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"n-p Correlations in the $^{12}\text{C}(d,\text{pn})^{12}\text{C}$ Reaction:  
A Search for Proximity Scattering and 
Singlet Deuteron Contributions"

by

Julian Sandler

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<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. EXPERIMENTAL APPARATUS AND PROCEDURES</td>
<td>6</td>
</tr>
<tr>
<td>A. Reaction Chambers</td>
<td>6</td>
</tr>
<tr>
<td>B. Targets</td>
<td>8</td>
</tr>
<tr>
<td>C. Detectors</td>
<td>9</td>
</tr>
<tr>
<td>D. Pulse Shape Discrimination</td>
<td>12</td>
</tr>
<tr>
<td>E. Charged Particle-neutron Time-of-Flight Coincidence Circuitry</td>
<td>25</td>
</tr>
<tr>
<td>F. Calibrations and Detector Efficiencies</td>
<td>25</td>
</tr>
<tr>
<td>III. REACTION THEORY</td>
<td>33</td>
</tr>
<tr>
<td>A. Kinematics for Simultaneous Breakup</td>
<td>34</td>
</tr>
<tr>
<td>B. Kinematics for Sequential Decay</td>
<td>37</td>
</tr>
<tr>
<td>C. Proximity Scattering</td>
<td>42</td>
</tr>
<tr>
<td>IV. EXPERIMENTAL DATA</td>
<td>47</td>
</tr>
<tr>
<td>V. RESULTS AND CONCLUSIONS</td>
<td>52</td>
</tr>
<tr>
<td>VI. APPENDICES</td>
<td>56</td>
</tr>
<tr>
<td>A. Derivation of Zero-Crossover Time Expression for the P.S.D. System</td>
<td>56</td>
</tr>
<tr>
<td>B. Experimental Detection Efficiency for the Neutron Counter</td>
<td>58</td>
</tr>
<tr>
<td>C. Theoretical Detection Efficiency for the Neutron Counter</td>
<td>62</td>
</tr>
</tbody>
</table>
D. Kinematics for Simultaneous Breakup

E. Kinematics for Sequential Decay

F. The Density of States Functions

G. Kinematical Limits on Proximity
Scattering from a Classical Approach

VII. REFERENCES

VIII. ACKNOWLEDGEMENTS
I. INTRODUCTION

In attempting to reach an understanding of the nature of nuclear interactions, much attention has been given in the past few years to the study of 3-body, final state interactions. Technological advances have greatly facilitated not only the acquisition, but also the analysis and interpretation of the data. Undeterred by the complexities of even the 3-nucleon problem\(^1,2,3\), experimentalists have extended their studies to more involved systems.

The \(^{12}\text{C}(d,\text{pn})^{12}\text{C}\) reaction was studied by Pitts et al.\(^4\), and from an interpretation of the proton spectra alone, the sequential decay via excited states in \(^{13}\text{N}\) was established. Current interest in this reaction has also been stimulated by the studies of Lang et al.\(^5\), who reported indications of "proximity scattering" in the reaction mechanisms.


2.

Investigation of this reaction would also yield information on a possible "singlet deuteron" final state interaction. In the complex momentum plane, the virtual singlet state of the deuteron is described by an S matrix pole on the negative k axis, and its direct physical observation is thus precluded. The n-p-singlet phase shifts are however influenced by the existence of this virtual singlet state, and they exhibit a maximum of ~40° near a relative energy of 40-60 keV. Coupled with phase-space, this gives rise to a broad peak in the 3-body cross-section. One can thus consider a quasi-particle consisting of a neutron and a proton in a $^1S_0$ configuration which behaves like a short-lived particle. Simpson $^1$, and Jackson $^2$ observed experimental evidence for singlet deuteron interactions in the p+d-p+p+n reaction, while Cohen et al. $^6$ have recently studied their production in proton induced reactions on $^7$Li, $^9$Be, and $^{13}$C. In all these systems, conservation of isospin does not preclude the formation of the singlet deuteron. However, in the d + $^{12}$C reaction, singlet deuteron production would violate isospin selection rules, and would be of particular experimental and theoretical interest.

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In this respect, Bonner et al. \(^7\), found clear evidence in the \(d + ^{12}\text{C}\) reaction for compound nucleus resonances in \(^{14}\text{N}^*\) having both \(T = 0\) and \(T = 1\) isospin admixtures, while the isospin-nonconserving reaction, \(^{12}\text{C}(d, \alpha) \rightarrow ^{10}\text{Be}^*\), has been extensively studied both experimentally\(^8\) and theoretically\(^9\). The data exhibited an excitation function like that of a compound nucleus in the region of giant resonances, while the angular distributions were indicative of a direct-interaction mechanism.

By studying the \(^{12}\text{C}(d, pn) \rightarrow ^{12}\text{C}\) reaction in a kinematically complete experiment, the contributions through proximity scattering and singlet deuteron production could be determined.

For a three-body final state, there are nine degrees of freedom. Conservation of energy, and linear momentum reduce the complete specification of the system to a knowledge of five independent scalar variables. In the centre-of-mass system, this would describe the final state adequately. However, the transformation to the laboratory system introduces a quadratic dependence between the kinetic energies of


the final state particles, and six variables are therefore necessary to specify the system completely. In general, the energies and angles of emission of two of the final state particles are measured, and the coincident spectra of these two particles will be constrained to lie on a calculable locus in the $E_1 - E_2$ plane.

If the reaction proceeds directly with the simultaneous emission of the three final state particles, the distribution along this locus will be modulated only by the available phase space. If, however, the reaction proceeds via sequential decay, or if there is a final state interaction between the particles, there will be an intensification of the yield where the relative momentum of these two particles corresponds to the excitation energy of the "composite state." As will be shown in section III C, proximity scattering will also produce an increased yield along the locus, and the relative momentum at which this enhancement occurs is directly related to the energy of the incident particle.

The observed distribution of the yield along the permitted locus for the three body reaction under various kinematical and geometrical conditions thus enables one to determine the reaction mechanisms involved.
The purpose of undertaking this study of the $d + ^{12}C \rightarrow p + n + ^{12}C$ reaction was therefore:

i) to refine and develop experimental techniques permitting coincidence studies of charged particles and neutrons resulting from a three body final state interaction,

ii) to resolve the ambiguity of the experimental data published thus far on proximity scattering contributions to this reaction, and

iii) to seek experimental evidence for singlet deuteron production.
II. EXPERIMENTAL APPARATUS AND PROCEDURES

The low yield characteristics of p-n coincidence studies necessitated extreme care with regard to background radiation. Specifically, much attention was given to the scattering chamber design, and the beam was monitored continuously to minimize instability effects. Electronic circuits, ideally suited for nanosecond time-of-flight applications, have now become commercially available, and the techniques developed to make such measurements routine will be described in detail.

A. Reaction Chambers. Both deuteron and proton beams, the latter primarily for alignment purposes, were used extensively in these studies, and were provided by the Rice University Tandem Van der Graaff accelerator.

For the three body investigations, as well as for the absolute neutron detector efficiency and pulse shape discrimination studies, the scattering chamber used was that designed by Velkeley. Figure 1 is a schematic representation of this chamber, which is essentially a stainless steel spherical shell, 12 inches in diameter, with walls .050" thick. The beam entrance port is a 4 inch diameter hole, flanged to accept a brass plate. Two stainless steel rods,
Figure 1. A schematic diagram of the "Velkely" chamber designed to minimize multiple scattering and background contributions.
positioned on either side of the beam axis, extend to the centre of the sphere from this plate, and support the target holder, the charged particle detector, which may be positioned externally to the desired angle, a viewing quartz, and an insulated tantallum collimating slit. A second such disk, its slit diameter being 1.5 mm., was situated 38.5 cms before the target holder. To further minimize gamma and neutron background, the beam was transported 15 meters beyond the scattering chamber, and deposited in a heavily shielded concrete beam dump housing. In addition, lead sheeting was used to line the 4" beam pipes. With both collimating slits in position, the total slit current was minimized, and the target slit was then rotated 180° about a horizontal plane out of the beam line. Continuous monitoring of the remaining slit current and frequent checks helped to maintain a stable, clean situation. It should be noted that the neutron counter was effectively shielded from the upstream collimating slit.

The beam current was monitored in a Faraday cup in the conventional manner, and though its accuracy was not critical, evaluation with a standard cell and resistor showed its absolute error to be less than 2%.
For relative neutron detection efficiency measurements, a gaseous deuterium target was used, and the appropriate chamber was that constructed by Bonner\textsuperscript{10}) \cite{c.f., Figure 2}. The gas cell length was 3 cms, with an entrance foil of .00008" nickel, and the seal was effected by pressure contact with an "O" ring. The target-gas pressure was maintained at 19.2 p.s.i. For a 3.0 MeV incident deuteron, the energy loss in traversing the nickel foil is 195 KeV\textsuperscript{11}). Referring again to the tabulations of Marion\textsuperscript{11)}, the specific energy loss of such a beam in the gaseous target is 79 KeV at the target centre. Thus, the mean deuteron beam energy at the centre of the gas target was 2.726 MeV. A neutron beam was then obtained by means of the d(d,n)\textsuperscript{3}He reaction, whose Q-value is \(3.26844 \pm 0.00042\) MeV\textsuperscript{11)}, and the 0° neutron energy was therefore 5.19 MeV, the relevant cross-section being 54.7 \(+\ .2 \) mb/sterad. according to Bradley and Fowler\textsuperscript{11)}.

B. Targets. Self-supporting carbon foils were used to study the \(^{12}\text{C}(d,pn)^{12}\text{C}\) reaction. These were supplied by the Yissum Research Development Company, and were 100 \(\mu\text{g/cm}^2\) thick.

\textsuperscript{11}) Nuclear Data Tables, Part IV (1960).
Figure 2. The "Bonner" gas-target chamber: Being insulated, the target chamber served also as a Faraday cup.
For the absolute detector efficiency measurements, solid deuterated polyethylene targets, nominally 200 \( \mu \text{g/cm}^2 \) thick, were used. The hydrogen composition of the polyethylene was 98.3\% \(^2\text{H}\), and the foils were prepared by depositing the polyethylene, dissolved in boiling carbon tetrachloride, onto a warm water surface.

As has been described already, deuterium gas was used as the target source for neutrons in the relative efficiency measurements. For elastically scattering the neutrons, a cylindrical organic scintillator, type Pilot B*)\(^{(*)}\), 1" in diameter and 1" high, was used. This was optically coupled to an Amprex 56 AVP**) photomultiplier tube, the details of which have previously been described by Bonner\(^{1)}\).

C. Detectors. The solid state detectors used were standard silicon surface-barrier detectors supplied by Ortec Inc.***)\(^{(***)}\), and selected to satisfy nanosecond timing requirements. To minimize charge collection times, only totally depleted detectors, with low resistivity characteristics, were considered.


***) Supplied by Ortec, Inc., 100 Midland Road, Oak Ridge, Tennessee, 37830.
The detectors were encased in holders provided with a circular tantalum slit telescope. In these measurements, the slit apertures were 3/32" and 3/64" at 3 inches and 5 inches from the target respectively, and the solid angle subtended by the counters was thus $0.69 \times 10^{-4}$ steradians.

For neutron detection, an organic scintillator, type NE218\textsuperscript{*}), encapsulated in a thin-walled aluminum cell, 5" in diameter, and 3" deep, was used.

The scintillator was optically coupled to a dual Amperex 56-AVP\textsuperscript{**}) photomultiplier tube assembly [c.f. Figure 3.] To reduce background contributions, aluminum was used exclusively in the construction of the housing, and bulk was eliminated without sacrificing structural stability. Each photo-tube was individually spring-mounted, and access to the dynode circuitry was possible externally without breaking the light seal. The entire detector was enclosed in an aluminum cylinder, 6-1/4" in diameter, its walls being .037" thick. Netic and co-netic magnetic shielding was placed on this inside rim, and this extended 3" beyond the photo-cathode.

\*\ Supplied by Nuclear Enterprises, 935 Terminal Way, San Carlos, California 94070.

Figure 3. The liquid-scintillator and photo-tube assembly.
NEUTRON COUNTER

Pulse-Shape Discrimination Circuitry

56AVP 56AVP

Light-Tight Aluminum Can

NE 218
3\"x5\"

Netic-Conetic Magnetic Shielding
In Figure 4, the photo-tube circuitry is reproduced. This circuit was designed for maximum stability, timing, and optimization of the dynode signals. Among its features are a set of zener diodes between the cathode, the focusing dynode, and the acceleration plates. Small resistors were introduced in series with the last 5 dynodes to reduce inductance effects. There are one anode and two dynode outputs for each tube, permitting both timing and pulse height analysis of the scintillations. To insure the matched response of the two tubes, a high-voltage divider was incorporated in the design, and thus, from a single supply, the photomultipliers could be operated at potentials varying by 200 volts. Provision was also made for a compressed air inlet to ensure adequate cooling of the circuit.

To minimize the detection of neutrons and γ-rays impinging on the detector from directions other than that directly from the target, the detector shield constructed by Bonner et al. was used [Figure 5]. This shield was suspended from an overhead crane assembly which pivoted about the target position. Thus, flight paths from 1 to 6 metres with an angular position accuracy of 0.1° were readily provided.
Figure 4. Circuit diagram of the photo-multiplier dynode voltage chain.
Figure 5. The neutron detector shield used to minimize the detection of neutrons and gamma rays incident on the detector from directions other than that directly from the target.
D. Pulse Shape Discrimination. Possibly the most valuable tool in enhancing the quality of fast neutron spectroscopy is "pulse shape discrimination." This technique identifies the radiation incident on a scintillator by its characteristic decay, and is particularly effective in neutron time-of-flight applications in reducing the effects of γ-radiation without substantially lowering the neutron detection efficiency.

Wright\textsuperscript{12)}, in 1956, reported that the scintillations produced in anthracene crystals by α-particles and electrons exhibited different decays, and innumerable methods of utilizing this characteristic of organic\textsuperscript{13-15)}, and inorganic scintillators\textsuperscript{16)} have since been proposed. Owen\textsuperscript{14)} showed that both the fast and the slow (exponential) components in the light pulses emitted by the scintillator were, in fact, independent of the incident radiation. The differences are in the intensity of the long lived components relative to

\textsuperscript{13)} F. D. Brooks, Prog. in Nucl. Phys. \textbf{5} (1956) 252.
that of the initial fast spike, i.e.:

\[ R_\alpha < R_p < R_e \]

where

\[ R = \frac{\int_\tau^{T} I(t) \, dt}{\int_\tau^{\infty} I(t) \, dt} \quad \tau \sim 10 \text{ ns} \]

The technique suggested by Brooks\(^{13}\) compared the total light output during the scintillation with that during the initial spike. A recent modification of this system\(^{17}\) is reported to resolve neutrons and \(\gamma\)-rays effectively at a pulse height corresponding to about 50 keV electron energy. Deadtime limitations, as well as the necessity for using two outputs of the photomultiplier, give rise to difficulties, and this system is also highly sensitive to gain shifts.

In a later method, first proposed by Owens\(^{18}\), the photomultiplier was operated such that the potential difference between its anode and last dynode was small. Thus, during the initial intense fast spike of the light pulse, space charge saturation was possible. By a suitable adjustment of the circuit parameters, a positive signal for neutron,


\(^{18}\) R. B. Owen, Nucleonics 17 (1959) 72.
but not gamma induced scintillations is given by the last
dynode. The primary disadvantages of this system are that
the anode signal is no longer available for other applica-
tions, the circuit is suitable only for neutrons above 500
KeV; and its dynamic range is $\sim 10:1$.

Many advances towards overcoming these disadvantages
have been reported in the literature, prominent amongst
which are Retheimeir's circuit\textsuperscript{19)}, which used only passive
elements, and also provided an anode timing signal, and
Etchelor et al.\textsuperscript{15)}, who extended the effective range of the
Owen circuit.

In developing a pulse shape discrimination circuit for
use in this laboratory, we attempted to meet the following
specifications:

i) the $\gamma$-suppression ratio was not to be achieved at
the expense of the counter's neutron detection efficiency,

ii) the dynamic range for neutron detection should be
from 100 KeV to 10 MeV,

iii) the system should be capable of count rates in
excess of $10^4$ c.p.s.,

iv) the $\gamma$-ray discrimination threshold should be low
($\sim 250$ KeV),

\textsuperscript{19)} J. Retheimeir, H. J. Boersma, and C. C. Jonkers, Nucl.
v) the timing characteristics of the anode should not be influenced by the system,

vi) the system should be insensitive to gain and temperature drifts and

vii) the system should be functional with a minimum of adjustments.

From a review of the literature, the method of zero-crossover detection suggested by Alexander and Goulding\textsuperscript{20)} seemed particularly promising. In this technique, the charge at a photomultiplier dynode is integrated, and then twice differentiated, thus producing a base-line crossover. The timing of this zero-crossover with reference to the start of the pulse is dependent on the scintillation decay components, but is amplitude-invariant for pulses of identical shape. This principle had been the basis of a system developed by Rupaal\textsuperscript{21)}, who reported the following quantitative results. $90.0 \pm 0.5\%$ of a mono-energetic [2.9 MeV] neutron beam were detected by the system while $95.8 \pm 0.7\%$ of the $\gamma$-rays from a $^{60}$Co source, and $98.3 \pm 0.5\%$ of the $\gamma$-rays from a Ra-Th source were rejected by the P.S.D. All these

\textsuperscript{20)} T. K. Alexander, and F. S. Goulding, Nucl. Instr. and Meth. \textbf{13} (1961) 244.

\textsuperscript{21)} A. S. Rupaal, Nucl. Instr. and Meth. \textbf{49} (1967) 145.
percentages are relative to count rates when the P.S.D. was not applied, and refer to a γ-ray threshold of 400 kev.

i) Scintillation theory and pulse shape discrimination.

The theory of the interaction of nuclear particles with scintillators, and the subsequent luminescence has been extensively reviewed by Birks\textsuperscript{22}). This mechanism in organic scintillators will be considered here briefly in order to obtain a fuller understanding of the process, particularly with regard to pulse shape discrimination.

The luminescence from an organic scintillator is characterized by three discrete types, fluorescence, phosphorescence, and delayed fluorescence. Fluorescence is the mechanism which gives rise to the fast decay component ($\tau_f \approx 3.9 \times 10^{-9}$ seconds for N. E. 218) in organic scintillations, and the intensity ($I$) of this emission decays exponentially with time according to:

$$I = I_0 e^{-t/\tau}$$

where

$I_0 =$ the initial intensity

$\tau =$ fluorescence decay time.

A metastable triplet state \( \text{M} \) may exist in certain scintillators below the principal singlet excited state \( S_1 \) of the \( \pi \) orbital electrons. Thus, instead of a radiative transition from this \( S_1 \) level to the ground state, as in fluorescence [c.f. Figure 6], a non-radiative conversion may populate this metastable state \( \text{M} \). It should be noted that absorption transitions from \( S_0 \) to the triplet states are spin forbidden. This metastable state is long-lived, its radiative lifetime typically being \( > 10^{-4} \) seconds, and one possible de-excitation channel is a radiative transition to the ground state \( S_0 \), a process termed phosphorescence.

A further luminescence process is possible. Even at normal temperatures, molecules in \( \text{M} \) may acquire sufficient thermal excitation to return to the \( S_1 \) level, and normal fluorescence may follow. This mechanism, delayed fluorescence, has a spectrum similar in form to that of the conventional fluorescence, but its decay time is far larger \( (\sim 10^{-7} \) seconds). This process in particular is responsible for the slower component in the light output pulse.

For particles producing a higher ionization and excitation density within the scintillator, an "ionization
Figure 6. A simplified diagram of the π-orbital electron excitation levels involved in the scintillation process.
Principal singlet excited state

Inter-system crossing

Principal triplet excited state

Phosphorescence

Fluorescence

Absorption

Ground state $S_0$

Principal singlet excited state $S_1$
"quenching" effect has been observed \(^{23}\). However, only the fast scintillation component is affected directly by this quenching, and thus the relative intensities of the fast and slow components in the luminescence depend on the nature of the ionizing particle. The shape of the scintillation pulse is therefore characteristic of the incident radiation.

Extensive experimental measurements indicate that this indeed is the principal mechanism responsible for the pulse shape discrimination properties of certain organic scintillators, including N.E. 218. Birks and King \(^{24}\) have further refined the theory of the decay components, and their model is in excellent agreement with the observed properties. It would appear, however, that little progress has been made in applying this knowledge to the formulation of a scintillator capable of even superior pulse shape discrimination between γ-rays and neutrons.

ii) Technique and circuitry. In this system, a dynode current pulse (Figure 7a) is integrated with a time constant longer than the scintillation time constants of interest (Figure 7b). This pulse is then delay-line clipped, re-

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\(^{24}\) J. B. Birks and T. A. King,
Figures 7 and 8. Wave forms and a block diagram of the pulse shape discrimination system.
sulting in a 1µs rectangular pulse, and split into two
cannels, each of which incorporates a R-C differentiating
etwork. The time constants of these networks are carefully
chosen such that for one channel (the "start" channel), all
scintillation pulses cross through zero during the fast
spike of the pulses' trailing edge (Figure 7d). However,
in the "stop" channel, a larger time-constant is used so
that γ induced scintillations still cross over during the
fast spike, while pulses having a greater proportion of
slow decay components (e.g. neutron induced), cross over at
a later time (Figure 7e).

By using zero cross-over discriminators, the time
difference between the base-line crossover of the output
pulses in the two channels may be measured with a time-to-
amplitude converter (T.A.C.). Since this time interval is
measured from the trailing edge of the rectangular pulse,
it is extremely sensitive to the relative amounts of fast
and slow components in the dynode pulse. For a current
pulse of the form given in equation i), (page 20), the
time difference, t, measured by the T. A. C. is proportional
to [c.f. Appendix A.]:
\[ I = I_0 \left( e^{-t/\tau_f} + I_{os} e^{-t/\tau_s} \right) \]  

\[ t \propto \tau_s \ln \left[ \frac{Q_0}{Q} \left( 1 - e^{-\tau_d/\tau_s} \right) \right] \frac{\tau_{RC}}{\tau_d} \]  

where 

\[ Q_s = \text{slow component of the scintillation pulse charge} \]

\[ Q = \text{total scintillation pulse charge} \]

\[ \tau_s = \text{time constant of the slow component} \]

\[ \tau_d = \text{delay line time constant} \]

\[ \tau_{RC} = \text{time constant of the RC differentiating network.} \]

For a \( \gamma \)-ray scintillation, the T.A.C. pulse is small, while for those initiated by incident neutrons, it is relatively large, and a means of identifying the nature of the incident radiation is thus provided. It should be noted that cosmic-ray induced pulses will have characteristics similar to those for \( \gamma \)-rays, as will noise pulses, and the system should therefore discriminate against these pulses as well.

Commercially available modules were used extensively, however the F.E.T. preamplifier, delay-line shaping amplifier, and integrally discriminated linear gate were designed specifically for this system. In Figure 8, a block diagram of the electronics associated with the pulse shape discrimination is given.
From the 11th dynode of the photomultiplier, the current pulse is fed into the F.E.T. preamplifier [c.f. Figure 9], which serves not only as an integrator \( \tau_{\text{int.}} = 20 \mu s \), but also matches the high input impedance to a 100 ohm load. The single delay line shaping amplifier [c.f. Figure 10] provides not only two linear outputs suitable for pulse height analysis, but also RC differentiates the input signal for the P.S.D. system. It was found experimentally that the anode signal normally used for timing applications adequately performed the function of the "start" signal. Only one RC network is thus necessary. A resistive step attenuator was incorporated into the input enabling the dynamic range of interest to be changed readily.

The crossover point of this P.S.D. output was determined in a fast zero-crossing discriminator, and served as the stop signal for the time-to-amplitude converter. The T.A.C. output passes through a variable discriminator, and serves as a gate signal. This linear gate [c.f. Figure 11], was expressly designed for this purpose by James Buchanan. Featuring an integrated circuit differential discriminator, it accepts bipolar 1v. analog input signals, and may be operated in an open, gated, or closed mode. It is designed for positive 6v. gate pulses, and two potentiometers vary
Figure 9. Circuit diagram of the F.E.T. preamplifier.
Figure 10. The delay-line clipped amplifier designed to provide suitable pulse height analysis and pulse shape discrimination signals from the integrated dynode signal.
i) All resistor values in ohms.
ii) Operational amplifiers are NIMAMP model MA30.
Figure 11. Circuit diagram of the linear gate and differential discriminator gate input designed for the P.S.D. system.
Discriminator/Linear gate.

Designed by T. Buchanan.
January, 1988

All diodes −1N4148.
All 500Ω 0.1% ±1%.
P1: Threshold, 10 turn, 1K
P2: Gate width, 0mA off.
the gate threshold and width. By means of a front panel switch, either a coincidence, or anti-coincidence gate requirement can be selected. For pulse shape discrimination against gammas, the module is used in a gated, anti-coincidence mode.

iii) Experimental performance. In order to evaluate the system, an experimental circuit as shown in Figure 12 was used. Two signals, the integrated dynode pulse height, and the P.S.D. time-to-amplitude converter, were fed into the Rice University 1800 computer analyzer. These signals also served as inputs into the multiple coincidence module which provided the gating signal for the analyzers. Furthermore, to reduce the photomultiplier tube noise level, a fast coincidence \( \tau_{\text{cma}} = 20 \text{ ns} \) was required of the two delay-line clipped anode signals. Each such signal was fed into a zero-crossover discriminator, the outputs of which were effectively stretched to 12 ns by means of fast discriminators, before being fed into the fast coincidence unit. This pulse served as a third input into the multiple coincidence unit.

In Figure 13, the response of the system to a plutonium-beryllium source is shown. In this 2-dimensional display, the T.A.C. output is along the x-axis, while the
Figure 12. A block diagram of the electronics used for evaluating the effectiveness of the P.S.D. system.
Figure 13. A contour plot of the P.S.D. system's response to a plutonium-beryllium source. In this 2-dimensional display, the T.A.C. output is along the x-axis, while the y-axis corresponds to the integrated dynode pulse height.
y-axis corresponds to the dynode pulse height. On this contour plot, the distinction between neutrons and γ-rays is evident. The maximum neutron energy associated with a Pu-Be source is 10.6 MeV, and thus the system is effective from the detector's threshold to beyond 10 MeV neutrons. For large dynode pulses, saturation effects in the delay-line shaped amplifier distort the response. However, the interference caused by this effect was minimal in the energy range of interest, and where necessary, this could be accounted for in off-line analysis of the data.

The broadening of the neutron time peak is thought to be due primarily to the increased dispersion in the crossover point associated with the pulse's small slope. It can be shown that the time resolution is inversely proportional to this slope\textsuperscript{25}. The non-symmetrical nature of the peak was at first considered to be a pile-up phenomenon. Indeed, at higher count rates, there was a shift in the neutron time peak towards that of the γ's [c.f. Figure 14], but it would appear as if there are other contributing factors. Similar observations have been reported by Hollandsworth and Bucher\textsuperscript{25}.

Figure 14. The shift in the pulse shape discrimination T.A.C. output with high count rates (20,000 c.p.s.)
The pulse height spectrum for a $^{60}$Co source with and without P.S.D. is shown in Figure 15. These spectra were gated by the anode and P.S.D. signals in coincidence, the discriminator threshold being zero in the case of the latter. The effect of the system's bias level is evident, and this was measured to correspond to an incident $\gamma$ energy of 125 kev.

In evaluating the neutron response of the system, neutron detection efficiencies, as described in section II-F, were repeated with the linear gate open and gated. The ratio of these two efficiencies thus gave the neutron acceptance of this system.

Quantitatively, these results may be summarized as follows. When $97.8 \pm .2\%$ of the 2.75 MeV neutrons from the $d(d,n)^3$He reaction were accepted by the system, $96.8 \pm .2\%$ of the $\gamma$-rays above 125 kev from a $^{60}$Co source were rejected. All these percentages are relative to count rates without pulse shape discrimination.

Many of the circuits previously reported have shown a strong count rate dependence. This aspect of the system was also investigated, but in the range 700 c.p.s. to 31,000 c.p.s., the $\gamma$ rejection ratio for $^{60}$Co decreased only from 96.8% to 96.2%. [c.f. Figure 16]. The system's count rate capabilities appear even higher, but neither the linear
Figure 15. The response of the detector with and without P.S.D. to a $^{60}$Co source.
Figure 16. The gamma-rejection efficiency of the system as a function of count-rate.
amplifiers, nor the computer-analyzer system have this capability.

The response of the circuit to different integration and differentiation time constants does not seem to correlate completely with accepted values of the scintillation pulse decay components. The behaviour is far more consistent with the presence of a decay component of approximately 1 μs. It would thus be interesting to investigate more completely the nature of the scintillator's light pulses.

E. Charged particle-neutron time-of-flight coincidence circuitry. In Figure 17, a block diagram of the electronics used in charged particle-neutron time-of-flight circuitry is given.

Four parameter data, consisting of the P.S.D. time-to-amplitude converter output, the dynode pulse height, the solid state detector, and the time-of-flight T.A.C. spectrum were recorded in the on-line computer analyzer system. By accumulating the data in this manner, the pulse shape discrimination system could be utilized to its fullest.

Certain features of this circuitry were incorporated specifically into our system, and have enhanced its performance significantly. Reference has been made to the technique for reducing photomultiplier tube-noise by re-
Figure 17. A block diagram of the electronics used in 3-body data accumulation, and in the detector efficiency measurements.
quiring a fast coincidence between the two delay-line clipped anode signals. Because of the large dimensions of the scintillator, it was felt that timing should be effected by the first photo-tube producing an anode pulse and meeting the above coincidence restriction. This approach improved the timing characteristics of the neutron counter by a factor of 1.4, and the overall timing resolution of the neutron detector system, over a dynamic range of 100:1, was 1.75 ns. Another problem was the need to delay fast logic signals up to 1 μs with a minimum of signal degradation and time jitter. By using a variable width dual discriminator, variable delays could be introduced into the circuit with less than 100 ps jitter.

For determining relative detection efficiency by means of n-p elastic scattering, the circuitry was analogous to the above, except that the time-of-flight stop signal was now provided by the scattering scintillator.

F. Calibrations and Detector Efficiencies. The solid state detectors were calibrated by taking free spectra, and identifying the reaction products through a knowledge of the two-body kinematics. In Figure 18, the kinematic relationships for the principal two-body reactions arising from deuterons on $^{12}\text{C}$ is represented graphically for a bombarding
Figure 18. Energy as a function of the (laboratory) detection angle for various 2-body reactions from $d+{}^{12}\text{C}$ at an incident energy of 9.0 MeV.
energy of 9.0 MeV. The presence of hydrogen as an impurity on the target foil is of particular value, as the elastically scattered deuterons' energy is an extremely sensitive function of the angular setting [c.f. Figure 18]. By iteration, it was thus possible to determine the detector position with respect to the beam with an accuracy of 0.1°. A typical free spectrum is reproduced in Figure 19. It should be noted that the threshold, above which timing signals from the surface barrier detector were obtained with 100% efficiency, was 200 KeV.

Determination of the time-to-amplitude converter calibration was effected with standard delay lines, and the integral linearity of the module was better than 0.2%.

Figure 20 is an example of a time-of-flight spectrum obtained by this system, and the time resolution (f.w.h.m.), in this case for the 2.75 MeV neutron from the d(d, 3He)n reaction, over a flight path of 1.5 metres, is 3.75 ns. The time-of-flight of a particle is directly related to its energy by the equation

\[ t = 7.92 \sqrt{\frac{A}{E}} \] nanoseconds/metre

where E is the energy of the particle in MeV, and A is its mass number.
Figure 19. A free charged-particle spectrum from the $d+^{12}C$ reactions for an incident deuteron energy of 9.0 MeV at a detection angle of 30°.
Figure 20. A time-of-flight T.A.C. spectrum obtained from the d(d, $^3$He) reaction requiring charged particle-neutron counter coincidences.
$d(d, ^3\text{He})n$

$E_d = 3$ MeV

$f-p = 80$ cm

$\theta_n = 103^\circ$

$1.903$ ch/ns
Thus \[ \frac{dE}{E} = \frac{2}{71.92} \sqrt{\frac{E}{A}} \times \frac{1}{d} \ dt = 5.2\%. \]

The neutron detection efficiency was yet another parameter of importance. Since this is dependent on the detector's bias, particularly at energies near the counter's threshold, care was taken to reproduce the bias settings accurately. As a further test on the system, an absolute detector efficiency measurement, both with and without pulse shape discrimination, was repeated prior to each run.

For this determination, the \( d(d,^3\text{He})n \) reaction was used. By recording the charged particle spectra of the freeing, and the \(^3\text{He}-n\) coincidences simultaneously, the ratio of the counts in the \(^3\text{He}\) peak measured the efficiency of the detector for that particular neutron energy directly. Kine-

matically, optimum conditions were obtained with an incident energy of 3.0 MeV and the solid state detector at 30°. The neutron thus emerges at a (laboratory) angle of 103° with an energy of 2.75 MeV. In these measurements, care was taken to ensure that the neutron counter subtended a solid angle larger than that of the solid state detector. Furthermore, dead-time, and background corrections in the charged-particle free spectrum were of importance.
The above technique, unfortunately, is unsuitable for determining the neutron detection efficiency below 2 MeV. For this, the n-p scattering technique, as described by Jackson et al. 26, was used. In Figure 21, the geometrical arrangement for these measurements is shown. To monitor the neutron flux, a BF$_3$ proportional counter was used.

In Appendix B, it is shown that the relative neutron efficiency, $\text{Eff}_r(E_n)$, satisfies the following relation:

$$
\text{Eff}_r(E_n) \propto \frac{C}{M} \frac{1}{A(E_n, \theta)} \frac{1}{\cos \theta}
$$

where:

- $C$ = total number of elastically scattered neutrons detected by the counter at a laboratory angle $\theta$ relative to the beam direction,

- $M$ = total number of counts recorded by the monitor,

$A(E_n, \theta)$ = a geometrical factor accounting for attenuation in the scatterer of the scattered neutrons.

These relative efficiency measurements are shown in Figure 22, while in Figure 23, these values normalized to

Figure 21. The experimental geometry for the relative neutron detection efficiency measurements using n-p elastic scattering.
**Figure 22.** The relative efficiency obtained from the n-p scattering measurements. $A$ is the ratio of the number of counts in the n-p peak to the number of counts in the monitor long counter.

$B$ is the result of applying the correlation due to the geometrical factor $[A(E_n, \theta)]$, and the variation in the laboratory differential cross-section for n-p scattering.
Figure 23. The absolute neutron detector efficiency.
the absolute detector efficiency at $E_n = 2.75 \text{ MeV}$ are displayed.

The derivation of the theoretical neutron detection efficiency is outlined in Appendix C. Since neutrons are detected primarily through elastic scattering by protons in the organic scintillator, the number of such recoil protons must be calculated in determining this detection efficiency. If $N_{rp}$ represents the number of recoil protons resulting from $n-p$ scattering alone, and $N_{rpc}$ represents the contribution from neutrons scattered initially from carbon, and then undergoing secondary scattering with protons, the efficiency, if every such recoil proton were seen by the system, would be given by:

$$\text{Eff}_{\text{th.}} = \frac{N_{rp} + N_{rpc}}{N_o}$$

where: $N_o =$ number of neutrons incident on the detector.

An inherent bias level, below which the protons are not seen, exists however in the system. This bias may readily be accounted for, and the observed detector efficiency is thus given theoretically by:

$$\text{Eff}_{E_n} = \left[1 - \frac{E_{\text{bias}}}{E_n}\right] \frac{N_{rp} + N_{rpc}}{N_o}$$
\( E_{\text{bias}} \) is the effective neutron energy corresponding to the bias level, and its value was chosen to give the best fit to the experimentally measured values [c.f. Figure 23].

This bias level was also determined experimentally. For \( \gamma \)-rays, the Compton peak is readily identified, and, after background corrections, half the maximum peak value corresponds to 1.05 times the maximum Compton energy\(^{27}\). In Figure 24, the detector's response to \(^{137}\)Cs is shown, and the \( \gamma \)-ray bias is seen to be < 20 keV. Although the light output from an organic scintillator is linearly related to the energy of the incident gamma radiation, such a relationship is not valid for neutron induced scintillations. Joseph \textit{et al.}\(^{28}\), have investigated this extensively for N.E. 213, and have obtained the following empirical relation between the recoil proton energy, \( E_p \), and the light output, \( L_p \):

\[
L_p = 0.26 E_p + 0.03 E_p^2 \quad 0 < E_p < 4 \text{ MeV}
\]

\[
= 0.60 E_p - 0.90 \quad 4 \text{ MeV} < E_p < 14 \text{ MeV}
\]


Figure 24. The response of the detector to a $^{137}$Cs source.
The equivalent neutron threshold for the detector is therefore $\sim 75$ KeV, a value consistent with our theoretical efficiency calculations.

It may be noted that the system has been used in time-of-flight application where the neutron energy varied from 150 KeV to 17 MeV, thus rather gratifyingly meeting the original objective of a dynamic range of 100:1.
III. REACTION THEORY

By measuring the energies, and angles of emission of two of the final state particles in the $^{12}_C + d \rightarrow p + n + ^{12}_C$ reaction, the kinematics of the reaction are determined completely, and as stated in the introduction, the distribution of events along the allowed kinematical locus in the $E_1 - E_2$ plane is indicative of the reaction mechanism. For deuterons incident on $^{12}_C$, several 3-body reaction channels are possible, and these have been summarized in Table 1.

The formalism for the theoretical treatment of the 3-body problem has been developed by Watson\textsuperscript{29)}, Migdal\textsuperscript{30)} and Phillips\textsuperscript{31)} and for a detailed discussion of the various contributing processes in 3-body breakup studies, reference should be made to the work of Simpson\textsuperscript{1)} and Niiler\textsuperscript{32)}, amongst others. In the studies reported here, particular

\begin{itemize}
  \item\textsuperscript{29)} K. M. Watson, Phys. Rev. \textbf{88}, 1163 (1952).
  \item\textsuperscript{1)} W. D. Simpson, Ph.D. thesis, Rice University (1966) unpublished.
\end{itemize}
Table 1. Reaction mechanisms possible in the d+^{12}C 3-body interaction.
TABLE 1

\[ d + ^{12}C \rightarrow ^{14}N^* \]

- \( p+n+^{12}C \) simultaneous breakup

- \( p+^{13}C^* \rightarrow p+n+^{12}C \)
  - sequential decay via \( ^{13}C^* \)

- \( n+^{13}N^* \rightarrow p+n+^{12}C \)
  - sequential decay via \( ^{13}N^* \)

- "rescattering and interference"
  - \( p+n+^{12}C \)

- \( p+^{13}C^* \rightarrow p+n+^{12}C \)
  - final state interactions via \( ^{13}C^* \)

- \( n+^{13}N^* \rightarrow p+n+^{12}C \)
  - final state interactions via \( ^{13}N^* \)
emphasis has been given to determining singlet deuteron, and
proximity scattering contributions to the reaction mechanism,
and the experimental geometry was carefully chosen to en-
hance these effects. In this section, therefore, theoretical
considerations will be directed to the contributions from
simultaneous breakup, sequential decay, singlet deuteron,
and proximity scattering mechanisms [c.f. Figure 25]. It is
reasonable to expect that the remaining processes should be
of negligible importance.

A. Kinematics for Simultaneous Breakup. The most convenient
manner for studying a system with three particles in the
final state is to determine the energies, and momentum
directions of two of these particles. In the energy-energy
plane, following such a 3-body breakup, the kinematically
allowed loci are a function of the energy and mass of the
incident particle, the reaction Q-value, the masses of the
final state particles, and the angles of the detectors alone.

These loci may be calculated directly from the conserv-
ation of energy and linear momentum. In the laboratory
system, the reaction is conveniently described by Figure 26.
$\vec{p}_i$ ($i = 1, 2, 3$) is the momentum of the $i^{th}$ final state par-
ticle, and $\theta_i$ is the angle between $\vec{p}_i$ and the incident beam
momentum, $\vec{p}_0$. $\theta_i$ measures the angle about the beam axis ($z$)
Figure 25. Graphical representation of
a) simultaneous breakup,
b) sequential decay,
c) sequential decay and
proximity scattering,
d) simultaneous breakup followed
by a final interaction.
Figure 26. A laboratory momentum diagram for the three particles in the final state. The $\bar{z}$ axis corresponds to the incident beam axis. [after Simpson¹]
from the plane containing the beam, and the x-axis. The kinetic energies and masses of the particles are \( T_i \) and \( m_i \) respectively, \((i = 0, 1, 2, 3)\), and the energy necessary for the 3-particle breakup is represented by \( Q \). As shown in Appendix D, the solution for \( p_2 \) as a function of \( p_1 \) for a particular laboratory geometry is given by:

\[
p_2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

where

\[
A = \frac{1}{2} \left[ \frac{1}{m_2} + \frac{1}{m_3} \right]
\]

\[
B = \frac{1}{m_3} [p_1 \cos \theta_1 \cos \theta_2
\]

\[
+ p_1 \sin \theta_1 \sin \theta_2 \cos (\theta_1 - \theta_2) - p_0 \cos \theta_2]
\]

\[
c = \frac{p_0^2}{2} \left[ \frac{1}{m_3} - \frac{1}{m_0} \right] + \frac{p_1^2}{2} \left[ \frac{1}{m_1} + \frac{1}{m_3} \right] - \frac{1}{m_3} [p_0 p_1 \cos \theta_1] - Q
\]

Thus, for a given value of \( p_1 \), there are in general two values of \( p_2 \), and the loci of possible solutions may be calculated for the experimentally observable system in the \( T_2 \) versus \( T_1 \) plane. [By convention, subscript 1 is associated with the parameters of that particle seen by detector 1, and similarly, subscript 2 refers to detector 2.]
For a reaction proceeding completely via simultaneous breakup [c.f. Figure 25a], there are no further kinematic restrictions, and the distribution of the reaction products along the locus will be modulated only by the phase-space. The calculation of the phase-space available to a 3-body breakup as observed in the laboratory is fundamental for a correct interpretation of such data. A complete derivation is included in the work of Simpson\(^1\) whose results are summarized here. The probability of detecting particle 1 in a solid angle \(\Omega_1\) with momentum between \(p_1\) and \(p_1 + dp_1\), when particle 2 is detected in a solid angle \(\Omega_2\), its momentum being between \(p_2\) and \(p_2 + dp_2\) is characterized by a distribution, \(N(p_1, \Omega_1, \Omega_2)\).

For 3-body breakup restricted to a plane, this function is given by:

\[
N(p_1, \Omega_1, \Omega_2) = \frac{m_3 p_1^2 [p_2(p_1, \theta_{12})]^2}{|p_2(p_1, \theta_{12}) (\frac{m_2 + m_3}{m_2}) - p_0 \cos \theta_2 + p_1 \cos \theta_{12}|}
\]

\(i\)

where \(p_2(p_1, \theta_{12})\) is the solution of the 3-body kinematics for \(p_2\) as a function of \(p_1\) and \(\theta_{12}\) [c.f. p. 32 and Appendix D].

That fraction, \(N_1\), of the total phase-space, \(N_T\), for an energy increment \(\Delta T_1\) over solid angles \(\Delta \Omega_1\) and \(\Delta \Omega_2\) is thus given by:
\[ N_1 = \frac{\Delta T_1 \Delta \Omega_1 \Delta \Omega_2}{N_T} \frac{N(T_1, \Omega_1, \Omega_2) \, dT_1, d\Omega_1, d\Omega_2}{N_1(\Omega_1, \Omega_2)} \]

where

\[ N(T_1, \Omega_1, \Omega_2) = \frac{\partial p_1}{\partial T_1} N(p_1, \Omega_1, \Omega_2) = \frac{m_1}{p_1} N(p_1, \Omega_1, \Omega_2) \]

and

\[ N_T = \int_{p_1} \int_{\Omega_1} \int_{\Omega_2} N(p_1, \Omega_1, \Omega_2) \, dp_1 \, d\Omega_1 \, d\Omega_2 \]

\[ = \frac{1}{2} [2\pi]^3 \left[ \frac{m_1 m_2 m_3}{m_1 + m_2 + m_3} \right]^{3/2} \left[ \frac{m_1 T_0}{m_1 + m_2 + m_3} + Q \right]^2. \]

\[ N_1(T_1, \Omega_1, \Omega_2) \] is a smoothly varying function, and is shown in Figure 27 for one geometry relevant to these \(^{12}\text{C}(d, pn)^{12}\text{C}\) studies.

\textbf{B. Kinematics for Sequential Decay.} Each point on the kinematically allowed locus for a simultaneous breakup process corresponds to a particular "reaction channel" for each of the sequential decay, and final state interaction modes [c.f. Figure 25b, and d]. If one of these channels is favored by the reaction, it will manifest itself by an increase in the yield along the locus. Thus, if the intermediate composite nucleus is in a state, either real or virtual, this should be seen on the experimental locus provided the reaction channel is not restricted by selection rules.
**Figure 27.** Phase-space calculated for the $^{12}_{\text{C}}(d,\text{pn})^{12}_{\text{C}}$ reaction at an incident energy of 5.0 MeV, and laboratory detection angles of 35° for both detectors. The axis of projection is that of the charged particle detector, and the experimental resolution and angular acceptances of the detectors have been included.
The internal energy of this intermediate system may be calculated, and this gives rise to a further restraint on the kinematics. Using the notation of the previous section, and with $E_{ij}$ the internal energy of the composite $[i+j]$ system when particle $k$ is emitted first, the following relations may be derived [c.f. Appendix E]:

\[ E_{23} = T_0 + Q - T_1 - \frac{1}{2m_{23}} \left[ p_0^2 + p_1^2 - 2p_0p_1 \cos \theta_1 \right] \]

\[ E_{13}^\pm = T_0 + Q - T_2^\pm - \frac{1}{2m_{13}} \left[ p_0^2 + p_2^2 - 2p_0p_2^{\pm} \cos \theta_2 \right] \]

\[ E_{12}^\pm = T_0 + Q - \frac{1}{2m_{12}} \left[ (p_1 \cos \theta_1 + p_2^{\pm} \cos \theta_2)^2 \right. \]
\[ \left. + (p_1 \sin \theta_1 - p_2^{\pm} \sin \theta_2)^2 \right] - \frac{1}{2m_{23}} \left[ (p_0 - p_1 \cos \theta_1 - p_2^{\pm} \cos \theta_2)^2 \right. \]
\[ \left. + (p_1 \sin \theta_1 - p_2^{\pm} \sin \theta_2)^2 \right] . \]

If a resonance exists in a composite system, this will produce peaking in the observed yield at that point on the locus corresponding to this resonant energy in the intermediate system.

Sequential decay or final state interactions may be described by the density of states function suggested by
Phillips et al.\textsuperscript{31}) This formalism is similar to that proposed by Watson\textsuperscript{29}), but the hard-sphere phase shifts are also taken into account. The 3-body reaction mechanism is characterized by a sequence of two-body interactions, and the cross-section for the sequential decay process (c.f. Figure 25b):
\[ b + t \to l + (2-3) \to l + 2 + 3 \]
is given by:
\[ \sigma_{\text{seq}}(E_{\text{lab}}) = \rho(k,a_{\perp}) \ |M|^{2} J(k) \]  
where \( \rho(k,a_{\perp}) \) is the appropriate density of states function, \( |M|^{2} \) is the matrix element described by the interaction Hamiltonian, and the initial and final state vectors; \( J(k) \) is the transformation from the centre-of-mass to the laboratory system.

These factors have been derived in Appendix F. If the final-state two body wave function is normalized within a large box of radius \( R \) (\( R \to \infty \)), the renormalized density of states for the n-p singlet interaction reduces to the following form:

\[ \rho(k,a_1) = \frac{u}{\hbar^2} \left[ \frac{d}{dk} (\delta + \phi) - \frac{1}{2k} \sin^2(\delta + \phi) \right] \] 

where

\[ \phi = ka_1 \] is the hard-sphere phase shift

\[ a_1 \] is the p-n interaction radius

\[ \delta \] is the nucleon-nucleon phase-shift obtained from the effective range expansion. Because of the low internal energies of the system, contributions for non-zero angular momenta have not been included in this expression.

If the final-state wave function is energy-independent within the interaction radius, \( a_1 \), equation i) may be reduced further,

\[ \rho(k,a_1) = \frac{2ua_1}{\hbar^2} \left[ \frac{\sin^2(\delta + \phi)}{p} \right] \]

Here, \( p \) represents the penetrability.

For comparison, the Watson form of the density of states is given by:

\[ \rho(k,a_1) = \frac{2ua_1}{\hbar^2} \left[ \frac{\sin^2 \delta}{p} \right] \]

These functions have been evaluated for both the singlet and triplet n-p interactions [c.f. Figures 28 and 29], and all three forms give good qualitative agreement with respect to both the position of the peak and its width.
Figures 28 and 29. Density of states calculations for the singlet and triplet n-p system. The various forms and parameters are indicated on the diagrams. [after Simpson]
n-p
Density of States
(Singlet)

A - PGB (Renorm.)
B \(-\left(\frac{2\mu g_1}{\hbar^2}\right) \frac{\sin^2(\beta+\phi)}{P}\)
C \(-\left(\frac{2\mu g_1}{\pi\hbar^2}\right) \frac{\sin^2(\beta)}{P}\)

r_0 = 2.49 f
\alpha = -23.806 f
\alpha_1 = 2.5 f
\[ n-p \]

Density of States (Triplet)

\[ A = \text{PGB (Renorm.)} \]
\[ B = \left( \frac{2\mu a_s}{\pi a^2} \right) \frac{\sin^2(\delta + \phi)}{P} \]

- \( r_0 = 1.65f \)
- \( a = 5.37f \)
- \( a_1 = 2.5f \)
Furthermore, for low relative energies, the singlet density-of-states function is considerably larger than that corresponding to the triplet n-p interaction. Koltveit and Nagatani	extsuperscript{33)} have also analyzed this system applying the distorted wave Born approximation, with similar results.

It is relevant to note that the density of states here tends to zero as the n-p relative energy tends to zero. This is determined, however, in the \(^{12}\text{C}-\text{d}^*\) recoil centre-of-mass system (where \(\text{d}^*\) refers to the n-p system, which may be regarded as a composite quasi-particle). As indicated in Equation i), [page 39], the observed cross-section includes a transformation from this centre-of-mass system to that of the laboratory. In particular, the Jacobian transforming from the \(^{12}\text{C}-\text{d}^*\) recoil centre-of-mass to the overall 3-body system centre-of-mass is singular at zero relative energy, and therefore, the experimentally observed n-p interaction exhibits a maximum here. This theoretical yield, determined from the density of states as transformed to the laboratory system, and the phase space, is shown in Figure 30 for a bombarding energy of 5.0 MeV, and proton and neutron detection angles of 35° on the same side of the beam.

Figure 30. The n-p singlet interaction resulting from the $^{12}\text{C(d,pn)}^{12}\text{C}$ reaction at an incident energy of 5.0 MeV, and laboratory angles of 35° for both detectors. An integration over the angles and experimental resolution have been included in this calculation.
The calculations for sequential decay via discrete states in either the $^{13}\text{C}^*$ or $^{13}\text{N}^*$ intermediate system is exactly analogous to the above procedure, the appropriate initial and final state vectors, and interaction Hamiltonians being the only modifications necessary in evaluating Equation 1).

C. Proximity Scattering. In a sequential decay process, following the decay of the intermediate nucleus, it may be kinematically possible for two of the outgoing particles to approach sufficiently close to permit a further interaction, generally elastic scattering, to occur. This mechanism is represented diagramatically in Figure 25c, and has been termed rescattering, or in the absence of a discrete "rescattering resonance," proximity scattering. Fox\textsuperscript{34)} suggested that this process might provide a tool for measuring radiative lifetimes of the composite nucleus, and theoretical aspects of this problem have also been studied by Kacser and Aitchinson\textsuperscript{35)}, Schmid\textsuperscript{36)}, and Wood\textsuperscript{37)}. Furthermore, experi-

\textsuperscript{34)} R. Fox, Phys. Rev. 125, 311 (1962).


\textsuperscript{37)} L. E. Wood, University of Maryland technical report No. 798 (1968).
mental evidence for this effect has been reported in the
$^{12}\text{C}(d,\text{pn})^{12}\text{C}$ by Lang et al. \textsuperscript{38}, and Bohne et al. \textsuperscript{39} who as-
ccribed a lifetime of $(.7 \pm .1) \times 10^{-20}$ secs for the unresolved
3.51 and 3.56 MeV doublet in $^{13}\text{N*}$.

The conditions for proximity scattering to be possible
can readily be visualized in a classical concept. Referring
to the triangle diagram representing this process in
Figure 25, after the decay of the intermediate (2-3)* reso-
nance, particle 2' must be emitted in the same direction
(in the overall centre-of-mass system) as particle 1', and
with a velocity sufficient to permit these nucleons to
interact. These restrictions have been derived in Appendix
G, and several important observations follow. For proximity
scattering between particle 1 and 2 subsequent to sequential
decay \textit{via} (2-3)* resonance, i.e.:

\[
b + t \rightarrow 1 + (2-3)^* \text{seq-decay} \\
\rightarrow (1-2)^* \text{prox. scattering} \rightarrow 1+2+3 \quad \text{i})
\]

the relative energy of particles 1 and 2, $E_{12}$, has a unique
value determined by the masses $m_p, m_t, m_1, m_2$ and $m_3$, the exci-


tation energy \( E_{23} \) in the \((2-3)^*\) system, the energy of bombardment, and the 3-body breakup Q-values:

\[
E_{12c1}^{12} = \frac{1}{2} \frac{m_1 m_2}{[m_1 + m_2]} \left[ \frac{2m_3 E_{23}}{m_2 (m_2 + m_3)} \right]^{1/2}
- \frac{2(m_1 + m_2 + m_3)}{m_1 (m_2 + m_3)} (W - E_{23})^{1/2} \right] \]

where \( W = E_b^1 + E_t^1 + Q \).

\( E_b^1, E_t^1 \) are the energies of particles b and t in the overall centre-of-mass system.

There is no angular dependence in Equation ii), and proximity scattering is thus characterized on the \( E_1 - E_2 \) kinematical locus by an internal energy in exactly the same manner as sequential decay and final state interaction mechanisms.

Furthermore, the range of incident energies for which the process is possible is well defined, and is given by:

\[
E_{b_{\min}} = \frac{(m_b + m_t)}{m_t} [E_{23} - Q] \quad \text{iii)}
\]

\[
E_{b_{\max}} = \frac{(m_b + m_t)}{m_t} \left[ \frac{m_1 m_3 + m_2 (m_1 + m_2 + m_3)}{m_2 (m_1 + m_2 + m_3)} E_{23} - Q \right] \quad \text{iv)}
\]

For the particular case reported here, where proximity scattering follows sequential decay via the 3.51-3.56 MeV
doublet in $^{13}\text{N}^*$, the deuteron bombardment energy, $E_d$, is restricted to $4.49 \text{ MeV} < E_d < 6.11 \text{ MeV}$. The internal energy for the n-p system at which this process occurs is summarized as a function of the incident energy in Table 2.

These expressions have been derived from a purely classical approach. The quantum mechanical analogue has been investigated by Wood\textsuperscript{37),} who showed that the relative energy of the particles involved in proximity scattering could assume any value compatible with energy and momentum conservation. A logarithmic singularity exists however at the classical value, and the classical conditions governing the proximity scattering mechanism are valid.

If an attempt to relate the probability of the proximity scattering to the lifetime, $\tau_o$, of the intermediate resonance is made, a basic conflict between the classical and quantal treatments arises. Classically, this probability is proportional to $\tau_o^{-2}$ [ref. 38], while quantum mechanically, the dominant contribution is proportional to $\tau_o^{-1}$ [ref. 37]. Furthermore, as the relative energy of the particles tends to zero, from a quantal approach, the probability of proximity scattering remains finite. It is thus

\textsuperscript{37) L. E. Wood, University of Maryland technical report No. 798 (1968).}
Table 2. The n-p relative energy for proximity scattering as a function of incident energy, $E_{\text{lab}}$, for the $^{12}\text{C}(d, pn)^{12}\text{C}$ reaction. The intermediate resonance is the 3.56 level in $^{13}\text{N}^*$. 
<table>
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<th>$E_{\text{lab}}$ (MeV)</th>
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questionable whether any meaningful lifetime of the composite nucleus may be obtained from a classical treatment of the problem [c.f. Ref. 38, 39].

Even more significantly, Schmid\textsuperscript{36}) has shown that while the proximity scattering, or rescattering singularities do affect the distribution of events in the Dalitz plot, producing a weak rescattering band, its projection into an invariant mass plot cannot result in a peak.

As stated in the introduction, a prime motivation in undertaking a study of the $^{12}\text{C} + d \rightarrow p+n^{12}\text{C}$ reaction was an attempt to resolve the ambiguity posed by the theoretical considerations of proximity scattering, an ambiguity which one might now more fully appreciate.

\textsuperscript{36) C. Schmid, Phys. Rev. 154, 1363 (1967).}
\textsuperscript{37) L. E. Wood, University of Maryland technical report No. 798 (1968).}
\textsuperscript{38) J. Lang, R. Muller, R. Bosch, and P. Marmier, Nucl. Phys. 88, 576 (1966).}
IV. EXPERIMENTAL DATA

The reaction $^{12}\text{C}(d, pn)^{12}\text{C}$ was observed at deuteron bombarding energies of 5.00, 5.15, 5.39, 5.50, 9.20 and 9.85 MeV. To enhance the probability of detecting singlet deuteron and proximity scattering contributions to the reaction, both the proton and neutron detectors were placed at the same laboratory angle. The relative energy of the n-p system thus spanned the range from 0 to several MeV.

Following their interaction, the proton and neutron are restricted to a cone in the laboratory system whose half-angle, $\theta_{1/2}$, is determined by the internal energy of the n-p system, $E_{1-2}$, and the system centre-of-mass velocity, $v'_{1-2}$. Actually:

$$\sin\theta_{1/2} = \left[ \frac{2E_{1-2}}{m_p + m_n} \right]^{1/2} \times \frac{1}{v'_{1-2}}$$  

In Table 3, the experimental parameters relevant to each run have been shown. By increasing the detector solid angles, equivalent data could be accumulated far more rapidly without significantly impairing the experimental resolution.
Table 3. Parameters under which data were accumulated in the $^{12}\text{C}(\text{d, pn})^{12}\text{C}$ reaction in this study.
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<th>$E_b$ (MeV)</th>
<th>$\theta_{S.S.}$ (degrees)</th>
<th>$\theta_n$ (degrees)</th>
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<th>$\Omega_n$ sr.$\times 10^{-3}$</th>
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The studies at 5.00, 5.15, 5.39 and 5.5 MeV were directed primarily towards evidence for proximity scattering. Both Lang et al.\textsuperscript{38)} and Bohne et al.\textsuperscript{39)} had reported data at a bombarding energy of 5.39 MeV, and laboratory detection angles of 90° for both detectors. By observing this reaction at this same energy, but different laboratory angles, as well as other energies at which proximity scattering was kinematically possible, very strong evidence for proximity scattering would be obtained if the peak observed tracked systematically with the kinematics. It should be emphasized that the singlet deuteron interaction (peaking at minimum relative energy), is also possible whenever proximity scattering may occur.

The positions of 3-body loci in the $E_1-E_2$ plane are characteristic of the reaction Q-values, and contributions from target contaminants are normally well separated from the locus of interest. In studies involving the incident particle breakup however, the loci for nuclei of similar mass are virtually indistinguishable. Since natural carbon targets contain 1.11% $^{13}$C, data for the "Zurich geometry"
[c.f. Ref. 38] were also taken with a 55.4% enriched $^{13}$C target.

At incident deuteron energies above 6.1 MeV, proximity scattering, from a classical approach, is no longer feasible but the n-p interaction is not kinematically restricted. At a bombarding energy of 9.2 MeV, an extremely interesting isospin violating resonance in the $^{12}\text{C}(d,\alpha)^{10}\text{Be}$ reaction occurs. Meyer-Schützmeister et al. 8) have studied this extensively, and have also reported two further resonances at deuteron laboratory energies of 12.8 and 14.5 MeV. If an increase in the singlet deuteron yield were observed over this resonance, this would not only provide unambiguous evidence for singlet deuteron contributions to this reaction, but would also demonstrate experimentally the T = 1 aspect of the singlet deuteron. Data were therefore also accumulated at 9.20 and 9.85 MeV at laboratory angles of 40°.

The experimental technique used in these experiments has been detailed in chapter II. Figure 31 is a two-dimensional plot of the raw data taken at a bombarding energy of 5.15 MeV, laboratory angles of 35° for both de-

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Figures 31 and 32. A two-dimensional plot of the experimental data at an incident energy of 5.15 MeV and detector angles of 35°. The x-axis corresponds to the S.S. energy, while the y-axis represents the time-of-flight [neutron energy]. Background has been subtracted in the second figure in the manner indicated in the text.
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THE DATA HAVE BEEN RESTRICTED TO LIE WITHIN THE SPECIFIED 2-D WINDOW.
detectors, and a flight path of 1.30 metres. The calculated
kinematic locus has been included. This same data, with
accidental charged particle-γ coincidences rejected, and
background subtracted, are shown in Figure 32. In addition
to the desired locus, a further band is evident. In a very
true sense, this corresponds to the γ-flash seen in neutron
"pulsed-beam" experiments. It arises from a "real" charged-
particle-γ coincidence resulting from the decay of the ex-
cited residual nucleus. From this γ-flash, the inherent
and inserted delay in the timing electronics can be
accurately determined. Moreover, the walk associated with
the charged particle detector timing signal can be estab-
lished, and corrected for.

Since the energy of the neutron is derived from its
time-of-flight, a reliable means of real-time background
determination is readily available. By its very nature,
accidental background is random in time, and therefore, by
averaging the yield for a given charged particle energy over
a region where no real (including 2-body) coincidences may
occur, the accidental background for that increment in the
S.S. energy may be obtained.

In all instances, the yields along the loci were pro-
jected on the charged particle, (\(E_1\)), axis, and these are
shown in Figures 33-41. Statistical errors, including those arising from the background subtraction, have been included, and the positions of possible sequential decay and final state interactions have been indicated.
Figures 33-41. Projections of the $^{12}\text{C}(d,pn)^{12}\text{C}$ loci onto the charged particle energy axes. Sequential decay and final state interactions have been indicated. The statistical errors include those arising from the background subtraction.
$E_P = 5.0$ MeV
$\theta_P = 35^\circ$
$\theta_N = 35^\circ$
$E_p = 5.5 \text{ MeV}$

$\theta_p = 35^\circ$

$\theta_N = 35^\circ$
$^{12}\text{C} (d, p \pi^{+})^{12}\text{C}$

$E_d = 5.39$ MeV

$\theta_p = 90^\circ$  $\theta_n = -90^\circ$
$E_p = 5.39$ MeV
$\theta_p = 90^\circ$
$\theta_N = 90^\circ$

Enriched $^{13}C$ target
$E_p = 5.39 \text{ MeV}$

$\theta_p = 35^\circ$

$\theta_N = 35^\circ$
V. RESULTS AND CONCLUSIONS

For all the geometries studied, the reaction mechanisms involved are clearly dominated by sequential decay via the 3.51 and 3.56 MeV doublet in $^{13}_N$. Where kinematical conditions were adequate, transitions through $^{13}_N*_{2.37}$ are also observed, though their intensity is a factor of 2 less than that of $^{13}_N*_{3.51}$. Only at a bombarding energy of 5.39 MeV, and laboratory detection angles of 90° was any significant decay through $^{13}_C*_{6.86}$ observed. [c.f. Figure 36] This is in contradiction with the reported work of Lang et al. 38), and may indicate that their timing threshold on the charged particle detector was higher than stated in their text.

By comparison, Figure 37 is data accumulated with a higher threshold in order to reduce excessive coincidences of minimal interest, and sequential decay via $^{13}_C*_{6.86}$ is not evident.

At bombarding energies of 9.2 and 9.85 MeV, similar considerations apply, but far more resonance levels are now seen.

---

Of far more interest is the slight, but nevertheless, statistically significant yield increases observed in the data taken at incident energies between 5.0 and 5.5 MeV. Though the effect is small, being generally two orders of magnitude less than the $^{13}_{N*} 3.51$ sequential decay, its very presence is highly rewarding, in that singlet deuteron interactions in isospin forbidden reaction channels have, to our knowledge, not been reported. In this respect preliminary analysis of the $d + d \rightarrow p + p + (2n)$ reaction in these laboratories indicate that the singlet deuteron mechanism may also contribute to this reaction 40).

It would also appear that the magnitude of this effect increases with decreasing bombarding energy. If indeed this yield is due to the singlet n-p interaction, an explanation of the isospin violation is necessary. The Hamiltonian representing the Coulomb interaction does not conserve isospin, and this may well be the mechanism responsible for the "spin flip." It is therefore reasonable to expect that as the incident projectile remains under the influence of the target Coulomb field for longer periods of time, the extent of this "spin flip" will be increased.

40) W. von Witsch, private communication.
If that data reported at 90° and a bombarding energy of 5.39 MeV\textsuperscript{38,39} are indeed due to proximity scattering, it is very difficult to reconcile this with the data shown in Figure 35. Here the reaction was studied at a similar bombarding energy, but at detector angles of 35°. Since the data have been normalized to the very resonance giving rise to the proximity scattering, the mechanism should not exhibit any striking angular dependence. At the p-n relative energy characteristic of proximity scattering, the yield at 35° is virtually non-existent. There is also very little evidence for the effect in the data at 5.0, 5.15, and 5.5 MeV, all kinematical regions at which the mechanism is possible.

The data shown in Figures 37 and 38 present the greatest dilemma. To interpret this as evidence for proximity scattering would be tempting, but a small bump seen at only one angle pair, and one specific bombarding energy cannot truly be said to be indisputable evidence for a mechanism as controversial as this. It is true that there may be a fair contribution from singlet deuterons, but this also seems inadequate to offer a satisfactory explanation. It is interesting to note that data accumulated with an enriched \textsuperscript{13}C target did indeed show greater yields precisely
in the region of interest. The larger error bars result from a radically increased background from $^{13}\text{C}(d,n)^{14}\text{N}$ reactions. It is clear that sequential decay contributions from the natural abundance of $^{13}\text{C}$ cannot be overlooked in this instance.

The reaction as observed at 9.2 and 9.85 MeV proved singularly disappointing. As stated previously, this corresponds to an isospin violating resonance in the $^{12}\text{C}(d,\alpha_2)^{10}\text{B}^*_1.74$ channel. Both on [c.f. Figure 39], and off [c.f. Figure 40] this resonance, no significant formation of the singlet deuteron was seen. This seems to indicate that the mechanism for the formation of the singlet n-p system is non-compound, an observation consistent with that of Otte\textsuperscript{41)}, who has studied the $p + ^{13}\text{C} \rightarrow p + n + ^{12}\text{C}$ over a wide range of energies.

It is evident that singlet deuteron formation in isospin-forbidden reaction channels poses the nuclear physicist with innumerable questions. Its very occurrence in the $^{12}\text{C}(d,pn)^{12}\text{C}$ reaction is encouraging, but what is the true nature of the mechanism involved, and as important, what is the fundamental significance of this effect to the mechanisms of nuclear interactions?

\textsuperscript{41) V. A. Otte, private communication.}
APPENDIX A.

DERIVATION OF THE ZERO-CROSSOVER TIME EXPRESSION

If the dynode current pulse is integrated with a long time constant

\[ Q = I_f \tau_f + I_s \tau_s = Q_s + Q_f \]

where \( Q = \) total pulse charge

\( Q_s = \) slow component of the scintillation pulse charge

It is however, more convenient, to consider the charge as an explicit function of time, i.e.:

\[ Q(t) = Q - Q_s e^{-t/\tau_s} + Q_f e^{-t/\tau_f} \]

Since we are interested in the integrated pulse at \( t = 1 \mu\text{sec} \), the fast component in the above expression may be overlooked to a first approximation \( [e^{-t/\tau_f} \approx 10^{-175} \text{ for } t = 1 \mu\text{sec}; \tau_f = 2.5 \text{ns.}] \)

Applying the Laplace transformation:

\[ Q(s) = \frac{Q}{s} - \frac{Q_s}{s + 1/\tau_s} \]

The operation of single delay-line differentiation may be represented by:
\[ 1 - e^{-\tau_d s} \]

where: \( \tau_d = 2 \times \text{delay-line length} \)

whilst the RC-differentiation may be represented by:

\[ s + \frac{1}{\tau_{rc.}} \]

where \( \tau_{rc.} = R \times C \).

Thus, the Laplace transform of the doubly differentiated pulse is

\[ Q''(s) = \left[ \frac{Q}{s} - \frac{Q_s}{s + 1/\tau_s} \right] \left[ 1 - e^{-\tau_d s} \right] \left[ s + \frac{1}{\tau_{rc.}} \right]. \]

Its inverse is:

\[ Q''(t) = \frac{Q}{\tau_{rc.}} + Q_s \left( 1 - e^{\frac{\tau_d}{\tau_s}} \right) \left( \frac{1}{\tau_s} - \frac{1}{\tau_{rc.}} \right) e^{-t/\tau_s} \]

Equating this to zero, in order to obtain the zero-crossover point:

\[ t \propto \tau_s \ln \left[ \frac{Q_s}{Q} \left( 1 - e^{\frac{-\tau_d}{\tau_s}} \right) \right] \frac{\tau_{rc.}}{\tau_d} \]
APPENDIX B.

EXPERIMENTAL DETECTION EFFICIENCY FOR THE NEUTRON COUNTER

Let $\frac{d\sigma}{d\Omega} (\theta)_{np}$ = the differential cross-section for the elastic scattering of neutrons by protons at a lab angle $\theta$ relative to the beam direction,

$C$ = total number of elastically scattered neutrons detected by the counter,

$M$ = total number of counts recorded by the monitor,

$\text{Eff}_r(E_n) = \text{relative efficiency of the counter for detecting neutrons of energy } E_n$,

$A(E_n,\theta) = \text{a geometrical factor to account for attenuation in the scatterer of the scattered neutrons.}$

Thus

$$\frac{C}{M} = \frac{d\sigma}{d\Omega} (\theta)_{np} A(E_n,\theta) \text{Eff}_r(E_n) .$$

However, the n-p scattering is isotropic in the centre-of-mass system for energies below about 15 MeV\textsuperscript{42}), therefore,

$$\frac{d\sigma}{d\Omega} (\theta)_{np} \propto \cos\theta .$$

The relative neutron efficiency thus satisfies the following relation,

\textsuperscript{42} J. B. Marion and J. L. Fowler, Fast Neutron Physics, Part I (Interscience Pub. 1960) Chapt. 2-B.
Figure 42. The scatterer geometry and defining relations as used in the calculation of the geometric factor, $A(E_n, \theta)$.
\[ \text{Eff}_{r}(E_n) \propto \frac{C}{M A(E_n, \Theta)} \frac{1}{\cos \Theta} \]

In order to calculate the geometrical correction factor \( A_{n, n', \Theta} \), for calculation purposes, let the scatterer be replaced by a 1" cube, with one face normal to the incident neutron beam, and let multiple scattering events above the second order be neglected. In Figure 42, the defining relations and reference axes necessary for this calculation are indicated. The z-axis is along the direction of the beam whilst the x-axis is perpendicular to the z-axis and the plane containing the detector and the beam. Furthermore, let:

\begin{align*}
N_p & = \text{number of hydrogen atoms per c.c. in the scatterer} \\
N_c & = \text{number of carbon atoms per c.c. in the scatterer} \\
A_p(E_n) & = \text{total n-p cross-section for a neutron of energy of } E_n \\
A_c(E_n) & = \text{total n-^{12}C cross-section for neutrons of energy } E_n \\
N_o & = \text{number of neutrons per cm}^2 \text{ initially present in the incident neutron beam} \\
L & = \text{the distance from a point of primary scattering to the edge of the scatterer in the direction } \Theta \\
E_n' & = \text{the neutron energy after the initial scattering} \\
a & = N_c A_c(E_n) + N_p A_p(E_n)' \\
a' & = N_c A_c(E_n) + N_p A_p(E_n)' 
\end{align*}
Then, if $N$ is the total number of neutrons per cm$^2$ as a function of $z$,

$$dN = -aN \, dz$$

$$N = N_0 e^{-az}$$

Thus, the geometrical factor $A(E_n, \theta)$ is determined by:

$$A(E_n, \theta) = \int_{x=-\xi}^{\xi} \int_{z=-\lambda}^{\xi} e^{-a z} e^{-a' z'} dx dz$$

This integration has been programmed for computer evaluation by Joseph$^{43)}$ and his results are summarized here. There are 4 cases of relevance.

i). If $0 < \theta < \eta$

$$A = \frac{\sin \theta}{a \, a'} [1 - e^{-2a \lambda}] - \frac{\sin \theta \, e^{-2a' \lambda \cos \theta}}{a' (a-a'/\cos \theta)} [1 - e^{-(a-a'/\cos \theta) 2 \lambda}]$$

$$+ \frac{(2 \xi - 2 \lambda \tan \theta) \, e^{-2a' \lambda \cos \theta}}{a - (a'/\cos \theta)} - 2 \xi \, e^{-2a \lambda}$$

$$+ \frac{e^{-2a' \cos \theta}}{(a-a'/\cos \theta)^2} [1 - e^{-(a-a'/\cos \theta) 2 \lambda}] .$$

ii). If $\eta < \theta < \pi/2$

\[
A = \frac{\sin \theta}{a} \left[ 1 - e^{-2a'\xi / \sin \theta} \right] + \frac{e^{-2a\xi \tan \theta}}{a(a-a'/\cos \theta)} \left[ 1 - e^{-(a-a'/\cos \theta)2\xi / \tan \theta} \right]
\]

\[
- \frac{2\xi e^{-2a\lambda}}{a-a'/\cos \theta} - \frac{e^{-2a\lambda \tan \theta}}{(a-a'/\cos \theta)^2} \left[ 1 - e^{-(a-a'/\cos \theta)2\xi / \tan \theta} \right].
\]

iii). If \( \pi/2 < \theta < \pi - \eta \)

\[
A = -\frac{\sin \theta e^{-2a\lambda}}{a} \frac{a'}{a'} \left[ 1 - e^{-2a'\xi / \sin \theta} \right] + \frac{2\xi}{(a-a'/\cos \theta)} \frac{\tan \theta}{a(a-a'/\cos \theta)} \left[ 1 - e^{-(a-a'/\cos \theta)2\xi / \tan \theta} \right]
\]

iv). If \( (\pi-\eta) < \theta < \pi \)

\[
A = \frac{\sin \theta}{a} \frac{a}{a'} \left[ 1 - e^{-2a\lambda} \right] - \frac{\sin \theta}{a'} \frac{(a-a'/\cos \theta)2\lambda}{a(a-a'/\cos \theta)} \left[ 1 - e^{-(a-a'/\cos \theta)2\lambda} \right]
\]

\[
+ \frac{2\xi - (2\xi + 2\lambda \tan \theta)e^{-(a-a'/\cos \theta)2\lambda}}{(a-a'/\cos \theta)}
\]

\[
- \frac{\tan \theta}{(a-a'/\cos \theta)^2} \left[ 1 - e^{-(a-a'/\cos \theta)2\lambda} \right].
\]
APPENDIX C.

THEORETICAL DETECTION EFFICIENCY FOR THE NEUTRON COUNTER

Neutrons are detected in an organic scintillator principally through elastic scattering by protons in the scintillator, and to calculate their detection efficiency theoretically, the number of such recoil protons must be calculated. Considering primary n-p scattering alone, this number \( N_{rp} \) is given by:

\[
N_{rp} = \int_0^s N_o N_p A_p (E_n) \, dz
\]

[c.f. Appendix B.]

where \( s \) is the length of the scintillator.

Thus

\[
N_{rp} = \frac{1}{a} \, N_o N_p A_p (E_n) \left( 1 - e^{-as} \right)
\]

There will be a further contribution to the number of recoil protons in the scintillator arising from neutrons scattering initially off carbon, and then undergoing secondary scattering with protons. This number, \( N_{rcpc} \), is given by:

\[
N_{rcpc} = \int_0^{s'} \int_0^{s''} N_o e^{-az} N_o A_c (E_n) \, N_p A_p (E_n') \, dz \, dz'
\]

\[
= \frac{1}{a} \, N_o N_p N_c A_p (E_n') \, A_c (E_n) \left( 1 - e^{-as} \right) \left( 1 - e^{-a'z'} \right)
\]
where \( t' \) is some characteristic distance presented by the scintillator to neutrons initially scattered off carbon. For these calculations a distance of 2.0 cm was chosen. This corresponds to the distance a 1 MeV neutron beam would traverse in the scintillator before being reduced to slightly less than half its intensity.

Thus, if every such recoil proton were seen by the detector system, the efficiency would be given by:

\[
\text{Eff} = \frac{(N_{\text{rp}} + N_{\text{rpc}})}{N_0}
\]

However, there exists an inherent bias level in the system, below which the protons are not seen. If the relationship between the neutron energy and its associated recoil proton pulse height is linear, this bias effect may readily be accounted for: Thus, the observed detector efficiency is given theoretically by:

\[
\text{Eff}(E_n) = (1 - \frac{E_{\text{bias}}}{E_n}) \frac{(N_{\text{rp}} + N_{\text{rpc}})}{N_0}
\]

\( E_{\text{bias}} \) is the neutron energy corresponding to the detector bias level; and to a first approximation, the linearity of the energy-pulse height function is a valid assumption.
APPENDIX D.

KINEMATICS FOR SIMULTANEOUS BREAKUP

In determining the permitted loci of the 3-body breakup in the $E_1 - E_2$ plane, it is convenient to use the coordinate system shown in Figure 26. $\vec{p}_i$ ($i = 1, 2, 3$) is the laboratory momentum of the $i$th final state particle, $\theta_i$ is the angle between $\vec{p}_i$ and the incident beam momentum, $\vec{p}_o$, and $\phi_i$ measures the angle about the beam axis $z$ from the plane containing the beam and the x-axis. The kinetic energies and masses of the particles are $T_i$ and $m_i$ respectively ($i = 0, 1, 2, 3$), and the energy necessary for the 3-particle breakup is represented by $Q$.

Applying the conservation of energy,

$$T_o + Q = \Sigma_{i=1}^{3} T_i$$  \hspace{1cm} (i)$$

$$\frac{\vec{p}_o^2}{2m_o} + Q = \Sigma_{i=1}^{3} \frac{\vec{p}_i^2}{2m_i}$$  \hspace{1cm} (ii)$$

From the conservation of momentum:

$$\vec{p}_o = \Sigma_{i=1}^{3} \vec{p}_i$$  \hspace{1cm} (iii)$$
This vector equation may also be written in terms of its three components. There are thus 4 linearly independent equations, and a solution for $p_2$ as a function of $p_1$ for a particular set of laboratory angles, $\theta_1$, $\theta_2$, $\phi_1$, and $\phi_2$ may thus be derived.

Rewriting equation iii) explicitly,

1. $p_o = \sum_{i=1}^{3} p_i \cos \theta_i \quad \text{(iii-a)}$
2. $0 = \sum_{i=1}^{3} p_i \sin \theta_i \sin \phi_i \quad \text{(iii-b)}$
3. $0 = \sum_{i=1}^{3} p_i \sin \theta_i \cos \phi_i \quad \text{(iii-c)}$

Eliminating $\phi_3$ between Equations iii-b) and iii-c):

$$p_3^2 \sin^2 \theta_3 = [p_1 \sin \theta_1 \sin \phi_1 + p_2 \sin \theta_2 \sin \phi_2]^2$$

$$+ [p_1 \sin \theta_1 \cos \phi_1 + p_2 \sin \theta_2 \cos \phi_2]^2 \quad \text{(iv)}$$

Combining Equations iii-a) and v), and substituting this into Equation ii) yields:

$$p_2^3 \left[ \frac{1}{2m_2} + \frac{1}{2m_3} \right] + p_2 \times \frac{1}{m_3} \left[ p_1 \cos \theta_1 \cos \phi_2 \right.$$}

$$+ p_1 \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2) - p_0 \cos \phi_2 \left. \right]$$

$$+ p_1^2 \left[ \frac{1}{2m_1} + \frac{1}{2m_3} \right] + p_0^2 \left[ \frac{1}{2m_3} - \frac{1}{2m_0} \right] - \frac{1}{m_3} p_0 p_1 \cos \theta_1 - Q = 0. \quad \text{(v)}$$
These equations have now been reduced to that of a general quadratic expression, whose result is given by:

\[ p_2 = -\frac{B \pm \sqrt{B^2 - 4AC}}{2A} \]

where:

\[ A = \frac{1}{2} \left[ \frac{1}{m_2} + \frac{1}{m_3} \right] \]

\[ B = \frac{1}{m_3} \left[ p_1 \cos \theta_1 \cos \theta_2 + p_1 \sin \theta_1 \sin \theta_2 \cos (\theta_1 - \theta_2) - p_\circ \cos \theta_2 \right] \]

\[ C = \frac{p_\circ^2}{2} \left[ \frac{1}{m_3} - \frac{1}{m_\circ} \right] + \frac{p_1^2}{2} \left[ \frac{1}{m_1} + \frac{1}{m_3} \right] - \frac{1}{m_3} p_\circ p_1 \cos \theta_1 - Q. \]
APPENDIX E.

KINETICS FOR SEQUENTIAL DECAY

The internal energy of the intermediate system in a sequential decay process may be calculated, and this gives rise to a further restraint on the kinematics. If $Q$ is the energy necessary for the 3-body breakup reaction and $E_{ij}$ is the internal energy in the intermediate system of $(i+j)$ when particle $k$ is emitted first [$i,j,k = 1,2,3$, $i \neq j \neq k \neq i$], the following 2 relations are obtained from the conservation laws.

\[ T_o + Q = T_{jk} + E_{jk} + T_i \quad \text{i)} \]
\[ \bar{p}_o = \bar{p}_i + \bar{p}_{jk} \quad \text{ii)} \]

Rewriting the momentum conservation relationship in components:

\[ p_o = p_i \cos \theta_i + p_{jk} \cos \theta_{jk} \quad \text{iii)} \]
\[ 0 = p_i \sin \theta_i \cos \phi_i + p_{jk} \sin \theta_{jk} \cos \phi_{jk} \quad \text{iv)} \]
\[ 0 = p_i \sin \theta_i \sin \phi_i + p_{jk} \sin \theta_{jk} \sin \phi_{jk} \quad \text{v)} \]
Since the two-body reaction is restricted to a plane, 
\[ \theta_i = \theta_{jk} + \pi. \]
Eliminating \( \theta_i \) and \( \theta_{jk} \) between Equations iv) and v):

\[ p_i^2 \sin^2 \theta_i = p_{jk}^2 \sin^2 \theta_{jk}. \]

Eliminating \( \theta_{jk} \) by using Equations iii) and vi):

\[ p_{jk}^2 = p_i^2 \sin^2 \theta_i + (p_o - p_i \cos \theta_i)^2 \]
\[ = p_o^2 + p_i^2 - 2p_o p_i \cos \theta_i. \]

Furthermore:

\[ T_{jk} = \frac{p_{jk}^2}{2m_{jk}}. \]

(from Equation i),

\[ E_{jk} = T_o + Q - T_i - \frac{1}{2m_{jk}} \left[ p_o^2 + p_i^2 - 2p_o p_i \cos \theta_i \right] \]

vii)

Particle 1 has been chosen as a reference particle, and it is clear that while \( E_{23} \) will be a single valued function of \( T_1 \), \( E_{12} \) and \( E_{13} \) will be double valued. These will be written as \( E_{12}^+, (E_{13}^+) \) and \( E_{12}^-, (E_{13}^-) \) according as \( p_2^+ \) or \( p_2^- \) is chosen.
Explicitly:

\[ E_{23} = T_0 + Q - T_1 - \frac{1}{2m_{33}} \left( p_0^2 + p_1^2 - 2p_0 p_1 \cos \theta_1 \right) \]

\[ E_{13^\pm} = T_0 + Q - T_{2^\pm} - \frac{1}{2m_{13}} \left[ p_0^2 + p_{2^\pm}^2 - 2p_0 p_{2^\pm} \cos \theta_2 \right] \]

\[ E_{12^\pm} = T_0 + Q - \frac{1}{2m_{12}} \left[ C_{\pm}^2 + D_{\pm}^2 \right] - \frac{1}{2m_{23}} \left[ p_0 - C_{\pm} \right]^2 + D_{\pm}^2 \]

where \[ C_{\pm} = p_1 \cos \theta_1 + p_{2^\pm} \cos \theta_2 \]

\[ D_{\pm} = p_1 \sin \theta_1 - p_{2^\pm} \sin \theta_2 \].
APPENDIX F.

THE DENSITY OF STATES FUNCTIONS

For a n-p interaction, there is no Coulomb interaction to be considered, and the renormalized density of states is given by:

\[ \rho(k, a_1) = \frac{\mu}{\pi n^2 k} \left[ \frac{d}{dk} (\delta + \varnothing) - \frac{1}{2k} \sin^2 (\delta + \varnothing) \right] \]

where

\( \delta \) is the p-n phase shift,
\( a_1 \) is the p-n interaction radius, and
\( \varnothing \) is the hard sphere phase shift.

The effective range expansion can be applied to determine \( \delta \), i.e.

\[ k \cot \delta = -\frac{1}{a_1} + \frac{1}{2} r_o k^2. \]

Here, \( r_o \) is the effective range of the n-p interaction.

Also, \( \varnothing = \tan^{-1} \left[ \frac{F_\ell}{G_\ell} \right] \) where \( F_\ell \) and \( G_\ell \) are the regular and irregular solutions to the Schrödinger equation for the system. Since there is no Coulomb interaction, these solutions are Bessel functions, and because of the low energies of the system, only \( L = 0 \) angular momentum contributions
need be considered.

Thus:

\[ F_0 \sim \sin k r_0 \]
\[ G_0 \sim \cos k r_0 \]

\[ \theta = \tan^{-1} \left[ \frac{\sin kr_0}{\cos kr_0} \right] \]

If the radius of interaction is set equal to the effective range:

\[ \theta = k a_1 \]
APPENDIX G.

KINEMATICAL LIMITS ON PROXIMITY SCATTERING FROM
A CLASSICAL APPROACH

Consider a generalized 3-body reaction proceeding via
sequential decay, with $Q$ being the energy associated with
the complete reaction, i.e.:

\[
\begin{align*}
\text{b+t} & \rightarrow \text{l' +(2-3)* seq. decay} \\
& \rightarrow (1-2)* \text{prox. scatt.} + 3 \\
& \rightarrow 1 + 2 + 3
\end{align*}
\]

i)

It is assumed that the lifetime of the (2-3)* composite system is large compared with the times necessary for
a particle to traverse a nucleus ($10^{-22}$ s). Relativistic
and Coulomb potential effects have not been taken into
consideration.

The energy available in the overall centre-of-mass
system is $W$, where

\[
W = \frac{m_t}{m_b + m_t} E_{\text{lab}} + Q .
\]

ii)

Thus, the velocity of particle $l'$ in the centre-of-mass
system before rescattering is:
\[ v_1' = \left[ \frac{2(m_2 + m_3)}{m_1(m_1 + m_2 + m_3)} (W - E_{23}) \right]^{1/2} \quad \text{iii) iii)} \]

where \( E_{23} \) is the internal energy of the sequential decay resonance.

Transforming to a coordinate system in which the \((2-3)^*\) system is at rest, (the R-system), the velocity of particle 1' is now:

\[ v_1^R = \frac{m_1 + m_2 + m_3}{m_2 + m_3} v_1' \quad \text{iv) iv)} \]

\[ = \left[ \frac{2(m_1 + m_2 + m_3)}{m_1(m_2 + m_3)} (W - E_{23}) \right]^{1/2} \quad \text{iva) iva)} \]

Similarly, the velocity of particle 2 in this reference system is:

\[ v_2^R = \left[ \frac{2m_3}{m_2(m_2 + m_3)} E_{23} \right]^{1/2} \]

Thus, the relative energy, \( E_{12}^{\text{Cl}} \), before rescattering is given by:

\[ E_{12} = \frac{1}{2} \mu_{12} \left[ v_2^R - v_1^R \right]^2 \]

\[ = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} \left[ \left( \frac{2m_3}{m_2(m_2 + m_3)} E_{23} \right) \right]^{1/2} \]

\[ - \left( \frac{2(m_1 + m_2 + m_3)}{m_1(m_2 + m_3)} (W - E_{23}) \right)^{1/2} \]}
If particle 2 is emitted in the direction of particle 1, and if its velocity is greater, these two particles may approach sufficiently close to interact, and proximity scattering may occur.

From Equation iva), the minimum energy, $E_{b_{\text{min}}}$, necessary to populate the intermediate resonance is determined by:

$$ W = E_{23} $$

i.e.

$$ \frac{m_t}{m_b + m_t} E_{b_{\text{lab}}} + Q = E_{23} $$

$$ E_{b_{\text{min}}} = \frac{m_b + m_t}{m_t} [E_{23} - Q] $$

Similarly, in order for particle 2 to interact with particle 1,

$$ V_2^R > V_1^R $$

$$ \frac{2m_3}{m_2 (m_2 + m_3)} E_{23} > \frac{2(m_1 + m_2 + m_3)}{m_1 (m_2 + m_3)} (W - E_{23}) $$

$$ E_{b_{\text{max}}} = \frac{m_b + m_t}{m_b} \left[ \frac{m_1 m_3 + m_2 (m_1 + m_2 + m_3)}{m_2 (m_1 + m_2 + m_3)} E_{23} - Q \right] $$
VII. REFERENCES


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41. V. A. Otte, private communication.


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