HARDY, Jr., Donald McCoy, 1944-
LIGHT NUCLEI INTERACTIONS WITH A POLARIZED
\( ^3\text{He} \) TARGET.

Rice University, Ph.D., 1970
Physics, nuclear

University Microfilms, A XEROX Company, Ann Arbor, Michigan
RICE UNIVERSITY

Light Nuclei Interactions with
a Polarized $^3$He Target

by

Donald McCoy Hardy, Jr.

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Thesis Director's signature

Houston, Texas

April 1970
Abstract

Light Nuclei Interactions with a Polarized $^3$He Target

by

Donald McCoy Hardy Jr.

The asymmetries in the scattering of $^4$He particles from $^3$He particles have been measured at $79.3^\circ$ c.m. and $114.0^\circ$ c.m. in the energy range 7.5 to 18.0 MeV. A phase shift analysis of $^4$He-$^3$He scattering incorporating the presently available polarization data has been performed for $^3$He laboratory beam energies ranging from 5.7 to 13.5 MeV. Data for the scattering of $^3$He from polarized $^3$He at $66.0^\circ$ c.m. in the energy range 9.3 to 17.5 MeV are also reported. Asymmetries in the elastic scattering of deuterons from polarized $^3$He at five center of mass scattering angles in the energy range 4.8 to 11.9 MeV have also been measured, as well as the proton asymmetries at two laboratory angles for the $^3$He(d,p)$^4$He reaction conducted with a polarized $^3$He target. Comparisons are made of the existing polarization data for d-$^3$He elastic scattering and for the $^3$He(d,p)$^4$He reaction.
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I. Introduction

Among the many types of experiments which can be employed in studying the nature of the nuclear force and the properties of various nuclei are those involving spin dependent effects. Evidence for the importance of spin dependence in the nuclear force can be found in the structure of the simplest bound system of nucleons, the deuteron, where the $J = 1$ triplet state is bound but the $J = 0$ singlet state is not. In addition, in order to account for the observed electric quadrupole moment and magnetic dipole moment of the deuteron, a spin-spin (or tensor) potential has been assumed. (En 66). Additional evidence is found in nucleon-nucleon scattering, where a spin-orbit and a tensor interaction have been used to explain the observed polarization effects (Gr 67). In more complicated nuclear systems, evidence for the spin dependence of the nuclear force can be found in the successes of the shell model description, in which a strong spin-orbit force is assumed (En 66). Recently, experiments involving polarization effects have been increasing rapidly in number and in variety. Such experiments supplement the extensive information which can be obtained through other means by providing information on the spin dependence of nuclear interactions.
When an unpolarized beam is scattered from an unpolarized target, the resulting polarization of the scattered beam, $\mathbf{P}$, is either parallel or antiparallel to the vector $\mathbf{k}_\text{in} \times \mathbf{k}_\text{out}$ where $\mathbf{k}_\text{in}$ and $\mathbf{k}_\text{out}$ are the directions of the incident particle before and after scattering, respectively. According to the Basel convention (Ba 60) the sign of $\mathbf{P}$ is defined to be positive when $\mathbf{P}$ is parallel to $\mathbf{k}_\text{in} \times \mathbf{k}_\text{out}$. For the case of spin $\frac{1}{2}$ particles the magnitude of the polarization is given by $P = (N_U - N_D) / (N_U + N_D)$ where $N_U$ and $N_D$ are the numbers of particles with spins parallel and antiparallel to the direction of $\mathbf{k}_\text{in} \times \mathbf{k}_\text{out}$, respectively. In describing the scattering of polarized beams, left (or right) scattering is defined as scattering for which the vector $\mathbf{k}_\text{in} \times \mathbf{k}_\text{out}$ is parallel (or antiparallel) to the polarization direction of the incident beam. The left-right scattering asymmetry is then given by $A = (N_L - N_R) / (N_L + N_R)$ where $N_L$ and $N_R$ are the numbers of particles scattered to the left and right, respectively.

The asymmetry produced in the scattering of a completely polarized beam from an unpolarized target is known as the analyzing power of the scatterer. For a given scattering process it can be shown (Bl 52, Wo 52) that under the assumption of time reversal invariance the polarization produced in the scattering of an unpolarized beam from
an unpolarized target is always equal to the analyzing power in an elastic scattering interaction. An analogous relationship holds for the case of inelastic scattering, where it can be shown on the basis of other symmetry principles (Bi 59) that the analyzing power of a reaction is equal to the polarization produced in the inverse reaction.

The description of the polarization of spin 1 particles is more complex than that of spin $\frac{1}{2}$ particles, and has been treated by several authors (La 55, Da 67). In addition to the vector polarization which a system of spin $\frac{1}{2}$ particles can possess, a system of spin 1 particles can possess tensor polarization. The vector polarization of a system of spin 1 particles is given by $\langle \vec{S} \rangle$ where $\vec{S}$ is the spin 1 operator. The tensor polarization is related to the expectation values of terms quadratic in the components of the spin 1 operator. In general, polarization experiments involving spin 1 particles may therefore involve the determination of both vector and tensor polarization. A recent discussion of the polarization effects involved in the scattering of polarized spin $\frac{1}{2}$ particles from unpolarized spin 1 particles has been given by Seyler (Se 69). General discussions of the polarization phenomena of spin $\frac{1}{2}$ particles are given
in References Wo 52, Wo 54 and Ba 67.

The relationships between the analyzing power and the polarization produced in the scattering of an unpolarized beam from an unpolarized target make it possible to measure the polarization in a given scattering process either by double scattering techniques or through experiments conducted with a polarized beam or target. In a double scattering experiment, the polarization produced by the first scatter is determined by measuring the asymmetry produced in a second scatter. The second scatterer is referred to as the analyzer. The asymmetry in the second scattering is equal to the polarization produced by the first scattering times the analyzing power of the second scatterer, and polarization experiments are sometimes undertaken merely to establish the analyzing power of particular scattering processes.

The practical problems associated with a double scattering experiment include a high sensitivity to the scattering geometry as well as the inherently low intensities and kinematic energy losses involved in two successive nuclear scattering processes. These problems can be avoided by using a polarized beam or target in a single scattering experiment. The beam or target polarization is usually reversible, and data can then be taken
so that many geometrical factors cancel identically (see appendix C).

The recent development (Wa 62, Co 63, Ga 65) of optical pumping in $^3$He gas has made possible the construction of polarized $^3$He targets for nuclear scattering experiments (Ph 62, Ba 65, Ba 69). As a means of studying nuclear processes, such targets combine the simplicity of a spin $\frac{1}{2}$ particle with the features of a three nucleon system. This work describes several types of experiments conducted with a polarized $^3$He target. In section II, data are presented for the scattering of $^4$He from polarized $^3$He. A region of high polarization was investigated as a possible polarization analyzer in $^4$He - $^3$He scattering, and a phase shift analysis incorporating the presently available polarization data is reported. Data for the scattering of $^3$He from polarized $^3$He are also discussed in section II. These data were obtained at the California Institute of Technology in collaboration with T. A. Tombrello and Y. S. Chen. In section III, data obtained at Rice University for the elastic scattering of deuterons from polarized $^3$He are given, as well as polarization data for the $^3$He(d,p)$^4$He reaction conducted with a polarized $^3$He target. A comparison is made of existing polarization data for these processes. A simple relationship between
the deuteron polarization and the $^3$He polarization in elastic scattering becomes evident when the data of section III are compared with recently reported deuteron polarization measurements (Pl 69). This effect is analogous to effects previously observed in the (d, p) reaction (Ba 65).

Material essential in the analysis of the data reported in sections II and III, or related to the process of optical pumping and the construction of polarized $^3$He targets is given in the appendices. Appendix A contains a discussion of the method of partial wave analysis for the scattering of spin $\frac{1}{2}$ particles from spin zero particles and a derivation of the formulas for the cross section and the polarization; appendix B gives a discussion of the method of optical pumping and the determination of the target polarization; appendix C gives the method of calculating laboratory asymmetries and errors; appendix D contains a discussion of background corrections; appendix E describes the optical pumping apparatus and gives details of the target construction.
II. The Scattering of $^4\text{He}$ and $^3\text{He}$ from Polarized $^3\text{He}$

A. Discussion

The structure of the $^7\text{Be}$ nucleus has been studied previously through a number of $^3\text{He} - ^4\text{He}$ and $p - ^6\text{Li}$ experiments which did not involve polarization measurements (Wa 56, Ha 63, Je 63, Mc 63, To 63, Ba 64, Sp 67). As a result of these experiments much of the level structure of $^7\text{Be}$ below an excitation energy of 11 MeV has been well established. The presently known energy levels of the $^7\text{Be}$ nucleus are shown in Fig. 1. An interesting conclusion drawn from these studies is that the first two $5/2^-$ levels of $^7\text{Be}$, which occur close to one another in energy, appear distinctly as a $^3\text{He} - ^4\text{He}$ cluster and as a $p - ^6\text{Li}$ cluster (Ph 64). The second $5/2^-$ level occurs at an excitation energy of 7.21 MeV in the $^7\text{Be}$ compound nucleus and is prominent in $p - ^6\text{Li}$ scattering. The first $5/2^-$ level occurs approximately 500 keV below the second and is indicated by a broad anomaly in $^3\text{He} - ^4\text{He}$ elastic scattering. The presence of this level is not indicated in either the elastic or inelastic $p - ^6\text{Li}$ scattering data. Previous phase shift analyses of $^3\text{He} - ^4\text{He}$ scattering (To 63, Ba 64, Sp 67) indicated that a number of regions of high $^3\text{He}$ polarization were to be expected and it was suggested that this scattering process might
Figure 1

Energy level diagram of the $^7$Be nucleus. The energy values are plotted vertically in MeV, based on the ground state as zero. Values of the total angular momentum and parity are indicated for each level. Typical elastic scattering excitation functions for reactions in which $^7$Be is the compound nucleus are also shown, with bombarding energies in the laboratory system plotted vertically and the differential cross sections plotted horizontally.
therefore be useful as an analyzer of $^3$He polarization.

In order to supplement the previous studies of the $^7$Be nucleus and to investigate a region that was potentially useful as a polarization analyzer, a measurement of the polarization in $^3$He - $^4$He scattering at a laboratory angle of 33° was undertaken through a $^3$He($^4$He,$^4$He)$^3$He experiment conducted with a polarized $^3$He target. As a result, the $^3$He polarization as a function of energy at two center of mass angles was measured for $^7$Be excitation energies ranging from approximately 5.0 to 9.3 MeV. This measurement is complimentary to other polarization measurements which have recently been reported for both p - $^6$Li and $^3$He - $^4$He scattering. For the case of p - $^6$Li scattering, polarization measurements and phase shift analysis have been reported by Brown and Petitjean (Br 68) and by Petitjean et al. (Pe 69). Double scattering measurements of $^3$He - $^4$He scattering have been reported by Armstrong et al. (Ar 69) who obtained a polarization angular distribution at 13 MeV $^3$He laboratory energy as well as the polarization at $\theta_{cm} = 40.1^\circ$ at three different energies. Polarization measurements are particularly helpful in that they can often provide a sensitive test of phase shifts which were derived on the basis of cross section measurements.
alone. Therefore, the polarization data reported in this section, the polarization data of Armstrong et al. (Ar 69) and the differential cross section measurements of Spiger and Tombrello (Sp 67) were combined in a phase shift analysis of \(^3\text{He} - ^4\text{He}\) scattering. On the basis of the derived phase shifts, a new \(^3\text{He}\) polarization contour map has been prepared and a comparison has been made of several regions which may be employed as \(^3\text{He}\) polarization analyzers. The results of the present phase shift analysis corroborate previous level assignments for the \(^7\text{Be}\) nucleus; in particular, no evidence of the second \(^{5/2}\)\(^-\) \(^7\text{Be}\) level has been seen in the elastic parameters.

Data on the scattering of \(^3\text{He}\) from polarized \(^3\text{He}\) for beam energies ranging from 9.3 to 17.5 MeV will also be reported in this section. The data were taken at a center of mass angle of 66° and indicate that the \(^3\text{He}\) polarization is near zero. These results may be compared with the analysis of Bacher et al. (Ba 68b) who studied the scattering of \(^3\text{He}\) from unpolarized \(^3\text{He}\) for beam energies up to 19 MeV and reported good agreement with the resonating group calculations of Thompson and Tang (Th 67). Both the \(^3\text{He} - ^3\text{He}\) phase shift analysis of Ref. Ba 68b and the calculations of Ref. Th 67 attribute changes in the shapes of the excitation curves and angular distributions
for beam energies above 12 MeV to a broad \( P \)-wave resonance. In these analyses, no splitting of any of the partial waves was assumed and the absence of any polarization effects at \( \theta_{cm} = 66^\circ \) is not inconsistent with these assumptions.

B. Experiment

1) Polarized \(^3\text{He}\) Target

The \(^3\text{He}\) target was polarized by the method of optical pumping (Co 63). The optical pumping apparatus used was the same as that reported by Baker et al. (Ba 69). A brief discussion of the optical pumping apparatus as well as an extensive discussion of the target construction is given in appendix E. The scattering chamber which was employed allowed the observation of left-right scattering asymmetries at a single laboratory angle of \( 33^\circ \). The scattered particles were collimated by two slits; one formed in the glass work of the scattering chamber and one placed just before the charged particle detector. The overall center of mass rms angular resolution, due to beam collimations, multiple scattering in the entrance foil of the chamber and collimation of the scattered particles varied from \( 1.85^\circ \) to \( 3.34^\circ \) and is given along with the asymmetry data in Tables I and II.
The target polarization was determined by optical measurements as described in appendix B, where the definitions of the parameters discussed below are also given. Within the accuracy of the optical measurements, the ratio of the optical signals, $\Delta I/I$, did not vary during the time required to obtain data at a given energy. However, the values of $\Delta I/I$ ranged from 0.67 to 0.52 during the several days in which data were taken. The value of the parameter $\rho$ was less than 0.010. The values used for $a$, $b$ and $c$ were 0.28, 0.10 and 0.30, respectively. The parameter $f$ was assumed to be 0.7. This value of $f$ gives an apparent target polarization midway between that obtained for $f = 0.5$ and $f = 1.0$. All of the data reported here assume this value of $f$. As discussed in appendix B and in Ref. Ba 69, the uncertainty in the value of $f$ results in a systematic uncertainty in the target polarization. As a result, all of the experimental asymmetries given in Tables I and II may be multiplied by a single factor ranging from 0.85 to 1.15. However, considering the values reported in Table I for $^4$He - $^3$He scattering at $\Theta_{cm} = 79.3^\circ$, which are already near maximum, a multiplying factor as large as 1.15 would be very improbable. Also, Fig. 2 indicates reasonable agreement with the double scattering data of Ref. Ar 69, and the proper
systematic correction is probably smaller than 15%.

2) Beam

The CIT-ONR tandem accelerator furnished $^4$He and $^3$He beams which had characteristic energy uncertainties of $\pm 25$ keV over the energy ranges used. The aluminum entrance foil of the scattering chamber had a thickness of $(2.30 \pm 0.05)$ mg/cm$^2$, and the energies of the incident $^4$He or $^3$He beams were corrected for their energy losses in the entrance foil. The uncertainty in the thickness of the entrance foil resulted in an additional systematic uncertainty of approximately $\pm 15$ keV in the beam energy.

C. $^4$He + $^3$He Asymmetry Measurements

Data were obtained at eighteen $^4$He beam energies ranging from 8.5 to 18.5 MeV. In terms of an experiment involving a $^3$He beam and a $^4$He target, the equivalent $^3$He beam energies inside the scattering chamber ranged from 5.65 to 13.5 MeV. Beam currents were typically 0.45 $\mu$A for alpha beams below 13 MeV and 0.15 $\mu$A for beams above 13 MeV. Scattered $^4$He particles and recoil $^3$He particles were detected at $\theta_{\text{lab}} = 33^\circ$ by a pair of silicon surface barrier detectors which were placed symmetrically about the beam axis and in the plane perpendicular to the direction of the $^3$He target polarization. The $^4$He particles,
which were of lower energy than the recoil $^3\text{He}$ particles, corresponded to elastic scattering at $\Theta_{\text{cm}} = 79.3^\circ$. The $^3\text{He}$ particles corresponded to elastic scattering at $\Theta_{\text{cm}}$ equal $114.0^\circ$. The left-right scattering asymmetries were calculated according to the procedure given in appendix C and then corrected for target polarization. These results yield the equivalent of the $^3\text{He}$ polarization at $\Theta_{\text{cm}} = 79.3^\circ$ and $114.0^\circ$ in an unpolarized experiment, and are presented in Table I and in Fig. 2.

Pulses from the two detectors were processed in such a way that no corrections were necessary due to dead time in the electronics. At $\Theta_{\text{lab}} = 33^\circ$ the $^4\text{He}$ and $^3\text{He}$ particles are well separated in energy and in the pulse height spectra obtained, the separation between the peaks due to the two species of particles resulted in peak-to-valley ratios which were typically in excess of 50:1. The number of $^4\text{He}$ events was determined by simply summing the spectrum over the region of the $^4\text{He}$ peak and then subtracting the estimated number of background events which were included in the peak integration. The number of $^3\text{He}$ events was obtained in a similar manner. The separation between the $^4\text{He}$ and $^3\text{He}$ peaks was sufficiently good so that the number of $^3\text{He}$ events included as background in the $^4\text{He}$ integrations (or vice versa) was negligible.
Table I

Experimental asymmetries, $\theta_3$, and random errors, $\Delta \theta_3$, versus $^4\text{He}$ laboratory beam energy, $E_4$, for $^3\text{He}(^4\text{He},^4\text{He})^3\text{He}$ scattering at center of mass angles of 79.3° and 114.0°. The overall center of mass rms angular resolution, $\Delta \theta_{\text{c.m.}}$, is also given. The asterisks indicate data points for which the random errors were increased as described in the text. The random errors do not include the possible systematic error discussed in the text.

$$\theta_{\text{c.m.}} = 79.3^\circ$$

<table>
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<tr>
<th>$E_4$ (MeV)</th>
<th>$\Delta \theta_{\text{c.m.}}$ (Degrees)</th>
<th>$\theta_3$</th>
<th>$\Delta \theta_3$</th>
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<tr>
<td>7.50</td>
<td>3.34</td>
<td>0.86</td>
<td>± 0.028</td>
</tr>
<tr>
<td>8.04</td>
<td>3.25</td>
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<td>8.57</td>
<td>3.15</td>
<td>0.61</td>
<td>0.021</td>
</tr>
<tr>
<td>9.10</td>
<td>3.07</td>
<td>0.71</td>
<td>0.020</td>
</tr>
<tr>
<td>9.64*</td>
<td>3.00</td>
<td>0.80</td>
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<tr>
<td>10.17*</td>
<td>2.92</td>
<td>0.84</td>
<td>0.060</td>
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<td>10.69</td>
<td>2.85</td>
<td>0.75</td>
<td>0.024</td>
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<tr>
<td>11.22</td>
<td>2.80</td>
<td>0.11</td>
<td>0.022</td>
</tr>
<tr>
<td>11.74</td>
<td>2.75</td>
<td>-0.59</td>
<td>0.023</td>
</tr>
<tr>
<td>12.26</td>
<td>2.72</td>
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<td>12.78*</td>
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<td>17.94</td>
<td>2.44</td>
<td>-0.16</td>
<td>0.050</td>
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Table I (Continued)

\( \Theta_{\text{c.m.}} = 114.0^\circ \)

<table>
<thead>
<tr>
<th>( E_4 ) (MeV)</th>
<th>( \Delta \Theta ) (_{\text{c.m.}} ) (Degrees)</th>
<th>( \rho_3 )</th>
<th>( \Delta \rho_3 )</th>
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<tr>
<td>7.50</td>
<td>2.53</td>
<td>0.29</td>
<td>± 0.055</td>
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<td>8.04</td>
<td>2.45</td>
<td>0.31</td>
<td>0.055</td>
</tr>
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<td>8.57</td>
<td>2.37</td>
<td>0.25</td>
<td>0.046</td>
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<td>9.10</td>
<td>2.30</td>
<td>0.24</td>
<td>0.037</td>
</tr>
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<td>9.64</td>
<td>2.25</td>
<td>0.20</td>
<td>0.032</td>
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Figure 2

Experimental asymmetries for $^3\text{He}(^4\text{He}, ^4\text{He})^3\text{He}$ scattering at center of mass angles of $79.3^\circ$ and $114.0^\circ$ as a function of the equivalent $^3\text{He}$ laboratory energy are plotted as open or closed circles. The error bars do not include the possible systematic error discussed in the text. The open circles indicate data points for which the random errors were increased as described in the text. The single point at $79.3^\circ$ c.m., plotted as a square, is an interpolation of the data of Ref. Ar 69. The solid curves are the $^3\text{He}$ polarization calculated from the phase shifts of this work and the dashed curve gives the polarization at $114.0^\circ$ c.m. predicted by the phase shifts of Ref. Sp 67.
Figure 2

79.3° c.m.

114.0° c.m.

ASYMMETRY

$^3$He LAB ENERGY (MeV)
There were, however, other sources of background present in the $^4$He peaks. The estimated number of background events included in the integration of these peaks decreased from 1.8% of the number of elastic scattering events at the lowest energy point to no background at the three highest energy points. The maximum error introduced by performing the background subtraction was in all cases much less than the uncertainty due to statistics alone. There were no background events included in the $^3$He peak integrations.

The method of data collection and data analysis is discussed in detail in appendix C. Definitions of the quantities $P_3$, $\rho_3$ and $\rho_0$ discussed below are also given in appendix C. For each center of mass scattering angle values of the nominally zero quantity $P_3 \rho_0$ were calculated at each energy. A $\chi^2$ test was then employed to determine if the $P_3 \rho_0$ distributions thus obtained were consistent with zero, as discussed in appendix C. The distribution of $P_3 \rho_0$ values corresponding to $\theta_{cm} = 114.0^\circ$ was consistent with zero within statistics, but the distribution for $\theta_{cm} = 79.3^\circ$ was not. A study of the 79.3$^\circ$ data showed that four of the eighteen data points had values of $P_3 \rho_0$ which were more than 2.25 statistical standard deviations from zero. No irregularities were
found in the peak integrations or background estimates for these points and the values of $P_3^G$ for these four energies were consistent with the general trend of the other data. Therefore, rather than delete these data, the uncertainty in both $P_3^G$ and $P_3^R$ at these energies was arbitrarily increased by setting it equal to $P_3^R$. These four energy points are plotted as open circles in Fig. 2 and are indicated by asterisks in Table 1.

The values of $P_3^G$ obtained from the asymmetry data were divided by $P_3$, the target polarization as determined from the optical measurements, and resulting values of $P_3$ are given in Table I and are plotted in Fig. 2.

D. $^3$He + $^3$He Asymmetry Measurements

Data were obtained at eight laboratory beam energies ranging from 9.3 to 17.5 MeV. The $^3$He beam currents were typically 0.45 $\mu A$ below 13 MeV and 0.15 $\mu A$ above 13 MeV. The experimental apparatus, particle detection system, experimental procedure and data analysis employed was the same as that used in measuring the $^4$He - $^3$He asymmetries. The results obtained yield the equivalent of the $^3$He polarization at $\Theta_{cm} = 66^\circ$ in an unpolarized beam and target experiment. The distribution of $P_3^R$ values was consistent with zero within statistics. The estimated number of background events decreased from 11.0%
of the number of elastic scattering events at the lowest energy point to 3.3% at the highest energy point. The maximum error introduced by performing the background subtraction was in all cases less than the uncertainty due to statistics alone. Values of $\rho$ were calculated as in the $^4\text{He} - ^3\text{He}$ data analysis and are listed in Table II and plotted in Fig. 3. Only one of the data points, that for 13.44 MeV, suggests that the polarization is non-zero. No irregularities were found in the peak integrations or background estimates for this data point, and the value of $P_3$ for this datum is small. However, there is nothing in the data of Ref. Ba 68b to indicate that the polarization should be changing with energy near 13.4 MeV, and in view of the other polarization data presented, the polarization at $\theta_{\text{cm}} = 66^\circ$ may be regarded as near zero throughout the energy range studied. In fact, even if it is assumed that the polarization is identically zero in this range, there is a 0.17 probability of obtaining a data set having a value of $\chi^2$ greater than that calculated for the data of Table II.

E. $^4\text{He} - ^3\text{He}$ Phase Shifts

A phase shift analysis of $^4\text{He} - ^3\text{He}$ scattering was performed for equivalent $^3\text{He}$ lab energies ranging from
Table II

Experimental asymmetries, $\rho_3$, and random errors, $\Delta \rho_3$, versus $^3$He laboratory beam energy, $E_3$, for $^3$He($^3$He, $^3$He)$^3$He scattering at a center of mass angle of 66.0°. The overall center of mass rms angular resolution, $\Delta \theta_{\text{c.m.}}$, is also given. The random errors do not include the possible systematic error discussed in the text.

$$\theta_{\text{c.m.}} = 66.0^\circ$$

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Experimental asymmetries, $\rho_3$, for $^3\text{He}(^3\text{He},^3\text{He})^3\text{He}$ scattering at a center of mass angle of 66.0° are plotted as a function of the $^3\text{He}$ laboratory beam energy, $E_3$. 
5.69 to 13.47 MeV and the derived phase shift parameters are given in Table III and are shown in Fig. 4. The differential cross section measurements of Spiger and Tombrello (Sp 67), the double scattering measurements of Armstrong et al. (Ar 69) and the data of this section were combined in the phase shift searches. The method of partial wave analysis for the scattering of spin $\frac{1}{2}$ particles from spin 0 particles is discussed in appendix A, where the coherent and incoherent scattering amplitudes are derived in terms of the phase shifts of the various partial waves.

The parameters used in the analysis included the real and imaginary parts of the complex $s$-, $p$-, $d$- and $f$-wave phase shifts. Below the first proton threshold for the $^3\text{He}(^4\text{He}, p)^6\text{Li}$ reaction at 7.0 MeV only the seven real parameters were used. Initial values for the phase shifts were determined from the phase shifts of Ref. Sp 67.

Several patterns of variation were tried in the analysis before the method described below was accepted. It appears that the existing data are not sufficient to specify all of the possible parameters uniquely, and predictions of the polarization based only on the phase shifts and which are not verified by experiment should be used with caution. By varying first the real phase shifts of the $f$-waves,
Table III

Derived phase shift parameters for $^4\text{He-}^3\text{He}$ scattering as a function of the equivalent $^3\text{He}$ laboratory beam energy, $E_3$, for each value of $J^\pi$ used in the analysis. The real parts of the complex phase shifts, $\delta_J^\pi$, are given in degrees. The imaginary parts of the phase shifts, $\gamma_J^\pi$, are given in terms of the damping parameters $\alpha_J^\pi = e^{-2\gamma_J^\pi}$.

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<th>3/2-</th>
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</table>
Figure 4
Derived real phase shifts for $^4\text{He-}^3\text{He}$ scattering are plotted as a function of the equivalent $^3\text{He}$ laboratory beam energy, $E_3$, for each value of $J^\pi$ used in the analysis except $J^\pi = 3/2^+$ and $5/2^+$. The real phase shifts for $J^\pi = 3/2^+$ and $5/2^+$ are near zero and are given in Table III along with the inelastic parameters. Note that the negative of the $1/2^+$ real phase shift has been plotted. The lines serve only to connect the values obtained at different energies.
then those of the s- and p-waves, and then those of the d-waves and by executing this cycle twice, good fits were obtained below the inelastic threshold. The search routine used was a gradient search in which the free parameters were adjusted in accordance with the gradient of $\chi^2$ with respect to those parameters. This method was attempted above the inelastic threshold where initial values of both the real and imaginary parts of the phase shifts were determined from the parameters of Ref. Sp 67. The inelasticity in the $5/2^-$ channel was held equal to that of Ref. Sp 67 since this parameter was in accordance with existing $^6$Li(p, $^3$He)$^4$He data (Ma 56, Mc 62). Fits to the data were obtained, but the imaginary parts of the phase shifts did not vary smoothly with energy. However, the results obtained suggested that i) the imaginary parts of the $1/2^+$ and $3/2^-$ phase shifts were approximately the same and ii) the imaginary parts of $3/2^+$ and $5/2^+$ phase shifts were approximately the same. In an effort to reduce the number of free parameters and to impose energy continuity on the imaginary parts of the phase shifts, values for the imaginary parts of the phase shifts were chosen which had a smooth variation with energy, which were consistent with the trends noted above, and which were such that the calculated total reaction cross
section was in agreement with that used by Spiger and Tombrello (Sp 67). With the values of the imaginary parts of the phase shifts thus fixed, the s-, p-, d- and f-wave real phase shifts were varied simultaneously. Satisfactory fits to the data were then obtained above the inelastic threshold and these results were energy continuous with parameters found below the inelastic threshold. Phase shift searches above 13.47 MeV, where no polarization data exist, were conducted in a similar manner using just the data of Ref. Sp 67. Results were obtained for energies up to 16 MeV and are continuous in energy with the parameters reported here.

Figure 5 shows a polarization contour map based on the derived phase shift parameters. Also shown on the map are the positions of the measured values of polarization reported here and Ref. Ar 69.

The existence of a region of high polarization at 79.3° c.m. which persists over a broad range of energy has been experimentally verified. It is interesting to compare the usefulness of this region as a polarization analyzer for 3He ions whose energies are over 13 MeV with other regions. The figure of merit used is the product

\[ M = t \left( P^2 \left( \frac{d\sigma_{\text{LAB}}}{d\Omega_{\text{LAB}}} \right) \right) \sin \Theta_{\text{LAB}} \Delta \Theta_{\text{LAB}} \]
Figure 5

Contour map of $^3$He polarization (Basel convention) versus center of mass scattering angle and equivalent $^3$He laboratory energy. The numbers not labeling a contour line give the calculated values of the polarization at the positions of their decimal points. The triangles indicate the points at which asymmetries were measured in this work, and the squares indicate the points at which measurements were made by Armstrong et al. (Ar 69). For energies of 13.2 MeV and above the polarizations given here were calculated from phase shifts somewhat different from those of Table III. Polarizations calculated from the phase shifts of Table III for these energies give results similar but not identical to those shown here.
where the term in brackets is an estimate of the average over the region in question of the product of the square of the analyzing power and the laboratory differential cross section; $\theta_{\text{lab}}$ and $\Delta \theta_{\text{lab}}$ are, respectively, the central angle and angular spread of the region; and $t$ is the thickness of the target corresponding to the range of energy included in the region. Since $(\sin \theta_{\text{lab}})(\Delta \theta_{\text{lab}})$ is proportional to the solid angle, the figure of merit $M$ gives a measure of the overall counting rate times the square of the analyzing power.

The values of $M$ for several regions are listed in Table IV. It should be noted that in Regions I and II the sign of the laboratory asymmetries of the recoil $^4\text{He}$ and scattered $^3\text{He}$ particles is the same, and the figure of merit is substantially increased when both particles are observed. However, Region I suffers from a practical standpoint from being at relatively low energies and shares the difficulty with Region II of occurring at rather small laboratory scattering angles. Region III, on the other hand, not only has a high figure of merit but also should be easily incorporated into a "venetian blind" polarimeter such as that described by Lush et al. (Lu 64).
Table IV

A comparison of several regions which may be useful as a $^3$He polarization analyzer. The figure of Merit $M$ gives a measure of the overall counting rate times the square of the analyzing power.

<table>
<thead>
<tr>
<th>Region</th>
<th>$\Theta_{\text{lab}}$ (degrees)</th>
<th>$\Theta_{\text{c.m.}}$ (degrees)</th>
<th>$^3$He Lab Energy (MeV)</th>
<th>Particle Detected</th>
<th>Figure of Merit (relative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ia</td>
<td>17-23</td>
<td>30-40</td>
<td>7.5-8.5</td>
<td>$^3$He</td>
<td>20</td>
</tr>
<tr>
<td>Ib</td>
<td>17-23</td>
<td>135-145</td>
<td>7.5-8.5</td>
<td>$^4$He</td>
<td>13</td>
</tr>
<tr>
<td>IIa</td>
<td>17-23</td>
<td>30-40</td>
<td>12.5-13.5</td>
<td>$^3$He</td>
<td>8</td>
</tr>
<tr>
<td>IIb</td>
<td>17-23</td>
<td>135-145</td>
<td>12.5-13.5</td>
<td>$^4$He</td>
<td>4</td>
</tr>
<tr>
<td>III</td>
<td>45-52</td>
<td>77-88</td>
<td>9.0-12.5</td>
<td>$^3$He</td>
<td>28</td>
</tr>
</tbody>
</table>
III. Deuteron Elastic Scattering and the (d,p) Reaction with a Polarized $^3$He Target

A. Discussion

The structure of the $^5$Li nucleus for excitation energies up to 25 MeV has been examined through a wide variety of experiments (La 66). Besides the $3/2^-$ ground state and the very broad $1/2^-$ first excited state, a $3/2^+$ state occurs at 16.65 MeV and a broad state of uncertain spin and parity exists near 20 MeV. An energy level diagram of the $^5$Li nucleus is shown in Fig. 6.

Perhaps the most numerous experiments have involved $p + ^4$He elastic scattering measurements. Many measurements of both the cross sections and the polarizations for this elastic interaction have been made and a number of phase shift analyses have been reported. Several studies of the $^3$He(d,p)$^4$He reaction (Ja 57, Ba 65) and of d - $^3$He elastic scattering (To 67) have also been made, but until recently no polarization data were available for d- $^3$He elastic scattering. New data are now available concerning spin dependent effects in both d- $^3$He elastic scattering and in the $^3$He(d,p)$^4$He reaction. Plattner and Keller (Pl 69) have reported measurements of the deuteron vector polarization in d- $^3$He elastic scattering and of the deuteron vector analyzing power in the $^3$He(d,p)$^4$He reaction.
Figure 6

Energy level diagram of the $^5$Li nucleus. The energy values are plotted vertically in MeV, based on the ground state as zero. Values for the total angular momentum and parity are indicated for each level.

Typical elastic scattering excitation functions for reactions in which $^5$Li is the compound nucleus are also shown, with bombarding energies in the laboratory system plotted vertically and the differential cross sections plotted horizontally. The $\frac{3}{2}^+$ level is cross-hatched to indicate that it is very broad.
which were obtained with a vector polarized deuteron beam. This section describes measurements conducted with a polarized \(^3\text{He}\) target that yield the equivalent of the \(^3\text{He}\) polarization in \(d - ^3\text{He}\) elastic scattering and of the proton asymmetry when the \(^3\text{He}(d,p)\)\(^4\text{He}\) reaction is initiated with a polarized \(^3\text{He}\) target. The latter augment earlier measurements of Baker et al. (Ba 65) and show a similarity to reaction data of Plattner and Keller (Pl 69). In the elastic scattering, there is a striking similarity between the deuteron vector polarization in Ref. Pl 69 and the \(^3\text{He}\) polarization reported here. The similarities in the various polarization effects which are observed for the \(^3\text{He}(d,p)\)\(^4\text{He}\) reaction have already received the attention of theorists (Ta 65, Du66), although a satisfactory explanation has not yet emerged. The disclosure of analogous effects in the elastic scattering may suggest new avenues of investigation to theorists presently concerned with the complicated mass five problem.

B. Experiment

1) Polarized \(^3\text{He}\) Target

The \(^3\text{He}\) target was polarized by the method of optical pumping; the optical pumping apparatus used is described in appendix E. Two scattering chambers were constructed
according to the procedure given in appendix E. The first chamber allowed the observation of scattering asymmetries at a laboratory angle of 33.0° and the second chamber allowed observations to be made at laboratory angles of 42.5° and 80.0°. The scattered particles were collimated as in section II. The overall center of mass rms angular resolution, due to beam collimation, multiple scattering in the entrance foils of the chambers, and collimation of the scattered particles varied from 1.0° to 2.0° and is given in Table VII.

The target polarization was determined by optical measurements as discussed in appendix B; the parameters discussed below are also defined in appendix B. The values of ΔI/I varied from 0.40 to 0.61 during the course of the experiment. The values of α, β, c, ρ and f were assumed to be the same as those given in section II, part B. As before, the choice of f = 0.7 gives a target polarization midway between that obtained for f = 0.5 and f = 1.0. The uncertainty in f results in a systematic uncertainty in the target polarization, and as a consequence, all of the experimental asymmetries given in Tables V and VI may be multiplied by a single factor ranging from 0.85 to 1.15. However, the optical pumping apparatus used was identical to that of section II and
therefore, for the reasons given there, the systematic uncertainty is probably less than 15%.

2) Beam

The Rice tandem accelerator supplied deuteron beams with energies from 5.0 to 12.0 MeV. The characteristic energy uncertainty was $\pm 50$ kev over the energy range used. The aluminum entrance foils of the first and second scattering chambers had thickness of $(2.33\pm0.5)\text{mg/cm}^2$ and $(2.29\pm0.5)\text{mg/cm}^2$, respectively, and the energies of the incident deuteron beams were corrected for energy losses in the foils. The uncertainty in the thicknesses of the entrance foils resulted in an additional systematic uncertainty of approximately $\pm 7$ keV in the beam energy.

C. Data

The deuteron laboratory beam energies inside the scattering chambers ranged from 4.8 to 11.9 MeV. Elastic scattering data were collected for center of mass scattering angles of $54.3^\circ$, $68.6^\circ$, $96.0^\circ$, $114.0^\circ$ and $121.0^\circ$. Data for the $^3\text{He}(d,p)^4\text{He}$ reaction were obtained at laboratory scattering angles of $33.0^\circ$ and $42.5^\circ$ (the corresponding center of mass angles ranged from $102^\circ$ to $112^\circ$ and from $119^\circ$ to $128^\circ$, respectively). Beam currents ranged from 0.15 $\mu$A to 2.5 $\mu$A during the several periods in
which data were taken. Scattered deuterons, recoil $^3\text{He}$ particles and $^4\text{He}$ particles from the $(d,p)$ reaction were detected at $\theta_{\text{lab}} = 33.0^\circ$, 42.5$^\circ$ and 80.0$^\circ$ by pairs of silicon surface barrier detectors which were placed symmetrically about the beam axis in the plane perpendicular to the direction of the $^3\text{He}$ target polarization. At both $\theta_{\text{lab}} = 33.0^\circ$ and $\theta_{\text{lab}} = 42.5^\circ$, $\Delta E - E$ counters were used in conjunction with coincidence and anti-coincidence gating to achieve particle identification and a reduction of background counts. This technique was not used at $\theta_{\text{lab}} = 80.0^\circ$ where it was possible to observe the elastically scattered deuterons without employing coincidence gating. Pulses from the detectors were processed in such a manner that no correction due to dead time in the electronics was necessary. The left-right scattering asymmetries were calculated and corrected for target polarization. For elastic scattering, these results yield the equivalent of the $^3\text{He}$ polarization in an unpolarized beam and target experiment, and are presented in Table V and in Fig. 7. For the $^3\text{He}(d,p)^4\text{He}$ reaction, the data as presented give the proton asymmetry when a polarized $^3\text{He}$ target is employed, and are given in Table VI and in Figure 8.

The number of scattered particles was obtained by summing the pulse height spectra over the regions cor-
Table V

Experimental asymmetries, $\rho_3$, and random errors, $\Delta \rho_3$, versus deuteron laboratory beam energy, $E_d$, for $^3\text{He}(d,d)^3\text{He}$ scattering at the center of mass angles indicated. The random errors do not include the systematic error discussed in the text. The double asterisks indicate data points for which only two counting intervals were used, as described in the text. The numbers in parentheses indicate the range of the percentage background present in the spectra.

<table>
<thead>
<tr>
<th>$E_d$ (MeV)</th>
<th>54.3°c.m.</th>
<th>68.6°c.m.</th>
<th>96.0°c.m.</th>
<th>114.0°c.m.</th>
<th>121.0°c.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_3$</td>
<td>$\Delta \rho_3$</td>
<td>$\rho_3$</td>
<td>$\Delta \rho_3$</td>
<td>$\rho_3$</td>
</tr>
<tr>
<td>4.78</td>
<td>0.12 ± 0.018</td>
<td></td>
<td>0.12 ± 0.036</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.30</td>
<td>0.16</td>
<td>0.019</td>
<td>0.16</td>
<td>0.031</td>
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</tr>
<tr>
<td>5.81</td>
<td>0.13</td>
<td>0.015</td>
<td>0.19</td>
<td>0.029</td>
<td></td>
</tr>
<tr>
<td>6.83</td>
<td>0.06</td>
<td>0.012</td>
<td>0.27</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>7.34</td>
<td>0.32</td>
<td>0.020</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.84</td>
<td>-0.01</td>
<td>0.006</td>
<td>0.30</td>
<td>0.029</td>
<td>-0.14 ± 0.005</td>
</tr>
<tr>
<td>8.35</td>
<td></td>
<td></td>
<td>0.28</td>
<td>0.028</td>
<td>-0.07 ± 0.020</td>
</tr>
<tr>
<td>8.86</td>
<td></td>
<td></td>
<td>0.34</td>
<td>0.027</td>
<td>-0.07</td>
</tr>
<tr>
<td>9.36</td>
<td></td>
<td></td>
<td>0.31</td>
<td>0.030</td>
<td>-0.05</td>
</tr>
<tr>
<td>9.87</td>
<td></td>
<td></td>
<td>0.34</td>
<td>0.029</td>
<td>-0.03</td>
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</table>
### Table V continued

<table>
<thead>
<tr>
<th>$E_d$ (MeV)</th>
<th>54.3° c.m. (0%)</th>
<th>68.6° c.m. (4%-16%)</th>
<th>96.0° c.m. (8%-20%)</th>
<th>114.0° c.m. (0%)</th>
<th>121.0° c.m. (35%-75%)</th>
</tr>
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<tr>
<td></td>
<td>$\rho_2$</td>
<td>$\Delta \rho_2$</td>
<td>$\rho_3$</td>
<td>$\Delta \rho_3$</td>
<td>$\rho_3$</td>
</tr>
<tr>
<td>10.37</td>
<td>-0.14 ± 0.022</td>
<td>0.39 ± 0.030</td>
<td>-0.04 ± 0.013</td>
<td>-0.25 ± 0.016</td>
<td>-0.34 ± 0.042</td>
</tr>
<tr>
<td>10.88</td>
<td>-0.20</td>
<td>0.037**</td>
<td>0.37</td>
<td>0.029</td>
<td>-0.06</td>
</tr>
<tr>
<td>11.38</td>
<td>-0.17</td>
<td>0.040**</td>
<td>0.38</td>
<td>0.033</td>
<td>-0.02</td>
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<tr>
<td>11.88</td>
<td>-0.22</td>
<td>0.040**</td>
<td>0.37</td>
<td>0.043</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

44
Experimental asymmetries, $\rho_3$, for $^3\text{He}(d,d)^3\text{He}$ scattering are plotted as a function of the deuteron laboratory energy, $E_d$, for the center of mass angles indicated. The error bars do not include the possible systematic error discussed in the text. The triangles indicate data points for which only two counting intervals were used, as described in the text. The squares represent the deuteron vector polarization, at 67.5° c.m. as given by Ref. Pl 70.
Figure 7 $^3\text{He}(d,d)^3\text{He}$

$\theta_{c.m.} = 54.3^\circ$

$\theta_{c.m.} = 68.6^\circ$

$\theta_{c.m.} = 96.0^\circ$

$\theta_{c.m.} = 114.0^\circ$

$\theta_{c.m.} = 121.0^\circ$
Experimental asymmetries, $\rho_3$, and random errors, $\Delta \rho_3$, versus deuteron laboratory beam energy, $E_d$, for the $^3\text{He}(d,p)^4\text{He}$ reaction. The data as presented give the proton asymmetries in this reaction at laboratory angles of 33.0° and 42.5°. The range of center of mass angles corresponding to each laboratory angle is also given. The random errors do not include the possible systematic error discussed in the text, and the double asterisks indicate data points for which only two counting intervals were used, as described in the text. The numbers in parentheses indicate the range of the percentage background present in the spectra.

<table>
<thead>
<tr>
<th>$E_d$ (MeV)</th>
<th>$\rho_3$</th>
<th>$\Delta \rho_3$</th>
<th>$\rho_3$</th>
<th>$\Delta \rho_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>119°≤θc.m.≤128°</td>
<td>(0%)</td>
<td>102°≤θc.m.≤112°</td>
<td>(2%-8%)</td>
</tr>
<tr>
<td>4.78</td>
<td>0.10 ± 0.068</td>
<td></td>
<td>0.13 ± 0.120</td>
<td></td>
</tr>
<tr>
<td>5.30</td>
<td>0.20 ± 0.070</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>5.81</td>
<td>0.24 ± 0.062</td>
<td></td>
<td>0.20 ± 0.119</td>
<td></td>
</tr>
<tr>
<td>6.31</td>
<td>0.36 ± 0.065</td>
<td></td>
<td>0.56 ± 0.126</td>
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<tr>
<td>6.83</td>
<td>0.29 ± 0.075</td>
<td></td>
<td>0.34 ± 0.051</td>
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</tr>
<tr>
<td>7.34</td>
<td>0.27 ± 0.052</td>
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<td>0.44 ± 0.079</td>
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<td>0.44 ± 0.079</td>
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<td>0.27 ± 0.077</td>
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<td>0.35 ± 0.090</td>
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<tr>
<td>8.35</td>
<td>0.51 ± 0.056</td>
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<td>0.28 ± 0.093</td>
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<tr>
<td>9.87</td>
<td>0.50 ± 0.097</td>
<td></td>
<td>0.50 ± 0.097</td>
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</tr>
<tr>
<td>10.37</td>
<td>0.37 ± 0.096</td>
<td></td>
<td>0.34 ± 0.164</td>
<td></td>
</tr>
<tr>
<td>10.88</td>
<td>0.35 ± 0.141**</td>
<td></td>
<td>0.21 ± 0.095</td>
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</tr>
<tr>
<td>11.38</td>
<td>0.28 ± 0.166**</td>
<td></td>
<td>0.20 ± 0.117</td>
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</tr>
<tr>
<td>11.88</td>
<td>0.20 ± 0.174**</td>
<td></td>
<td>0.20 ± 0.197</td>
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</table>
Figure 8

Experimental asymmetries, $\mathcal{P}_3$, for the $^3\text{He}(d,p)^4\text{He}$ reaction are plotted as a function of the deuteron laboratory energy, $E_d$, for laboratory scattering angles of 33.0° and 42.5°. The data as presented give the proton asymmetries in the (d,p) reaction at these laboratory angles. The error bars do not include the possible systematic error discussed in the text. The triangles indicate data points for which only two counting intervals were used, as described in the text.
Figure 8

$^3\text{He}(d,p)^4\text{He}$

$\theta_{\text{LAB}} = 42.5^\circ$

$\theta_{\text{LAB}} = 33.0^\circ$
Table VII

Overall center of mass rms angular resolution versus deuteron laboratory beam energy, $E_d$, for each center of mass angle studied in $d-^3He$ elastic scattering and for each laboratory angle studied in the $(d,p)$ reaction.

<table>
<thead>
<tr>
<th>$E_d$ (MeV)</th>
<th>54.3°c.m.</th>
<th>68.6°c.m.</th>
<th>96.0°c.m.</th>
<th>114.0°c.m.</th>
<th>121.0°c.m.</th>
<th>$\theta_{lab}=33°$</th>
<th>$\theta_{lab}=42.5°$</th>
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<td>1.90</td>
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<td>1.77</td>
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<td></td>
<td>1.50</td>
<td>1.42</td>
</tr>
<tr>
<td>11.38</td>
<td>1.40</td>
<td>1.40</td>
<td>1.75</td>
<td>1.75</td>
<td>1.00</td>
<td>1.50</td>
<td>1.40</td>
</tr>
<tr>
<td>11.88</td>
<td>1.40</td>
<td>1.40</td>
<td>1.75</td>
<td>1.75</td>
<td>1.00</td>
<td>1.50</td>
<td>1.40</td>
</tr>
</tbody>
</table>
responding to the various particles and estimating the number of background events included in these peak integrations. A linear estimate of the background level was used in arriving at the number of background events. The range of background that was present in the pulse height spectra is indicated in Tables V and VI where the amount of background is expressed as a percentage of the number of scattering events. Even though very large percentage backgrounds were present in the spectra corresponding to \( \Theta_{\text{cm}} = 121.0^\circ \), the data analysis described below indicated that the only adverse effect due to the background was a reduction in the statistical accuracy of the asymmetry data.

The method of data collection and data analysis followed that of appendix C except for the data indicated by double asterisks in Tables V and VI and plotted as triangles in Figures 7 and 8. For these data points, only two of the four counting intervals discussed in appendix C were used. Definitions of the parameters \( P_3 \), \( \Theta_0 \) and \( \Theta_3 \) discussed below are also given in appendix C.

Values of the nominally zero quantity \( P_3 \Theta_0 \) were calculated for all of the data prior to correcting for background, and those data points which had values of \( P_3 \Theta_0 \)
1.75 standard deviations or more from zero were discarded. Where necessary, the data were then corrected for background according to procedure one of appendix D and new values of \( P_3 \overline{Q}_0 \) were obtained. When this method of correcting for background is used, an incorrect estimate of the background can result in calculated values of \( P_3 \overline{Q}_0 \) that are not consistent with zero within statistics.

This was not, however, found to be the case. All of the new values of \( P_3 \overline{Q}_0 \) were within 1.75 standard deviations of zero. The data for \( \Theta_{\text{cm}} = 96^\circ \), where the asymmetries are near zero, and the data for \( \Theta_{\text{cm}} = 121^\circ \), which included very large percentage backgrounds, were then corrected for background according to procedure two of appendix D.

The values of \( P_3 \overline{Q}_3 \) obtained from the asymmetry data were divided by \( P_3 \), the target polarization as determined from the optical measurements, and resulting values of \( \overline{Q}_3 \) are presented in Tables V and VI and in Figures 7 and 8.

D. Comparisons with Other Data

It is interesting to compare the elastic and inelastic polarization data presented here with that of Plattner and Keller (Pl 69) and of Brown and Haeberli (Br 63). Data for the deuteron vector polarization, \( \langle iT_{11} \rangle \), produced in an initially unpolarized \( d - ^3\text{He} \) elastic scattering experiment are shown as squares in Fig. 7.
The parameter $\langle iT_{11} \rangle$ is the deuteron vector polarization in the spherical tensor representation and is proportional to $\langle S_y \rangle$, where $S_y$ is the component of the spin operator normal to the scattering plane. These measurements were obtained by Plattner (Pl 70) in an experiment conducted with a vector polarized deuteron beam. The data of Plattner correspond to a center of mass scattering angle of 67.5° and, for the purpose of comparison, are plotted along with the values of $\zeta_3$ for $\Theta_{cm} = 68.6^\circ$. The values of $\zeta_3$ give the $^3$He polarization produced in the scattering of unpolarized deuterons from unpolarized $^3$He. As can be seen from Fig. 7, the energy dependence of the parameters $\zeta_3$ and $\langle iT_{11} \rangle$ is remarkably similar for center of mass scattering angles near 68°. Other more striking similarities are seen when a comparison is made of the angular distributions of these parameters. Such a comparison is given in Fig. 9, which was adapted from Ref. Pl 69. The full circles are the values of $\langle iT_{11} \rangle$ as given by Plattner and Keller (Pl 69); the solid line serves only to guide the eye. Interpolations of the $\zeta_3$ data reported in this section are plotted as open circles. The error bars assigned to these data points are representative of the uncertainties in the data used to perform the interpolations and serve only as an estimate
Figure 9

A comparison of the angular distributions of the $^3$He polarization, $\rho_3$, and the deuteron vector polarization, $\langle i \hat{T}_{||} \rangle$, in $^3$He(d,d)$^3$He scattering at deuteron laboratory beam energies of 6.0, 8.0, and 10.0 MeV. The full circles are the values of $\langle i \hat{T}_{||} \rangle$ as given by Ref. Pl 69; the solid line serves only to guide the eye. The open circles are interpolations of the $\rho_3$ data given in this section; the error bars only give an estimate of the accuracy of these interpolations.
Figure 9

$^3\text{He}(d,d)^3\text{He}$

10.00 ± 0.03 MeV

$<i T_{II}>$

8.00 ± 0.03 MeV

$<i T_{II}>$

6.00 ± 0.03 MeV

$<i T_{II}>$

$\theta_{cm}$
of the accuracy of these interpolations. Until the present, such a comparison of the polarization effects in deuteron-\(^{3}\)He elastic scattering was not available. On the basis of the data presented in Figures 7 and 9 it appears that the approximate relationship \(\langle iT_{11} \rangle \simeq P_3\) may hold for deuteron-\(^{3}\)He elastic scattering in the energy range investigated. Because of the complexity involved in describing the interaction of spin 1 particles with spin \(\frac{1}{2}\) particles, it is difficult to provide a theoretical interpretation of this relationship. However, it is possible that the existence of such a relationship may suggest useful simplifications in the analysis of \(d-^{3}\)He scattering data.

Comparisons can also be made of the various polarization effects observed in the \(^{3}\)He(d,p)\(^{4}\)He reaction. The proton polarization, \(P\), produced when this reaction is initiated with an unpolarized beam and target have been measured by Brown and Haeberli (Br 63). The data presented in this section for the proton asymmetry, \(A\), obtained when a polarized \(^{3}\)He target and unpolarized beam are used supplement the earlier measurements of Baker et al.-(Ba 65). Measurements of the proton asymmetries produced when this reaction is conducted with a vector polarized deuteron beam and an unpolarized target,
\( P_d \), are given in Ref. Pl 69. The relationship \( A \approx -P \) was noted in Ref. Ba 65. It was shown by Tanifuji (Ta 65) that this relationship can not be obtained by conventional direct-interaction theories, but that it can be obtained by assuming a tensor \( p - ^3 \text{He} \) interaction and/or a D-state deuteron admixture. However, it was also pointed out in Ref. Ba 65 that \( A \) and \( P \) have zeros at distinctly different angles and therefore even those models which predict \( A = -P \) are inadequate. As yet, a satisfactory explanation of these data is not available. The recent work of Plattner and Keller (Pl 69) has disclosed the additional relation \( P_d \approx A \approx -P \). A comparison of these parameters is shown in Fig. 10, which was also adapted from Ref. Pl 69. Interpolations of the values of \( A \) reported here are plotted as open circles; the error bars are only estimates of the uncertainties. The full circles give the values of \( A \) previously reported by Baker et al. (Ba 65) and the solid line represents the values of \( P_d \) reported in Ref. Pl 69. The dashed curve gives the negative of the proton polarization, \( P \), as given in Ref. Br 63. The new values of \( A \) agree well with those previously reported and are not inconsistent with the assumption \( P_d \approx A \approx -P \). The existence of such effects is not easily explained, but as Tanifuji (Ta 65) has shown, their disclosure can
Figure 10

A comparison of the angular distributions of the polarization parameters $P_d$, $A$ and $-P$ (defined in the text) for the $^3\text{He}(d,p)^4\text{He}$ reaction at deuteron laboratory beam energies of 6.0, 8.0 and 10.0 MeV. The solid lines represent the values of $P_d$ reported in Ref. Pl 69, the dashed curves give the values of $-P$ as reported in Ref. Br 63, the full circles give the values of $A$ reported in Ref. Ba 65, and the open circles represent values of $A$ based on interpolations of the data of this section. The error bars only give an estimate of the accuracy of these interpolations.
Figure 10

$^3\text{He}(d,p)^4\text{He}$

4.0 MeV

6.0 MeV

8.0 MeV

10.0 MeV

$\theta_{cm}$

$\theta_{cm}$
lead to the suggestion of interactions which might other-
wise remain unconsidered.
Appendix A: The Scattering Spin 1/2 Particles from Spin 0 Particles

The wave function $\psi$ which describes the scattering of a spinless particle by a potential $V$ may be represented for large values of $r$ by the sum of an incident plane wave and a spherical wave scattered outward from the origin.

$$\psi = e^{ikz} + S(\theta) \frac{e^{ikr}}{r}$$

The differential elastic scattering cross section is then given by

$$\sigma(\theta) = \frac{d\sigma}{d\Omega} = |S(\theta)|^2$$

If the beam consists of spin 1/2 particles incident on a spinless target, then the wave function in the asymptotic region may be written as

$$\psi = \left[ e^{ikz} + S(\theta, \phi) \frac{e^{ikr}}{r} \right] \chi$$

$$\chi = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad S(\theta, \phi) = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}$$

where $\chi$ is the spin function which describes the incident beam and $S(\theta, \phi)$ is the scattering matrix. The numbers $a_1$ and $a_2$ are the amplitudes corresponding, respectively, to polarization along or opposed to the direction of the initial momentum vector (which is co-linear with the z-axis).
In terms of the normalized spin eigenfunctions of $\sigma_z$:

$$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

the wave function $\psi$ consists of the two functions:

$$\psi_1 = a_1 \left[ e^{ikz} \alpha + \frac{e^{ikr}}{r} (S_{11} \alpha + S_{12} \beta) \right]$$

$$\psi_2 = a_2 \left[ e^{ikz} \beta + \frac{e^{ikr}}{r} (S_{12} \alpha + S_{22} \beta) \right]$$

The general form of the matrix $S(\Theta, \Phi)$ can be deduced by the procedure given below, which is based on the treatment of Mertzbacher (Me 61a). The Hamiltonian $H$ is assumed to be a scalar invariant under rotations and reflections.

If $A$ is an operator which commutes with $H$ and $\psi$ is an eigenfunction of $H$, then $A\psi$ is also an eigenfunction of $H$. The operator $\mathbf{J}_z = \hbar \left( \frac{\hat{r}}{i} \frac{\partial}{\partial \phi} + \frac{1}{2} \sigma_z \right)$ commutes with $H$ because $H$ is invariant under rotations about the $z$-axis. Therefore if $\psi_1$ and $\psi_2$ are eigenfunctions of $H$ they must also be eigenfunctions of $\mathbf{J}_z$. By requiring that

$$\mathbf{J}_z \psi_1 = \hbar \left( \frac{1}{i} \frac{\partial}{\partial \phi} + \frac{1}{2} \sigma_z \right) \psi_1 = + \frac{\hbar}{2} \psi_1$$

$$\mathbf{J}_z \psi_2 = \hbar \left( \frac{1}{i} \frac{\partial}{\partial \phi} + \frac{1}{2} \sigma_z \right) \psi_2 = - \frac{\hbar}{2} \psi_2$$

it can be shown that $S_{11}$ and $S_{22}$ must be functions of $\Theta$ only and that $S_{12}$ and $S_{21}$ must have the form:

$$S_{12} = \text{(function of } \Theta) \cdot e^{-i\Phi}$$

$$S_{21} = \text{(function of } \Theta) \cdot e^{+i\Phi}$$
For example, evaluating $J_2 \psi$ yields
\[ a_i \left[ \frac{\hbar}{i} e^{ikz} \alpha + \frac{\hbar}{i} \left( \frac{1}{2} \frac{\partial S_{11}}{\partial \phi} + \frac{1}{2} S_{11} \right) \frac{e^{ikr}}{r} \alpha \\
+ \frac{\hbar}{i} \left( \frac{1}{2} \frac{\partial S_{21}}{\partial \phi} - \frac{1}{2} S_{21} \right) \frac{e^{ikr}}{r} \beta \right] \]

The requirement $J_2 \psi = \frac{\hbar}{2} \psi$ implies that
\[ \frac{\hbar}{i} \left( \frac{1}{2} \frac{\partial S_{11}}{\partial \phi} + \frac{1}{2} S_{11} \right) = \frac{\hbar}{2} S_{11} \]
\[ \frac{\hbar}{i} \left( \frac{1}{2} \frac{\partial S_{21}}{\partial \phi} - \frac{1}{2} S_{21} \right) = \frac{\hbar}{2} S_{21} \]

Therefore
\[ \frac{\partial S_{11}}{\partial \phi} = 0 \quad \Rightarrow \quad S_{11} = g(\theta) \]
\[ \frac{\partial S_{21}}{\partial \phi} = i S_{21} \quad \Rightarrow \quad S_{21} = \pm h(\theta) e^{i \phi} \]

Similar results are obtained for $S_{22}$ and $S_{12}$. In addition, $H$ is invariant under a reflection in any coordinate plane.

It can be shown (Me 61b) that the operator for reflection in the yz-plane is $P_x \sigma_x$ where $P_x$ changes $x$ into $-x$ and
\[ \sigma_x \alpha = \beta \quad \sigma_x \beta = \alpha \]

The application of $P_x \sigma_x$ to $\psi$ will change $e^{ikz} \alpha$ into $e^{-ikz} \beta$ and will leave $\exp(ikr)/r$ invariant. The resulting function must be an eigenfunction of $H$, and the effect of $P_x \sigma_x$ is to transform $\psi$ into $\psi$. Thus
\[
\frac{a_z}{a_1} P_x \sigma_x \Psi_i = a_z \left[ \text{e}^{ikz} \beta + \frac{i}{r} \text{e}^{ikr} (\beta P_x S_{11} + \alpha P_x S_{21}) \right]
\]

\[
= a_z \left[ \text{e}^{ikz} \beta + \frac{i}{r} \text{e}^{ikr} (\beta S_{22} + \alpha S_{11}) \right]
\]

Taking \( S_{11} = g(\theta) \) and \( S_{21} = -h(\theta) \text{e}^{i\phi} \) (where the minus sign is chosen for convenience) and noting that in spherical polar coordinates \( P_x \) changes \( \phi \) into \( \pi - \phi \) but does not change \( \theta \), it is seen that

\[
P_x S_{11} = P_x g(\theta) = g(\theta) = S_{22}
\]

\[
P_x S_{21} = P_x (-h(\theta) \text{e}^{i\phi}) = h(\theta) \text{e}^{-i\phi} = S_{12}
\]

Therefore the general form of the scattering matrix for this case is

\[
S(\theta, \phi) = \begin{pmatrix} g(\theta) & h(\theta) \text{e}^{-i\phi} \\ -h(\theta) \text{e}^{i\phi} & g(\theta) \end{pmatrix}
\]

\[
= g(\theta) I + i h(\theta) \left[ \sigma_x \sin \phi + \sigma_y \cos \phi \right]
\]

where \( I \) is the identity matrix

\[
I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

and where

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

are the Pauli spin matrices. The unit vector
\[ \hat{n} = (\sin \phi, \cos \phi, 0) \]

is normal to the scattering plane and S may be re-written as

\[ S = g(\phi) I + i h(\phi) \hat{n} \cdot \vec{\sigma} \]

By using this general form for S, expressions for the differential elastic scattering cross section \( \sigma(\phi) \) and the polarization \( \vec{P}(\phi) \) can be derived in terms of \( g(\phi) \) and \( h(\phi) \).

\[ \sigma(\phi) = \begin{bmatrix} S \chi \end{bmatrix}^+ \begin{bmatrix} S \chi \end{bmatrix} = \chi^+ S^+ S \chi \]

\[ \vec{P}(\phi) = \begin{bmatrix} S \chi \end{bmatrix}^+ \vec{\sigma} \begin{bmatrix} S \chi \end{bmatrix} / \sigma(\phi) \]

= \chi^+ S^+ \vec{\sigma} S \chi / \sigma(\phi)

Evaluating \( S^+ S \) gives

\[ S^+ S = (g^* - i l^* \hat{n} \cdot \vec{\sigma})(g + i l \hat{n} \cdot \vec{\sigma}) \]

= \|g\|^2 + \|l\|^2 (\hat{n} \cdot \vec{\sigma})^2 + i (g^* l - g l^*) \hat{n} \cdot \vec{\sigma} \]

Using the relation \((\vec{\sigma} \cdot \vec{\alpha})(\vec{\sigma} \cdot \vec{\beta}) = \vec{\alpha} \cdot \vec{\beta} + i \vec{\sigma} \cdot (\vec{\alpha} \times \vec{\beta})\)

shows that \((\hat{n} \cdot \vec{\sigma})^2 = 1\). Therefore

\[ \sigma(\phi) = \chi^+ S^+ S \chi \]

= \|g\|^2 + \|l\|^2 + i (g^* l - g l^*) \hat{n} \cdot \vec{P}_0 \]

where \( \vec{P}_0 = \chi^+ \vec{\sigma} \chi \) is the incident polarization vector.

This formula shows that the cross section depends on the azimuthal angle only through the angle between the incident polarization vector and the normal to the scattering plane.

If the direction of \( \vec{P}_0 \) is perpendicular to the z-axis
and the incident beam is completely polarized \( |\vec{P}_o| = 1 \), then according to the Basel convention (Ba 60), left scattering corresponds to \( \hat{n} \cdot \vec{P}_o = +1 \) and right scattering corresponds to \( \hat{n} \cdot \vec{P}_o = -1 \). The cross sections for left and right scattering are

\[
\sigma_L = |g|^2 + |h|^2 + i(g^*h - g h^*)
\]
\[
\sigma_R = |g|^2 + |h|^2 - i(g^*h - g h^*)
\]

The left-right scattering asymmetry is given by

\[
A(\theta) = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{i(g^*h - g h^*)}{|g|^2 + |h|^2}
\]

The polarization \( \vec{P}(\theta) \) for an arbitrary incident polarization \( \vec{P}_o \) can be evaluated (Me 62) in a manner similar to the evaluation of \( \sigma(\theta) \) and is given by

\[
\vec{P}(\theta) = \frac{a \vec{P}_o + b \hat{n} + c \hat{n} \times \vec{P}_o}{|g|^2 + |h|^2 + i(g^*h - g h^*) \hat{n} \cdot \vec{P}_o}
\]

\[
a = |g|^2 - |h|^2
\]

\[
b = i(g^*h - g h^*) + 2|lh|^2 \hat{n} \cdot \vec{P}_o
\]

\[
c = -(g^*h + gh^*)
\]

If the incident beam is completely unpolarized, \( \vec{P}_o = 0 \), then the scattered beam is polarized along the direction of \( \hat{n} \).
\[ P(\theta) \rightarrow \hat{\mathcal{P}}(\theta) = \left[ i(\gamma \lambda - g \lambda^*) \right] / (|g|^2 + |\lambda|^2) \]

The function \( \mathcal{P}(\theta) \) is identical to \( Q(\theta) \), and thus the (maximum) scattering asymmetry obtained when a completely polarized beam is used is equal to the spin polarization obtained when an unpolarized beam is used.

The functions \( g(\theta) \) and \( h(\theta) \) are known as the coherent and incoherent scattering amplitudes, respectively. This terminology comes from the fact that \( g(\theta) \) is the amplitude for scattering events in which the spin projection is not changed. For pure Coulomb scattering of protons (or nuclei) in which \( v^2/c^2 \ll 1 \), the spin is unaffected (Wo 49). Thus, if the total Hamiltonian includes both Coulomb and non-Coulomb interactions, then only that portion of the scattering which leaves the spin projection unchanged can have a definite phase relationship with the Coulomb scattering and therefore be coherent. The function \( h(\theta) \) represents scattering events in which the spin projection has changed, and these events are therefore incoherent with respect to Coulomb scattering.

It is often advantageous to use the method of partial wave analysis to parameterize the functions \( g(\theta) \) and \( h(\theta) \) in terms of the phase shifts of the various partial waves. Such a parameterization, based on the derivation of
Melkanoff et al. (Me 62), will now be given. The case of an uncharged incident particle is considered first, and then the results are modified to include the Coulomb interaction.

1) Uncharged Incident Particles

The incident wave, representing a flux of \( v \) particles per unit time per unit area, is

\[
\psi_i = e^{ikz} \chi
\]

where \( v \) is the relative velocity, \( k = \sqrt{2 \mu E/h^2} \) is the wave number, \( E \) is the energy in the center of mass system, and the incident spin function in terms of previously defined quantities is

\[
\chi = a_1 \alpha + a_2 \beta
\]

The incident plane wave can be expanded in terms of spherical waves (Me 61c) and the resulting partial wave expansion is

\[
\sum_{l=0}^{\infty} (2l+1)(l)! j^l(kr) \left[ \frac{4\pi}{2l+1} \right]^{1/2} Y^0_l(\theta, \phi) \left[ a_1 \alpha + a_2 \beta \right]
\]

where \( j^l(kr) \) is the regular spherical Bessel function of order \( l \), the normalized spherical harmonics are

\[
Y^m_l(\theta, \phi) = (-1)^{m+|m|} \left[ \frac{2l+1}{4\pi} \right]^{1/2} \left[ \frac{(l-|m|)!}{(l+|m|)!} \right]^{1/2} P^{|m|}_l(\cos \theta) e^{im\phi}
\]
and \( P_1^m (\cos \theta) \) are the associated Legendre polynomials.

The product functions \( Y^o_\ell \alpha \) and \( Y^o_\ell \beta \) that appear in the expansion of \( \psi_i \) can be re-written in terms of the functions \( Y_{j, l, s}^{m_j} \) which are simultaneous eigenfunctions of \( J^2, L^2, S^2 \) and \( J_z \) where

\[
\mathbf{J} = \mathbf{L} + \mathbf{S} \quad \mathbf{S} = \frac{\mathbf{R}}{2} \frac{\mathbf{D}}{2}
\]

The possible values of \( j = l + \frac{1}{2} \) and \( j = l - \frac{1}{2} \), and because only \( Y^o_\ell \) occur in the expansion of \( \psi_i \), the values of \( m_j \) are limited to \( m_j = \pm \frac{1}{2} \).

The \( Y_{j, l, s}^{m_j} \) are constructed by using the Clebsh-Gordon coefficients

\[
Y_{l+\frac{1}{2}, l, s}^{m_j} = \left[ \frac{l + m_j + \frac{1}{2}}{2l + 1} \right]^{\frac{1}{2}} Y^o_l \alpha \left[ \frac{l - m_j + \frac{1}{2}}{2l + 1} \right]^{\frac{1}{2}} Y^o_l \beta
\]

\[
Y_{l-\frac{1}{2}, l, s}^{m_j} = -\left[ \frac{l - m_j + \frac{1}{2}}{2l + 1} \right]^{\frac{1}{2}} Y^o_l \alpha \left[ \frac{l + m_j + \frac{1}{2}}{2l + 1} \right]^{\frac{1}{2}} Y^o_l \beta
\]

When \( Y^o_\ell \alpha \) and \( Y^o_\ell \beta \) are written in terms of the \( Y \)'s, the expansion for \( \psi_i \) separates into a series corresponding to \( j = l + \frac{1}{2} \) and a series corresponding to \( j = l - \frac{1}{2} \).

\[
\psi_i = \sqrt{4\pi} \sum_{l=0}^{\infty} (l+1)^{\frac{1}{2}} (i)^l J_L(kr) \left[ a_1 Y^\frac{1}{2}_l \left( \psi_{l+\frac{1}{2}, l, s} \right) + a_2 Y^{-\frac{1}{2}}_l \right] + \sqrt{4\pi} \sum_{l=0}^{\infty} (l+1)^{\frac{1}{2}} (i)^l J_L(kr) \left[ -a_1 Y^{-\frac{1}{2}}_l \left( \psi_{l-\frac{1}{2}, l, s} \right) + a_2 Y^{-\frac{1}{2}}_l \right]
\]

The total wave function is

\[
\psi_T = \psi_i + \psi_s
\]
where $\Psi_s$ is the scattered wave function. In form, $\Psi_T$ differs from $\Psi_i$ only in its radial dependence, and can be written as

$$
\Psi_T = \sqrt{4\pi} \sum_{l=0}^{\infty} (l+1)^{\frac{3}{2}} (i)^l \Psi^+_{l}(r) \left[ a_1 J_{l+\frac{1}{2}, l_s} + a_2 J_{l-\frac{1}{2}, l_s} \right]
$$

$$
+ \sqrt{4\pi} \sum_{l=0}^{\infty} (l+1)^{\frac{3}{2}} (i)^l \Psi^-_{l}(r) \left[ -a_1 J_{l-\frac{1}{2}, l_s} + a_2 J_{l+\frac{1}{2}, l_s} \right]
$$

where $\Psi^\pm_{l}(r)$ are the radial functions associated with $j = l \pm \frac{1}{2}$, respectively. The radial functions $\Psi^\pm_{l}$ must reduce to $J_{l}(kr)$ when there is no interaction. Furthermore, only the outgoing part of $\Psi_T$ can be modified by the interaction. These conditions can be satisfied if $\Psi^\pm_{l}(r)$ are given by

$$
\Psi^\pm_{l}(r) = \frac{1}{kr} e^{i \delta^\pm_{l}} \sin \left( k r - \frac{\ell \pi}{2} + \delta^\pm_{l} \right)
$$

where the complex phase shift $\delta^\pm_{l}$ represents the effects of the interaction. For no interaction $\delta^\pm_{l} = 0$ and it is seen that $\Psi^\pm_{l}$ reduces to asymptotic form of $J_{l}(kr)$

$$
J_{l}(kr) \sim \frac{1}{kr} \sin \left( k r - \frac{\ell \pi}{2} \right) \quad r \to \infty
$$

The expression given above for $\Psi^\pm_{l}(kr)$ represents a sum of ingoing and outgoing spherical waves

$$
\Psi^\pm_{l}(kr) \sim \frac{1}{2 i k r} \left[ e^{i (k r - \ell \pi/2)} - e^{-i (k r - \ell \pi/2)} \right]
$$
and the function $\psi_{\pm}^\pm(r)$ is obtained by modifying the outgoing wave by a factor $e^{2i\delta_{l}^\pm}$

$$\frac{1}{2i\alpha_{l}^\pm} \left[ e^{i(kr - l\pi/2)} - e^{-i(kr - l\pi/2)} \right]$$

$$= \frac{1}{2i\alpha_{l}^\pm} \left[ e^{i(kr - l\pi/2 + \delta_{l}^\pm)} - e^{-i(kr - l\pi/2 + \delta_{l}^\pm)} \right]$$

$$= \psi_{l}^\pm(r)$$

The expressions for $\psi_{l}^\pm$ can now be substituted into the expansion of $\psi_{l}$, and the expansion of $\psi_{l}$ can be subtracted to yield

$$\psi_{S} = \sqrt{4\pi} \sum_{l=0}^{\infty} (l+1)^{1/2} (i)^{l} e^{-\frac{i\pi}{2}} \frac{e^{ikr}}{kr} \frac{1}{2i} (e^{2i\delta_{l}^+} - 1) |y_+\rangle$$

$$+ \sqrt{4\pi} \sum_{l=0}^{\infty} (l+1)^{1/2} (i)^{l} e^{-\frac{i\pi}{2}} \frac{e^{ikr}}{kr} \frac{1}{2i} (e^{2i\delta_{l}^-} - 1) |y_-\rangle$$

where

$$|y_+\rangle = a_1 \psi_{l+\delta, r, s}^{1/2} + a_2 \psi_{l+\delta, r, s}^{-1/2}$$

$$|y_-\rangle = -a_1 \psi_{l-\delta, r, s}^{1/2} + a_2 \psi_{l-\delta, r, s}^{-1/2}$$

It is convenient to define

$$C_{l}^\pm = \frac{1}{2i} \left( e^{2i\delta_{l}^\pm} - 1 \right)$$

and to express $|y_+\rangle$ and $|y_-\rangle$ in terms of $\alpha, \beta$ and $\psi_{l} (\cos \theta) e^{im\phi}$ by employing the Clebsh–Gordon
coefficients and the definitions of the normalized spherical harmonics. After collecting terms and simplifying the formula for $\psi_s$ becomes

$$\psi_s = \frac{e^{ikr}}{kr} \left\{ g(\theta) \left[ a_1 \alpha + a_2 \beta \right] + h(\theta) \left[ a_2 e^{-i\phi} \alpha - a_1 e^{+i\phi} \beta \right] \right\}$$

where the functions $g(\theta)$ and $h(\theta)$ are given by

$$g(\theta) = \frac{1}{k} \sum_{\ell=0}^{\infty} \left[ (\ell+1) C_\ell^+ + \ell C_\ell^- \right] P_\ell^0 (\cos \theta)$$

$$h(\theta) = \frac{1}{k} \sum_{\ell=0}^{\infty} \left[ C_\ell^- - C_\ell^+ \right] P_\ell^1 (\cos \theta)$$

It is easily verified that $\psi_s$ can be written as

$$\psi_s = S(\theta, \phi) \chi$$

where $S$ is the previously discussed scattering matrix and $\chi$ is the incident spin function.

2) Inclusion of the Coulomb Interaction

The formulas for $g(\theta)$ and $h(\theta)$ derived above assume that the incident particle is uncharged, and must be modified if the Coulomb interaction is present. If the incident particle has charge $z_1 e$ and the target has charge $z_2 e$, then a Coulomb potential term

$$V_c = \frac{z_1 z_2 e^2}{r}$$
must be added to the potential. The appropriate formulas for \( g(\theta) \) and \( h(\theta) \) can now be derived by simply replacing the incident plane wave, \( \psi_i \), by \( \psi_c \chi \) where \( \psi_c \) is the solution of the Schrödinger equation

\[
\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + \frac{Z_+ e^2}{r} - E \right] \psi_c = 0
\]

which corresponds to the scattering of two point charges. It is important to note, however, that \( \psi_c \) consists of both a distorted incoming wave and a scattered wave due to the point charge distribution.

The function \( \psi_c \) is known to be

\[
\psi_c = \Gamma(1 + i\eta) e^{-\eta \frac{\pi}{2}} e^{i k \xi} F(-i\eta, 1, i k \xi)
\]

where

\[
\eta = \mu \frac{Z_+ e^2}{\hbar^2 k} \quad \xi = r - z
\]

and \( F \) is the confluent hypergeometric function.

The asymptotic form of \( \psi_c \) is

\[
\psi_c \sim e^{i \left[ k^2 - \eta \ln k(r-z) \right]} \left[ 1 - \frac{\eta^2}{i k(r-z)} \right] + \frac{1}{r} f_c(\theta) e^{i \left[ k^2 - \eta \ln 2kr \right]}
\]

where the Rutherford scattering amplitude is

\[
f_c(\theta) = \frac{-\eta}{2k \sin^2 \theta^*} \in^{-i \eta \ln (\sin^2 \theta^*)} + i \frac{\eta}{\sin^2 \theta^*}
\]

\[
\sigma_c = \text{arg} \left( 1 + i \eta \right)
\]
The partial wave expansion of $\psi_c$ is

$$
\psi_c = \sum_{l=0}^{\infty} (2l+1) (i)^l e^{i\delta_2} \frac{1}{kr} F_\ell(\eta, kr) \left[ \frac{\pi}{2l+1} \right]^{\frac{1}{2}} Y_\ell^0(\theta, \phi)
$$

where $F_\ell(n, kr)$ is the regular Coulomb function and the Coulomb phase shift is

$$
\delta_2 = \arctan (l + 1 + i\eta)
$$

The incident wave is replaced by $\psi_e = \psi_c \chi$, which, by analogy with the previous derivation, can be written as

$$
\psi_e = \sqrt{4\pi} \sum_{l=0}^{\infty} (2l+1) (i)^l e^{i\delta_2} \frac{1}{kr} F_\ell(\eta, kr) |y+\rangle + \sqrt{4\pi} \sum_{l=0}^{\infty} (2l) (i)^l e^{i\delta_2} \frac{1}{kr} F_\ell(\eta, kr) |y-\rangle
$$

The total wave function is $\psi_T = \psi_e + \psi_S$ where $\psi_S$ represents only non-Coulomb scattering, and $\psi_T$ must be identical to $\psi_e$ except for the radial functions, which are replaced by $\psi_e^+(r)$ and $\psi_e^-(r)$. The general form of $\psi_\ell^\pm$ is determined by requiring that $\psi_\ell^+(r)$ reduce to $F_\ell(n, kr)/kr$ when the nuclear potential is zero, and that only the outgoing wave be modified by the nuclear interaction. In the asymptotic region

$$
F_\ell(\eta, kr) \sim \sin \left( kr - \eta \ln 2kr - \frac{\pi}{2} + \delta_2 \right)
$$

and $\psi_\ell^\pm$ are chosen to be

$$
\psi_\ell^\pm(r) \sim \frac{1}{kr} e^{i\delta_\ell^\pm} \sin \left( kr - \eta \ln 2kr - \frac{\pi}{2} + \delta_\ell + \delta_\ell^\pm \right)
$$
The function corresponding to nuclear scattering alone is \( \psi_S = \psi_r - \psi_e \) and is given by

\[
\begin{align*}
\psi_S &= \sqrt{4\pi} \sum_{l=0}^{\infty} (l+1)^2 (i)^l e^{-\frac{il\pi}{2k\rho}} e^{i(k\nu - \nu\ln 2k\nu)} \frac{e^{z i\delta_k}}{C^+_l} |y+\rangle \\
&\quad + \sqrt{4\pi} \sum_{l=0}^{\infty} (l)^2 (i)^l e^{-\frac{il\pi}{2k\rho}} e^{i(k\nu - \nu\ln 2k\nu)} \frac{e^{z i\delta_k}}{C^-_l} |y-\rangle
\end{align*}
\]

where \( C^+_l \) and \( |y\pm\rangle \) have been previously defined. This can be re-written as

\[
\begin{align*}
\psi_S &= \frac{1}{\gamma} e^{i(k\nu - \nu\ln 2k\nu)} \left\{ A(\theta) \left[ a_1 \alpha + a_2 \beta \right] \\
&\quad + B(\theta) \left[ a_2 e^{-i\phi} \beta - a_1 e^{i\phi} \alpha \right] \right\}
\end{align*}
\]

where

\[
A(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} e^{z i\delta_k} \left[ (l+1) C^+_l + l C^-_l \right] P_l(\cos \theta)
\]

\[
B(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} e^{z i\delta_k} \left[ C^+_l - C^-_l \right] P^1_l(\cos \theta)
\]

The total wave function is now formed by adding to \( \psi_S \) the following expression for \( \psi_e = \psi_c \chi = \psi_c [a_1 \alpha + a_2 \beta] \)

\[
\psi_e = \left\{ e^{i(k\nu - \nu\ln 2k\nu)} \left[ 1 - \frac{\nu^2}{i(k\nu - z)} \right] + \frac{1}{i} f_c(\theta) e^{i(k\nu - \nu\ln 2k\nu)} \right\} \chi
\]

The result is
\[ \psi_{\uparrow} = \left\{ e^{i(kz - \gamma \ln k(r-z))\left[1 - \frac{\eta^2}{ik(r-z)}\right]} \right\} \chi \]

\[ + \frac{1}{r} e^{i(kr - \gamma \ln zkr)} \left\{ g(\theta)[a_1 \chi + a_2 \beta] + h(\theta)[a_2 e^{-i\phi_2} \beta - a_1 e^{i\phi_1} \chi]\right\} \]

where

\[ g(\theta) = f_c(\theta) + A(\theta) \]

\[ h(\theta) = B(\theta) \]

As before, it can be easily shown that the total outgoing scattered wave, which now represents both Coulomb scattering and nuclear scattering, can be written in terms of the scattering matrix and the incident spin function

\[ \frac{1}{r} e^{i(kr - \gamma \ln zkr)} S(\theta, \phi) \chi \]

The differential elastic scattering cross section and the polarization may be written in terms of the complex phase shifts, \( \delta_{\ell}^\pm \), by using the formulas previously derived for \( \delta(\theta) \) and \( \mathbf{P}(\theta) \).

The imaginary parts of the phase shifts parameterize the inelastic processes. If \( \gamma_{\ell}^\pm \) is the imaginary part of \( \delta_{\ell}^\pm \), then the corresponding damping parameter is defined as

\[ \alpha_{\ell}^\pm = e^{-2 \gamma_{\ell}^\pm} \]
In terms of the damping parameters, the total reaction cross section, \( \sigma_T \), representing the net effect of all inelastic processes, is given by
\[
\sigma_T = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \left\{ (l+1)[1-(\alpha_l^t)^2] + \lambda[l-(\alpha_l^s)^2] \right\}
\]

The expression for \( \sigma_T \) can be derived from the general formulas of Lane and Thomas (La 58) or of Preston (Pr 62). Such a derivation has been given by Rich (Ri 67) and, for the sake of completeness, is summarized below.

The collision matrix element
\[
\mathcal{U}_{\alpha's'\ell'\alpha s \ell}^J
\]
gives the probability amplitude for a reaction of total angular momentum \( J \) which goes from channel \( \alpha s \ell \) to channel \( \alpha's'\ell' \). The notation follows that of Ref. Pr 62; \( \ell \) and \( s \) are the orbital angular momentum and channel spin quantum numbers, respectively, and \( \alpha \) designates the two body partition being considered. In the case of pure elastic scattering, there is only one exit channel and \( \mathcal{U}_J^J \) is given by
\[
\mathcal{U}_J^J = \mathcal{C} e^{2i\delta^+_{\ell}}
\]
where phase shift \( \delta^+_{\ell} \) must be real. In the inelastic case, there are in general many open reaction channels. However, for the specific purpose of calculating the total reaction cross section, the net effect of all open reaction
channels may be represented by a single exit channel.

The collision matrix may then be written as a $2 \times 2$ unitary
and symmetric matrix (Pr 62). Denoting the off diagonal
elements by a subscript $0$ and the real part of $\delta^\pm_\ell$ by $\Delta^\pm_\ell$, the collision matrix is of the form

\[
\begin{pmatrix}
\alpha^\pm_\ell e^{2i\Delta^\pm_\ell} & \alpha^J_0 e^{2i\delta^J_0} \\
\alpha^J_0 e^{2i\delta^J_0} & \alpha^\pm_\ell e^{2i\Delta^\pm_\ell}
\end{pmatrix}
\]

where the unitary condition implies that

\[
(\alpha^\pm_\ell)^2 + (\alpha^J_0)^2 = 1
\]

The effects of the elastic and reaction exit channels are
represented by the diagonal and off-diagonal elements,
respectively. The total cross section for the reaction

$\alphao < \alphao'$ is (Pr 62)

\[
\sigma_{\alphao, \alphao'} = \frac{\pi}{k^2} \sum_{J1J2S1S2} \frac{2J+1}{(2S_1+1)(2S_2+1)} \left| U^{\mathcal{J}}_{\alphao' S1', \alphao S2} \right|^2
\]

where $s_1$ and $s_2$ are the spins of the beam and target,
respectively. For the case considered, $s_1 = \frac{1}{2}$, $s_2 = 0,$
and $J = \ell \pm \frac{1}{2}$. Therefore, when $\alphao'$ is taken to represent
all open reaction channels,

\[
\sigma_T = \frac{\pi}{k^2} \sum_{J, \ell} \frac{1}{2}(2J+1) \left| U^{\mathcal{J}}_{\alphao' \alphao} \right|^2
\]
and

\[ | \mathcal{U}_{\alpha', \alpha}^J |^2 = | \alpha_0^J e^{2i \delta_0^J} |^2 \]

\[ = 1 - (\alpha_{\pm}^J)^2 \]

By summing over \( J = \ell \pm \frac{1}{2} \), the previously given formula for \( \sigma_T \) is obtained.
Appendix B: Optical Pumping and the Optical Measurement of Polarization

In the method of optical pumping (Co 63, Ga 65), circularly polarized resonantance radiation is used to polarize metastable $^3$He atoms which in turn polarized ground state $^3$He atoms. A weak electric discharge is used to continuously produce $^3$He atoms in the $^2S_1$ metastable state. These atoms are excited from the $^3S_1$ to the $^3P_0$ state by $1.08 / \lambda$ wavelength light as shown in Fig. 11. This light is provided by a $^4$He lamp and is circularly polarized by passing it through a linear polarizer and a quarter wave plate. Due to the selection rule $\Delta M_F = \pm 1$, atoms in either low or high hyperfine magnetic sublevels can be selectively excited to the $^3P_0$ by controlling the sense of the circularly polarized pumping light. The atoms excited to the $^3P_0$ state decay to all of the hyperfine magnetic sublevels of the $^3S_1$ state. As a result, the proportion of $^3S_1$ atoms having either low or high hyperfine magnetic quantum numbers is increased and one obtains $^3S_1$ atoms polarized either parallel or antiparallel to the direction of the pumping light. A weak magnetic field is used to maintain the direction of the polarization. By adiabatically rotating
Figure 11
Energy level diagram of the $^3$He atom in an external magnetic field (not to scale). The numbers $n_1, n_2, \ldots, n_6$ represent the level populations of the indicated magnetic sublevels.
Figure 11

IONIZATION

\[ 2^3P_0 \]
\[ \sim 1 \text{ cm}^{-1} \]
\[ 2^3P_{1/2} \]
\[ 4 \times 10^4 \text{ cm}^{-1} \]
\[ 1.6 \times 10^5 \text{ cm}^{-1} \]
\[ 9233 \text{ cm}^{-1} \]
\[ 2^1S_0 \]
\[ 2^3S_1 \]
\[ 0.22 \text{ cm}^{-1} \]

\[ F = 1/2 \]
\[ + 1/2 \rightarrow n_6 \]
\[ -1/2 \rightarrow n_5 \]
\[ + 3/2 \rightarrow n_4 \]
\[ + 1/2 \rightarrow n_3 \]
\[ -1/2 \rightarrow n_2 \]
\[ -3/2 \rightarrow n_1 \]

\[ 1^1S_0 \]
the magnetic field to the opposite direction, the direction of the $^3\text{He}$ spins can be reversed. If the sense of the circularly polarized light is also reversed, the pumping light will tend to maintain the polarization in this new direction.

Large target polarizations are obtained through collisions of the polarized $^3S_1$ metastables with ground state atoms. Metastability exchange collisions can occur in which the hyperfine magnetic quantum number of the polarized metastable atoms changes by $\pm 1$ and the hyperfine magnetic quantum number of the ground state atom changes by $\pm 1$. Such a collision results in the polarization of the nuclear spin of the $^3\text{He}$ ground state atom and the depolarization of the metastable atom. The metastable atom can then be polarized again through the optical pumping process. Since the cross section for metastability exchange collisions is very large ($\sim 4 \times 10^{-16} \text{ cm}^2$ at room temperature) the metastable and ground state systems are tightly coupled and processes which tend to polarize or depolarize one system will similarly effect the other. The presence of impurities in the $^3\text{He}$ gas reduces the number of metastable atoms, and magnetic field gradients tend to relax the metastable polarization. Consequently, these effects reduce the ground state polarization.
A method for measuring the target polarization by using optical signals has been presented by Colgrove et al. (Co 63). In this method an optical measurement of the metastable polarization is made, and because of the tight coupling between the ground state and metastable systems, such a measurement is equivalent to measuring the ground state polarization. The method used to monitor the target polarization is the same as that of Ref. Co 63 except that the nuclear polarization is reversed, not destroyed. The optical measurement consists of determining

\[
\frac{\Delta I}{I} = \frac{I(P) - I(-P)}{I(P)}
\]

where \(I(P)\) is the absorption of \(^3P_0 - ^3S_1\) light by the metastable system for a given target polarization \(P\).

The quantity \(I\) is measured by turning the weak discharge off and on and recording the change in the transmitted pumping light. When the weak discharge is off, no pumping light is absorbed because there are no metastable atoms (the relaxation time for metastables is approximately \(2 \times 10^{-4}\) sec.). The ground state polarization remains constant over a period of time much longer than that required to measure \(I(P)\). The quantity \(\Delta I\) is measured by recording the change in the transmitted pumping light as the \(^3\)He polarization is rotated to the opposite direction.
by smoothly reversing the direction of the magnetic field. It was found that polarization losses occurred during the measurement of \( \Delta I \). A correction for these losses was made according to the procedure described by McSherry (Mc 69).

The relationship between \( \Delta I/I \) and \( P \) can be established as follows: The metastable polarization \( P \) has been related to the level populations, \( n_i \), of the pertinent magnetic sublevels of the \( ^3 \)He atoms in Ref. Co 63. The levels to which \( n_i, \ i = 1,6 \), refer are indicated in Fig. 11.

The following steady state solution is given in Ref. Co 63.

\[
\begin{align*}
n_1 &= \left[ (1-P)^3 / (b + 2P^2) \right] n \\
n_2 &= n_5 = \left[ (1+P)(1-P)^2 / (b + 2P^2) \right] n \\
n_3 &= n_6 = \left[ (1+P)^2 (1-P) / (b + 2P^2) \right] n \\
n_4 &= \left[ (1+P)^3 / (b + 2P^2) \right] n \\
\end{align*}
\]

\[ n_1 + n_2 + n_3 + n_4 + n_5 + n_6 = n \]

The quantity \( I(P) \) can be expressed in terms of the \( n_i \) above and the parameters \( a, b, c, \rho \) and \( f \). The quantities \( a, b \) and \( c \) are the electric dipole transition probabilities
\[(2^3S_1 - 2^3P_0)\] given below (Co 63, Sc 66):

<table>
<thead>
<tr>
<th>Magnetic Sublevel</th>
<th>Original</th>
<th>Final</th>
<th>Transition Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F = 3/2)</td>
<td>(M_F = -3/2)</td>
<td>-1/2</td>
<td>(a = 0.28)</td>
</tr>
<tr>
<td>3/2</td>
<td>-1/2</td>
<td>1/2</td>
<td>(b = 0.10)</td>
</tr>
<tr>
<td>1/2</td>
<td>-1/2</td>
<td>1/2</td>
<td>(c = 0.30)</td>
</tr>
</tbody>
</table>

The quantity \(\rho\) is the percent circular polarization of the pumping light, and \(f\) is the ratio of the illumination of the \(F = 3/2\) sublevels to the total illumination of both the \(F = 3/2\) and the \(F = 1/2\) sublevels. The relative illumination of these sublevels depends on the spectral properties of the \(^4\)He lamp used and the uncertainty in the value of \(f\) results in a systematic uncertainty in the target polarization (Ba 69).

For \(\Delta M_F = 1\) transitions, the light absorbed by the metastables is

\[
I(P) = n_1 f (1-\rho) a + n_2 f (1-\rho) b + n_3 f \rho b + n_4 f \rho a + n_5 (1-f) (1-\rho) c + n_6 (1-f) \rho c
\]

\[
= a \left[ n_1 (1-\rho) f + n_4 \rho f \right] + b \left[ n_2 (1-\rho) f + n_3 \rho f \right] + c \left[ n_2 (1-\rho) (1-f) + n_3 \rho (1-f) \right]
\]
By using the equations for \( n_1 \) one obtains

\[
I (P) = \frac{n}{b + 2p^2} \left\{ aP \left[ (1-P)^3 + \rho P (6 + 2 \rho^2) \right] + [bP + c(1-P)](1+P)(1-P) \left[ (1-P) + 2\rho P \right] \right\}
\]

from which an equation for \( \Delta I/I \) can be readily obtained.
Appendix C: Calculation of Laboratory Asymmetries

In measuring the left-right scattering asymmetry at a given energy (Ba 69), data are generally collected for all possible combinations of magnetic field direction and target polarization direction. In the following discussion the direction of the magnetic field or of the target polarization will arbitrarily be designated as either up or down, where these terms are only meant to imply one direction as opposed to the other.

For each laboratory scattering angle a pair of counters are placed symmetrically about the beam axis, and in the plane perpendicular to the direction of the target polarization. The number of particles detected at each counter is assumed to be dependent upon the beam integration I, the effective solid angle \( \Omega \), an arbitrary parameter \( M \) representing any effect due to the magnetic field, and the cross section \( \delta^- \). For a polarized target, left scattering is defined according to the Basel convention (Ba 60) as scattering for which the vector \( \vec{k}_{\text{in}} \times \vec{k}_{\text{out}} \) is parallel to the direction of the target polarization, where \( \vec{k}_{\text{in}} \) is the direction of the incident particle and \( \vec{k}_{\text{out}} \) is the direction which the incident particle has after it has been scattered. The scattering geometry
Figure 12

The scattering geometry employed in measuring left-right scattering asymmetries. The vectors $\vec{k}_{\text{in}}$ and $\vec{k}_{\text{out}}$ are defined in the text. The unit vector $\vec{p}_3$ gives the direction of the $^3$He target polarization.
is shown in Fig. 12. The cross sections for left and right scattering are defined in terms of the unpolarized cross section $\sigma_0$ as follows

$$\sigma(\theta, -\pi/2) = \sigma_L(\theta) = \sigma_0(\theta)(1 + P_3 Q_3)$$
$$\sigma(\theta, +\pi/2) = \sigma_R(\theta) = \sigma_0(\theta)(1 - P_3 Q_3)$$

where $P_3$ is the target polarization and $Q_3$ is the analyzing power for scattering at an angle $\theta$. For the target condition in which the magnetic field is up, the numbers of particles detected at the two counters are given by

$$N_A = I_A \sum_A M_{uA} \sigma_0 (1 + P_3 Q_3)$$
$$N_B = I_B \sum_B M_{uB} \sigma_0 (1 - P_3 Q_3)$$

where the subscripts $A$ and $B$ are used to designate the two counters, the subscript $U$ designates the magnetic field direction, and it has been assumed that left-scattering is observed in counter $A$ when the target polarization is up.

In order to allow cancellation of the factors $\sum$ and $M$, the data are taken during four periods which correspond to the four possible combinations of magnetic field direction and target polarization direction.
As a result, eight numbers, \( N_j \), are obtained. The target polarization is then written as
\[
P_3^{(i)} = P_3 \left(1 + \delta_i\right)
\]
i = 1, 4

\[
\delta_1 + \delta_2 + \delta_3 + \delta_4 = 0
\]

where \( P_3^{(i)} \) is the target polarization for the \( i \)th period.

The analyzing power is expressed as
\[
Q_3^{(A)} = Q_3 \left(1 + \delta'\right)
\]
\[
Q_3^{(B)} = Q_3 \left(1 - \delta'\right)
\]

where \( Q_3^{(A)} \) and \( Q_3^{(B)} \) give the effective values of the analyzing power for scattering into counters A and B, respectively. This allows for the possibility that the mean scattering angle is not the same for counters A and B. Expressions for the eight \( N \)'s may now be written for the various target conditions, and these are given in Table VIII.

Using Table VIII the ratio \( R \) can be computed, where
\[
R = \frac{N_1 N_4 N_5 N_9}{N_2 N_3 N_6 N_7}
\]

\[
= \left[\frac{1 + \frac{P_3}{1 - P_3} Q_3}{1 + \frac{P_3}{1 - P_3} Q_3}\right]^4 \left[1 + f(Q_3 \delta', P_3 \delta_i)\right]
\]

The function \( f(Q_3 \delta', P_3 \delta_i) \) includes only second and higher order terms in \( Q_3 \delta' \) and \( P_3 \delta_i \). Usually \( Q_3 \delta' \)
and \( P_3 \delta_i \) are all small, and \( f(Q_3 \delta', P_3 \delta_i) \) may be
Table VIII

The eight types of data counts used in calculating the left-right scattering asymmetries. The direction of the target polarization or of the magnetic field is arbitrarily designated as either up or down as discussed in the text. The column headed TP refers to the target polarization and the column headed MF refers to the magnetic field direction.

<table>
<thead>
<tr>
<th>Data Counts</th>
<th>TP</th>
<th>MF</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_1 = \mathcal{I}_1 \mathcal{I}<em>A M</em>{UA} g_0 (1 + \rho_3^{(4)} \rho_3^{(A)}) )</td>
<td>Up</td>
<td>Up</td>
</tr>
<tr>
<td>( N_2 = \mathcal{I}_1 \mathcal{I}<em>B M</em>{UB} g_0 (1 - \rho_3^{(4)} \rho_3^{(B)}) )</td>
<td></td>
<td>Up</td>
</tr>
<tr>
<td>( N_3 = \mathcal{I}_2 \mathcal{I}<em>A M</em>{DA} g_0 (1 - \rho_3^{(2)} \rho_3^{(A)}) )</td>
<td>Down</td>
<td>Down</td>
</tr>
<tr>
<td>( N_4 = \mathcal{I}_2 \mathcal{I}<em>B M</em>{DB} g_0 (1 + \rho_3^{(2)} \rho_3^{(B)}) )</td>
<td></td>
<td>Down</td>
</tr>
<tr>
<td>( N_5 = \mathcal{I}_3 \mathcal{I}<em>A M</em>{DA} g_0 (1 + \rho_3^{(3)} \rho_3^{(A)}) )</td>
<td>Up</td>
<td>Down</td>
</tr>
<tr>
<td>( N_6 = \mathcal{I}_3 \mathcal{I}<em>B M</em>{DB} g_0 (1 - \rho_3^{(3)} \rho_3^{(B)}) )</td>
<td></td>
<td>Up</td>
</tr>
<tr>
<td>( N_7 = \mathcal{I}_4 \mathcal{I}<em>A M</em>{UA} g_0 (1 - \rho_3^{(4)} \rho_3^{(A)}) )</td>
<td>Down</td>
<td>Up</td>
</tr>
<tr>
<td>( N_8 = \mathcal{I}_4 \mathcal{I}<em>B M</em>{UB} g_0 (1 + \rho_3^{(4)} \rho_3^{(B)}) )</td>
<td></td>
<td>Up</td>
</tr>
</tbody>
</table>
neglected. Then

\[ P_3 \, Q_3 = \left( R^{\frac{1}{4}} - 1 \right) / \left( R^{\frac{1}{4}} + 1 \right) \]

The data may be tested for self consistency by forming the ratio

\[ R_0 = \frac{N_1 \, N_3 \, N_6 \, N_8}{N_2 \, N_4 \, N_5 \, N_7} \]

in which all effects, including those due to target polarization, cancel identically for an ideal experiment. The nominally zero quantity \( P_3 \, Q_0 \) is defined as follows

\[ P_3 \, Q_0 = \left( R_0^{\frac{1}{4}} - 1 \right) / \left( R_0^{\frac{1}{4}} + 1 \right) = \varepsilon \]

Because the equation defining \( P_3 \, Q_0 \) has the same form as the equation for \( P_3 \, Q_3 \), the derivations of the statistical error in \( P_3 \, Q_3 \) presented here and in appendix D can also be used to determine the error in \( P_3 \, Q_0 \) by simply replacing \( R \) by \( R_0 \).

If no correction for background is necessary, the statistical uncertainty in \( P_3 \, Q_3 \) is calculated as follows:

Let

\[ \alpha = P_3 \, Q_3 \]

Then

\[ \Delta \alpha = \frac{\partial \alpha}{\partial R} \Delta R \]
\[ \frac{\Delta R}{R} = \left[ \sum_{i=1}^{3} \left( \frac{\Delta N_i}{N_i} \right)^2 \right]^{1/2} \]

\[ \Delta N_i = \left( N_i \right)^{1/2} \]

Therefore

\[ \Delta Q = \frac{R^{1/4}}{2(1+R^{1/4})^2} \left[ \sum_{i=1}^{3} \frac{4}{N_i} \right]^{1/2} \]

When it is necessary to correct the data for background the formula for \( \Delta Q \) must be modified according to the procedure given in appendix D.

The quality of the asymmetry data may be tested by studying the deviation from zero of \( \varepsilon \), the experimental value of \( P_3 \, \Omega_0 \). For each center of mass scattering angle, the value of \( \chi^2 \) where

\[ \chi^2 = \sum_{i=1}^{n} \left( \frac{\varepsilon_i \sigma_i}{\sigma_i} \right)^2 \]

\( n \) = number of \( \varepsilon \)'s

\( \sigma_i \) = statistical uncertainty in \( \varepsilon_i \)

can be calculated to determine if the distribution of \( \varepsilon \)'s is consistent with zero within statistics.
Appendix D: Correction for Background

Two background correction procedures will be discussed and the appropriate formulas for $\Delta A$, the statistical uncertainty in $A$, will be derived. (Bak:68). As in appendix C, the quantity $A$ is given by

$$A = P_3 Q_3 = \frac{(R^{I_4} - 1)}{(R^{I_4} + 1)}$$

where $P_3$, $Q_3$ and $R$ are defined in appendix C. The possible effects of the background correction on the calculated value of $A$ will also be discussed.

The quantities relevant to the calculation of $A$ or $\Delta A$ are indicated in Fig. 13 which represents a pulse height spectrum in which background events are present. It is to be understood that $N$, $F_N$, $F_B$ and $F_H$ all carry a subscript $j$. The subscript $j$ ranges from 1 to 8 and corresponds to the eight types of data counts discussed in appendix C.

Procedure One

The quantity $C = N + F_N$ is measured where $F_N$ is the number of background counts associated with $N$. Quantities of the form $N = C - F_N$ are then used to calculate $A$ according to the procedure given in appendix C. The statistical uncertainty in $C$ is determined by the statistical uncertainty in $F_N$. The statistical
Pulse height spectrum in which background events are present. The quantities $N$, $F_N$, $F_B$, and $F_H$ are the numbers of counts in each of the indicated regions.
uncertainty in $F_N$ is determined by the statistical fluctuation in $F_N^2$ and by the statistical fluctuation of the quantity $T = F_B + F_H$ which is used to estimate the background level. Therefore

$$
\left( \frac{\Delta F_N}{F_N} \right)^2 = \left( \frac{\Delta T}{T} \right)^2 + \frac{1}{F_N} = \frac{1+\delta^T}{\delta^T F_N}
$$

where

$$\delta^T = \frac{T}{F_N^2}
$$

The statistical uncertainty in $N$ can be found from

$$
(\Delta N)^2 = (\Delta C)^2 + (\Delta F_N)^2
$$

$$
= F_N \left( \frac{1+2\delta}{\delta} \right) + N
$$

$$
= N \left[ 1 + \rho \left( \frac{1+2\delta}{\delta} \right) \right]
$$

where

$$\rho = \frac{F_N}{N}
$$

The appropriate formula for $\Delta Q$ is obtained by substituting $\Delta N_j$ as given above into the derivation of $\Delta Q$ presented in appendix C and is

$$
\Delta Q = \frac{R^{\frac{1}{4}} S}{2 (1+R^{\frac{1}{4}})^2}
$$
where

\[ S = \left\{ \sum_{j=1}^{8} \left[ \frac{1 + P_i \left( \frac{1 + 2 \alpha_i}{N_j} \right)}{N_j} \right] \right\}^{1/2} \]

This formula for \( \Delta \alpha \) may also be used to determine the statistical uncertainty in \( P_3 \alpha_0 \) by merely replacing \( R \) by \( R_0 \). The quantities \( R_0 \) and \( P_3 \alpha_0 \) are defined in appendix C.

This expression for \( \Delta \alpha \) gives the uncertainty in \( \alpha \) due to statistical effects alone and, as such, represents the minimum uncertainty in \( \alpha \). The errors inherent in estimating \( F_N \) may exceed the statistical errors, and it is possible that the errors involved in performing the background subtraction \( N = C - F_N \) may change the sign of \( \alpha \). This is most probable when the fractional background \( F_N/N \) is large or when the value of \( \alpha \) is near zero.

**Procedure Two**

When \( F_N/N \) is large or \( \alpha \) is very small, it is preferable to employ the background correction procedure presented below. The value of \( \alpha' \) is calculated by substituting the values of \( C_j \) for \( N_j \) in the formula for \( \alpha \) given in appendix C. The value of \( \alpha \) may then be related to \( \alpha' \) through a multiplicative factor, as will be shown. It should not be possible to change the sign of \( \alpha \) by this procedure.
The factor relating \( a \) and \( a' \) will now be derived, as well as the appropriate statistical uncertainty, \( \Delta a \).

For simplicity, consider the case in which data are collected for both directions of target polarization but for only one magnetic field direction. The numbers of counts obtained in counters A and B are then given by

\[
A^+ = I_2 \Omega_A \sigma_0 \left[ (1+a) + F_A \right] \\
A^- = I_1 \Omega_A \sigma_0 \left[ (1-a) + F_A \right] \\
B^+ = I_2 \Omega_B \sigma_0 \left[ (1-a) + F_B \right] \\
B^- = I_1 \Omega_B \sigma_0 \left[ (1+a) + F_B \right]
\]

where \( I_1, I_2 \) are the beam integrations, \( \Omega_A, \Omega_B \) are the effective solid angles, \( a = P_3 Q_3 \) is the laboratory asymmetry, and \( F_A, F_B \) are the fractional backgrounds in counters A and B assuming that the background is not a function of the polarization direction. Now let

\[
F_A = F - \delta \\
F_B = F + \delta
\]

The expression corresponding to the ratio \( R \) of appendix C then becomes \( R \) where

\[
R = \frac{A^+B^-}{A^-B^+} = \frac{(1+F+a-\delta)(1+F+a+\delta)}{(1+F-a-\delta)(1+F-a+\delta)}
\]
Now define $a'$ and $\delta'$ such that
\[ a' = \frac{a}{1 + F}, \quad \delta' = \frac{\delta}{1 + F} \]
then
\[ Q = \frac{(1 + a' - \delta')(1 + a' + \delta')}{(1 - a' - \delta')(1 - a' + \delta')} \]
\[ = \frac{(1 + a')^2 - (\delta')^2}{(1 - a')^2 - (\delta')^2} \]
\[ = \left( \frac{1 + a'}{1 - a'} \right)^2 + \frac{4a'(\delta')^2}{(1 - a')^4} + Q(\delta') \]
where $Q(\delta')$ includes only terms of order four or higher in $\delta'$. Therefore if $\delta'$ is very small, all but the first term in the expansion of $Q$ may be neglected and the values of $a'$ and $a$ are given by
\[ a' = \frac{R^{\frac{1}{2}} - 1}{R^{\frac{1}{2}} + 1} \quad a = (1 + F)a' \]

The statistical error in $a$ is determined by the statistical uncertainty of $a'$ and the statistical uncertainty in the factor $(1 + F)$. The statistical error in $a'$ is determined by using the formula for $\Delta a$ given in appendix C, with, of course, the numbers $C_j$ substituted for the numbers $N_j$. The statistical error in $(1 + F)$ is determined
as follows:

Let

\[ \frac{f}{C} = 1 + F = 1 + \frac{F_N}{N} = \frac{C}{C - F_N} \]

where the mean fractional background \( F \) is given by \( F = \frac{F_N}{N} \), and the best estimate of the quantities \( f \) and \( F \) are obtained by re-defining \( N, F_N \) and \( C \) as

\[ N \rightarrow \sum_{j=1}^{3} N_j, \quad F_N \rightarrow \sum_{j=1}^{3} F_{N_j}, \quad C \rightarrow \sum_{j=1}^{3} C_j \]

Then

\[ (\Delta f)^2 = (\frac{\partial f}{\partial C})^2 (\Delta C)^2 + (\frac{\partial f}{\partial F_N})^2 (\Delta F_N)^2 \]

\[ = \frac{F_N^2 C}{(C - F_N)^4} + \frac{C^2 (\frac{1 + \delta}{\delta}) F_N}{(C - F_N)^4} \]

\[ = \frac{F_N [F_N + C (\frac{1 + \delta}{\delta})]}{C [C - F_N]^2} \]

Therefore

\[ (\frac{\Delta a}{a})^2 = (\frac{\Delta a'}{a'})^2 + (\frac{\Delta f}{f})^2 \]

may be used to calculate the statistical error in \( a \) when this method of correcting for background is used.

The expression corresponding to the ratio \( R_o \) of appendix C is \( P_0 \) where
\[ Q_0 = \frac{A^+ B^+}{A^- B^-} \]

The procedure given above for calculating \( A \) and \( \Delta A \) may be used to calculate \( P_3 Q_0 \) and the statistical error in \( P_3 Q_0 \) by merely replacing \( Q \) by \( Q_0 \). An expansion of \( Q_0 \) shows that when terms quadratic in \( \delta \) are negligible, \( Q_0 \) will be equal to one and \( P_3 Q_0 \) should therefore equal zero.
Appendix E: Polarized $^3$He Target

1) Discussion

In recent years, polarized $^3$He nuclear targets employing the technique of optical pumping have come into use. This appendix describes one such target which is particularly simple in design and which has demonstrated its utility in a variety of nuclear scattering experiments (Ba 65, Ba 69, Mc 69b, Ha 70). A brief description of the associated apparatus necessary to perform optical pumping as well as the procedure to be used in cleaning and filling the target will be given. The reader is referred to the literature for more extensive discussions of these topics (Co 63, Ba 69, Fi 69). The essential features of the target and its construction will be described with attention being given to differences in the design of this target and similar targets reported elsewhere (Ba 65, Ba 69). The limitations of the present design will be discussed, and suggestions for future modifications will be made.

2) Optical Pumping Apparatus

In the process of optical pumping a weak electric discharge continuously produces $^3$He atoms in the metastable $2^3S_1$ state. Circularly polarized radiation is used to polarize the metastable atoms, which in turn polarize
the much more numerous ground state atoms.

The optical pumping apparatus employed is essentially the same as that discussed at length in Ref. Ba 69 and is shown in Fig. 14. Pumping light is provided by a $^4$He lamp which consists of a button-shaped cavity connected to a gas reservoir. A $^4$He pressure of 10-20 Torr is used. The lamp is excited by a 100 MHz oscillator at power levels ranging from 200-300 W. A concave mirror directs the light toward the chamber. A linear polarizer and a quarter wave plate are used to circularly polarize the pumping radiation, which is in the infrared. The linear polarizer can be rotated with respect to the quarter-wave plate to change the sense of the circular polarization.

Low pressure, high volume air supplied by a squirrel cage blower is used to cool the $^4$He lamp, the linear polarizer and the power tubes of the 100Mhz oscillator. The amount of light transmitted through the chamber is monitored by a PbS infrared detector; changes in the amount of light transmitted for various target conditions are used to determine the target polarization (see appendix B).

A 500 kHz oscillator and electrodes are used to produce a weak discharge in the $^3$He target gas. Helmholtz coils provide a uniform magnetic field of a few gauss in the region of the chamber and along the optical axis. The
direction of the field, and thus the direction of the
$^3$He polarization, can be smoothly reversed.

3) Glass Scattering Chamber

The experience at Rice University (Co 63, Ba 69)
in both atomic and nuclear applications of $^3$He optical
pumping has indicated that it is preferable to contain
the $^3$He gas in glass rather than metallic containers.
Glass containers present fewer degassing problems, and
make it simple to maintain a uniform weak discharge in
the $^3$He gas. Therefore the $^3$He target gas is contained
and optically pumped inside a glass scattering chamber.
It is usually necessary to attach thin foils to the body
of the glass chamber in order to allow the entrance and
exit of nuclear particles. There are particular problems
associated with this requirement and they are discussed
later. One of the conditions necessary to achieve optical
pumping is that magnetic field inhomogeneities in the
vicinity of the target not exceed approximately one part
in $10^4$. For this reason the target must be constructed
exclusively of non-magnetic materials. Because even some
commercially supplied brass and aluminum is slightly
magnetic, it is desirable to test even supposedly non-
ferrous metals before using them in the construction of
a target.
The glass scattering chamber consists of a spherical pyrex bulb 5.0 cm in diameter to which glass tubes (Tu 70) with flanged ends are attached; the central scattering angle is largely determined by the positioning of the glass tubes. The precise angular positioning of these tubes during the construction of the chamber is facilitated by the use of the metal jig shown in Fig.15. Metal blocks are mounted in slots machined in a metallic ring. Brass rods with outside diameters very nearly equal to the inside diameter of the glass tubes pass through holes drilled in the centers of the blocks, and are used to define the positions of the glass tubes during the construction of the chamber. These rods may also be used to check the geometry of the chamber after the glass work has been annealed. Fig.15 also illustrates the manner in which collimating slits are formed in the glass work of the chamber. Rectangular ends are machined on the appropriate brass rods and during the process of attaching the glass tubes to the pyrex bulb, the glass is softened and crimped around the brass at the points where these tubes join the body of the chamber. A second collimating slit is placed just in front of the charged particle detectors. (This slit is positioned by the metal foil holders, as shown in Fig.16.) The central scattering
Metal jig used in the construction of the glass scattering chamber.
angle is determined by the positioning of these two slits and measurements have indicated that this angle can be set with an angular accuracy of $\pm 0.6^\circ$. The length of the glass tubes and the dimensions of the slits are chosen such that, for a given laboratory angular resolution, the counting efficiency $E = \Omega \ell$ is maximized. Here $\Omega$ is the solid angle subtended by the second slit and $\ell$ is an estimate of the effective target thickness for scattering into the solid angle $\Omega$. A ball and socket joint and ordinary high vacuum stopcock lubricated with low vapor pressure silicone grease are attached to the chamber so that the chamber can be easily attached to or removed from the vacuum system which is used in the cleaning and filling operations (See Fig.17).

High purity of the $^3$He gas is necessary to achieve optical pumping. The pressure of the $^3$He target gas is typically 2-4 Torr and the partial pressure of impurities should be much less than $10^{-4}$ Torr. The introduction of impurities into the chamber due to leaks at the foils or from the outgassing of the chamber walls is a major problem. The foil mounting method described below has allowed thin foils to be mounted quickly and reliably on the glass scattering chamber and represents a significant improvement over previously used techniques (Ba 69).
Foil holders are made from two aluminum disks 2.86 cm in diameter with 0.63 cm diameter holes drilled through their centers. Foils are mounted between these disks, which are held together by a number of screws. An indium gasket made from flattened 0.064 cm indium wire (Wi 70) provides the vacuum seal. After foils have been mounted in the foil holders and leak tested, the foil holders are then attached to the chamber by other screws, as illustrated in Fig. 16. A second indium gasket made of 0.154 cm wire forms the vacuum seal between the foil holder and flanged ends of the glass tubes. By rigidly mounting the foils between two disks and then attaching these disks to the chamber, failures due to leaks at the foil itself or at the interface between the foil holder and the chamber have been greatly reduced. In most applications, aluminum foils of approximately 2.2 mg/cm$^2$ thickness have been used. For the observation of heavy ions and/or very low energies thinner foils are required. Nickel foils may not be used because they are magnetic, and mylar foils are permeable to helium. Thin mica foils (Fo 70) ($\sim$ 1.0 mg/cm$^2$) can be used, but the mounting of these foils is complicated by the fact that the mica tends to crack as the indium gasket is compressed between the aluminum disks. This problem can be reduced by
The detector chamber, the foil holder, and the aluminum fitting used in attaching the foil holder to the flanged glass tube.
assembling the foil holders on a hot plate set at a temperature just below the melting point of indium (156° C). This softens the indium so that it flows readily as the screws are tightened. It is necessary to make the mica foils opaque so that light does not enter the detectors; this can be accomplished by evaporating a thin aluminum film onto the mica prior to assembly in the foil holders. It is possible by this technique to assemble mica foils as thin as 1.0 mg/cm² which will support a pressure differential of one atmosphere over an aperture of 0.63 cm. Thinner foils could presumably be used if the aperture were reduced, or if provision were made for reducing the pressure differential across the foil. Maintaining a reduced pressure differential across the foil at all times would, however, complicate the cleaning and use of the target.

An aluminum fitting conically tapered to match the flanged end of the glass tube is employed in attaching the foil holders to the glass tube. (Fig. 16) Teflon tape provides a cushion between the aluminum and glass, and epoxy is used to produce a vacuum seal at this point.

Solid state detectors are mounted in aluminum detector chambers which allow the detectors to be operated in vacuum and also shield the detectors from the rf sources
of the optical pumping apparatus. One version of these chambers, in which two transmission type detectors can be mounted for $\Delta E - E$ measurements, is also shown in Fig. 16.

The apparatus which is used to adjust the alignment of the chamber with respect to the beam axis is illustrated in Fig. 17. This figure also shows the manner in which the entrance foil holders are modified in order to allow the chamber to be attached to the alignment apparatus.

The principal limitation of this design is that with a given chamber, scattering may only be observed at certain fixed laboratory angles. However, chambers of this design have the advantage of being far easier to construct than chambers that allow the observation of wide angular regions and as a result have proven to be quite useful. The minimum laboratory scattering angle is limited at present by the size of the tapered aluminum fittings and it would be possible to observe scattering at laboratory angles of less than $25^\circ$ by fabricating smaller fittings. The size of these fittings could be reduced if smaller flanged glass tubes were used. To our knowledge, smaller tubes are not available commercially, but could be manufactured if necessary.
The apparatus used to adjust the alignment of the chamber with respect to the beam axis, and the modified entrance foil holder used to attach the chamber to the alignment apparatus.
4) Cleaning and Filling the Target

The procedure used in cleaning and filling the target follows that of Ref. Co 63 and Ref. Fi 69. The chamber is evacuated and then filled with reagent grade $^4\text{He}$ gas. A bright discharge is produced using 2450 MHz rf power. High voltage tesla coils are used to excite discharges in the glass arms and at the metallic foils. The level of impurities produced in the $^4\text{He}$ gas by the outgassing of the chamber walls is estimated by observing the discharge with a low resolution spectroscope. When the gas has become impure, the chamber is evacuated ($\sim10^{-6}$ Torr) and the procedure is repeated. When a chamber has been thoroughly cleaned only the atomic $^4\text{He}$ lines can be seen in the bright discharge, even after a continuous discharge of twenty minutes or more. With discharges of this length, the glass arms become sufficiently hot to endanger melting the indium gaskets and care must be taken to see that the vacuum seals are not broken in this manner.

After the chamber has been cleaned, $^3\text{He}$ gas at a pressure of approximately 3 Torr is purified by passing it through a liquid helium cold trap and admitted to the chamber through the high vacuum stopcock attached to the chamber.
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Tu 70  Sentinel flanged glass pipe, obtained from the Sentinel Glass Co., Hatboro, Pa.
Wi 70  Obtained from the Indium Corporation of America, Utica, New York.
Acknowledgements

I would like to thank Dr. Stephen D. Baker not only for his guidance and participation throughout the entirety of the work described in this thesis, but also for his contagious enthusiasm, his friendship and his patient explanations, too numerous to mention. I am also sincerely grateful for the assistance of Diana McSherry, R. J. Spiger, T. A. Tombrello and Y. S. Chen, who were coexperimenterers during various parts of the work described in this thesis. My wife, Marilyn, I thank for countless sacrifices cheerfully made in the name of Physics, as well as for her indispen-sable help in the preparation of the text. The experiment described in Section II was conducted at the California Institute of Technology and was supported in part by the National Science Foundation, the Office of Naval Research, and the U. S. Atomic Energy Commission. The financial support of Rice University and of the government, in the form of a National Defense Education Act Fellowship, is also gratefully acknowledged.