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Domain Wall Velocity in Magnetic Materials

by

David S. Bartran

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I. INTRODUCTION

Except in permanent magnet applications, magnetic flux reversal processes are of ultimate importance in device behavior. Moving domain boundaries, or walls, constitute the dominant mode of flux reversal. Although wall motion phenomenon have been used in many practical devices such as transformers and other devices which require high permeability, only very simplified and intuitive concepts have been necessary to understand the theory of wall motion.

Recently, devices have been developed for computer memory and logic applications in which information is stored, manipulated, and shifted from one location to another in the form of magnetic domains [1,2]. To utilize these devices and to develop others, a more complete knowledge of the theory of wall motion and its relationship with the experimental situation is required.

The first definitive experiments on the propagation of domain boundaries were reported by Sixtus and Tonks in 1931 [3]. Since then, several important developments in the analytical description of planar wall motion have been made.

In 1935, Landau and Lifshitz [4] developed the dynamic torque equations together with a phenomenological damping
term and presented the first correct although approximate micromagnetic calculation of a moving Bloch wall in a uniaxial, anisotropic ferromagnet. The inertial properties of the wall were later estimated by Döring [5] and by Kittel [6].

In 1956, L. R. Walker, as quoted by Dillon [7] in 1963, discovered an exact solution to the moving wall problem and showed that the steady-state, velocity-field relationship was nonlinear. It was predicted that for large drive fields the static wall configuration (although preserved in functional form) was distorted by an intense wall demagnetizing field which is created as the spins in the wall precess about the applied field and out of the wall plane. This distortion has been descriptively called "wall contraction" because the wall width shrinks with increasing velocity. This effect is reflected in a nonlinear velocity-field relationship which is characterised by a velocity peak and a region of negative, differential mobility.

In 1959, Bean and DeBlois [8] analyzed wall motion in the limit that the spins in the wall do not precess out of the wall plane. Their results show that a domain wall experiences "pseudo relativistic" effects as its velocity is increased. These effects were described as a kind of "Lorentz contraction" with a limiting wall velocity.

In 1961, Hagedorn and Gyorgy [9] reported limiting wall
velocities in low loss Yttrium-Iron-Garnet and proposed a model in which the wall leaves a wake of fluctuating spins. Dillon [7] later discussed their results and concluded that their data and model corresponds to fields beyond the limits of the Walker solution. Similar conclusions were reached by Asti, et. al. [10] concerning nonlinear velocity-field behavior in $Ba_0.6Fe_2O_3$.

In 1963, Enz [11] included the wall demagnetizing field (to the first order) in a calculation similar to Bean's and confirmed the prediction of limiting wall velocities and wall contraction. In this same year, Walker's exact solution was published and found to apply for a limited range of drive fields.

In 1967, Palmer and Willoughby [12] considered the velocity of domain walls in materials with cubic anisotropy. They were able to predict limiting velocities as well as the velocity-field behavior in $Ni_{0.75}Fe_{2.25}O_4$ by resorting to numerical techniques; however, the experimental results indicated no such behavior.

Feldtkeller [13], in 1968, recognizing the onset of wall contraction in the velocity-field behavior of domain walls in permalloy thin films, extended Enz's calculation to include an applied field and damping term. However, this calculation (made independently of Walker's contribution) is only approximate and was not substantially verified by the available
experimental results.

With the advent of bubble domain, memory and logic devices, the unusual velocity-field behavior similar to the phenomenon reported by Hagedorn and Gyorgy posed potential limitations on ultimate operating speeds of these devices. As a result, renewed and vigorous interest in the theory of wall motion developed.

In 1971, Slonzewski [14] analyzed wall motion in bubble materials. He developed approximate solutions, describing non-planar walls in materials with large anisotropy fields. He also developed a model to describe wall motion for drives beyond the range of Walker's solution. Simultaneously, Walker's description of wall motion was re-examined by Schlömann[15] and generalized to include the case of orthorhombic anisotropy in a review paper by Hagedorn [16]. Schlömann [17] also demonstrated the relationship of Walker's solution to linear spin wave theory and analyzed [18] wall contraction in one-dimensional thin film walls with a self-consistent variational technique.

During the development of the theory, most of the experiments, reported in the literature, have dealt either with permalloy and related alloys [19-23] or with bubble materials [16]. The data in the first case corresponds to drive fields less than 5 oe and is well explained by the linear velocity-field relationship predicted by Landau and
Lifshitz [4] and an appropriate model of wall coercive force [13, 24]. The data in the second case corresponds to such large drive fields that the Walker theory does not apply. As discussed by Dillon [7] and iterated by Hagedorn [16], the velocity behavior in bubble materials is not yet completely understood; however, several models have been proposed [9, 14, 25]. In either case, the Walker theory was neither experimentally confirmed nor disproved.

Recently (1972), the theory has been extended with an analytic estimate of transient wall contraction [26], and the effects of steady-state wall contraction have been observed in Bloch wall permalloy thin films [27]. The observed velocity-field behavior of thin film walls bears a strong resemblance with the theory of one-dimensional walls in bulk samples in spite of the fact that planar thin film Bloch wall configurations are two-dimensional [28, 29].

To correlate further the theory and experiment, the transient effects predicted by Bourne and Bartran in reference [26] are considered. But, first it is informative to explore the similarities between one-dimensional bulk wall configurations for which analytic theory is available and thin film wall configurations.

**Bulk and Thin Film Walls**

The thin film wall is characterized by extensive flux closure at the film surfaces. Instead of the wall magnetization
being constrained in the wall plane, as it is in a self-
consistent, one-dimensional, thin film wall calculation, the
two-dimensional configuration is Neel-like at the surfaces
of the sample and Bloch-like inside. The Neel components of
the wall on the upper and lower surfaces of the sample are
of opposite polarity.

The terms Bloch and Neel, applied to descriptions of
wall configuration in honor of the individuals concerned,
indicate whether or not the wall magnetization is in the wall
plane (Bloch) or normal to the wall plane (Neel).

The Neel and Bloch-like components of the wall cooperate
to form an approximately divergence-free configuration of the
magnetization. This results in a considerably reduced
influence of the sample surfaces on the Bloch-like interior
of the wall. In other words, the Bloch component is not
exposed to the intense demagnetizing field found in a one-
dimensional thin film wall, and is more accurately described
by the theory of a one-dimensional bulk wall than by the
theory of a one-dimensional thin film wall. In addition,
the demagnetizing field of the moving wall, which is created
as the wall spins precess out of the wall plane, is reduced
because of the finite wall cross-section [30].

To account for the reduction in wall demagnetizing field
a phenomenological scaling factor, analogous to a demagnetizing
factor for the case of uniform wall magnetization [30], may be
introduced. The static wall width parameter, $s_0$, contained in the bulk theory, may also be treated as an adjustable parameter to further modify the theory. An exact description of thin film wall behavior is not expected since wall contraction in two-dimensional wall configurations may not have a one-to-one, quantitative relationship with the one-dimensional theory. Nevertheless, a meaningful comparison of the theory and experiment is possible.

A variational calculation of a moving, one-dimensional thin film wall [18] shows that Bloch walls in 1200 Å permalloy films exhibit negligible wall contraction. This result obtains because the wall energy, being dominated by demagnetizing energies in the static wall as well as in the moving wall, does not vary much as the wall velocity is increased. Consequently, the degree of wall distortion in the moving wall, relative to the static wall, is much smaller than expected for a bulk-like wall in the same material.

The fact that considerable wall contraction does occur [27], at least in permalloy materials, is an indication that the Bloch component of stationary walls is not dominated by demagnetizing fields. This agrees in part with the two-dimensional wall calculations and with the phenomenological modification of the theory.
II. The Theory

II.a The Equations of Motion

The dynamic torque equation, with Gilbert damping, is given in equation \((1a)\). The local torque, equation \((1b)\), acting on the magnetization is due to Zeemann, magnetostatic, anisotropy, and exchange contributions and is derivable from the magnetic energy, equation \((1c)\), of the system. The coordinates are shown in Fig.1.

\[
\frac{1}{8} \frac{\delta \mathbf{M}}{\delta t} - \frac{\alpha}{\gamma M} \left( \mathbf{M} \times \frac{\delta \mathbf{M}}{\delta t} \right) = -\mathbf{T} \tag{1a}
\]

where

\[
\mathbf{T} = \frac{\alpha_0}{\sin \theta} \frac{\delta \omega}{\delta \phi} - \alpha_0 \frac{\delta \omega}{\delta \theta}, \tag{1b}
\]

\[
\omega = -H_M \cos \theta - H_y M \sin \theta \sin \phi + K \sin^2 \theta \]
\[
+ A \left[ (\nabla \theta)^2 + \sin^2 \theta (\nabla \phi)^2 \right], \tag{1c}
\]

and the unit vectors \(\alpha_\theta\) and \(\alpha_\phi\) correspond to the \(\theta\) and \(\phi\) directions respectfully.

The magnetization vector is \(\mathbf{M}\), the exchange constant is \(A\), the gyromagnetic ratio is \(\gamma\), the viscous damping parameter is \(\alpha\), and the uniaxial anisotropy constant is \(K\). The fields \(H_M\) and \(H_y\), are the total fields acting along the wall and perpendicular to the wall plane and may include applied as well as demagnetizing fields.

The functional derivatives in equation \((1b)\) are defined
in terms of the following partial derivatives:

\[ \frac{\delta w}{\delta \phi} = \frac{\partial w}{\partial \phi} - v \cdot \{ \frac{\partial w}{\partial (\phi \theta)} \} \quad (2a) \]

and

\[ \frac{\delta w}{\delta \theta} = \frac{\partial w}{\partial \theta} - v \cdot \{ \frac{\partial w}{\partial (\phi \theta)} \} \quad . \quad (2b) \]

It is worth noting that \[ \frac{\partial M}{\partial t} \] but that \[ \frac{\partial M}{\partial t} \] is obtained in the following manner:

\[ \lim_{\Delta t \to 0} \frac{M_{\phi \theta} + M_{\sin \theta \phi}}{\Delta t} \]

or

\[ \frac{\partial M}{\partial t} = M \left\{ \frac{\partial \phi_{\theta}}{\partial t} + \sin \theta \frac{\partial \phi}{\partial t} \right\} . \quad (3b) \]

Further, \[ M \frac{\partial M}{\partial t} = M^2 \left\{ \frac{\partial \phi}{\partial t} - \sin \theta \frac{\partial \phi}{\partial t} \right\} . \quad (4) \]

After carrying out the various operations indicated in equations (1) and (2) together with equations (3) and (4), the torque equations are obtained in more explicit form:

\[ \frac{M_{\sin \theta \phi}}{\partial t} + \frac{\alpha M}{\partial t} \sin^2 \theta \frac{\partial \phi}{\partial t} = H \delta M \sin \theta \cos \phi \]

\[ + 4A\sin \theta \cos \theta (v \phi - v \theta) \]

\[ + 2A\sin^2 \theta \nu^2 \phi \]

\[ (5a) \]

\[ \frac{M_{\sin \theta \phi} - \alpha M_{\phi}}{\partial t} = H \delta M \sin \theta - H \mu \cos \phi \sin \theta \cos \theta \]

\[ + 2K \sin \theta \cos \theta (v \phi - v \theta) \]

\[ - 2A \nu^2 \theta . \quad (5b) \]

These coupled, nonlinear, partial differential equations do not permit analytic descriptions of wall motion except in very idealized cases. The following sections, review the theory from the classical static wall calculation to the
recently developed theory of transient wall contraction.

II.b One-Dimensional Walls

A domain wall is assumed to exist "a priori" with the boundary conditions that $\theta$ vary from $0^\circ$ to $180^\circ$ as $y$ goes from $-\infty$ to $+\infty$.

For a one-dimensional wall, the spin configuration is described in terms of a single positional coordinate, $y$; hence,

$$\nabla \phi = \frac{\partial \phi}{\partial y}, \quad \nabla^2 \phi = \frac{\partial^2 \phi}{\partial y^2}$$  \hspace{1cm} (6a)

and

$$\nabla \theta = \frac{\partial \theta}{\partial y}, \quad \nabla^2 \theta = \frac{\partial^2 \theta}{\partial y^2}.$$  \hspace{1cm} (6b)

Landau and Lifshitz [4] showed that a wall demagnetizing field arises if the wall spins do not lie in the wall plane. In a moving wall, this demagnetizing field, $H_y$, arises because of the divergence of magnetization in the wall created as the wall spins precess out of the wall plane in response to a field applied along the wall and in the easy axis direction.

From magnetostatics, the demagnetizing fields, $H$, may be defined in terms of a fictitious magnetic charge, $\rho_m$.

$$\nabla \cdot H = \frac{\rho_m}{\mu_0}$$ \hspace{1cm} (7a)

Using Gauss' theorem, equation (7a) may be written as:

$$\int_{\text{wall plane}} H \cdot n \, ds = \int_{\text{wall plane}} H_y \, ds = \int_{\text{wall plane}} ds \int_{\text{wall plane}} \frac{\rho_m}{\mu_0} \, dy.$$ \hspace{1cm} (7b)
where $\mathbf{n} = \hat{a}_y$ is the wall normal.

The vector magnetization is given by

$$\mathbf{M} = M (\sin \theta \cos \phi \hat{a}_x + \sin \theta \sin \phi \hat{a}_y + \cos \theta \hat{a}_z) .$$  \hspace{1cm} (7c)

Combining equation (7c) with equation (7b) and using $\mathbf{P}_m = \mathbf{v} \cdot \mathbf{M}$, the wall demagnetizing field is obtained as

$$H_y = -\frac{M}{\mu_0} \int_0^1 \left( \sin \theta \sin \phi \right) dy = -\frac{M}{\mu_0} \sin \theta \sin \phi .$$ \hspace{1cm} (8a)

According to previous discussion, a reduction in the wall demagnetizing field is expected in thin film samples. To account for this, phenomenologically, a scaling factor, $\beta$, is introduced in equation (8a) with the result:

$$H_y = -\frac{\beta M}{\mu_0} \sin \theta \sin \phi .$$ \hspace{1cm} (8b)

A bulk wall corresponds to $\beta = 1.0$.

In addition to the assumption of a one-dimensional wall, it is further assumed that $\mathbf{v} \phi$ and $\nabla^2 \phi$ are negligible. This assumption appears to be necessary for closed form analytic descriptions of wall motion.
III. Wall Calculations

III.a Static Wall Configuration [4]

For a stationary wall, \( \frac{\partial \phi}{\partial t} \) and \( \frac{\partial^2 \phi}{\partial t^2} \) are zero as is the easy axis field, \( H_z \). Since interactions of the domain wall with lattice defects etc. have not been introduced, a non-zero easy axis field causes those domains aligned with field to expand at the expense of the oppositely directed domains thus contradicting the assumption of a stationary wall.

Equation (5), with \( \nabla \phi = 0 \) and \( \nabla^2 \phi = 0 \), combined with equation (8b) reduces to:

\[
\phi = -\frac{BM^2}{2} \sin^2 \theta \sin \phi \cos \phi \quad (9a)
\]

and

\[
\phi = \frac{BM^2}{2} \sin \theta \cos \theta \sin^2 \phi + 2K \sin \phi \cos \phi - 2A \frac{\partial^2 \phi}{\partial y^2}. \quad (9b)
\]

An immediate consequence of equation (9a) is that for the static one-dimensional wall, \( \phi = 0 \); hence, \( H_y \), the wall demagnetizing field, is also zero. Using this condition on \( \phi \) in equation (9b), the following differential equation is obtained:

\[
\phi = 2K \sin \theta \cos \theta - 2A \frac{\partial^2 \phi}{\partial y^2} \quad (10)
\]

which must be satisfied for all \( 0^\circ \leq \theta \leq 180^\circ \).

The following trial function satisfies equation (10):

\[
\frac{\partial \phi}{\partial y} = C \sin \theta \quad (11a)
\]
where $C$ is a constant and

$$\frac{\Delta^2 \theta}{\Delta y^2} = C \cos \theta \frac{\Delta \theta}{\Delta y} = C^2 \sin \theta \cos \theta.$$  \hfill (11b)

Upon substitution of equation (11b) into equation (10), the following result is obtained:

$$0 = 2k \sin \theta \cos \theta - 2AC^2 \sin \theta \cos \theta,$$  \hfill (12)

and a solution is possible for

$$C^2 = \frac{K}{A}.$$  \hfill (13)

The solution of equation (10) is obtained by integration of the trial function, equation (11a), and subsequent substitution of equation (13); hence,

$$\int \frac{d\theta}{\sin \theta} = \sqrt{\frac{K}{A}} y$$  \hfill (14a)

or

$$\ln \left\{ \tan \frac{\theta}{2} \right\} = \sqrt{\frac{K}{A}} y.$$  \hfill (14b)

As $\theta$ approaches $0^\circ$, $y$ approaches $-\infty$, and as $\theta$ approaches $180^\circ$, $y$ approaches $+\infty$. Hence, the boundary conditions are indeed satisfied.

A static wall width parameter, $S_o$, may be defined in terms of $\frac{\Delta \theta}{\Delta y}$ evaluated at the wall center. From equation (11a) and equation (13) with $\theta = 90^\circ$, the domain wall width is determined as:

$$S_o = \pi \left/ \left( \frac{\Delta \theta}{\Delta y} \right) \right|_{c} = \pi \sqrt{\frac{A}{K}}.$$  \hfill (15)
The exchange constant, A, represents the tendency for adjacent spins to align themselves, and the anisotropy constant, K, represents the tendency for the magnetization to lie in the easy directions, i.e. in either the \( \theta = 0^\circ \) or \( \theta = 180^\circ \) directions. Hence, small wall widths are associated with a small exchange interaction or with a large, uniaxial, magnetic anisotropy.

III.b Approximate Wall Velocity \([4]\)

If a static field is applied along the wall, then the domains favored by the field expand at the expense of the others until flux reversal is complete, assuming that this occurs with constant velocity and occurs such that the wall retains its static configuration i.e. \( \phi \sim 0^\circ, \nabla \phi = 0, \nabla^2 \phi = 0 \) and \( \frac{\partial \phi}{\partial t} = 0 \), then equations (5), (8c) and (10) combine to yield

\[
\frac{MSin \theta}{\alpha} \frac{\partial \theta}{\partial t} = -\frac{BM^2 Sin^2 \theta Sin \phi Cos \phi}{\mu_0}
\]

\[
-\frac{\alpha M}{\mu_0} \frac{\partial \phi}{\partial t} = H_z M Sin \theta + \frac{BM^2}{\mu_0} Sin \theta Cos \theta Sin^2 \phi
\]

\[
+2K Sin \theta Cos \theta - 2A \frac{\partial \phi}{\partial y^2}
\]

(16a)

(16b)

Since it is assumed that \( \phi \sim 0^\circ \), these equations are self-consistent only in the limit of small angular velocity. If equation (16b) is combined with equation (10), that is, combined with the requirements for a static wall configuration, then

\[
-\frac{\alpha M}{\mu_0} \frac{\partial \phi}{\partial t} = H_z M Sin \theta.
\]

(17)

In a moving coordinate system, an observer moving with
the wall would see a time invariant structure. This requires that

\[ \frac{d\theta}{dt} = \frac{d\theta}{dy} \frac{dy}{dt} + \frac{d\theta}{dt} = 0; \]  

(18)

hence, the velocity of the moving coordinates is

\[ V = -\frac{d\theta}{dy}. \]  

(19a)

Combining and evaluating equations (11a), (13), (17), and (19a) at the wall center, the following estimate of the wall velocity is made:

\[ V = \frac{\chi}{\alpha \sqrt{\kappa}} H_z, \]  

(19b)

and the structure of the wall is given by

\[ \ln \left\{ \tan \theta_2 \right\} = \sqrt{\kappa} \left[ y - Vt \right]. \]  

(20)

Often wall mobility, defined as \( \frac{\partial V}{\partial H_z} \), is a convenient parameter characterizing the domain walls in a given sample. From equation (19b) the mobility is expressed as:

\[ G = \frac{\partial V}{\partial H_z} = \frac{\chi}{\alpha \sqrt{\kappa}} = \frac{\chi}{\alpha \sqrt{\kappa}}. \]  

(21)

Since \( \alpha \), the damping parameter, occurs in the denominator, large wall mobilities are generally indicative of low damping
for given anisotropy and exchange parameters.

Notice that the reduction in wall demagnetizing field is not reflected in the wall velocity, equation (19b), or in its structure, equation (20); hence, these solutions apply only if the wall distortion due to the demagnetizing field is negligible.

III.c Exact, Steady-State Wall Velocity[7]

In the previous analysis, an inconsistency arose because it was assumed that the wall retained its static configuration. A more general solution may be obtained if the functions $\frac{de}{dt}$ and $\frac{de}{dy}$ found in the approximate analysis are used as trial functions. For example:

\[
\frac{de}{dt} = c_1 \sin \theta \\
\frac{de}{dy} = c_2 \sin \theta .
\]

(22a)  
(22b)

Combining these functions with equations (5) and equation (8b) the following obtains:

\[
\frac{M}{\gamma} \sin^2 \theta \phi_1 + \frac{\alpha M}{\gamma} \sin^2 \theta \phi_2 = -\frac{BM^2}{\mu_0} \sin^2 \theta \sin \phi \cos \phi
\]

(23a)

\[
\frac{M}{\gamma} \sin \phi_1 - \frac{\alpha M}{\gamma} \sin \theta \phi_1 = H_E M \sin \theta + \frac{BM^2}{\mu_0} \sin \theta \cos \phi \sin \phi
\]

\[+ 2K \sin \theta \cos \phi - 2AC \sin \theta \cos \phi .
\]

(23b)
Equating terms of like functional form, an equivalent set of equations is determined.

\[
\frac{M}{\epsilon} \frac{\partial C_1}{\partial t} + \frac{\alpha M}{\epsilon} \frac{\partial \phi}{\partial t} = -\frac{\beta M^2}{\mu_0} \sin \phi \cos \phi ; \sin^2 \theta \text{ terms} \quad (24a)
\]

\[
\frac{M}{\gamma} \frac{\partial \phi}{\partial t} - \frac{\alpha M}{\gamma} C_1 = H_z M ; \sin \theta \text{ terms} \quad (24b)
\]

\[
\frac{\beta M^2}{\mu_0} \sin^2 \phi + 2K - 2AC_z^2 = 0 ; \sin \phi \cos \phi \text{ terms} \quad (24c)
\]

Eliminating \( \frac{\partial \phi}{\partial t} \) from equations (24a) and (24b) yields:

\[
(1 + \alpha^2) C_1 = - \left[ \gamma \alpha H_z + \gamma \beta M / \mu_0 \sin \phi \cos \phi \right] \quad (25a)
\]

and \( (1 + \alpha^2) \frac{\partial \phi}{\partial t} + \gamma \alpha \beta M \sin \phi \cos \phi = \gamma H_z \quad (25b) \)

while equation (24c) yields \( C_z \) directly:

\[
C_z = \left( \frac{\beta M^2 \sin^2 \phi + H_k}{H_z} \right) / \left( 2A \right) \quad (25c)
\]

where \( H_k \), the anisotropy field, is defined as \( 2K/M \).

Since \( \nabla \phi = 0 \), the partial derivative \( \frac{\partial \phi}{\partial t} \) becomes an ordinary derivative \( \frac{1}{\epsilon} \frac{\partial \phi}{\partial t} \), and in the limit of uniform, steady state flux reversal, \( \frac{\partial \phi}{\partial t} = 0 \). Hence, from equation (25b) the following condition must be satisfied for uniform steady-state motion:

\[
H_z = \alpha \beta M / \mu_0 \sin \phi \cos \phi \quad (26)
\]
This result places an upper bound on the applied field since the maximum magnitude of $\sin \phi \cos \phi$ is $\frac{1}{2}$, that is,

$$|H_z| \leq \frac{1}{2} \alpha \frac{BM}{\mu_0}. \quad (27)$$

The velocity of the wall is given by equation (19a). Combining equations (22), (25a), and (25c) yields the velocity in terms of the angle $\phi$:

$$V(\phi) = -C_1 C_2 = \frac{\sqrt{2A}}{1 + \alpha^2} \frac{(\beta H_z \gamma (BM/\mu_0) \sin \phi \cos \phi)}{(BM/\mu_0 \sin^2 \phi + H_k)^{1/2}}. \quad (28)$$

From equation (26) and from several trigonometric identities, one finds that:

$$\sin^2 \phi = \frac{1}{2} \left[ 1 - \sqrt{1 - \sin^2 2 \phi} \right] \quad (29a)$$

or

$$\sin^2 \phi = \frac{1}{2} \left[ 1 - \sqrt{1 - \left( \frac{2H_z}{\alpha BM/\mu_0} \right)^2} \right]. \quad (29b)$$

Explicit expressions for $C_1$ and $C_2$ are obtained by combining equation (26) and equation (29b) with equation (25a) for $C_1$, and with equation (25c) for $C_2$.

$$C_1 = \frac{\alpha}{\gamma} H_z \quad (30a)$$

$$C_2 = \sqrt{\frac{K}{A}} \left\{ 1 + \frac{BM/\mu_0}{2H_k} \left[ 1 - \sqrt{1 - \left( \frac{2H_z}{\alpha BM/\mu_0} \right)^2} \right] \right\}^{1/2} \quad (30b)$$

These together with equation (28) and equation (15) yield
the wall velocity directly:

\[ V = \frac{2}{\alpha} H_x S_0 \left\{ 1 + \frac{BM/\mu_0}{2H_K} \left( 1 - \sqrt{1 - \left( \frac{2H_x}{\alpha \beta M/\mu_0} \right)^2} \right)^{\frac{1}{2}} \right\}^{-\frac{1}{2}} \tag{31} \]

The wall structure is obtained by integrating equation (22b) as with equation (14) and equation (20), and the traveling wave solution is:

\[ \ln \left( \tan \theta_2 \right) = C_2 (y - \nu t) \tag{32} \]

where \( C_2 \) is given by equation (30b).

In the limit of \( H_x \) approaching zero,

\[ C_2 = \sqrt{\frac{K}{H}} \tag{33} \]

and equation (31) and (32) reduce to the approximate solutions of Landau and Lifshitz.

At the wall center

\[ \left( \frac{\partial \theta}{\partial y} \right)_{C} \equiv C_2 \tag{34} \]

and the dynamic wall width may be defined as in equations (15) and (16).

\[ S = S_0 \left\{ 1 + \frac{BM/\mu_0}{2H_K} \left( 1 - \sqrt{1 - \left( \frac{2H_x}{\alpha \beta M/\mu_0} \right)^2} \right) \right\}^{-\frac{1}{2}} \tag{35} \]
where $\delta_0 = \pi \sqrt{A K}$. The wall width, $\delta$, is a decreasing function of drive field; hence, the origin of the term "wall contraction".

The velocity expression, equation (31), is shown in Fig. 2, on normalized coordinates with $h_k = \mu_0 H_k / \beta M$ as a parameter.

Several important features of the velocity-field curve are immediately apparent: (1) at low drives the velocity-field curve is linear, (2) at high drives, the solution contains an upper bound on the drive field beyond which the analytic solution does not apply, and (3) for intermediate drives, the velocity-field curve is nonlinear with a velocity peak and a region of negative differential wall mobility. An upper bound on the velocity is also present. Further, materials with large anisotropy are characterized by a reduction in the nonlinearity and a shift of the velocity peak to larger normalized drive fields.

The question of stability in the negative differential mobility region is important. Slonczewski [14] shows that initially planar walls characterized by $\frac{\partial V}{\partial H_2} < 0$ are only conditionally stable. He argues that, if a planar wall is made to have a small bulge, then the resulting magnetostatic fields together with $\frac{\partial V}{\partial H_2} < 0$ cause this protuberance to grow with time; hence, an instability. In interrupted pulse experiments, this growth may be suppressed if the amplitude
of the protuberance is less than the randomizing influence of the wall coercive force acting on the wall after the field is removed.

The coordinates of the velocity peak [shown below] are found by equating \( \frac{\partial V}{\partial H_z} = 0 \).

\[
V_{pk} = \sqrt{\frac{LA}{\mu_0}} \left[ \left( 1 + h_K \right)^{k_z} - h_K^{k_z} \right] \leq \sqrt{\frac{LA}{\mu_0}} \beta \tag{36a}
\]

\[
\left( \frac{h_z}{h_K} \right)_{v_{pk}} = \alpha h_K^{-2\beta} \left( 1 + h_K \right)^{V_{pk}} \left[ \left( 1 + h_K \right)^{k_z} - h_K^{k_z} \right] \tag{36b}
\]

The upper bound on the drive field is:

\[
(H_z)_m = \frac{1}{2} \alpha \left( \frac{BM}{\mu_0} \right) \tag{36c}
\]

and the corresponding wall velocity is:

\[
V_m = \sqrt{\frac{LA}{\mu_0}} \left( 2 + 4h_K \right)^{-k_z} \tag{36d}
\]

Both \( H_z \) and \( H_K \) are normalized by \( \frac{BM}{\mu_0} \) to obtain \( h_z \) and \( h_K \) respectively. Because of this common normalization

\[
\left( \frac{h_z}{h_K} \right)_{v_{pk}} = \left( \frac{H_z}{H_K} \right)_{v_{pk}} \tag{36d}
\]
III.d A Sample Calculation

For bulk permalloy,

\[
\begin{align*}
\beta &= 1.0 \\
M &= 1.0 \text{ Wb/m}^2 \\
\alpha &= 0.01 \\
A &= 10^{-11} \text{ J/m} \\
K &= 200 \text{ J/m}^3 \\
\gamma &= 2.21 \times 10^5 \text{ rad. (A/m)}^{-1} \text{ S}^{-1}
\end{align*}
\]

and the fields and velocities predicted in equations (36) are:

\[
H_K = 5 \text{ oe} \quad ; \quad h_K = 5 \times 10^{-4}
\]

\[
(H_z)_{vpk} = 75 \text{ oe} \quad ; \quad V_{pk} = 8.2 \times 10^4 \text{ cm/s}
\]

\[
(H_z)_{m} = 50 \text{ oe} \quad ; \quad V_{m} = 5.8 \times 10^4 \text{ cm/s}
\]

Although the theory is consistent with the MKS system of units, the fields and velocities are converted to oersteds (1 oe=80 A/m) and cm/s respectively in keeping with traditional usage.

Also of interest is the maximum amount of wall contraction defined in terms of a ratio of the minimum wall width to the static wall width. From equation (35) with \(H_z = (H_z)_m\), this ratio is expressed as:

\[
\frac{S_{\text{min}}}{S_0} = \left(1 + \frac{8mH_0}{2H_K}\right)^{-\frac{1}{2}} \sim \frac{1}{30}.
\]  
(37)
Hence, for a static wall width of \( S_0 = 7000 \AA \) characteristic of bulk permalloy, the fully contracted wall has a width of \( S = 2300 \AA \).

In practice the velocity peak may not be observed since the corresponding field is so large that other processes are expected to dominate the flux reversal. These processes may consist of non-planar domain boundary configurations induced by eddy current damping [31] (for bulk samples only) or rotational processes which may occur for drives greater than the anisotropy field, \( H_K \).

III.e. "Exact", Transient Wall Velocity [26]

To analyze the acceleration of the wall exactly, numerical procedures must be used because of terms involving \( \nabla \phi \) and \( \nabla^2 \phi \); however, an approximate but analytic solution to the torque equations is possible if the asymmetry in the wall configuration is neglected. This assumption is equivalent to \( \nabla \phi = 0 \), and the trial functions for the exact, steady-state solution may be used if the coefficients \( C_1 \) and \( C_2 \) of equation (22) are permitted to be functions of time but not of position.

These trial functions represent the first term in an expansion of an exact transient solution, and as such, they yield a first order description of transient wall contraction. The quantitative significance of higher order terms is difficult to assess because they must necessarily describe
asymmetry in the wall and may not be analytically estimated.

Since the trial functions of the transient analysis are identical with those of the steady-state analysis, equations (19), (25a), (25b), and (25c) apply.

For a drive field, of duration T, with a magnitude less than the upper bound expressed in equation (27), equation (25b) may be integrated by means of "separation of variables". With $\phi(t=0) = 0$, the solution has the form:

$$\tan \phi(t) = \frac{2h_z}{\alpha} \left(1 - e^{-nt}\right), \ t \leq T$$  \hspace{1cm} (38a)

where

$$n = \frac{n \alpha \beta M/M_0}{1 + \alpha^2} \left[1 - \left(\frac{2h_z}{\alpha}\right)\right]^{1/2}$$

and

$$h_z = \mu_0 H_z / \beta M$$

and

$$\phi(t-T) = \tan(\phi_T) e^{-\frac{h_z \beta M / \mu_0}{1 + \alpha^2} (t-T)}, t \geq T$$  \hspace{1cm} (38b)

where $\phi_T$ is the wall angle just prior to the removal of the field.

The instantaneous velocity of the wall is given by equation (28) with the solution for $\phi(t)$ appropriate to the particular time interval. The instantaneous velocity of the wall is shown in Fig.3 with normalized drive field as a parameter. The time scale is normalized by $[\alpha \beta M / \mu_0]^{-1}$, with $\alpha^2 \ll 1$, and the curves correspond to $h_z = 0.01$ which is
appropriate to a thin film wall in permalloy with $\phi = 0.05$ as discussed in chapter VI.b.

The overshoots on the leading and trailing edges of the velocity transient are expected, and are a direct result of the functional dependence of wall velocity on $\phi$ (see equation (28)).

The average wall velocity, defined in terms of net displacement and field duration, is actually measured in interrupted pulse experiments and is of practical interest. The predicted curves of average velocity versus normalized drive field with the field duration as a parameter are shown in Fig.4. These curves are obtained by numerical integration of the analytic velocity expression, equation (28), together with the appropriate solution for $\phi(t)$, that is, equation (38a) or equation (38b). Simpson's integration rule is used.

The curves of Fig.4 approach the Walker solution when the field duration is greater than $20 [2 \phi \beta M/\mu_0]^{-1}$ seconds. As the field duration is reduced, the velocity overshoots, indicated in Fig.3, move closer together and result in an increased average velocity.

For field durations on the order of and less than $[2 \phi \beta M/\mu_0]^{-1}$, the wall angle, $\phi$, remains small throughout the transient, and the average velocity versus field curve is approximately linear, with a slope approaching the linear mobility of the steady-state solution. The low drive behavior
of the wall is not affected by the field duration.

The above results apply to a wall moving in an idealized material in which no defects are present.
IV. The Coercive Force Interaction

IV.a The Model

Baldwin [32] shows that the interaction of a moving wall with defects is inherently nonconservative, and he proposes a spring model of wall coercive force. In this model, a real flexible wall is replaced by an equivalent but rigid, planar wall with the defect-wall interaction approximated by a breakable spring. The spring is linear for extensions less than the defect range, $\Delta$, which is characteristic of the wall coercive force mechanism.

A wall moving through a large number of defects, which are randomly positioned but otherwise identical, is connected by a spring to each defect in its wake, for a distance $\Delta$. Defects not yet making contact with the wall do not act upon it.

If the density of defects is very large, the moving wall is continuously breaking springs, and the average, total, defect pressure exerted on the wall is $2M \mathcal{H}_c$, where $\mathcal{H}_c$ is defined as the wall coercive force.

A more complete model of wall coercive force includes both nonconservative and purely conservative interactions as discussed by Baldwin [24] and Feldtkeller [13]. The nonconservative spring model, however, represents the dominant features of the coercive force interaction.

The effect of wall coercive force may be included as a field acting along the wall; hence, the equations of motion
corresponding to equation (25b) and equation (28) are written as:

\[
(1+\alpha^2) \frac{d\phi}{dt} + (\gamma \alpha \beta M/\mu_o) \sin \phi \cos \phi = \gamma [H_z - H_c (\frac{y-y_0}{\beta \mu_z})]
\]  

(39a)

\[
\frac{dy}{dt} = \frac{\gamma \sqrt{2A}}{1+\alpha^2} \frac{\left\{ \alpha \left[ H_z - H_c \left( \frac{y-y_0}{\beta \mu_z} \right) \right] + \frac{\beta M}{\mu_o} \sin \phi \cos \phi \right\}}{\left\{ \frac{\beta M}{\mu_o} \sin^2 \phi + H_k \right\}^{1/2}}
\]  

(39b)

for \(|y-y_0| \leq A/2\),

and

\[
(1+\alpha^2) \frac{d\phi}{dt} + (\gamma \alpha \beta M/\mu_o) \sin \phi \cos \phi = \gamma [H_z - H_c]
\]  

(40a)

\[
\frac{dy}{dt} = \frac{\gamma \sqrt{2A}}{1+\alpha^2} \frac{\left\{ \alpha \left[ H_z - H_c \right] + \frac{\beta M}{\mu_o} \sin \phi \cos \phi \right\}}{\left\{ \frac{\beta M}{\mu_o} \sin^2 \phi + H_k \right\}^{1/2}}
\]  

(40b)

for \(|y-y_0| > A/2\).

These equations are nonlinearly coupled and do not appear to have analytic solutions. To obtain average velocity curves analogous to those of Fig.4, equation (39) and equation (40) may be solved numerically; however, it is instructive to examine the effect of the coercive force interaction in the linearized equations of motion.
IV.b The Linearized Equations

Equations (39) and (40) may be separately linearized and combined to form two, second order differential equations. For example, in the limit of $\sin \phi \approx \phi$ and $\cos \phi \approx 1$ as well as $\alpha^2 \ll 1$, equation (39a) and (39b) reduce to:

\[
\frac{d\phi}{dt} + \gamma \omega \beta \frac{M}{\mu_0} \phi = \gamma \left[ H_z - H_c \left( \frac{y - y_o}{\Delta z} \right) \right] \quad (41a)
\]

\[
\frac{dy}{dt} = \gamma \frac{S_o}{\delta} \beta \frac{M}{\mu_0} \phi \quad (41b)
\]

Differentiating equation (41b) with respect to time and afterwards substituting equations (41a) and (41b), the following differential equation is obtained:

\[
m \frac{d^2y}{dt^2} + \frac{2M}{G} \frac{dy}{dt} + 2M \frac{H_c}{\Delta z} \left( \frac{y - y_o}{\Delta z} \right) = 2M H_z, \quad |y - y_o| \leq \Delta z \quad (42a)
\]

where $m$ (the wall mass) = $\frac{2M}{\gamma \omega \beta \frac{M}{\mu_0}}$ and $G$ (the wall mobility) = $\frac{2 \beta \frac{M}{\mu_0}}{\Delta z}$ are defined in terms of the static wall width, $S_o$.

In a similar fashion, equations (40a) and (40b) may be linearized and combined to obtain:

\[
m \frac{d^2y}{dt^2} + \frac{M}{G} \frac{dy}{dt} = 2M (H_z - H_c), \quad |y - y_o| > \Delta z \quad (42b)
\]

In the limit of $\Delta \to 0$, the wall coercive force is a constant during the motion of the wall and equation (42b)
applies with \( |y-y_*| > 0 \). This equation has a simple solution for a constant drive field of duration \( T \).

With \( \frac{dy}{dt} = V(t) \) and \( V(0) = 0 \), the instantaneous wall velocity satisfying the equation of motion is

\[
V(t) = G(H_e - H_c)(1 - e^{-\frac{2M}{mG}t}) , \quad t \leq T,
\]  

(43a)

and

\[
V(t-T) = [G(H_e-H_c)(1-e^{-\frac{2M}{mG}T}) + GH_c]e^{-\frac{2M}{mG}(t-T)} - GH_c, t > T.
\]  

(43b)

The cessation of motion corresponds to \( V(t'-T) = 0 \), and from (43b) this instant in time is given as

\[
t' - T = \frac{mG}{2M} \ln \left\{ \frac{(H_e - H_c)(1-e^{-\frac{2M}{mG}T}) + H_c}{H_c} \right\}.
\]  

(44)

The net wall displacement is determined by integration:

\[
y_{net} = \int_0^T V(t) \, dt + \int_T^{t'} V(t-T) \, dt , \quad t' > T
\]  

(45a)

or

\[
y_{net} = G(H_e - H_c)T - GH_c(t'-T),
\]  

(45b)

and the average velocity may subsequently be expressed as \( \bar{y}_{net}/T \) or

\[
\langle V \rangle = G(H_e - H_c) - GH_c\left(\frac{t'-T}{T}\right) , \quad t' > T.
\]  

(46)

In the limit of long field duration, the average velocity reduces to the traditionally accepted result. For short durations, the velocity is reduced by the second term in
equation (46), and since the threshold field remains fixed and equal to the wall coercive force, this implies a reduction in the slope or mobility of the average velocity curve.

Another, limiting case of equations (42a) and (42b) corresponds to negligible wall inertia, and the describing equations reduce to:

\[
\frac{dy}{dt} + G H_c \left( \frac{y_{\text{y}}}{H} \right) = G H_{\text{y}} , \quad |y - y_{\text{y}}| < \Delta z
\]  

\[
\text{and} \quad \frac{dy}{dt} = G (H_{\text{y}} - H_c) , \quad |y - y_{\text{y}}| > \Delta z .
\]  

The solutions of these equations are:

\[
y(t) = \frac{A}{2} \frac{H_y}{H_c} \left( 1 - e^{-\frac{G H_c t}{H}} \right) , \quad y < \frac{\Delta z}{2} , \quad t \leq T ,
\]  

\[
\text{and} \quad y(t-t_0) = \frac{A}{2} + G (H_{\text{y}} - H_c) (t-t_0) , \quad y > \frac{\Delta z}{2} , \quad t \leq T .
\]

The coercive constraint of the wall is broken at 
\[t = t_0\] , and from equation (48a) this instant is determined when \[y = \frac{\Delta z}{2}\] ; hence,

\[
t_0 = \frac{\Delta z}{2} \ln \left\{ \frac{H_{\text{y}}}{H_{\text{y}} - H_c} \right\} .
\]  

The average velocity may be obtained from equation
(48b) as:

\[ \langle v \rangle = \frac{\gamma w^2}{T} = \left( \gamma (T-t_0) - \frac{A_2}{2} \right) / T \]

or

\[ \langle v \rangle = G (H^*_2 - H_c)(1 - \frac{t_0}{T}) \cdot \]

Although this result predicts a weak nonlinearity in the velocity-field curve, the major implication of this result is more clearly evident in the limit of \( H^*_2 \gg H_c \), for which

\[ t_0 = \frac{A_2}{G H^*_2} \]

and

\[ \langle v \rangle = G (H^*_2 - H_c - \frac{A_2}{T}) \cdot \]

It is apparent from this result that the primary effect of the coercive force interaction is manifested as a pulse-width dependent threshold field which approaches the static threshold in the limit of long field durations. The increase in threshold field arises because the maximum excursion of the wall must exceed a critical distance before irreversible displacement may occur.

When wall mass and the defect range are retained, analytic solutions are possible but cumbersome so a fourth order, Runge-Kutta integration method is used instead. The average velocity versus field curves obtained from equations (42a) and (42b) are shown in Fig.5 with field duration as a parameter. The wall mass, etc., indicated in the figure are
order of magnitude estimates corresponding to a thin film Bloch wall and are chosen to fit experimental data presented in Chapter VI. The predicted increase in threshold field and the decreased wall mobility are consistent with previous discussion.

IV.c The "Exact" Equations

When wall contraction effects are important, then equations (39) and (40) must be solved. A fourth order Runge-Kutta method is used to integrate the simultaneous equations and the results are displayed in Fig.6 as average velocity versus field magnitude with field duration as a parameter. The steady-state velocity curve is from equation (31) with the coercive force included as a constant.

The introduction of wall coercivity causes several important changes in the average velocity curves. Not only is an increase in the threshold field and a decrease in apparent wall mobility predicted as the field duration is reduced, but also for a fixed drive field in the vicinity of the velocity peak, the average velocity is predicted to increase, saturate, and decrease as the field duration is reduced. The predicted wall mobility, normalized to its steady-state magnitude, versus field duration is shown in Fig.7 with normalized defect range as a parameter. These curves are obtained from predicted average, velocity-field curves.
By comparing the results of the piece-wise linear equations of motion with the results of the "exact" equations, the following conclusion is reached: the decrease in apparent wall mobility and the increase in threshold field predicted to occur for short duration drive fields is a characteristic of the coercive force interaction.

To date (1973), only the increase in motion threshold field [33,34] and a slight increase in average velocity [34] for short duration fields have been reported in the literature.
V. Experimental Techniques
V.a General Procedures

Interrupted pulse techniques are used to determine the velocity-field behavior of planar Bloch walls in permalloy films. The experiment consists of applying a train of short duration easy axis field pulses until the domain boundary moves a specified distance (~100μm). The wall is then reset to its approximate, original position by partially demagnetizing the sample with a sinusoidal easy axis field and the experiment continued for various magnitudes and durations of the drive field. The average velocity of the wall is defined as the displacement per pulse divided by the field duration. Each data point is an average of several measurements.

Only planar walls parallel to the easy axis and located in a region of uniform wall coercive force are used. If care is not exercised in this respect, the data is characterized by excessive randomness.

The longitudinal Kerr effect is used to make the domains visible and the wall displacements are measured with a reading telescope to within 5μm. Further, the ambient fields (other than purposefully applied) are reduced below 5 millioersteds by field canceling coils.

V.b Drive Field Apparatus

To generate the necessary drive fields, a pulse
generator is designed which uses a high pressure mercury-wetted switch in a coaxial mount. Single, well defined rectangular pulses are generated in matched loads by discharging an appropriate length of charged pulse forming cable. Pulses of 1 ns duration with .25 ns rise and fall times are easily produced. The drive fields are formed in a small, shorted strip-line and a 50 ohm resistor is used to match the shorted line to the pulse generator. The combined strip-line and matching resistor is 3 cm long and is usable for pulse, rise times down to 0.2 ns.

For optical access, a single port (1 cm x 1 cm) is located in the top conductor directly above the sample. The sample on glass slide, ~0.5 mm thick, is mounted inside the strip-line on the bottom conductor. The field uniformity under the port is sufficient for samples 5 mm in diameter. The field calibration obtained by Middelhoek's technique [19] is 3.5 oe per kilovolt of charging potential (5 kilovolts maximum).

Most of the velocity data is taken with drive fields comparable to the anisotropy field, and for such fields, diffuse boundary propagation emanating from the sample edges may occur [35]. However, it is observed that when the sample is carefully demagnetized, the threshold field for diffuse switching processes is somewhat increased. In this way meaningful data on 180° walls may be observed for
drives exceeding the anisotropy field. For these large drives, the scatter in the data appears to depend on the degree of sample demagnetization.

V.c Sample Preparation

The permalloy samples are evaporated by conventional techniques. A melt of 81% Ni, 19% Fe (by weight) is heated by electron bombardment, and the vapors are condensed onto heated (100°C-200°C) glass substrates in the presence of a 30 oe applied field. The deposition rate is \( \sim 30\,\text{Å/s} \) at a pressure of \( 10^{-6} \) Torr during the evaporation. Two hours after the evaporation, the samples are allowed to cool to room temperature and are removed.

An anti-reflection layer of SiO, which improves domain contrast for the Kerr effect, is subsequently deposited but with the samples at room temperature. The deposition of SiO is terminated when the light reflected from the sample reaches a minimum.

Prior to evaporation (of permalloy) the substrates are (1) cleaned ultrasonically in xylene then acetone for several seconds to remove gross contamination, (2) immediately rinsed in running deionized water and finally degreased with hot isopropyl alcohol vapor, and (3) baked under vacuum at 200°C. If the glass is allowed to dry before degreasing, water spots are formed which result in regions of poor film adherence. After removal of the substrates from the degreaser,
a visual check is made, and if dust particles, etc. are observed, the substrate is recycled through the cleaning routine. Sometimes it is necessary to wash the substrates with a mild detergent solution. Also, once the cleaning is started, the glass is handled only with clean tweezers.
VI. Experimental Results

VI.a Steady-State Wall Velocity

Several velocity-field curves, obtained from interrupted pulse experiments, are shown in Fig.8. The field duration is shown in the legend with the sample properties. The curves are characterized by a peak in the average velocity and a region of negative differential mobility. These features bear a strong resemblance with the Walker description of wall contraction (see Fig.2).

These velocity curves appear to be independent of field duration up to 200 ns. However, for the drives required to observe the nonlinear velocity behavior only a few pulses of 200 ns duration are needed to switch the entire sample. Good data is very difficult to obtain under such conditions.

In some of the samples it is possible to observe meaningful data for drives beyond the region of negative mobility. For these drives, the velocity is approximately proportional to the drive field.

As discussed earlier, the motion is only conditionally stable in the negative mobility region; however, because of the randomizing influence of wall coercive force acting on the wall after the drive field is removed, the unstable motion appears to be suppressed. Although drive fields of large duration may permit the instability to develop, other reversal processes such as diffuse boundary propagation [35]
may switch the sample before it (the instability) develops. For example, in permalloy samples the wave front velocity of diffuse boundary motion is on the order of $10^6$ cm/s while planar wall velocity is on the order of $10^4$ cm/s; hence a 200 ns pulse permits the diffuse boundary to move 2 mm while the planar wall only moves $20 \mu m$. For a 5 mm diameter sample, this situation often prevents meaningful observation of the planar wall motion.

The observed magnitude of the velocity peak is $\sim 10^4$ cm/s and does not appear to be a strong function of the measurable film parameters.

The location of the velocity peak, $(H_z)_{vpk}$, has a more definite relationship with the anisotropy field, $H_K$, and in the limit of $H_K \ll \frac{\beta M}{\mu_0}$, equation (36b), in un-normalized form reduces to:

$$
\left( \frac{H_z}{H_K} \right)_{vpk} = \alpha \left( \frac{\beta M}{\mu_0} \right)^{3/4} H_K^{-3/4}.
$$

(52)

The observed $(H_z/H_K)_{vpk}$ versus $H_K$ is shown in Fig. 9 for a number of NiFe and CoNiFe samples. Since the location of the velocity peak depends on the saturation magnetization, $M$, a correction of $[\text{Mpermalloy}/M]$ is made to account for the increased $M$ of the CoNiFe samples relative to the NiFe samples. The nominal magnetization of the NiFe samples is $M = 1 \text{ wb/m}^2$ and that of the CoNiFe samples is $1.08 < M < 1.28$. 
The data in Fig. 9 is characterized by a $-3/4$ exponent as predicted by the theory. However, this agreement may be somewhat deceiving because a shift in the velocity to larger fields due to a nonconservative wall coercive force is not taken into account.

In the steady-state limit, the coercive force interaction may be treated as a constant field opposing the motion of the wall, and the location of the velocity peak is shown as $(H_z v_{pk} - H_c^2/4H_K)$ versus $H_K$ in Fig.10. A correction for permalloy is included as before. The solid curve is a fit of equation (36b) and an exponent of $-3/4$ still describes the data at least for samples with low anisotropy field.

VI.b Sample Calculations, Revisited

The linear mobility, $G$, may be expressed in terms of the wall width, $S_0$, by combining equation (15) and equation (21).

$$G = \frac{a}{\alpha} \frac{S_0}{\pi}$$  \hspace{1cm} (53)

Assuming a wall width of 1500Å [29], for a Bloch wall in a 1500Å permalloy film, and a nominal initial wall mobility of $7 \times 10^3$ cm/oe-s [19-23] the following estimate of the wall damping parameter may be made.

$$\alpha = .012$$  \hspace{1cm} (54)
The curve fit in Fig. 10, implies that \( \alpha \beta^{3/4} = 0.00255 \); hence, the scaling parameter is estimated to be

\[
\beta = 0.127
\]

which is considerably less than the bulk wall limit of \( \beta = 1.0 \). This estimate may apply only for velocities in the vicinity of the velocity peak.

The estimate of \( \alpha = .04 \) and \( \beta = .04 \) in reference [27] is based on a wall mobility of \( 2 \times 10^3 \) cm/oe-s which is characteristic of the slope of the velocity curves near the velocity peak and not the low drive behavior. It was incorrectly concluded that the slope of the curves just below the velocity peak were more representative of the actual wall mobility. The present estimate of \( \alpha \) and \( \beta \) are more realistic.

The velocities and fields determined in equation (36a) through equation (36d) may now be re-calculated with the estimate of \( \beta \) in equation (55).

\[
\begin{align*}
H_K &= 5 \text{ oe} \\
V_p &= 2.9 \times 10^4 \text{ cm/s} \\
V_m &= 7.0 \times 10^4 \text{ cm/s} \\
G &= 3.3 \times 10^4 \text{ cm/oe-s}
\end{align*}
\]
If in addition to the scaled demagnetizing field, the static wall width of the bulk theory is reduced to that of the thin film wall, a more self-consistent comparison is achieved. This additional scaling is easily accomplished by reducing the term $\sqrt{A}$ wherever it occurs, by a factor of $(S_{\text{thin film}}/S_{\text{bulk}})$. In the present case this correction becomes:

$$\frac{S_{\text{thin film}}}{S_{\text{bulk}}} = \frac{1500\text{Å}}{7000\text{Å}} = 0.214$$

The previously calculated velocities are influenced by this adjustment but not the fields, and the new estimates are:

$$V_{pk} = 6.25 \times 10^3 \text{cm/s}$$
$$V_m = 4.4 \times 10^3 \text{cm/s}$$
$$G = 7 \times 10^3 \text{cm/tesla}$$

With the wall width correction, the modified theory is self-consistent in regards to the initial mobility, but the predicted $V_{pk}$ is considerably lower than the observed magnitude of $10^4 \text{ cm/s}$. Another criteria for a comparison of the theory and data ignores the precise location of the velocity peak and negative mobility region to obtain a better fit for fields
less than the observed \((H_z)_{vpk}\). For example, the results of such a criteria yield \(S_o=1200\text{Å}, \alpha=.01,\) and \(B=0.7,\)
\(v_{pk}=10^{14}\text{ cm/s, } (H_z)_{vpk}-H_c=11\text{ oer.}\) and \((H_z)_{m}-H_c=35\text{ oer.}\)

Another parameter of interest is the wall mass (see equation (42)), and it is estimated as

\[
m=\frac{2\mu_0}{\varphi^2} \frac{H_c}{S_o} = 8.5 \times 10^{-9} \text{ kg/m}^2\]

(56)

where \(\varphi=0.127\) and \(S_o=1500\text{Å}.

This mass compares with \(m>10^{-7}\text{ kg/m}^2\) predicted by one-dimensional thin film wall calculations for \(1500\text{Å}\) permalloy films [18,30] and with \(m=0.2 \times 10^{-9}\text{ kg/m}^2\) for a bulk wall.
Clearly other choices for \(\varphi\) and \(S_o\) are possible but until the dynamics of the actual thin film wall are more accurately described by the theory only order of magnitude estimates are meaningful.

VI.c Transient Wall Velocity

A family of observed velocity-field curves is shown in Fig.11 with field duration as a parameter. For field durations greater than 20 ns, the velocity curves correspond to the steady-state limit. For field durations less than 20 ns, the magnitude of the average velocity in the vicinity of the velocity peak increases concurrently with the disappearance of the negative mobility region. For smaller field durations, the threshold for net motion rapidly
increases and the initial slope of the average velocity curves sharply decreases.

Middelhoek [19] observed that the wall mobility was unchanged for field durations down to 20 ns. Others have reported an increased threshold field [33,34] for short duration drive fields. The present data is consistent with these previous observations.

The gradual disappearance of the velocity peak is understood since overshoots (see Fig.3) on the leading and trailing edges of the velocity transient merge as the field duration is reduced. This merger eliminates the negative mobility region and causes the average velocity of the peak to increase. The increased threshold field and the reduced wall mobility are predicted if a nonconservative spring model of wall coercive force is included.

For very short duration fields, the overshoots in the velocity transient do not have time to develop and the motion is approximately described by the piecewise linear equations of motion (42).

The wall mobility versus field duration is shown in Fig.12 for several permalloy films with the sample properties indicated in the legend.

The normalized, excess threshold field, \( \frac{(H^2)_{4k} - H_{c}}{H_c} \), versus field duration is shown in Fig.13. This data corresponds to the same three samples used in Fig.12. The
solid line (in Fig. 13) is obtained from the theoretical result shown in Fig. 5 and it reasonably agrees with the data for short duration fields. The theory predicts a sharp decrease in threshold fields for field durations exceeding 5 ns while the data appears to satisfy a more-or-less reciprocal relationship for a much larger range of applied fields. A possible explanation of this result may be found in a more complicated model of wall coercive force.

The observed velocity curves including the wall mobility and threshold fields agree reasonably well with the results in Fig. 5a for \( \Delta \gamma = 2000 \text{Å} \) and \( m = 1.0 \times 10^{-8} \text{kg/m}^2 \). This critical distance compares with \( 100 \text{Å} < \Delta \gamma < 4000 \text{Å} \) reported in the literature [33, 34].

It is difficult to make a quantitative comparison of the transient theory with the data; however an estimate of the wall time constant, \( \left[ \gamma \alpha (\beta M/\mu_0) \right]^{-1} \), may be made. For \( \alpha = 0.012 \) and \( \beta = 0.127 \), the time constant is estimated as 3.7 ns. This estimate is much larger than the \( \sim 0.5 \) ns expected for bulk permalloy or expected for uniform rotational switching [36]. It should be noted that \( \beta = 0.127 \) is characteristic of the velocity peak, and not representative of the low drive behavior. For example, using \( \beta = 0.7 \) and \( \alpha = 0.01 \) the steady-state, low drive data is fitted by the theory quite well as discussed in the previous section, and
the corresponding wall time constant is estimated as 0.8 ns. A time constant on the order of 1 ns appears representative of the samples studied.
VII. Conclusions

The transient theory developed in reference [26], in which a real one-dimensional wall accelerating to its equilibrium velocity is approximated by a symmetrical wall configuration, predicts constructive interference of the overshoots on the leading and trailing edges of the velocity transient. This interference is confirmed experimentally suggesting that the symmetrical wall approximation is adequate for drives below the Walker limit, that is, \( H_2 < \frac{1}{2} \alpha g M / \mu_0 \).

For drives greater than 20 ns, the observed velocity-field curves are generally in the steady-state limit and exhibit the nonlinear behavior predicted by the Walker theory [7]. Other effects reflected in the wall mobility and threshold field dependence on field duration are explained if a suitable model of wall coercivity is included in the theory.

Attempts to modify the bulk wall theory, to account for flux closure in the thin film wall and its observed wall width, yield only qualitative agreement with the observed velocity curve, taken as a whole. A more accurate representation of the dynamic thin film wall is needed to improve the agreement between the theory and experiment.
REFERENCES


Fig. 1: The wall coordinates with the polar angles $(\Theta, \phi)$ describing the orientation of the magnetization vector.

Fig. 2: The normalized velocity-field curves predicted by the Walker theory with $h_k = H_k / \beta M$ as a parameter.
Fig. 3: Typical velocity transients predicted by the "exact" theory. The drive field magnitude $2h_z/\alpha = 2\mu_0 h_z/\alpha s M$ is shown as a parameter, and normalized velocity and time coordinates are used. The duration of the drive field is 5.0 units of time.
Fig. 4: The normalized, average, velocity-field curves with normalized field duration as a parameter.
Fig. 5a: The average velocity curves predicted by the piecewise linear theory with field duration as a parameter.

Fig. 5b: The slope of the velocity curve (Fig. 5a) versus field duration.

$m = 1.0 \times 10^{-8} \text{ kg/m}^2$

$\Delta/2 = 200 \mu\text{m}$

$G = 10^4 \text{ cm/oe-s}$

$H_c = 1.0 \text{ oe}$
Fig. 6: The normalized, average, velocity-field curves with normalized field duration as a parameter. Baldwin's coercive force model is assumed.
Fig. 7: The mobility of the average velocity curves (see Fig. 6) versus normalized field duration with normalized defect range as a parameter. The mobility is normalized to its steady-state magnitude.
Fig. 8: Observed, velocity-field curves for several permalloy samples. The average velocity is defined as the net wall displacement divided by the field duration.

Field Duration: \( T \)
Sample Composition: 80:20 NiFe

- \( H_c = 1.3 \text{ oe}, H_K = 2.5 \text{ oe}, T = 30 \text{ ns} \)
- \( H_c = 1.2 \text{ oe}, H_K = 3.4 \text{ oe}, T = 30 \text{ ns} \)
- \( H_c = 2.6 \text{ oe}, H_K = 4.0 \text{ oe}, T = 30 \text{ ns} \)
- \( H_c = 1.3 \text{ oe}, H_K = 5.3 \text{ oe}, T = 50 \text{ ns} \)
Fig. 9: The location of the velocity peak, \((H_{v})_{\text{peak}} / H_K\), versus the anisotropy field. No correction is made for the coercive force. The slope of \(-3/4\) is predicted by the theory in equation (36b).
Fig. 10: The location of the velocity peak as in Fig. 9, but with a correction for a purely nonconservative wall coercive force, $H_c$. The solid curve is a fit of equation (36b) for $\alpha (\frac{2M}{\mu_0})^{3/2} = 2.55$. 

- Slope $= -3/4$ for low $H_k$ 
- NiFe ($M = 1$ wb/m²) 
- CoNiFe (peak disappears gradually)
Fig. 11: Observed velocity-field curves with field duration, $T$, as a parameter. Most samples show very little transient behavior for field durations exceeding 20 ns.
Fig. 12: Observed wall mobility versus field duration for several permalloy samples. This data is taken from the slopes of the curves in Fig. 11.
Fig. 13: The normalized excess threshold field versus field duration for several samples with properties indicated in the legend. The threshold-field predicted by the linearized theory and obtained from Fig. 5 is shown as a solid line.