ($\tau_{1/2} = 8$ min) was produced by the $^{40}\text{Ca(d,}\alpha)$ reaction;
therefore similar absorption curves (Figure 11) were
measured of the decay of these nuclei by bombarding a self-
supporting carbon or calcium target. After beta-decay
equilibrium was reached (approximately three halflives),
the target was placed in a holder under a solid-state counter.
FIGURE 11

Beta-ray absorption curves. The relative yields of the beta decays of various nuclei are shown versus aluminum absorber thickness. The following beta endpoint energies are represented: (1) 1.2 MeV from $^{13}_{\text{N}}$, (2) 1.74 MeV from $^{17}_{\text{F}}$, (3) 2.68 MeV from $^{38}_{\text{K}}$, (4) 5.03 MeV from $^{38}_{\text{K}^*}$, and (5) 5.5 MeV from $^{41}_{\text{Sc}}$. 
While measuring the yield as a function of absorber thickness, both the counting period and the period for absorber change were timed in order that corrections (Appendix D) could be made for the fact that the source halflife was on the order of the total length of the measurement. The presence of $^{13}\text{N}$ or $^{38}\text{K}$ was verified by a subsequent decay-curve measurement and analysis.

The results of the absorption-curve measurements show that essentially all betas from competing reactions, except those from $^{38}\text{K}^*$, can be completely absorbed by about 0.8 $\text{g/cm}^2$ of aluminum in front of the counting systems. With this absorber, about 50% of the scandium beta yield is lost. In addition, it was found from these measurements that if 0.1 $\text{g/cm}^2$ of absorber were placed between the counters, the Compton electrons from annihilation radiation produced in one counter could not penetrate the second, thereby causing a coincidence event. With this exception, results were insensitive to whether the absorbers were placed in front of the counters or between them.

The results of the efforts to reduce background must be viewed in the light of the relative cross sections, or more directly, the relative yields for each of the competing reactions. The table below summarizes the predictions of the relative beta yields from recoils formed in competing reactions as calculated from known differential cross sections at 4 MeV for a target of typical composition (Appendix E).


<table>
<thead>
<tr>
<th>Parent</th>
<th>Halflife</th>
<th>$E_{\text{max}}(\beta^+)$</th>
<th>$\sigma(\sim300)$</th>
<th>Relative Yield</th>
<th>Zero Absorber</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{41}$Sc</td>
<td>0.6 sec</td>
<td>5.5 MeV</td>
<td>12 mb/sr</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$^{38}$K*</td>
<td>0.95 sec</td>
<td>5.0 &quot;</td>
<td>0.36 &quot;</td>
<td>.05</td>
<td></td>
</tr>
<tr>
<td>$^{38}$K</td>
<td>7.76 min</td>
<td>2.7 &quot;</td>
<td>58 &quot;</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>$^{17}$F</td>
<td>66 sec</td>
<td>1.7 &quot;</td>
<td>300 &quot;</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>$^{13}$N</td>
<td>9.96 min</td>
<td>1.2 &quot;</td>
<td>55 &quot;</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Since the relative yields of all but one of the competing reactions is much greater than the scandium beta yield, the necessity for the use of an absorber of at least 0.8 g/cm² is evident. To confirm the effectiveness of an absorber of this amount in the discrimination against unwanted activities, additional measurements in the actual recoil geometry used in the polarization experiment was necessary. These tests involved the measurement and analysis of the decay of the combined activity from a typical target as a function of absorber thickness and are discussed in the following section.

**Tests of the effectiveness of background reduction.**

A set of measurements of the beta yield from those nuclei implanted on the catcher foil was made in the polarimeter chamber to confirm the results of efforts to reduce background. Decay curves of the total target activity, including remaining background due to gamma rays, were measured as a function of absorber thickness at deuteron energies
of 4 and 6 MeV. In these studies, the measurements were statistically inadequate to separate the $^{41}$Sc and the $^{38}K^*$ contributions to the total activity. The analysis of the decay curves was therefore used only to separate the combined $^{41}$Sc and $^{38}K^*$ decays from the lower energy background activity. The results of the analysis of the data, as presented in Figure 12, indicate an absorber thickness of 0.7 g/cm$^2$ is sufficient to eliminate the majority of the lower energy beta radiation. Viewed from the standpoint of relative yields, the results at the two energies are quantitatively different, reflecting a more favorable scandium (plus potassium) to background ratio at a bombarding energy of 6 MeV.

To study the effects of the magnetic field on the relative yields of scandium and background, the measurements at 6 MeV were repeated in the presence of a six kilogauss field. These results, also shown in Figure 12, indicate a negligible effect.

The measurements shown in Figure 12, as derived from the analysis of decay curves at each absorber thickness, reflect different admixtures of $^{41}$Sc plus $^{38}K^*$ and background radiation than if the measurements had been made in the rapid-switching mode of operation appropriate to the polarization studies. The behavior characteristic of the rapid mode can be derived from the following relation:

$$\frac{Y_1}{Y_2} = CR_0$$
FIGURE 12

Results of decay-curve analyses as a function of absorber thickness at 4 and 6 MeV. The solid lines and circles represent the relative yields of $^{41}\text{Sc}$ and $^{38}\text{K}^*$ betas combined and of other background at the beginning of the decay curve. The dashed lines represent the conversion of these ratios to those that would be obtained if the rapid switching mode of operation had been used. The open circles represent a repetition of the measurements at 6 MeV with the magnetic field on.
where \( Y_1/Y_2 \) is the ratio of the scandium to background beta yield in the rapid-switching mode (\( Y_1 \) includes the \(^{38}\text{K}^*\) contribution in this case), \( R_0 \) is the corresponding ratio at the beginning of the decay curve, and \( C \) is the proportionality factor relating these quantities which is calculated in Appendix C. The ratios \( Y_1/Y_2 \) for each absorber are also included in Figure 12.

The efforts discussed in this section have been successful in reducing the background from gamma radiation and low energy betas from competing reactions. However, the background due to the decay of \(^{38}\text{K}^*\) can only be reduced by an appropriate choice of beam energy where the ratio of cross sections for the reactions producing \(^{41}\text{Sc}\) and \(^{38}\text{K}^*\) is favorable to the formation of scandium. This problem is analyzed in Section B below where the decay curves measured were statistically adequate to determine the relative contributions of the decay of these two nuclei to the total yield as a function of deuteron beam energy.

B. Dependence of Yields upon Beam Energy and Target Thickness

The beta yield from the scandium recoils was measured at 4 MeV as a function of target thickness to determine the recoil efficiency of the heavy ions. Then the magnitudes of the yields from the decay of scandium, potassium and background were measured as a function of beam energy to
determine an energy where the detection of the scandium beta yield is optimum.

Measurement of recoil efficiencies. The optimum target thickness for efficient recoil of \(~200-400\) keV scandium (see recoil energy distributions in Appendix F) depends upon the range and degree of multiple scattering of the ions in the target. While integration of the energy loss curve (Do 70) reveals a penetration of about 25 to 30 \(\text{ug/cm}^2\) for every 100 keV of recoil energy, it was calculated that multiple scattering of the scandium ions becomes very pronounced after the path length in the target reaches the order of tens of \(\text{ug/cm}^2\).

A measurement of the total activity from ions recoiled onto the catcher foil was made for each of several thin calcium targets of varying thicknesses and for a self-supporting target of about 320 \(\text{ug/cm}^2\). An absorber of 0.9 \(\text{g/cm}^2\) was used at the detector in order to reduce the background to the lowest possible level, and the measurement was made in the rapid-switching mode. The yield, as shown in Figure 13, tends to rise linearly at first; but as the target becomes thicker, it tends toward a saturation level. Thus targets thinner than about 50 \(\text{ug/cm}^2\) make use of the entire recoil range of the scandium ions, while thicker targets do not contribute proportionately to an increased recoil yield. This trend is in agreement with the previous studies which indicate that the average
FIGURE 13

Total coincidence beta versus target thickness.
scattering angle of a scandium ion in a CaO environment reaches the order of the seven-degree resolution (angular acceptance of the recoil collimator) of the present experiment when the target thickness reaches about 10 µg/cm². The results of this measurement indicate an optimum target thickness between 30 and 70 µg/cm².

**Measurement of the recoil yield versus beam energy.**

The total coincidence yield of radiations emanating from the catcher foil was measured using a 30 µg/cm² target, in 500 keV steps, over the deuteron energy range from 4 to 6 MeV. The experimental procedure consisted of measuring a decay curve over a 4 second period at each of these energies in order to determine the individual contributions of the short-lived beta emitters, $^{41}$Sc and $^{39}$K*, and of other background. These results were converted to the absolute yields corresponding to operation in the rapid-switching mode and are presented in Figure 14.

There are two obvious trends in the results of these measurements which will lead to a final choice of beam energy. As the beam energy is increased from 4 to 6 MeV, the absolute scandium beta yield increases by a factor of two, while the relative yield from the decay of scandium drops from 74% to 56%. Since the quantity to be measured, the beta-decay asymmetry as defined in Chapter II, is not dependent on either of these trends, it suffices to study the effects of both on the accuracy $\Delta A$ of the final
measurement. This quantity is given in Appendix B as:

\[ \Delta A = \frac{(1 + \frac{1}{2} R_b^2)^{\frac{1}{2}}}{1 - R_b} \] [Total yield per run]^{-\frac{1}{2}}

\( R_b \) is the relative background yield, and the total yield includes any background contribution. The variation of \( \Delta A \) with beam energy is summarized in the chart below using the results given in Figure 14.

<table>
<thead>
<tr>
<th>Beam Energy</th>
<th>Normalized Total Yield</th>
<th>Relative Background</th>
<th>( \Delta A )</th>
<th>( \Delta A ) (at 4 MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>1.00</td>
<td>26.3%</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>4.5</td>
<td>1.25</td>
<td>31.6%</td>
<td></td>
<td>0.97</td>
</tr>
<tr>
<td>5.0</td>
<td>1.58</td>
<td>36.8%</td>
<td></td>
<td>0.94</td>
</tr>
<tr>
<td>6.0</td>
<td>2.38</td>
<td>43.8%</td>
<td></td>
<td>0.88</td>
</tr>
</tbody>
</table>

It can be seen from the evaluation of \( \Delta A \) that while 6 MeV is the optimum beam energy, the improvement over running at 4 MeV is very slight.

C. A Determination of the Charge State of the Recoils.

The determination of the charge state of the scandium ions as they emerge from the calcium target was necessary before the design of the recoil collimator could be finalized. If these recoils are charged, their trajectories will be curved; and, as a consequence, a smaller aperture is required to insure that all are deposited on the catcher foil. Those that are embedded short of the catcher con-
FIGURE 14

The absolute yields of betas from scandium, potassium, and other longer-lived background measured over a small energy range.
tribute only to background as a result of depolarization mechanisms in the chamber wall.

This measurement made use of the scattering chamber and 6-inch pole face electromagnet described in Chapter III. The chamber was modified so that the recoil trajectory would extend the full six inches of the magnetic field area. The recoil beam was very tightly collimated to maximize sensitivity to the effects of the magnetic field on the ion's path. A measurement of the recoil yield as a function of field strength by a beta counter placed at the end of the tube then gives an indication of the charge state distribution.

**The design of the apparatus.** A special recoil tube assembly was designed for the charge state experiment, and was attached to the scattering chamber in place of the usual recoil catcher tube. As shown in Figure 15, this tube contains three 1/8 inch lead slits which define the recoil beam. The scandium ions recoil through the slit system and are stopped in a 2 mil gold foil at the end of the tube where two solid state counters operating in coincidence were mounted. Aluminum absorbers are included in front of the counters as well as between; and, in addition, the counters are shielded from the rear. As shown in the figure, any curvature of the recoil trajectory results in a reduction of the yield. Note that the leading edge of the first slit defines the midpoint of the recoil path.
FIGURE 15

Apparatus for the charge state measurement.
**Calculation of expected yields.** To interpret the results of the experiment, the dependence of the recoil yield upon the magnetic field strength was calculated. (See Appendix F.) The calculation was performed by determining the portion of the recoil beam intercepted by the first slit in the recoil path as the magnetic field strength (or magnet current) was increased. This was, in turn, based upon a determination of the recoil velocity distribution at each beam energy as given by the reaction kinematics. The results, as shown in Figure 16, were used in the analysis of the data.

**Experimental results and analysis.** The measurement of the charge state of the scandium recoils was performed at 4 and 6 MeV bombarding energies using the apparatus and procedure outlined above. The length of each run was determined by integration of the beam current collected in the Faraday cup; most runs were repeated several times to check for consistency. Targets of 20-30 ug/cm² were used along with an absorber of 0.9 g/cm². The results of these measurements are shown in Figures 17 and 18.

The background level was determined by repeating the yield versus field measurements with a 1 mil mylar recoil absorber placed over the entrance of the recoil collimator. Thus all heavy ions were stopped about 4 inches from the gold catcher foil. Only beta and gamma background radiation could penetrate this absorber. As indicated by the peaking
FIGURE 16

Results of the calculation of the recoil yield versus magnetic field strength for the three most likely charge states of the recoiling ions at bombarding energies of 4 and 6 MeV.
Ch. 0 + Background

Ch. 1

Ch. 2

Ch. 3

$E_d = 6$ MeV

Ch. 0 + Background

Ch. 1

Ch. 2

Ch. 3

$E_d = 4$ MeV
FIGURE 17

Charge state data and analysis at 4 MeV bombarding energy. The error bars are the statistical error in the data. The circles represent the total yield, the squares the background yield. The upper solid line is the fit to the total yield with charge state one recoils plus background contributing. The lower solid line is the background level resulting from this analysis. The dashed line is the fit with charge state two recoils and background contributing; whereas, the dotted line represents charge state three plus background.
FIGURE 18

Charge state data and analysis at 6 MeV bombarding energy. The symbols and curves are the same as in the preceding figure.
at zero field in the run at 6 MeV, it is obvious that there
is an increasing interference from betas coming from the
target area, because this background is immediately reduced
by small values of magnetic field.

Analysis was accomplished by fitting the data with a
function of the type:

\[ Y = af_a + bf_b \]

via a least squares method similar to that described in
Appendix C. \( Y \) is the total beta yield, \( f_a \) is one of the
ionized charge state functions shown in Figure 16, \( a \) is
the relative amount of this function compared to \( b \), the
amount of background plus neutral charge state function \( f_b \).
At 6 MeV, \( f_b \) was not constant, but shaped according to the
data to account for the increased beta background flux at
this energy. The results of the fitting of the data are
summarized in the table below. The relative yields are
characteristic of the value at zero field.

The last column in the table gives the probability
that a correct or adequate fit to the data will give an \( X^2 \)
differing from 1 by an amount equal to or greater than the
value of \( X^2 \) given in the analysis. Thus it can be concluded
that in both cases, the existence of charge states two and
three is highly unlikely.

The fits to the data are given in Figures 17 and 18.
Note that in each case the calculated background level
agrees with the experimental data very closely, therefore the possibility of the existence of the neutral charge state is very unlikely.

Results of Data Analysis

<table>
<thead>
<tr>
<th>Energy</th>
<th>Components</th>
<th>Relative Yields</th>
<th>Chi-Square</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 MeV</td>
<td>Ch. 1 + const. 0.45 ± 0.01 0.55 ± 0.003</td>
<td>1.036 80%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ch. 2 + const. 0.38 0.01 0.62 0.002</td>
<td>4.567 1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ch. 3 + const. 0.33 0.01 0.67 0.002</td>
<td>7.233 1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 MeV</td>
<td>Ch. 1 + const. 0.33 0.01 0.67 0.004</td>
<td>0.893 80%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ch. 2 + const. 0.26 0.01 0.74 0.002</td>
<td>3.823 1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ch. 3 + const. 0.25 0.01 0.75 0.002</td>
<td>5.873 1%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of this measurement may be explained in the following manner. The scandium nuclei are produced from calcium ions in a CaO cubic lattice. Since the calcium is presumably in a doubly positive ionic state, the scandium is produced by the (d,n) reaction triply ionized. The subsequent gain or loss of electrons by the scandium depends upon the recoil velocity and length of time spent in the target. It has been shown by Zaidens (Za 62) that, to a first approximation, the probability for the gain of an electron is equal to the probability for loss of an electron when the ion velocity reaches the orbital velocity of the electron in question. From these arguments, the
most probable charge state turns out to be the singly ionized configuration, provided the ion resides in the target for periods long enough to allow charge state equilibrium to be reached. Charge states 0, 2 and 3 are, of course, also likely to be present, but in reduced proportion.

However, the transit time of the recoil in the target is only about $10^{-13}$ seconds, which may be too short to allow the required number of atomic transitions to take place. Thus another possibility is that the calcium atom is more likely to be in a neutral state in the CaO lattice, thereby explaining the experimental results without regard to atomic transitions.
CHAPTER V
Asymmetry Measurements

The performance of the polarimeter system was investigated by studying the asymmetries in the radiations detected when operating at a deuteron bombarding energy of 6 MeV. In order to determine the magnitude of the extraneous asymmetries present in the system, measurements were first made using a mylar stopping foil where no asymmetry due to nuclear polarization of the scandium recoils would be expected. This included the measurement of the magnitudes of the asymmetry due to the construction and alignment of the chamber, the asymmetry in the flux of background radiation, the instrumental or electronic asymmetries, and the changes in these asymmetries owing to the presence and reversal of the direction of the magnetic field. Once the combined effect of these asymmetries was established, the measurement was repeated using an aluminum foil where the possibility for the maintenance of nuclear polarization existed. Comparison of the two results would then furnish an indication of any asymmetry due to the polarization of the decaying scandium nuclei.

The purpose of these measurements was to establish an accurate method for detection of the scandium beta-decay asymmetry, in the presence of background radiation, that would ultimately enable a measurement of the magnetic
moment of the $^{41}$Sc ground state to be made, as described in Chapter I.

Finally, a procedure was developed by which both the nuclear polarization and its time of relaxation could be measured simultaneously.

The expression for the laboratory asymmetry $A$ given in Chapter II included only the scandium beta decay without regard to background. Since a relative background level of about 45% of the total yield has been observed at this bombarding energy, the terms in the expression for $A$ must be redefined to include this effect. Let $A'$ be the measured asymmetry, including background, defined by:

$$A' = \frac{T_r - T_l}{T_r + T_l} \pm \left[ T_r + T_l \right]^{-\frac{1}{2}}$$

where $B$ is the total background count detected in both systems such that:

$$T_r = S_r + \frac{1}{2}B$$

$$T_l = S_l + \frac{1}{2}B$$

The experimentally observed asymmetry $A'$ is related to the asymmetry $A$ in just the scandium decay as follows:

$$A' = \frac{S_r - S_l}{S_r + S_l + B} = \frac{S_r - S_l}{(S_r + S_l)/R_S} = R_S A$$

where $R_S$ is the fraction of the total yield due to the scandium beta decay. The relation between the error in the two asymmetries, $A$ and $A'$, as derived in Appendix B, is:
\[ \Delta A = \frac{(1 + \frac{1}{2} R_D^2)^{\frac{1}{2}}}{1 - R_D} \Delta A' \]

where \( R_D \) is the relative amount of background yield in the measurements which determine \( A' \).

The direct measurements of the laboratory asymmetry \( A' \) involved the accumulation of data with the detector assemblies located on either side of the catcher foil while operating in the rapid-switching mode. This method, as previously described, allowed a sufficiently rapid acquisition of the data to determine \( A' \) with an accuracy of about 1% in a run of reasonable duration.

The two experimental measurements, along with a discussion of the analysis of the data, will be described in detail in this chapter. The final section will contain the conclusions and further implications of the experiment.

A. Experimental Measurements and Results

The performance of the polarimeter system. The first measurements of the asymmetry \( A' \) in the right-left counting rates were performed without the presence of the magnetic field to determine the magnitude of the extraneous instrumental and chamber asymmetries. Before actual data acquisition was begun, care was taken to insure that a change in the right-left count ratios could not be induced by different tuning conditions on the deuteron beam. With the tight beam collimation described in Chapter III, all
count ratios repeated within statistics regardless of the most extreme tuning conditions.

To explore the magnitudes of the counting system asymmetry and any asymmetries in the construction of the catcher foil, an experimental procedure involving successive reversals of the positions of the counters and rotation of the catcher foil was adopted. The asymmetry, calculated for each of the four runs from equation (V-1), is shown in the table below for two stopping foils.

<table>
<thead>
<tr>
<th>Stopping Foil</th>
<th>Field Direction</th>
<th>Counter Position</th>
<th>Foil Position</th>
<th>Asymmetry A' (%)</th>
<th>Error A' (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mylar</td>
<td>off</td>
<td>normal</td>
<td>normal</td>
<td>+0.21</td>
<td>1.04</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>normal</td>
<td>-2.83</td>
<td>1.05</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>reverse</td>
<td>+0.87</td>
<td>1.03</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>normal</td>
<td>reverse</td>
<td>-1.64</td>
<td>1.02</td>
</tr>
<tr>
<td>average</td>
<td></td>
<td></td>
<td></td>
<td>-0.85 ± 0.52</td>
<td></td>
</tr>
<tr>
<td>aluminum</td>
<td>off</td>
<td>normal</td>
<td>normal</td>
<td>+0.79</td>
<td>1.04</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>normal</td>
<td>+0.11</td>
<td>1.04</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>reverse</td>
<td>-0.01</td>
<td>1.02</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>normal</td>
<td>reverse</td>
<td>-1.72</td>
<td>1.03</td>
</tr>
<tr>
<td>average</td>
<td></td>
<td></td>
<td></td>
<td>-0.21 ± 0.52</td>
<td></td>
</tr>
<tr>
<td>average of both foils</td>
<td></td>
<td></td>
<td></td>
<td>-0.53 ± 0.37</td>
<td></td>
</tr>
</tbody>
</table>

The results of the above measurements indicate no asymmetries in the counter systems or catcher foil that are inconsistent with the statistical accuracy of the individual
runs. The average of both of the results gives a net extraneous asymmetry of -0.53%, a result well within the 1% accuracy desired.

These measurements were repeated both with the magnetic field on and in the reverse direction to determine the effects of the magnetic field on the accuracy of the measurement. The results are presented in the following table.

<table>
<thead>
<tr>
<th>Stopping Foil</th>
<th>Field Direction</th>
<th>Counter Position</th>
<th>Foil Position</th>
<th>Asymmetry A' (%)</th>
<th>Error A' (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mylar</td>
<td>right</td>
<td>normal</td>
<td>normal</td>
<td>-0.27</td>
<td>1.19</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>normal</td>
<td>-1.27</td>
<td>1.19</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>reverse</td>
<td>+2.50</td>
<td>1.22</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>normal</td>
<td>reverse</td>
<td>+1.34</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>average</td>
<td>+ 0.58 ± 0.60</td>
<td></td>
</tr>
<tr>
<td>aluminum</td>
<td>right</td>
<td>normal</td>
<td>normal</td>
<td>+0.15</td>
<td>1.18</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>normal</td>
<td>+2.28</td>
<td>1.20</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>reverse</td>
<td>+0.56</td>
<td>1.23</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>normal</td>
<td>reverse</td>
<td>-0.10</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>average</td>
<td>+0.86 ± 0.60</td>
<td></td>
</tr>
<tr>
<td>average of both foils</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mylar</td>
<td>left</td>
<td>normal</td>
<td>normal</td>
<td>-2.70</td>
<td>1.12</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>normal</td>
<td>-0.92</td>
<td>1.11</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>reverse</td>
<td>+0.02</td>
<td>1.11</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>normal</td>
<td>normal</td>
<td>-2.27</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>average</td>
<td>-1.47</td>
<td>0.56</td>
</tr>
<tr>
<td>aluminum</td>
<td>left</td>
<td>normal</td>
<td>normal</td>
<td>-0.42</td>
<td>1.10</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>normal</td>
<td>-0.80</td>
<td>1.12</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>reverse</td>
<td>reverse</td>
<td>-0.23</td>
<td>1.12</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>normal</td>
<td>reverse</td>
<td>-3.01</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>average</td>
<td>-1.12 ± 0.56</td>
<td></td>
</tr>
<tr>
<td>average of both foils</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results are presented in the following table.
The results of these measurements indicate that for each field condition, the average laboratory asymmetries for the two foils are equal within statistics. Since the asymmetry in the beta decay of recoils stopped in the aluminum can possibly include the effects of nuclear polarization, while that measured with the mylar should not, the overlap of the two results within uncertainties indicate the non-existence of beta-decay asymmetry in the scandium decay. This is true for both directions of the magnetic field.

The above results do, however, exhibit the effects of the magnetic field on the measurement. If the average asymmetry for both foils of each of the field-on conditions is compared to the average for the field-off condition, it can be seen that the asymmetry shifts by +1.30% when the field is on to the right and by -0.77% when the field is reversed. Thus the total effect of the magnetic field is to change the asymmetry by 2.07%.

It is evident from the measurements presented in this section that the presence of the magnetic field induces the major source of asymmetry unrelated to the existence of nuclear polarization.

Analysis of direct asymmetry measurements. The following procedure for analysis of the data tabulated above was developed by extending the method of Sugimoto, et al. In this procedure, all asymmetries other than that owing to
the possible nuclear polarization cancel. This procedure and analysis requires measurements of the type described above.

For each catcher foil and for each field condition, data were accumulated in four separate blocks, reversing the relative position of the counting assemblies and rotating the catcher foil as indicated in the last section. The relation between the counts per run observed in the right and left systems can be expressed in terms of the various asymmetries present in the system in the following manner:

\[(V-4) \quad T_r = \epsilon \lambda \eta \rho \ T_1\]

where \( \epsilon \) = the relative detection efficiency of the two counting systems
\( \lambda \) = the relative asymmetry in the absorption of the catcher foil
\( \rho \) = the relative asymmetry due to scandium beta-decay anisotropy
\( \eta \) = the relative asymmetry of all other sources.

Thus if all coefficients in (V-4) are unity, \( T_r = T_1 \). If this is not the case, then upon reversal of the counters only,

\[T_r = \frac{\lambda \eta \rho}{\epsilon} \ T_1\]

upon a rotation of the catcher foil, \( T_r = \frac{\eta \rho}{\epsilon \lambda} \ T_1 \), etc.
Forming the right-left count ratios, \( R_i = \frac{T_{ri}}{T_{li}} \) for each of the four possible combinations of counter and catcher positions leads to the following results.

<table>
<thead>
<tr>
<th>Counter Position</th>
<th>Foil Position</th>
<th>Ratio of Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal</td>
<td>normal</td>
<td>( R_1 = \epsilon \lambda \eta \rho )</td>
</tr>
<tr>
<td>reverse</td>
<td>normal</td>
<td>( R_2 = \lambda \eta \rho / \epsilon )</td>
</tr>
<tr>
<td>reverse</td>
<td>reverse</td>
<td>( R_3 = \eta \rho / \epsilon \lambda )</td>
</tr>
<tr>
<td>normal</td>
<td>reverse</td>
<td>( R_4 = \epsilon \eta \rho / \lambda )</td>
</tr>
</tbody>
</table>

If the following product is formed, the result is independent of counter-catcher asymmetries:

\[
(V-5) \quad R = R_1 \cdot R_2 \cdot R_3 \cdot R_4 = (\eta \rho)^4
\]

When the mylar catcher is used, the polarization is expected to be totally relaxed before detection, thus \( \rho = 1 \); but for a suitably chosen catcher foil, possibly aluminum, for example, polarization will be maintained and \( \rho \neq 1 \). The value of \( \eta \), however, for both catcher foils remains the same. Thus the ratio

\[
(V-6) \quad \frac{R_{al}^{1/4}(\text{aluminum})}{R_{my}^{1/4}(\text{mylar})} = \frac{(\eta \rho)^{4}(\text{aluminum})^{1/4}}{(\eta \rho)^{4}(\text{mylar})} = \rho_{al}
\]

where \( \rho_{al} \) is reduced by any symmetrical background contributions that may be present. Finally, the laboratory asymmetry \( A' \), characteristic only of the beta-decay anisotropy but diluted by background, is given by
\[ A' = \frac{\rho_{al} - 1}{\rho_{al} + 1} \]

It is shown in Appendix B that the error in \( A' \) is given by the following expression after the four asymmetry measurements are made with both a mylar and an aluminum foil:

\[ \Delta A' = \frac{\rho_{al}}{2(\rho_{al} + 1)^2} \left[ \sum_{i=1}^{8} \frac{1}{a_i} \text{(aluminum)} + \sum_{i=1}^{8} \frac{1}{a_i} \text{(mylar)} \right]^\frac{1}{2} \]

where the \( a_i \) represent the number of counts in the right or left detector in each of the measurements summarized in the tables of data presented in the preceding section.

The product factor \( R_i^k \) for the measurements with both foils for each of the three field conditions, calculated from equation (V-5), is presented in Table III. Equations (V-6), (V-7), and (V-8) are also evaluated to determine \( \rho_{al}, A', \) and \( \Delta A' \). The data presented show a small positive asymmetry for all three field conditions; but in view of the associated errors, the results are consistent with zero asymmetry.

The small values of \( A' \) observed with the field on can be the result of several effects. As will be seen later, a much larger than desirable relative background yield was present which, as can be seen in equation (V-1), can effectively dilute the value of \( A' \) for a given value of the scandium beta-decay asymmetry \( A \). Another possibility is
TABLE III
The Product Factor $R^4$
Calculated Asymmetries and Errors

<table>
<thead>
<tr>
<th>Foil</th>
<th>Field On--Right</th>
<th>Field Off</th>
<th>Field On--Left</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mylar</td>
<td>1.0116</td>
<td>0.9832</td>
<td>0.9771</td>
</tr>
<tr>
<td>Aluminum</td>
<td>1.0173</td>
<td>0.9959</td>
<td>0.9779</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.0057</td>
<td>1.0129</td>
<td>1.0070</td>
</tr>
<tr>
<td>$A'$</td>
<td>+ 0.28%</td>
<td>+ 0.64%</td>
<td>+ 0.35%</td>
</tr>
<tr>
<td>$\Delta A'$</td>
<td>± 0.83%</td>
<td>± 0.80%</td>
<td>± 0.76%</td>
</tr>
</tbody>
</table>
that the relaxation time of the nuclear polarization may be short compared to the scandium half-life in the particular stopping medium used. Since, as a result, many scandium decays would be detected from unpolarized nuclei, the magnitude of A, and therefore of the observed A', would be less than indicative of an unrelaxed nuclear polarization. Finally, such a small observed effect may result because the net nuclear polarization may itself be substantially smaller than the value at the time of formation, estimated in Chapter I to be 8%.

However, it is of interest to proceed with the analysis of the measured asymmetry, both to illustrate the procedure that would be followed had the value of A' been substantially different from zero, and to gain information concerning the restrictions on the magnitudes of the relaxation time $\tau_r$ and the initial polarization $P_0$ imposed by the accuracy of the measurement.

Since a value of A' resulting from a genuine polarization effect would be independent of the direction of the magnetic field, the two results given in Table III for the field-on conditions may be averaged to give

$$A' = +0.32\% \pm 0.56\%$$

Equations (V-2) and (V-3) can now be used to calculate the corresponding values of the asymmetry and its uncertainty, $A \pm \Delta A$, in the scandium beta decay. The quantity
$R_s$ appearing in (V-2) is the fraction of the total measured yield due to the $^{41}$Sc decay and, for this measurement, has the value 0.37. The value was determined by analysis of the rate of decay of radiations from the target after activation, as will be described below. The values that result are:

$$A \pm \Delta A = +0.86% \pm 1.65\%$$

or equivalently:

$$-0.79\% \leq A \leq +2.51\%$$

which is, of course, still consistent with $A = 0$.

The above data cannot predict the values of $P_o$ and $\tau_r$ individually, but do limit the combinations of $P_o$ and $\tau_r$, through equation (V-9) below, to values consistent with the above results. The relationship between the beta asymmetry $A$, the initial recoil polarization $P_o$, and the relaxation time $\tau_r$, including the effects of beam switching, is as follows:

(V-9) \[ A = P_o (1+\cos \alpha) \left( \frac{\lambda}{\lambda'} \right)^2 \frac{1-e^{-\lambda'T}}{1-e^{-\lambda\tau}} \frac{1-e^{-\lambda'T}}{1-e^{-\lambda'\tau}} \frac{e^{-\lambda't_1}}{e^{-\lambda't_2}} \frac{e^{-\lambda't_1}}{e^{-\lambda't_2}} \]

where $\lambda' = \lambda + 1/\tau_r$, and the other parameters involved have been previously defined in Chapter II. This expression is derived in Appendix A.
This equation can be evaluated to determine the sets of values of \((\tau_r, P_o)\) consistent with the values of the asymmetry \(A\) determined in these measurements. The results of this evaluation are given in Figure 19. Any point \((\tau_r, P_o)\) lying outside the limits defined by the two curves is excluded by the experimental results.

A further restriction may be placed on the set of values of \((\tau_r, P_o)\) allowed by the experimental data by simply excluding values of \(P_o\) greater than 8\%, in accordance with the predictions given in Chapter I.

While \(A'\) was measured to the stipulated value of uncertainty of 1\%, the presence of a larger than desirable component of background led to an uncertainty in \(A\) which, in turn, offers little restrictions on the allowed set \((\tau_r, P_o)\). For example, a polarization as high as 6\% with a relaxation time as long as 250 ms would give a result consistent with zero asymmetry. Or, if the relaxation time were long (\(\sim 1\) sec), polarizations as large as 2.5\% would still be consistent with \(A = 0\). Thus the measurement, as it stands, cannot resolve the question of existence of nuclear polarization.

If it is recognized that the large component of background is the source of weakness in the measurement, then efforts can certainly be made to improve it. If the level were reduced to the value of 45\% of the total yield as achieved in the experiments discussed in Chapter IV, then
FIGURE 19

Probable values of the relaxation time $\tau_r$ and initial polarization $P_o$. Any point $(\tau_r, P_o)$ lying within the limits indicated is consistent with the data. The dashed lines at $\pm 8\%$ represent the maximum value of $P_o$ as estimated in Chapter I.
the results, for the same magnitude of scandium beta yield, would have been:

\[-0.43\% \leq A \leq +2.15\%\]

which would limit the polarization to about 5% at a relaxation time of 250 ms and 2% for longer relaxation times.

The source of the large background in this particular measurement is attributed to an excessive buildup of contaminants on the target during the long period of time (about 20 hours of run time) preceding the actual measurements, during which many preliminary experiments and checks were being performed. The remedy for this difficulty is to either change to fresh targets frequently (for example after each series of measurements using a mylar and a metal foil), or to reposition the target such that a fresh portion can be exposed to the deuteron beam at regular intervals. With such a precaution, it is believed that the relative background level can be reduced still further than 45% of the total yield, thus again improving the degree of restrictions placed on the values of the relaxation time and initial polarization.

It is clear from the above experimental results that, even if the background level is reduced, experimental circumstances must be improved if an asymmetry \(A'\) as large as the objective of 2% stated in Chapter I is to be achieved. Since the value of \(P_o\) cannot be changed, the only recourse
is to seek recoil stopping materials in which the relaxation time might be more favorable. Indeed, for the purpose of an eventual measurement of the scandium magnetic moment, only the existence of an asymmetry clearly different from zero is required. Thus the direction of effort in future studies must be toward finding materials that do yield a net asymmetry in the scandium beta decay of this magnitude.

Assuming that in some such material a net asymmetry \( A \geq 2\% \) could be established suitable for the magnetic moment measurement, then it would also be of interest to determine the actual polarization of the implanted nuclei and the relaxation time in this medium. The procedure of measurement by which this could be accomplished is described in the following section. Since aluminum showed no asymmetry different from zero, another foil, namely gold, was used to illustrate this method.

The measurement of the polarization relaxation. A measurement of the time dependence of the experimental asymmetry may be made to establish the existence of polarization by observing a relaxation of the asymmetry with time. This may be accomplished by the simultaneous operation of two multiscalars, each accumulating data from the counting system on one side of the target. The same procedure used in the measurements of the decay curves described in preceding chapter was followed in obtaining these results. Each multiscalar channel has a variable
dwell time; data are accumulated over a total time interval which can also be varied. From these data, the asymmetry $A'$ can be calculated channel by channel as a function of time. Though it requires a longer run time than in the previous set of measurements, this method is relatively insensitive to long-term fluctuations in instrumental or extraneous asymmetries.

For the relaxation measurements carried out with the gold catcher foil, the sequence of operation was to bombard the target for one-half second, then measure the decay radiations, with a dwell time of 8 ms, over the subsequent three and one-half seconds with the beam switched off. This cycle was repeated over a period of 18 hours, limited only by the practical necessities with respect to available accelerator time. The measurements furnished, however, a quantity of data minimally sufficient to exhibit any time dependence of the asymmetry.

The data gathered in this manner for both the right and left counting systems are exhibited in the two upper panels of Figure 20. The curves show the total contribution from the two short-lived components, the $^{41}$Sc and $^{38}$K* decay radiations, and from an essentially constant background. In addition, assuming polarization of the scandium nuclei, they contain an enhancement of the scandium beta yield in one counting system and a suppression of the yield in the other, with respect to the yield that
FIGURE 20

Decay curves in the right and left counting systems measured in the determination of the time dependence of the asymmetry. The fit to the data indicated in each of the upper graphs is one-half of that resulting from analysis of the sum of both sets of data shown in the lower graph.
would be present if the scandium nuclei were unpolarized. This enhanced or suppressed component of the yield will decay with the relaxation time $r$. In order to determine the relative amounts of betas or other radiations from $^{41}$Sc, $^{38}$K*, and background in the measured yields, the sets of data from the right and left counting systems were combined by adding the yields channel by channel, and the result of this operation is exhibited in the lower panel of Figure 20. In so doing, any time-dependent asymmetries in the data associated with a possible polarization of the scandium nuclei are canceled to first order. The fraction of the initial yield contributed by the decay of $^{41}$Sc is 37%, a value once again smaller than desired, but resulting, in this case, because of the use of the same contaminated target as in the previous set of measurements.

The counting statistics per channel of the data exhibited in Figure 20 were too poor to justify a calculation of the asymmetry A' for each channel using equation (V-1). Instead, A' was calculated after summing data contained within every ten channels, thus changing the time resolution of A' from 8 ms per point to 80 ms per point, still sufficient to exhibit any time dependence of A', provided, of course, that the relaxation time is on the order of or greater than 80 ms. The results of this reduction of the data is presented in Figure 21.
FIGURE 21

The time dependence of the experimental asymmetry calculated after summing every ten channels of the data presented in Figure 20. The fit is that which results in a minimum in Chi-square.
The data are seen to scatter about a net negative asymmetry of about 4%, with possibly a trend toward a value of half this magnitude in the first 250 ms. The fact that this extraneous or chamber asymmetry is not zero reflects the less stringent beam collimating condition, effected in the interest of increasing the yield per unit time. This non-zero net asymmetry does not imply any inaccuracy in the results, but simply determines a new reference level with respect to which the time-dependent changes in the asymmetry are measured.

The behavior of the data during the first few channels, while suggestive of a time-dependence of the asymmetry, is statistically too inconclusive to establish this effect. Once again, it is of interest to proceed with the analysis of these data as if the evidence for a time dependence of A' were more explicit in order to demonstrate the method by which the values of $P_0$ and $\tau_r$ can be extracted.

In order to analyze these data for the polarization and relaxation time of scandium nuclei in the gold catcher foil, it is necessary to incorporate these quantities in the analytical description of the right and left decay curves. In addition, of course, the $^8K^*$ and other background contributions must also be included in this description.

The required expressions are derived in Appendix A, and are explicitly for the right and left scandium yields, $S_r$ and $S_l$, equal to:
\[ S_r(t) = \frac{T R_s e^{\lambda R t}}{2} \left[ e^{-\lambda S (t-t_0)} + P(t_0, P_0) e^{-\lambda S (t-t_0)} \right] (1+AS) \]

\[ S_l(t) = \frac{T R_s e^{\lambda R t}}{2} \left[ e^{-\lambda S (t-t_0)} - P(t_0, P_0) e^{-\lambda S (t-t_0)} \right] (1-AS) \]

where \( T \) is the total yield at time \( t = t_0 \), the beginning of the counting period, \( R_s \) is the fraction of the total yield contributed by scandium at this time, \( P(t_0, P_0) \) is the net nuclear polarization remaining at time \( t_0 \), and \( AS \) is the total asymmetry in the scandium decay owing to effects other than the nuclear polarization; and for the right and left \(^{38}K^*\) yields, \( P_r \) and \( P_l \), equal:

\[ P_r(t) = \frac{T R_P e^{\lambda P t}}{2} e^{-\lambda P (t-t_0)} (1+AP) \]

\[ P_l(t) = \frac{T R_P e^{\lambda P t}}{2} e^{-\lambda P (t-t_0)} (1-AP) \]

where \( R_P \) is the fraction of the total yield contributed by potassium, and \( AP \) is the total asymmetry in the potassium decay; and for the right and left background yields, \( B_r \) and \( B_l \), equal to:

\[ B_r(t) = \frac{T R_B e^{\lambda B t}}{2} e^{-\lambda B (t-t_0)} (1+AB) \]

\[ B_l(t) = \frac{T R_B e^{\lambda B t}}{2} e^{-\lambda B (t-t_0)} (1-AB) \]

where \( R_B \) is the fraction of the total yield contributed by background, and \( AB \) is the total asymmetry in the background.
decays. In the case of the background radiation contribution, a time-dependent term has been included to allow for the possibility of a slow rate of decay characteristic of one of the longer-lived sources of background, for example the positrons from the decay of $^{17}F$ ($\tau_{\frac{1}{2}} = 66$ sec).

Of the parameters appearing in the foregoing expression, $T$, $R_s$, $R_p$, and $R_d$ have been determined in the decay-curve analysis described above. The remaining parameters $A_s$, $A_p$, and $A_B$ describing the asymmetries arising from effects other than owing to the polarization from the scandium can be determined from the long term behavior of the decay-curves after a time in which the nuclear polarization is fully relaxed. Further, it is physically plausible to assume that all three of these parameters exhibit the same behavior, thus can be set equal. In application to the above data shown in Figure 20, the analysis lead to a background asymmetry of 3.5%.

Then the time dependence of the asymmetry is expressed in terms of the above relations by:

$$A'(t) = \frac{S_r + \frac{P_r}{r} + B_r - S_1 - \frac{P_1}{r} - B_1}{S_r + \frac{P_r}{r} + B_r + S_1 + \frac{P_1}{r} + B_1}$$

To complete the analysis for the values of $P_o$ and $\tau_r$, a grid search was conducted by varying $\tau_r$ and $P_o$ after fixing $A_s$, $A_p$, and $A_B$ at the above value. Because the data in the first 48 channels showed some dependence of $A'$ on
time, only the data in these channels were included in the search.

The Chi-square function is defined by:

$$\chi^2 = \sum_{i=1}^{N} \frac{(A'(t) - A'_i)^2}{\Delta A'_i^2}$$

$$\Delta A'_i = \left[ \frac{T_1 + T_2}{T_1 T_2} \right]^{-\frac{1}{2}}$$

where $A'(t)$ = the calculated value of the asymmetry,

$A_i$ = the corresponding experimental value, and

$N$ = the number of data points.

A minimum in Chi-square was found at $\tau = 80$ ms and $P_o = 14\%$. However no physical significance is attached to these results; they are presented in Figure 21 as an illustration only. It can be pointed out, however, that these results are again positive for the polarization and are not incompatible with the data taken with the aluminum catcher foil. But this can only be viewed as suggestive of an effect in the data and not as evidence for it, and one must conclude that the existence of polarization has not been established.
B. Summary

The investigations of the sources of asymmetry in the beta decay of scandium recoil nuclei have been carried to a point where certain refinements in the experimental technique are necessary in order to establish evidence for or against the existence of nuclear polarization. This will require a much more extensive investigation of a variety of catcher foil materials where physical properties are such as to allow relaxation times in excess of 100 ms.

The measurements carried out so far have been crippled by an excessive level of background, a situation not without remedy as already pointed out. In addition to a frequent changing of targets, it would very likely be advisable to also change catcher foils at appropriate intervals to alleviate the possibility of loss of polarization due to radiation damage in the stopping medium.

Further efforts to reduce the effects of relaxation mechanisms pointed out in Chapter I include the possibilities of cooling the stopping area, and the use of highly-pure single crystals as catcher foils. In addition, the strength of the externally applied magnetic field may be increased in view of the possible loss of polarization of the recoils in flight due to a Larmor precession resulting from an inadequate decoupling of the nuclear spin I from the atomic spin J.
APPENDIX A

Calculation of Experimental Asymmetries

The quantity observed experimentally in the measurement of the beta-decay asymmetry of $^{41}$Sc recoil nuclei was defined in Chapter II as:

$$A = \frac{S_r - S_l}{S_r + S_l} \times \left[ S_r + S_l \right]^{-\frac{1}{2}}$$

where $S_r$ and $S_l$ are the counts per run in the right and left beta-counting systems respectively. To relate this asymmetry to the nuclear polarization $P$, we must integrate the beta intensity distribution discussed in Chapter I over the finite angular range of the counting systems. This angular range, as shown in Figure 8, is approximated by a cone of half-angle $\alpha$.

1. Integration of the Intensity Distribution

The intensity distribution is:

$$I(\theta) = I_0 \left( 1 + P \cos \theta \right)$$

where $I_0$ is the total number of betas emitted per steradian during a given run, that is:

$$I_0 = \frac{\gamma_o}{4\pi}$$

where $\gamma_o$ is the total beta yield per run.

Performing the integration, we have:

$$S_r = \int_0^\alpha I(\theta) d\alpha$$
\[ S_{\theta} = \frac{Y_0}{2} \left[ 1 - \cos \alpha - P(\cos^2 \alpha - 1) \right] \]

\[ S_{\phi} = \int_{180^\circ}^{180^\circ} I(\theta) \sin \theta \, d\theta \]

\[ = \frac{Y_0}{2} \left[ 1 - \cos \alpha + P(\cos^2 \alpha - 1) \right] \]

Then the asymmetry becomes:

\[ A = \frac{S_{\phi} - S_{\theta}}{S_{\theta} + S_{\phi}} \]

\[ = \frac{\frac{Y_0}{2} \left[ 1 - \cos \alpha - P(\cos^2 \alpha - 1) - 1 + \cos \alpha - P(\cos^2 \alpha - 1) \right]}{\frac{Y_0}{2} \left[ 1 - \cos \alpha - P(\cos^2 \alpha - 1) + 1 - \cos \alpha + P(\cos^2 \alpha - 1) \right]} \]

\[ = P(1 + \cos \alpha) \]

And the error in the asymmetry is:

\[ \Delta A = \left[ S_{\phi} - S_{\theta} \right]^{-\frac{1}{2}} \]

\[ = \left\{ \frac{Y_0}{2} \left[ 1 - \cos \alpha - P(\cos^2 \alpha - 1) + 1 - \cos \alpha + P(\cos^2 \alpha - 1) \right] \right\}^{-\frac{1}{2}} \]

\[ = \left[ \frac{Y_0}{2} (1 - \cos \alpha) \right]^{-\frac{1}{2}} \]

2. Beam-pulsing and Relaxation Corrections

Two additional facts must now be considered. The first is that after a given recoil is produced, its polarization relaxes exponentially with a lifetime \( \tau_r \) as given by:

\[ P = P_0 e^{-t/\tau_r} \]

where \( P_0 \) is the initial value of the nuclear polarization.

A second correction that must be made is due to the fact that the beam is being alternately switched in con-
juncture with the counting systems. Thus a relationship must be derived showing the dependence of \( A \) and \( \Delta A \) upon these factors. The following diagram will be used in the derivation.

Where:

\[ T_r = \text{time at end of } n^{th} \text{ beam-on cycle,} \]

\[ \tau_r = \text{time at end of } n^{th} \text{ beam-off cycle,} \]

\[ \tau = \text{total cycle time,} \]

\[ T = \text{total bombardment time,} \]

\[ t_1 = \text{time between beam off and counter on, and} \]

\[ t_2 - t_1 = \text{counting time each cycle.} \]

Now define the following quantities:

\[ Y(t') = \text{the number of radioactive nuclei formed between } t' \text{ and } t' + dt'. \] This quantity can be assumed constant over all values of \( t' \).

\[ Y(t) = \text{the number radioactive nuclei left of those formed at } t' \text{ at time } t > t'. \]
If $\lambda$ is the decay constant of the radioactive nucleus, then:

$$\gamma(t) = \gamma(t') e^{-\lambda(t-t')}$$

The beta yield per unit time in $t$, resulting from those nuclei formed at $t'$ is:

$$\frac{d\gamma(t)}{dt} = \lambda \gamma(t') e^{-\lambda(t-t')}$$

The beta yield in the right counting system between $t'$ and $t$ due to $\gamma(t')$ formed at $t'$ is:

$$\int_{t'=t^*}^{t} \lambda \gamma(t') e^{-\lambda(t-t')} \left[ 1 - \cos \alpha + \rho e^{-(t-t')/\tau} (1 - \cos^2 \alpha) \right] dt$$

The total beta yield due to all nuclei formed in the $n$th cycle between $t' = \tau_{n-1}$ and $t' = T_n$ but counting from $t = T_n + T_2$ to $t = T_n + T_2$ is:

$$\int_{t'=\tau_{n-1}}^{t=T_n} \int_{t'=\tau_{n-1}}^{t=T_n+T_2} \lambda \gamma(t') e^{-\lambda(t-t')} \left[ 1 - \cos \alpha + \rho e^{-(t-t')/\tau} (1 - \cos^2 \alpha) \right] dt \, dt'$$

$$= \int_{t'=t}^{t=T_n} \int_{t'=t}^{t=T_n+T_2} I(t',t) \, dt \, dt'$$

Now the total observed beta yield over the $n$th cycle must also include all $\gamma(t')$ formed during previous cycles. If we assume that the reaction is at equilibrium, (i.e. the average number of nuclei formed per unit time equals the
average number decaying per unit time) then the total yield \( \eta(\omega) \) observed during the \( n \)th cycle is the following sum:

\[
\eta(\omega) = \int_{\tau_{n-1}}^{T_n} \int_{T_n + \tau_1}^{T_{n+\tau_2}} I(t', t) \, dt \, dt' + \int_{\tau_{n+\tau_2}}^{T_{n+\tau_3}} \int_{T_n + \tau_2}^{T_{n+\tau_3}} I(t', t) \, dt \, dt'
\]

+ \cdots

\[
= \sum_{k=0}^{\infty} \int_{\tau_{n-k}}^{T_{n-k}} \int_{T_{n-k} + \tau_1}^{T_{n-k} + \tau_2} I(t', t) \, dt \, dt'
\]

Now recall that \( Y(t') = \) a constant which we shall call \( Y_0 \). Or we may define:

\( Y_0 = \) the number of radioactive nuclei formed per unit time while the beam is on target.

We now have:

\[
\eta(\omega) = \sum_{k=0}^{\infty} \int_{t'}^{t} \int_{t}^{\frac{1}{\lambda + t'}} Y_0 e^{-\lambda(t'-t')/\tau_2} \left[ 1 - \cos \alpha + P_0 e^{-\frac{t-t'}{\tau_2}} (1 - \cos^2 \alpha) \right] \, dt \, dt'
\]

\[
= \frac{Y_0}{2} \sum_{k} \int_{t'}^{t} \int_{t}^{t'} \lambda e^{-\lambda(t'-t')/(1-\cos \alpha)} \, dt \, dt'
\]

+ \frac{Y_0}{2} \sum_{k} \int_{t'}^{t} \int_{t}^{t'} \lambda e^{-(\lambda + t'/\tau_2)} (t-t') P_0 (1 - \cos^2 \alpha) \, dt \, dt'

Let \( \lambda' = \lambda + 1/\tau_2 \)

\[
\eta(\omega) = \frac{Y_0}{2} \sum_{k} \int_{t'}^{t} \int_{t}^{t'} \lambda e^{-\lambda(t'-t')/\tau_2} \, dt \, dt'
\]

+ \frac{1}{\lambda'} P_0 Y_0 (1 - \cos^2 \alpha) \sum_{k} \int_{t'}^{t} \int_{t}^{t'} \lambda' e^{-\lambda'(t'-t')} \, dt \, dt'
Now we must evaluate the following integral and infinite sum:

\[ \sum_{k=0}^{\infty} \int_{t' = \tau_{n-1-k}}^{T_n - k} \int_{t = \tau_n + t_1}^{t'} \lambda e^{-\lambda(t-t')} \, dt \, dt' \]

Integrating over t:

\[ \sum_{k} \int_{t'} e^{-\lambda(t'-t)} \bigg|_{t' = \tau_n + t_1}^{t'} \, dt' \]

\[ = \sum_{k} \int_{t'} e^{-\lambda(\tau_n + t_1 - t')} - e^{-\lambda(\tau_n + t_2 - t')} \, dt' \]

\[ = e^{-\lambda \tau_n} \left[ e^{-\lambda t_1} - e^{-\lambda t_2} \right] \sum_{k} \int_{t'} \tau_{n-1-k} e^{\lambda t'} \, dt' \]

Integrating over t':

\[ e^{-\lambda \tau_n} \left[ e^{-\lambda t_1} - e^{-\lambda t_2} \right] \sum_{k} \frac{1}{\lambda} e^{-\lambda t'} \int_{\tau_{n-1-k}}^{T_n - k} \]

\[ = \frac{1}{\lambda} e^{-\lambda \tau_n} \left[ e^{-\lambda t_1} - e^{-\lambda t_2} \right] \sum_{k} \left( e^{-\lambda(\tau_n - k)} - e^{-\lambda(\tau_{n-1-k})} \right) \]

\[ = \frac{1}{\lambda} \left[ e^{-\lambda t_1} - e^{-\lambda t_2} \right] \sum_{k} \left[ e^{-\lambda(T_n - \tau_{n-k})} - e^{-\lambda(T_n - \tau_{n-1-k})} \right] \]

Now note that:

\[ T_n - \tau_{n-1-k} = T + k \tau \]

\[ T_n - \tau_{n-k} = k \tau \]

Thus the infinite sum becomes:

\[ \sum_{k=0}^{\infty} \left[ e^{-\lambda k \tau} - e^{-\lambda(T + k \tau)} \right] \]
\[
(1 - e^{-\lambda T}) \sum_{k=0}^{\infty} e^{-\lambda k T}
\]

This sum is merely a geometrical progression. The ratio term is:

\[
r = \frac{e^{-\lambda (k+1) T}}{e^{-\lambda k T}} = e^{-\lambda T}
\]

The first term is \(a = 1\) and the number of terms is \(p = m+1\). Then the sum is:

\[
a \cdot \frac{r^p - 1}{r - 1} = \frac{1 - (e^{-\lambda T})^{m+1}}{e^{-\lambda T} - 1} = \frac{e^{-\lambda (m+1) T} - 1}{e^{-\lambda T} - 1}
\]

As \(m \to \infty\) the infinite sum approaches:

\[
(1 - e^{-\lambda T}) \cdot \frac{1}{e^{-\lambda T} - 1} = \frac{1 - e^{-\lambda T}}{1 - e^{-\lambda T}}
\]

Thus the integral is:

\[
\frac{1}{\lambda} \left[ e^{-\lambda t_1} - e^{-\lambda t_2} \right] \frac{(1 - e^{-\lambda T})}{(1 - e^{-\lambda T})}
\]

Substituting this result into the original expression for the total beta yield in the right counter:
\[ \eta(\infty) = \frac{V_o (1-cos\theta)}{\lambda} \left( \frac{1 - e^{-\lambda \tau}}{1 - e^{-\lambda T}} \right) (e^{-\lambda \eta_r} - e^{-\lambda \eta_i}) + \frac{P_o V_o (1 - cos^2 \theta)}{2} \frac{\lambda}{\lambda'} \left( \frac{1 - e^{-\lambda' \tau}}{1 - e^{-\lambda' T}} \right) (e^{-\lambda' \eta_r} - e^{-\lambda' \eta_i}) \]

Or:

\[ \eta(\infty) = \frac{V_o}{2} \left[ B + C \right] \quad \text{(right counter)} \]

= \eta_r

A similar calculation for the left counter yields:

\[ \eta(\infty) = \frac{V_o}{2} \left[ B - C \right] \quad \text{(left counter)} \]

= \eta_i

Now if \( p \) is the number of counting cycles during a given run, we can calculate the total yields in the two sets of counters in the following manner. First let:

\[ \eta_r = \text{number of betas observed in the right counter per cycle, and} \]

\[ p \eta_r = \text{number of betas observed in the right counter per run.} \]

If \( Y_o \) is the number of nuclei formed per unit time during bombarding periods, then:

\[ Y_o T = \text{number of nuclei formed (and recoiled onto the catcher foil) per cycle} \]

\[ p Y_o T = \text{number of nuclei formed per run} \]

= \( Y_f \)
Now: \( Y_o = \eta_r \frac{2}{B+c} = \eta_l \frac{2}{B-c} \)

and: \( Y_f = Y_o \rho T = \eta_r \rho T \frac{2}{B+c} = \eta_l \rho T \frac{2}{B-c} \)

But: \( S_r = \rho \eta_r \) and \( S_l = \rho \eta_l \)

Thus: \( Y_f = S_r T \frac{2}{B+c} = S_l T \frac{2}{B-c} \)

and: \( S_r = \frac{Y_f}{2T} (B+C) \)

\( S_l = \frac{Y_f}{2T} (B-C) \)

The expression for the asymmetry then becomes:

\[
A = \frac{S_r - S_l}{S_r + S_l} = \frac{Y_f}{2T} \frac{(B+C - B+C)}{(B+C + B-C)}
\]

\[
= \frac{Y_f}{2T} \frac{(B+C - B-C)}{(B+C + B-C)}
\]

\[
= \frac{C}{B} = P_0 (1+\cos \alpha) \left( \frac{\Lambda}{\Lambda'} \right)^2 \frac{(1-e^{-\Lambda' T}) (1-e^{-\Lambda T})}{(1-e^{-\Lambda T}) (1-e^{-\Lambda' T})} \frac{(e^{-\Lambda' T} - e^{-\Lambda T})}{(e^{-\Lambda T} - e^{-\Lambda' T})}
\]

The expression for the error in \( A \) becomes:

\[
\Delta A \propto \left[ S_r + S_l \right]^{-\nu_1}
\]

\[
= \left[ \frac{Y_f}{2T} (B+C + B-C) \right]^{-\nu_2} = \left[ \frac{Y_f}{2T} B \right]^{-\nu_2}
\]

\[
= \left[ \frac{Y_f}{2} \frac{(1-\cos \alpha)}{\Lambda} \frac{(1-e^{-\Lambda T})}{(1-e^{-\Lambda' T})} (e^{-\Lambda T} - e^{-\Lambda' T}) \right]^{-\nu_2}
\]
It can be shown (Do 70) that if a deuteron beam is incident upon a $^{40}$Ca target that the number of nuclei recoiled during a given run can be expressed in the following manner:

$$Y = \frac{1}{2.66 \cdot 10^{-7}} \frac{\sigma(\theta, E)}{A} Q \Omega t \left[ f_b(E_0) f_{re}(E_r, \varepsilon) \right]$$

where:

$\sigma(\theta, E) = \text{lab. differential cross section, which}$

$\text{depends upon the recoil lab. angle } \theta \text{ and the}$

$\text{beam energy } E, \text{ expressed in millibarns per}$

$\text{steradian,}$

$t = \text{target thickness expressed in milligrams per}$

$\text{square centimeter,}$

$f_b = \text{fraction by which the detected beta yield is}$

$\text{reduced by the setting of a baseline. This}$

$\text{fraction depends upon the endpoint energy } E_0$

$\text{of the beta decay,}$

$A = \text{atomic weight of target nuclei,}$

$Q = \text{integrated beam current per run expressed in}$

$\text{microcoulombs,}$

$\Omega = \text{recoil solid angle in steradians, and}$

$f_{re} = \text{recoil efficiency which depends upon the recoil}$

$\text{energy } E_r \text{ and the target thickness.}$

The magnitudes of the above quantities are discussed in the text.
3. Optimization of Pulsing Parameters

As previously mentioned in the description of the procedures of the various experiments of this work, it was desirable to optimize the production of scandium during a given time period by proper adjustment of the timing parameters. More specifically, this involved increasing the beta yield per unit time for a given maximum beam intensity. The total beta yield per run is given by:

\[ S_b + S_f = \frac{\varphi f}{\tau} \frac{(1 - \cos \omega)}{\lambda} \frac{(1 - e^{-\lambda T})}{(1 - e^{-\lambda \tau})} (e^{-\lambda \tau_1} - e^{-\lambda \tau_2}) \]

\( \varphi f \) depends upon the rate at which the target is bombarded, or:

\[ \varphi f = \text{constant} \cdot Q \]

\[ = \text{constant} \cdot I \cdot \frac{T}{\tau} \cdot \text{run-time} \]

where \( I = \) the steady-state (unpulsed) beam intensity. Thus the quantity to be maximized is:

\[ \frac{S_b + S_f}{\text{time}} = \text{constant} \cdot \frac{IT}{\tau} \frac{(1 - \cos \omega)}{\lambda T} \frac{(1 - e^{-\lambda T})}{(1 - e^{-\lambda \tau})} (e^{-\lambda \tau_1} - e^{-\lambda \tau_2}) \]

or merely:

\[ \sim \frac{1}{\tau} \frac{(1 - e^{-\lambda T})}{(1 - e^{-\lambda \tau})} (e^{-\lambda \tau_1} - e^{-\lambda \tau_2}) \]

\( = \) Yield Factor

Two conclusions can be drawn immediately from this equation. First the delay \( t_1 \) between beam off and counters on should be as short as possible. With the transistorized
pulser this minimum setting was .0075 seconds. The parameter $t_2$ can range from $t_1$ to within .0075 seconds of the full length of the beam-off period. Obviously the best value is that which is as long as possible. Thus:

$$t_2 = \bar{t} - T - 0.0075 \text{ (sec)}$$

With $t_1$ and $t_2$ fixed, $\tau$ and $T$ can be varied over the range of those times available with the pulser. In Figure 22 the Yield factor is evaluated, with $\tau$ varied continuously from 120 ms to 360 ms, and $T$ expressed as a fraction $f$ of $\tau$ ranging from 0.1 to 0.9. The results indicate that this fraction should be 0.5, and that $\tau$ should be as long as possible for a maximum yield per unit time.

To properly adjust the timing parameters, an investigation of the ratio $A/\Delta A$ was performed by evaluating the expressions derived in this appendix over all possible values of timing parameters. Again, $Y_f$ was expressed in terms of the time required for a run instead of in terms of the total charge collected. It was found that $A/\Delta A$ was maximized for the same conditions giving a maximum yield per unit time, except when $\tau_r << \tau_2$. In this situation $\Delta A \approx A$, and the asymmetry is essentially not measurable. No change in any of the pulsing parameters could produce such a change in $A/\Delta A$ as to render the experiment feasible. Therefore, the following parameters, which maximize the yield per unit time, were chosen as optimum:
FIGURE 22

The yield factor versus $\tau$ for various ratios

$T/\tau = f$. 
\[ \tau = 360 \text{ ms} \]
\[ T = 180 \text{ ms} \]
\[ t_1 = 7.5 \text{ ms} \]
\[ t_2 = 172.5 \text{ ms} \]

4. Evaluation of the asymmetry Expression

The dependence of the experimental asymmetry upon the relaxation time is illustrated in Figure 23, where the expression derived earlier is evaluated using the optimum pulsing parameters given above. In addition, the effects of the presence of background radiation on the magnitude of the asymmetry is shown by use of the following relation given in Chapter V:

\[ A' = A \cdot R_s \]

where \( A \) = the asymmetry in the scandium beta decay,
\( A' \) = the asymmetry when diluted by the addition of background radiation, and
\( R_s \) = the relative scandium beta yield per run.

The value of the asymmetry is based on an assumed initial nuclear polarization of 8\% (Chapter I).

5. Calculation of the Asymmetry Time-dependence

A special form of the asymmetry expression must now be derived in order to evaluate the results of the measurement its time dependence with the mechanical timer. This
FIGURE 23

The magnitude of the observed asymmetry $A'$ versus the ratio of the polarization relaxation time to the $^{41}$Sc beta-decay lifetime $\tau_1 = 1.44 \tau_\frac{1}{2}$. 
experiment, as described in Chapter VI, involved the use of
two multiscalar systems which measured the yield versus time over
a 3.54 second period on either side of the catcher foil
after a bombarding period of 0.442 seconds. From these
data the relaxation time of the polarization could be seen
by a calculation of the asymmetry channel by channel using:

\[ \mathcal{A}' = \frac{T_r - T_l}{T_r + T_l} \]

\[ \Delta \mathcal{A}' = \left( \frac{1}{T_r + T_l} \right)^{-\frac{1}{2}} \]

where \( T_r \) and \( T_l \) are the counts, including background, in
corresponding channels in the right and left multiscaling
systems. These data can be fit with an expression for the
observed asymmetry to determine \( P_0 \) and \( \tau_r' \). This expression
will now be derived.

The data in each multiscalar were analyzed using the
decay-curve analysis procedure described in Appendix C.
It was found that the total yield was composed of betas
from \(^{41}\text{Sc}\) and \(^{38}\text{K}^*\) plus an essentially constant background.
That the background is assumed constant can be justified by
noting that the major sources of background radiation, as
mentioned in Chapter IV, had half-lives on the order of a
minute or longer, therefore their contribution to the yield
can be assumed constant over the four second cycle period. It must also be noted that the halflives of the scandium and potassium decays are short compared to the length of the measuring period, thus their contributions to the yield are essentially zero near the end of the counting period.

The following two differential equations apply to either scandium or potassium.

(beam off) \[ \frac{dN(t)}{dt} = -\lambda N(t) \]

(beam on) \[ \frac{dN(t)}{dt} = Y - \lambda N(t) \]

where \( N(t) \) = the number of radioactive nuclei present at time \( t \),

\( Y \) = the number of radioactive nuclei formed per unit time during bombardment,

\( \lambda \) = the decay constant of the radioactive nucleus,

\( t = 0 \) is the beginning of the bombarding period, and,

\( t = t_o \) is the time of "beam off" and counters on.

The above equations have the following solutions.

\[ N(t) = N(t_o) e^{-\lambda (t-t_o)} \]  \hspace{1cm} (beam off)

\[ N(t) = C e^{-\lambda t} + \frac{Y}{\lambda} \]  \hspace{1cm} (beam on)

where \( C \) = a constant.
Now if \( N(0) = 0 \), as assumed above, we have:

\[
\text{at } t = 0 \quad 0 = C + \gamma / \lambda
\]

or

\[
N(t) = -\gamma / \lambda \ e^{-\lambda t} + \gamma / \lambda
\]

\[
= \gamma / \lambda (1 - e^{-\lambda t}) \quad \text{(beam on)}
\]

Now the beta yield per unit time is defined as the activity \( X(t) \):

\[
X(t) = \lambda N(t)
\]

Thus:

\[
X(t) = \gamma (t_0) e^{-\lambda (t - t_0)} \quad \text{(beam off)}
\]

\[
X(t) = \gamma (1 - e^{-\lambda t}) \quad \text{(beam on)}
\]

Now if \( Y(t') \) is the number of radioactive nuclei recoiled per unit time onto the catcher foil between \( t' \) and \( t' + dt' \), then at \( t > t' \), the total (infinitesimal) activity resulting from these (infinitesimal) amount of) recoils is:

\[
\text{\lambda times the number left at time t}
= \lambda \ Y(t') \ dt' \ e^{-\lambda (t - t_0)}
\]

More explicitly, in the right counting system the activity is:

\[
\lambda \ Y(t') \ dt' \ e^{-\lambda (t - t')/\kappa} \left[ (1 - \cos \omega) + P e^{-(t - t')/\kappa_0 (1 - \cos \omega)} \right]
\]

The actual activity at time \( t \) in the right counting system is then the integral of this quantity over all \( t' \), where \( t > t_0 \).
\[ X_r(t) = \int_{-\infty}^{t'} Y_{\epsilon}\ e^{-\lambda(t-\infty)} \ \left[ (1-\cos\alpha + P_0 e^{-(t-t')/r_0} (1-\cos\alpha) \right] dt' \]

Now, as before, \( Y(t') = Y \), a constant over all \( t' \).
Therefore:
\[
X_r(t) = \frac{\lambda Y}{2} \left[ (1-\cos\alpha) \int_0^{t_0} e^{-\lambda(t-t')} dt' \right.
+ P_0 (1-\cos^2\alpha) \int_0^{t_0} e^{-\lambda'(t-t')} dt' \]
\[ \left. \right] \]

where \( \lambda' = \lambda + 1/r_0 \).

Evaluating these integrals as before:
\[
X_r(t) = \frac{Y}{2} \left[ (1-\cos\alpha) (1-e^{-\lambda t_0}) e^{-\lambda(t-t_0)} \right.
+ P_0 \frac{\lambda}{\lambda'} (1-\cos^2\alpha) (1-e^{-\lambda' t_0}) e^{-\lambda'(t-t_0)} \]
\[ \left. \right] \]

This expression can be evaluated at \( t = t_0 \).
\[
X_r(t_0) = \frac{Y}{2} \left[ (1-\cos\alpha) (1-e^{-\lambda t_0}) + P_0 \frac{\lambda}{\lambda'} (1-\cos^2\alpha) (1-e^{-\lambda' t_0}) \right] \]
\[ \times \frac{Y}{2} (1-\cos\alpha) (1-e^{-\lambda t_0}) \left[ 1 + P_0 \frac{\lambda}{\lambda'} (1-\cos^2\alpha) (1-e^{-\lambda' t_0}) \right] \]
\[ = R(t_0)/2 \left[ 1 + P(t_0) \right] \]

where \( P(t_0) = \) the net polarization remaining at \( t_0 \), and
\( R(t_0) = \) the activity from the catcher foil at \( t_0 \).

Now define:
\( S_r(t) = \) scandium beta activity in the right counting system,
\( P_r(t) = \) potassium beta activity in the right counting system,
\( B_r(t) = \) background activity in the right counting system, etc.
Also define:

\( A_S = \) the chamber asymmetry of the scandium recoils,

\( A_P = \) the chamber asymmetry of the potassium recoils, and

\( A_B = \) the chamber asymmetry of the background.

Now the following yields can be written:

\[
S_r(t) = \frac{S(t_0)}{2} \left( e^{-\lambda_s (t - t_0)} + P_o(t_0) e^{-\lambda_s (t - t_0)} \right) \left( 1 + A_S \right)
\]

\[
S_f(t) = \ldots \quad \ldots \quad \ldots \quad \ldots
\]

\[
P_r(t) = \frac{P(t_0)}{2} \left( e^{-\lambda_P (t - t_0)} \right) \left( 1 + A_P \right)
\]

\[
P_f(t) = \ldots \quad \ldots \quad \ldots \quad \ldots
\]

\[
B_r(t) = \frac{B(t_0)}{2} \left( e^{-\lambda_b (t - t_0)} \right) \left( 1 + A_B \right)
\]

\[
B_f(t) = \ldots \quad \ldots \quad \ldots \quad \ldots
\]

Note that \( S(t_0) \) and \( P(t_0) \) include only activity built up during the preceding bombarding period, while \( B(t_0) \) includes activity produced over a large number of preceding cycles.

The laboratory asymmetry can now be expressed as a function of time in the following manner.

\[
A'(t) = \frac{S_r(t) + P_r(t) + B_r(t) - S_f(t) - P_f(t) - B_f(t)}{S_r(t) + P_r(t) + B_r(t) + S_f(t) + P_f(t) + B_f(t)}
\]

The activities at \( t_0 \) are precisely the results of the decay curve analysis performed on the sum of the two sets of data.

At this point a correction must be made to allow for the period of time \( t_1 \) between beam off and beam on. The
activities at \( t_o \) resulting from the decay curve analysis are really:

\[
\begin{align*}
S (t_o + t_1) &= \mathcal{T} R_S e^{\lambda_S t_1} \\
P (t_o + t_1) &= \mathcal{T} R_P e^{\lambda_P t_1} \\
B (t_o + t_1) &= \mathcal{T} R_B e^{\lambda_B t_1}
\end{align*}
\]

Then

\[
\begin{align*}
S (t_o) &= S (t_o + t_1) e^{\lambda_S t_1} = \mathcal{T} R_S e^{\lambda_S t_1} \\
P (t_o) &= P (t_o + t_1) e^{\lambda_P t_1} = \mathcal{T} R_P e^{\lambda_P t_1} \\
B (t_o) &= B (t_o + t_1) e^{\lambda_B t_1} = \mathcal{T} R_B e^{\lambda_B t_1}
\end{align*}
\]

where \( \mathcal{T} \) = the total yield (on both sides) at \( t = t_o + t_1 \),

\( R_S \) = the relative scandium beta yield at \( t = t_o + t_1 \),

etc.

Thus the individual yields may be written:

\[
S_b (t) = \mathcal{T} \frac{R_S}{2} e^{\frac{\lambda_S t_1}{2}} (e^{-\lambda_S (t-t_o)} + P(t_o) e^{-\lambda_S (t-t_o)}) (1 + A_S)
\]

etc.

where \( t \) is evaluated from \( t_o + t_1 \) to the end of the counting cycle.

The above expression for \( A'(t) \) is evaluated, as illustrated in Figure 24, for a typical experimental situation for relaxation times ranging from 100 to 1000 ms. The following parameter values were used, characteristic of the decay of scandium and potassium and a fluorine beta background at 6 MeV bombarding energy.

\[
\begin{align*}
R_S &= 0.70 & \lambda_S &= 1.16/\text{sec} & AS &= +0.02 & t_o &= 0.442 \text{ sec} \\
R_P &= 0.20 & \lambda_P &= 0.73/\text{sec} & AP &= +0.02 & t_1 &= 0.040 \text{ sec} \\
R_B &= 0.10 & \lambda_B &= 0.01/\text{sec} & AB &= -0.02 & 0.008 \text{ sec/channel}
\end{align*}
\]
FIGURE 24

Evaluation of the asymmetry time dependence.
APPENDIX B
Error Analysis

The expression for the experimentally observed asymmetry has been given as:

\[ A = \frac{S_\perp - S_\parallel}{S_\perp + S_\parallel} \]

The general rule for the calculation of the most probable error is:

\[ \Delta A = \left[ \left( \frac{\partial A}{\partial S_\perp} \Delta S_\perp \right)^2 + \left( \frac{\partial A}{\partial S_\parallel} \Delta S_\parallel \right)^2 \right]^{1/2} \]

However, since \( A \) is not dependent upon \( S = S_r + S_\perp \) we treat this quantity as a constant, and the error in \( A \) is:

\[ \Delta A = \left[ \left( \frac{\sqrt{S_\perp}}{S_\perp + S_\parallel} \right)^2 + \left( \frac{\sqrt{S_\parallel}}{S_\perp + S_\parallel} \right)^2 \right]^{1/2} \]
\[ = \left[ \frac{1}{S_\perp + S_\parallel} \right]^{1/2} \]

Now if a background level \( B \) in both counting systems is added, the experimentally observed quantity is \( A' \):

\[ A' = \left[ \frac{T_r - T_d}{T_r + T_d} \right] \quad ; \quad \Delta A' = \left[ \frac{T_r + T_d}{T_r - T_d} \right]^{-1/2} \]

where:

\[ T_r = S_r + \frac{1}{2} B \]
\[ T_d = S_d + \frac{1}{2} B \]

But \( B = R_b T \), where \( R_b \) is the relative magnitude of the background from the decay-curve analysis, and \( T = T_r + T_d \).
\[ S_r = T_r - \frac{1}{2} R_b T \]
\[ S_I = T_I - \frac{1}{2} R_b T \]

Then the individual errors are:
\[ \Delta S_r = \left[ \left( \frac{2S_r}{S_r} \Delta T_r \right)^2 + \left( \frac{2S_r}{S_I} \Delta T \right)^2 \right]^{1/2} \]
\[ = \left[ T_r + \frac{1}{4} R_b T \right]^{1/2} \]

and:
\[ \Delta S_I = \left[ T_I + \frac{1}{4} R_b T \right]^{1/2} \]

Substituting into the original expression for \( \Delta A \):
\[ \Delta A = \left[ \frac{T_r + \frac{1}{4} R_b T}{(S_r + S_I)^2} + \frac{T_I + \frac{1}{4} R_b T}{(S_r + S_I)^2} \right]^{1/2} \]

Now substitute:
\[ S_r + S_I = S = R_s \cdot T = R_s \left( T_r + T_I \right) \]

where \( R_s \) is the relative scandium yield. Thus:
\[ \Delta A = \left[ \frac{T_r + T_I + \frac{1}{2} R_b ^2 \left( T_r + T_I \right)}{R_s \left( T_r + T_I \right)^2} \right]^{1/2} \]
\[ = \left[ 1 + \frac{1}{4} R_b ^2 \frac{1}{R_s ^2} \right]^{1/2} \frac{1}{(T_r + T_I)} \]
\[ = \left[ 1 + \frac{1}{4} R_b ^2 \right]^{1/2} \Delta A' \]

Now recall that:
\[ \Delta A' = \left[ \frac{1}{T_r + T_I} \right]^{1/2} = \left[ \frac{1}{(S_r + S_I) / R_s} \right]^{1/2} \]
\[ = R_s^{1/2} \Delta A_{no} \]
where $\Delta A_{no}$ is the error if no background were present.

After letting $R_s = 1 - R_b$, the expression for $\Delta A$ becomes:

$$\Delta A = \left[ \frac{1 + \frac{1}{2} R_b^2}{1 - \frac{1}{2} R_b^2} \right]^{1/2} \Delta A_{no}$$

The actual asymmetry measurements described in Chapter V were made by determining the quantity $\rho$ which was defined as:

$$\rho = \left[ \frac{R_{(aluminum)}}{R_{(molybdenum)}} \right]^{1/4} \equiv Q^{1/4}$$

Each $R$ was the product of four count ratios:

$$R = \frac{a_1}{a_s} \cdot \frac{a_3}{a_4} \cdot \frac{a_5}{a_6} \cdot \frac{a_7}{a_8}$$

Thus:

$$\left( \frac{\Delta R}{R} \right)^2 = \frac{1}{\rho} \left( \sum_{i=1}^{q} \left( \frac{\Delta a_i}{a_i} \right)^2 \right)$$

$$= \frac{1}{\rho} \left( \sum_{i=1}^{q} \frac{1}{a_i} \right)$$

Then the error in $Q$ is:

$$\left( \frac{\Delta Q}{Q} \right)^2 = \left( \frac{\Delta R (A1)}{R (A1)} \right)^2 + \left( \frac{\Delta R (M1)}{R (M1)} \right)^2$$

$$= \frac{1}{\rho} \left( \sum_{i=1}^{q} \frac{1}{a_i} (\Delta a_i) \right) + \frac{1}{\rho} \left( \sum_{i=1}^{q} \frac{1}{a_i} (M1) \right)$$

The laboratory asymmetry was shown to be:

$$A' = \frac{\rho - 1}{\rho + 1}$$

$$= \frac{Q^{1/4} - 1}{Q^{1/4} + 1}$$
Therefore the error in the asymmetry is:

\[
\Delta A' = \frac{2A'}{2A} \Delta A
\]

\[
= \frac{(Q^{\nu_4} + 1)^{\nu_4} Q^{-3/4} - (Q^{\nu_4} - 1)^{\nu_4} Q^{-3/4}}{(Q^{\nu_4} + 1)^2} \Delta q
\]

\[
= \frac{\Delta q}{2Q^{3/4}(Q^{\nu_4} + 1)}
\]

\[
= \frac{Q^{\nu_4}}{2(Q^{\nu_4} + 1)^2} \frac{\Delta q}{Q}
\]

\[
= \frac{\rho}{2(\rho + 1)^2} \left[ \sum_{i=1}^{q} \frac{q_i}{a_i} (\Delta i) + \sum_{i=1}^{q} \frac{1}{a_i} (M_i) \right]^{1/2}
\]
APPENDIX C

Decay-curve Analysis

The measurements of this experiment involved the counting of the composite activity from several beta-emitting nuclei. It was necessary to determine the relative contributions of each of the constituents, more specifically to separate the 0.6 sec decay of $^{41}\text{Sc}$ from competing reactions and constant background. As described in Chapter III, measurements of the total radiation from an activated target were made using a 512 or a 2048 channel multiscalar to define a decay curve. Then the separation was accomplished by use of a standard least-squares fitting technique. (See, for example, Mathews and Walker, Mathematical Methods of Physics.)

A computer code was written to perform this analysis which basically consisted of fitting the composite decay curve with a linear combination of $n$ functions:

$$g_i = \sum_{i=1}^{n} a_i f_{i}$$

where

$$f_{i}(t) = \exp\left(-\lambda_i t_i\right)$$

Each of the functions $f_{i}$ represents the exponential decay of one of the constituents with decay constant $\lambda_i$. $t_i$ is the time of the $i^{th}$ counting period and $a_j$ is the initial yield amplitude of the $j^{th}$ component.

The most probable fit to $N$ data points is determined by requiring
\[ \sum_{i=1}^{N} \frac{(y_i - \bar{y}_i)^2}{\sigma_i^2} = \text{a minimum} \]

where \( \sigma_i = \sqrt{y_i} \) is the standard deviation of the \( i \)th data point. The adequacy of the fit could be assessed by calculating the value of Chi-square \( (X^2) \), which is defined:

\[ X^2 = \frac{1}{D} \sum_{i=1}^{N} \frac{(y_i - \bar{y}_i)^2}{y_i} \]

where \( D = \text{the number of degrees of freedom} = N - n \).

In this case, with \( N-n \gtrsim 500 \), the most probable fit would give \( X^2 = 1 \). Standard Chi-square tables were consulted to determine the probability of getting a \( X^2 \) that is different from 1.

Readily available in this analysis was the standard deviation or probable error in each of the initial yield amplitudes \( a_j \).

Most significant in this experiment was the use of this technique to separate the short-lived decay of the first excited state of \( ^{38}\text{K} \) \( (E_{\text{max}}(B^+) = 5.03 \text{ MeV}, \tau_{1/2} = 0.946 \text{ sec}) \) from that of \( ^{41}\text{Sc} \). Since the halflives of the other competing reactions were long compared to a few seconds, separation from the short-lived decays was relatively easy, taking only about 100 counts per channel in the first few channels. However, it was found that approximately 1000 counts per channel were necessary to accurately separate the 0.6 and 0.9 second decays.
To check the sensitivity or accuracy of the least squares technique for the determination of the relative contribution of the two short-lived beta emitters, a Monte Carlo analysis was performed. A random number generator was used to create a fictitious decay curve consisting of decay components believed to be present in the real experiment. A computer code was written that would calculate a 440 point decay curve, which would then be randomly distributed about its mean according to a gaussian probability. As a check for true randomness, a set of 2000 numbers was randomly distributed about a mean value of 100. These numbers were then grouped in units of \( \frac{1}{2} \sigma \) (\( \sigma = 10 \)) on either side of the mean and compared to a true gaussian distribution. The results are shown in the following table.

<table>
<thead>
<tr>
<th>Group</th>
<th>True Gaussian Positive</th>
<th>True Gaussian Negative</th>
<th>Actual Numbers Positive</th>
<th>Actual Numbers Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - ( \frac{1}{2} \sigma )</td>
<td>382.9</td>
<td>382.9</td>
<td>373</td>
<td>384</td>
</tr>
<tr>
<td>( \frac{1}{2} \sigma - \sigma )</td>
<td>300.2</td>
<td>300.2</td>
<td>307</td>
<td>309</td>
</tr>
<tr>
<td>( \sigma - 3/2 \sigma )</td>
<td>183.7</td>
<td>183.7</td>
<td>200</td>
<td>161</td>
</tr>
<tr>
<td>3/2 ( \sigma - 2\sigma )</td>
<td>88.1</td>
<td>88.1</td>
<td>91</td>
<td>95</td>
</tr>
<tr>
<td>2( \sigma - 5/2 \sigma )</td>
<td>33.1</td>
<td>33.1</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>5/2 ( \sigma - \infty )</td>
<td>12.4</td>
<td>12.4</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

The above quantities were calculated by integrating:

\[
p(a\sigma, r, b\sigma) = \int_{a\sigma}^{b\sigma} \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{(x - X_0)^2}{2\sigma^2}\right) \, dx
\]

where \( X_0 = 100 \), \( \sigma = 10 \).
An initial decay curve analysis performed upon a typical data set gave the following results:

<table>
<thead>
<tr>
<th>Component</th>
<th>Halflife</th>
<th>Abundance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{41}$Sc</td>
<td>0.6 sec</td>
<td>60%</td>
</tr>
<tr>
<td>$^{38}$K*</td>
<td>0.946 sec</td>
<td>30%</td>
</tr>
</tbody>
</table>

other long-lived components and background 10%

Two sets of fictitious data were created using 90% short-lived components and 10% constant background. The first included $^{38}$K* along with the $^{41}$Sc decay and the second did not. Then these data were analyzed both with and without $^{38}$K*. These results, along with the results of fits to the actual data, are shown below.

<table>
<thead>
<tr>
<th>Fictitious Data Components</th>
<th>Results of Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>%Sc %K % Bkgd % Sc Error %K Error %B Error Chi-Square</td>
<td>90 0 10 90.008 0.516 9.991 0.052 0.997 91.706 1.804 -1.684 1.345 9.988 0.066 0.996</td>
</tr>
</tbody>
</table>

Results of Fits to Actual Data

<table>
<thead>
<tr>
<th>Data Set A</th>
<th>Data Set B</th>
</tr>
</thead>
<tbody>
<tr>
<td>89.956 0.467 10.043 0.047 2.298</td>
<td>91.247 0.448 8.752 0.043 2.611</td>
</tr>
<tr>
<td>55.968 1.849 33.903 1.395 10.128 0.065 0.848</td>
<td>58.211 1.744 33.022 1.304 8.765 0.059 1.153</td>
</tr>
</tbody>
</table>

From a standard Chi-square table, it can be determined that for 440 degrees of freedom the value of $\chi^2$ should be:
\[ x^2 = 1 \pm 0.0674 \quad 68.27\% \text{ of the time} \]
\[ x^2 = 1 \pm 0.1348 \quad 95.45\% \text{ of the time} \]

Therefore a \( x^2 > 1.13 \) or \( < 0.87 \) is highly unlikely. Thus results of the analysis of the fictitious data above show that a good fit cannot be obtained if the \(^{38}\text{K}^*\) decay is present in the data, but not included in the analysis. The same behavior is evident in the fits to the actual data, therefore the results of this method of analysis can be assumed correct.

The beam pulsing diagram showing the relationship between the various timing parameters given in Chapter II also applies to data accumulated with the mechanical timer. In the measurement of the typical decay curve, it was desirable to adequately define the magnitude of the background yield as well as that of the short-lived components. Therefore the timing parameters were adjusted such that the counting period was about 90% of the total cycle time. The following chart gives the values of the pulsing parameters used for each of the possible cycle times. These were measured by using the timer to gate a scalar which was driven at 100K counts per second. The accuracy of the measurement was better than 0.3%.
Cam Timer Pulsing Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>4 sec</th>
<th>10 sec</th>
<th>21.3 sec</th>
<th>36 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>0.442</td>
<td>1.106</td>
<td>2.36</td>
<td>3.98</td>
</tr>
<tr>
<td>τ</td>
<td>4.</td>
<td>10.</td>
<td>21.3</td>
<td>86</td>
</tr>
<tr>
<td>t₁</td>
<td>0.134</td>
<td>0.0336</td>
<td>0.0716</td>
<td>0.121</td>
</tr>
<tr>
<td>t₂</td>
<td>3.54</td>
<td>8.86</td>
<td>18.9</td>
<td>31.88</td>
</tr>
</tbody>
</table>

A typical decay-curve analysis is presented in Figure 25. The data are that of set A, used previously in this appendix. The curve is composed of 440 points with a dwell time of 0.020 seconds per channel.

The yields of the individual constituents of the composite decay curve given as results of the least squares analysis apply to first multiscalar channel of the decay curve. However, the desired quantities are those characteristic of the rapid switching mode of data accumulation, as this method was used in the final polarization experiment. Thus it was necessary to derive relationships between both the relative yields and the absolute yields in the two modes of operation.

The data accumulated in both the right and left counting telescopes were summed when decay curves were measured in order to achieve a greater degree of statistical accuracy. Using the results of Appendix A, we can write an expression for the total number of counts per run S as:
FIGURE 25

A semi-log plot of a typical decay-curve, with results of a two-exponential plus constant background fit. The data are normalized to the number of counts in the first channel, and the dwell time is 0.020 seconds per channel.
\[ S = S_L + S_I = \frac{V_f}{2T} (B + C) + \frac{V_f}{2T} (B - C) \]

\[ = \frac{V_fB}{T} \]

Substituting the expression for B:

\[ S = \frac{\sqrt{2}}{T} (1 - \cos \alpha) \frac{1}{\lambda} \left( \frac{1 - e^{-\lambda T}}{1 - e^{-\lambda T}} \right) \left( e^{-\lambda t_1} - e^{-\lambda t_2} \right) \]

Since this equation applies to the total number of counts per run regardless of whether the transistorized pulser (T.P.) or the mechanical timer (M.T.) is used, one may write:

\[ S(T.P.) = \frac{\left[ \frac{1 - e^{-\lambda T}}{1 - e^{-\lambda T}} \left( e^{-\lambda t_1} - e^{-\lambda t_2} \right) \right]}{\left[ \frac{1 - e^{-\lambda T}}{1 - e^{-\lambda T}} \left( e^{-\lambda t_1} - e^{-\lambda t_2} \right) \right]}_{T.P.} \]

\[ S(M.T.) = \frac{\left[ \frac{1 - e^{-\lambda T}}{1 - e^{-\lambda T}} \left( e^{-\lambda t_1} - e^{-\lambda t_2} \right) \right]}{\left[ \frac{1 - e^{-\lambda T}}{1 - e^{-\lambda T}} \left( e^{-\lambda t_1} - e^{-\lambda t_2} \right) \right]}_{M.T.} \]

\[ = \left[ B \right]_{T.P.} \bigg/ \left[ B \right]_{C.T.} \]

where the timing parameters are evaluated according to which of the timers was used.

The total yield \( S(M.T.) \) must now be related to the yield in the first channel of the decay curve. This result is simply the sum of the yields of the particular component in all 440 channels:

\[ S(M.T.) = \sum_{n=1}^{440} Y_1 e^{-\lambda t n} \]

where \( Y_1 \) = the first channel yield, 
\( t \) = the time spent in each channel, and 
\( n \) = the channel number.
This sum is simply a geometric series, therefore the result is:

\[ S(M.t.) = Y_1 e^{-\lambda t} \frac{1-e^{-\lambda t}}{1-e^{-\lambda t}} = [C] Y_1 \]

Thus we have an expression for the absolute yield per run in the rapid pulsing mode as a function of the absolute yield in the first channel of the decay curve.

\[ S(T.P.) = \frac{[B]}{[B]} [T.P.] [C] Y_1 \]

Once absolute yields are calculated, then relative yields follow directly.

The terms in this equation are evaluated in the chart below. The time per channel t was such as to extend 500 channels over the full cycle of the mechanical timer. The counting period, however, included only 440 of these channels.

**Evaluation of Conversion Factors**

<table>
<thead>
<tr>
<th>Source</th>
<th>Half-life (sec)</th>
<th>4 sec</th>
<th>10 sec</th>
<th>21.3 sec</th>
<th>36 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{41})Sc</td>
<td>0.6</td>
<td>0.596 105.8 0.841 42.8 1.447</td>
<td>-</td>
<td>2.438</td>
<td>-</td>
</tr>
<tr>
<td>(^{38})K*</td>
<td>0.946</td>
<td>0.554 157.2 0.685 67.6 1.014</td>
<td>-</td>
<td>1.591</td>
<td>-</td>
</tr>
<tr>
<td>(^{17})F</td>
<td>66.</td>
<td>0.520 431.9 0.519 420.2 0.519</td>
<td>-</td>
<td>0.520</td>
<td>-</td>
</tr>
<tr>
<td>(^{38})K</td>
<td>460.2</td>
<td>0.520 438.8 0.519 437.1 0.519</td>
<td>-</td>
<td>0.520</td>
<td>-</td>
</tr>
</tbody>
</table>

The relationship between the relative yields of two components in both switching modes can be derived simply by
taking ratios of the above factors in order to arrive at a value for \( K \) in the following expression:

\[
\left[ \frac{S_{(1)}}{S_{(2)}} \right]_{\gamma, \rho} = k \left[ \frac{\gamma_{1}(1)}{\gamma_{2}(1)} \right]_{M, \gamma}.
\]

\( K \) is evaluated for the cases of interest in the chart below.

**Values of the Constant \( K \)**

<table>
<thead>
<tr>
<th>Ratio</th>
<th>4 sec</th>
<th>10 sec</th>
<th>21.3 sec</th>
<th>36 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {^{41}\text{Sc}}/^{38}\text{K}^* )</td>
<td>0.726</td>
<td>0.780</td>
<td>0.905</td>
<td>1.003</td>
</tr>
<tr>
<td>( {^{41}\text{Sc}}/^{17}\text{F} )</td>
<td>0.282</td>
<td>0.166</td>
<td>0.141</td>
<td>0.150</td>
</tr>
<tr>
<td>( {^{41}\text{Sc}}/^{38}\text{K} )</td>
<td>0.278</td>
<td>0.160</td>
<td>0.130</td>
<td>0.131</td>
</tr>
</tbody>
</table>
APPENDIX D

Count-Rate Corrections for a Rapidly Decaying Source

The determination of the absorption curves for $^{13}\text{N}$ and $^{38}\text{K}$ in Chapter IV involved a series of measurements of the activity of a rapidly decaying source. To compare the count rate during successive runs, a correction had to be made for the fact that the decay rate was not constant.

For a decaying sample:

$$N(t) = N(t_0) e^{-\lambda t}$$

when $t_0 = 0$

where: $N(t_0) =$ the number of radioactive nuclei present at time $t_0$,

$N(t) =$ the number of radioactive nuclei still present at time $t$, and

$\lambda =$ the decay constant of the source.

The experimental measurements involved a counting period of $\tau$ seconds and a resting period of $\tau$ seconds, etc. Between $t_a$ and $t_b$, $t_a < t_b$, the beta yield is:

$$N(t_a) - N(t_b) = N(t_0) \left[ e^{-\lambda t_a} - e^{-\lambda t_b} \right]$$

If $t_b = t_a + \tau$, then the yield for a measurement beginning at $t_a$ is:

$$N(t_a) - N(t_a + \tau) = N(t_0) \left[ e^{-\lambda t_a} - e^{-\lambda (t_a + \tau)} \right] = N(t_0) \left[ e^{-\lambda t_a} (1 - e^{-\lambda \tau}) \right]$$
Let \( t_0 = 0 \) = beginning of first counting period, and
\[ t_n = \text{end of } n^{\text{th}} \text{ period} = n \tau. \]

The yield during the first period is:
\[
\Psi_1 = N(t_0) - N(t_i) = N(t_0) \left[ e^{-\lambda t_0} - e^{-\lambda t_i} \right]
= N(t_0) \left[ 1 - e^{-\lambda \tau} \right]
\]

The yield during the \( n^{\text{th}} \) period is:
\[
\Psi_n = N(t_{n-1}) - N(t_n) = N(t_0) \left[ e^{-\lambda t_{n-1}} - e^{-\lambda t_n} \right]
= N(t_0) \left[ e^{-\lambda (n-1) \tau} - e^{-\lambda n \tau} \right]
= N(t_0) e^{-\lambda (n-1) \tau} \left[ 1 - e^{-\lambda \tau} \right]
\]

Thus:
\[
\frac{\Psi_n}{\Psi_1} = e^{-\lambda (n-1) \tau}
\]

Since data were taken only during odd numbered periods:
\[ n = 1, 3, 5 \ldots \]

The formula can now be evaluated, with the use of the following:

<table>
<thead>
<tr>
<th>Source</th>
<th>Halflife</th>
<th>Decay Constant</th>
<th>Counting Period ( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{38}\text{K-B}^+)</td>
<td>7.67 min</td>
<td>0.1505 \cdot 10^{-2}/sec</td>
<td>100 sec</td>
</tr>
<tr>
<td>(^{13}\text{N-B}^+)</td>
<td>9.96 min</td>
<td>0.116 \cdot 10^{-2}/sec</td>
<td>200 sec</td>
</tr>
</tbody>
</table>

Substituting, one gets the following correction factors:
<table>
<thead>
<tr>
<th>Period (n)</th>
<th>$C_n^{38K}$</th>
<th>$C_n^{13N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>.7401</td>
<td>.6288</td>
</tr>
<tr>
<td>5</td>
<td>.5477</td>
<td>.3853</td>
</tr>
<tr>
<td>7</td>
<td>.4054</td>
<td>.2486</td>
</tr>
<tr>
<td>9</td>
<td>.3000</td>
<td>.1563</td>
</tr>
<tr>
<td>11</td>
<td>.2220</td>
<td>.09827</td>
</tr>
<tr>
<td>13</td>
<td>.1643</td>
<td>.06179</td>
</tr>
<tr>
<td>15</td>
<td>.1216</td>
<td>.03885</td>
</tr>
<tr>
<td>17</td>
<td>.08999</td>
<td>.02443</td>
</tr>
<tr>
<td>19</td>
<td>.06660</td>
<td>.01536</td>
</tr>
<tr>
<td>21</td>
<td>.04929</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>.03648</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>.02700</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX E

Beta Yields from Competing Reactions

The efforts of the subsidiary experiments concerning the absorption of betas from the competing reactions listed in Chapter IV can be summarized by a calculation of the expected beta yields per hour from each reaction. This will involve the use of measured cross sections at a 4 MeV bombarding energy and the absorption curves presented in Figure 11 of Chapter IV.

The total beta yield in both counting systems for any of the competing reactions producing beta-active products is:

\[ N_r + N_1 = \frac{N_s}{T} \left( \frac{1 - \cos \alpha}{\lambda} \frac{1 - e^{-\lambda T}}{1 - e^{-\lambda \tau}} \right) \left( e^{-\lambda t_1} - e^{-\lambda t_2} \right) \]

where

\[ N_s = \frac{1}{2.66 \cdot 10^{-7}} \frac{i(\theta)}{R} Q \Omega R t f_b f_r e \]

The factor \( f_b \) corrects for the fraction of the beta spectra absorbed by the aluminum disks; the remainder of the terms are as defined in Appendix A. The values of the following parameters are applicable to each reaction:

\[ T = 180 \text{ ms} \quad \alpha = 26 \text{ degrees} \]
\[ \tau = 360 \text{ ms} \quad \Omega_\tau = 5.17 \cdot 10^{-3} \text{ steradians} \]
\[ t_1 = 7.5 \text{ ms} \]
\[ t_2 = 172.5 \text{ ms} \]
The values of the parameters particular to each reaction are summarized below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$^{41}\text{Sc}$</th>
<th>$^{38}\text{K}^*$</th>
<th>$^{38}\text{K}$</th>
<th>$^{17}\text{F}$</th>
<th>$^{13}\text{N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ (sec)$^{-1}$</td>
<td>1.555</td>
<td>0.733</td>
<td>0.00151</td>
<td>0.0105</td>
<td>0.00116</td>
</tr>
<tr>
<td>$\sigma(\theta)$ (mb/sr)</td>
<td>12</td>
<td>0.36</td>
<td>58</td>
<td>300</td>
<td>55</td>
</tr>
<tr>
<td>$t$ (mg/cm$^2$)</td>
<td>$30 \cdot 10^{-3}$</td>
<td>$30 \cdot 10^{-3}$</td>
<td>$30 \cdot 10^{-3}$</td>
<td>$12 \cdot 10^{-3}$</td>
<td>$12 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$f_{re}$</td>
<td>60%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The differential cross sections for the production of the beta-decaying states of the parent nuclei of the competing reactions include not only the particular state in question, but also all other excited states that gamma decay to the beta-unstable states. The cross sections for the production of the ground state of $^{41}\text{Sc}$ is taken from Figure 3. Since all excited states of this nucleus are proton unstable, only the ground state need be included. The same is true for the production of $^{13}\text{N}$, where a ground state cross section of about 55 mb/sr was calculated by application of kinematics to neutron distributions measured by Sawers, et al. (Sa 66). A ground state cross section of about 100 mb/sr and one of about 200 mb/sr for the first excited state of $^{17}\text{F}$ was calculated in the same manner from the neutron data of Loudin and Nilsson (Lo 70). All higher states in $^{17}\text{F}$ are proton unstable.
No calculation of this nature using known alpha particle distributions from the $^{38}$K and $^{38}$K* reactions can be performed because of the many excited states that lie below the threshold for proton emission which gamma decay to the ground state or the first excited state. Instead, these cross sections were estimated using the results of the decay-curve analyses summarized in Chapter IV. The $^{38}$K* cross section was calculated from the $^{41}$Sc to $^{38}$K* ratio at 4 MeV given in Figure 14. Then the $^{38}$K ground state could be estimated from Figure 12 by requiring agreement with the background yields versus absorber, after subtracting the calculated fluorine beta yield and the ambient chamber background.

A typical target thickness of 30 $\mu$g/cm$^2$ of calcium was chosen for the calculation. The amount of oxygen present would then be 12 $\mu$g/cm$^2$ for the fully oxidized condition. The 12 $\mu$g/cm$^2$ of carbon included the target backing plus a 2 $\mu$g/cm$^2$ buildup during a typical series of runs.

A 60% recoil efficiency for scandium ions in a 30 $\mu$g/cm$^2$ target was estimated using the results of the yield versus target thickness measurement presented in Figure 12. The efficiency for the recoil of the other parent nuclei could be estimated by approximating the cross section for large angle scattering by the Rutherford cross section (Bl 66). This approximation is good for recoil energies of about 1 MeV or higher. Thus the multiple
scattering of the other ions could be compared to that of scandium by considering the values of \((Z/\text{recoil energy})^2\). This comparison indicates that the scattering probability for these ions is only a few percent of that for scandium, thus \(f_{\text{re}}\) was taken to be 100%.

The effective baseline setting \(f_b\) for each beta decay, except that of \(^{17}\text{F}\), was taken from the absorption curves of Figure 11. The absorption of betas from fluorine was determined by interpolation, since the accuracy of the corresponding absorption curve was very poor. The results of this calculation of the yields per microcoulomb of beam of the various competing reactions versus absorber thickness is given in Figure 26. These yields are characteristic of a typical run at 4 MeV. Yields at higher energy can be readily obtained from the data presented in Figure 14 of Chapter IV. These calculations indicate an optimum absorber thickness of about 0.7 or 0.8 g/cm\(^2\) of aluminum at this beam energy.
FIGURE 26

Expected yield per microcoulomb of beam versus absorber thickness for each of the competing reactions at 4 MeV.
APPENDIX F

Expected Yields for the Charge State Determination

The determination of the charge state of the scandium recoil ions is the subject of the latter portion of Chapter III. To interpret the results of the experiment, the dependence of the yield upon the magnetic field strength was calculated. A typical recoil trajectory is indicated in the left of Figure 27. An ion, with a given recoil velocity and in a given charge state, travels a distance $d = 6$ inches from the target to the catcher. If the magnetic field is on, there is a deflection $x$ of this ion at the midpoint of its course as it moves with a radius of curvature $R$. The deflection is then:

$$x = R - y$$

$$= R - \left[ R^2 - \left(\frac{d}{2}\right)^2 \right]^{\frac{1}{2}}$$

If we assume that the 1/8 inch diameter beam of recoils is homogeneous, that is, the particle density is constant, we can represent the interception of the beam by the first slit in the right of Figure 27. The upper circle represents the cross section of the beam at the position of the slit, and the lower circle represents the slit itself. The shaded area is then half the portion of the beam that passes through the slit. If we consider the bottom circle, the area of the upper half is:
FIGURE 27

Apparatus for the charge-state measurement.
\[ \frac{\pi r^2}{2} \quad \text{where } r = 1/8 \text{ inch} \]

The area subtended by \( \theta \) on both sides is:

\[ 2 \pi r^2 \frac{\theta \text{(radians)}}{2\pi} = \theta \text{(radians)} \cdot r^2 \]

The area subtended by \( \phi \) and the circle diameter on both sides is:

\[ 2 \cdot \frac{s}{2} \sqrt{r^2 - s^2} = s \sqrt{r^2 - s^2} \]

Thus the shaded area is:

\[ \frac{\pi r^2}{2} - \theta \text{ (radians)} \cdot r^2 - s \sqrt{r^2 - s^2} \]

\[ = \frac{\pi r^2}{2} - r^2 \sin^{-1}(\frac{s}{r}) - s \sqrt{r^2 - s^2} \]

The total area shared by both circles is twice this. Also \( s = \frac{1}{2} \) the deflection \( x \) of the recoil beam. Therefore the fraction of the original beam passing through the middle slit is:

\[ (F-2) \quad \text{Ratio} = 1 - \frac{2}{\pi} \sin^{-1}(\frac{x}{2r}) - \frac{x}{\pi r^2} \sqrt{r^2 - (x^2/4)} \]

To evaluate this expression for each charge state at various values of the magnetic field, the value of the radius of curvature must be known. This, of course, depends upon the recoil velocity, which must be expressed in terms of a velocity or energy distribution function. Note that the radius of curvature is dependent upon the recoil velocity in the following manner:
\[ R = \frac{m v c}{Be} \]

where \( m \) is the mass of the ion, \( v \) is its velocity, \( c \) is the speed of light, \( B \) is the strength of the magnetic field, and \( e \) is the ionic charge.

It must be assumed that the energy distribution of the recoils is the same as that for the initial products of the reaction. This is only partially valid due to the effects of multiple scattering. It must be further assumed that the energy loss in the target is negligible compared to the recoil energy. Then the calculation of the energy (or velocity) distribution function can be accomplished through use of measured neutron differential cross sections at 4 MeV (Gr 70) and at 6 MeV (Le 68). These distributions, as presented in Figures 28 and 29, were integrated numerically in ten-degree steps in the neutron center of mass system. Then the average recoil energy of the corresponding scandium recoils was computed from the kinematics of the reaction. The partially integrated cross sections were divided by the length of the corresponding energy interval and the results are presented in mb/MeV in Figures 28 and 29.

Thus for each of the four possible charge states, the yield versus magnetic field function could be computed by evaluating the equations (F-1) and (F-2) for each numerical step in the recoil energy distribution function. A com-
FIGURE 28

Neutron differential cross section in the center of mass system from the $^{40}\text{Ca}(d,n)$ reaction, and the resulting $^{41}\text{Sc}$ recoil energy distribution at a bombarding energy of 4 MeV.
FIGURE 29

Neutron differential cross section in the center of mass system from the $^{40}$Ca(d,n) reaction, and the resulting $^{41}$Sc recoil energy distribution at a bombarding energy of 6 MeV.
puter code was written to perform this operation, and the results are given in the table below.

Yield Functions for Charge State Measurements

<table>
<thead>
<tr>
<th>Magnet Current (amps)</th>
<th>Field Strength (kG)</th>
<th>Percent Yield 4 MeV</th>
<th>Percent Yield 6 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ch. 1</td>
<td>Ch. 2</td>
</tr>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>0.05</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.42</td>
<td>87.7</td>
<td>76.0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.93</td>
<td>72.5</td>
<td>49.0</td>
</tr>
<tr>
<td>0.3</td>
<td>1.38</td>
<td>59.8</td>
<td>24.0</td>
</tr>
<tr>
<td>0.4</td>
<td>1.85</td>
<td>48.0</td>
<td>13.3</td>
</tr>
<tr>
<td>0.6</td>
<td>2.78</td>
<td>24.0</td>
<td>1.0</td>
</tr>
<tr>
<td>0.8</td>
<td>3.70</td>
<td>13.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>4.55</td>
<td>5.0</td>
<td>&quot;</td>
</tr>
<tr>
<td>1.3</td>
<td>5.77</td>
<td>0.4</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

At 6 MeV the background was not a constant, but shaped according to the data to account for the increased beta background flux at this energy (see Chapter IV).
APPENDIX G

Calculation of the Beta-Decay Asymmetry Parameter

The intensity distribution of beta rays emitted from polarized nuclei can be described by the following relation, as given in Chapter I:

\[ I = I_0 \left( 1 + \frac{v_e}{c} \right) \frac{<I_2>}{I} \ A \cos \theta \]

To evaluate this expression, it is necessary to determine the value of the asymmetry parameter \( A \), characteristic of the type of beta transition involved in the \(^{41}\)Sc positron decay. The following is a summary of the results of a theoretical formalism presented by H. F. Schopper (Sc 66) concerning beta intensity distributions from polarized nuclei.

For pure Fermi or pure Gamow-Teller transitions, the asymmetry parameter is given by:

\[ A_F = 0 \]

\[ A_{GT} = \pm \lambda_1 \frac{\mu_{TT} - \mu_{AA}}{k_{TT} - k_{AA}} \]

upper sign for \( B^- \) decay

lower sign for \( B^+ \) decay

where

\[ \lambda_1 \begin{cases} 
1 & \Delta I = -1 \\
\frac{1}{I+1} & for \ \Delta I = 0 \\
\frac{-1}{I+1} & \Delta I = +1 
\end{cases} \]

(I = the spin of the radioactive nucleus.)
and
\[ k_{ij} = k_{ij}^* = c_i c_j^* + c_i' c_j'^* + d_i d_j^* + d_i' d_j'^* \]
\[ l_{ij} = l_{ij}^* = c_i' c_j^* + c_i c_j'^* + d_i d_j^* + d_i' d_j'^* \]
\[ i,j = s, v, t, a, p \]

The constants $C_i$ represent the relative strengths of the five possible forms of the weak interaction, in the above mentioned formalism—scalar ($S$), vector ($V$), tensor ($T$), axial vector ($A$), and pseudoscalar ($P$). To allow for a possible parity violation in these transitions, another set of such transitions are defined with relative strengths denoted by $C_i'$. Furthermore, additional interactions, whose relative strengths are described by $D_i$ and $D_i'$, are included to allow for non-conservation of Lepton charge.

Several conclusions with respect to the values of these (complex) constants have been extracted from experimental work on the form of the beta interaction. These are:

\[ C_V = + C_V' \quad C_A = + C_A' \]
\[ C_A = (1.175 \pm 0.02) C_V e^{i \phi} \quad \phi = 180^\circ \pm 8^\circ \]
\[ c_i = c_i' = 0 \quad \text{for} \quad i = s, t, p \]
\[ d_i = d_i' = 0 \quad \text{for} \quad i = s, v, t, a, p \]

Thus it is found that:
\[ A_G = \mp \lambda \Gamma \]

For mixed transitions, the beta-decay asymmetry parameter has the following form:
\[ A = A_{67} \epsilon + 2 \left( \frac{I}{I+1} \right)^{1/2} \sqrt{\frac{\epsilon}{1-\epsilon}} \]

where:

\[ N = \frac{R_e (I_{57} - I_{VA})}{\kappa_{77} + \kappa_{AA}} \]
\[ \epsilon = \frac{1}{(x^2/R) + 1} \]
\[ \chi = \frac{|M_F|}{|M_{GT}|} \]
\[ R = \frac{\kappa_{77} + \kappa_{AA}}{\kappa_{ss} + \kappa_{vv}} \]

\( N \) and \( R \) reduce to:

\[ R = (1.175)^2 \]
\[ N = +1 \]

The matrix elements \( |M_F| \) and \( |M_{GT}| \) of the Fermi and Gamow-Teller transitions are calculated by Mayer and Jensen (Ma 55) using single particle wavefunctions to describe the initial and final nuclear states. Since each of these nuclei, \(^{41}\)Sc and \(^{41}\)Ca, are considered to possess a "good" single-particle ground-state configuration, these results should be an acceptable approximation. These are:

\[ |M_F|^2 = 1 \quad |M_{GT}|^2 = 1.28 \]

Thus:

\[ \chi^2 = \frac{1}{1.28} \quad \epsilon = 0.639 \]

Therefore the asymmetry parameter can be readily calculated for the case \( I = 7/2, \Delta I = 0 \) with the result:

\[ A = 0.99 \]
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Fu 69  Fuller and Cohen, Nuclear Data Tables A5 (1969) 433.
Gr 70  L. R. Greenwood, Rice University, private communication.


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