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CYLINDRICAL IMPLATION DRIVEN PROJECTILE LAUNCHER

by

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Nomenclature

\( a \) - speed of sound

\( A \) - similarity parameter

\( A_b \) - base area of projectile

\( A_r \) - area as a function of \( r \)

\( B \) - proportionality constant in equation 27

\( C \) - constant of equation 60

\( C_1 \) - constant of equation B12

\( D_f \) - denominator of equation 17

\( e_{int} \) - internal energy per unit mass

\( f \) - nondimensional pressure

\( f_1 \) - similarity variable for pressure

\( m \) - exponent for equation 27

\( m_p \) - mass of the projectile

\( M \) - Mach number

\( N_f \) - numerator of equation 17

\( p \) - pressure

\( P \) - momentum

\( r \) - radial distance from axis of cylinder

\( r_b \) - base radius of projectile

\( R \) - radial distance of shock from the axis of cylinder

\( t \) - time

\( u \) - velocity of gas behind shock wave

\( U_s \) - velocity of shock wave

\( v_p \) - velocity of projectile
w - width of projectile
\gamma - ratio of specific heats
\delta - exponent for power law variation of pressure with shock radius
\phi - nondimensional velocity of gas
\phi_1 - similarity variable for velocity
\rho - density
\psi - nondimensional density
\nu - parameter determining axial or spherical symmetry

Subscripts

c - conditions at the singularity
o - undisturbed conditions
s - conditions directly behind the shock wave
1 - conditions behind the implosion
2 - conditions behind the reflected shock wave
Introduction

A method of producing extremely short duration, 1 usec, light pulses for an interferometric photography study of shock wave structure was required. It was felt that this could be accomplished by using a closed-open-closed electro-mechanical shutter driven by a cylindrical implosion. The speed with which this type shutter moves must be known in order to obtain the exposure time for a given opening.

In order to obtain the velocity of the shutter, the impulse imparted to it must be known. The evaluation of the impulse can be made by analyzing the implosion and reflected shock waves to determine the integral

$$ P = \int p A_r \, dt $$

Cylindrical implosions have been investigated in a quasi-similar manner by B. H. K. Lee\(^{(1, 2)}\) and J. H. Lee\(^{(2)}\). These studies allowed nonsimilarities of either a weak shock wave or heat release at the shock front as in detonation waves. V. F. D'Iachenko and V. S. Imshennik\(^{(3)}\) attacked this problem using a very non-similar type solution considering viscous and thermal dissipation near the origin in a fully ionized plasma. Shock waves moving in a channel of variable cross-section are treated by R. F. Chisnell\(^{(4)}\) for a uniform medium ahead of the shock and by G. B. Whitham\(^{(5)}\) for a nonuniform medium. R. B. Payne\(^{(6)}\) uses a finite difference numerical solution with an artificial diffusion to solve the equations of motion behind the shock wave. As the origin is approached, the diffusion of the shock wave prevents the pressure and shock velocity from becoming infinite although the flow behind the shock
wave becomes self-similar. G. Guderly\(^7\) has made a similarity solution for strong cylindrical and spherical implosions which have no outer boundary. R. F. Flagg\(^8\) has approached the problem of a spherical implosion within a finite boundary from a similarity point of view. However, he did not investigate his similarity parameter to see if it was a constant as required.

In this thesis a similarity type solution is employed to reduce the partial differential equations of motion to ordinary differential equations. For a finite radius enclosure with a finite energy addition with all dissipative effects neglected, the similarity parameter is shown to remain constant as the origin is approached.

The case of a strong cylindrical explosion has been investigated by Shao-Chi Lin\(^9\) as an extension of the work of G. I. Taylor\(^10\) on a spherical blast wave. Both assumed a similarity type solution and showed the similarity parameter to be constant. These studies were conducted assuming the gas medium into which the blast wave was propagating was uniform. This will not be the case in the present study as the explosive phase occurs after reflection of the inbound implosion. Also, with the model of reflection employed here, the reflected shock wave is not assumed strong and the similarity type solution cannot be employed during the explosive phase. Here, for the explosive phase, the shock wave is assumed to propagate into a nonuniform medium at a constant Mach number.

Knowing the pressure through the implosive and explosive phases, the impulse is calculated; this is equal to the momentum imparted to the projectile. This analysis yields a constant of proportionality between the projectile velocity and the square root of the energy input.
mental work by Basil N. Antar\textsuperscript{(14)}, although limited in its applicability, is compared with the velocities predicted by this theory and shows a similar functional form.
**Derivation Of Equations**

In this section, the equations governing the flow behind the implosion are derived. A similarity approach is used to determine the non-dimensional distributions of pressure, density, and velocity behind the shock wave assuming that the Mach number of the shock is large. Using the similarity constant, the time for the shock wave to propagate from one radius to another is derived. A finite enclosure is investigated in which the implosion is initiated by a finite energy addition. The similarity constant, $A$, is shown to be truly a constant as the shock radius approaches zero for the case of a finite energy within the cylinder.
Implosion Phase Equations

This section will evaluate the flow behind a cylindrically imploding shock wave using the assumption that the axis of the implosion is infinite in extent such that there are no end effects. The shock wave is also assumed to be an axially symmetric converging cylindrical shock wave. The gas behind this shock wave is to have no dissociation or ionization while the shock Mach number is large compared to 1. The gas is also assumed to have a constant ratio of specific heats, i.e., to be calorically perfect. All dissipative effects are neglected.

The equations governing this type of flow are then

Continuity Equation

\[ \frac{\partial \rho}{\partial t} + \frac{u \partial \rho}{\partial r} + \rho \left( \frac{\partial u}{\partial r} + \frac{u}{r} \right) = 0 \]

Momentum Equation

\[ \frac{\partial u}{\partial t} + \frac{u \partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 \]

Energy Equation

\[ \left( \frac{\partial}{\partial t} + \frac{u \partial}{\partial r} \right) \frac{p}{\rho^\gamma} = 0 \]

State Equation

\[ e_{\text{int}} = \frac{p}{(\gamma - 1) \rho} \]

The pressure, velocity, and density distributions behind the shock wave are assumed to behave in a similar manner. The values of pressure and velocity, while having a similar distribution behind the shock wave, have a power law dependence on the shock radius. The governing relations are:
(5) \[ \frac{p}{p_o} = \frac{f_1(n)}{\delta R} \]

(6) \[ u = \frac{\phi_1(n)}{\delta R/2} \]

(7) \[ \frac{\rho}{\rho_o} = \psi(n) \]

where \( \eta = r/R \geq 1 \)

Using these similarity relations and noting that \( r \) is independent of time while \( R \) is a function of time, the continuity, momentum and energy equations, (1), (2), and (3), may be rewritten by use of the chain rule

\[ \frac{\partial}{\partial r} = \frac{1}{R} \frac{\partial}{\partial \eta} \]

\[ \frac{\partial}{\partial t} = -\frac{n}{R} \frac{dR}{dt} \frac{\partial}{\partial \eta} \]

(8) \[ R^{\delta/2} \frac{dR}{dt} = \frac{\phi_1}{\eta} + \frac{\psi}{\psi} \left( \frac{\phi_1^*}{\eta} + \frac{\psi^*}{\eta} \right) \]

(9) \[ R^{\delta/2} \frac{dR}{dt} = \frac{\phi_1 \phi_1^* + (p_0 f_1^*)/(\rho_0 \psi)}{\phi_1^* \eta + \phi_1 \delta/2} \]

(10) \[ R^{\delta/2} \frac{dR}{dt} = \frac{\phi_1 (f_1^* - \gamma f_1 \psi^*/\psi)}{\eta f_1^* - \eta f_1 \psi^*/\psi + \delta f_1} \]

where the primes denote differentiation with respect to \( \eta \).

The assumption of similarity for \( f_1, \phi_1, \) and \( \psi \) can hold in equations (8), (9), and (10) if and only if the left hand side of these equations is a constant. That is, if

(11) \[ R^{\delta/2} \frac{dR}{dt} = A \]

Using this relation, \( f_1 \) and \( \phi_1 \) can be nondimensionalized in the following manner

(12) \[ f(\eta) = \frac{f_1(n)a^2}{A^2} \]
\[ \phi(\eta) = \frac{\phi_1(\eta)}{A} \]

These two relations are now utilized to nondimensionalize equations (8), (9), and (10) yielding

\[ \frac{\psi'}{\psi} (\eta - \phi) = \phi' + \frac{\phi}{\eta} \]

\[ \phi' (\eta - \phi) + \frac{\phi \delta}{2} = \frac{f'}{\gamma \psi} \]

\[ \frac{\gamma f' \psi'}{\psi} (\eta - \phi) = f'(\eta - \phi) + \delta f \]

R. F. Flagg \(^{(8)}\) derived a similar set of equations using a symmetry parameter, \(\nu\), which was 2 for axial symmetry and 3 for spherical symmetry. His equations reduce to (14), (15), and (16) for the case of axial symmetry.

Equations (14), (15), and (16) may be solved simultaneously to obtain \(f', \phi', \text{ and } \psi'\). This gives three non-linear first order differential equations which can be solved numerically once the boundary conditions are known. From equation (14)

\[ \phi' = \frac{\psi'}{\psi} (\eta - \phi) - \frac{\phi}{\eta} \]

Inserting this in equation (15) gives

\[ \frac{\psi'}{\psi} = \frac{f'}{\gamma \psi} - \frac{\phi \delta}{2} + \frac{\phi}{\eta} (\eta - \phi) \]

\[ (\eta - \phi)^2 \]

Using the above in equation (16) yields

\[ f' = \frac{f[-\delta \eta + \phi(\delta - \gamma \delta/2 + \gamma) - \gamma \phi^2/\eta]}{(\eta - \phi)^2 - f'/\psi} \]

\(f'\) is a function only of the nondimensional variables \(f, \phi, \text{ and } \psi\), and does not depend on their derivatives. For any point at which \(f, \phi, \text{ and } \psi\) are known, \(f'\) can be calculated and used as a known quantity for
calculating \( \phi' \) and \( \psi' \). The relations for these two quantities are

\[
(18) \quad \phi' = \frac{\frac{\phi'}{\gamma} - \frac{\phi}{\gamma} \delta}{\frac{\phi}{\gamma} - \frac{\phi'}{\gamma} - \frac{\phi}{\gamma} + \frac{\phi}{\gamma}}
\]

\[
(19) \quad \psi' = \frac{\psi(\phi + \phi'/\gamma)}{\frac{\phi}{\gamma} - \phi + \frac{\phi}{\gamma}}
\]

These equations are solved numerically by a Runge-Kutta technique for which the computer program is given in Appendix A. The initial conditions for the solution are obtained from the Rankine-Hugoniot equations as the Mach number becomes large. These boundary conditions are

\[
(20) \quad f(1) = \frac{2\gamma}{\gamma + 1}
\]

\[
(21) \quad \phi(1) = \frac{2}{\gamma + 1}
\]

\[
(22) \quad \psi(1) = \frac{\gamma + 1}{\gamma - 1}
\]

It should be noted that there is a singularity in equation (17). This equation is required to be regular at the singularity, i.e., the numerator must go to zero at the same instant the denominator reaches zero. This produces the correct value for \( \delta \), since it is the only factor not determined from the denominator. Evaluation at the singularity yields:

\[
D_f = (\eta_c - \phi_c)^2 - f_c/\psi_c = 0
\]

\[
(23) \quad \eta_c = \phi_c \pm \sqrt{f_c/\psi_c}
\]

It is seen that only the positive sign is of significance as \( \eta \geq 1 \) and \( \phi \leq 1 \) at all times and positions behind the shock wave. This gives

\[
(24) \quad \eta_c = \phi_c + \sqrt{f_c/\psi_c}
\]

Evaluating the numerator gives

\[
N_f = f[-\delta \eta + \phi(\delta + \gamma - \gamma \delta/2) - \gamma \phi^2/\eta] = 0
\]
\[ \delta = \frac{2\gamma \phi_c (\phi_c - \eta_c)}{\eta_c (2\phi_c - 2\eta_c - \gamma \phi_c)} \]

Referring to Flagg's study, he requires the solution to be regular by solving for \( \eta \) which makes the numerator zero. Flagg solves equation (25), which is quadratic in \( \eta \), for \( \eta \) and obtains an upper bound on \( \delta \) by requiring the two zeroes to coincide. It is then pointed out that the upper bound for the spherical case is exactly twice that for the cylindrical case, but it is not shown to hold for the actual \( \delta \). Here this result is derived for the actual \( \delta \) which is to be used. From Flagg:
\[ f(-\delta \eta + \phi[\delta + (\nu - 1)\gamma - \gamma \delta/2] - (\nu - 1)\gamma \phi^2/\eta) = 0 \]
which, when solved for \( \delta \), gives
\[ \delta = \frac{2\gamma \phi_c (\nu - 1)(\phi_c - \eta_c)}{\eta_c (2\phi_c - 2\eta_c - \gamma \phi_c)} \]

For spherical symmetry, the factor \( \nu - 1 \) is twice that for axial symmetry showing that \( \delta \) for a cylindrical implosion is one-half that for a spherical implosion. Flagg also produced a relation for \( \delta \) of a spherical implosion by curve fitting. This relation gives \( \delta \) within 1% accuracy in its region of applicability. Using Flagg's equation for \( \delta \) given below, a starting value for \( \delta \) in the cylindrical case is obtained by multiplying by one half. Flagg's equation for \( \delta \) is:
\[ \delta = B(\gamma - 1)^m \]
where \( B \) and \( m \) are constants with the following values over the given ranges of \( \gamma \):

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( B )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 to 1.2</td>
<td>1.080</td>
<td>0.323</td>
</tr>
<tr>
<td>1.2 to 1.6</td>
<td>1.035</td>
<td>0.296</td>
</tr>
<tr>
<td>1.6 to 2.0</td>
<td>1.000</td>
<td>0.234</td>
</tr>
</tbody>
</table>
The value of \( \delta \) obtained after alteration to a cylindrical implosion is then used in the numerical solution for \( f \), \( \phi \), and \( \psi \). This value of \( \delta \) is checked against that found by equation (25) at the singularity, and if it is sufficiently accurate, the solution is continued, otherwise \( \delta \) is replaced by that found using equation (25) and the procedure is repeated.

With \( f' \) regular at the singularity, it can be evaluated by applying L' Hospital's Rule at this point. This gives

\[
\lim_{\eta \to \eta_c} \frac{f'}{f} = \lim_{\eta \to \eta_c} \frac{d}{d\eta} \frac{\{f[-\delta \eta + \phi(\delta + \gamma - \gamma \delta/2) - \gamma \psi^2/\eta]\}}{\frac{d}{d\eta} \{-(\eta - \phi)^2 - f/\psi\}}
\]

\[
f' \bigg|_{\eta_c} = \lim_{\eta \to \eta_c} \{ -\psi[-\delta \eta + \phi(\delta + \gamma - \gamma \delta/2) - \gamma \psi^2/\eta]\}
\]

taking the limit produces

(28) \( f' \bigg|_{\eta_c} = 0 \)

Equation (28) allows the numerical solution to proceed past the singularity occurring at \( \eta_c \). If the solution is continued to large values of \( \eta \) and is plotted on log-log paper, it is seen that the values of \( f \), \( \phi \), and \( \psi \) asymptotically approach straight lines. The equations for these asymptotes are found to be

(29) \( f(\eta) \to f_\infty \eta^{-\delta} \)

(30) \( \phi(\eta) \to \phi_\infty \eta^{-\delta/2} \)

(31) \( \psi(\eta) \to \psi_\infty \)

The solutions for \( f \), \( \phi \), and \( \psi \) for values of \( \gamma \) of 1.12, 1.4, and 2.0 are plotted in figures 1, 2, and 3 respectively. The asymptotes are clearly visible in these figures.

With an approach similar to that of G. I. Taylor\(^{(10)}\), the time for
the implosion to propagate from one radius to another can be calculated. This method involves the similarity constant, \( A \), which is utilized in the following manner. From equation (11), it is known

\[
A = \frac{dR}{dt} R^{\delta / 2}
\]

\[
R^{\delta / 2} dR = A \ dt
\]

\[
\int_{R_0}^{R} R^{\delta / 2} dR = \int_{0}^{t} A \ dt
\]

(32)

\[
t = \frac{2 + \delta}{2} \frac{R^2}{A} - \frac{2 + \delta}{2} \frac{R_0^2}{A}
\]

The similarity constant also gives a simple relation for the pressure, velocity, and density when \( f, \phi, \) and \( \psi \) are dimensionalyzed. These functions are

\[
p = f p_0 \frac{u^2}{a_0^2}
\]

\[
U_s = \frac{dR}{dt} = \frac{A}{R^{\delta / 2}}
\]

(33)

\[
p = f \frac{p_0 A^2}{a_0^2 R^\delta}
\]

\[
u = \phi U_s
\]

(34)

\[
u = \phi A/R^{\delta / 2}
\]

(35) \( \rho = \psi p_0 \)

Equations (33), (34), and (35) give \( p, u, \) and \( \rho \) behind the shock wave as a function of \( \eta \) for a given shock radius, \( R \).

For an implosion generated in a cylinder of finite radius and length by the explosion of a foil liner after an instantaneous energy discharge,
the analysis presented will hold if the similarity constant and the energy contained within the cylinder can be shown to remain constant as the implosion approaches the axis. The energy between the shock wave and the outer wall is the sum of the internal energy of the gas and its kinetic energy. This is given by

\[ E_{\text{tot}} = 2\pi \nu \int_{R}^{R_0} \left[ \frac{p}{\gamma - 1} + \frac{\nu u^2}{2} \right] r \, dr \]

\[ = 2 \pi \nu R^{2-\delta} \rho_0 A^2 \int_{1}^{\eta_0} \left[ \frac{f}{\gamma (\gamma - 1)} + \frac{\psi \phi^2}{2} \right] n \, dn \]

(36)

\[ A^2 = \frac{E_{\text{tot}}}{2 \pi R^{2-\delta} \nu_0} \int_{1}^{\eta_0} \left[ \frac{f}{\gamma (\gamma - 1)} + \frac{\psi \phi^2}{2} \right] n \, dn \]

where \( E_{\text{tot}} \) is the energy transferred to the gas.

If the energy discharged in initiating the shock wave is much larger than the energy of the gas inside the cylinder, \( E_{\text{tot}} \) will be this discharge. The energy addition will be a constant within the cylinder as there is no other energy addition and all dissipative effects are assumed negligible. \( A \) can be shown to be a constant if the denominator of equation (36) is a constant as \( R \) approaches zero. As \( R \) goes to zero, \( \eta_0 \) will become larger and larger eventually approaching infinity. The tail of the integral will become dominate in this region, and the functions \( f, \phi, \) and \( \psi \) may be replaced by their asymptotes such that

\[ A^2 = \lim_{R \to 0} \frac{E_{\text{tot}}}{2 \pi R^{2-\delta} \nu_0} \int_{1}^{\eta_0} \frac{\nu \phi^2}{2} \left[ \frac{f}{\gamma (\gamma - 1)} + \frac{\psi \phi^2}{2} \right] n \, dn \]

\[ = \lim_{R \to 0} \frac{E_{\text{tot}}}{2 \pi R^{2-\delta} \nu_0} \left[ \frac{f}{\gamma (\gamma - 1)} + \frac{\psi \phi^2}{2} \right] \left[ \frac{\eta_0^{2-\delta}}{2-\delta} \right] \]

Replacing \( \eta \) by \( r/R \) gives
\[ A^2 = \frac{E_{\text{tot}}(2 - \delta)}{2 \pi \rho_0} w \left[ \frac{f_\infty}{\gamma (\gamma - 1)} + \frac{\psi_\infty \phi_\infty^2}{2} \right] R_0^{2-\delta} \]

(37)

\[ A = \pm \left( \frac{E_{\text{tot}}(2 - \delta)}{2 \pi \rho_0} w \left[ \frac{f_\infty}{\gamma (\gamma - 1)} + \frac{\psi_\infty \phi_\infty^2}{2} \right] R_0^{2-\delta} \right)^{1/2} \]

Examination of equation (11), and noting that \( dR/dt \) must be negative as the implosion propagates toward the axis shows that only the negative sign is valid which produces:

(38)

\[ A = - \left( \frac{E_{\text{tot}}(2 - \delta)}{2 \pi \rho_0} w \left[ \frac{f_\infty}{\gamma (\gamma - 1)} + \frac{\psi_\infty \phi_\infty^2}{2} \right] R_0^{2-\delta} \right)^{1/2} \]

This shows that \( A \) is indeed a constant for finite energy addition as the implosion approaches the axis of the cylinder.
The Nature of the Reflection at the Axis of the Cylinder

The reflection of a cylindrical implosion can be explained relatively simply. As the implosion approaches the axis, it becomes increasingly strong such that at the origin, in the classical case, it will become infinitely strong. The interaction of an infinitely strong cylindrical shock wave is viewed in this study as in figure 4. When taken over an infinitesimal angle dθ, the curved shock wave can be approximated by a plane shock wave. Directly opposite the element dθ there is another element with exactly the same properties due to the axial symmetry. As these two elements approach each other radially, they increase in the same manner. Their separation will diminish until at the axis they meet. This view presents the cylindrical reflection over an infinitesimal element as the reflection of two infinitely strong shock waves which has been investigated at great length.

The pressure rise across the reflected shock wave due to the interaction of these two strong plane shocks is given by

\[ \frac{p_2}{p_1} = \frac{3\gamma - 1}{\gamma - 1} \]  

(39)

The pressure rise across the reflected wave is seen to be greater than or equal to three for any value of \( \gamma \). For values of \( \gamma \) of interest, this type of reflection produces a weak shock wave on the outbound or "explosive" phase. The Mach number of this wave can then be calculated from the weak shock relations and is:

\[ \frac{p_2}{p_1} = \frac{2\gamma}{\gamma + 1} \frac{M_2^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1} \]
(40) \[ M_2 = \left[ \frac{2\gamma}{\gamma - 1} \right]^{1/2} \]

The speed with which the shock wave will propagate relative to the axis of the cylinder can be obtained from the product of the Mach number of the shock wave relative to the gas and the speed of sound behind the implosion and taking into account the speed of the gas relative to the axis. The speed of sound behind the implosion is related to that of the stationary gas by

(41) \[ \frac{a_1}{a_0} = \left[ \frac{T_1}{T_0} \right]^{1/2} \]

The temperature ratio is found from the Rankine-Hugoniot relations, and for high Mach numbers is

(42) \[ \frac{T_1}{T_0} = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} M_{\text{imp}}^2 \]

Using equation (42) in (41) yields

(43) \[ a_1 = \left[ \frac{2\gamma(\gamma - 1)}{\gamma + 1} \right]^{1/2} \frac{|A|}{\delta/2} \]

The speed of propagation of the shock wave is

\[ U_{\text{exp}} = M_2 a_1 + u_1 \]

(44) \[ U_{\text{exp}} = \frac{2(1 - \gamma)}{\gamma + 1} \frac{A}{\delta/2} \]

The density and velocity of the gas behind the exploding shock wave can be obtained from the collision of two infinitely strong shock waves in a similar manner.
Momentum Transfer to a Projectile

In order to launch a projectile using a cylindrical implosion, only one-half of a cylinder can be used. The geometry employed is shown in figure 5. The analysis is the same, except the energy used to produce the implosion must be multiplied by a factor of two due to its input into one half the volume of a cylinder.

In this analysis, it is assumed that the projectile is in contact with the diaphragm used to contain the gas within the cylinder for the entire first cycle of the shock wave, i.e., the implosive phase from the cylinder wall and the explosive phase to the cylinder wall. During the explosive phase, the shock wave will be considered to have a constant Mach number and propagate into a medium whose properties are given by those directly behind the imploding shock wave at that radius.

With these simplifying assumptions, the momentum transfer to the projectile can be obtained. It is calculated by taking the time integral of the pressure multiplied by the area, and is given by the expression

\[(45) \quad P = \int pA_\tau \, dt\]

where both the pressure and the area are functions of time and \( P \) is the momentum. The total momentum transfer will contain contributions from both the implosive and explosive phases. The implosive phase momentum transfer will arise from the high pressure behind the imploding shock wave as it moves over the base area of the projectile as shown in figure 6a. The pressure and area are seen to be functions of the radial distance. The expression for the pressure is given by equation (33), and the expression for the area is
\[ A_{\text{imp}} = A_b (1 - r/r_b) \]

where \( A_b = 2w r_b \). The time can be converted to a function of the radial distance by employing equation (11) such that one obtains

\[ dt = \frac{r^{\delta/2}}{A} \, dr \]

Using equations (33), (46), and (47) in equation (45) and noting that \( f(1) \) is given by equation (20) gives the implosive momentum transfer as

\[ P_{\text{imp}} = \int_{r_b}^{0} \frac{2 \gamma p_o A^2}{(\gamma + 1)a_o^2 r^5} A_b (1 - r/r_b) \frac{r^{\delta/2}}{A} \, dr \]

\[ P_{\text{imp}} = - \frac{8 \gamma A a_o r_b^{1-\delta/2}}{(\gamma + 1)(4 - \delta)(2 - \delta)} \]

The momentum transfer from the explosive phase has three components. Two of these components arise from the time integral of the pressure multiplied by the area as shown in figure 6b. The two components of this integral arise from the pressure behind the reflected shock wave and that in front of the shock wave that remains from the implosion. The third component is from the integral of the pressure over the area of the projectile multiplied by the time for the exploding wave to propagate from the edge of the shutter to the cylinder wall.

The first component, that behind the exploding shock wave as it propagates to the edge of the projectile, is obtained in a similar manner to that for the imploding momentum transfer. The pressure is given by the product of equations (33) and (39) and as a function of \( r \) is given by

\[ P_2 = \frac{(3 \gamma - 1)}{(\gamma - 1)} \frac{2 \gamma p_o A^2}{(\gamma + 1)a_o^2 r^\delta} \]

The area as a function of \( r \) for this component is given by
\( A_{\text{exp}}(r) = A_b \frac{r}{r_b} \)

The incremental time, \( dt \), is found by dividing an incremental distance, \( dr \), by equation (44) for the speed of the shock wave relative to the axis and is

\[ dt = \frac{(\gamma + 1)r^\delta/2}{2(1 - \gamma)A} \]

The momentum transfer becomes

\[ P_{\text{exp}} = \int_0^{r_b} \frac{(3\gamma - 1)}{(\gamma - 1)} \frac{2\rho_o A^2 A_b}{(\gamma + 1)r^\delta} \frac{r (\gamma + 1)r^\delta/2}{2(1 - \gamma)A} \, dr \]

\[ P_{\text{exp}} = -\frac{2(3\gamma - 1)}{(\gamma - 1)^2} \frac{\rho_o A A_b r_b^{1-\delta/2}}{(4 - \delta)} \]

The second component of the explosive phase momentum transfer is obtained from equations (33), (51) and the following equation for the area

\[ A_{\text{exp}}(r) = A_b (1 - r/r_b) \]

The momentum transfer calculated using this is

\[ P_{\text{exp}} = \int_0^{r_b} \frac{2\gamma \rho_o A^2}{(\gamma + 1)r^\delta} A_b (1 - r/r_b) \frac{(\gamma + 1)r^\delta/2}{2(1 - \gamma)A} \, dr \]

\[ P_{\text{exp}} = -\frac{4\rho_o A A_b r_b^{1-\delta/2}}{(\gamma - 1)(2 - \delta)(4 - \delta)} \]

The third contribution to the momentum transfer is obtained from integration of the pressure behind the exploding shock wave over the area of the base of the projectile and multiplication of this result by the time of propagation to the wall from the edge of the projectile. The pressure relation is given by the product of equations (33) and (39) and the incremental area, \( dA \), is given by
\begin{equation}
\frac{\partial A}{\partial r} = \frac{A_b}{r_b} \, dr
\end{equation}

The integral of the pressure over the area is seen to be

\begin{equation}
\int_0^{r_b} p_2 dA = \int_0^{r_b} \frac{(3\gamma - 1)}{(\gamma - 1)} \frac{2\gamma p_o A^2}{(\gamma + 1) a_o^2 r^\delta} \frac{A_b}{r_b} \, dr
\end{equation}

\begin{equation}
= \frac{2(3\gamma - 1)}{(\gamma - 1)(\gamma + 1)} \frac{\rho_o A^2 A_b}{(1 - \delta)} \frac{r_\delta}{r_b}
\end{equation}

The time is calculated from the integral of \( dt \) and is

\begin{equation}
\int_{t_b}^{t_o} dt = \int_{r_b}^{R_o} \frac{r^{\delta/2}}{2(1 - \gamma)A} \, dr
\end{equation}

\begin{equation}
t_o - t_b = \frac{(\gamma + 1)(R_o^{1+\delta/2} - r_b^{1+\delta/2})}{2(1 - \gamma)A(1 + \delta/2)}
\end{equation}

The momentum contribution is calculated to be

\begin{equation}
P_{\text{exp}_3} = \int_0^{r_b} p dA \int_{t_b}^{t_o} dt
\end{equation}

\begin{equation}
P_{\text{exp}_3} = -\frac{2(3\gamma - 1)}{(\gamma - 1)^2} \frac{\rho_o A A_b}{(1 - \delta)(2 + \delta)} \frac{R_o^{1+\delta/2} - r_b^{1+\delta/2}}{r_b^\delta}
\end{equation}

The total momentum transferred to the projectile can be calculated by summing the individual components from the implosive and explosive phases. These components are given by equations (48), (52), (54), and (58). This total gives the total momentum transferred to the projectile

\begin{equation}
P_{\text{tot}} = P_{\text{imp}} + P_{\text{exp}_1} + P_{\text{exp}_2} + P_{\text{exp}_3}
\end{equation}

\begin{equation}
P_{\text{tot}} = \rho_o A A_b \left[ -\frac{8r_b^{1-\delta/2}}{(\gamma + 1)(4 - \delta)(2 - \delta)} - \frac{2(3\gamma - 1)}{(\gamma - 1)^2} \frac{r_b^{1-\delta/2}}{(4 - \delta)} \right]
\end{equation}
From equation (38), it is seen that \( A \) is directly proportional to the square root of the energy addition. The constant of proportionality is a function of the ratio of specific heats, the density of the gas initially in the chamber, and the dimensions of the chamber. Knowing this and noting that \( A \) is the only variable in equation (59), the momentum transfer can be rewritten as

\[
\text{(60) } P = CE_{\text{tot}}^{1/2}
\]

Equation (60) gives the momentum transferred to the projectile by the cylindrical implosion and its reflection at the axis of symmetry. The momentum of the projectile is given by

\[
\text{(61) } P_p = m_p v_p
\]

By use of equations (60) and (61), the velocity of the projectile can be calculated as a function of the energy addition. This gives

\[
\text{(62) } v_p = \frac{CE_{\text{tot}}^{1/2}}{m_p}
\]

Thus, the velocity of a projectile launched by a cylindrical implosion is directly proportional to the square root of the energy addition used to initiate the implosion. In the event that a capacitor bank is used as the energy source to initiate the imploding shock wave, the velocity of the projectile will be directly proportional to the voltage on the capacitor if the capacitance is held constant.
Summary And Results

This analysis assumed a similarity profile for pressure, velocity and density behind an imploding shock wave. These profiles are plotted in a nondimensional form as a function of the nondimensional distance from the shock wave on log-log paper in figures 1, 2, and 3 for various values of $\gamma$. These curves are seen to approach the asymptotes given by equations (29), (30), and (31).

The energy contained within the gas between the shock wave and the outer wall is shown to be a constant, the energy input, as the outer wall becomes infinitely far from the shock wave in nondimensional coordinates. This is markedly different from that of G. Guderly\(^7\), but it must be recalled that he assumed no outer boundary, thus, producing infinite energy.

The reflection at the axis of the cylinder is viewed over an infinitesimal angle, $d\theta$, as the interaction of two infinitely strong plane shock waves. After reflection, the shock wave is assumed to propagate at a constant Mach number to the cylinder wall. The conditions ahead of the reflected shock are nonuniform and given by the conditions behind the implosion at that radius.

The momentum transfer found using this analysis gives a constant of proportionality between the velocity at which a projectile can be propelled and the energy input to the gas within the launcher. If the kinetic energy of the projectile is found, there will be direct proportionality between it and the input energy. The proportionality constant between kinetic energy and input energy can be viewed as an efficiency for energy transfer
by this type of device.

For the launcher of figure 5, the velocity with which an 80 mg projectile may be expelled is plotted in figures 7 and 8 as a function of voltage and energy respectively. Figure 7 assumes an energy source composed of a 15 μf capacitor bank charged to the voltage as shown.

There were no experiments conducted to determine the accuracy of this model of an implosion within a cylinder and its subsequent reflection at the axis. Thus, no evaluation of this model is possible. Some experimental data was obtained from Basil N. Antar\(^{(11)}\) for projectile velocities obtained from a launcher of the configuration shown in figure 5. This data is also plotted in figures 7 and 8. This data is of questionable applicability since the chamber is far from being infinitely long and a wire was used to initiate the implosion rather than a foil liner. A functional agreement is seen, however, between theory and experiment.
Proposed Future Studies

Future experimental work is needed to determine the validity of this analysis of a cylindrical implosion. It is hoped that an implosion chamber more nearly approximating an infinitely long cylinder can be produced, along with a means of initiating a symmetric implosion. Instrumentation of this chamber should include pressure transducers capable of reacting quickly enough to determine pressure-time relations within the chamber. Also desired is a method of determining contact time of the projectile with the diaphragm.

A further theoretical investigation should include the effect of a real gas inside the chamber limiting the pressure rise as the origin is approached. Also of interest for continued investigation is the propagation of the reflected wave into a non-uniform medium as this reflected shock wave is decreasing in intensity.

These considerations will give a more complete understanding of the problem investigated here, and the experimental work will show the validity of the assumptions made.
$f, \phi, \psi$ vs $\eta$

Figure 1

\[ \gamma = 1.12 \]
\[ \delta = 0.27508457 \]
\[ f_* = 2.4703560 \]
\[ \phi_* = 0.78000132 \]
\[ \psi_* = 62.652023 \]
Figure 2

\[ f, \phi, \psi \text{ vs } \eta \]

\[ \gamma = 1.4 \]
\[ \delta = 0.39422835 \]
\[ f_\infty = 2.0543514 \]
\[ \phi_\infty = 0.67115372 \]
\[ \psi_\infty = 12.875206 \]
Figure 3
Figure 4
Figure 6
Velocity vs Voltage

Energy source - 15μf capacitor bank

○ - 4.5 mil wire
△ - 8.0 mil wire

Figure 7
Velocity vs Energy

Energy source – 15μf capacitor bank

○ - 4.5 mil wire
△ - 8.0 mil wire

Figure 8
Bibliography


Appendix A

This section describes the computer program utilized to solve the simultaneous differential equations for f, \( \phi \), and \( \psi \). A ratio of specific heats, \( \gamma \), is assumed and an initial value of \( \delta \) is calculated using equation (27). The initial conditions on f, \( \phi \), and \( \psi \) are calculated by equations (20), (21) and (22) respectively.

With the value of \( \delta \) obtained above and the initial values for f, \( \phi \), and \( \psi \), the differential equations may be numerically integrated forward. This forward type integration is well suited to a Runge-Kutta type solution, therefore, this method of solution is used. The numerical solution is initiated, and at each point the denominator is inspected to see if the singularity has been reached. The integration is continued until the singularity is encountered. At this point the value of \( \delta \) being used is compared to that calculated at the singularity. If the two values of \( \delta \) are within 0.05\%, the solution is continued. If, however, the values of \( \delta \) are not within this accuracy, \( \delta \) is replaced by that calculated at the singularity, and the solution is begun again with the new value of \( \delta \). This procedure is repeated until \( \delta \) is sufficiently accurate.

The solution is terminated when \( \eta \) obtains the value of 100. At this point, the values for the constants in the equations for the asymptotes are calculated. The constants, along with the values for \( \gamma \) and \( \delta \) and the dimensions of the implosion chamber are then used to calculate the proportionality constant C in equation (60) relating input energy and projectile momentum. For a given projectile mass, the velocity is then calculated for input energies from 0 to 750 joules. The program terminates
after the velocity and energy are output.

The program is written in Algol to allow variable names to accurately denote the variables they represent. The program was executed on Rice University's Burrough's B-5500 computer system.

A flow chart is included to show the program flow. A listing may be obtained upon request.
Begin

Choose $\gamma$

Initialize $\delta$

Increment $\delta$

Is $\delta$ sufficiently accurate?

Yes $\Rightarrow$ Continue integration

$\eta \leq 100$?

Yes $\Rightarrow$ Continue integration

No $\Rightarrow$ Initialize $f, \phi, \psi$

Has $\delta$ been checked and found to be sufficiently accurate?

Yes $\Rightarrow$ Increment $\delta$

No $\Rightarrow$ Has singularity been passed?

Yes $\Rightarrow$ Continue integration

No $\Rightarrow$ Initialize $f, \phi, \psi$
Calculate $f_\infty$, $\phi_\infty$, $\psi_\infty$

Calculate $C$

Calculate $C/m$

DO 750 Volt = 0 → 10000, 750

Calculate $E$

Calculate $V$

Output Volt, $E$, $V$

End
Appendix B

For the case of the spherical implosion, only the continuity equation is altered. It takes the form

\[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \left( \frac{\partial u}{\partial r} + \frac{2u}{r} \right) = 0 \]

This equation, along with the momentum, energy, and state equations in the text, defines the flow behind the imploding shock wave. Nondimensionalizing as before gives

\[ \frac{\psi'}{\psi} \left( \eta - \phi \right) = \phi' + \frac{2\phi}{\eta} \]

The momentum and energy equations retain their nondimensional form as before. Solution for \( f' \), \( \phi' \), and \( \psi' \) yields

\[ f' = \frac{f[-\delta \eta + \phi(\delta + 2\gamma - \gamma \delta/2) - 2\gamma \phi^2/\eta]}{(\eta - \phi)^2 - f/\psi} \]

\[ \phi' = \frac{f'}{\psi} - \frac{\phi \delta}{\eta - \phi} \]

\[ \psi' = \frac{\psi(\phi' + \phi/\eta)}{\eta - \phi} \]

These three simultaneous, nonlinear ordinary differential equations may now be solved numerically up to and past the singularity in (B3) by requiring the solution to be regular at the singularity. This is satisfied when \( \delta \) has the value

\[ \delta = \frac{4\gamma \phi_c (\phi_c - \eta_c)}{\eta_c \left( 2\phi_c - 2\eta_c - \gamma \phi_c \right)} \]

The similarity constant, \( A \), can be shown once again to remain finite at the origin by integrating the energy, both kinetic and internal, between the shock front and the spherical wall.
\[ F_{\text{tot}} = 4\pi \int_R^{R^0} \left( \frac{p}{\gamma - 1} + \frac{\rho u^2}{2} \right) r^2 dr \]

\[ = 4\pi R^{3-\delta} \rho_o A^2 \int_1^{\eta^0} \left[ \frac{f}{\gamma(\gamma - 1)} + \frac{\psi^2}{2} \right] \eta^2 d\eta \]

(B7)

\[ A^2 = \frac{E_{\text{tot}}}{4\pi R^{3-\delta} \rho_o} \int_1^{\eta^0} \left[ \frac{f}{\gamma(\gamma - 1)} + \frac{\psi^2}{2} \right] \eta^2 d\eta \]

This must hold as the radius of the shock wave approaches zero. At this point, the functions \( f, \phi, \) and \( \psi \) can be replaced by their asymptotes giving

\[ A^2 = \lim_{R \to 0} \left\{ \frac{E_{\text{tot}}}{4\pi R^{3-\delta} \rho_o} \int_1^{\eta^{\infty}} \left[ \frac{f}{\gamma(\gamma - 1)} + \frac{\psi^2}{2} \right] \eta^{2-\delta} d\eta \right\} \]

carrying out the integration and replacing \( \eta \) by \( r/R \) yields

\[ A = -\left\{ \frac{E_{\text{tot}} (3-\delta)}{4\pi R^{3-\delta} \rho_o} \left[ \frac{f}{\gamma(\gamma - 1)} + \frac{\psi^2}{2} \right] \right\}^{1/2} \]

The reflection at the origin for this case can be viewed in a similar manner to that of the cylindrical case. The spherical surface can be approximated by a plane over an infinitesimal solid angle \( d\omega \). This gives the interaction of two infinitely strong plane shock waves at the origin as in the cylindrical case, and the relations obtained before are repeated.

The momentum transfer to a projectile located at the center of a hemispherical launcher is calculated using the four associated components which are found to be

(B8)

\[ P_{\text{imp}} = -\frac{16\rho_o A A_b r_b^{1-\delta/2}}{(\gamma + 1)(2 - \delta)(6 - \delta)} \]

(B9)

\[ P_{\text{exp}} = -\frac{2(3\gamma - 1) \rho_o A A_b r_b^{1-\delta/2}}{(\gamma - 1)^2 (6 - \delta)} \]
\[ P_{\text{exp}2} = - \frac{8 \rho_0 A_b r_b^{1-\delta/2}}{(\gamma - 1)(2 - \delta)(6 - \delta)} \]
\[ P_{\text{exp}3} = - \frac{4(3\gamma - 1)}{(\gamma - 1)^2} \frac{\rho_0 A_b}{(2 - \delta)(2 + \delta)} \frac{(R_o^{1+\delta/2} - r_b^{1+\delta/2})}{r_b^{\delta}} \]

where \( A_b = \pi r_b^2 \). The total momentum transfer is the sum of these components. This sum yields

\[ P_{\text{tot}} = C_{1} E_1 \]

From this, the velocity is seen to be

\[ v_p = \frac{C_1 E_1^{1/2}}{m_p} \]

Comparison of the projectile velocities for cylindrical and spherical implosions in which the base areas of the projectiles and the energy per unit volume of the driving chambers are equal shows that the velocity obtained with the spherical chamber is approximately 30 times that obtained with a cylindrical chamber.