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SIMULATED ECONOMIC CONTROLS
OVER RESEARCH AND DEVELOPMENT

by

Charles E. Fisk, Jr.

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

Doctor of Philosophy

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Chapter 1

AN APPROACH TO SOME UNANSWERED PROBLEMS
OF RESEARCH MANAGEMENT

1.1 Selecting Research Projects

Virtually all articles written in the last decade on the economics of research and development (R&D) can be divided into two groups, namely (i) prescriptions of rules that firms and governmental agencies ought to follow in allocating resources among research projects, and (ii) evaluations of governmental controls over research firms.¹

In 1969 the editor of IEEE Transactions on Engineering Management (one of the leading publishers of methods for selecting R&D projects) stated that "most of the models in the literature have never been, and probably will not be,

¹/All statements in this dissertation concerning the literature on the economics of R&D apply to the following set of books and articles: all books listed in the dissertation's references; all articles on R&D referenced in A KWIC Index in Operations Research (IBM, 1967), in the Bibliography on Simulation (IBM, 1966), and in all indexes published by the American Economic Association; and all articles on R&D published in the last decade (through December 1971) in the Journal of Business, the Journal of Finance, IEEE Transactions on Engineering Management, Management Science, Operations Research, the Operational Research Quarterly, Automation and Remote Control (USSR Academy of Sciences), Economics and Mathematical Methods (USSR Academy of Sciences), and Engineering Cybernetics (USSR Academy of Sciences). Books and articles are referenced in the dissertation's text by authors' names, and the full citations appear in the dissertation's reference list.
used for decision making." The argument that such models are seldom used is documented further by Baker and Pound (1964), and by Dory and Lord (1970). Baker and Pound explain the apparent sterility of R&D budgeting models by saying that "many of the formal methods are weak because they leave out, or treat inadequately, too many important considerations," and that "there is insufficient evidence to demonstrate whether the methods can be used, and if they are feasible, whether or not the methods are desirable."

The decision models formulated by Howard (1960, 1971), Raiffa (1969), Fishburn (1964), and many others are, however, sufficiently general to allow a realistic mathematical formulation of any R&D portfolio selection problem. But such models suffer from an apparently incurable "curse of dimensionality".\(^2\) Thus although realistic models can be formulated, they have not been solved; and conversely, only grossly unrealistic models of optimal economic controls over R&D have seemed computationally feasible.

\(^2\)In particular, Howard proposes a "value-iteration" algorithm that essentially determines a profit-maximizing mapping of a state space onto an action space. The state of research in a firm can easily be described mathematically: for example, one state might be a value of the vector \(s(1), s(2), \ldots, s(n)\), where \(s(i) = 1\) if the \(i\)th of \(n\) potential research tasks has been completed successfully, and \(s(i) = 0\) otherwise. In reality, \(n\) could easily be as large as 100, and thus Howard's algorithm would necessarily be applied to a system involving trillions of states, which is to say that the algorithm would fail.
Marschak (1967), who has apparently recognized that prospects for finding optimal rules for research funding are quite dim, suggested that researchers might profitably compare various rules of thumb for selecting R&D projects. Such a comparison involves first characterizing mathematically the process actually used by managers in selecting projects, and then demonstrating conditions under which some rules work better than others. Bonini (1963) developed a digital simulation model describing the decision processes in a typical manufacturing firm; but according to Baker and Pound, "the current knowledge of the R&D selection process tends to be sketchy and poorly organized."\(^3\)

Chapter 2 of this dissertation draws from several recent works a set of facts and hypotheses about how firms select R&D projects; and from these observations, Chapter 3 synthesizes a simulation model of a budgeting process. Chapter 4 then compares various rules for selecting projects, and for funding research departments. Also, an appendix to Chapter 4 suggests parallels between problems of (i) project

\(^3\)This view has been repeated by Mansfield and Brandenburg (1966), who state that "there have been surprisingly few detailed studies of the R&D activities of the firm. Consequently, although we know something about the factors influencing the firm's total expenditures on R&D, little is known about the allocation of funds among projects, the characteristics of projects that are undertaken, and the probable outcome of these projects."
selection, (ii) assigning military weapons to targets, and (iii) stratified random sampling.

A novel feature of the work in Chapters 3 and 4 is that all algorithms developed there were programmed and executed in a computer language called APL (i.e., A Programming Language), which Iverson (1962) proposed originally as a system of mathematical notation. Documentation and testing in APL overcomes a long-standing criticism by applied mathematicians -- notably Woolsey (1972) -- that many mathematical programming algorithms really do not work, even though the algorithms' convergence to an optimal solution has been proved.

1.2 Governing Research Firms

Many economists have used hypotheses about research funding as arguments either for or against anti-trust policies. Recently, Grabowski (1968) attempted an empirical test of Schumpeter's 1947 hypothesis that large firms are responsible for most industrial inventive activity. According to Grabowski,

Few of the studies that have tested Schumpeter's hypothesis ... suggest why this hypothesis is apparently valid for some industries and not for others. And statistical studies going beyond this question, to try to relate R&D expenditures to firm profit expectations and the availability of funds as in other investment decisions, are rare ... .
Grabowski begins his study with a sample of data from 16 firms in the chemical industry, 15 in petroleum refining, and 10 in drugs. As a measure for the intensity of a firm's R&D effort in relation to the firm's size, he chooses the (research/sales) ratio $R_{it}/S_{it}$, where $R_{it}$ denotes R&D expenditures by the $i$th firm (in one of three industries) during the $t$th year, where $t = 1, 2, 3, 4$ corresponding to the period 1959-62 for which data is available. He then attempts to find variables $x = (x_1, x_2, \ldots, x_n)$ such that a linear regression of $(R_{it}/S_{it})$ on $x$ yields a high $R^2$ value, and such that each regressor's $t$ ratio indicates significance. To the extent that Grabowski's work represents the current "state of the art" in explaining the funding of research projects, the following questions seem important.

A research project (e.g., an attempt to find a cure for arthritis) can be described partially by a probability-of-success function that maps (i) time rates of expenditures on the project onto (ii) probabilities of technical success. This is to say that some research projects can be viewed as lotteries. At the start of a firm's budgeting period (or fiscal year), the firm perceives an "inventory" of potential research projects; and by following some decision rule, the firm allocates its research facilities among the projects in inventory. Governmental regulation of research (e.g., regulation through patent laws, anti-trust policies,
depreciation allowances, etc.) alters the "richness" of an inventory of projects. How, then, will a firm's pattern of research funding vary with the characteristics of the project inventory?

Regression approaches such as Grabowski's fail to link (i) the kinds of research that firms do, with (ii) government policies regulating research. With reference to Grabowski's notation, a change in governmental policy (e.g., a reduction in patent lives for certain drugs) may not affect \( \frac{R_{it}}{S_{it}} \); but the policy change may cause firms to neglect entirely research projects that might have been funded by the private sector.

Even if attention is focused only on the \( \frac{R_{it}}{S_{it}} \) ratio and not on the pattern of research funding, the approach of trying different regression equations until one stumbles onto the "right" equation does not seem promising, since one could examine infinitely many different sets of regressors and functional forms. In particular, Grabowski's regression equation explaining research funding failed to explain over 70% of the variation observed in research expenditures within the petroleum industry.

Normally one does not cast blindly about for regressors and functional forms. Grabowski, for example, chooses his regressors on the basis of a somewhat amorphous theory of profit maximization. But as his results testify,
the theory provides weak guidance for empirical research. If a model is to be at all testable, it must be formulated in terms of concepts that correspond to those for which a typical firm generates data. In this regard, Bonini pointed out the limitations of classical microeconomic theory, and attempted to provide a testable theory explaining the behavior of a manufacturing firm. But no effort such as Bonini's has been made to explain the behavior of a research firm.

Chapter 5 of this dissertation shows how the simulation model developed in Chapter 3 can be used to establish testable hypotheses about reactions of research firms to changes in various public policies. In particular, the model is exercised to suggest how a firm might rationally distribute the value of a new grant among current research projects. Also, Chapter 5 examines the contention that larger firms benefit from their abilities to spread the risks of research over many projects, and thus that such firms will profitably underwrite projects that are intolerably risky for smaller firms. Chapter 6 concludes by suggesting topics for further research on rules for project funding, and on public policies toward R&D firms.
Chapter 2

AN OVERVIEW OF THE R&D BUDGETING PROCESS

2.1 Introduction

In studying a mathematical model of a complicated process, one can easily become so bogged down in notation and in technical details that one never fully grasps the essential character of the model -- which is to say that one never understands what the model is attempting to describe, and which of the model's assumptions can easily be modified. This chapter attempts to avoid the morass of notation by presenting the basic features of a system for economic control over R&D projects. A detailed version of the model appears in Chapter 3.

Several important assumptions embodied in the model described in this chapter are drawn from the Mansfield and Brandenburg study of an equipment manufacturer's system of control for R&D. Certain aspects of the manufacturer's system, however, are obviously peculiar in the sense that few firms could be imagined to have adopted similar processes.¹

¹In particular, Mansfield and Brandenburg state that one aspect of the manufacturer's budgeting system consisted of reporting by "project analysts" on the "merit" of each research proposal. The figure of merit equalled \((g_{trG})/(E+D)\), where \(g\) is the project evaluation group's estimate of the extra annual profit if the project is technically and commercially successful, \(t\) is their estimate of the number of years that this profit stream will continue, \(G\) is their
These oddities are omitted from the model, which describes a class of budgeting processes that is sufficiently general to include, for example, research control problems faced by pharmaceutical manufacturers.\textsuperscript{2/} There are obviously limitations on the model's generality, and where these occur they are noted.

2.2 The Process of Allocations within a Research Department

A partial organization chart for the manufacturer studied by Mansfield and Brandenburg appears in Figure 2.1. The firm's central research laboratory is organized by departments concerned with the study of materials science, metallurgy and ceramics, etc. Each department has a single manager, who reports to the central laboratory managers, who in turn report to the firm's top management. Larger firms might exhibit a further decomposition of research departments into research branches; but aside from the degree of decomposition, the manufacturer's departmental organization of research is obviously common to many firms.

\[ \text{estimate of the probability of commercial success, } r \text{ is a department manager's preliminary estimate of the probability of technical success, } E \text{ is his preliminary estimate of the cost of the R&D, and } D \text{ is a research division's estimate of the cost of putting the research result into practice. If the merit value is greater than four, the proposal is given an "A" rating, etc. Clearly, this mechanism of evaluation is not common to many firms.} \]

\textsuperscript{2/}Models for pharmaceutical research funding were proposed by Asher (1962), and by Atkinson and Bobis (1969).
Figure 2.1: a partial organization chart for the equipment manufacturer studied by Mansfield and Brandenburg
The manufacturer chooses a portfolio of R&D projects for each year. The $i$th of $n$ possible projects consists of a lottery \( \{ v_i, p_i(v_i | t_{il}, \ldots, t_{im}) \} \) in which $v_i$ denotes the expected discounted net return that the firm will gain if at least one of $m$ research departments succeeds on the project. If no department succeeds, the firm gains no return. This characterization of a project is common to many firms. For example, one project might be an attempt to find a cure (or retardation) for rheumatoid arthritis.\(^3\)

A success on this project would definitely yield a monetary return, and research firms do calculate values for such conditional returns.

Concerning the binary outcome of projects, Mansfield and Brandenburg wrote that "... the firm seemed to act as if there were only two outcomes of a project -- technical success or technical failure." Certainly one can imagine projects with a spectrum of possible outcomes; but an important point to be drawn from the Mansfield-Brandenburg study (as well as from the contributions by Weingartner (1966), Hamilton (1969), Atkinson and Bobis (1969), and Asher (1962)) is that binary projects do constitute an important subset of real-world investment proposals.

\[^3\] Sanders (1968) surveys current research on arthritis.
The term \( P(V_i, T_{i1}, \ldots, T_{im}) \) denotes the probability of technical success by at least one department, given that the \( j \)th department \((j = 1, \ldots, m)\) allocates a fraction \( T_{ij} \) of its total resources for the coming year to the \( i \)th project. Thus the lottery concept introduced above assumes that the probability of success on one project depends entirely on the resources allocated to that project, and not on the allocations to other projects. This assumption is not sufficiently general to encompass all research efforts. Rather, the independence assumption is a restatement of one aspect of the model developed by Mansfield and Brandenburg. In contrast, some of the models referenced by Weingartner (1966) do not assume this type of independence among projects.

Different departments work on the same project by trying different approaches. For example, before 1950 cortisone could not be produced in large quantities, and different researchers experimented with entirely distinct methods of synthesizing the drug. Dean and Hauser (1967) and Abernathy and Rosenbloom (1969) give other examples of "parallel" research efforts.

The concept of a research department's budget for a given year requires attention. Mansfield and Brandenburg assume that each department manager "has in mind some total budget for his department and that he fixes $C_1 \ldots $
[the total budget allocation for the next year to a particular project] so as to maximize the sum of the expected discounted profit from all his projects."

Actually, however, at the start of a fiscal year t, a department manager must have two budgets; namely (i) a fixed value $B_j$ of total allocable research facilities, and (ii) a sum of money that he can allocate during the coming year in order to determine the configuration of his department for the (t+1) year. The notion of dual budgets follows from a few obvious facts.

First, money is not -- as R&D budgeting models commonly assume -- a kind of miracle jelly that a firm distributes over research projects. Rather, at the start of a fiscal year any department has a given configuration of resources for research: the jth department employs scientists X, Y, and Z, who use the firm's Brand C computer, Brand D centrifuge, and so forth. Moreover, this configuration will remain fixed for the coming fiscal year. In particular, buildings cannot be disposed of immediately; a department's equipment can have only a nominal resale value; at least six months will be required to hire new employees; old employees, if they are to be fired, must be given time to find new jobs; and so forth for other obvious factors that tend to petrify a department's configuration in the short run.
Thus for the coming year, the allocation of departmental resources for research must refer to the assignment of existing men and machines to do certain jobs. This problem of assignment can be called the year's allocation problem, in contrast to the reconfiguration question that will be discussed below. A value of $T_{ij}$, therefore, represents for the $j$th department a rough measure of the importance of the $i$th research task for the coming year. Equivalently, $T_{ij}$ can be interpreted as the fraction of $S_{B_j}$ (i.e., the value of the $j$th department's fixed configuration for the coming year) that will be devoted to work on the $i$th of $n$ tasks.

The second budget (whose amount $S_{C_i}$ may well differ from $S_{B_j}$) refers to the amount of funds that a department manager is allowed to commit for research during the $(t+1)$ year. Within the limit imposed by this second budget, the manager hires new employees (or decides to retain old ones) for $(t+1)$. Similarly, equipment rental contracts are established for $(t+1)$, and so forth for other commitments. Obviously, new employees will not first be available for assignment on precisely the first day of the $(t+1)$ fiscal year, and rental contracts will not always coincide exactly with the fiscal year; but without doing great violence to reality, one can assume -- for an arbitrary research department -- the allocation process illustrated in Figures 2.2 and 2.3.
\begin{verbatim}
\$BUDGETS[\ldots]$

\$BUDGETS$

[1]  \text{ILLUSTRATES THE PROCESS OF A RESEARCH}
[2]  \text{DEPARTMENT'S ALLOCATION OF RESOURCES.}
[3]  \text{ICOND1:B+IBUDGET}
[4]  \text{ICOND2:C+NBUDET}
[5]  \text{T\leftarrow{}0}
[6]  \text{TSTART:T+T+1}
[7]  \text{ALLOCATE:ALLOCATIONS+ALLOC B}
[8]  \text{RECONFIGURE:EXPPATTERN+FUND C}
[9]  \text{CHANCE:OUTCOMES+NATURE ALLOCATIONS}
[10] \text{BOSS:C+BOSS OUTCOMES}
[11] \text{B\leftarrow{}EXPPATTERN}
[12] \text{\rightarrow{}(T\leq{}TMAX)/TSTART}
\end{verbatim}

\textbf{Figure 2.3:} an APL program illustrating the process of allocations to research
Figure 2.2 is a conventional block diagram of the department's allocation process, and Figure 2.3 illustrates the same process as an APL program.\footnote{"APL" stands for A Programming Language, which was proposed originally as a system of mathematical notation by K. Iverson (1962), and has recently been implemented as a computer language. Because of their fidelity to mathematical systems, APL programs are sometimes used to explain algorithms.} The steps in the process are:

(a) At the start of an arbitrary fiscal year $t$, there exist two initial conditions; namely (i) the research department's existing configuration of resources, whose value is $B_j$; and (ii) a sum $C_j$, which the department manager can commit for research facilities to be used during the year $(t+1)$.

(b) As suggested by Mansfield and Brandenburg, at the start of $t$, the firm has an inventory of $n$ research projects, some of which have been proposed by research departments, and some by the firm's operating divisions. According to some rule (say the rule ALOC, as shown on line 7 of Figure 2.3), the department's resources are targeted against projects.

(c) During the year $t$, a rule (e.g., the FUND rule in Figure 2.3) commits the budget $C_j$ in a way that will determine the department's configuration for $(t+1)$.

(d) As a result of a chance mechanism (e.g., Nature's decision), the firm observes certain outcomes from each department's research efforts in $t$. 

\footnote{"APL" stands for A Programming Language, which was proposed originally as a system of mathematical notation by K. Iverson (1962), and has recently been implemented as a computer language. Because of their fidelity to mathematical systems, APL programs are sometimes used to explain algorithms.}
(e) Acting on the basis of these outcomes, the firm's central research management determines $C_j$ for $(t+1)$. We assume that the firm's total research budget is determined exogenously for any year. In modelling the R&D budgeting process, our concern is with the distribution of a total research budget, and not the breakdown of the firm's total expenditures among research, advertising, etc.

(f) The value of $B_j$ for $(t+1)$ becomes the value $C_j$ that was committed during $t$.

(g) The time counter $t$ is incremented by unity, and the process cycles. The iterations terminate for an arbitrarily determined time value $T_{MAX}$.

2.3 The Firm's Allocation Decision

As suggested above, at the start of the $t$th year the configurations of each of the firm's research departments will be fixed, and the firm will be confronted with a set of $n$ potential research projects $\{V_i : P_i(V_i | T_{il'}, \ldots, T_{im'})\}$. The allocation decision thus consists of choosing a value for the matrix $(T_{ij})$ such that $z$, the expected value of research, will be maximized subject to the constraints

\begin{equation}
\sum_{i=1}^{n} T_{ij} \leq B_j \text{ for all } j, \text{ and } T_{ij} \geq 0
\end{equation}

In order to describe the way in which department managers attempt to task resources to do research, Mansfield and Brandenburgh assumed that each department manager allocates
his department's resources among projects in a way that he thinks will maximize

\[ Z_j = \sum_{i=1}^{n} V_i P_{ij} \]

where the probability-of-success function (PSF) has the form

\[ P_{ij} = 1 - \exp(-K_{ij} T_{ij}) \]

in which the parameter \( K_{ij} \) reflects the \( j \text{th} \) department's capability to succeed on the \( i \text{th} \) project. If \( X_{ij} \) measures the minimum amount of resources that the \( j \text{th} \) department would have to commit to the \( i \text{th} \) project in order to succeed on it, then \( 1/K_{ij} = E(X_{ij}) \).

The firm studied by Mansfield and Brandenburg did not necessarily target resources against projects in the pattern suggested by the department managers. We assume in describing a typical firm's behavior that resources will be allocated roughly in the pattern suggested by department managers, which is to say that the firm will control the pattern of resource allocations by communicating values of projects to department managers, and then letting each manager allocate the resources of his department. A survey by Cordtz (1971) indicated that this sort of control is now used by a number of research firms.

Concerning their assumption of exponential probability-of-success functions (as in 2.3 above), Mansfield and Brandenburg argued as follows.
Taking a particular project, it seems obvious that the probability of completing the project in a given time interval is an increasing function of the length of the time interval and the annual rate of expenditure on the project. Moreover, it is also obvious that the probability tends to zero as either of these variables tends to zero. Given these conditions, the simplest assumption that meets them is that the probability is proportional to the product of the two variables. This model is a reasonable approximation for R&D where the probability of success, holding the time interval and the rate of expenditures constant, does not vary greatly with time. In effect, it views R&D as a search process where the ground previously explored is always so small, relative to the total, that the probability of success varies little within a reasonably short span of time. Although not all R&D is of this type, this model is perhaps as good a first approximation as any of comparable simplicity.

A somewhat different -- and perhaps more persuasive -- rationale for the exponential PSF is the following. If a department manager has been told to devote, say, 80% of his department's resources for the next year to a particular project, then obviously the department must begin its work by trying an approach: for example, if a medical research team has been assigned to develop a drug having certain properties, then a first approach might be that of testing each of 10 compounds; if these fail, another ten would be tried, and so on for successive attempts. Similarly in the area of computer development, a research team could attempt to improve the execution speed of a given programming language by (i) writing a better compiler for the language, or (ii) by designing computer hardware with a different machine language.
If each possible approach to a project offers a probability of success that is (i) the same for many other possible approaches, and (ii) independent of approaches that have been tried unsuccessfully, then it can be shown -- as in this chapter's appendix -- that probability-of-success functions must be members of the exponential family of distribution functions. This is to say that the existence of a virtually bottomless supply of independent, equally promising approaches to a project should immediately confine our attention to the exponential family of PSFs.

If we adopt the exponential PSF assumption, then the firm's allocation problem becomes that of choosing a value for the matrix \( T_{ij} \) that maximizes the objective function

\[
Z = \sum_{i=1}^{N} V_i (1 - \exp(-\sum_{j=1}^{M} K_{ij} T_{ij}))
\]

subject to the constraints

\[
\sum_{i=1}^{N} T_{ij} \leq B_j \text{ for all } j, \text{ and }
\]

\[
T_{ij} \geq 0 \text{ for all } i \text{ and } j
\]

Evidently a firm that solves its allocation problem by (i) telling its department managers that values \( V_i \) measure the relative importance of projects, and (ii) allowing each department manager to target his department's resources against the projects in inventory will not necessarily achieve an optimal solution to the programming
problem posed above. But as a scheme for describing how firms do in fact solve the problem of targeting resources against projects, the Mansfield-Brandenburg assumption of steps (i) and (ii) above seems reasonable on \textit{a priori} grounds. There is ample evidence (e.g., Cordtz (1971) and Dory (1970)) that a firm's central coordination of its research departments may not involve telling each departmental manager exactly how his resources should be employed, which is to say that department managers may well be given considerable freedom in assigning resources. Moreover, it is not unreasonable to suppose that each department manager will allocate resources in a way that he thinks will yield the greatest profit for the next fiscal year.

2.4 \textbf{The Firm's Reconfiguration Decision}

At the start of an arbitrary fiscal year, possibly FY 1970, we assume that the firm's department managers begin to make financial commitments for resources that will be available to do research in 1971. In particular, equipment rental contracts will be signed, new employees will be offered jobs, etc. Thus at the start of 1970, the firm's central research management must determine values for the variables \((x_i, i = 1, \ldots, m)\) and \((y_j, j = 1, \ldots, p)\), where \(x_i\) denotes the total budget allocated to the \(i\)th of the firm's current research departments, and \(y_j\) is the
budget to be allocated to the $j$th of $p$ potential research departments.

At the start of 1970, the firm may perceive various opportunities for diversification: new department managers might be hired, and might bring with them new teams of scientists; old departments might be split, and so on. Thus we assume that at the start of an arbitrary year the firm can choose from a set (possibly the null set) of opportunities to diversify, and that various amounts can be invested in each diversification proposal.

For each of the old departments, the feasible values of $x_i$ are constrained as follows. Usually a lower bound $XLB_i$ is determined by a decision rule that the central research management has established in order to mitigate the risk of destroying potentially profitable research departments. For example, Department A might have failed in 1969 to produce profitable results, and this failure might have dimmed the central management's hopes for future profits from the department; nevertheless, management may be unwilling to abolish the department for 1971, since the 1969 failure might have occurred with low probability.

An upper bound $XUB_i$ on $x_i$ reflects the maximum amount that the firm contemplates investing in the $i$th department at the start of a year. Like the value of $XLB_i$ for a given year, the value of $XUB_i$ is usually related to the levels at which the department has been funded in the past. In
particular, the firm might consider funding the $i^{th}$ department within an interval of 10% about the level at which the department was funded in the previous year. The fact that firms do actually determine departmental budgets in this way is fairly well established. (For documentation of this sort of behavior, see Bonini.)

The rationale for establishing upper bounds on funding levels might first be that many departments can assimilate new funds only at a limited rate. Moreover, the upper limit acts to constrain the risk inherent in funding a department: for example, the firm has past information on the returns that a particular department has yielded in response to investments in the department; but for investments far outside the range of past inputs, the firm has no basis for predicting how the department will perform. Thus we observe the common pattern of either a gradual increase in the budget of a department that seems relatively profitable, or a gradual phasing out of what seems to be an unprofitable group.

Each of the $(y_j)$ variables pertaining to the potential departments will also be constrained above and below. In the case of new departments, however, there may be substantial set-up costs. This is to say that the existing departments may have competitive advantages over potential entrants, in the sense that much of the firm's viable research equipment might have a negligible book value,
while the cost of establishing a new department might be substantial. Thus the decision to add a new department represents a kind of fixed charge problem: the firm must first decide whether to add the department at all, and then at what level within the feasible interval \((YLB_j, YUB_j)\) to fund the new department.

A model of the reconfiguration decision might be the programming problem of choosing values for the variables \((x_i), (y_j),\) and \((z_j)\) that will maximize

\[
R = \sum_{i=1}^{M} c_i x_i + \sum_{j=1}^{P} d_j z_j y_j
\]

subject to the constraints

\[
\sum_{i} x_i + \sum_{j} y_j z_j \leq B, \tag{2.8}
\]

\[
x_L i \leq x_i \leq x_U i \text{ for all } i, \tag{2.9}
\]

\[
y_L j \leq y_j \leq y_U j \text{ for all } j, \tag{2.10}
\]

\[
z_j = (0,1) \text{ for all } j, \tag{2.11}
\]

where \(B\) denotes the exogenously-determined total research budget.

The objective function coefficients \((c_i)\) and \((d_j)\) can be determined from the following assumptions. Possibly for each of the five past years, the firm has recorded some performance measure for each existing research department. In particular, if in 1968 the \(i\)th department had been assigned to work on projects \(A, B,\) and \(C,\) and if for that year the department's record of successes on the projects was \((0,1,0),\) then the firm might have attributed the
expected market value \( V_B \) of the B project to the department. Thus at the start of 1970, we assume that the firm has compiled a matrix \( \text{RHIST}(i,t) \), \( i = 1, \ldots, m \), and \( t = 1, \ldots, 5 \), denotes the return attributed to the \( i \)th department for the fiscal year \( (1970 - (6 - t)) \). Similarly, a matrix \( \text{CHIST} \) of dimension \((m \times 5)\) records the past costs of the firm's research departments. Also by assumption, each potential department exhibits a history of attributed returns \( \text{PRHIST}(j, t = 1, \ldots, 5) \), and a vector of past costs \( \text{PCHIST}(j, t = 1, 5) \).

For each department (new or old) the objective function coefficients can then be determined by a weighted linear regression of returns on costs. In particular, let \( W \) denote a \((5 \times 5)\) matrix whose only non-zero elements are on the main diagonal, and whose \( t \)th diagonal element denotes a subjective "weight" assigned by the central research management to an arbitrary department's returns and costs for the \((1970 - (6 - t))\) year. Also, let \( C \) be the \((5 \times 2)\) matrix whose first column consists of unity values, and whose second column is the vector \( \text{CHIST}(i, t = 1, \ldots, 5) \); and let \( R \) be the vector \( \text{RHIST}(i, t = 1, \ldots, 5) \). Then the objective function coefficient \( c_i \) can be determined from the regression coefficients

\[
(2.12) \quad \beta = (C'W^{-1}C)^{-1}C'W^{-1}R.
\]
The rationale for weighting past observations on costs and returns is this: the central research management may believe that an observation taken, say, four years ago indicates less about a department's potential profitability than more recent observations. Thus \( W(2,2) \) might be set at \( .75(W(5,5)) \). Of course, if management thinks that all past observations are equally important in assessing a research department's potential profitability, then \( W \) could be the identity matrix.

It is important to note that although the quantity \( c_i x_i \) can be interpreted as the range value of an expected return function \( \text{RET}_i(x_i) \), we have not assumed that the return function is linear over all values of \( x_i \). Instead, the assumption is that over the domain \( (XLB_i, XUB_u) \) the return function can be approximated by a linear segment. This assumption is illustrated in Figure 2.4, which depicts the following sequence of events.

For 1964 the amount indicated in Figure 2.4 by \( x_i = 1 \) was allocated to the \( i \)th department; and at the end of 1965, the indicated return was attributed to the department. In 1965 the amount \( x_i = 2 \) was spent by the department, and the corresponding return was recorded, and so forth through 1968. Notice carefully the lag structure involved in the funding: at the start of 1964, the \( i \)th department's value of resources for 1965 is determined; and it is the return from this value that is associated with the amount determined at the start of 1964.
Figure 2.4: an illustration of the assumed return function for a particular research department
For 1969 an amount indexed by \( x_1 = 6 \) has been allocated, but at the start of 1970 the return from \( x_1 = 6 \) has not been observed. Over the domain indicated by the feasible region for 1970, the firm assumes that the weighted least-squares line \( B \) approximated the true (but unknown) expected return function \( A \). At the start of 1971, when the return from the 1969 allocation has been recorded as shown, the approximating line is \( C \) rather than \( B \), since the oldest observation has been dropped from consideration and the 1969 observation has been added. On the basis of the \( C \) approximation, the firm determines the size of the budget that the \( i \)th department manager is allowed to commit during 1971 in order to determine the department's configuration for 1972.

2.5 A Department Manager's Budget Allocation Decision

The decision process by which a department manager commits his total budget during a fiscal year is exogenous to our model, which is to say -- for example -- that we do not attempt to explain why Manager \( A \) decides to rent a Brand \( B \) computer rather than some other kind. Our model views each department manager as a lottery: in particular, at the start of 1969 the firm "bets" some amount \$x_1\) on the \( i \)th manager's capabilities. Then at the start of 1970, the firm observes (i) that as a result of the manager's
allocation decisions during 1969, the \textit{ith} department exhibits a particular configuration of research facilities; and (ii) that the firm can select projects from a revised inventory, in which some of the projects have been proposed by the \textit{ith} department.

Each project in inventory can be characterized -- explained in Section 2.3 above -- as a lottery 
\[ \{v_i, 1 - \exp(-\sum K_{ij} T_{ij})\}, \] where the \(K_{ij}\) values reflect the \textit{stated} capabilities of each research department with respect to the \textit{ith} project. Thus, as Chapter 3 explains in detail, the operating characteristics of the \textit{jth} department manager \( (j = 1, \ldots, m) \) can be characterized by several parameters whose interpretations are as follows.

(a) A manager may be more capable than his colleagues, perhaps in the sense that he hires scientists who suggest extremely profitable projects for the firm. This sort of relative superiority can be characterized by various "mean factors".

(b) One manager may be more erratic than others, in the sense that his department's capabilities may differ greatly among projects. For example, one department may be capable of doing research only on projects that its scientists suggest, while another department may be able to respond to project suggestions from the firm's operating divisions. This variability in a department's capabilities can be characterized by certain "variance factors".
(c) A manager may be either chronically optimistic or pessimistic in stating his department's capabilities. This phenomenon can be characterized by certain additions to the stated \((K_{ij})\) values.

2.6 **Summary**

Certain aspects of the model developed in this chapter could obviously be modified in order to describe mathematically the way a particular firm associates points in a "parameter space" with points in a decision space. For example, the firm's organization may have more layers of departments and sub-departments than shown in Figure 2.1; or alternative probability-of-success functions could be substituted for exponential distributions; or chance constraints could be added to the description of each department manager's allocation problem.

The following aspects, however, could not be changed.

(a) The firm must make allocation decisions, in the sense that employees must be "targeted" against research projects.

(b) The firm must also determine a strategy for reconfiguration.

(c) The allocation and reconfiguration decisions can be made independently in this sense: for a given configuration, many allocation decisions are possible; and the allocation in one period does not necessarily imply a reconfiguration for the next period.
The three central features stated above prompt the following questions, which motivate the algorithms developed in subsequent chapters.

(a) Methods of mathematical programming might be used to improve the allocation decision. If the firm's objective function for allocation has the form of 2.4 above, then the SUMT algorithm developed by Fiacco and McCormick could -- in principle -- supply an optimal allocation decision. In practice, however, a firm's allocation problem could easily involve at least 1000 variables (e.g., 10 departments and 100 projects), and in 1969 McCormick stated that the SUMT algorithm had never solved a problem of this size. Moreover, an optimal solution to the allocation problem (versus the heuristic solution that firms achieve by allowing each department manager to allocate resources in what seems to him an optimal pattern) would not necessarily add even one or two percent to the firm's average profits over several years. So there are two problems of allocation, namely (i) how a research firm can solve its allocation problems, and (ii) whether optimal solutions would increase profits appreciably.

(b) Which of infinitely many possible reconfiguration rules should a firm follow? Methods of digital simulation can be used to compare the profitability of reconfiguration
rules only if an algorithm for rapidly solving allocation problems is available.

(c) Given that a firm intends to use particular rules for reconfiguration and allocation, what is an optimal scale for the firm's total research program?

(d) Any public policy imposed on a research firm would operate directly on some parameters that the firm regards as exogenous to its allocation and reconfiguration decisions, and indirectly on the decisions themselves. Thus, how would a firm that behaves according to the model developed in this chapter respond to parametric variations caused by changes in patent laws, anti-trust laws, and other public policies?
Appendix

The rationale for assuming exponential probability-of-success functions follow immediately from two lemmas on real-valued functions. Historical references for these lemmas can be found in Thomasian (1969).

To begin the proof, let $g(x)$ be the probability that a project will require more than $x$ in order for a given team to succeed on the project within a specified planning horizon. Thus $g(r) = P(F_r)$, etc. By assumption,

(2) \[ g(r + t) = g(r)g(t) \text{ for all } r \text{ and } t \geq 0, \text{ and} \]

(3) \[ g(x) \leq 1, \text{ } x \geq 0. \]

We also assume that $g(x)$ is not identically zero -- i.e., that there exists $x_\circ$ such that $g(x_\circ) > 0$, where $x_\circ > 0$.

Given these assumptions, it can be shown that for all $x > 0$, $g(x) > 0$, and that there exists a finite $M$ such that $|\log(g(x))| \leq M$ for all $\{x|0 < x \leq 1\}$. The proof is that for any positive integer $n$,

(4) \[ g(nx_\circ) = g(x_\circ) \ldots g(x_\circ) = \{g(x_\circ)\}^n > 0, \text{ so that} \]

(5) \[ 0 < g(nx_\circ) = g(x)g(nx_\circ - x) \text{ for } 0 < x < nx_\circ. \]

Thus, since $n$ is arbitrary, $g(x) > 0$ for all $x > 0$. Also,

(6) \[ g(1) = g(x)g(1 - x) \leq g(x) \text{ if } 0 < x < 1. \text{ Thus} \]

(7) \[ \log(g(1)) \leq \log(g(x)) \leq \log(1) \]

for $0 < x < 1$; and since $g(1) > 0$, $|\log(g(x))|$ is bounded on the unit interval.
The second lemma asserts that if \( f \) is a real-valued function defined for all positive reals and bounded on the unit interval -- i.e., if
\[
|f(x)| \leq M \quad \text{for} \quad 0 < x \leq 1,
\]
and if it is true that
\[
f(x + y) = f(x) + f(y) \quad \text{for all} \quad x, y > 0,
\]
then
\[
f(x) = f(1)x \quad \text{for all} \quad x > 0.
\]

The proof for the second lemma is as follows: by assumption,

\[
\{f(x + y) - f(1)(x + y)\} = \{f(x) - f(1)x\} + \{f(y) - f(1)y\}
\]
for \( x \) and \( y \) positive. We can then define \( F(x) \) as \( f(x) - f(1)x \) for all positive \( x \), such that \( F(x) \) is bounded on the unit interval, and

\[
F(x + y) = F(x) + F(y) \quad \text{for} \quad x, y > 0.
\]

Now consider an arbitrary positive sequence of values \( \{x_i\} \), such that
\[
F(\sum_{i=1}^{n} x_i) = F(\sum_{i=1}^{n-1} x_i) + F(x_n)
\]
\[
= F(\sum_{i=1}^{n-2} x_i) + F(x_{n-1}) + F(x_n)\ldots
\]
\[
= \sum_{i=1}^{n} F(x_i).
\]

If \( x_2 = x_3 = \ldots = x_n = 1 \), then

\[
F(x_1 + (n-1)) = F(x_1) \quad \text{for all} \quad x_1 > 0.
\]
and all \((n-1) = 1, 2, \ldots\). Thus \(F\) is periodic, with period 1; and since \(F\) is bounded on the unit interval,
\[(16) \quad |F(x)| \leq M' \text{ for all } x > 0\]
If \(x_1 = x_2 = \ldots = x\), then \(F(nx) = nF(x)\). So if for some \(x_0 > 0\), \(F(x_0) \neq 0\), then
\[(17) \quad |F(nx_0)| = n|F(x_0)| > M'.\]
since for all \(M'\) we can choose \(n\) large enough so that (17) will hold. But (17) contradicts the boundedness of \(F\), so we must have \(F(x)\) identically zero for all positive \(x\). So, by definition of \(F\),
\[(18) \quad f(x) = f(1)x;\]
and from the definition of \(f(x)\), we have
\[(19) \quad \log(g(x)) = \log(g(1))x;\]
and since for any logarithm base \(b\) we have \(\log_b y = (\log_b e)(\ln y)\) for all positive \(y\),
\[(20) \quad g(x) = \exp((\ln(g(1)))x),\]
which was to be shown.

Since \(g(1)\) denotes the probability that the project will fail if $1 is invested in it, the constant term \(K_{ij}\) can be interpreted as the natural logarithm of the probability that the \(j\)th team will fail on the \(i\)th project, assuming that \(T_{ij} = 1\).
Chapter 3

A MODEL OF ECONOMIC CONTROLS
OVER RESEARCH

3.1 Introduction

This chapter explains how the budgeting process described above in Chapter 2 can be simulated by a computer. The simulation program's order of execution is explained, and each major step in the program is illustrated in a flowchart. Moreover, the entire program is listed fully as a sequence of APL subroutines.1/ Variables exogenous to the program are underlined in the explanation, but not in the subroutine listings.

3.2 The Program DRIVER

The first program to be executed is DRIVER, which is listed in Figure 3.1. This program merely establishes several initial conditions and then repeatedly calls the program MAIN1, thereby compiling a value for the returns matrix RETM whose dimensions are \((NITER \times HORZ)\). The element \(RETM(1,1)\) denotes the total return earned in the first of \(HORZ\) years by an R&D firm that begins operating under a given set of initial conditions, and that succeeds

---

1/ Because of their fidelity to mathematical notation, A Programming Language programs have been called executable flowcharts. For detailed explanations of the APL code, see Berry (1968), Patkin (1968), and Gilman and Rose (1970).
DRIVER[]\[1] D\[2] RETM\[3] START ITERATIONS.
\[8] RHIST+IRH
\[9] CHIST+ICH
\[10] N\[11] N1+N
\[12] K+IM
\[13] TBVEC+ITBVEC
\[14] BUD+IBUD
\[15] TKMVEC+ITKMVEC
\[16] TKVVEC+ITKVVEC
\[19] MAIN1
\[20] =>(TIME<(HORZ-0.5))/TST
\[21] =>(IT<(NITER-0.5))/ITST

Figure 3.1: a listing of the DRIVER program
on certain projects in the first year. The value of 
RETM(1,2) is the return earned in the second year, and 
so forth for the first row (i.e., for the row indexed by 
IT = 1). When the time index TIME exceeds HORZ, the 
iteration counter IT is incremented by unity, and -- given 
the same initial conditions from which the first row of 
the returns matrix was compiled -- the firm begins again 
in the first year of a sample run through time. This 
process continues until IT exceeds NITER, at which point 
the program stops.

3.3 Generating Project Values and PSF Parameters

The MAIN1 program is listed in Figure 3.2, and flow-
charted in Figure 3.3. When DRIVER calls MAIN1, it in 
turn calls the subroutine VGENER1, which generates N 
project values \{V(1),...,V(N)\} from a probability distri-
bution. The initial value of N is set in DRIVER as IN;
and a normal distribution with mean VMEAN and variance 
VVAR is used to generate the values, although many 
different generators for random variables could be 
substituted for the normal generator function R that is 
called -- as shown in Figure 3.4 -- by VGENER1.

Once the project values have been generated, MAIN1 
calls KMTXER1, which establishes a matrix of the K(i,j) 
values that determine each department's probability-of-
success functions with respect to M projects (where M is
Figure 3.2: a listing of the MAIN1 program
START

Generate project values $V_1, \ldots, V_n$.

Generate probability-of-success function parameters $\{K_{ij}; \, i=1, \ldots, n; \, j=1, \ldots, m\}$.

Solve the resource allocation problem, and thus compute a matrix of allocations $\{T_{ij}; \, i=1, \ldots, n; \, j=1, \ldots, m\}$.

Generate true values for $n$ projects.

Generate true values for the parameters $\{K_{ij}\}$.

Determine each department's success or failure on each project, and thus the firm's total return for the year.

Figure 3.3: a partial flowchart of the MAIN1 program (continued on the next page)
Update the matrices RHIST and CHIST, where the rightmost column of RHIST lists the returns most recently attributed to each department, and CHIST is similarly defined as a history of departmental budgets.

Generate cost and return histories for potential research departments.

Generate means and variances for the expected errors in judging each potential department's capabilities.

Set bounds on budgets for existing and potential departments.

Establish objective function coefficients for existing and potential departments, in order to determine parameters for the reconfiguration decision.

Figure 3.3: a partial flowchart of the MAIN1 program (continued on the next page)
Figure 3.3: a partial flowchart of the MAIN1 program
\( \text{VGENER1}() \)
\( \text{VG}+\text{VGENER1} \)
\[ \text{GENERATES } V[i] \text{ VALUES.} \]
\[ \text{VVEC}+n \text{ LGNER(VMEAN, VVAR)} \]
\( \text{LGENR}() \)
\( \text{LGN}+n \text{ LGNER PRMS; VARY; MY; NV} \)
\[ \text{GENERATES } n \text{ LOGNORMAL VALUES, WITH MEAN} \]
\[ \text{PRMS}[1] \text{ AND VARIANCE } \text{PRMS}[2] \]
\[ \text{VARY}+\phi((\text{PRMS}[2]+(\text{PRMS}[1]*2)+1) \]
\[ \text{MY}+(\phi\text{PRMS}[1])-(0.5 \times \text{VARY}) \]
\[ \text{NV}+N R 0 1 \]
\[ \text{LGNN}+(\text{MY}+(\text{VARY} \times 0.5) \times \text{NV}) \]
\( \text{VR}() \)
\( \text{R}+N R V \)
\[ \text{R}+V[1]+(V[2] \times 0.5) \times (1002 \times (?NpR) \times R) \times (2 \times (\Theta R+?NpR+8388608) \times 0.5 \]
\( \text{KMTXER1}() \)
\( \text{KMT}+\text{KMTXER1; NV; VEXP; RATS} \)
\[ \text{GENERATES THE KMTX.} \]
\[ \text{NV}+N \times M \]
\[ \text{VEXP}+(M,N) \times \text{VVEC} \]
\[ \text{RATS}+(N,M) \times \text{NV LGNER(RMEAN, RVAR)} \]
\[ \text{KMT}+(1-\text{PROB}) \times (\text{VEXP} \times \text{RATS}) \]

Figure 3.4: listings for the VGENER1, R, KMTXER1, and LGNER subroutines
initially set for each run through time as $T(i,j)$. As is apparent in Figure 3.4, KMTXER1 generates a typical $K(i,j)$ value in the following way. A ratio -- say the value .75 -- is selected from a lognormal distribution with mean $RMEAN$ and variance $RVAR$; and $K(i,j)$ is set at the value such that the $j$th department can succeed -- with probability $PROB$ -- on the $i$th project by allocating the value of resources $T(i,j) = .75 \times V(i)$.

It is important to note that the $K(i,j)$ values generated via the above process correspond to the capabilities that the firm believes its departments have with respect to projects. As will be explained below, the actual capabilities may differ substantially from those reflected by the $K(i,j)$. Moreover, although it is assumed that -- on the average -- all departments appear equally capable, their actual average abilities to succeed may differ.

3.4 Targeting Resources against Projects

As explained in Chapter 2, the following mechanism for targeting resources against projects is assumed. At the start of an arbitrary fiscal year, each department manager commands a configuration of resources that remain fixed for the coming year. Moreover, the central research manager presumably tells each department manager an estimate of the expected value $V_i$ of a technical success on each of $n$ projects in inventory. Each department manager
then determines a rough set of priorities, which for an arbitrary department \( j \) can be measured by a realization of the vector \((T_{ij}, ..., T_{nj})\), where the accounting value \( B_j \) of the department's resources is known, and the ratio \( T_{ij}/B_j \) measures the relative priority assigned by the \( j \)th project currently in inventory.

By assumption, the \( j \)th manager acts roughly as if he were attempting to maximize his department's contribution to the firm's profits for the coming fiscal year. To model this behavior, it is assumed that the \( j \)th manager solves the problem of choosing \((T_{ij}, ..., T_{nj})\) to maximize

\[
(3.1) \quad z_j = \sum_{i=1}^{n} V_i (1 - \exp(K_{ij}T_{ij}))
\]

\[
(3.2) \quad \sum_{i=1}^{n} T_{ij} \leq B_j \text{ and}
\]

\[
(3.3) \quad T_{ij} \geq 0 \text{ for all } i.
\]

The SOTPER algorithm that is displayed in Figure 3.5 and is called in line 6 of MAIN1 solves the above optimization problem for each department \( j = 1, ..., m \). The steps in the algorithm are the following.

(a) Set the vector \( T = (T_i) = 0 \), and drop the \( j \) subscript.

(b) Calculate the vector \( D = (\partial z/\partial T_1, ..., \partial z/\partial T_n) \), where each partial derivative is evaluated at zero, such that \( \partial z/\partial T_i = V_i K_i \).
Figure 3.5: a listing of the SOFTER algorithm
(c) Set the vector $\text{ORDER} = (o_1, \ldots, o_n)$, where $o_i$ is the index value of the $i$th largest component of $D$.

(d) Set the vector $\text{DORD} = D(\text{ORDER})$.

(e) Set the project counter $a = 0$.

(f) Set $a = a + 1$.

(g) Define the vector $\text{INDX}$ as the first $a$ components of $\text{ORDER}$.

(h) Set the vector $\text{THAT} = (t_1, \ldots, t_n) = 0$.

(i) Define $\text{KONST} = \text{DORD}(a + 1)$.

(j) Solve the equation

$$t_i = (\ln V_i + \ln K_i - \ln \text{KONST})/K_i$$

for $t_i$, for all $i$ in $\text{INDX}$.

(k) Set $\text{COND1} = 1$ if $\Sigma t_i < B$, and 0 otherwise.

(l) Set $\text{COND2} = 1$ if $a < (n-1)$, and 0 otherwise.

(m) If $\text{COND1} = \text{COND2} = 1$, branch to (f) above.

(n) If $\text{COND1} = 0$ and $\text{COND2} = 1$, branch to (p) below.

(o) Set $\text{INDX} = \text{ORDER}$.

(p) Where all summations are taken over $i$ in $\text{INDX}$, solve

$$\ln \phi = (\Sigma (\ln V_i)/K_i) + \Sigma ((\ln K_i)/K_i - B)/(1/\Sigma (1/K_i))$$

for $\phi$.

(q) Set $\text{THAT} = 0$.

(s) For all $i$ in $\text{INDX}$, solve (3.4) above for $t_i$, and then set $T = \text{THAT}$.

(t) Calculate the value of the objective function corresponding to $T$, and terminate.
The fact that the SOPTER algorithm converges in a finite number of steps to the assumed allocation decision for each department is obvious from the following considerations. First, the non-negativity constraints on the \((T_i)\) are never violated; and when the algorithm terminates, the budget constraint is obviously satisfied. Second, in determining each department's allocation, the algorithm will definitely terminate in \(n\) steps or less, since the first partial derivative of \(z_j\) with respect to \(T_{ij}\) for each \(i\) is monotonically decreasing. Each iteration includes a variable in the INDX set, and possibly terminates execution by branching to \((p)\); and when INDX contains all variables, a branch to \((p)\) is certain. Finally, the relevant subset of the necessary and sufficient Kuhn-Tucker conditions\(^2\) for the problem is as follows.

For each department \(i\), there exists a scalar \(\phi\) and a realization of \((T_{ij}, \ldots, T_{nj})\) — say the realization \((x_1, x_2, \ldots, x_n)\) — such that for all \(x_i < 0,\)
\[
(3.6) \quad \frac{\partial z_j}{\partial x_i} = \phi \frac{\partial g_j}{\partial x_i} = \phi,
\]
where \(g_j(T_{ij}, \ldots, T_{nj})\) denotes the \(j\)th department's budget constraint.

For all \(x_i = 0,\)
\[
(3.7) \quad \frac{\partial z_j}{\partial x_i} < \phi \frac{\partial g_j}{\partial x_i} = \phi x_i = 0
\]

\(^2\) For a concise derivation and statement of the Kuhn-Tucker conditions, see Fiacco and McCormick (1968).
since $\phi$ will obviously be positive. The SOPTER algorithm terminates at (and only at) the $\phi$ value and the realization $(x_1, x_2, \ldots, x_n)$ required by the Kuhn-Tucker conditions.

3.5 Generating True Project Values and PSF Parameters

After the allocation problem has been solved, line 8 of MAIN1 invokes the TRUVER subroutine (listed in Figure 3.6) in order to generate "true" values for the projects. An actual value for a project is set as a multiple of the assumed value $V(i)$, where the multiplier is a normal variate with mean $TMF$ and variance $TVF$. The rationale for the simulation algorithm's distinction between presumed and true project values is that the firm allocates its resources partly on the basis of an assumed set of project values (or "utilities", or "priorities"); but obviously the firm can misjudge these values. Thus the TRUVER subroutine models the firm's errors in estimation.

Similarly, the firm bases its allocations partly on estimates of its departments' abilities to succeed on the research projects in inventory; but these estimates can be wrong, possibly in a systematic way. As shown in Figure 3.6, the TRUKER subroutine simulates errors in estimating the $K(i,j)$ parameters that determine each department's probability-of-success functions. A true $K(i,j)$ value is a multiple of the assumed value, where the multiplier is a lognormal random variate with mean
\[ \text{\textsc{Truver}}() \]
\[ \text{\textsc{Tru}+\textsc{Truver};nv} \]
[1] \text { A DETERMINES TRUE PROJECT VALUES.}
[2] \text { TVVEC+VVEC×(N \& (TMF,TVF))} \\
\]
\[ \text{\textsc{Truker}}() \]
\[ \text{\textsc{Trk}+\textsc{Truker};col} \]
[1] \text { A GENERATES THE TRUE KMTX.}
[2] \text { COL+0} 
[3] \text { TKMTX+(N,M)\&0} 
[4] \text { CS:COL+COL+1} 
[5] \text { TKMTX[;COL]+KMTX[;COL]×(N LGNER(TKVMVEC[COL],TKVVEC[ COL])]} 
[6] \text { →(COL<\&M-0.5)/CS} \\
\]
\[ \text{\textsc{Rlzer}}() \]
\[ \text{\textsc{Rl}+\textsc{Va} \textsc{R} \textsc{Lzer} \textsc{R} \textsc{k}} \]
[1] \text { A GENERATES REALIZED PROFIT, ATTRIBUTED PROFITS.}
[2] \text { MTMS+(N,M)\&(RNER(∗,RK)))} 
[3] \text { HITS+(MTMX\&MTMS)} 
[4] \text { RZ+(\&HITS)+,×VA} 
[5] \text { RETM[IT;TIME]+RZ} 
[6] \text { ATR+(\&HITS)+,×VA} \\
\]
\[ \text{\textsc{Rner}}() \]
\[ \text{\textsc{Rne}+\textsc{Rner vec};vals} \]
[1] \text { A GENERATES A VECTOR OF EXPONENTIALLY DISTRIBUTED}
[2] \text { A OBSERVATIONS, GIVEN A VECTOR \textsc{vec} OF MEANS.}
[3] \text { VALS+(pVEC/10000)\&10000} 
[4] \text { RNE+(−VEC)×(\&VALS)} \\

Figure 3.6: listings of the \textsc{Truver}, \textsc{Truker}, \textsc{Rlzer}, and \textsc{Rner} algorithms
TKMVEC(j) and variance TKVVEC(j). These two parameters can be set at different values for different j values, in order to model the fact that although all departments might appear equal in average capabilities, some departments might actually perform much better than others, and some might exhibit greater variability in performance. The initial values for the two vectors are ITKMVEC and ITKVVEC.

3.6 Determining Results from Research

Once the true project values and PSF parameters have been generated, MAIN1 next invokes the RLZER subroutine (listed in Figure 3.6), which records results from the firm's research efforts, and which also attributes returns to each research department. RLZER functions by first generating a matrix MTMS of minimum \( T_{ij} \) values, each from the exponential distribution determined by a true \( K_{ij} \) value. Each value of MTMS is then compared with the corresponding element in TMTX -- i.e., with the (i,j) allocation set by the SOFTER subroutine. If \( MTMS(i,j) < TMTX(i,j) \), then the jth department fails on the ith project, and a zero is recorded in the (i,j) position of a binary-valued matrix HITS. Conversely, a success by j on i implies that HITS(i,j) = 1.

If at least one department succeeds on the ith project, the firm gains the true value TVVEC(i). The sum of the true gains is recorded in a returns matrix as
the element RETM(i,j). Moreover, the RLZER program compiles a vector ATR of returns attributed to departments. The ATR(i) value for a typical department is defined as the dot product of the jth column of HITS, and the vector TVVEC that lists true project values. Clearly, the sum over ATR can be greater than the firm's total return RETM(i,j), and thus the last ATR reflects only the performance of each department in relation to its peers.

3.7 Updating Histories for Casts and Returns

Two matrices -- IRH and ICH -- are specified as initial values for RHIST and CHIST, for each simulation run over the number of years specified by HORZ. The rightmost column of IRH lists returns attributed to each department for the first year before the simulation run through HORZ years begins (e.g., if a firm wanted to use the simulation model to forecast profits for the period 1972 through 1976, then HORZ would be 5, and the rightmost column of IRM would be returns attributed for 1971; the second column (from the right) of IRH would refer to returns attributed for 1970, and similarly for each of the other NPAST columns of IRH, where NPAST denotes the number of years for which the firm maintains a history of attributed returns in order to forecast future performances.) ICH is defined analogously as an initial value for the cost history CHIST.
The rightmost column of CHIST is then examined to see whether any department has been funded below a critical level CRIT. When a department's funding has fallen below the level at which the department can continue to function at all, the department disappears from the firm. This feature of the model can be used to rule out the possibility that a department could be kept in a kind of suspended animation at zero funding levels for several years, and could then be revived for immediate service.

In addition to the above updating process, a further modification in the firm's state occurs through the NSER subroutine appearing in Figure 3.7. It is assumed that each year a number of potential research departments are perceived by the firm: for example, departments in other firms might be "pirated", new departments could be formed within the firm, and so forth for other obvious possibilities. Thus in the third step of the NSER subroutine, NN (the number of potential departments) is generated from a Poisson distribution with mean LM. The Poisson generating function is the POIER subroutine in Figure 3.7.

The return history for each potential department is a random vector of NFAST components, each of which comes from a lognormal distribution with mean IRM and variance IRH. Similarly, each component of the PCHIST matrix — whose rows are cost histories for the potential departments —
\textbf{RCTER}[]\textbf{v}
\textbf{v RCT+TBVEC RCTER AR;BIN}
[1] a UPDATES THE RHIST AND CHIST MATRICES.
[2] RHIST+RHIST[;1:14]
[3] CHIST+CHIST[;1:14]
[4] RHIST+RHIST COLCAT AR
[5] CHIST+CHIST COLCAT TBVEC
[6] BIN+TBVEC>CRIT
[8] RHIST+BIN/[1] RHIST
[9] CHIST+BIN/[1] CHIST
[10] TKMVEC+BIN/TKMVEC
\textbf{v COLCAT}[]\textbf{v}
\textbf{v R+X COLCAT V}
[1] a STICKS A COLUMN ON THE RIGHT OF A MATRIX X.
[2] R+((mX)[(2)m1],0)\textbf{v}
[3] R[;1]((mR)[(2)]+v
\textbf{v NSER}[]\textbf{v}
\textbf{v NS+NSER;NN}
[1] a SETS UP PROVISIONAL CHIST, RHIST MATRICES
[2] a FOR THE NEW TEAMS ON THE HORIZON.
[3] NN+POIER LM
[4] +(NN=0.5)/PUNT
[5] PRHIST+(NN,NPAST)p((NNxNPAST) LINDER(IHM,IRV))
[6] PCHIST+(NN,NPAST)p((NNxNPAST) LINDER(ICM,ICV))
[7] PNT+NN
[8] +0
[9] TUNT;PNT+0
\textbf{v POIER}[]\textbf{v}
\textbf{v POI+POIER LM;I;VAL;SUM}
[1] a GENERATES A POISSON VARIABLE WITH
[2] a MEAN AND VAR OF LM.
[3] VAL+((?10000)+10000
[4] SUM+0
[5] I-1
[6] IST;I+I+1
[8] +(VAL\leq SUM)/QUIT
[9] +IST
[10] QUIT:POI+I
\textbf{v}

**Figure 3.7:** listings of the RCTER, COLCAT, NSER, and POIER subroutines
is a lognormal variate with mean IC\textsubscript{M} and variance IC\textsubscript{V}. The accounting method used to attribute returns to the firm's existing departments is presumably used also to compile return histories for potential departments, so that the two classes can be compared as candidates for further funding. (For example, a potential department might now be a research group at another firm, and last year the group succeeded on projects that yielded a certain return to the rival firm: this amount would then be a component of the potential department's return history.)

With each potential department comes not only a known history of returns and costs, but also a set of "prevarication factors" unknown to the firm. Once a new department is added to the firm, the K(i,j) values for the department are drawn from the same probability distribution that is assumed for all existing departments -- which is to say that all departmental managers either propose new projects or claim abilities to succeed on projects suggested by others, such that all existing departments appear equally profitable on the average with respect to all projects. In fact, however, some departments might contribute much less than others to the firm's earnings. As explained above in Section 3.4, this difference in average abilities is modeled via the assumption of different values in the TKMVEC and TKVVEC lists.
Thus for each potential department, the PERVAR subroutine (listed in Figure 3.8) generates two values, one from a lognormal distribution with mean $E_{M1}$ and variance $E_{M2}$, and one from a second lognormal distribution with parameters $(E_{V1}, E_{V2})$. If the firm accepts a potential department, the first lognormal value is catenated on the right of the TKMVEC list, and the second on the right of TKVVEC. To model a budgeting process in which there is no chance that potential departments would misjudge their own abilities with respect to any project, one would set $(E_{M1}, E_{M2})$ at $(1,0)$, and $(E_{V1}, E_{V2})$ at $(0,0)$. To model other assumptions about the accuracy with which potential departments judge their own abilities, one would set the parameters at values other than $(1,0)$ and $(0,0)$.

3.8 Establishing the Reconfiguration Problem

As explained above in Chapter 2, it is assumed that the firm neither cuts nor expands an existing department's budget by more than $PCN_{1}$ per cent of the previous year's funding for the department. The rationale for the lower bounds might be that a department's previous failures have occurred with low probability, such that the department should not be eradicated. The assumption of an upper bound derives from the fact that many departments cannot assimilate huge immediate increases in funding. New
\( \text{PERVAR}() \)\n\( \text{PER} + \text{PERVAR} \)
[1] \( \text{SETS UP ERROR MEANS AND VARIANCES} \)
[2] \( \text{FOR PROVISIONAL DEPARTMENTS.} \)
[3] \( \text{EM} + \text{PNT} \ \text{LGNER} (\text{EM1}, \text{EM2}) \)
[4] \( \text{EV} + \text{PNT} \ \text{LGNER} (\text{EV1}, \text{EV2}) \)

\( \text{UDNER}() \)\n\( \text{UDN} + \text{UDNER}; \text{V1}; \text{V2}; \text{PCV1}; \text{PCV2} \)
[1] \( \text{ESTABLISHES UPPER AND LOWER FUNDING} \)
[2] \( \text{LEVELS FOR BOTH NEW AND OLD TEAMS.} \)
[3] \( \text{V1} + \text{CHIST}[;5] \)
[4] \( \text{PCV1} + \text{PCN1} \times \text{V1} \)
[5] \( \text{OUBDS} + \text{V1} + \text{PCV1} \)
[6] \( \text{OLBDS} + \text{V1} - \text{PCV1} \)
[7] \( \text{OUBDS} + (\text{OUBDS} \geq 0) \times \text{OUBDS} \)
[8] \( \text{OLBDS} + (\text{OLBDS} \geq 0) \times \text{OLBDS} \)
[9] \( \text{\( \rightarrow \) (PNT} < 0.5) / 0 \)
[10] \( \text{V2} + \text{PCHIST}[;5] \)
[11] \( \text{PCV2} + \text{PCN2} \)
[12] \( \text{NUBDS} + \text{V2} + \text{PCV2} \)
[13] \( \text{NLBDS} + \text{V2} - \text{PCV2} \)
[14] \( \text{NUBDS} + (\text{NUBDS} \geq 0) \times \text{NUBDS} \)
[15] \( \text{NLBDS} + (\text{NLBDS} \geq 0) \times \text{NLBDS} \)

Figure 3.8: listings of the PERVAR and UDNER subroutines
departments each require a minimum level of funding if the firm chooses to establish them; and, like existing departments, the potential entrants cannot assimilate increases above certain levels of funding. The control parameter for bounds on funding potential departments is PCN2.

In line 3 of the UDNER subroutine (listed in Figure 3.8) the vector V1 is assigned the levels at which each existing department has been funded most recently, and then in lines 4, 5, and 6 the upper and lower bounds are established. If no new department is available, then bounds on the funding for the new departments are set.

Once the funding bounds have been determined, the F1ER function (shown in Figure 3.9) determines objective function coefficients for the firm's reconfiguration decision problem. For the firm's existing research departments, the coefficients calculated are stored in the COLD vector, as indicated in the MAIN1 program; and also as indicated in MAIN1, the coefficients for potential departments appear as elements in CNEW -- if the firm has perceived any new departments.

A typical coefficient COLD(j), j = 1,...,M, is determined as follows. For the jth department the firm presumably has kept a record of the earnings attributed to the department for, say, the past five years. This
Figure 3.9: listings of the FLER, FITER, and JINV subroutines
value appears as the $j$th row of the RHIST matrix. Similarly, the firm's allocations to the $j$th department are stored in the $j$th row of the CHIST matrix. Given this information, the firm's central research management wants to estimate the ratio ($\Delta$ earned by the $j$th department)/($\Delta$ spent on the $j$th department), where this ratio is assumed constant over the range of funding determined in the UDNER subroutine for the $j$th department.

Thus the objective function coefficient for the department is determined from a weighted linear regression of RHIST($j$, t = 1, ..., 5) on CHIST($j$, t = 1, ..., 5), where the weights in the vector $W$ reflect the firm's judgment about the relative values of past observations on costs and returns as indicators of the department's potential earnings. In particular, a value of $W = (.25, .25, .50, .75, 1)$ reflects a judgment that the earnings and costs recorded five years ago are less indicative of the department's potential profitability than more recent observations.

As indicated in line 8 of FLER, for each department the weighted least squares fit is performed by the subroutine FITER appearing in Figure 3.9. Within FITER, the required matrix inversion is done by the Gauss-Jordan inversion routine that is listed in Figure 3.9 as the subroutine JINV.

Once the objective function coefficient has been determined for each existing department, the MAIN1 program
checks to determine whether the firm has perceived any potential departments. If so, both the returns matrix PRHIST and the cost matrix PCHIST have been determined; and thus the subroutine FLER is again applied in order to determine the vector of objective function coefficients CNEW.

3.9 Choosing the New Departments to be Funded

As explained in Chapter 2, the firm's reconfiguration decision involves two steps; namely (i) selecting which potential departments will be funded at all, and (ii) determining funding levels for both the old departments and the new ones that have been selected for funding. The first problem is solved by the function SETER that appears in Figure 3.10. In addition, SETER determines certain initial conditions that face the firm at the start of the next fiscal year.

Line 2 of SETER determines the number of projects in inventory at the start of the next period. The number of projects N1 comes from a Poisson distribution with mean (\textit{ENP}+3). If the firm has perceived no potential departments, SETER terminates on line 3. Otherwise, the vector CNEW of objective function coefficients for the "candidate" departments is sorted in descending order, such that CNORD(1) is the largest element in CNEW, etc.
\$SETER[]\$
\$SET+SETER;ORD;CNORD;LORD;UORD;CWORST;FLOAT;I;NV1;NV2
[1] a SETS UP VECTORS FOR THE LP PROBLEM.
[2] N1+(POIER ENP)
[3] →(PNT<0.5)/0
[4] ORD+\$CNEW
[5] CNORD+CNEW[ORD]
[6] LORD+NLBDS[ORD]
[7] UORD+NUBDS[ORD]
[8] CWORST+L/COLD
[9] FLOAT+BUD- (+/OLBDS)
[10] I=0
[12] →(FLOAT<ORD[I])/ICHK
[13] →(CNORD[I]<CWORST)/BDS
[14] M=M+1
[15] ADTMS+ADTMS+1
[16] RHI:ST+RHI:ST ROWCAT PRHIST[I;]
[17] CHI:ST+CHI:ST ROWCAT PCHIST[I;]
[18] TKMVEC+TKMVEC,EM[I]
[19] TKVVEC+TKVVEC,EV[I]
[20] OLBD:SS+OLBDS,LORD[I]
[21] OUBDS+OUBDS,UORD[I]
[22] COLD+COLD,CNORD[I]
[23] FLOAT+FLOAT-UORD[I]
[24] ICHK:→(I<(pCNEW)-0.5))/IST
[25] BDS:LB+OLBDS
[26] UB+OUBDS
[27] C+COLD
\$
\$
\$ROWCAT[]\$
\$R+X ROWCAT V
[1] a STICKS A ROW ON THE BOTTOM OF A MATRIX X.
[2] R+(1 0 +pX)p(,.X),V
\$

Figure 3.10: listings of the SETER and ROWCAT subroutines
The order of sorting CNEW is then used to sort the upper and lower funding bounds for the candidate departments.

The lowest objective function coefficient for the firm's existing departments is stored as CWORST. The difference between (i) the firm's total research budget, and (ii) the sum of the minimum funding levels for the existing departments is assigned to FLOAT. Beginning with line 11 of SETER a loop is entered: the counter variable I runs from 1 through the number of candidate departments. For a typical value of I, the candidate department with the Ith largest objective function coefficient is examined first to see whether the firm can afford to fund the department at its minimum level. If the candidate is too expensive, the next most promising department is examined, and so on.

If the firm can afford the Ith candidate, then the candidate's objective function coefficient is compared with the worst coefficient for the firm's existing departments. If CNORD(I) is less than CWORST, then SETER terminates. Otherwise, the Ith candidate is accepted for funding. The number of departments is then incremented by unity, the number of departments added is incremented, the matrices RHIST, CHIST, TKMVEC, TKVVEC, OLBDS, OUBDS, and COLD are updated to reflect the new department's entrance. Finally, FLOAT is decremented by the minimum amount at which the new department must be funded.
3.10 **Determining Funding Levels**

When SETER terminates, the firm then presumably determines a funding level for each department by solving the linear programming problem of choosing the allocations vector \( \mathbf{x} \) to maximize

\[
(3.8) \quad z = \sum_{j=1}^{M} c_j x_j
\]

subject to

\[
(3.9) \quad \text{LB}_j \leq x_j \leq \text{UB}_j, \quad j = 1, \ldots, M, \text{ and}
\]

\[
(3.10) \quad x_j \geq 0 \quad \text{for all } j, \text{ and}
\]

\[
(3.11) \quad \sum_{j=1}^{M} x_j \leq B,
\]

where \((c_1, c_2, \ldots, c_M)\) denotes the vector COLD of objective function coefficients determined in the SETER subroutine. The program RSLN listed in Figure 3.11 solves the above problem.

RSLN begins by funding each department at its lowest level \( \text{LB}_j \), \( j = 1, \ldots, M \). According to the subroutines SETER and UDNER, the sum of the lower bounds is less than or equal to the total budget \( B \). The positive difference is then allocated sequentially among departments: the department with the largest \( c_j \) value is considered first, and its funding level is increased until either (i) the total budget is exhausted, or (ii) the upper bound \( \text{UB}_j \) is reached. If funds remain after the most promising department
\[ RSLN[] \]
\[ RS+RSLN; I; NDX; SP; P; Q; DIF; LP1; UB1; C1; B1 \]

1. SOLVES THE LP RECONFIGURATION PROBLEM.
2. \[ ((+/LB)>BUD)\vee((+/UR)<BUD)/YELL \]
3. \[ Q+P \]
4. \[ NDX+P_{i\in C} \]
5. \[ C_{i\in C}(NDX) \]
6. \[ UB1+UB(NDX) \]
7. \[ LB1+LB(NDX) \]
8. \[ B1+BUD-(+/LB) \]
9. \[ DIF+UB1-LB1 \]
10. \[ SOL+LB \]
11. \[ I+0 \]
12. \[ IST:I+I+1 \]
13. \[ SP+NDX[I] \]
14. \[ +(B1\leq DIF[I])/QUIT \]
15. \[ SOL[SP]=UB1[I] \]
16. \[ B1+B1-DIF[I] \]
17. \[ +(I<(Q-0.5))/IST \]
18. \[ TVVEC+SOL \]
19. \[ \rightarrow 0 \]
20. \[ QUIT:SOL[SP]=LB1[I]+B1 \]
21. \[ TVVEC+SOL \]
22. \[ \rightarrow 0 \]
23. \[ YELL:'BAD BOUNDS' \]

\[ \n \]

**Figure 3.11:** a listing of the RSLN subroutine
has been funded at its upper limit, the department having
the next largest $c_j$ value is considered, and so on. From
the design of UDNER and SETER, it is obvious that the
total budget constraint forces RSLN to terminate for some
department $j = 1, 2, \ldots, M$.

3.11 Restarting the Budgeting Cycle

Once the RSLN subroutine has terminated, the budgeting
cycle begins again: the MAIN1 program returns control to
DRIVER, which either (i) terminates the model's operation
if the iteration counter IT exceeds the specified number
of iterations NITER, or (ii) recalls MAIN1 to begin with
the terminal conditions (e.g., the department funding
levels, the histories of costs and returns, etc.) that
were established in the previous execution of MAIN1, if
the time counter TIME is less than the number of years
HORIZ for which the firm's progress is to be recorded,
or (iii) resets the initial conditions, resets the time
counter to unity, and recalls MAIN1 in order to generate
another sample of the firm's behavior over the time
horizon of HORIZ years.

3.12 Summary

The NITER parameter in DRIVER can be set at an
arbitrarily large value, in order to compile empirical
frequency distributions over computed variables such as
total returns from investments in research. Thus the
simulation algorithm can be viewed simply as an operator that associates (i) probability distributions over some variables with (ii) assumed distributions for other variables such as project values. All parameters of the model are summarized in the program PARMER (listed in Figure 3.12), which identifies and prints any given set of parameter values.
\$PARMER[]\$
\$ P+PARMER
[1] a LISTS PARAMETERS.
[2] 'NUMBER OF YEARS IN THE SAMPLING PERIOD...' ;HORZ
[3] 'NUMBER OF ITERATIONS...' ;NITER
[4] 'NUMBER OF DEPARTMENTS...' ;IN
[5] 'NUMBER OF PROJECTS...' ;IN
[6] 'IMEAN , IVAR FOR PAST RETURNS...' ;IRM , IRV
[7] 'IMEAN , IVAR FOR PAST COSTS...' ;ICM , ICV
[8] 'NUMBER OF YEARS IN COST AND RETURN HISTORIES...' ;
NPAST
[9] 'INITIAL BUDGETS FOR EACH DEPARTMENT...' ;ITBVEC
[10] 'TOTAL BUDGET FOR RESEARCH...' ;IBUD
[12] 'CONSTANT PROBABILITY OF SUCCESS...' ;PROP
[13] a FOR EACH DEPARTMENT AND EACH PROJECT,
[14] a KMTX[I;J] IS CHOSEN SUCH THAT THE
[15] a DEPARTMENT CAN SUCCEED--WITH PROBABILITY
[16] a PROB--ON THE PROJECT BY ALLOCATING
[17] a RESOURCES OF V[I;J]*RATS[I;J].
[18] 'MEAN AND VARIANCE FOR RATIOS...' ;RMEAN , RVAR
[19] a TO DETERMINE THE TRUE V[I] FOR A PROJECT,
[20] a VVEC[I] IS MULTIPLIED BY A FACTOR.
[21] 'FACTOR MEAN AND VARIANCE...' ;TMF , TVF
[22] a TO DETERMINE THE TRUE X[I;J] VALUES,
[23] a EACH KMTX[I;J] IS MULTIPLIED BY A
[24] a FACTOR.
[25] 'DEPARTMENT KFACTOR MEANS...' ;ITKMEVEC
[26] 'DEPARTMENT KFACTOR VARIANCES...' ;ITKVVEC
[27] 'EXPECTED NUMBER OF POTENTIAL DEPARTMENTS...' ;LM
[28] a EACH POTENTIAL DEPARTMENT HAS ITS OWN
[29] a KFACTOR MEAN AND VARIANCE.
[30] 'MEAN AND VARIANCE FOR KFACTOR MEANS OF'
[31] 'POTENTIAL DEPARTMENTS...' ;EM1 , EM2
[32] 'FUNDING LEVEL AT WHICH A DEPARTMENT'
[33] 'DISAPPEARS...' ;CRIT
[34] 'MEAN AND VARIANCE FOR KFACTOR VARIANCES'
[35] 'OF POTENTIAL DEPARTMENTS...' ;EV1 , EV2
[36] 'PERCENTAGE FUNDING BOUND FOR EXISTING'
[37] 'DEPARTMENTS...' ;PCN1
[38] 'PERCENTAGE FUNDING BOUND FOR POTENTIAL'
[39] 'DEPARTMENTS...' ;PCN2
[40] 'RECONFIGURATION WEIGHTS...' ;W
[41] 'EXPECTED NUMBER OF PROJECTS...' ;3+ENP

Figure 3.12: a listing of the PARMER subroutine
Chapter 4

PRESCRIPTIONS FOR MANAGING RESEARCH

4.1 Introduction

Section 4.2 presents a nonlinear programming algorithm called OPTER that can be applied to the allocation problem posed above in Chapter 2. (Recall that the SOPTER code developed in Chapter 3 merely models the assumption that a firm allows each department's manager to allocate resources among research projects -- so the SOPTER solution to an allocation problem might be grossly suboptimal.)

Lasdon (1970), Wagner (1969), Geoffrion (1969), and Fiacco and McCormick (1968) survey algorithms that in principle solve the allocation problem; and thus the OPTER method certainly does not apply to a class of problems that operations researchers have never considered. Moreover, the basic idea underlying the OPTER program is nothing more than the equimarginal principle of economics: in order to choose a value for the vector \((x_1, x_2, \ldots, x_n)\) that maximizes a concave objective function \(z(x_1, x_2, \ldots, x_n)\) subject to certain constraints, the program initially sets each \(x_i\) at zero; then selects the variable that offers the greatest marginal gain in \(z\); then increases the "most attractive" variable's value
until a second variable seems equally attractive at the margin; then increases the two best variables' values until a third seems equally attractive; and then continues this process until constraints prohibit further increases.

In attempting to apply the equimarginal principle to the allocation problem, however, one encounters several difficulties with computation and with the "bookkeeping" involved in programming a machine to remember which variables should be increased, what values the increases should assume, etc. And after these details have been resolved, there remains the question of whether a simple, intuitively appealing application of the principle yields a solution that is actually optimal. Finally -- and perhaps most importantly -- there is the question of whether an application of the principle allows one to solve non-trivial versions of the allocation problem in reasonable times. These issues motivate the discussion in Section 4.2, which explains the OPTER code, proves that solutions derived from the code are optimal, and demonstrates that certain nonlinear programs involving more than 50 variables can be solved in less than 20 seconds.

Although the OPTER algorithm achieves its virtues of speed, accuracy, and simplicity at the considerable expense of being much more specialized than most methods of nonlinear programming, a survey of recent literature indicates that operation researchers are attempting to
solve several diverse problems to which the OPTER code could easily be applied. In particular, Passy (1971) suggests a geometric programming algorithm for solving a "targeting" problem posed by Manne (1958). The first section of this chapter's appendix suggests an improvement in Manne's formulation is a restatement of the allocation problem established above in Chapter 2.

The second section of the appendix shows how the OPTER algorithm can be modified in order to solve a statistical problem posed by Srikantan (1963), and reconsidered by Sananthanan (1971). Srikantan's problem has the same form as a mathematical program formulated by Moses (1972) as a problem of allocating servers to networks (e.g., telephone networks) whose nodes and/or arcs fail stochastically. Moreover, as Hadley (1964) points out, many dynamic programming problems require a choice of \((x_1, x_2, \ldots, x_n)\) that maximizes

\[
(4.1) \quad x = \sum_{j=1}^{M} f_i(x_i), \text{ subject to } \\
(4.2) \quad \sum_{j=1}^{n} a_j x_j \leq b, \quad a_j > 0, \text{ and } \\
(4.3) \quad x_j \in \{\text{all non-negative integers}\}.
\]

If the functions \(f_i(x_i)\) are concave and twice differentiable everwhere, then the continuous version of the problem falls
within the class suggested by Srikanatan. For example, the two Dean-Hauser models (1967) for "optimal funding of materiel concepts" and "optimal system funding" take certain probability-of-success functions as inputs; and apparently in the applications considered by Dean and Hauser, the input functions can be approximated closely by differentiable concave functions, such that the two models are amenable to the modified OPTER algorithm.  

Similarly, Hamilton (1969) postulates an "all series" problem of allocating funds for research, and an "all parallel" problem. These are essentially the Dean-Hauser problems, except that Hamilton simply assumes gamma distribution functions as representing probabilities of success. In order to develop a workable algorithm, he uses exponential distribution functions. For the exponential case, the OPTER code solves both problems.

Despite the fact that the OPTER algorithm is superior to the sub-optimizing procedure described by the SOPTER code, a firm might be unwise to abandon the traditional approach of letting departmental managers allocate the

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1/ Dean and Hauser relied on dynamic programming, and reported that computational difficulties forced them to use a coarser grid than they had planned. Had they approximated certain functions with smooth curves, their problem sizes would have been well within the computational power of the OPTER code, and the errors in approximation might have been much less pernicious than the errors induced by the grid they used.
resources at their disposal. Either of the following scenarios might be true.

(a) To use a "centralized" allocation mechanism such as the OPTER algorithm, a firm's central research management would first have to ask departmental managers to estimate parameters for probability-of-success functions. Each departmental manager thinks he knows the optimal allocation for his department's resources; and so, within certain limits, he changes his best estimates of the parameters in a way that he believes will induce the central management to allocate resources properly. Moreover, even if each manager reports the true parameter values, the OPTER allocates resources only slightly better than the SOPTER method; and this advantage is more than destroyed when the centralized planning mechanism induces moderate lies from managers. Thus by relying on central planning, the firm wastes the time managers spend in supplying parameter estimates, and ultimately achieves an allocation worse than that which could have been achieved by letting the managers rule their own shops.

(b) In contrast to the above scenario, the truth might be that the decentralized procedure works miserably in comparison with centralized planning: departments do far too much "parallel" research on some projects, and neglect others. Thus even if managers do fudge
parameter estimates, the end result of centralized planning is superior.

Section 4.3 below presents simulated comparisons of OPTER and SOPTER, which indicate that the decentralized procedure works remarkably well, and that apparently small errors in parameter estimates substantially degrade the performance of a centralized mechanism.\(^2\)

As indicated above in Chapter 2, the issue of centralized versus decentralized allocations relates only to one aspect of the research budgeting process. Thus the fourth section of this chapter presents simulated comparisons of two strategies for reconfiguration. The arguments against a "liberal" reconfiguration rule -- which motivate the discussion in Section 4.4 -- are as follows.

In favor of a conservative rule, one might observe that some departments are sporadically

\(^2\) As the surveys by Baker and Pound (1964), cetron et al. (1967), and Gear et al. (1971) indicate, dozens of methods for allocating resources among research projects have been proposed; but apparently no one has investigated whether the proposed methods work appreciably better than the "rough and ready" procedures that firms actually use, and whether biases that might be induced by centralized planning obviate any advantage to be gained by adopting a new method. Mansfield and Brandenburg had observed this in 1966, when they wrote: "Unfortunately, the discussion of selection techniques [for research projects] seldom, if ever, contain any treatment of the sensitivity of the results to errors in estimates of [probabilities of technical success]."
profitable, such that a liberal rule (which views a department's earnings in the recent past as the best indicator of future profitability) might unwisely eliminate a firm's most profitable research efforts. Moreover, in adding a new department the firm is essentially buying a risky, untried product, so that a liberal rule would greatly increase the chance of wasting money on unprofitable research groups.

In addition to the questions of allocation and reconfiguration, there is a "scale" problem for research management: even if a firm uses a good set of rules for allocating resources and funding departments, the firm's entire research program may be unprofitable because the total budget for research is the wrong amount. Thus Section 4.5 concludes this chapter by demonstrating how the methods of simulation and optimization can be linked in order to suggest a profitable scale for research funding.

4.2 The OPTER Algorithm for Resource Allocation

The allocation problem posed above in Chapter 2 is that of choosing a value of the matrix \( (T_{ij}, i = 1, \ldots, N; j=1,\ldots,M) \) to maximize

\[
(4.4) \quad z = \sum_{i=1}^{N} V_i P_i,
\]

where the \((V_i, i=1,\ldots,N)\) denote \(N\) known project values, and the \((P_i, i=1,\ldots,N)\) denote corresponding probabilities
of gaining the values. The probability that the \( j \)th of \( M \) research groups will succeed on the \( i \)th project is assumed to be

\[
P_{ij}(T_{ij}) = 1 - \exp(-K_{ij}T_{ij}),
\]

where the values \( K_{ij} \) are known constants. Thus \( P_i \) becomes

\[
P_i = 1 - \exp(-\sum_{j=1}^{M} K_{ij}T_{ij}).
\]

The \( (T_{ij}) \) values are constrained as

\[
\sum_{i=1}^{N} T_{ij} \leq B_j, \ j = 1, \ldots, M, \text{ and}
\]

\[
T_{ij} \geq 0 \text{ for all } (i,j),
\]

where the \( (B_j) \) values are known budgets for the \( M \) departments. The ratio \( (T_{ij}/B_j) \) can be interpreted as the fraction of the \( j \)th department's total resources (for some time period) that is assigned to the \( i \)th project.

The SOPTER code described above in Chapter 3 yields sub-optimal solutions to the nonlinear programming problem of allocating resources to research projects. In contrast, the OPTER code shown in Figure 4.1 yields optimal allocations. In conventional notation, the steps in the algorithm are as follows.

(a) Set the matrix of resource allocations \( (T_{ij}) = 0 \).

(b) Set the vector \( ZOLD = (z_1, \ldots, z_M) = 0 \).

(c) Set \( j \), the index of research teams, \( = 0 \).

(d) Set \( j = j + 1 \).
\[\text{OPTER}[]\]
\[\text{OPT}+\text{OPTER}\]
[1] \(\text{TMTX}+=(N,M)p0\)
[2] \(\text{ZOLD}+=mp0\)
[3] \(\text{CVAL}+=mp0\)
[4] \(\text{PASS}:=j+0\)
[5] \(\text{COLS}:=j+1\)
[6] \(\text{TMTX}[,;j]+Np0\)
[7] \(\text{PR}+\text{KMTX}x\text{TMTX}\)
[8] \(\text{DZIN}+\text{KMTX}[;j]x\text{VVEC}x((+-/[2] PR))\)
[9] \(\text{ORDER}+\text{DZIN}\)
[10] \(\text{DORD}+\text{DZIN}[\text{ORDER}]\)
[11] \(A:=0\)
[12] \(\text{AS}:A+A+1\)
[13] \(\text{KONST}+\text{DORD}[A+1]\)
[14] \(\text{INDX}+A+\text{ORDER}\)
[15] \(\text{THAT}+Np0\)
[16] \(\text{K1}+\text{KMTX}[\text{INDX};j]\)
[17] \(\text{P1}+\text{PR}[\text{INDX};]\)
[18] \(\text{V1}+\text{VVEC}[\text{INDX}]\)
[19] \(\text{THAT}[\text{INDX}]+((*\text{V1}+(*\text{K1}+(-\text{KONST})+(-/[2] P1)))*K1\)
[20] \(\text{COND1}+((+/\text{THAT})<\text{TBVEC}[J])\)
[21] \(\text{COND2}+((A>((N-2)+1E^{-5}))\)
[22] \(+(\text{COND1}^\text{COND2})/\text{AS}\)
[23] \(+(\text{COND1}^\text{COND2})/\text{PHIBAR}\)
[24] \(+(\text{INDR}^\text{COUT})/\text{INDXSET}\)
[25] \(+(\text{COND1}^\text{COUT})/\text{PHIBAR}\)
[26] \(\text{INDXSET}:\text{INDX}+\text{ORDER}\)
[27] \(\text{K1}+\text{KMTX}[\text{INDX};j]\)
[28] \(\text{P1}+\text{PR}[\text{INDX};]\)
[29] \(\text{V1}+\text{VVEC}[\text{INDX}]\)
[30] \(\text{PHIBAR}:B++/(+\text{K1})\)
[31] \(\text{T1}++/((\text{V1}+\text{K1})\)
[32] \(\text{T2}++/((\text{K1}+\text{K1})\)
[33] \(\text{P2}++/[2] P1\)
[34] \(\text{T3}++/\text{P2}^\text{K1}\)
[35] \(\text{PB}++((\text{T1}+\text{T2}+(-\text{T3})+(-\text{TBVEC}[J]))+B)\)
[36] \(\text{CVAL}[J]+\text{PB}\)
[37] \(\text{THAT}[\text{INDX}]+((\text{V1}+(*\text{K1})+(-\text{PB})+(-\text{P2}))^\text{K1}\)
[38] \(\text{TMTX}[\text{ORDER};J]+\text{THAT}[\text{ORDER}]\)
[39] \(\text{ZOLD}[J]+\text{VVEC}+.x(1-(*+/[2](-\text{KMTX}^\text{TMTX})))\)
[40] \(\text{ZMIN}:+/\text{ZOLD}\)
[41] \(\text{ZDIF}+\text{ZOLD}^\text{ZMIN}\)
[42] \(+(+/\text{ZDIF}<0,1)/\text{PRINT}\)
[43] \(+(J<(N-1E^{-5}))/\text{COLS}\)
[44] \(\text{PASS}\)
[45] \(\text{OPT}+\text{TMTX}\)
[46] \(\text{PRINT}:+(\text{OS1}=0)/0\)
[47] \('\text{OPTIMAL TIMES WERE...'}\);\text{TMTX}\)
[48] \('\text{OPTIMAL OBJECTIVE FUNCTION WAS...'}';\text{ZOLD}[J]\

**Figure 4.1:** a listing of the OPTER algorithm
(e) Set the $j$ column of $(T_{ij}) = 0$: this amounts to assuming that none of the $j$th team's resources have been allocated to any project.

(f) Note that

$$
\frac{\partial z}{\partial T_{ij}} = K_{ij} V_i \left( \exp \left( - \sum_{j=1}^{M} K_{ij} T_{ij} \right) \right);
$$

given the assigned values for $j$, $(T_{ij})$, and $(K_{ij})$, compute the values of $(\partial z/\partial T_{1j}, \ldots, \partial z/\partial T_{Nj})$, and store this string as DZIN.

(g) Define a vector ORDER = $(i_1, \ldots, i_N)$, such that $i_1$ is the index of the largest element in DZIN, $i_2$ is the index of the second largest element, etc.

(h) Order the elements in DZIN in descending order, and call the resulting string DORD.

(i) Set $a$, a counter of projects, = 0.

(j) Set $a = a + 1$.

(k) Select the scalar value KONST = DORD(a+1).

(l) Define the vector INDX as the first $a$ components of ORDER. Thus INDX defines a set $\psi$ of projects, given the current value of $j$, which can be denoted by $j^*$.

(m) Set the vector THAT = $(t_1, \ldots, t_N) = 0$.

(n) Define the vector $K_1$ as the INDX values of the $j$th column of the matrix $(K_{ij})$.

(o) Where the matrix $PR = (K_{ij}, T_{ij})$, define the matrix $P_1$ as the INDX rows of $PR$. 
(p) Define $V_l$ as the INDX elements of the vector of project returns $(V_1, \ldots, V_N)$.

(q) Note that from (4.9) above,

\[\ln(\partial z/\partial T_{ij}) = \ln V_i + \ln K_{ij} - \sum_{j=1}^{M} K_{ij} T_{ij};\]

and so, given the assigned values of $(T_{ij}, j \neq j^*)$, KONST, and INDX, the values of $T_{ij}$ -- for $i$ in $\Psi$ and for $j = j^*$ -- that force $\partial z/\partial T_{ij} = \text{KONST}$ can be found from the equation

\[t_i = (\ln V_i + \ln K_{ij^*} - \ln \text{KONST} - \sum_{j \neq j^*} K_{ij} T_{ij})/K_{ij^*}.\]

(r) Note that for the project counter value $a = 1$, the computation described in steps (e) - (q) above has the following economic interpretation: first we assume that none of the $j$th team's resources have been allocated; then we select the project whose marginal value product is greatest in terms of the $j$th team's possible investments. (That is, we choose the project whose value of $\partial z/\partial T_{ij}$ is greatest, where we have assumed (i) that the resources of all other teams have previously been allocated in some way, and (ii) that for the $j$ team under consideration, $T_{ij} = 0$ for all $i$.) We then begin investing in the "best" project until the marginal value product (MVP) of investment here is just equal to the MVP of investing a small amount in the next-best project. As the firm increases its investment in the best project, we can be sure that its MVP will decline, since
\( (4.12) \quad \frac{\partial^2 z}{\partial T_{ij}} = -(K_{ij} T_{ij} \exp(- \sum_{j=1}^{M} K_{ij} T_{ij})) \leq 0. \)

Thus by increasing the jth team's investment in the best project, we eventually reach a level of investment at which the firm is indifferent between investing a marginal dollar in either of the two best projects. At this point, we check to see whether the jth team has exceeded its budget limit \( B_j \). We also check to see whether more than one "untapped" project remain for the jth team -- i.e., we check to determine the number of THAT values that remain set at zero.

(s) If the budget has not been exceeded, and if more than one untapped project remains, program control reverts to step (j) above. This amounts to investing in the two best projects, until the firm is indifferent at the margin between these two projects and the third-best project. Given this investment pattern, the checks in step (r) are repeated.

(t) If the budget has been exceeded, but there remains more than one untapped project, we then ask what level of investment in each of the tapped projects would equate their marginal value products at a level higher than the best MVP offered by the best untapped project, and would just meet the budget constraint imposed by \( B_j \). The appropriate common MVP can be called \( \phi \), and can be inferred from the budget requirement that
\[ (4.13) \quad \sum_{i \in \Psi} T_{ij} = B_{j^*}, \text{ since from (4.11) above we have} \]

\[ (4.14) \quad t_i = (\ln V_i + \ln K_{ij^*} - \ln \phi - \sum_{j \neq j^*} \frac{K_{ij} T_{ij}}{K_{ij^*}}) / K_{ij^*}. \]

If the \( (\partial x / \partial T_{ij^*}) \) values -- for \( i \in \Psi \) -- are to be equal to \( \phi \), then from (4.13) and (4.14),

\[ (4.15) \quad B_{j^*} = \sum_{i \in \Psi} \left\{ \ln V_i + \ln K_{ij^*} - \ln \phi - \sum_{j \neq j^*} \frac{K_{ij} T_{ij}}{K_{ij^*}} \right\} / K_{ij^*}. \]

which can be solved for \( \phi \). That is,

\[ (4.16) \quad \ln \phi = \left( \frac{\sum_{i \in \Psi} (\ln V_i / K_{ij^*}) + \sum_{i \in \Psi} (\ln K_{ij^*} / K_{ij^*})}{\sum_{i \in \Psi} (\sum_{j \neq j^*} \frac{K_{ij} T_{ij}}{K_{ij^*}} / K_{ij^*} - B_{j^*})} / (1 / \sum_{i \in \Psi} K_{ij^*}). \]

Given a value of \( \phi \), we can use (4.11) to determine appropriate levels of investment for the \( j^* \) group's resources.

(u) If the budget constraint has not been exceeded, but only one untapped project remains for the \( j^{th} \) team, then program control sets the INDX vector equal to the entire ORDER vector; then control moves to a sequence of steps in which we solve for \( \phi \), as above in step (t).

In this case, all projects become tapped, and all marginal value products are equated.

(v) If the budget constraint is exceeded and only one untapped project remains, then control again reverts to the calculation for \( \phi \).

(w) We now assume that the resources of the \( j^{th} \) team have been invested, such that the INDX values of \( \text{THAT} \) become
the INDX values of the \textit{jth} column of \((T_{ij})\). Given that the matrix of resource allocations has assumed a new value, we calculate the value of the objective function, and store this value in ZOLD as \(z_j\).

(x) We then check to see whether the components of ZOLD differ by any significant amount; if they do not, the algorithm has converged, and its results are printed. If the ZOLD values do differ significantly, control reverts either to an increased \(j\) value, signifying that the \((j+1)\) column of \((T_{ij})\) is to be considered, or -- if \(j = M\) -- to the \(j\) value of 1, indicating that the first group's resources should be reallocated.

The OPTER algorithm stops at -- and only at -- a Kuhn-Tucker point, which is both necessary and sufficient for optimality, since the objective function is concave, and the feasible region is convex. In particular, since the algorithm never leaves the feasible region, the relevant KT conditions are that there exists a vector \((u_1, \ldots, u_m)\) such that:

\begin{equation}
(4.17) \quad \text{if } T_{ij}^* > 0, \frac{\partial z(T^*)}{\partial T_{ij}} = \sum_{k=1}^{M} u_k \frac{\partial g_k(T^*)}{\partial T_{ij}},
\end{equation}

where \(T^*\) indicates that the objective function \(z(.)\) and all constraint derivatives are evaluated at the "candidate" optimum point, and where \(g_k(.)\) denotes the \textit{kth} of the problem's \textit{m} resource constraints; and
(4.18) if $T_{ij}^* = 0$, $\frac{\partial z(T^*)}{\partial T_{ij}} - \sum_{k=1}^{M} u_k \frac{\partial g_k(T^*)}{\partial T_{ij}} \leq 0$; and

(4.19) if $u_k > 0$, $g_k(T^*) - B_j = 0$; and

(4.20) if $u_k = 0$, $g_k(T^*) - B_j \leq 0$; and

(4.21) $u_k \geq 0$ for $k = 1, 2, \ldots, m$.

If we now consider an arbitrary value of $j$, we can recall from the algorithm's description that iterations stop when and only when there exists some subset of $i$ values -- say the set $I_j$ -- such that

(4.22) $\frac{\partial z}{\partial T_{ij}} = \frac{\partial z}{\partial T_{kj}}$ for all $(i,k)$ in $I_j$, where this common value of partial derivatives can be called $u_j$, and where for all pairs $(i,j)$ such that $i$ is not a member of $I_j$,

(4.23) $\frac{\partial z}{\partial T_{ij}} \leq u_j$.

We also recall that for all $i$ in $I_j$, $T_{ij}$ is positive; and for all $i$ not in $I_j$, $T_{ij}$ is zero. Moreover, $\frac{\partial g_j}{\partial T_{ij}}$ will be unity for all $i$ corresponding to a given $j$, and

$\frac{\partial g_k}{\partial T_{ij}} = 0$ for $j \neq k$. So

(4.24) $\sum_{k=1}^{M} u_k \frac{\partial g_k(T^*)}{\partial T_{ij}}$

which appears in the first two KT conditions stated above, will be $u_j$ for all $(i,j)$ and for all vectors $u$.

Thus, given the way we have defined $u_j$, we know that for all positive $T_{ij}$, the first KT condition will be satisfied.
And since for all zero values of $T_{ij}$ we know that $u_j \geq \partial z/\partial T_{ij}$, the second KT condition is automatically satisfied. Also, all $u_j$ will be positive according to our definition, and all constraints will hold as equalities, by construction of the algorithm -- so our proof is complete.

The following observations indicate that OPTER necessarily converges to a KT point, although not in a predictable number of iterations. A new objective function value $z(t)$, $t = 1, 2, \ldots$, is calculated each time the values in a column of the $(T_{ij})$ matrix are recomputed. We know from the algorithm's construction that

$$z(1) \leq z(2) \leq z(3) \leq \ldots$$

Let a cycle of the algorithm be defined as a sequence $\{z(t), z(t+1), \ldots, z(t+2m)\}$, where $m$ is the number of columns in $(T_{ij})$. If some elements in a cycle are unequal, then the cycle has improved the objective function value. And if all elements in any cycle are equal (within an arbitrarily small tolerance limit), then the value of $(T_{ij})$ corresponding to the cycle is optimal.

The optimality follows from the fact that immediately after $z(t)$ has been set, there is -- for some column $j^*$, and for some set $\psi$ of $i$ values -- a $\phi$ value such that

$$\phi = \partial z/\partial T_{ij^*} = \partial z/\partial T_{k j^*}$$

for all $(i, k)$ in $\psi$. 
and

(4.27) \( \phi \geq \partial z/\partial T_{ij} \) for all \( i \) not in \( \psi \).

If \( z(t+m) = z(t) \), no changes in the columns of \( (T_{ij}) \) corresponding to \( \{(t+1), (t+2), \ldots, (t+m-1)\} \) have perturbed the \( \psi, \phi, \) and \( (T_{ij}) \) values corresponding to \( z(t) \).

Similarly, if \( z(t+1) = z(t+m+1) \), the optimality conditions for the \((j^*+1)\) column remain intact. When optimality conditions remain intact for all \( j \), a KT point has been reached. Thus we never expect a cycle such that

(4.28) \( z(t) = z(t+1) = \ldots = z(t+2m) \),

where the \( (T_{ij}) \) value corresponding to the cycle is not an optimal point.

Figure 4.2 shows a sample of convergence times obtained by applying OPTER to allocation problems involving between 20 and 60 variables. The parameters defining each of the 9 problems solved were generated randomly from certain probability distributions, and for each problem the convergence tolerance limit on the \( z(t) \) values was less than .1% of the optimal objective function value. The solution times are much larger than they would be if the OPTER algorithm had been programmed efficiently, since the speed of execution in APL is usually between 20 and 30 times slower than in FORTRAN. Thus an allocation problem involving 60 variables could probably be solved in less than one second.
Figure 4.2: times required for OPTER to solve problems of various sizes
4.3 Centralized versus Decentralized Allocations

Fifty-seven comparisons of centralized versus decentralized procedures for allocating resources among research projects have indicated (i) the expected value of research planned centrally is probably less than 1.25 times the value of decentrally planned research; (ii) the central planning mechanism scores relatively best when research projects are low-risk ventures; and (iii) especially when projects are quite risky -- given the firm's total budget for research -- errors in parameter estimates that might be induced by the centralized procedure cause the decentralized method to perform relatively best.

The comparisons were done via the GR23ER program (listed in Figure 4.3), which functions as follows.

(a) Before GR23ER is invoked, two matrices -- namely GRAPH2 and GRAPH3 -- are each dimensioned to have 19 rows and 2 columns. The first column of each is the vector (.05,.10,...,.95).

(b) A row-pointer IT is set at unity, and the variable PROB assumes the value .05.

(c) The number of projects among which resources are to be allocated is chosen as 3 plus a Poisson random variate with mean 8.

(d) The number of research departments is chosen as 3 plus a Poisson random variate with mean 4.
\[ \forall \text{GR23ER}[]\forall \text{GR23+GR23ER;}IT \]

1. COMPARES THE OPTER AND SOFTER ALGORITHMS.
2. \[ IT+0 \]
3. \[ ITST;IT+IT+1 \]
4. \[ PROB+GRAPH2[IT;1] \]
5. \[ N+3+POIER 8 \]
6. \[ M+3+POIER 4 \]
7. \[ BUD=1000+(1 \ R)(0 \ 4000) \]
8. \[ PVEC+=7(Mp1000) \]
9. \[ PVEC+PVEC: (+/PVEC) \]
10. \[ TVEC+BUD*PVEC \]
11. \[ VVEC+3000+(N \ R)(0 \ 100000) \]
12. \[ KMTXR1 \]
13. \[ SOFTER \]
14. \[ GRV1+FOV \]
15. \[ OPTER \]
16. \[ GRV2+ZOLD[J] \]
17. \[ GRAPH2[IT;2]+(GRV2+GRV1)1 \]
18. \[ KMTX+KMTX*(N,M)\ R((N\times M) \ R(1 \ 0.1)) \]
19. \[ GRV3+VVEC+.\times((1-(\times+[2](h-KMTX\times TMTX)))) \]
20. \[ GRAPH3[IT;2]+(GRV3+GRV1)1 \]
21. \[ \rightarrow (IT=19)/0 \]
22. \[ \rightarrow ITST \]

Figure 4.3: a listing of the GR23ER subroutine
(e) The total budget for research is chosen as 1000 plus a normal random variate with a mean of zero and a variance of 4000.

(f) The distribution of the total budget among research departments is selected by first drawing a value for each department randomly from the integers \(1, 2, \ldots, 1000\), then normalizing the set of values, and then multiplying each normalized value by the total budget.

(g) Each project value is selected as 3000 plus a normal random variate having a mean of zero and a variance of 100,000.

(h) The matrix of probability-of-success parameters \(K_{ij}\) is selected. Typically, \(K_{ij}\) assumes a value such that the \(j\)th department will fail -- with the probability PROB -- if it allocates a value \(R_{ij} \times V_i\) to the \(i\)th project. Each element in the \(R_{ij}\) matrix is a lognormal variate with a mean of .5 and a variance of 1.

(i) The SOPTER algorithm solves the allocation problem defined by steps (1) - (12) in the GR23ER program, and the resulting objective function value is stored in the variable GRV1.

(j) The OPTER algorithm solves the same problem, and the optimal value is stored in GRV2.

(k) The value \(\{(GRV2/GRV1) - 1\}\) is stored in the \((IT, 2)\) position of the GRAPH2 matrix.
(l) The \((K_{ij})\) matrix is multiplied in parallel by a matrix \((S_{ij})\), whose typical value is a normal variate having a mean of 1 and a variance of .1.

(m) A revised objective function value is computed from the matrix of allocations \((T_{ij})\) determined by the OPTER algorithm, and from the revised \((K_{ij})\) matrix. This value is stored as GRV3.

(n) The value \([(GRV3/GRV1) - 1]\) is stored in the \((IT,3)\) position of GRAPH3.

(o) If \(IT\) is less than 19, control reverts to step (b) above, where \(IT\) is incremented and steps (c) - (n) are repeated. Otherwise, the program stops.

The results of two independent computations of the GRAPH2 matrix are plotted in Figure 4.4, where the horizontal axes measure probabilities of failure, and the vertical axes refer to values of the variable \([(GRV2/GRV1) - 1]\). Although the GR23ER algorithm selects parameters for allocation problems from a reasonably large space \((\text{e.g., departmental capabilities can be similar for all projects, or can vary widely})\), the two plots reveal clearly that the OPTER algorithm works relatively best when a firm undertakes few risks in its research, and that for a significant subset of parameter values the OPTER algorithm -- which is a method of central planning for resource allocations -- does not work appreciably better than the approach of allowing each
Figure 4.4: relations between the risks of research and the superiority of central planning
department's manager to allocate the resources at his disposal.

When the \((K_{ij})\) values are perturbed slightly (as they might be if managers alter their best judgments in order to induce the central management to allocate resources in the "right" way), then -- as shown by the plot of GRAPH3 in Figure 4.5 -- the decentralized approach becomes most attractive in some cases. Thus although the SOPTER algorithm would seem to be a poor choice, in fact its results often compare favorably with those from the OPTER code.

4.4 A Comparison of Rules for Reconfiguration

By changing certain initial conditions for the DRIVER program described above in Chapter 3, one can compare alternative rules for reconfiguration. A set of conditions that includes a "liberal" rule for reconfiguration appears in Figure 4.6. Several noteworthy aspects of the conditions are the following.

(a) The DRIVER program will compile a returns matrix having 10 rows and 10 columns, where each row can be viewed as a future time sequence of earnings that "Nature" might deal to a firm, given that the firm reconfigures its research departments according to a particular adaptive rule implied by a value of the weight vector \(w\) and by a setting of the bound parameter \(PCNL\). A \(w\) value of \(.05, \ldots\)
Figure 4.5: an illustration of the effects of errors in parameter estimates.
Figure 4.6: parameter settings corresponding to a liberal rule for reconfiguration
.07, .2, .5, 1) indicates that the firm forecasts each department's potential profitability on the basis of earnings that have been attributed to the department for the past five years. Earnings attributed five years ago, however, have little influence (e.g., a weight of .05) on the forecast, in comparison to earnings attributed most recently.

(b) A PCNL value of .3 implies that the firm never cuts or expands any of its departments' budgets by more than 30% in one year. Under the 30% rule, a department that performs badly in relation to other departments can be virtually eliminated in a few years. In contrast, the budget of a relatively profitable department can expand quite rapidly.

(c) At the start of each "draw" of returns for the next ten years, the firm has four departments. Although each department will claim on the average to be as profitable as others, two of the existing departments are in fact superior to the others. Moreover, although the potential departments the firm will perceive in the ten-year sampling period will appear on the average to be as capable as existing departments, the candidates will actually tend to be inferior. This is not to say that all candidates will be inferior, or that the firm can discern which candidates are better than others; rather, the assumption
implied by the given set of initial conditions is that
the firm will be selecting new departments from a population
whose average member would be a poor choice.

(d) The stochastic process implied by the conditions
is extremely noisy: for example, project values have a
variance of 100,000 and a mean of 50 in a lognormal
distribution. As will be apparent, however, even this
magnitude of noise is not large enough to obscure at
least two important differences in results from alternative
rules for reconfiguration.

The DRIVER program was executed once with the initial
conditions discussed above, and once with a variant of the
conditions. In the second case, the \( \bar{W} \) vector value was
\( (1,1,1,1,1) \), and the \( PCN1 \) value was \( .1 \). These changes
correspond to a shift toward conservatism in the firm's
strategy for reconfiguration: in any year, no department's
budget is cut or expanded by more than 10%; and returns
attributed in each of the five past years have the same
weights as predictions of future profits.

Column averages (i.e., average returns per year) for
the returns matrices compiled by the two executions of
the DRIVER program are plotted in Figure 4.7. It is
important to note that of the ten years in the sampling
period, only the last five are relevant in comparing the
two rules, since the first five years serve to generate
actual histories of costs and returns. (Recall that each
Figure 4.7: A comparison of annual returns from liberal and conservative rules for reconfiguration
execution of DRIVER requires inputs of histories for costs and attributed returns; and these inputs are not necessarily drawn from the return-generating process implied by settings of parameters such as the mean and variance for project values.)

The two runs indicate that the liberal rule performs better on the average than the more conservative approach. But when standard deviations of yearly returns are plotted as in Figure 4.8, it becomes clear that the liberal rule causes greater dispersions in earnings. A trace on each of the program executions yielded no clear explanation of the results observed for the two rules. In some years of the sampling period, the firm's better departments failed to produce net profits, and the liberal rule cut their budgets severely (and unwisely). Also, the liberal rule added more inherently unprofitable departments to the firm than did the conservative approach; but the liberal approach more rapidly corrected the mistakes it had caused.

Perhaps the most interesting aspect of the comparison was that different effects of the rules could be observed in a fairly small sample (100 observations in each run), even though the processes sampled were extremely noisy. This suggests that digital simulation of adaptive controls might be effective in cases where ignorance forces a firm's
Figure 4.8: A comparison of risks corresponding to liberal and conservative rules for reconfiguration
management to specify high-entropy probability distributions over important variates such as project values.  \(^3\) 

4.5 The Optimal Scale for Funding Research

If a firm's research program has been unprofitable, the difficulty could be that the total budget for research has been set at the wrong level in relation to certain characteristics of the research problems that the firm is attempting to solve. The SCALER program listed in Figure 4.9 can be used to determine an optimal total budget (i.e., an optimal scale) for a class of research projects characterized by (i) the maximum number of projects that the firm might perceive in a future year, (ii) the mean and variance of project values, and (iii) the "richness" of future project inventories, which is described by values for the variables RMEAN, RVAR, and PROB, each of which is interpreted above in Chapter 3. In particular, a rich, low-risk class of projects might be described by the values RMEAN = .1, RVAR = .0001, and PROB = .9, indicating that the firm could typically expect to succeed -- with a probability of .9 -- on a project whose value is \(V_i\), if the firm were to allocate resources worth (.1)(\(V_i\)) to the project.

The steps in the SCALER program are as follows.

\(^3\)/Entropy can be viewed as a measure of ignorance or uncertainty. See Shannon (1948).
\`\nSCALER[N]V
V SC=SCALER;M1;WTS;J1
[1]  \textit{\small a EXAMINES SCALE EFFECTS.}
[2]  RETM=(NRUNS,NBUD)p0
[3]  M1+1
[4]  MST=M1+M1+1
[5]  'THE COLUMN IS...';M1+1
[6]  AIN+INB+(M1×BINC)
[7]  J1+0
[8]  JST=J1+J1+1
[9]  'THE ROW IS...';J1
[10]  WTS=?4p100
[12]  TBVEC=AIN+WTS
[13]  VGENER1
[14]  KMTKXER1
[15]  OPTER
[16]  RETM[J1;M1+1]+ZOLD[J]
[17]  →(J1<(NRUNS-0.5))/JST
[18]  →((M1+1)<(NBUD-0.5))/MST
\`\n
Figure 4.9: a listing of the SCALER program
(a) A returns matrix RETM is dimensional to have \( NRUNS \) rows and \( NBUD \) columns, and a counter \( M1 \) is set at zero.

(b) The variable AIN measures a firm's total budget for research; and AIN is set initially at the value \( INB \).

(c) A counter \( J1 \) is set at unity, and the distribution of AIN among \( M \) research departments is selected randomly.

(d) The subroutine VGENER1 -- which is listed above in Chapter 3 -- generates \( N \) project values, each having a mean \( VMEAN \) and a variance \( VVAR \).

(e) The KMTXHR1 subroutine generates the \( (K_{ij}) \) matrix of probability-of-success parameters.

(f) The OPTER algorithm solves the allocation problem implied by (i) the distribution of AIN among \( M \) departments, (ii) the project values \( (V_1, V_2, \ldots, V_n) \), and (iii) the \( (K_{ij}) \) matrix.

(g) The objective function value is stored in the \( (J1, M1 + 1) \) position of the returns matrix RETM.

(h) If the row counter \( J1 \) has not exceeded \( NRUNS \), then \( J1 \) is incremented by unity, the distribution of AIN among the departments is again chosen at random, new project values are drawn, a new \( (K_{ij}) \) matrix is generated, and the new problem is solved.

(i) When \( J1 \) exceeds \( NRUNS \), the column counter \( (M1+1) \) is incremented, and the total budget is increased from its current value by the amount \( BINCR \). Another column of
RETM is completed via the process described above, and so forth for \texttt{NBUD} columns.

Once a value of RETM has been compiled, the mean of each column can be viewed as an average annual return corresponding to a given total budget. The column averages derived from one execution of the SCALER program are plotted in the first graph of Figure 4.10. The parameter settings corresponding to these averages are \( N = 7; \ M = 4; \ \texttt{VMEAN} = 20; \ \texttt{VVAR} = 100; \ \texttt{PROB} = .5; \ \texttt{RMEAN} = .5; \ \texttt{RVAR} = .01; \ \texttt{NBUD} = 10; \ \text{and} \ \texttt{NRUNS} = 10. \) These parameters characterize a typical project perceived by a firm as having an expected net value of zero, such that the firm can expect a positive return from research only by selecting unusually attractive projects.

When the number of projects perceived is doubled to 14, a second execution of SCALER yields the second graph in Figure 4.10. Additional executions (whose results are not plotted) were done with \( \texttt{VMEAN} = 40, \) and \( N = 7, \) and with \( \texttt{VMEAN} = 10, \) and \( N = 7. \) The four runs (each of which generated 100 sample points) were designed to examine how the optimal scale for research shifts in response to (i) changes in the values of a project inventory, and (ii) changes in the arrival rate of projects with fixed average characteristics.

The column averages of the returns matrix compiled by each execution behaved according to the exponential relation
Figure 4.10: results from two executions of the SCALER program
(4.29) \[ \text{TR}(x) = \alpha \{1 - \exp(-\beta x)\} \], where \( x \) denotes the total budget for research, and the values \((\alpha, \beta)\) are regression parameters.\(^4\)

The "true" value of \( \alpha \) for any execution is obviously \( (\text{VMEAN} \times N) \), and thus one might suspect that \( \beta \) would remain constant as \( \alpha \) is doubled or halved. Actually, however, \( \beta \) definitely changes with movements in \( \alpha \). For example, when \( \text{VMEAN} = 20 \), and \( N = 7 \), then \( \beta \) turns out to be approximately \( .01209 \); but when \( N \) is increased to 14, then \( \beta \) becomes \( .0059 \). When the number of projects is left at 7 and the average value is doubled to 40, however, \( \beta \) remains constant at about \( .0059 \).

For a given value of \((\alpha, \beta)\), the optimal total budget for research is the value of \( x \) that solves
\[ (4.30) \quad 1 = \alpha \beta \exp(-\beta x). \]
For an \( \alpha \) value of 70, the optimal budget turns out to be about 22. If \( \alpha \) is doubled to 140, then \( \beta \) changes such that \( x \) becomes about 43.5; and when \( \alpha \) is again doubled to 280, \( x \) becomes approximately 85.

\(^4\)In each case, the exponential relation fit the column means quite well: if \((r_1, \ldots, r_{10})\) denotes the actual column averages from one execution, and \((s_1, \ldots, s_{10})\) denotes the values estimated from the regression equation, then the vector \((r_1/s_1, \ldots, r_{10}/s_{10})\) indicates goodness of fit; and for all executions, the average absolute deviation of this vector's elements away from the unit vector \((1, \ldots, 1)\) was less than \( .04 \).
These results suggest the following hypotheses about returns to scale.

(a) A change in expected project values moves the optimal budget by the same amount as a proportionate change in the annual flow rate of projects perceived by a firm. Thus in terms of returns from research, a firm might be indifferent between the alternatives of (i) seeking more projects from a given population, and (ii) seeking projects from a more lucrative population.

(b) The optimal budget changes in constant proportion to the product of (i) the expected value of projects perceived by the firm, and (ii) the maximum number of projects perceived.
Appendix

A4.1 Applications of OPTER to Targeting Problems

Manne (1958) posed the following problem of assigning guns to targets.

It is assumed that there are \( m \) guns, labelled \( i = 1, 2, \ldots, m \), and that these are to be assigned to \( n \) targets, indicated by the subscript \( j = 1, 2, \ldots, n \). The objective is stated as one of minimizing the expected value of the surviving targets. More formally, if one lets \( x(i, j) \) denote the probability with which we assign the \( i \)th gun to the \( j \)th target, and \( p(i, j) \) the conditional probability that the \( i \)th gun will destroy the \( j \)th target -- given that the target survives all other guns, then the problem becomes one of selecting values for the \( x(i, j) \) so as to minimize

\[
(1) \quad \sum_{j=1}^{n} \sum_{i=1}^{m} a_j \prod_{i=1}^{m} (1 - p_{ij} x_{ij}), \text{ subject to }
\]

\[
(2) \quad \sum_{i=1}^{m} x_{ij} = 1 \text{ for } i = 1, \ldots, m; \text{ and }
\]

\[
(3) \quad x_{ij} \geq 0 \text{ for all } (i, j).
\]

Manne's problem has the same form as the resource allocation problem posed above in Chapter 2, except that the probability-of-success functions are linear over the interval \((0,1)\). Manne does not attempt to justify the linear form; and in fact, the exponential form seems more appropriate in formulating a targeting problem. If \( p(a|t) \) denotes the probability that a gun will destroy a target in \( a \) units of time, given that the gun has failed
against the target in $t$ units, then -- if the gun's accuracy
does not improve with successive shots -- we have
\[ p(a|t) = p(a|t+h) \] for all $(a, t, h) \geq 0$

This condition implies (as shown in the appendix to
Chapter 2) that the probability-of-success functions must
be members of the exponential family.

Manne stated that he could not solve the targeting
problem he posed, but suggested that a related problem
could be solved via linear programming. Passy (1971)
claimed that Manne's problem could be cast as a geometric
programming problem; but Passy provided no evidence that
his algorithm could solve non-trivial versions of the
problem in reasonable times. Thus a model that can be
solved by the OPTER algorithm seems relatively attractive
as a formulation for a targeting problem in which there
is no feedback of information (on hits) to the attacker.

As reported by Bracken and McCormick (1968), in 1965
Mylander formulated the following problem of assigning
weapons. The variables to be determined are arrayed in
a matrix $(x_{ij})$, whose typical element $x_{ij}$ is the number
of weapons of type $i$ assigned to target $j$, where $i = 1, \ldots, p,$
and $j = 1, \ldots, q$. Limitations on the number of weapons
assigned to targets are
\[ \sum_{j=1}^{q} x_{ij} \leq a_i, \ i = 1, \ldots, p, \text{ and} \]
\[
\sum_{i=1}^{P} x_{ij} \geq b_j, \quad j = 1, \ldots, q,
\]

where \(a_i\) denotes the total number of weapons of the \(i\)th type available, and \(b_j\) is the minimum number of weapons of all types assigned to target \(j\). The objective function to be maximized is

\[
\sum_{j=1}^{q} u_j \left\{ 1 - \prod_{i=1}^{P} \exp(x_{ij} \ln a_{ij}) \right\}
\]

where \(u_j\) is the military value of target \(j\), and \(a_{ij}\) is the probability that the \(j\)th target will be undamaged by an attack using one unit of weapon \(i\).

Since one can define \(K_{ij} = (-\ln a_{ij})\) for all \((i,j)\), the objective function has the form assumed by the OPTER algorithm. Thus if \(b_j = 0\) for all \(j\), the OPTER code applies to the continuous version of Mylander's problem.\(^1\) Moreover, the hypotheses developed above in Chapter 4 concerning (i) centralized versus decentralized control, and (ii) the optimal scale for research both apply directly to the weapons assignment problem (e.g., one class of weapons considered by Mylander is "fighter bombers"; and the results of Chapter 4 indicate that military objectives might be served best by allowing a commander of fighter bombers to allocate his weapons without regard to the allocation of long-range bombers and other kinds of weapons).

\(^1\) In order to obtain the integer solution demanded by the problem's formulation, Mylander solved the continuous version and then rounded the solution to nearest integers.
A4.2 Applications of OPTER to Problems of Stratified Sampling

Srikantan (1963) suggested that certain problems of choosing the sizes of strata in samples fall within the class of nonlinear programs requiring a choice of \((x_1, \ldots, x_n)\) to minimize

\[
(8) \quad z = \sum_{i=1}^{n} f_i(x_i), \text{ subject to } \\
(9) \quad \sum_{i=1}^{n} c_i x_i \leq C (c_i > 0 \text{ for all } i), \text{ and } \\
(10) \quad a_i \leq x_i \leq b_i, (a_i, b_i) > 0 \text{ for all } i,
\]

where \(d^2 f_i(x_i)/dx_i\) is assumed positive for all \(i\). Sananthanan (1971) reconsiders this problem, and proposes a second algorithm for solving it as well as a class of problems involving multi-stage stratified sampling.

A simplified version of the OPTER algorithm solves Srikantan's problem. The steps in the method are the following.

(a) Introduce the transformation \(c_i x_i = X_i\), and redefine \(a_i\) as \(a_i/c_i\), and \(b_i\) as \(b_i/c_i\). Also, recast the problem as that of maximizing \((-z)\).

(b) Set the solution vector \(X = (X_1, \ldots, X_n) = (a_1, \ldots, a_n)\) and define the set \(S(1)\) as the index values \((i = 1, \ldots, n)\). Define the sets \(S(2), S(3),\) and \(S(4)\) as null sets.

(c) Evaluate the derivatives \((df_i/dX_i)\) at \((a_1, \ldots, a_n)\).

In terms of this evaluation, arrange the derivative functions
in descending order, and use \{D_1(X_1), D_2(X_2), \ldots, D_n(X_n)\} to denote the rearranged functions. Set the iteration counter \(i\) at zero.

(d) Set \(i = i + 1\), and add \(i\) to the set \(S(2)\). Remove \(S(2)\) from \(S(1)\). For each \(j\) in \(S(2)\), set the \(X_j\) value such that \(D_j(X_j) = D_{i+1}(X_{i+1} = A_{i+1})\). If inverses of the derivative functions can be written as formulas, the appropriate \(X_j\) values can be determined by substitution; and if not, the values can be found via a simple search technique such as interval bisection.

(e) For all \(j\) in \(S(2)\), set \(X_j = \min(X_j, B_j)\). For each \(X_j\) that has been decreased to \(B_j\), delete the \(j\) value from \(S(2)\) and add it to \(S(4)\).

(f) If \((\Sigma X_i) = C\), the current \(X\) value is optimal.

(g) If \((\Sigma X_i) < C\), and if \(i = (n-1)\), set \(\lambda_1 = D_n(X_n = A_n)\); set \(\lambda_2 = \min \{D_i(B_i), i = 1, \ldots, n\}\); add \(i\) to \(S(2)\); and go to (j) below.

(h) If \((\Sigma X_i) < C\), and if \(i < (n-1)\), add \(S(4)\) to \(S(3)\); nullify \(S(4)\); and go to (d) above.

(i) If \((\Sigma X_i) > C\), set \(\lambda_1 = D_i(A_i)\), and set \(\lambda_2 = D_{i+1}(A_{i+1})\).

(j) Set \(\lambda_3 = \lambda_2 + .5(\lambda_1 - \lambda_2)\).

(k) For all \(j\) in \(\{S(2) + S(4)\}\), find the \(X_j\) such that \(\lambda_3 = D_j(X_j)\). For these \(j\), set \(X_j = \min(B_j, X_j)\); and then compare \((\Sigma X_i)\) with \(C\). If \((\Sigma X_i) = C\) -- within an arbitrarily small tolerance range -- then the current solution is optimal.
If \((\Sigma X_i) > C\), set \(\lambda_2 = \lambda_3\); and go to (j) above. Otherwise, set \(\lambda_1 = \lambda_3\), and go to (j). If \((\Sigma B_i) < C\), the problem has no solution.

The stopping point \(X\) for the above algorithm is a Kuhn-Tucker point for Srikantan's problem, as can be seen from the following considerations. If we define \(S(1)\) as the set of \(i\) such that the elements \(X_i\) of \(X\) are equal to \(A_i\), and \(S(2)\) as the set of \(i\) such that \(A_i < X_i < B_i\), and \(S(3)\) as the \(i\) values for which \(X_i = B_i\), then from the algorithm's construction it is true that for all \(i\) in \(S(2)\), the derivatives \(\frac{df_i(X_i)}{dx_i}\) are equal to a common value \(\lambda\); and for \(i\) in \(S(1)\) each derivative is less than \(\lambda\); and for \(i\) in \(S(3)\), \(\frac{df_i(X_i)}{dx_i}\) is greater than \(\lambda\).

The problem's constraint functions can be defined as 
\[g_j(X), \ j = 1, \ldots, 2n+1, \text{ where } g_j(X) = X_j \text{ for } j = 1, \ldots, n;\]
\[g_j(X) = -X_j \text{ for } j = n+1, \ldots, 2n; \text{ and } g_j(X) = (\Sigma X_i) \text{ for } j = (2n+1).\]
For any set of Lagrange multipliers \((u_1, \ldots, u_{2n+1})\), and for any \(i\),
\[
(11) \sum_{j=1}^{2n+1} u_j \left[ \frac{\partial g_j(X)}{X_i} \right] = u_i - u_{i+n} + u_{2n+1}
\]
Thus if for \(i\) in \(S(1)\) we define \((u_i, u_{i+n})\) as the pair \(\{0, \lambda - \frac{df_i(A_i)}{dx_i}\}\), and for \(i\) in \(S(2)\) we define \((u_i, u_{i+n})\) as \((0,0)\), and for \(i\) in \(S(3)\) we set \((u_i, u_{i+n})\) at \(\{\frac{df_i(B_i)}{dx_i} - \lambda, 0\}\), then if \(u_{2n+1}\) is set at \(\lambda\), for all \(i\) it is true that
\( \frac{\partial z}{\partial x_i} = \sum_{j=1}^{2n+1} u_i \{ \frac{\partial g_j(x)}{\partial x_j} \}. \)

Moreover, the above definition of the \((u_i)\) satisfies the complimentary slackness aspect of the Kuhn-Tucker conditions. Clearly, then, \(x\) is optimal.
5.1 Introduction

In its manifold relations with research firms, a government can rarely exercise direct controls over the firms' behavior; rather, the government can only change certain parameters in a firm's decision process, and then await reactions. This chapter suggests several ways in which a model of a research firm's allocation decision can be used to predict effects of certain public policies.

Section 5.2 examines the problem of estimating the volume of work a research firm will actually do in return for a government contract. In particular, let us suppose that an agency needs a certain kind of software package in order to program its computers to store and retrieve large amounts of data; and so the agency grants some firm $C$ in order to develop the package. As such contracts are usually understood by the government's contract monitors, the firm allocates some fraction of $C$ to administrative expenses, and the rest to research on the desired product.

The firm might, however, view the contract somewhat differently: the value $C$ (less administrative expenses) is actually an addition to the firm's total research budget,
and can be allocated to any one of $n$ projects upon which the firm is working -- which is to say that the firm's allocation of resources to satisfy a contract can apparently bear no relation to the contract's amount. Thus the question of interest in Section 5.2 is how a firm might actually allocate different contractual payments among given research projects.

Section 5.3 examines an aspect of the long-standing issue of public versus private research. The series of articles by Sanders (1968) on arthritis research suggests that private firms definitely allocate resources to risky ventures involved in finding retardants for certain arthritic diseases. But in view of the prevalence and severity of these diseases, one suspects that private industry is not devoting the amount of resources that a most profitable scale of investigation would dictate. Nelson (1965) has offered various explanations of why "market incentives tend to cause business firms to spend much less than is socially desirable on research and development exploring advanced concepts and designs".

No one, of course, has defined clearly the amount of research on arthritis that is "socially desirable", and thus it is impossible to test Nelson's conjectures. It is possible, however, to demonstrate with a simple model why a firm might willingly undertake risky projects -- as pharmaceutical manufacturers do -- and yet fund these projects at much less than the amount for which expected marginal revenues and costs are equal.
Section 5.4 discusses another aspect of public policy, namely the extent to which a proliferation of research departments actually improves the profitability of a firm's research program. Mansfield (1965) has argued on empirical grounds that holding the size of firms constant, the number of significant inventions carried out by a firm seems to be highly correlated with the size of its R&D expenditures. Thus, although the productivity of an individual R&D project is obviously very uncertain, it seems that there is a close relationship over the long run between the amount a firm spends on R&D and the total number of inventions it produces.

One would suspect -- without a great deal of empirical work -- a positive correlation between inputs of expenditures and outputs of inventions. What is not clear, however, is whether some sort of actuarial principle insures increasing returns to a "scale" defined by the number of a firm's research groups. In particular, let us consider the manufacturer studied by Mansfield and Brandenburg (1966): each year the firm is bombarded by potential research projects, some that are inherently unprofitable, and some that would be lucrative if the firm had the right configuration of research groups. Having the right group at the right time is, of course, a chance event -- so, by adding more departments, the firm might do little to increase the number of fortuitous "collisions" between research talents and projects, especially in view of the fact that new
departments might tend to overlap in terms of capabilities. Or, the addition of new departments might produce a kind of synergy that a relatively small firm could not match.

5.2 Traces of Allocations to Specific Projects

The reconfiguration rules discussed above in Chapter 4 are methods of accommodating a firm's structure to stochastic inflows of projects and potential departments. Such rules operate partly on the basis of returns attributed in the past to research departments; and thus the rules might comprise a "retrospective" class. In contrast, a heuristic rule might determine departmental budgets by comparing the different incremental expected returns that a small amount added to each department's budget would provide. The problem to be considered in this section regarding heuristic rules is as follows.

At some point in time, a firm has a given configuration, a given set of projects, and known -- or presumed -- capabilities of each research department to succeed within a year on each project. The firm gains a contractual award of $C. How, then, would the firm allocate the award among existing projects? In particular, a firm might now be attempting to develop 15 different kinds of software packages, the $i$th of which offers the firm an expected value of $V_i$. A new contract may stipulate that the firm should spend an additional $C$ on developing the tenth package. Once the firm has the contract,
however, the values \( \{V_i, i=1,\ldots,15\} \) may remain unchanged (e.g., the firm still thinks that if it could develop the tenth package, it could sell it for a million dollars). Thus the firm would actually distribute the award over several projects, and would "charge" work done on these to the contract for work on the tenth package. The question of interest is then how the pattern of charges would be determined, and how the pattern would vary with different sizes of contractual awards.\(^1\)

The SQDER program shown in Figure 5.1 embodies one plausible heuristic rule for reconfiguration. The program steps are as follows.

(a) The subroutine VGENERI generates a set of project values, according to parameter values for \( \text{VMEAN} \) and \( \text{VVAR} \).

(b) The KMTXER1 program sets a value for the matrix \( (K_{ij}) \), given values for \( \text{RMEAN}, \text{RVAR}, \) and \( \text{PROB} \).

(c) A matrix SQG is dimensioned as \( (20 \times 4) \), and the first column is set at the value \( (40,41,\ldots,59) \).

(d) An initial budget of 40 is divided randomly among \( M \) research departments.

---

\(^1\) One might argue that an agency should not attempt to support a firm's research on a problem, but should instead wait until the problem has been solved and then buy the resulting product. This section does not suggest that the practice of directly supporting research is in any sense optimal, but that in fact the practice exists on a substantial scale. See, for example, the articles by Klein (1965) and Cherington (1965) on military R&D.
\*V\*SQDER[\*]V

\*V SQ+SQDER

[1]  \*a COMPUTES AMOUNTS ALLOCATED TO A PROJECT.
[2]  VGENER1
[3]  KMTXER1
[4]  SQG+ 20 4 p0
[5]  SQG[;1]+40+(0,(19)×1)
[6]  WTS+(10 p1000)
[7]  WTS+WTS+(+/WTS)
[8]  TBVEC+WTS×40
[9]  J2f+1
[10]  \*SOLVE:SOPTER
[12]  BEST+1+\*\*LNPV
[14]  J2f+J2f+1
[15]  +(J2f<20.5)/\*SOLVE

\*V

\textbf{Figure 5.1:} a listing of the SQDER program
(e) The SOPTER algorithm solves the allocation problem defined by the steps (a) - (d) above.

(f) The optimal allocations for the first three projects are summed over departments, and the sums are recorded in the first row of the SQG matrix.

(g) The optimization algorithm associates a Lagrange multiplier -- which can be interpreted as $\frac{\partial z^*}{\partial B_j}$ -- with each department. The SQDER program increments the firm's budget by one unit (i.e., by $1/40$ of the initial budget), and allocates this increase to the department having the largest $\frac{\partial z^*}{\partial B_j}$ value. In other words, the "most promising" department gets the first small increase in the total budget.

(h) The revised allocation problem is again solved, and new $\frac{\partial z^*}{\partial B_j}$ values are computed. The next small budget increment goes to the department that now appears most promising.

(i) The new problem is again solved, the optimal allocations for the first three projects are again stored in the SQG matrix, the budget is again incremented, and so on for subsequent iterations.

A budgetary "track" computed by the SQDER program for one project appears in Figure 5.2. Initial conditions for the program were $N = 15$, $M = 10$, $V\text{MEAN} = 10$, $V\text{VAR} = 100,000$, $R\text{MEAN} = .5$, $R\text{VAR} = .1$, and $\text{PROB} = .5$. These conditions were chosen to yield substantial differences in project values, and in the research departments' relative capabilities.
Figure 5.2: allocations to a project, as a function of a firm's total research budget
to solve each research task. From Figure 5.1, one is inclined to believe that expansion path of allocations to a project is really linear, and that small apparent deviations from a linear form are caused by minor aberrations in one or more of the many programs used to derive and plot the graph.

The track for the second project, however, shows -- as in Figure 5.3 -- unmistakable steps: as the budget is increased over some intervals, the project gets no additional funding; but at certain critical points, allocations to the project jump markedly. Allocations compiled by the same execution of the SQDER program for a third project -- as well as allocations compiled on two subsequent runs -- varied in pattern between the step-function form and the almost-linear case.

For each of the nine projects examined in three separate executions, a linear regression equation was estimated. The slopes of the equations differed greatly among the projects. Interestingly, however, each slope was quite close to the initial ratio of the project's total allocation divided by the firm's total budget for research.

The observed correspondences suggest as a rule of thumb that one might estimate the fraction of a contractual award that would go to a particular project by calculating the initial fraction of the firm's budget that is allocated to the project. This is, of course, only a rough guideline:
Figure 5.3: allocations to a project, as a function of a firm's total research budget.
if a firm were given a contract to develop an APL file management system, but the firm had funded no prior work on such a system, then -- in contradiction to the rule -- the firm would do some work to pacify the grantor.

Nevertheless, the economic sense of the rule remains intact. If the firm had allocated none of its resources to the project, then either (i) the firm saw no value that might accrue from the research effort, or (ii) the firm thought it could gain more money by working on other projects, given its configuration of research talents. If there is no link between the contract and the relative value that the firm perceives for work specified in the contract, then whatever work the firm does to satisfy the grantor might well amount to nothing more than "progress reports", "phase proposals", etc.

If the contract's amount is as large as, say, twice the firm's initial budget for research, then clearly the firm might be induced to push its funding for a project away from the zero level. The most interesting aspect of the simulations described above, however, is the fact that even as the total budget was increased by half its initial amount, projects that were initially "small" to the firm remained small.

5.3 Advantages of Sub-Optimal Funding

As indicated above in Chapter 4, there is an optimal
scale determined by certain characteristics of the project flow into the firm. Apparently, however, there are good reasons why a firm would want to fund research at much less than an optimal scale. These reasons can be demonstrated by executing the FAILER program that is listed in Figure 5.4.

The program operates as follows.

(a) The SCALER program discussed above in Chapter 4 is invoked. (Recall that the SCALER program merely draws elements from a sequence of populations, where the sequence is ordered by total budgets for research. Given a research budget, a typical element in the population is a vector (TBVEC, KMTX, VVEC, R) where TBVEC = (B_1, ..., B_m) is a randomly distributed list of departmental budgets, KMTX = (K_{ij}; i=1, ..., N; j=1, ..., M) is a random matrix generated via the KMTXERL program, VVEC = (V_1, ..., V_N) is a random vector generated by the VGENERL program, and R is the optimal expected return associated with TBVEC, KMTX, and VVEC. Figuratively speaking, then, the firm postulates a total research budget for each of -- say -- the next five years, and then samples points in a population of conceivable annual results from the budget setting.)

(b) Given the returns matrix RETM, the FAILER program computes for each budget level (i.e., for each column in RETM) the probability that the firm's research effort would fail in the sense of producing a return less than the total budget.
\$V\text{FAILER}[]V\$

\$V FA+\text{FAILER};BV;BVM;RAVGM\$

[1] \text{COMPUTES PROBABILITIES OF LOSS}

[2] \text{CORRESPONDING TO DIFFERENT TOTAL BUDGETS.}

[3] \text{SCALER}

[4] \text{BV}+1+(0, (19) \times 10)

[5] \text{BVM} \leftarrow 10 \ 10 \ pBV

[6] \text{PFAIL} \leftarrow 10 \ 2 \ p0

[7] \text{PFAIL}[:,:,1] \leftarrow BV

[8] \text{PFAIL}[:,2] \leftarrow (+/[1] (R\text{ETM}<BVM)) \times 10

[9] \text{RAVGM} \leftarrow 10 \ 10 \ p(+/[1] \ R\text{ETM}) \times 10

[10] \text{SIGM} \leftarrow (+/[1] ((RAVGM - R\text{ETM})*2)) \times 0.5

\$V$

\text{Figure 5.4: a listing of the FAILER program}
(c) In addition, the program computes a standard deviation of returns for each budget level.

Since the probability distribution for project values is held constant for all budget levels, one might suspect that the variance of optimal expected returns would appear roughly constant (or would decrease) over budgets, and that the probability of failure would decrease as the firm approaches an optimal scale for funding. In particular, the column averages of a \((10 \times 10)\) returns matrix compiled by one execution of the SCALER subroutine within the FAILER program are plotted in Figure 5.5; and -- although noise has caused some deviations of the means away from the exponential form suggested above in Chapter 4 -- the optimal scale for funding appears to be somewhere in the interval \((45, 65)\), such that the risk of failure would apparently diminish as the firm increases its budget from zero to within the interval.

The two graphs in Figure 5.6 indicate, however, that the probability of failure increases as the firm approaches an optimal scale. Moreover, the variance of annual optimal expected returns also increases with additions to the research budget. Since the graph of total returns is virtually parallel to the total cost function for a reasonably wide range of budgets, the slight gain in total expected profits that the firm would achieve by moving from a total budget of 11 to 45 hardly seems worth the increased probability of failure.
Figure 5.5: returns associated with total budgets
Figure 5.6: problems of failure and standard deviations of returns associated with total budgets
A trace on executions of FAILER indicates at least one explanation for the increases in variances of annual returns. When the firm's total budget is relatively small, virtually any realization of the variables \((V_1, \ldots, V_n)\) and \((K_{ij})\) offers the firm an expectation of a net return, which is to say that the OPTER algorithm can usually succeed in searching among projects to find a profitable pattern of allocations. As the firm increases its research budget, it "tools up" to exploit projects that offer large returns for large inputs of research effort. The efficacy of a move toward more research capabilities depends, then, on the arrival of "large" projects: the best solution that the OPTER algorithm can find becomes increasingly dependent on the whims of the VGENER1 and KMTXER1 codes -- and thus the variance of returns increases, even though additions to the research budget do not alter characteristics of the project flow.

5.4 Tests for Synergy

The SYNER1 program listed in Figure 5.7 can be used to examine the hypothesis that the rate of total return (i.e., \(\Delta TR^*/\Delta INPUT\)) increases as a firm expands by increasing (i) its budget for research, (ii) the number of its research departments, and (iii) the number of projects it works on. The steps in the program are as follows.
\$SYNER1[[]]\$
\$SY1+SYNER1$

[1] a LOOKS FOR SYNERGY.
[2] N+9
[3] M+4
[4] CCT+1
[5] SMX+ 10 10 p0
[6] SYN1+ 10 3 p0
[7] SYN1[;1]+40+(0,(19)x10)
[8] SYN1[;2]+SYN1[;1]
[9] GOCO:N+N+1
[10] M+M+1
[11] RCT+1
[12] GORO:VGENER1
[13] XMTXER1
[14] WTS+?(N\(\geq\)1000)
[15] WTS+WTS+(+/WTS)
[16] TAVEC+SYN1[CCT;1]xWTS
[17] OPTER
[18] SMX[RCT;CCT]+ZOLD[J]
[19] RCT+RCT+1
[20] +(RCT<10.5)/GORO
[21] CCT+CCT+1
[22] +(CCT<10.5)/GOCO
[23] SYN1[;3]+(+/[1] SMX)*10

Figure 5.7: a listing of the SYNER1 program
(a) The maximum number of projects is set initially at 9, and the number of research departments at 3.

(b) At the start of each iteration, both the number of projects and the number of departments are incremented by unity. Each additional department costs an average of 10 monetary units, and each additional project has an expected value of 20 units, such that each addition of a project and a department appears -- on the average -- to be a net gain to the firm.

(c) Given the maximum number of projects, the VGENER1 subroutine generates a realization of project values from a lognormal distribution with a mean of 20 and a variance of 1000.

(d) The KMTXER1 subroutine generates a realization of the matrix of probability-of-success parameters \( K_{ij} \), given the parameter values \( \text{RMEAN} = .5, \text{RVAR} = .1, \) and \( \text{PROB} = .5 \). This setting of parameters, together with the value variance of 1000, commands the SYNER1 program to sample values and research capabilities from a reasonably large space: although the departments presumably overlap in their capabilities, the degree of duplication can be large or small, depending on the realization generated by KMTXER1.

(e) The firm's total budget for research (which is set initially at 40, and is incremented by 10 on each iteration) is divided randomly among the research departments.
This models the fact that in a future year the firm could find itself in a situation where its least capable department at the time could command virtually all of the firm's budget for research.

(f) The OPTER program solves the allocation problem defined in the steps above; and the optimal expected value of research is stored in a column of the SMX matrix.

(g) Each column of SMX corresponds to a fixed set of values for the total research budget, the number of departments, and the maximum number of projects. Since each column has 10 positions, the SYNER1 program continues looping back to the step at which new project values are generated, until 10 optimal expected values have been compiled.

(h) Once one column has been completed, another iteration is begun by increasing the research effort's size, as described above. These iterations continue over 10 budget levels, such that 100 optimal expected values have been generated when the SYNER1 program stops.

Average computed returns associated with different total budget levels are plotted in Figure 5.8. Apparently the duplications in departmental capabilities that were programmed in SYNER1 were almost exactly balanced by increments in the total budget and in the number of projects, since average total returns increased linearly with budget increases. There is no indication of synergy (in the sense
Figure 5.8: average optimal expected returns associated with budget levels
of increases in $\delta R^*/\delta\text{INPUT}$, nor is there an optimal scale of investment: as long as the firm continues buying new projects with a mean of 20 by spending a mean of 10 on new departments, the firm's average expected net profits increase.

A careful examination of the SMX matrix, however, reveals a reason why a firm might not expand its research efforts indefinitely, even if it could continually assimilate departments that are on the average quite profitable. Figure 5.9 shows the standard deviations in optimal expected returns corresponding to various total budgets. The trend shows that even though a firm continues to draw project values from the same distribution as it increases its research efforts, the variance of optimal expected returns increases substantially. Risk preferences, then, could constrain the firm's expansion.

Economists (e.g., Nelson (1965)) have often viewed research firms as insurance businesses. As an insurance firm grows by writing more policies, adding new agents, etc., the firm's costs -- and hence its net profits -- become more predictable. A cursory view of research projects implies the same thing: as a research firm underwrites more projects (i.e., policies), returns from research also become more predictable. And just as large insurance firms can assume risks that small firms cannot, large research firms can presumably undertake projects whose risks
Figure 5.9: standard deviations of returns associated with budget levels
would stupefy small businesses. A short step from this view is the conclusion that only the largest firm of all -- the U. S. Government -- can assume the risks inherent in many worthwhile research projects.

Thus it is interesting to see that returns from research can become less predictable as a firm increases its research budget. Although the example of decreasing predictability comes from a special model of a research firm, the paradigm suggests (i) that the actuarial principles pertaining to research management are not necessarily those of the insurance industry, and (ii) that perhaps some law of increasing uncertainty constrains the amounts firms spend on research. This is not to say that firms calculate carefully the risks of research, but that firms might have observed the effects of such a law, just as perhaps insurors who knew nothing about the law of large numbers once observed that larger insurance firms could predict their profits better than smaller ones.

Empirical studies of research have corroborated the hypothesis that "the low-cost, high-risk exploratory work [on a research project is] commonly initiated by a small company or independent inventor, with the lower-risk, higher-cost development work taken over by a large company."^{2/}

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^{2/} Nelson et al. (1970).
This is not what one would expect if larger firms were --
as in the insurance industry -- better qualified to underwrite
risky projects.
Chapter 6

SUMMARY AND DIRECTIONS FOR FUTURE RESEARCH

Section 4.2 and the appendix to Chapter 4 show how an elementary principle of economics can be programmed in order to solve a special class of nonlinear programming problems. Apparently, the OPTER algorithm is much more efficient and accurate than many older methods of convex programming (e.g., Hartley's convex simplex method, which is explained by Hadley (1964)); but the extent to which the OPTER code's efficiency exceeds that of, say, the SUMT algorithm\(^1\) is not clear. Thus it would be interesting to compare the codes by first programming the OPTER algorithm in FORTRAN, and then running sample problems involving hundreds of variables.

If the OPTER approach could be shown to be superior to existing algorithms, then the method could profitably be used for a wide variety of problems. In particular, the set of branch-and-bound algorithms is burgeoning at what must be an exponential rate\(^2\); and the efficiency of

\(^1\)See Fiacco and McCormick (1968), and especially their discussion of factorable programs.

\(^2\)For an indication of how rapidly this field has grown, see the references provided by Ochoa-Rosso (1968).
many of these codes depends crucially on the speed with which one can solve continuous sub-problems. Occasionally the sub-problems have a form amenable to the OPTER code.

For example, the Jones-Scland bid evaluation problem (1969) has the form of a dynamic programming problem, but cannot be solved by dynamic programming because of dimensionality difficulties. Thus the authors suggest a branch-and-bound algorithm whose sub-problems are linear programs. These, however, have a special form to which the RSLN subroutine (developed in Chapter 3) can be applied.3/ In this case, the speed of the branch-and-bound algorithm can be improved by using a subroutine that is "tailored" to the structure of sub-problems.

Section 4.3 presents simulated comparisons between optimal and heuristic methods of allocating resources. This sort of comparison obviously could -- and should -- be done for many proposed methods of capital budgeting4/, for two reasons. First, the process of implementing any system of capital budgeting can be extremely costly in terms of the time managers must spend in "formalizing" the problem, estimating parameters, etc. Second, the optimal

3/ The RSLN program is one version of the method suggested in the appendix to Chapter 4.

4/ In particular, each of the methods surveyed by Weingartner (1966) might be tested against heuristic alternatives.
budget might not be appreciably better than what a firm could achieve via a simple, inexpensive rule of thumb.

There are, of course, capital budgeting models for which no heuristic methods can be suggested (e.g., the models by Watters (1967), Brandenburg and Stedry (1966), and Brockoff (1969)). But when no heuristic alternatives to a proposed method are available, one suspects either that the formal model misrepresents reality, or that real problems are special cases of the proposed model: if a model actually applies to an existing problem, then a firm has somehow solved the problem in the past. Past methods of solution can be coded and thus compared to proposed approaches.

Section 4.4 shows how different adaptive rules for budgeting can be compared. The method of "embedding" an allocation model in a larger decision process and then simulating results from alternative control policies could be used for various budgeting models. For example, the Brandenburg - Stedry model (1966) assumes fixed "laboratory budgets" for future years. In the context of this model, a reconfiguration problem would be that of deciding how the budgets should be determined adaptively on the basis of past experience.

The scale problems examined in Sections 4.5, 5.3, and 5.4 could be elaborated in many ways. In particular, values of the research projects could be linked to other
activities in a firm (e.g., if a pharmaceutical firm develops a new analgesic, the product's value could depend on advertising expenses, allocations to production departments, and the like). Moreover, one could work with alternative "lottery" definitions of research projects, where the exponential form suggested in Chapter 2 is incorrect.

An example of an alternative lottery concept appears in the following problem, which has been proposed by the U.S. Environmental Protection Agency. Each solid arc in the network shown in Figure 6.1 corresponds to a "deterministic" activity such as the construction of a part for a machine. The technology for building the part is known; but, by increasing the amount of money allocated to the activity, the time required to build the machine can be reduced according to a known time-cost trade-off function. A prize $V_i$ can be won only if each deterministic activity has been completed by a specified future date. Moreover, as is usual in critical path analysis, a given deterministic activity can be started only after each of its predecessor activities has been completed.

Each dotted line denotes a research activity (e.g., an attempt to invent -- or improve -- a machine component). Presumably, each research activity -- if undertaken -- will require a known amount of money; and after a time interval of known length, it will be possible to say whether the
Figure 6.1: a network illustrating some scheduling problems involved in R&D budgeting
research activity has succeeded or failed. A known probability of success is associated with undertaking any given research activity. The prize $V_i$ can be won only if at least one research activity in each "cluster" of activities is successful; and each activity in a particular cluster can be started only after the cluster's predecessor activities have been completed.

From the above assumptions, one problem of allocation becomes that of choosing (i) how much money to allocate to each deterministic activity, and (ii) which research activities to fund, such that one maximizes the probability of winning $V_i$ within a given time, subject to a total budget constraint.

Many interesting variants of the above problem can be formulated. For example, each cluster of research activities might be a much more involved network than the sample parallel networks shown in Figure 6.1. Also, instead of one budget constraint we might impose a set of "annual" budget constraints. The problem could be further re-formulated as an attempt to find a optimal adaptive rule for funding the activities (e.g., "start at $t=0$ by funding research activities $X$, $Y$, $Z$; if after six months each of these attempts has failed, then fund activities $P$, $Q$, and $R$."). Several authors (e.g., Burkov (1967)) have developed algorithms for variants of the network-lottery budgeting problem; but realistically large versions of the
problem have never been solved, despite their obvious applicability to the development of devices for controlling pollution, constructing new military machines, and the like.

Aside from working on new algorithms for allocating funds to research projects, one could attempt to test empirically the hypothesis generated in Section 5.2, which asserts that some research firms redistribute contract awards among departments in order to maintain roughly fixed proportions of a total budget allocated to current projects. This is not to say that the Mansfield-Brandenburg model is universally appropriate, but that the fixed-proportions hypothesis is an interesting starting point for investigating the actual budgetary processes of contract research firms.
References


 Mathematical Methods of Network Planning (Nauka).