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GENERAL EQUILIBRIUM APPROACH

by

Yusuke Horiguchi

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CHAPTER I

Introduction
This study concerns the process in which the firm makes its financial decisions and how changes in various internal and external factors exert their influence upon the outcomes of these decisions. How much to invest, how to finance, and how to distribute the proceeds of investment between retention and dividends are the particular aspects of the corporate financial problem with which we shall deal. Underlying our treatment of these problems is the belief that a meaningful answer to them can be achieved only by examining and incorporating the interdependence of the investment, financing and dividend decisions into the analysis.

Currently there is no widely accepted theoretical apparatus linking these three decisions. Economists' usual assumptions of perfect certainty and perfect markets, as well as their identification of a business firm with an individual entrepreneur, assume away all the interesting problems of the financing and dividend decisions. The former assures the single-valued cost of capital schedule's independence from the amount and mix of financing. The latter makes the market value of the firm, whose maximization is assumed to be the objective of the firm, merely a present value of the future, yet certain, earning stream. The problem of the distribution of each period's earnings between dividends and retention simply reduces to a part of the investment problem. Dividends, being a residual, are spent for consumption or purchase of financial asset(s), depending upon the balance between the time preference and the promised yield.

Finance specialists have long recognized the importance of uncertainty and consequently the importance of differences in the various instruments of financing. Furthermore, they have recognized the prevalence of publicly owned corporations in which the functions of management, an operational decision taker, are separated from those of owners, reward receivers for
their committed capital. From this it follows that the market value of
the firm can no longer be identified with the present value of the future
earning stream. Potential effects of the dividend decision on the market
value has as well been recognized. However, their unfamiliarity with
more advanced methods of theoretical research and their general lack of
interest in theory construction has precluded them from establishing a
unified theory of corporate finance.

It is only recently that joint research by economists and finance
specialists began. Modigliani-Miller's controversial papers* on the
cost of capital problem and the irrelevance of the dividend decision have
compelled both finance specialists and economists to reevaluate their en-
tire theoretical constructs. Unfortunately, however, their approach to
the problems of corporate finance, i.e., segregating the investment, fi-
nancing, and dividend decisions, set the direction of subsequent research
in the field toward a partial equilibrium analysis. Questions were asked
in the following typical sequence: (1) Given the cost of capital schedule
and the predetermined dividend decision, how much should the firm invest?
(2) Given the predetermined investment decision, what is the best method
of financing? (3) Given the predetermined investment and financing, what
is the optimal dividend decision?

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*F. Modigliani and M. H. Miller, "The Cost of Capital, Corporation
Finance and the Theory of Investment," The American Economic Review,
Vol. XLVIII, No. 3 (June 1958) and "Dividend Policy, Growth and the
Valuation of Shares," Journal of Business of the University of Chicago,
Vol. XXXIV, No. 4 (October 1961).
Recognition of the fact that such variables as the share price, the capital budget, the dividend payout and the financial structure are jointly determined in the real world has since inspired a more realistic, general equilibrium approach to these problems. Initial efforts, however, remain unsatisfactory. Lerner-Carleton's* analysis failed to advance the matter much because of their essential abstraction from the problem of debt employment and their arbitrary specification of the potential effect of the rate of retention on the cost of capital. Eli Schwartz's** attempt to integrate the problem of debt employment with the capital budget and the share price was not convincing due to his neglect of the dividend problem and his assumption of a constant risk class, regardless of the size and the composition of investments.

Elaboration of our belief into a theoretical construct necessarily involves the explicit recognition of the importance of: (1) the dividend decision on the amount and mix of financing as well as on the current satisfaction of investors, (2) the amount of financing on the business risk characteristics of the firm, which partly determine the financial risk characteristics of the firm and (3) debt employment on the business risk of the firm through its effect on the amount of financing, as well as on the financial risk. These are the primary factors which link and


necessitate the simultaneous solving of the firm's three financial decisions.

Our task then is to establish a theoretical model which simultaneously determines these decisions with full recognition of their interdependence. We develop a basic, yet hypothetical, one-period model in chapter II. This is designed to show how the amount and mix of financing is optimally determined, given the available net cash flow to be distributed between dividends and retention. In the subsequent two chapters our concern will be an application of the model to the analysis of the firm's long-run financial decisions. An investigation of the optimal time path characteristics of the three financial decisions reveals the internal constraint, in the form of a prespecified dividend payout, that the firm's long-run policy imposes on its one-period decisions. The one-period decision thus becomes a "short-run" decision in the traditional economic sense. Accordingly, the topic of the final chapter of our analysis is the derivation of a theory of the firm's short-run investment behavior.
CHAPTER II

One-Period Analysis
In this chapter, an attempt will be made to establish a hypothesis which explains how the firm makes its investment, financing* and dividend decisions** simultaneously so as to attain its objective to be specified. We treat a hypothetical situation with a time horizon of one period.

Our basic assumptions are:

(1) the available amount of net cash flow, which was obtained in the previous period and is to be distributed between dividend and retained earnings, is given;

(2) the unit cost schedule of debt is given;

(3) the firm and the investors in the market have identical expectations of the economic and industrial conditions which will prevail in the period***;

(4) the value of other relevant variables for the analysis is taken to be given implicitly; and

(5) the objective of the firm is to maximize the level of utility**** of investors in the market, where the level of utility is assumed to

*Financing method available to the firm is assumed to be either the use of internal funds or that of debt. New equity issue is excluded. Since the firm in reality regards this method of financing extremely expensive in comparison to the other methods because of the flotation cost, rightward shift of supply curve and fear of dilution of earnings, they rarely use this method. Therefore, this abstraction has almost no practical importance.

**We may call simultaneous investment, financing and dividend decisions of the firm a total financial decision.

***Dispensing with this assumption will be the primary objective of our analysis in the next chapter.

****A utility function will be specified shortly.
depend only on the total financial decision of the firm in the period under consideration.*

*This assumption will be maintained throughout the entire analysis of this paper.
Summary* of Our Hypothesis

The firm attempts to determine its total financial decision so as to maximize the level of utility of investors in the market, given the available amount of net cash flow (NCF) and the expectation of the economic and industrial situations. The level of utility is assumed to be a function of the magnitude of total risk (TR), defined as a sum of total business risk (TBR) and financial risk (FR), and of the amount of dividend (D).** The magnitude of TBR is assumed to depend upon the amount of capital to be invested which consists of retained earnings (RE) and borrowing (B). The magnitude of FR is assumed to be a function of the magnitude of TBR and B. It follows, then, that the magnitude of FR is a function of RE and B. The magnitude of TR, a sum of TBR and FR, thus also depends upon RE and B. With the identity that NCF (given) = RE + D, which implies RE = NCF - D, we can state that TR is a function of D and B. Therefore, the firm's optimal total financial decision to attain maximum utility of investors reduces, in the final analysis, to the determination of optimal D and B.*** The optimization is hypothesized to be made in the following manner.

We first introduce the hypothesis that the firm, if D is taken to be given tentatively (which implies that RE is given), determines B such

---

*We shall summarize our analysis of this chapter at the outset for the purpose of introducing the basic framework of the entire argument. We shall use, freely, the concepts yet to be introduced.

**It will be clear, shortly, that this statement and the statement in the assumption (5) are consistent.

***TR is, therefore, an auxiliary variable, through the determination of which we can find out the investment and financing decisions of the firm if we prespecify the amount of D.
that TR will be minimized. We note that, with the assumption that RE is tentatively taken to be given, the optimal B determines both the size of investment and the method of its financing. This is an intermediate optimization to attain the optimum optimorum of utility maximization.

We can repeat this intermediate optimization at each and every possible level of RE. RE can take on the value between zero and the available NCF. With different amounts of RE, the firm determines different optimal B to minimize TR. Different optimal B produce the different magnitude of resulting TR. We recognize that as the amount of RE increases, TR, as the result of optimal borrowing, decreases; but at a decreasing rate. We can state, with an economic assertion that \( \frac{\partial U}{\partial TR} < 0 \), that \( \frac{\partial U}{\partial RE} > 0 \). We note, however, that any increase in RE implies an identical amount of decrease in D because of the assumption that NCF is fixed. It is asserted that \( \frac{\partial U}{\partial D} > 0 \). The optimal distribution of NCF between RE and D, thus, rests on this trade-off between a greater D and a smaller TR. Among attainable combinations* of D and TR, the firm will choose one that maximizes utility of investors in the market. This is the final stage of the optimization. The determination of the optimal D implies that of RE. With this optimal RE, we can find the optimal B, by referring back to the intermediate stage of optimization, which determines the optimal investment and financing decisions.

*The boundary of attainable combinations can be derived as the result of repeated intermediate optimization, at each and every level of RE.
Detail

I. Simultaneous determination of optimal investment and financing decisions -- intermediate optimization.*

In reality, firms desire to invest, if they can, by using internally generated funds.** This is true because they regard internal funds as the cheapest source of capital. They try to avoid the employment of debt capital because the fixed debt charge, which must be paid from uncertain and fluctuating cash inflow, gives them a strong feeling of risk. They consider the use of debt capital only when they do not have sufficient internal funds to finance the required amount of investment. We shall develop a hypothesis which explains a typical investment and the financing behavior of the firm. This hypothesis will provide an economic rationale. It will also offer a key to the solution of the optimal total financial decision of the firm.

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*At this stage of analysis, we assume that the optimal distribution of available NCF between RE and D has been already made. RE will be, thus, treated as a given number.

**See Gordon Donaldson, Corporate Debt Capacity (Boston; Division of Research, Graduate School of Business Administration, Harvard University, 1961).
Development of the Hypothesis

In his general theory,* Keynes hypothesises that the amount of investment of the economy is determined at the intersection of the marginal efficiency of investment schedule and the prevailing market rate of interest which is assumed to be independent of the amount borrowed. He explains also that the internal rate of return decreases as the amount of investment increases because of the Law of Diminishing Marginal Value Productivity. This line of reasoning is sufficiently sound when our concern is with the investment of the economy as a whole. Following this line of reasoning, however, cannot be justified at the micro level of investment behavior of individual firms. When we consider the investment behavior of individual firms, especially that of oligopolistic firms that account for so high a proportion of our economic activity, we have to recognize the necessity of looking at the matter from somewhat different aspects.

Let us think about the situation where a firm in an industry, which is, say, oligopolistic, considers a small amount of investment for a period. Does this small amount of investment guarantee a higher internal rate of return than greater amounts of investment which could be undertaken? It certainly does not; because if the other firms in this industry undertake a large amount of investment in order to improve the cost and the quality of products, it is very probable that the firm under consideration will lose its market share and possibly even the absolute volume of sales. The firm will not only suffer a damage over a short

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period of time but also will be required an extended effort to recover the loss. Then the rate of return on this small amount of investment will probably be very low, perhaps negative, and will probably be considerably lower than the rate of return on the sufficient amount of investment to maintain its competitive position. The range of possible variation of return that the firm has to expect on this small amount of investment may be very large. This consideration of the competitive risk is the first important factor we have to recognize in analyzing the investment and financing behavior of the individual firm.

Another factor we must consider is that the firm in this dynamic and fluctuating economy has to adjust its amount of investment to the possible fluctuation of the economic environment. The firm has to take into account the danger of "investing too much," increasing its operating leverage* that will subject the firm to a riskier position with respect to the business fluctuation.**

We can relate each of these two types of "business risk"*** to the

---

*For the explanation of the concept of the operating leverage and its effect on the "riskiness" of the firm with respect to the fluctuation in expected sales volume, see J. F. Weston and E. F. Brigham, Managerial Finance, 3rd edition (Holt, Rinehart and Winston, 1969). Note that the "riskiness" here refers to the possible range of fluctuation in EBIT due to the fluctuation in the sales volume.

**We can sum up these considerations by saying that the firm in this severely competitive economy (we may call it oligopolistic, in general, which is the predominant feature of our present economy), therefore has to steer a course between the cyclical risk of investing too much and the competitive risk of investing too little. ct. James S. Duesenberry, Business Cycle and Economic Growth (New York: McGraw-Hill, 1958).

***For the distinction between business risk and financial risk, see James C. Van Horn.
amount of investment to be considered, given the currently existing stock of capital.* The following two business risk schedules will be generated.

![Diagram of BR 1 and BR 2 risk schedules](image)

II-1

Business risk 1 (BR 1), risk associated with "investing too little," diminishes as the amount of investment to be undertaken increases but at a decreasing rate, i.e., \( \frac{dBR_1}{dI} < 0 \) and \( \frac{d^2BR_1}{dI^2} < 0 \). Business risk 2 (BR 2), risk associated with "investing too much," increases as the amount of investment to be undertaken increases and at an increasing rate, i.e., \( \frac{dBR_2}{dI} > 0 \) and \( \frac{d^2BR_2}{dI^2} > 0 \).** We denote these two curves in figure II-1 by business risk schedules; BR 1 schedule and BR 2 schedule, respectively.

We now consider primary determinants of the relative position and

---

*It is assumed that these two types of business risk are comparable in the same unit of measure; or we may create something that might be called a disutility function having those two types of business risk as its argument.

**The assumption about the sign of the second derivatives reflect a usual economic assertion.
shape of these business risk schedules. The relative position of BR 1 schedule is primarily determined by the relative intenity of competitive pressure that the firm perceives. Therefore, if the market share of the firm had been increasing at a faster rate than the situation reflected in figure II-1, for example, the schedule would shift downward such that \[ \frac{dBR_1}{dI} \] and \[ \frac{d^2BR_1}{dI^2} \] would both be smaller than in the original situation.*

The relative position and shape of BR 2 schedule are determined, primarily, by the future prospect of the economy and industry. This determines the expected range of future sales volume, given the market share that the firm desires to maintain or acquire and maintain. The future prospect depends upon the general level and trend of the general economic activity and of the expansion of the industry in the recent past, or upon the current attitude of the monetary authority, etc. Therefore, if for example, the larger increase in the demand for the product of the industry than the original situation reflected in figure II-1 had been experienced in the recent past, then the schedule would shift downward, such that \[ \frac{dBR_2}{dI} \] and \[ \frac{d^2BR_2}{dI^2} \] would both be smaller than in the original situation.

We discussed the primary determinants of the relative position and shape of business risk schedules to clarify the concepts of these schedules. The argument made here will become a basis for establishing a new theory of investment in chapter V.

*Since our primary concern in the subsequent discussions will be with the changes in the position and shape of BR 1 schedule, we can neglect the potentially important effect of the target market share on the absolute position and shape of BR 1 schedule.
Having constructed two business risk schedules and discussed their properties and primary determinants, we are now ready to introduce total business risk schedule into our analysis. By vertically summing these two business risk schedules, we can construct a total business risk schedule (TBR schedule). This function relates the magnitude of total business risk, a sum of these two types of business risk, to the amount of investment to be undertaken, given the existing stock of capital.*

TBR schedule is seen in the figure below to slope downward at first, to have a minimum point corresponding to some level of investment**, and then to slope upward.

---

*Economically meaningful variable, strictly speaking, is not the amount of capital assets in existence, but the magnitude of return and the range of its fluctuation that the firm has experienced with them.

**At that level of investment, \( \frac{dBR_1}{dI} = \frac{dBR_2}{dI} \) holds. At the level of investment smaller (greater) than that, \( \left| \frac{dBR_1}{dI} \right| > \frac{dBR_2}{dI} \left( \frac{dBR_1}{dI} \right) < \frac{dBR_2}{dI} \) holds.
Another set of information necessary for establishing our hypothesis explaining the simultaneous determination of optimal investment and financing decision is the financial risk schedule (FR schedule). By relating the intensity of financial risk perception to the various amounts of borrowing to be incurred by the firm, given the total business risk schedule that the firm faces, we can construct what we call the financial risk schedule. Note first that the intensity of financial risk is a function of two factors: (1) the amount of borrowing (B), and (2) the magnitude of total business risk*, i.e., \( FR = FR(TBR, B) \). We can state, in general, that \( \frac{\partial FR}{\partial TBR} > 0 \) and that \( \frac{\partial FR}{\partial B} > 0 \). We now construct the financial risk schedule, given TBR schedule and the amount of available retained earnings, and then summarize its properties in what follows.**

We note that each FR curve*** describes a functional relationship between the intensity of financial risk and the amount of borrowing, given some level of total business risk. FR, for example, is drawn, given TBR \( (I_o) \)****, with respect to the change in the amount of borrowing. These FR curves are assumed to have the following properties, reflecting the plausible economic considerations: (1) \( \frac{dFR}{dB} > 0 \) and \( \frac{d^2FR}{dB^2} > 0 \) everywhere and (2) if TBR \( (I_j) > TBR (I_1) \), then we have corresponding

---

*The ambiguity that might be involved in the statement will be clarified shortly.

**In the following discussion of this chapter, we will assume, merely for the simplicity of exposition, that the firm has no currently outstanding debt in its capital structure. The situation with a positive currently outstanding debt will be analyzed in great detail in chapter V. Note, however, that what follows in this chapter will be perfectly valid regardless of the amount of currently outstanding debt.

***See figure II-3 on the next page.

****We read it as TBR, being a function of I, evaluated at \( I_o \).
fr\(_1\) and fr\(_2\) curves such that at each and every level of borrowing to be incurred, \(\frac{df_{1\text{fr}}}{dB} > \frac{df_{2\text{fr}}}{dB}\) and \(\frac{d^2f_{1\text{fr}}}{dB^2} > \frac{d^2f_{2\text{fr}}}{dB^2}\).

In terms of economic meaning, these properties imply, respectively:

(1) given the level of total business risk, financial risk increases at an increasing rate as the amount of borrowing increases; (2) the marginal increment of financial risk due to the marginal increase in borrowing in the period is greater with greater total business risk, and the difference enlarges as the amount of borrowing increases.

The construction of FR schedule is made in the following manner, given the above discussion on fr curves.

(1) Draw fr curves for every level of TBR on the given TBR schedule, and find the relevant point on each fr curve. We shall illustrate the relevant point. For example, fr\(_x\) (B\(_x\)) is a resulting level of financial risk if the firm employs B\(_x\) to finance I\(_x\), being combined with the pre-specified amount of retained earnings.

(2) We connect the relevant point of all fr curves. Then we will have an FR schedule corresponding to the given TBR schedule and RE.
We discuss a property of FR schedule. Let the total derivative of FR with respect to the amount of borrowing be denoted by $\frac{dFR}{dB}$. Then $\frac{dFR}{dB} = \frac{\partial FR}{\partial B} + \frac{\partial FR}{\partial TBR} \frac{dTBR}{dI} \frac{dI}{dB}$, because of our assertion that the intensity of financial risk is a function of the magnitude of total business risk and the amount of borrowing. We note that $\frac{\partial FR}{\partial B} > 0$ and $\frac{\partial FR}{\partial TBR} \frac{dTBR}{dI} \frac{dI}{dB} < 0$. At the low level of borrowing, the effect of the first term is greatly offset by that of the second term. But as the amount of borrowing increases, the effect of the first term becomes too

*Note again that the intensity of financial risk depends both on (1) the level of total business risk and (2) the amount of borrowing.

**It is a total effect of the marginal increase in the amount of borrowing on the intensity of financial risk.

*** $\frac{\partial FR}{\partial B}$ is a change along an FR curve and $\frac{\partial FR}{\partial TBR} \frac{dTBR}{dI} \frac{dI}{dB}$ is a change due to the shift in FR curves from one corresponding TBR (I) to that corresponding to TBR (I + I).
great to be offset to a significant degree by the second term. This is 
so for the following two reasons. One reason is that for any given level 
of TBR, \( \frac{dFR}{dB} \) increases at an increasing rate as the amount of borrowing 
increases. The second reason is that \( \frac{dTBR}{dI} \) decreases as an amount of 
investment increases, because of the properties of BR 1 and BR 2 schedule 
already discussed. The slope of TBR schedule, which partly determines 
the relative shape and position of FR schedule, will be an important 
consideration in our analysis of the firm's investment behavior in 
chapter V. We can in general state that if \( \frac{dTBR}{dI} \) is relatively small 
over a relevant range of investment, FR schedule is expected to be such 
that \( \frac{dFR}{dB} \) is large because of the dominance of \( \frac{dFR}{dB} \). Furthermore, 
if \( \frac{dTBR}{dI} \) is relatively large, then we expect FR schedule to be such 
that \( \frac{dFR}{dB} \) is small because of the significance of \( \frac{dTBR}{dTBR} \frac{dFR}{dI} \frac{dI}{dB} \).

Given the above discussion on the various properties of these sche-
dules, explained with easily acceptable economic reasoning, we are now 
ready to state an hypothesis to explain the simultaneous determination 
of investment and financing decisions of the individual firm.
Total Risk Minimization Hypothesis

Basic Outline of the Hypothesis

A firm, confronted with a total business risk schedule and in addition the financial risk schedule, determines the amount of investment* and the method of financing it so that total risk (TR)**, defined as a sum of total business risk and financial risk, will be minimized.***

*The composition of this total amount of investment is optimally determined by assumption.

***We define, also, the total risk schedule (TR schedule) as a function which relates the magnitude of total risk with each and every level of investment with concomitant financing.

***The true variable is the amount of borrowing, as explained in the summary, with the assumption that RE is tentatively given.
(1) Suppose that the firm has enough retained earnings* to finance $I_\ast$, which is the level of investment that minimizes total business risk. Then the amount of investment to be undertaken will be $I_\ast$, and the amount of borrowing to be employed will be zero. By so determining, the firm, under this assumption, minimizes total risk.

(2) If the prespecified amount of available retained earnings is not enough to finance $I_\ast$, the decision will be made as follows. Given the total business risk schedule and the prespecified amount of available

---

*The amount of retained earnings is taken tentatively as given.
retained earnings, we can construct the corresponding FR schedule. Then, given these two schedules, we can construct TR schedule as their vertical sum. Then the amount of investment and of borrowing will be determined so that total risk is minimized.* Therefore, the amount of investment will be \( I_{x^*} \), instead of \( I_x \). The amount of borrowing will be \( B_{x^*} \) which, in combination with the available retained earnings, will be used to finance \( I_{x^*} \). The optimality condition in the marginal terms is stated as the equivalence of the marginal increment of financial risk and the marginal decrease in total business risk at \( I_{x^*} \). The trade-off between the reduction in total business risk and the increment in financial risk is the key to the optimal investment and financing decisions.

By paying particular attention to the close interdependence of the firm's investment and financing decisions and to their simultaneous determination, we established a total risk minimization hypothesis. This hypothesis not only explains how the investment and financing decisions are made simultaneously, but also finds an economic rationale for the employment of debt capital as the reduction of total risk.** Furthermore, the hypothesis will constitute the basis for the analysis of simultaneous investment, financing and dividend decisions in the next section and for establishing our new theory of "investment behavior" of the firm in chapter V.

*See figure II-5.

**The employment of debt capital has been taken as a factor to increase "risk" in the literature.
II. Optimal distribution of the available net cash flow between dividend and retained earnings* -- final optimization.

We see, first, that the attainable smallness of total risk resulting from the optimal simultaneous investment and financing decisions depends crucially on the amount of the available retained earnings. Given the total business risk schedule that the firm faces, we can draw a family of FR schedules according to the amount of available retained earnings.

---

*This implies the simultaneous determination of investment, financing and dividend decisions, as will be clear by the end of the discussion in this section.
It must be obvious why FR schedule becomes steeper* as the amount of retained earnings decreases, given the TBR schedule. We remember the way we constructed FR schedule from fr curves. Point "a" in the figure is a relevant point on fr\(_i\), corresponding to TBR (I\(_i\)), which is a resulting magnitude of total business risk when the firm invests I\(_i\) (by RE\(_i\) and \(\bar{d}\)). Point "b" is a relevant point on fr\(_j\), corresponding to TBR (I\(_j\)), which is a resulting total business risk when the firm invests I\(_j\) (by RE\(_2\) and \(\bar{d}\)). It is clear that the same amount of debt, \(\bar{d}\), can result in different financial risks depending upon the different amounts of retained earnings available.

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*This term means that the magnitude of financial risk measured at the same level of debt, for all levels of debt, is greater when the firm has RE\(_i\) than when it has RE\(_j\) where RE\(_i\) < RE\(_j\), given TBR schedule.
It also becomes obvious that the optimal investment and financing decisions result in the different optimal sizes of investment and amounts of debt, depending upon the available amount of retained earnings. It can be further shown that, cet. par., the greater amount of available retained earnings implies the smaller magnitude of total risk as a consequence of optimal simultaneous investment and financing decisions.

![Diagram](image)

Compare the case where the firm has RE₁ and the case where the firm has RE₂. Resulting TR for the case of RE₂ is necessarily greater than the case of RE₁. Compare, next, the case of RE₂ and the case of RE₃. Suppose that the optimal amount of investment in the case of RE₂ is I₂* and that in the case of RE₃ is I₂ or less than I₂. Then the resulting TR for the case of RE₃ is necessarily greater. Suppose that the optimal amount of investment in both cases is greater than I₂, then because of the

*Note that when the firm has RE₂ which can just finance I₂, where I₂ is smaller than the amount of I that minimizes resulting TBR, the firm will never invest less amount than I₂.
properties of FR schedules corresponding to RE_2 and RE_3, respectively, it is obvious that the resulting TR for the case of RE_3 is greater. This reasoning applies for all RE_1 and RE_j.

We can now construct the locus of points which combine each and every amount of the available retained earnings, given the prespecified amount of available net cash flow obtained in the previous period, with the magnitude of total risk resulting from the optimal investment and financing decisions given the corresponding amount of available retained earnings.

We can easily verify what is implied by the shape of this locus. Because of the properties of BR 1 and BR 2 schedules*, TBR schedule, sum of these

\*i.e., \( \frac{dBR_2}{dt} \) decreases as the amount of investment to be undertaken increases, and \( \frac{dBR_2}{dI} \) increases as the amount of investment increases.
two schedules, has such a property that \( \left| \frac{dTBR}{dI} \right| \) diminishes as the amount of investment to be undertaken increases. Then, because of the relationship between TBR schedule and FR schedule, the difference in the steepness of FR schedules corresponding to RE\(_1\) and RE\(_{1+1}\)*, respectively, decreases as RE increases. These considerations lead us to the conclusion that the marginal contribution of the marginal increment in RE measured in terms of the induced marginal decline in the magnitude of total risk diminishes as the available amount of RE increases.

Now given the prespecified amount of available net cash flow which will be distributed between dividend and retained earnings, we can redraw the figure in the following manner which contains exactly the same information as the above figure does.

*Refer to the figure II-8 to understand what RE\(_1\) and RE\(_{1+1}\) are.

**We are assuming that the available amount of net cash flow is less than can finance the amount of investment which minimizes TBR.
We call this locus an "opportunity locus." The opportunity locus is one of two sets of information we need to have in order to determine the optimal allocation of available net cash flow between dividend and retained earnings.

The other set of information we need to have is the preference of the investors in the market for the magnitude of TR and the amount of dividend to be paid out. Where we assume that the investors in the market are risk averse and that the marginal rate of substitution between TR and greater D diminishes in either direction.

Preference map is expected to shape as following.
By combining these two sets of information, we can find the solution to the problem of the optimal allocation of available net cash flow between dividend and retained earnings.

By subtracting $D_*$ from the prespecified amount of available net cash flow, we find the optimal level of retention. Then, given this optimal amount of retained earnings, we will find the optimal amount of investment and the optimal amount of debt by referring back to the first stage* of this analysis. These optimal investment and financing decisions, given the optimal level of retention, needless to say, will result in the magnitude of TR which corresponds to $D_*$ on the opportunity locus, because of the way we constructed the opportunity locus.

This is the optimal total financial decision of the firm** where, given the TER schedule and the prespecified amount of available net

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*This refers to the intermediate optimization.

**Remember that we are assuming that the time horizon is one period.
cash flow, the firm determines its optimal investment, financing and dividend decisions simultaneously so as to give investors a maximum attainable satisfaction.

We could conduct a comparative static analysis at this stage to examine the changes in the optimal financial decision with respect to some important parametrical changes, which necessarily involve the changes in the optimal dividend decision without knowing which we cannot infer the changes in the optimal investment and financing decisions because of our theoretical construction. But given the commonly observed phenomena that the firms usually appear to attempt, at least, to stabilize the dividend payment in the form of either constant level, constant growth rate or (constant) target rate, it would be, we feel, merely a mental exercise to analyze the change in the optimal dividend decision in one period analysis, without further analysis on the optimal dividend policy.

We should not, however, make an assumption that the dividend decision, being one of three important financial decisions we are concerned with, is absolutely predetermined in one period analysis without justifying it based on theoretical grounds.* This is where we have to go to the long-run analysis where an attempt will be made to find out how the firm determines its optimal time path of investment, financing and dividend decisions so as to attain its objective.

*It is amazing to find out how little literature has been written on how the firm determines its dividend policy, why it attempts to stabilize it once determining it and when it considers very seriously the change in the dividend policy, though a number of empirical works has been conducted without theoretical ground as well as great many sketchy, informal discussions on the matter.
CHAPTER III

Preliminaries for Long Run Analysis
I. Comment on the Conventional Analysis of Long-Run Financial Decision

Before we embark on long-run analysis, we shall pay attention to the organizational change associated with the development of the corporation system in the history of capitalism. We shall specifically analyze the economic consequence of "owners of publicly owned corporations being essentially outsiders." Failure to make such analysis seems to have been one important cause of keeping the theory of stock valuation and of long-run financial decisions of the firm from making sense.

Robin Marris, in his "The Economic Theory of Managerial Capitalism,"* clearly pointed out the importance of recognizing the above mentioned economic consequence.

The essence of traditional method of economic organization is the unification of the function of risk carrying, reward receiving and operational decision taking in one individual. By combining ownership with management, the person who carries much of the risk also makes most of the decisions determining its extent. As owner he receives rewards of success and is therefore motivated to optimize the balance between boldness and prudence. Economic theory has often overlooked the interdependence between risk and organization. Risk is seen arising mainly from exogenous uncertainty, for example, from uncertainty in the future demand for the product of a particular asset. The shape of the resulting probability distribution of returns is therefore outside the control of anyone within the firm. Risk taking, in this concept, consists only in choosing between lists of projects, with given, unalterable probability distributions. Real commercial life is much more flexible. If the demand for one product turns out badly, a nimble decision taker may restore his position by changing the line of production or at least he may minimize losses by timely retrenchment. A slow decision taker, on the other hand, may fall into a vicious spiral of financial decline; by acting too late, he incurs losses inducing actions causing further losses and so on. Thus, the probability distribution of financial results is not only differently shaped from the exogenous parent

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distribution (because in the latter, distributions are independent, in the former set) but also depends intimately on decision taking skill and will power. It is difficult for an outsider to assess the ability and integrity of a particular professional manager or team. The assessment itself involves considerable uncertainty and represents perhaps the greater part of the risk of an investment. As such it must be paid for.

Considering Marris' statement, we shall examine the conventional analysis of long-run financial decision making of the firm.*

In all of the theoretical writings in the field, the objective of the firm has been assumed to be the maximization of the market price of the shares in the current period. They have assumed additionally that the market price of the shares is the present value of either expected future net earning, dividend or net cash flow stream. The firm's optimization was defined as its decision making in the current period (time zero) to determine the future time path of investment, financing and dividend policy such that either of these expected future streams as results of the planned decision maximizes the current market price of the shares.

We shall carefully scrutinize the inconsistency in the setting of their analysis described above, based on the fact whose important economic consequence is remarked clearly by Marris. What the firm does in their writings is to determine, from the perspective of time zero, its investment, financing and dividend decisions over the time horizon between time zero and infinite future such that the current market price of the shares is to be maximized. Who, however, determines the price of,

or the worth of holding of, the shares in the final analysis? It is investors in the market who determine the worth of shares to them,* not the management of the firm. Investors determine the worth of holding shares to them by formulating the expectation as to the future stream of the relevant variable(s), based on the past history and future guess as well as the current development. Though the firm can possibly determine the whole future course of its decisions at time zero (current period), the only thing that the investors in the market can observe is, at the most, the partial results of the decision made at time zero for the current period. There is no way for them to know what the management of the firm has actually decided concerning the investment, financing and dividend policy over the infinite future based on the management's own expectation as to the future. All that the investors can do about the future, being essentially outsiders, is to formulate expectations of the future performance of the firm which will depend upon both the firm's financial decisions over its time horizon and the economic and industrial circumstances in the future. Their expectation is a function of the past and current performance of the firm as well as of the investor's guess about the future economic and industrial environment. In turn, the past performance is the joint product of the firm's decision in the past and its particular exogenous factors, while the current performance is a product both of current and past decisions and of their respective economic-industrial climates. The investor's guess about the future performance of the firm is formulated from his guess about these two determining factors. There is no way in which the firm's current decisions for the future

*They do so by capitalizing "expected" future stream of some relevant variable(s), which we do not know what it is (they are) yet.
financial policy can affect the formulation of the investor's expectations. The current market price of shares is determined, therefore, not by the future investment, financing and dividend policy of the firm.

In conclusion, the assumption made by other writers as to the objective of the firm, to maximize the current market price of the shares, and what they attempt to analyze as the optimization by the firm, that is, to decide the time path of investment, financing and dividend policy between the current period and the infinite future, is mutually inconsistent.

The consistent ways of analyzing the problem of optimal financing decision of the firm seem to be following. One possible way is:

(1) We assume that the objective of the firm is to maximize the current market price of the share. Then,

(2) What to be analyzed is the determination of the optimal investment, financing and dividend decisions of the firm for the current period. The criterion of the optimality is that this particular set of current decisions, given the past history of the firm, will make the investor's expectation of the future performance of the firm such as to maximize the current market price of the shares. Another possible way is:

(1) We make the objective of our analysis as examining the "long-run" optimal financial decision of the firm where the firm determines the paths of investment, financing and dividend policies for the relevant future time horizon such that some consistent objective of the firm will be attained.

(2) We assume, then, the objective of the firm in making its long-run financial decision from its current period perspective is to maximize
\[ P = \int_0^T \tilde{p}(t) \, dt \] and we further add the constraint to be stated as, subject to \( \tilde{p}^{(i-1)} \leq \tilde{p}^{(i)} \) for all \( i \). \( \tilde{p}(t) \) is the market price of the shares expected by the firm to prevail at \( t^{***} \), \( r \) is some relevant time horizon and time zero is an arbitrary time period in which the firm is assumed to make its investment, financing and dividend decisions over the horizon.

By taking the latter course, we shall analyze long-run financial decision making of the firm in the following chapter.

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*The maximization of \( P \) is merely a more clear cut presentation of the following ambiguous statement made in the finance literature. "The objective of the firm is to maximize the long-run value per share of the common stock on the market."

**It may be stated that this constraint simply follows the criterion maintained in the finance literature that any financial plan of the firm in any period must pass, i.e., "Will the financial plan raise the market value of the firm's share? If so, it is worth undertaking. If not, it is not worth undertaking." In our subsequent theoretical development, however, the validity of the constraint will be explained in a great detail in relation to be maximization of \( \).

***This expectation is made at time zero, i.e., at the time of long-run decision making.
II. Revision of Our Theory

We discussed how unrealistic it is to assume that the investors in the market know exactly what the management of the firm has determined concerning its investment, financing and dividend policies over the infinite future, as has been assumed by the other writers. We have to recognize that the above criticism based on the fact of investors being outsiders, unfortunately, directly applies in essence to the theory we developed in the previous chapter. We assumed in the previous chapter that the total business risk schedule, financial risk schedule and total risk schedule constructed in the mind of the management of the firm and those constructed in the mind of investors in the market are identical. This assumption is very implausible in light of what was argued above. Let us first see what this assumption implies. This assumption required:

(1) that the investors in the market as well as the management of the firm have sufficient detailed information as to the value of relevant variables such as demand condition, cost situation, competitive situation and other numerous variables which describe the whole facets of the firm's activity and its surroundings, (2) that both formulate an identical estimation as to the values of these variables resulting from each and every level of investment with concomitant financing to be undertaken and further (3) that both perceive exactly identical magnitude of feeling of risk corresponding to each and every level of investment cum financing, based on the expected magnitude of some objective measure (or indicators) of the risk position of the firm. This expected magnitude is calculated (or estimated) based on the estimated resulting values of those relevant
variables corresponding to each and every level of investment cum financing.*

As has been argued in the discussion in the previous section of this chapter, the investors in the market, being essentially outsiders though they are referred to as the (potential) owners of the firm, have such a limited access to the necessary information concerning the firm's activity that they are not able to calculate in a meaningful manner resulting values of numerous variables as a consequence of each and every level of investment cum financing. It is, in addition, very doubtful that the risk perception of the investors and that of the management are identical. It is, therefore, extremely misleading to assume an equivalence of the risk schedules. The analysis of the firm's financial decision based on total risk and dividend must break down. We have to, then, revise our theory developed in the previous chapter to make it more meaningful. The revision of our theory will be made by taking several steps.

We first determine what the best measure (or indicator) is which constitutes the objective basis for total business risk and total risk schedules constructed in the mind of the management of the firm.** The most plausible objective measures among various alternatives are the coefficient of variation of cash flow correspondence to each and every level of investment to be undertaken for the total business risk schedule and

*See the appendix to this chapter.

**The existence of some concrete measure of risk which constitutes the basis for the risk schedules in the mind of the management of the firm must be assumed in order for the discussion to be carried analytically.
the coefficient of variation of net cash flow corresponding to each and every level of investment cum financing for the total risk schedule, respectively. Then we can state that the firm, indetermining simultaneously the investment, financing and dividend policy for the period under consideration, is given a choice represented by the opportunity locus. This locus consists of all the possible "efficient" combinations of the amount of dividend to be paid out by the management of the firm in the period from the available net cash flow (obtained in the previous period) and the coefficient of variation of net cash flow; that is, the net cash flow which the management of the firm expects to obtain during the period.

We next reconsider the utility function of the investors in the market. We have assumed up to now that the level of utility the investors derive from holding the share(s) of the firm depends on what they "expect" as the results of the decisions made by the management of the firm (or, we would rather state, that the investors in the market were concerned with the decisions made by the firm which gives the investors some idea about the results of the operation of the firm, where its operation is based on the decisions made by the firm). Taking now into consideration the fact discussed above; that is, that the investors in the market, being essentially outsiders, have only a limited and incomplete version of the information required to be had if they are to calculate in the meaningful manner the expected outcomes as the results of each and every level of investment with concomitant financing, we will, instead, assume that the
level of utility that the investors in the market derive by holding the share(s) of the firm depends on the set of "the actual results" of the decisions made by the firm for the period and the operation of the period based on the decisions. "The actual results" refer to the net cash flow realized during the period and the amount of dividend paid out in the period out of net cash flow* obtained in the previous period. Then we can draw the preference map of the investors in the market as follows, assuming only that \( \frac{\partial U}{\partial D} > 0 \), \( \frac{\partial U}{\partial NCF} > 0 \) and the marginal rate of substitution diminishes in either direction.

The task of management of the firm, being given the opportunity locus in terms of the coefficient of variation of net cash flow (based on the expectation held by the management of the firm) and the amount of dividend to be paid out, becomes, then, to choose the simultaneous dividend, investment and financing policies for the period which the management of the firm expects to produce the results which will maximize the satisfaction of investors, given their preference map. The decision making will be conducted by the management in the following way.

The coefficient of variation of net cash flow consists of two components, which are, in ordinary definition, the standard deviation (numerator) and the expected value (denominator) of the random variable with a specified probability distribution. In our analysis, however, they are (1) possible downward deviation of net cash flow and (2) desired amount of net cash flow, both of which are assessed by the management with respect to each and every level of investment with concomitant financing. We will still denote (1) as \( \sigma \) and (2) as \( \mu \), following the statistical convention. We defined the coefficient of variation of net cash flow which is expected by the management of the firm to be realized during the period in the above informal and practical manner in order to avoid the assumption, most commonly made in the finance literature, of the existence of the complete probability distribution of the uncertain outcome (net cash flow in this case) at each and every level of investment cum financing contemplated by the firm.

Given \( \sigma \) and \( \mu \) of net cash flow, which is expected by the management of the firm to be realized as a result of the optimal investment and financing decisions with each and every possible level of retained earnings, we can construct two opportunity loci in the dimension of net
cash flow and dividend. They are (1) the optimistic opportunity locus which is the locus of all the efficient combinations of dividend and $\mu^*$, and (2) the pessimistic opportunity locus which is the locus of all the efficient combinations of dividend and $\mu - \sigma^*$. The management, with these

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*Each level of $\mu$ on the opportunity locus is the amount of net cash flow that the firm hopes to earn during the period as a result of optimal investment and financing decisions, given corresponding level of dividend.*

**Each level of $\mu - \sigma$ on the opportunity locus is the conservative estimate of the net cash flow that the firm expects to earn during the period as a result of optimal investment and financing decisions, given corresponding level of dividend.*
two opportunity loci, has to determine the optimal unique D and corresponding investment and financing decisions, which are expected by the management of the firm to maximize the investor's satisfaction.

If the firm determines its dividend policy as $D_*$ (implying corresponding investment and financing decisions) basing its decision on the optimistic opportunity locus, it expects that the investors in the market will obtain the level of utility represented by I. But if the true situation* is such that the pessimistic opportunity locus is the correct description of the firm's opportunity, then the level of utility that the investors will obtain, given that dividend policy, will be only II which is lower than $I'$; that is, the level of utility the investors will obtain

*This is, of course, unknown to the firm at the time of its decision.
if the firm determines its dividend policy as \( D^* \) basing its decision on
the pessimistic opportunity locus and if that is the correct description
of the firm's opportunity. But if the firm bases its decision on the
pessimistic opportunity locus and determines its dividend policy as \( D^* \),
while the optimistic opportunity locus describes correctly the firm's
ture opportunity, then the investors will obtain the level of utility
represented by \( II' \), instead of that represented by \( I \) which could be ob-
tained if the management expected its opportunity correctly.

How does the firm determine its optimal unique \( D \) and corresponding
optimal investment and financing decisions under this situation? We
can think of two alternative hypotheses for the firm's decision making
under this complicated situation. Both, however, have more or less identi-
tical economic implication and consequence; and therefore, we will discuss
only one in detail.* The first hypothesis is that the firm will construct
what we call an "anticipated opportunity locus," between those two ex-
treme opportunity loci, on which it will base its financial decision.
The "anticipated opportunity locus" is defined as the locus of all the
effective combinations of dividend and \( \mu - \alpha \cdot \sigma \), to be called an anticipated net cash flow, where \( \alpha \) takes on the value between zero and one
and its final value is determined as follows.

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*In both hypotheses, we assume that the management of the firm thinks
that either the optimistic opportunity locus or the pessimistic opportuni-
ty locus describes the unknown true opportunity of the firm, with the
probabilities \( P \) and \( 1 - P \), attached by the management to the occurrence
of each of these two events, since we are assuming that \( \sigma \) and \( \mu \) (remember
that they are defined in our own way) are the only things that the
firm is concerned with concerning the unknown net cash flow and cash flow
to be obtained during the period corresponding to each and every level of
investment cum financing with various levels of retained earnings; or we
would say, since \( \mu \) and \( \sigma \) in our definitions are the only thing we can
assume that the firm can meaningfully figure out.
Let $\alpha = \alpha_1$, and we look at the situation.

The firm, basing its financial decision now on this anticipated opportunity locus, determines its optimal dividend decision represented by $D^*$ and also the corresponding investment and financing decisions. The firm expects that the investors in the market will obtain either the level of satisfaction represented by $\text{II''}$ or that represented by $\text{III''}$, for we assume that the firm expects either the optimistic opportunity locus or the pessimistic opportunity locus to describe the unknown true opportunity of the firm for the period. We recognize that the maximum obtainable level of satisfaction of the investors in this case is smaller than when the firm bases its financial decision solely on the optimistic opportunity locus and is greater than when it bases that solely on the pessimistic opportunity locus. We also recognize that the range of possible deviation of resulting level of satisfaction this case is smaller.
than where the firm bases its financial decision solely on the optimistic opportunity locus and is greater than when the firm bases its financial decision solely on the pessimistic opportunity locus.

We can generalize this information by stating that the greater the value of $\lambda$ becomes, the greater both the maximum obtainable level of satisfaction and the range of the possible deviation of level of satisfaction of the investors become. By assuming that the management of the firm attaches the probabilities to the two possible events, i.e., (1) that the optimistic opportunity locus describes the unknown true opportunity of the firm and (2) that the pessimistic opportunity locus describes the unknown true opportunity of the firm, where the magnitude of the probability that the management of the firm attaches to each of these depends on the confidence that it has as to the likelihood of each event, then we can put the determination of the optimal $\lambda$ by the management of the firm in the familiar analytical apparatus of "choice between risk and return."
Though we cannot know the precise shape of the efficiency locus unless we know the shapes of preference map of investors, two opportunity loci of the firm and the probabilities that the management of the firm attaches to the occurrence of each of possible events, we draw it in the above manner by assuming that the marginal productivity of risk taking measured in terms of marginal increment of expected value of the level of utility diminishes as the assumed risk becomes greater. We have assumed also that the management of the firm is risk adverse. Since the above figure is self-explanatory other than the assumptions stated above, we will simply state that this is the way the management of the firm determines the optimal value of $\alpha$.
Given this optimal value of $\alpha$, $\alpha^*$, we go to the final stage of the firm's decision making under the first hypothesis. Having determined $\alpha^*$, the management of the firm will now determine the optimal dividend policy and corresponding optimal investment and financing policies, basing its decisions on the optimal anticipated opportunity locus which is the locus of all the effective combinations of $D$ and $\mu - \alpha^*\sigma$, that will maximize the investor's satisfaction.

The firm finally determines its optimal dividend, $D^*$, and corresponding optimal investment and financing decisions in this manner. We note that the anticipated net cash flow corresponding to $D^*$ on the optimal anticipated opportunity locus constitutes one of the critical elements in the decision making of the firm for the next period, just as the decisions of the firm for this period depended crucially on the net cash flow obtained in the previous period.**

**This will be discussed in detail in our analysis of long-run financial decision making of the firm.
We note that the optimal value of \( \lambda \) depends upon the degree of risk averseness, on the shape of two opportunity loci and on the confidence that the management has as to the likelihood of each of the events which, with the preference map of the investors, determines the shape of this binomial probability distribution of the unknown event.\(^*\)

The other hypothesis may be stated as follows. The firm first calculates the mean value and the standard deviation of the level of utility that the management of the firm expects the investors in the market to obtain\(^**\), corresponding to each and every possible dividend decision and related optimal investment and financing decisions, where the amount of dividend is allowed to take the value between \( D^* \) and \( D^1 \) in the figure III-4. Each and every possible amount of dividend payment, then, takes on the value, in general form, \( D^1 + \lambda (D^* - D^1) \) where \( \lambda \) is between zero and one. Then the optimal \( \lambda \) and there the optimal dividend and corresponding optimal investment and financing decisions will be determined in essentially the same manner as in the previous hypothesis by means of analysis of choice between risk and return.

We will, in the subsequent discussion, make use of the former hypothesis, rather than the latter, which will help us see the link between the decision making for one period and that for next period more clearly,\(^*\)

\(^*\)Figuring out the resulting change in the value of \( \lambda \) due to the change in the last element, we will encounter the familiar complication with income and substitution effects, and the precise knowledge of the preference map becomes necessary to know the result.

\(^**\)We are still assuming that the management of the firm thinks that either the optimistic or pessimistic opportunity locus describes the true unknown opportunity of the firm and attaches the respective probabilities to the occurrence of each of these possible events.
the link that will become the key to the extension of our one period analysis to the long-run analysis.
Appendix I.

We argued that the investors in the market have such a limited access to the necessary information to calculate in a meaningful manner the estimate of the resulting net cash flow and its possible downward deviation corresponding to each and every level of investment with concomitant financing, given the various possible level of retained earnings available, that it is very misleading to assume that they construct identical risk schedules in their minds to those constructed in the mind of management. This argument will be supported further by the following supplementary discussion.*

In the entire capital budgeting procedures (this term means the determination of the size, composition and financing of the investment to be undertaken), nothing is of greater importance than a reliable estimate of the cost saving and/or revenue increases that will be achieved from the prospective outlay of capital funds. This estimate needs accurate information as to the values of numerous variables which describe whole facets of the firm's activity, internal or external, based on which the management of the firm attempts to formulate the plausible expectation as to the future values of these variables as a consequence of the firm's investment of the period, where this formulated plausible expectation is the basis of the estimate of the benefit to be achieved by the investment with concomitant financing to be undertaken. Examples of the items that may affect the benefits derived from capital expenditures are,

*We are indebted, in the discussion developed here, to the textbook written by Weston and Brigham. c.f. Weston, F. and Brigham, E., Managerial Finance (New York: Holt, Rinehart and Winston), chapter 7.
internally, changes in quality and quantity of direct labor, in amount and cost of scrap and rework time and in maintenance, expense, downtime, safety, flexibility and so forth, and externally the change in the demand situation both independent of and depending on capital expenditure to be undertaken, the other firm's expected capital expenditure, attitude of monetary authority which affects the unit explicit cost of debt (including annual sinking fund costs) and so forth. So many variables are involved that it is almost impossible even for the firm, who has far better access to necessary information than investors in the market, to make precise generalizations. This process of formulating necessary data and making plausible estimate of benefits and costs of capital expenditure is not a routine clerical work to be performed on a mechanical basis. It requires continuous monitoring and evaluation of estimates by individuals competent to make such evaluations -- engineers, economists, accountants, cost analysts, market researchers and other qualified persons.

We believe that this discussion further supports our argument made above.
CHAPTER IV

Long-Run Analysis
I. Introduction to Long-Run Analysis

A. Decision scheme in one period financial decision making, link between the financial decision for one period with that for the next period, decision scheme in long-run (multiperiods) financial decision making.

Having completed the analysis of one period financial decision making of the firm, we are now ready to analyze the long-run (multiperiods) financial decision making of the firm. In order to assure the continuity of the discussion and also in order to reveal the basic difference between one period and multiperiod (long-run) financial decision making as clearly as possible, we will make a brief remark on one period financial decision making scheme, on link between the decision for one period and that for the next period, and on long-run financial decision making scheme.

When the time horizon was one period, the firm's financial decision scheme was as follows:

(1) given the coefficient of variation of cash flow schedule (TBR schedule in the first version of our theory);

(2) given the explicit annual unit cost schedule of the borrowing (including the sinking funds cost);

(3) given the available amount of net cash flow which was obtained in the previous period;

(4) the firm derives the opportunity locus which is the locus of all the efficient combinations of the amount of dividend to be paid out from the available net cash flow and of the magnitude of the coefficient of variation of net cash flow as a consequence of the optimal investment and financing decisions given that amount of dividend (the magnitude of TR in the first version);
(5) given the investor's utility function whose arguments are the amount of dividend which has been paid out in the period and the level of net cash flow which has been realized in the period, both as results of the firm's financial decision for the period.

(6) the firm determines the optimal $\lambda^*, \lambda'$, to construct an anticipated opportunity locus which is the locus of all the efficient combinations of the amount of dividend and the anticipated net cash flow defined as $\mu - \lambda' \cdot \sigma'$.

(7) the firm, then, determines the optimal dividend and corresponding optimal investment and financing decisions, by combining the utility map and the anticipated opportunity locus.

We note that, as consequences of the optimal financial decision for the period, the management of the firm has its anticipation of the resulting net cash flow ($\mu - \lambda' \cdot \sigma'$ of net cash flow) and of the resulting cash flow ($\mu - \lambda' \cdot \sigma'$ of cash flow) for the period. The resulting anticipated cash flow will constitute the most crucial determinants of the anticipated cash flow schedule for the next period**, while the resulting anticipated net cash flow will determine the

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*We note that the necessary sets of information to construct the anticipated opportunity locus are (1) anticipated cash flow schedule which relates each level of investment with a corresponding anticipated cash flow, (2) unit cost schedule for the borrowing, (3) available amount of net cash flow.

**Unless the firm foresees in the next period some radical changes in its external and internal circumstances, so radical that the results of the decisions for this period are thought to have nothing to do with the situation the firm will face in the next period, it is very reasonable to consider that the resulting cash flow anticipated by the firm to be accruing to then existing stock of capital becomes critical basis for calculating cash flow anticipated by the firm to be accruing to the various new levels of stock of capital that the firm contemplates to attain by its next period's investment.
anticipated opportunity locus for the next period, in conjunction with the anticipated cash flow schedule for the next period and expected unit cost schedule of borrowing.

What we find, therefore, is that the anticipated results of the financial decision for this period constitute the critical basis for the financial decision for the next period. The decisions for any two periods are related in this way, i.e., the different sets of financial decision for this period will produce different sets of financial situations (on the basis of which the firm makes its financial decision for the next period) by resulting in different sets of anticipated values of various important variables.* This link between the decision for one period and for the next period continues in the future.

Because of this fact of the interrelation of one period with another, the long-run financial decision making of the firm becomes far more complicated than its one-period decision making. The anticipated cash flow schedule (or the coefficient of variation of cash flow schedule, or TBR schedule)** and the anticipated available amount of net cash flow at the beginning of each period which will be distributed between the dividend payment and retained earnings, both exogenously given in one period financial decision making, are no longer exogenous in the long-run decision making. Time paths of these variables are determined also by the firm's

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*This fact makes our model inherently dynamic.

**These terms are now used interchangeably to convey the same concept.
current decision, at least, partially.* It is, therefore, no longer possible to analyze the decision making step by step as we did in our one-period analysis (described by the above discussion on the one-period financial decision scheme). Instead the time paths of all the variables concerned must be simultaneously determined. In one-period decision the investment, financing and dividend decisions were endogenous variables and the TBR schedule and the available amount of net cash flow were exogenous variables. In the long-run decision, however, all these variables become endogenous variables, whose value must be determined simultaneously within the system. This is the fundamental difference between one-period and long-run financial decision makings. Investment, financing and dividend decisions in any one period must be made by the firm fully taking into account the important effect of their anticipated results on the decisions to be made for the subsequent periods, as well as the satisfaction of the investors in that period. The complication due to this interdependence of any two periods and its economic consequence will be analyzed fully in the illustrative period analysis** in the subsequent section.

B. Remark on our assumption concerning the objective of the firm in long-run.

As we have stated previously, we will in the following sections of the chapter, attempt to analyze the optimal total financial decision of the firm in the long-run, with the assumption that the firm attempts to optimize its time paths of investment, financing and dividend decisions

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*We, of course, do not fail to recognize the totally external forces as determinants of time paths of these variables. But the firm can change its time paths, within the limit imposed by the totally external forces, by its own decision.

**For the use and the definition of the period analysis, refer to W. Baumol, Economic Dynamics.
so as to maximize \( \mathcal{P} = \int_0^T \tilde{P}(t) dt \), subject to \( \tilde{P}(i-l) \leq \tilde{P}(i) \) for all \( i \) over the chosen time horizon. The basic verification of this objective was given in the section of the previous chapter where we criticized the conventional treatment of the long-run financial decision of the firm. A brief remark will be made concerning this assumption on the objective of the firm in this section. We attempt to give it an additional justification by comparing it with an alternative objective. The alternative objective is the maximization of the sum of the present value of the utility stream that the firm expects investors in the market will obtain over the time horizon where the amount of utility at each and every period is assumed as before to be a function only of the amount of dividend which has been paid and the level of net cash flow which has been realized in the period.**

The basic difficulty this alternative objective encounters if that it cannot incorporate the effect of the changes over time in the values of arguments of the utility function on the level of utility that the investors in the market obtain. Suppose that the investors in the market observe that the firm pays out \( D_i \) out of \( NCF_{i-1} \), and earns \( NCF_i \), in the \( i \)th period. Is it plausible to think that the investors in the market would feel exactly the same level of psychic satisfaction from this combination of \( D_i \) and \( NCF_i \) regardless of the values of \( D_{i-1} \) (and/or that of more distant past) and \( NCF_{i-1} \) (and/or that of more distant past), i.e., whether \( D_{i-1} \) and \( NCF_{i-1} \) are such that \( D_{i-1} > D_i \) and \( NCF_{i-1} > NCF_i \), or

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*This assumed objective is also consistent with what to be analyzed in the long-run analysis.

**This assumption is used quite frequently in the literature on the capital accumulation and dynamic portfolio model. e.g., K. J. Arrow, application of control theory to economic growth.
D_i > D_{i-1} and NCF_i > NCF_{i-1}? Investors, very possibly, feed different levels of satisfaction from the same combination of D_i and NCF_i, depending on the level of D_{i-1} and NCF_{i-1}. It seems, however, extremely difficult to incorporate the effect of the changes in the values of arguments over time into the utility function itself.

This difficulty is resolved simply by adopting the assumption that the objective of the firm is to maximize \( P = \int_0^\tau \tilde{\tau}(t) \, dt \), subject to \( \tilde{\tau}(i-1) = \tilde{\tau}(i) \) for all \( i \) over the chosen time horizon, with our hypothesis on the stock valuation now to be explicitly stated as

\[
\tau(t) = \varphi \left[ \int_t^{i' \tau} e^{-r(t-s)} \tilde{\tau}(s) \, ds \right].
\]

\( \varphi(\cdot) \) is the price of the shares which prevails at the \( i \)th period; \( e^{r(t)} \) is the level of utility that the investors in the market expect to obtain, having their expectation made at time \( i \) as to \( D_t \) and NCF_t over the periods between \( i \) and \( i + \tau' \); \( r(t) \) is the time discount factor which is assumed to consist of two components, say \( r(t) = \alpha + \beta(t) \) where \( \alpha \) is the discount factor due to the pure time preference and \( \beta(t) \) is the discount factor due to the uncertainty involved in estimating the outcomes in the unknown future and we assume that \( \beta > 0 \). The hypothesis states that the market price of the shares is some function \( \varphi \) of the present value of the future utility stream that the investors in the market expect to obtain, discounted by an appropriate capitalization rate. We have to distinguish clearly between \( \varphi(\cdot) \) and \( \tilde{\tau}(\cdot) \). \( \varphi(\cdot) \) is the market price of the shares that prevails in the market at time \( i \), being determined, in the final analysis, by the expectation made by the investors. \( \tilde{\tau}(\cdot) \), on the other hand, is the price of the shares that the firm expects, at time zero, to prevail at time \( i \), projected by the firm by knowing (or the management of the firm is assumed
to believe that it knows) the shapes of the functions $\varphi[\cdot]$, $r(\cdot)$ and $\tilde{U}(\cdot)$ * and the way the investors formulate their expectations.

This objective function obviously, and as will be discussed in greater detail in the subsequent section, incorporates the importance of both the level of D and NCF at each period and the changes** in their levels from one period to another. As a conclusion of this section, we will, then, state that we will attempt to analyze the firm's optimal financial decision in the long-run, where the firm attempts to determine the time paths of its investment, financing and dividend decisions such that the expected time path of $\tilde{\gamma}$ will maximize $\mathcal{E} = \int_0^\infty \tilde{\gamma}(t) dt$, subject to $\tilde{\gamma}(1-t) \leq \tilde{\gamma}(1)$ for all $i$.

In the following section we will analyze the economic consequence of the complicated interdependence between the decision for any one period and that for next period by means of illustrative period analysis. We will draw one important conclusion from the analysis concerning the inter-temporal trade-off which the firm must take into consideration in its determination of optimal time paths of its investment, financing and dividend decisions. The intertemporal trade-off and the already discussed intratemporal trade-off, in conjunction with the implications of the stock price determination hypothesis in relation to the objective function cum constraint, will reduce the infinite number of attainable sets of time

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*We make an assumption for simplicity that $\varphi[\cdot]$, $r(\cdot)$, and $U(\cdot)$ are invariant over time, without affecting the conclusions of our analysis.

**The importance of changes in D and NCF over periods of time, in terms of their influence on the market price of the shares, is what prevents the maximization of the level of utility in each and every period from being necessarily the optimal financial policy of the firm in the long run.
paths of investment, financing and dividend decisions that the firm can take to the effective subset of attainable sets of time paths of its investment, financing and dividend decisions. From this subset the firm should choose one that will maximize \( P = \int_0^T \tilde{P}(t) \, dt \), subject to \( \tilde{P}(i) = \tilde{P}(i)^* \) for all \( i \). Then in Appendix, we will develop a model of corporate expansion with which we will both conduct the comparative dynamic analysis in order to examine the changes in the optimal long-run financial decisions with respect to some important parametrical changes and clarify some ambiguities (or unpreciseness) which might be involved in our arguments in the immediately following section because of the heuristic exposition we will have to make there.

* \( \tilde{P}(i-1) / s_{P(i-1)} = \tilde{P}(i-1) / s_{P(i)} \) would be a more realistic assumption if we consider general stock market price moves. But, since general stock market price moves are governed by more speculative factors, which are not our primary concern, we shall simply maintain our original assumption.
II. Period Analysis

As has been discussed in the previous chapter, the financial decisions for one period and that for the next period are closely linked together.* This interdependence imposes upon the firm an additional complication particular to its long-run financial decision making. In this section, we will illustrate, by means of period analysis**, the important inter-temporal trade-off which is "the" most crucial complication arising from the interdependence relationship.

Suppose that the firm is faced with the anticipated opportunity locus in the period zero such as drawn below.

Suppose also that if the firm chooses A on the anticipated opportunity locus as its decision for the period zero for the reason that this choice

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*More specifically, we stated that the resulting anticipated cash flow and anticipated net cash flow as a set of consequences of the decision for the period will determine the anticipated opportunity locus for the next period, via determining the anticipated cash flow schedule and the available amount of net cash flow for the next period.

**The ambiguity which might be involved in the discussion in this section due to the heuristic approach to be taken will be cleared in the discussion on the corporate expansion model in the following section.
is anticipated to give the highest level of the satisfaction to the investors in the period, then the anticipated opportunity locus for the period one is such as drawn below. (in dotted line)

\[ IV-(II)-2 \]

We can then see that it is impossible for the firm to maintain the dividend payment at the level of \( D^*_0 \) in period one. Furthermore, we notice that the firm has to greatly reduce the dividend payment in period one in order to sustain the level of resulting anticipated net cash flow at \( ANCF^*_0 \) in the period one.

Suppose next that the firm, instead of \( A \), chooses \( B \) as its decision for the period zero, where \( B \) is assumed to be such that the anticipated opportunity locus for the period one will be the same as that for the period zero.

\[ IV-(II)-3 \]

The reasons for \( (1)' \) being located to the outside of \( (1) \) are: (a) the anticipated net cash flow in the period one resulting from the decision \( B \) is greater than that resulting from the decision \( A \); and (b) the reason
(a) implies that the anticipated cash flow for the period zero resulting from the decision B is greater than that resulting from the decision A. This in turn implies, cet.par. (that can be assumed this case) that the anticipated cash flow schedule for period one corresponding to the decision B is such that the anticipated amount of cash flow at each and every level of investment to be contemplated for period one is greater than that corresponding to the decision A. We see, then, that the firm can sustain both the dividend and anticipated net cash flow in period one at the same level as in period zero.

Suppose now that the firm chooses C, instead, as a decision for period zero, where C is assumed to be such that the anticipated opportunity locus for the period one will be such as shown below, denoted by (1)". The reasons for (1)" being located to the outside of (1)' are the same as above.
Then the firm can, obviously, increase the dividend payout or the level of anticipated net cash flow or both of them at once in period one, i.e., growth becomes possible.

Let us suppose, given the above situation, that the firm chooses $E$ as its decision for period one (where the difference between $C$ and $E$ is simply a greater dividend payment for $E$), where $E$ is assumed to be such that the anticipated opportunity locus for period two is the same as that for period one, i.e., $(1)^*$. We denote it by $(2)$.

Then, the only way in which the firm can maintain the growth rate "$g$" of dividend will be to greatly decrease the net cash flow for period two** as can be seen in the picture.

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*This assumption can be verified by the same reasoning as used for the comparison for $(1)$ and $(1)'$, and for $(1)'$ and $(1)'". The only reservation is that it might be that we should not assume strict ceteris paribus as was done above.

**This greatly decreased level of the anticipated cash flow for the period two in comparison to that for the period one (and zero) implies that the anticipated opportunity locus for the period three will be to the far inside of $(2)$ (and $(1)'"), so that the maintenance of the growth rate of dividend payout, $g$, will become impossible.
Suppose now that the firm chooses F, instead of E, as its decision for period one under the same situation, where F is assumed to be such that the anticipated opportunity locus for period two will be such as shown below, denoted by (2)'. The difference between C and F is that both dividend and the resulting anticipated net cash flow at F are greater than those at C in more or less the same proportion, i.e.,

\[
\frac{D_F}{D_C} \approx \frac{ANCF_F}{ANCF_C},
\]

while the difference between C and E was merely a greater dividend at E than at C.

Then the firm can maintain the growth rate, \( g' \), of both dividend and the anticipated net cash flow in the second period where we assume that the decision is H. This growing anticipated net cash flow implies that the firm can maintain the growth of both dividend and anticipated cash flow in the future, too. If the firm chooses for period one a point on the left of F, then the potential of growth would be even greater. Note, however, that the growth of dividend and anticipated net cash flow in the future periods is realizable only if the firm relinquishes current dividend payment.
We have presented only the instructive illustrations which suggest to us such an important conclusion about the intertemporal trade-off. The conclusion is as follows.

In the one-period financial decision, a decrease in the resulting anticipated net cash flow was both a necessary and sufficient condition for an increase in the dividend payment to be made, i.e., the increase in dividend was made possible only at the expense of the decreased anticipated net cash flow. In the long-run, however, an increase in dividend payment in any one period is made possible at the expense not only of a decrease in the anticipated net cash flow of that period, but also of a decrease in the opportunity of the future represented in terms of future level of dividend and anticipated net cash flow. A decrease in the amount of anticipated net cash flow in one period is no longer a sufficient condition for an increase in the "level stream" of dividend, though it still is a necessary condition for an increase in dividend in that period. In order for the firm to maintain a level of dividend stream over time, a certain level of anticipated net cash flow stream must be maintained. In order for the firm to maintain a growth rate of dividend over time, more or less the same growth rate of anticipated net cash flow stream is required. Otherwise, as illustrated, it will either eventually become necessary for the firm to reduce dividend in order to maintain the level or growth rate of anticipated net cash flow, or it will eventually become impossible to sustain the level or the growth rate of
dividend.* There is, therefore, a definite limit to the level or the
growth rate of dividend stream that can be maintained over an extended
period of time.** We can then state that, among an infinite number of
attainable expansion paths, where the expansion path is defined as the
combined time paths*** of dividend stream and anticipated net cash flow
stream, there exists only a limited number of attainable expansion paths
in which the level or the growth rate of dividend is maintained. Impli-
cit, then, is the maintenance of the level or the growth rate of the
anticipated net cash flow stream, where its growth rate must be more or
less the same as that of dividend stream.

But why is the maintenance of the constant level or the growth rate
of dividend and of the anticipated net cash flow streams important? This
is the topic we will discuss in the next section.

*A full justification of this statement will be given in the corporate
expansion model to be developed in the next section; but it is very easy
to justify it on the informal basis if we think about what less and less
proportion of anticipated net cash flow to be ploughed back for reinvest-
ment implies in our model; that is, that the opportunity locus will even-
tually start shifting inwardly and the growth will become impossible.

**We neither have discussed nor are discussing the economic signifi-
cance of the maintenance of the level or the growth rate of dividend and
anticipated net cash flow.

***This combined time path, as must be clear from our one-period analysis,
summarizes the time paths of investment, financing and dividend decisions
over the time horizon.
III. Maintenance of the constant level or the growth rate of dividend and of anticipated net cash flow streams as a necessary condition in the long-run financial decision of the firm.

Our hypothesis of stock valuation* states that the market price of the shares is a function of the present value of the expected utility stream, where the level of expected utility is a function of the expected amount of dividend and net cash flow.** We might assume that this utility function is of Friedman-Savage type***, where the expected utility itself is a random variable, being a function of two random variables, the amount of dividend and of net cash flow. Thus we assume that the investors in the market at each and every period of time have (subjective) probability distributions in their minds defined over the possible outcome of the amount of dividend $D_t$ and the amount of net cash flow $NCF_t$ for all $t$.**** Or we might very well assume that, instead of having complete probability distributions over $D_t$ and $NCF_t$ for all $t$ at each and every period of time, investors in the market have a rough estimate of $D_t$ and $NCF_t$ over

\[ P(i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t} \mu_t \left( \bar{D}_t, \bar{NCF}_t \right) dt \]

* i.e., $P(i) = \varphi \left[ \int_{-\infty}^{\infty} e^{-t} \mu_t \left( \bar{D}_t, \bar{NCF}_t \right) dt \right]$

** Here the expectation is that held by investors.


****We note that (1) subjective probability distributions held in the mind of any investor at any time $K$, over $D_t$ and $NCF_t$ do not have to be same as those over $D_{t+i}$ and $NCF_{t+i}$ for any $i$, (2) subjective probability distributions formulated in the mind of any investor at time $K$ and $K+j$ for any $j$, over $D_t$ and $NCF_t$ do not have to be same at all.
their time horizon, which is of unique values.* As a consequence of
this assumption, the utility function takes on the unique values corre-
sponding to each and every estimated value of \( D_t \) and \( NCF_t \), totally inde-
pendent of the probability consideration. With the first assumption an
unexpected decrease, for example, in either \( D \) or \( NCF \) from \( D_{k-1} \) (\( NCF_{k-1} \))
to \( D_k \) (\( NCF_k \)) has two possible effects on the subjective probability dis-
tributors, held by the investors at time \( K \) over \( D_t \) and \( NCF_t \) for all
subsequent \( t \): (1) the expected value of \( D_t \) (\( NCF_t \)) for any subsequent
\( t \) (or at least some subsequent \( t \)) is lowered, and (2) the variance (or
standard deviation) is possibly greater, due to the evidence of this
fluctuation, which, since we assume that the investors are risk averse,
has the same effect as the lowered expected value on the level of utility.
An unexpected increase in either \( D \) or \( NCF \) from \( D_{k-1} \) (\( NCF_{k-1} \)) to \( D_k \) (\( NCF_k \))
has an opposite effect, though we are not certain about the intensity of
its effect in comparison to that of an unexpected decrease. In the case
of the second assumption, an unexpected decrease for example, in either
\( D \) or \( NCF \) from \( D_{k-1} \) (\( NCF_{k-1} \)) to \( D_k \) (\( NCF_k \)) will have an effect of lowering
the estimated value of \( D_t \) (\( NCF_t \)) for any subsequent \( t \) (or at least some
\( t \)), in comparison to the estimation held at time \( k-1 \). We can see the
opposite effect in the case of an unexpected increase in either or both
of them.**

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*We call them "estimated values" in order not to confuse them with
"expected value" of random variable with a specified probability distri-
bution. We are not concerned with how they reach these unique estimated
values of uncertain outcome. See I. R. Hicks, Value and Capital (New York:

**Concerning this point, refer to the literature on the level of aspira-
p. 48-49; R. M. Cyert and J. G. March, A Behavioral Theory of the Firm,
Whichever assumption we make as to the utility function, therefore, the changes in the arguments of utility function have, at least qualitatively, the same effects on the expected future streams of utility, and therefore on the market price of the shares. For simplicity we will take the latter assumption though we note that the conclusion to be derived is independent of this choice of the assumption.

We will now examine with the above assumption the importance of the maintenance of the level or growth rate of dividend and anticipated net cash flow in relation to the attainment of the objective of the firm. Suppose that the firm fails to maintain the level or growth rate of dividend or net cash flow at time $K$ which has been maintained until time $K-1$ without failure. Then we expect that the estimated value of $D_t (NCF_t)$ for any subsequent $t$ (or at least some $t$), where estimation is formulated at time $K$, must be smaller than that estimated at time $K-1$ until when the level or growth rate was perfectly maintained. This downward change in estimated value of $D_t (NCF_t)$ in the future period(s), of course, through its effect on the expected future utility stream, results in the market price decline which is obvious from our hypothesis of the stock price determination. Recognizing this fact of, or at least large possibility of, the consequent market price decline, the management of the firm must make such a set of long-run investment, financing and dividend policies that insures, as its planned results, the constant level or growth rate of dividend and anticipated net cash flow stream over the time horizon according to the future expectation held by the management of the firm. This is what the management of the firm must do in order not to violate the constraint, $\tilde{P}(i-1) \leq \tilde{P}(i)$ for all $i$ in the time horizon, though it never guarantees either that the actually realized future dividend and
net cash flow streams will be such that the constant level or growth rate be maintained or that the actually realized future time path of the market price of the shares will be such that \( \tilde{P}(t) \leq \tilde{P}(i) \) for all \( i \) even if the constant level or growth rate of both dividend and net cash flow stream be perfectly maintained.* In other words, the maintenance of the constant level or growth rate of planned dividend and anticipated net cash flow streams is a necessary condition that the management of the firm has to satisfy in its choice of its long-run investment, financing and dividend policy, in order to satisfy the constraint.

We concluded above that the maintenance of a constant level or growth rate of planned dividend and anticipated net cash flow stream is a necessary condition for satisfying the constraint. We now have to examine the plausibility of this constraint itself more thoroughly. Arguing that it is important for the firm, in its attempt to maximize \( J = \int_0^T \tilde{P}(t) dt \), to do nothing that lowers, or is expected to lower, the market price of shares, we will give sufficient justification for incorporating this assumption in our analysis. Now the price decline is, as stated above, solely due to the downward change in the investor's estimation of the future stream of dividend and net cash flow caused by the observed decrease** in dividend or net cash flow in the period, when the price

*We are not arguing, either, that the failure in the maintenance of the constant level or growth rate of dividend or net cash flow necessarily induces a market price decline as its result; but it is of great possibility so that the firm must avoid the failure.

**Decrease refers to that in either level or growth rate.
decline occurs.* Each individual investor changes his estimation in a unique manner even if the cause of change, such as decline in dividend or that in net cash flow in the period, is same to all investors. The initial price decline, if it occurs, results from the market force generated by the change in the estimation held by the majority of the investors in the market, each of whom may very well have a different change in his estimation from the others'. There may very well be a number of investors who think that an observed decline in dividend or net cash flow is just a temporary phenomenon for this firm. There may very well be a number of investors who are not quite sure about how to interpret this observed decline. But this price decline, which is observed by all the investors in the market as evidence of the downward change in the estimation held by the majority of people in the market, gives the "market assurance" of decline in the future level or growth rate of dividend or net cash flow stream. This market assurance gives those who initially were not too sure about how to interpret the observed decline in dividend or net cash flow a very concrete justification for their beginning to estimate, also, a decreased future level or growth rate of dividend or net cash flow. The market assurance also furnishes those who initially thought of the decline as temporary with the necessary evidence for reconsideration, while offering each of those whose change in the estimation caused the initial price decline a guarantee that his newly formulated estimation was correct in the sense that the majority of

*Or it may be caused by the changes in their guess as to the future economic and industrial prospects.
the investors in the market agreed with him.* Therefore the price
decline will, or is expected by the firm to, generate an additional
market force which will lower the market price further. This process
will repeat itself until some strong evidence appears to change the
direction of their estimation, such as a definite increase in the level
or growth rate of dividend or net cash flow, or other changes which are
external to the firm but which give the definite sign of upturn of the
economy or industry.

Now if we hypothesize further, given the assumption that the invest-
tors are risk averse, that the elasticity of estimation with respect to
a downward change in the level or growth rate of dividend or net cash
flow is greater than that with respect to an upward change in them, then
we see how costly is the failure to maintain the constant level or growth
rate of dividend or net cash flow in any period which generates the
market force to lower the market price where this decline in the market
price in turn generates the additional market force to lower the price
further, in light of the objective of the firm that is to maximize
\[ \mathcal{E} = \int_{0}^{\tau} \tilde{\mathcal{P}}(t) \, dt. \]

This above argument gives a sufficient justification for incorporating
this constraint in our analysis as a plausible assumption.

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*The fact that the majority of the investors in the market accepted
one's future estimation as correct does not, of course, mean that his
estimation will turn out to be correct in the future; but considering the
unknown future, the market opinion (majority opinion) has such a tremen-
dous ability (power) of assurance.
IV. Target Dividend Payout Rate, As a Corollary to the Above Discussed Necessary Condition

We discussed in the previous sections that the firm, as a necessary condition in making its long-run financial decision, has to maintain a constant level or growth rate of both dividend and anticipated net cash flow streams over the time horizon and that in order that the firm maintain the constant level or growth rate of both streams, both streams must grow at more or less the same growth rate (including zero growth rate, i.e., a constant level). This conclusion implies that the firm determines, upon its making a long-run financial decision, the "target dividend payout rate" which the firm plans to maintain over the time horizon, where the target dividend payout rate is defined as the ratio of dividend to the anticipated net cash flow that the firm plans to maintain in an attempt to maximize \( P \) subject to \( \hat{P}^{(i-1)} \leq \hat{P}^{(i)} \) for all \( i \). We will now take up the target dividend rate.

The first remark we will make concerns the definition of the target dividend payout rate. In the conventional finance literature, the target dividend rate is defined as the ratio of the amount of dividend to be paid out to the amount of either net cash flow or of net earnings, actually realized, that the firm desires to maintain over time. There would be no economic rationale if the firm followed the target dividend payout ratio defined this way. In our definition the target dividend payout rate is the ratio of planned dividend payment to the planned anticipated net cash flow that the firm desires to maintain over its time horizon. The firm determines the ratio making a long-run financial decision at time zero with the objective to maximize \( P \) subject to

\[ \hat{P}^{(i-1)} \leq P^{(i)} \text{ for all } i. \]
suggested that the gap between the target rate (which the firm somehow determines) and the actual payout rate is a transitory phenomenon and the gap is tapped after a certain length of adjustment period. In our hypothesis, the gap between the target rate (in our definition) and the actual payout rate in each period of time is what the firm expects to occur upon its planning; no effort is made to tap the gap from period to period. The firm determines the target dividend payout rate as a ratio of dividend to the "anticipated" net cash flow, which, the firm knows, will not usually coincide with the realized net cash flow because the former is simply a rough approximation of the future net cash flow, whose estimation involves a high degree of uncertainty. The firm will not adjust its dividend payment from period to period according to the fluctuating (realized) net cash flow in order to maintain the target dividend payout rate defined in our way. The firm lets the actual payout rate fluctuate by paying out in each period the specified amount of dividend which the firm has already decided upon when determining the long-run financial policy. This is why we observe that the actual dividend payout rate fluctuates from period to period. Fluctuation is not because of the time lag required for readjustments, explained as a transitory phenomena in the conventional literature. As long as the variation of realized net cash flow is within the expected range, i.e., between $\mu$ and $\mu - \sigma$, corresponding to the amount of dividend to be paid out, the firm will not concern itself with adjusting the amount of dividend payment.*

*For its definition, refer back to chapter III, Revision of our Theory.

**This statement will be elaborated in the subsequent part of this section.
The second remark we will make is that "the target dividend payout rate" defined in our way must not be identified as the "dividend policy (decision)," as is usually the case in the conventional literature. The target dividend payout rate is the planned result of the firm's long-run total financial decision. That is, the firm determines its time paths of investment, financing and dividend decisions so that a planned target dividend payout rate will be maintained over the time horizon, as a corollary to the necessary condition we discussed above, where the target rate is expected by the firm to maximize \( P \). The dividend decision is merely one of three important decisions which, as a whole, determine the optimal target dividend rate. The target dividend payout rate to be determined by the firm is not a dividend decision. Confusion between the determination of the target dividend rate and the dividend decision in conjunction with their belief in the existence of the target dividend rate defined in their way has led many important writers in the field to believe that, in the long-run financial decision, the dividend decision has top priority among the three. There seems to be no economic reason why the dividend decision has top priority over the two other decisions. In our hypothesis the dividend decision is simply one of three important decisions, all of which must be simultaneously determined so that the constant level or growth rate* of both dividend and anticipated net cash flow is to be maintained over the time horizon. This implies, then, that the firm determines the target dividend payout rate when making the long-run financial decision, as the planned result of the decision.

*The growth rates of dividend and anticipated net cash flow, as has been repeatedly stated, must be more or less the same, in order for the rate of growth of both to be maintained constant.
We have to understand clearly, then, that the change in the target dividend rate is not the change in the dividend policy, but the change in the long-run financial policy as a whole, where the change in the long-run financial policy is considered by the firm in the following manner.

We have been hypothesizing that the management of the firm in its making a long-run financial decision at period zero assesses the opportunity locus of each and every period corresponding to each and every possible time path of investment, financing and dividend decisions. Recall that the opportunity locus has been defined as a locus of all the possible effective combinations of the amount of dividend and the magnitude of coefficient of variation of net cash flow which was defined in our own way. The firm, we can then assert, expects that the realized net cash flow at any period \( t \) will be within the range between \( \mu_t \) and \( (\mu - \sigma)_t \) corresponding to \( D_t \). The firm also expects that the anticipated net cash flow, \( (\mu - \alpha \sigma)_t \) corresponding to \( D_t \), will not usually coincide with the realized net cash flow as has been stated above. Unless the realized net cash flow deviates outside the expected range, the firm will not consider the change in the long-run total financial decision even if the realized net cash flow deviates from the anticipated net cash flow, in any period. There are only two situations in which the firm considers seriously a change in the long-run financial policy; (1) when the realized net cash flow has deviated outside the expected range, (2) when it is foreseen that the net cash flow to be realized in the near future, given the dividend payment already planned, will be outside the expected range.

*The change in the long-run financial policy does not necessarily involve the change in target dividend payout rate.*
This is when the firm reexamines the whole internal and external situations and reconsiders the long-run financial planning, in our hypothesis. The change in the long-run financial decision may very well manifest itself in the change in the target dividend payout rate, but this does not necessarily have to be the case. But when the target dividend rate changes, it definitely means that the firm changed its long-run financial policy. Note that the change in the target dividend rate is not the change in the dividend policy only, but is the change in the long-run total financial policy.

Having understood what the target dividend payout rate must mean, we will examine the factors possibly influencing the firm's determination of the target dividend payout rate in the appendix to this chapter. What are the main reasons for the different firms having different target dividend rates and for the firms changing the target dividend payout rate, which, as must be clear by now, implies the change in the time paths of investment, financing and dividend decisions of the firm, i.e., the change in the long-run financial policy? This has been an important but yet unsolved problem, and this is what we attempt to analyze. For this purpose, we will establish a dynamic corporate expansion model by means of which we will conduct a comparative dynamic analysis to see how the changes in some important parameters affect the determination of the target dividend payout rate. In addition, this corporate expansion model with only one very plausible assumption will substantiate the arguments we have been making, which possibly involved some ambiguity because of the heuristic exposition we had to make, by giving them a precise reasoning.
Appendix to Chapter IV

(1) Corporate Expansion Model

In this section of the appendix, we will develop a model of corporate expansion, which will be used for the subsequent comparative dynamic analysis and for the clarification of some ambiguities (or unpreciseness) which might have been involved in our previous arguments.

As has been seen in the above period analysis, the anticipated opportunity locus* that the firm expects it will face in the period $i+1$ as well as the amount of net cash flow available for the period $i+1$ depends crucially upon the decision to be made for the period $i$. That is, both depend on what point on the anticipated opportunity locus of the period $i$ the firm chooses as a decision for the period $i$ which, as we saw, determines the investment, financing and dividend decisions for the period $i$.

Our basic model is, therefore, inherently dynamic, and the future course of the development of the firm, thus, depends critically on the time path of the firm's decision as well as the expected future economic and industrial outlook. The economic and industrial environment and the decision to be made by the firm are interdependent. Totally external

*Note that the amount of the anticipated net cash flow available for the period $i+1$ which the firm expects will be obtained in the period $i$ is .5, one of three principal determinants of the anticipated opportunity locus that the firm expects it will face in the period $i+1$. The others are, as discussed before, (1) the amount of anticipated cash flow, expected by the firm to be resulting from the decision for the period $i$, which characterizes the resulting total business risk position of the firm at the end of the period $i$; and it is very reasonable to think that the total business risk schedule (anticipated cash flow schedule) the firm expects it will face in the period $i+1$ is primarily determined by the total business risk position of the firm at the end of the period $i$, and (2) the explicit unit cost schedule of borrowing.
elements in the economic and industrial environment* impose an absolute limit on the range of the choice that the firm makes in a period and therefore on the extent to which the firm's decision can influence the range of choice for the following period, but the firm's decision for the period that determines which to choose within the absolute limit will in turn determine, in conjunction with totally external elements, the manner in which the absolute limit will be imposed for the subsequent period.

More concretely, the totally external elements set the limit to, say, the maximum volume of sales that the firm expects to be attainable, the maximum cost reduction that the firm expects to be possible, etc., in a period, but the firm's decision will in turn influence the limit that the totally external element imposes for the subsequent period through its effect on the competitive position, on the improvement in the ability to cover the fixed cost, etc. Our model to be developed will be based on the consideration for this interdependence.

We will make an assumption concerning both the absolute limit that the totally external factors impose for a period and also the way the firm's decision to be made for the period influences the absolute limit to be imposed for the subsequent period. The assumption we will make is as to the average rate of return on the retention (to be defined precisely, shortly), whose various implications will be clarified so as to verify the assumption. We define the average rate of return on retention as the

*Totally external elements in the economic and industrial environment are meant to be factors such as population, general state of science and technology as well as institutional and political elements that the firm's decision cannot influence to a meaningful extent.
ratio of the total anticipated net cash flow to be obtained during the period \( i \) to the amount of retention* to be required in the period \( i \) in order to generate that total amount of anticipated net cash flow.**

We distinguish clearly, in order to avoid confusion, between our definition and the definition usually made in the literature. The average rate of return (either certain or expected) on the investment is usually defined as the ratio of the return accruing to the newly acquired capital stock to the amount of investment to be required to purchase that capital stock; the return expected to be accruing to the old capital stock is assumed to be kept intact, totally independent of newly undertaken

*This amount of retained earnings will be combined with a proper amount of borrowing to finance the level of investment which is expected to generate that amount of anticipated net cash flow, as we saw in one period analysis.

**This rate, we assume, can be evaluated at each and every amount of retention.
investment. We, on the other hand, defined it as the ratio of the anticipated net cash flow accruing to the total capital stock, comprising the old and new capital stock, to the amount of retention to be used, combined with a proper amount of borrowing, to purchase that new capital stock needed in addition to the old capital stock in order to secure that amount of anticipated net cash flow. This way, we can avoid the assumption that the rate of return on the old capital stock is kept intact independent of the newly undertaken investment.

The average rate of return on retention, of course, is the tangent of the slope $\alpha$. The assumption we make is that the average rate of return on retention evaluated at each and every "rate" of retention is the same for the periods $i$ and $i+1$ for all $i$ over the time horizon, regardless of the amount of the anticipated net cash flow available in each of the periods, which can be reinvested.

This is "the" only assumption we make to establish our model of corporate expansion, whose economic implications will be explained in order to verify the assumption. Prior to discussing its implications, we note that this assumption can be stated in terms of the marginal rate of return on retention. We define it as the marginal increment in the total amount of anticipated net cash flow (accruing to the total capital stock comprising the old and new capital stock) to be obtained in the $i$th period due to the marginal increment in the "amount" of retention, which, as before, is to be combined with the proper amount of incremental borrowing. Our assumption can be restated, then, as that the marginal rate of return on retention evaluated at each and every "rate" of retention is the same for the periods $i$ and $i+1$ for all $i$ over the time horizon, regardless of the amount of anticipated net cash flow available in each
of the periods which can be reinvested. That this statement follows from the assumption we made in terms of the average rate of retention can be verified as follows. Let the amount of anticipated net cash flow available for the period \( i \) and period \( i + 1 \) be \( Q_i \) and \( Q_{i+1} \), which can be reinvested. We can in general write the relation between \( Q_i \) and \( Q_{i+1} \) as \( Q_{i+1} = \alpha \cdot Q_i \). Then at each and every "rate" of retention, the "amount" of retention for \( i \) and \( i + 1 \) has the following relation.

<table>
<thead>
<tr>
<th>&quot;Amount&quot; of Retention at &quot;Rate&quot; of Retention</th>
<th>Period ( i )</th>
<th>Period ( i + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 )</td>
<td>( q_1 )</td>
<td>( \alpha \cdot q_1 )</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>( q_2 )</td>
<td>( \alpha \cdot q_2 )</td>
</tr>
<tr>
<td>( r_k )</td>
<td>( q_k )</td>
<td>( \alpha \cdot q_k )</td>
</tr>
<tr>
<td>( r ) (in general)</td>
<td>( q ) (in general)</td>
<td>( \alpha \cdot q ) (in general)</td>
</tr>
</tbody>
</table>

By defining the anticipated net cash flow functions for periods \( i \) and \( i + 1 \), \( f(q) \) and \( p(\cdot q) \), respectively, which map the available amount of anticipated net cash flow for the period into the anticipated net cash flow to be obtained during the period, we can write the assumption as

\[
\frac{f(q)}{q} = \frac{p(\alpha \cdot q)}{\alpha \cdot q} \tag{1}
\]

i.e., the average rate of return on retention evaluated at each and every "rate" of retention for \( i \) and \( i + 1 \) are same for all \( i \). Then, given (1), we have

\[
\alpha \cdot q^1 \cdot f(q) = \alpha \cdot p(\alpha \cdot q) \tag{2}
\]

\[
\alpha \cdot f(q) = p(\alpha \cdot q) \tag{2'}
\]

Let the value of \( p(\cdot) \) function be denoted by \( y \).

\[
y = p(\alpha \cdot q) = \alpha \cdot f(q) \tag{3}
\]
Then,
\[ \frac{dy}{dq} = p'(\alpha \cdot q) \cdot \alpha = \alpha \cdot f'(q) \] (4)
\[ p'(\alpha \cdot q) = f'(q) \] (4)

where (4)' states that the marginal rate of return on retention evaluated at each and every "rate" of retention for \( i \) and \( i + 1 \) are same.

Let us now explain the implication of the assumption. The first implication of this assumption is that if the firm chooses (a) on figure IV-(V)-2 as its decision for the period \( i \), where (a) implies that the firm expects it will have the same amount of anticipated net cash flow available for the period \( i + 1 \) as the period \( i \), then the anticipated opportunity locus for the period \( i + 1 \) is expected to be identical to that for the period \( i \). We can verify the plausibility of this implication of the assumption as follows.

The amount of the anticipated net cash flow available for the period \( i \) is a result of the decision made for the period \( i - 1 \). The other result of the decision made for the period \( i - 1 \) which concerns the firm is the anticipated cash flow, which, as we stressed before, characterizes the total business risk position of the firm at the end of the period \( i - 1 \).
This anticipated cash flow constitutes the primary basis for the TBR schedule (anticipated cash flow schedule) that the firm expects it will face in the period i, unless there is expected* to be any sign of very radical changes in the economic and industrial environment between the period i - 1 and the period i.

By combining the amount of anticipated net cash flow available for the period i with the anticipated cash flow schedule for the period i which is constructed, based crucially on the amount of the anticipated cash flow expected to be resulting in the period i - 1, the firm derives the anticipated opportunity locus that the firm expects it will face in the period i, which we see in the picture above.

Now, if the decision for the period i is (a), the firm expects that it will obtain the same amount of the anticipated net cash flow during the period i as during i - 1, which will be available for the period i + 1. But what this implies is that the firm expects it will obtain more or less the same amount of anticipated cash flow during the period i as during i - 1, unless there is expected to be some sign of very radical change in the explicit unit cost schedule of borrowing between the period i - 1 and period i. This amount of the anticipated cash flow, characterizing the total business risk position of the firm at the end of the period i, will constitute the primary basis for the TBR schedule (anticipated cash flow schedule) that the firm expects it will face in the period i + 1. Given its expectation stated above, that is, that the anticipated cash flow to be obtained during the period i - 1 and during the period i are more or less the same, the firm expects, then, that the

*Note that the expectation is held at the period zero, time of long-run financial decision making.
TBR schedule (anticipated cash flow schedule) for the period $i$ and that for the period $i + 1$ will be more or less identical. Then given this expectation, in conjunction with its expectation that the amount of net cash flow available for the period $i$ and the period $i + 1$ will be the same, the firm naturally expects that the anticipated opportunity locus it faces in the period $i$ and in the period $i + 1$ will be more or less identical.

The second implication of the assumption is

\[ D_{i+1} \cdot \max \text{ if the decision for the period } i \text{ is (a), abbreviated as } D_{i+1} \cdot \max(a). \]

\[ D_{i+1} \cdot \max = ANCF_{i-1}, \text{ if the decision for the period } i \text{ is (b), abbreviated as } D_{i+1} \cdot \max(b). \]

(1)-(3)

that if the firm's decision for the period $i$ is (b)* on figure IV-(V)-3,

*The decision (b) is characterized by the retention rate $1 - \tan B$ where $\tan B$ is the dividend payout rate; the decision (a) is characterized by the retention rate $1 - \tan A$ where $\tan A$ is the dividend rate. Note, $\tan A > \tan B$. 


instead of (a), which means that the firm expects that the amount of resulting anticipated net cash flow to be obtained during the period \( i \) is \( 1 + g \) times as great as the amount of the anticipated net cash flow that the firm expects will be obtained during the period \( i - 1 \).* Then the anticipated opportunity locus that the firm expects it will face in the period \( i + 1 \) is outside of that for the period \( i \), having the identical shape to that for the period \( i \). How far "outside" depends upon the growth rate \( g \), where \( g \) is equal to the multiple of the retention rate, \( 1 - \tan B \), and the average rate of return evaluated at that retention rate less \( l \).

The anticipated opportunity locus that the firm expects it will face in the period \( i + 1 \) as a result of the decision (b) made for the period \( i \) is, then, such that the amount of the anticipated net cash flow that the firm expects it will obtain during the period \( i + 1 \) is \( 1 + g \) times as great as that which the firm expects it will obtain during the period \( i ** \), at each and every "rate" of retention.

Verification of this implication of the assumption can be made by following the reasoning we used in order to justify the first implication. We first note that the decision (b) made for the period \( i \) results in the anticipated net cash flow to be obtained during the period \( i \) which is \( 1 + g \) times as great as that which is expected to be obtained during the

---

*This amount is in turn equal to the anticipated net cash flow that the firm expects it would obtain during the period \( i \) if the decision were (a).

**The amount of anticipated net cash flow that the firm expects it will obtain during the period \( i \) is, in turn, same as that which the firm expects it would obtain during the period \( i + 1 \) if it chose (a) as its decision for the period \( i \), at each and every rate of retention.
period $i - 1$. This implies that the anticipated cash flow, expected to be obtained during the period $i$, resulting from the decision (b) is greater than that which is expected to be obtained during the period $i - 1$. That the anticipated cash flow expected to be obtained during the period $i$ as a result of the decision (b) is greater than that which is expected to be obtained during the period $i - 1$ implies, taking that the first implication of the assumption is plausible, that the anticipated cash flow resulting from each and every level of investment to be undertaken in the period $i + 1$ is greater than that in the period $i$, at each and every relevant level of investment. Combining this with the firm’s expectation that the anticipated net cash flow to be available for the period $i + 1$, as a result of the decision (b) is $1 + g$ times as great as that to be available for the period $i$, we can justify this implication of the assumption, though we recognize the "amount" of retention at each and every "rate" of retention in the period $i + 1$ is $1 + g$ times as great as that in the period $i$ and that the implicit assertion has been made that the marginal rate of return on retention diminishes as the amount of retention increases, cet. par.

In terms of the marginal rate of return on retention, we can state this implication as follows. If the firm chooses (b) as its decision for the period $i$, which means that the firm expects that it will obtain $1 + g$ times as great an anticipated net cash flow during the period $i$ as that during the period $i - 1$, the anticipated opportunity locus for the period $i + 1$ will be, for the reasons stated above, such that the marginal rate of return on retention is same as that in the period $i$, at each and "rate" of retention. This occurs even though the "amount" of retention in the period $i + 1$ is $1 + g$ times as great as that in the period $i$ at
each and every rate of retention.* In other words, in order for the marginal rate of return on retention to diminish by the same magnitude, \( 1 + g \) times as large an amount of retention can be made in the period \( i + 1 \) as in the period \( i \).**

Another implication of the assumption is that no matter what time path of its financial decision the firm takes, the maximum attainable growth "rate" per period is constant over the time horizon, whose value is the average rate of return evaluated at the maximum retention (either rate or amount) less one for all period \( i \). This is an absolute limit that the totally external factors impose on the firm's range of choice in a period, and hence on the extent to which the firm can influence by its decision for a period the way the absolute limit will be imposed by the totally external factors in the subsequent period.

Having discussed the plausibility of the assumption that the anticipated opportunity locus for the period \( i \) and that for the period \( i + 1 \), for any \( i \), are the same in terms of the average rate of return on retention or in terms of the marginal rate of return on retention, evaluated at each and every "rate" of retention. We now proceed to see how the firm can secure the constant level or the constant growth rate of dividend and anticipated net cash flow over the time horizon, either of which we have shown to be a necessary condition to satisfy the constraint. In other words, we will construct the effective subset of attainable expansion

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*The marginal rate of return on retention evaluated at the same "amount" of retention is greater in the period \( i + 1 \) than in the period \( i \) at each and every amount of retention.

**We could have compared the situation resulting from the decision (b) with that from (a) in order to make the point discussed above.
paths from which the firm is to choose one that maximizes $\delta$. According to the assumption made above, the constant level of dividend and anticipated net cash flow over the time horizon can be secured by choosing and maintaining a dividend payout rate that is expected to result in the same amount of anticipated net cash flow in a period as that expected to be obtained in the previous period.

For example, if the firm chooses point (a) on the anticipated opportunity locus in the period $i$ as its decision for the period $i$, expecting to obtain the same anticipated net cash flow in period $i$ as in period $i - 1$, and if the firm maintains the dividend rate which is expressed by $\tan A$, the situation depicted in the figure will repeat itself over the time horizon. The constant level of dividend and anticipated net cash flow will be expected to be secured. Now if the firm chooses point (b) on the anticipated opportunity locus in the period $i$ as its decision for the period $i$, expecting to obtain, in period $i$, $l + g$ times as great an anticipated net cash flow as in period $i - 1$, and if the firm maintains the dividend rate expressed by $\tan B$, this rate of growth of anticipated net cash flow and dividend will be expected to be sustained.
The firm, then, given the absolute limits imposed by totally external factors, will choose the target dividend rate that maximizes \( P \). The optimal decision depends, as previously explained, upon the investors' utility function, their time preference, and the growth potential which is indicated by the average rate of return on retention evaluated at every "rate" of retention.

Having constructed the corporate expansion model with an assumption found to be plausible, we now conduct the comparative dynamic analysis to examine effects of changes of some parameters upon the target dividend rate, which summarizes the time paths of investment, financial and dividend decisions. The situation to be analyzed is as follows. The firm, at time zero, attempts to make an optimal long-run total financial decision which can be summarized by the target dividend rate. The firm is confronted, in period zero, with the given amount of available net cash flow and with the opportunity locus which changes in subsequent periods.
only by a scale factor, i.e., the opportunity locus for period zero repre-
sents the opportunities available to the firm and the absolute limit im-
posed by totally external factors over the time horizon. We will infer
the probable directional change in the target dividend rate with respect
to some parameter changes.
Appendix (II) - Comparative Dynamics

In this section, we shall examine effects of changes of some parameters upon the target dividend rate. As we have already shown, the firm's optimal financial decision in the long-run depends upon the investor's utility function, their time preference, and the shape and the position of the opportunity locus whose property in terms of the average rate of return on retention (or marginal rate of retention) represents the firm's growth potential. We admittedly lack information about the shape of the utility function and its time preference pattern, but we can still predict the directional change in the target dividend payout rate with respect to parameter changes.

Suppose that the original situation the firm faces is as depicted in figure (II)-1, where the anticipated opportunity locus is drawn based upon given business risk schedules and amount of net cash flow available in the period zero. This characterizes the economic and industrial environment that the firm will face over the time horizon which imposes the absolute limit to the firm's potential. It should be stressed that the anticipated opportunity locus of period zero differs from the anticipated opportunity locus of any other periods only by a scale factor, i.e., it typically describes the limit set by totally external factors and it further prescribes the range of the possible advances within this limit. The maximum growth rate in any period is \((\tan S) - 1\). The maximum permissible dividend payout rate to satisfy the constraint* is \(\tan\).

*At least the constant level of dividend and anticipated net cash flow must be maintained.
Assuming that the firm found the optimal target dividend payout rate under the situation depicted in the above figure, we shall now attempt to deduce what the directional change in the target dividend payout rate would be for a change in: (1) the competitiveness of the industry, and (2) the expected industry demand. We first analyze the effect of an increase in the competitiveness of the industry, ceter. par.

The changes in the properties of the anticipated opportunity locus due to the increase in the competitiveness of the industry can be summarized.
as follows. (1) The average rate of return on retention evaluated at every rate of retention is smaller than the original anticipated opportunity locus. The maximum rate of growth that the firm can attain in any period is smaller than previously, i.e., \((\tan S) - 1 > (\tan S') - 1\).

(2) The maximum allowable dividend payout rate (or minimum required rate of retention) is smaller (greater) than previously, i.e., \(\tan A > \tan A'\).

(3) The marginal rate of return on retention evaluated at every rate of retention increases. This is indicated by the steeper slope of the dotted opportunity locus. This implies that the marginal increment in the growth due to a marginal increment in the rate of retention is greater than previously.

From this above observation, we can predict the change in the target dividend payout rate without knowing the initial optimal target payout rate. The firm is obliged to have a higher retention rate merely to maintain the constant level of dividend and anticipated net cash flow. Additionally, since the marginal rate of return on retention is higher at every rate of retention, the marginal opportunity cost of the marginal increment in the dividend payout in terms of the foregone growth potential is higher than originally. Thus the firm must be willing to retain more. Therefore, we can conclude that the target dividend payout rate will decrease as the competitiveness of the industry increases.

Next we analyze the effect of an increase in the expected industry demand, cet. par. The change is described in figure (II)-2. The changes in the anticipated opportunity locus due to an increase in the expected industry demand can be summarized as follows.

(1) The average rate of return on retention evaluated at every rate of retention is greater than previously. Then the maximum rate of growth
that the firm can attain in any period is now greater than originally, i.e., \((\tan S') - 1 > (\tan S) - 1\).

(2) The maximum allowable dividend payout rate (or the minimum required rate of retention) to maintain at least the constant level of dividend and the anticipated net cash flow is greater (smaller) than previously, i.e., \(\tan A' > \tan A\).

(3) The marginal rate of return on retention evaluated at every rate of retention increases, i.e., the marginal increment in the growth potential due to the marginal increment in the rate of retention increases for every rate of retention.

In this case, the directional change in the target dividend payout rate is indeterminate, because the forces described in (1) - (3) above, are working in the opposite directions. The marginal opportunity cost of dividend in terms of the foregone growth potential is greater than previously, and therefore the "substitution" effect must work so as to reduce the dividend payout rate. The minimum required rate of retention is smaller than originally, and therefore the "income" effect must
work so as to increase the dividend payout rate.

This conclusion is at odds with other writers, who maintain that the "better investment opportunity"* unambiguously increases the target dividend payout rate.

The analysis of preceding cases can be used to analyze the situation of a so-called "rapidly growing company." A "rapidly growing company" is a firm found in an industry whose demand is expanding very rapidly so that it has a good opportunity for expansion. But since the good opportunity is available for all firms, not just for "the firm," the competition to acquire the best opportunities may be intense. Especially if we consider that the source of rapidly expanding demand is usually the development of new products for which improvement in the quality and the reduction in the production costs is possible, we expect intense competition for the best opportunity. These considerations are summarized in the following pictures.

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*This expression, though ambiguously stated, seems to mean a higher expected industry demand.
The changes in the properties of the anticipated opportunity locus can be summarized as follows. (1) The maximum attainable growth in any period is greater than previously. (2) The minimum required rate of retention is approximately the same, since the greater competitive pressure and the increased demand for the product work in opposite directions in this respect, i.e., \( \tan A \approx \tan A' \). (3) The marginal rate of return on retention evaluated for every rate of retention greatly increases, which implies that the marginal increment in the growth rate caused by a marginal increment in the retention increases greatly. The greater competitive pressure and the greater industry demand both work in the same direction in this respect.

Thus we conclude that the firm will definitely retain a greater fraction of the anticipated net cash flow stream (the target dividend payout rate will decrease), since the substitution effect dominates. A super normal rate of growth can be obtained because of the high retention rate and the high average rate of return on retention evaluated at high rates of retention. This can be contrasted with the first situation. There the firm was shown to retain the greater fraction of the anticipated net cash flow than the firms in the less competitive industry, the rate of growth expected to be obtained was less than the firms in the less competitive industry. It should be pointed out that what causes the target dividend rate to be unambiguously smaller is the higher competitive pressure, not the rosier opportunity of investment.

We could further analyze the probable changes in the target dividend payout rate with respect to changes in some other parameters such as change in the structure of individual income tax (on dividend, capital gains), change in the time preference, etc., but since the directional
change can be predicted more or less accurately without a formal analysis, we will not go any further.
VII. Toward a short-run optimal financial behavior of the firm. (Conclusion of the long-run analysis.)

The long-run analysis showed that maximizing \( P \) subject to
\[
\hat{P}(i) = \hat{P}(i-1) \leq \hat{P}(i)
\]
for all \( i \) determines the firm's investment, financing and dividend decision time paths so that a constant level or growth rate of dividend and the anticipated net cash flow stream is secured.* This implies that the firm determines the optimal target dividend rate in the long-run financial decision making.

With the conclusions derived from the long-run analysis, we can make the following statement concerning the financial behavior of the firm in each one period in its time horizon. The firm pays the dividend prescribed by its long-run financial decision, and then makes the investment and financing decisions that will minimize total risk. The decisions, hopefully, result in the actual net cash flow equaling the anticipated net cash flow for the period. The firm, however, is well aware that the resulting actual net cash flow deviates from the anticipated net cash flow for the period because the anticipated net cash flow is merely a rough approximation (though that is the best that the firm can make) of

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*In light of our corporate expansion model, where the assumption necessary to set up the model was verified, we can easily see why the same rate of growth of the anticipated net cash flow and dividend stream are necessary to maintain a constant growth rate of both streams. That dividend stream cannot grow at any constant rate unless the anticipated net cash flow grows at the same rate and is easily seen. That the anticipated net cash flow cannot grow at any constant rate unless the constant dividend payout rate is maintained (which implies that dividend must grow at the same rate) can be easily verified if we assume, as we do, the diminishing marginal rate of return on retention and recognize that the marginal rate of growth with respect to the amount of retention is the marginal rate of return on retention divided by the available amount of the anticipated net cash flow.
the resulting net cash flow. The estimate required in the long-run financial decision making involves high degree of uncertainty. The deviation of the actual net cash flow from the anticipated net cash flow does not induce the firm to take any action as long as it stays within the expected range, i.e., between $\mu$ and $\mu - \sigma$ corresponding to the prespecified amount of dividend, because this fluctuation of the resulting net cash flow within the expected range is expected and well taken into account when the firm determines its long-run financial policy. It is only when the firm foresees in its one period financial decision making that the resulting net cash flow will deviate outside the expected range if that prespecified amount of dividend is to be paid out, that the firm will reexamine the whole situation and consider the changes in the structural long-run policy. Whenever the new long-run policy involves, the different optimal target dividend rate as its planned result depends upon the cause of foreseen deviation. Once its new long-run policy is determined, the firm again determines its one period financial decision in the manner described above.

What must be recognized is that the firm's long-run financial decision, by prespecifying the dividend payment, imposes an internal constraint upon its one period financial decision. Note, however, very clearly that this never implies that the dividend decision has a top priority in one period financial decision over the other decisions. Prespecified amount of the dividend is merely one of the planned results of the firm's long-run total financial decision.

The firm, given this prespecified amount of dividend, determines its investment and financing decisions for the period such that the total risk will be minimized. Thus, the firm's financial decision in one period
becomes that in short-run in traditional economic sense, and we can analyze it by our hypothesis of total risk minimization with the given prespecified amount of dividend. Thus, to determine the firm's optimal financial decision for any one period does no longer require knowledge of the shape of the investors' utility function which was necessary in our original one-period analysis. We are now in a position to derive a workable theory of so called "investment behavior of the firm" based on the hypothesis of total risk minimization. It is workable in a sense that we can empirically test it. We shall derive the theory by the comparative static analysis to examine effects of various parameter changes on the size of investment to be undertaken. These effects could not be examined in our originally stated one-period analysis, because unless we knew the required change in the dividend decision, we could not determine the induced changes in the investment and financing decisions. The derivation of a theory of "short-run investment behavior of the firm" by comparative statics is the next topic.
CHAPTER V

Short-Run Investment Behavior of the Firm
In this chapter we shall derive a short-run investment behavior of the firm by conducting a comparative static analysis based on our hypothesis that the firm, in the short-run, determines its investment and financing decisions, given the predetermined amount of dividends, such that total risk will be minimized. We shall be concerned with the effect of the change in such parameters as the firm's expectation on the industry demand, the level of excess capacity, competitive pressure, the amount of the available retained earnings and the level of the currently existing debt in the firm's capital structure. The change in these parameters will result in the shift in BR 1, BR 2 and FR schedules. By tracing the shift of them corresponding to the change in these parameters, we will find out the relationship between the size of investment and these parameters.

---------- Comparative static analysis ----------

Suppose that the original situation of the firm, prior to any change in parameters, is as shown in figure V-1, where the firm, given BR 1, BR 2 (hence TBR), FR (hence TR), and RE, determines its optimal size of investment, I*, and optimal borrowing, B*, which will finance I*, combined with RE. Given this original situation*, we conduct our comparative static analysis.

(1) We first analyze the effect of the change in the expectation held

*These original schedules are constructed in the mind of the management of the firm based on its expectation concerning the various aspects of the economic, industrial and internal (within the firm) development which is formulated based on the information on those aspects. In our comparative static analysis, we change the expectation on one and only one aspect (or the information on one and only one aspect) at a time and see its effect on the size of investment.
by the management on the industry demand. The change in its expectation can be thought to be brought about for various possible reasons such as a change in the level of demand for the product in the recent past, a change in the price(s) of the substitute good(s), a change in the attitude of monetary authority and so forth.*

Let us suppose that the industry demand curve shifted upward in the recent past (in comparison to the original demand curve) for whatever reasons, and that the firm, as its result, expected a greater general demand for the product for this period than it originally expected. The new situation is depicted in figure V-1 in dotted curves. BR 2 schedule

*Note that "a change" here is a change from the original situation (which gives the firm an original set of information and hence an original set of expectations), not a change over time.
shifts downward such that \( \frac{dR_2}{dI} \) > \( \frac{dR_1}{dI} \). TBR schedule shifts accordingly and FR schedule shifts correspondingly.*

The new optimal size of investment is \( I^*_1 \), where \( I^*_1 > I^*_0 \) and the new optimal amount of borrowing is \( B^*_1 \), where \( B^*_1 > B^*_0 \). We can therefore conclude that the increase in the expected general level of demand for the product of the industry, cet. par., will increase the optimal size of investment and hence that of borrowing (since the available amount of retained earnings is kept unchanged).**

*The manner in which these schedules shift corresponding to various parameter changes have been analyzed in detail in chapter III.

**This conclusion is unambiguously stated, based on our observation that (1) and (2) and remembering that the slope of TBR and FR must be equal at the optimum.
(2) We shall analyze the effect of the change in the competitive pressure. Suppose that the firm has been experiencing a decrease in its market share in the recent past.* We describe the situation in figure V-2.

We see that \( \left| \frac{dBR_1}{dI} \right| > \left| \frac{dBR_s}{dI} \right| \) and, therefore, that \( \left| \frac{dTBR_s}{dI} \right| > \left| \frac{dTBR_o}{dI} \right| \). We need to pay attention to the way in which FR schedule shifts. As we have pointed out in chapter III, the magnitude of financial risk depends on both the magnitude of total business risk and the amount of borrowing. Given the same amount of retained earnings as the original situation and the greater magnitude of total business risk than the original situation corresponding to each and every level of investment and hence borrowing, we draw a new FR schedule such that the magnitude of financial risk corresponding to each and every level of borrowing (hence of investment) is greater than the original FR schedule. Our concern, however, is not the general level of the magnitude of financial risk but the rate of change in the magnitude of financial risk caused by the successive marginal increment in the amount of borrowing (and hence in the size of investment).

Comparing the new TBR schedule with the original TBR schedule we recognize that \( \left| \frac{dTBR_s}{dI} \right| > \left| \frac{dTBR_o}{dI} \right| \) over a relevant range of investment. This implies that \( \frac{dFR}{dTBR} \) is offset by \( \frac{dTBR}{dI} \cdot \frac{dI}{dB} \) to a greater extent in the new situation than originally. It is especially so when the amount of borrowing (and hence that of investment) is relatively small, because

*More precisely, a greater decrease (or a smaller increase) than the original situation in the market share in the recent past.
the difference between \( \frac{dTBR_I}{dI} \) and \( \frac{dTBR_O}{dI} \) is large.*,**,** It is for this reason that we draw FR, schedule such that, though the general level is higher than the original FR.

Then given (a) that \( \frac{dTBR_I}{dI} \) > \( \frac{dTBR_O}{dI} \), and (b) that \( \frac{dFR_I}{dB} \) < \( \frac{dFR_O}{dB} \)

over the relevant range**, we can conclude that the decrease in the market share in the recent past, etc. par., impels the firm to invest greater amount in the period. Given the prespecified amount of retained earnings, this also implies that the firm will borrow more under this situation.

(3) We shall analyze the effect of the change in the extent of excess capacity. Suppose that the firm has been experiencing a greater extent of excess capacity than the original situation. We can describe the

*See chapter II for the discussion on the matter.

**We may very well expect a big jump in the magnitude of financial risk between no borrowing and an infinitesimal amount of borrowing in this case, because of the extremely high total business risk corresponding to a small amount of investment. This is the situation depicted in the figure.

***We recognize that \( \frac{dFR_I[B, TBR.(I)]}{dB} > \frac{dFR_O[B, TBR.(I)]}{dB} \) at every B (implying that, at every I). The difference between them, however, is small because: (1) though, when I is small, the difference between TBR, and TBR, is large, B is small, and a property of the basic fr curves insures that the difference in \( \frac{dFR}{dB} \) corresponding to different magnitude of TBR is small when B is small; (2) when B is large, I is also large, and the difference between \( \frac{dTBR_I}{dI} \) and \( \frac{dTBR_O}{dI} \) is small at a large I.

****We note that at the point of optimality, i.e., the size of investment and the corresponding amount of borrowing at which the total risk is minimized, \( \frac{dTBR}{dI} = \frac{dFR}{dB} \), where \( \frac{dFR}{dB} = \frac{dFR}{dI} \) + \( \frac{dFR}{dI} \) \( \frac{dTBR}{dI} \) \( dB \) must hold.
situation as follows.

BR schedule shifts upward such that \( \frac{dBR}{d\lambda} > \frac{dBR_o}{d\lambda} \) and \( \frac{dBR}{d\beta} > \frac{dBR_o}{d\beta} \).

TBR schedule shifts accordingly, such that \( \left| \frac{dTBR}{d\lambda} \right| < \left| \frac{dTBR_o}{d\lambda} \right| \) over the relevant range of investment. Correspondingly, FR schedule shifts upward such that \( \frac{dFR}{d\beta} > \frac{dFR_o}{d\beta} \). This is so because \( \left| \frac{dFR}{d\lambda} \right| < \left| \frac{dFR_o}{d\lambda} \right| \) as well as \( \frac{dFR}{d\beta} > \frac{dFR_o}{d\beta} \).

Then given (a) that TBR has a less steep downward slope than TBR_o and (b) that FR has a steeper upward slope, the optimal amount of investment, at which \( \frac{dTBR}{d\lambda} = \frac{dFR}{d\beta} \) must hold to minimize the magnitude of total risk, is smaller under the new situation than in the original situation.

Given the prespecified amount of retained earnings, this also implies that the amount of borrowing to be employed will be smaller.
(4) Concerning the effect of the change in the available amount of retained earnings, cet. par., on the size of investment, we can conclude without detailed analysis that the greater amount of retained earnings results in the greater amount of investment, from our theoretical construct.* We pay specific attention to its effect on the amount of borrowing to be employed. As we have discussed, the TBR schedule has such properties that \( \frac{d_{TBR}}{d I} < 0 \) but \( \frac{d^2_{TBR}}{d I^2} > 0 \) over the relevant range of investment because of our assumption that \( \frac{d_{BR}}{d I} < 0, \frac{d_{BR}^2}{d I^2} > 0 \) \( \frac{d_{BR}}{d I} > 0 \), and \( \frac{d_{BR}^2}{d I^2} > 0 \). Therefore, \[ \left| \frac{d_{ER}}{d_{TBR}} \frac{d_{TBR}}{d I} \frac{d I}{d B} \right| \] evaluated at the same amount of borrowing, given TBR schedule, decreases as the amount of available retained earnings becomes greater. This consideration in conjunction with the property of TBR schedule leads us to think** that the amount of borrowing will not increase in proportion to the increase in the available amount of retained earnings. Therefore, we can infer that the size of investment does not increase in proportion to the increase in the amount of retained earnings.***

(5) We finally analyze the effect of the change in the amount of currently outstanding debt on the size of the investment. Since we have been abstracting from the existence of currently outstanding debt in our analysis until now, we must discuss, first, how we can incorporate it into our basic analytical apparatus. For this purpose we shall treat the

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*See chapter II, Final Optimization.

**Though we cannot definitely conclude about the difference in the magnitude of \( \frac{d_{PR}}{d B} \) of PR schedules corresponding to various amounts of retained earnings (evaluated at the same amount of borrowing), unless we know the difference between \( \frac{d_{ER}}{d_{TBR}} \frac{d_{TBR}}{d I} \frac{d I}{d B} \) and \( \frac{d_{ER}}{d_{TBR}} \frac{d_{TBR}}{d I} \frac{d I}{d B} \).

***It might be possible that the increase in the amount of retained earnings even decreases the amount of borrowing to be employed.
following simplest hypothetical case. Suppose that the firm has such an amount of currently outstanding debt (denoted by d in figure V-4) that is just as large as the available amount of retained earnings, i.e., the firm can, if it wants, pay off all the debt outstanding in the period under consideration.

Then given the total business risk schedule, we can construct the FR schedule in the following manner. First we assume that the firm pays off all the currently outstanding debts by its available retained earnings. Then we let the firm start borrowing to finance investment for the period. Therefore, the FR schedule starts from the origin. We note that the financial risk is perceived, given the TBR schedule, based on the amount of the resulting total outstanding debt, S, consisting of the currently outstanding debt and the borrowing to be made in this
period; the latter can be either negative, zero, or positive.* We also note that two following decisions will result in the same magnitude of financial rate (as a matter of course); (1) the firm does not pay back any fraction of its outstanding debt at all and invests all the available retained earnings, and (2) the firm pays off all its outstanding debts and then invests by borrowing as much as the available amount of retained earnings. In either case the magnitude of financial risk is $\alpha$ in figure V-4. Now, given TBR schedule and FR schedule, the firm as before, determines its size of investment and the amount of borrowing such that total risk will be minimized. The firm will, then, either (1) invest more than the available amount of retained earnings can finance by borrowing a positive amount, (2) invest just as much as the available retained earnings can finance or (3) pay back some fraction (or all) of the currently outstanding debt and invest less than the available retained earnings can finance, depending on the slopes of TBR and FR schedules.

In our example, optimal decision turns out to be the case (3), where the optimal size of investment is less than the available retained earnings and the firm pays off $(RE - I_k)$ of the currently outstanding debt, i.e., the amount of borrowing in this period is negative.

The extension of the analysis into the other conceivable situations is straightforward. Let us now suppose that the amount of the currently outstanding debt is smaller than the available amount of retained earnings. Given TBR schedule, we can construct FR schedule as depicted.

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*We stated in the previous analyses that the magnitude of financial risk perceived depends on the magnitude of total business risk and the amount of borrowing to be made in the period, but it was because there was no currently existing debt by assumption.
In this case, the FR schedule starts from RE - d. This is so because we can, as before, assume that the firm pays off all its currently outstanding debts and then we let it start borrowing. Given the origin of the schedule, we can construct FR schedule by connecting the relevant point on each and every FR schedules as before.*

We now treat the situation where the amount of currently outstanding debt is greater than the amount of retained earnings available for the period.

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*Remember that the magnitude of financial risk, given the magnitude of total business risk, depends on the amount of total outstanding debts which are the sum of the amount of the currently outstanding debts and the amount of new borrowing to be made.
In this case we first draw fr curve corresponding to total business risk evaluated at $I = 0$, i.e., $TBR(0)$. The origin of the curve is the point $\xi$ which characterizes zero resulting total outstanding debt. The relevant point on this fr curve is the origin of the relevant portion* of the FR schedule for this situation. We can see that even if the firm pays off as large a fraction of its currently outstanding debt as the total amount of available retained earnings, there will still exist a positive outstanding debt and therefore positive financial risk. FR schedule then can be constructed as before by connecting the relevant point of each and every fr curves, all of which start from the point

*The portion of FR schedule corresponding to $I = 0$. 
Given TBR schedule, we can draw two FR schedules corresponding to two different amounts of currently outstanding debts, \( d_0 \) and \( d_1 \). We first draw \( FR \) and \( FR_1 \) curves, given TBR (0), starting from \( \xi_0 \) and \( \xi_1 \), respectively. These two curves are identical, since these are drawn corresponding to the same magnitude of total business risk, except for their origins. We take the relevant point of each of these curves, which is the point on each corresponding to \( I = 0 \). These relevant points, \( p \) and \( q \), are the origins of the relevant portion of \( FR_0 \) schedule and \( FR_1 \) schedule, respectively. Then, the magnitude of financial risk corresponding to \( I = 0 \) is seen to be greater on \( FR_1 \) schedule than on \( FR_0 \) schedule. This must be so because of a property of \( FR \) curves, spelled out in chapter III.*

We can also easily verify that \( \frac{dFR_1}{d\delta} > \frac{dFR_0}{d\delta} \) at every level of investment.** This is so because of the following reason. \( \frac{dFR}{d\delta} \) consists of two components, i.e., \( \frac{dFR}{dTBK} \frac{dTBK}{dI} \frac{dI}{d\delta} \) and \( \frac{dFR}{d\delta} \), since financial risk depends on total business risk and the amount of resulting total outstanding debt. Now the magnitude of the first component at the same level of investment is same along both \( FR_0 \) and \( FR_1 \) schedule, since both \( FR \) schedules are constructed corresponding to the same TBR schedule. \( \frac{dFR}{d\delta} \) at the same level of investment, however, is greater along \( FR_1 \) than along \( FR_0 \), since the amount of resulting total outstanding debt to

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*Given the magnitude of total business risk, \( \frac{dI}{d\delta} \) (\( \frac{dI}{d\delta} \) in chapter II).

**Strictly speaking, we should say the derivatives \( \frac{dFR_0}{d\delta} \) and \( \frac{dFR_1}{d\delta} \) evaluated at \( S_0 \) and \( S_1 \) where \( S_0 \) and \( S_1 \) correspond to the same level of investment.
be needed to finance the same amount of investment is greater for the new situation than the original situation.

We can therefore conclude that the firm, in an attempt to minimize total risk, will invest less in the new situation than in the original situation.* This, given the assumption that the amount of available retained earnings is kept unchanged, implies that the amount of borrowing to be made in the new situation is smaller than in the original situation. In our illustration, the firm invests more than the available retained earnings can finance (i.e., it will incur a positive borrowing in the period) in the original situation, but it invests less than the available retained earnings can finance (i.e., it will incur a negative borrowing, by paying back some fraction of the currently outstanding debt, in the period) in the new situation.**

This seems to be the first formal attempt to answer the question concerning the effect of the change in the amount of currently outstanding debt on the size of investment, at least to the knowledge of the present author, except for the work of Modigliani-Miller, whose famous proposition implies their answer to this question as a corollary.***

Duesenberry states his viewpoint on the matter in his "Economic Growth

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*This is easily verified, knowing that (1) at the optimal \( \frac{dTR}{dI} \), must hold, (2) TRR schedule is identical in both situations and (3) FR\(_1\) is steeper than FR\(_0\) at each and every level of investment.

**In this part of the analysis, we are implicitly assuming that the unit cost schedule of borrowing is invariant over time.

and Business Cycle,* but his underlying idea of opportunity cost of retained earnings as the foregone opportunity of paying off the outstanding debt is still conjectural, though interesting. Mayer, Kuh and Lock Anderson** also state their views on the matter, but they do not give arguments to support their views.


-- Summary of the Chapter --

By means of comparative static analysis based on our hypothesis of total risk minimization, we derived a theory of so called "investment behavior" of the firm. We drew a set of conclusions about the sign of the partial derivatives of the size of investment with respect to the various arguments which we think, according to our hypothesis of total risk minimization, are the primary determinants of the size of investment. We, at the same time (as a byproduct of the analysis), derived a set of conclusions about the sign of the partial derivatives of the amount of (new) borrowing to be employed with respect to those arguments. We admit that the empirical implementation of our theory takes a tremendous amount of work and efforts to be expended. We, however, believe that our theory is logically sufficiently sound and also the assumptions reflect the reality sufficiently close, to make the result of the future empirical test of it very meaningful, however the result turns out.
CHAPTER VI

A Concluding Note
As stated at the outset, our study was directed toward establishing a more realistic theory of corporate financial decisions. We pursued our objective with the belief that we could offer a meaningful explanation of the firm's financial behavior only if we examined and incorporated into our analysis the interdependence of the firm's investment, financing and dividend decisions. The development and application of a general equilibrium model, based on this belief, we believe has cast a new light on various aspects of corporate financial behavior.

Our model explicitly answered two aspects of the financing problem for which the partial equilibrium approach was found lacking. There does exist an optimal method of financing current investments, and the amount of currently existing debt influences the size of the capital budget and the methods of financing. However, we failed to give an explicit answer to another aspect of the problem; that is, that of the existence of an optimal capital structure on the firm's balance sheet, given the existing asset structure. Our answers to the other two aspects of the financing problem, however, did lend implicit support to the existence of an optimal capital structure. An optimal capital structure, it should be noted, does not imply that a particular debt to equity ratio is desired by the firm. Rather, it is a result of external pressure which requires the firm to obtain a portion of its total capital through the use of debt instruments. External pressure refers, as must be clear by now, to that pressure arising from competition and from the investors' demand for current dividends. Recognition of the motives for debt employment seems to be the key to explicitly answering the optimal capital structure problem.
Our general equilibrium model provided a theory of the firm's investment behavior which served to unify assorted past empirical findings into a comprehensive theoretical construct. It also enabled us to establish a plausible hypothesis about the manner in which the amount of dividends to be paid out affects the level of the investors' utility as well as the stock price.

Finally, our theoretical framework seems to be applicable to the study of business cycle problems. That is, our analysis of the effect of the change in external and internal factors on the corporate financial behavior seems to be adequate to explain the changing corporate financial behavior patterns over various phases of business cycle.

We believe that this area is potentially one of great interest and are hopeful that this study will serve to stimulate further research.


