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SEARCH FOR HYPERFINE LINE
FROM SINGLY IONIZED HELIUM-3
IN GALACTIC H-II REGIONS

by

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I) **Introduction**

Since the advent of radio astronomy (Jansky 1932, 1933), information on galactic and extra-galactic objects has been accumulating rapidly. Continuum surveys at low frequencies provided the early maps of the radio sky with limited resolution. Now higher frequency antennas and low noise receivers have extended these surveys above 10 GHz. Interferometric techniques now provide resolution approaching that obtainable at optical wave-lengths, and very long baseline interferometry with remote recording of data has provided resolutions of 0.001 seconds of arc. Sources with sizes comparable to the radius of the earth's orbit about the sun have been resolved. The various fields of experimental astronomy (radio, infrared, optical, x-ray and others) compliment each other in giving detailed evidence for the physical processes occuring in the solar system, galaxy and universe. Recent discoveries in each field have given considerable impetus to the other fields, and astronomy and astrophysics in general.

In 1951 the observation of the H-I (neutral hydrogen) hyperfine transition at 21 cm (Ewen and Purcell) added an extraordinary tool to radio astronomy. Subsequent observations of the 21 cm line have mapped the structure, rotation, and turbulence of our galaxy and other nearby galaxies such as M 31. Once the hydrogen 21-cm line was observed, the search for other lines was started. The
The deuterium hyperfine transition was not observed (Weinreb 1962), but hydrogen and He$^4$ recombination lines have been studied extensively in galactic H-II (singly ionized hydrogen) regions as originally suggested by Kardashev (1959). These extended regions of hydrogen gas in the vicinity of hot ultraviolet stars are predominately ionized because of the intense stellar radiation.

The physical parameters in these very diffuse plasma have been extensively analyzed by the techniques of radio astronomy. Continuum and recombination line measurements can give the electron temperature, electron densities, turbulence velocities and emission measures (Mezger and Hoglund 1967). Since there is negligible obscuration by intervening matter at radio wavelengths, the surveys of H-II regions can reveal many regions that are not known from optical work. In this way the distribution of H-II regions can be related to the spiral features of our galaxy (Mezger 1970). A H-II region outside the galaxy, the 30 Doradus nebula, has been observed at 6 cm in the Large Magellanic Cloud (Mezger et al. 1970). In addition to hydrogen recombination lines, lines have been observed from helium and carbon. The analysis of the hydrogen and helium recombination lines has yielded the helium abundance in a number of H-II regions (Palmer et al. 1969). Thus recombination line studies give a continuing means for the study of our galaxy.

Augmenting the continuing recombination line studies
have been spectacular discoveries of spectral lines from interstellar molecules. The search for the OH radical continued for six years until accurate line frequencies and sensitive equipment finally enabled it to be detected in absorption against Cas A (Weinreb et al. 1963)*. Most of the early measurements were in absorption against galactic radio sources. But in 1965 strong, narrow-line, polarized emission OH sources were found. These emission sources have intensities that vary by three orders of magnitude (Menon 1967). It was found through very long baseline interferometry that some of the more intense sources are on the order of 1000 A. U. in diameter, or of solar system dimensions. Thus, these sources may be newly formed stars or protostars (Mezger and Robinson 1968). Barret and Wilson (1968) have reported OH emission from infrared stars, notably NML Cygnus (Neugebauer et al. 1965). These strong OH emission sources have been grouped tentatively into 3 classes: those associated with thermal radio sources (Mezger et al. 1967), with supernova remnants, and with infrared sources. Using this classification scheme Turner (1969, 1970) searched for and found greater than 50 new OH sources. The nature of the anomalous emission has been explained by maser action with an infrared pumping mechanism (Turner 1967 and Townes 1970). The maser action explains the narrow, intense lines (some with

*See Ball (1969) for a discussion of the history of OH observations.
brightness temperatures greater than $10^6$ °K) and the observed strong polarization.

Because of the abundance of OH emission sources, Snyder and Buhl (1969) examined the feasibility of searching for H$_2$O. Subsequently interstellar H$_2$O was discovered by Cheug et al. (1969). Most of the H$_2$O sources were associated with known OH emission sources (Turner et al. 1970). Numerous other molecular lines have been found from basic organic compounds, having up to 5-6 atoms.

These atomic and molecular line observations have given information on the abundance of He$^4$, on the abundance ratio of C$^{12}$ and C$^{13}$, on physical processes in the interstellar medium, etc. However, little is known on the interstellar abundance of He$^3$. The upper limit for the He$^3$ and He$^4$ ratio has been set at $10^{-2}$ for the Orion Nebula with optical measurements (Vaughn 1968), but this limit is above that predicted by most theories. Hence, a more sensitive technique was needed to search for He$^3$.

This thesis details the search for the ground state hyperfine line from singly ionized helium-3. The line frequency has been measured in the laboratory (Fortson et al. 1966) and calculated theoretically (Goldwire 1967) as $\nu$=8665.650 MHz (3.46 cm). This line is the ionized He$^3$ counterpart to the neutral hydrogen 21 cm line.

The magnetic dipole transition from $F=1 \rightarrow F=0$ of ground state (He$^3$)$^+$ has an Einstein coefficient of $1.95 \times 10^{-12}$ sec$^{-1}$, which corresponds to a radiative
lifetime of 16,300 years. Goldwire (1967) has shown that electron scattering in H-II regions will cause the hyperfine levels to be populated with a spin temperature equal to the nebular electron temperature. Since $T_e \gg \frac{h\nu_o}{k}$, the levels are populated according to their statistical weights. Primary sources of interstellar He$^3$ are primeval production in a "big bang" universe or subsequent production in stellar and galactic evolution. Present observed upper limits will be related to the cosmological parameters for a "big bang" universe. Then the production mechanism in stellar evolution is examined to relate the He$^3$ to H ratio to the expected He$^3$ production. A generalized model for galactic evolution including the pertinent stellar birth rates and time dependences has been developed by R. J. Talbot and W. D. Arnett of Rice University. This scheme has been applied to the problem of He$^3$ production and is used to compare the observed upper limits to the parameters in the galactic evolution model.

Observations were made in August 1968, and November 1968, and in November 1970 on the 140 foot fully steerable telescope at the National Radio Astronomy Observatory in Greenbank, West Virginia. The 3.5 cm receiver was designed and built at Rice University under the direction of Mr. J. L. Halpain (1970). It uses a gaseous helium refrigerator at 18 °K to cool a two-stage parametric amplifier and associated components. The overall noise
temperature of the entire receiving system including the 140 foot telescope was 90 °K.

During the August 1968 observations 13 sources were observed. Virgo A was the primary calibration source, and eleven H-II regions were studied. The galactic center was observed briefly for the possibility of absorption lines but none were found. The November 1968 run did not give any useful He\(^3\) data because of receiver failure, but some hydrogen and helium recombination lines were observed (Kalshoven 1971). However, the November 1970 observing period was quite productive. Only the four primary sources (W3A, Orion A, M17, W51B) were observed in 1970 in order to obtain the best signal to noise in the allotted observing time. Unfortunately no He\(^3\) signal was detected, but stringent new upper limits for the He\(^3\) cosmic abundance can be set for the H-II regions that were observed. The quality of the observational data and resulting very low limits is emphasized by the detection, with good signal to noise ratio, of a hydrogen recombination line, \(H_{171,7}^*\). This line, with \(\Delta N=7\), has an antenna temperature of only 0.05 °K in M17. This is as weak as any line that has been reported in the radio astronomy literature.

For each observing period preliminary processing of the data taken on the site in Green Bank was done

\(H_{171,7}^*\) corresponds to the transition in hydrogen from \(n=178, n=171\), in analogy to the notation for low order lines \(H_9\alpha\), \(H_{11}\beta\), etc.
at the N. R. A. O. computer in Charlottesville, Virginia, but the major part of the data reduction was done at Rice University. Various data reduction techniques were developed and utilized. These techniques have given a detailed weighting to the data so that the maximum amount of information was obtained from the observations.

In the following chapter, the physical properties of the \((\text{He}^3)^+\) ion that are pertinent to this investigation are reviewed in detail, as are the factors governing the signal strength expected at the telescope receiver. In Chapter III, cosmological theories and models of stellar and galactic evolution are discussed to provide a theoretical framework in which the observational results can be interpreted. The details of the observational equipment, observational procedures, and data reduction techniques are fully described in Chapter IV.

In chapter V, the final data is presented and the limiting \(\text{He}^3\) abundance is derived from this final data. The results are discussed in Chapter VI, where they are compared to theory. The limiting \(\text{He}^3\) to \(H\) ratio of \(4 \times 10^{-5}\) is discussed with respect to observed ratios in other regions, and possible improvements in the observable limits are contemplated.
II) **Helium-3 Hyperfine Line**

In order to estimate what abundances of He$^3$ are detectable with present day techniques, the hyperfine line from singly ionized He$^3$ is discussed in regard to its expected intensity. In particular the signal strength from a "typical" H-II region is estimated to show the possible sensitivity of this search.

2.1 **Description of Hyperfine Line**

Singly ionized helium-3 is a simple hydrogenic system with $Z=2$ and a ground state binding energy of -54.4 eV. The ground state has no fine structure since it is a $^2S_{1/2}$ state, but it does exhibit hyperfine splitting due to the interaction of the electron and nuclear magnetic moments. For the ground state electron $J = L + S = 1/2$ and for the helium-3 nucleus $I = 1/2$. The resultant $J$ and $I$ interact through magnetic dipole fields to give a resultant spin $F = J + I$. In particular, $F = 0$ or 1, giving rise to singlet and triplet hyperfine states. The singlet state has the higher relative energy because of the negative value of the nuclear magnetic moment.

The hyperfine energy splitting has been calculated by Fermi (1930) for the nS state of hydrogen atoms and is
\[ \Delta E = \frac{8\pi}{3} \mu_B \cdot |\psi_n(0)|^2 \times \begin{cases} 
1, & F = I + 1/2 \\
-I+1, & F = I - 1/2 
\end{cases} \]

Where \( \mu_B \) = Bohr magneton, \( \mu \) is the magnetic moment of the nucleus (\( \mu = -2.1276 \) nuclear magnetons), and \( \psi_n(0) \) is the value for the non-relativistic nS state wave function at the nucleus. The zero magnetic field energy splitting between the two 1S states of \((\text{He}^3)^+\) corresponds to a frequency of \( \nu_0 = 8657 \) MHz. The splitting is shown in Figure 2.1.

This calculation only takes into account the finite mass of the nucleus. Radiative corrections to the static moment of the free electron, quantum electrodynamical effects, corrections due to nuclear motion and structure, and other effects have been studied by Goldwire (1967 a). He determined \( \nu_0(1S) = 86656 50900 \pm 1900 \) Hz. This compared very well with the experimentally measured value determined by Fortson et al. (1966). This experimental value was \( \nu_0(1S) = 86656 49905 \pm 50 \) Hz. Thus the frequency of the transition is very well known.

The hyperfine transition occurs by a magnetic dipole transition which has an Einstein spontaneous emission coefficient given by

\[ A_m (i,f) = \frac{1}{(2F_i + 1)} \frac{64\pi \nu^3}{3hc^3} S_m (i,f) \]
FIGURE 2.1

(He$^3$)$^+$ Hyperfine Splitting
(Condon and Shortley 1964), where the line strength is given by

\[ S_m(i,f) = \sum_{m_F(i)} \sum_{m_F(f)} |\langle i, m_F(i) | M | f, m_F(f) \rangle|^2, \]

\[ M = -\frac{e}{2mc} (\mathbf{L} + 2\mathbf{S} - \gamma \mathbf{L}) \]

For helium-3 this has been evaluated by Goldwire (1967 b) to give \( \gamma = -0.0023 \) and \( S_m(0,1) = 3 \left( 1 + \frac{\gamma}{2} \right)^2 \left( \frac{\hbar e}{2mc} \right)^2. \)

This yields a value for \( A_m(0 \rightarrow 1) = 1.950 \times 10^{-12} \text{ sec}^{-1}, \)
and a radiative lifetime

\[ \tau = \frac{1}{A_m} = \frac{0.513 \times 10^{11} \text{ sec}}{3.16 \times 10^7 \text{ sec/yr}} = 16,300 \text{ years}. \]

This is substantially less than the \( \tau = 11 \text{ million years} \) for the H-I hyperfine transition. The difference is due to the \( v^3 \) dependence of \( A_m(i, f) \) and the statistical weighting factors.
2.2 **Excitation of Helium in H-II Regions**

In order to establish the feasibility of a search for the \((\text{He}^3)^+\) hyperfine transition in galactic H-II regions we must first establish that excitation conditions are favorable for finding helium in the singly ionized state. Goldwire has shown that this is indeed the case in most diffuse galactic H-II regions and planetary nebulae.

Starting with Strömgren (1939), studies have shown, for a uniform medium, that there is a spherical shell of ionized hydrogen around the hot ultraviolet (O or B) stars, with a sharp drop in the fraction of hydrogen that is ionized beyond a certain critical distance from the star. This critical distance defines the radius of the Strömgren sphere. In regions also containing He, there are corresponding spheres for \(\text{He}^+\) and \(\text{He}^{++}\). In regions very close to a hot star helium is doubly ionized, but strong absorption of ionizing radiation in that region precludes maintenance of \(\text{He}^{++}\) at greater distances. Consequently, there is a large spherical shell in which any helium is predominately singly ionized.

Recently Rubin (1969) and Palmer et al. (1969) have shown that there is negligible \(\text{He}^{++}\) present in our primary sources (\(\text{He}^{++}/\text{H}^+ < 5 \times 10^{-5}\)), and that the radii of the helium and hydrogen Strömgren spheres are equal. These results do not depend on a particular model
for the H-II region but only on the type of exciting star. Stars hotter than 07 have equal radii for the H+ and He+ Stromgren spheres.

In H-II regions various processes contribute to the relative populations of the singlet and triplet hyperfine states of ground state helium-3. Goldwire (1967 b) has shown the dominating mechanism to be spin exchange collisions with electrons. Other processes are absorption and re-emission of photons, recombination, and resonant scattering of He-I Lyman alpha. Goldwire (1967 b) has shown that the effect of electron spin exchange collisions was to make the spin temperature, $T_s$, defined by

$$\frac{N_0}{N_1} = \frac{g_0}{g_1} \exp \left( -\frac{h\nu_0}{kT_s} \right)$$

equal to the kinetic or electron temperature. Since typical electron temperatures in H-II regions range from 4,000 to 10,000 degrees Kelvin and $h\nu_0/k = 0.413$ °K, the singlet and triplet hyperfine states are populated according to their statistical weights of 1 and 3 respectively.

In order to determine the amount of radiation that this hyperfine transition will produce, it is necessary to find the brightness temperature due to an emission line. When the equation of transfer,

$$\frac{dT_B}{ds} + T_B \cdot K(s) \cdot \rho(s) = T_e(s) \cdot K(s) \cdot \rho(s),$$

is solved, assuming local thermodynamic equilibrium and a uniform temperature, $T_e$, the observed brightness temperature is
\[ T_B \text{ (observed)} = T_B(0)e^{-\tau} + T_e(1-e^{-\tau}). \]

The optical depth, \( \tau \), is a function of frequency and is defined as

\[ \tau = \int_0^L K(s)\rho(s)ds, \]

where \( K(s) \) is the volume absorption coefficient at a point \( s \), \( \rho(s) \) is the mass density, and \( L \) is the length of the emitting region. At the center frequency of the line, the brightness temperature due to the line and continuum emissions is

\[ T_{L+C} = T_e(1-e^{-(\tau C + \tau L)}), \]

assuming the brightness temperature from behind the cloud, \( T_B(0) \), is zero. For a nearby frequency where no line is present the brightness temperature is only due to continuum emission

\[ T_C = T_e(1-e^{-\tau C}). \]

Since the line optical depth is much less than one, the line brightness temperature is

\[ T_L \equiv T_{L+C} - T_C = T_e \tau_L e^{-\tau_C}. \]

This equation applies to hydrogen and helium recombination lines and most other emission lines as well as to the \( \text{(He}^3) ^+ \) hyperfine line. The continuum optical depth is due to free-free transitions or thermal bremsstrahlung in the nebula. At 8666 MHz the continuum is optically thin, \( \tau_C \ll 1 \), so that

\[ T_L = T_e \tau_L. \]
The optical depth for the \((\text{He}^3)^+\) hyperfine transition has been derived by Goldwire (1967 b)

\[
\tau_L(\nu_0) = \frac{\hbar c^2 A_m N_0}{8\pi^3/2 k T_s \nu_0 \Delta \nu}.
\]

Where, 
- \(A_m\) = Radiative Einstein coefficient,
- \(N_0\) = Number of \((\text{He}^3)^+\) atoms in singlet ground state per \(\text{cm}^2\) along line of sight, and
- \(\Delta \nu = 1/e\) half-width of line determined by Doppler broadening.

Numerically, \(\tau_L(\nu_0) = 5.4 \times 10^{-14} \frac{N_0}{T_s \Delta \nu}\).

Hence, \(T_L = 5.4 \times 10^{-14} \frac{N_0}{\Delta \nu}\).

The number of \((\text{He}^3)^+\) atoms along the line of sight is what is to be determined by this experiment. The expected Doppler broadening of the \((\text{He}^3)^+\) hyperfine line can be calculated by parameters derived by recombination line observations. Two major factors cause a broadening in the line shape. These are the random thermal motions of the high temperature electrons and turbulent motions within the gas cloud. Each of these motions gives rise to an equivalent velocity which causes Doppler broadening. The most probable velocity of particles with mass \(M\) and temperature \(T\) is \(v_1 = \frac{2 k T}{M}\). The most probable velocity due to turbulence is related to the root-mean-square turbulence velocity by

\[
v_2 = \frac{2}{\sqrt{3}} v_{\text{rms}}.
\]
These quantities have been studied in H-II regions by analyzing the shapes of hydrogen and helium recombination lines. Each of the broadening mechanisms leads to a Gaussian line shape. The effect of two Gaussian shaped functions is given by the convolution of the two Gaussian functions. The result is the function:

\[
f(\nu) = (\pi\Delta\nu^2)^{-1/2} \times \exp \left[ -\left( \frac{\nu - \nu_0}{\Delta\nu} \right)^2 \right]
\]

where \( \Delta\nu = \nu_0 \left( \nu_1^2 + \nu_2^2 \right)^{1/2} / c \).

Since an observable brightness temperature is 0.02 °K, and \( \Delta\nu \approx 500 \) KHz, then

\[
N(\text{He}^3) = \Delta\nu \cdot T_L \approx 2 \times 10^{17} \text{ cm}^{-2}.
\]

\[
\approx 5.4 \times 10^{-14}
\]

In the primary H-II regions that were observed, the number of hydrogen atoms along the line of sight was typically

\[
N(H) = 4 \times 10^{21} \text{ cm}^{-2}.
\]

Hence, the detectable He\(^3\) to H abundance ratio is about

\[
\frac{N(\text{He}^3)}{N(H)} \approx 5 \times 10^{-5}.
\]

The actual ratios are discussed in Chapters V and VI.
III) **Helium-3 Production**

In Chapter V the resulting $\text{He}^3$ abundance limits are found from the observation data.

We wish to discuss the meanings of possible observed limits: first in terms of the abundance of original $\text{He}^3$ in the primordial gas cloud from which the galaxy condensed, and, second, in terms of the processes which may have enhanced or diminished any initial $\text{He}^3$.

Among the several models of the universe, the most widely studied and accepted is some form of the primordial fireball. This model consists of a mass singularity at time equal zero with initial temperature and density of $10^{12}$ °K and $10^6$ gm/cm$^3$. This singularity contains all the matter and radiation in the universe with nuclei broken down into protons, neutrons, electrons, etc. As this primordial fireball rapidly expands, the matter cools. During the cooling, reactions occur between the particles to create light nuclei such as hydrogen, deuterium, tritium, helium-3, helium-4, lithium-7, and beryllium-7. These reactions have been completed after approximately $10^3$ seconds, when the matter has cooled to $400 \times 10^6$ °K. Wagoner (1967) has studied the net amounts of each nuclear species produced for various models of a general-relativistic universe. For an isotropic universe the net mass fraction of $\text{He}^3$ is $2 \times 10^{-4}$ to $3 \times 10^{-6}$, depending on the average density of the universe today.
(\rho > 10^{-31} \text{ gm/cm}^3). The above models considered an equal number of electron neutrinos and antineutrinos. If an excess of neutrinos or antineutrinos existed in the primordial fireball this would change the relative abundance of protons and neutrons and consequently the final elemental abundances. For no excess of either neutrino or antineutrino, the He\textsuperscript{3} mass fraction was $10^{-5}$, with a sharp increase to $10^{-2}$ if there was a large excess of electron antineutrinos. Finally, the He\textsuperscript{3} mass fraction for various expansion rates of the fireball was considered. This gave a maximum He\textsuperscript{3} mass fraction of $10^{-3}$. All of these results are based on fairly simple models, but the results do indicate what possible levels of He\textsuperscript{3} could have been formed in a primordial fireball.

Whatever the initial history of the universe, eventually great clouds of matter (masses = $10^6 - 10^{13} \text{ M}_\odot$) condense to give the various forms of galaxies. Within each of these galaxies star formation and evolution goes on continuously. Since our galaxy is about 10 billion years old and the solar system is less than 5 billion years old, the matter from which the sun and the planets was formed had gone through 5 billion years of stellar and galactic evolution. Hence it is necessary to consider how this processing has changed any initial He\textsuperscript{3} abundances.

When a star ignites and starts burning hydrogen in its core, a number of nuclear reactions occur which create and destroy He\textsuperscript{3}. These various reactions determine the
abundance of He$^3$ that is formed in a star. This abundance is solely determined by the nuclear reaction rates in main sequence stars. Iben (1965 a, 1965 b, 1966 a, 1966 b, 1966 c, 1967) has numerically calculated the evolution of population II* stars from their formation until they evolve off of the main sequence and become red-giant stars, just before their death. For the heavier stars ($M>3M_\odot$), the end may come about in the form of a nova or supernova, or for the lighter stars ($M<3M_\odot$), by shedding the stellar envelope to form a planetary nebula. The dynamics and energetics of these processes are not well understood, but the end result appears to be the formation of a white dwarf or neutron star from the original stellar core and the dispersion of the stellar envelope and intermediate layers into the interstellar medium. Super-novae eject this material with velocities of 1000 km/sec. Presumably this material will mix completely with the interstellar medium to be incorporated into succeeding generations of stars.

The nuclear reactions which create and destroy helium-3 in main sequence stars are those of the proton-proton cycle

$$H^1 + H^1 \rightarrow D^2 + B^+ + \nu$$
$$H^1 + D^2 \rightarrow He^3 + \gamma$$
$$He^3 + He^3 \rightarrow 2H + He^4$$

*Population II stars are the younger stars in a galaxy which generally have higher metal and helium abundance than the older population I stars.
\[ \text{He}^3 + \text{He}^4 \rightarrow \text{Be}^7 + \gamma. \]

If \( H, D^2, \text{He}^3, \) and \( \text{He}^4 \) represent the number densities of hydrogen, deuterium, helium-3, and helium-4 in the stellar interior, the helium-3 density is given by:

\[ \frac{d\text{He}^3}{dt} = \lambda_{12} \text{HD} - \lambda_{33} (\text{He}^3)^2 - \lambda_{34} \text{He}^3 \text{He}^4. \]

Because of the low coulomb barrier for proton-proton and proton-deuterium reactions, the deuterium is in equilibrium, so,

\[ \frac{dD}{dt} = \lambda_{11} \frac{H^2}{2} - \lambda_{12} \text{HD} = 0. \]

then

\[ \lambda_{11} \frac{H^2}{2} = \lambda_{12} \text{HD}. \]

Hence, the change in the \( \text{He}^3 \) abundance depends only on the abundances of hydrogen, helium-3 and helium-4. The reaction rates, \( \lambda_{12}, \lambda_{33}, \lambda_{34} \) are the cross-section, velocity products averaged over a Maxwellian velocity distribution. The reaction rate has the form

\[ \lambda_{ij}(\text{cm}^3/\text{sec}) = 1.297 \times 10^{-15} \left[ \frac{Z_i Z_j}{A} \right]^{1/3} T_6^{-2/3} \times e^{-\frac{\tau S(E_0)}{f}}, \]

where, \( Z_i, Z_j \) are the nuclear charges of the two interacting species,

\[ A = \frac{A_i A_j}{A_i + A_j} \]

\( T_6 \) is the temperature in units of \( 10^6 \) °K,

\[ \tau = 42.60 \left( Z_i^2 Z_j^2 A \right)^{1/3} T_6^{-1/3}, \]

is the penetration factor for the repulsive Coulomb barrier,

\( S(E_0) \) (kev-barns) is the non-resonant cross section and,

\( f \) is an enhancement factor due to electron screening which lowers the effective coulomb barrier of the reaction.
When the equations are recast in terms of the mass fractions $X_i, X_j$, they take the form

$$\frac{dX_i}{dt} = \varepsilon_{11}X_1^2 - \varepsilon_{33}X_3^2 - \varepsilon_{34}X_3X_4,$$

where $X_1, X_3, X_4$ are the hydrogen, helium-3 and helium-4 mass fractions,

$$\varepsilon_{ij} = \rho N_\Lambda \lambda_{ij} = \frac{\rho}{T_6^{2/3}} \exp \left[ A_{ij} - B_{ij}/T_6 - C_{ij} \sqrt{\rho/T_6^3} \right].$$

The appropriate reaction rate parameters, $A_{ij}, B_{ij}, C_{ij}$, are given in Table 3.1.

Because of the strong temperature dependence of the reaction rates the He$^3$ rapidly comes into equilibrium in the stellar core where the temperature is hottest. In equilibrium $\frac{dX_3}{dt} = 0$, so

$$\varepsilon_{11}X_1^2 = \varepsilon_{33}X_3^2 + \varepsilon_{34}X_3X_4.$$

Hence, the equilibrium He$^3$ fraction, $X_{3e}$, is

$$X_{3e} = \left[ \sqrt{\frac{2}{\varepsilon_{34}X_4^2 + 4\varepsilon_{33}\varepsilon_{11}X_1^2}} - \frac{\varepsilon_{34}X_4^2}{2\varepsilon_{33}X_1} \right]^{1/2}$$

$$= \left( \frac{\varepsilon_{11}X_1^2}{\varepsilon_{33}} \right)^{1/2} \times \left[ 1 + \frac{\varepsilon_{34}X_4^2}{4\varepsilon_{11}\varepsilon_{33}X_1^2} \right]^{1/2} - \frac{\varepsilon_{34}X_4}{2\sqrt{\varepsilon_{11}\varepsilon_{33}X_1}}.$$

The first factor is the He$^3$ equilibrium fraction when the He$^3$ (He$^4, \gamma$)He$^7$ reaction is not important because of a low He$^4$ abundance. The helium-3 equilibrium is plotted in

*The mass fraction $X_i$ of a species $i$ is related to the number density, $N_i$ and mass density, $\rho$, by $X_i = N_iA_i/N_\Lambda \rho$, where $A_i$ is the atomic number of species $i$ and $N_\Lambda (gm^{-1})$ is Avogadro's number.
### Table 3.1

**Rate Constants for He$^3$ Reactions**

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$i,j$</th>
<th>$A_{ij}$</th>
<th>$B_{ij}$</th>
<th>$C_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H (p, e^+ v) D^2$</td>
<td>1,1</td>
<td>-28.342</td>
<td>33.810</td>
<td>0.262</td>
</tr>
<tr>
<td>$\text{He}^3 (\text{He}^3, 2p) \text{He}^4$</td>
<td>3,3</td>
<td>28.218</td>
<td>122.773</td>
<td>1.050</td>
</tr>
<tr>
<td>$\text{He}^4 (\text{He}^3, \gamma) \text{Be}^7$</td>
<td>3,4</td>
<td>18.619</td>
<td>128.275</td>
<td>1.050</td>
</tr>
</tbody>
</table>
Figure 3.1 He

Equilibrium Value
Figure 3.1 as a function of temperature for various mixtures of H and He\(^4\).

Although helium-3 is in equilibrium at the center of the star the equilibrium abundance that it obtains is small as seen from Figure 3.1. For main sequence stars the central temperature increases with increasing stellar mass. This point is illustrated in Table 3.2, where the central temperatures are given for the start, \( T_{Ci} \), and end, \( T_{Ce} \), of main-sequence hydrogen burning. Thus the greatest amount of He\(^3\) is produced in low mass stars. Iben (1967) has found that \( q_3 \equiv \frac{M(\text{He}^3)}{M_*} = 4.7 \times 10^{-4} \left[ \frac{M_*}{M_\odot} \right]^{-2.1} \).

Since his cross-section factor, \( S(E_0) \), for the destruction reaction, \( \text{He}^3 + \text{He}^3 \rightarrow 2\text{H} + \text{He}^4 \), is a factor of 5 too low, the helium-3 production which he found, has been reduced by a factor of \( \sqrt{5} \).

When the abundance of He\(^3\) is determined as a function of the position within a star it appears as in Figure 3.2. The He\(^3\) mass fraction is plotted as a function of \( q \), where \( q \) is the fractional mass of the star from 0 to 1.0. This figure is for a 1.5 \( M_\odot \) star, but the same features are true for all main-sequence stars which produce significant helium-3. The He\(^3\) mass fraction, \( X_3 \), peaks at \( q \approx 0.6 \) with a half-width of \( \Delta q \approx 0.2 \). All of the heights and widths are presented in Table 3.2 for various stellar masses.

Since the differential equation for the He\(^3\) mass fraction has the form \( \frac{dX}{dT} = a + bX + cX^2 \), it can be simply
<table>
<thead>
<tr>
<th>$\frac{M}{M_\odot}$</th>
<th>$T_{ci}$</th>
<th>$T_{ce}$</th>
<th>$t_{ms}$</th>
<th>$\Delta q$</th>
<th>$X_3$(peak)</th>
<th>$q_{un}$</th>
<th>$q_{CE}$</th>
<th>$X_3$(CE)</th>
<th>$q_3$(total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>14</td>
<td>28</td>
<td>10195</td>
<td>0.18</td>
<td>2400$\times 10^{-6}$</td>
<td>0.334</td>
<td>0.708</td>
<td>660$\times 10^{-6}$</td>
<td>470$\times 10^{-6}$</td>
</tr>
<tr>
<td>1.25</td>
<td>17</td>
<td>24</td>
<td>4060</td>
<td>0.21</td>
<td>1300</td>
<td>0.324</td>
<td>0.768</td>
<td>400</td>
<td>310</td>
</tr>
<tr>
<td>1.50</td>
<td>19</td>
<td>25</td>
<td>2000</td>
<td>0.20</td>
<td>880</td>
<td>0.314</td>
<td>0.734</td>
<td>270</td>
<td>200</td>
</tr>
<tr>
<td>2.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.7</td>
<td>130</td>
<td>89</td>
</tr>
<tr>
<td>3.00</td>
<td>24</td>
<td>30</td>
<td>225</td>
<td>0.27</td>
<td>67</td>
<td>0.332</td>
<td>0.651</td>
<td>75</td>
<td>49</td>
</tr>
<tr>
<td>5.00</td>
<td>27</td>
<td>34</td>
<td>65</td>
<td>0.20</td>
<td>58</td>
<td>0.335</td>
<td>0.72</td>
<td>25</td>
<td>18</td>
</tr>
<tr>
<td>9.00</td>
<td>31</td>
<td>38</td>
<td>21</td>
<td>0.26</td>
<td>19</td>
<td>0.364</td>
<td>0.765</td>
<td>7.6</td>
<td>5.8</td>
</tr>
<tr>
<td>15.00</td>
<td>34</td>
<td>50</td>
<td>10</td>
<td>0.27</td>
<td>7.6</td>
<td>0.346</td>
<td>0.683</td>
<td>2.2</td>
<td>1.5</td>
</tr>
<tr>
<td>$\infty$</td>
<td>34</td>
<td>50</td>
<td>10</td>
<td></td>
<td></td>
<td>0.35</td>
<td>0.7</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE 3.2**

He$^3$ Abundance Versus Stellar Mass
FIGURE 3.2 He$^3$ Abundance in 1.5 $M_\odot$ Star
(after Iben 1967).
integrated to give
\[ X_3(t) = \frac{d}{2 |c|} \tanh \left[ \frac{dt}{2} - \tanh^{-1} \left( \frac{2cX_3 + b}{d} \right) \right]^{b/2c} \]

where, \( d^2 = b^2 - 4ac = \varepsilon_{34}^2 X_4^2 + 4\varepsilon_{11} X_1^2 \varepsilon_{33}, \)

and \( X_{3i} \) is the initial abundance of helium-3. The quantity \( \tau_e = 2/d \), is an effective time constant which determines how quickly helium-3 comes into equilibrium. This time constant divided by the density is plotted in Figure 3.3 for various mixtures of hydrogen and helium-4 as a function of temperature.

For high temperatures the time constant \( \tau_e \) is very short compared with a stellar lifetime. So at the core of the star \( \text{He}^3 \) is in equilibrium but with a very low abundance. Towards the intermediate layers of the star the temperature has dropped, but the \( \text{He}^3 \) is still in equilibrium at a higher abundance level. In the outer regions of the star the temperature is so low that \( \text{He}^3 \) does not have time to reach its high equilibrium value. Thus, the \( \text{He}^3 \) mass fraction is low in the inner and outer zones of the star. This qualitatively explains the \( \text{He}^3 \) abundance peak near \( q = 0.6 \).

During the main-sequence hydrogen burning, a stars structure is remarkably static with gradual composition changes in the stellar core where the energy production is occurring. Its structure is similar to that shown in Figure 3.4. There is a general increase in the central temperature and pressure which accelerates the hydrogen
FIGURE 3.3

$\tau_{3e}$ (years)

$X_1 = 1 \quad X_1 = X_4 = 0.5$

He$^3$ Time Constant
FIGURE 3.4  Stellar Structure During Core Hydrogen Burning. 

$T, \rho$ of 1.5 $M_\odot$ Star on Main Sequence at 

$t = 0.94 \times 10^9$ years  (After Iben 1967).
burning in the core. As hydrogen is exhausted in the core, shell hydrogen burning starts. The energy required to maintain the temperatures and densities necessary for shell hydrogen burning is supplied by gravitational energy from an overall contraction of the star. For population II stars, when central temperatures and densities become high enough the energy production mechanism in the shell changes from the proton-proton cycle to the carbon-nitrogen cycle (See Clayton 1968). In either energy production cycle hydrogen is converted to helium-4.

At this stage the star has a high-density core, predominately He\(^4\), with a shell of burning hydrogen around it. The density distribution has become very peaked towards the center. The outer portion of the star (60-70%) has become very tenuous (\(\rho < 1 \text{ g/m/cm}^3\)), so that the star now has a very large radius. Since a star's luminosity is given by

\[ L = 4\pi R^2 \sigma T_e^4 \]

where \(R\) is its radius, \(T_e\) the effective surface temperature, and \(\sigma\) the Stephan-Boltzman constant, the large increase in radius causes the luminosity to increase markedly. Since the overall contraction preceding this phase has lowered the surface temperature to about 4500 °K, this phase is called the red-giant phase in stellar evolution. The tenuous envelope is cool enough so that the matter is no longer fully ionized. The presence of bound electrons causes the matter to become quite opaque so that the
energy generated in shell hydrogen burning can not be efficiently transferred to the stellar surface by radiative processes. Hence, convective mixing occurs to transfer the energy by macroscopic motions of the convective envelope. The mass fraction of the star in the convective envelope, $q_{CE}$, is about 70%. The actual values from Iben's studies of stellar evolution are presented in Table 3.2.

Within this convective envelope the various species of nuclei are mixed so that there is a uniform composition. Since He$^3$ was formed in the intermediate layers of the star, all of the He$^3$ is contained in the convective envelope. The mixing dilutes the produced He$^3$ to a level, $(X_3)_{CE}$, which is given in Table 3.2.

Because of the temperature and density distribution in the star during core hydrogen burning any He$^3$ in the outer 30% of the star is not enhanced or diminished by nuclear reactions. Thus, any He$^3$ in the gas from which the star was formed will not be processed in the outer region of the star. However, during convective mixing the initial He$^3$ will be diluted by $\approx 3/7$. This is schematically shown in Figure 3.5-a, b. Part (a) shows the He$^3$ distribution before convective mixing and (b) after convective mixing.

Once the star has gone through the red-giant stage with convective mixing, the question of how much of the star is ejected into the interstellar medium arises. It is assumed that all of the outer portion of the star is returned to
FIGURE 3.5a  He$^3$ Production and Convective Mixing
(After Core Hydrogen Burning.)
**FIGURE 3.5b** He³ Production and Convective Mixing.

*(After Convective Mixing.)*
the interstellar medium leaving only a dense body such as
a white dwarf or a neutron star of an approximate mass of
0.7 M_☉. Let the net fraction of the total stellar mass,
in which He³ is unprocessed and which is ejected be Q_{33}
and the mass fraction that is ejected as He³ processed
from hydrogen be Q_{13}. Because of uncertainty in the
dynamics of a star when it disrupts and in the structure
of the residual body, the mass of the residual body is not
known. It is only estimated from Schönberg-Chandrasekhar
(1942) limit for white dwarfs with complete electron degener-
acy or from the 0.7 M_☉ limit for a neutron star
(Oppenheimer and Volkoff 1939). The actual mass of the
residual body can strongly effect the amount of He³ ejected
by low mass stars (M_☉<2.5 M_☉). These stars are the ones
which produce the most He³, so this point will determine
the net amount of He³ observed. The quantities, Q_{13} and
Q_{33}, are shown in Figure 3.6 and 3.7 with four choices
for the residual core (M_R = 0.7, 0.8, 0.9, 1.0 M_☉).
These figures show how sensitively the net amount of He³
ejected into the interstellar gas depends on the details
of a star's death.

Once the element production from a single star is
known, it is possible to consider the effect of stellar
evolution on galactic evolution and the present observed
abundances. There have been a number of studies (Salpeter
1959, Schmidt 1959, 1963) of galactic evolution. These
studies used a one-zone model in which a fixed mass of
matter is evolved through a number of generations of stars with the amount of matter in the gas gradually decreasing as it is used up to form new stars.

When a certain portion of gas is formed into stars, stars of varying masses are made. The relative number of stars which have a particular mass is given by the luminosity function, $\psi(M)$. This function is derived from observations on young galactic clusters in which star formation is now going on. Using the data of Salpeter (1959), the luminosity function has the approximate form

$$\psi(M) = 6.3 \times 10^{-3} \text{ pc}^{-3} \left( \frac{M}{M_0} \right)^{-1.57},$$

where the dimensions of $\psi$ are number of stars of mass $M$ formed per cubic parsec ($3.09 \times 10^{13}$ km). Following Salpeter's formulation of a galactic evolution model, the total mass of $\text{He}^3$ that has been formed is given by

$$M_{\text{He}^3 \text{ (total)}} = \int_0^\infty M_{\text{He}^3}(M) \cdot N_B(M,t) \, dM,$$

where,

$$M_{\text{He}^3}(M) \equiv \text{mass of He}^3 \text{ formed by a star of mass } M = 4.7 \times 10^{-4} \times M^{-1.1},$$

and

$$N_B(M,t) \equiv \text{number of stars of mass } M \text{ that have gone through stellar evolution and have broken up. It is given by}$$

$$N_B(M,t) = N \int_0^{t_0 + t_1} M^g(+) \psi(M) \, dt,$$

$$t_1 = 10 M^{-r}, \ r = 4,$$

where $t_1$ is the lifetime of a mass $M$ star in units
of $10^9$ years. The mass of galactic material in the form of gas at time is $M(t)$. Using these definitions the total mass of $^3\text{He}$ produced is

$$M_{^3\text{He}}(\text{total}) = N \int_0^\infty \int_0^{M_{^3\text{He}}(M)} \left[ \frac{t_0-t_1}{\psi(M)} \right] \, dt \, dM.$$  

Reversing the order of integration gives

$$M_{^3\text{He}}(\text{total}) = N \int_0^{t_0} M(t) \left[ 3.0 \times 10^{-6} \int_0^{\infty} \frac{1}{M^{1.1} M^{5.7} dM} dt \right] \left[ M = \left( \frac{t_0-t_1}{10} \right)^{-1/r} \right]$$

$$= N(3.0 \times 10^{-6}) \int_0^{t_0} M(t) \left[ \int_0^{\infty} M^{2.7} dM \right] dt$$

$$= N(3.0 \times 10^{-6}) Mg(0) \left[ \frac{1}{1.67} \int_0^{t_0} e^{-1(t)} \left( \frac{t_0-t_1}{10} \right)^{1.67} dt. \right]$$  

The normalization constant, $N$, is equal to 8.3, and the mass of gas at time $t$ is related to the initial mass $Mg(0)$ by

$$Mg(t) = Mg(0)e^{-1(t)},$$

where $1(t) = t/\tau_1 + (t/\tau_2)^2 + \ldots$.

Letting $r = 4$ gives

$$M_{^3\text{He}}(\text{total}) = 1.5 \times 10^{-5} Mg(0) \int_0^{t_0} e^{-1(t)} \left( \frac{t_0-t_1}{10} \right)^{0.42} dt.$$  

This can easily be numerically integrated. If $\tau_1 = 1.8,$
\( \tau_2 = \infty \), then the integral has the value 1.35 for \( t_0 = 7 \) billion years. This gives

\[ M_{He^3}(\text{total}) = 2.0 \times 10^{-5} \text{ Mg(0)}. \]

Thus, the mass fraction of He\(^3\) in this galactic model is

\[ X_3(t_0) = 2.0 \times 10^{-5}. \]

Changing the value of the luminosities functions, the stellar lifetimes, or the gas decay time can change the resulting He\(^3\) mass fraction by factors of two or three. Nevertheless, even this simple model produces He\(^3\) abundances close to those observed in the solar system (\( X_3 = 5 \times 10^{-5} \)).

Recently R. J. Talbot and W. D. Arnett have developed a detailed model of galactic evolution in which they follow the time dependence of the abundance for a number of nuclear species (H, He\(^3\), He\(^4\), C, O, N, Fe, etc.). They are preparing several papers (Talbot and Arnett 1971, Arnett and Talbot 1971) in which they discuss the dependence of observed abundances on various model parameters.

A result of their analysis is a time dependence for \( X_3 \) which has an approximate solution of the form

\[ X_3(t) = X_{3i} e^{-t/\tau} + X_{3e}(1 - e^{-t/\tau}), \]

where \( X_{3i} \) is an initial He\(^3\) abundance, and

\( X_{3e} \) is an equilibrium value for the \( X_3 \) mass fraction in the galactic gas. For the conditions in the solar neighborhood \( t/\tau \approx 5 \) so \( e^{-t/\tau} \approx 0.01 \). The destruction
of any initial $\text{He}^3$ is accomplished by the rapid turnover of matter in high mass stars which have very short lifetimes. The $\text{He}^3$ equilibrium mass fraction is $1 - 4 \times 10^{-5}$, depending on the particular choice of parameters in their model. Thus their detailed calculations agree with predictions of simplier models.

In summary, Helium-3 is produced in most models for a primordial fireball in varying degrees. It is also an important link in the proton-proton cycle for stellar energy sources and is consequently produced in main-sequence stars during the core hydrogen burning phase. When the effects of stellar evolution are accumulated on a galactic scale, it appears that the $\text{He}^3$ in the interstellar gas will have reached an equilibrium situation for the present age of our galaxy. This equilibrium will unfortunately mask any information on the initial abundance of $\text{He}^3$ in the universe.
IV) Observations and Data Reduction

4.1) General Description of Observational System and Techniques

The three basic components of a radio telescope are the physical structure of the antenna, the receiving apparatus and preliminary amplifiers, and the signal processing and recording equipment. For our observations the telescope employed was the 140 foot, fully-steerable paraboloid at the National Radio Astronomy Observatory in Green Bank, West Virginia. It is a prime focus telescope with $f/D = 0.43$. The gain of the antenna was 69 dB.

The 3.5 cm receiver, housed at the prime focus of the telescope, was designed and built especially for this experiment in the Space Science Department of Rice University. Development and construction was supported by National Science Foundation Grant GP8054. The overall system noise temperature on the antenna was 90 $^\circ$K. The linearly polarized feed was a rectangular wave-guide horn with a 15 dB taper at the edge of the dish. The first stage of the receiver consists of two cryogenically cooled parametric amplifiers, operating in series to give a net gain of 26 dB. These are followed by a tunnel diode amplifier and a mixer-preamp which gives the 150 MHz intermediate frequency (IF).
The local oscillator (LO) frequency for the mixer is generated in the telescope control room. The LO frequency would be at 8665.650 MHz minus 150.0 MHz for the helium-3 line if the source was at rest with respect to the telescope. Since there is relative motion between the source and the telescope, all of the LO frequencies have to be corrected for Doppler shifts of the spectral lines. These shifts are composed of two components: the velocity of the source with respect to the local standard of rest ($V_{\text{LSR}}$), and the component of the observer's motion in the direction of the source. The $V_{\text{LRS}}$ is a constant which is independent of time, but the observer's projected motion has to be computed frequently to allow for the earth's rotational velocity and variations in the earth's orbital velocity. The $V_{\text{LSR}}$ for each source has already been determined from hydrogen recombination line surveys. The source velocities which we used were from the H$_{109\alpha}$ survey by Reifenstein et al. (1970). The 1970 observations used the computer routine DOPSET (Gordon and Manchester 1970) to tabulate the required LO frequencies. They were updated every two hours of observing to keep the line fixed within 5 KHz.

The IF signal processing equipment is located in the base of the telescope near the operator's console. Figure 4.1 shows the system components and their interactions. The line receiver in this figure is the fifty-channel filter used in August 1968. The IF signal is split between a NRAO standard receiver and the spectral line receiver.
Figure 4.1: Observational System
The NRAO standard receiver measures total signal power from the 3.5 cm receiver over a 20 MHz band centered at the line frequency. In addition, the difference in power between a signal and a reference is measured by a synchronous detector. Both the switching and the detection are slaved to computer control.

In radio astronomy measurements are commonly made by periodically switching between a signal and comparison. This switching allows the detection of weak features, either in the continuum or in spectral features. The three basic switching modes are beam, load, and frequency switching. With beam switching there are two feed horns which look at adjacent portions of the sky. This is useful when doing continuum work since the source temperature can be compared to the nearby 'cold-sky' temperature. Load switching (also known as Dicke switching) involves comparing the source continuum temperature with that of a matched load (internal to the receiver front end) which is at a known temperature. Frequency switching is extensively used in the observation of spectral lines. A signal band containing the spectral line is compared to a nearby (in frequency) band containing no significant lines. Because the signal and reference bands are typically chosen to be within 10 MHz of one another, the effects of atmospheric extinction (moisture, clouds) cancel out. Since these effects are very time dependent and not easily predicted, the continual cancellation of these effects improves the quality of
observations considerably. For frequency switching, only one feed horn is used and the signal seen by the parametric and tunnel diode amplifiers does not change. Instead, the local oscillator frequency which is fed to the mixer is changed. Thus signals from two frequency bands are processed by the spectral line receiver. The on-line computer controls the switching and handles the data appropriately (Appendix 7.1).

Once the equipment was installed at the prime focus and the telescope positioned on a source the front-end receiver and spectral-line receiver were tested by observing some hydrogen recombination lines ($\mathrm{H_{1148}}$ $\mathrm{H_{1304}}$) near the ($\mathrm{He^3}$)$^+$ line frequency. Their detection insured the proper functioning of the receiving system and the accuracy of the Doppler shift calculations.
4.2) **140 Foot Telescope Parameters at 3.5 cm.**

The 140 foot diameter radio telescope at the National Radio Astronomy Observatory is a solid surface paraboloid which is equatorially mounted and fully steerable in declination and hour angle. Its construction and basic features are described by Small (1965). The pointing errors are fairly well understood and accounted for, within 0.3 arc, by correction curves. The characteristics of the telescope as a function of wavelength have been analyzed by Mezger et al. (1966).

The telescope parameters at 3.46 cm are summarized in Table 4.1. The quantities in the table are the effective area of the telescope ($A_e$), the aperture efficiency ($\eta$), the beam widths in E and H-planes of the telescope beam ($\Theta_E$, $\Theta_H$), the main beam solid angle ($\Omega_M$), the beam efficiency ($\epsilon_B$). All of the quantities are functions of zenith angle.

The calibration sources, Virgo A and Taurus A, were used to determine $A_e$ which gives the aperture efficiency and the antenna solid angle, $\Omega_A$. The quasar, 3C273, is effectively a point source at 9 GHz, so it was used to determine $\Theta_E$, $\Theta_H$, $\Omega_M$.

The theory and techniques for determining these quantities is presented below. The flux, $S$ (watts/(m²-steradian -Hz)), from a source is

$$S = \frac{2kT_B}{\lambda^2} \int_{\text{source}} \psi(\Theta, \phi) \, d\Omega = \frac{2k}{\lambda^2} T_B \cdot \Omega_S,$$
<table>
<thead>
<tr>
<th>Zenith angle (degrees)</th>
<th>$A_e$ (m$^2$)</th>
<th>$\eta$</th>
<th>$\Theta_E$</th>
<th>$\Theta_H$</th>
<th>$\Omega_M$ sterad.</th>
<th>$\Omega_A$ sterad.</th>
<th>$\varepsilon_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>760 - 3.7Z</td>
<td>0.532</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$1.57 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($1 - 0.0042Z$)</td>
</tr>
<tr>
<td>16°</td>
<td>712</td>
<td>0.490</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$1.68 \times 10^{-6}$</td>
</tr>
<tr>
<td>26°</td>
<td>682</td>
<td>0.476</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$1.75 \times 10^{-6}$</td>
</tr>
<tr>
<td>36°</td>
<td>*652</td>
<td>*0.455</td>
<td>3.45</td>
<td>3.55</td>
<td>1.18 $\times 10^{-6}$</td>
<td>*1.83 $\times 10^{-6}$</td>
<td>0.645</td>
</tr>
<tr>
<td>42°</td>
<td>624</td>
<td>0.457</td>
<td>3.55</td>
<td>3.65</td>
<td>1.24 $\times 10^{-6}$</td>
<td>1.92 $\times 10^{-6}$</td>
<td>0.638</td>
</tr>
<tr>
<td>48°</td>
<td>616</td>
<td>0.420</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.94 $\times 10^{-6}$</td>
</tr>
<tr>
<td>65°</td>
<td>*565</td>
<td>*0.394</td>
<td>3.60</td>
<td>3.75</td>
<td>1.30 $\times 10^{-6}$</td>
<td>*2.12 $\times 10^{-6}$</td>
<td>0.613</td>
</tr>
</tbody>
</table>

* Calculated from fitted function.

**TABLE 4.1**

**TELESCOPE PARAMETERS**
where, $T_B (°K)$ is maximum source brightness temperature, 
$\Omega_s$ is source solid angle, and $\Psi (\Theta, \Phi)$ is the source distribution function over the entire sphere, primarily a source near $\Psi (0,0)$, but including everything within $4\pi$ steradians. The high gain (69 dB) of the antenna is what determines the integration limits. The observed flux from a source is

$$S_{\text{obs}} = \frac{2k}{\lambda^2} \int T_B (\Theta, \Phi) P_A (\Theta, \Phi) d\Omega = \frac{2k}{\lambda^2} T_B \Omega'_s .$$

The antenna beam pattern, $P_A (\Theta, \Phi)$, is normalized so that $P_A (0,0) = 1$.

$$\Omega_M \equiv \int P_A (\Theta, \Phi) d\Omega = \text{main beam size, (steradians)}$$

$$\Omega_s \equiv \int \Psi (\Theta, \Phi) d\Omega = \text{source solid angle.}$$

$$\Omega'_s \equiv \int \Psi (\Theta, \Phi) P_A (\Theta, \Phi) d\Omega = \text{effective source solid angle.}$$

Many sources are adequately represented by a two-dimensional Gaussian distribution:

$$\Psi (\Theta, \Phi) = \exp \left[ -4\ln 2 \left( \frac{\Theta^2 + \Phi^2}{\Delta \Theta^2} \right) \right],$$

where $\Delta \Theta, \Delta \Phi$, are the full-widths at half-maximum along the $\Theta$, and $\Phi$ axes. In addition, the beam shape for a parabolic antenna is well represented by a two-dimension Gaussian function,
\[ P_M (\Theta,\phi) = \exp \left[ -4\ln 2 \left( \frac{\Theta^2}{E^2} + \frac{\phi^2}{H^2} \right) \right] . \]

Using this representation

\[ \Omega_S = \frac{\pi}{4\ln 2} \Delta \alpha \Delta \delta , \]

\[ \Omega_M = \frac{\pi}{4\ln 2} \Theta_E \Theta_H , \]

and \[ \Omega'_S = \Omega_S \Omega_M / \Omega_{obs} , \]

where, \( \Omega_{obs} \) is the observed source size defined by

\[ \Omega_{obs} = \frac{\pi}{4\ln 2} \Delta(H,\alpha) (E,\delta) \]

\[ \Delta(H,\alpha) = \left( \Theta_H^2 + \Delta \alpha^2 \right)^{1/2} , \]

\[ \Delta(E,\delta) = \left( \Theta_E^2 + \Delta \delta^2 \right)^{1/2} . \]

All these quantities have been defined in order to relate the readily observed quantities to theory. Since the observed antenna temperature \( T_A(^\circ K) \) is defined by

\[ T_A = A_e \cdot S_{obs} / 2k , \]

the observed antenna temperature is easily related to the flux \( S \).

\[ S = 2k \cdot \frac{T_A}{A_e} \cdot \frac{\Omega_{obs}}{\Omega_M} . \]

The main beam size was determined by doing \( E \) and \( H \)-plane scans across the point source 3C 273. These scans are done by aligning the receiver polarization (\( E \)-plane) along the declination or right ascension axis of the telescope, then
monitoring the receiver total power on a chart recorder as the antenna beam is moved (in declination or right ascension) at a constant rate across the source. This gives a Gaussian shaped pattern whose half-power, full-width is $\theta_E$ or $\theta_H$. When this type of scan is done on a non-point source, the quantities $\Lambda(H, \alpha)$ and $\Lambda(E, \delta)$ are measured.

By using the standard calibration sources (Baars et al. 1965), whose flux is well known over a wide frequency range, the effective area of the telescope can be determined from the above equation. The definition $A_E = \lambda^2 / \Omega_A$ is then used to find the total antenna beam solid angle, $\Omega_A$. This gives the quantities

$$\varepsilon_B \equiv \frac{\Omega_M}{\Omega_A} = \text{beam efficiency},$$

$$\eta = \frac{A_e}{A_{phy}} = \text{aperture efficiency},$$

where $A_{phy} = 1430 \text{m}^2$ = physical area of the telescope.
4.3) Low Temperature Receiver

The 3.5-cm receiver, designed and built at Rice University under the direction of J. L. Halpain (1970) has a two-stage, cooled parametric amplifier as its principle feature. The receiver development was funded by the National Science Foundation and it is to be turned over to the National Radio Astronomy Observatory after this investigation is finished.

The receiver was designed to observe the \((\text{He}^3)_1^+\) hyperfine line at 8665.6 MHz (3.46 cm), but can be used from 8400 to 8900 MHz with proper tuning of the parametric amplifiers. Figure 4.2 shows a block diagram of the receiver with the noise calibration circuit indicated.

The parametric amplifiers and their associated ferrite circulators are in a vacuum chamber where they are cooled to 18 °K by an A. D. Little 204LS helium refrigerator. To eliminate thermal loss the input signal comes through a choke coupling into the evacuated paramp chamber. The variable capacitance diodes (varactors) have a typical zero voltage capacitance of 0.36 pF and a cut off frequency of 400 GHz at -6V bias. The paramp's are pumped by a 31 GHz klystron with approximately 10 mW of power on each diode. This results in an idler (pump minus signal) frequency of 22.3 GHz. The paramp's are operated at 13 dB gain each and are followed by a tunnel diode amplifier with 16 dB gain. Then, after the signal power is split, half of it goes to
a 150 MHz mixer with a gain of 20 dB. Thus the total system gain is 55 dB, including losses in the lines.

The local oscillator (LO) for the mixer is derived from a high precision, Hewlett-Packard, frequency synthesizer between 200 and 300 MHz. This frequency is multiplied by six and amplified by a traveling-wave tube in the LO cabinet to give a 0.8 Watt signal at 1703 MHz in the telescope control room. This signal is sent up to the 3.5 cm receiver and its frequency multiplied by five (5) to give the 2 mW of power required by the mixer at 8516 MHz.

Stability of the paramp gains is a very important factor in good system performance. Pump power variations are the primary source of gain changes in most paramps. A 0.01 dB change in pump power can cause a 0.1 dB change in paramp gain (Kraus 1966, Section 7-3c). The control circuits use the sensitive property of the varactor diodes that makes them draw more current when the pump power is increased. A voltage proportional to the diode current (≈0.1 μA) is fed to the pump control circuit, which in turn controls pump power by means of voltage variable attenuators in the pump power lines. Figure 4.3 shows the paramp control circuits and the signal flow through the paramps.

Diode currents, pump power level, vacuum, and helium refrigerator temperature can be monitored at the instrument rack in the control room of the 140 foot telescope. This rack also contains the klystron high voltage power supply, and an oscilloscope to monitor the bandpass of the receiver.
**PARAMETRIC AMPLIFIER ASSEMBLY**

Figure 4.3
The receiver band pass must be tuned to be as flat and as broad as possible in order to make the baseline for the spectral line receiver flat and balanced. Tuning is accomplished by using the 8.4-8.9 GHz, sweep source illustrated in Figure 4.4 or by varying the LO to observe the IF response. The LO method is discussed at the end of this section. The shape of the paramp response can be changed by adjusting the diode current, pump power, or by adjusting the klystron reflector voltage (this changes the pump frequency slightly). The sweep source can be controlled to give a sweep 200 to 500 MHz wide in the range from 8.4 to 8.9 GHz. There is a control circuit to give a level output (±2%), and two frequency markers calibrated within 2 MHz.

Coaxial transfer switches (CTS) perform various functions in the sweep source and noise calibration circuits. The coaxial transfer switch is a four-port device made for switching 3 mm coaxial lines. Figure 4.4 has a diagram of the transfer switch. When the switch is in position 1 the ports joined by solid lines are connected together with only 0.2 dB loss and a 60 dB isolation from the other ports. In position 2, the ports joined by broken lines are connected together. When the CTS-2 is in position 2, the swept output is fed into the video detector. This is displayed on an oscilloscope located in the instrument rack in the control room. This display allows calibration of the paramp gain. The CTS-2 is now switched to position 1. The swept signal is injected into the paramps via a
mechanical attenuator, an unfired noise lamp and a 22 dB cross-guide coupler (see Figure 4.2). The mechanical attenuator is adjusted to make the total gains and losses equal zero with the proper paramp gain (usually 26 dB). Then the output from the TDA is visible through the video detector. This picture of the RF bandpass may be used to make fine adjustments to smooth out or broaden the bandpass.

The block diagram of Figure 4.2 was used for the August 1968 observations, but it has at least one bad feature. The CTS-1 was wired so that either the calibration noise lamp or the injection noise lamp is input to the 22 dB coupler. They cannot both be injected at the same time. The system should be able to inject noise and calibration signal simultaneously, so the circuit was subsequently rewired to allow for this. Noise was injected during off source observations to simulate the source continuum, thus more accurately approximating source conditions. This results in an improved baseline correction, although slightly higher system temperature.

The varactor diode current is very sensitive to the RF power being received. Because the pump power control loop is locked to the diode DC current, an increase in power at 8.7 GHz decreases the pump power at 32 GHz, and consequently, the paramp gain. Thus, during calibration and testing, the power from the calibration sweep circuit may affect the diode current and consequently the pump power. Therefore, the bandpass displayed in the control
room may not represent the actual paramp gain. A superior procedure was adopted in which the local oscillator was swept; in this case the receiver total power gives a picture of the bandpass including the IF section of the receiver. Incorporated in the NRAO local oscillator is a periodic voltage ramp which may be used to sweep any digit of the local oscillator frequency. Usually the 100 KHz digit is swept to give a sweep of 30 MHz at the RF frequency. The entire band pass may be viewed or any section of it may be seen by varying the extent of the LO sweep. Since the LO settings that were used ranged from 283 to 284 MHz, this was usually the sweep range. The analog output from the total power section of the NRAO standard receiver was the vertical signal for the oscilloscope and the time base was triggered by the LO sweep generator.
4.4) Spectral Line Receivers

4.4.1) Fifty Channel Filter Receiver

When the IF at 150 MHz is brought down to the control room for further processing, the signal is amplified and split, half going to a NRAO standard receiver to monitor total power and half going to the 50 channel filter receiver. Figure 4.1 shows a block diagram of the receiver and its interactions with the 50 channel receiver, local oscillator, and DDP-116 computer.

The 50 channel filter receiver consists of fifty adjacent 100 KHz wide filters which cover a 5 MHz band. The local oscillator is adjusted so that spectral bandpass will be centered at 150 MHz in the IF. In that case, when processed by the 50 channel receiver, the line will be centered between channels 25 and 26. In the 50 channel receiver the 150 MHz signal is reduced to 5.5 MHz by using a mixer and a second LO at 144.5 MHz. The 5 MHz band from 3 to 8 MHz is fed into the 50 channel filter bank. There, the outputs from the tuned-filter, detector circuits are integrated for 45 ms and then sampled by analog to digital converters associated with the DDP-116 computer. In the frequency-switching observing mode the DDP-116 computer switches between signal and reference frequency bands at a 10 Hz rate. The computer processes the data, as explained
in detail in Appendix 7.1, and accumulates data for ten seconds before writing a record on magnetic tape. This record contains information on apparent telescope position in right ascension and declination, local sidereal time, eastern standard time, local oscillator frequencies, total power for the 5 MHz band and power level for each of the 50 channels in terms of the noise lamp calibration temperature, $T_{cal}$.

The 50 channel output is updated on an oscilloscope display every 10 seconds.

Data acquisition is done by the DDP-116 computer in the following manner. Under computer control the local oscillator is switched at a 10 Hz rate between the signal and reference frequencies, typically, 3 MHz apart. The local oscillator was switched between a signal frequency, $f_0$, and two reference frequencies $f_1$ and $f_2$. The switching sequence was $f_0$, $f_1$, $f_0$, $f_2$, $f_0$, $f_1$, etc. For our scans $f_1 = f_2$. For periods of 50 ms, the 50 channel receiver looks at the signal or reference band. The signal from each 100 KHz filter is integrated for 45 ms and then read into the computer through an analog to digital converter. Every 2 seconds the calibration temperature is added to the signal band for 50 ms. This is done to periodically calibrate the system temperature, to compensate for long term changes in the receiver gain. After 10 seconds of data processing a magnetic tape record is written. Since a tape can contain approximately one day's data, the tapes are sent to NRAO's main computer center at
Charlottesville, Virginia every morning for further processing. Appendix 7.1 describes this processing.

Printed records of the 50 channel data are available at the end of every 10 second scan and at the end of the scan. Records were collected at the end of every scan for our observations. In addition to the printer, a teletype was used for communication between observer and the computer. Scan labels (20 characters), observer labels, and scales for the oscilloscope display are accepted by the teletype input to the computer. Computer calculations are not affected by different oscilloscope scales. The observer and scan labels are identification phrases which are written on each magnetic tape record until a new label is entered at the teletype.

The observational records are magnetic tape data, printed numeric data, pictures of oscilloscope displays, (see Figure 4.5), teletype entries, an analog record of the receiver total power, and the telescope operator's log. Total power from the NRAO standard receiver is recorded on a chart recorder at the telescope operator's console. This record is used by the telescope operator in pointing the telescope at a source by finding a peak in total power, in focusing the receiver, and in doing scans in right ascension and declination in order to determine the antenna beam width or source extent. The telescope operator's log sheets contain the name of source, position in terms of right ascension and declination, scan start and stop times
FIGURE 4.5 Oscilloscope Display for Fifty-Channel Receiver.
August 9, 1968 - H\textsubscript{148} line from M 17.
in local sidereal time, LO frequencies, and various settings of the receivers.

4.4.2) **Digital Autocorrelation Receiver**

The digital autocorrelation receiver is used in a manner analogous to that for using the fifty-channel filter receiver. The same system block diagram of Figure 4.1 applies, with the filter receiver replaced by the autocorrelator receiver.

When the IF signal is split in the NRAO standard receiver, half of the power goes to the NRAO Model-II Autocorrelator Receiver (Shalloway et al. 1968). This spectral line receiver can be used in series (receiver A with 384 channels) or in parallel (receivers A and B operating independently with 192 channels each). Receiver C, having 29 channels, samples the same IF signal as receiver A but with less resolution. The total bandwidth for each receiver is \(10.000 \text{ MHz}/2^M\), where \(M = 0\) to 8 for A and B and \(M = 0\) to 4 for C. Receivers A and B are independent so they can have different bandwidths and can sample different spectral lines, different polarizations, etc. The resolution of each receiver is

\[ \Delta f = 1.21 \text{ B/N}, \]

where B is the bandwidth and N the number of channels. The root-mean-square fluctuations for a channel are given by
\[ \Delta T = 3.06 \, T_S \left( ^\circ K \right) / \sqrt{\tau (\text{sec}) \cdot \Delta f (\text{Hz})}. \]

This is 50% more noise than a filter receiver with the same bandwidth. Since receivers A and B sample the same IF with the same bandwidth, they provide almost independent spectra. Thus, when receivers A and B are then averaged together, the noise decreases by a factor of 1.3 instead of the factor of \( \sqrt{2} \) by which the noise would decrease if receivers A and B were totally independent. This almost independent sampling partially compensates for the inherently less sensitive method of the digital auto-correlation receiver.

Although the autocorrelator receiver is more complicated, and noisier than a filter receiver of equivalent resolution, these disadvantages are offset by the versatility of the autocorrelator. The positive features of the autocorrelator are: the various bandwidths, which can be easily changed to fit a particular observer's needs; the independent receivers A and B, which are frequently used in work on polarized OH lines to sample orthogonal polarizations in order to determine the Stokes' polarization parameters for the line emission. Another important feature is the insensitivity of the line receiver to power changes in the input signal. Thus, small gain changes (<0.5dB) do not effect the spectra which is determined by the autocorrelation receiver.

The steps in which the data is processed are as follows:
first the 150 MHz IF is mixed down to place the spectral bandpass in the range from 0 to B. This signal is put through a low pass filter to reject high-frequency components, then the signal is clipped to produce a series of plus and minus rectangular pulses of constant amplitude instead of the input signal with sinusoidal components. The clipped signal is positive whenever the input signal is positive and negative whenever the input signal is negative. The symmetry of the clipping is controlled so that the number of positive and negative pulses is the same within 0.1%. This clipping preserves frequency information but loses absolute amplitude information. The only integration of the signal is supplied by the inherent RC time (≈ 0.6 nanoseconds) of the clipping circuit. The clipped signal is then sampled at a rate exactly equal to twice the bandwidth. The sampling rate is \( \Delta t = 1/2B \). Thus, a 10 MHz bandwidth signal is sampled \( 2 \times 10^7 \) times a second or every 50 nanoseconds. The sampling converts the clipped signal into a bit-string of zeros and ones compatible with digital logic circuits.

An autocorrelation is then performed between the current bit and the previous \((N-1)^*\) bits. Table 4.1 is a truth table for one-bit autocorrelation. The current bit, corresponding to zero lag, is compared to the bits for various lag times, \( \tau_i = i \cdot \Delta t \), using the truth table to

\*N is the number of channels in the receiver.
**TABLE 4.2**

TRUTH TABLE FOR ONE-BIT CORRELATION

<table>
<thead>
<tr>
<th>BIT FOR ZERO-LAG</th>
<th>BIT FOR ith-LAG</th>
<th>SUMMED INTO ith ACCUMULATOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
determine what number is added into the accumulator for each time lag. Each accumulator has a 28 bit word length to store the correlations. After an autocorrelation is done each bit is shifted to the next register (the $\tau_0$ bit to the $\tau_1$ register, etc.) and the next sample is stored in the $\tau_0$ register. The $\tau_{N-1}$ bit is discarded since it is no longer needed. Then an autocorrelation is done and the procedure repeated.

The receiver dumps the 413 accumulators to the on-line computer every 10 seconds. Because of the autocorrelation method, the $\tau_0$ accumulator for each receiver contains the total number of bits that were sampled. The total number of samples is divided into each accumulator for a receiver and 0.5000 subtracted from each accumulator. This subtraction is why the very low asymmetry ($<0.001$) is necessary when the signal is clipped. This gives the normalized autocorrelation function, $\rho'(\tau_1)$, for the clipped input signal where $\rho'(0) = 1$. Because of the hard clipping the normalized autocorrelation, $\rho(\tau_1)$ for the unclipped input signal is (Van Vleck 1943, Weinreb 1963)

$$\rho(\tau_1) = \sin \left[ \frac{\pi \rho'(\tau_1)}{2} \right].$$

Then a Fourier cosine transform is done to give the spectral shape

$$P(v_J) = \rho(0) + 2 \sum_{1}^{N-1} \rho(\tau_1) \cos \left[ \frac{I \cdot J \cdot \pi}{N} \right],$$
\[ \nu_J = \nu_{\text{RF}} + \frac{B}{N} \left( J - \frac{N-1}{2} \right) \, , \]

where, \( \nu_{\text{RF}} \) is the center of the receiver bandpass before it is mixed down to 150 MHz.

The bandpass \( P(\nu_J) \) is determined for both a signal and comparison spectra, and the normalized ratio \( R(\nu_J) \) found. The ratio is computed to account for gain variations in the bandpass shape.

\[ R(\nu_J) = \left( \frac{P_{\text{sig.}}(\nu_J)}{P_{\text{comp}}(\nu_J)} \right) - 1. \]

This "quotient" spectra is then written on magnetic tape to be combined with other data for the observing period. Section 4.6 shows some sample spectra and discusses how the data is processed.
4.5) **Observing Techniques**

4.5.1) **Frequency Switching**

In August and November 1968, frequency switching with a 5 MHz bandwidth was used for our spectral line observations. Frequency switching is done by switching the local oscillator (LO) frequency between three frequencies. The three frequencies \(f_0, f_1, f_2\) are observed in this sequence: \(f_0, f_1, f_0, f_2, f_0, f_1\), etc. For our observations \(f_1 = f_2\), so only one comparison frequency was used. The signal \(f_0\) and comparison \(f_1\) bands are observed for equal amounts of time. When using a filter receiver the comparison spectra is subtracted from the signal spectra to account for channel to channel gain variations and variations in total power received (due to receiver gain changes, weather conditions, etc.). When using the digital autocorrelation receiver the ratio of signal spectra to comparison spectra is found (see Section 4.4.2).

The LO frequencies can be chosen so that the desired spectral line is in both the signal and reference frequency bands. In this case the signal-to-noise ratio for the spectral line is optimized, although the analysis of the data is more complex. Section 4.6 describes the analysis techniques used on this type of data.

The helium-3 line shapes expected with our three
frequency switching combinations are shown in Figure 4.6. This figure does not show the high-order hydrogen recombination lines which are significant in the stronger sources, (Orion, M17, W49, W51). The recombination lines are shown in Figure 4.7, on a relative scale, from -12 MHz to 10 MHz relative to 8665.650 MHz.

The 5 MHz bandwidth, used in August 1968, was wide enough to include the helium-3 line in both bands, but this does not leave many channels with which to establish a baseline. When possible, the widest bandwidth, consistent with resolving the anticipated lines, should be used, both to establish the best baseline and to decrease each channel's signal-to-noise ratio. The main problem with wide bandwidths are the increased curvatures in the final output. These curvatures are due to limited RF bandwidths. The 3dB bandwidth of the 3.5 cm receiver was 50 MHz.

The detailed observations for frequency switching were done as follows. When a source was acquired the antenna power was maximized by peaking the focus position of the receiver box. Then a series of observations, each typically 15 minutes long, was done both on and off source. The off source observations were usually done 3 minutes of right ascension west of the source position. The sequence was typically: OFF, ON, ON, OFF, ON, ON, OFF, etc. Thus each ON observation was adjacent to an OFF. The same LO frequencies were switched, both on and off source.
FIGURE 4.6 Helium-3 Frequency Combinations
FIGURE 4.7 Hydrogen Lines near the He$^3$ II Hyperfine Transition
The off source observations were used to correct for any irregularities in the receiver baseline. These include the curvatures caused by frequency switching and the channel to channel variation of a filter receiver. This correction is particularly important for the observations with a filter receiver because of the channel to channel variations in sensitivity. If a autocorrelation receiver is used and curvatures are small, the off source observations may not be necessary. This is possible since there are no channel to channel gain variations, and slight curvatures can be accounted for in the data reduction.

Since the total off source time was about half that on source, the off spectra are noiser by a factor of \(\sqrt{2}\). This was compensated for by smoothing (see Appendix 7.2) the OFF spectra before it is subtracted from the ON spectra.

This correction was done for both the filter and digital autocorrelation receivers. The ratio necessary for the digital autocorrelation receiver is already computed from the signal and comparison frequency bands.

4.5.2) Total Power Technique

In November 1970 the total power technique or "antenna switching" (Ball 1969) was used with the 413 channel digital autocorrelation receiver. A 10 MHz bandwidth was used with receivers A and B both sampling the same IF signal. The total power technique does not involve a continual switching
between a signal and comparison frequency. Only one local oscillator frequency is used. The selected band is observed continuously with $T_{\text{cal}}$ inserted periodically to calibrate the system. Because the LO is not switched, all the on source time samples the desired line. Consequently, the total power technique is twice as sensitive as frequency switching.

To illustrate this compare a pair of OFF-ON measurements for the total power method with a pair using frequency switching. Let the system temperature be the same both off and on source, with $\tau$ the equal time spent off and on source. The noise for an OFF-ON pair from the total power method is

$$\Delta T_{TP} = 3T_s/\sqrt{(\Delta f \cdot 2\tau)}.$$  

The noise is equivalent to that of a switched receiver with an integration time of $2\tau$. The noise from both the OFF and ON for the frequency switched receiver is

$$\Delta T_{FS} = 3T_s/\sqrt{(\Delta f \cdot \tau)}.$$  

When the difference is taken $\Delta T_{FS}$ increases by 2. So

$$\frac{\Delta T_{FS}}{\Delta T_{TP}} = \frac{3\sqrt{2}T_s}{\sqrt{(\Delta f \cdot \tau)}} \div \frac{3T_s}{\sqrt{(\Delta f \cdot 2\tau)}} = 2.$$  

The sensitivity of frequency switching can be improved, by including the line in both signal and comparison bands and by only doing ON observations, so that the methods are comparable in sensitivity. However, the total power method is still superior because of the less
complex baseline. The total power method requires a system whose gain is quite stable over the period between off source measurements. Minor gain changes (<0.2dB) do not effect the measurements made with a digital autocorrelation receiver since it only determines the bandpass shape, not the absolute amplitude.

Because of reflections between the receiver and the telescope surface a sine wave modulation is present on the baseline (Weinreb 1967). Its frequency is \( c/2L = 7.5 \text{ MHz} \) for the 140 foot telescope, where \( L \) is the focal length of the telescope. In order to minimize its effect on the observed data, spectra were taken at \( \pm \lambda/8 \) from the peak focus position. \( \lambda \) is the observational wavelength of 3.46 cm.

When a source was acquired the focus peak was found, then the focus setting changed by \( \pm \lambda/8 \). The total power level was noted on the analog chart recorder* and the telescope moved off source (3 minutes of time west of the source peak) and noise injected until the source continuum temperature was simulated or the maximum amount of noise was injected**. Then a 15 minute OFF was done. Afterwards

---

*This analog record had two pens, one the total IF power from the NRAO standard receiver and the other the total power from receiver A of the autocorrelator.

**The noise injection had been calibrated versus the calibration noise temperature. The noise injection settings were recorded for each scan and these settings and the calibration curves used to find source continuum temperatures as a function of hour angle.
the noise injection was turned off, the telescope repeated
on the source (the continuum was peaked, the focus was not
changed), and a 15 minute ON done. After this the focus
was moved -\(\lambda/4\) to be at peak focus minus \(\lambda/8\), then the OFF,
ON sequence was repeated. The focus was repeated every
hour, since it varies as a function of hour angle (Mezger
et al. 1966). The LO setting was changed every two hours
to account for Doppler shifts.

It was determined during our 1970 run that changing the
IF attenuator settings on the NRAO autocorrelator receiver
between an OFF and an ON caused baseline irregularities
such as drastic curvatures near the edge of the bandpass.
After consultation with NRAO technical personal it was
decided that no attenuator adjustments would be made
between a set of OFF and ON observations, even on Orion
where a 15% imbalance existed. The imbalance occurred
because of the limited amount of noise which could be
injected. The baseline irregularities may have been due
to variations in the IF impedance match, within the
correlator, when the attenuators were changed. When no
settings were changed between on off and on source
observations, very straight baselines with little slope
were obtained

During the 1970 data reduction some +\(\lambda/8\) observations
were averaged together and a set of -\(\lambda/8\) observations were
averaged together. This gave two spectra for each source,
each one-eighth of a wavelength from the focal peak. To
see the magnitude of the sine wave the two spectra were subtracted, instead of being added, and the results examined for the sine wave. No wave was evident with peak-to-peak amplitude greater than 0.1 °K. This absence is due to several factors: a scattering structure has been placed at the vertex of the 140 foot telescope; the feed horn was carefully matched to have a standing-wave-ratio (SWR) <1.02, and the SWR for the system input was only 1.2. Even though the system input has a non-negligible SWR the signal reflected comes principally from the input circulator at 20-40 °K, and consequently the reflected power is an order of magnitude lower than that from a room temperature circulator.
4.6) **Data Reduction**

4.6.1) **Weighting of Data**

Data reduction is concerned with analyzing the data which is collected by the spectral line receiver, processed and formatted by the on-line computer and written on digital magnetic tape for further processing. The particular observing technique and spectral line receiver determine the detailed form of the data, but general techniques apply to further processing.

The following discussion describes how observations are combined to improve the signal-to-noise ratio, how the combined data is analyzed to determine what lines were observed, and how well determined the final numbers are.

The principle innovation in our analysis technique provides a detailed weighting for each observation, considering both integration time, the total system temperature, and the source antenna temperature for each observation. The total system temperature includes contributions from the receiver, background antenna temperature and the source continuum antenna temperature.

Only the November 1970 data reduction is discussed in detail, although the same type of analysis was applied to both the August 1968 and November 1970 observations. The August 1968 data reduction is outlined in Appendix 7.1.
During an observation using the digital autocorrelation receiver, the autocorrelation function is dumped to the online computer every ten seconds. A number of dump cycles is accumulated within the computer*, then a fast Fourier transform is done to give the bandpass shape. Numerous settings are recorded on magnetic tape along with: P(I,J), the bandpass for channel I, record J, and data for determining T_S(J), the system temperature for that record. The P(I,J) are normalized so that for each receiver
\[ \sum_{I=1}^{N} P(I,J) = N, \]
where N is the number of channels in that receiver (N = 29, 192 or 384 depending on the particular mode in which the autocorrelator is being used).

The NRAO standard reduction programs weight by time only. The off source reference bandpass shape is
\[ R(I) = \frac{1}{M} \sum_{J=1}^{M} P(I,J); \]
where M is the number of records (minutes) in the observations, I is channel number. From this shape and the ON observation the temperature spectra T_K(I), for observation K is obtained.

*The integration time for each record is dialed in at the 140 foot telescope operator's console. This time must be greater than 30 seconds in units of 10 seconds. Sixty seconds was used for the 1970 observations.
\[
T_K(I) = \frac{1}{M} \sum_{J=1}^{M_K} \left( \frac{T_{ON}(J) \cdot S(I,J)}{K(I)} \right) \cdot T_{ON}
\]

where \(T_{ON}(J)\) is the on source total system temperature for record \(J\), and \(S(I,G)\) is the signal (ON) bandpass for channels \(I\), record \(J\) of the \(K^{th}\) observation.

This average, \(T_K(I)\), is the output of the NRAO program TPOWER2. Subsequent steps weight the spectra together by time.

\[
T(I) = \frac{\sum_{K=1}^{N_{obs}} T_K(I) \cdot M_K}{\sum_{K=1}^{N_{obs}} M_K}
\]

Weighting strictly by integration time has been the standard technique for radio astronomy data. Since

\[
\Delta T_{rms} = \sigma_{rms} \propto T_s / \sqrt{\tau}, \quad \sigma^{-2} \propto \tau / T_s^2.
\]

For statistical purposes, observations which have a Gaussian distribution, but a parent distribution characterized by different standard deviations, \(\sigma_K\), have a mean calculated by:

\[
\bar{X} = \left( \sum_{K=1}^{N} \frac{X_K}{\sigma_K^2} \right) \div \left( \sum_{K=1}^{N} \frac{1}{\sigma_K^2} \right), \quad K = 1, N
\]

observations. In weighting strictly by integration time, there is a tacit assumption that the total system temperature is exactly the same for each observation. There are many reasons why this is not true. The observed source
continuum temperature varies as a function of hour angle due to changes in antenna efficiency; the background antenna temperature increases as zenith angle increases; and the receiver temperature may vary from observation to observation because of system changes*. Hence, a more thorough analysis should use a true weight

$$W_K = 1/\sigma_K^2 \tau_K / T_S^2,$$

for each observation and each channel.

The following technique was used to determine the $\sigma(I)$ for each channel for the $K$th OFF or ON observation.

$$P_K(I) = \frac{1}{M} \sum_{J=1}^{M} P(I,J),$$

$$P_K^2(I) = \frac{1}{M} \sum_{J=1}^{M} (P^2(I,J),$$

$$\sigma(I) = T_S \cdot \left( \left[ \frac{P_K^2(I) - P_K(I) \cdot P_K(I)}{M-1} \right] \right)^{1/2},$$

where $$T_S = \frac{1}{M} \sum_{J} T_S(J).$$

Thus, for each OFF or ON observation, the following is determined for each of the 413 channels:

$$T(I) = T_S \cdot P_K(I),$$

and $\sigma(I)$. The "quotient" spectra, $T_Q$ is then found from the OFF and ON measurements.

*For example: during the November 1970 observations, the last two days of observations were made with only one parametric amplifier. This doubled the system temperature and made it more unstable.
$$\bar{T}_Q(I) = \frac{T_{ON}(I)}{\bar{T}_{OFF}(I)/\bar{T}_{OFF}} - \bar{T}_{ON},$$

$$\sigma_Q(I) = \frac{T_{ON}(I)}{\bar{T}_{OFF}(I)/\bar{T}_{OFF}} \left[ \left( \frac{\sigma_{OFF}(I)}{\bar{T}_{OFF}(I)} \right)^2 + \left( \frac{\sigma_{ON}(I)}{\bar{T}_{ON}(I)} \right)^2 \right]^{1/2}$$

where \( \bar{T}_{OFF}, \bar{T}_{ON} \) are the total system temperatures off and on source. Figure 4.8 shows \( P(I) \) an off source observation for all 413 channels of the autocorrelation receiver. Then \( \sigma(I) \) is presented in Figure 4.9. As is seen in the figures, the uncertainties increase rapidly for the edge channels of each receiver. But the scattering in these points is also great. Thus, this scatter can be accounted for by doing a channel by channel weighting, which will give less weight to the edge channels.

Since the source continuum \( (T_C) \) and line \( (T_L) \) temperature varies as a function of hour angle (Section 4.2), and since even measurements made at the same hour angle can have different temperatures, it is important to also take these variations into account. The off source observations were balanced with noise injection so \( T_C \) may be determined as

$$T_C = (T_{ON} - T_{OFF}) + T_{NT},$$

where \( T_{OFF} \) is total OFF system temperature including noise,

\( T_{ON} \) is total ON system temperature, and

\( T_{NT} \) is noise injection temperature determined from
FIGURE 4.8 Digital Autocorrelation Receiver Bandpass.
calibrated settings. Each OFF-ON pair should be scaled by \(1/T_C\) to give the ratio \(T'_K(I) = T_K(I)/T_C(K)\) for each channel. This ratio would then be constant even if \(T_C\) varied, since \(T_L\) would vary proportionally. Because of the scaling, \(\sigma_K(I)\) should be changed to \(\sigma'_K(I) = \sigma_K(I)/T_C(K)\) where \(T_C(K)\) is a function of the observation \(K\).

The various observations are weighted together channel by channel as

\[
T(I) = \sum_{K=1}^{N_{\text{obs}}} \frac{T'_K(I)}{(\sigma'_K(K))^2},
\]

\[
= \sum_{K=1}^{N_{\text{obs}}} \frac{\left(T_K(I) \cdot T_C(K)\right)}{\sigma_K^2(I)}, \quad I = 1,413 \text{ channels}.
\]

The resulting spectra can be converted to line temperatures by scaling by the continuum temperature at transit.

4.6.2) Analysis of Weighted Data

After the data was averaged together using our weighting scheme, the resulting spectra was analyzed with curve fitting routines. Two basic routines were used: a non-linear technique to analyze Gaussian-shaped line profiles and/or sine wave ripples in the baseline, while including polynomial terms to account for broad baseline shapes; and a multiple linear-regression algorithm which can examine a spectra for various components, including baseline shapes, and selectively fit to the components which are
statistically significant.

The non-linear technique has been adapted from Bevington (1969). Starting from some set of initial values for the functional parameters, \( A_j \), the fitting function \( Y(X_i, A_j) \) is expanded to first order with Taylor's expansion

\[
Y(X_i, A_j) = Y(X_i, A_j) \sum_{j=1}^{N} \frac{\partial Y(X_i, A_j)}{\partial A_j} \cdot \delta A_j,
\]

where, \( X_i \) are the independent variables. This set of equations is linear in \( \delta A_j \), so a linear least squares technique is used to find the parameter increments, \( \delta A_j \).

The quality of the fit is represented by chi-squared

\[
\chi^2 = \sum_{i=1}^{N} \left( \frac{1}{\sigma_i^2} \right) \left( Y(X_i) - Y(X_i, A_j) \right)^2
\]

where \( Y(X_i) \) are the data points, with uncertainties \( \sigma_i \). The non-linear curve fitting routine finds the best fit by minimizing \( \chi^2 \). Specifically the algorithm of Marquardt (1963) is used to optimize the iterative procedure. This algorithm combines the features of a gradient search to find the direction of maximum change in \( \chi^2 \) towards a minimum, and a fit to an analytic expansion of \( \chi^2 \) about its minimum. The gradient search is dominant during the first iterations, while the fit to the analytic expansion is used for the later iterations.

Functions of the form

\[
Y(X, A_j) = A_1 \exp(-4ln2(X-A_2)^2/A_3^2) \]

\[
+ A_4 \exp(-4ln2(X-A_5)^2/A_6^2)
\]
\[ + A_7 + A_8 \cdot X + A_9 X^2 + \ldots, \]

and
\[ Y(X, A_j) = A_1 \sin \left( \frac{2\pi(X-A_2)}{A_3} \right) \]
\[ + A_4 \sin \left( \frac{2\pi(X-A_5)}{A_6} \right) \]
\[ + A_7 + A_8 \cdot X + A_9 \cdot X^2 + \ldots, \]

were frequently used to fit Gaussian shaped recombination lines or to find the significance of sine waves in the baseline.*

The multiple regression routine has been adapted from IBM Share program 3205, coded from Efroyson (1960). The independent variables, \( F_j \), and the dependent variable, \( Y_j \), are arranged in an \( N \times M \) matrix. The total number of variables including the dependent variable is \( M \), while \( N \) is the number of data points. The matrix is in the form
\[
(F_1, F_2, F_3, \ldots, F_{M-1}, Y),
\]
where each function, \( F_j \), is a column vector with \( N \) elements.

A fit of the form
\[ Y'(I) = B_1 F_1(I) + \ldots + B_{M-1} F_{N-1}(I) \]
is made. The regression is linear since \( Y'(I) \) is a linear combination of the \( F_j(I) \). The reduced chi-squared is

*These ripples were due to reflections in the 150 MHz IF line coming from the receiver to the telescope control room (\( L = 100m \)). This IF line has been replaced by a well-matched line.
\[ \chi^2_R = \frac{1}{\eta} \sum_{I=1}^{N} \left( \frac{Y(I) - Y'(I)}{\sigma_I} \right)^2, \]

where, \( \eta \), is the number of degrees of freedom which equals \( N \) minus the number of variables that are included in the fit. This quantity, \( \chi^2_R \), is minimized by the regression analysis. Some of the coefficients, \( B_j \), will be zero since only those terms are included which significantly lower \( \chi^2_R \).

The types of functions that are included in the fitting procedure are presented in Figure 4.10. They are polynomials, sine wave, hydrogen recombination line shape and the helium-3 line shape. Figure 4.11 shows: a) the initial data for M17, with 3 channels grouped together, and a polynomial baseline fit; b) the data minus the polynomial fit, along with the fitted hydrogen recombination line spectra. The grouping technique is discussed in Appendix 7.2.

Although the intensity coefficient of the hydrogen recombination line spectra is an independent variable, the resulting intensity for the \( H_{171,7} \) line can be compared with the observed intensity of the \( H_{1148} \) line to see if they are in the expected ratio. The data are presented in Chapter V.

From the final \( \chi^2 \) for a fit the scatter of the data about the fit is given by

\[ S^2 = \chi^2 / \sum_{i=1}^{N} 1/\sigma_i^2, \]
FIGURE 4.10

Linear Regression Fitting Functions
FIGURE 4.11

M17 Data and Fit

a) M17 data and fitted polynomial baseline. Antenna temperature versus frequency relative to the 50 MHz IF.

Line at 3.1 MHz is $H_{171,7}$.

Line at -3.35 MHz is $H_{213,14}$.

$\text{He}^3$ line would be at -1.50 MHz.
M17 Data and Fit

b) Polynomial baseline removed.
where $S$ is an estimate to the standard deviation. The quantity, $S$, is used to determine the limiting $\text{He}^3$ line intensity that could have been detected.
V) **Results**

The data from each observing period were processed and analyzed as described in Chapter IV. For both the August 1968 and November 1970 observing period, each off or on source observation gave a temperature spectra with an uncertainty for each channel, which was derived from the data. The on and off source spectra were then combined to lessen baseline irregularities. For each source, these corrected spectra were combined, using the detailed weighting technique discussed in Section 4.6, to give the final spectra for each source.

Tables 5.1 and 5.2 give pertinent data on all the sources which were observed in August 1968 and November 1970. The source parameters in Table 5.1 are: the source position for epoch 1950.0 in right ascension (RA) and declination (Dec); the new galactic coordinates (l\¹°, b\¹°) in degrees; the actual source size (Δα, Δδ) in minutes of arc; the velocity of the source with respect to the local standard of rest (V\(_{\text{LSR}}\)) in km/sec; the root-mean-square turbulence velocity of the source (V\(_{\text{rms}}\)); the electron temperature (T\(_{e}\)) derived from a spherical source model; the source diameter (D) in parsecs (pc); the electron density (N\(_{e}\)(cm\(^{-3}\))) assuming a spherical, uniform source; and the emission measure ξ. The emission measure defines the electron density because of the definition

\[
ξ \text{ (kpc/cm}^6\text{)} = \int_0^D N_e^2 ds.
\]
<table>
<thead>
<tr>
<th>Source</th>
<th>R. A. (1950)</th>
<th>Dec. (1950)</th>
<th>Gal. Coord.</th>
<th>Source Size (min of arc)</th>
<th>VLSR# (km/sec)</th>
<th>VLSR* (km/sec)</th>
<th>$V_{\text{rms}}$ (km/sec)</th>
<th>$T_e$ (K)</th>
<th>D (pc)</th>
<th>$N_e$ (cm$^{-3}$)</th>
<th>$\xi$ (kpc/cm$^6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W3 A</td>
<td>2$^{h}$21$^m$39$^s$</td>
<td>61$^\circ$53'00&quot;</td>
<td>133$^\circ$7</td>
<td>1.2</td>
<td>2.9 4.7</td>
<td>-43.5</td>
<td>-42.3</td>
<td>17.0±0.9</td>
<td>6800±700</td>
<td>3.3</td>
<td>330</td>
</tr>
<tr>
<td>W3 B</td>
<td>2 22 49</td>
<td>62 02 00</td>
<td>133.8</td>
<td>1.4</td>
<td>4.6 4.6</td>
<td>-50.4</td>
<td>7.2</td>
<td>9500</td>
<td>5.1</td>
<td>168</td>
<td>283</td>
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<tr>
<td>Orion A</td>
<td>5 32 44</td>
<td>-5 24 54</td>
<td>209.0</td>
<td>-19.4</td>
<td>3.8 4.3</td>
<td>-2.0</td>
<td>-2.7</td>
<td>17.8±1.0</td>
<td>7000±800</td>
<td>0.6</td>
<td>1700</td>
</tr>
<tr>
<td>Orion B</td>
<td>5 38 58</td>
<td>-1 54 12</td>
<td>206.5</td>
<td>-16.4</td>
<td>3.6 3.0</td>
<td>4.4</td>
<td>7.0</td>
<td>11.0±1.6</td>
<td>7200±800</td>
<td>0.6</td>
<td>790</td>
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<td>69 55 06</td>
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<td>40.6</td>
<td>7.0 10.2</td>
<td>-5.1</td>
<td>-2.5</td>
<td>12.6±1.5</td>
<td>6400±1000</td>
<td>1.6</td>
<td>320</td>
</tr>
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<td>NGC 6334</td>
<td>17 11 3</td>
<td>-35 54 54</td>
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<td>11.7 13.4</td>
<td>-5.9</td>
<td>-2.9</td>
<td>20.6±1.2</td>
<td>6100±830</td>
<td>1.8</td>
<td>330</td>
</tr>
<tr>
<td>W22 B</td>
<td>17 22 21</td>
<td>-34 17 36</td>
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<td>0.7</td>
<td>14.1 18.2</td>
<td>27.9</td>
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<td>18.0±2.8</td>
<td>6100±1500</td>
<td>12.8</td>
<td>48</td>
</tr>
<tr>
<td>Sag A</td>
<td>17 42 28</td>
<td>-28 59 00</td>
<td>0.0</td>
<td>0.0</td>
<td>3.7 3.7</td>
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<td>92.3</td>
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<td>4.9 7.3</td>
<td>17.6</td>
<td>17.2</td>
<td>24.8±0.8</td>
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<td>30.7</td>
<td>-0.1</td>
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<td>88.6</td>
<td>92.3</td>
<td>20.2±1.6</td>
<td>5600±860</td>
<td>15.8</td>
<td>150</td>
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<td>19 07 44</td>
<td>9 01 42</td>
<td>43.2</td>
<td>0.0</td>
<td>1.5 2.0</td>
<td>7.4</td>
<td>8.6</td>
<td>16.5±2.0</td>
<td>7700±1100</td>
<td>7.0</td>
<td>430</td>
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<td>W51 B</td>
<td>19 21 19</td>
<td>14 25 42</td>
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<td>59.1</td>
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<td>18.8±1.4</td>
<td>7600±1100</td>
<td>7.1</td>
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**Table 5.1 Source Parameters**

*(From Mezger and Hoglund, 1967)*
*(From Reifenstein, 1968)*
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<tr>
<th>Source</th>
<th>Gal. Coord. $^\text{I}$</th>
<th>Obs. Time</th>
<th>$T_a$</th>
<th>$\Delta v_{\text{HP Obs.}}$</th>
<th>$\Delta v_{\text{HP Cal.}}$</th>
<th>$\Delta v_{\text{HP He}}$</th>
<th>$T_{6\text{cm}}$</th>
<th>$T_{3.5\text{cm}}$</th>
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<tr>
<td></td>
<td>$^\text{II}$</td>
<td>ON</td>
<td>OFF</td>
<td>$^\text{K}$</td>
<td>KHz</td>
<td>KHz</td>
<td>KHz</td>
<td>°K</td>
</tr>
<tr>
<td>W 3 A</td>
<td>133.7 1.2</td>
<td>3.2 2.0</td>
<td>0.157</td>
<td>772</td>
<td>835</td>
<td>726</td>
<td>14.0</td>
<td>7.5</td>
</tr>
<tr>
<td>W 3 B</td>
<td>133.8 1.4</td>
<td>10.6 5.6</td>
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<td>-----</td>
<td>663</td>
<td>448</td>
<td>-----</td>
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<td>.838</td>
<td>807</td>
<td>865</td>
<td>757</td>
<td>76.3</td>
<td>40.0</td>
</tr>
<tr>
<td>Orion B</td>
<td>206.5 -16.4</td>
<td>2.6 1.0</td>
<td>-----</td>
<td>-----</td>
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<td>527</td>
<td>11.8</td>
<td>7.0</td>
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<tr>
<td>NGC 6334</td>
<td>351.4 0.7</td>
<td>0.5 0.2</td>
<td>.154</td>
<td>540</td>
<td>698</td>
<td>571</td>
<td>17.6</td>
<td>5.2</td>
</tr>
<tr>
<td>W 22 B</td>
<td>353.2 0.7</td>
<td>0.5 0.5</td>
<td>.200</td>
<td>802</td>
<td>840</td>
<td>740</td>
<td>15.5</td>
<td>4.6</td>
</tr>
<tr>
<td>M 16</td>
<td>16.9 0.8</td>
<td>1.0 0.6</td>
<td>-----</td>
<td>.15</td>
<td>850</td>
<td>756</td>
<td>4.3</td>
<td>1.2</td>
</tr>
<tr>
<td>M 17</td>
<td>15.0 -0.7</td>
<td>2.0 0.8</td>
<td>.452</td>
<td>1050</td>
<td>1090</td>
<td>1015</td>
<td>79.1</td>
<td>33.0</td>
</tr>
<tr>
<td>W 43</td>
<td>30.7 -0.1</td>
<td>5.3 1.8</td>
<td>.308</td>
<td>802</td>
<td>916</td>
<td>837</td>
<td>16.1</td>
<td>7.1</td>
</tr>
<tr>
<td>W 49 A</td>
<td>43.2 0.0</td>
<td>2.4 1.2</td>
<td>.114</td>
<td>750</td>
<td>844</td>
<td>720</td>
<td>12.8</td>
<td>9.6</td>
</tr>
<tr>
<td>W 51 B</td>
<td>49.5 -0.4</td>
<td>7.8 5.0</td>
<td>.364</td>
<td>958</td>
<td>909</td>
<td>801</td>
<td>23.9</td>
<td>15.0</td>
</tr>
</tbody>
</table>

**TABLE 5.2**

Observational Parameters for August 1968
These quantities were found from the H\textsubscript{109α} recombination line survey by Reifenstein et al. (1970).

The observational parameters for August 1968 are presented in Table 5.2. The parameters for each source are: the source galactic coordinates; the He\textsuperscript{3} observation time both on and off source; the observed H\textsubscript{114β} line temperature (T\textsubscript{114β}); the observed H\textsubscript{114β} line width (Δν\textsubscript{HP}); the hydrogen line width calculated (see Section 2.2) from the electron temperature (T\textsubscript{e}) and turbulence velocities in Table 5.1; the He\textsuperscript{3} line width calculated from T\textsubscript{e} and V\textsubscript{rms}; the source continuum antenna temperature found at 6 cm by Reifenstein et al. (1970); and the source antenna temperature at 3.5 cm, extrapolated from the 6-cm data.

The August 1968 observations used frequency switching with three different spacings between the signal and reference band (see Figure 4.6). These different combinations were combined as explained in Appendix 7.1. A three channel grouping was done to give the data presented in Figure 5.1. The 4σ error bars are a measure of the peak-to-peak scatter, ΔT\textsubscript{pp}, in the final data points. Further grouping was done to optimize the signal-to-noise ratio. The ΔT\textsubscript{pp} from this optimum grouping is given in Table 5.3. This ΔT\textsubscript{pp} is the minimum line temperature that could have been detected. The 4σ limit allows for statistical scatter in the data and uncertainties in the baseline fit. For statistical scatter with a Gaussian distribution, the
FIGURE V-1a  300 KHz Data for all Sources
<table>
<thead>
<tr>
<th>SOURCE</th>
<th>$T_C$ ($^\circ$K)</th>
<th>$T_{A(H_{14}\beta}$ ($^\circ$K)</th>
<th>$\tau_{ON}$ (HRS)</th>
<th>$\tau_{OFF}$ (HRS)</th>
<th>$\Delta T_{pp}$ (THEORY) (m $^\circ$K)</th>
<th>$\Delta T_{pp}$ (m $^\circ$K)</th>
<th>$\Delta T_b$ (m $^\circ$K)</th>
<th>$\frac{N(He^3)}{N(H)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W3B</td>
<td>7.5</td>
<td></td>
<td>10.6</td>
<td>5.6</td>
<td>7.0</td>
<td>8</td>
<td>17</td>
<td>3.6x10^-5</td>
</tr>
<tr>
<td>Orion B</td>
<td>7.0</td>
<td></td>
<td>2.6</td>
<td>1.0</td>
<td>14.3</td>
<td>15</td>
<td>33</td>
<td>18</td>
</tr>
<tr>
<td>NGC 6334</td>
<td>5.2</td>
<td>0.154</td>
<td>0.5</td>
<td>0.2</td>
<td>31.9</td>
<td>40</td>
<td>62</td>
<td>27</td>
</tr>
<tr>
<td>W22 B</td>
<td>4.6</td>
<td>0.200</td>
<td>0.5</td>
<td>0.2</td>
<td>31.7</td>
<td>17</td>
<td>25</td>
<td>*22.</td>
</tr>
<tr>
<td>M16</td>
<td>1.2</td>
<td>0.150</td>
<td>1.0</td>
<td>0.6</td>
<td>21.8</td>
<td>16</td>
<td>22</td>
<td>*15.</td>
</tr>
<tr>
<td>W43</td>
<td>7.1</td>
<td>0.308</td>
<td>5.3</td>
<td>1.8</td>
<td>10.2</td>
<td>12</td>
<td>22</td>
<td>4.5</td>
</tr>
<tr>
<td>W49 A</td>
<td>9.6</td>
<td>0.114</td>
<td>2.4</td>
<td>1.2</td>
<td>15.3</td>
<td>19</td>
<td>130</td>
<td>13.</td>
</tr>
</tbody>
</table>

*Calculated for theoretical $\Delta T_{pp}$.

TABLE 5.3

August 1968 RESULTS
± 2σ limit gives a 95% confidence level for the limit.

In addition to the above analysis, a detailed weighting and fitting was done on M17. The grouped data for one frequency switching combination was fit to a polynomial baseline, in addition to the complex helium-3 and hydrogen spectral shapes. The fitted shapes are shown in Figure 5.2a, with the data points and the final fits shown in 5.2b. The error bars on the points are plus and minus one sigma. This marginal detection of the (He$^3$)$^+$ hyperfine line was not confirmed by the higher quality data of November 1970. The "detection" may have been due to baseline irregularities or to the complex baseline caused by frequency switching. The detailed weighting and fitting has not been applied to other sources observed in August 1968, because it is estimated that the He$^3$ limits of Table 5.3 would not change by more than 15% if the analysis was done.

In November 1970 only one frequency band, centered at 8667.150 MHz, was used, with no frequency switching. Both off and on source measurements were made at this one frequency. The (He$^3$)$^+$ hyperfine line was at -1.500 MHz relative to the center of the 10 MHz receiver bandwidth. In addition, the hydrogen recombination line, H171,7, was in the observed band at +3.065 MHz (see Figure 4.7). All of the frequencies used in the November 1970 data reduction were relative to 8667.150 MHz.
3-CHANNEL GROUPING

$T_A(\circ K)$

$10^{-3} \circ K$

$\alpha$-FITTED BASELINES

$\beta$-FINAL FIT

FIGURE IV-2
The data from the four H-II regions observed in November 1970 (W3A, Orion A, M17, and W51B) was subjected to a linear regression analysis as detailed in Section 4.6. Only a polynomial baseline and the hydrogen recombination line shape were found to be statistically significant fitting functions. The spectra with the polynomial baseline subtracted are shown in Figures 5.3-5.7 for a 4 channel grouping (channel spacing is 10 MHz/192 = 52.1 KHz). The figures are the spectra and residuals from W3A, Orion A, M17, W51B and Virgo A respectively. The first four sources are the primary H-II regions of our search. The calibration source, Virgo A, is a non-thermal source with only continuum emission. This source was observed with the same procedures used on the H-II regions, in order to check the observing techniques. Also, the data from Virgo A was run in parallel to the H-II region data analysis.

The November 1970 data analysis yields the antenna temperature for the $H_{171.7}$ line and the scatter, $S$, of the residual points. The scatter, $S$, is defined by

$$S^2 = \frac{1}{\eta} \left\{ \sum_{I=1}^{N} [T(I) - T_{fit}(I)]^2 / \sigma^2(I) \right\} + \sum_{I=1}^{N} [1/\sigma^2(I)],$$

where, $\eta$, is the number of degrees of freedom, $T(I)$, the data points; $T_{fit}(I)$, the fitted function, and $\sigma(I)$ the uncertainty in each point. The upper limit to the antenna temperature for the $({\text{He}}^3)^+$ hyperfine line emission is $\Delta T_{pp} = 4S$. This limit, $\Delta T_{pp}$, is given in Table 5.4 for
<table>
<thead>
<tr>
<th>Source</th>
<th>$T_C$ °K</th>
<th>$T_{171.7}$ m°K</th>
<th>$T_{ON}$ HRS</th>
<th>$\Delta T_{rms}^{*}$ Theory m°K</th>
<th>$\Delta T_{pp} = 4 \cdot S$ m°K</th>
<th>$\frac{N(\text{He}^3)}{N(\text{H})}$ LIMIT</th>
<th>$\frac{N(\text{He}^4)}{N(\text{H})}$ #</th>
<th>$\frac{N(\text{He}^3)}{N(\text{He}^4)}$ LIMIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>W3A</td>
<td>11</td>
<td>10±2</td>
<td>6.6</td>
<td>6.3</td>
<td>5.9</td>
<td>5.7x10⁻⁵</td>
<td>0.081</td>
<td>7.0x10⁻⁴</td>
</tr>
<tr>
<td>Orion A</td>
<td>45</td>
<td>36±7</td>
<td>11.9</td>
<td>6.5</td>
<td>15.8</td>
<td>14x10⁻⁵</td>
<td>0.083</td>
<td>17</td>
</tr>
<tr>
<td>M17</td>
<td>34</td>
<td>53±4</td>
<td>10.8</td>
<td>6.2</td>
<td>11.0</td>
<td>5.3x10⁻⁵</td>
<td>0.090</td>
<td>5.9</td>
</tr>
<tr>
<td>W51 B</td>
<td>18</td>
<td>24±3</td>
<td>7.3</td>
<td>6.5</td>
<td>13.4</td>
<td>7.1x10⁻⁵</td>
<td>0.081</td>
<td>8.8</td>
</tr>
<tr>
<td>Virgo</td>
<td>11</td>
<td>——</td>
<td>4.7</td>
<td>7.5</td>
<td>7.2</td>
<td>——</td>
<td>——</td>
<td>——</td>
</tr>
</tbody>
</table>

**TABLE 5.4**

November 1970 Results

*Calculated for $T_R = 80°K$, with $T_{total} = T_R + T_C$.

#From Palmer et al. (1969).
FIGURE 5.3

November 1970 Data

-W3A.
FIGURE 5.4

November 1970 Data
-Orion A
FIGURE 5.5

November 1970 Data

M17
FIGURE 5.6

November 1970 Data

W51
FIGURE 5.7

November 1970 Data

Virgo
a fit with twelve channel grouping. This gives a resolution of 755 KHz, which optimizes the signal-to-noise ratio for the sources of the November 1970 observations (see Appendix 7.2).

The limiting antenna temperatures for the \((\text{He}^3)^+\) hyperfine line emission are listed in Tables 5.3 and 5.4 for August 1968 and November 1970. Table 5.3 contains data on the sources observed only in August 1968, since the limits for the remaining sources were lowered in November 1970. These limits, \(\Delta T_{pp}\), are antenna temperatures. The limiting brightness temperatures, \(\Delta T_B\), are related to the antenna temperatures, \(\Delta T_{pp}\), as shown in Section 4.2. That discussion shows

\[
\Delta T_B = \frac{\lambda^2}{K_e} \frac{\Omega_{\text{obs}}}{\Omega_M \Omega_S} \Delta T_{pp},
\]

\[
= \frac{\Omega_A}{\Omega_M} \frac{\Omega_{\text{obs}}}{\Omega_S} \cdot \Delta T_{pp},
\]

\[
= \frac{1}{\varepsilon_B} \left[ 1 + (\theta_H/\Delta \alpha)^2 \right]^{1/2} \cdot \left[ 1 + (\theta_E/\Delta \delta)^2 \right]^{1/2} \Delta T_{pp},
\]

where, \(\varepsilon_B\) is the beam efficiency, \(\theta_H, \theta_E\) the beam half-power widths, and \(\Delta \alpha, \Delta \delta\) the source widths in right ascension and declination. As was shown in Section 2.2, this limiting brightness temperature is directly related to the maximum number of \((\text{He}^3)^+\) ions along the line of sight, within the antenna beam.
\[ N[(\text{He}^3)^+] = 1.3 \times 10^{13} \Delta T_B \cdot \Delta \nu_{HP}, \]

where, \( \Delta T_B \) is in degrees Kelvin, \( \Delta \nu_{HP} \) in Hertz, and \( N(\text{He}^3)^+ \) in atoms per square centimeter.

To find the helium-3 to hydrogen ratio, it is necessary to determine the number of hydrogen atoms along the line of sight for each source. Within each source the electron density is related to the H and He\(^4\) number densities by

\[ N_e = N(H) + N(\text{He}^4), \]

\[ = N(H) \left[ 1 + N(\text{He}^4)/N(H) \right]. \]

This assumes that all of the hydrogen and helium in the source is singly ionized. The quantity, \( N(\text{He}^4)/N(H) \), has been found from studies of adjacent hydrogen and helium-4 recombination lines such as \( \text{H}_109\alpha \), \( \text{He}_{109}\alpha \) (Palmer et al. 1969). Other ionized atoms which contribute to the number of electrons are not abundant enough to be significant. The number of electrons along the line of sight is found by assuming that the source depth is equal to the diameter \( D \). Since the helium-3 and hydrogen ionization zones have the same depth, the number densities of \( \text{He}^3 \) and H are related by

\[ \frac{N(\text{He}^3)}{N(H)} = 1.3 \times 10^{13} \Delta T_B \cdot \Delta \nu_{HP} \frac{[1 + N(\text{He}^4)/N(H)]}{N_e \cdot D}. \]

where \( N_e \) is the electron density in cm\(^{-3}\), and \( D \) the
source diameter in centimeters. When \( T_B \) is in \((10^{-3} \, \text{°K})\), \( \Delta v_{\text{HP}} \) in MHz, and \( N_e \cdot D \) in units of \(10^{21} \, \text{cm}^{-2} \), the ratio becomes

\[
\frac{N(\text{He}^3)}{N(\text{H})} = 1.3 \times 10^{-5} \frac{\Delta T_B \cdot \Delta v_{\text{HP}}}{N_e \cdot D} \left[ 1 + \frac{N(\text{He}^4)}{N(\text{H})} \right].
\]

This relationship was used to find the upper limit for the \( \text{He}^3 \) to \( \text{H} \) number ratio. This ratio is given in Table 5.3 for the August 1968 observations and in Table 5.4 for November 1970. In addition to this data, for November 1970, the observed \( \text{H}_{114\beta} \) line temperature is extrapolated in Table 5.5 to predict the \( \text{H}_{171,7} \) line temperature assuming local thermodynamic equilibrium (LTE). If the line widths are the same for both lines the line temperatures are related by

\[
\frac{T_{171,7}}{T_{114\beta}} = \frac{N_{178}}{N_{116}} \frac{A_{178,171}}{A_{116,114}},
\]

where \( A_{178,171}, A_{116,114} \), are the Einstein coefficients for the transitions \( 178 \rightarrow 171 \) and \( 116 \rightarrow 114 \) respectively and \( N_{178}/N_{116} \) is the ratio of atoms in the \( n = 178 \) state to those in the \( n = 116 \) state. This ratio is given by

\[
\frac{N_{178}}{N_{116}} = \frac{2 \cdot (178)^2}{2 \cdot (116)^2} \exp \left[ \frac{-(E_{178} - E_{116})}{kT_e} \right].
\]

Since the energy difference between the 178 and 116 states \( (E_{178} - E_{116}) \) is so small, the exponential factor equals one. Combining these results gives the result
<table>
<thead>
<tr>
<th>Source</th>
<th>T148 (°K)</th>
<th>T171,7 Predicted (mK)</th>
<th>T171,7 Measured (mK)</th>
<th>R Predicted</th>
<th>R Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>W3A</td>
<td>0.196</td>
<td>0.838</td>
<td>75</td>
<td>2.1</td>
<td>0.75</td>
</tr>
<tr>
<td>Orion A</td>
<td>0.452</td>
<td>40</td>
<td>55±4</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>M17</td>
<td>0.364</td>
<td>33</td>
<td>24±3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 5.5**

H171,7 Line Measurements
for LTE as

$$T_{171,7} = 0.0894 \cdot T_{1148}.$$ 

The implications of the $\text{He}^3$ limits and on the observed $\text{H}_171,7$ line temperatures are discussed in the following chapter.
VI) Conclusions

The resulting helium-3 hydrogen limits for the sources observed only in August 1968 range from $3.6 \times 10^{-5}$ for W3B and $4.5 \times 10^{-5}$ for W43 to $27 \times 10^{-5}$ for NGC 6334. The lowest limits result from long integration times which lower the limiting brightness temperature, and high hydrogen concentrations along the line of sight. For 1970 the limits for W3A and M17 are $5.7 \times 10^{-5}$ respectively. This is from an optimum grouping of twelve channels and is the 4 S limit. All of these limits have uncertainties of about 20%, with half of these uncertainties due to errors in the antenna temperature calibration and the remaining errors due to unknowns in the source parameters $N_e$ and $D$.

One other attempt at measuring the $(\text{He}^3)^+$ hyperfine line has been made by Seling and Heiles (1968). However they were somewhat optimistic in reporting their results. Their upper limit to the helium-3 line temperature corresponded to a helium-3 to hydrogen ratio of $(3.7 \pm 5.5) \times 10^{-5}$. They quoted a result of $3.7 \times 10^{-5} = 4 \times 10^{-5}$. It appears that a more realistic limit is $4 \sigma = 20 \times 10^{-5}$, or a factor of 5 larger than the quoted result.

Our observed limits are a factor of 1.5 to 2.0 larger than the observed $\text{He}^3$ to $\text{He}^4$ ratio of $2.7 \times 10^{-4}$ for the solar system abundances (Goldwire and Goss 1967b). This value was determined from meteorite samples and represents the elemental ratio 4.6 billion years ago when the solar
system was formed. The limits in the observed H-II regions are for the present time. Thus, a limit is set on the amount of He\(^{3}\) production that has occurred in the last 5 billion years. This limit falls within the behavior of the galactic evolution model developed by Talbot and Arnett. Their results for the galactic gas in the solar neighborhood indicate that the He\(^{3}\) abundance would have slowly increased over the last 4.6 billion years to be about 50% greater than the present amount. Unfortunately our experiment has not been able to confirm this result, which is just under the observation limit.

However, even the solar system abundance of He\(^{3}\) would be detectable with a factor of five improvement in sensitivity.

A larger telescope would be needed to resolve the most intense portions of the H-II regions, and a receiver with a 30 °K noise temperature is needed to utilize the observing time as effective as possible. With this type of system the limits of detection would be determined by instrumental and baseline effects.

The H\(_{171,7}\) line measurements show a significant deviation from the local thermodynamic equilibrium (LTE). Since this is the highest order recombination line to ever have been measured, these results will be very useful in doing a non-LTE analysis of H-II regions.

In summary, the limiting helium-3 to hydrogen number ratio has been set at 4 X 10\(^{-5}\). This is consistent with the
production of $\text{He}^3$ in low and intermediate mass stars over a time of several billion years.
VII) Appendices

7.1) Fifty-Channel Receiver Data

This section discusses the processing done by the DDP-116 computer to the on-line, frequency-switched data. First the effects of channel to channel gain variations, and receiver gain changes are considered. Then the subsequent data reduction techniques are discussed. These include: weighting by time, weighting by $1/\sigma^2$, and editing of the data. The details of the fifty-channel receiver and the on-line processing are discussed by Ballister (1968).

7.1.1) On-Line Processing

Receiver and source temperatures are defined below

\[ T_R = \text{Receiver and antenna noise temperature}, \]
\[ T_{\text{Cal}} = \text{Calibration noise temperature}, \]
\[ T_0 = \text{Source antenna temperature at signal frequency, } f_0, \]
\[ T_1 = \text{Source antenna temperature at reference frequency, } f_1, \]
\[ G_0 = \text{Gain at signal frequency, } f_0, \]
and \[ G_1 = (1+\alpha) G_0 = \text{Gain at reference frequency, } f_1. \]

The quantities measured in each of the 50 channels are
\[ A = G_0 \left(T_R + T_0\right) = \text{Power from signal band}, \]
\[ B = G_1 \left(T_R + T_1\right) = \text{Power from reference band}, \]
and
\[ C = G_0 \left(1 + \alpha\right) \left(T_R + T_0 + T_{\text{cal}}\right) = \text{Power from signal band and calibration temperature}. \]

The change in receiver gain due to injecting calibration noise is accounted for by \(1 + \beta\). This gain change occurred because the paramp control circuits were not isolated from the noise lamp circuits.

The computer assumes \(G_0 = G_1 = G\) for each channel. The quantity \((A-B)/B\) is found for each channel. Also for each channel \(B/(C-A)\) is calculated and averaged over the 50 channels to give an average \([B/(C-A)]\).

The quantity
\[
\left(\frac{A-B}{B}\right) \left(\frac{B}{C-A}\right) = \frac{T_0 - T_1}{T_{\text{cal}}} \]

is the output for each channel in the magnetic tape record and oscilloscope display. Since \(G_0 \neq G_1\), for a real system, it must be determined what \(\frac{A-B}{B} \left(\frac{B}{C-A}\right)\) represents. For one channel
\[
\frac{B}{C-A} = \frac{G_0 \left(1+\alpha\right) \left(T_R + T_1\right)}{G_0 \left[\beta \left(T_R + T_0\right) + T_{\text{cal}}\right]} = \frac{(1+\alpha) \left(T_R + T_1\right)}{T_{\text{cal}}} \]

Thus the actual \(T'_{\text{cal}}\) is composed of a term due to the receiver gain change and the intended calibration noise, \(T_{\text{cal}}\). Also for one channel
\[
\frac{A-B}{B} = \frac{G_0 \left(T_R + T_0\right) - (1+\alpha) G_0 \left(T_R + T_1\right)}{G_0 \left(1+\alpha\right) \left(T_R + T_1\right)}, \]

which gives
\[
\frac{A-B}{B} = \frac{(T_0 - T_1) - \alpha(T_R + T_1)}{(1+\alpha)(T_R + T_1)}.
\]

If the quantity \( B/(C-A) \) is averaged, then

\[
\overline{\frac{B}{C-A}} = \frac{(1+\alpha)(T_R + T_1)}{T_{cal}}.
\]

Assuming

\[
(1+\alpha)(T_R + T_1) =
\]
gives

\[
T_L = \left(\frac{A-B}{B}\right)\overline{\frac{B}{C-A}} = \frac{T_0 - T_1 - \alpha(T_R + T_1)}{T_{cal}}.
\]

For most scans

\[
\left| \left(\frac{A-B}{B}\right)\overline{\frac{B}{C-A}} \right| < 10\%
\]

so the imbalance was less than 0.1 \( T'_{cal} \) \( < 1 \) °K. Thus \( \alpha \) is usually small. The gain change, \( \alpha \), is a function of the channel, so a modulation of the gain due to VSWR effects in the feed or circulators can cause non-random effects in the observed baseline. These effects appear to have been small for our scans, except for channels 43 and 49. These channels had a significant gain variation between the signal and reference frequency bands. This was probably due to effects in the fifty-channel receiver. Fortunately this variation was the same amount for off and on source observations, so subtracting the OFF from the ON corrected this effect. But that is the purpose of the OFF observations.
7.1.2) **Subsequent Data Reduction**

The first data analysis of the August 1968 observations involved weighting records by time only. For each off or on source observation, the mean temperature was found for each channel from the individual 10 second records. Since each OFF was used with two ON's, the total off source time was half that spent on source and the noise of the OFF was $\sqrt{2}$ times that of the ON's. To decrease this noise, the OFF's were treated with a seven-channel smoothing function of approximately Gaussian shape. This Gaussian shape had a width of $\sqrt{3}$ channels to decrease the effective resolution of the fifty channel receiver from 100 kHz to $\sqrt{[(100)^2 + 300]} = 200$ KHz.

Increasing the effective channel width by a factor of two, decreases the noise by $\sqrt{2}$ since

$$\Delta T = 1/\sqrt{\Delta f}.$$  

Then the smoothed off (SOFF) was subtracted from the on source observations. This procedure gave satisfactory results except for channels 43 and 49, which had unusually high gain variations between signal and reference. The smoothing procedure distributes this gain variation to the adjacent channels so that the gain variations in the ON are not as well accounted for when the ON-OFF is computed. Figure 7.1 illustrates this point very well. The data is for a $H_{1148}$ line observation on M17. The four parts of the figure are: (a), the on source observation; (b), the
FIGURE 7.1 Effects of Smoothing on Fifty-Channel Data.
off source observation; (c), the ON-OFF with a linear baseline subtracted; and (d), the ON-SOFF with a linear baseline subtracted. The gain variations in channels 43, and 49 are shown by the circled points in (a), (b), and (d). Even though there are significant deviations from the baseline for those two points, the ON-OFF shows no sign of this effect. For this data the on source time was 15 minutes and the off source time 5 minutes. When the OFF is smoothed the resulting ON-SOFF gives better quality line shape but it does not account for the deviant channels. When using the ON-SOFF to determine the line temperature and width, points 43 and 49 were not used in the fitting routine.

After this weighting by time was done for each frequency switched combination (see Section 4.5.1 and Figure 4.6) on a source, the results from the three combinations had to be put together to obtain the best signal to noise ratio. The (He$^3$)$^+$ hyperfine line was not in the same channel for the three combinations so less than fifty channels overlapped for the weighting. There was no appropriate integration time with which to weight the various combinations together. Therefore, the deviation of the baseline points about the linear fit was used as a sigma with which to combine the data. The linear fit was done only on the channels not expected to contain the He$^3$ line. This technique yields the data for each source which was presented in Figure 5.1.

The utility of weighting by $1/\sigma^2$ suggested that this method should be used throughout the analysis of the August
1968 data. The technique was applied by computing $T_{ik}$, and $\sigma_{ik}$ for each channel $i$ of the $k^{th}$ off or on observation, where

$$T_{ik} = \frac{1}{N} \sum_{j} T_{ijk}, \quad i = 1/N \text{ records}$$

$$\bar{T}_{ik}^Z = \frac{1}{N} \sum_{j} T_{ijk}^2,$$

and

$$\sigma_{ik}^2 = (\bar{T}_{ik} - \bar{T}_{ik}^Z)(N-1).$$

The number of records, $N$, in an observation was 90 for a 15 minute observation. Then the off and on source observations were combined to give a corrected spectra

$$\bar{T}_{ik} = \bar{T}_{ik}(\text{ON}) - \bar{T}_{ik}(\text{OFF}),$$

and

$$(\sigma_{ik}')^2 = [\sigma_{ik}^2(\text{ON}) + \sigma_{ik}^2(\text{OFF})].$$

Then the observations for a particular frequency-switched combination for a source were combined to give

$$T_i = \sum_{k=1}^{M_{\text{obs}}} \frac{\bar{T}_{ik}/(\sigma_{ik}')^2}{1/(\sigma_{ik}')^2},$$

and

$$\sigma_{i}^{-2} = \sum_{k=1}^{M_{\text{obs}}} [1/(\sigma_{ik}')^2].$$

This data was then subjected to multiple linear regression analysis. The results of this type of fitting is illustrated for M17 in Figure 5.2.

Editing of each observation was tried but it was not found to appreciably change the final data. The editing was done after the quantities $\bar{T}_{ik}$, and $\sigma_{ik}$ had been deter-
mined for each observation. Then the individual data points were checked to see if

$$|T_{ijk} - T_{ik}| > 3.2 \sigma_{ik}.$$  

A record in which any point met this criteria was rejected. After editing, the \( T \) and \( \sigma \) for an observation was recalculated using only good records. The factor 3.2 was chosen to reject only those points which were very deviant. Only about 1% of the records were rejected by editing. The technique is not worth the time, unless some very suspect data is being evaluated.
7.2) **Data Optimization**

The first part of this appendix discusses the smoothing of data, and considers the optimum techniques. Part two of this section then describes the generation of numerical random "noise" which has a Gaussian distribution. This "noise" is used in testing curve fitting and smoothing schemes.

7.2.1) **Data Smoothing and Grouping**

The spectral data obtained in radio astronomy is often smoothed to reduce the noise fluctuations, and hopefully, to improve the signal-to-noise ratio. This section discusses various smoothing and grouping techniques; how they are used; and what effects these techniques have on line shapes, etc.

For a discrete set of data, various forms of smoothing are done, which can all be represented by the following operation

$$D_i = (D*S)_i = \frac{\sum_{k=\text{MAX}(1,i-J)}^{\text{MIN}(N,i+J)} D_k S_{k-i}}{\sum_{k=\text{MAX}(1,i-J)}^{\text{MIN}(N,i+J)} S_{k-i}}$$

where, $D_i$, $i=1,N$ are the data points for the $N$ observations (channels, etc.), $S_i$, $j = -J, \ldots, -1,0,0,\ldots, +J$ are the
(2J+1) points for the discrete smoothing operation. The
discrete convolution of the data and the smoothing function
is represented by D * S. The smoothed data is D. The sum-
mation limits are determined so that 1 ≤ k ≤ N, whatever the
number of channels in the smoothing function S.

Some commonly used weighting functions are

\[ S_j = 0.25, 0.5, 0.25 \text{ [hanning weighting]}, \]

\[ S_j = 0.23, 0.54, 0.23 \text{ [hamming weighting]}, \]

(see Blackman and Tukey 1958), and

\[ S_j = \frac{1}{2J+1}, \frac{1}{2J+1}, \ldots, \frac{1}{2J+1}, \frac{1}{2J+1} \text{ [running mean]}. \]

The utility of any smoothing function ties in how well
the noise is reduced, and what extraneous features are
introduced by the convolution. For radio astronomy data,
even more important are the effects of the smoothing on
line shapes being sought in the data. In radio astronomy
typical line shapes are Gaussian or Lorentzian. These
line shapes may not retain their character when treated
with an arbitrary smoothing function. As an example see
Figure 7.2 which shows a Gaussian smoothed with a rectangu-
lar function of unit area

\[ S(x) = \frac{1}{\Delta s}, |x| < \Delta s/2, \]

\[ = 0, |x| > \Delta s/2, \]

where \( \Delta s \) is the width of the function. The convolution
was done numerically for \( \Delta s/\Delta x = 0, 0.4, 0.6, 1.0, 1.2, 
1.5, 2.0 \). The convolution was performed on a Hewlett-
Figure 7.2 Convolutions of a Gaussian with Rectangular Functions
Packard 9100A Calculator using the 9125A Calculator Plotter to draw the convoluted functions. The curve for $\Delta s/\Delta x = 0$ represents the original Gaussian. The dashed curve is a Gaussian whose height equals that of the convoluted function for $\Delta s/\Delta x = 2.0$. This convoluted function has a wider half-power width than the Gaussian function and the wings of the convoluted function disappear more quickly than those of the Gaussian. Although this was an extreme example of smoothing, it demonstrates how a Gaussian function can be deformed by smoothing. This consideration is especially important if spectral features which are composites of Gaussian and Lorentzian functions are being analyzed.

Channel grouping is the same as doing a running mean, but with less number of final data points. If smoothing is done on $N$ data points the smoothed data contains $N$ points which are no longer independent. Channel grouping does improve the signal-to-noise while only retaining those points which are independent. These independent points were then subjected to the statistical curve fitting techniques described in Section 4.6.

Let a spectral feature with an expected half-power width $\Delta \nu$ be observed with an instrumental resolution $\Delta f$. Any smoothing which is applied to the data will increase the line width and the instrumental width $\Delta f$. If the spectral line, instrument and smoothing functions all have Gaussian shapes then the effective line width and resolution after smoothing are
\[ \Delta v' = \sqrt{(\Delta v^2 + \Delta s^2)}, \]
\[ \Delta f' = \sqrt{(\Delta f^2 + \Delta s^2)}, \]

where \( \Delta s \) is the half-power width of the smoothing function. The above equations are approximately true for non-Gaussian shapes. Since the area under a curve is constant the line signal, \( s \), varies inversely as the line width \( \Delta v' \). But the noise varies inversely as the square root of the instrumental bandwidth. Thus, the signal to noise is

\[ \frac{S}{N} = \frac{\sqrt{\Delta f'}}{\Delta v'}. \]

In order to optimize this quantity the derivation of \( s/N \) is taken with respect to \( \Delta s \). Setting this to zero gives

\[ \Delta s = \sqrt{(\Delta v^2 - 2\Delta f^2)}. \]

For most measurements

\[ \Delta v \gg \Delta f \]

so

\[ \Delta s = \Delta v. \]

Thus for most experiments the optimum signal to noise ratio is obtained by smoothing or grouping so that the smoothing or grouping function is the same width as the anticipated line width.

7.2.2) Random Noise Generation by Numerical Techniques

The power residue method is used to generate numbers with a uniform distribution over the interval \((-1/2, 1/2)\), and these numbers are used in generating random numbers with a Gaussian distribution.
The first step in the power residue method is to select an odd integer \( x \) and multiply it by a carefully selected odd integer \( \gamma \). The product \( x \cdot \gamma \) is divided by an integer \( m \) and the remainder designated as \( z_1 \). In the notation of number theory \( x \cdot \gamma = z_1 \pmod{m} \). (For instance, \( 11 = 3 \pmod{8} \); \( 37 = 2 \pmod{7} \).) Thus, \( 0 < z_1 < m \). The integers \( x \), \( \gamma \), \( m \) are chosen so that the greatest common divisor of \( (x \) and \( m \) and \( (\gamma \) and \( m \) is 1. The number \( n \) on the interval \((-1/2, + 1/2)\) is found by \( n = (z_1/m) - 1/2 \). Further terms in the sequence are derived by finding the quantities \( \frac{z_1}{x} \cdot \frac{x}{z_2} = \frac{z_2}{m} \) and \( n_2 = \frac{z_2}{m} - 1/2 \), etc.

When using a computer which operates in binary arithmetic \( m \) is selected as a power of 2. Thus \( x \) and \( \gamma \) need only be odd in order to have no common factors with \( m \). The multiplier \( x \) should be selected so that it is an odd integer of the form \( 8t \pm 3 \). In that case there will be \( m/4 \) terms in the sequence before it starts repeating. The inverse of \( x \) should also be studied. The inverse, \( x^{-1} \), is defined by \( x \cdot x^{-1} = 1 \pmod{m} \). The multiplier \( x \) should be selected to be as large as possible as long as \( x^{-1} > x \). This insures that the numbers will appear randomly on the interval \((-1/2, + 1/2)\) (IBM Form C20-8011).

The Burroughs 5500 computer at Rice University has a 39 bit integer so \( m \) was chosen as \( 2^{39} \). The multiplier \( x \) was chosen to be \( 2^{36} + 3 = 6 \ 87194 \ 76739 \). Its inverse \( \pmod{m} \) is \( 11 \ 45324 \ 61227 \). The multiplying and finding of the remainder are done in double precision arithmetic which uses
78 bits. This is done to insure that no bits are lost due to round-off or truncation.

Once numbers are available with a uniform distribution, the question of how these can be used to simulate noise arises. Theoretical noise has a Gaussian distribution:

\[ P(x) = (2\pi \sigma^2)^{-1/2} \exp\left(-\frac{(x-\bar{x})^2}{2\sigma^2}\right), \]

where \( \bar{x} \) is the mean of the distribution and is the standard deviation.

If two numbers which are randomly distributed on the interval \((-1/2, 1/2)\) are added together, the resultant number no longer has a uniform distribution. The resultant distribution is the convolution of the initial functions.

Defining \( F_1(x) = \begin{cases} 0, & 1/2 < |x| \\ 1, & 0 \leq |x| < 1/2 \end{cases} \), then

\[ F_2(x) = \int_{-\infty}^{+\infty} F_1(u) F_1(x-u) \, du = \begin{cases} 0, & 1 < |x| \\ 1 - |x|, & 0 \leq |x| < 1 \end{cases}. \]

The sum of three random numbers has a distribution \( F_3(x) \), where

\[ F_3(x) = \int_{-\infty}^{+\infty} F_2(u) F_1(x-u) \, du. \]

The sum of \( K \) random numbers has a distribution whose standard deviation is \((K/12)^{1/2}\). The first three distributions are sketched in Figure 7.3 along with the distribution for \( K = 12 \).

As a consequence of the central limit theorem the distribution rapidly approaches a Gaussian (Hamming 1962). This is seen as Figure 7.3. The normalized Gaussian function
\[ G_\kappa (\gamma) = \sqrt{\frac{6}{\kappa \pi}} \, e^{-\frac{\gamma^2}{\kappa}} \]

\[ \Delta A_\kappa = \int_{-\infty}^\infty |F_\kappa (\gamma) - G_\kappa (\gamma)| \, d\gamma \]

\[ \Delta A_2 = 8.8\% \]

\[ \Delta A_3 = 5.2\% \]

\[ \Delta A_4 = 3.8\% \]

\[ \Delta A_{12} = 1.2\% \]

FIGURE 7.3. Central limit functions
with $\sigma = \sqrt{K/12}$ is also presented in Figure 7.3. For the computer calculation of Gaussian shaped noise, $K = 12$ was selected to give $\sigma = 1$.

The numbers generated by this process were tested to determine how well they simulated true noise. Sets of 420 numbers with a Gaussian distribution, a mean of zero, and $\sigma = 1$ were generated on the computer. Then various sized groups of points were averaged together. The standard deviations of these new groups of points were calculated and checked to see if they went as $N^{-1/2}$, where $N$ is the number of points averaged together. The results from a number of sets of data are shown in Figure 7.4. The points fulfill the theoretical expectations within reasonable limits.
VIII) References


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