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AN ECONOMETRIC MODEL OF URBAN BUS TRANSIT OPERATIONS

by

GARY R. NELSON

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

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ABSTRACT

AN ECONOMETRIC MODEL OF URBAN BUS TRANSIT OPERATIONS

by Gary R. Nelson

This thesis develops and estimates a simultaneous-equations model of the urban bus transit market. The econometric model consists of a demand equation and two equations which represent supply phenomena. The supply equations are derived from a model of the transit market in which the number of passengers, the transit fare, and a measure of the level of transit service are determined. For empirical purposes, the bus transit market is defined as an urbanized area in which no rapid-rail service exists. This study is limited to those markets served by a single bus transit firm. The parameters of the model are estimated from two cross-sections of data on bus firms and urbanized areas: 51 transit markets in 1968 and 44 transit markets in 1960.

The analysis of the economic behavior of transit firms and regulators is an important aspect of this thesis, inasmuch as the subject has not previously been dealt with in the literature in urban transportation. Regulation which is purely "profit regulation" permits the firm to have considerable flexibility in both the fare and the scale of transit operations.
The firm which maximizes either profit or revenue seeks to operate at the maximum level of service. This scale of operations does not result in maximum passengers, chiefly because the firm is able to attain this scale only by setting an extremely high transit fare. Statistical estimates of supply behavior lead to a rejection of the hypotheses of maximum profit or maximum revenue subject only to profit regulation.

Estimates of the cross-section demand function indicate that transit passengers may be highly responsive to variations in the fare and the level of transit service. The estimated effect of fare increases on transit ridership is contrary to other estimates appearing in the literature. The estimated average elasticities in 1960 (-0.81) and in 1968 (-0.67) are over twice as great as the rule-of-thumb used by the transit industry (-0.30). The results also give some indication of the relationship of transit demand to the characteristics of transit markets. Transit ridership appears to be lower, ceteris paribus, in markets with large percentages of either old or young persons or with large percentages of households earning less than $3,000 per year in 1960.
FOREWORD

This thesis was written at the Institute for Defense Analyses (IDA), Arlington, Virginia, primarily between February 1971 and May 1972. During this period I have benefited from many conversations with my colleagues at IDA, many of whom read and commented on this work in its early stages of development. I am particularly indebted to Dr. John D. Wells, Dr. J. Hayden Boyd, and Dr. Henry M. Peskin for their suggestions and advice during the course of this research.

I would also like to thank the members of my thesis committee--Dr. Charles E. McLure, Jr., Dr. Raymond J. Struyk, and Dr. Ralph Conant--who provided encouragement during the completion of this research and who were instrumental in the shaping of the final document.

Mrs. Julie Wilfang deserves acknowledgement because of her extremely able performance under the duress of typing and re-typing this thesis in the final days before the deadline. Finally, I would like to express my gratitude to my wife, Carol, for her patience and support during the course of this thesis.

G. R. N.
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I. INTRODUCTION

Mass transportation in urban areas has been a declining industry for the past 25 years. Patronage of urban transit systems has fallen sharply since the peak years of World War II.\(^1\) Part of this decline is due to the growth in automobile ownership and to changing patterns of urban land use which have resulted in the rise of suburban residential areas and the decline in the population densities of core cities.\(^2\) The trends in urban transit use have probably been self-reinforcing. Part of the decline in patronage is due to the deteriorating levels of service and the sharply rising fares made necessary by shrinking transit revenues. The bus transit industry, with limited possibilities for technological substitution between capital and labor, has been particularly affected by the increase in wage rates during the past 25 years.

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The problems of urban mass transportation, along with other urban problems—such as housing and crime, have attracted the interest of all levels of government. This is reflected in the trend toward government ownership and operation of local transit properties, in Federal grants for the construction and purchase of transit facilities, and in numerous government-sponsored studies of the problems of urban mass transportation. Despite a considerable amount of research into urban transportation during the past 15 years, knowledge of the economics of the urban transportation market is extremely uneven. A large number of studies have focused on the demand for urban transportation. (Some of the more significant contributions to the economics literature are cited in this Chapter.) The economics literature has also been enriched by studies of the cost and production of urban transportation service. Strangely lacking, however, are economic studies based on the actual behavior of suppliers of urban transit service. The supply and pricing of urban transportation has usually been approached from the standpoint of recommending optimal policy rather than determining actual practices.\(^3\) The large number of cities in the United States with urban transit service would seem to encourage the study of the economic behavior of transit firms, regulatory

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bodies, and other institutions which might influence the supply and pricing of urban transportation. The prevalence of bus transit in the U.S. suggests this industry as a prime candidate for study.

This thesis is concerned with the economic behavior of both users and suppliers of bus transit service. The existence of urbanized areas as self-contained markets for urban mass transportation provides a framework for constructing an economic model of the supply and demand for bus transit service. The analysis of the supply side of the transit market represents an original attempt to model the choices and decisions facing urban transportation firms. A starting point for this analysis is the economic behavior of the regulated firm. Empirical verification is sought for both supply and demand hypotheses in data on transit operations from approximately 50 urbanized areas in the United States.

A. THE BUS TRANSIT MODEL

1. The Topic of the Thesis

This thesis develops a simultaneous-equations model of an urban bus transit market and estimates the parameters of the model from cross-section data on bus firms and transit markets in the United States. The transit market is defined geographically as an urbanized area, consisting of a central city and the surrounding non-rural environs. The sample of
transit markets is restricted to those served by a single bus firm and in which no competing rapid-rail service exists. Thus, the quantity of transit service supplied consists of the output of a single firm. The prevalence of bus transit throughout the United States and the practice of granting exclusive franchises would make it possible—if data were available—to include in this thesis a majority of urbanized areas with a population of more than 50,000.

Data on operations of bus transit firms were made available by the American Transit Association, the national association for urban public transit firms. Two sets of data were developed for this thesis: a sample of 51 bus firms and transit markets for 1968 and a sample of 44 bus firms and transit markets for 1960.

2. The Structure of the Transit Model

The model for an urban bus transit market consists of three simultaneous equations. The equations are probabilistic in that they include some stochastic, or random, variables. A discussion of the stochastic properties of the model, which require special techniques of statistical estimation, is delayed until Chapter V.

4. In the few markets with more than one urban bus firm, the market is carved up so that the various firms serve nonoverlapping routes. This type of market organization is known in the industry as "regulated competition."
The demand equation represents a market demand function for bus transit. In this equation, transit usage (measured in terms of number of passengers) is a function of the bus fare, the characteristics of bus service, the population and population density of the transit market, age and income distribution, and other variables. This equation reflects the behavior of consumers of transit service. The second equation in the model assumes that cost and revenue for the firm have a fixed relationship which may be determined by the maximum allowed rate of return on capital or the existing policy of subsidizing transit. This equation may represent the behavior of transit regulators and other agencies which influence local transit operations. The final equation in the model is a supply-response equation, which relates an index of the level of transit service to such variables as the number of passenger trips, the population of the transit market, and the marginal cost of bus service. The supply equation is based on the hypotheses about the behavior of transit firms, although the level of transit operations may also be influenced by regulators and other organizations which have an interest in local transit service.

3. The Variables in the Transit Model

Variables employed in the transit model are classified into two groups. Endogenous variables are those variables whose values are determined within the context of the model.
There are three endogenous variables in the bus transit model: transit passengers, transit fare, and a measure of the level of transit service. Three endogenous variables are necessary to find a solution to the three-equation transit model. Other variables in the model are exogenous variables. The values of these variables may vary across firms and transit markets, but in terms of supply and demand within a single market these values may be treated as constants. The population of the transit market, for instance, is essentially a datum, because it is assumed that population does not change as a result of transit operations.

It is usually the case in econometric studies that the availability of data pertinent to economic concepts places sharp limits on the ability to use economic theory in empirical studies. Some difficulties do arise in the measure of the price of bus service and in the two measures of quantity—the number of passengers and the level of transit service, but the availability of these operating data does permit the model to be developed through the use of economic theory. In fact, the existence of two quantity measures adds an extra degree of richness to the usual supply-demand analysis.

In the transit model the bus fare is measured by the average revenue per passenger. Transit usage is measured by the annual number of revenue (paying) passengers. The level
of transit service is the annual number of bus miles of transit operations. The use of bus-miles to represent aggregate characteristics of transit service is the least satisfactory aspect of this representation of a bus transit system. The case for using bus-miles rests on the assumption that the only characteristics of bus service which vary significantly are the route-miles (linear mileage of routes served) and the frequency of service. Other aspects of service which affect demand, such as passenger comfort, bus speed, schedule reliability, may be endemic to the bus mode of urban transportation and may vary very little among bus systems. The average frequency of service within a system is the average number of times per day or per year that buses serve each point in the system. Since one definition of average frequency of service is bus-miles divided by route-miles, total bus-miles equal route-miles times the average frequency of service. Increases either in route-miles or in average frequency of service result in proportional increases in bus-miles.

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5. The model does not explicitly recognize the structure of transit fares or the distribution in the lengths of transit trips. The use of average revenue instead of a fare structure and the number of passenger trips instead of passenger miles makes some implicit assumptions about transit systems. Variations in average revenue per passenger is assumed to represent variation in the entire structure of fares. Variation in passenger trips is taken to represent proportional increases or decreases in trips of all lengths.
4. **Empirical Findings**

The generality and validity of the results of this study, as is usually the case in empirical work, are subject to certain limitations and qualifications. First, unless the model and the econometric equations are properly specified, the statistical estimates may provide very little evidence on the supply and demand hypotheses under consideration in this study. Second, the data used in this study may contain errors and, therefore, yield misleading results. Finally, either the sample of transit firms and markets or the two years chosen for the study may not be typical of the U.S. bus transit industry.

The following results are reported, given these qualifications, as findings of this study of the urban bus transit industry:

(1) The supply results offer evidence on hypotheses about the supply behavior of transit firms. The hypotheses that the firms in the industry act to maximize profit or maximize revenue do not provide good explanations for the supply responses observed in the cross-section sample of firms and markets. Furthermore, supply objectives for highly subsidized transit operations appear to be somewhat different from the supply objectives for other transit systems.
(2) Demand estimates give some evidence that the elasticity of demand with respect to transit fares may be as large in absolute value as -0.70. This is considerably larger than the industry's rule-of-thumb, which is approximately -0.30.

(3) Demand estimates also indicate that transit patronage is highly responsive to variations in bus-miles. Thus, both of the policy instruments available to transit firms and local governments—the transit fare and the level of transit service—may be quite effective in increasing or reducing transit ridership.

(4) The results of this study tend to indicate that bus patronage is lower in urbanized areas with large percentages of old and young persons and large percentages of households earning less than $3,000 in 1960.

B. THE TRANSIT MODEL IN THE CONTEXT OF THE ECONOMICS LITERATURE

This thesis makes an attempt to contribute to the economics literature on urban transit in three distinct areas:

(1) An original approach to estimating the demand for bus transit, using cross-section data on urban bus firms and transit markets.
(2) A formal analysis of trade-offs between lower fares and improved transit service, using the theory of the regulated firm to generate supply functions for urban bus transit service.

(3) The derivation and estimation of a simultaneous-equations model of an urban bus transit market, making it possible to analyze the effect of such phenomena as Federal subsidies for urban mass transit.

This thesis overlaps the economics literature on urban transportation in two primary areas. First, a considerable empirical literature already exists on the demand for urban transportation. Second, some of the very recent theoretical literature on the economics of the firm subject to regulatory constraint provides a starting point for the development of the supply side of the transit market. Due to the originality of the supply analysis in this thesis there are no obvious parallels in the urban transportation literature.

1. The Demand for Transit

Studies of the demand for transit may usually be classified into one of three categories: aggregate demand models, based on the total use of mass transit in an urbanized area; interzonal demand models, which seek to predict urban travel by transit and by private automobile between pairs of zones within
the urbanized area; and binary choice models of individual or household behavior which estimate the choice of transportation mode, usually between transit and the private automobile.

**Individual Demand Model**

The most numerous and in many ways the most interesting economic studies of the demand for transit are the binary choice models of individual behavior. Of these, the works of Warner, Beesley, Quarmby and Lisco⁶ are of particular importance. Each of these studies focuses on the choice between transit and automobile use. Emphasis is given to the work trip as opposed to trips for other purposes. These studies are interesting in that individual economic behavior is modeled and that results can also be interpreted in these terms. It is not surprising that researchers have turned to modeling the individual demand for transit given the richness of microeconomic data on this phenomenon.

Researchers have been able to measure costs of using alternative modes quite precisely and to rely on survey data to obtain minute variations in total travel time. (Lisco's work is particularly noteworthy in objectively measuring travel time by alternative modes for individual commuters.)

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Detailed presentation of the binary choice model would constitute a digression; consequently, this section is limited to a discussion of the results. Only the results of Warner, Lisco, and Quarmby can be interpreted to yield the fare-elasticity of the demand for transit. Warner estimated this elasticity to be -0.2, while Lisco estimated a 20 percent fare increase would lead to decrease in ridership of about 7.5 percent, implying an arc-elasticity of -.375. Quarmby gives two estimates for the arc-elasticity of transit demand with respect to fare in the city of Leeds. For a 50 percent fare increase the arc-elasticity is -0.74. For conversion to a free transit system, the arc-elasticity is -0.22. Quarmby's results seem to indicate a higher average elasticity than Warner's or Lisco's.

7. The formula for arc-elasticity is:

\[ \eta = \frac{D_1 - D_2}{\frac{1}{2}(D_1 + D_2)} \frac{F_1 - F_2}{\frac{1}{2}(D_1 + D_2)} \]

where \( D_1 \) and \( F_1 \) are new and old passenger levels and \( D_2 \) and \( F_2 \) are new and old fare levels. With the formula

\[ \eta = \frac{D_1 - D_2}{D_2} \frac{F_1 - F_2}{F_2} \]

the arc-elasticities are -0.59 and -0.43.
The explicit purpose of the studies by Beesley and Lisco and a major purpose of the Quarmby study is the valuation of time spent in traveling. Each study estimated the value of travel time at less than one-half the wage rate. Quarmby's and Beesley's results for commuters in the United Kingdom estimated the value of travel time from about one-fourth to one-third of the wage rate. Lisco's study of Chicago suburbanites found a value of time of $2.67 per hour, about 47 percent of the mean hourly earnings of the commuters. An interesting additional finding by Quarmby is that out-of-vehicle travel time may be valued in excess of twice the rate of in-vehicle travel time.

**Interzonal Demand Model**

The best-known interzonal demand model has been developed by Kraft and others at Charles River Associates (CRA), and applied to data from the Boston urbanized area.

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for transit and the demand for private automobile use are a function of estimated in-vehicle and out-of-vehicle time and cost and also population variables, median income, and car ownership. Time and cost data for both transit and auto use are included in each equation. McGillivray has pointed out the similarity of this interzonal demand model to the "abstract mode" model developed by Quandt and Baumol and applied to inter-city travel. Quandt and Baumol develop the idea that transportation modes are perfect substitutes and that demand is a function of price, travel time, comfort and other factors which can be defined independently of the mode. Data from travel on all modes can be used as observations in estimating the abstract-mode demand functions. A difference between the Quandt-Baumol model and the Boston-area study by Kraft et al. is that the CRA study estimated separate demand functions for transit and auto use.

Kraft et al. found the elasticity of transit demand with respect to travel cost to be only -0.19 while the elasticity with respect to total in-vehicle and out-of-vehicle trip time to be -1.099. The auto-use demand equation shows larger absolute elasticities, but travel time again has a larger

impact than travel cost. The estimating technique raises questions about the reliability of the results. Regression analysis is performed using a quadratic programming technique which constrains direct time and cost elasticities to be non-positive and cross-elasticities to be non-negative. As a result, no auto variable entered the equation for work transit trips. A further characteristic of this quadratic programming technique is that standard deviations are not available for parameter estimates. Estimates were made using logarithmic and linear forms of all variables simultaneously, which may tend to create problems of collinearity. Finally, the $R^2$ for the work transit trip equation was only 0.35, hardly impressive, given the number of variables used.

**Market Demand Model**

The most direct and in many ways the easiest study to perform is a cross-section study of demand, relating aggregate transit ridership to the fare, the characteristics of service, and characteristics of the transit market. This approach to estimating the market demand for bus transit is adopted in this thesis. Strangely enough, there appear to be no studies in either the economics literature or the transportation literature which share this approach. Two earlier studies by Schnore

and Adams\textsuperscript{11} relate transit use in different cities to such variables as size, population density, income, and land use distribution but do not include a price variable for transit service. The lack of any attempt to estimate a complete structural demand equation for transit might be attributable to three factors: (1) American Transit Association data on the operations of firms are not generally available to researchers.\textsuperscript{12} (2) It may have been recognized that transit fare and level of service are not exogenous variables, and thus estimation of the demand function would require specification of the supply side of the transit market. (3) Most large-scale transportation studies have focused on a single urban transportation market. Such studies as the Chicago Area Transportation Study have provided data for several studies of the demand for urban transportation. (See footnote 6.)

2. Possible Shortcomings in Existing Empirical Studies of Demand

It should be a "metatheorem" of econometrics that the better the data and the more relevant the model, the narrower is the range of questions addressed by a study. The existing empirical work on the demand for urban transit appears to bear this out. The binary choice models allow a trade-off between


\textsuperscript{12} The Institute for Defense Analyses, at which this study is being done, is a member of the American Transit Association.
transportation mode, but the total number of trips demanded remains constant. This must lead to an underestimate of the response to increases in fare and changes in the level of service. Beesley, Lisco and Quarmby study only the behavior of commuters, but it seems quite likely that non-work trips are more elastic with respect to fare and service levels than work trips. (The CRA estimates of transit demand show a fare elasticity of -.32 for non-work trips and -.19 for work trips.) Hence, it is not clear what the binary choice studies reveal about the response of total transit ridership to changes in fares and service levels. Very likely the modal-split estimates of the demand for work trips constitute an underestimate of the responsiveness of ridership.

A further problem is the fact that all studies which concentrate on a single transit market observe transportation decisions which occur under a given structure of transit fares for each mode of mass transit. The fare-elasticity is imputed from changes in the length of the trip, which may increase the total fare for the trip, or from variations in automobile costs such as parking fees. In the cross-section sample of transit markets chosen for this study, the base fare varies from 10 cents to over 40 cents.

In using demand estimates based on travel time to aid in transportation planning, travel time must be related to characteristics of transportation service. This relationship
requires detailed knowledge both of the workings of the transit system and the distribution of origins and destinations of transit riders. For many of the policy issues in urban transportation, it is far simpler to have a direct operational link between a service characteristic such as bus-miles and the number of transit riders.

3. The Supply of Transit Service and the Behavior of the Firm

Trade-offs Between Lower Fares and Improved Service

The crucial aspect of the supply side of the transit market is the insufficiency of traditional profit regulation to determine output and pricing in the transit industry. This indeterminacy can be attributed to the existence of an additional degree of freedom in the model of the transit market, in the form of two measures of the quantity of transit service (passengers and bus-miles). The firm may combine fare and level of service in various proportions to satisfy profit regulation. The regulation may either set a maximum rate of return on capital or, more likely in the case of bus transit, set a minimum ratio of operating costs to operating revenues. Moreover, the firm is likely to have a wide range of supply options. At one extreme, the firm may serve only the most heavily traveled routes only during peak hours, operate always with full buses, and thereby charge a minimal fare. Higher levels of service can be supported with higher fares. Under reasonable assumptions an upper bound exists on the level
of service which can be supported by transit passengers through the fare box.

It is certainly not an original concept that trade-offs exist between lower fares and improved levels of transit service.\textsuperscript{13} This model, however, represents the first systematic attempt to analyze this problem from the viewpoint of the supplier of urban mass transit service.

**The Economic Behavior of the Regulated Firm**

A useful approach to the behavior of the transit supplier is suggested by the recent literature on the behavior of the firm under regulation. The pioneering work of Averch and Johnson and the comprehensive study of Bailey and Malone provide an analysis of the resource allocation of the firm under regulatory constraint.\textsuperscript{14} Both of these studies explore the subject within a mathematical framework. The firm allocates resources and sets prices to maximize some objective such as profit or total revenue subject to a constraint imposed by the regulator. The application of this approach to the bus transit firm does lead to

\textsuperscript{13} For instance, see Domencich and Kraft, *Free Transit*, op. cit., p. 2. The idea that various combinations of service levels and fares can be stable equilibria in the competitive taxi industry is explored by George L. Stevens of the University of North Carolina in an unpublished paper.

conditions which determine the fare, the level of transit service, and the number of transit passengers.

Transit regulator and other public agencies may take a direct hand in determining not only fares and routes but actual schedules for bus companies to follow.\textsuperscript{15} Thus, the regulator rather than the firm may determine which combination of transit fare and service level is chosen by the firm. This suggests that performance characteristics, such as number of passenger trips or number of bus-miles, rather than profit-oriented objectives may be the most important factors in the supply and pricing of bus transit service.

The conditions for maximizing either profit or certain performance characteristics subject to regulatory constraint can be used to generate a type of supply function for the bus transit industry.\textsuperscript{16} The function expresses the level of service, measured by bus-miles, in terms of the population and land area of the transit market, the number of passenger trips, the marginal cost of transit service, and the constraint on profit imposed by the regulator. Estimates of the parameters of this function can be used to test hypotheses about the behavior of the bus transit firm.

\textsuperscript{15} The activities of transit regulators are explored by R. L. Banks and Associates, Inc., Study and Evaluation of Local Transit Regulation and Regulatory Bodies, DOT-UT-75 (Washington: 1971).

\textsuperscript{16} Since the function is partially based on marginal revenue conditions, it is not independent of demand, and thus is not a supply function in the purest sense. For simplicity of expression, it will be referred to as a supply function.
C. APPLICATIONS OF THE BUS TRANSIT MODEL

Statistical estimation of the bus transit model can serve three distinct purposes. First, the results may contribute to knowledge about the economic behavior of transit users and firms and the workings of the transit market. Second, estimates of individual parameters, such as the fare-elasticity of transit demand, have an obvious application to real problems faced by transit regulators and other policymakers. Finally, the transit model constitutes a system of simultaneous equations which can be used to evaluate the effects of certain phenomena on the entire transit market. The complete model can be used to estimate changes in ridership, level of service and fare which result from policies, changes in cost, or other factors.

Congress has asked the Department of Transportation (DOT) to evaluate the possibility of Federal operating subsidies for local transit firms. In the opinion of Congress (S-280, 92nd Congress),

immediate substantial Federal assistance is needed on an interim basis to enable many mass transportation systems to continue to provide vital service during the period required to overhaul and revitalize mass transportation operation.

Numerous subsidy mechanisms have been proposed, but the two proposals considered most seriously are subsidies applied as a fraction of operating cost and subsidies which provide a given amount of support per revenue passenger.
Statistical estimates of the bus transit model can be used to provide a quantitative assessment of the impact of the various subsidy proposals on transit fares, levels of service, and number of passengers.

D. ORGANIZATION OF THE THESIS

The remainder of the thesis is organized into five chapters. Chapter II formulates a demand function for a bus transit market. Chapter III develops supply functions for bus transit based on alternative hypotheses concerning transit supply objectives. Chapter IV suggests how ownership or regulation of bus transit by different types of institutions might affect the transit market. Chapter V presents and analyzes the statistical results of the study. Simultaneous-equation estimating techniques are used to estimate the parameters of the equations in the bus transit model. Chapter VI applies the model and the statistical results to the problem of evaluating the impact of operating subsidies on transit markets.
II. THE DEMAND FOR BUS TRANSIT

The market demand for bus transit is the aggregate of individual demand functions. In classical economic theory the demand for any service is a function of the price of that service, the prices of substitutes, the prices of complementary goods and services, and the level of income. Lisco, Warner, and Quarmby, whose studies were cited in Chapter I, have constructed models of individual transportation decisions which closely follow classical demand theory in including income and the cost of using alternative modes of transportation. This thesis attempts to estimate the market demand for bus transit from cross-section data on transit systems. The sample of transit systems exhibits variation in the fare and the level of transit service. Although data on income distribution and other variables are available, the prices of goods and services which are substitutes and complements for urban bus transit are not available. One reason is that the study specifically excludes markets with competing rail and bus systems, but more importantly, data on such items as automobile parking fees were not available for the cross-section sample of transit markets. In many cases, prices relevant for transportation decisions vary enormously throughout the urbanized area, and thus a distribution of prices occurs rather than a single price. Parking fees constitute a case in point.
This Chapter develops a demand function for bus transit in which the number of passengers per time period is a function of the fare and the level of transit service (measured by bus-miles). Other variables which may affect the demand for bus transit are included in the market demand function: the population and land area of the urbanized area, variables expressing automobile availability and highway capacity, the income distribution of households, and the age distribution of the population.

A. DEMAND AND TRANSIT SERVICE

One hypothesis of this thesis is that additional transit service leads to an increase in the number of transit passengers. The effect of transit service on the demand for transit can be explained in terms of its effect in reducing average travel time for potential users of transit service. Studies of individual demand and interzonal demand have shown that travel time has an important bearing on urban transportation decisions.\(^1\) Those studies which have estimated the value of travel time find it to be equal to one-fourth to one-half of the wage rate. Quarmby's study of commuters in the city of Leeds finds that walking time and waiting time are valued considerably more highly than in-vehicle time.\(^2\) This is particularly interesting

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1. See Domencich and Kraft, Lisco, Quarmby, Warner, and Beesley, op. cit.

2. Quarmby, op. cit., p. 291.
since these time components are most sensitive to increases in the level of service.

The components of total trip time for the individual depend on the type of trip and the average speed of transit vehicles and particularly on the location of transit stops, the frequency of service, and the location of routes. While a factor such as vehicle speed may be endemic to the bus mode, service characteristics will vary from individual to individual, depending on his point of departure, his destination, and his preferred time of arrival or departure. Improvement in these characteristics should be responsive to increases in transit service. The proliferation of routes and the addition of buses to existing routes serve to reduce the time cost of transit use. Since time can be given a monetary value, the improvement of service may constitute a de facto decrease in the fare. Consequently, if total service is increased, the number of transit passengers would be expected to increase.

The demand for transit in this cross-section study is defined as the aggregate demand in a transit market and is measured by the number of passenger trips. Transit passenger trips are a function of the average fare and the total bus-miles of transit service.\(^3\) The measurement of transit service

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3. Annual data are reported to the American Transit Association (ATA) by member firms and published in ATA Transit Operating Reports, Part II, "Motor Bus Operations."
by total bus-miles involves strong assumptions. In using bus-miles as a proxy for service, we are assuming that scheduling and route assignment practices are similar among firms and that demand for urban transportation in urbanized areas is similar in terms of the distribution of times and destinations for urban trips. Thus, in two cities of equal population and land area, a given number of bus-miles per year implies roughly similar service characteristics in the two transit markets. Other service characteristics, such as route miles and frequency of service, can be obtained, but these data are less reliable than bus-miles and are reported by fewer firms.

1. **A Priori Considerations**

The shape of the relationship assumed to exist between demand and bus-miles depends on several assumptions:

- The firm allocates a given amount of service to maximize the number of transit riders. In so doing, bus trips are added first where the greatest net increment in ridership occurs; second, where

---

4. Among the more obvious shortcomings of bus-miles is the inclusion of "dead mileage" incurred when buses are not servicing routes. This shortcoming becomes relatively unimportant if the proportion of dead mileage to total mileage is relatively constant across systems.

5. A measure such as seat-miles of service was also rejected. At the relatively low passenger densities in the industry (three to four boardings per bus-mile), additional bus capacity may have little effect on service characteristics.
the second greatest increment occurs, and so on.

- This method of allocating service implies diminishing incremental gains in passengers per additional unit of service in the absence of certain external effects which are assumed to be nonexistent. The incremental number of passengers from an additional bus-mile declines as total bus-miles increase. The absence of external effects implies that additional service along one route in the system does not increase passengers on some unconnected route. 6

6. Although some case for the existence of external effects has been made, it does not appear to be a phenomenon of overriding significance for aggregate demand.

In an unpublished paper on economies of scale in transit, Herbert Mohring has pointed out that additional service on a particular route may decrease the expected waiting time. These decreases in cost may lead to increasing increments in ridership as service becomes more frequent. Mohring's result depends on the randomness of bus service, or at least randomness from the viewpoint of the user. If increasing returns are large, there is a large payoff to efforts by the firm to publish schedules and to follow them. Furthermore, the Mohring result applies on a route-by-route basis. One would expect the firm, in the process of allocating service, to concentrate on service routes to take advantage of these increasing returns as a result of this allocation process.
2. **A Specific Functional Form**

The demand function used in this paper has a declining elasticity of demand with respect to additional bus-miles. The demand function is

\[ D = \exp \left[ -\alpha_1 \left( \frac{B}{S} \right)^{-\alpha'} \right] f(\cdot), \]  

where \( f(\cdot) \) includes all other variables in the function. \( D \) is demand, \( B \) is bus-miles, \( S \) is a measure of the size of the transit market, and \( f(\cdot) \) is a function of the other variables discussed in Sections B and C. The size variable, \( S \), may stand for either population (POP) or land area in square miles (AREA). A general definition of size which includes both population and land area is \( S = \text{POP}^\lambda \text{AREA}^{1-\lambda} \), where \( \lambda \) may vary from zero to one. Both parameters \( \alpha_1 \) and \( \alpha' \) are expected to be positive.

This exponential function exhibits a declining service-elasticity of demand as the quantity of bus-service increases relative to the size of the market. This elasticity is

\[ \varepsilon_B = \alpha_1 \alpha' \left( \frac{B}{S} \right)^{-\alpha_1}. \]  

The attractiveness of this function is attributable to the property of a declining service elasticity. The rate of decline in elasticity is regulated by the parameter \( \alpha' \). At very small \( \alpha' \) the service-elasticity tends to remain constant.
Figure 1 is a graph of passengers as a function of bus service. Passengers are an increasing function of the level of service but asymptotically approach an upper bound as service continues to increase. Although demand exhibits decreasing elasticity as service increases, there are increases in the incremental number of passengers in the dotted portion of the curve between the origin and $B_0$. Demand elasticity, although diminishing in this interval, is still larger than one; thus, the increasing returns.  

7. Although a demand function can be specified which does satisfy a priori considerations in every interval of $B/S$, the weakness in using a function which meets these considerations in an open-ended interval may not be too serious. Furthermore, there is no obvious situation in which any transit firm would be willing to supply a level of service less than $B_0$, because the marginal gains in revenue should exceed the marginal costs of bus service. The compensating strengths of this exponential function are substantial. Briefly, use of this demand function leads to a simple and testable set of supply hypotheses. Also, most of the parameters of the function can be estimated using linear regression techniques. One function which does satisfy all a priori requirements is

$$D = \left[1 - \exp \left(-\alpha \frac{B}{S}\right)\right]^\alpha f(\cdot).$$

A drawback to the use of this function is that it is not possible to get analytical solutions for supply behavior under certain objectives. Thus, since a primary focus of this paper is empirical knowledge of the behavior of the firm, use of this function is, in a sense, self-defeating. Another drawback is the necessity of using non-linear regression to estimate the parameters of the demand function.
B. DEMAND AND FARE

Higher fares reduce passenger trips by making other modes of transportation more attractive and by reducing the total quantity of urban travel. The bus transit model is a model of a single-fare transit system. Although many systems have a structure of fares which may depend on the length of the trip (interzonal charges), the time of day (shoppers' specials), or the age of the passenger (youth and senior-citizen discounts), the fare referred to in this model is the average revenue per
paying passenger. Despite the existence of a structure of fares, it is still true that heavily traveled routes subsidize lightly traveled routes and service during rush hours subsidizes non-rush-hour service.

The total revenue function facing the firm is a unimodal curve with a single local maximum (Figure 2). This is the basic a priori consideration about the effect of fare on demand. At a very low fare, increases in the fare lead to

FIGURE 2. Unimodal Total Revenue Function
increases in total revenue. Beyond some critical fare, however, additional fare increases are more than offset by declines in ridership, and total revenue declines. At this critical fare, \( F^* \), total revenue is a maximum. Another way to state this assumption is that the elasticity of demand with respect to the fare increases from below one as the fare increases, and at some fare \( (F^*) \) the elasticity becomes greater than one. Total revenue is a maximum where the fare-elasticity of demand equals one.

Although a number of demand functions satisfy this condition, a simple function that is easy to adapt to the transit model is the following:

\[
D = e^{-\alpha_2 F} g(\cdot) \quad (3)
\]

\( F \) is fare, and \( g(\cdot) \) is a function of bus-miles and the variables discussed in Section C. The elasticity of demand is equal simply to \(-\alpha_2 F\), and this elasticity is unity where \( F = \frac{1}{\alpha_2} \).

In Figure 3, the rectangle represents maximum total revenue \( F^* \cdot D^* \).

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8. The most common demand function of this type is the straight-line function: \( D = (\alpha_1 - \alpha_2 F) g(\cdot) \). At a zero fare, demand equals \( \alpha_1 \) and at a fare of \( \alpha_1/\alpha_2 \) no one is willing to use bus transit service. The drawback of this function is that it requires nonlinear estimation of the parameters and that it is difficult to use in the supply analysis of Chapter III.
The transit industry's rule-of-thumb, known as the Simpson and Curtin formula, is that the fare-elasticity of demand is a constant with a value \(-0.3\). In light of the assumption that the elasticity varies with the fare, an elasticity of \(-0.3\) would apply only at one fare. A property of the demand function developed in this section is that elasticity is proportional to the fare. If the fare elasticity is \(-0.3\) at a fare of 25 cents, then at a fare of 50 cents the elasticity is \(-0.6\).
C. THE LOGARITHMIC FORM OF THE DEMAND FUNCTION

Estimates of the parameters of the demand function are based on linear regression of a logarithmic form of the demand function. The complete demand function takes the following form:

\[
\ln D = \alpha_0 - \alpha_1 \left( \frac{B}{S} \right)^{-\alpha} - \alpha_2 F + \alpha_3 \ln \text{POP} \\
+ \alpha_4 \ln \text{AREA} + \alpha_5 \ln \text{AUTOS} + \alpha_6 \ln \text{HWAY} \\
+ \alpha_7 \text{INC}_3 + \alpha_8 \text{INC}_{10} + \alpha_9 \text{AGE}_{18} + \alpha_{10} \text{AGE}_{65} \tag{4}
\]

where

POP = population of the urbanized area,
AREA = land area (square miles) of the urbanized area,
AUTOS = automobiles available per capita,
HWAY = population per unit of urban highway capacity,
INC$_3$ = proportion of households with income of less than $3,000 per year (1960),
INC$_{10}$ = proportion of households with income in excess of $10,000 per year (1960),
AGE$_{18}$ = proportion of population 18 years of age or younger (1960),
AGE$_{65}$ = proportion of population 65 years of age or older (1960),
\(\ln\) = natural logarithm (to the base \(e = 2.71\ldots\)).
In actual estimates of the demand function, the bus-service variable is always defined in terms of population. (Thus, for $S = \text{POP}^\lambda \text{AREA}^{1-\lambda}$, $\lambda = 1$.) Land area and population, however, both appear as separate variables in the equation. Age and income data for 1960 are used in both sets of estimates. Although income and age distributions are definitely different in 1968 from the 1960 figures, it seems unlikely that the relative ranks of cities changed greatly during this eight-year period. Further discussion of the definitions of the variables and a listing of sources of data appear in Appendix B.

1. Population and Land Area ($\ln \text{POP}, \ln \text{AREA}$)

The coefficients of $\ln \text{POP}$ and $\ln \text{AREA}$ describe what happens to demand as population increases and population density changes.\(^9\) Suppose all other variables remain constant, including bus-miles per capita ($B/S$). If population and land area both increase by one percent (i.e., density remains constant), demand increases by $\alpha_3 + \alpha_4$ percent. If only population increases, demand increases by $\alpha_3$ percent.

---

9. Since population and land area are highly correlated, it may appear that including both variables in the model introduces a problem of collinearity. $\ln \text{POP}$ and $\ln \text{AREA}$ are less highly correlated than POP and AREA. Collinearity does not create a problem in estimating the effect of market size in either demand or supply functions, because when population and land area both vary, the standard error of the effect is not large.
2. **Automobiles and Highway Capacity** *(in AUTOS, in HWAY)*

Other things equal, an increase in the availability of automobiles should decrease the demand for bus transit. Increases in highway capacity, however, may decrease travel time for both autos and bus transit. Thus, while additional highway construction may increase the demand for urban transportation, the predicted effect on choice of mode is indeterminate.

3. **The Income Distribution of Households and the Age Distribution of the Population** *(INC$_3$, INC$_{10}$, AGE$_{18}$, AGE$_{65}$)*

The demand for bus transit is a composite for the demand for urban transportation and the choice of mode. Factors which appear to make bus transit relatively more attractive than automobiles may actually lead to decreases in the demand for bus transit if the factor simultaneously reduces the demand for urban transportation. The old, the young, and the poor must rely on bus transit. Members of the labor force, most of whom do not fall in these categories, usually make at least two urban trips daily. Estimates of these effects of age distribution and income distribution provide a test of the relative strengths of these two factors.

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10. Using these variables as logarithms makes it impossible to derive a consistent meaning for the behavior of individuals in the omitted parts of the income and age distributions. However, if the variables are in nonlogarithmic form an increase in the proportion of the population between 19 and 64 or an increase in the proportion of household earnings between $3,000 and $10,000 per year can be calculated from the coefficients of AGE$_{18}$, AGE$_{65}$, INC$_3$, and INC$_{10}$. 
On the basis of 1960 Census data, each transit market is given an age distribution of the population and an income distribution for households. The age groupings are 18 or younger, 19 to 64, and 65 or older. The income distribution groups are less than $3,000 per year, $3,000 to $10,000 per year, and more than $10,000. The proportion falling into the two extremes of each distribution are included as variables in the demand function.
III. SUPPLY ASPECTS OF THE TRANSIT MARKET

The supply side of the model of the transit market encompasses a wide range of economic phenomena, including cost concepts and cost functions for bus transit service (Section A), the existence of financial constraints which typically take the form of profit regulation (Section B), and the supply and pricing of transit service by bus transit firms (Sections C and D). An analysis which includes all of these aspects of transit supply leads to the development of a bus transit supply function. Estimates of the supply function, which is derived in Section E, provide a test of various hypotheses concerning supply behavior.

A. THE COST OF BUS TRANSIT SERVICE

The concept of cost used here is the total cost of transit operations borne by the firm.¹ This definition includes

¹ The total social cost of transit operations may differ from the private cost incurred by the firm. This notion of private cost excludes the time cost of users of the transit system, specifically the value of portal-to-portal time spent in urban travel. The value of travel time appears (at least implicitly) in the demand function in this model of transit operations. Social cost may differ from private cost if the cost of building and maintaining freeways and roads which is allocable to bus transit is different from the amount paid by the firm in road-use taxes on tires and fuel. The externalities of urban transportation, particularly congestion and pollution, are another excluded element of the social cost of transportation. The net effect of urban transit operations may result in a decrease in congestion and pollution through a reduction in automobile use. Nevertheless, bus transit service does contribute to both air pollution and traffic congestion, even if these are outweighed by the beneficial effects.
depreciation and an interest charge on total capital, in addition to what is commonly called operating costs. If capital costs are partially paid for by a Department of Transportation capital grant, then this subsidy reduces total costs\(^2\) under this definition. Additional details of capital costs and operating costs, as well as estimates of a cost function for bus transit appear in Appendix A.

The average cost per bus-mile in any transit operation is assumed to be a constant. This assumption implies that total cost is proportional to the level of service with no economies of scale or inelasticities in the supply of capital or labor. The parameters of the cost function estimated in Appendix A show the elasticity of cost with respect to bus-miles to be 0.982 in 1968 and 1.013 in 1960. Neither estimate is significantly different from 1.0, the elasticity prevailing under constant returns to scale. Thus, increasing bus-miles by 100 percent would increase costs by approximately 100 percent. Although the results of this paper do not depend on a strict proportionality between cost and quantity of service, this simplifying assumption facilitates analysis.

\(^2\) The Department of Transportation through the Urban Mass Transportation Administration began a program of capital grants to urban transportation projects in February 1965. By the end of 1970, over $700 million in Federal funds were committed for 155 capital grant projects.
The cost function for bus transit may more clearly resemble the cost function for a taxi fleet than the cost functions for other modes of mass transit such as rapid rail. The major elements of bus costs are roughly proportional to bus-miles of service.\textsuperscript{3} Rail costs include costs of stations, track, and right-of-way. These costs are less than proportional to vehicle miles, resulting in economies of scale.

B. A COST-REVENUE RESTRICTION

The bus transit market is a model involving three variables--bus-miles, average fare, and bus passenger trips. Cost per bus-mile, population, and other variables are constants in a given market. Three independent relationships are necessary to obtain a mathematical solution of the transit model. One such relationship is the demand function explained in Chapter II.

\[ D = D (B, F). \] (5)

In this model, the remaining relationships are related to the behavior of transit firms and regulating agencies, referred to

\textsuperscript{3} Bus costs are predominantly drivers' wages (40 percent); fuel (10 percent); maintenance (15 percent); insurance, advertising, and taxes (12 percent); administrative costs (11 percent); and vehicle costs (10 percent). Vehicle costs consist of depreciation (10 percent of estimated value) and interest or normal profit (77 percent of total estimated value). Station costs are only 1 percent. Vehicle cost items are discussed in detail in Appendix A. Other items of operating costs (reduced by 10 percent to reflect the inclusion of vehicle costs) are taken from J. D. Wells and Sharron Thomas, Economic Characteristics of the Urban Bus Transit Industry, 1971, TDA.
here as supply behavior. One possible hypothesis which, presumably, applies to nonregulated industries is that the firm sets the fare and the level of service to maximize profit. The formal regulation of privately owned firms and the nature of the ownership of publicly owned firms suggests that unconstrained profit maximization is not an appropriate model for the bus transit industry.

It is likely that most firms in the bus transit industry operate under some financial constraint, restricting the size of either the surplus or deficit. For privately owned firms, the binding constraint may be the maximum profit permitted by the regulatory mechanism. Regulators usually control profit by limiting either the rate of return on capital or the operating ratio (operating costs divided by operating revenue). Informal aspects of the mechanism, such as the lag in permitting fare increases or service reduction, may also be very important determinants of the allowed profitability of transit operation. For publicly owned firms, a constraint may apply to the maximum loss which the public authority is willing or able to subsidize. Independent transit authorities may have no resources with which to subsidize transit operations or may have requirements imposed by bond underwriters that revenues must cover all operating costs. City-run firms may be able to tap the general revenues of the city to subsidize transit service.
These various constraints are summarized by the following requirement placed on transit firms:

\[ k_o = \frac{cB}{FD} \]  \hspace{1cm} (6)

\( k_o \) is the cost-revenue ratio; \( c \) is the cost per bus-mile. The cost-revenue ratio is distinct from the operating ratio in including interest and normal profit as well as depreciation. The requirement of this second relationship in the transit model is that the cost-revenue ratio, \( k \), must equal some predetermined value, \( k_o \). In the empirical hypothesis discussed in Chapter IV and estimated in Chapter V, the value of \( k_o \) is determined by the types of institutions which regulate and own the bus firms.

The cost-revenue restriction is identical to setting a maximum return on capital or a minimum operating ratio. If capital and labor are not substitutable in the production function for bus transit, and if firms operate at the regulatory constraint, these conditions are easy to verify. If \( R \) is total revenue \((R = F \cdot D)\), \( L \) is labor, \( K \) is capital, \( w \) is the wage rate, and \( r \) is "normal" profit per unit of capital, the three policies can be expressed as follows:

rate-of-return regulation--

\[ \rho_o \leq \frac{R - wL}{K} , \]  \hspace{1cm} (7)

operating-ratio regulation--

\[ h_o \geq \frac{wL}{R} , \]  \hspace{1cm} (8)
cost-revenue restriction--

\[ k_o = \frac{rk + wL}{R} \]  \hspace{1cm} (9)

If the firm operates at the regulatory constraint \( \rho_o \) of the constraint \( h_o \), the inequality becomes an equality. If capital and labor are not substitutable, capital and labor are applied in fixed proportions in the production process. Assume this proportion is \( s = K/L \), or \( K = sL \). It is simple to show that the cost-revenue restriction can be related to \( \rho_o \) and \( h_o \) using only the constants \( r, w, \) and \( s \). In particular,

\[ \frac{w/s}{r + w/s} \cdot k_o = h_o \]  \hspace{1cm} (10)

and

\[ \frac{r + w/s}{k_o} - w/s = \rho_o \]  \hspace{1cm} (11)

Thus, under these conditions, the regulation of the rate of return on capital, the operating ratio, and the cost-revenue can be made equivalent.

If capital and labor are substitutable, regulation either by rate of return or the operating ratio may lead to a different level of output or a different capital-labor ratio from that achieved by restricting the cost-revenue ratio.\(^4\) The slowly

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changing nature of the bus transportation technology in the United States during the past two decades suggests that the degree of substitutability between capital and labor may indeed be quite low. There is also some empirical evidence that tends to support this assumption.\(^5\) In this case, the representation of other types of regulation with the cost-revenue ratio may be a very close approximation.

C. ALTERNATIVE HYPOTHESES OF TRANSIT SUPPLY BEHAVIOR

1. Supply Options in the Transit Market

A distinguishing feature of the model of the bus transit market is the wide range of supply options that are consistent with the demand function, the cost function, and the financial constraint placed on transit firms. Figure 4 shows a locus of these supply options in terms of bus-miles and number of transit passengers. The transit fare increases along this locus in a clockwise direction. At a minimal level of operations the firm may serve only the most heavily traveled routes only during rush hour and operate almost exclusively with full buses. With these

\(^{5}\) Although the bus transit technology in the U.S. may be described as one driver, one bus, substitution may involve nondriver personnel and nonbus capital. Furthermore, by varying the size of the bus or the number of shifts using the same bus, the capital-labor ratio can be varied. Some as yet unpublished IDA econometric work has yielded estimates of the elasticity of substitution (ES) in bus transit. If fixed proportions prevail, ES equals 0.0. If perfect substitutability exists, ES equals ∞. The estimate of ES for the bus transit industry is 0.27. At this value of ES, capital and labor are relatively insubstitutable.
high passenger densities the firm is able to charge only a minimal fare. As long as the fare-elasticity of demand is less than one (in absolute terms), an increase in fare yields additional revenue, permitting the firm to provide additional bus-miles without reducing the cost-revenue ratio. One result which is demonstrated in this section is that the maximum level of service occurs where the fare-elasticity $\xi_F$ equals -1.0. Quite interestingly, in terms of the incentives of bus transit supply, the maximum number of passengers does not occur at the maximum level of service. Maximum passengers occur at some lower level of
service where the increase in passengers as a result of the increase in service just offsets the decline in ridership resulting from an increase in the fare. The dotted portion of the locus of transit supply options represents a combination of low levels of service and high fares which is uniformly inferior to the other points on the supply locus.

2. **Maximum Bus-Miles**

In recent years there has been a substantial interest among economists in the theory of the firm under regulation. A typical assumption is that the regulator limits the earnings of the firm by specifying a maximum rate of return on capital or a minimum ratio of cost to revenue. Except for this constraint, the firm is free to pursue an objective such as profit maximization or revenue maximization.6

Section B has shown that the cost-revenue ratio, the operating ratio, and the rate of return on capital are equivalent regulations under assumptions that are at least approximately

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6. In the pioneering study of the behavior of the firm under regulation, H. Averch and L. Johnson in "Behavior of the Firm Under Regulatory Constraint," American Economic Review, LII (December 1962), show that profit-maximizing firms operating under a maximum rate of return on capital may tend to overcapitalize and to expand the rate base by pursuing unprofitable ventures. Most relevant to our model are the results of E. Bailey and J. Malone, op. cit. If the regulator sets a minimum cost-revenue ratio, the profit-maximizing and revenue-maximizing firm "will want to operate where marginal revenue is zero, even if it means increasing costs by some arbitrary means" (p. 140). Thus, the firm is prepared to increase allowable costs by any method in order to increase total revenue.
satisfied in the bus transit industry. The simplest case to analyze concerning the motivation of the firm is that of revenue maximization. Since the financial constraint requires cost and revenue to remain in a fixed proportion, revenue is maximized where cost is maximized. Maximum cost occurs at maximum bus-miles. Hence, the firm which maximizes revenue subject to the financial constraint acts to maximize bus-miles.

If the firm is allowed by the financial constraint to earn at least a normal profit, the case of the profit maximizer is identical to that of the revenue-maximizer. The firm maximizes profit by maximizing total revenue because profit is limited by the financial constraint to a fixed proportion of total revenue. Thus, if the supplier's incentive is to maximize profit, the fare and level of service are adjusted to yield maximum bus-miles.\(^7\)

Other factors may tend to support maximum service as the eventual, if not the optimal, outcome. Maximum bus-miles may yield maximum employment and thus be the preferred position of the transit union. Second, in a market where demand is declining relative to cost, if the adjustments occur chiefly through fare increases rather than service reductions, transit operations will inevitably arrive in a situation where no other fare increases are possible. The service provided at this point is the maximum attainable at the allowed ratio of cost to revenue.

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7. If the cost-revenue ratio does not allow the firm to cover all costs and earn a normal profit, the firm maximizes profit (or minimizes losses) in the long run by halting operations completely.
Maximum bus-miles subject to the cost-revenue restriction is found by maximizing

\[ B = \frac{k_o}{c} F \cdot D \]  \hspace{1cm} (12)

\[ \frac{\partial B}{\partial F} = \frac{k_o}{c} (D + F \frac{\partial D}{\partial F}) = 0. \]  \hspace{1cm} (13)

\[ F \frac{\partial D}{\partial F} = -D \]  \hspace{1cm} (14)

\[ \frac{F}{D} \frac{\partial D}{\partial F} = -1 = \xi_F. \]  \hspace{1cm} (15)

The fare at which the fare-elasticity is equal to 1.0 also maximizes total revenue. As indicated previously, maximum bus-miles and maximum total revenue occur at the same fare and bus-miles.

3. Maximization of Passenger Trips

In many ways, maximizing passengers is a more desirable outcome than maximizing bus-miles. There is greater consumption of transit service. The fare for transit users is less than the fare at maximum bus-miles, although characteristics of service are not as good. Finally, the total cost of transit service is less, at least if cost is measured only in terms of inputs provided by the firm (excluding social costs). There does not appear to be any economic incentive for the private firm to want to operate at this point unless this is preferred by the regulator or another influential agency. Thus, the realization of this
outcome hinges on the ability and desire of the regulatory authorities and other public agencies to control the fare and the level of transit service.

A formulation for maximizing ridership using the Lagrangian multiplier $\lambda$ is

$$\Lambda = D(B,F) - \lambda \left( k_o - \frac{CB}{F \cdot D(B,F)} \right).$$  \hfill (16)

Differentiating $\Lambda$ with respect to $B, F$, and $\lambda$ and setting these partial derivatives to zero yields:

$$\frac{\partial \Lambda}{\partial F} = \frac{\partial D}{\partial F} + \lambda \frac{\partial}{\partial F} \left( \frac{CB}{F \cdot D} \right) = 0,$$  \hfill (17)

$$\frac{\partial \Lambda}{\partial B} = \frac{\partial D}{\partial B} + \lambda \frac{\partial}{\partial B} \left( \frac{CB}{F \cdot D} \right) = 0,$$  \hfill (18)

$$\frac{\partial \Lambda}{\partial \lambda} = k_o - \frac{CB}{F \cdot D} = 0.$$  \hfill (19)

If $\lambda$ is eliminated from the first two expressions,

$$\frac{\frac{\partial D}{\partial F}}{\frac{\partial}{\partial F} \left( \frac{CB}{F \cdot D} \right)} = \frac{\frac{\partial D}{\partial B}}{\frac{\partial}{\partial B} \left( \frac{CB}{F \cdot D} \right)}.$$  \hfill (20)

Carrying out the differentiation in the denominators and using the third expression from above yields:

$$-k_o \left( \frac{1}{F} + \frac{1}{D} \frac{\partial D}{\partial F} \right) = k_o \left( \frac{1}{F} - \frac{1}{D} \frac{\partial D}{\partial B} \right).$$  \hfill (21)
This yields
\[
- \frac{F}{D} \frac{\partial D}{\partial F} (1 + \frac{F}{D} \frac{\partial D}{\partial F}) = \frac{B}{D} \frac{\partial D}{\partial B} (1 - \frac{B}{D} \frac{\partial D}{\partial B}) .
\]  
(22)

The fare-elasticity is \( \frac{F}{D} \frac{\partial D}{\partial F} \), and the service-elasticity is
\( \frac{B}{D} \frac{\partial D}{\partial B} \). Therefore,
\[
\frac{-\epsilon_F}{1 + \epsilon_F} = \frac{\epsilon_B}{1 - \epsilon_B} .
\]  
(23)

Or,
\[
-\epsilon_F = \epsilon_B .
\]  
(24)

Thus, for a given cost-revenue ratio, the maximum number of passengers in a bus transit system occurs where fare and bus-miles are set such that the elasticity of demand with respect to the fare is equal to the elasticity of demand with respect to bus-miles. This is an intuitively plausible result. The maximum number of transit passengers occurs where the percentage reduction in passengers due to a one percent increase in the fare

---

8. Second-order conditions for a maximum are satisfied if
\[
B^2 \frac{\partial^2 D}{\partial B^2} + 2BF \frac{\partial^2 D}{\partial B \partial F} + F^2 \frac{\partial^2 D}{\partial F^2} > 0 .
\]

It is easy to show that this relatively weak condition is met by the demand function derived in Section B.
is exactly balanced by the percentage increase in passengers due to a one percent increase in bus-miles. The one percent change in both fare and bus-miles, \textit{ceteris paribus}, leaves the cost-revenue ratio unchanged.

4. \textbf{Profit Maximization}

The analysis of supply objectives featuring maximum passengers and maximum profit subject to regulatory constraint invites comparison with other hypotheses in which a financial constraint does not play a major role. An obvious choice of such a comparison is the case of unconstrained profit maximization, which is analyzed in this subsection. The case of free transit, where the transit fare is zero, is examined in the following subsection.

The pricing condition for profit maximization is identical with the pricing condition for maximum bus-miles; however, the profit-maximization criterion for setting the level of service insures that a system run by an unconstrained profit maximizer will have a lower level of service and fewer passengers than the system where the profit maximizer is subject to regulatory constraint.

The problem of profit maximization can be formally stated:

\[
\max_{B, F} \pi = F \cdot D(B, F) - cB \quad \quad \quad (25)
\]

The firm sets the fare such that the marginal revenue from an additional increase in fare is equal to zero. The firm sets the level of service such that the marginal revenue of an additional
bus-mile is equal to the marginal cost of an additional bus-mile, c. These two conditions are:

\[ F \frac{\partial D}{\partial F} = \epsilon_F = -1 \]  \hspace{1cm} (26)

and

\[ F \frac{\partial D}{\partial B} = c. \]  \hspace{1cm} (27)

Note that the former condition is the same as the condition for maximizing service.\(^9\) It is simple to show that the profit maximizer provides a lower level of service than the service maximizer. If under maximum service the constraint is point \( k_0 = 1 \), or total cost equals total revenue, then that constraint can be written:

\[ F \frac{D}{B} = c. \]  \hspace{1cm} (28)

In a market with diminishing marginal returns per bus-mile, \( \frac{\partial D}{\partial B} \) will always be smaller than \( F \frac{D}{B} \). Thus, the profit maximizer, although he uses the same pricing principle, will establish a lower level of service.

---

9. In a conventional market where demand is \( Q = Q(P) \) and cost is \( c \cdot Q \), the single marginal condition is that:

\[ PQ'(P) = c. \]
5. **Free Transit**

The cause of free transit has attracted a number of adherents in recent years. The case for free transit is often built on the argument that unless the system is at capacity an increase in the number of passengers does not materially increase the costs of providing transit service. Thus, as the fare is lowered, there is a net increase in benefits to the users of transit service but in the absence of capacity restrictions, no offsetting increase in the real costs of transit service. Suppose the fare is zero, and the subsidy provided by the government is $S$. Then a free transit system with bus-miles $B = S/c$ can be created.

At the level of subsidy $S$ which government is willing to provide, a free transit policy may not yield the maximum ridership. If the number of passengers lost by raising the fare from zero to one cent is less than the number of passengers that can be gained by spending this revenue on additional bus service, then greater ridership can be obtained by increasing the fare and providing additional transit service. In particular, this condition is:\(^{(11)}\)

---

10. The best-known study of free transit is the Charles River Associates study of Boston, **op. cit.**

11. The general condition at a nonzero fare is that

\[
\frac{-\frac{\partial D}{\partial F}}{D + F} > \frac{\frac{\partial D}{\partial B}}{c - \frac{\partial D}{\partial B}} F.
\]

This condition can be derived from the Lagrangian formulation:

\[
V = D(B,F) + \lambda[S - F \cdot D(B,F) + cB].
\]
- \frac{\partial D}{\partial F} D > \frac{\partial D}{\partial B} c. \quad (29)

6. A General Objective Function $D^\theta B^{1-\theta}$

Maximum bus-miles and maximum passengers are special cases of maximizing the "supply objective" $D^\theta B^{1-\theta}$. At $\theta = 0$, bus-miles are maximized. At $\theta = 1$, passenger trips are maximized. It is highly probable that transit fares and levels of service are not determined by any of the simple patterns of behavior suggested previously. The welter of private firms, government agencies, labor unions, and consumer groups which may influence the operation of public transit introduces the possibility of economic behavior based on the interaction of these various groups. Rather than begin the fruitful but arduous task of modeling these interactions, economic behavior in the transit market is investigated with the aid of this general, but quite artificial, transit supply objective. As the parameter $\theta$ is varied, the maximum solution moves around the transit supply locus (Figure 5). This permits us to describe each interval on the supply locus in terms of maximizing some supply objective.

The objective function $D^\theta B^{1-\theta}$ may be interpreted as a family of indifference curves, representing not the preferences of the firm or the consumer but the collective preference of all institutions and individuals which influence the supply and pricing of bus transit service. The parameter $\theta$ is not restricted exclusively
FIGURE 5. A Comparison of Optimal Solutions Under Different Supply Objectives

to values between zero and one. At values of $\theta$ greater than one the indifference curves assign a negative value to additional bus service. The result is that the fare and the level of service are both below the levels which maximize transit passengers. 12

12. This result may be reasonable if bus operations contribute significantly to congestion or pollution. A lower fare and a lower level of service permits a higher ratio of passengers to bus-miles. This economizes on the inefficient production of bus-miles.
Maximum $D^\theta B^{1-\theta}$ occurs at the maximum of the Lagrangian expression

$$\Lambda = D^\theta B^{1-\theta} - \lambda \left( k_o - \frac{CB}{F \cdot D} \right) \quad (30)$$

Solution follows the lines of the solution in Section 3 to the problem of maximizing $D$ subject to the cost-revenue constraint. The condition for a maximum involves the fare-elasticity, the service-elasticity and the parameter $\theta$:

$$-\varepsilon_F = \theta \varepsilon_B + (1 - \theta). \quad (31)$$

Note that for $\theta = 1$, $-\varepsilon_F = \varepsilon_B$, and for $\theta = 0$, $\varepsilon_F = -1$.

Variation of the value of $\theta$ is a useful way to demonstrate the relevance of different points on the transit supply locus. There are five possible ranges of values for $\theta$: $\theta = 0$, $\theta = 1$, $0 < \theta < 1$, $\theta > 1$, and $\theta < 0$. For a value of $\theta$ in each of these intervals we will reach a maximum at some point on the locus. For a value of $\theta$ in each of these intervals. The condition for an optimum

$$-\varepsilon_F = \theta \varepsilon_B + (1 - \theta) \quad (32)$$

enables us to solve also for the range in values of the fare-elasticity and the service-elasticity of demand in each of these five intervals. These values appear in Figure 6.

13. Except for very large or very small values of $\theta$, which may approach a maximum in the neighborhood of the origin.
(1) In the first interval, both demand and bus-miles are increasing. The effect of increases in the fare on passengers is more than outweighed by the effect of increases in bus-miles. Thus, the service-elasticity is greater than the fare-elasticity. Both lie between 0 and 1.0 in absolute value. The meaning of the parameter $\theta > 1$ is that the supply objective rewards ridership, $D$, but penalizes additional service, $B$. Hence the bias toward a solution with relatively low fares, but relatively low levels of service.

(2) This is the point of maximum ridership.

(3) This interval is the reverse of (1). The effect of increases in fare more than outweighs the increases in service. Consequently, demand is declining. A point on this interval may be optimal if the objective values bus service in its own right, independent of its effect on passengers.

(4) This is the point of maximum service.

(5) In this interval the fare-elasticity of demand is greater than one. Both ridership and revenue would increase if the fare were lowered. This interval can contain the optimum only if the objective penalizes ridership.
D. A VARYING SUPPLY OBJECTIVE

The choice between lower fares and higher levels of service, which is made in selecting a point on the transit supply locus, may vary with changes in the cost-revenue ratio. As the cost-revenue ratio increases, due perhaps to government subsidies, the character of the transit solution may change. If a firm is to maintain a position of maximum ridership on the supply locus as the cost-revenue ratio increases from $k_o$ to $k_1$, then it must both decrease the fare and increase bus-miles such that $\xi_B$ is equal to $-\xi_F$ at the new ratio. If the firm varies from a position of maximizing ridership, the supply objective, in effect, is changing. One way to analyze this effect is to include the cost-revenue restriction in the supply objective. Two special forms of this general supply objective are of interest:

$$D^\theta B^{1-\theta} U(k)$$

(33)

and

$$D^\theta(k)B^{1-\theta}(k).$$

(34)

In the former case, the firm adopts the same relative decisions concerning ridership and service, regardless of the cost-revenue ratio. The relationship of the fare-elasticity of demand to the service-elasticity will remain:

$$-\xi_F = \theta\xi_B + (1 - \theta).$$

(35)

As the cost-revenue ratio increases, fares are reduced and service is expanded. Nevertheless, the relationship of the two
elasticities remain constant. Thus, we may say, supply decisions are neutral with respect to changes in the operating ratio.

In the latter case, the relative elasticities may vary as the cost-revenue ratio changes. If \( \theta \) is an increasing function of \( k \), ridership becomes more important as the cost-revenue ratio increases. In this circumstance, changes that occur as the cost-revenue-ratio increases tend to emphasize fare reduction at the expense of service expansion. This is a case of fare bias. If \( \theta \) is a decreasing function of \( k \), increases in the cost-revenue ratio tend to be translated into an expansion of bus service. This is called service bias.

Figure 7 shows an example of neutral response, fare bias, and service bias in the context of an increase in the cost-revenue ratio.

FIGURE 7. Supply Responses Under a Variable Operating Ratio as Cost-Revenue Ratio Increases from \( k_1 \) to \( k_0 \)
E. BUS TRANSIT SUPPLY FUNCTIONS

This section develops a supply function for bus transit service whose coefficients can be estimated from cross-section data. The following supply function is used:

\[ \ln B = \beta_0 + \beta_1 \ln \text{POP} + \beta_2 \ln \text{AREA} + \beta_3 \ln D + \beta_4 \ln c + \beta_5 \ln k_0 \quad (36) \]

An equivalent function can be derived from this equation in which the fare replaces bus-miles on the left side of the equality. The definition of the cost-revenue ratio can be used to make the transformation:

\[ \ln F = \beta_0 + \beta_1 \ln \text{POP} + \beta_2 \ln \text{AREA} + (\beta_3 - 1) \ln D + (\beta_4 - 1) \ln c + (\beta_5 - 1) \ln k_0 \quad (37) \]

This thesis emphasizes the supply function in terms of bus-miles, because it seems to contain greater intuitive meaning.

While this supply function will be given a formal interpretation later in this section, it might be useful to give a heuristic explanation of what it means. The effect of increases in market size variables, population and land area, is to reduce the degree of market saturation by transit service which already exists. These increases may induce increases in the level of transit service to restore the old level of market saturation. Passenger trips, cost per bus-mile and the cost-revenue ratio may all have similar effects on the supply of bus transit service.
Independent changes in each of these variables causes the firm to be out of equilibrium with respect to the cost-revenue ratio. If passenger trips or the cost-revenue ratio increase or if the cost per bus-mile decreases, the firm must act to restore the cost-revenue ratio to its required value either by providing additional bus-miles or by reducing the transit fare. The response depends on the supply objectives of the firm. Under performance objectives such as maximum service or maximum bus-miles the firm is indifferent to whether variation occurs in the number of passengers, costs per bus-mile or the cost-revenue ratio: it responds symmetrically to changes in each of the variables.

This section shows that supply functions developed from the conditions derived in the preceding section are algebraically equivalent to equation (36). The supply function derived in this thesis is a quasi-supply function in that the behavior of the supplier depends on the values of the demand elasticities. The demand function from Chapter II is used in this derivation.

1. Maximum Bus-Miles

As derived on page 48, the condition for maximum bus-miles is \( \xi_F = -1 \). According to the demand function developed in Chapter II, the fare elasticity of demand is \( \xi_F = -\alpha_2 F \).

Thus,

\[
F = \frac{1}{\alpha_2}
\]  

(38)
Solving the cost-revenue ratio formula for bus-miles and substituting the optimal value for the fare yields

\[ B = \alpha_2^{-1} Dc^{-1} k_0. \] (39)

Thus, the bus-miles the firm is willing to supply under service maximization is directly proportional to demand and the cost-revenue constraint and inversely proportional to the cost per bus-mile. Note that this is linear in the logarithm of the variables. Supply response is unaffected by variables measuring the size of the transit market.

2. Maximum Passengers

As shown on page 50, the condition for maximum ridership is \(-\varepsilon_F = \varepsilon_B\). The elasticity of demand with respect to bus-miles is

\[ \varepsilon_B = \alpha_1 \alpha' \left( \frac{B}{\text{POP}^\lambda \text{AREA}^{1-\lambda}} \right)^{-\alpha'}. \] (40)

The condition becomes

\[ \alpha_2 F = \alpha_1 \alpha' \left( \frac{B}{\text{POP}^\lambda \text{AREA}^{1-\lambda}} \right)^{-\alpha'}. \] (41)

Substituting for \(F\) in the cost-revenue constraint and solving for \(B\), yields

\[ B = \left( \frac{\alpha_1 \alpha'}{\alpha_2} \right) \frac{1}{\text{POP}^{\lambda \alpha'} \text{AREA}^{(1-\lambda) \alpha'}} \frac{\lambda \alpha'}{1+\alpha'} D \frac{1}{1+\alpha'} c \frac{1}{1+\alpha'} k_0 \] (42)
This supply function is linear in the logarithm of the variables. The parameter $\alpha'$ represents the rate of decline in the service-elasticity of demand. This function predicts that bus-miles supplied by the firm are proportional to simultaneous increases in POP, AREA, and D, because the sum of the exponents of these variables,

$$\lambda \frac{\alpha'}{1+\alpha'} + (1-\lambda) \frac{\alpha'}{1+\alpha'} + \frac{1}{1+\alpha'}$$

(43)

is equal to one. If the rate of decline of demand elasticity is large ($\alpha'$ large) then the size of the urbanized area is relatively more important than demand. If $\alpha'$ is small, however, demand becomes relatively more important. The quantity of bus-miles the firm is willing to supply is inversely related to the cost per bus-mile and directly related to the regulatory constraint. However, these relationships are less than proportional.

3. **Approximate Supply Functions for General Supply Hypotheses**

As demonstrated in Section C, maximum bus-miles and maximum passenger trips are two important but special cases of more general supply objectives. In particular, maximization of $D^\theta B^{1-\theta}$ is suggested as a supply objective which does not vary with changes in the cost-revenue ratio $k_o$. Unfortunately, in maximizing this objective subject to the cost-revenue restriction, it is not possible to obtain an analytical solution
for B in terms of the other variables. However, both maximum bus-miles ($\theta = 1$) and maximum passenger trips ($\theta = 0$) yield geometric (or multiplicative) functions of the same set of variables. Since the general objective is a geometric combination of bus-miles and passenger trips, a geometric combination of the solutions to the two special cases is suggested as an approximate solution to the general case $D^\theta B^{1-\theta}$. The parameter $\theta$ is used to make a linear interpolation of the exponents in Eqs. (39) and (42). The exponent of the demand variable $D$ becomes

$$\beta_3 = (1-\theta) \cdot 1 + \theta \frac{1}{1+\alpha}. \quad (44)$$

This approximate solution is written

$$B = \left( \frac{1}{\alpha_2} \right)^{1-\theta} \left( \frac{\alpha_1}{\alpha_2} \right)^{\theta} \frac{1}{1+\alpha} \left( \frac{\text{POP}^\lambda \text{AREA}^{1-\lambda}}{\text{POP}^\lambda \text{AREA}^{1-\lambda}} \right)^{\theta} \frac{\alpha'}{1+\alpha'} \left( Dc^{-1}k_o \right)^{(1-\theta) + \theta \frac{1}{1+\alpha}} \quad (45)$$

At $\theta = 0$, this expression equals the solution for maximum bus-miles; at $\theta = 1$, it equals the solution for maximum passenger trips.

---

14. In the case of $D^\theta B^{1-\theta}$ the optimal level of service $B$ is given by a solution to

$$B = -k_1 Dc^{-1} \left[ \theta \frac{\alpha'}{\alpha_2} \left( \frac{\text{POP}^\lambda B \text{AREA}^{1-\lambda}}{\text{POP}^\lambda \text{AREA}^{1-\lambda}} \right)^{-\alpha'} + \frac{(1-\theta)}{\alpha_2} \right].$$
The objective $D^\theta B^{1-\theta}$ implies that the supply objective does not vary with respect to the cost-revenue ratio. If the cost-revenue ratio is allowed to increase (e.g., through a program of subsidies), this objective implies that the greater relative costs are absorbed through reductions in fares and increases in service such that the same relative trade-offs exist between passenger trips and bus-miles.\(^{15}\) Higher cost-revenue ratios may lead to a shift in the supply objective which emphasizes lower fares. This is defined in the previous subsection as a case of fare bias. If higher cost-revenue ratios lead to a shift in objective toward greater service levels, this is defined as service bias. Each exponent in the supply function represents an elasticity of bus-miles. Under maximum service, maximum ridership, or maximum $D^\theta B^{1-\theta}$ the supply elasticities with respect to trips demanded $(D)$, the inverse of unit costs $(c^{-1})$, and the cost-revenue ratio $(k_o)$, are all equal. In the case of service bias where a higher cost-revenue ratio implies a shift in emphasis toward bus-miles, the supply elasticity with respect to the cost-revenue ratio should be larger than the other two elasticities. In the case of fare bias, this elasticity should be smaller.

\(^{15}\) Or at least, fare is reduced and service increased relative to what would have occurred if the cost-revenue ratio had not been allowed to rise. In the case of rising costs per bus-mile, an increase in the ratio of costs to revenues occurs without fare reduction or service augmentation.
4. **An Empirical Comparison of Supply Hypotheses**

Table 1 provides a comparison of the two "specific" supply objectives and the three "general" supply objectives. All of the objectives except maximum bus-miles depend crucially on the demand parameter $\alpha'$, which is related to the service-elasticity of demand. In Table 1

$$v = \frac{1}{1+\alpha'}$$  \hspace{1cm} (46)

and

$$x = (1-\theta) + \theta \frac{1}{1+\alpha'}$$, \hspace{1cm} (47)

where $\theta$ is the parameter in the supply objective $D^\theta B^{1-\theta}$. The elasticity of demand with respect to bus miles is

$$\epsilon_B = \alpha' \left( \frac{B}{\text{POP}^\lambda \text{AREA}^{1-\lambda}} \right)^{-\alpha'}$$ \hspace{1cm} (48)

If $\alpha'$ equals 1.0, a doubling of bus-miles cuts the service-elasticity in half. If $\alpha'$ takes on the smaller value of 0.3, doubling bus-miles reduces the elasticity of demand with respect to bus service by only about 20 percent. The larger the value of $\alpha'$, the smaller the response of bus-miles to changes in the variables of the supply function under any objective except maximum bus-miles. This is an intuitively reasonable result. If the transit market becomes rapidly saturated with bus service
Table 1
SUPPLY PARAMETERS UNDER ALTERNATIVE SUPPLY OBJECTIVES

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Special Objectives</th>
<th>General Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Maximum Bus-Miles</td>
<td>Maximum Passengers</td>
</tr>
<tr>
<td>dm POP</td>
<td>$\beta_1$</td>
<td>0</td>
<td>$\lambda (1-v)$</td>
</tr>
<tr>
<td>dm AREA</td>
<td>$\beta_2$</td>
<td>0</td>
<td>$(1-\lambda)(1-v)$</td>
</tr>
<tr>
<td>dm D</td>
<td>$\beta_3$</td>
<td>1</td>
<td>$v$</td>
</tr>
<tr>
<td>dm c</td>
<td>$\beta_4$</td>
<td>-1</td>
<td>$-v$</td>
</tr>
<tr>
<td>dm $k_0$</td>
<td>$\beta_5$</td>
<td>1</td>
<td>$v$</td>
</tr>
</tbody>
</table>

($\alpha'$ is large), the transit supplier may be justifiably hesitant to increase transit service. Instead, he relies primarily on increases or decreases in the transit fare.

A problem in testing the various supply hypotheses is the absence of any objective knowledge about the demand parameter $\alpha'$. One consequence of this lack of information is the ability to distinguish between the hypothesis of maximum passengers and the general supply objective $D^\theta B^1-\theta$, where $\theta > 0$.

Columns two and three of Table 1 are identical for the purposes of statistical testing because both $x$ and $v$ are unknown positive parameters. Thus, maximum passengers and maximum of some
combination of passengers and bus-miles are statistically indistinguishable. Only in the case of maximum bus-miles is a completely testable hypothesis presented. This testable hypothesis predicts that bus-miles do not depend on the market size variables and increase proportionally with an increase in cost per bus-mile.

The hypothesis can also be tested that changes in the cost-revenue ratio have the same absolute effect on bus-miles as variation in passengers or in cost per bus-mile. The hypothesis of neutrality can be tested against the hypotheses of fare bias or service bias with respect to changes in the cost-revenue ratio by testing whether the parameter \( y \) in Table 1 is equal to zero. If \( y \) is zero, the firm responds symmetrically to an increase in passengers, a decrease in cost, or an increase in the cost-revenue ratio.
IV. THE COST-REVENUE RATIO

In this cross-section study of transit firms we assume that the cost-revenue ratio is determined by the types of institutions which own and regulate the bus transit operations. Transit operations may be publicly owned or privately owned. Among the five classifications of transit firms three are private firms and two are publicly owned operations, as described below:

A. PRIVATE/POWER

A small, but significant, percentage of privately owned transit operations are owned by much larger companies which also supply electric and gas power. These firms are public utilities on two counts, and both transit operations and power are subject to regulation. In all but one case, both operations are regulated by the same agency. In that exception the local government, which regulates transit operations, contracts with the power company to receive electricity. The importance of power-company-owned transit operations is the opportunity which is presented for subsidizing transit operations out of the profits from supplying power.

B. PRIVATE/LOCAL AND PRIVATE/STATE

The remaining private firms are regulated by either a local government agency or the state public utilities commission. In the case of state regulation, one may expect rather mechanical
regulation based on profit. The local regulatory authority may be more sensitive to service and fare than to the profitability of transit operations. The local agency may also be more responsive to the wishes of the community or certain members of the community.

C. PUBLIC/TRANSIT AUTHORITY AND PUBLIC/CITY GOVERNMENT

One class of publicly owned firms is under the direct control of the municipal government. Authority over fares and schedules may rest with the city manager or the city council. Other public firms are operated by a transit authority which at least is nominally independent of the municipal government. Municipal governments may be more responsive to public protests against increases in fares or reductions in service. Government accounts may be such that there is no incentive for the managers of the firm to operate out of the fare box. Transit authorities may at least have a thin layer of insulation against political pressure. More importantly, such firms may be legally unable to operate at a deficit. This is particularly true if bond obligations of the transit authority place restrictions on the cost-revenue ratio.

The cost-revenue equation is simply:

\[ \ln k_0 = \gamma_1 \text{POWER} + \gamma_2 \text{LOCAL} + \gamma_3 \text{STATE} \]

\[ + \gamma_4 \text{CITY} + \gamma_5 \text{AUTH} \]  

(49)
The variables are dummy variables, which equal one if a firm is in that category and are zero otherwise. The coefficients simply represent the means of the logarithms of the cost-revenue ratio of firms in the appropriate categories.

Table 2 indicates the number of firms falling in each category for the two samples used in the study. The great increase in the proportion of publicly owned firms reflects overall trends in the industry.

<table>
<thead>
<tr>
<th>Type of Firm</th>
<th>1968</th>
<th>1960</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private/Power</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Private/Local</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>Private/State</td>
<td>15</td>
<td>21</td>
</tr>
<tr>
<td>Public/City</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Public/Authority</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>51</td>
<td>44</td>
</tr>
</tbody>
</table>
V. EMPIRICAL ESTIMATES

The model of the transit market consists of three equations:

\[ \ln D = \alpha_0 - \alpha_1 \left( \frac{B}{\text{POP}^\lambda \text{AREA}^{1-\lambda}} \right)^{-\alpha'} + \alpha_2 F + \alpha_3 \ln \text{POP} + \alpha_4 \ln \text{AREA} \]

\[ + \alpha_5 \ln \text{AUTOS} + \alpha_6 \ln \text{HWAY} + \alpha_7 \text{INC}_3 + \alpha_8 \text{INC}_{10} \]

\[ + \alpha_9 \text{AGE}_{18} + \alpha_{10} \text{AGE}_{65} + \epsilon_1 . \]  \hspace{1cm} (50)

\[ \ln B = \beta_0 + \beta_1 \ln \text{POP} + \beta_2 \ln \text{AREA} + \beta_3 \ln D + \beta_4 \ln c \]

\[ + \beta_5 \ln k + \epsilon_2 . \]  \hspace{1cm} (51)

\[ \ln k = \gamma_1 \text{POWER} + \gamma_2 \text{LOCAL} + \gamma_3 \text{STATE} + \gamma_4 \text{CITY} \]

\[ + \gamma_5 \text{AUTH} + \epsilon_3 \]  \hspace{1cm} (52)

Furthermore, there is the identity,

\[ \ln k = \ln c + \ln B - \ln F - \ln D , \]  \hspace{1cm} (53)
which defines the cost-revenue ratio.\footnote{No distinction is made here between a "target" ratio \( k_0 \) and an "attained" ratio \( k \). If this distinction is made, equation (52) could be replaced by

\[
\ln k_0 = \gamma_1 \text{POWER} + \gamma_2 \text{LOCAL} + \gamma_3 \text{STATE} + \gamma_4 \text{CITY} \\
+ \gamma_5 \text{AUTH} + \epsilon_3' \tag{54}
\]

and

\[
\ln k = \ln k_0 + \epsilon_4 \tag{55}
\]

In (55), the logarithm of the attained ratio equals the logarithm of the target ratio plus a random component. Substituting (54) into (55) yields equation (52) where \( \epsilon_3 = \epsilon_3' + \epsilon_4 \). Thus, in determining the cost-revenue ratio, there are random factors both in setting the target and attaining the target.}
treated as an endogenous variable for econometric purposes because its random variation is related to the random variation in the level of service and the number of transit passengers, both of which are endogenous variables. The remaining variables in the model are classified as independent, or exogenous, variables.

Estimation of the parameters of the demand equation and the supply equation is complicated by the presence of three endogenous variables in each equation. The endogenous variables are correlated with the error terms in each equation. As a result, ordinary least squares (OLS) leads to inconsistent estimates of the parameters. Two-stage least squares (2SLS) is used to estimate the parameters of the demand and supply equations.\(^2\)\(^3\) In this approach, endogenous variables on the right side of the equation are replaced with estimates that


\textbf{3.} Simultaneous-estimation techniques fall into two categories. "Full-information" techniques, such as three-stage least squares or full-information maximum likelihood, estimate all the parameters of the model simultaneously. Single-equation techniques, such as 2SLS or limited-information maximum likelihood (LIML), estimate parameters for a single equation in the model. Full-information techniques, although very powerful, tend to be more subject to specification error than single-equation techniques. However, the major reason for not using full-information techniques is that the full system of equations is nonlinear in the parameters, although each single-
are assumed to be uncorrelated with the error term in the equation. Since the cost-revenue ratio equation does not contain other endogenous variables, OLS is appropriate for estimating the parameters of this equation.

Empirical estimates of equations in the bus transit model are presented in this chapter for the cost-revenue ratio, the supply equation, and the demand equation in that order. Table 3 contains values of the principal variables in the model for a sub-sample of 25 firms in 1968.

A. THE COST-REVENUE RATIO

The coefficients of the cost-revenue equation appear in Table 4. In both 1968 and 1960 this simple classification of transit firms explains one-half of the total squared variation in the logarithm of the cost-revenue ratio (adjusted for degrees of freedom). Each coefficient represents the mean value of equation is linear in the parameters. (Note how the variable B enters differently in the demand equation and the supply equation.) 2SLS is preferred to LIML because there are convenient normalization rules to follow in each equation. (2SLS requires that one endogenous variable be selected for the left side of the equation.) Furthermore, there is no information about the variances of $\epsilon_1$, $\epsilon_2$, $\epsilon_3$. An advantage of LIML is that it is possible to utilize information about these variances in estimating the coefficients of the equation. All four techniques mentioned here are biased for finite samples. Each has the asymptotic property of consistency, which implies that the distribution of the estimate collapses to the true value of the parameter as the sample sizes become infinite.
<table>
<thead>
<tr>
<th>Location</th>
<th>Classification</th>
<th>Population</th>
<th>Bus-Miles Per Capita</th>
<th>Fare (cents)</th>
<th>Cost-Revenue Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Fitchburg &amp; Leominster, Mass.</td>
<td>Private/State</td>
<td>65,000</td>
<td>12.6</td>
<td>15.8</td>
<td>1.04</td>
</tr>
<tr>
<td>2. Portland, Me.</td>
<td>Private/State</td>
<td>116,000</td>
<td>11.7</td>
<td>23.4</td>
<td>.92</td>
</tr>
<tr>
<td>3. Buffalo, N.Y.</td>
<td>Private/Local</td>
<td>930,000</td>
<td>15.7</td>
<td>24.9</td>
<td>.98</td>
</tr>
<tr>
<td>4. Harrisburg, Pa.</td>
<td>Private/State</td>
<td>236,000</td>
<td>6.4</td>
<td>22.2</td>
<td>.95</td>
</tr>
<tr>
<td>5. New Castle, Pa.</td>
<td>Authority</td>
<td>61,000</td>
<td>7.3</td>
<td>21.5</td>
<td>1.60</td>
</tr>
<tr>
<td>6. Spartanburg, S.C.</td>
<td>Power</td>
<td>73,000</td>
<td>8.3</td>
<td>21.1</td>
<td>1.13</td>
</tr>
<tr>
<td>7. Savannah, Ga.</td>
<td>Authority</td>
<td>171,000</td>
<td>13.8</td>
<td>16.0</td>
<td>.97</td>
</tr>
<tr>
<td>8. Columbia, S.C.</td>
<td>Power</td>
<td>260,000</td>
<td>6.2</td>
<td>15.7</td>
<td>1.35</td>
</tr>
<tr>
<td>9. Charleston, S.C.</td>
<td>Power</td>
<td>226,000</td>
<td>7.7</td>
<td>14.4</td>
<td>1.28</td>
</tr>
<tr>
<td>10. Chattanooga, Tenn.</td>
<td>Private/Local</td>
<td>252,000</td>
<td>7.3</td>
<td>30.5</td>
<td>1.01</td>
</tr>
<tr>
<td>11. Cincinnati, O.</td>
<td>Private/Local</td>
<td>1,148,000</td>
<td>9.5</td>
<td>30.3</td>
<td>.99</td>
</tr>
<tr>
<td>12. Columbus, O.</td>
<td>Private/Local</td>
<td>755,000</td>
<td>10.1</td>
<td>29.2</td>
<td>.96</td>
</tr>
<tr>
<td>13. Kansas City, Kan.-Mo.</td>
<td>Authority</td>
<td>1,181,000</td>
<td>7.4</td>
<td>33.5</td>
<td>.97</td>
</tr>
<tr>
<td>14. St. Louis, Mo.</td>
<td>Authority</td>
<td>2,064,000</td>
<td>11.5</td>
<td>30.0</td>
<td>.96</td>
</tr>
<tr>
<td>15. Lafayette, La.</td>
<td>City</td>
<td>83,000</td>
<td>4.2</td>
<td>13.3</td>
<td>1.22</td>
</tr>
<tr>
<td>16. Fresno, Calif.</td>
<td>City</td>
<td>280,000</td>
<td>5.0</td>
<td>21.3</td>
<td>1.51</td>
</tr>
<tr>
<td>17. Flint, Mich.</td>
<td>City</td>
<td>357,000</td>
<td>3.8</td>
<td>31.2</td>
<td>1.37</td>
</tr>
<tr>
<td>18. Springfield, Mo.</td>
<td>City</td>
<td>119,000</td>
<td>9.5</td>
<td>16.2</td>
<td>1.72</td>
</tr>
<tr>
<td>19. New Orleans, La.</td>
<td>Power</td>
<td>1,100,000</td>
<td>13.2</td>
<td>10.3</td>
<td>1.87</td>
</tr>
<tr>
<td>20. San Antonio, Tex.</td>
<td>City</td>
<td>765,000</td>
<td>9.9</td>
<td>20.2</td>
<td>.89</td>
</tr>
</tbody>
</table>
Table 4
ESTIMATED COEFFICIENTS OF THE COST-REVENUE EQUATION
(With Standard Deviations)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1968</th>
<th></th>
<th>1960</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\ln k)</td>
<td>k</td>
<td>(\ln k)</td>
<td>k</td>
</tr>
<tr>
<td><strong>Dependent Variable</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private/Power</td>
<td>0.350</td>
<td>1.42</td>
<td>0.230</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td></td>
<td>(0.040)</td>
<td></td>
</tr>
<tr>
<td>Private/Local</td>
<td>-0.001</td>
<td>1.00</td>
<td>-0.017</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td></td>
<td>(0.022)</td>
<td></td>
</tr>
<tr>
<td>Private/State</td>
<td>0.028</td>
<td>1.03</td>
<td>-0.006</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td></td>
<td>(0.028)</td>
<td></td>
</tr>
<tr>
<td>Public/City</td>
<td>0.283</td>
<td>1.33</td>
<td>-0.128(^{a})</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td></td>
<td>(0.085)</td>
<td></td>
</tr>
<tr>
<td>Public/Authority</td>
<td>0.087</td>
<td>1.09</td>
<td>0.015(^{b})</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td></td>
<td>(0.062)</td>
<td></td>
</tr>
<tr>
<td><strong>R(^2)(adjusted)</strong></td>
<td>0.493</td>
<td></td>
<td>0.504</td>
<td></td>
</tr>
<tr>
<td><strong>Standard Error of</strong></td>
<td>0.144</td>
<td></td>
<td>0.082</td>
<td></td>
</tr>
<tr>
<td><strong>Regression</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Number of Observations</strong></td>
<td>51</td>
<td></td>
<td>44</td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\) Contains only one firm.
\(^{b}\) Contains only two firms.
$\ln k$ (the logarithm of the cost-revenue ratio) for firms belonging to that category. Since $\ln 1 = 0$, a negative coefficient represents a ratio of total cost to total revenue which is less than 1.0, and a positive value represents a cost-revenue ratio greater than 1.0.

In the 1968 sample, private firms owned by power companies and public firms operated by city governments have cost-revenue ratios significantly greater than 1.0. The average cost-revenue ratios corresponding to these estimates are 1.42 for power companies and 1.33 for city-run operations. For both categories, total costs, including depreciation and normal profit, are at least one-third greater than total revenues. For power companies, a form of cross-subsidization may exist in which users of electricity and natural gas subsidize users of transit service. The general tax revenues of the municipality may be the source of subsidy for city-run operations. In other categories, the mean of $\ln k$ is not significantly different from zero. Costs, therefore, are not significantly greater than revenues for these firms. Private firms regulated by local governments, in fact, are successful on average in covering total costs. For state-regulated firms and firms operated by transit authorities, costs are slightly greater than revenues in this sample.
The primary differences between the 1960 and 1968 results are attributable to differences in the institutional composition of the transit industry. The 1960 sample is marked by the virtual absence of publicly owned transit firms. Only two transit-authority firms and one city-run firm appear among the 44 firms in that year. It is interesting to note, however, that the single city-run transit operation has the best cost-revenue ratio of any firm in either year \((k = 0.88)\).

Among private operations, the situation is similar to 1968. The estimated coefficient of firms owned by power companies corresponds to a cost-revenue ratio of 1.26. Private firms under both local and state regulation tend to cover total costs.

1. Alternative Estimates of the Ratio of Total Cost to Total Revenue

The hypothesis that the ratio of total cost to total revenue depends only on the classification of firms by ownership and regulation constitutes an institutional theory of the cost-revenue ratio. Other hypotheses about the ratio of cost to revenue are possible. The ratio may be influenced by other exogenous variables in the model, such as the cost per bus-mile. It is also possible that the structure of the transit model may be entirely different, such that the cost-revenue ratio is determined by demand and bus miles, in a manner
parallel to the supply equation of Chapter III. It is difficult to estimate such a relationship, even if one were derived, because the cost-revenue ratio is defined in terms of the fare, passenger trips, and bus-miles.

Table 5 reports the results of regressing the logarithm of the cost-revenue ratio against all the exogenous variables in the model. This reduced-form equation can give evidence of the effect of both endogenous and exogenous variables on the cost-revenue ratio. The effect of an endogenous variable such as quantity of passenger trips may show its effect through a variable like population which appears in the demand function. The coefficients in Table 5, however, offer support to the following institutional hypotheses about the cost-revenue ratio:

(1) The coefficients of the institutional variables are not significantly different from the results reported in Table 4.

(2) Only one other variable enters at the one percent level of significance. \( \ln \text{HWAY} \) is the logarithm of the ratio of population to highway capacity in an urbanized area. (See Appendix B for definition of highway capacity.) A high ratio of population to urban highway capacity is associated with a high
Table 5
AN ALTERNATIVE ESTIMATE OF THE RATIO OF TOTAL COST TO TOTAL REVENUE

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Definition</th>
<th>1968</th>
<th>1960</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable</td>
<td>Ln k</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Independent Variable</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER</td>
<td>Power company</td>
<td>.326&lt;sup&gt;a&lt;/sup&gt;</td>
<td>.189&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>STATE</td>
<td>State regulation</td>
<td>.005</td>
<td>-.004</td>
</tr>
<tr>
<td>CITY</td>
<td>City ownership</td>
<td>.276&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-.203&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>AUTH</td>
<td>Transit authority</td>
<td>-.002</td>
<td>.031</td>
</tr>
<tr>
<td>Ln c</td>
<td>Cost per bus-mile</td>
<td>.076</td>
<td>.325&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Ln POP</td>
<td>Population</td>
<td>-.148</td>
<td>-.040</td>
</tr>
<tr>
<td>Ln AREA</td>
<td>Land area</td>
<td>.081</td>
<td>.053</td>
</tr>
<tr>
<td>Ln AUTOS</td>
<td>Automobiles</td>
<td>.224</td>
<td>.068</td>
</tr>
<tr>
<td>Ln HWAY</td>
<td>Highway capacity</td>
<td>.217&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td>INC&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Under $3,000</td>
<td>.118</td>
<td>.893</td>
</tr>
<tr>
<td>INC&lt;sub&gt;10&lt;/sub&gt;</td>
<td>Over $10,000</td>
<td>1.46</td>
<td>.081</td>
</tr>
<tr>
<td>AGE&lt;sub&gt;18&lt;/sub&gt;</td>
<td>18 and under</td>
<td>.502</td>
<td>1.41</td>
</tr>
<tr>
<td>AGE&lt;sub&gt;65&lt;/sub&gt;</td>
<td>65 and under</td>
<td>2.46</td>
<td>1.45</td>
</tr>
<tr>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td></td>
<td>.717</td>
<td>.508</td>
</tr>
<tr>
<td>Standard error</td>
<td></td>
<td>.120</td>
<td>.082</td>
</tr>
<tr>
<td>Number of observations</td>
<td></td>
<td>51</td>
<td>44</td>
</tr>
</tbody>
</table>

a. Significant at the one percent level (two-tailed test, 37 degrees of freedom).

b. Significant at the five percent level (two-tailed test, 37 degrees of freedom).
ratio of total cost to total revenue in 1968.\(^4\) Although this is an interesting effect, \(\ln \text{HWAY}\) is probably not acting as a surrogate for demand. As Section C shows, the coefficient of this variable is not significantly different from zero in the demand function.

(3) Finally, the cost per bus-mile has a small and insignificant effect on the ratio of total cost to total revenue in the 1968 estimates. The effect is larger in 1960 and is significant at the five percent level.

2. **Conclusions from the Cost-Revenue Equation**

In summary, the ratio of total cost to total revenue for a bus transit firm depends largely on the institutional structure of the transit industry. Not all publicly owned firms have high cost-revenue ratios, and not all firms with high cost-revenue ratios are publicly owned. In particular, publicly owned firms operated as part of the municipal government and privately owned firms operated by power companies appear willing to provide internal subsidies for transit operations.

Cost per bus-mile has a weak and insignificant effect on the cost-revenue ratio in 1968 but a somewhat stronger effect in 1960.

---

4. No urban highway data is available for 1960.
Also, variables that have a significant effect on the number of passenger trips demanded do not affect the cost-revenue ratio. This may indicate that transit markets with high ratios of total cost to total revenue are not marked by deficient demand.

B. THE SUPPLY EQUATION

Estimates of the supply equation provide general evidence on the relationship of bus-miles of transit service to variables representing the size of the transit market, the number of passenger trips demanded, the cost per bus-mile, and the ratio of cost to revenue. The estimates of these parameters provide specific evidence on two empirical issues about the supply behavior of the bus transit industry. One issue is whether bus firms provide maximum bus-miles as opposed to maximum passenger trips or some combination of ridership and service. The second issue is whether firms with high cost-revenue ratios make the same choice between maximum ridership and maximum service or between low fares and high levels of service. Table 6 contains 2SLS estimates of the supply equation. This statistical technique is used to avoid the effects of correlation between the random term \( \epsilon_2 \) and the variables \( \ln k_o \) and \( \ln D \). \( \ln k_o \) is replaced by an estimate based on the coefficients of the cost-revenue equation (Table 4). The estimate of \( \ln D \) is based on a regression of \( \ln D \) on all exogenous variables in the model.\(^5\)

---

5. An alternative method in which \( \ln k_o \) is regressed on all exogenous variables in the model does not yield results which are substantially different from those in Table 5.
Table 6
2SLS ESTIMATES OF THE SUPPLY EQUATION

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1968</th>
<th>1960</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable</td>
<td>( \ln B )</td>
<td>( \ln B )</td>
</tr>
<tr>
<td>Independent Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.42</td>
<td>-1.05</td>
</tr>
<tr>
<td>( \ln POP ) Population</td>
<td>.248</td>
<td>.055</td>
</tr>
<tr>
<td>( \ln AREA ) Land area</td>
<td>.055</td>
<td>.008</td>
</tr>
<tr>
<td>( \ln D ) Passenger trips</td>
<td>.727</td>
<td>.927</td>
</tr>
<tr>
<td>( \ln c ) Cost per bus-mile</td>
<td>-.601</td>
<td>-.446</td>
</tr>
<tr>
<td>( \ln k ) Cost-revenue ratio</td>
<td>-.065</td>
<td>-.511</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.982</td>
<td>.971</td>
</tr>
<tr>
<td>Standard error</td>
<td>.170</td>
<td>.133</td>
</tr>
<tr>
<td>Number of observations</td>
<td>51</td>
<td>44</td>
</tr>
</tbody>
</table>

The supply equation has the form

\( \ln B = \beta_0 + \beta_1 \ln POP + \beta_2 \ln AREA + \beta_3 \ln D + \beta_4 \ln c + \beta_5 \ln k \). \hspace{1cm} (56)

Under maximum bus-miles, bus service is proportional to the quantity of trips demanded and is not affected independently by market size. Furthermore, bus-miles is inversely proportional to the cost per bus-mile. This suggests three tests of the maximum bus-miles hypothesis against the alternative hypothesis.
that passenger trips or some combination of passenger trips
and bus-miles are maximized:

<table>
<thead>
<tr>
<th>Maximum Bus-Miles</th>
<th>Maximum Ridership, etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1 + \beta_2 = 0$</td>
<td>$\beta_1 + \beta_2 &gt; 0$</td>
</tr>
<tr>
<td>$\beta_3 - 1 = 0$</td>
<td>$\beta_3 - 1 &lt; 0$</td>
</tr>
<tr>
<td>$\beta_4 + 1 = 0$</td>
<td>$\beta_4 + 1 &gt; 0$</td>
</tr>
</tbody>
</table>

Evidence on the responsiveness of supply objectives to
variation in the cost-revenue is based on a comparison of the
coefficient of $\ln k$ with the coefficient of $\ln D$ and the coeffi-
cient of $-\ln c$. Under a neutral response the three coefficients
should be equal. Under fare-bias $\ln k$ has a smaller effect on
bus-miles than the other variables (and a larger effect on the
average fare). Under service-bias $\ln k$ has a larger effect than
the other variables. The following statistical tests are used:

<table>
<thead>
<tr>
<th>Neutral</th>
<th>Fare-biased</th>
<th>Service-biased</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_5 - \beta_3 = 0$</td>
<td>$\beta_5 - \beta_3 &lt; 0$</td>
<td>$\beta_5 - \beta_3 &gt; 0$</td>
</tr>
<tr>
<td>$\beta_5 + \beta_4 = 0$</td>
<td>$\beta_5 + \beta_4 &lt; 0$</td>
<td>$\beta_5 + \beta_4 &gt; 0$</td>
</tr>
<tr>
<td>$\beta_3 + \beta_4 = 0$</td>
<td>$\beta_3 + \beta_4 = 0$</td>
<td>$\beta_3 + \beta_4 = 0$</td>
</tr>
</tbody>
</table>

The third test indicates whether the evidence is consistent with
any of the three hypotheses.
Statistical inference is based on the assumption that the ratio of the estimated coefficient to the estimated standard deviation has approximately the $t$-distribution. In view of some controversy surrounding the distribution of 2SLS estimators, we report the outcome of tests using both 45 and 5 degrees of freedom in 1968 and both 38 and 4 degrees of freedom in 1960.\footnote{6}

1. 1968 Supply Estimates

The supply estimates for the 1968 sample lead us to reject the hypothesis of service-maximization. The outcomes of the three tests of this hypothesis are as follows:

---

6. The distribution of 2SLS estimates continues to be a point of some controversy. The most common practice has been to assume the estimates are distributed like estimates in classical normal least-squares. Thus, the estimated coefficient is assumed to be normally distributed; the estimated variance is assumed to be $\hat{\sigma}^2$, with $T - K - 1$ degrees of freedom. $T$ is the number of observations; $K$ is the number of right-side variables (dependent and exogenous). The ratio of the estimated coefficient to the square root of the estimated variance has the $t$-distribution with $T - K - 1$ degrees of freedom. If the error terms in the model are normally distributed, it has been shown that 2SLS estimators are asymptotically normal. Thus, some grounds exist for using the $t$-statistic with 2SLS estimators.

Although it is common practice to use $T - K - 1$ degrees of freedom, P. Dhrymes, "Alternative Asymptotic Tests of Significance and Related Aspects of 2SLS and 3SLS Estimated Parameters," Review of Economic Studies, XXXVI (2), April, 1969, has recently shown that tests on coefficients using 2SLS have the asymptotic $t$-distribution with degrees of freedom equal to the degree of over-identification of the equation. The degree of over-identification in this model is the number of exogenous variables excluded from the equation minus the number of "extra" dependent variables included in the equation. The degree of over-identification is five in 1968 and four in 1960.
a. The effect of population and land area on bus-miles supplied:

Maximum bus-miles hypothesis: $\beta_1 + \beta_2 = 0$
Alternative hypothesis: $\beta_1 + \beta_2 > 0$

$b_1 + b_2 = .303$ standard deviation = .095 $t$ - ratio = 3.19

Level of significance:

0.5 percent with 45 degrees of freedom
2.5 percent with 5 degrees of freedom

b. The effect of demand on bus-miles supplied:

Maximum bus-miles: $\beta_3 - 1 = 0$
Alternative: $\beta_3 - 1 < 0$

$b_3 - 1 = -.273$ standard deviation = -.095 $t$ - ratio = 2.87

Level of significance:

0.5 percent with 45 degrees of freedom
2.5 percent with 5 degrees of freedom

c. The effect of cost per bus-mile on bus-miles supplied:

Maximum bus-miles: $\beta_4 + 1 = 0$
Alternative: $\beta_4 + 1 > 0$

$b_4 + 1 = .399$ standard deviation = .164 $t$ - ratio = 2.43

Level of significance:

1 percent with 45 degrees of freedom
5 percent with 5 degrees of freedom
The coefficient of each variable is an estimate of the elasticity of supply with respect to that variable. The tests show that the elasticity with respect to demand and cost per unit is less than one in absolute terms. The elasticity of supply with respect to market size is significantly greater than zero. Thus, the firms in the sample exhibit some of the characteristics of ridership maximization and similar objectives because changes in demand and cost affect not only service, but presumably transit fare as well.

The hypothesis that the cost-revenue ratio has no impact on the relative preference for low fares or high levels of service can also be rejected. The evidence clearly supports the hypothesis of fare-bias—that is, the firm tends to provide lower fares rather than greater levels of service at high cost-revenue ratios. The outcomes of three tests of the hypothesis of neutrality are as follows:

a. The relative effects of demand and the cost-revenue ratio on bus-miles supplied:

<table>
<thead>
<tr>
<th>Neutrality:</th>
<th>$\beta_5 - \beta_3 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service or fare bias:</td>
<td>$\beta_5 - \beta_3 \neq 0$</td>
</tr>
<tr>
<td>$b_5 - b_3 = -0.792$</td>
<td>standard deviation = 0.184</td>
</tr>
<tr>
<td>$t$ - ratio = -4.30</td>
<td></td>
</tr>
</tbody>
</table>

Level of significance:

0.1 percent with 45 degrees of freedom
1 percent with 5 degrees of freedom
b. The relative effects of cost per bus-mile and the cost-revenue ratio on bus-miles supplied:

- Neutrality: \( \beta_5 + \beta_4 = 0 \)
- Service or fare bias: \( \beta_5 + \beta_4 \neq 0 \)

\[ b_5 + b_4 = -0.666 \quad \text{standard deviation} = 0.230 \]
\[ t - \text{ratio} = -2.90 \]

Level of significance:
- 1 percent with 45 degrees of freedom
- 5 percent with 5 degrees of freedom

c. The relative effects of cost per bus-mile and demand on bus-miles supplied:

- Neutrality and service or fare bias: \( \beta_3 + \beta_4 = 0 \)
- Alternative: \( \beta_3 + \beta_4 \neq 0 \)

\[ b_3 + b_4 = 0.126 \quad \text{standard deviation} = 0.140 \]
\[ t - \text{ratio} = 0.90 \]

Not significant.

Thus, the cost-revenue ratio has a significantly smaller effect on bus-miles than either demand or cost per bus-mile. The relative effects of demand and cost per bus-mile are not significantly different. Estimates of a fare equation underline the results of this section. This represents an equivalent but alternative formulation of supply behavior.

\begin{equation}
\ln F = 1.013 + 0.055 \ln \text{POP} + 0.133 \ln \text{AREA} - 0.083 \ln D \\
(1.13) \quad (.159) \quad (.080) \quad (.106)
\end{equation}

\[ + 0.034 \ln c - 0.948 \ln k_o \]
\[ (.184) \quad (.212) \quad (57) \]
Thus, the negative impact of the cost-revenue ratio on the transit fare is far larger than the effects of either demand or cost.

2. **1960 Supply Estimates**

The results of tests based on the 1960 estimates are not so clear-cut as the 1968 results. Service maximization can be rejected on only one of the three tests. The impact of the cost-revenue ratio on bus-miles supplied is, however, significantly different from the impact of demand and cost. The outcomes of the six tests are as follows:

Maximum bus-miles:

a. \( \beta_1 + \beta_2 = 0 \)

\[
\begin{align*}
  b_1 + b_2 &= .063 \\
  \text{standard deviation} &= .130 \\
  t - \text{ratio} &= 0.48
\end{align*}
\]

Not significant.

b. \( \beta_3 - l = 0 \)

\[
\begin{align*}
  b_3 - l &= .073 \\
  \text{standard deviation} &= .130 \\
  t - \text{ratio} &= 0.56
\end{align*}
\]

Not significant.

c. \( \beta_4 + l = 0 \)

\[
\begin{align*}
  b_4 + l &= .554 \\
  \text{standard deviation} &= .165 \\
  t - \text{ratio} &= 3.36
\end{align*}
\]

Level of significance:

- 0.5 percent with 38 degrees of freedom
- 2.5 percent with 4 degrees of freedom
Neutrality:

d. $\beta_5 - \beta_3 = 0$
   
   \[ b_5 - b_3 = -1.438 \]  \quad \text{standard deviation} = 0.278
   
   \[ t - \text{ratio} = -5.17 \]

   Level of significance:
   
   0.1 percent with 38 degrees of freedom
   
   1 percent with 4 degrees of freedom

e. $\beta_5 + \beta_4 = 0$
   
   \[ b_5 + b_4 = -0.956 \]  \quad \text{standard deviation} = 0.284
   
   \[ t - \text{ratio} = -3.37 \]

   Level of significance:
   
   1 percent with 38 degrees of freedom
   
   5 percent with 4 degrees of freedom

f. $\beta_3 + \beta_4 = 0$
   
   \[ b_3 + b_4 = 0.482 \]  \quad \text{standard deviation} = 2.82
   
   \[ t - \text{ratio} = 2.82 \]

   Level of significance:
   
   1 percent with 38 degrees of freedom
   
   5 percent with 4 degrees of freedom.

The 1960 supply estimates imply that higher cost-revenue ratios not only are biased toward lower fares, but actually decrease the quantity of service supplied by the firm. Thus, increases in the cost-revenue ratio correspond to more than proportional decreases in transit fares. The 1960 estimate of the fare equation is:
\[ \ln F = -0.967 + 0.040 \ln \text{POP} + 0.013 \ln \text{AREA} - 0.072 \ln D \\
(0.548) \quad (0.123) \quad (0.067) \quad (0.116) \]
\[ + 0.601 \ln c - 1.59 \ln k_0. \]
\[ (0.171) \quad (0.243) \]

(58)

3. **Conclusions from Estimates of the Supply Equation**

The evidence from both 1968 and 1960 strongly supports the hypothesis that higher cost-revenue ratios imply lower fares rather than higher levels of service. Accordingly, the supply objective shifts away from the objective of maximum bus-miles as the cost-revenue ratio grows. The evidence is less strong on the general question of maximum ridership or maximum service. In 1968 it is possible to reject the hypothesis of maximum service on the basis of a significant effect of market size on bus-miles supplied and on the basis that the effects of passenger trips demanded and the cost per bus-mile are less than proportional. In 1960 the market size coefficients and the passenger trips coefficient support the hypothesis of maximum bus-miles. Only the coefficient of the cost variable leads to a rejection of this hypothesis.

C. **CONDITIONAL ESTIMATES OF THE DEMAND FUNCTION**

Attempts to estimate the market demand function for bus transit face a number of difficulties. One of the biggest problems is the difficulty of specifying the prices of substitutes and complements; consequently, the results are somewhat different from the conventional model of demand for a good or service.
Also, 1960 census data on the age distribution of the population and the income distribution of households must be applied to bus transit markets in 1968. Fortunately, the Department of Transportation has made estimates of the 1968 population and land area of all urbanized areas in the U.S. Despite these and other shortcomings the estimates of the demand function for bus transit apparently constitute a unique attempt to estimate the responsiveness of transit demand to fare, where observations on ridership are taken from transit systems with different fares. Interzonal demand studies and individual demand studies have been based on necessity on transit decisions occurring under a given structure of fares. Also, estimates of transit demand as a function of the level of bus service yield information which might be quite useful in the formulation of transit policy.

Another estimation problem is the algebraic form of the bus-service variable in the demand equation. This term appears as

\[ -\alpha_1 \left( \frac{B}{\text{POP}^\lambda \ \text{AREA}^{1-\lambda}} \right)^{-\alpha'} \]

Neither \( \alpha' \) nor \( \lambda \) can be estimated using linear regression techniques and attempts at non-linear regression failed to converge. The estimates of the parameters of the demand equation which appear in Table 7 assume that \( \alpha' = 0.3 \) and \( \lambda = 1.0 \). Experimentation with different values of \( \alpha' \) indicates that only \( \alpha_1 \) varies significantly. This is due to the large covariance
between estimates of $\alpha_1$ and $\alpha'$. The choice of $\lambda = 1.0$ implies that population is the most relevant measure of market size. This choice is at least partly supported by estimates of the supply equation in which the coefficient of land area is never statistically significant.

**Table 7**

**CONDITIONAL 2SLS ESTIMATES OF THE DEMAND FUNCTION WITH TESTS THAT THE COEFFICIENTS EQUAL ZERO**

<table>
<thead>
<tr>
<th>Right-Side Variable</th>
<th>1968 Coefficients</th>
<th>1960 Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level of Significance</td>
<td>Level of Significance</td>
</tr>
<tr>
<td></td>
<td>40 degrees of freedom</td>
<td>3 degrees of freedom</td>
</tr>
<tr>
<td></td>
<td>38 degrees of freedom</td>
<td>2 degrees of freedom</td>
</tr>
<tr>
<td>Constant</td>
<td>$-0.3$</td>
<td>$a$</td>
</tr>
<tr>
<td>$-\left(\frac{8}{POP}\right)$</td>
<td>8.81 (2.00)</td>
<td>0.1% 5%</td>
</tr>
<tr>
<td></td>
<td>6.54 (1.12)</td>
<td>0.1% 5%</td>
</tr>
<tr>
<td>F</td>
<td>-3.06 (1.60)</td>
<td>10 -</td>
</tr>
<tr>
<td></td>
<td>-4.52 (1.22)</td>
<td>0.1 10</td>
</tr>
<tr>
<td>$\ln$ POP</td>
<td>1.10 (.13)</td>
<td>0.1 1</td>
</tr>
<tr>
<td></td>
<td>1.11 (.064)</td>
<td>0.1 1</td>
</tr>
<tr>
<td>$\ln$ AREA</td>
<td>.0208 (.11)</td>
<td>- -</td>
</tr>
<tr>
<td></td>
<td>.0021 (.063)</td>
<td>- -</td>
</tr>
<tr>
<td>$\ln$ AUTOS</td>
<td>-.175 (.40)</td>
<td>- -</td>
</tr>
<tr>
<td></td>
<td>-.106 (.11)</td>
<td>- -</td>
</tr>
<tr>
<td>$\ln$ HWAY</td>
<td>.156 (.16)</td>
<td>- -</td>
</tr>
<tr>
<td>INC$^3$</td>
<td>-3.02 (1.03)</td>
<td>1 10</td>
</tr>
<tr>
<td></td>
<td>-1.61 (1.08)</td>
<td>- -</td>
</tr>
<tr>
<td>INC$^{10}$</td>
<td>-3.57 (1.97)</td>
<td>10 -</td>
</tr>
<tr>
<td></td>
<td>-0.40 (1.20)</td>
<td>- -</td>
</tr>
<tr>
<td>AGE$^{18}$</td>
<td>-5.95 (2.44)</td>
<td>5 10</td>
</tr>
<tr>
<td></td>
<td>-1.74 (1.14)</td>
<td>- -</td>
</tr>
<tr>
<td>AGE$^{65}$</td>
<td>-8.17 (3.42)</td>
<td>5 10</td>
</tr>
<tr>
<td></td>
<td>-0.87 (1.61)</td>
<td>- -</td>
</tr>
</tbody>
</table>

| $R^2$ (adjusted) | .976 | .986 |
| Standard Error of Regression | .227 | .113 |
| Number of Observations | 51 | 44 |

a. Constant terms are not comparable because of different units of measurement.
In discussing demand estimates, we discuss results from both 1968 and 1960 at the same time.

1. The Impact of Service and Fare on Demand

The two most important variables in the demand function, at least from the standpoint of the ability of the firm or the regulator to influence demand, are fare and bus-miles. An increase in the fare is expected to decrease the number of persons willing to use bus transit, while increases in service are expected to increase the number of passenger trips.

The elasticity of demand with respect to bus-miles is assumed to be

\[ -0.3 \alpha_1 \left( \frac{B}{POP} \right) \]

Estimates of \( \alpha_1 \) are significantly greater than zero for both 1968 and 1960; thus, we conclude that an increase in bus service leads to an increase in the number of persons willing to use bus service at the prevailing fare. The estimated coefficients, 8.81 in 1968 and 6.54 in 1960, indicate elasticities of demand somewhat larger than expected. At the mean levels of service prevailing in 1968 and 1960, the service elasticities of demand are 1.35 and .92, respectively. Under service maximization, ridership maximization, or other types of motivation discussed in Chapter III the firm will operate where the service-elasticity is greater than one. These estimated elasticities are not, however, significantly greater than one.
Increases in fares lead to decreases in passenger trips by making other modes of travel more attractive and by decreasing the demand for urban transportation. Estimates of the effects of fares on ridership tend to confirm the assumptions, although the estimated coefficient in 1968 has a lower level of significance than the coefficient for 1960. At the mean fares of 22 cents in 1968 and 18 cents in 1960, the fare-elasticities are -.67 and -.81, respectively. The industry's rule-of-thumb is -0.3, substantially smaller than these estimates.

Table 8 contains estimates of the service and fare parameters in 1968 under six different functional forms. This provides a check on the high service-elasticity of demand and the insignificant fare-elasticity of demand estimated from this sample. The other parameter estimates in the demand equation are not included in Table 8. Eq. (1) is the result reported in Table 7 where the parameter $\alpha'$ is assumed to be 0.3. In Eq. (2) $\alpha' = 1.0$. Increases in service have a stronger effect in reducing the service-elasticity of demand at an $\alpha'$ of 1.0 than at 0.3. The elasticity is still greater than one at the mean level of service. Both Eqs. (3) and (4) tend to confirm the results of Eq. (1). The coefficient of $\ln B$ is the service-elasticity and is greater than one. In Eq. (4), the coefficient of the fare variable is also an elasticity and is approximately the same as the elasticity at the mean fare (-.67) in Eq. (1). Equations (5) and (6) take the service variable from Eq. (1). The fare variable is weighted by an algebraic root of the land
### Table 8
2SLS ESTIMATES OF ALTERNATIVE FORMS OF THE DEMAND FUNCTION, 1968

<table>
<thead>
<tr>
<th>Equation</th>
<th>( \ln D )</th>
<th>( \ln B )</th>
<th>( \ln F )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>8.81 ((B/POP)^{-0.3})</td>
<td>-</td>
<td>3.06 (F)</td>
<td>(R^2 = .976)</td>
</tr>
<tr>
<td>2.</td>
<td>10.96 ((B/POP)^{-1.0})</td>
<td>-</td>
<td>2.91 (F)</td>
<td>(R^2 = .973)</td>
</tr>
<tr>
<td>3.</td>
<td>1.39 (\ln B)</td>
<td>-</td>
<td>3.17 (F)</td>
<td>(R^2 = .976)</td>
</tr>
<tr>
<td>4.</td>
<td>1.32 (\ln B)</td>
<td>-</td>
<td>0.76 (\ln F)</td>
<td>(R^2 = .979)</td>
</tr>
<tr>
<td>5.</td>
<td>6.49 ((B/POP)^{-0.3})</td>
<td>-</td>
<td>(61.3 \frac{F}{AREA^{0.5}})</td>
<td>(R^2 = .968)</td>
</tr>
<tr>
<td>6.</td>
<td>8.12 ((B/POP)^{-0.3})</td>
<td>-</td>
<td>(13.4 \frac{F}{AREA^{0.25}})</td>
<td>(R^2 = .979)</td>
</tr>
</tbody>
</table>

area in these equations. This attempts to adjust the average fare for differences in the average length of a bus trip. In Eqs. (4) to (6), the fare parameter is significantly greater than zero. The \(R^2\) is better in (4) and (6) than in (1). The problem of an unexpectedly large service-elasticity of demand, however, is not eliminated by these changes in the functional form.
2. **The Response of Passengers to Increases in the Size of the Market**

The coefficients of $\ln \text{POP}$ and $\ln \text{AREA}$ describe what happens to passengers if population, land area, bus-miles, automobile ownership, and highway capacity grow in the same proportion. Thus, the bus-service variable, $\ln \text{AUTOS}$, and $\ln \text{HWAY}$—all expressed in per capita terms—do not vary. The sum of the estimated coefficients ($\beta_3 + \beta_4$) is 1.12 in 1968 and 1.11 in 1960. As we move from small to large cities, the demand for transit increases approximately 10 percent faster than population and land area. In 1960, the sum of these estimated coefficients is significantly greater than one at the five percent level (with 34 degrees of freedom).

3. **Automobiles and Highway Capacity**

Automobiles provide the chief substitute for bus transit in the urbanized areas included in this study. Increases in highway capacity, *ceteris paribus*, reduce the time required for both auto and bus travel. Neither variable has a significant effect on transit passengers in these estimates of the demand function.

4. **Socio-Economic Characteristics: The Income Distribution and the Age Distribution**

Among the arguments put forth for maintaining and subsidizing bus transit operations is that such actions aid low-income groups. The old, the young, and the poor are less likely to have automobile transportation available and more likely to
rely on mass transit. It has been generally recognized for some
time that the major use of urban mass transportation is for the
work trip. Consequently, the typical user is a member of the
labor force and most likely to be neither young, nor old, nor
poor. Meyer, Kain, and Wohl\(^7\) report the results of a 1953
Detroit survey in which 60 percent of transit trips were work
trips and a more recent Philadelphia survey in which about 80
percent of rail travelers were going either to or from work.
Consequently, general subsidies for urban transportation appear
unlikely to effect a redistribution of income in favor of low-
income groups, although special fares for "worthy" groups and
improved off-peak bus service may well aid riders who are not
members of the labor force.

Estimates of the market demand for bus transit as a
function of the income and age distribution tend to support
these conclusions. Urbanized areas with a higher proportion of
the population over 65 or under 18 or with a higher proportion
of households earnings less than $3,000 per year (1960) have
fewer transit riders than urbanized areas which are otherwise
identical. The 1968 estimates are far stronger than the 1960
results in this regard. The 1968 results also give some weak
support to the modal choice results that higher-income members
of the labor force are more likely to use the automobile than

\(^{7}\) J. R. Meyer, J. F. Kain, and M. Wohl, The Urban
mass transportation. Urbanized areas with a high proportion of households earning more than $10,000 per year seem to have fewer transit riders than other cities, although this result is in no sense statistically significant.

Table 9 reports the estimated effects on transit ridership of changes in the income and age distribution. These estimates are based on the 1968 demand results. The absolute size of these effects is so large that it raises a question of whether the demand estimates are picking up differences in the demand for urban transportation or whether some other factor

<table>
<thead>
<tr>
<th>A 1% increase in:</th>
<th>Leads to an increase in ridership of:</th>
<th>t-ratio</th>
<th>Level of Significance 40 degrees of freedom</th>
<th>Level of Significance 3 degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>% under $3,000</td>
<td>-2.31%</td>
<td>-2.92</td>
<td>1%</td>
<td>10%</td>
</tr>
<tr>
<td>% between $3,000 and $10,000</td>
<td>3.24</td>
<td>2.47</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>% over $10,000</td>
<td>-2.82</td>
<td>-1.58</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>% 18 and under</td>
<td>-4.72</td>
<td>-2.05</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>% between 18 and 65</td>
<td>6.51</td>
<td>2.84</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>% over 65</td>
<td>-5.79</td>
<td>-1.82</td>
<td>10</td>
<td>-</td>
</tr>
</tbody>
</table>
enters the results. For instance, a 1.0 percent increase in the population between 18 and 65 increases ridership by 6.5 percent. An increase in the proportion of working-age adults may have disproportionate effects on transit ridership not only because adults consume more urban travel than children or the aged but because the absence of children may free adults, particularly women, to participate in the labor force. Nevertheless, the 6.5 percent response appears too large to be explained only by labor-force participation.

D. A SUMMARY OF EMPIRICAL FINDINGS

This summary of the five major empirical findings of the study begins with a caveat concerning these empirical results. The discussion of the results assumes that demand and supply functions are properly specified and that the observations of transit operations in the cities of the sample consist of points on the demand and supply functions. However, the demand and supply estimates may merely capture differences between cities rather than differences in transit behavior related in a systematic way to differences in the values of the exogenous variables of the model. In this case, the results of this thesis carry no credibility.

1. The Fare-Elasticity of Demand

The demand estimates give some evidence--considerably stronger for the 1960 data than for the 1968 data--that the average fare-elasticity of demand may be as high as -0.70. The
industry's rule-of-thumb (the Simpson and Curtin formula) and estimates based on disaggregated models indicate a fare-elasticity of about -0.30. Most of the disaggregated models examine the demand response only in terms of switches between transit and auto use, and thus underestimate the total response to change in fare.

A fare-elasticity of -0.70 instead of -0.30 should be a matter of great significance for transit policy. Suppose the firm requires a five percent increase in total revenue to cover operating and capital costs (including normal profit). According to the Simpson and Curtin formula, a fare increase slightly in excess of seven percent will yield the additional revenue. If the elasticity if -0.70, a fare increase of nearly 17 percent is required. In the former case, ridership declines by two percent, while in the latter case, ridership declines by nearly twelve percent. If the decline in transit ridership results in greater automobile use, there might be significant externalities in the form of congestion and pollution. In the face of these factors, local governments may prefer to provide subsidies for transit riders than permit further fare increases.

The problem for local governments may be accentuated if the fare elasticity is estimated as -0.30 but is really -0.70. A ten percent fare increase is expected to result in a three percent decline in passengers. In the months after the fare increase, the decline amounts to seven percent. If the unexpected
loss of passengers is attributed to a shift in demand as opposed to a movement along the demand function, local governments may conclude that ridership is declining because of tastes or preference for automobiles rather than to a factor which is easily controllable through transit subsidies.

2. **The Level of Service**

Demand estimates also indicate that transit passengers are highly responsive to variations in bus-miles. Thus, both of the policy instruments available to transit firms and local governments--the transit fare and the level of service--may be quite effective in increasing or decreasing transit riders.

3. **Bus Patronage Among Income and Age Groups**

Arguments for bus transit subsidies occasionally rely on distributional arguments, that improved service or lower fares will primarily aid the old, the young and the poor, who cannot rely on the private automobile for urban transportation. The results of this paper tend to indicate that bus patronage is lower in urbanized areas with large percentages of old or young persons and large percentages of households earning less than $3,000 in 1960. These results lend support to an alternative thesis, for which other evidence is readily available, that transit usage is dominated by work trips and that general transit subsidies may predominantly benefit members of the labor force.
4. **Profit-Maximization or Revenue-Maximization under Regulation**

Supply estimates for the 1968 data make it possible to reject the hypothesis that transit supply behavior is represented by the objectives of maximizing either profit or revenue subject to regulatory constraint. Given the large number of alternative hypotheses, including some which may not have been considered here, this study does not suggest a supply hypothesis which is best supported by the evidence. The lack of any consistent pattern in the 1960 supply results also prevents the selection of any "best" supply hypothesis.

5. **Response to Variations in the Regulatory Constraint**

Performance-oriented supply objectives such as maximum bus-miles and maximum passenger trips predict that the firm will respond the same way to increases in passengers, decreases in costs or increases in the ratio of total cost to total revenue. Both 1960 and 1968 supply estimates show that higher cost-revenue ratios tend to be associated with lower fares and that more passengers or lower costs tend to be associated with improved levels of service. One indication from these results is that performance objectives may vary as the cost-revenue ratio varies. Transit operations with high cost-revenue rates may be associated with low fares, while unsubsidized operations may be more likely to emphasize service levels.
VI. THE IMPACT OF SUBSIDIES ON THE BUS TRANSIT MARKET

This thesis has concentrated on a technical treatment of a rather simple model of the market for bus transit service. The model can be readily extended and broadened in several ways. For instance, some of the most interesting results stem from the presence of two quantity measures, passengers and bus-miles, in the transit model. If this phenomenon carries over to other markets, such as microwave communications or intercity air travel, some of the results of this thesis may apply to these markets as well. One obvious extension of the bus transit model is the formulation of the more general cost function necessary to include rapid-rail service in the transit model. Another obvious extension of this thesis is to disaggregate the bus transit model to include separate demand and cost functions for peak and off-peak urban travel.

The theoretical analysis and the statistical estimation of behavior in the transit market, however, have far outstripped the practical application of these results. The first five Chapters of this thesis contain a rich lode of potential applications for bus transit policy. This Chapter discusses one application, the general subject of subsidies for urban transit, and evaluates the impact of proposed Federal subsidies on the market for bus transit service. Subsidies have now become a fact of life for
urban mass transportation, and recently Congress requested the Administration to examine the possibility of Federal operating subsidies to cover transit deficits. An analysis of the subsidy question is a good application of the transit model, since it requires a thorough knowledge of the workings of the transit market. This includes the demand-response of transit users to changing fares and levels of service and the supply-response of transit firms to changing ridership and costs.

The decline in urban mass transportation use since the artificially high levels of World War II has forced some very difficult decisions on local governments in their roles as operators or regulators of transit systems. A significant factor in the decline of transit patronage has been the growing decentralization of cities, both in terms of job locations and residences. As a result of this decentralization process, traffic densities in terms of transit passengers per hour have tended to decline along the major travel corridors in metropolitan areas.¹ The costs of an urban mass transportation system have become spread over fewer and fewer riders. Since most transit systems have been operated by private firms guaranteed a normal profit, the decline in ridership has brought forth requests to regulatory authorities for either increases in fares or reductions in service. Increases in operating costs, chiefly due to rising

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¹ Meyer, Kain, and Wohl, op. cit., Chapters 2,3.
wages, have engendered similar requests. The choice of either a fare increase or a service reduction promises an even further reduction in transit ridership.

The self-reinforcing aspect of passenger losses in the transit market has led local governments to consider subsidies as alternatives to fare increases or service reductions. Subsidies would replace the lost revenue or match increases in operating costs without leading to further losses of transit patronage. A large number of transit systems now receive subsidies, and the Federal government is contemplating a fairly large program of operating subsidies for local transit operations. A Federal program of capital grants for urban transit has existed since 1964.

A. EXISTING TRANSIT SUBSIDIES

1. Subsidies for Publicly Owned Transit Operations

The desire to avoid fare increases and losses of transit ridership has probably been a major factor in the rapid growth of publicly owned transportation firms. A sample of 44 firms in 1960 reveals that only seven percent are publicly owned, whereas in a similar sample in 1968, 35 percent of the firms are publicly owned. A survey indicates that 76 transit systems were subject to public acquisition between January 1, 1960 and January 1, 1970.² Public acquisition has probably been

². R. L. Banks & Assoc., op. cit., Appendix IV.
significantly aided by Federal grants and loans to local agencies for the purchase of privately owned transit properties.

The trend toward public ownership has increased both direct and indirect subsidies for transit operations. Publicly owned properties operated as an integral part of a municipal government may have little constraint to earn sufficient revenue to cover total cost. In eight such systems in 1968, operating costs plus capital costs exceeded total revenues by about one-third, on average. In some systems, like Baltimore, transit managers are expected to meet operating costs out of operating revenues but capital costs are fully subsidized. Publicly owned systems operated by quasi-independent transit authorities, unless they have taxing powers, may not have the financial flexibility to provide direct subsidies for urban transit. An exception exists for authorities (such as in New York) which can use revenues generated from other sources (the bridges, tunnels, and airports) to subsidize urban transit. Independent transit authorities may also serve as effective conduits for grants or subsidies from other governmental agencies.

An indirect subsidy serves to reduce the cost of transit operations. Total indirect subsidies for publicly owned transit systems may be as large as direct subsidies. Publicly owned operations are exempt from local and state property taxes and state taxes on fuel. The low rates of interest available to municipal and state governments because of Federal income tax
law also constitute a major subsidy for publicly owned transit operations. Evidence indicates that the cost per bus-mile of publicly owned transit properties is 10 percent less than that of private firms, controlling for differences in wage rates and age of equipment. Most of this 10 percent differential is due to indirect subsidies. (See Appendix A for details on cost functions for bus transit operations.)

2. Other Non-Federal Subsidies for Transit Operations

Publicly owned systems are not the exclusive beneficiary of transit subsidies. All transit firms may receive a subsidy in the form of urban highway services, if the cost of building and maintaining urban roads which is allocable to transit vehicles exceeds taxes on fuel and tires. In both the Washington, D. C. and New York metropolitan areas exclusive bus lanes are used by vehicles from privately owned companies. Although a case for such a practice can be built on the grounds of economic efficiency, the practice does constitute a subsidy for private transit firms. Privately owned firms may also receive direct subsidies from local governments. Many cities make direct payments to transit companies for transporting school children at reduced fares. Washington, D. C. is currently considering a $3 million subsidy for a privately owned transit company to avoid an increase in fare above 40 cents.

The case of transit firms operated by suppliers of electricity and natural gas is an interesting example of a private
subsidization scheme. Eight such firms in 1968 had ratios of total cost to total revenue averaging 1.42. The firm is allowed extra earnings on power operations to cover the deficit in transit operations. The process is simplified by the fact that in almost all cases a single agency regulates both transit operations and the supply of electricity and natural gas. Customers of the power operations subsidize users of transit operations. This process of cross-subsidization is similar to an excise tax on utility bills earmarked for transit operations. Often the users paying the tax live in rural areas or in a different city from the users of transit operations. 3

3. Federal Subsidies for Urban Transit

A program of direct Federal subsidies for urban transit was initiated by the Urban Mass Transportation Act of 1964. 4 This legislation authorized Federal grants and loans for capital purchases available on a matching basis to local governments. The Federal grant can provide up to two-thirds of the funds necessary for capital projects. Between 1964 and 1971 over $700 million in Federal funds were committed for 155 capital grant projects. As has been noted previously, Federal funds were instrumental in the public acquisition of a number of private transit properties.

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3. The Wisconsin Public Service Corporation, for instance, provides power throughout Wisconsin but operates a transit system only in the city of Green Bay.

4. Indirect subsidies may have existed for years in the form of Federal support for local highway construction.
In the Urban Mass Transportation Act of 1971, Congress has asked the Department of Transportation to evaluate Federal assistance for urban transit operations which would take the form of operating subsidies. The nature and timing of the subsidy proposal are undoubtedly related to the financial problems of urban mass transit systems, marked by the growing deficits of publicly owned properties and the apparently ceaseless requests for fare increases or service reductions by private firms. Despite the shrinkage in ridership, there still exists a large body of transit users who suffer whenever fares are increased or service is reduced. This group has placed demands on urban governments for transportation services which local agencies have not had the financial resources to meet. This has led local agencies to seek help from Congress. Another factor in the political appeal of subsidies for urban transit is the fact that the transit industry was once healthy and profitable. The decline in the fortunes of urban transit may have made a case in equity for transit subsidies.  

B. THE ECONOMIC RATIONALE FOR TRANSIT SUBSIDIES

Economic arguments in support of subsidies for urban transit have an entirely different tone from the political and financial factors underlying the proposal for Federal operating

5. The preamble to proposed subsidy legislation notes "that for many years the mass transportation industry satisfied the transportation needs of urban areas of the country capably and profitably."
The most convincing arguments for transit subsidies cite the existence of economies of scale in urban mass transportation, the fact that increased use of mass transit may reduce some of the external costs associated with urban transportation, and the fact that the use of private automobiles in urban areas is already subsidized.

In addition to offering a justification for transit subsidies, economies of scale also provide an explanation for the financial straits of the urban transit industry. Economies of scale in urban bus transit exist with respect to transit passengers but not with respect to bus-miles or other measures of the level of transit service. (See Appendix A.) The incremental cost of an additional passenger is much less than the average cost per passenger, if the scale of transit operations is held constant. The source of economies of scale in bus transit is the large capacity of transit vehicles. Seating capacity of many buses is in excess of 45 persons and total capacity may range as high as 80 persons. Peak-hour transit service, where buses are largely full, may contain very limited economies of scale because


7. The cost of an additional passenger consists of the time delay necessary to take on an extra passenger and the loss of comfort, if any, suffered by other passengers.
the system is operating near capacity. The incremental cost of an additional passenger at capacity may exceed the average cost per passenger because some other potential passenger must be eliminated. Off-peak transit service, however, possesses considerable economies of scale because of the excess capacity in the system at these times.

Economies of scale with respect to transit passengers have never been fully exploited because transit passengers have annually paid fares on the basis of the average cost per passenger, thus assuring that total revenue covers total cost. Setting transit fares on the basis of incremental costs would not yield sufficient revenue to cover costs, and subsidies would therefore be required. Operating subsidies designed to permit incremental-cost pricing would lead to a greater number of transit riders at only a slight increase in terms of real costs. Incremental-cost pricing at a given scale of operations does not solve the problem of determining the proper scale of operations. The incremental cost of an additional bus-mile, for instance, has been shown in Appendix A to be approximately the same as the average cost per bus-mile. In rail rapid transit economies of scale exist with respect to car-miles as well as with respect to passengers. The basis for providing subsidies to bus and rail transit at a given level of service appears firm, but is far less firm in the case of bus transit, where the level of service is allowed to vary.
Average-cost pricing for bus transit service will lead to fare reductions over time in systems where the growth in passengers outstrips the growth in cost. The existence of economies of scale implies a decline in the average cost per passenger. However, since World War II, bus transit systems have moved in the other direction. The decline in passengers, due initially to exogenous factors, has tended to increase the average cost per passenger because the cost savings from losing passengers are very small. The increase in fares resulting from a policy of average-cost pricing causes an even further decline in transit riders. Thus, the financial problems of urban transit and the pressure for Federal subsidies are tied very closely to the existence of economies of scale.

The existence of external costs in urban transportation or the existence of subsidies for automobile users are probably best treated with taxes or with technology rather than with subsidies for bus transit. In the absence of such actions, bus transit subsidies may become justified. The major external costs associated with urban transportation involve traffic congestion and air pollution. Air pollution is a well-known example of an external cost. Traffic congestion is an externality because the individual driver is able to impose delays on other drivers without bearing the costs of the delay. Increased use of bus transit, if it occurs at the expense of automobile use, may reduce traffic congestion during rush hours and reduce air
pollution resulting from engine emissions. Direct ways of solving pollution and congestion problems include requiring pollution control equipment on automobiles and levying congestion tolls during rush hour traffic. The practice of reserving freeway lanes exclusively for buses is an interesting technological approach to the problem of congestion. Nevertheless, operating subsidies for bus transit service may lead to some reduction in pollution and congestion.

The "countervailing subsidy" argument holds that transit subsidies are needed to counteract subsidies received by users of automobiles. Meyer, Kain, and Wohl have shown that automobile users do not receive substantial subsidies in the form of highway services, since their taxes on fuel and tires pay nearly the full cost for the construction and maintenance of urban roads. This finding is open to question on two grounds. Urban freeways and surface arterials may involve immense social costs which are not covered in highway construction and maintenance appropriations. Also, off-peak users of urban highways subsidize users during peak hours since both groups pay the same fuel taxes but the highways were constructed to meet peak-hour requirements. A major source of subsidy for automobile users may be in the form of subsidized parking provided by employers for their employees. Employers who provide free parking for employees do not typically provide compensating benefits for employees who choose transit
to come to work. This does not by itself, however, provide a strong argument for operating subsidies for bus transit.

In summary, the case for operating subsidies in urban transit appears to contain considerable merit, on the basis of economies of scale in transit ridership and on the basis of pollution and congestion costs which are largely associated with automobile traffic.

C. OPERATING SUBSIDIES

1. The Subsidy Mechanism

The operating subsidy currently under study has the stated purpose of providing emergency financial support to transit firms on an interim basis. This implies that the measure is designed for firms with operating deficits. The subsidy mechanisms under primary consideration, however, would spread benefits to all firms, whether or not they have operating deficits. A major justification for such a policy is that transit deficits are at least partially the result of subsidy arrangements already in existence. The analysis in Chapter V finds that the ratio of total cost to total revenue depends almost exclusively on the

8. The Federal government, as an employer, is among the worst offenders at subsidizing auto users. In 1971, for instance, the Federal government began condemnation proceedings against the lease in the parking garage of the building which houses much of the Department of Transportation, including the Urban Mass Transportation Administration. The government then provided parking for Federal employees at $6 per month instead of the market rate of $30 per month.
institutional structure of the transit market, rather than on variables such as cost per bus-mile or population which might indicate some fundamental economic reasons for transit deficits in bus systems. The absolute size of transit deficits in a few systems are such that rapid rail transit service in New York, Chicago, and Boston would absorb well over one-half of any subsidy based exclusively on existing deficits. Furthermore, a policy of subsidizing deficits would reward the special arrangements between power companies and bus transit systems. There seems to be considerable question whether such systems should be singled out for favorable treatment. While deficits may primarily reflect special local policies, there is even more reason for not having a subsidy which covers operating deficits *per se*. Such a policy would tend to destroy the remaining financial integrity of the transit industry by encouraging the creation and enlargement of deficits within the transit industry.

The subsidy mechanisms receiving the most serious consideration would reward transit systems on the basis of some formula relating the amount of the operating subsidy to some measure of the scale of operations. Two subsidy mechanisms under examination in this Chapter are a cost subsidy, which sets the amount as a fraction of total costs, and a fare-subsidy, which provides a given amount of support per revenue (paying) passenger. As of late spring, 1972, the Department of Transportation appeared on the verge of recommending the latter formula.
2. Qualitative Effects of Cost and Fare Subsidies

The most important feature of a subsidy for urban bus transit seems to be the promotion of greater patronage and levels of service for all bus transit systems rather than assurance of continued survival of a few endangered systems. Although circumstances may be quite different for rail systems, there is no evidence that bus firms facing large deficits can not eliminate these deficits by raising fares or by sharply reducing the levels of service. Indeed, the existence of deficits for bus systems is evidence that local governments prefer deficits to either increased fares or lower levels of service.

Cost subsidies and fare subsidies constitute changes in the fundamental economic conditions of the transit market. A cost subsidy has the effect of causing a downward shift in the cost function of the bus transit firm. A fare subsidy constitutes an increase in the demand for transit from the viewpoint of the firm because the number of persons willing to ride transit for the same net revenues has increased. The assumption is made that the firm responds to these stimuli just as it would respond to natural shifts in cost or demand.

A number of factors may influence the response of the firm to subsidies for bus transit operations. The effect of the subsidy on transit operations depends on the proportion of the subsidy passed on to transit users in the form of lower fares or increased levels of service. The remaining portion of the subsidy
goes to reduce the ratio of total costs to total revenue. In the long-run it seems possible that all or nearly all of the subsidy is passed along to transit users, if not directly then perhaps through foregoing future increases in fares or reductions in service. These actions would have been necessary in the absence of the Federal subsidy. In the short run, the actions of firms and regulators are more difficult to predict. Presumably, regulators will force transit firms to remain at the same cost-revenue ratio.

The effect of operating subsidies on transit operations may also depend on the specific form of the subsidy mechanism. Transit firms may not react to a decrease in cost in the same way as to an increase in demand. The cost subsidy implies a decline in the incremental cost of a unit of transit service, while the fare subsidy implies an increase in the number of passengers occurring at the same net revenue to the firm. Whereas the cost subsidy may spur the firm only in the direction of improved levels of service, the fare subsidy may induce the firm to seek increased passengers, both through improved service and through lower transit fares.

An important determinant of the impact of the operating subsidy is the strength of the supply and demand elasticities in the transit market. The response of transit riders to lower fares or improved service levels and the response of transit firms to an increase in passengers or a decrease in the incremental
cost of transit service are governing factors in the ability of subsidies to increase patronage or levels of service in urban bus transit markets. The econometric model developed and estimated in Chapters II through V provides both estimates of the relevant elasticities and a framework for examining the effect of the subsidy. Appendix D derives the algebraic relationships implied by the model, relating changes in passengers, bus-miles, and the transit fare to the subsidy.

3. Quantitative Effects of Cost and Fare Subsidies

This section presents and analyzes the quantitative effects on urban bus transit of a one percent cost subsidy and a one percent fare subsidy. The results assume that the entire subsidy is passed along to transit users in the form of lower fares or improved levels of service. The nine sets of results appearing in Table 10 and Table 11 are based on combinations of three different assumptions about supply elasticities and three different assumptions about demand elasticities. Table 10 contains the results of the cost subsidy and Table 11 contains the results of the fare subsidy.

The three assumptions about demand elasticities include the case where the fare-elasticity and the service-elasticity are large and equal. The values of the fare-elasticity (−.75) and the service-elasticity (.75) are in about the same range as estimates found in this study. The second set of elasticities used (−.30 and .30) are small and equal indicating that patronage
Table 10
THE EFFECTS OF A ONE PERCENT COST SUBSIDY

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Table 11
THE EFFECTS OF A ONE PERCENT FARE SUBSIDY

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<th>Supply Elasticities</th>
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is less responsive to changes in the fare and the levels of service. The third set of elasticities contains a small fare-elasticity (-.20) and a moderately large service-elasticity (.50). This case is particularly interesting in that it seems to reflect the results found by Kraft et al. in the Charles River Associates study of the Boston area. That study found transit ridership much more responsive to travel time than to travel cost.

The supply parameters indicate to what extent transit firms respond to increases in passengers and decreases in cost by improving the level of service and to what extent by reducing fares. The firm which attempts to maximize bus-miles responds exclusively through adjustments in service, and the supply elasticities are 1.000 with respect to passengers and -1.000 with respect to cost. The supply estimates in this thesis for the 1968 sample find elasticities of .727 and -.601, which indicate a tendency for the firm to use fare adjustments as well as service adjustments. The third case assumes that response to extra passengers or lower cost occurs mainly through fare adjustment. These elasticities are .300 and -.300.

The most important result in Tables 10 and 11 is that the demand elasticities are the crucial parameters in the transit model. First, if the fare-elasticity is equal to the service-elasticity in absolute terms, the increase in passengers is independent of both the subsidy mechanism and the supply elasticities. This result is obvious from the equations derived in Appendix D. With the large elasticities, which are representative
of the statistical results of this thesis, a one percent cost subsidy and a one percent fare subsidy lead to a three percent increase in transit passengers under each assumption about the supply elasticities. The impact of subsidies on the transit market varies dramatically with the size of the demand elasticities. The increase in passengers is seven times as great where the demand elasticities are .75 and -.75 as in the case where the elasticities are .30 and -.30.

The increase in passengers in the case where demand elasticities are unequal in value (.50 and -.20) depend on both the subsidy mechanism and the supply elasticities. In this particular case the response of passengers to increases in service is greater than the response in passengers to reductions in the fare. Thus, the cost subsidy has a greater impact on the number of passengers than the fare subsidy. Moreover, the supply response which emphasizes bus-miles is more effective in increasing passengers than the supply responses which involve fare adjustments. In fact, the increase in passengers resulting from a one percent subsidy varies from .28 and 1.00, depending on the subsidy mechanism and the nature of the supply response.

The one percent cost subsidy and the one percent fare subsidy are "equal cost" policies only if the ratio of total cost to revenue is equal to 1.0. In systems where local subsidies are provided, the cost-revenue ratio \( k_o \) may be considerably in excess of 1.0. If \( k_o \) is 1.2, for instance, an "equal cost"
cost subsidy leads to an increase in passengers of only 2.50 percent under the large demand elasticities. Thus, if the majority of systems have cost-revenue ratios in excess of 1.0, fare subsidies may have a greater per dollar effectiveness than cost subsidies.

The results in Tables 10 and 11 conform to expectations. The cost subsidy, which has the effect of making bus-miles relatively cheaper, yields a greater increase in bus-miles than the fare subsidy but results in a smaller decrease in fare. The supply assumptions yield similar results. The large supply elasticities lead to relatively large responses in terms of bus-miles; the small supply elasticities result primarily in lower fares.

D. GENERAL GUIDELINES FOR ELIMINATING DEFICITS IN URBAN BUS TRANSIT

The primary purpose of Federal operating subsidies, it appears, is to reduce transit deficits without resort to policies which would further reduce transit ridership. The operating subsidy enables the firm to forego fare increases and service reductions which would have such an effect. The economic rationale for subsidies is the existence of economies of scale in the number of bus transit passengers and the congestion and pollution associated with high levels of automobile use. The impact of operating subsidies has been shown in this Chapter to depend very heavily on the values of two demand parameters--
the fare-elasticity and the service-elasticity. The impact of particular subsidies on transit ridership depends on the magnitudes of these elasticities and whether the fare-elasticity is larger or smaller than the service-elasticity.

If the major purpose of transit policy is to prevent the further loss of transit riders, it is possible to construct some general policy guidelines which serve this purpose. Table 12 outlines the major policy alternatives and the circumstances under which each alternative is relatively more effective in preventing the loss of transit riders.

The recommended actions are based on the values of the service-elasticity of demand and the fare-elasticity of demand. Small elasticities range between 0 and 0.50 in absolute value, whereas large elasticities range between 0.50 and 1.00. There are four possible actions available in each case: raise fares, reduce levels of service, subsidize costs, subsidize fares. In each case two of the four actions appear relatively attractive, and two appear relatively unattractive.

1. **Large Fare-Elasticity, Large Service-Elasticity**

   In the case of large demand elasticities subsidies can prevent the loss of large numbers of transit riders. At elasticities of .75 in absolute value a one percent subsidy increases passengers by 3.0 percent. If a cost subsidy or fare subsidy of 10 percent is necessary to eliminate the transit deficit, this can prevent a loss in passengers as great as 30 percent.
Table 12
POLICY GUIDELINES FOR ELIMINATING TRANSIT DEFICITS

<table>
<thead>
<tr>
<th>Large Fare-Elasticity</th>
<th>Small Fare-Elasticity</th>
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<tr>
<td>Large Service-Elasticity</td>
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<td>2. Fare Subsidy</td>
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2. **Small Fare-Elasticity, Small Service-Elasticity**

In this case the number of passengers is not highly sensitive to either the fare or the level of service. At elasticities of 0.30 in absolute value results from the previous section indicate a 10 percent cost or fare subsidy would prevent a reduction in transit passengers of only 4.3 percent. The subsidy therefore is not an especially effective policy. Due to the insensitivity of demand to the fare and service levels either fare increases or reductions in service can be used to eliminate the deficit without seriously affecting transit ridership.

3. **Small Fare-Elasticity, Large Service-Elasticity**

In the case of unequal elasticities the fare and the level of service are not combined in a way which maximizes passengers. Chapter III has shown that maximum passengers
imply that the fare-elasticity is equal to the service-elasticity in absolute value. Consequently, policies designed to minimize the loss in ridership should take advantage of the fact that the firm is not maximizing passengers. If the fare-elasticity is smaller than the service-elasticity, an increase in the fare would minimize the loss of passengers. If a subsidy is adopted, a cost subsidy would maximize the gain in passengers by encouraging the production of additional bus-miles.

4. **Large Fare-Elasticity, Small Service-Elasticity**

This case is analogous to the preceding case, but the opposite policies are given as guidelines. With a large fare-elasticity but a small service-elasticity service reductions result in a smaller loss in passengers than fare increases. If a subsidy is adopted, a fare subsidy is recommended because it encourages lower fares, which may result in a substantial increase in the number of passengers.

The estimates of the demand for bus transit and the analysis of operating subsidies in this Chapter build a case for the use of either cost subsidies or fare subsidies in preventing a large loss in transit riders. The mean fare-elasticity and the mean service-elasticity exceed 0.50 in both the 1968 and 1960 estimates of the demand function. The very wide confidence intervals, especially on the estimates of the fare-elasticity, prevent a strong recommendation of the subsidy policy. The
binary choice models of the demand for transit and the inter-
zonal model developed by Kraft et al. yield smaller estimates of
the fare-elasticity of demand. If, as the results by Kraft et
al. seem to indicate, passengers are more responsive to improve-
ments in service than reductions in fare, the guidelines recom-
mand either an increase in fares or the use of a cost subsidy,
which would encourage the production of bus-miles. These re-
results would not recommend the fare subsidy, the subsidy
mechanism being most seriously considered by the Department of
Transportation.

5. Summary

This Chapter has provided a brief overview of the political
and economic factors underlying subsidies for urban mass trans-
portation. In evaluating operating subsidies for urban mass
transit, the effectiveness of a subsidy in promoting transit
ridership has been shown to depend on the responsiveness of
transit riders to reduced fares and improved levels of service.
The statistical findings of this thesis tend to give a rather
sanguine view of operating subsidies for urban mass transporta-
tion.

It should also be clear from this chapter that more
research is needed on the market for bus transit service.
Better estimates of the demand parameters are a necessity if
policies are to be recommended with any confidence. It would
also be desirable to place the economic rationale for transit
subsidies on a firm empirical basis and determine that subsidy
policies serve the interest of economic efficiency in addition
to serving the interest of good politics.
Appendix A
THE COST OF BUS TRANSIT OPERATIONS

In the models of supply behavior included in the text, the firm acts as though all costs are variable and the cost per bus mile is a constant, regardless of the scale of operations. This Appendix discusses the cost of transit operations in some detail, including an evaluation of the assumptions of the model. The Appendix is organized into four sections: (1) the definition of transit costs as the term is used in this paper, (2) a discussion of operating and capital costs, (3) creation of a formula for capital costs, and (4) estimation of the parameters of a cost function. In estimating the cost function we are particularly interested in the existence of cost economies and diseconomies for larger firms and in the effects of wage rates and bus characteristics (fleet age, seating capacity) on total cost.

A1. THE COST OF TRANSIT OPERATIONS TO THE FIRM

The concept of cost relevant for this paper is the cost of transit operations borne by the firm. This includes the direct cost of operating buses (drivers' wages, fuel, oil, tires), maintenance of the equipment, administrative expense, insurance, taxes, rents associated with offices and bus barns, and the costs of bus capital.
No portion of the cost of freeways, arterials, roads, or traffic-control devices is included in transit costs unless it appears in a road-use tax or similar levy. 1 In this respect, bus transit systems differ from rail systems. Rail companies must provide and maintain their own roadbed rights-of-way. In effect, they must provide their own "social overhead capital," while bus companies may share government-built roads with other modes of transportation. 2 In terms of vehicle miles, there are economies of scale for rail which do not exist for bus systems because the stations, roadbeds, and rights-of-way of a rail system constitute a large element of fixed cost. In a rail system, costs do not increase proportionally with vehicle miles unless track mileage also increases proportionally.

The operation of vehicles in a crowded urban environment creates externalities of various kinds. Exhaust fumes may foul the air, leading to adverse health effects or at least to a decline in the utility of urban residents and workers. An additional vehicle on a crowded freeway or city street makes an incremental contribution to congestion and imposes delays on other urban travelers. These are real costs, but neither is borne directly by the bus transit firm. Proponents of transit

1. An unusual kind of "road-use tax" occurs in some northern towns where bus transit companies have the responsibility for snow removal.

2. In the case of exclusive freeway lanes for buses, the cost is not paid by the bus company.
argue that additional bus transit service leads to a reduction in both air pollution and congestion. Increased bus service, by attracting additional ridership, is supposed to result in a net decline in vehicular traffic. Consequently, air pollution and congestion diminish. In this case, the omission of externalities from cost considerations may overstate the real cost of transit operations.

A2. OPERATING AND CAPITAL COSTS

Costs are developed for two samples of firms (1968 and 1960) on the basis of data supplied by the firms to the American Transit Association (ATA). Total costs consist of operating costs and capital costs. Operating costs consist of wages and salaries, rents, fuel and lubricants, tires, materials, taxes, and other items used and paid for in the course of transit operations. The definition is the same as that used by the ATA, with the exception that depreciation and amortization are included here as capital cost rather than an operating cost.

The important question is whether bus transit costs are fixed or variable during the period of analysis. The period of analysis appropriate for this model is the medium-run period where capital items like the bus fleet are variable but where other kinds of capital may be fixed. Almost all transit costs are variable under this definition. Items which are directly connected to bus-miles like drivers' wages, fuel, and tires
are obviously variable in the extreme short-run. Many overhead items consist predominantly of personnel costs and should also vary in the short-run. Bus capital costs are not considered a fixed cost because of the mobility of this capital equipment and because of the active market for used buses. Interest and depreciation in plant and property probably should be considered a fixed cost. Much of the total value of this item, however, may lie in urban property which is a fungible commodity and which can be converted to alternative uses within a relatively brief period of time. Capital costs attributable to plant and property amount to no more than one percent of total bus transit costs, in any event.

Wells' "Economic Characteristics of the Urban Bus Transit Industry: 1960-1969" provides a breakdown of operating cost by function (Table A.1). Drivers' wages make up the largest single item in operating costs (44 percent). Other labor costs occur in equipment, maintenance, and garage expense and in administrative and general expense and may swell the wages and salaries portion of operating cost to 65 percent.

Firms incur capital costs because capital equipment declines in value (depreciates) with use and age and because of the opportunity cost of their financial investment in equipment. This return consists of interest payment or "normal" profit, depending on the method of finance used by the firm. Capital

3. J. D. Wells and Sharron Thomas, op. cit.
Table A.1
DISTRIBUTION OF OPERATING EXPENSES

<table>
<thead>
<tr>
<th>Category Name</th>
<th>Percent of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1960</td>
</tr>
<tr>
<td>Total operating costs</td>
<td>100.00</td>
</tr>
<tr>
<td>Equipment, maintenance and garage</td>
<td>20.50</td>
</tr>
<tr>
<td>Transportation</td>
<td>52.61</td>
</tr>
<tr>
<td>Drivers' and helpers' wages, etc.</td>
<td>(42.14)(^a)</td>
</tr>
<tr>
<td>Station</td>
<td>.64</td>
</tr>
<tr>
<td>Traffic, advertising, etc.</td>
<td>.95</td>
</tr>
<tr>
<td>Insurance and safety</td>
<td>5.65</td>
</tr>
<tr>
<td>Administrative and general</td>
<td>10.71</td>
</tr>
<tr>
<td>Operating taxes and licenses</td>
<td>8.43</td>
</tr>
<tr>
<td>Operating rents, net</td>
<td>.51</td>
</tr>
<tr>
<td>Number of firms in sample</td>
<td>107</td>
</tr>
</tbody>
</table>

a. Included in Transportation Expense.

employed in the production of transit service includes buses, the bus "barn" (building and equipment), offices, and stations, and not all of the cost of capital services is included under capital cost. Maintenance expense is part of the cost of using capital goods; yet this is included in operating expenses. Finally, to the extent that the firm leases buildings and offices, these rents are included in operating costs rather than capital costs.

To obtain capital costs for the firm, we estimate the value of the capital stock and apply a consistent formula for depreciation, interest, and normal profit to all firms in the
sample. One difficulty with the ATA data on depreciation and amortization is that firms use different schedules for buses. Thus, there is no comparability among firms. Furthermore, some publicly owned firms report no depreciation data whatsoever. The estimate of the value of the capital stock of the firm is based on data reported to the ATA on the number of buses operated by the firm and their year, model, and seating capacity.

Basing capital stock estimates only on bus capital tends to understate both the capital and the capital costs of the firm. Although actual data on capital and assets are scanty, there is evidence that the capital stock owned by the firm consists overwhelmingly of bus capital. Meyer, Kain, and Wohl, for instance, estimated the investment in yards and shops per bus to be one-seventh of the cost per bus. Because of the longer life of yards and shops (50 years vs. 11 years) other capital costs are only one-thirteenth as great as the cost of buses. Second, operating cost data indicate that one-half of one percent of costs are for operating rents. Some of the non-bus capital costs are undoubtedly covered in this category. Third, some yards and shops are quite old and, thus, the capital cost of these is reduced proportionally. The total underestimate of capital costs may be one-thirteenth, (eight percent) or smaller. Since capital costs amount to only ten percent of total costs, the overall error would be smaller than one percent.
A3. THE CAPITAL COST FORMULA

The cost of capital services depends on the value of the capital stock, the rate of depreciation, and the rate of return or rate of interest reflecting opportunity costs. We use a "declining-balance" formula in which depreciation is a constant proportion of the current value of the asset. (Depreciation which is a constant proportion of the initial value of the asset is a "straight-line" formula.) The opportunity cost of the investment is measured by the long-term loan rate of interest available to the firm. This is perhaps a conservative estimate, since it assumes the firm uses the funds to reduce borrowing rather than make an investment which yields some higher rate of return.

For a single bus the capital cost is the product of the value of the bus \( V \) and the depreciation rate plus the interest rate:

\[ V(S,A)(\delta + r). \]  

\[ \text{(Al)} \]

---

4. Considerable development in the theory of the user's cost of capital services has occurred in recent years. This section represented a simplified version of this theory. For the economist and ambitious layman wishing to investigate this subject in greater depth, a good summary of the present state of the art is: U.S. Department of Labor, Bureau of Labor Statistics, Capital Stocks, Production Functions, and Investment Functions for Selected Input-Output Sectors (Washington, D. C., 1970), Report No. 355. This report was prepared by Jack Faucett Associates.
The value \( (V) \) is a function of the age \( (A) \) and seating capacity \( (S) \) of the bus. Given the kind of depreciation formula used here, the value of the bus declines exponentially with age. If \( V_0(S) \) is the value of a new bus, the capital cost is:

\[
V_0(S) e^{-\delta A} (\delta + r).
\] (A2)

Ideally, capital costs for a bus fleet should be the sum of costs for individual buses. As an alternative to this long, involved computation, we assume that all fleets are composed of identical buses, each of which has an age and seating capacity equal to the average for that fleet. Total capital costs equal the product of the number of buses and the capital cost for the "typical" bus.

A number of firms in the 1968 sample have been recipients of capital grants from UMTA. UMTA capital grants which pay up to two-thirds the purchase price of capital goods reduce the capital costs of the firm. We adjusted for this by defining \( (s) \) the proportion of capital purchased under the UMTA program. This adjustment factor is \( (1 - 0.67s) \), where 0.67 represents the maximum proportion of the total cost which can be paid by UMTA.

Where \( n \) is the number of buses in the fleet, the capital costs are defined as:

\[
C_X = n(1 - 0.67s) V_0(S) e^{-\delta A} (\delta + r).
\] (A3)
The rate of depreciation ($\delta$) is 0.10. Thus, the value of the bus declines about 10 percent per year. Under the declining-balance formula, the bus has fallen to half its original value after seven years. The long-term borrowing rate of interest depends on whether the firm is publicly or privately owned.

The debt obligations of publicly owned bus firms are tax exempt and bear a substantially lower rate of interest. In 1960, high-grade municipal bonds had a yield of 3.73 percent, while grade Baa private bonds had a yield of 5.19 percent. In 1968, the differential was even greater: 4.51 for public and 6.94 for private. For a private firm in 1968, capital costs amounted to about 17 percent (10 percent plus 6.94 percent) of the estimated value of the capital stock.

It is surprisingly difficult to establish the average price of a new bus. Models differ in price, although the seating capacities may be the same. There is a wide range of optional equipment available such as air-conditioning and power steering which can increase the price of a bus by almost $10,000. Finally, the list price apparently differs substantially from the price at which buses are actually sold. To get an estimate of the average price actually paid for a new bus, we analyzed 24 fleet purchases made under the capital grants program between 1965 and 1970. We found the average purchase price for a 45-passenger bus to be $29,000. (The range was from $24,000 to $33,000.) Reducing or increasing
seating capacity from this standard size affected the price by about $1,000 per seat. This differential is larger than the differential on the price lists, indicating perhaps that purchasers of larger buses are less likely to buy a spartan model and more likely to purchase optional equipment. The formula for the value of a new bus is:

\[ V(S) = 29,000 + 1,000 (S - 45). \]  

(A4)

This formula was applied to both 1968 and 1960 data. The wholesale price index for transportation equipment increased only from 98.8 to 102.8 during this period.

Capital costs were computed for all firms in the two samples and added to operating costs. On average, these capital costs were approximately 10 percent of total costs. The range was rather large. In one firm with an entirely new fleet of large buses, capital costs were almost 20 percent of total costs. In other firms with extremely old fleets, the ratio was as low as three percent.

A4. COST FUNCTIONS FOR BUS TRANSIT OPERATIONS

The issues in the cost of transit operations center on the existence of economies of scale for large firms, the impact of wage rates on the cost of bus transit, and the effect of fleet characteristics on costs. The tool used in economic analysis to bring evidence to bear on these issues is the cost function. In this Appendix we present econometric results
based on estimates of this cost function:

\[
\ln C = \delta_0 + \delta_1 \ln B + \delta_2 \ln w + \delta_3 \ln VEL + \delta_4 A + \delta_5 S + \delta_6 PUB + \delta_7 s ,
\]

where

- \( C \) = total costs,
- \( B \) = bus-miles,
- \( w \) = hourly wage rate of operating personnel,
- \( VEL \) = bus-miles per bus-hour attained by the firm,
- \( A \) = average age of fleet,
- \( S \) = average seats per bus,
- \( PUB \) = 1 for publicly owned firm; 0 otherwise,
- \( s \) = proportion of fleet purchased with capital grant.

Table A.2 presents estimates of the coefficients of the cost function. We note the following results:

- Evidence indicates the lack of substantial economies or diseconomies of scale. Increasing transit operations by increasing bus-miles by 100 percent increases costs by 98 percent and 101 percent, according to our estimates. There is no evidence supporting the hypothesis that larger bus systems have lower costs per bus-mile. Apparently, our assumption of a constant cost per bus-mile is not wrong.

- In both 1968 and 1960 higher wage rates have very strong effects on total costs. Although labor is only 60 percent of total costs, a 100 percent increase in wage rates

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Table A.2
COST FUNCTIONS FOR 1968 AND 1960

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1968</th>
<th>1960</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable</strong></td>
<td><strong>$\ln$ Total Cost</strong></td>
<td><strong>$\ln$ Total Cost</strong></td>
</tr>
<tr>
<td>Constant</td>
<td>0.930</td>
<td>0.864</td>
</tr>
<tr>
<td></td>
<td>(0.557)</td>
<td>(0.417)</td>
</tr>
<tr>
<td>$\ln$ B</td>
<td>0.982</td>
<td>1.013</td>
</tr>
<tr>
<td></td>
<td>(0.0327)</td>
<td>(0.0223)</td>
</tr>
<tr>
<td>$\ln$ w</td>
<td>0.883</td>
<td>0.785</td>
</tr>
<tr>
<td></td>
<td>(0.181)</td>
<td>(0.120)</td>
</tr>
<tr>
<td>$\ln$ VEL</td>
<td>-0.779</td>
<td>-0.862</td>
</tr>
<tr>
<td></td>
<td>(0.173)</td>
<td>(0.161)</td>
</tr>
<tr>
<td>A</td>
<td>-0.00084</td>
<td>-0.00375</td>
</tr>
<tr>
<td></td>
<td>(0.0055)</td>
<td>(0.00538)</td>
</tr>
<tr>
<td>S</td>
<td>0.0010</td>
<td>0.00197</td>
</tr>
<tr>
<td></td>
<td>(0.0053)</td>
<td>(0.00474)</td>
</tr>
<tr>
<td>SUB</td>
<td>-0.059</td>
<td>-0.0954</td>
</tr>
<tr>
<td></td>
<td>(0.153)</td>
<td>(0.04444)</td>
</tr>
<tr>
<td>PUB</td>
<td>-0.106</td>
<td>-0.106</td>
</tr>
<tr>
<td></td>
<td>(0.0466)</td>
<td>(0.04444)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.990</td>
<td>.994</td>
</tr>
<tr>
<td>Observations</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td>Standard error</td>
<td>.121</td>
<td>.082</td>
</tr>
</tbody>
</table>

increases costs by 88 and 78 percent. The unusual strength of the wage-rate coefficients may be due either to fringe benefits or to the fact that high wage rates tend to occur in larger transit markets where capital equipment is more intensively utilized.
The fringe-benefit effect is important if employees who bargain successfully for higher wages also have larger pensions and longer vacations than other groups of transit employees. In this case, the cost of fringe benefits per dollar of wages increases with the wage rate. Employees in large cities tend to have higher wage rates than employees in other systems. In larger systems, labor costs may be a higher proportion of total costs, not only because of higher wages, but also because buses tend to be operated more intensively in these larger systems. As Wells and Thomas demonstrate, large systems tend to have more employees per bus than smaller systems.6

- Differences in miles per bus-hour do have a strong effect on costs. Coefficients of -0.78 and -0.86, however, indicate that the decrease in costs may be slightly less than proportional to the increase in average velocity although the difference is not statistically significant.

- Newer buses, larger buses, and unsubsidized buses are more costly from the standpoint of interest and depreciation. Estimates of the cost coefficients of $A$, $S$, and $s$ bear this out. On the basis of the capital cost formula,

6. Wells and Thomas, op. cit.
an extra year of age decreases the value of capital and thus capital cost by about ten percent. Total costs should decrease about one percent per year as the bus fleet ages because capital costs are ten percent of total costs. This can be offset if older buses are less productive or require more maintenance. The coefficient of fleet age is negative but not as large as predicted. Instead of a coefficient $-0.0100$ we have $-0.0008$ and $-0.0038$. The differences show only a very weak statistical significance but indicate that older buses require more maintenance or are otherwise less productive.

- Because of lower interest rates, capital costs of public firms should be 10 to 20 percent lower than for private firms. Thus, total costs should be one to two percent lower. Estimates of the cost function reveal, however, that the total cost of transit operations for public firms is 10 percent lower than the cost of transit operations for private firms. This difference unquestionably warrants further investigation.

A5. ALTERNATIVE DERIVATION OF THE COST FUNCTION

The roots of the cost function are imbedded in the production function. In fact, it is often possible to derive the cost function from the production function if it is assumed that
the firm produces transit service in a way that minimizes cost.

One of the simplest two-factor production functions is the Cobb-Douglas:

$$Q = \gamma K^\alpha L^\beta.$$  \hspace{1cm} (A6)

$Q$ is output; $K$ and $L$ are factors of production. The parameter $\gamma$ is called an efficiency parameter. $\alpha$ and $\beta$ are called share parameters. $\alpha + \beta$ is less than one, diseconomies of scale exist. If greater than one, economies of scale exist.

If the firm employs $K$ and $L$ such that the ratios of the costs of a unit of the factors is equal to the ratio of the marginal products, the cost function is:

$$C = \gamma - \frac{1}{\alpha+\beta} \frac{1}{\alpha+\beta} P_K \frac{\alpha}{\alpha+\beta} \frac{\beta}{\alpha+\beta} P_L \frac{\beta}{\alpha}. \hspace{1cm} (A7)$$

$C$ is total cost; $P_K$ is the price per unit of $K$; $P_L$ is the price per unit of $L$. Note that this function is linear in the logarithms of the variables.

We can draw an analogy between this cost function and the cost function for bus transit. The quantity is $B$, bus miles. One factor price can be the wage rate $w$. Another factor price depends on the age ($A$) and size ($S$) of the capital equipment and also on whether the firm is public or private ($FUB$).

Finally, perhaps the efficiency parameter $\gamma$ may depend on the

---

attainable miles per bus-hour in the bus system (VEL). (The CES production function, which permits factors to be less substitutable than in the Cobb-Douglas case, yields a similar result.) If we are willing to assume the production function for bus transit is the Cobb-Douglas function and that firms minimize cost in the manner described, it is possible to estimate the parameters of the production function from the cost function.
Appendix B

VARIABLES IN THE BUS TRANSIT MODEL

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Annual revenue passengers</td>
<td>ATA&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>B</td>
<td>Bus-miles per year</td>
<td>ATA&lt;sup&gt;a,b&lt;/sup&gt;</td>
</tr>
<tr>
<td>F</td>
<td>Average revenue per passenger</td>
<td>ATA&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>POP</td>
<td>Population of urbanized area</td>
<td>DOT: 1968&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Census: 1960&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>AREA</td>
<td>Land area of urbanized area</td>
<td>DOT: 1968&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Census: 1960&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>INC&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Percentage of households earning less than $3,000 per year (1959)</td>
<td>Census&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>INC&lt;sub&gt;10&lt;/sub&gt;</td>
<td>Percentage of households earning more than $10,000 per year (1959)</td>
<td>Census&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>AGE&lt;sub&gt;18&lt;/sub&gt;</td>
<td>Percentage of population 18 years of age or under (1960)</td>
<td>Census&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>AGE&lt;sub&gt;65&lt;/sub&gt;</td>
<td>Percentage of population 65 years of age or over (1960)</td>
<td>Census&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>AUTOS</td>
<td>Automobiles per capita</td>
<td>Rand McNally: 1968&lt;sup&gt;e&lt;/sup&gt;</td>
</tr>
<tr>
<td>HWAY</td>
<td>Population per unit of highway capacity&lt;sup&gt;f&lt;/sup&gt;</td>
<td>DOT</td>
</tr>
</tbody>
</table>


b. A problem exists in defining bus-miles of transit operations because almost all bus firms have charter operations. Operating data includes both charter and transit operations. For most firms in the sample, charter is less than three percent of passenger revenue, but for several firms in both 1960 and
1968, charter revenue was over 10 percent of passenger revenue. Consequently, it was necessary to adjust bus-miles to revenue mileage due to charter operations. In doing so, we made two assumptions: (1) the cost per bus-mile is the same for charter and passenger operations, (2) the revenue from charter operations exactly covers the fully apportioned cost of transit operations. In effect, we multiplied bus miles by the factor $(1 - \frac{CR}{TC})$, where CR is charter revenue and TC is total operating and capital costs.

c. The urbanized area is supposed to be the geographical area which is urban-in-fact. The city measures only the legal boundaries of the central city. The SMSA (Standard Metropolitan Statistical Area) is a county or multi-county unit over most of the country. Unfortunately, the Bureau of the Census and the Department of Transportation have disagreed in the past on the definition of an urbanized area. Our population and land-area data are based on the Census definition in 1960 and the DOT definition in 1968. Significant differences in the two definitions are most prevalent in the large urban agglomerations. Thus, DOT recognizes the Bay Area, the Puget Sound area, and the tri-state New York area as unified urbanized areas, while the Census tends to break these areas down into the individual cities. Since our data is based on smaller cities almost exclusively, it is felt that disparity between the two definitions does not have a serious impact on our estimates. DOT has made estimates of population and land area of urbanized areas for 1968 which are used in this study.


The income distribution of households in urbanized areas came from Ibid., Table 152, pp. 1-333. The age distribution of the population is found in Ibid., Table 63, pp. 1-181. For some of the smaller cities, data is found only in the state volumes, Table 13.

The data on automobile availability by urbanized area in Census data, supplied in this case by DOT.

e. Automobile ownership for 1968 is taken from auto registrations reported in Rand McNally & Co., Commercial Atlas and Marketing Guide, 101st Ed. (New York: 1970). In using these data, it was necessary to use autos per capita on a county basis rather than on an urbanized area basis.
f. DOT has devised a set of formulas for estimating the capacity of urban highway systems. Capacity is based on the mileage of freeways and of surface arterials in the urbanized area. Capacity per freeway mile is estimated to be 8720 autos per hour, regardless of the size of the urbanized area. The capacity of surface arterials was found to vary with the size of the urbanized area, varying from 2225 vehicles per hour for smaller cities to 2760 vehicles per hour for the largest cities.
Appendix C

1968 AND 1960 SAMPLES OF BUS FIRMS AND URBANIZED AREAS

Each of the 51 firms in the 1968 sample and the 44 firms in the 1960 sample (Table C.1) is assigned to one of the following categories:

- Private ownership/power company operation (Power)
- Private ownership/local regulation (Private-Local)
- Private ownership/state regulation (Private-State)
- Public ownership/city council control (City)
- Public ownership/transit authority control (Authority).

<table>
<thead>
<tr>
<th>Company Name</th>
<th>Location</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitzhugh &amp; Leominster St. Railway Co.</td>
<td>Fitzhugh-Leominster, Mass.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Greater Portland Transportation Co.</td>
<td>Portland, Me., Manchester, N.H.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Manchester Transit, Inc.</td>
<td>New Bedford, Mass.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Union St. Railway Company</td>
<td>Buffalo, N.Y.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Erie Metropolitan Transit Authority</td>
<td>Erie, Pa.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Harrisburg Railways Company</td>
<td>Harrisburg, Pa.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Niagara Frontier Transit System, Inc.</td>
<td>New Castle, Pa.</td>
<td>Private-State</td>
</tr>
<tr>
<td>New Castle Transportation Authority</td>
<td>Raleigh, N.C.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Raleigh City Coach Lines, Inc.</td>
<td>Charlotte, N.C.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Charlotte City Coach Lines, Inc.</td>
<td>Greensboro, N.C.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Duke Power Company</td>
<td>Spartanburg, S.C.</td>
<td>Power</td>
</tr>
<tr>
<td>Duke Power Company</td>
<td>Durham, N.C.</td>
<td>Power</td>
</tr>
<tr>
<td>Duke Power Company</td>
<td>Greensville, S.C.</td>
<td>Power</td>
</tr>
<tr>
<td>Greenville City Coach Lines, Inc.</td>
<td>Jacksonville, Fla.</td>
<td>Power</td>
</tr>
<tr>
<td>Jacksonville Coach Co.</td>
<td>Savannah, Ga.</td>
<td>Power</td>
</tr>
<tr>
<td>Savannah Transit Authority</td>
<td>Chattanooga, Tenn.</td>
<td>Power</td>
</tr>
<tr>
<td>Southern Coach Lines, Inc.</td>
<td>Asheville, N.C.</td>
<td>Power</td>
</tr>
<tr>
<td>Asheville Transit Authority</td>
<td>Akron, Ohio</td>
<td>Power</td>
</tr>
<tr>
<td>Akron Transportation Company</td>
<td>Charleston, Va.</td>
<td>Power</td>
</tr>
<tr>
<td>Charleston Transit Company</td>
<td>Wheeling, W.Va.</td>
<td>Power</td>
</tr>
<tr>
<td>Co-operative Transit Company</td>
<td>Lafayette, Ind.</td>
<td>Power</td>
</tr>
<tr>
<td>Greater Lafayette Bus Co., Inc.</td>
<td>Milwaukee, Wis.</td>
<td>Power</td>
</tr>
<tr>
<td>Milwaukee &amp; suburban Transportation Corp.</td>
<td>Green Bay, Wis.</td>
<td>Power</td>
</tr>
<tr>
<td>Wisconsin Public Service Corp.</td>
<td>Oshkosh, Wis.</td>
<td>Power</td>
</tr>
<tr>
<td>Tiffin City Transit, Inc.</td>
<td>Little Rock, Ark.</td>
<td>Power</td>
</tr>
<tr>
<td>Interstate Power Company</td>
<td>Dubuque, Iowa Div.</td>
<td>Power</td>
</tr>
<tr>
<td>Kansas City Area Transportation Authority</td>
<td>Rep. City of Des Moines</td>
<td>Power</td>
</tr>
<tr>
<td>B6-State System Transit Authority</td>
<td>Fort Worth, Texas</td>
<td>Power</td>
</tr>
<tr>
<td>City Bus Company</td>
<td>Lubbock, Texas</td>
<td>Power</td>
</tr>
<tr>
<td>Fort Worth Transit Co., Inc.</td>
<td>San Antonio, Texas</td>
<td>Power</td>
</tr>
<tr>
<td>City of Lafayette Municipal Transit</td>
<td>San Angelo, Texas</td>
<td>Power</td>
</tr>
<tr>
<td>Albuquerque Transit System</td>
<td>Albuquerque, N.M.</td>
<td>Power</td>
</tr>
<tr>
<td>Sacramento City Transit Authority</td>
<td>San Diego, Calif.</td>
<td>Power</td>
</tr>
<tr>
<td>Flint City Coach Lines, Inc.</td>
<td>Flint, Mich.</td>
<td>Power</td>
</tr>
<tr>
<td>Grand Rapid Transit Authority</td>
<td>Grand Rapid, Mich.</td>
<td>Power</td>
</tr>
<tr>
<td>Baltimore Transit Company</td>
<td>Baltimore, Md.</td>
<td>Power</td>
</tr>
<tr>
<td>Memphis Transit Authority</td>
<td>Memphis, Tenn.</td>
<td>Power</td>
</tr>
<tr>
<td>Metropolitan Dade County Transit Authority</td>
<td>Miami, Fla.</td>
<td>Power</td>
</tr>
<tr>
<td>South Carolina Electric &amp; Gas Co.</td>
<td>Columbia, S.C.</td>
<td>Private-State</td>
</tr>
<tr>
<td>South Carolina Electric &amp; Gas Co.</td>
<td>Charleston, S.C.</td>
<td>Private-State</td>
</tr>
<tr>
<td>Nashville Transit Company</td>
<td>Tennessee</td>
<td>Private-State</td>
</tr>
<tr>
<td>Columbus Transit System</td>
<td>Cincinnati, O.</td>
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Appendix D

THE USE OF THE TRANSIT MODEL AS A TOOL TO ANALYZE SUBSIDIES

The subsidy mechanisms under consideration in Chapter VI include a cost subsidy, which pays a certain percentage of total transit costs, and a fare subsidy, which pays a certain fixed amount per transit passenger. The impact of each subsidy mechanism can be evaluated by tracing its effects through the model of the transit market. The form of the model most readily applicable to this problem is the total differential of the demand equation, the supply equation, and the definition of the cost-revenue ratio:

\[
\frac{dD}{D} = \alpha_1 \alpha' \left( \frac{B}{F} \right)^{-\alpha'} \frac{dB}{B} - \alpha_2 \frac{dF}{F} \quad (D1)
\]

\[
\frac{dB}{B} = \beta_3 \frac{dD}{D} + \beta_4 \frac{dc}{c} + \beta_5 \frac{dk}{k} \quad (D2)
\]

\[
\frac{dk}{k} = \frac{dB}{B} + \frac{dc}{c} - \frac{dF}{F} - \frac{dD}{D} \quad (D3)
\]

All other variables are exogenous and remain constant. The effective cost per bus-mile c may vary only through subsidy. The cost-revenue ratio is assumed not to vary as a result of the subsidy. Thus, all benefits are passed along to users through lower fares or improved levels of service.
The model used for evaluating the effects of subsidies can be further simplified. The demand equation contains an expression for the service-elasticity of demand,

$$\varepsilon_B = \alpha \alpha' \left( \frac{B}{P} \right)^{-\alpha'},$$

(D4)

and the fare-elasticity of demand,

$$\varepsilon_F = -\alpha_2 F.$$

(D5)

The parameters $\beta_3$ and $\beta_4$ represent elasticities with respect to passengers and cost per bus-mile, $\eta_D$ and $\eta_C$. A final assumption is that this model based on infinitesimals gives approximately correct answers if small but finite changes occur in the variables. Thus, the model actually used in evaluating subsidies can be written

$$\frac{\Delta D}{D} = \varepsilon_B \frac{\Delta B}{B} + \varepsilon_F \frac{\Delta F}{F}$$

(D6)

$$\frac{\Delta B}{B} = \eta_D \frac{\Delta D}{D} + \eta_C \frac{\Delta C}{C}$$

(D7)

$$0 = \frac{\Delta B}{B} + \frac{\Delta C}{C} - \frac{\Delta F}{F} - \frac{\Delta D}{D}$$

(D8)

The procedure for making numerical calculations assumes that these elasticities are constants—an assumption which permits solution of the equations without resort to computer simulation.
D1. The Impact of a Cost Subsidy

The impact of a cost subsidy on the number of passengers, the level of service, and the transit fare can be found by solving the system of equations (D6) - (D8) for the relative changes in these three variables as a function of the proportion of total costs subsidized. The following solution is found:

\[
\frac{\Delta D}{D} = \frac{\Delta c}{c} \frac{\epsilon_F (1 + \eta_c) + \eta_c \epsilon_B}{1 - \eta_D} \frac{\epsilon_B + \epsilon_F (1 - \eta_D)}{1 - \eta_D} \quad (D9)
\]

\[
\frac{\Delta B}{B} = \frac{\Delta c}{c} \frac{\epsilon_F (\eta_D + \eta_c) + \eta_c}{1 - \eta_D} \frac{\epsilon_B + \epsilon_F (1 - \eta_D)}{1 - \eta_D} \quad (D10)
\]

\[
\frac{\Delta F}{F} = \frac{\Delta c}{c} \frac{1 - \eta_D \epsilon_B + \eta_c (1 - \epsilon_B)}{1 - \eta_D} \frac{\epsilon_B + \epsilon_F (1 - \eta_D)}{1 - \eta_D} \quad (D11)
\]

The changes in response to the cost subsidy are directly proportional to the size of the subsidy. One requirement for a stable solution is that the denominator in the solution is positive. This occurs whenever the weighted average of the absolute values of the fare-elasticity of demand and the service-elasticity of demand is less than one.

D2. The Impact of a Fare Subsidy

A fare subsidy is somewhat more complicated to analyze. There are two methods of paying a fare subsidy, each of which requires a slightly different formulation of the problem. If the subsidy is paid by distributing tokens to consumers, both
the fare subsidy and the change in the nominal fare charged by the firm enter the demand equation (D6). If the subsidy is paid directly to the firm, only the change in nominal fare enters the demand equation, but the change in nominal fare and the fare subsidy enter the cost-revenue equation (D8). It is simple to demonstrate that the two procedures are equivalent, leading to the same net fare for transit users, the same number of passengers and the same level of service.

The effect of a fare subsidy on the transit market depends on the size of the subsidy (-ΔF*) and the supply and demand parameters:

\[
\frac{\Delta D}{D} = \frac{\Delta F^*}{F} \frac{\xi_F}{1 - \eta_D \xi_B + \xi_F (1 - \eta_D)} \tag{D12}
\]

\[
\frac{\Delta B}{B} = \frac{\Delta F^*}{F} \frac{\eta_D \xi_F}{1 - \eta_D \xi_B + \xi_F (1 - \eta_D)} \tag{D13}
\]

\[
\frac{\Delta F}{F} = \frac{\Delta F^*}{F} \frac{1 - \eta_D}{1 - \eta_D \xi_B + \xi_F (1 - \eta_D)} \tag{D14}
\]

The denominator in these solutions is identical to the denominator in the expressions indicating the impact of a cost subsidy. Since ΔF* and ξF are negative, a fare subsidy leads to an increase in both passengers and bus-miles. The change in the effective fare is composed of the fare subsidy plus a further reduction in the nominal fare induced by the subsidy. It is interesting
that the fare subsidy induces the firm to lower the fare by an amount greater than the subsidy. This enables the firm to increase passengers and, therefore, receive an even larger subsidy.
BIBLIOGRAPHY

Books


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