SINEX III, Charles Helm, 1943-
ANGULAR CORRELATION STUDIES OF THE Ca^{40} (pp,γ)Ca^{40} REACTION.

Rice University, Ph.D., 1970
Physics, nuclear

University Microfilms, A XEROX Company, Ann Arbor, Michigan
RICE UNIVERSITY

Angular Correlation Studies of the 
$^{40}$Ca$(pp'\gamma)^{40}$Ca Reaction

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Thesis Director’s signature:

Houston, Texas

May, 1970
To Cile
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I. INTRODUCTION

The doubly magic nucleus of $^{40}\text{Ca}$ has been the subject of extensive experimental and theoretical investigations during recent years in an attempt to understand its structure. Experiments have shown that the simple spherical shell model is unable to explain the behavior of the $^{40}\text{Ca}$ nucleus, despite the fact that both the proton and the neutron shells are closed. The experimentally known levels of $^{40}\text{Ca}$ are shown in figure 10 (pg. 88). The simple shell model is unable to account for the large number of observed levels, and the predicted levels do not agree with either the sequence of spins or the energies of levels in the known spectrum. As an example of these difficulties, spherical shell model calculations predict only one $3^{-}$ level below 7 MeV excitation energy in $^{40}\text{Ca}$, while there are three such levels in the actual spectrum.

Other difficulties arise in the gamma ray decay modes for the positive parity excited states of $^{40}\text{Ca}$. Many of these transitions exhibit enhancements in transition strengths of an order of magnitude or more over the predictions of the simple spherical shell model. Finally there are a number of $0^{+}$, $2^{+}$, $4^{+}$ sequences of excited states in the
Ca$^{40}$ nucleus that are suggestive of some collective behavior for the positive parity levels. There is also an accumulating amount of evidence, including the results of the present work, that this collective behavior applies to the negative parity levels as well.

In an attempt to understand this behavior, a number of extensions of the shell model to include collective rotational motion have been proposed. One of the most promising of these models has been introduced by Gerace and Green\textsuperscript{1,2,3}). Their treatment recognized the fact that the energies of single particle states in a spherical nucleus are changed as the nucleus suffers a deformation. This effect comes from the fact that the spherical potential field of a nucleus becomes spheroidal as the nucleus is deformed and has been worked out in detail by Nilsson\textsuperscript{4}). His work showed in particular that as the nuclear deformation parameter B is increased to approximately 0.2, the 1f$^{7/2}$ and 2p$^{3/2}$ single particle states come closer to the closed 2s$^{1/2}$ - 1d$^{3/2}$ states in energy than in the spherical nucleus.

Gerace and Green\textsuperscript{1)}, using this fact, suggested that exciting two and four particles from the Ca$^{40}$ core to the 2p-1f shell in a deformed nucleus might produce two particle-two hole (2p-2h) and (4p-4h) deformed states.
enough in energy to mix significantly with the spherical ground state wave function of the undeformed nucleus. They predict a (2p-2h) 0+ excited level about 8 MeV above the Ca$_{40}$ ground state and a (4p-4h) 0+ at 4 MeV with rotational bands built on each 0+.

Residual two-body interactions then mix the wave functions of these deformed states with the spherical ground state and perturb the energies of the 0+, 2+ and 4+ levels to produce the final observed energies in the Ca$_{40}$ spectrum. Gerace and Green attack the problem in reverse at this point and identify the 2+ levels seen at 3.90 MeV and 6.90 MeV as the final positions of the 2+ levels of the (4p-4h) and (2p-2h) bands after mixing. The matrix elements of the states in each rotational band, before the two-body interaction is applied, can then be calculated. From these matrix elements, the perturbed positions of the two 0+ and 4+ levels can be calculated. The 0+ ground state level is chiefly a (0p-0h) excitation in this model.

The results of this theoretical treatment are shown in figure 26. The 4+ levels are predicted at energies of 5.25 and 8.0 MeV and may be identified with the levels observed at 5.28 and 7.928 MeV. The 0+ levels are predicted at 3.5 and 7.2 MeV and may be identified with the levels
observed at 3.35 and 7.33 MeV, the latter recently seen by Leenhouts\textsuperscript{5}). Gerace and Green also calculated a number of gamma ray transition strengths for decays from these levels that have been found in fairly good agreement with theory\textsuperscript{6,7}).

The success of this initial work led Gerace and Green to extend their calculations to include (8p−8h) excitations to explain a third rotational band starting at 5.213 MeV. A (4p−4h) K = 2 band was included to explain the observed gamma ray transition from the 2\textsuperscript{+} level at 5.25 MeV to the 2\textsuperscript{+} level at 3.90 MeV; intraband transitions of this type are explained by the mixing of multiparticle-multihole states of different intrinsic character by residual two-body interactions. Again their theoretical predictions find good agreement with theory\textsuperscript{7}).

All of the positive parity levels in Ca\textsuperscript{40} up to 7 MeV excitation energy may be fairly well interpreted in terms of the Gerace and Green model. This present work finds an additional 2\textsuperscript{+} level at 7.114 MeV that is tentatively proposed as the head of another (8p−8h) K = 2 band on the basis of its observed gamma ray decay modes. A schematic diagram of all of these levels is given in figure 26 (pg.147).
The negative parity levels of Ca$^{40}$ are less thoroughly understood and presently only a few of the many known negative parity levels can be fit by any theory. A number of shell model calculations using (1p-1h) excitations based on the K39 ground state have been carried out. Gillet and Sanderson$^8$) made calculations of this type to obtain spectroscopic factors for the Ca$^{40}$ excited states, but experiments with the K$^{39}$(He$^3$,d)Ca$^{40}$ $^9,10$) reaction find only fair agreement with the predictions. Similar calculations by Dieperink, et al.$^{11}$) predict gamma ray transition strengths that are well below the experimentally observed strengths for E2 gamma ray decays. Additionally, all of these calculations have trouble in predicting enough levels above 6 MeV to fit the known spectrum.

Probably the best theoretical treatment to date has been carried out by Gerace and Green$^3$) using (1p-1h) and (3p-3h) deformed states interacting with the Ca$^{40}$ ground state wave function, analogous to their treatment of the positive parity levels. Although their predicted energies are usually too high, their prediction of the negative parity level order is in good agreement with the known spectrum up to 6.5 MeV. With slight changes in level ordering past this point, their predictions will also explain the
two new negative parity levels found in this present work. Gerace and Green also calculate a number of gamma ray transition strengths from this model that find quite good agreement with experiment\textsuperscript{12}).

With the complexity of present theoretical treatments, merely the observation of a level with the correct spin and parity at the predicted energy is no longer an adequate verification of a theory. The types of interaction forces used in the calculations are strongly reflected in the predicted gamma ray decay modes and strengths; therefore lifetime measurements and gamma ray transition strengths (the probability of a particular gamma ray decay) take on a new importance in experimental work. These experiments have been carried out by a number of workers for most of the known positive parity levels and, as mentioned, find fairly good agreement with the predictions of theory.

Similar calculations are now needed for the negative parity levels in Ca\textsuperscript{40}. At present, only the few lower lying levels have accurate measurements for lifetimes. The purpose of this present work is to partially remedy this deficiency. Decay modes of two of the remaining levels about 7 MeV were studied. Angular correlation measurements in the Litherland and Ferguson geometry II\textsuperscript{13}) were performed
to determine the spins of these levels as well as the mixing ratios and branching ratios for their gamma ray decay. These ratios are important together with the lifetime in determining the partial width of a level for gamma ray decay via a particular type of gamma ray (E2 gamma rays, for example). The lifetimes were measured with a Doppler shift attenuation method. The transition strengths obtained from this experiment, together with previously measured values for lower lying excited states, are compared in section V.

The present experimental work also illustrates the power of angular correlation measurements for making unambiguous spin assignments to an unknown level. A number of different types of correlation measurements were investigated, and methods of simultaneously fitting data from several different measurements to obtain unambiguous assignments for the parameters of a level are discussed.
II. EXPERIMENTAL APPARATUS AND PROCEDURES

A. Experimental Philosophy

The angular correlations used in this present work were performed in the Litherland and Ferguson geometry II described in detail in section III13). In this geometry, the proton from a $^{40}\text{Ca} \rightarrow (pp'\gamma)\text{Ca}^{40}$ reaction is detected in an annular particle detector located at 180° with respect to the beam axis. This detector has a small hole in the center to allow the accelerator beam to pass through to the target chamber. Several experimental difficulties are caused by this arrangement.

The particle detector is close to the beam axis and can detect protons backscattered through 180° from the beam stopper located beyond the target chamber. The accelerator beam must be tightly collimated near the particle detector to prevent any direct beam from striking the back face of the detector. At the high beam energies necessary in this experiment (greater than 10 MeV), this collimation near the chamber can then act as a strong source of background gamma radiation.

Order of magnitude calculations to illustrate these problems using typical experimental parameters obtained
from experience are performed below. The particle and gamma ray detector counting rates from the target and from the background sources will be calculated and compared to the actual rates obtained in practice.

A Ca\(^{40}\) target of 750 \(\mu \text{gm/cm}^2\) thickness was found to be suitable for this experiment. The accelerator beam currents used were on the order of 30 nanoamps of protons at 10 MeV and approximately 75% transmission through the collimating system could be achieved. Distances from the target were 3.6 cm. for the particle detector and 40 cm. for the gamma ray detector. The exact positions of collimators and aperture sizes are given in part B of this section, and the solid angles subtended by the detectors were .0246 steradians (1/500 of a sphere) for the particle detector and .072 steradians (1/150 of a sphere) for the gamma ray detector. If we take NaI(Tl) efficiencies for 1 MeV gamma rays as approximately 50% and assume only 1 gamma ray emitted per excited state, the gamma ray counting rate should be 1.5 times the particle counting rate from the target.

If we estimate the average total inelastic cross-section as 50 millibarns/excited state and 20 excited states as significantly populated at 10 MeV with isotropic scattering distributions, the expected particle count rate is 4800
counts/second (4000 counts/second observed). The gamma ray count rate for target associated gamma rays is then 7200 counts/second (6000 counts/second observed).

For a lead beam stopper 250 cm. downstream, the solid angle subtended by the particle detector is reduced a factor of $1/3600$. The range of 10 MeV protons in lead is about 0.3 mm., or about 100 times the thickness of the Ca$^{40}$ target. If identical cross-sections for scattering from lead and Ca$^{40}$ are assumed and Rutherford scattering neglected (the errors introduced by these assumptions tend to cancel), the reduction in solid angle and increase in yield give a count rate of 130 counts/second from the beam stopper (110 counts/second observed).

A major source of gamma ray background radiation is the collimating slit located near the target chamber. This slit cuts about 7 nanoamps out of the 30 nanoamp beam; the solid angle subtended by the gamma ray detector for this slit is the same as for the target. There is about 1 inch of lead shielding between this slit and the gamma ray detector; assuming all gamma rays produced at the collimating slit are 0.5 MeV, this lead reduces the gamma ray count rate from the slit by a factor 1/60. A gamma ray count rate of 5000 counts/second is then calculated (3000 counts/
second observed). The observed value includes the gamma ray yield from the beam stopper, which has a calculated value of 250 counts/second. Probably one reason the calculated values exceed the measured count rates is that the assumption all of the beam is collimated by the slit nearest the chamber is unfair and in fact the first slit, 1 meter away from the gamma ray detector, should take a fair proportion of the 7 nanoamps to be collimated. If this fraction is taken as 1/2, and the remaining 3.5 nanoamps is stopped on the slit near the chamber, the revised gamma ray background rate is 3300 counts/second. This rate is in better agreement with the observed value.

It is apparent from these calculations that the important sources of background radiation are the collimator slits near the chamber for gamma rays and the beam stopper for particles. The calculations also indicate the best way to reduce background is by reducing the detector solid angle by collimating well before the target chamber and stopping the beam well past the target chamber. Lead shielding is then useful for the necessary collimation near the chamber. In a well designed experiment, these methods can produce target to background counting rates in excess of 2/1 for gamma rays and 40/1 for particles.
B. Experimental Apparatus

The Rice University tandem accelerator supplied an energy-analyzed beam of protons for this experiment. The basic arrangement of experimental apparatus is shown in figure 1 (pg.14), including beam collimators, the target chamber, detector mounts, and the beam stopper. Particle-gamma ray angular correlations performed in the Litherland-Ferguson geometry \(1^{13}\) require a particle detector at 0° or 180° to the beam axis. In practice, an annular-ring solid state detector is placed at the entrance to the target chamber and a tightly collimated beam passed through a small aperture in its center.

The collimators used to provide this tight collimation near the particle detector must also produce a minimum of background gamma radiation. This is an especially demanding requirement with the large volume gamma ray detectors used in this experiment. The high proton energies required (10 MeV) complicate the problem, since above 5 MeV most materials are strong sources of background gamma radiation.

1. Target Chamber

The target chamber was a brass cylinder 10 cm. in diameter and 13.5 cm. deep. The walls of the chamber were
kept to a thickness of 0.24 cm. to minimize gamma ray absorption between the target and the gamma ray detectors. A brass target holder was supported in the center of the chamber by rods to the top and bottom of the chamber and could be rotated to any angular position to keep the target holder out of the path of detected gamma rays.

The beam emerged from the target chamber into a lead-lined flight tube and was stopped 2.6 meters away in a lead beam stopper. The purpose of the flight tube was to reduce the solid angles subtended by the particle and gamma ray detectors for backscattered protons and gamma radiation produced when the beam was stopped. The first 10 cm. of flight tube after the chamber was made as small as possible in diameter. This allowed gamma ray detectors to be rotated to small forward angles, as close as 25° to the beam axis, in order to cover as large an angular range as possible in the angular correlation measurements.

The target chamber was entirely supported by an iron pipe mounted to a floor plate. This plate had leveling screws to allow adjusting the pipe to a vertical position. The top of the pipe had an adjustable bracket that locked around the collimator tube just before it entered the chamber and allowed adjustments of the collimator tube-chamber
Figure 1

Target Chamber Assembly
assembly in both a horizontal and vertical direction. A similar assembly supported the other end of the input beam tube, 0.9 meters before the chamber. This collimator tube-chamber assembly was completely decoupled from the rest of the system by bellows in front of the collimator tube and behind the chamber. This arrangement allowed precise alignment of the chamber with the accelerator beam. Diffusion pumps were mounted before and after the chamber assembly to allow pumping on both sides of the beam collimator slits.

A large aluminum table, 1.4 by 1.2 meters, was constructed beneath the chamber assembly to allow mounting of gamma ray detectors. The chamber supporting pipe ran through a hole in the table top to keep the chamber and table mechanically decoupled and allow the table to be independently leveled. Two rotating arms, pivoted at the center of the table, allowed for mounting and rotating the gamma ray detectors in a horizontal plane on either side of the target chamber. The table was centered under the target chamber after the latter had been aligned. The angle of the gamma ray detectors was read from a scale attached to the table top which had been rotated so that 0° on the scale was lined up with the accelerator beam axis. This arrangement allowed placing up to several hundred pounds of lead shield-
ing on the table without disturbing the target chamber alignment.

2. Beam Collimation

The beam collimating system was composed of two separate systems illustrated in figure 1 (pg.14). The first system was a number of cylindrical lead plugs, 7.7 cm. long with a 1 cm. hole down the center, spaced along the collimator tube. Two of these plugs at the front and at the rear of the collimating tube also had provision for mounting collimating slits at their forward ends. This system is shown in detail in figure 2 (pg.18). These collimating slits were made of thin lead sheet, 0.1 cm. thick, with 5/64" holes drilled through their center. Lead was chosen to minimize gamma ray background produced by beam actually striking the slits. This system formed the actual beam collimating system for the experiment, with the lead plugs providing shielding for the gamma ray detectors from gamma radiation produced at the slits. The 0.7 meter separation between the front and back collimating slits allowed only a reasonably parallel beam to pass on to the target chamber.

The second collimating system was mounted in a brass support that also held the particle detector. The purpose
of this assembly was to prevent any direct beam from striking the back surface of the particle detector and is shown in detail in figure 2 (pg. 18). A short length of tantalum tube was fitted in this mount and projected through the hole in the annular detector to provide the actual shielding. Two scraping slits upstream, each with a 6/64" hole, prevented any beam halo from scraping the sides of the tantalum tube. Additional shielding is provided on the target side of the detector by a tantalum ring and collar. These shields improve particle resolution by preventing backscattered protons from striking the inner and outer edges of the detector. The average angle of the exposed detector surface with this arrangement was $174^\circ$ measured from the target.

Careful tuning of the accelerator allowed up to 95% beam transmission through the target chamber. All parts of the collimator assembly could be removed through the target chamber by the use of pull rods without disturbing chamber alignment. This was very useful in aligning the chamber with scintillating slits replacing the collimating slits. All beam tubes leading to the target chamber and the flight tube away were lead lined to reduce gamma ray background.
Figure 2
Beam Collimators and Particle Mount
C. Targets

Self-supporting targets of natural metallic calcium (97% Ca$^{40}$) were prepared by vacuum evaporation of natural calcium from a tantalum boat onto a clean aluminum sheet. Flexing this sheet then caused a self-supporting foil of calcium to break loose. Target thicknesses were on the order of 750 $\mu$gm/cm$^2$, corresponding to a 21 keV energy loss for 10 MeV protons. The target thickness was determined by measuring the elastic proton yield from calcium at an energy of 4.26 MeV, where the elastic cross-section is constant over a wide energy range and its value well known.

The calcium foil was mounted over a 0.75 cm. hole in a tantalum target blank which was then placed in the brass target holder. The entire target holder assembly was insulated and biased 300 volts to ground to reduce electron emission from the target caused by the beam passing through the target. The troublesome contaminants were O$^{16}$, from oxidation of the calcium during target preparation, transport to and subsequent use in the target chamber, and C$^{12}$ deposited on the target from the vacuum system.
D. Detectors

The particle detector was an ORTEC annular surface barrier detector with 100 mm$^2$ total active area. The resistivity was 21,000 ohm-cm. with a depletion depth of 1000 microns, capable of stopping 12 MeV protons. The quoted alpha resolution was 35 keV FWHM (6 MeV alphas) with a noise width of 27 keV FWHM. The best resolution obtained in this experiment, including the effects of the detector, target thickness, kinematic broadening due to the finite detector size (4 keV), beam spread and electronics was 32.5 keV FWHM for 5.7 MeV protons. A typical particle spectrum from a calcium target for a 10.81 MeV beam is shown in figure 3 (pg.22). The elastic peaks from calcium, oxygen and carbon are shown and the proton inelastic groups from levels in Ca$^{40}$ are labeled with a subscript in the order in which they appear.

The gamma ray detector was a 12.5 cm. by 12.5 cm. long NaI(Tl) crystal optically coupled to an 11 stage photomultiplier tube. The crystal was surrounded with a 0.3 cm. thick lead shield to reduce gamma ray background coming into the sides of the detector and mounted with the crystal face toward the target unshielded. The entire detector assembly
was mounted on a rotating arm previously described with the detector face from 25 to 40 cm. from the target. A typical gamma ray spectrum for 0.75 MeV and 3.737 MeV gamma rays for this detector is shown in figure 11a (pg.93). The total system resolution is 80 keV FWHM for the 0.75 MeV gamma ray full energy peak. On occasion, two of these detectors, similar in performance, were used in this experiment.

The Li-Ge detector was a 25 cm$^3$ cylindrical Li-Ge crystal with the 5 cm. long axis mounted lengthwise in the cold arm of a dewar and cooled with liquid nitrogen. The detector was placed 10 cm. from the target. The high resolution of this detector was utilized to make careful studies of the gamma ray decay modes of the Ca$^{40}$ excited states investigated and perform Doppler shift measurements on selected gamma rays. A typical gamma ray spectrum for 0.75 MeV and 3.737 MeV gamma rays for this detector is shown in figure 11b (pg.93), taken with a low noise FET preamplifier. The total system resolution for the 0.754 MeV gamma ray was 15 keV FWHM.

The solid angle subtended by the NaI(Tl) detector in this experiment was .072 steradians compared to .0825 steradians for the Li-Ge detector. After correcting the gamma ray yields in the spectra of figure 11 for the duration of
Figure 3

Particle spectrum from $^{40}$Ca$(pp')^{40}$Ca

$E_{\text{beam}} = 10.81$ MeV
Ca$^{40}$ (P, P') Ca$^{40}$

$E_{\text{BEAM}} = 10.813$ MeV
the experiment (5.3 hours for the NaI(Tl) and 24 hours for the Li-Ge) and different target thicknesses, the Li-Ge detector was found to have an efficiency of 14% that of the NaI(Tl) for 0.754 MeV gamma rays. For 3.737 MeV gamma rays and using only the FEP in the NaI(Tl) spectrum, the efficiency was 4.04% for the FEP in Li-Ge and 5.3% for the second escape peak. The Li-Ge detector had a relative efficiency ratio of 8/1 for the detection of 0.754 MeV gamma rays versus 3.737 MeV gamma rays, taking FEP yields only.

E. Electronics

The Rice University IBM 1800 on-line computer was used to store data during the experiment. This computer has four 1024 channel ADC's (analog-to-digital converters) and is capable of being operated in a multiparameter mode. This operation makes use of four gate circuits in the computer interfacing. Each gate can be associated with any combination of the ADC's. A positive gating signal allows the computer to analyze analog signals at the associated ADC's. The digital outputs, together with a gate identification tag, are then streamed sequentially onto magnetic tape for storage.
This versatility is used experimentally by gating the computer when a proton and gamma ray are detected in coincidence and recording the energies of both the proton and gamma ray. Up to four separate coincidence requirements may be made simultaneously, allowing one to record up to two true coincidence and two accidental coincidence requirements in the same experiment. Later analysis allows windows to be set around proton groups of interest and the associated gamma ray spectra investigated. This technique removes the difficulties of setting individual particle windows before the experiment and shortens the experimental time required by recording data on all inelastic protons from the excited states of Ca$^{40}$ in coincidence with subsequent gamma rays simultaneously.

Two major types of electronic circuits were used to provide suitable signals and gates for the computer. For all experiments involving double correlations, requiring a coincidence between a proton and a single gamma ray, a TAC (time-to-amplitude converter) coincidence circuit was used. For the triple correlation experiments, requiring a coincidence between a proton and two gamma rays, a conventional fast-slow coincidence circuit was used.
The block diagram of a TAC coincidence circuit is shown in figure 4 (pg.26). After suitable preamplification, the particle and gamma ray signals are taken to main linear amplifiers where double-delay line clipping provides a bipolar output. This output is suitably delayed and taken to a computer ADC. Another amplifier output is taken to a single channel analyzer where a base line and window set limits on the energies of the particle and gamma ray to be stored.

A third amplifier output is taken to an ORTEC timing single channel analyzer whose function is to produce a sharp timing pulse for the coincidence requirement. In a bipolar pulse produced by delay line clipping, the voltage passes through zero at the same time T after the beginning of the pulse, independent of the pulse amplitude. Use is made of this fact by setting a multivibrator in the timing unit which is then triggered at the zero-crossover point in voltage. The output is differentiated to give a short negative spike suitable for timing. A discriminator is included to avoid triggering the unit on low level noise. A separate timing analyzer is used for the particle and gamma ray sides, and delays in the units are adjusted so that gamma ray timing spike occurs about 200 nanoseconds after the proton timing spike for a true coincidence requirement.
Figure 4

TAC Electronics
The proton timing spike is used to start a TAC unit, and the gamma ray timing spike is used to stop the same unit. The TAC produces an output pulse whose voltage is proportional to the time separation between the start and stop signals. If no stop signal occurs within 300 nanoseconds after starting, the unit produces no output.

A typical TAC spectrum for a NaI(Tl) detector is shown in figure 5a (pg.27). The flat background from 0 to 300 nanoseconds is due to accidental coincidences where there is no time relation between particle and gamma ray signals. Superimposed on this flat background is a true coincidence peak, from 150 to 210 nanoseconds, composed of all proton and gamma ray coincidences from the same excited state in Ca$^{40}$ where the time relation between the signals is constant. The resolving time of the circuit, equivalently the width of the true peak, is 30 nanoseconds, FWHM. The asymmetric shape is due to time walk of the gamma ray timing signal over the 10/1 dynamic range of energies.

The TAC output is amplified in a main linear amplifier and outputs taken to a third ADC and a third single channel analyzer. The three single channel analyzers are placed in slow coincidence with one another in a Cosmic fast-slow coincidence circuit and the output used to gate the computer.
Figure 5

a. NaI(W1) TAC Time Spectrum
b. Li-Ge TAC Time Spectrum
The operation of the circuit requires detection of a proton and gamma ray within 300 nanoseconds of one another to produce a TAC output. If the particle and gamma ray signals are also within the appropriate energy range, the triple slow coincidence circuit will fire and gate open the computer. The proton, gamma ray and TAC signals will then be converted to a channel number between 0 and 1024 and stored sequentially on magnetic tape with a gate identification tag. Programs and methods of analysis of data in this format will be discussed in a later section.

The TAC type of circuit is not suitable for triple coincidence experiments where a proton signal must be placed in coincidence with two gamma ray signals. In these cases, the circuit was replaced by a conventional fast-slow coincidence circuit shown in figure 6 (pg. 30). Again the proton and gamma ray signals are amplified, double-delay line clipped, and outputs taken to three computer ADC's after suitable delay.

Outputs are also taken to three Cosmic zero-crossover units. These crossover units produce fast timing pulses in a manner similar to the timing single channel analyzers previously discussed. A discriminator is included to avoid producing timing pulses from low level noise. The fast timing
signal is a short negative pulse, adjustable in width from 0 to 90 nanoseconds and variable in delay with respect to the input pulse. A single channel analyzer within the same unit is used to set a base line and window around the energy range of interest.

Each zero crossover unit produces two output pulses, a fast timing pulse and a slow pulse-height signal. The fast timing signals from the three crossover units are patched into a conventional fast coincidence circuit and are placed in coincidence with one another by the variable delays in the individual units. The three slow pulse-height signals are patched into a slow coincidence unit, together with the fast coincidence output. When a triple fast coincidence is detected, and all three signals fall in an acceptable energy range, the combination fast-slow coincidence unit produces an output pulse that is used to gate the computer. The resolving time of the circuit is set by the variable width of the fast timing pulses and was adjusted to be 80 nanoseconds, FWHM. The shape of the resolving curve is shown in figure 7a (pg.32) for the two NaI(Tl) detectors in coincidence with one another and in figure 7b (pg.32) for the proton detector in coincidence with one of the NaI(Tl) detectors.
When the computer receives this gating pulse, it converts the proton and two gamma ray signals to digital channel numbers between 0 and 1024 and stores them sequentially on magnetic tape with a gate identification tag. Accidental correction is obtained by constructing an identical circuit with an additional 400 nanoseconds of delay added to the fast signals from the movable gamma ray detector. This second circuit passes only accidental coincidences to the computer, and data from the two circuits are kept separate by using another gate input to the computer. Since data are separated on tape by the different gate tags written with each event, this allows the same ADC's to be used for the proton and gamma ray signals. Analysis programs for data in this format will be discussed in a later section.

Since some type of normalization is required in an angular correlation experiment, a free particle spectrum was recorded in a Nuclear Data multichannel analyzer throughout the experiment. This technique provided a separate normalization for each Ca\(^{40}\) excited state investigated, independent of target thickness variations, beam fluctuations, and variations in yield due to accelerator energy fluctuations. The Nuclear Data analyzer averaged approximately 20% dead time with 5% variations due to beam current fluctuations. Since
no dead time correction was available for the Nuclear Data, charge integration off the insulated beam stopper was used as a check on the multichannel analyzer normalization. The ratio of Nuclear Data yield for the elastic group of interest to charge was calculated for each data point measured. The rms. error of this series of numbers was calculated and was usually about 5%, reflecting the dead time corrections. This error was included in the final data analysis in lieu of an actual dead time correction.

In the Doppler shift measurements, the gamma ray range of interest included gamma rays up to 4 MeV in energy. To make use of the high resolution inherent in Li-Ge detectors, the TAC type of circuit was used with the gamma ray ADC converting into 2048 channels. This arrangement gave a final energy calibration of 2.1 keV/channel.

The same zero crossover timing previously discussed was used for the Li-Ge detector. The resulting TAC time spectrum is shown in figure 5b (pg.27). The resolving time is 65 nanoseconds at FWHM and 120 nanoseconds to extinction. This resolving time is noticeably poorer than the 30 nanoseconds FWHM obtained for the NaI(Tl) detectors and, together with the very asymmetric shape, is characteristic of Li-Ge detectors with zero-crossover timing. The large charge
collection time in a Li-Ge crystal (typically 50 µseconds to half-maximum charge) and corresponding statistical fluctuations in the crossover point are chiefly responsible for this effect.

Special timing units for Li-Ge detectors are now available that perform suitable shaping on the pulse to provide a zero crossover signal taken from a constant fraction of the pulse height, resulting in a timing signal relatively free of this time walk. Typical time resolutions obtainable with these constant fraction devices are on the order of a few tens of nanoseconds at FWHM. This reduction in time resolution allows significantly higher count rates in the detector while maintaining a good true-to-accidental coincidence ratio. Unfortunately such equipment was not available at the time of this experiment.

F. Experimental Conditions

With careful tuning of the tandem accelerator, it was possible to pass up to 95% of the proton beam through the collimator assembly and into the target chamber. The stripping gas in the accelerator terminal was then reduced to lower the beam current to a value suitable for the experiment without disturbing the beam focus.
A beam current through the chamber of 15-25 nanoamps was found to be optimum for this experiment. This current was sufficiently low to prevent pileup in the particle and gamma ray detectors and yet provided a fairly high coincidence counting rate. Typical singles counting rates (no coincidence requirements) were 4000 counts/second for the particles, 5000 counts/second for the Li-Ge detector, and 12,000 counts/second for the NaI(Tl) detector. The gamma ray counting rates for background radiation from the collimator assembly and beam stopper, with no target in place, were on the order of 20 to 30% of the singles rates above.

The particle-NaI(Tl) double correlation experiments performed with the TAC circuit had a total coincidence counting rate over the full 300 nanosecond range of 28 counts/second; half of these were trues. The true to accidental rate in the time peak alone with this circuit was 5/1. The correlation was performed at angles of 30°, 40°, 50°, 60°, and 90°. Each data point at an angle lasted approximately 1-1/2 hours, and the entire series of five angles was measured three times in a random order. The three data points at each angle were then summed in forming the final angular correlation function. This procedure was used to offset any long range drifts in the electronics.
The triple correlation experiments performed with the fast-slow circuit had a coincidence counting rate of 0.22 counts/second with a true to accidental ratio of 7.1/1. The correlation was performed at angles of 30°, 60° and 90° with the second NaI(Tl) detector fixed at 90°. Each data point at an angle lasted approximately 8 hours, and the series of three angles was measured three times in a random order. The data from all measurements at each angle were again summed to form the final correlation function.

The Doppler shift measurements with the Li-Ge detector and the TAC circuit had a coincidence counting rate over the full 400 nanosecond range of 11 counts/second. Again half of these were trues with a true to accidental ratio of 6/1 in the time peak alone. Data were taken at angles of 30° and 120° to maximize the Doppler shift and each data point lasted approximately three hours. The two angles were measured alternately eight times and the data summed at each angle to produce the final peak shapes. The centers of the gamma ray peaks of interest were determined by a Gaussian fitting program to be described in a later section.

Prior to the coincidence experiments, an excitation function was measured for the excited states in Ca^{40}. The range of energies covered was 9.0 to 11.75 MeV in 8 keV
steps. A self-supporting target of natural calcium with a thickness of 1.44 mg/cm$^2$, corresponding to a 42 keV energy loss for 9 to 12 MeV protons, was used. The results of this excitation function for the excited state in Ca$^{40}$ at 6.75 MeV is shown in figure 8 (pg.39). From this curve, an energy of 10.81 MeV was chosen to maximize the yield of this excited state.

G. Data Reduction

During the experiment, each coincidence event was written as three words on magnetic tape. Each word corresponded to a channel number between 0 and 1024 produced by the associated ADC. In addition, each event had a gate identification tag written on tape before the three ADC words. By convention, the first and second ADC's represented the particle and movable gamma ray detectors respectively. The third ADC represented the TAC spectra for double correlation experiments or the fixed gamma ray detector for triple correlation experiments. The same "zoom lens" program, designated Trip, was used to analyze data in either tape format.

Since this experiment is a two-body reaction, all inelastic protons from a particular excited state in Ca$^{40}$ will have the same energy and fall in the same few channels in
the particle ADC. On the other hand, the storage of the associated gamma rays by the gamma ray ADC will resemble a typical NaI(Tl) (or Li-Ge) spectrum, with a full energy peak, Compton edge, and escape peaks for gamma rays of sufficiently high energies. If the excited state decays by branching or cascades, several different gamma ray spectra will be superimposed on one another. The output of the TAC ADC will have a distribution as the one shown in figure 5 (pg. 27).

The basic operation of Trip is to search through each coincidence event stored on tape through the length of the run. If the gate tag associated with an event matches a tag read in on a control card, that event is further analyzed. If not, Trip continues on to the next event without analysis. In the event of a match, the set of three ADC channel numbers is read from the tape. Two of these words are then compared to channel limits read into the program on control cards. If the two words so inspected fall within the desired channel limits, the third word is stored in a 0 to 1024 channel area depending on its channel number. At the end of the tape, the program prints out this 1024 channel area to give the spectrum stored in the third ADC. If desired, this 1024 channel spectrum may be compressed in size by lumping
Figure 8

Excitation Function

6.75 MeV Level in Ca-40
together every two, three, four, etc. channels at the time of analysis.

The first step in analysis is to construct the particle and TAC spectra for all coincidence events recorded. Typical TAC spectra from this operation are shown in figure 5 (pg. 27). Then particle channel limits are set around a particle group of interest, TAC channel limits are set around the true time peak (figure 5), and the gamma ray spectrum associated with these conditions produced by rezooming the tape. A typical gamma ray spectrum produced by this procedure is shown in figure 11 (pg. 93). This procedure gives all gamma rays detected in true coincidence with all particles from a particular excited state in Ca$^{40}$. The number of gamma ray counts in this spectrum, usually taken to include the full energy peak and both escape peaks, gives the gamma ray yield at that particular angle.

Accidental correction is handled by rezooming the data with TAC channel limits set for a range equal to that used for the true time peak, but in the flat accidental region of the spectrum. Since the time widths of these two regions are identical, the number of accidentals in the two regions is also identical to within statistics. A subtraction of the two gamma ray spectra channel by channel leaves a spec-
trum with only true gamma ray coincidences. This subtraction is handled within the program by searching over the two regions of the TAC spectrum simultaneously, and only the true gamma ray spectrum is printed out. Up to five particle groups may be analyzed simultaneously by Trip in this manner.

A slightly different mode of operation is used in the case of triple correlations. Here the third ADC contains a gamma ray spectrum from the fixed gamma ray detector. For the case of a double gamma ray cascade, a window is placed around the full energy peak of the higher energy gamma ray in the cascade and the spectrum of the moving gamma ray detector in coincidence is printed out. Accidental correction is handled by zooming the data twice over the two gate identification tags previously discussed. A second triple correlation geometry may also be investigated by placing this gamma ray window on the moving detector and looking at the lower energy gamma ray in coincidence in the fixed gamma ray detector.

Repetition of this procedure for each angle measured provides the angular correlation function for gamma rays from a particular excited state in Ca^{40}. The spectra at each angle were summed by hand to produce the final correlation function.
Since the data should follow Poisson statistics, the errors in the data were taken to be \( \sqrt{N} \), where \( N \) is the number of events in the summation. The data at each angle were normalized to the particle singles spectrum from the Nuclear Data analyzer. The accuracy of this normalization was calculated from the ratio of particle singles yield for the group of interest to the charge collected for each data point, and the rms. error of this series of numbers computed. This error was taken as the total error of the monitor and usually was on the order of several percent. This error was combined with the statistical error of the data, on the order of 2 to 7\%, to give the total error in the final correlation function.

In a number of cases studied, a gamma ray of interest was superimposed on the spectrum of a higher energy gamma ray from the same cascade. For these cases, the higher energy gamma ray was stripped out of the spectrum. This process was carried out by placing windows around the full energy peak of the higher energy gamma ray and around the gamma ray region of interest. A stripping factor \( F \), representing the ratio of counts in the region of interest to the counts in the full energy peak, was obtained from analysis of lower excited states in \( \text{Ca}^{40} \) where only the interfering
gamma ray was present. The stripping factor applied to the full energy peak for the excited state of interest then gave the interfering yield in the desired region. The difference between this calculated yield and the observed yield was then taken as the yield of the lower energy gamma ray for the correlation function. The errors in the stripping factors were on the order of 2% and were included in the final error analysis. This procedure will be discussed in more detail in later sections for specific excited states where it was used.
III. THEORY AND PROGRAMS

A. Development of Correlation Theory

The basis of all correlation theory is that the emission probability for some type of radiation from a decaying nucleus is generally dependent on the angle between the direction of emission and the nuclear spin axis. For the special case of a radioactive sample with all the nuclei oriented randomly in space, this angular dependence averages out to isotropy. Nuclear reactions often produce an ensemble of non-randomly oriented nuclei and in these cases an anisotropic distribution of subsequent particle or gamma ray emission may be observed. The shape of this anisotropy can then be analyzed to yield information about the decay processes involved and some parameters such as spins and parities of excited states in the decaying nucleus. This type of information can often be related back to the nuclear reaction processes occurring in the framework of a particular reaction model.

As a general rule, a nuclear reaction can be considered to produce an excited nucleus that decays by the rapid emission of two or more successive radiations $R_1, R_2, \ldots, R_1$. This viewpoint includes the final recoiling nucleus in the ground
state as one of the radiations. By convention, an angular distribution $W(\theta)$ refers to measuring the relative probability $W(\theta)d\Omega$ of detecting one of these radiations emitted into a solid angle $d\Omega$ with respect to an axis in space, generally taken to be the beam axis, leaving all other radiations undetected.

Angular correlations are restricted to measurements on nuclei decaying by emission of at least two radiations in addition to the recoiling nucleus. Here the relative probability $W(\theta)d\Omega$ of detecting $R_2$ emitted into a solid angle $d\Omega$ with respect to the direction of emission of $R_1$ is measured, with $R_1$ and $R_2$ being detected in a coincidence experiment.

In this present work, a "double correlation" refers to a single coincidence requirement between two radiations while a "triple correlation" refers to a double coincidence requirement between three radiations. Experimentally the use of detectors of finite size restricts the measurement to determining some value $W(\theta)$ averaged over the angular range subtended by the detector. This averaging effect must be considered in the data analysis.

Historically the discovery of nuclear physics was made through the study of angular distributions. Rutherford's
work in 1911 on the scattering of alpha particles by gold nuclei is the simplest example of an angular distribution where no quantum mechanical treatment is necessary and the forces are well known \(^{14}\).

Correlation work at the present time is usually restricted to production of a nucleus in an excited state via a nuclear reaction and investigation of the angular distributions or correlations of radiations emitted as the nucleus decays back to the ground state. One of the pioneering theoretical efforts on this type of problem was carried out by Hamilton in 1940 \(^{15}\). Hamilton used perturbation theory to study double correlations between gamma rays for a limited range of gamma ray multipolarity values. He obtained a \(W(\theta)\) expression for this \(\gamma-\gamma\) process in terms of a summation of matrix elements formed by using the interaction Hamiltonian operator for gamma ray emission. This summation was over all magnetic substates at every stage of decay. This original formulation was quite cumbersome to evaluate and of limited usefulness.

The first major improvement in correlation theory was made by Gardner \(^{16}\) in 1942 and extended by Racah \(^{17}\) soon afterwards. By introducing Racah algebra to the theory, the summation over magnetic substates could be expressed in a
closed form and effectively eliminated by the use of Racah coefficients. The calculation of theoretical distributions was greatly speeded up by this simplification. This improvement has also been extended to the case of triple correlations by the use of the Wigner 9-j symbols.

In the quantum mechanical treatment of correlations, considerable simplification results from taking the arbitrary quantization axis along the direction of emission of one of the radiations, say $R_1$. The eigenfunctions of a plane wave can then be used to describe that radiation in a fairly straightforward procedure. The subsequent treatment of the other radiation $R_2$ with respect to this special axis is still quite complicated and cannot be considered separately from the first radiation. Racah extended the theory in 1951 to take advantage of the symmetries inherent in quantizing along the direction of emission for each radiation. Racah introduced a separate coordinate system for each radiation in which it could be described by plane wave eigenfunctions. By the use of the transformation properties of these functions, the final wave function for each radiation $R_1$ and $R_2$ along some arbitrary $z$-axis (say the beam axis) could be found as a summation over the easily calculated plane wave eigenfunctions.
This separate quantization of radiations had far reaching consequences for correlation theory. All radiations could now be treated independently and on an equal footing with any other radiation. The introduction of radiation parameters was another consequence. These parameters describe the behavior of a detector to a particular type of radiation with no reference to an experimental coordinate system. This dependence is contained elsewhere in the formalism as a rotation matrix. Each transition of a decay cascade could then be factored into a closed form, and the whole correlation function \( W(\theta) \) expressed as a product of these independent factors summed over suitable variables.

This factorization property allows the correlation function for a decay of any complexity to be easily calculated by simply multiplying in new factors for each additional branch of the cascade desired. Whether the new branch is a gamma ray or a particle decay determines only the type of factor used, not the basic form of the correlation. Wenesar and Hamilton\(^{19}\) and Raboy and Krohn\(^{20}\) extended this work soon after to include factors for intermediate unobserved radiations in a cascade decay.

The final major contribution to the general theory came with the introduction of statistical tensors by Fano\(^{21}\).\)
and Coester and Jauch\textsuperscript{22}) in 1951 and 1953. In measuring angular correlations, one is dealing with an ensemble of nuclei about which very limited experimental information is available. While there is not sufficient information to treat individual nuclei with a complete quantum mechanical description, the entire system can still be well described statistically.

This type of treatment is best handled in terms of density matrices $<a_\alpha|\rho|a_\alpha>$ which describe the probability of observing a decaying nucleus in a certain quantum mechanical state with spin $\bar{a}$ and magnetic substate $\alpha$. A simple linear transformation of these density matrices (see equation 4) yields the statistical tensors $\rho_k(a_\alpha)$. These tensors completely specify statistically the spin states of the nuclei observed experimentally and have simple rotational transformation properties.

The introduction of statistical tensors greatly simplified correlation theory by allowing the product nucleus from a nuclear reaction, before any decay radiations are emitted, to be described as a linear sum of statistical tensors. All details of the specific reaction mechanism, usually involving a large number of unknowns entering as nonlinear terms, can be compressed into a few linear unknowns that can be deter-
mined as part of the data analysis. This expression in linear unknowns simplifies data analysis since the theory of least squares fitting to a linear system of equations is well developed and can easily be adapted to modern high speed computers.

Subsequent work by Biedenharn and Rose\textsuperscript{23}) in 1953 and Devons and Goldfarb\textsuperscript{24}) in 1957 did much to unify the theory in its present form. The general correlation functions for a number of specific cases were presented and standard conventions proposed for such quantities as gamma ray phase factors. Biedenharn and Rose also included specific factors for those cases where one of the radiations emitted in a decay was a particle.

With the basic theory standardized, a number of workers tabulated extensive tables of angular correlation coefficients. Some of the most notable compilations were produced by Ferentz and Rosenzweig\textsuperscript{25}) in 1953, Ferguson and Rutledge\textsuperscript{26}) in 1957, and Sharp \textit{et al.}\textsuperscript{27}) in 1953. The first two tabulations were for $\gamma-\gamma$ coincidence work, while the tabulation of Sharp \textit{et al.}\textsuperscript{27}) was for the analysis of particle angular distributions.

Specific problems such as polarization effects in a correlation, either as gamma ray distributions from an
ensemble of polarized nuclei or polarization of gamma rays emitted in a decay, were included in the theory in 1953 by Simon and Welton. The effects on angular correlations by perturbations of the decaying nuclei in atomic fields was investigated by Abragam and Pound in 1953. Their results apply to long-lived nuclear states (lifetimes greater than $10^{-10}$ seconds) and are of limited usefulness in the study of gamma ray decays from short-lived excited states.

In 1961, Litherland and Ferguson realized that considerable simplification in correlation expressions resulted by making the experiment symmetric about the beam axis. This technique applies to a very large class of nuclear reactions of the type $X(a,b)Y^*$ where the light particle $b$ is either undetected or detected in an annular detector located at $0^\circ$ or $180^\circ$ along the beam axis.

In either case, the recoiling excited nucleus $Y^*$ with spin $\tilde{a}$ can be described statistically by the probability of observing a certain projection $\alpha$ of its spin along the $\hat{z}$-axis, taken to be the beam axis. From basic quantum mechanics, a system of angular momentum $\tilde{a}$ has $(2a+1)$ possible $z$-projections called $\alpha$. This observed projection defines the magnetic substate of the system and has units $\hbar$, where $\alpha$ has the range $a$, $(a-1)$, $\ldots$, $(a-1)$, $-a$. The probability
of observing a nucleus with a particular projection $\alpha$ is just the density matrix $\langle a\alpha | \rho | a\alpha \rangle$, commonly called the population parameter $P(\alpha)$. An aligned system is one in which equal numbers of nuclei point in opposite directions along the $z$-axis, i.e. $P(\alpha) = P(-\alpha)$.

For such an axially symmetric system, the recoiling nuclei $Y^*$ are aligned and can be described in terms of $(a+1)$ unknown population parameters $P(\alpha)$. A further restriction results because these unknowns must be positive; this requirement is usually imposed during the data analysis. Litherland and Ferguson also noted that detection of the outgoing light particle $B$ along the beam axis produced the condition that $P(\alpha) = 0$ for all but the lowest values of $\alpha$; equivalently, only nuclei with the lowest $z$-projections can emit particles along the beam axis. This effect is due to conservation of angular momentum and is discussed in greater detail in part B of this section. Litherland and Ferguson finally defined a set of seven geometries for $\gamma-\gamma$ correlation work that would yield sufficient independent information to allow all of the possible unknowns to be determined unambiguously.

Tables of correlation coefficients applicable to aligned systems described in terms of population parameters have been tabulated by Litherland and Ferguson$^{13}$, Smith$^{30}$, and
Warburton and Poletti\textsuperscript{31}). These tables have the limitation that they apply only to cascades of no more than two gamma rays for $\gamma-\gamma$ correlations and to gamma rays with multipolarities no higher than quadrupole. These coefficients moreover do not completely factor each gamma ray decay into a separate link.

This deficiency has recently been eliminated by Watson and Harris\textsuperscript{32}). They extended the theoretical formalism to complete separation of each step in a gamma ray cascade. This extension allows any number of intermediate unobserved cascades in a $\gamma-\gamma$ correlation to be handled easily. Watson and Harris tabulate an extensive collection of coefficients suitable to their factored notation up to octupole multipolarity for gamma rays. They calculated these coefficients for either the population parameter or statistical tensor formalism.

This present paper includes the necessary formulas for calculating $\gamma-\gamma$ correlations in the cases where the first gamma ray in a cascade is unobserved. Since correlation expressions are best described in terms of population parameters of the level emitting the first observed gamma ray, it is necessary to relate these population parameters back to those of the initial state where only the few lowest
parameters are nonzero.

Correlation theory at this stage of development is a strong spectroscopic tool for determining the spins and decay parameters of nuclear levels. Complete tables of coefficients exist so one may easily evaluate theoretical correlations in terms of a few unknowns for comparison to experimental results and perform this comparison rapidly on modern computers. In the event a measurement does not yield a unique spin assignment, the parameters determined by the first experiment can be used to predict the correlation function in another geometry. This process can easily be carried out for a number of geometries in order to find one where the ambiguities of the first experiment are no longer present. A second experiment in this new geometry coupled with the results of the first experiment would then allow a unique spin assignment. The ability to carry out calculations of this form with a computer in a short amount of time makes correlation measurements a particularly effective tool.

B. Theory of Angular Correlations

The analysis of gamma ray angular correlations from the nuclear reaction $X + a \rightarrow Y^* + b$, $Y^* \rightarrow Y + \gamma$ is considerably simplified when the reaction product $b$ is detected at
0° or 180° along the beam axis. This simplification results from the nuclei Y* being aligned with respect to the beam axis as mentioned in section A and has been extensively discussed by Litherland and Ferguson. This present section will develop the theory and formulas for particle-gamma double correlations and particle-gamma-gamma triple correlations measured in the Litherland and Ferguson geometry II, where the light product b is detected. The formalism will follow that used by Watson and Harris.

The value of the Litherland and Ferguson geometry II lies in the fact that the observed recoiling nuclei Y* are aligned with only the lowest magnetic substates α populated. This is most easily seen by considering a particle of spin \( \vec{J} \) moving along the beam axis, defined as the z-axis. This particle must have the z-projection of its spin on this axis as one of the \((2J + 1)\) magnetic substates \( m_J \). Since orbital angular momentum \( \vec{l} \) is defined classically as \((\vec{r} \times \vec{p})\), the z-projection of the orbital angular momentum \( l_z \) of the incoming particle a, or \( l_2 \) of the outgoing particle b, must be zero. The z-components of angular momentum add algebraically, so the maximum magnetic substate of Y* that can be observed is given by \( m_{Y^*} = \max. (|m_a| + |m_b| + |m_X|) \). Thus for inelastic proton scattering by Ca\(^{40}\), only the sub-
states 0, ±1 occurring in the excited Ca\textsuperscript{40} nucleus can be seen by detecting the scattered proton along the beam axis.

An annular particle detector is used at 180° with respect to the beam axis in this experiment, so the apparatus is axially symmetric about this z-axis. There is also invariance of the experiment under reflection in an x-y plane through the target. These two symmetries limit the \( Y^* \) nuclei to be aligned nuclei. Equivalently, if \( P(\alpha) \) is the fraction of nuclei \( Y^* \) populating a magnetic substate \( \alpha \) in \( Y^* \), then \( P(\alpha) = P(-\alpha) \). A third symmetry comes from the use of a cylindrical gamma ray detector. This leaves the experiment insensitive to detecting the polarization of gamma rays, and specifically there is no observable difference between electric and magnetic multipole gamma radiation.

An angular correlation \( W(\theta) \) is defined as the number of proton-gamma ray coincidences/nuclear reaction with the gamma ray detector at an angle \( \theta \) to the beam axis (figure 1). These gamma rays result from the decay of a level in \( Y^* \) of spin \( \bar{a} \) to a level of spin \( \bar{b} \) in \( Y \), shown in figure 9a (pg. 58). Level \( \bar{a} \) is populated directly by inelastic protons, while level \( \bar{b} \) can be either the ground state or a lower excited state in \( Y \). The gamma radiation has a multipole value \( \bar{L} \) within the range \( |\bar{a}-\bar{b}| \leq \bar{L} \leq |\bar{a}+\bar{b}| \) and an electric (\( E \)) or mag-
netic (M) character depending on the parities of states \( \tilde{a} \) and \( \tilde{b} \). The type of radiation allowed (EL or ML) is fixed by conservation of parity and the fact that the parity of an E gamma ray goes as \((-)^L\) and the parity of an M gamma ray goes as \((-)^{L+1}\). There can also be mixed radiative decay of multipoles \( L \) (E or M) and \( L' \) (M or E) = \( L+1 \) in the \( \tilde{a} \rightarrow \tilde{b} \) transition. Usually only the two lowest multipoles \( L = |\tilde{a}-\tilde{b}| \) and \( L' = \tilde{L}+1 \) are of importance.

A more general type of gamma ray decay is shown in figure 9b (pg. 58). Here a gamma ray cascade leads from state \( \tilde{a} \) to a lower excited state \( \tilde{b} \) which in turn decays to a third state \( \tilde{c} \). The gamma ray in the \( \tilde{b} \rightarrow \tilde{c} \) transition has its own set of multipoles \( L_2, L'_2 \). For this case, the correlation may be performed over either the first gamma ray or over the second gamma ray with the first gamma ray unobserved. Watson and Harris extend the theory to the case where both gamma rays in this cascade are detected in coincidence. Their theory also extends to the case shown in figure 9c (pg. 58), where there are one or more unobserved gamma rays \( L_1, L'_1 \) between the two of interest.

A greater degree of complexity results in the case shown in figure 9d (pg. 58), where a double correlation is performed on gamma rays \( L_1, L'_1 \) and \( L_2, L'_2 \), possibly with
Figure 9

Types of Gamma Ray Decays

a. Simple decay

b. Simple cascade

c. Cascade with unobserved intermediate gamma rays

d. Cascade with unobserved intermediate and initial gamma rays
unobserved intermediate gamma rays $L_i$, $L'_i$, and the first
gamma ray transition $L_j$, $L'_j$ is unobserved. The extension of
the theory to this case is discussed later in this section.

The formalism of the general theory is best illus-
trated by a gamma ray cascade of the type shown in figure
9b (pg.58). The angular momenta are related in the follow-
ing way\textsuperscript{33}:
\begin{align*}
\vec{\alpha} & \rightarrow \vec{\epsilon} \ast \vec{\ell}_1 \\
\vec{\epsilon} & \rightarrow \vec{\epsilon} \ast \vec{\ell}_2
\end{align*}
(1)

Every emission in the cascade, the two gamma rays and the re-
coil nucleus of spin $\vec{c}$, can be described by a statistical
tensor $p_{kX}(LL')$ that gives the probability of occurrence of
such an emission. Multiplying this tensor by an efficiency
tensor $e^*_{kX}(LL')$, related to the probability of detecting the
particle or gamma ray, gives the probability of seeing that
branch of the cascade. The product of such terms for each
branch of the cascade, summed over all possible variables,
gives the total probability of seeing a coincidence event,
which is just the correlation function $W(\theta)$:

\begin{equation}
W(\theta) = \sum p_{kX z}(cc') \rho_{kX n}(L_1 L_1') \rho_{kX z}(L_2 L_2') e^*_{kX n}(cc') e^*_{kX z}(L_1 L_1') e^*_{kX z}(L_2 L_2')
\end{equation}
(2)
Finally the equation for $W$ is transformed back to an expression in terms of only efficiency tensors for all emission processes and the statistical tensors for the first state $\bar{a}$ by repeated application of the relation:

$$\rho_{k_3k_4}(\omega) \rho_{k_5k_6}(\ell_{\Sigma \ell_{\Sigma}}) = \sum \rho_{k_3k_4}(\omega) \rho_{k_5k_6}(\ell_{\Sigma \ell_{\Sigma}}) \mathcal{E} \hat{k}_e \hat{k}_z \cdot$$

$$\left[ {c_{\Sigma \ell_{\Sigma}} b \choose c'_{\Sigma \ell_{\Sigma}} b'} \right] <c_{\Sigma \ell_{\Sigma}} b \bar{b}> <c'_{\Sigma \ell_{\Sigma}} b \bar{b}>^*$$

(3)

where $\hat{J} = (2J+1)^{1/2}$.

Expressions for the efficiency tensors can be obtained from consideration of the actual detectors used in the experiment and contain all of the angular dependence of the correlation. The statistical tensor for the initial state $\bar{a}$ is related to the population parameters by:

$$\rho_{k_0}(\bar{a}) = \sum_{\alpha > 0} (-)^{\alpha - \alpha} (a_\alpha a_{-\alpha} | k_0) P(\alpha)$$

(4)

These population parameters contain all of the information on the reaction mechanism forming state $\bar{a}$ and the usual practice is to leave them as unknowns to be determined from the experiment.

This technique can be used for gamma ray cascades of any complexity, but usually count rate factors limit experi-
ments to at most two gamma rays in coincidence with a particle.

1. Particle-Gamma Ray Double Correlations

The theoretical correlation function for a single gamma ray in coincidence with a particle for a decay of the type shown in figure 9a (pg. 58) is:

\[
W(\theta) = \sum_{K}^{\text{MAX}} A_K Q_K P_K(\theta) = \sum_{K}^{\text{EVEN}} A_K P_K(\theta) \tag{5}
\]

where

\[
A_K = \hat{K} \frac{E_{K0}(aL_1L_2b) + E_{K0}(aL_1L_2b')} S_1 + E_{K0}(aL_1L_2b) \cdot S_1^2}{(L_1 \cdot S_1^2)} \tag{6}
\]

The \(E_{K0}^{0}\) are coefficients tabulated by Harris and Watson and are functions of Clebsch-Gordan and Wigner 9-j coefficients. \(\delta_1\) is the mixing ratio, or the ratio of reduced gamma ray matrix elements \(<b|L_1|a>\) for the gamma ray transition:

\[
\delta_1 = \frac{<b|L_1''a>}{<b|L_1''a>} \tag{7}
\]

For a decay to a spin 0 state, \(\delta_1\) is identically zero.

The \(P_K(\theta)\) in equation 5 are Legendre polynomials describing the position of the gamma ray detector. The \(Q_K\) terms
are attenuation coefficients, corresponding to the fact that a finite size detector is used. Since no available tables list the $Q_k$ coefficients for $k > 4$ and the present work required terms to $Q_6$, an approximation formula derived by Rose $^{34}$ was used. Consider a gamma ray detector at an angle $\theta$ to the beam whose face subtends an effective angle $2\xi$. Assuming any interaction is counted and the detection efficiency $\varepsilon(\varnothing)$ is constant for all $\varnothing$ between 0 and $\xi$ and zero elsewhere, Rose calculates for cylindrical detectors:

$$Q_k = \left[ P_{k-1} (\cos \xi) - \cos (\xi) P_k (\cos \xi) \right] \cdot \left[ (k+1)(1-\cos(\xi)) \right]^{k-1}$$

$$\cot \xi = \frac{1}{r} (h + \frac{0.35}{\varepsilon})$$

(8)

Here $h$ is the target-to-detector face, $r$ is the detector radius, and $\tau$ is the total absorption coefficient for gamma rays in NaI(Tl). This approximation is valid to within 1% for $h > 2r$. For consistency, this equation was used to calculate all $Q_k$ terms used in the analysis.

From triangle inequalities in the Wigner 9-j coefficients in the $E_{0}^{0}$ terms, the summation over $k$ in equation 5 is limited by:

$$k_{\text{max.}} = \min (2a, 2L_{\text{max.}})$$

(9)
The observation of aligned nuclei and Clebsch-Gordan symmetries also limit the \( k \) summation to even values of \( k \).

For the cascade decay of figure 9b (pg. 58) where the first gamma ray \( L_1 \), \( L_1' \) is unobserved and the correlation performed on the second gamma ray of the cascade, a slightly more involved formula is used. Here the mixing ratio \( \delta_1 \) of the unobserved gamma ray enters the equations as well as \( \delta_2 \), the mixing ratio of the second observed gamma ray. The resulting distribution is given by:

\[
W(\theta, \delta; e) = \sum_{\kappa} P(\kappa) \sum_{K \text{ even}}^{K_{\text{max}}} A_{\kappa K} \left\{ \frac{E_{a K}(a L_1 L_1') + \delta_1^2 E_{b K}(a L_1 L_1')}{1 + \delta_1^2} \right\} P_K(\theta)
\]

(10)

\[
A_{\kappa K} = Q_{K} \{ \frac{h_K(b L_2 L_2) + \delta_2^2 h_K(b L_2 L_2') + \delta_2^4 (b L_2 L_2')}{1 + \delta_2^2} \}
\]

The \( h_K \) coefficients are also tabulated by Watson and Harris and, together with the \( Q_K \) coefficients, apply to the second gamma ray. The summation over \( k \) in equation 10 is limited by equation 9 applied separately to each branch of the cascade. The \( P(\kappa) \) are the same for either gamma ray observed, so the two correlations in equations 5 and 10 offer two independent measurements for the spin \( \bar{a} \). Usually previous measurements fix the spins of states \( \bar{b} \) and \( \bar{c} \) and the mixing ratio \( \delta_2 \).
2. Particle-Gamma-Gamma Triple Correlations

A triple coincidence experiment detects the initial gamma ray of a cascade of the type shown in figure 9b (pg. 58) in a detector at an angle $\theta_1$ and the cascade gamma ray in another detector at an angle $\theta_2$, $\phi$. Here $\phi$ is the necessary rotation about the z-axis to bring the two detectors together with $\theta_1 = \theta_2$. The correlation distribution for this experiment is given by:

$$W(\theta_1, \theta_2, \phi) = \sum_{n=-N}^{N} G_{\text{M}}(a_{l1}b_{l2}c_{l3}) H_{\text{M}}(b_{l2}c_{l3})$$

$$(11)$$

where

$$H_{\text{M}}(b_{l2}c_{l3}) = \frac{h_{\text{M}}(b_{l2}c_{l3}) + b_2 h_{\text{M}}(b_{l2}l_{2}c_{l3}) + b_2^2 h_{\text{M}}(b_{l2}l_{1}c_{l3})}{(1 + b_2^2)}$$

$$(12)$$

and

$$G_{\text{M}}(a_{l1}b_{l2}c_{l3}) = \frac{E_{\text{M}}(a_{l1}b_{l2}c_{l3}) + b_1 E_{\text{M}}(a_{l1}l_{1}b_{l2}c_{l3}) + b_1^2 E_{\text{M}}(a_{l1}l_{1}l_{1}c_{l3})}{(1 + b_1^2)}$$

$$X_{\text{KM}}(\theta_1, \theta_2, \phi)$$

$$(13)$$

The $X_{\text{KM}}^N$ are angle functions involving the associated Legendre polynomials and are given by:

$$X_{\text{KM}}^N(\theta_1, \theta_2, \phi) = \left[\frac{(2M+1)(2K+1)}{(K+N)!(M+N)!}\right]^{1/2} P_{K}^N(\theta_1) P_{M}^N(\theta_2) \cos N \phi$$

$$(14)$$
The summation over \( K \) refers to the first gamma ray and the summation over \( M \) refers to the second gamma ray. Each is limited by equation 9 and restricted to even values for physical reasons previously discussed. Any intermediate unobserved gamma ray, such as the case shown in figure 9c (pg. 58), is included by multiplying each \( H_M \) in equation 11 by a \( U_M(\text{bl}ic) \) given by:

\[
U_M(\text{bl}ic) = \frac{U_M(\text{bl}i\text{li}c) + s_i^2 U_M(\text{bl}i\text{li}c)}{1 + s_i^2}
\]  

(15)

Again there is no change in the \( P(\alpha) \) factors; they refer to the initial state of spin \( \bar{a} \).

Normally one detector is held fixed while the other is varied during the experiment. In the multiparameter system, described in part E of section II, all gamma ray energies in both detectors are recorded during the experiment. During later analysis, a window can be placed around the full energy peak of a gamma ray in either detector and the resulting spectra in the other detector "zoomed" out. This allows two geometries to be studied in triples in a single experiment. Geometry I refers to the case where the cascade gamma ray is observed in the fixed detector and the initial gamma ray is observed in the moving detector (in terms of equation 11, \( \theta_1 \) variable, \( \theta_2 \) fixed). Geometry II is the reverse of
this case, with $\theta_1$ fixed, $\theta_2$ variable.

This provides a very powerful technique for determining spins, since doubles and triples experiments together give four independent correlation functions describing the decay of a state of spin $\tilde{a}$ with the same $P(\alpha)$ unknowns for each of the functions. Even a measurement at only three angles would give 12 data values to determine two population parameters (for the case of Ca$^{40}$) and one or two mixing ratios. This highly overdetermined set of equations usually allows a unique spin assignment.

The final degree of complexity involves transitions of the type shown in figure 9d (pg. 58) where the initial gamma ray in a triple correlation is unobserved. The state with spin $\tilde{A}$ has only the lowest population parameters nonzero as previously discussed, but state $\tilde{a}$ has all $(2a+1)$ parameters populated. Since the general triple correlation formula of equation 11 is written in terms of the $P(\alpha)$ for the first observed state $\tilde{a}$, a transformation must be made. The necessary transform is:

$$P_{a}(\alpha) = \frac{1}{2 - \delta_{\alpha \bar{\alpha}}} \sum_{a, a', \bar{\alpha}} (-)^{a-a'} \left( \langle \alpha, a, a'| k \rangle \tilde{A} \sum_{k} \langle \bar{\alpha}, a, \bar{\alpha}| k \rangle \right) P_{a}(\alpha)$$

$$= (\tilde{a} A - B| k \rangle \tilde{A} \sum_{k} (\tilde{a} A - B| k \rangle \tilde{A} \sum_{k} \langle \bar{\alpha}, a, \bar{\alpha}| k \rangle \right) P_{a}(\alpha)$$

(16)
where \( p_a(\alpha) \) refers to state \( \tilde{a} \) and \( p_A(B) \) refers to state \( \tilde{A} \).
It is apparent from the symmetries of the experiment that for state \( \tilde{a} \), \( p_a(\alpha) = p_a(-\alpha) \) and the system remains aligned even with an unobserved gamma ray transition.

C. Fitting Programs

After the experimental correlation function \( V(\theta_i) \) was determined with the associated errors \( E(\theta_i) \) at each angle \( \theta_i \), a number of fitting programs were used to analyze the data. The basic technique in all of the programs was to assume a spin \( \tilde{a} \) (it is assumed that lower spins \( \tilde{b}, \tilde{c}, \) etc. and mixing ratios for lower gamma rays in a cascade are already known) and read the appropriate theoretical coefficients into the computer. The presence of nonlinear terms in the formulae, such as the mixing ratio \( \delta \), was handled by assuming a value for \( \delta \), calculating a fit to the data, and then stepping to another value of \( \delta \) until the full range was covered. Since \( \delta \) has a range from \(-\infty \) to \(-\infty \), this stepping was done non-linearly by making the substitution \( \theta = \tan^{-1}\delta \) and varying \( \theta \) from 80° to -80° in 5° steps.

The fitting program then calculates the theoretical distribution in terms of the population parameters:
\[ W_{\text{theory}}(\Theta, \alpha) = A(\alpha \Theta_1, \alpha = 0)P(0) + A(\alpha \Theta_1, \alpha = 1)P(1) \]  

(17)

Equation 17 may also be written in matrix form:

\[ W = AP \]  

(18)

where \( W \) is a column matrix with elements \( W(\Theta_1) \), \( A \) is a rectangular matrix with elements \( A(\Theta_1, \alpha) \) where \( \Theta_1 \) labels the rows and \( \alpha \) labels the columns and \( P \) is a column matrix with \( P(\alpha) \) representing the unknowns. For the cases studied in this work, \( P \) was a two element matrix with terms \( P(0) \) and \( P(1) \) representing the population parameters to be determined.

The degree of fit between data and theory is best described by the chi-squared value obtained from the method of least squares, where:

\[ \chi^2 = \sum_i \left[ \frac{V(\Theta_1) - W(\Theta_1)}{E(\Theta_1)} \right]^2 \]  

(19)

The best fit is obtained by minimizing \( \chi^2 \) by proper choice of the unknowns \( P(\alpha) \) in \( W(\Theta_1) \). This is accomplished by setting up the two normal equations for \( \alpha = 0 \) and \( \alpha = 1 \):

\[ \frac{\partial \chi^2}{\partial P(\alpha)} = 0 \]  

(20)
Straightforward algebraic manipulation then gives:

\[(\tilde{A} \omega A) P = \tilde{A} \omega V\]  \hspace{1cm} (21)

where \(w\) is a square matrix with elements given by:

\[\omega_{ij} = \delta_{ij} / (E(\Theta_i) \cdot E(\Theta_j))\]  \hspace{1cm} (22)

The matrix \(\tilde{A}\) is the transpose of \(A\), and the combination \(\tilde{A}wA\) is the normal matrix \(N\). This is a \(2 \times 2\) square matrix and may easily be inverted to obtain the unknown \(P(\alpha)\) giving the best \(\chi^2\) fit:

\[P = N^{-1} \tilde{A} \omega V\]  \hspace{1cm} (23)

The error in the fitted parameters \(P(\alpha)\) is obtained from the inverse of the normal matrix \(N^{-1}\) by the relation:

\[\epsilon_\alpha = (N^{-1}_{\alpha \alpha})^{1/2}\]  \hspace{1cm} (24)

where \(\epsilon_\alpha\) is the standard deviation of the parameter \(P(\alpha)\).

The chi-squared value of this best fit is obtained by calculation of \(\chi^2\) from equation 19 and normalization to the number of degrees of freedom \(M\) to obtain \(\chi^2 = \chi^2'/M\). \(M\) is the number of data points measured less the number of fitted parameters. The computer prints out this \(\chi^2\) value with the values of the \(P(\alpha)\) and steps to the next value of \(\Theta\). This
series of calculations forms the \((\chi^2 - \Theta)\) graphs used later in the data analysis.

The expectation value of this \(\chi^2\) is unity; the actual relation is given by \(\frac{\chi^2}{\nu} = 1. \pm \sqrt{2/\nu}\). The consistency of the data is determined from the probability that \(\chi^2\) can exceed the value actually found and still be the correct fit. This probability \(P\) is given in the form of a graph for various degrees of freedom by Wapstra\(^{35}\). Equivalently, the probability that a better fit could be found by repeating the experiment is given by \((1-P)\). Normally a probability of 1% or less is taken to exclude that case as a possibly correct fit.

The appearance of the \(P(\alpha)\) as linear terms only in the normal equations (equation 20) guarantee that \(\chi^2\) has a single unique minimum for any particular set of data for a particular \(\delta\) in \(P(\alpha)\) space. The worst that can happen is that \(\chi^2\) is independent of one of the \(P(\alpha)\) terms because of zero coefficients in the \(A\) matrix. The normal equations method of finding this minimum thus finds the only minimum and changing either of the \(P(\alpha)\) terms from its optimum value causes \(\chi^2\) to increase.

A common problem that does occur in fitting is that the minimum in \(\chi^2\) corresponds to one of the \(P(\alpha)\) having a negative value. This is a physically unreal situation and corres-
ponds to statistical errors in the data points $V(\theta_i)$. In these cases, the best $\chi^2$ value for a real solution is obtained by setting the negative $P(\alpha)$ term identically to zero and resolving the normal equations in terms of the remaining $P(\alpha)$.

This fitting procedure uses an overdetermined set of equations and ideally will uniquely determine the spin and mixing ratio for a particular state. This will be seen as a significant dip in $\chi^2$ at a particular value of mixing ratio $\delta$ in the $(\chi^2-\theta)$ graphs for that spin choice $\bar{a}$. Typically the $\chi^2$ at the dip would be on the order of 1 while all other $\chi^2$ values are greater than 3 or 4. Ambiguities in the form of several dips in $\chi^2$ at different values of $\delta$ for the same spin $\bar{a}$, or dips in $\chi^2$ for different values of spin, can and do occur. These ambiguities can often be resolved by simultaneously fitting two or more independent correlations. Programs that do this simultaneous fitting will be discussed in a later section.

1. Fits of Double Correlations

The program used to fit single gamma ray correlations of the type in equations 5 or 10 was designated AC5 and was written for the Rice University IBM 1800 computer. The
initial step of the program was to use a matrix inversion technique to fit the data to a curve of the form:

\[
W(\theta) = \sum_{k \text{ even}}^{k_{\text{max}}} a_k P_k(\cos \theta)
\]  

(25)

The values of \( a_k \) corresponding to the fit, their associated errors and the \( \chi^2 \) value of the fit were then printed out. The number of degrees of freedom in this case is \( M = \# \text{ of angles} - (k_{\text{max}}/2 + 1) \) and was used to normalize equation 19. The program is capable of handling fits up to a \( P_8(\cos \theta) \) polynomial.

A preliminary analysis can be made at this point by asserting that states with spin \( \tilde{J} \) should have \( a_k = 0 \) for all \( k > 2J \). Then a non-zero \( a_k \) term in a fit would imply that level had a spin \( J \geq k'/2 \). The reverse argument does not hold, and a number of instances occur where a high spin state has only the \( a_0 \) and \( a_2 \) coefficients non-zero.

The program then fits the data with various spin assumptions in the manner already described. Since the input data are usually unnormalized (equation 25 has an \( a_0 \) term different from unity), the \( P(\theta) \) values from the fit are also unnormalized. This condition can be removed by requiring the relation:
to be applied after the fitting is complete.

2. Simultaneous Fits of Double Correlations

For gamma ray transitions of the type in figure 9b (pg. 58), the AC5 program produces two \((\chi^2 - \theta)\) graphs, one for each gamma ray. Ambiguities in these single fits may often be eliminated by fitting both gamma rays simultaneously and requiring the best fit to both sets of data have the same \(P(\alpha)\) values. This takes advantage of the physical situation that the population parameters are independent of the later decay of a state. The program used for this fitting was designated Cile9 and was also written for the IBM 1800 computer.

Since this fitting involves two sets of unnormalized data, the requirement to be imposed is that for the best fit:

\[
\frac{P(\alpha)}{P'(\alpha)} = \frac{P(\alpha+1)}{P'(\alpha+1)} = \cdots = \frac{P(\alpha_{\text{MAX}})}{P'(\alpha_{\text{MAX}})}
\]

(27)

where \(P(\alpha)\) are the parameters for one set of gamma ray data and \(P'(\alpha)\) are the parameters for the other set. This requirement is best imposed by adding a third unknown \(K\) through
the transformation:

\[ P'(\omega) = \frac{P(\omega)}{K} \]  \hspace{1cm} (28)

and solving for \( K \) as part of the data analysis. Programming difficulties are simplified by using the following equality:

\[ \left( \frac{kV_i - a_i P(\omega) - b_i P(\omega)}{KE_i} \right)^2 = \left( \frac{V_i - a_i \frac{P(\omega)}{K} - b_i \frac{P(\omega)}{K}}{E_i} \right)^2 \]  \hspace{1cm} (29)

The data and errors in the second gamma ray of the cascade are to be multiplied by the unknown \( K \). This unknown enters the normal equations nonlinearly, as seen from the relation for \( \frac{\partial \chi^2}{\partial K} = 0 \):

\[ 0 = L \sum_i \left[ \frac{V_i(a_i P(\omega) + b_i P(\omega))}{(E_i)^2} - \frac{1}{K^2} \sum_i \left( \frac{a_i P(\omega) - b_i P(\omega)}{E_i} \right)^2 \right] \]  \hspace{1cm} (30)

Here the summation is over only the angles measured for the second gamma ray. For \( P(0) \) and \( P(1) \) fixed, it is seen that \( \chi^2 \) for \( K = 0 \) is infinitely large and \( \frac{\partial \chi^2}{\partial K} \) has an infinitely negative slope. At the other extreme, \( \chi^2 \) at \( K = \infty \) is finite with zero slope. So equation 30 predicts a unique minimum in \( \chi^2 \) as a function of \( K \) which, at the very worst, could come at \( K = \infty \). \( \chi^2 \) is a monotonically increasing quantity on either side of the minimum.
By assuming an arbitrary value for K, solving for \( P(\alpha) \) by matrix inversion, taking \( \chi^2 \) as the sum of the \( \chi^2 \) from each set of gamma ray data and then stepping K to another value and finding the new \( \chi^2 \) sum, the change in \( \chi^2 \) tells whether the step in K was towards or away from the minimum. Cile9 is designed to step back and forth over this minimum in decreasing steps in K until the minimum is located to one-thousandth of a part in K. If one of the \( P(\alpha) \) is negative, the best \( \chi^2 \) for a real solution is found by repeating the process with that \( P(\alpha) \) held identically zero.

3. Simultaneous Fits for Double and Triple Correlations

In the event that a simultaneous fit of a doubles correlation of the type shown in figure 9b (pg. 58) does not resolve all possible spin ambiguities, a triple correlation experiment may be required. Usually Cile9 will leave only two or three spin possibilities and angles may be chosen for a triples experiment to maximize the differences between these possibilities. Previous discussion pointed out that the multiparameter system allowed doing two geometries in triples simultaneously. If gamma ray windows are applied over equal energy ranges between the two geometries, then
theory alone will predict the relative normalization between the two sets of data. Equivalently, we could use Cile9 to simultaneously fit both sets of data with the requirement that $K$ be unity.

A more powerful technique would be to simultaneously fit both of the double correlation experiments with both of the triple correlations and add a second factor $K'$ to relate the triples data to the doubles data. The mathematical treatment of this case is identical to the discussion for Cile9, and again there is a single unique minimum in $\chi^2$ space as a function of the population parameters and the two normalization factors $K$ and $K'$.

The fitting program to simultaneously fit all four sets of data was designated Cile4 and was also designed to run on the IBM 1800 computer. The basic technique was to assume starting values for $K$ and $K'$ and locate a minimum by stepping $K$ in the manner previously discussed. With this new value of $K$, $K'$ is then stepped to locate its minimum in $\chi^2$ space. Usually this process changes the position of the $K$ minimum in $\chi^2$ space, and $K$ must be stepped again to find its new $\chi^2$ minimum. This process is continued, alternately stepping $K$ and $K'$, until the unique minimum is found in $\chi^2$ space. This $\chi^2$ is again the sum of the $\chi^2$ for each of the
four sets of gamma ray data. The process converges fairly rapidly to the best fit. If one of the $P(\alpha)$ is negative at the best fit, again the best physical fit is found by repeating the process with that $P(\alpha)$ held identically zero. It should also be noted that the normalization factors $K$ and $K'$ are counted as fitted parameters in determining the number of degrees of freedom in the $\chi^2$ calculations.

D. Doppler Shift Theory

The Doppler shift method of measuring lifetimes of excited states depends on the fact that a gamma ray emitted by a nucleus moving with some velocity $\vec{v}$ toward or away from a detector suffers a Doppler shift in its energy. If $E_0$ is the energy of a gamma ray emitted by a nucleus at rest in the laboratory system, and a nucleus is recoiling with some velocity $v(t)$ at the time of decay at an angle $\theta$ with respect to the detector, then the observed gamma ray energy will be:

$$E_\gamma = E_0 \left(1 + \frac{v(t)}{c} \cos \theta \right)$$

(31)

where $c$ is the velocity of light. This is a non-relativistic formula and is valid only for velocities less than a few percent of the speed of light. Typical recoil velocities in
the present experiment for $^{40}\text{Ca}$ nuclei were on the order of $10^6$ meters/second and satisfy this requirement.

One of the particular advantages of the Litherland and Ferguson geometry II for this type of measurement is that all of the recoiling nuclei are moving along the beam axis (within $3^\circ$) with maximum initial velocity, so that $\theta$ becomes the angle of the gamma ray detector. If two angles $\theta_1$ and $\theta_2$ are chosen on opposite sides of $90^\circ$, the observed Doppler shift will be given by:

$$\Delta \varepsilon = \frac{E_0 v(t)}{c} \left( \cos \theta_1 - \cos \theta_2 \right)$$

(32)

where $\theta_1$ is the forward angle. For the angles of $30^\circ$ and $120^\circ$ measured in this experiment and assuming $v(t)$ is the initial recoil velocity, typical Doppler shifts are on the order of $12$ keV/MeV, an easily measured energy shift with the present Li-Ge detectors. Since the stopping time of recoil nuclei within a solid are on the order of $10^{-11}$ seconds, the useful range for the Doppler shift method is $10^{-11}$ to $10^{-14}$ seconds. For times shorter than this, no slowing down occurs and the full Doppler shift will be measured, giving only a lower limit on the lifetime.
While the experiment is straightforward, the calculation of a lifetime from a measured shift is more involved, depending on a good knowledge of the stopping mechanisms for recoil nuclei in a solid. The matter is further complicated by the fact that the recoiling nuclei are decaying at all times according to the radioactivity law, and we measure an average Doppler shift in effect. This situation is referred to as an attenuated Doppler shift measurement. The Doppler shift actually measured is an averaged quantity given by the equation:

$$\Delta E_{\text{obs.}} = \frac{1}{N_0} \sum_{\text{TIME}} \Delta N(t) \Delta E(t) \Delta t$$  \hspace{1cm} (33)

where $N_0$ is the original number of recoiling nuclei, $\Delta N(t)$ is the number of nuclei decaying within the time period $(t)$ to $(t+dt)$ and $\Delta E(t)$ is the Doppler shift corresponding to those $\Delta N(t)$ nuclei given by equation 32. By the radioactivity decay law:

$$N(t) = N_0 e^{-\frac{t}{\text{half-life}}}$$  \hspace{1cm} (34)

Differentiation of $N(t)$ in equation 34 and substitution of the result and equation 32 into equation 33 yields:
\[ \Delta E_{\text{obs.}} = \frac{E_0}{c \tau_m} \left[ \cos \theta_1 - \cos \theta_2 \right] \int_0^\infty v(t) e^{-t/\tau_m} \, dt \] (35)

Experimental results are often quoted as the ratio of observed Doppler shift to maximum possible Doppler shift. This ratio \( F \) is an attenuation coefficient given by:

\[ F = \frac{1}{v_s \tau_m} \int_0^\infty v(t) e^{-t/\tau_m} \, dt \] (36)

With a knowledge of \( v(t) \), a measurement of \( F \) then gives \( \tau_m \), the mean lifetime for the excited state.

A recoiling nucleus loses energy in two ways: by interactions with atomic electrons producing ionized and excited atoms in the stopping material and by elastic scattering with the nuclei of the stopping material. The electron stopping is more or less proportional to the ionic velocity for recoil velocities less than \( 10^7 \) meters/second. The nuclear stopping becomes the dominant effect at low recoil velocities (being equal to the electron stopping at approximately .300 MeV recoil energy or \( 1.2 \times 10^6 \) M./second for Ca\(^{40}\)) and produces large angular deflections in the path of the recoil nucleus. Stopping cross-sections for these effects have been derived by Lindhard et al.\(^{37}\) and extended by
Blaugrund\textsuperscript{38}) who considers the large angular deflections and derives an expression for $v(t)$ of the recoiling nucleus in terms of the original direction of motion (the beam axis).

The importance of Blaugrund's work may be seen by considering the effects of large angle deflections in the path of a recoiling nucleus. For a given scattering angle $\theta$, the scattering is axially symmetric about the beam axis. So the increased Doppler shift of a gamma ray emitted by a recoil nucleus scattered toward the detector is balanced by the decreased Doppler shift from a recoil nucleus scattered away from the detector. The result is just a Doppler broadening of the gamma ray line shape and does not affect the Doppler shift measurement. On the other hand, this scattering averaged over all possible scattering angles results in a lower net velocity along the beam axis than expected and hence a Doppler shift. This effect is important for the accuracy of the measurement and fortunately has been allowed for in the work of Blaugrund.

Another source of error in Doppler shift measurements may come from recoil nuclei escaping from the target before they have emitted a gamma ray and subsequently decaying in vacuum with a higher velocity than expected. This effect causes an increase in the measured Doppler shift, but may be
minimized by using a thick target. The range of recoil Ca nuclei for the range of energies used in this present work was approximately 140 \( \mu \text{gm/cm}^2 \). Since the target thickness was 1400 \( \mu \text{gm/cm}^2 \) and the recoil nuclei must escape within about the first 10% of their path to exhibit a strong Doppler shift, the number of nuclei that could distort the measurement in this manner was less than 1%. This is a negligible effect and is consequently ignored.

The lifetime of an excited state is obtained by integrating equation 36 using formulas given by Blaugrund. This calculation has been carried out on the Rice IBM 1800 computer by numerical integration for a variety of recoil energies. The electronic stopping contribution was increased 20% over the value calculated by Lindhard et al. to be more in agreement with the experimental values of the electronic stopping power. A plot of the slowing down curve obtained from these calculations relating the observed fraction of Doppler shift to the lifetime is shown in figure 18 (pg. 114).

E. Gaussian Fitting Program for Doppler Shift Measurements

One of the problems encountered in the Doppler shift work was accurately defining the center of a gamma ray peak
with a peak height only a few hundred counts above background. Consequently the peak shape was often distorted by statistical fluctuations in the number of counts, rendering visual estimates of the center position unreliable.

A more quantitative measure of peak positions was obtained by assuming that a gamma ray peak could be described by a gaussian curve. A gaussian fitting program was then used to fit a symmetrical gaussian curve to the data.\(^{41}\)

Order of magnitude calculations indicated that the shape of the gamma ray angular correlation averaged over the angle subtended by the gamma ray detector (± 8°) would not shift the center of the peak by more than 0.1 keV. This was a negligible effect and consequently a skewed gaussian fit to the data was unnecessary. The fitting program also includes corrections for the fact that the gamma ray peak may be superimposed on a large background, usually the flat Compton edge of a higher energy gamma ray.

The fitting procedure used described the gamma ray spectrum \(Y(x)\), where \(Y\) is the number of counts measured in channel \(x\) of the spectrum, by a gaussian line shape superimposed on a flat background:

\[
Y(x) = u_1 + u_2 e^{-\frac{(x-x_0)^2}{2 \sigma^2}}
\]

(38)
where \( w \) is the square of the standard deviation \( \sigma \) for the

Gaussian peak and is equivalent to \( \frac{1}{2} (x')^2/0.693 \), where

\( 2x' \) is the FWHM of the Gaussian peak\(^{41} \).

Initial values were assumed for the peak center \( x_0 \) and

width \( x' \) and the amplitude coefficients \( U_1 \) and \( U_2 \) calculated

by matrix inversion to yield the best fit to the data using

the linear least squares method. The \( \chi^2 \) of this fit is cal-

culated from equation 19, normalized to the number of data

points used in the fit. Normally 30 channels on either side

of the peak were used to provide a good background
determination.

For a given \( x_0 \), the peak width was measured by oscil-

lating steps in the width of decreasing size, similar to the

method used in Cile9, to locate the minimum \( \chi^2 \). For this new

width, \( x_0 \) was then varied a fixed amount and the new width

for the best fit redetermined. This procedure was continued

until the best \( \chi^2 \) fit to the data had been obtained. Typical

fits obtained with this program are shown in figure 17 (pg.

113). The accuracy of the program in determining the center of

a peak was estimated to be approximately \( 1/4 \) channel for a

peak with a 5 to 8 channel width at FWHM.

In conjunction with the Gaussian peak fitter, a gain

shifter program was used to reduce a number of gamma ray
spectra taken with different energy calibrations to a single spectrum with a new calibration. This procedure allowed a large number of gamma ray spectra taken at different times (with different energy calibrations) to be added together to improve the statistics, a procedure especially useful in searching for weak gamma ray cascades or in Doppler shift measurements to correct small drifts in the electronic gains before using the Gaussian peak fitter.

The gain shifter requires the original spectrum to be calibrated by a linear least squares method to a calibration of the form \( E(x) = a \cdot x + b \), where \( x \) is the channel number and \( E(x) \) is the energy corresponding to that channel. Each channel \( x_i \) in the original spectrum can then be assigned an energy range \( \Delta E_i \), with energy \( E_i \) at the lower edge and energy \( E_{i+1} = E_i + a \) at the upper edge of the channel.

The original spectrum is then superimposed over a new spectrum with \( a' \) equal to the desired calibration and \( b' = 0 \). The counts in a particular channel of the old spectrum are then placed into a channel of the new spectrum of the same energy proportional to the amount of overlap between the channels. For example, if the old spectrum has \( N \) counts in a channel covering an energy range 1000 to 1004 keV and the new spectrum has a channel covering the range 999-1001 keV,
then 0.25 N counts will go into this new channel, 0.5 N counts will go into the next higher channel with the range 1001 to 1003 keV, and 0.25 N counts will go into the next higher channel of the new spectrum with the range 1003-1005 keV. This procedure is carried out over the full range of the old spectrum. The accuracy of the gain shifter may be determined by the conservation of total number of counts between the old and the new spectra, since some counts may be lost due to round-off errors in the computer. Typically the agreement between the two spectra was on the order of 1 count lost/20,000 counts gain-shifted over a range of 2000 channels.
IV. DISCUSSION OF EXPERIMENTAL RESULTS

The energy level diagram of states in Ca$^{40}$ up to 7.5 MeV is shown in figure 10 (pg.88). The energies, gamma ray decay modes, spins, parities and branching and mixing ratios are entered when known. Each state is also referenced with a series of numbers to indicate the papers in the bibliography that definitely establish these parameters. The levels of interest in the present work are the 4.49 MeV level, the 6.75 MeV level, and 7.11 MeV level. The $5^-$ state at 4.49 MeV will be discussed first. Since the spin and parity of this level are well known, this treatment will illustrate the methods used to analyze data and serve in addition as a check on the accuracy and procedure of this experiment.

A. 4.49 MeV Level

1. Previous Work

The 4.49 MeV level decays entirely to the 3.73 MeV level ($J = 3^-$) which in turn decays entirely to the ground state.

The properties of the 4.49 MeV level have been extensively studied in a number of reactions. Stripping experi-
Figure 10

Energy Level Diagram of Ca$^{40}$
ments with the \( K^{39}(\text{He}^3,d)\text{Ca}^{40} \) reaction by Erskine\(^9\) and Seth et al.\(^{10}\) both place the 4.49 MeV level in the \( (d_{3/2}^{-1}f_{7/2}) \) configuration based on an \( l = 3 \) stripping pattern and assign a \( J^\pi = 5^- \) on the basis of spectroscopic factors. Gamma ray angular correlation measurements with the \( \text{Ca}^{40}(pp'\gamma)\text{Ca}^{40} \) reaction by Poletti and Grace\(^{42}\) and the \( K^{39}(p\gamma)\text{Ca}^{40} \) reaction by Leenhouts\(^{44}\) and Leenhouts and Endt\(^{43}\) give the spin as 5 (though Poletti and Grace do not completely exclude a \( J = 3 \) possibility) with predominantly E2 decay to the 3.73 MeV level. Leenhouts\(^{44}\) also includes a gamma ray polarization measurement to fix the parity as negative. Table I summarizes the gamma ray mixing ratios found in these experiments, including the results of this present work:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Spin</th>
<th>( \delta = \frac{M3}{E2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leenhouts et al.(^{43})</td>
<td>5</td>
<td>.05 ± .03</td>
</tr>
<tr>
<td>Leenhouts(^{44})</td>
<td>5^-</td>
<td>.01 ± .02</td>
</tr>
<tr>
<td>Poletti and Grace(^{42})</td>
<td>5(3)</td>
<td>.05 ± .05</td>
</tr>
<tr>
<td>present work</td>
<td>5</td>
<td>.08 ± .08</td>
</tr>
</tbody>
</table>

Although all of the measurements in Table I except the first are consistent with a \( \delta = 0 \) mixing ratio, the fact that the values always cluster on the positive side of zero
is suggestive that there may indeed be some weak M3 mixture in the gamma ray decay of this level. Such a mixture will also be required to explain data to be presented later in this thesis. The significance of a non-zero mixing ratio will be discussed in more detail in section V.

Lifetime measurements on the 4.49 MeV level have been performed using the Doppler shift attenuation method on the Ca\(^{40}\)(pp'\(\gamma\))Ca\(^{40}\) reaction by MacDonald, et al.\(^{45}\) and the K\(^{39}\)(p\(\gamma\))Ca\(^{40}\) reaction by Lindeman, et al.\(^{47}\) Their results are listed in Table II, where F is the fraction of Doppler shift observed compared to the maximum possible Doppler shift if the recoil nucleus decayed before any slowing down occurred:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>F</th>
<th>(\tau_M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MacDonald et al.(^{45})</td>
<td>0</td>
<td>&gt;1500 fs.</td>
</tr>
<tr>
<td>Lindeman, et al.(^{47})</td>
<td>.10 ± .15</td>
<td>250 fs.</td>
</tr>
</tbody>
</table>

The 4.49 MeV level is clearly a long-lived state with little or no Doppler shifting of the 0.754 MeV gamma ray.

Preliminary considerations of the experimental coincidence requirements will enable us to exclude extreme spin assignments. Accepting the spin and parity of the 3.73 MeV
level as $3^-$, the decay of the 4.49 MeV level would require gamma radiation with $L = 3$ for a spin of 0 or $\geq 6$. On the basis of the extreme single particle transition lifetime, the faster E3 radiation that would be present if the 4.49 MeV level had positive parity would have a lifetime of approximately $10^{-4}$ seconds for a 0.754 MeV gamma ray$^{54}$. Since the coincidence resolving time of this experiment was $10^{-7}$ seconds, an enhancement of approximately 1000 for E3 radiation would be required to observe the 0.754 MeV gamma ray in coincidence. Such an enhancement is considered extremely unlikely since no such cases have been reported. Thus further analysis will need to consider spins only in the range of $J = 1$ to 5. The following data analysis will be conducted independently of the known results previously discussed.

2. Decay Scheme

The 4.49 MeV level produces a relatively uncomplicated gamma ray spectrum shown in figures 11a and 11b (pg. 93). Figure 11a is a NaI(Tl) gamma ray spectrum at 30% in coincidence with inelastic protons from the 4.49 MeV level. Accidental gamma ray coincidences have been subtracted out of the spectrum. The dominant features are the peaks at .511 MeV,
.754 MeV, 3.22 MeV, and 3.737 MeV. The .754 MeV peak is associated with the full energy peak (FEP) of the .754 MeV initial gamma ray superimposed on the spectrum of the cascade 3.73 MeV gamma ray. This cascade gamma ray gives rise to the FEP at 3.737 MeV and the first and second escape peaks at 3.22 MeV and 2.715 MeV from pair production by the 3.737 MeV gamma ray. The large peak at 0.511 MeV is associated with annihilation gamma rays. Since accidental correction does not remove this 0.55 MeV peak, the major source of this radiation is interpreted as pair production from coincident 3.73 MeV gamma rays stopping in lead shielding around the target chamber and detector with one of the resulting annihilation gamma rays being detected.

Figure 11b (pg. 93) is a Li-Ge gamma ray spectrum at 30° taken in coincidence with inelastic protons from the 4.49 MeV level at the same beam energy. Accidental gamma ray coincidences have also been subtracted out of this spectrum. The dominant features are a weak 3.73 MeV FEP, a fairly conspicuous Compton edge at 3.49 MeV and a weak first escape peak at 3.22 MeV with a strong second escape peak at 2.715 MeV from pair production. Superimposed on this spectrum is the 0.754 MeV gamma ray spectrum with a strong 0.754 MeV FEP and a Compton edge at 0.574 MeV. The
Figure 11
4.49 MeV Level Gamma Ray Spectra

a. NaI(Tl) at 30°
b. Ll-Ge at 30°
0.511 MeV peak is present for the reasons discussed above.

3. Correlation Measurements

The yield for the 0.754 MeV gamma ray was calculated at each angle from the number of events in the region marked C of figure 11a (pg. 93). The number of events in region D of the same figure, corresponding to Compton scattering of 3.73 MeV gamma rays, was then subtracted from this yield. The resulting 0.75 MeV gamma ray yield was then normalized to the yield of the multichannel analyzer for inelastic protons from the 4.49 MeV level and the resulting correlation function plotted in figure 12a (pg. 95) with the associated errors. The significance of the continuous curves in figure 12a (pg. 95) will be discussed later in this section. The advantages of plotting the gamma ray correlation yield \( W(\theta) \) against \( \cos^2 \theta \) is that a spin 1 level has a correlation dependence only on \( \cos^2 \theta \), equivalently \( P_2(\cos \theta) \), and would form a straight line on such a graph. This property serves as a preliminary aid in the data analysis.

The AC5 program was now used to fit this experimental 0.75 MeV gamma ray correlation to theoretical correlations for different spin possibilities treating the mixing ratio, \( \delta \), and population parameters, \( P(\alpha) \), as unknowns. The \( \chi^2 \)
Figure 12

4.49 MeV Level Angular Correlations

a. 0.75 MeV gamma ray correlation
b. 3.73 MeV gamma ray correlation
c. Triple correlation, geometry I
d. Triple correlation, geometry II
values of the best fits are plotted against $\theta$, where $\theta = \tan^{-1} \delta$, in figure 13a (pg. 98). Spin 4 is not shown since its lowest $\chi^2$ value was 5.4. Generally if a $\chi^2$ value exceeds the 1% probability level, corresponding to $\chi^2 = 3.8$ in figure 13a, the spin giving that fit is considered excluded. As previously discussed, a $\chi^2$ falling at some probability level $p$ gives the probability that a correct fit could have a $\chi^2$ worse than that value. The probability levels are marked in percentages on the right side of the $(\chi^2 - \theta)$ graphs. Spins 1 and 2 are also excluded as fitting to the 1% probability level at best, and consequently only spin 2 is shown as representative.

Spins 3 and 5 have quite good fits for mixing ratio values of $-65^\circ$ and $7.5^\circ$, respectively. In an attempt to resolve this ambiguity, the 3.73 MeV gamma ray was analyzed as the second gamma ray in a cascade, treating the 0.75 gamma ray as unobserved. The resulting correlation function and associated errors are plotted in figure 12b.

The fits obtained with the AC5 program for this "skip correlation" are shown in figure 13b (pg. 98), where the mixing ratio $\delta$ still refers to the initial $4.49 \rightarrow 3.73$ gamma ray transition in the cascade. Since skip correlations have a theoretical dependence only on even powers of $\delta$, figure 13b
is plotted using $|\theta|$ from 0° to 80°. Spins 2, 3 and 5 all fit well below the 1% probability level while spin 1, not shown, fits slightly below the 1% level. Consequently, the results of fitting this skip correlation do not by themselves remove any of the spin ambiguities.

Since the correct spin choice must fit with the same mixing ratio $\delta$ for either correlation, only regions of overlap between figures 13a (pg. 98) and 13b are of importance. For example, spin 3 fits quite well in figure 13a at $\theta = -65°$, while in figure 13b spin 3 fits only to $\chi^2 = 7.5$ at $|\theta| = 65°$. This lack of agreement would indicate that a spin 3 assignment with a mixing ratio $\theta = -65°$ would not be correct.

This type of comparison can be made quantitative by simultaneously fitting both angular correlations with the same mixing ratio and set of population parameters. Such fitting requirements are possible since whichever gamma ray is observed has no effect on the properties of the emitting level. This initial level must be described with a unique set of population parameters.

The results of this simultaneous fitting with the Cile9 program are shown in figure 13c. Spin 5 is now preferred over spin 3 by a factor of 16 in probability with the best $\chi^2$ at 7.5° and 0°, respectively. This is in sharp contrast
Figure 13

$\chi^2$-0) Fits of Correlation Data

4.49 MeV Level

a. 0.75 MeV gamma ray single fit

b. 3.73 MeV gamma ray single fit

c. 0.75-3.73 MeV gamma rays simultaneous fit

d. Doubles-triples simultaneous fit
to the results of figure 13a where spin 3 actually fit better at $-65^\circ$ than spin 5 at $7.5^\circ$. Simultaneous fitting has now totally eliminated one ambiguity. On the other hand, the $J = 3$ fit at $0^\circ$ has now improved, coming down to a $\chi^2 = 2$. This is a consequence of the extremely good $J = 3$ fit in figure 13b with $\chi^2 = .063$ at $0^\circ$ averaging with the poorer fit of figure 13a. This behavior is generally true of simultaneous fitting with the simultaneous fit $\chi^2$ lying between the best $\chi^2$ values of the two independent fits.

To resolve this discrepancy between spins 3 and 5, triple coincidence data in geometries I and II were simultaneously fit with the 0.75 MeV and 3.73 MeV gamma ray correlations already discussed. The results of this "four-fold" simultaneous fitting are shown in figure 13d. Spin 3 is now definitely excluded with a best $\chi^2$ value of 4.1 while spin 5 still fits to the 20% probability level with a $\chi^2$ of 1.45. Most of the contribution to this $\chi^2$ value comes from the triples data where the statistics are quite poor, with each angle having only 100 coincidence counts on the average. The discrimination is still sufficiently sharp to totally exclude the spin 3 possibility completely.

On the other hand, spin 2 still fits to the 1% level at $25^\circ$ in figure 13d. On the basis of the "four-fold"
fitting, spin 5 is then preferred over spin 2 by a factor of only 20 in probability. This is in contrast to the simultaneous fitting of double correlations only in figure 13c where the spin 5 to spin 2 preference is a factor of 80 in probability. This behavior is again due to the averaging characteristics of simultaneous fitting and should be viewed with a certain amount of caution. In particular, the results of figure 13c are sufficient to exclude spin 2 over spin 5, and the lesser degree of exclusion from the results of figure 13d with data of poorer statistics should not diminish this fact.

The results of fitting a spin 1 are not shown in figure 13d, but the minimum $\chi^2$ is 20, sufficiently high to completely exclude spin 1. This leaves spin 5 as the only possibility for the 4.49 MeV level. Taking an average between the mixing ratios for figures 13c and 13d, the final mixing ratio found in this experiment is $5^\circ \pm 5^\circ$, or equivalently $0.08 \pm 0.08$, in fairly good agreement with the other values quoted in Table I. The error used here is taken as the region of the ($\chi^2$-$\theta$) graph where the probability level is within half of the best value.

The values of population parameters and mixing ratios corresponding to the best fits obtained in figure 13d for
spins 2, 3 and 5 were used to calculate the continuous curves shown in figures 12a–12d (pg. 95). These curves represent the best theoretical fit to the data.

This example illustrates the use of simultaneous fitting and the addition of triple correlation data to eliminate spin and mixing ratio ambiguities. By adding more and more independent pieces of data to the fit, each having to be described by the same few population parameters and mixing ratio, the ambiguities are successively eliminated. The success of this approach for the 4.49 MeV level verifies the accuracy of the present experiment and the validity of the data analysis methods used. The analysis of higher excited states in the Ca\(^{40}\) nucleus can now be performed using these methods and the results accepted with a high degree of confidence.

B. 6.75 MeV Level

1. Previous Work

The 6.75 MeV level is one of the few remaining levels below 7 MeV excitation energy in the Ca\(^{40}\) nucleus that has not been given a definite spin assignment. Poletti and Grace\(^{42}\) report its decay as 100% to the 3.73 MeV level. Both Erskine\(^{9}\) and Seth et al.\(^{10}\) observe the 6.75 MeV level in the K\(^{39}\)(He\(^{3}\),d)Ca\(^{40}\) reaction and limit its spin to
0, 1, 2 or 3 with negative parity based on an \( l = 1 \) stripping pattern. Both authors place the 6.75 MeV level in the \( \left( d_{3/2}^{-1} p_{3/2} \right) \) configuration and Seth et al.\(^{10}\) propose a \( J = (0,2)^- \) based on spectroscopic factors and shell model considerations.

2. Decay Scheme

A NaI(Tl) gamma ray spectrum at 30° in coincidence with \( \alpha \)-inelastic protons from the 6.75 MeV level is shown in figure 14a (pg.104). Again accidental gamma ray coincidences have been subtracted from the spectrum. The dominating features are the 3.73 MeV FEP and the large peak in the 3.0 to 3.2 MeV region, composed of the 3.02 MeV gamma ray FEP superimposed on the first and second escape peaks of the 3.73 MeV gamma ray. The 0.511 MeV peak is present for the same reasons discussed in the analysis of the 4.49 MeV level.

This spectrum can be compared to a Li-Ge gamma ray coincidence spectrum taken at 30° and shown in figure 14b (pg.104). The presence of the 3.737 MeV and 3.02 MeV FEP's and first and second escape peaks at energies 0.511 MeV and 1.022 MeV lower confirm the decay to the 3.73 MeV 3^- level. The absence of any other peaks in these spectra are in agreement with Poletti and Grace's\(^{42}\) claim of a 100% decay to the 3^- level.
3. Correlation Measurements

Since the 3.02 MeV gamma ray FEP is merged with the first and second escape peaks from the 3.73 MeV gamma ray in the NaI(Tl) spectrum, spectrum stripping was necessary to extract the 3.02 MeV gamma ray yields for correlation measurements. This stripping was accomplished in the following manner: The gamma ray spectra at all angles measured in coincidence with particles from the 3.73 MeV excited state in Ca$^{40}$ are analyzed. These spectra give the shape for a pure 3.73 MeV gamma ray, so the necessary stripping factor F can be immediately obtained by taking the ratio of Compton yield (in the 0.6 to 3.5 MeV range) to the FEP yield. For the detector used in this experiment, F was found to have a value 2.22 ± .035.

At each angle in the 6.75 MeV level coincidence spectra the 3.73 MeV FEP is multiplied by this F to obtain the 3.73 MeV gamma ray Compton contribution in the (0.6 to 3.5 MeV) region. The difference between the observed yield in this region and this contribution is the yield due to the 3.02 MeV gamma ray and can be normalized to the particle monitor to produce the correlation function shown in figure 15a (pg.106). The errors associated with this correlation function include the additional errors introduced by this
Figure 14
6.75 MeV Level
Gamma Ray Spectra
a. NaI(Tl) at 30°
b. Li-Ge at 30°
method of spectrum stripping.

The 3.73 gamma ray yields can immediately be normalized to the particle monitor to yield the 3.73 skip correlation shown in figure 15b (pg. 106) with the associated errors.

Applying the same type of lifetime arguments used in the discussion of the 4.49 MeV level, lifetimes predicted by the single particle model are $10^{-8}$ seconds and $5 \times 10^{-7}$ seconds for 3.02 MeV gamma rays assuming E3 and M3 multipoles$^{54}$. Neither possibility is definitely excluded by the experimental resolving time, so spins in the range $J = 0$ to 6 must be considered. Despite the spin limitations imposed by the work of Erskine$^{9}$ and Seth et al.$^{10}$, all of these spin possibilities were tested in the present analysis for completeness.

The results of fitting the 3.02 MeV gamma ray with the AC5 program are shown in figure 16a (pg. 109). Spins 0, 5 and 6 had minimum $\chi^2$ values of 10, 5.11 and 13.64, respectively. These values are all considerably above the 1% probability level and consequently are not shown in the figure. Spin 4 fits quite well in a limited region about -20° while spins 1, 2 and 3 have acceptable fits over a variety of ranges of the mixing ratio $\delta$. 
Figure 15

6.75 MeV Level Angular Correlations

a. 3.02 MeV gamma ray correlations
b. 3.73 MeV gamma ray correlations
c. Triple correlation, geometry I
d. Triple correlation, geometry II
In an attempt to resolve these ambiguities, the 3.73 MeV skip correlation was fitted with the AC5 program, treating the 3.02 MeV gamma ray as unobserved. The results are shown in figure 16b (pg. 109). Spin 4 is excluded with a minimum $\chi^2$ value of 7.5, but spins 1, 2 and 3 still have quite acceptable fits.

Simultaneous fitting of the 3.02 MeV and 3.73 MeV correlations with the Cile9 program does not significantly improve the situation. Spins 1, 2 and 3 all still fit below the 1% level, seen in figure 16c. Some changes are evident from figure 16a however; the spin 3 fit at 0° is excluded and the broad minima in the spin 1 fits are replaced by sharper dips. Spin 2 is relatively unchanged by the simultaneous fitting, still having acceptable $\chi^2$ values over a broad range of mixing ratios from 20° to 70°.

These ambiguities were finally resolved by performing a triple coincidence correlation experiment in the two geometries previously discussed. Geometry I detects the 3.73 MeV gamma ray in a fixed detector at 90° while Geometry II detects the 3.02 MeV gamma ray in the fixed detector at 90°. Both detectors were in the plane and angles of 30°, 60° and 90° were measured. The resulting correlation functions and associated errors are shown in figures
15c and 15d (pg. 106). Each of the six data points had approximately 150 coincidence counts.

The results of "four-fold" simultaneous fitting of these data with the two double correlations using the Cile4 program are shown in figure 16d (pg. 109). Spins 1 and 3 are now completely excluded while spin 2 still has a good fit with $\chi^2 = 1$ at 40°. The spin of the 6.75 MeV level is thus unambiguously $J = 2$ with a mixing ratio $\delta = 0.84 \pm 0.22$, taking the range of $\chi^2$ at one-half the probability level of the best fit.

It is interesting to note that the triple correlation measurement not only produces an unambiguous spin assignment, but also places a considerable restriction on the possible range of mixing ratios. In particular, figure 16c showed spin 2 as having the best fit for a mixing ratio of 60° while the final results of figure 16d completely exclude this value.

4. Lifetime Measurements

The lifetime of the 6.75 MeV level was measured by the Doppler shift attenuation method described in section II. Using a TAC coincidence circuit, particle-gamma ray coincidences were taken with a 2.1 keV/channel calibration on the
Figure 16

$\chi^2 - \theta$ Fits of Correlation Data

6.75 MeV Level

a. 3.02 MeV gamma ray single fit

b. 3.73 MeV gamma ray single fit

c. 3.02-3.73 MeV gamma rays simultaneous fit

d. Doubles-triples simultaneous fit
gamma ray side. Measurements were made at angles of 30°, 45°, and 120°. Each measurement lasted 3 hours and the series of angles was repeated 8 times in a random order. This procedure was used to minimize the effects of gain shifts in the electronics during the time necessary for the experiment.

The possibility of such electronic gain shifts was investigated in the data analysis by observing gamma rays emitted by long-lived excited states. These gamma rays suffer no Doppler shift, so any variation in their peak positions must be due to gain shifts. Plots of all gamma rays in coincidence with any inelastic proton from Ca$^{40}$ were obtained for each three hour data point. Since many of the excited states in Ca$^{40}$ cascade through the long-lived 3.737 MeV and 4.491 MeV states, strong gamma ray peaks at 3.7371 MeV, 2.7151 MeV, and 0.7547 MeV were observed in these gamma ray "shadow plots" with no Doppler shift. Another strong gamma ray peak suitable for calibration was the 0.511 MeV peak from annihilation radiation where the nature of the process guarantees no Doppler shift.

These four gamma ray peaks were fitted by gaussian functions using a gaussian fitting program previously described to accurately locate their center positions. A
straight line energy calibration of the form \( a \cdot x + b = E(x) \) 
\( x = \) gamma ray channel number) was then obtained by linear least squares fitting for these four gamma rays for each three hour data point. The absence of any Doppler shift for these gamma rays allows one to associate differences in the calibration coefficients "a" and "b" between data points with electronic gain shifts. Typically these shifts were on the order of four keV or less throughout the experiment.

The gain shifting program previously described was then used to reduce all of the spectra at each angle to a new spectrum with a constant 2 keV/channel calibration. The summed, gain shifted spectra are shown in figure 17 for the 30° and 120° angles. The FEP and second escape peak for both the 3.02 MeV and 3.737 MeV gamma rays are shown. These peaks, including the 0.511 MeV peak not shown, were then refit with the gaussian fitting program to determine their new center positions. The final gaussian fits are shown as solid lines in figure 17 (pg. 113) for comparison to the data.

Recalibration of these gain shifted spectra, using the 0.511 MeV, the 2.7151 MeV and 3.7371 MeV gamma rays, yielded the Doppler shifted energy of the 3.03 MeV gamma ray. The observed Doppler shifts in keV for the 3.02 MeV gamma ray and the 2.00 MeV second escape peak are listed in Table III.
The corresponding fraction $F$ of full Doppler shift measured is also tabulated.

<table>
<thead>
<tr>
<th>Angular Range</th>
<th>2.00 Second Escape Peak</th>
<th>3.02 Full Energy Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30^\circ$-$120^\circ$</td>
<td>$21 \pm 2.2$ .87 $\pm$ .09</td>
<td>$18.6 \pm 2.5$ .76 $\pm$ .10</td>
</tr>
<tr>
<td>$45^\circ$-$120^\circ$</td>
<td>$16.6 \pm 2.2$ .77 $\pm$ .10</td>
<td>$16.5 \pm 2.5$ .76 $\pm$ 1.2</td>
</tr>
</tbody>
</table>

Including both the errors in the fitted calibration coefficients "a" and "b" and the inability of the gaussian fitting program to exactly define the center of a peak (to within $1/4$ channel), a final value for the fraction of Doppler shift observed was obtained with $F = .788 \pm .055$.

From the observed Doppler shift, the energy of the 3.02 MeV initial gamma ray was calculated to be $(3015.0 \pm 1.7$ keV). This yields an energy for the level of $6752.1 \pm 1.8$ keV, using the value of $3737.1 \pm .3$ given for the cascade gamma ray by Kashy et al.$^{49}$

Referring to the slowing down curve shown in figure 18 (pg. 114), this fraction of Doppler shift corresponds to a lifetime $205 \pm 50 \times 10^{-15}$ seconds. The physical significance of this lifetime in terms of theoretical models for
Figure 17
Doppler Shift Measurements
6.75 MeV Level
a. Li-Ge detector at 30°
b. Li-Ge detector at 120°
Figure 18
Doppler Shift Slowing Down Curve
the Ca\(^{40}\) nucleus is discussed in section V. Similar measurements on the 3.90 MeV \(2^+\) excited state yield a lifetime of \(74 \pm 65 \times 10^{-15}\) seconds \((F = .92 \pm .07)\), in good agreement with the accepted value of \(70 \times 10^{-15}\) seconds. This agreement gives strong support to the accuracy of the present lifetime measurements.

C. 7.114 MeV Doublet

1. Previous Work

The excited state at 7.114 MeV has been investigated in a number of experiments, but as yet no definite spin-parity assignments have been made and verified. Preliminary analysis of a recent experiment by Tellez \textit{et al.}\(^{51}\) suggests the level is a narrowly separated doublet, which further complicates the situation.

A large part of the difficulty in understanding this level comes from a lack of knowledge about its decay scheme. Tellez \textit{et al.}\(^{51}\) using the Ca\(^{40}\) \((pp'\gamma)\)Ca\(^{40}\) reaction, proposes five gamma ray cascades for this level, shown in figure 19a (pg. 116) with a letter to represent the intensity of that cascade. The letter "a" corresponds to the most intense branch and on down the line. Tellez makes no effort to separate the gamma rays from each component of the doublet.
Figure 19

Gamma Ray Decays from 7.114 MeV Level

a. Results of Tellez$^{51}$
b. Results of Lindeman et al.$^{47}$
c. Results of Dolan, et al.$^{53}$
d. Results of present experiment
Other work on this level has been done by Lindeman et al. \(^{47}\) using the \(^{39}\text{K}(\gamma\gamma)^{40}\text{Ca}\) capture reaction to study the 3.377 MeV gamma ray decay to the 3.737 MeV level. Doppler shift attenuation measurements on this gamma ray yield a lifetime of 430 ± 130 fs. Lindeman et al. \(^{47}\) also report a gamma ray decay scheme composed of two branches, shown in figure 19b (pg. 116), but make no mention of the ground state decay or 3.907 MeV cascade claimed by Tellez et al. \(^{51}\).

The 30%–70% branching ratios quoted by Lindeman et al. \(^{47}\) disagree with the results of Dolan et al. \(^{55}\) who used the \(^{50}\text{Ca}(\gamma\gamma)^{40}\text{Ca}\) reaction. Their gamma ray decay scheme is shown in figure 19c and includes a 17% branch to the 5.613 MeV level, in agreement with the claims of Tellez et al. Again they make no mention of the ground state decay or 3.907 MeV cascade. These three different decay schemes are very suggestive of a doublet with possibly a positive parity component and a negative parity component. In general, positive parity levels in \(^{40}\text{Ca}\) decay to positive parity levels, and negative parity levels to negative parity levels. (figure 10, pg. 88) A simple interpretation would then be the negative parity component decays to the 3.737 MeV, the 4.491 MeV, and the 5.613 MeV levels. The 5.613 branch would be quite weak, which would explain its absence in the
decay scheme proposed by Lindeman et al.\textsuperscript{47} The positive parity component is then responsible for the ground state decay and 3.907 MeV cascade reported by Tellez et al.\textsuperscript{51} and was not populated in the work of Lindeman et al.\textsuperscript{47} and Dolan et al.\textsuperscript{55}.

Spin information on the 7.114 MeV level is equally confusing. Blum et al.\textsuperscript{56} using the Ca\textsuperscript{40}(e\textsuperscript{−},e\textsuperscript{−})Ca\textsuperscript{40} reaction assigned a $J^\pi = 2^+$, Erskine\textsuperscript{9} and Seth et al.\textsuperscript{10} both observe the 7.114 MeV level in the $^3\text{He}^3$(d)Ca\textsuperscript{40} reaction and assign it to the $d_{3/2}^{-1}p_{3/2}^{-1}$ configuration with negative parity on the basis of an $l = 1$ stripping pattern. Again these experiments are consistent with the existence of a doublet at 7.114 MeV having positive and negative parity components with the decay schemes proposed earlier.

2. Gamma Decays
In an attempt to resolve these discrepancies, the gamma ray decays of the 7.114 MeV level were studied at three different beam energies using the Ca\textsuperscript{40}(pp')Ca\textsuperscript{40} reaction. A NaI(Tl) detector was used to investigate gamma rays in the energy range of 0–7.5 and a Li-Ge detector for higher resolution work in the 0–4.0 MeV energy range. The results are shown in figures 20 (pg.120) and 21 (pg.121).
Figure 20a (pg.120) shows a NaI(Tl) gamma ray spectrum at 30° in coincidence with inelastic protons from the 7.114 MeV level. Figure 20b is a Li-Ge detector gamma ray spectrum at 90° under similar conditions. The beam energy for both spectra was 10.81 MeV. Figure 21a (pg.121) is another NaI(Tl) coincidence spectrum taken at a beam energy of 9.86 MeV with the detector at angles of 30°, 45°, 60°, 75°, and 90°. The data at 9.86 MeV were summed over all angles to improve the statistics. Finally figure 21b is a Li-Ge gamma ray coincidence spectrum at 90° taken at a beam energy of 11.29 MeV.

All four spectra show relatively strong gamma rays with energies of 0.75 MeV, 3.38 MeV, and 3.73 MeV and a weaker 2.63 MeV gamma ray. Although the 3.38 MeV FEP and the 3.22 MeV first escape peak are merged in the NaI(Tl) spectra, these peaks are clearly resolved in both of the Li-Ge spectra. These gamma rays confirm the decays to the 3− and 5− levels previously reported. The ground state decay reported by Tellez et al.51) is also confirmed by both of the NaI(Tl) gamma ray spectra. Due to amplifier saturation at 7.1 MeV in the spectrum of figure 20a (pg.120), part of the gamma ray FEP is lost and consequently no measure of relative populations of the two components of the
Figure 20

7.114 MeV Level

Gamma Ray Spectra

a. NaI(Tl) at 30° $E_{\text{beam}} = 10.81$ MeV

b. Li-Ge at 90° $E_{\text{beam}} = 10.81$ MeV
Figure 21

7.114 MeV Level
Gamma Ray Spectra

a. NaI(Tl summed 30° to 90°
\[ E_{\text{beam}} = 9.86 \text{ MeV} \]

b. Li-Ge at 90° \[ E_{\text{beam}} = 11.29 \text{ MeV} \]
proposed doublet is possible.

The Li-Ge spectrum of figure 21b (pg.121) also shows a 1.305 MeV gamma ray and a weaker 3.90 MeV FEP. These are suggestive of a previously unreported decay through the $0^+$ level at 5.213 MeV. The 1.90 MeV initial gamma ray of this decay is not seen in this spectrum, possibly due to angular correlation effects. The absence of these gamma rays in the Li-Ge spectrum in figure 20b lends support to the suggestion of a doublet at 7.114 MeV.

If the transition to the 5.213 MeV $0^+$ level exists, theoretical considerations (discussed in section V) suggest another transition to the $2^+$ level at 5.628 MeV. Careful examination of both of the NaI(Tl) spectra show weak peaks in the gamma ray energy regions around 1.5 MeV and 1.9 MeV and a 1.3 MeV peak in figure 21a that is absent in figure 20a. The two Li-Ge spectra may also show evidence for a weak gamma ray in the energy range 1.485 to 1.501 MeV. There is no indication of the 5.63 MeV gamma ray from the decay of the 5.628 MeV level, but the FEP efficiency for this gamma ray is down one-half from the efficiency for detecting the 1.5 MeV gamma ray. Consequently this high energy gamma ray may be present, but so weakly that it is lost in the statistics of the Compton region of the 7.11 MeV gamma ray.
Careful consideration of all of this evidence together with the errors in the gamma ray energies (approximately 5 keV for these spectra) and the errors in the energies of the states involved in these cascades (approximately 5-10 keV) shows that the present data seem to support a weak cascade through the 5.613 MeV level, consistent with the results of Tellez et al.\textsuperscript{51} and Dolan et al.\textsuperscript{55} This conclusion is made on the existence of the weak 1.5 and 1.9 MeV gamma rays in the NaI(Tl) spectrum of figure 20a, where the corresponding Li-Ge spectrum of figure 20b (pg.120) does not show any evidence for the 3.90-1.30 MeV gamma rays. These gamma rays, seen in figure 21b (pg.121), are then associated with the other component of a doublet that is only weakly populated at the 10.81 MeV beam energy. A 5.628 MeV branch from this other assumed component is consistent with the present data, although there is no convincing evidence for its presence.

The decay scheme suggested by the present work is shown in figure 19d (pg.116). The uncertainty about the existence of the 5.628 MeV branch is reflected in the use of dotted lines. The 7.11 MeV gamma ray and proposed 1.90 MeV gamma ray are assumed to originate from the same component of a doublet, which most probably has positive parity.
The remaining gamma rays can then be associated with the decay of the negative parity component seen by Erskine\textsuperscript{9}) and Seth et al.\textsuperscript{10})

3. Branching Ratio Measurements

Branching ratio measurements were performed on the proposed negative parity component of the doublet using both Li-Ge and NaI(Tl) detectors at a beam energy of 10.81 MeV. The basic technique in branching ratio measurements is to carry out an angular correlation measurement on the level of interest. One gamma ray from each cascade is then fitted to an expansion of the form:

\[ W(\Theta) = \sum_{k \text{ even}} a_k P_k(\cos \Theta) \]  \hspace{1cm} (39)

The \( a_0 \) coefficient in this expansion represents the relative intensity of that cascade, since:

\[ a_0 = \int_{\text{SPHERE}} W(\Theta) \, d\Omega \]  \hspace{1cm} (40)

from the orthogonality of the Legendre polynomials. Dividing \( a_0 \) by the detector efficiency for that gamma ray energy then gives the relative yield of that cascade from which the branching ratios can be calculated.
In the NaI(Tl) measurements, the 5.613 MeV branch was assumed negligible. The 3.737 FEP yield is then composed of gamma rays associated with either the 3\(^{-}\) or the 5\(^{-}\) branch. The NaI(Tl) detector efficiency \(K\) for 0.75 MeV and 3.73 MeV gamma rays was accurately measured by reversing the technique described above for the 4.49 MeV level where the branching ratio is known to be 100\%. Measuring an \(a_0\) for the 0.754 MeV gamma ray correlation in the decay of the 7.114 MeV level and multiplying by \(K\) then gives the fraction of 3.73 MeV gamma rays associated with the cascade through the 5\(^{-}\) level. The remainder of the 3.73 MeV gamma rays must then belong to the 3\(^{-}\) cascade. Since all necessary coefficients are measured in the experiment, the resulting branching ratio is expected to be fairly reliable, subject to the assumption of a negligible decay through the 5.613 MeV level.

In the Li-Ge measurements, the \(a_0\) coefficients for the initial gamma ray in each cascade were measured, including the suspected 1.50 MeV gamma ray. The Li-Ge detector efficiencies and target chamber wall absorption effects were included in the branching ratio calculation. Since the statistical errors on the \(a_0\) coefficients were quite high and the Li-Ge detector efficiencies have some uncertainty, the resulting branching ratios are to be viewed with a
certain amount of caution. The results are tabulated in Table IV:

Table IV

7.114 MeV Branching Ratios

<table>
<thead>
<tr>
<th>detector</th>
<th>per cent branch to:</th>
<th>3.737</th>
<th>4.491</th>
<th>5.613</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaI(Tl)</td>
<td>74.3 ± 3.8</td>
<td>25.7 ± 1.9</td>
<td>assumed 0</td>
<td></td>
</tr>
<tr>
<td>Li-Ge</td>
<td>68.5 ± 6.8</td>
<td>31.5 ± 4.3</td>
<td>assumed 0</td>
<td></td>
</tr>
<tr>
<td>Li-Ge</td>
<td>62.2 ± 6.1</td>
<td>28.8 ± 3.9</td>
<td>8.8 ± 1.6</td>
<td></td>
</tr>
</tbody>
</table>

The fair agreement between the NaI(Tl) and Li-Ge measurements for the assumption of no 5.613 MeV branch lends confidence to the Li-Ge results. Thus the calculated 8.8% branch for the 5.613 MeV level can be taken as a fairly reliable upper limit. This value is seen to be less than half of the 17% branch reported by Dolan et al. 55)

4. Angular Correlation Measurements

Angular correlation measurements were performed on the 7.114 MeV ground state transition at two different beam energies, 9.86 MeV and 10.81 MeV, using NaI(Tl) detectors. Since there is no evidence for other gamma rays of energies greater than 3.90 MeV, the 7.114 MeV gamma ray yield was summed down to 4.0 MeV in order to improve the statistics. The correlation functions with their associated errors were
formed in the usual manner and are shown in figures 22a (pg. 129) and 22b.

The AC5 program was then used to analyze these correlations. Since this decay goes by a pure multipole gamma ray, there is no mixing ratio to vary. Spins higher than 3 were not tested since it was felt an \( L = 4 \) gamma ray could not compete favorably with lower order multipole cascades. Spin 0 is excluded by the observation of the gamma ray in the first place. The results of this fitting for spins 1, 2 and 3 are shown in Tables V and VI:

**Table V**

7.114 MeV Gamma Ray
9.86 MeV

<table>
<thead>
<tr>
<th>spin</th>
<th>( P(0) )</th>
<th>( P(1) )</th>
<th>( \chi^2 )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.226 ± .055</td>
<td>.387 ± .034</td>
<td>6.2</td>
<td>&lt;.1%</td>
</tr>
<tr>
<td>2</td>
<td>.210 ± .033</td>
<td>.395 ± .020</td>
<td>.55</td>
<td>65.0%</td>
</tr>
<tr>
<td>3</td>
<td>.475 ± .035</td>
<td>.262 ± .023</td>
<td>23.1</td>
<td>&lt;.1%</td>
</tr>
</tbody>
</table>

**Table VI**

7.114 MeV Gamma Ray
10.81 MeV

<table>
<thead>
<tr>
<th>spin</th>
<th>( P(0) )</th>
<th>( P(1) )</th>
<th>( \chi^2 )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.160 ± .031</td>
<td>.420 ± .016</td>
<td>3.68</td>
<td>1.5%</td>
</tr>
<tr>
<td>2</td>
<td>.295 ± .017</td>
<td>.352 ± .012</td>
<td>1.94</td>
<td>12.0%</td>
</tr>
<tr>
<td>3</td>
<td>.420 ± .018</td>
<td>.289 ± .012</td>
<td>42.34</td>
<td>&lt;.1%</td>
</tr>
</tbody>
</table>
Figure 22

7.114 MeV Level Angular Correlations

a. 7.114 MeV gamma ray correlations
   $E_{\text{beam}} = 9.86$ MeV

b. 7.114 MeV gamma ray correlations
   $E_{\text{beam}} = 10.81$ MeV

c. 3.44 MeV gamma ray correlations

d. 0.75 MeV gamma ray correlations
From these two independent measurements, the spin of the level associated with the ground state decay is unambiguously $J = 2$. This is in good agreement with the $2^+$ spin reported by Blum et al.\textsuperscript{56}) The best fits to the data for each spin tested are shown as continuous curves in figures 22a (pg.128) and 22b.

If the 7.114 MeV level were a singlet, the observed decay to the $5^-$ level would then require an E3 gamma ray, using Blum et al.'s\textsuperscript{56}) positive parity assignment. The extreme single particle model predicts a lifetime of $10^{-7}$ seconds for such a gamma ray, just at the limits of the experimental resolving time. So the observation of the $5^-$ branch and the presence of a negative parity state seen by Erskine\textsuperscript{9}) and Seth et al.\textsuperscript{10}) are strong evidence for the existence of a doublet.

5. Lifetime Measurements

To resolve this controversy, Doppler shift attenuation measurements were made on the 3.38 MeV and 2.62 MeV gamma rays. The method of analysis was identical to that used in the treatment of the 6.75 MeV level and the results are shown in Table VII. Figure 23 (pg.131) shows the data and gaussian fits obtained for the gain-shifted spectra at 45° and 120°.
Table VII
7.114 MeV Level
Doppler Shifts

<table>
<thead>
<tr>
<th>Angular range</th>
<th>2.36 MeV</th>
<th>3.37 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°-120°</td>
<td>22.6 ± 3.2</td>
<td>.82 ± .11</td>
</tr>
<tr>
<td>45°-120°</td>
<td>19.8 ± 3.2</td>
<td>.82 ± .13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angular range</th>
<th>2.62 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°-120°</td>
<td>13.5 ± 3.3</td>
</tr>
<tr>
<td>45°-120°</td>
<td>12.6 ± 3.3</td>
</tr>
</tbody>
</table>

Using the slowing-down curve shown in figure 19 again, these shifts yield lifetimes summarized in Table VIII. The energy of the emitting level and the initial Doppler shifting gamma ray are included, calculated from unshifted gamma ray data taken at 90°.

Table VIII
7.114 MeV Level
Lifetimes and Energies

<table>
<thead>
<tr>
<th>Gamma Ray observed E(keV)</th>
<th>Observed F</th>
<th>Lifetime (x10⁻¹⁵ sec.)</th>
<th>Energy of initial state (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3376.2 ± 2.4</td>
<td>.765 ± .062</td>
<td>234 ± 64</td>
<td>7113.3 ± 2.5</td>
</tr>
<tr>
<td>2622.0 ± 2.3</td>
<td>.642 ± .115</td>
<td>390 ± 150</td>
<td>7113.1 ± 2.5</td>
</tr>
</tbody>
</table>
Figure 23

Doppler Shift Measurements

6.75 MeV Level

a. Li-Ge detector at 45°

b. Li-Ge detector at 120°
The 3.376 MeV gamma ray shows a substantial Doppler shift and should provide a fairly good measure of the lifetime of its parent level. The poorer statistics available on the 2.622 MeV gamma ray limit its interpretation as being compatible with coming from the same parent level as the 3.376 MeV gamma ray. This relation is supported by the agreement in both the calculated energy of the parent level and the Doppler shifts for the two gamma rays. These measured lifetimes are approximately half of the value measured by Lindeman et al.\textsuperscript{47}, but it is felt the present experiment is a more accurate determination of the lifetime.

The observed Doppler shift of the 2.622 MeV gamma ray indicates a lifetime at least $10^6$ times shorter than the order of magnitude estimate given by the single particle lifetime, if the parent level were a $J = 2$ state. Such an enhancement is unprecedented, so the 7.114 MeV gamma ray and the 2.622 MeV gamma ray must come from different states. A doublet does indeed exist at 7.114 MeV, and on the basis of the gamma ray decay spectra shown earlier and the agreement in Table VIII, it is reasonable to assume the 3.376 MeV gamma ray and the 2.622 MeV gamma ray have the same parent level.
6. Angular Correlation Measurements

To further verify this conclusion, angular correlations were measured for the 3.376 MeV gamma ray and the 0.754 MeV gamma ray, treating the initial 2.622 MeV gamma ray as unobserved. Spectrum stripping similar to that used for the analysis of the 6.75 MeV level was used. The resulting correlation functions and associated errors are shown in figures 22c and 22d (pg. 128).

The results of fitting these two correlations with the AC5 program are shown in figures 24a and 24b (pg. 135). Since spins 1 and 2 have been ruled out by lifetime considerations and the $^{39}\text{K} (\text{He}^3,\text{d})^{40}\text{Ca}$ work of Erskine$^9$ and Seth et al.$^{10}$ excluded spins with $J > 3$, only spin 3 was tested. The 3.376 MeV gamma ray correlation fits to good $\chi^2$ values less than 1 for mixing ratios of 20° and -75°. In fitting the "skip correlation" for the 0.75 MeV gamma ray, a knowledge of the mixing ratio $\delta_4$ describing the $5^- \rightarrow 3^-$ transition is necessary. Figure 24b shows four fits obtained assuming different mixing ratios of 0°, 2.5°, 5° and 7.5°. As discussed earlier in the analysis of the 4.49 MeV level, this present work seems to require a non-zero mixing ratio for this gamma ray on the order of 5°. This earlier result is confirmed
here by the poor fit with $\chi^2$ barely coming below the 1% probability level for 0° mixing. The fit rapidly improves as $\delta_4$ is increased up to 5°, coming down to the 10% probability level, and then seems to remain constant. A value of 5° will be assumed for $\delta_4$ in the subsequent data analysis. In all cases, this best fit occurs for zero $\delta_2$, the mixing ratio of the unobserved 2.622 MeV gamma ray. This is exactly the value expected for a $3^- \rightarrow 5^-$ transition, where $\delta_2 = M3/E2$.

The real ambiguity now lies in the two possible mixing ratios found for the 3.376 MeV gamma ray. The obvious step to eliminate this problem would be a simultaneous fit of the 3.376 MeV and associated 3.737 MeV gamma rays. This introduces new difficulties since the 4.491 MeV branch also feeds the 3.737 MeV level as the third step in a cascade and the 5.613 MeV level, if populated, has two cascades through this $3^-$ level. So the observed 3.737 MeV correlation is at best a mixture of two separate correlations, and very likely four such components are present.

Assuming all contributions to the 3.737 MeV FEP yield come from the same $3^-$ level at 7.114 MeV, the same set of population parameters must describe each of the 3.737 MeV correlation components. The Cile9 program was modified to add all of the theoretical contributions for the 3.737 MeV
Figure 24

($\chi^2 - \theta$) Fits of Correlation Data

7.114 MeV Level

a. 3.37 MeV gamma ray single fit

b. 0.75 MeV gamma ray single fit

c. 3.37-3.73 MeV gamma rays simultaneous fit
peak together for an assumed decay scheme for the 7.11 MeV level and compare the result to the observed composite 3.737 MeV correlation while simultaneously fitting the 3.376 MeV correlation. Two cases were tested; one assuming no 5.613 MeV branch and one assuming a 17% branch to that level, taking Dolan et al.'s\(^56\) branching ratio as the worst case. The results of this fitting are shown in figures 24c and 24d (pg.135).

For no 5.613 MeV branch, a 75%-25% branching ratio was assumed (Table IV) and all values of \(\delta_2\), the mixing ratio for the unobserved 2.622 MeV gamma ray, were tested for each value of the mixing ratio \(\delta\) for the 3.736 MeV gamma ray. This simultaneous fitting eliminates the ambiguity as seen in figure 24c (pg.135), where \(J = 3\) now fits for \(\theta = -75^\circ\), \(\delta_2 = 0^\circ\). The previous \(J = 3\) fit at \(\theta = 20^\circ\) is totally excluded.

The same method was used for the 0.75-3.73 MeV simultaneous fit shown in figure 24d. Again the only fit coming below the 1% probability level occurs for \(\theta = 80^\circ\) \(\delta_2 = 0^\circ\). These two fits in figures 24c and 24d are consistent in that each predicts the same mixing ratios \(\delta\) and \(\delta_2\).

The possibility of a 5.613 MeV branch was then included in the 3.376 MeV simultaneous fit in figure 24c. The branch-
ing ratios of Dolan et al.\textsuperscript{56} were assumed and the decay properties of the 5.613 MeV level taken as those in figure 10. The 2.622 MeV gamma ray mixing ratio $\delta_2$ was now assumed fixed at 0° and $\delta_3$, the mixing ratio for the 1.501 MeV gamma ray feeding the 5.613 MeV level, was varied over all possible values for each value of $\delta$ tested. The modified Cile9 program was again used treating the 3.737 MeV correlation with four components. The best fit was found to be at mixing ratios $\theta = -72.5°$ with $\delta_3 = 0°$, with no ambiguities.

It is interesting to note the great similarity between the fits in figure 24c for a 0% and a 17% branch through the 5.613 MeV level. This similarity comes from the fact that each successive gamma ray correlation in a cascade is damped out and becomes less anisotropic. The 3.737 MeV gamma ray contributions for the 4.491 MeV and 5.613 MeV branches correspond to at least the third gamma ray in a cascade and have the same weak $l. + \approx 1/2 P_2(\cos\theta)$ dependence on angle. In practice, this similarity makes the actual decay scheme unimportant for even large branches through the 5.613 MeV level. Regardless of the origin of the 3.737 MeV gamma rays, the simultaneous fit with the 3.376 MeV correlation will fit only for $\theta = -75° \pm 10°$. 
To summarize the analysis, the level at 7.114 MeV is a doublet with the decay schemes shown in figure 19d. The two components are a $2^+$ level and a $3^-$ level. The $3^-$ is a fairly short-lived state and has its major decay modes to the $3^-$ and $5^-$ levels. There is some evidence for a weaker third branch to the $4^-$ level at 5.613 MeV. Mixing ratios have been unambiguously determined as $\delta = -75^\circ \pm 10^\circ$, $\delta_2 = 0^\circ \pm 20^\circ$, and $\delta_3 = 35^\circ$.

This corresponds to a $\delta_3 = 0.7$ for an E2/M1, which is an acceptable value for a $3^- \rightarrow 4^-$ transition. No errors are quoted on $\delta_3$ since the $\chi^2$ curve for it was quite flat, due to the dampening effect previously discussed.
V. CONCLUSIONS

A. Experimental Summary

It has been seen in the discussion of the experimental results that the present work verifies the $J = 5$ spin assignment to the 4.49 MeV level and, supported by the summary of work in Table I and the data analysis on the 7.114 MeV level, strongly suggests some M3 mixing in the dominantly E2 gamma ray transition from this level. Taking an average of the values listed in Table I and computing the rms error, a value for the mixing ratio of $0.0475 \pm 0.078$ is obtained. A unique spin-mixing ratio assignment has been made to the level at 6.75 MeV and lifetime measurements performed. Finally the presence of a doublet at 7.113 MeV has been established with spin assignments to both members and lifetime measurements on the negative parity level of the doublet. These results are summarized in Table VIII. The parity assignments in this table are based on evidence presented in the experimental discussion of section IV and other work on Ca$^{40}$ previously discussed. Errors in the measurements are listed below the measured values.
### Table VIII

#### Experimental Results

<table>
<thead>
<tr>
<th>Energy of level (keV)</th>
<th>$J^\pi$</th>
<th>Energy of gamma-ray decay (keV)</th>
<th>% branch to level at $\delta$ (x10$^{-15}$ sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4491.</td>
<td>5(-)</td>
<td>*</td>
<td>100% to 3737, 3$^-$, 0.08 $^+$0.08</td>
</tr>
<tr>
<td>6752.1</td>
<td>2(-)</td>
<td>3015.0 3$^+$1.7</td>
<td>100% to 3737, 3$^-$, .84 $^+$0.22, 205 $^+$50</td>
</tr>
<tr>
<td>7113.1</td>
<td>3(-)</td>
<td>3376.2 $^+$2.4</td>
<td>75% to 3737, 3$^-$, (-3.73 $^+$234 $^+$64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2622.0</td>
<td></td>
<td></td>
<td>25% to 4491, 5$^+$2.3, 0 $^+$0.36, 390 $^+$150</td>
</tr>
<tr>
<td>(1.498)</td>
<td></td>
<td></td>
<td>8% to 5613, 4$^-$, .70 $^+$0.8</td>
</tr>
<tr>
<td>(7114.</td>
<td>2(+*)</td>
<td>*</td>
<td>a% to gnd, 0$^+$</td>
</tr>
<tr>
<td></td>
<td>$^+$10.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>b% to 5.201, 0$^+$</td>
</tr>
</tbody>
</table>

* - not measured in present work

#### B. Theoretical Predictions

1. **Negative Parity Levels**

The negative parity levels predicted by Gerace and Green$^3$ on the basis of (3p-3h) deformed state calculations are shown in figure 25 (pg. 142). The experimentally known
levels are included for comparison, as well as the levels calculated by Dieperink et al.\textsuperscript{11} using (1p-1h) states. The latter predictions are included as representative of the results predicted by simpler shell model calculations on Ca\textsuperscript{40} without using (3p-3h) excitations.

It is immediately obvious that the (1p-1h) calculation not only leaves a large gap in the level structure from 6 to 7.5 MeV, but predicts the wrong level order. On the other hand, the deformed state calculations of Gerace and Green\textsuperscript{3} predict the correct level order up to 6.5 MeV, although the predicted energies are higher than the observed energies for all but the lowest excited states.

If the order of the 2\textsuperscript{−}, 4\textsuperscript{−} pair predicted at 7 MeV by Gerace and Green could be changed without disturbing the basic features of their model, the predicted 2\textsuperscript{−} can then be associated with the 2\textsuperscript{−} found at 6.75 MeV by the present work. This would seem to be an acceptable proposal since the final energies calculated by Gerace and Green depend rather strongly on a number of the parameters used in the theory. The 3\textsuperscript{−} state predicted at 8.2 MeV might then be tentatively equated to the 3\textsuperscript{−} seen at 7.114 MeV in this present work.

This leaves the 4\textsuperscript{−} predicted at 7.2 MeV without a known experimental level, unless it corresponds to the T = 1 4\textsuperscript{−}.
Figure 25

Negative Parity Levels in Ca
level at 7.6 MeV. This is considered unlikely, since it would then be the one state predicted at an energy lower than the associated level in Ca$^{40}$. There are a number of unassigned levels left in Ca$^{40}$ that could be identified with this predicted 4$^{-}$. Probably the most promising is the level at 6.93 MeV since it is now the last unidentified level still below the predicted energy of 7.2 MeV. There are also a number of levels above 7.2 MeV of unknown spin-parity that would also be possible candidates.

Additional support for the calculations of Gerace and Green comes from a comparison of the known B(E2) values (equivalently, the measure of the gamma ray transition strength for a particular decay) to the predicted values using the wave functions calculated by Gerace and Green. This summary was prepared by MacDonald et al.$^{12}$ and is presented in Table IX. A few of the predictions based on the calculations of Dieperink et al.$^{11}$ are included for comparison. The subscript on the spins in Table IX indicates the order in which that level appears in the spectrum of the nucleus, counting only levels of the same spin. The B(E2)'s are calculated from the relation$^{54}$:

$$\Gamma_0 = \frac{4\pi}{75} \left( \frac{E_2}{\hbar c} \right)^5 B(E2)$$

(41)
where \( c \) is the velocity of light, \( h \) is Planck's constant, and \( \Gamma_p \) is the partial width of the level for an E2 gamma ray transition. \( \Gamma_p \) is calculated from the full width of the level by taking the percentage E2 present, obtained by a measurement of the mixing ratio in the particular branch of interest. The full width \( \Gamma \) is obtained from the lifetime of the level by the relation \( \Gamma \tau_m = h \).

Table IX

B(E2) Values

<table>
<thead>
<tr>
<th>Transition</th>
<th>Experiment(^{12})</th>
<th>Theory(^{12})</th>
<th>Dieperink et al.(^{11})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5^- \rightarrow 3^- ) (_1^1 )</td>
<td>8.4 ± 0.3</td>
<td>7.6</td>
<td>0.17</td>
</tr>
<tr>
<td>( 4^- \rightarrow 3^- ) (_1^1 )</td>
<td>1.9 ± 0.8</td>
<td>2.5</td>
<td>0.35</td>
</tr>
<tr>
<td>( 4^- \rightarrow 5^- ) (_1^1 )</td>
<td>48. ± 23.</td>
<td>53.6</td>
<td>15.</td>
</tr>
<tr>
<td>( 1^- \rightarrow 3^- ) (_1^1 )</td>
<td>&lt;25.0</td>
<td>22.0</td>
<td></td>
</tr>
<tr>
<td>( 2^- \rightarrow 3^- ) (_1^1 )</td>
<td>37.0 ± 5.0</td>
<td>34.9</td>
<td>4.7</td>
</tr>
<tr>
<td>( 3^- \rightarrow 3^- ) (_1^1 )</td>
<td>&lt;1.0</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>( 2^- \rightarrow 5^- ) (_1^1 )</td>
<td>72.0 ± 18.0</td>
<td>72.0</td>
<td></td>
</tr>
<tr>
<td>( 3^- \rightarrow 3^- ) (_1^1 )</td>
<td>10.6 ± 3.6</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>( 3^- \rightarrow 5^- ) (_1^1 )</td>
<td>4.1 ± 2.0</td>
<td>4.7</td>
<td></td>
</tr>
<tr>
<td>( 2^- \rightarrow 3^- ) (_2^1 )</td>
<td>6.26 ± 1.9</td>
<td>no comparison</td>
<td></td>
</tr>
<tr>
<td>( 3^- \rightarrow 3^- ) (_4^1 )</td>
<td>5.23 ± 1.6</td>
<td>no comparison</td>
<td></td>
</tr>
<tr>
<td>( 3^- \rightarrow 5^- ) (_4^1 )</td>
<td>7.0 ± 2.1</td>
<td>no comparison</td>
<td></td>
</tr>
</tbody>
</table>
The excellent agreement between experiment and theory for the values listed by MacDonald et al.\textsuperscript{12} are impressive support for the predictions of Gerace and Green, while the B(E2) values predicted by Dieperink et al.\textsuperscript{11} are consistently too low. The B(E2)'s measured in this present work fall within the range of other known values, giving promising indications that these new levels can also be fitted into the work of Gerace and Green\textsuperscript{3}.

2. Positive Parity Levels

The positive parity level predictions of Gerace and Green\textsuperscript{3} are shown in figure 26 together with the experimentally known levels for comparison. The (2p-2h), (4p-4h) $K = 0,2$ and proposed (8p-8h)$K = 0$ bands account for all of the known positive parity levels up to 7 MeV excitation energy. There is remarkable agreement between the experimental and predicted energies. Numerous tabulations\textsuperscript{1,2,3,45,46} also exist comparing the B(E2) predictions for gamma ray transitions between these levels to the experimental values. Again the agreement is quite good.

The (4p-4h) $K = 2$ band proposed by Gerace and Green is of special interest in view of the results of the present experiment. This band has a level sequence $2^+, 3^+, 4^+$.
with the $2^+$ head of the band showing decays to the $0^+$ and $2^+$
levels in the (4p–4h) $K = 0$ band and the $0^+$ ground state.
The decay of the recently identified $2^+$ level at 7.114 MeV
looks very much the same, with a strong transition to the
$0^+$ ground state and a weaker decay to the $0^+$ level at 5.213
MeV. On this basis, the $2^+$ at 7.114 is tentatively proposed
as the head of a (8p–8h) $K = 2$ band to explain its known de-
cay to the 5.213 MeV level. If this interpretation is cor-
rect, there should also be a decay to the 5.628 MeV $2^+$
level in the (8p–8h) $K = 0$ band. While the present work
does not confirm such a decay, the data does not rule out
such a possibility. Additional work will be required to con-
firm or deny this proposed assignment.

C. Proposals

Angular correlations coupled with the use of the Doppler
shift attenuation method for measuring lifetimes have been
shown to form a powerful spectroscopic tool for investigating
excited levels in nuclei. Angular correlations, both doubles
and triples, may be performed in several geometries to make
an unambiguous spin-mixing ratio assignment to a level of
interest. Methods of decomposing complicated gamma ray
spectra and simultaneously fitting correlation data from a


Figure 26

Positive Parity Levels in Ca$^{40}$
number of different experiments have been developed. This information together with lifetime measurements on the level allow the B(E2) values (or in general, the B(E2rM,L) values) to be determined. As seen from the theoretical discussion, these quantities are quite important in comparing experimental results to theoretical predictions.

Problems of immediate interest would be a closer examination of the decay of the 7.114 MeV $2^+$ level. The presence of a decay to the $2^+$ 5.628 MeV level should be verified or denied. Lifetime and branching ratio measurements on the known decays to the ground state and the 5.213 MeV level can also be performed to obtained values for the B(E2)'s.

Equally interesting would be a search for the predicted $4^-$ level in the 7 MeV region. A number of candidates are available for this state, but the most promising is the level at 6.93 MeV. This is the middle member of a triplet, but both of the other members decay 100% by ground state transitions, so the necessary spectrum stripping to obtain correlations for the cascades of the middle member should be fairly uncomplicated.
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41. L. R. Greenwood, private communication.


ACKNOWLEDGEMENTS

I wish to thank Dr. C. M. Class for his suggestion of this project and his guidance of the various phases of the work to a successful completion. A large measure of thanks must also go to Dr. R. S. Cox for his assistance in the experimental work and subsequent data analysis. I also wish to acknowledge the many helpful and enlightening discussions with both Dr. C. M. Class and Dr. R. S. Cox throughout this work.

I wish to acknowledge the help of R. W. Dougherty in the Doppler shift calculations and the contribution of Dr. L. R. Greenwood, who wrote the Gaussian fitting program used in the Doppler shift analysis.

I would like to thank the staff of the Rice University Physics Department machine shop, and particularly Mr. Pete DeVries, for their part in building the experimental apparatus. I also appreciate the assistance of Mr. James Buchanan in designing the electronics circuits and Mr. Hugh Jones in writing the computer tape analysis programs.

I would like to express my appreciation to Rice University and to the U. S. Atomic Energy Commission for the financial support that made this work possible. I also
wish to thank Mrs. Mary Comerford for her part in typing this thesis.

Finally I wish to express my gratitude to my wife Cile, for her patience and support during the last three years, and particularly for her assistance in the final preparation of this thesis.