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by

Po-ching Liu

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ABSTRACT

THE GALVANOMAGNETIC EFFECT ON
FERMI SURFACE AND AC AND DC MEASUREMENTS
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By treating the Fermi surface as a rigid unit, the galvanomagnetic-coefficients $\sigma_{xx}(J_x)$ and $\sigma_{xy}(R_H, J_y \text{ or } \varphi_{xy})$ were calculated for spherical, cubic and octahedral Fermi surfaces. The shape factors ($\delta$) 1, 2, and $\frac{2}{3}$ are associated with the expression $\alpha \tau H_z = \frac{eH}{mc} \tau$ in this semiclassical approach of calculating $\sigma_{xx}$ and $\sigma_{xy}(R_H)$. The Fermi-surface parameters of molybdenum were described and calculated; the Fermi surface of molybdenum is closed and charge carriers are compensated. Using the Hall-coefficient value of polycrystalline molybdenum as a guideline and with the aid of semiclassical approach it was estimated that $\left( \frac{\alpha}{m_e} \right) \sim 0.3 \left( \frac{\tau}{m_h} \right)$.

For the measurements, a sample holder was designed and was successfully operated at low temperature and high magnetic field. With the aid of a PAR HR-8 Lock-in Amplifier an AC circuit was designed, and measurements were made on $\frac{1}{4}$" diameter thin molybdenum slabs ($\sim$10 mil) by Van der Pauw's method at
$T = 77^\circ K$ and $300^\circ K$. Large amount of inductive components were identified, and the corresponding inductance was found to be $0.355 \mu H$ in a series of frequency test. For future operation a compensator including a phase shifter was designed and incorporated into the measuring system.

DC measurements were made on molybdenum and molybdenum-rhenium alloys at both room and liquid-nitrogen temperatures by a Honeywell 2773 six-digit potentiometer and a Keithley 148 nanovoltmeter. No detectable magnetoresistance was found in both molybdenum and molybdenum-rhenium alloys. This could be due to the random arrangement of component grains in the specimens. The measured resistivities increase linearly from 0% Re alloy to 25%-Re alloy. Thus, it is justified that alloying rhenium atoms to molybdenum does not increase the effective number of conduction electrons but enhances the amount of scattering centers and affect both the relaxation time $\tau$ and effective mass $m^*$ in hole and electronic bands, though the rhenium atom carries one more s-electron than molybdenum.
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1. Introduction:

Since Pippard\textsuperscript{48} first proposed the "dog-boned" Fermi surface model for copper, investigation of the metallic Fermi surface deviation from the free-electron model has been of great interest to physicists and material scientists. A considerable amount of discussion of Fermi surface is contained in Harrison's book\textsuperscript{1}. Calculation of Fermi surface parameters of metals and the effect of alloying on Fermi surface is frequently found in recent literature\textsuperscript{2,3,4}. With progress in the technology of preparing purer crystals and the improvement in electronic equipment, measurement of Fermi surfaces has become more readily feasible. Among the techniques applied in the Fermi-surface study are: cyclotron resonance, anomalous skin effect, magneto-acoustic effect, and the galvanomagnetic effect. Measurements by the first three methods in general require complex electromechanical instrumentation and very often microwave operation is required. Measurement by galvanomagnetic effect is simpler as far as instrumentation is concerned, but accompanied by a small signal to noise ratio.

Some of the recent measurements of galvanomagnetic effects on single-crystal metals and alloys are found for the Hall effect in copper\textsuperscript{6,7}, vanadium\textsuperscript{8}, chromium-vanadium\textsuperscript{9}, and for the magnetoresistance in molybdenum\textsuperscript{10,11}. Methods used in the measurements were mostly by DC and sometimes AC\textsuperscript{12}, although
seldomly pulsed methods\textsuperscript{13} were used. The phenomenological discussion of galvanomagnetic effect can be found in Beer's\textsuperscript{14} and Kalayush's\textsuperscript{15} work and the quantum-mechanical approach to the electronic properties of metals and alloys in Laugreth's\textsuperscript{16}, Aldea's\textsuperscript{17} and Stern's\textsuperscript{18} publications. The bridge between these two different approaches is to define the galvanomagnetic transport coefficients in terms of the electronic state on the Fermi surface.

In this investigation we treat the Fermi surface as a rigid unit and regard the galvanomagnetic effect as to displace the Fermi surface in the reciprocal lattice space. On this basis, the expressions for resistivity and Hall coefficient are derived for several different Fermi geometries. Both magnetoresistance and Hall coefficient have been selected for this investigation because of the simplicity in the equipment required in the measurement. Owing to the inherent requirement for the thinness of the sample in Hall measurement and the difficulty in obtaining large quantity of some metallic single crystal, we selected Van der Pauw's geometry as a guideline of transport-coefficient measurement. Molybdenum and molybdenum-rhenium alloys were selected for our study because molybdenum is inexpensively commercially available and single-crystal molybdenum-rhenium alloys have successfully been produced in the Material Science Laboratory of Rice University.
AC measurements were made on molybdenum polycrystalline sample because AC method was thought to superior in detecting true signal and subject to less drift. DC measurements were made and have determined the magnetoresistance and Hall coefficients of molybdenum and molybdenum-rhenium alloys at $T = 77^\circ K$ and $T = 300^\circ K$. 
2. Review of Galvanomagnetic Transport Coefficients: Hall Effect and Magnetoresistance

2-1. Phenomenological Description:

The phenomenological equations that govern the galvanomagnetic and thermomagnetic effects are given by

\[ E_i = \rho_{ik} J_k + \varepsilon_{ik} G_k + \frac{1}{e} \frac{\partial \xi}{\partial x} \]  \hspace{1cm} (2-1)
\[ W_i = -\pi_{ik} J_k + \lambda_{ik} G_k + \frac{\xi}{e} J_i \]  \hspace{1cm} (2-2)

where
- \( \rho_{ik} \) = electrical resistivity tensor,
- \( \lambda_{ik} \) = thermal conductivity tensor,
- \( \varepsilon_{ik} \) = absolute thermoelectric power tensor,
- \( \pi_{ik} \) = Peltier tensor,
- \( \xi \) = chemical potential of the electrons,
- \( e \) = electronic charge (negative number),
- \( W_i \) = heat current density,
- \( J_i \) = electronic current density,
- \( G_k \) = temperature gradient.

If we define

\[ E_i^* = E_i - \frac{1}{e} \frac{\partial \xi}{\partial x} \]
\[ W_i^* = W_i - \frac{\xi}{e} J_i \]

Then eqs. (2-1) and (2-2) reduce to

\[ E_i^* = \rho_{ik} J_k + \varepsilon_{ik} G_k \] \hspace{1cm} (2-3)
\[ W_i^* = -\pi_{ik} J_k + \lambda_{ik} G_k \] \hspace{1cm} (2-4)

For homogeneous medium and uniform chemical potential, \( E_i^* \) and \( W_i^* \) become \( E_i \) and \( W_i \), i.e., conventional electrical field intensity and heat current density.
Hall coefficient $R_H$ and the resistivity $\rho_{xx}$ in the presence of magnetic field are defined from eqs. (2-3) and (2-4) as:

$$R_H = \frac{E_y}{J_x B_z} = \frac{\rho_{yx}}{B_z} = \frac{1}{\sigma_y B_z}$$

$$\rho_{xx} = \frac{E_x}{J_x}$$

where $B_z$ = applied magnetic induction.

2-2. Single-charge Carrier Model:

For a rectangular sample of the dimensions shown in Fig. 2-1 and assuming the charge carriers are free electrons,

![Fig. 2-1 Hall field on rectangular sample.](image)

the induced Hall voltage can be found to be

$$V_y = \frac{E_y b}{Y_y} = R_H B_z I_x / d \quad \text{(2-5)}$$

If $t = 0.1 \text{ mm}$, $I_x = 1 \text{ amp}$, $B_z = 1 \text{ KG}$, and $R_H = 20 \cdot 10^{-11} \text{ V-m}^3/\text{amp-w}$, then $V_y = 0.2 \mu \text{V}$

or equivalently $V_y = 0.1 \mu \text{V}$ for 10 mil thick sample. For $l = 1 \text{ in}$, $b = 5 \text{ mm}$ the longitudinal voltage drop \(\sim 10,000 \mu \text{V}\), therefore 1 mil of misalignment would cause about 10 $\mu \text{V}$ difference at Hall probes.
On the same free-electron model, the longitudinal conductivity is found to be \( \sigma_{xx} = \frac{n e^2 \tau}{m} \) independent of the magnetic field,

where \( n \) = free electron concentration, 
\( \tau \) = relaxation time, 
\( m \) = free electron mass.

Therefore, it is seen that the Hall voltage is odd with respect to both current density \( J_x \) and magnetic induction \( B_z \) and the longitudinal voltage drop is odd with respect to \( J_x \) but even with respect to \( B_z \). The two signals can be differentiated by reversing the field orientation.

2-3. Two-Band Model:

If the charge carriers in the medium consist of two types of charge carriers, say electrons and holes, the Hall coefficient\(^{19}\) is calculated as:

\[
R_H = \frac{1}{e} \cdot \frac{\frac{\sigma_e^2}{n_e} - \frac{\sigma_h^2}{n_h} + \frac{B^2}{e^2} \frac{\sigma_e^2 \sigma_h^2}{\sigma_e^2 + \sigma_h^2} \frac{(n_e - n_h)^2}{n_e^2 n_h^2}}{\frac{(\sigma_e + \sigma_h)^2}{\sigma_e^2 + \sigma_h^2} + \frac{B^2}{e^2} \frac{\sigma_e^2 \sigma_h^2}{\sigma_e^2 + \sigma_h^2} \frac{(n_e^2 - n_h^2)}{n_e^2 n_h^2}}}
\]

---(2-6)

where \( \sigma_e = \frac{n_e e^2 \tau_e}{m_e} \) = conductivity due to electrons, 
\( \sigma_h = \frac{n_h e^2 \tau_h}{m_h} \) = conductivity due to holes, 
\( \tau_e \) = relaxation time of electrons, 
\( \tau_h \) = relaxation time of holes, 
subscripts e and h refer to quantities related to elec-
trons and holes, respectively.

If \( n_e \neq n_h \)

\[
R_H \sim \begin{cases} 
\frac{1}{e} \cdot \frac{\sigma_e^2}{n_e} \frac{n_h}{\sigma_e + \sigma_h^2} & \text{---- small magnetic field} \\
\frac{1}{e} \frac{1}{n_e - n_h} & \text{---- high magnetic field}
\end{cases}
\]

If \( n_e = n_h = n_0 \)

\[
R_H = \frac{1}{e} \frac{\sigma_e - \sigma_h}{\sigma_e + \sigma_h} \text{ ---- independent of magnetic field} \quad \text{----(2-7b)}
\]

The change in resistance due to a magnetic field \( B \), to the first approximation, is

\[
\frac{\Delta \rho_{xx}}{\rho_{xx}} = \frac{\sigma_e \sigma_h}{e^2 (\sigma_e + \sigma_h)^2} \left( \frac{\sigma_e}{n_e} + \frac{\sigma_h}{n_h} \right)^2 B^2
\]

---(2-8)

\[
\left| 1 + \frac{\sigma_e^2 \sigma_h^2}{e^2 (\sigma_e + \sigma_h)^2} \frac{(n_e - n_h)^2}{n_e^2 n_h^2} \right| B^2
\]

If \( n_e \neq n_h \)

\[
\frac{\Delta \rho_{xx}}{\rho_{xx}} \sim \frac{n_e^2 n_h^2}{\sigma_e \sigma_h (n_e - n_h)^2} \text{ for large field} \quad \text{----(2-9)}
\]

If \( n_e = n_h = n_0 \)

\[
\frac{\Delta \rho_{xx}}{\rho_{xx}} = \frac{\sigma_e \sigma_h}{n_0^2 e^2} B^2 \quad \text{--------------------------(2-10)}
\]

Thus, it is seen that both Hall coefficient and magnetoresistance are even function of the field. However, the Hall voltage is an odd function of magnetic field, while the magnetoresistance is an even function of magnetic field.

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3. Semiclassical Approach to The Galvanomagnetic Effect on Fermi Surface

3-1. Free Electron Theory:

The electronic motion in solid can be described by Boltzmann’s transport equation:

$$\frac{\partial f}{\partial t} = \mathbf{\dot{\kappa}} \cdot \nabla_{\mathbf{k}} f + \mathbf{\chi} \cdot \mathbf{\nabla} f \quad \text{(3-1)}$$

where \( f(\mathbf{r}, \mathbf{k}, t) \) = distribution function of electrons as a function of displacement and wave vector. The general solution to equation (3-1) is given as:

\[
\phi = \mathbf{\kappa} \cdot \mathbf{\dot{\kappa}} = \frac{\mathcal{c} \left[ \mathbf{P} - \mathbf{\alpha 2} \mathbf{H} \times \mathbf{P} + (\mathbf{\alpha 2})^2 \mathbf{H} \cdot \mathbf{P} \right]}{1 + (\mathbf{\alpha 2})^2} \times \frac{\hbar}{m} \quad \text{(3-2a)}
\]

\[
\phi = - \frac{\mathbf{f} - f_0}{\partial f_0} \cdot \mathbf{\dot{f}} \quad \text{(3-2b)}
\]

\[\mathcal{E} = e \mathcal{E} - (e \cdot \eta) \nabla \ln T \quad \text{(3-2c)}\]

\(\mathcal{E}\) = externally applied electric field,
\(\mathcal{H}\) = externally applied magnetic field,
\(f_0\) = electronic distribution function without externally applied field,
\(f\) = electronic distribution function after the application of \(\mathcal{E}\) and \(\mathcal{H}\) fields,
\(\epsilon\) = total energy of an electron,
\(\eta\) = Fermi level,
\(T\) = temperature.
Confining ourselves to the galvanomagnetic effects only and under the condition of weak magnetic field, \( H_z \ll 100 \text{ KG} \), we then have \( \alpha z H_z \ll 1 \), and \( \nabla T = 0 \), the solution of the Boltzmann's transport equation is further reduced to eq. (3-2b).

\[
f = f_0 - \frac{e E_x z}{\hbar} \left( \frac{\partial f_0}{\partial K_x} \right) + \frac{e E_x z}{\hbar} (\alpha z H_z) \left( \frac{\partial f_0}{\partial K_y} \right) \tag{3-2b}
\]

The expression of the new distribution function \( f \) has a simple meaning, i.e., the externally applied electric field will shift the distribution function in the \( K_x \) direction by an amount \( \Delta K_x = \frac{e E_x z}{\hbar} \). The externally applied magnetic field \( (H_z) \) interacts with the electric field \( E_x \) and shifts the distribution function in the \( K_y \) direction by an amount of \( \Delta K_y = -\frac{e E_x z}{\hbar} \). Therefore the Fermi surface will shift by the amounts \( \Delta K_x \) and \( \Delta K_y \) with respect to the original position.

3-2. Calculation of Transport Coefficients \( \sigma_{xx} (J_x, P_{xx}), \sigma_{xy} (J_y, P_{xy}) \), \( R_H \) or \( \rho_{xy} \) for Different Geometries of Fermi Surface:

Assuming that the electric field \( (E_x) \) is applied in the \( X \)-direction, and the magnetic field \( (H_z) \) in the \( Z \)-direction, the transport coefficients are related by:

\[
J_x = \sigma_{xx} E_x \tag{3-3a}
\]
\[
J_y = \sigma_{xy} E_y = -\sigma_{yx} E_y \tag{3-3b}
\]
\[
\rho_{xx} = \frac{E_x}{J_x} = \frac{1}{\sigma_{xx}} \tag{3-4a}
\]
\[
R_H = -\frac{E_x J_y}{J_x^2 H_z} \tag{3-4b}
\]
where \[ j(x, t) = \frac{e}{4\pi^3} \int \mathcal{V}_k f(x, k, t) dk \] \[ = \text{electric current density vector.} \]

Eqs. (3-3a), (3-3b) and (3-4a) follow from basic definition, eq. (3-4b) is defined by the magnetoconductive approach\textsuperscript{21} by assuming the transverse Y direction is infinite.

\[ \Delta k_x = \frac{eE_x \tau}{\hbar} \]
\[ \Delta k_y = \frac{eE_y \tau}{\hbar} (\Delta \tau H_x) \]

**Fig. 3-1 Relation between Vol(f) and Vol(f').**

Referring to Fig. 3-1

Let \[ \text{Vol} (f'_o) = \text{Volume of undisturbed Fermi sphere} = I + II \]
\[ \text{Vol} (f) = \text{Volume of shifted Fermi sphere} = II + III \]
\[ \text{Vol} (I) = \text{Volume of Region I} \]

Then \[ \text{Vol} (f) = \text{Vol} (I) + \text{Vol} (II) + \text{Vol} (III) - \text{Vol} (I) \]
\[ = \text{Vol} (f'_o) + \text{Vol} (III) - \text{Vol} (I) \]

Also \[ n = \frac{2}{3\pi^3} \frac{4\pi}{3} K_o^3 = \frac{K_o^3}{3\pi^2} \] \[ = \text{(3-6a)} \]

First we would like to calculate the resistivity and Hall coefficient for spherical Fermi surface (free electron
model) by making use of the concept in eq. (3-2b). From Fig. 3-1 we approximate eq. (3-5) as follows:

\[
\int \mathbf{v}_k f(\mathbf{r}, \mathbf{K}, t) d\mathbf{K} = <\mathbf{v}_k> \int f_0 \text{Vol}(f_0) + <\mathbf{v}_k> \text{Vol}(\mathbf{III}) - <\mathbf{v}_k> \text{Vol}(\mathbf{I})
\]

\[
= <\mathbf{v}_k> \text{Vol}(\mathbf{III}) - <\mathbf{v}_k> \text{Vol}(\mathbf{I}) \quad \text{(3-6b)}
\]

Referring to Appendix B and Appendix C,

\[
\text{Vol}(\mathbf{I}) = \text{Vol}(\mathbf{III}) \sim \pi k_0^2 \Delta K_x
\]

\[
<v_x>_{\mathbf{III}} \sim \frac{2}{3} v_0 \quad <v_y>_{\mathbf{III}} \sim -\frac{2}{3} v_0 \tau H_z
\]

because of symmetry \(<v_x>_{\mathbf{I}} = -<v_x>_{\mathbf{III}}\)

\[
J_x = \frac{e}{4\pi^3} (<v_x>_{\mathbf{III}} \text{Vol}(\mathbf{III}) - <v_x>_{\mathbf{I}} \text{Vol}(\mathbf{I}))
\]

\[
= \frac{e}{2\pi^3} <v_x>_{\mathbf{III}} \text{Vol}(\mathbf{III})
\]

\[
= \frac{e}{2\pi^3} \left( \frac{2v_0}{3} \right) k_0^2 \Delta K_x \quad \text{-----------------------(3-7)}
\]

So \(\sigma_{xx} = \frac{ne^2 \tau}{m}\)

Similarly

\[
J_y = \frac{e}{4\pi^3} (<v_y>_{\mathbf{III}} \text{Vol}(\mathbf{III}) - <v_y>_{\mathbf{I}} \text{Vol}(\mathbf{I}))
\]

\[
= -\frac{e}{2\pi^3} <v_y>_{\mathbf{I}} \text{Vol}(\mathbf{I})
\]

\[
= -\frac{e}{2\pi^2} \left( \frac{2v_0}{3} \right) \tau H_z \cdot k_0^2 \Delta K_x
\]

\[
= -\frac{ne^2 \tau}{m} \cdot \frac{e \tau H_z E_x}{mc} \quad \text{-----------------------(3-8)}
\]

\[
\sigma_{xy} = \frac{ne^2 \tau}{m} \cdot \frac{e \tau H_z}{mc} = \frac{ne^2 \tau}{m} \tau H_z
\]

Substituting the values of eqs. (3-7) and (3-8) into eq. (3-4b)

\[
R_H = \frac{-E_x J_y}{J_x^2 H_z} = \frac{1}{nec} \quad \text{-----------------------(3-9)}
\]

This is the familiar result of the free electron model.
Fig. 3-2 Displacement of cubic and octahedral Fermi surfaces for electric field $E_x$ and magnetic field $H_x$.

\[ x' = x - \Delta x \]
\[ y' = y + \Delta y \]
\[ x'' = x\cos\theta + y\sin\theta = (x - \Delta x)\cos\theta + (y + \Delta y)\sin\theta \]
\[ y'' = -x\sin\theta + y\cos\theta = -(x - \Delta x)\sin\theta + (y + \Delta y)\cos\theta \]
\[ x''' = x\cos\theta + y\sin\theta + (-\Delta x\cos\theta + \Delta y\sin\theta) \]
\[ y''' = -x\sin\theta + y\cos\theta + (\Delta x\sin\theta + \Delta y\cos\theta) \]

$\bigtriangleup EFGH = \text{Vol}(\mathcal{F})$

$\bigtriangleup ABCD = \text{Vol}(\mathcal{G})$
As a further application of the concept of the Fermi surface in eq. (3-2b) we calculate the magnetoresistance and the Hall coefficient for the case of the cubic Fermi surface. This corresponds approximately to the constant-energy surface of a heavy hole in Germanium\textsuperscript{22}. Assuming that the applied electric field makes an arbitrary angle with the edges of the cube as shown in Fig. 3-2. It is clear

\[ \text{Vol}(f) = \text{Vol}(f_0) + \text{Vol}(8) + \text{Vol}(5) + \text{Vol}(9) - \text{Vol}(7) - \text{Vol}(3) - \text{Vol}(10) \]

Since \text{Vol}(5) and \text{Vol}(3) are negligibly small compared with other terms,

\[ \text{Vol}(f) = \text{Vol}(f_0) + \text{Vol}(8) + \text{Vol}(9) - \text{Vol}(7) - \text{Vol}(10) \]

\[ J(x, t) = \frac{e}{4\pi^2} (\langle v_x \rangle \text{Vol}(8) + \langle v_y \rangle \text{Vol}(9) - \langle v_z \rangle \text{Vol}(7) - \langle v_\theta \rangle \text{Vol}(10)) \]

Since \[ \langle v_x \rangle = v_o \cos \theta \]
\[ \langle v_y \rangle = v_o \sin \theta \]
\[ \langle v_\theta \rangle = -v_o \sin \theta \]
\[ \langle v_z \rangle = -v_o \cos \theta \]

\[ n = \frac{2}{3} (2K_0)^3 = \frac{2K_0^3}{\pi^3} \]
\[ v_o = \frac{\hbar K_0}{m^*} \]

It follows that

\[ J_x = \frac{e}{4\pi^3} \cdot 2 \cdot (\langle v_x \rangle \text{Vol}(8) + \langle v_y \rangle \text{Vol}(9)) \]

\[ = \frac{e}{2\pi^3} (2K_0)^2 v_o \cos \theta (\Delta K_x \cos \theta - \Delta K_y \sin \theta) \]

\[ + v_o \sin \theta (\Delta K_x \sin \theta + \Delta K_y \cos \theta) \]

\[ = \frac{e}{2\pi^3} (2K_0)^2 v_o \Delta K_x = \frac{ne^2}{m^*} E_x \]

\[ \text{------------- (3-10)} \]

Similarly we see that

\[ J_y = -\frac{e}{4\pi^3} (2K_0)^2 2 \langle v_y \rangle \text{Vol}(7) + \langle v_y \rangle \text{Vol}(10) \]
with $\theta = 0$

$$\overrightarrow{MC} = \frac{k^0}{\sqrt{2}} \hat{i}_1 - \frac{k^0}{\sqrt{2}} \hat{i}_2 - k^0 \hat{i}_3$$

$$\overrightarrow{MB} = \frac{k^0}{\sqrt{2}} \hat{i}_1 + \frac{k^0}{\sqrt{2}} \hat{i}_2 - k^0 \hat{i}_3$$

$$\overrightarrow{MC} \times \overrightarrow{MB} = k^2 [\sqrt{2} \hat{i}_1 + \hat{i}_3]$$

$$|\overrightarrow{MC} \times \overrightarrow{MB}| = \sqrt{3} k^2$$

Area of $\triangle MCB = \frac{\sqrt{3}}{2} k^2$

Unit normal of $\triangle MDC = \frac{1}{k^2} (\sqrt{2} \sin \theta \hat{i}_1 - \sqrt{2} \cos \theta \hat{i}_2 + \hat{i}_3)$

Unit normal of $\triangle MBC = \frac{1}{k^2} (\sqrt{2} \cos \theta \hat{i}_1 + \sqrt{2} \sin \theta \hat{i}_2 + \hat{i}_3)$

Unit Normal of $\triangle MAB = \frac{1}{k^2} (-\sqrt{2} \sin \theta \hat{i}_1 + \sqrt{2} \cos \theta \hat{i}_2 + \hat{i}_3)$

Unit normal of $\triangle MAD = \frac{1}{k^2} (-\sqrt{2} \cos \theta \hat{i}_1 - \sqrt{2} \sin \theta \hat{i}_2 + \hat{i}_3)$

**Fig. 3-3a** One face of the Octahedron.

**Fig. 3-3b** Dimensions of Octahedral Fermi Surface.
\[ J_y = -\frac{e}{4\pi^3}(2K_o)^2v_o(\Delta K_x\sin\theta + \Delta K_y\cos\theta) \]
\[ -v_o\sin\theta(\Delta K_x\cos\theta - \Delta K_y\sin\theta) \]
\[ = -\frac{e}{2\pi^3}(2K_o)^2v_o\Delta K_y \]
\[ = -\frac{ne^2\ell}{m^*\tau H_z} \]

Therefore
\[ R_H = -\frac{e}{J_x^2H_z} = \frac{l}{ne\tau} \]

Since the light hole band in Germanium and part of the Fermi surface of molybdenum and tungsten are regarded to have an octahedral constant-energy surface, we use the same concept of the shift of the Fermi surface in eq. (3-2b) to calculate the magnetoresistance and Hall coefficient of a material with cubic constant energy surface.

Referring to Figs. 3-2, 3-3a, and 3-3b we carry out calculations parallel to the previous two cases:

\[ \text{Vol}(f) = \text{Vol}(f_o) + \text{Vol}(8) + \text{Vol}(5) + \text{Vol}(9) - \text{Vol}(10) \]
\[ - \text{Vol}(3) - \text{Vol}(7) \]
\[ \sim \text{Vol}(f_o) + \text{Vol}(8) + \text{Vol}(9) - \text{Vol}(10) - \text{Vol}(7) \]
\[ J = \frac{e}{4\pi^3}(\langle v_x \rangle_8 \text{Vol}(8) + \langle v_x \rangle_9 \text{Vol}(9) - \langle v_x \rangle_{10} \text{Vol}(10) \]
\[ - \langle v_x \rangle_7 \text{Vol}(7) \]

Since
\[ \langle v_x \rangle_8 = v\cos\theta \]
\[ \langle v_x \rangle_9 = v\sin\theta \]
\[ \langle v_y \rangle_{10} = -v\cos\theta \]
\[ \langle v_y \rangle_7 = -v\sin\theta \]
\[ v_\perp = \frac{\sqrt{2}}{\sqrt{3}}v_o = \text{component of velocity perpendicular to } H_z \]

So
\[ J_x = \frac{e}{2\pi^3}(\langle v_x \rangle_8 \text{Vol}(8) + \langle v_x \rangle_9 \text{Vol}(9)) \]
\[ J_x = \frac{e}{2 \pi^2} \left( \frac{\sqrt{3}}{2} K_0 \right)^2 \left[ v_\perp \cos \theta \left( \frac{1}{\sqrt{3}} (\sqrt{2} \cos \theta \hat{z}_1 + \sqrt{2} \sin \theta \hat{z}_2 + \hat{z}_3) \cdot (\Delta K_{x \hat{z}_1} + \Delta K_{y \hat{z}_2}) \right. \right. \\
\left. \left. + v_\perp \sin \theta \left( \frac{1}{\sqrt{3}} (\sqrt{2} \sin \theta \hat{z}_1 - \sqrt{2} \cos \theta \hat{z}_2 + \hat{z}_3) \cdot (\Delta K_{x \hat{z}_1} + \Delta K_{y \hat{z}_2}) \right) \right] \right] \\
= \frac{e}{2 \pi^2} \frac{\sqrt{3}}{2} K_0 \frac{2 \sqrt{2}}{3} (2 \cos^2 \theta (\Delta K_{z \hat{z}_2}) + 2 \sin^2 \theta (\Delta K_{x \hat{z}_1})) \\
= \frac{\sqrt{3}}{3} \frac{e}{2 \pi^2} V_0 K_0^2 \Delta K_x 
\tag{3-13} \]

and
\[ J_y = \frac{e}{4 \pi^2} (\langle v_y \rangle > Vol(8) + <v_x > g Vol(9) - <v_x > 10 Vol(10) \\
- <v_x > 7 Vol(7)) \\
= -\frac{e}{2 \pi^2} (\langle v_x > 10 Vol(10) + <v_x > 7 Vol(7)) \\
\]

Since
\[ <v_y > 8 = v_\perp \sin \theta \]
\[ <v_y > 9 = -v_\perp \cos \theta \]
\[ <v_y > 10 = -v_\perp \sin \theta \]
\[ <v_y > 7 = v_\perp \cos \theta \]

It follows that
\[ J_y = -\frac{e}{2 \pi^2} \frac{\sqrt{2}}{2} K_0^2 v_\perp \sin \theta \left( \frac{1}{\sqrt{3}} (\sqrt{2} \cos \theta \hat{z}_1 + \sqrt{2} \sin \theta \hat{z}_2 + \hat{z}_3) \cdot (\Delta K_{x \hat{z}_1} + \Delta K_{y \hat{z}_2}) \right) \]
\[ + v_\perp \sin \theta \left( \frac{1}{\sqrt{3}} (\sqrt{2} \sin \theta \hat{z}_1 - \sqrt{2} \cos \theta \hat{z}_2 + \hat{z}_3) \cdot (\Delta K_{x \hat{z}_1} + \Delta K_{y \hat{z}_2}) \right) \]
\[ = -\frac{\sqrt{2}}{2} \frac{e}{\pi^2} K_0^2 v_\perp \Delta K_y \\
= -\frac{i}{\sqrt{3}} \frac{e}{\pi^2} K_0^2 v_\perp \Delta K_y \n\tag{3-14} \]

If we define \( m^* \) the effective mass in the direction of the shortest distance from the origin to the Fermi surface,
Then
\[ v_0 = \frac{\hbar K_0}{\sqrt{3} \pi^2} \]
and
\[ n = \frac{2}{3 \pi^2} \frac{2}{3} K_0 (2K_0^2) = \frac{K_0^3}{3 \pi^3} \]

Substituting the values of \( v_0 \) and \( n \) into eq. (3-13) and eq. (3-14),

then
\[ J_x = \frac{ne^2 \zeta}{m^2} E_x \quad \text{---------}(3-15) \]
\[ J_y = -\frac{ne^2 \zeta}{m^2} E_x (\alpha \zeta H_z) \quad \text{---------}(3-16) \]

Therefore
\[ R_H = -\frac{E_x J_y}{J_x^2 H_z} = \frac{1}{nec} \quad \text{---------}(3-17) \]

3-3 Discussion:

It has long been recognized that the transport coefficients are closely related to the Fermi surface of single crystals under the influence of external electromagnetic field. Though the coefficients are defined in terms of the perturbed Fermi-Dirac distribution function \( f \), the actual calculation is, as listed in most literature, transformed to that related to \( \frac{\partial f}{\partial \varepsilon} \), i.e. the energy derivative of original distribution function.

Our discussion here is to go back to the original solution of the transport equation, which is solved exactly for spherical constant energy surfaces under the application of \( E_x \) and \( H_z \); the solution in this case, to the first approximation, is
\[ f = f_0 - \frac{eE_x}{\hbar} \frac{\partial f_0}{\partial K_x} + \frac{eE_x}{\hbar} (\alpha \tau H_z) \frac{\partial f_0}{\partial K_y} \]  \hspace{1cm} (3-5) \]

The perturbed Fermi surface is simply obtained by displacing the original Fermi surface by

\[ \Delta K_x = \frac{eE_x}{\hbar} \tau \]
\[ \Delta K_y = -\frac{eE_x}{\hbar} (\alpha \tau H_z) \]

Assuming that the shift of the Fermi surface under the application of \( E_x \) and \( H_z \) is general, we proceed to the calculation of \( \sigma_{xx}(J_x) \), \( \sigma_{xy}(J_y) \) and \( R_H \) for spherical, cubic and octahedral Fermi surfaces. The condition which was tacitly assumed was \( f = 1 \) inside the Fermi surface and \( f = 0 \) otherwise equivalent to the distribution function at \( T = 0^\circ K \). The result is

\[ \sigma_{xx} = \frac{ne^2 \tau}{m^*} \]
\[ \sigma_{xy} = \frac{ne^2 \tau}{m^*} (\alpha \tau H_z) \]
\[ R_H = \frac{1}{ne c} \]

where \( m \) is replaced by \( m^* \) in cubic and octahedral cases.

Comparing our results with those obtained by McClure's and Allgaier's methods (Appendix A), the two calculations differ on the value of \( R_H \) by the factor \( \frac{1}{2} \) and \( \frac{2}{3} \) for the cubic and octahedral Fermi surfaces respectively. This deviation stems from the assumption we have used: \( \alpha \tau H_z = \frac{e\tau H_z}{m^*c} \). This is recognized as the product of cyclotron-resonance frequency and relaxation time for spherical Fermi surfaces. Therefore, the \( m^* \) in the expression of \( \alpha \tau H_z \) can be interpreted either as relating to the effective mass \( m_{ij} \) or the cyclotron resonance
mass $m_H$. For the free electron model, $m_{ij}$ and $m_H$ are equal. For the cases of cubic and octahedral constant-energy surfaces, the effective mass varies on the Fermi surface from point to point. Therefore the direct substitution of $\alpha$ by $\frac{e}{m'tc}$ should be multiplied by shape factor ($\delta$) for different geometries of Fermi surfaces. If we were satisfied with these arguments, assuming $\alpha = \frac{e}{2m'tc}$ and $\alpha = \frac{2e}{3m'tc}$ for the cases of cubic and octahedral constant energy surfaces respectively, then the result in our treatment of the Fermi surface agrees with the more rigorous calculation.
4. Description of The Fermi Surface of Molybdenum and Effect of Alloying with Rhenium

4-1. Material Selection:

Molybdenum and molybdenum-rhenium alloys were selected for the study of the galvanomagnetic effect. These materials were chosen because molybdenum is inexpensively available in the form of rods and Rice University has an electron beam melter which has successfully produced single-crystal transition metals and alloys by the zone-refining method. Molybdenum-rhenium alloy single-crystals had been made by Dr. D. L. Davidson\textsuperscript{25}, containing 8, 15, and 25 nominal atomic percent of rhenium.

4-2. Fermi Surface of Molybdenum and Effect of Alloying with Rhenium

Molybdenum has the atomic number 42 and is known to have an atomic structure [Kr]\textsuperscript{4d\textsuperscript{5}s\textsuperscript{1}}. Molybdenum crystallizes in the BCC structure. Rhenium has the atomic number 75 and possesses the atomic structure [Xe]\textsuperscript{4f\textsuperscript{1}5d\textsuperscript{5}s\textsuperscript{2}}. It crystallizes in the HCP structure. From the phase diagram of Mo-Re alloys, the alloy consists of $\beta$ phase (BCC) below 2500\textdegree C and within 30% atomic percent of rhenium.
The Fermi surface of molybdenum was first proposed by Lomer\textsuperscript{26,27}. The three-dimensional diagram based on this model was sketched by Bezuglyi\textsuperscript{5}. Their results are reproduced in Figs. 4-1, 4-2, 4-3, 4-4 to show the (100) and (111) cross-section and the three-dimensional model of the Fermi surface.

![Fermi surface diagrams](image)

**Fig. 4-1** Fermi surface of molybdenum electron zones.  **Fig. 4-2** Fermi surface of molybdenum hole zones.

Thus, we see that the Fermi surface of molybdenum can be described as consisting of six ellipsoidal holes at points $N$ of the Brillouin Zone. Six electron lenses are formed by the intersection of the prolate spheroids and the rounded points of the octahedron, and necks are introduced in connecting the lenses and the electron octahedron at $\Gamma$. The volumes of holes and electrons, according to Sparlin\textsuperscript{4}, are nearly compensated, and the Fermi surface is closed.

Since the lattice parameter of Molybdenum is $3.1466 \text{Å}$, the lattice parameter in the reciprocal lattice space is
Fig. 4-3 [100] section of the Fermi Surface of Molybdenum, after Lomer.

Fig. 4-4 [110] section of the Fermi Surface of Molybdenum, after Lomer.
\[ |A| = \frac{4\pi}{a} = 3.9936 \text{ Å} \]

Since each unit cell in reciprocal lattice space contains four atoms, therefore

the volume of the Brillouin zone = \( \frac{1}{4} \left( \frac{4\pi}{a} \right)^3 \)

= \( 2 \left( \frac{2\pi}{a} \right)^3 = 15.9232 \text{ Å}^3 \)

total electronic volume = 1.80 Å\(^3\) = \( V_e \)

total hole volume = 1.65 Å\(^3\) = \( V_h \)

Thus, it is seen that \( V_e \) and \( V_h \) are approximately compensated.

\[ n_e \sim n_h = \frac{1}{2} (1.8 + 1.65) \text{ Å}^{-3} \]

= \( 1.725 \cdot \frac{2}{3\pi} \cdot 10^{24} \) electrons/cm\(^3\)

= \( 1.39 \cdot 10^{22} \) electrons/cm\(^3\)

Investigation of the Fermi surface model by the measurements of de Haas-van Alphen effect\(^4\), high frequency magneto-acoustic effect\(^28\), and cyclotron resonance\(^29\), all qualitatively agree with Lomer's proposed model.

To summarize, some of the properties of molybdenum and rhenium are listed as below:

<table>
<thead>
<tr>
<th>elements</th>
<th>Lattice Parameter</th>
<th>Lattice Structure</th>
<th>Electronic Structure</th>
<th>Density (gm/mil)</th>
<th>Atomic Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mo</td>
<td>3.1466 Å BCC</td>
<td>[Kr]4d(^5)5s(^1)</td>
<td>10.2</td>
<td>95.94</td>
<td></td>
</tr>
<tr>
<td>Re</td>
<td>2.7609 Å HCP</td>
<td>[Xe]4f(^1)5d(^5)6s(^2)</td>
<td>21.0</td>
<td>186.2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>elements</th>
<th>Atomic Concentration</th>
<th>Closest Atomic Distance</th>
<th>1st ioniz.(^{48})</th>
<th>2nd ioniz.(^{48})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mo</td>
<td>0.6400 ( \cdot 10^{23} )</td>
<td>2.725 Å</td>
<td>7.10 v</td>
<td>16.15 v</td>
</tr>
<tr>
<td>Re</td>
<td>0.6790 ( \cdot 10^{23} )</td>
<td>2.740 Å</td>
<td>7.87 v</td>
<td>16.6 v</td>
</tr>
</tbody>
</table>

Table 4-1 Some Properties of Mo & Re Elements
For molybdenum:

Volume of Brillouin zone = 15.9232 Å⁻³
Reciprocal lattice parameter = 3.9936 Å⁻¹
Average electronic volume = Average hole volume
= 1.725 Å⁻³ = 1.39 × 10⁻²² electrons/cm³
Average charge carrier (sum of electrons and holes)
contributed by an atom = \( \frac{2 \times 1.39 \times 10^{22}}{0.6400 \times 10^{33}} = 0.435 \)

Thus, it is seen that the distribution of charge carriers is far less than the sum of s- and d- electrons. Since molybdenum and rhenium are approximately equal in size, we would, off hand, expect that the atomic density of Mo-Re alloy will not differ significantly from that of molybdenum. Consequently, the average concentration of conduction electrons should increase linearly with rhenium concentration. However, this may not be the case because of the following considerations:

As was mentioned earlier, the number of effective charge carriers (sum of electrons and holes) in molybdenum per atom is 0.435, which is much less than the sum of s- and d- electrons. We would expect therefore that the conduction mechanism in molybdenum departs appreciably from that of the superposition of s- and d-bands. In addition, the similarity of the first ionization potential and second ionization potential between molybdenum and rhenium leads us to the supposition that when rhenium is introduced into molybdenum, each rhenium atom will contribute about the same number of conduction electrons as ---
the average molybdenum atom, making each rhenium ion core carry about the same positive charge as a molybdenum ion core. The total number of conduction electrons should therefore not differ appreciably between molybdenum and molybdenum-rhenium alloy.

4-3. Application of The Semiclassical Approach to The Investigation of Molybdenum:

The semiclassical calculation on single band charge carriers leads to the expressions:

\[ \sigma_{xx} = \frac{n e^2 \zeta}{m^*} \quad \sigma_{xy} = \frac{n e^2 \zeta}{m^*} \alpha \zeta T_H \]

\[ R_H = \frac{1}{neC} \quad \alpha = \mathcal{S} \cdot \frac{e}{m^*c} \]

\[ \mathcal{S} = \text{Shape factor for different geometries of Fermi surfaces.} \]

Referring to Sparlin's data, assuming that the electronic and hole volumes for molybdenum are exactly compensated, and approximating the electronic jack volume by an octahedron, we have the following parameters for molybdenum in reciprocal lattice space by referring to Fig. 4-1 and 4-2.

Electronic Volume = 1.725 A\(^{-3}\)

\[ = 1.3908 \times 10^{22} \text{ electrons/cm}^3 = n_o \]

Hole volume at N of Brillouin Zone = \[ \frac{0.71}{1.66} \times 1.725 \text{ A}^{-3} \]

\[ = 0.4277n_o = n_{hN} \]
Hole volume in reciprocal space $\vec{v} H$ of Brillouin

$$\text{Zone} = \frac{0.95}{1.66} \times 1.725 \text{Å}^{-3} = 0.5723 n_o = n_{hH}$$

Using the polycrystalline molybdenum data of Frank, $R_H = 18 \times 10^{-11} \text{m}^3/\text{amp-s}$ we would like to explore if the above results could be used. Because of the occurrence of electrons and holes at different bands, we must consider the difference between $\left( \frac{e}{m_e} \right)$ and $\left( \frac{e}{m_h} \right)$. The lumping of $\tau$ and $m$ into one factor amounts to assuming that anisotropy in $\tau$ and $m$, if any, comes from the same origin. Since electronic conduction and hole conduction comes from two different bands, we would let

$$\frac{e}{m_e} = x \frac{e}{m_h} \quad \text{-------------------(4-1)}$$

Then

$$J_y = (\text{Hall current due to } n_e) +$$

$$\left( \text{Hall current due to } n_{hH} \right) +$$

$$\left( \text{Hall current due to } n_{hN} \right)$$

$$= \frac{n_e e^2 \tau_h}{m_h} \frac{e}{m_h} \tau_h H_z (-x^2 + 0.4277 x^2 + 0.5733 x^2) E_x$$

$$= \frac{n_e e^2 \tau_h}{m_h} \frac{e}{m_h} \tau_h H_z (-\frac{2}{3} x^2 + 0.4277 \cdot 1 + 0.5733 \cdot \frac{2}{3}) E_x$$

$$\text{-------------------(4-2)}$$

The $x^2$ factor is introduced because the whole expression of $J_y$ contains $\left( \frac{e}{m} \right)^2$.

$$J_x = (\text{electronic current}) + (\text{hole current})$$

$$= \frac{n_e e^2 \tau_h}{m_h} (1 + x) E_x \quad \text{-------------------(4-3)}$$

Hence,

$$R_H = \frac{-E_x J_y}{J_x^2 H_z} = -\frac{0.6667 x^2 + 0.8082}{n_o e (1 + x^2)} \quad \text{-------------------(4-4)}$$
Substituting the value of $R_H$ and $n_o$ into eq. (4-4),

$$n_o eR_H = 0.4055$$

$$\alpha = 0.3426 \sim 0.3$$

So

$$\frac{\overline{z_e}}{m_e} / \frac{\overline{z_h}}{m_h} \sim 0.3$$

Therefore using the polycrystalline molybdenum data as a guide-line, we see that the difference between the ratio of $(\overline{z_e}/m_e)$ for electron and hole bands is about 0.3. It is seen that it is possible to utilize the semiclassical approach to the calculation of $R_H$, if $(\overline{z}/m)$ for the charge carrier is known in advance or, alternatively, we can calculate $(\overline{z}/m)$ of $R_H$. 
5. Method of Measurement

5-1. Selection of AC Method:

The selection of the AC method in the transport-coefficient measurement is based on the reasoning that spurious signals be minimized. As it is well known that there also exists thermoelectric\textsuperscript{30} and thermomagnetic effects. Heat is absorbed or emitted at a junction of different materials as a result of the Peltier effect, and the longitudinal current flow (X-direction) would cause transverse electric fields (Y-direction) because of the combination of the Nernst and Ettinghausen effect. Since the thermoelectric power for metals is of the order of $1 \sim 10 \ \mu \text{V}$ for a temperature difference of $1^\circ \text{K}$ across the sample, the above Nernst-Ettinghausen Effect would cause a spurious signal of the order of magnitude as the galvanomagnetic voltage in presence of the slightest nonuniformity in temperature on the sample. Utilizing low-frequency AC current would nullify the problem of the temperature gradient. It was for this reason that the AC method was utilized.

5-2 Selection of Sample Geometry:

The measurement of the Hall voltage was conventionally performed on thin rectangular slabs. With the main current flowing in the X-direction and magnetic induction in the Z-direction, the Hall voltage is measured as the voltage
differential in Y-direction with and without a magnetic field, as shown in the geometric arrangement of Fig. 2-1. Specially designed rectangular slabs are often used which have ear lobes protruding from the sides. The magnetoresistance is measured by monitoring the potential drop across the current electrodes.\textsuperscript{32}

However, the aformentioned method is generally utilized in polycrystalline metals or semiconductors; where in the former case the polycrystalline specimen can be mechanically worked to be sufficiently thin (≈10 mil), in the semiconductor case the signal is about three orders of magnitude larger than in the metal and therefore the requirement for very thin sample is relaxed, in the case of single-crystal metals, the preparation of single-crystal rectangular slabs is difficult because single crystals are generally available in the form of cylindrical rods. Obtaining rectangular slab at a certain crystal plane would require to cut the crystal twice along the crystal axis containing a diameter. This presents certain technical difficulties. On the other hand, a circular slab can be easily cut from a cylindrical crystal of known axis. Therefore, a different method was adopted to make use of the circular samples.

5-3 Van der Pauw's Geometry and Generalization:

In 1958 Van der Pauw\textsuperscript{33} made an analysis and found that
Fig. 5-1 The value of resistivity modulating function $f\left(\frac{R_{AB,CD}}{R_{BC,DA}}\right)$ after Van der Pauw.
<table>
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Table 5-1. Value of resistivity modulating function

\[ f \left( \frac{R_{AB,CD}}{R_{BC,DA}} \right) \quad \text{near} \quad \frac{R_{AB,CD}}{R_{BC,DA}} \sim 1 \]
<table>
<thead>
<tr>
<th>$\frac{R_{AB,CD}}{R_{BC,DA}}$</th>
<th>$f\left(\frac{R_{AB,CD}}{R_{BC,DA}}\right)$</th>
<th>$\frac{R_{AB,CD}}{R_{BC,DA}}$</th>
<th>$f\left(\frac{R_{AB,CD}}{R_{BC,DA}}\right)$</th>
</tr>
</thead>
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Table 5-1 (Continued)
For a thin sample of arbitrary shape and thickness d, and contacts A, B, C, and D placed on the periphery of the sample, the resistivity \( \rho \) can be found with the aid of the transfer resistances \( R_{AB, CD} \) and \( R_{BC, DA} \),

where

\[
R_{AB, CD} = \frac{V_{BC}}{I_{AB}} \quad R_{BC, DA} = \frac{V_{AD}}{I_{BC}}
\]

\[
\rho = \left( \frac{\pi d}{\ln 2} \right) \frac{1}{2} \left( R_{AB, CD} + R_{BC, DA} \right) \frac{f(R_{AB, CD})}{f(R_{BC, DA})} \quad \text{(5-1)}
\]

where \( f \left( \frac{R_{AB, CD}}{R_{BC, DA}} \right) \) satisfies eq. (5-2) below. It is plotted in Fig. 5-1 and tabulated in Table 5-1 for \( \frac{R_{AB, CD}}{R_{BC, DA}} = 1.00-2.00 \).

\[
\frac{R_{AB, CD} - R_{BC, DA}}{R_{AB, CD} + R_{BC, DA}} = f \left( \frac{R_{AB, CD}}{R_{BC, DA}} \right) \cosh^{-1} \left( \frac{\exp(\ln 2/f)}{2} \right) \quad \text{(5-2)}
\]

The validity of eq. (5-1) is further generalized to anisotropic material \(^{24} \) with \( \rho \) replaced by the geometric mean of diagonal components along the sample plane:

\[
\rho = \sqrt[3]{\rho_1 \rho_3} \quad \text{for anisotropic material.}
\]

Since Van der Pauw's method is applied mainly to thin slabs of material and is independent of the shape of the specimen, we can apply eq. (5-1) to the study of a circular sample. Also, for the analysis in the following paragraphs, we could use the arrangement of four contacts A, B, C, and D equally spaced along the circumference as showing in Fig. 5-2, to study the Hall effect by eq. (2-5).
Fig. 5-2 Point contact arrangement by Van der Pauw's method.

The following paragraphs will prove the validity of eq. (2-5) for the case of circular specimens:

The potential, \( V \), in a two-dimensional slab of thickness \( d \) at a distance \( r \) from a current source \( I \) is

\[
V = V_0 - \frac{\varphi I}{2 \pi d} \ln r,
\]

where \( V_0 \) is an arbitrary reference potential.

Referring to Fig. 5-2 and choosing the origin as the zero point, the potential at point \( P(r, \theta) \) is

\[
V' = -\frac{\varphi I}{2 \pi d} \ln r_1 + \frac{\varphi I}{2 \pi d} \ln r_2
\]

Because the current sources are located on the boundary, the image sources \( A' \) and \( C' \) also overlap \( A \) and \( C \), so that the
potential \( V \) at \( P \) should have the doubled value of eq. (5-3).

\[
V = \frac{\varphi I}{\pi d} \ln \left( \frac{r^2 - a^2}{r^2 - a^2 - 2 a \cos \theta} \right) \\
= \frac{\varphi I}{2\pi d} \left( \ln(r^2 + a^2 - 2a \cos \theta) - \ln(r^2 + a^2 + 2a \cos \theta) \right)
\]

-------------(5-4)

The electric field \( \mathbf{E} \) is

\[
\mathbf{E} = -\nabla V \\
= \frac{\varphi I}{\pi d} \left( \frac{r - a \cos \theta}{r^2 + a^2 - 2a \cos \theta} + \frac{r + a \cos \theta}{r^2 + a^2 + 2a \cos \theta} \right) \hat{\mathbf{r}} \\
\left( - \frac{a \sin \theta}{r^2 + a^2 - 2a \cos \theta} - \frac{a \sin \theta}{r^2 + a^2 + 2a \cos \theta} \right) \hat{\mathbf{\theta}}
\]

Along the \( Y \)-axis, \( \theta = 90^\circ \)

\[
\mathbf{E} = \frac{2 \varphi I}{\pi d} \cdot \frac{2}{r^2 + a^2} \hat{\mathbf{x}} \\
\mathbf{J} = \sigma \mathbf{E} = \frac{2 \varphi I a}{\pi d} \cdot \frac{1}{r^2 + a^2} \hat{\mathbf{x}}
\]

The Hall field along the \( Y \)-axis is

\[
E_{HY} = R_H J_x B_z 
\]

-------------------(5-5)

The Hall EMF is

\[
V_H = \int_a^d E_{HY} \, dy = \frac{R_H B_z I a}{\pi d} \int_a^d \frac{dy}{y^2 + a^2} \\
= \frac{R_H B_z I}{d} 
\]

---------------(5-6)

Hence, the validity of eq. (2-5) is proved for circular specimens.
5-4 Other Considerations:

The decision to select Van der Pauw's method in transport-coefficient measurements is based on the easiness in the preparation of the specimen. Frank\textsuperscript{36} made a thorough analysis on the geometric effects of the electrode and probe geometry. Due to the shorting effect around the electrodes, the ratio of the Hall voltage and measured value along a center line perpendicular to the current flow is

\[
\frac{V_m}{V_H} = \frac{8}{\pi^2} \sum_{k=0}^{\infty} (-1)^k \tanh \left( \frac{2k + 1}{2} \frac{\nu_i}{\nu_1} \right) \left( \frac{2k + 1}{2} \right)^2
\]

where $\nu_i$ and $\nu_1$ are values of the sample boundary on the curvilinear coordinates, for which the origin coincides with the geometric center. The curvilinear coordinates are determined by the equipotential lines and current flow lines through the sample. The value of the true Hall voltage in eq. (2-5) generally holds for linear electrodes when $\frac{B_z R_H}{\rho} \ll 1$, a condition which is satisfied in a metal.

Calculations of the ratio $\left( \frac{\nu_i}{\nu_1} \right)$ are obtained by solving the Laplace equation $\nabla^2 \nu = 0$ subject to the geometric boundary condition. The other set of orthogonal curves can be found from the conjugate function $\nu = c$. In general, solution of Laplace's equation of this type can be approached by computer-aided numerical analysis. Excellent references for computer-aided field analysis are found in the literature\textsuperscript{37, 38}.

\[36\]
The numerical value of \( \frac{V_m}{V_H} \) in eq. (5-6) has been plotted by Isenberg. It is seen that \( \frac{V_m}{V_H} \sim 1 \) for \( \frac{V}{V_i} > 4 \); thus, it does not seem necessary to use a specially sophisticated sample geometry other than rectangular and circular samples. Simplified sample preparation, therefore, dictates the use of circular specimens.
6. Sample Preparation and Instrumentation

6-1. Sample Preparation:

The samples used in the experiment were prepared in the form of circular slices. Abbreviations MOP and MR-100 stand for molybdenum polycrystal and molybdenum-rhenium single-crystal alloy of <100> axis orientation. The number "8" in BMR-100 means the atomic percentage of Re in the alloy. The molybdenum polycrystals were purchased as metallurgical rod from Westinghouse Corp. A photomicrograph of a MOP-series crystal is shown in Fig. 6-1. The Mo-Re (MR series) single-crystal alloy rods were grown by the floating-zone, electron-beam melting method in the Material Science Laboratory of Rice University by Dr. D. L. Davidson. All crystals were of \( \frac{1}{4} \)" diameter. Slices approximately 1 mm thick were cut from the crystals on a Servomet spark cutter. They were glued to the flat end of a stainless-steel rod; the rod was then locked in an epoxy guard ring ready for lapping. The whole assembly was then lapped by silicon-carbide abrasive up to grit number 600. Slices as thin as 2 to 5 mil can be produced by this method without causing mechanical deformation of the wafer. However, owing to the consideration of mechanical strength of the slices it is recommended that in future measurements single crystal slices should be at least 10—15 mil thick.
Fig. 6-1 Photomicrograph of metallurgical Molybdenum used in the measurements (200 X).

Fig. 6-2 Laue back reflection pattern of Sample 8MR-100-1.
The dimensions of various molybdenum and molybdenum-rhenium crystal samples produced by this method are shown in Table 6-1.

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<th>Thickness (mil)</th>
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<th>Thickness (mil)</th>
</tr>
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<tr>
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<td>8.6</td>
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</tbody>
</table>

Table 6-1 Typical values of sample thickness produced by lapping.

The single crystals of 8MR, 15MR and 25MR series of molybdenum-rhenium alloy slabs were examined by X-ray Laue back reflection. It is seen in Fig. 6-2 that polycrystal layers existed in the bulk single crystal. Annealing sample 8MR-100 at 900°C for 12 hours as an attempt to remove the polycrystalline layer was not successful. Because of the existence of polycrystal layers, the MR series specimens listed in Table 8-1 are regarded as polycrystals, though they were cut from single crystal rods originally. However, the surface imperfection was easily removed by electropolishing in a mixture of 87.5% methanol and 12.5% sulfuric acid.\(^ {40} \)
6-2. Magnetic Field Measurement:

All the measurements of the strength of magnetic field refer to 9.81 KG F. W. Bell standard magnet. The field was measured with BH-200 Hall-Pak field sensor, which has a sensitivity of 94 mv/amp-KG. The main current to the Hall-Pak is supplied by a HP 721A Power Supply through a 100Ω resistor, the field induced voltage is measured with HP 425A DC-Micro-Volt-Ammeter. The overall sensitivity of field measuring system is adjusted to 10 mv/KG. This voltage is then amplified internally in the HP 425A. The stability of the field is observed at different KG setting for a time period of five minutes without giving rise to visual change. The block diagram of the field detecting system is shown in Fig. 6-3. Because of the fringing effect, the magnitude of the magnetic field along the rim of the pole face is about 2-3% larger than that over the center.

Fig. 6-3 Field Detecting system.
6-3. Design of Sample Holder:

A sample holder was designed, as shown in Figs. 6-4 and 6-5. The tip of the sample holder and the connectors were made of Kel-F. This is a material of greater strength than Teflon suitable for machining and operation at low temperature. The sample was glued to the central slider. The relative position of the sample with respect to the probes was adjusted by the micrometer and the rotational movement of the micrometer. The rotational movement of the micrometer stem was translated into vertical displacement along a guide. The rigidity of the tip of the sample holder was maintained by clamping the three-rod support with two pieces of the clamper shown in Fig. 6-6. The position of the sample plane with respect to the top circular piece was easily adjusted to be parallel to a marker line beneath the top flange before clamping the rods. The circular flange sat on a matching platform. The rim of the platform was angularly engraved to an accuracy of 2° per division, so that the angle between the sample plane and the magnetic field could be controlled within ±1°. The three Kel-F connectors close to the top flange served also as thermal insulators, which reduced heat flow along the brass rods to a minimum.

Six binding posts were built on the top flange; four of them were connected to probes A, B, C and D on the Kel-F tip,
Fig. 6-4 The Assembly of Sample Holder.

Scale: Half Size

Tip of Sample Holder.

Slotted Junction: two pieces are rotatable with respect to each other.

Starret-63L Micrometer

Binding Post

Kel-F Connector

Gd Grounded Binding Post

Straight hole for mounting.

Straight hole for liquid nitrogen feed-in.
Fig. 6-5 Prospective view of the tip of sample holder.

Fig. 6-6 Crosssectional view of clamper. Each piece is 1" long and has 4 screw holes.

Scale: Full Size.
the other two (Gd) were grounded. Electrical connection between the binding posts and the copper electrodes was through teflon-wrapped, shielded wire. The shielding was connected to the ground. The coating on the stranded wire was examined to be nonferromagnetic. Points A, B, C and D on the sample were brought out to the top flange and thus became easily accessible.
7. Results and Discussions

7-1. AC Measurement on Molybdenum Slabs

The major instruments used in the AC measurement is a Krohn-Hite UF 101A Power Amplifier, PAR HR-8 Lock-in Amplifier and PAR Type B Preamplifier. The PA unit is used to supply the main current, The PAR combination function as an AC voltmeter (Linearity: ±0.1% of full scale), which has extremely narrow-band width and is sensitive to the phase change of the input signal. The Type B preamp. is specially designed for signals from low source impedance. The PAR combination is capable of measuring signals as low as \( \frac{1}{100} \) nv.

The resistivity coefficients \( R_{AB,CD} \& R_{BC,DA} \) are measured by a system connection as shown in Fig. 7-1. The 8Ω power resistors are used to limit the main current, the 10 mv drop drawn from a 1Ω standard resistor is calibrated so that exactly 1 amp is flowing through the sample. This signal can also be used to monitor the phase of the main current through the sample. Points A, B, C and D are the four contact points along the edge of the sample as shown in Fig. 5-2.

At 1 amp of main current, \( V_{AD} \) and \( V_{DC} \) were measured at 77° K and 300° K on sample MOP-10, as shown in Figs. 7-2 and 7-3. Calculations of the magnetoresistivity by Van der Pauw's
equation, eq. (5-1) is listed in Table 7-1 and replotted in Fig 7-4. The apparent magnetoresistivity at $77^0$ K can be further approximated by

$$\rho = 8.41 + 0.127 B^2 \mu \text{m-cm} \quad \text{(7-1)}$$

The room temperature data can be approximated by two line segments:

$$\rho = 9.0 \quad \mu \text{m-cm}$$
$$\rho = 0.7909(B - 1.5) \mu \text{m-cm} \quad \text{(7-2)}$$

The latter can also be viewed as a parabolic curve with a tendency to saturate. This is similar to the general observation of parabolic behavior of magnetoresistance\textsuperscript{19}. However, because of the kink in Fig. 7-3 and the change in phase reading as will be explained later, we doubted that there is reactive component contained in the measured signal.

The Hall coefficient was measured by recording $V_{BD}$ with $I_{AC} = 1$ amp. Because the two probes were nearly located on an equipotential line, the magnitude of $V_{BD}$ is an order of magnitude smaller than those of $V_{BD}$ and $V_{AD}$. At zero magnetic field, the plot of $V_{BD}$ vs frequency in Fig. 7-5 shows considerable variation with respect to frequency. This led to the suspicion that other components were mixed with the signal. An attempt to buck out the undesirable signal was shown in the arrangement of Fig. 7-6. This circuit differs from that of Fig. 7-1 by the addition of a compensating voltage in
series with the signal. The noise level from the compensator was recorded by measuring the output voltage shown in Fig. 7-7 with HP 355D Attenuator at full attenuation. It is seen that introduction of a noise level by a compensator of this nature will not add uncertainty in the signal by more than ±4 nV. With the compensator carefully adjusted for minimum magnitude of $V_{BD}$, the compensated $V_{BD}$ was recorded vs frequency as shown in Fig. 7-8. Comparison of Fig. 7-5 and Fig. 7-8 does not reveal complete compensation.

Since the compensation was not complete and the compensator noise level could be controlled to within ±4 nV, it appears that the compensating voltage source and the uncompensated signal $V_{BD}$ were not in phase or even in quadrature. Also the small minimum at 1 KG in Fig. 7-4, which was definitely reproducible, indicates that there could be some kind of RLC resonance situation.

The uncompensated $V_{AD}$ and $V_{DC}$ and phases $\phi(V_{AD})$ and $\phi(V_{DC})$ were therefore carefully examined in the frequency range $f = 20 - 2000$ cps. It is seen in Figs. 7-9, 7-10, 7-11 and 7-12 that at high frequency range $V_{AD}$ and $V_{DC}$ increase linearly with frequency and the phase angle approaches 90°. This indicates that the amount of noise mixed in the measurement was reactive and at frequency above 200 cps, this component was inductive. With frequency set to 40 cps, $V_{DC}, V_{AD}, \phi(V_{DC})$ and
ϕ(V_{AD}) were measured as a function of magnetic induction at T = 77⁰ K and T = 300⁰ K, also V_{BD} and ϕ(V_{BD}) were measured at T = 300⁰ K shown in Figs 7-13a,b; 7-14a,b; and 7-15a,b; It is seen in all these measurements that a reactive component was always present. It was only in very rare situation that the phase angle approached zero in the range B = 5-6 KG in Figs. 7-13a 7-13b.

To ascertain the reactive component, a series of frequency test was run on sample MOP-06 with B = 0,1,2,3, & 4 KG, at T = 300⁰ K, tabulated in Tables 7-2, 7-3, 7-4, 7-5 and 7-6, plotted in Figs. 7-16, 7-17, 7-18, 7-19 and 7-20. V_{AB} and V_{BC} in these measurements were taken from the top flange of the sample holder and contained a large contact-resistance voltage drop. Recording of the phases of V_{AB} & V_{BC} is sometimes difficult because of overscale in the measurement by PAR HR-8 Lock-in Amplifier and thereby could be obtained alternatively by monitoring the phase of the 10 mv signal. We see that all high frequency data extrapolate back to 2.23 μv at f = 1 cps corresponding to an inductance of \( L = \frac{2.23 \times 10^{-6}}{2 \times 1} = 0.355 \mu \text{h}. \) The abnormal phase peak that occurs around 80 cps reveals that the circuit approached resonance condition, or the reactive component was about to become zero. This correlate with the zero phase portion in Figs. 7-13a and b at 40 cps. It is therefore concluded that the recorded signal beyond 200 cps is mainly inductive.
7-2. Design of a Phase Shifter

The basic structure of an RC phase shifter can be found in Millman's book. Two arrangements of the phase shifters were considered; their circuits and transfer characteristics are shown in Figs. 7-21 and 7-22. Based on the consideration of the widest range of amplitude variation at $\phi = 90^0$ or $\phi = 270^0$ and impedance matching, a phase shifter was designed and added to the compensator shown in Fig. 7-24. The values of the components were carefully selected, so that all were available from local distributors. Referring to Fig. 7-24, the input impedance of the phase shifter was approximately $R = 2R$ in the range of interest, where $R$ is one of the components value shown in Fig. 7-23. Therefore, the phase shifting circuit will not load the secondary coil of the transformer. The output impedance varies from 15.9$\Omega$ at 100 cps to 1.59$\Omega$ at 1000 cps; thus, cascading the 60$\Omega$ HP 355D attenuator would not load the compensator output either. Based on the asymptotic plot in Fig. 7-21, the approximate amount of resistance for adjusting the phase angle was calculated and listed in Table 7-7. It would be extremely difficult without this Table to adjust the three dials and obtain the appropriate phase angle. It is recommended that in the future a 10-turn dial should be attached to the shaft of the potentiometer ($R$), so that resistance values can be accurately controlled. With circuit parameters chosen as shown in Fig. 7-23, the maximum amount of voltage
Fig. 7-23 New compensator with phase shifter added.

Stancor TA-46 transformer

R: Spectral Model 860 10-turn potentiometer.
R: Bourne Model 3501S-1-104 potentiometer.

from the compensator is expected to be no less than 3000 µV for phase angles in the range of 0°-180°. The 3-stage RC phase shifter is not the only circuit that could be used, a 2-stage RC circuit, though less flexible, would also meet the purpose. It is shown in Appendix D for reference.
7-3. Recommended Procedures for Further Measurement

With the new compensator (Fig. 7-23) installed similar to that of Fig. 7-6, the operating frequency recommended is 400-500 cps from previous analysis, and the procedures of operation can be listed as follows:

1. Monitor the phase of 10 mv signal; this will be the phase of the main current. As a check, this phase should be equal to the phase of the voltage drop across the sample such as $\phi(V_{AB})$ and $\phi(V_{BC})$ in Tables 7-2, 7-3, 7-4, 7-5, and 7-6.

2. Rotate the phase dial by 90° and adjust the phase and amplitude controls for best null setting.

3. Return the phase dial to the position in step 2 and record the true voltage.
Fig. 7-1 AC measurement without compensation.
f=70 cps T=77°K
Sample: MOP-10

Fig. 7-2 Apparent amplitudes of $V_{AD}$ & $V_{DC}$ vs magnetic induction.
$f = 70$ cps  $T = 300^\circ K$

Sample: MOP-10

Fig. 7-3 Apparent amplitudes of $V_{AD} \& V_{DC}$ vs magnetic induction.
T = 77°K  
Sample: MOP-10

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T = 300°K  
Sample: MOP-10

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Table 7-1: Calculation of apparent magnetoresitivity of Molybdenum polycrystal.

\[
\rho = \frac{\pi d}{\ln 2} \left( \frac{R_{AB,CD} + R_{BC,DA}}{2} \right) f\left( \frac{R_{AB,CD}}{R_{BC,DA}} \right)
\]

\[ d = 5.1 \text{ mil} \]
Fig. 7.4 Apparent resistivity of polycrystalline Molybdenum vs magnetic induction B.

Resistivity
Sample: MOP-10

T = 77° K
T = 300° K

Magnetic induction B

(μΩ cm)

Resistivity
Fig. 7-5 Uncompensated $V_{BD}$ as a function of frequency.

$V_{BD}$ (uncompensated) $\mu V$

- $T = 300^\circ K$
- $T = 77^\circ K$
Fig. 7-7 Noise level from the compensator as a function of frequency.
Fig. 7-8 Best compensation of $V_{BD}$ at different frequencies.
Uncompensated $V_{AD}$ as a function of frequency.
Fig. 7-10 Uncompensated $V_{DC}$ as a function of frequency.
Fig. 7-11 Phase plot of the uncompensated $V_{AD}$.

Fig. 7-12 Phase plot of the uncompensated $V_{DC}$. 

Frequency

10 2 4 6 8 100 2 4 6 8 1000 (cps)
Fig. 7: Amplitude and phase plots of $V_{AD}$ & $V_{DC}$ vs magnetic induction $B$. 

$V_{DC}$

$V_{AD}$
Voltage \( f=40 \text{ cps} \quad T=77^\circ\text{K} \)

Sample: MOP-08

\( \triangle \triangle \triangle V_{AD} \)
\( \circ \circ \circ V_{DC} \)

Amplitude and phase plots of \( V_{AD} \) & \( V_{DC} \) vs magnetic induction \( B \).

Phase \( f=40 \text{ cps} \quad T=77^\circ\text{K} \)

Sample: MOP-08

\( \triangle \triangle \triangle \phi(V_{AD}) \)
\( \circ \circ \circ \phi(V_{DC}) \)
Fig. 7-15a,b Amplitude and phase plots of $V_{BD}$ vs magnetic induction $B$.
Sample: MOP-06

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Table 7-3.
\[ B = 2 \text{ KG} \quad T = 300^\circ \text{K} \]

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B=3 KG

T=300°K

Sample: MOP-06
\( B = 4 \text{ KG} \quad T = 300^\circ \text{K} \)

Sample: MOP-06

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Table 7-6

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Fig. 7-16 Frequency test at B=0 KG.

Sample: MOP-06

B=0  T=300°K
Fig. 7-17 Frequency test at B = 1 KG.

Sample: MOP-06

B = 1 KG  T = 300°K
Fig. 7-18 Frequency test at B = 2 KG.
Fig. 7-19 Frequency test at B = 3 KG.

Sample: MOP-06

B = 3 KG  T = 300° K

V_{AD}  \Phi(V_{AD})  V_{DC}  \Phi(V_{DC})
Fig. 7-20 Frequency test at $B = 4$ KG.
Fig. 7-21 Asymptotic amplitude and phase plots of the transfer function $\frac{V_2}{V_1}$ of the Lag-phase shifter.
Fig. 7-22 Asymptotic amplitude and phase plots of the transfer function \( \frac{V_2}{V_1} \) of the Lead-phase shifter.
$Z_{in} = \frac{R}{3} \frac{\left(1 + \frac{R}{\alpha} \right)}{(1 + jRCW) \left(1 + \frac{jRCW}{1.555} \right) \left(1 + \frac{jRCW}{3.247} \right)}$}

Fig. 7-24 Asymptotic amplitude and phase plots of $Z_{in}$ of the phase shifter.
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Table 7-7 Approximate resistance vs phase angle
-guiderline for adjusting phase shifter.
8. DC Measurement on Molybdenum and Molybdenum-Rhenium Alloys

The Hall coefficient was measured by the DC method. In fact, the very first resistivity and Hall coefficient measurements were made by DC methods. Though DC measurement is generally thought to be more vulnerable to drift and AC pick-up, the improvement in the chopper input nanovoltmeter and the multi-digit potentiometer still make the DC method one of the best in the measurement of transport coefficients.

The donation of a Keithley 148 nanovoltmeter and a Honeywell 2773 six-digit potentiometer by the Jet Propulsion Laboratory made the DC measurement of the transport coefficient possible. A switching box was built for the DC measurement. Using the same sample holder developed for AC measurements and the switching box designed, the block diagram for the DC measurement is shown in Fig. 8-1. Referring to this diagram it is seen that the total current passing the sample was determined by monitoring the voltage drop across a 1Ω standard resistor which, in turn, was measured by extracting \( \frac{1}{10} \) of its value through a voltage divider of 10 precision resistors. Connection to the power supply and the sample holder was through banana plugs. The two pairs of the shielded Belden audio cable were of equal length to minimize any possible thermal voltage injection. Wire connection within the switching box consisted of both pressure contacts and soldering.
Fig. 8-1 DC measurement of transport coefficient by Van der Pauw's method.
With the input to the potentiometer shorted, the fluctuation was less than ±5 mv.

Both the resistivity and Hall coefficient of molybdenum and molybdenum-rhenium alloys were measured by Van der Pauw's method outlined in Sec. 5-3. The total current through the sample was adjusted to 1 amp so that the numerical reading of $V_{DC}$, $V_{AD}$ in volts is the same in ohms as $R_{AB,CD}$ and $R_{BC,DA}$ in ohms. The resistivity and Hall coefficient are calculated using eq. (5-1) and eq. (5-6) from the entries of the values for $V_{DC}$, $V_{AD}$ and $V_H$ given in Table 8-1 to Table 8-4. Each of the entries in $V_{DC}$, $V_{AD}$ and $V_H$ is a linear combination of $V(\pm, \pm), V(\pm, -), V(-, +)$ and $V(\pm, -)$ as explained later, where $V(\pm, -)$ indicates that the voltage is measured with the current flowing in the positive direction and the magnetic field in the negative direction following Kunzler's convention. The experimental data are taken starting $B = 0$ KG to $B = 8$ KG and back to $B = 0$ KG with the magnetic field being switched to positive and negative directions alternatively. The drifting component is eliminated by subtracting from each of the measured data $\Delta = \frac{1}{16} (V(B=0)_{\text{Final}} - V(B=0)_{\text{Initial}})$, the divisor 16 is introduced because there are 16 time intervals in the course of changing the magnetic field from $B = 0$ KG to $B = 8$ KG and then back to 0 KG. $\bar{V}$ and $\bar{R}_H$ are the average value of each column. The following equations outline the major steps in data calculation:
\[ V_{DC}(+, -) |_{BG} = V_{DC}(+, -) |_{BG} - \Delta \]

\[ \Delta = \frac{1}{166} \left[ V(B = 0) |_{Initial} \right] - \left[ V(B = 0) |_{Final} \right] \]

\[ V_{DC} = \frac{1}{4} \left( V_{DC}(+, +) + V_{DC}(+, -) - V_{DC}(-, +) - V_{DC}(-, -) \right) \]

\[ V_{AD} = \frac{1}{4} \left( V_{AD}(+, +) + V_{AD}(+, -) - V_{AD}(-, +) - V_{AD}(-, -) \right) \]

\[ V_{H} = \frac{1}{4} \left( V_{BD}(+, +) - V_{BD}(+, -) - V_{BD}(-, +) + V_{BD}(-, -) \right) \]

\[ \rho = \frac{\pi d}{\ln 2} \left( \frac{R_{AB, CD}}{2} + \frac{R_{BC, DA}}{2} \right) \left( \frac{R_{AB, CD}}{R_{BC, DA}} \right) \]

\[ R_{H} = \frac{V_{H} d}{B_{z} I} \]

Inspection of Table 8-1 to 8-4 indicates that within an accuracy of ±0.4% the resistivity is constant with respect to the magnetic field. Therefore within ±0.4% accuracy there is no magnetoresistance for Mo and Mo-Re samples. This seems to be a surprising fact contrary to the known peculiar shape of the Fermi surface of molybdenum. However, it is to be remembered that the measurement was made on polycrystalline specimens and that the random arrangement of component single crystal grains in the specimen could hide the directionality of the Fermi surface. The variation of the resistivity component with respect to Re content is plotted in Fig. 8-2. It is seen that the variation is linear. By the technique of linear regression \(^47\) the data can be approximated by

\[ \rho = 0.530 \cdot x + 6.06 \mu \Omega \cdot cm, \] at \( T = 300^0 \) K \quad \text{(8-1)} \]

35
\[ \varphi = 0.354 \cdot x + 0.567 \quad \text{at } T = 77^\circ K \quad ——— (8-2) \]

\( x \) = atomic percentage of Re in Mo.

From eqs. (8-1) and (8-2) it is obvious that the resistivity of Mo-Re alloys consists of two components; one that is independent of Re content, the other is linearly proportional to Re content. The ratio of Re-independent component \( \frac{0.567}{6.06} \sim 10.7 \) at \( T = 300^\circ K \) and \( T = 77^\circ K \) is rather temperature sensitive compared with that of the Re-dependent component \( \frac{0.354}{0.530} \sim 1.5 \). It seems that the Re-independent part comes from lattice scattering or electron-phonon interaction while the Re-dependent part comes from impurity scattering. The increase in resistivity with the addition of rhenium content further justifies the argument that the addition of rhenium atoms to molybdenum will not increase the conduction electron content, because of the similarity in atomic size and first and second ionization potentials between molybdenum and rhenium mentioned in Sec. 4-2. For if the concentration of the conduction electrons should increase because rhenium atoms carry one more s-electron than molybdenum, the resistivity would have decreased, but this is not the case in the measurement.

Referring to Fig. 8-3 and Fig. 8-4 the Hall coefficients of both molybdenum and molybdenum-rhenium alloys are constant with respect to the magnetic field up to the field of 8 KG. The variation of the Hall coefficient as a function of composition is plotted in Fig. 8-5. The sign of the measured Hall coeffi-
cient is positive in all measurements. The Hall coefficient of the polycrystalline molybdenum measured is $1.6 \times 10^{-10} \text{(MKS)}$. This numerical difference is probably caused by the degree of purity of the molybdenum samples used. It is seen that for compositions less than 15% Re, the Hall coefficient decreases linearly with increase in Re-concentration. The equations are given by linear regression as:

\[ R_H = -0.0462 \times 1 + 1.613 \times 10^{-10} \text{ at } T = 300^\circ \text{K} \quad (8-3) \]

\[ R_H = -0.0553 \times 1 + 1.423 \times 10^{-10} \text{ at } T = 77^\circ \text{K} \quad (8-4) \]

Above 15% Re the Hall coefficient shows a tendency toward saturation.

Because the measured Hall coefficients for molybdenum and molybdenum-rhenium alloy do not vary with respect to the magnetic field and the fact that the addition of rhenium atom is to increase the impurity scattering rather than change the effective number of charge carriers we adopt the two-band model mentioned in Sec. 2-3 to represent the Hall coefficient by

\[ R_H = \frac{1}{ne} \cdot \frac{\sigma_e - \sigma_h}{\sigma_e + \sigma_h} = \frac{1}{ne} \cdot \frac{1 - \frac{\sigma_h}{\sigma_e}}{1 + \frac{\sigma_h}{\sigma_e}} \quad (2-7b) \]

Using $n = 1.39 \times 10^{28}$ electrons/m$^3$ (Sec. 4-2) and assuming that the effective number of charge carriers remain the same for both molybdenum and molybdenum-rhenium alloys, as is justified, we calculate:
\[
\frac{\sigma_h}{\sigma_e} = \frac{1 - 0.222 R_H}{1 + 0.222 R_H} \\
\sigma_e = \frac{1}{\rho \left(1 + \frac{\sigma_h}{\sigma_e}\right)} \\
\sigma_h = \sigma_e \frac{\sigma_h}{\sigma_e} \\
\rho = \text{total resistivity} = \frac{1}{\sigma_e + \sigma_h}
\]

The results are tabulated in Table 8-5 and the electronic and hole conductivities are plotted in Fig. 8-6 and Fig. 8-7. The reciprocals of conductivities, i.e., resistivities, are plotted

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<th>$\frac{\sigma_h}{\sigma_e}$</th>
<th>($\mu\Omega \cdot \text{cm}$)</th>
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<th>$\sigma_h$</th>
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<td>(measured)</td>
<td>(measured)</td>
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<td>($10^6 \text{mho/cm}$)</td>
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|     | $T = 300^\circ \text{K}$         |                             |                          |           |           |
| 0   | 1.431                           | 0.517                      | 0.582                    | 1.133     | 0.5860    |
| 8   | 0.964                           | 0.647                      | 3.24                     | 0.187     | 0.1210    |
| 15  | 0.603                           | 0.764                      | 6.06                     | 0.0935    | 0.0714    |
| 25  | 0.376                           | 0.846                      | 9.28                     | 0.0584    | 0.0494    |

|     | $T = 77^\circ \text{K}$         |                             |                          |           |           |

Table 8-5 Calculation of electronic and hole conductivities.

in Fig. 8-8. From Fig. 8-5 and Fig. 8-6 it is seen that the decrease in the Hall coefficient is mainly caused by the reduction in the difference between electronic and hole conductivities rather than the increase of effective conduction
electrons. The linear dependence of electronic and hole resistivities on rhenium content in Fig. 8-8 shows that the effect of impurity scattering is reflected in both the electron and hole bands. This means that the difference between \( \frac{\sigma_e}{m_e} \) and \( \frac{\sigma_h}{m_h} \) becomes smaller. Exactly what happens to the band structure in the alloy with the addition of Re atoms remains a topic that deserves to be further probed.

To evaluate any possible magnetoresistance by the two band model in eq. (2-10) we pick the largest calculated conductivity pairs \( \sigma_e = 1.13 \times 10^8 \) mho/m, \( \sigma_h = 0.586 \times 10^8 \) mho/m in Table 8-5, and \( n = 1.39 \times 10^{28} \) electrons/m\(^3\) in Sec. 4-2. At \( B = 5 \) KG,
\[
\frac{\Delta \rho_{xx}}{\rho_{xx}} = \frac{\sigma_e \sigma_h}{n^2 e^2} B^2 \left( \frac{1.13 \times 10^8}{1.39 \times 10^{28}} \frac{0.586 \times 10^8}{1.6 \times 10^{-19}} \right) (0.5)^2
\]
\[
= 0.0745 \times 10^{-2} \approx 0.07\%
\]
This agrees with our measurement that no magnetoresistance was observed with an accuracy of \( \pm 0.4\%\). It appears that the temperature should be lowered below \( T = 77^\circ \) K in order to observe the magnetoresistance in the Mo-Re alloys.
Sample: MOP-07  \( d = 6.6 \text{ mil} \)

\[ T = 300^\circ K \]

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<th>( V_{DC}(\mu V) )</th>
<th>( V_{AD}(\mu V) )</th>
<th>( V_{H}(\mu V) )</th>
<th>( R_H10^{10}(\text{MKS}) )</th>
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<td>-0.706</td>
<td>1.479</td>
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</table>

\[ \overline{V}_{DC} = 90.311\mu V, \quad \overline{V}_{AD} = 65.468\mu V, \quad \varphi = 5.87 \mu \text{a-cm}, \quad \overline{R}_H10^{10} = 1.61(\text{MKS}) \]

Table 8-1

\[ T = 77^\circ K \]

<table>
<thead>
<tr>
<th>B(KG)</th>
<th>( V_{DC}(\mu V) )</th>
<th>( V_{AD}(\mu V) )</th>
<th>( V_{H}(\mu V) )</th>
<th>( R_H10^{10}(\text{MKS}) )</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>8.952</td>
<td>6.484</td>
<td>0.000</td>
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</tr>
<tr>
<td>1</td>
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</tr>
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<td>2</td>
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<td>6.483</td>
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<td>1.500</td>
</tr>
<tr>
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<td>6.490</td>
<td>-0.263</td>
<td>1.470</td>
</tr>
<tr>
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<td>6.490</td>
<td>-0.343</td>
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<td>6.497</td>
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<td>8</td>
<td>8.962</td>
<td>6.576</td>
<td>-0.680</td>
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\[ \overline{V}_{DC} = 8.950 \mu V, \quad \overline{V}_{AD} = 6.504 \mu V, \quad \varphi = 0.582 \mu \text{a-cm}, \quad \overline{R}_H10^{10} = 1.43(\text{MKS}) \]
Sample: 8MR-1 \( d = 12.3 \) mil
\( T = 300^\circ K \)

<table>
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<th>B (KG)</th>
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<th>( V_{AD} (\mu v) )</th>
<th>( V_H (\mu v) )</th>
<th>( R_H 10^{10} ) (MKS)</th>
</tr>
</thead>
<tbody>
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<td>0.000</td>
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</tr>
<tr>
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<td>69.820</td>
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<td>81.708</td>
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<td>81.713</td>
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<tr>
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<td>81.708</td>
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<td>81.705</td>
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<tr>
<td>8</td>
<td>69.810</td>
<td>81.707</td>
<td>-0.316</td>
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\( \bar{V}_{DC} = 69.812 \mu v, \quad \bar{V}_{AD} = 81.708 \mu v, \quad \rho = 10.7 \mu \Omega \cdot cm, \quad R_H 10^{10} \) (MKS) = 1.26

**Table 8-2**

\( T = 77^\circ K \)

<table>
<thead>
<tr>
<th>B (KG)</th>
<th>( V_{DC} (\mu v) )</th>
<th>( V_{AD} (\mu v) )</th>
<th>( V_H (\mu v) )</th>
<th>( R_H 10^{10} ) (MKS)</th>
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<tr>
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<td>25.316</td>
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<td>1.000</td>
</tr>
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<tr>
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<td>20.663</td>
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</tr>
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<td>0.898</td>
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\( \bar{V}_{DC} = 20.666 \mu v, \quad \bar{V}_{AD} = 25.310 \mu v, \quad \rho = 3.24 \mu \Omega \cdot cm, \quad R_H 10^{10} \) (MKS) = 0.964 (MKS)
Sample: 15MR-1  \(d = 8.4\) mil  

\[ T = 300^\circ K \]

<table>
<thead>
<tr>
<th>B (KG)</th>
<th>(V_{DC} (\mu V))</th>
<th>(V_{AD} (\mu V))</th>
<th>(V_H (\mu V))</th>
<th>(R_H 10^{-10} (MKS))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>173.274</td>
<td>0.000</td>
<td>----</td>
</tr>
<tr>
<td>1</td>
<td>116.556</td>
<td>173.279</td>
<td>-0.045</td>
<td>0.960</td>
</tr>
<tr>
<td>2</td>
<td>116.562</td>
<td>173.285</td>
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<td>0.949</td>
</tr>
<tr>
<td>3</td>
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<td>173.289</td>
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<td>116.488</td>
<td>173.300</td>
<td>-0.328</td>
<td>0.875</td>
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</tbody>
</table>

\(\bar{V}_{DC} = 116.538 \mu V\), \(\bar{V}_{AD} = 173.274 \mu V\), \(\rho = 13.8 \mu V\cdot cm\), \(R_H 10^{-10} = 0.911\) (MKS)

\(T = 77^\circ K\)

<table>
<thead>
<tr>
<th>B (KG)</th>
<th>(V_{DC} (\mu V))</th>
<th>(V_{AD} (\mu V))</th>
<th>(V_H (\mu V))</th>
<th>(R_H 10^{-10} (MKS))</th>
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<tbody>
<tr>
<td>0</td>
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<td>75.505</td>
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</tr>
<tr>
<td>1</td>
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<td>0.533</td>
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<tr>
<td>2</td>
<td>51.308</td>
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<td>-0.070</td>
<td>0.747</td>
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<td>-0.091</td>
<td>0.647</td>
</tr>
<tr>
<td>4</td>
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<td>75.555</td>
<td>-0.108</td>
<td>0.576</td>
</tr>
<tr>
<td>5</td>
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<tr>
<td>6</td>
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<td>75.537</td>
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<tr>
<td>7</td>
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<td>75.532</td>
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</tr>
<tr>
<td>8</td>
<td>51.312</td>
<td>75.564</td>
<td>-0.211</td>
<td>0.563</td>
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</table>

\(\bar{V}_{DC} = 51.309 \mu V\), \(\bar{V}_{AD} = 75.534 \mu V\), \(\rho = 6.06 \mu V\cdot cm\), \(R_H 10^{-10} = 0.603\) (MKS)
Sample: 25MR-1  \( d = 8.6 \) mil  
\( T = 300 \) K

<table>
<thead>
<tr>
<th>B(KG)</th>
<th>( V_{\text{DC}}(\mu\text{v}) )</th>
<th>( V_{\text{AD}}(\mu\text{v}) )</th>
<th>( V_{\text{H}}(\mu\text{v}) )</th>
<th>( R_h \times 10^{10} \text{(MKS)} )</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
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<td>0.633</td>
</tr>
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<tr>
<td>3</td>
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<td>171.458</td>
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</tr>
<tr>
<td>4</td>
<td>219.485</td>
<td>171.465</td>
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<td>0.612</td>
</tr>
<tr>
<td>5</td>
<td>219.498</td>
<td>171.560</td>
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<td>0.594</td>
</tr>
<tr>
<td>6</td>
<td>219.495</td>
<td>171.463</td>
<td>-0.161</td>
<td>0.586</td>
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<tr>
<td>7</td>
<td>219.488</td>
<td>171.453</td>
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<td>0.565</td>
</tr>
<tr>
<td>8</td>
<td>219.465</td>
<td>171.465</td>
<td>-0.214</td>
<td>0.584</td>
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</table>

\( \overline{V}_{\text{DC}} = 219.494 \mu\text{v}, \overline{V}_{\text{AD}} = 171.452 \mu\text{v}, \rho = 19.3 \mu\text{O-cm}, R_h \times 10^{10} \text{(MKS)} = 0.619 \)

Table 8-4

\( T = 77^\circ \text{K} \)

<table>
<thead>
<tr>
<th>B(KG)</th>
<th>( V_{\text{DC}}(\mu\text{v}) )</th>
<th>( V_{\text{AD}}(\mu\text{v}) )</th>
<th>( V_{\text{H}}(\mu\text{v}) )</th>
<th>( R_h \times 10^{10} \text{(MKS)} )</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
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</tr>
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<tr>
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<tr>
<td>8</td>
<td>102.598</td>
<td>85.147</td>
<td>-0.137</td>
<td>0.374</td>
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</table>

\( \overline{V}_{\text{DC}} = 102.713 \mu\text{v}, \overline{V}_{\text{AD}} = 85.299 \mu\text{v}, \rho = 9.28 \mu\text{O-cm}, R_h \times 10^{10} \text{(MKS)} = 0.376 \)
Fig. 8-2 Resistivity (\(\rho\)) of Mo and Mo-Re alloys as a function of alloy composition.

\[\rho = 0.530x + 6.06\] for \(T = 300^\circ K\)

\[\rho = 0.354x + 0.567\] for \(T = 77^\circ K\)
Alloys vs. Magnetic Induction (B) at T = 300° K.

Figure 8-3: Measured Hall Coefficient (R_H) of Mo and Mo-Re.
FIG. 8-4  Measured Hall Coefficient \( R_H \) of Mo and Mo-Re Alloys vs Magnetic Induction \( B \) at \( T = 77 \, \text{K} \).
Fig. 8-5 Hall coefficient of Mo and Mo-Re alloys as a function of alloy composition
Fig. 8-6 Electronic and hole conductivities of Mo & Mo-Re alloys at $T = 300^\circ$K.

Electronic conductivity:

$$\sigma_e = \frac{0.997}{x + 8.51}$$

Hole conductivity:

$$\sigma_h = \frac{0.974}{x + 8.9}$$
Fig. 8-7 Electronic and hole conductivities of Mo & Mo-Re alloys at $T = 77^\circ$ K.
$T = 300^\circ K$

$T = 77^\circ K$

**Fig. 8-8** Electronic and hole resistivities of Mo & Mo-Re alloys.

- $\rho_h = 1.027x + 19.446$
- $\rho_e = 1.003x + 8.534$
- $\rho_h = 0.744x + 2.127$
- $\rho_e = 0.659x + 0.605$
9. Conclusion:

1. Calculation of galvanomagnetic transport coefficients are rigorously formulated by the Shockley-Chambers integral formula (Appendix A), although, in actual, the evaluation is difficult. By extending the calculation to the free-electron model, we then treat the Fermi surface as a whole unit and generalizing the free electron theory that the effect of $E_x$ and $H_z$ are to shift the Fermi surface by $\Delta K_x = \frac{eE_x \zeta}{h}$, $\Delta K_y = -\frac{eE_x \zeta}{h} (\alpha z H_z)$ in the reciprocal lattice space. Evaluation of the magnetoresistance and Hall coefficient on this basis requires that

$$\alpha = \frac{e}{m^* c}, \quad \mathcal{J} = 1, \text{ for spherical Fermi surface}$$

$$\alpha = \frac{e}{m^* c} \cdot \frac{1}{2}, \quad \mathcal{J} = \frac{1}{2}, \text{ for cubic Fermi surface}$$

$$\alpha = \frac{e}{m^* c} \cdot \frac{2}{3}, \quad \mathcal{J} = \frac{2}{3}, \text{ for octahedral Fermi surface}$$

in order to meet the more elaborate calculation by Allgaier,$^{24}$ McClure$^{23}$ and Miyazawa$^{43}$ (Refer to Appendix A). This is justified because of the anisotropy of the effective mass for nonspherical Fermi surfaces. Therefore, the expression $\frac{e}{m^* c} \zeta H_z$ should be multiplied by a shape factor $\mathcal{J}$.

2. Since the Fermi surface of molybdenum consists of both hole and electron bands, application of the semiclassical approach to calculate $R_H$ requires the superposition of the
two bands. Assuming \( \frac{Z_e}{m_e} = x \frac{Z_h}{m_h} \) and using the value of the Hall coefficient of polycrystal molybdenum value as a guideline we find that \( \frac{Z_e}{m_e}/\frac{Z_h}{m_h} \approx 0.3 \). This difference in the characteristics of the electron and hole bands is reasonable. As a result, further development of Fermi-surface study and understanding of \( \gamma \) and \( m^* \) of metals makes the evaluation of Hall coefficients possible. However, at the present stage, the knowledge of the relaxation time and effective mass \( m^* \) in relation to the Fermi surface of metals are not well established. Numerical calculation of transport coefficients from Fermi surface parameters remains to be further explored.

3. Design of the sample holder using Kel-F material was successfully proved to be able to resist thermal shock. The angular position of the dial and the Kel-F connectors were designed so that they could easily match the helium dewar in the Material Science Laboratory.

4. Preparation of the specimen was accomplished by standard metallurgical processes, i.e., cutting by spark cutter followed by lapping and electropolishing. In our measurement on molybdenum polycrystalline specimens, pressure contact were used. However, soldering\(^7\)\(^1\) should be used in the future in order to protect the single crystal nature of the specimen.

5. Direct application of Van der Pauw's method to AC measurement of molybdenum samples showed that a large amount
of reactive components were present. A series of frequency
tests (Figs. 7-16, -17, -18, -19 and -20) show that the re-
response to frequency is linear above 200 cps, and that the
reactive element corresponds to an inductance of $L = 0.355 \mu H$.

6. By Van der Pauw's method DC measurement was made on
molybdenum and molybdenum-rhenium alloys at both room and
liquid nitrogen temperatures. Samples measured include 0%-Re
alloy (metallurgically pure molybdenum), 8%-Re alloy, 15%-Re
alloy and 25%-Re alloy. Within an accuracy of $\pm 0.4\%$ there is
no evidence of magnetoresistance for both Mo and Mo-Re sam-
ple the random arrangement of compo-
ment single crystal grains in polycrystalline samples. The
resistivities measured indicate a linear increase from 0%-Re
sample to 25%-Re sample. The variation of the Hall coeffi-
cient with respect to the magnetic field is measured to be
constant. Therefore it justifies the supposition that alloying
rhenium atoms, though with one more s-electron than molybdenum
atoms, to molybdenum does not increase the effective number
of conduction electrons but the effect shows in the increase
of scattering centers thus affect both the relaxation time
and effective mass $m^*$ in electronic and hole bands.
Appendix A

McClure's Approach to The Transport Coefficients $\sigma_{xx}$ and $\sigma_{xy}$

The conductivity tensor is generally given by the Shockley tube integral formula:

$$\sigma_{\alpha\beta} = \frac{1}{4\pi^2} \frac{e^2}{n^2} \int \frac{m_H^4}{\omega_H} \left\{ \int_0^{2\pi} \int_0^\infty V_{\alpha}(\phi, K_H) V_{\beta}(\phi - \phi', K_H) e^{\frac{-\phi'}{\omega_H}} d\phi d\phi' \right\} dK_H'$$

$$m_H^4 = \frac{\hbar^2}{2\pi} \frac{2A}{\varepsilon} \quad \omega_H = \frac{eH}{m_H^4 c}$$

$$\phi = \omega_H \frac{c_H}{eH} \int \frac{dK_s}{V_\perp}$$

$V_\perp$ = component of velocity perpendicular to magnetic field,

$dK_s$ = differentential element along the hodograph,

$A$ = area enclosed by a hodograph.

The evaluation of this equation over nonspherical Fermi surface is extremely difficult. McClure has introduced a formula to calculate the conductivity tensor component. This method is to expand the velocity on the hodograph (intersection of the Fermi sphere and a plane perpendicular to the magnetic field) as a Fourier series, with the period $T =$ time required for the electron to precess around the hodograph. The conductivity tensor can then be calculated as:

$$\sigma_{\alpha\beta} = \frac{e^2}{8\pi^3} \int d^3k \left( -\frac{\partial f_0}{\partial \varepsilon} \right) \tau \delta_{\alpha\beta} \quad \text{(A-1)}$$
\[ S_{xx} = \sum_{m=1}^{\infty} \frac{2|V_x(m)|^2}{1 + (m\tau z)^2} \]

\[ S_{xy} = \sum_{m=1}^{\infty} \left\{ \frac{V_x(m)V_y(m) + V_x(-m)V_y(-m)}{1 + (m\tau z)^2} + i \left[ \frac{V_x(-m)V_y(m) - V_x(m)V_y(-m)}{1 + (m\tau z)^2} \right] m \omega z \right\} \]

\[ \tau = \sum_{m=-\infty}^{\infty} \gamma (m) \exp(i\omega z) \]

\[ \omega = \frac{2 \pi e H}{\hbar c} \int \frac{dK_x}{\nu_F} \]

Assuming constant relaxation time \( \tau \), and Fermi-Dirac distribution, the volume integral in eq. (A-1) can be reduced to a surface integral

\[ \tilde{g}_{\alpha\beta} = \frac{e^2}{8\pi^3} \frac{\tau}{\hbar \nu_F} \int dA \tilde{S}_{\alpha\beta} \]

(A-2)

where \( V_F = \) velocity on the Fermi surface.

McClure's method is especially suitable for the calculation of spherical and cylindrical constant energy surfaces in which \( \omega \) is constant. Conductivity tensor components and Hall coefficient are calculated for spherical and cubic Fermi surfaces using eq. (A-2) for comparison with our derivation in Section 3-2.

A-1. Spherical Constant Energy Surface

\[ V_x = \frac{V_x}{2} (e^{i\omega z} + e^{-i\omega z}) \]

\[ S_{xx} = \frac{V_x^2}{2} \]

\[ S_{xy} = \frac{1}{2} \frac{V_x^2(\omega z)}{1 + (\omega z)^2} \sim \frac{V_x^2}{2} \omega z \]
Fig. A-1 Hodograph of spherical Fermi surface

\[ \int v_\perp^2 \, dA = \int_0^{2\pi} \int_0^\pi K_o^2 \sin \theta \, d\theta \, d\phi \left( V_o^2 \sin \theta \right) \]

Fermi surface \[ = \frac{6\pi}{3} K_o^2 V_o^2 \]

\[ n = \frac{1}{8\pi^3} \left( \frac{4\pi}{3} K_o^3 \right) = \frac{K_o^3}{6\pi^2} \]

\[ \sigma_{xx} = \frac{e^2}{8\pi^3} \cdot \frac{\tau}{V_o} \cdot \left( \frac{8\pi}{3} K_o^2 \right) \cdot \frac{V_o^2}{2} = \frac{ne^2\tau}{m} \]

\[ \sigma_{xy} = \frac{e^2}{8\pi^3} \cdot \frac{\tau}{V_o} \cdot \frac{8\pi}{3} K_o^2 \cdot \frac{V_o^2}{2} \cdot \omega \tau = \frac{ne^2\tau}{m} \omega \tau \]

\[ R_H = \frac{\sigma_{xy}}{(\sigma_{xx})^2 \pi} = \frac{1}{ne^2 c} \]

A-II. Cubic Constant Energy Surface

Calculations of cubic Fermi surfaces using McClure's method were made by Goldberg\textsuperscript{20}, etc. Similar calculations were made by Miyazawa\textsuperscript{43}, These calculations are summarized for comparison with our work:
The Fourier components of the velocity at hodograph are:

\[ v_x(m) = \frac{2V_0}{m \pi} \sin \frac{m \pi}{2} \cos \frac{m \pi}{4} \]

\[ v_y(m) = \frac{2V_0}{m \pi i} \sin \frac{m \pi}{2} \sin \frac{m \pi}{4} \]

\[ S_{x \times} = \frac{4V_0^2}{\pi^2} \sum_{m=1,3,5 \ldots} \frac{1}{m^2 \left[ 1 + (m\omega \tau)^2 \right]} \]

For \( \omega \tau \ll 1 \)

\[ = \frac{4V_0^2}{\pi^2} \left[ \frac{\pi^2}{8} - \frac{\pi(\omega \tau)}{2} \left( \coth \frac{\pi}{\omega \tau} - \frac{1}{2} \coth \frac{1}{2\omega \tau} \right) \right] \]

\[ = \frac{V_0^2}{2} \]

\[ S_{x \gamma} = \frac{4V_0^2}{\pi^2} (\omega \tau) \sum_{m=1,3,5 \ldots} \frac{(-1)^{m/2} - 1}{m \left[ 1 + (m\omega \tau)^2 \right]} \]

\[ = \frac{4V_0^2 \omega \tau}{\pi^2} \left\{ \frac{\pi}{4} - \frac{\pi}{2} \cdot \frac{\sinh \frac{\pi}{2\omega \tau}}{\sinh \frac{\pi}{\omega \tau}} \right\} \]

For \( \omega \tau \ll 1 \)

\[ = \frac{V_0^2 \omega \tau}{\pi} \]

\[ n = \frac{1}{8\pi^3} (2K_0)^3 = \frac{K_0^3}{\pi^3} \]

Because the transverse components of velocity on the two ends are zero, only four surfaces of the cube are involved in eq. (A-2)
\[
\sigma_{xx} = \frac{e^2}{8 \pi^3} \cdot \frac{e}{\hbar} \cdot 4 \kappa_0^2 \cdot 4 \cdot \frac{\nu_0^2}{2} = \frac{ne^2\tau}{m} \\
\sigma_{xy} = \frac{e^2}{(2\pi)^3} \cdot \frac{e}{\hbar} \cdot (2\kappa_0)^2 \cdot 4 \cdot \frac{\nu_0^2 \omega \tau}{\pi} \\
= \frac{2ne^2\tau}{\pi \cdot m} = \frac{ne^2\tau}{m} \cdot \frac{e\varepsilon H}{2mc} \\
R_H = \frac{\sigma_{xy}}{\sigma_{xx}^2 H} = \frac{1}{2ne\varepsilon} 
\]

A-III. Octahedral Constant Energy Surface

The calculation of conductivity coefficients for octahedral constant energy surface was also described by Miyazawa\textsuperscript{43}, using McClure's method. The result in a low-field limit is summarized as follows:

\[
\sigma_{xx} = \frac{ne^2\tau}{m} \\
\sigma_{xy} = \frac{ne^2\tau}{m} \cdot \frac{2}{3} \cdot \frac{e\varepsilon H}{mc} \\
R_H = -\frac{2}{3} \cdot \frac{1}{ne\varepsilon} 
\]
Appendix B

Calculation of The Volume Swept by a Sphere for Small Displacement

Fig. B-1 Fermi sphere shifted by $\Delta K_x$ and $\Delta K_y$.

Fig. B-2 Geometrical relations of various portions in Fig. B-1.

\[
\begin{align*}
V_A &= \int_{-\frac{1}{2} \Delta r}^{\frac{1}{2} \Delta r} \pi p^2 dK_z \\
&= \int_{-\frac{1}{2} \Delta r}^{\frac{1}{2} \Delta r} \pi (K_0^2 - K_z^2) dK_z \\
&= \left[ \pi K_0^2 K_z - \frac{\pi K_0^3}{3} \right]_{-\frac{1}{2} \Delta r}^{\frac{1}{2} \Delta r} \\
&= \frac{2\pi K_0^3}{3} + \frac{\pi K_0^2 \Delta r}{2} - \frac{\pi}{3} \left( \frac{\Delta r}{2} \right)^3 \\
V_B &= \int_{\frac{1}{2} \Delta r}^{K_0} \pi p^2 dK_z = \int_{\frac{1}{2} \Delta r}^{\frac{1}{2} \Delta r} \pi (K_0^2 - K_z^2) dK_z \\
&= \frac{2\pi K_0^3}{3} - \frac{\pi K_0^2 \Delta r}{2} + \frac{\pi (\Delta r)^3}{12} \\
V_I &= V_A - V_B = \pi K_0^2 \Delta r - \frac{\pi (\Delta r)^3}{12} \\
&\sim \pi K_0^2 \Delta r \\
\end{align*}
\]

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Appendix C

Calculation of The Average Velocity Over The Displaced Portion of Fermi Sphere

Fig. C-1 Finding the average velocity over a shifted Fermi sphere.

Sphere A:

\[
x^2 + y^2 + (z - c)^2 = K_o^2
\]

\[
r^2 - 2c \cdot \cos \theta \cdot r + (c^2 - K_o^2) = 0
\]

\[
r = c \cdot \cos \theta + \sqrt{K_o^2 - c^2 \sin^2 \theta}
\]

Sphere B:

\[
x^2 + y^2 + z^2 = K_o^2
\]

\[
r = K_o
\]

\[
r_A - r_B = c \cdot \cos \theta + K_o^2 - c^2 \sin^2 \theta - K_o
\]

\[
= c \cdot \cos \theta - K_o (1 - (1 - \frac{c^2}{K_o^2} \sin^2 \theta)^{\frac{1}{2}})
\]

\[
= c \cdot \cos \theta - \frac{1}{2} \cdot \frac{c}{K_o} (\sin^2 \theta)
\]

\[
\sim c \cdot \cos \theta
\]

Therefore, the thickness of the shaded layer is proportional
to \cos \theta$. Since $v_o$ is radial and is of constant amplitude, the average velocity in the direction of the displacement is

$$<v_z> = \frac{\int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} (v_o \cos \theta) \cdot (\cos \theta) \sin \theta \, d\theta}{\int_0^{2\pi} \cos \theta \cdot \sin \theta \, d\theta}$$

$$= \frac{2}{3} v_o \int_0^{\pi/2} \cos \theta \cdot \sin \theta \, d\theta$$

The average velocities in the $Y'$ and $Z'$ directions are

$$<v_{z'}> = <v_z> \cos \beta$$

$$<v_{y'}> = <v_z> \sin \beta$$

If $\beta$ is very small, in our case, say $\beta \sim \alpha \tau H_z \ll 1$

$$<v_{z'}> \sim <v_z> \frac{2}{3} v_o$$

$$<v_{y'}> \sim <v_z> \beta = \alpha \tau H_z \cdot \frac{2}{3} v_o.$$
Appendix D

Two Stage RC Phase Shifters

Fig. D-1 Asymptotic and phase plots of the transfer function $\frac{V_2}{V_1}$ of two stage R-C phase shifters.

$$\frac{V_2}{V_1} = \frac{(jRC\omega)^2}{(1 + jRC\omega/0.382)(1 + jRC\omega/2.618)}$$

Lead phase shifter

$$\frac{V_4}{V_1} = \frac{1}{(1 + jRC\omega/0.382)(1 + jRC\omega/2.618)}$$

Lag phase shifter
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