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REINFORCED CONCRETE CANTILEVER BEAMS
UNDER SLOW CYCLIC LOADINGS

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

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Thesis Director's signature:

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CHAPTER 1 - INTRODUCTION

1.1 Object and Scope

The purpose of the work reported herein was to determine the effect of load history on the strength and ductility of reinforced concrete cantilever beams. The research consisted of an experimental program in which 20 specimens were tested followed by the development of a method to predict the response of the test specimens.

The experimental program consisted of two monotonic loading tests, four tests with cyclic loading in one direction, and fourteen tests with cyclic loading in two directions. Measurements were taken at various locations on the test specimens to determine the distribution of curvature along the beam and the deformations of the reinforcement in the anchorage zone, as well as the load, end deflection, and end rotation. The influence of the following factors on the response of the specimens under a prescribed load history was studied: stirrup spacing, percentage of top and bottom reinforcement, and moment-shear ratio.

Using the results of the tests and the behavior of the component materials reported by other researchers, a method was developed to predict the behavior of the specimens as a function of the load history.

1.2 Previous Studies

In order to rationally design a reinforced concrete structure which may be subjected to earthquake motion, or to other cyclic load variations, the effect of the induced loading history must be understood. If the moment-rotation behavior can be established as a function of the load history, the redistribution of moments within the structure with cycling of load can be analysed. Previous related research is of two basic types, tests of the component materials and tests of structural members.
1.2.1 Materials Studies

Basic studies of plain concrete and reinforcing steel have been performed under cyclic loading. Singh, Gerstle, and Tulin\textsuperscript{12} performed concentric cyclic compressive loading tests of plain concrete cylinders in which additional deflection was added with each cycle. The material exhibited a shakedown limit (stress level beyond which strains in subsequent cycles do not stabilize) and a unique envelope stress-strain curve. Shah and Winter\textsuperscript{13} established a shakedown limit in a range of 88\% to 95\% of the ultimate capacity of plain concrete prisms. In both investigations the concrete stiffness upon reloading was gradually decreased as the number of cycles increased. Karsan\textsuperscript{1} found that the stress-strain relationship for plain concrete under a strain gradient and cyclic loading could be reliably predicted by the stress-strain relationship for concentric cyclic loads. He also found that repeated eccentric loading would not produce failure if the average stress across the section, $f_{cu}$, was less than about 0.5 $f'_{c}$.

The effect of cyclic loads on reinforcing steel was studied by Singh, Gerstle, and Tulin\textsuperscript{5}. The stress-strain relationship exhibited by the specimens tested were elasto-plastic in the first direction of loading and non-linear upon reversal of load, a phenomenon known as the Bauschinger effect. Morrow\textsuperscript{11} developed an expression for the stress-strain relationship of steel subjected to load reversal, but the test specimens were not elasto-plastic in the first direction of loading.

1.2.2 Studies of Structural Members

The properties exhibited by the component materials can be seen in the behavior of reinforced concrete structural members. Singh, Gerstle, and Tulin\textsuperscript{14} obtained moment-curvature relationships for nine singly
reinforced beams \((p' = 0)\) subjected to repeated cycles of loading in one direction. The beams exhibited essentially elasto-plastic behavior with a slight decrease in stiffness with an increasing number of cycles. Tests on doubly reinforced beams \((p' \neq 0)\) by Agrawal, Gerstle, and Tulin\(^{15}\) showed that the presence of compressive steel did not change the general elasto-plastic behavior under repeated loads in one direction. However, when reversed load was applied after the tension steel had yielded in the first direction, the behavior became markedly non-linear. This behavior was attributed to the Bauschinger effect in the reinforcing steel. The authors proposed a method of predicting the moment-curvature relationship for both repeated and reversed loading. No comparisons were made between the available ductility of the specimens under reversed loads and that of monotonically loaded specimens. The reversed loading was very small in magnitude compared to that in the first direction and only two reversed loads were applied. The length of the beam over which the computed curvature was applicable was not discussed.

Burns and Siess\(^3\) compared the strength and ductility of beams subjected to repeated and reversed loads. The strength was unimpaired by load reversals but the ductility was reduced. Only three beams were subjected to load reversal, and more tests are needed to show the trends quantitatively. Eighteen beams subjected to repeated loading in only one direction exhibited a decreased stiffness as the beam deflection increased. When the deflection reached about one-half of the ultimate deflection, the reloading stiffness became fairly stable at about one-half its initial value. This was attributed to a gradual deterioration of the concrete.

Tests on reinforced beams by Ruiz and Winter\(^4\) indicated that one directional load cycling near ultimate did not affect the rotation or
load carrying capacity. They observed limited increased deformations in consecutive cycles of equal load which were attributed to bond deterioration. The rotation capacity of the beams was found to be sensitive to the confinement provided by the compressive and transverse reinforcement and to the restraint against buckling of the compressive steel.

The addition of axial load on beams subjected to load reversal reduced the available ductility and altered the shape of the moment-curvature relationship as reported by Aoyama\textsuperscript{16}. Only three beams were tested and more tests are required to determine the effects of axial load.

Bertero and McClure\textsuperscript{17} studied the effect of load reversal on reinforced concrete frames. Three models of a one-story, single bay rigid frame were subjected to reversed cycles of loading and the results compared to those of two frames loaded monotonically. Apparently, the reduction in stiffness was proportional to the magnitude of the load and to the number of cycles. Anchorage problems which developed in the joints indicated that bond slip contributed significantly to the deformations of the frame.

Hanson and Conner\textsuperscript{18} conducted tests on full size beam-column joints to determine the joint reinforcement required to insure ultimate capacity for cast-in-place beams and columns subjected to multiple reversals of loading. They concluded that the inclusion of hoops in unconfined beam-column joints significantly improved the ability of a structure to withstand seismic deformation by improving the ductility capacity.

DeCossio and Rosenbluth\textsuperscript{19} presented photographic evidence of failures of reinforced concrete members during recent earthquakes in Mexico. From their observations they recommended the need for ductile behavior and better performance of joints.
1.2.3 Summary of Previous Research

On the basis of the work that has been performed in the area of behavior under cyclic loading of reinforced concrete members, it is clear that more information is needed. The following is a brief summary of the previous research in the area.

1. A substantial number of tests have been performed on specimens in which the load was cycled in one direction, and generally the effect of cycling was small.

2. Very few tests have been performed in which the load has been reversed, and available tests have not been coordinated nor have a range of variables been considered.

3. Most of the specimens tested were symmetric, i.e. the members did not depend on anchorage to develop the strength of the reinforcing steel.

4. In the method proposed (Reference 14) to predict the moment-curvature relationship under load reversal simplifying assumptions were made. Residual strains in the steel and residual cracks in the concrete were not included in the analyses. The stress-strain curve of the steel was constant regardless of the strain to which it had been subjected.

5. The variables which may affect the behavior of a member under load reversal have not been systematically studied.
CHAPTER 2 - TEST PROGRAM

2.1 Description of Test Specimen

The specimen chosen for this investigation is shown in Fig. 2.1. The geometry of the specimen was selected to simulate the connection of a beam to a column. The enlarged end section represents that portion of the connection common to a column and beam framing into a point in a structure. The length of the enlarged end or anchorage zone was chosen to assure that the strength of the longitudinal reinforcement was developed by anchorage of the straight portion of the bars. Hooks were provided at the ends of the bars to insure against an unforeseen anchorage failure. Considerations for the design of the anchorage in the enlarged section is discussed in detail by Ismail.

The specimen was bolted to a monolith which in turn was attached to the floor of the test bay (Fig. 2.2 - a). The enlarged end was held in place with channels attached to the monolith with large bolts which were anchored in the concrete when the monolith was cast. The nuts holding the channels were tightened with a torque wrench to produce approximately 1000 psi normal pressure on the enlarged end block to simulate a column load. A total of twenty specimens were tested under five load histories (Sec. 2.2.1). Table 2.1 lists all the specimens with designations describing the specimen and the load history. The parameters considered are discussed in the following section.

2.2 Variables

2.2.1 Load History

Three basic types of loads were applied to the test specimens: monotonic, repeated cycles in one direction, and reversed cycles. Two variations of the second and third loading histories were applied result-
ing in essentially five load histories as listed below.

M - Monotonic loading to failure

RP1, RP4 - Repeated loading in one direction with an increment of
deflection equal to either one or four times the yield
deflection added in each cycle.

RV5, RV10 - Reversed cycles of loading to a constant end deflection
in each direction of either 5 or 10 times the yield
deflection.

The load histories are shown in Fig. 2.3. Some difficulty arose in
determining the deflection limit under the reversed loading in the RV
load histories. In the first direction of loading the deflection incre-
ment was added to a well defined yield point. However, upon load reversal,
there was no well defined yield point. The point in the load-deflection
response from which the increment of deflection was added in the second-
ary direction of loading was defined as the point at which the tension
steel in that direction reached yield strain. The deflection limits were
chosen to produce roughly the same load in both directions and about the
same area under the load deflection curve in each direction after the
first cycle of loading.

2.2.2 Reinforcing Details

Three different longitudinal reinforcement details were used in
the test specimens. Nine specimens were reinforced with #8 bars top and
bottom, nine with #6 bars top and bottom, and two with #6 bars top and
#6 bars bottom. The reinforcement provided for each specimen is given
in Table 2.1.

2.2.3 Stirrup Spacing

All stirrups were fabricated from #3 deformed bars using a template
to insure consistent dimensions. The spacings of stirrups were established from two bases as follows.

1. To satisfy the requirements for shear of the ACI Building Code\textsuperscript{10} the minimum stirrup spacing of d/2 (5 in.) controlled for the specimens with 60 in. spans. No reduction in stirrup spacing was required for the specimens with 30 in. span with #6 reinforcing bars. However, the specimens with 30 in. span and #8 bars required a 4 in. stirrup spacing.

2. To provide for additional confinement of the concrete core, the stirrup spacing was reduced to 2 in.

A detail of the stirrups used in the test program is shown in Fig. 2.1.

2.2.4 Moment-Shear Ratio

The shear span for the monotonic and repeated loading tests and for ten of the fourteen reversed loading tests was 60 in. The remaining four specimens subjected to reversed loading were tested with a shear span of 30 in, thereby increasing the shear-moment ratio on the specimen and the magnitude of the shear on the beam cross-section.

2.2.5 Definitions and Specimen Designation

a. Primary Direction- The primary direction of loading in the reversed load histories refers to the direction in which the beam is first loaded. By this definition, all of the monotonic and repeated loadings were in the primary direction.

b. Secondary Direction- The secondary direction of loading refers to the reversed direction of load. Only specimens with an RV type load history were loaded in the secondary direction.

c. Specimen Designation- A specimen designation was devised to identify the important parameters considered in each test. The parameters
designated are bar size, stirrup size and spacing, load history, and shear span (only the 30 in. shear spans are designated; otherwise a 60 in. shear span is understood). An example of the notation is as follows.

```
88 - 34 - RV5 - 30
```

- shear span (30 in.)
- load history
- stirrup spacing (4 in.)
- stirrup size (#3 bars)
- bottom bar size (#8)
- top bar size (#8)

2.3 Materials

The concrete was designed using the ACI absolute volume method. Type III Portland cement (high early strength) was used throughout the test program. The coarse aggregate was a blend of 3/4 and 3/8 in. maximum size to obtain a satisfactory gradation. The fine aggregate had a fineness modulus of approximately 2.52. All aggregates, obtained from the Colorado River plains, were commercially available in the area. The compressive strengths of the concrete for each specimen are given in Table 2.1.

Deformed reinforcing bars having a nominal yield strength of 45 ksi were used throughout the test program. Coupons were tested for each bar to determine yield and ultimate strength, yield and ultimate strain, and strain hardening strain. The properties of the steel are included in Table 2.1.

2.4 Preparation of Test Specimens

Before casting the concrete, the longitudinal steel was bent to the desired hook geometry. The longitudinal bars were then instrumented with strain gages and the gage installations waterproofed. The longitudinal
steel and the stirrups were tied to form cages and placed in the form on 1\(\frac{1}{2}\) in. chairs at four locations to provide a consistent concrete cover for the bottom bars. The cover for the top bars was 1\(\frac{3}{4}\) in. in all specimens. The steel cage was then tied to the form with tie wire to prevent movement during casting. A cage consisting of #3 bars was provided in the enlarged portion of the specimen to provide minimal tensile capacity.

A two inch diameter hole was blocked out at the desired location on the beam for the loading spindle assembly (Fig. 2.1). Each specimen required three batches of concrete with three control cylinders cast from each batch. The concrete was mechanically vibrated internally and externally. The specimens were cured in the forms using wet burlap covered by a polyethylene film for three days. The specimens were then removed from the forms and cured an additional eleven days in the same manner. The specimens were then air cured until testing. Cylinders were cured in the same manner and tested on the same day as the specimens.

2.5 Instrumentation

A continuous record was maintained in all of the tests of load, end deflection, end rotation, and intermediate rotations. Strain gage data from the longitudinal steel in the anchorage zone and dial gage readings were recorded at intervals during the test. Continuous data was recorded on a Honeywell Visicorder with the load and end deflection also recorded on an X - Y plotter to give an immediate indication of the response. Strains were monitored with Budd digital strain equipment.

The load was measured with compressive load cells instrumented with strain gages. A separate load cell was required for each direction of loading as shown in Fig. 2.4. The load cells were calibrated on a testing machine several times during the span of the test program. A switching unit was used to change load cell signals quickly upon reversal of load.
The **end deflection** was monitored with a linear motion transducer. Since the maximum displacement of the loading frame was 40 in. (20 in. in each direction), a lever system was devised to reduce the amplitude to a value within the displacement limits of the transducer (Fig. 2.2-b). With a transducer maximum travel of \( \frac{1}{2} \) in., an 80 to 1 ratio was required between the horizontal length from the beam to the lever pivotal point \((L_1)\) and the horizontal length from the transducer to the pivotal point \((L_2)\) for a 40 in. deflection amplitude. However, the maximum deflection recorded in the test series was 20 in., hence \(L_1\) was reduced to provide greater sensitivity.

The **end rotation** (rotation near the free end) was measured by a transducer which monitored the relative rotation between the beam and a pendulum which remained vertical by gravity (Fig. 2.2-c). Friction in the pivotal point of the pendulum was minimized by using a bushing lubricated with graphite. The tangent of the relative rotation was recorded by the transducer from which the angle was determined.

The **intermediate rotations** were obtained by measuring longitudinal deformations between two points on brackets attached to the beam. The brackets were fixed to the specimen with \( \frac{1}{4} \) in. diameter threaded rods which penetrated completely the concrete core just inside the longitudinal steel. The brackets were spaced a 4, 5, and 6 in. respectively from the fixed end. Transducers were mounted on the brackets (Fig. 2.2-d), from which the axial deformations in the cross-section were determined.

The **fixed end rotation** or the rotation near the interface of the beam and the fixed end was obtained using dial gages which measured the tangent of the angle between the beam and the fixed end. To provide a point of reference on the beam, aluminum angles were fixed to the concrete.
surface with epoxy. This data was available only as long as the concrete to which the angles were affixed remained intact.

The deflection at a distance of 15 in. from the fixed end was measured with dial gages.

Strain gages were mounted on the longitudinal steel in the fixed end for determining the stress distribution in the anchorage zone. Strains were recorded at intervals on Budd digital strain balancing equipment.

2.6 Loading and Support Arrangements

The enlarged portion of the specimen was bolted to a concrete monolith in which eight 1\(\frac{1}{2}\) in. diameter threaded rods were anchored (Fig. 2.2-a). A steel channel spanning across the end block between each pair of threaded rods held the specimen in place atop the concrete monolith. All bearing surfaces were grouted with a layer of Hydrocal, a gypsum compound, to permit even distribution of bearing pressures. After the Hydrocal had hardened, the nuts on the threaded rods were tightened with a torque wrench to provide approximately 1000 psi vertical pressure on the fixed end. The frictional surfaces on the threads and at the base of the nut were oiled each time to provide a consistent frictional loss in tightening the nuts.

The load was applied to the specimen through a loading frame (Fig. 2.4) designed to apply vertical loads in either direction. The load was provided by four pairs of 10 kip capacity hydraulic jacks (F)* each with 10 in. travel in series, two pair for each direction of loading. A moveable cross beam (A) loaded at each end by the rams transferred the force through a load cell (D) to tension plates (E). The cross beam was maintained in a horizontal position with a system of pulleys (B) and

* The letter refers to the labels of Fig. 2.4
cables (C) utilizing a mechanism similar to that used in parallel drafting bars. Shear pins (J) transferred the force from the tension plates to the assembly which housed the horizontal track (G). The loading spindle which passed through the 2 in. diameter hole in the test specimen was provided with roller bearings at each end which rolled in the horizontal track. In this manner, the horizontal component of the load was eliminated. An identical system was used for loading in the opposite direction.

2.7 Loading Program

Load was applied to all specimens statically. The hydraulic pump was not capable of loading the specimen at a rate which would create significant inertial forces (i.e., the frequency of loading was much less than the beam's natural frequency). The maximum rate of loading of the system was \( \text{in-kip} \) about 1 \( \text{Sec} \). At this rate of loading about 40 seconds were required to deflect specimen 66-32-RV5 through a half-cycle of loading.

The loading was stopped for approximately 2-3 minutes at various times during testing to permit recording of data, marking cracks, and taking photographs. The intervals at which readings were taken depended upon the type of loading applied and on the response of the specimen.

Readings for the specimens under monotonic loading were taken at approximately 2 kip intervals before yield. Because the stiffness decreased rapidly after yield, readings were then taken at deflection increments of about one inch.

In the specimens subjected to repeated loads, readings were taken in the same manner as in the monotonic tests with each cycle being treated as if it were a monotonic test. However, the number of readings was reduced.
In the first half cycle of the reversed loading tests the same procedure was used as in the monotonic tests. Upon first reversal, readings were taken at 2 kip increments until the bottom steel yielded after which readings were taken at various increments of end deflection. In subsequent cycles, readings were taken at various deflection increments regardless of load and were fewer in number than in the first cycle.

When the behavior of a specimen under cyclic loading was stable through several cycles (i.e. reproduced the previous cycle), readings were not taken until the load-deflection behavior showed some deviation from the previous cycle.
CHAPTER 3 - TEST RESULTS

3.1 Preliminary Remarks

The purpose of the test program was to observe and interpret the behavior of the test specimens under the prescribed load histories. The behavioral relationships of interest in the test program are the effect of the load (or the moment) history on the deflection and rotation of the test specimen and on the apparent modes of failure. Observations of the behavior of the anchorage zone reinforcement are discussed in Appendix B.2 and by Ismail.\textsuperscript{8}

3.2 Behavior Under Monotonic Load

Two test specimens were subjected to monotonic loading to failure or until the deflection capacity of the loading frame was reached. Monotonic tests were performed to establish a standard of comparison with research performed by others and with specimens subjected to cyclic loading. Two tests were performed to determine the effect of steel percentage on the behavior and, more importantly, to compare with similarly reinforced specimens subjected to cyclic loads. Both test specimens had equal tension and compression reinforcement, one with 2-#6 bars, top and bottom, the other with the same arrangement using #8 bars.

Specimen 66-35-M was loaded to a 15.2 in. end deflection at which time it was unloaded, the loading frame adjusted to permit additional deflection, and reloaded to the deflection capacity of the test frame. The load-deflection curve (Fig. 3.1) exhibited a yield point at about 6 kips at which the stiffness decreases significantly. Beyond yield the curve showed a general decrease in stiffness as the deflection was increased with no loss of load carrying capacity.

The rotations measured at the four locations discussed in Sec. 2.5
are plotted cumulatively (i.e., $\theta_2$ is the total rotation at location 2) in Fig. 3.2. An indication of the curvature distribution is the incremental rotation of the intermediate points. So little rotation occurs between $\theta_3$ and $\theta_E$ that the difference is indistinguishable on the scale shown. This indicates that almost all of the rotation occurs in a plastic hinge with the unyielded portion of the beam behaving as a rigid body. Only the total rotation data $\theta_E$ was measured beyond a load of 9.2 kips as the transducers were removed to prevent damage. This procedure was required for all specimens, hence the intermediate rotations are not measured to failure.

The other monotonic test specimen (88-35-M) was deflected in almost the same manner as was 66-35-M (Fig. 3.3). However, in this case when the specimen was reloaded, it reached the peak load but failed shortly thereafter. The load decreased gradually for about one inch of end deflection and then decreased sharply to about 70% of the peak load at which time the load was removed. Observation of the test specimen at failure (Fig. 3.46-a) indicates a shear-compression failure followed by buckling of the compression steel. The concrete cover both at the top and bottom of the specimen had spalled leaving the compression steel unrestrained against buckling between stirrups. The spalling in the compressive zone was initiated at a load of 12 kips and was probably the result of slip between the compression steel and the concrete which produced a wedging and splitting effect.

The cumulative load-rotation curve (Fig. 3.4) indicates that most of the rotation occurred within the first 15 in. of the beam as was the case for 66-35-M. It is noteworthy that during the first two inches of end deflection, most of the total rotation was contributed in the first
9 in. of the beam (θ₂). As the deflection was increased, the third interval of rotation began to contribute an increasingly greater amount which implies that the plastic zone was spreading as the load was increased.

It was not surprising that the more heavily reinforced specimen failed whereas the other monotonically loaded specimen did not. With a larger amount of tension reinforcement, the cross-section was subjected to greater shear forces and coupled with correspondingly greater compressive forces, the shear-compression-buckling failure was more likely for the 88-35-M specimen.

3.3 Behavior Under Repeated Loading

Four specimens were tested under unidirectional cyclic loading in which end deflection was added with each cycle. Again the effect of the steel percentage was considered as well as the effect of the magnitude of the deflection increment.

Two specimens, reinforced with #6 bars top and bottom, were comparable to the monotonically loaded specimen, 66-35-M. Specimen 66-35-RP4 was subjected to an additional deflection of about 4Δy for five cycles at which time it was subjected to an additional deflection equal to the capacity of the loading frame (Fig. 3.5). Very little difference was observed between the envelope of the load-deflection curves of 66-35-RP4 and 66-35-M. Cycling slightly reduced the load carrying capacity and the stiffness of specimen 66-35-RP4 and a decrease in load was observed beyond a 17 in. end deflection. Observation of the test specimen (Fig. 3.46-b) at the maximum deflection indicated the beginning of an apparent shear-compression failure. Fig. 3.6 shows the cumulative rotations vs. cycle number and represents rotations at the peak end deflection of the cycle. This representation differs from the continuous
load-rotation curves (Figs. 3.2 and 3.4) and was chosen because of the
difficulty in plotting or interpreting the continuous curves for cyclic
loading. For convenience the end deflection is also included on the
abcissa. The load corresponding to the rotations of Fig. 3.6 may be
obtained from the deflection using Fig. 3.5. The growth of the incre-
mental rotation $\theta_3 - \theta_2$ of Fig. 3.6 is an indication of the spread of the
plastic hinge away from the fixed end.

When the deflection added with each cycle was reduced from $4\Delta_y$ to
$\Delta_y$ (66-35-RP1), the envelope of the load-deflection curve was almost
unchanged until an end deflection of about $1\frac{1}{4}$ in. was reached (Fig. 3.7).
Beyond this point the beam carried a reduced load with increased end
deflection as a result of a shear failure which led to buckling of the
compression steel (Fig. 3.46 - c).

The cumulative rotations at the deflection peaks of every fourth
cycle of 66-35-RP1 (Fig. 3.8) compared with 66-35-RP4 (Fig. 3.6) shows
that $\theta_1$ was substantially less for the former specimen at a comparable
end deflection. The difference was attributed to the slip in the fixed
end which was included in the measurement of $\theta_1$. When the contribution
of deformations in the anchorage zone to the end deflection was small,
more deformation was required of the beam to reach a given end deflection.
It appears that the greater deformations required of the beam portion of
66-35-RP1 eventually produced failure, whereas in 66-35-RP4 the increased
slip in the anchorage zone allowed the beam to withstand greater end
deflections without failure.

A second group of two specimens with #8 bars top and bottom,
comparable to specimen 88-35-M, was subjected to repeated loading. The
upper envelope of the load-deflection curve of 88-35-RP4 (Fig. 3.9) is
almost identical to that of the companion monotonically loaded specimen (Fig. 3,3). A reduction of load with additional end deflection occurred in the ninth cycle of loading at an end deflection of about 17 in. The mode of failure (Fig. 3.47-a) was shear-compression followed by buckling of the compression steel as for 88-35-M. The distribution of rotation for 88-35-RP4 (Fig. 3.10) includes also the rotation $\theta_{PE}$ measured at the face of the fixed end produced by bond slip in the anchorage zone. When the contribution of the second rotation interval $\theta_2 - \theta_1$ is compared with that of 66-35-RP4, it appears that the rotation for the more heavily reinforced specimen is greater. This implies that the plastic zone had extended over a greater length of the beam for comparable end deflections in the beams reinforced with #8 bars.

When the increment of deflection was reduced from $\Delta y$ to $\Delta y$, specimen 88-35-RP1 was able to withstand 20 in. of end deflection without a loss of strength (Figs. 3.11 and 3.47-b). However, the contribution to the end deflection due to slip in the anchorage zone was about 30% greater at an end deflection of 15 in. than that of 88-35-RP4, thus demanding less deformation in the beam. It is interesting that in the 66 series the smaller deflection increment (RP1) led to failure, whereas in the 88 series the larger (RP4) produced eventual failure. In both cases of failure, however, the slip in the anchorage zone contributed less to the end deflection than in the companion beam under repeated loading which did not fail.

All of the specimens subjected to repeated loading showed little, if any, loss of ductility compared with the monotonically loaded specimens. In those beams in which failure was produced, deformations in the anchorage zone were less than in the beams which did not fail.
Essentially the beam portion of the specimens was subjected to greater deformations in order to achieve a given end deflection. The ductility of the anchorage zone was an important factor in the overall ductility of the specimen and is discussed in greater detail by Ismail. 

3.4 Behavior Under Reversed Loading

3.4.1 Introduction

In order to determine the effect of steel percentage, shear span, and stirrup spacing on the behavior of specimens subjected to reversed cycles of loading, fourteen specimens were tested, six reinforced with #6 bars top and bottom, six with #8 bars top and bottom, and two with #8 bars top and #6 bars bottom. Load-deflection and curvature distribution curves are presented for all of the specimens. Several load-rotation curves are presented to show the rotation at intermediate points during the tests. For any set of section properties, a duplicate specimen was cast so that two load histories, RV5 and RV10, could be studied as discussed in Sec. 2.2.1. If the specimen did not give an indication of failure after about ten cycles of RV5 loading, the deflection was increased from $5\Delta_y$ to $7\Delta_y$ because the specimen was considered to have performed satisfactorily under the original loading.

3.4.2 Behavior Common to All Specimens Under Reversed Loads

All of the specimens subjected to reversed loading demonstrated the following behavior:

a. The primary direction of the first cycle of loading exhibited essentially bilinear behavior similar to that of the monotonic tests.

b. An increase in the magnitude of the deflection limits of the cycles reduced the number of cycles required to produce failure. For
example RV10 loading produced failure at a fewer number of cycles than did the RV5 loading for a given section geometry. This behavior was expected since larger end deflections were accompanied by higher loads and greater deformations in the cross-section.

c. An increase in the percentage of reinforcement (either p or p') reduced the number of cycles required to produce failure. Such behavior is not surprising because larger amounts of tension reinforcement produce greater compressive forces on the cross-section which lead to deterioration of the concrete.

3.4.3 Series 66, #6 Bars Top and Bottom

Six specimens with #6 bars top and bottom were tested: two with 5 in. stirrup spacing (66-35), two with 2 in. stirrup spacing (66-32), and two with a 30 in. span (66-35-RV-30). Figs. 3.13 through 3.24 show the load-deflection and rotation distribution curves for this group of specimens. Figs. 3.25 and 3.26 show the variation of the load-rotation behavior between the cycle peaks. In every case the larger deflection (RV10) resulted in failure after fewer cycles than were required to produce failure under the RV5 load history. All specimens showed a non-linear load-deflection relationship after load reversal in both directions of loading due primarily to the Bauschinger effect (App. C.2) and to the non-linear load-slip behavior of the reinforcement after load reversal. The addition of more stirrups (2 in. spacing compared with the ACI Building Code requirement of 5 in. spacing) increased the number of cycles to failure in every case but particularly under RV10 loading (Fig. 3.14 and 3.16). When the span was reduced from 60 in. to 30 in., the beam exhibited a concavity (dip) in its load deflection curve upon reversal of load. The stiffness in the primary direction of cycle 2
was significantly increased as the load increased. The temporary loss of stiffness was attributed to shear deformation and the reduced area of concrete in compression prior to closure of the flexural cracks formed in the previous direction of loading and was regained when the flexural cracks closed. The concavity in the load-deflection curve substantially reduced the energy absorbed in the cycle (i.e. the area under the load-deflection curve or hysteresis loop). This reduction was particularly noticeable in specimen 66-35-RV10-30 (Fig. 3.18) compared with 66-35-RV5-30 (Fig. 3.17).

The specimens with the 60 in. spans, 66-35, appear to have failed in shear followed by buckling of the compression steel (Fig. 3.47-c, 3.47-d). The shear failure was initiated along vertical cracks located roughly at the location of the stirrups. The concrete core cracked into segments (bounded by adjacent stirrups and the longitudinal steel) which were unbonded to the steel or to the adjacent segment of concrete. The shear was then transferred by the longitudinal steel with little shear restraint provided by the concrete.

When the stirrup was reduced, the length of the damaged portion of the beam was substantially reduced (Fig. 3.48-a, 3.48-b) and greater damage occurred in the fixed end.

Because the concrete at the face of the fixed end spalled leaving only the concrete core, the length of the beam had essentially been increased. However, the additional length of beam did not have stirrups since the first stirrup was located one inch from the original fixed end. An apparent shear failure occurred at the interface between the beam and the fixed end as indicated by the slope of the longitudinal steel at that point. Apparently the addition of stirrups enabled the
beam to withstand the shear but led to a shear failure in the anchorage zone. The same mode of failure occurred in 66-32-RV10.

When the span was reduced from 60 in. to 30 in., the shear was substantially increased. A shear failure followed by buckling of the compression steel occurred in both specimens (66-35-RV5-30 and 66-35-RV10-30) as shown in Fig. 3.48-c, 3.48-d.

The distribution of rotation (Fig. 3.19) along specimen 66-35-RV5 gives an indication that shear failure occurred. The total rotation of the beam decreased with a constant end deflection and indicated that the contribution of shear deflection to the end deflection was increasing. It is noteworthy that the end rotation in the secondary direction was less than \( \theta_3 \) as a result of the residual curvature in the beam produced from loading in the primary direction. A similar behavior was exhibited by 66-35-RV10 (Fig. 3.20).

The rotation distribution for specimen 66-32-RV5 (Fig. 3.21) shows that \( \theta_1 \) contributes a very high percentage of the total rotation which is in agreement with the observed short plastic zone and failure in the anchorage zone. The companion specimen (66-32-RV10) shows a relatively even distribution of rotation throughout the test (Fig. 3.22).

The rotation distributions (Fig. 3.23 and 3.24) of both specimens with 30 in. shear spans indicate the presence of large shear deformations. The points of inflection produced in the beam by large shear distortion combined with flexural deformation result in measured negative curvatures (e.g. \( \theta_3 < \theta_2 \)) and hence the intermediate values of rotation were not as meaningful as in the case of the longer specimens.

3.4.4 Series 88, #8 Bars, Top and Bottom

A second group of six specimens, with #8 bars top and bottom, was
subjected to the same program of loads as those previously discussed in order to observe the effect of a higher percentage of reinforcement on behavior under load reversals. The load-deflection and rotation distribution curves for this group of specimens appear in Figs. 3.27 through 3.38. In general, the variables (load history, stirrup spacing, moment-shear ratio) had the same effect on the 88 series as on the 66 series. All specimens subjected to RV10 loading failed in fewer cycles; closely spaced stirrups increased significantly the number of cycles to failure; and a decreased moment-shear ratio shortened considerably the life of the specimen.

The failure modes for the 88-35 specimens were in shear (Fig. 3.49-a and 3.49-b) and were similar to the 66-35 specimens. When the stirrup spacing was reduced, failure occurred in a shorter zone very near and including the fixed (Fig. 3.49-c and 3.49-d). The specimens with the 30 in. shear span (88-34-RV-30) failed in shear after severe concrete deterioration (Fig. 3.50-a and 3.50-b).

The rotation distributions for the 88 series were virtually the same as those in the 66 series. The primary difference was the reduction of $\theta_E$ with number of cycles as a result of the greater shear deformation. Comparisons of the total rotations for the intermediate points during the test (Fig. 3.39 and 3.40) shows that the reduction in rotation was increased with an increase in shear. As shear deformations were increased, less rotation was required of the beam to reach a given end deflection.

3.4.5 Series 86, #8 Bars Top, #6 Bars Bottom

The final group of specimens tested had unequal top and bottom steel (86-35-RV5 and 86-35-RV10). The load-deflection curves for these specimens (Fig. 3.41 and 3.42) indicate a substantial reduction in load in the secondary direction when the #6 bars are in tension. In the
primary direction after load reversal the concavity of the load-deflection curve is more pronounced than in any other series of tests which can be attributed to the wider residual flexural tensile cracks resulting from loading in the secondary direction and to the decreased stiffness in the #6 bars in compression due to large residual strains (App. C.2). The same general shape is exhibited by the load-rotation curve for specimen 86-35-RV10 (Fig. 3.43).

Failure of both beams in this series appears to have been initiated by buckling of the #6 bars (Fig. 3.50-c and 3.50-d). The stiffness of the compression steel was reduced by the Bauschinger effect in every specimen subjected to reversed loading. However, in the case of unbalanced reinforcing with a larger bar in tension, a greater compressive force was required in the smaller bar. Cycling of load spalled the concrete cover thus leaving the compressive steel unrestrained against buckling.

3.4.6 Summary

In summary the behavior of all of the test specimens subjected to load reversals differed significantly from that of specimens loaded in only one direction. Both the load-deflection and load-rotation relationships are non-linear because of the Bauschinger effect (App. C.2), shear deformations (App. D), closing of residual concrete cracks (App. A.3), and non-linear load-slip relationships in the anchorage zone (App. B.2).

The load history with the larger deflection limits (RV10) produced failure with fewer cycles than did the RV5 load history on specimens with the same section geometry.

A larger percentage of reinforcement (p or p') reduced the number of cycles to failure apparently by producing greater shear forces in the
Closely spaced stirrups significantly increased the number of cycles to failure apparently by increasing the shear capacity of the specimen.

A shorter shear span and hence a reduced moment-shear ratio reduced the number of cycles to failure in every case because the shear force was significantly increased. The shape of both the load-deflection and load-rotation relationships for the shorter spans were altered resulting in reduced energy absorption in each cycle.

The mode of failure for all specimens with equal top and bottom steel appeared to be in shear across planes of weakness which resulted directly from load reversals and would not otherwise be expected. Shear failure was followed by buckling of the compression bars usually between adjacent stirrups. The mode of failure for the two specimens with unequal top and bottom steel was buckling of the smaller bars. The large tensile force produced by the larger tension steel area required greater compressive forces than the smaller bars could withstand without buckling.
CHAPTER 4 - ANALYSIS OF TEST RESULTS

4.1 Introduction

The objective of this chapter is to predict the behavior of the test specimens as a function of the load history based on the material properties and on the phenomena observed in the experimental program. The moment-curvature relationship of a section is based on equilibrium and compatibility conditions and the stress-strain properties of the steel and concrete. Material behavior is a function of the load history and is discussed in the appendices. Deflection and rotation of the beam are based on the distribution of the curvature both in the beam and in the anchorage zone. The predicted curvature distribution in the beam is based primarily on test results reported previously. However, the curvature distribution depends on the loading history. The curvature in the anchorage zone is considered as a rotation at the interface of the beam and the fixed end and is based on measurements from the test program.

Because of the markedly different behavior exhibited by the test specimens at different stages in the load history, three cases will be considered in the analysis: monotonic and repeated loading, first load reversal, and subsequent load reversals. The monotonic and repeated loading case includes the first cycle, or primary direction, of the specimens subjected to reversed loading.

4.2 Analysis of Monotonic and Repeated Loading Cycles

4.2.1 Preliminary Remarks

In this section the load-deflection and moment-rotation characteristics of the monotonic and repeated loading cycles and the first cycle,
primary direction of the reversed loading tests will be analysed. Only the post-yield behavior is of concern in this analysis. Pre-yield behavior of similar beams is discussed by Burns\textsuperscript{3}, and the behavior of the anchorage zone before yield is discussed by Ismail\textsuperscript{8}. The determination of deformations at first yield is presented separately because a closed solution is possible, however, it could be included in the post-yield analysis.

4.2.2 Analysis at Yield

The first point of interest in the behavior of the test specimens occurs when the strain in the tension steel reaches yield. Beyond the yield point there is a substantial reduction in stiffness due to inelastic behavior of the materials.

Straight line theory, i.e. stresses in the concrete proportional to strains, is assumed to govern at loads up to yield thus permitting a closed solution. In every specimen of this program substantial compression steel insured linear behavior of the concrete up to yield. Young's modulus for concrete is taken from the ACI Building Code\textsuperscript{10} as

\[ E_c = 57,000 \frac{f_c^{0.5}}{} \text{ psi} \tag{4.1} \]

and for both the tension and compression steel

\[ E_s = 29 \times 10^6 \text{ psi} \tag{4.2} \]

The equilibrium and compatibility conditions are shown in Fig. 4.1 with the following assumptions.

1. Plane sections remain plane.
2. Concrete has a linear stress-strain relationship with a stiffness given by Eq. 4.1.
3. Concrete carries no tension.
4. Yield of the steel is known and the stiffness is given by Eq. 4.2.

The yield strain may be determined by assumption 4. \( \varepsilon_y = \sigma_y / E_s \).

The location of the neutral axis must then be determined from equilibrium and compatibility of the cross-section. With a linear relationship between stress and strain on both the steel and concrete, a closed solution for the location of the neutral axis may be determined.

\[
k = \sqrt{\left[ np + p'(n-1) \right]^2 + 2\left[ np + p' \frac{d}{d} (n-1) \right] - [np + p'(n-1)]} \quad 4.3
\]

where \( p = A_s / bd \)

\( p' = A_s^{*} / bd \)

\( n = E_s / E_c \)

Knowing \( \varepsilon_y \) and \( k \) the strain throughout the cross-section is completely defined by assumption 1. If the compression steel has yielded, Eq. 4.3 does not apply. At yield the moment \( M_y \) is given by the following equation.

\[
M_y = A_s f_y l_y \quad 4.4
\]

where \( A_s \) is the area of tensile steel, \( f_y \) is the steel stress at yield, and \( l_y \) is the distance from the tension steel to the resultant compressive force on the section.

The curvature at yield is shown in Fig. 4.1-a.

\[
\phi_y = \frac{\varepsilon_y}{d(1-k)} \quad 4.5
\]

The curvature is assumed to be distributed along the beam and in the anchorage zone according to Fig. 4.1-b. The rotation computed from the assumed distribution of curvature in the anchorage zone is in good agreement with the measured rotation at the face of the anchorage zone at
yield. The end deflection at yield based on this distribution of
curvature is

\[ \Delta_y = \phi_y \left( \frac{L^2}{3} + 12L + 78 \right) \text{ in.} \]  

where

\[ L = \text{length of beam in inches} \]
\[ \phi_y = \text{curvature from Eq. 4.4 in radians/ inch} \]

The rotation at any point \( x \) along the beam at yield is

\[ \theta_y = \int_0^x \phi_y \, dx + \theta_{FE} \text{, radians} \]

where \( \theta_{FE} \) is the area of curvature in the anchorage zone and is equal
to \( 12 \phi_y \). Comparisons between measured and predicted results at yield
are presented in Chapter 5.

**4.2.3 Post-Yield Analysis**

In order to determine the moment-curvature and load-deflection
relationships for post-yield conditions, the straight line analysis
previously used is not realistic. The following assumptions define the
material behavior in the post-yield range.

1. The stress-strain relationship for both unconfined and confined
   concrete subjected to a strain gradient is defined in Appendix A
   (App. A.1 and A.2 respectively).
2. Concrete carries no tension.
3. The stress-strain relationship for both the tension and compression
   steel is defined in Appendix C (App. C.1).
4. Slip between the tension steel and the surrounding concrete produces
   strains in the concrete at the level of the reinforcement which are
   not compatible with steel strains.
5. The compression steel buckles according to the tangent modulus
theory discussed in Appendix C (App. C.3).

6. Shear deformation is negligible.

A strain compatibility factor ($F$) is used to account for the slip which occurs between the tension reinforcement and the surrounding concrete in the beam (assumption 4). In effect, the assumption that plane sections remain plane used in the yield analysis is no longer valid. Burns used a value of $F = 0.85$ in his analysis and obtained satisfactory correlation with measured response. Hence the same value for $F$ will be used in this analysis.

The effect of shear deformation is probably negligible, and, in any case, no satisfactory method of computing shear deformations was available.

The procedure for the determination of forces on the cross-section in the post-yield range follows closely that used in the yield analysis. However, the concrete is inelastic, the tension reinforcement is beyond yield, and the compression reinforcing may be either elastic or inelastic.

To determine the moment-curvature relationship, computations were made at discrete points obtained by increasing the concrete compressive strains at the extreme fiber (at the section of highest moment) in increments of 0.001 from an initial value of 0.001. A closed solution for computing the neutral axis for a given concrete strain is very cumbersome so a trial and error approach was adopted. The procedure may be outlined briefly as follows. For a given extreme fiber strain in the concrete $\varepsilon_c$, the position of the neutral axis $k_p$ is assumed and the total tension and compression forces acting in the cross-section are computed. If the tension and compression forces are not in equilibrium, the location of the neutral axis is adjusted until the tension and
compression forces are within a specified tolerance. A typical strain
diagram for the cross-section in the post yield range is shown in Fig. 4.2.

The total compressive force in the concrete is made up of
contributions from both confined and unconfined concrete, the total force
computed as the sum of three forces. The first, $C_1$, is the concrete
force computed for the entire concrete area in compression considered as
unconfined. The second, $C_2$, is the compressive force contributed by
the confined concrete bound by the stirrups and the compression steel.
Finally, $C_3$ is the compressive force for the concrete bound by stirrups
and the compression steel computed as unconfined concrete. The total
concrete force consists of concrete force in the area bounded by the
stirrups $C_2$, and an unconfined concrete force $C_1 - C_3$ in the remaining
concrete compressive zone (Fig. 4.2).

The value of $C_1$ is computed from the expression

$$C_1 = k_1 k_3 f'_c b k_p d$$

where $k_p d$ is the depth to the neutral axis from the top fiber of the
section (Fig. 4.2) and $k_1 k_3$ is a measure of the average concrete stress
over the entire compressive area as defined in App. A.1. The centroid
of force $C_1$ is located at a distance

$$l_1 = d(1-k_2 k_p)$$

from the tension reinforcement. The numerical values of $k_1 k_3$ and $k_2$
are developed in App. A.1 as functions of the extreme fiber concrete strain.

The concrete force $C_2$ is developed in the confined concrete core
bounded by the inside of the stirrups and is given by
\[ C_2 = k_1 k_3 f'_1 b''(k_p d - d' + \frac{D}{2}) \]  

where

- \( b'' \) = width of the core measured to the inside of the stirrups
- \( f'_1 \) = strength of confined concrete as defined in App. A.2
- \( D \) = diameter of compression steel

The centroid of \( C_2 \) acts at a distance

\[ l_2 = d - d' + \frac{D}{2} - k_2(k_p d - d' + \frac{D}{2}) \]  

from the tension steel. The values of \( k_1 k_3 \) and \( k_2 \) are given in App. A.2 as functions of the concrete strain at the extreme fiber of the confined concrete zone. The third concrete force \( C_3 \) is that portion of the unconfined concrete force developed in the core which must be removed since it has already been included in the computation for \( C_2 \).

\[ C_3 = k_1 k_3 f''_c b''(k_p d - d' + \frac{D}{2}) \]  

with a centroid acting at a distance

\[ l_3 = d - d' + \frac{D}{2} - k_2(k_p d - d' + \frac{D}{2}) \]  

from the tension steel. The values of \( k_1 k_3 \) and \( k_2 \) are given in App. A.1 and are functions of the concrete strain at the extreme fiber of the confined zone.

The stress in the compression reinforcement is obtained from an idealized stress-strain diagram. Since the cross-sectional area occupied by the compression steel is included in the computations for the concrete force, the resultant steel force is

\[ C_s = A'_s (f_s - f_c) \]

- 33 -
where

\[ A_s' = \text{area of the compressive steel} \]
\[ f_s = \text{stress in the compressive steel corresponding to the strain at the centroid of the steel with a limitation based on the buckling capacity discussed in App. C.3} \]
\[ f_c = \text{stress in confined concrete corresponding to the strain at the level of the compression steel.} \]

The compressive steel force acts at a distance

\[ l_s = d - d' \quad 4.15 \]

from the tension steel. The only tensile force acting on the cross-section is that of the steel. Since the tensile and compressive forces are equal as required for equilibrium, the moment of the compressive force components about the tension steel is the moment of the cross-section and is given by

\[ M_p = C_1 l_{11} + C_2 l_{22} + C_s l_s - C_3 l_3 \quad 4.16 \]

The corresponding curvature is

\[ \phi_p = \frac{\varepsilon_c}{k_p d} \quad 4.17 \]

Using the above analysis, the moment-curvature relationship may be determined for any concrete extreme fiber strain as long as the forces on the cross-section satisfy equilibrium.

In order to compute end deflections and rotations the distribution of curvature along the beam must be determined. DeCossio\(^6\) and Sawyer\(^7\) recommend distributing the computed curvature over the length of the
member where the moment exceeds the yield moment plus an additional "plastification" zone of length \( d/2 \) (Fig. 4.3). As will be discussed later, the end deflections and rotations computed using this curvature distribution compare favorably with the measured values.

Deformations of the reinforcement in the anchorage zone also contribute to the end deflection. The deformations of the anchored reinforcement and the resulting rotation \( \theta_{FE} \) at the interface of the beam and the fixed end are discussed in Appendix B. The centroid of the rotation computed from the plastic curvature distribution is assumed to act at the interface of the beam and the fixed end.

With the curvature distribution shown in Fig. 4.3 the total end deflection is

\[
\Delta_e = \theta_{FE} L + \phi_p \left[ (\beta + \frac{d}{2}) L \right] + \phi_y \frac{(L - \beta - \frac{d}{2})^3}{3(L - \beta)} \]

where \( L \) = beam span

\[
\beta = L \left( 1 - \frac{M_y}{M_P} \right) = \text{yielded length of beam}
\]

The rotation at any point of the beam is

\[
\theta = \int_0^x \phi_p \, dx + \theta_{FE}
\]

Thus the load-deflection and moment-rotation relationships may be computed for any deformation for which the internal forces are in equilibrium. Comparisons of measured and computed results are presented in Chapter 5.

4.2.4 Analysis of Repeated Loading Tests

Examination of the experimental data obtained in this test program indicates that repetition of load in one direction has little effect on
the load-deflection and moment-rotation envelope curves. Repetitions
do not appear to reduce the rotation capacity of the member. For this
reason the procedure for monotonic loading analysis is used to determine
the envelope curve for both load-deflection and moment-rotation under
repeated loads. However, the unloading and reloading stiffness is
significantly altered with respect to that for first loading from zero
to yield. It appears that this stiffness is related primarily to the
end deflection at unloading. Although there is considerable scatter of
the data points (Fig. 4.4), there is a trend toward a decreasing
stiffness with increasing load. A reasonable estimate of the unloading
stiffness is

\[ \frac{k_r}{k_y} = \left( \frac{d_y}{d} \right)^{0.2} \quad 4.20 \]

where

- \( k_r \) = unloading stiffness
- \( k_y \) = stiffness of original loading curve
- \( d_y \) = end deflection at yield
- \( d \) = end deflection at unloading

The results indicated that there is a hysteresis loop when the specimen
is unloaded and reloaded although the reloading is not significantly
different from the unloading stiffness. For this reason Eq. 4.20 is
used to predict both unloading and reloading stiffness.

4.3 Analysis of Load Reversal

4.3.1 Preliminary Remarks

The behavior of the test specimens differs markedly after the load
has been reversed. The well defined yield point observed in the
monotonic tests is no longer present and non-linear behavior is observed.
The purpose of this section is to predict the behavior as a function of the load history. A separate analysis is made for the first load reversal and the subsequent reversals. When the load is first reversed, the materials have been subjected to loading in only one direction, whereas in subsequent reversals, the materials have undergone both tensile and compressive stresses.

4.3.2 Analysis of First Load Reversal

The procedure for predicting the behavior during the first reversal follows very closely that of the post yield analysis of the monotonically loaded specimens. The same assumptions are made with the following exceptions.

1. Because residual steel strains may produce open concrete cracks upon reversal of load, the stress-strain relationship for both confined and unconfined concrete subjected to a strain gradient is defined as in App. A,3.

2. The stress-strain relationship for both tension and compression steel is defined in App. C,2 to include the Bauschinger effect. The maximum concrete strain $\epsilon_c$ is increased in increments of 0.001 from an initial value of 0.001 as was the case of post yield analysis of the monotonic case. For a given value of $\epsilon_c$, the strains and resulting stresses are shown in Fig. 4,5.

The crack strain $\epsilon_{cr}$ in App. A,3 occurs as a result of loading in
the primary direction. Case A of Sketch 4a represents the strain at the peak load in the previous direction of loading. When the load is removed, a residual strain remains as shown in Case B. When moment is applied in the opposite direction, the resulting strain during the reversal is shown in Case C measured from the residual strains of Case B such that $\varepsilon_c$ now includes $\varepsilon_{cr}$ and $\varepsilon_s$ includes the residual steel strain. Before the stresses in the concrete can be developed for the strains shown in Case C, the residual crack in the concrete $\varepsilon_{cr}$ must first close. It is assumed that the crack closes simultaneously throughout the compression zone. This approach assumes there are an infinite number of cracks along the beam and that the position of the neutral axis is unchanged by load reversal as discussed in the following paragraph. However, cracks form at discrete intervals along the beam in reality. The crack strain is essentially the crack width divided by the spacing between cracks.

The value of $\varepsilon_c$ in Sketch 4a, Case A is determined from the analysis discussed in Sec. 4.2.3. The value of $\varepsilon_{cr}$ was chosen as a percentage of $\varepsilon_c$ since it represents the strain which results from the unloading of Case A. Several trial values for this percentage showed that a value of 40% gave the best agreement with test results. While a value of 40% may appear to be low, the use of a small value partially corrects the oversimplification that concrete carries no force until the crack closes. Forces in the concrete may be transferred at the crack before it closes for the following reasons:

1. Under shear deformation, the crack may not close in its original configuration. Sketch 4b (page 39) illustrates that in this case the concrete will begin carrying force when any contact is made
even though the crack has not closed throughout the compression zone.

Sketch 4b

2. Because the neutral axis when loading in one direction does not coincide with that of reversal of loading, the crack will not close simultaneously throughout the compressive zone. Sketch 4a is based on coinciding neutral axes for simplicity in explaining the residual crack phenomenon. Sketch 4c shows how the difference in the position of the neutral axes affects the closure of the crack. No stress will be developed in the concrete at any level until the crack strain is overcome by the compressive strain at that level (Sketch 4c, Case B). Obviously the crack will not close simultaneously throughout the compressive zone but will close first at the top fiber. As the rotations increase, closure of the cracks propagates downward but the crack may not close near the neutral axis.

The values of $k_1 k_2$ and $k_2$ may be obtained from App. A.2 which is based on the closing of residual cracks in the concrete. The magnitude
and location of the forces of the concrete are given by the expressions presented in Sec. 4.2.3 using the appropriate values of $k_1, k_2$, and $k_3$.

The steel force after load reversal is no longer uniquely related to the strain but is also related to the strain history. In App. C.2 the stress is completely defined in terms of strain and initial plastic strain. The initial plastic strain is defined as the residual strain which remains between points of successive zero stress. Thus when the load is reversed, it is necessary to determine the residual strain in both the tension and compression steel at the point when the load is reversed. The unloading stiffness of the steel, rather than being equal to the first loading stiffness from zero stress to yield, is considered to be a function of the maximum strain from which the steel is unloaded as implied by Morrow. Based on measured response, this residual strain is assumed to be 80% of the peak strain in the previous direction of loading.

The tension and compression forces in the steel shown in Fig. 4.4 are determined from the relationships given in App. C.2 with an initial plastic strain of 80% of the peak strain in the previous cycle. The area occupied by the compression steel has already been considered in the determination of the confined concrete force and therefore the concrete stress must be removed as in Eq. 4.14 for $C_s$. The moment and curvature are given by expressions similar to those given in Sec. 4.2.3.

$M_r = C_1 l_1 + C_2 l_2 - C_3 l_3 + C_s l_s \quad 4.21$

$\phi_r = \frac{\varepsilon c}{k_r d} \quad 4.22$

The numerical values, however, are quite different because the forces required for equilibrium change significantly.
The rotation and end deflection computations for the first load reversal must in some way include the effect of the load history. The beam and the anchorage zone have both been damaged as a result of the previous loading. The approach taken in this analysis is to assume that a portion of the beam has been "softened" by the previous loading. Observation of the test specimens indicates that the beam is damaged considerably in the region of highest moment for a length which coincides approximately with the yielded length in the previous direction of loading. The length of the softened region measured from the fixed end is

\[ \gamma = L \left( 1 - \frac{M_y}{M_m} \right) \]  \hspace{1cm} (4.23)

where \( M_m \) = maximum moment to which the beam has previously been subjected.

\( M_y \) = yield moment in the same direction of loading that produced \( M_m \).

The computed maximum curvature is assumed to act over this length \( \gamma \) or over the length \( \beta + d/2 \) (where \( \beta \) is defined in Eq. 4.18) whichever is greater. In this way the curvature is distributed over the entire yielded length as computed for the previous direction of loading \( \gamma \) unless the load in the secondary direction produces a yielded length which exceeds \( \gamma \) (Fig. 4.6). If \( \gamma > \beta + d/2 \), the curvature computed by Eq. 4.21 is assumed to be distributed as shown in Fig. 4.3. The centroid of the rotation computed from the plastic curvatures is assumed to act at the interface of the beam and the fixed end. The end deflection based on the assumed curvature distribution is
\[
\Delta_E = \theta_{FE}L + \phi_r \gamma L + \phi_y \frac{(L - \gamma)^3}{3(L - \beta)}, \quad \gamma \geq \beta + \frac{d}{2}
\]

\[
\Delta_E = \theta_{FE}L + \phi_r (\beta + \frac{d}{2}) + \phi_y \left[ \frac{(L - \beta - \frac{d}{2})^3}{3(L - \beta)} \right], \quad \gamma < \beta + \frac{d}{2}
\]

and is measured from the deflection at zero load.

The rotation at a point along the beam is

\[
\theta = \frac{\int_0^x \phi_r \, dx + \theta_{FE}}{x}
\]

Comparisons between computed and measured quantities appear in Chapter 5.

4.3.3 Analysis of Subsequent Load Reversals

The procedure for computing the behavior in all subsequent load reversals is almost the same as for the first reversal. The only difference is the determination of the effective concrete cross-section. Test results indicated that the first complete cycle of loading causes the concrete cover to spall leaving the cross-section shown in Fig. 4.7. Only two concrete forces must be considered, one for the confined and one for the unconfined concrete. The resultant force in the unconfined concrete is developed over the shaded area outside the stirrup shown in Fig. 4.7. The values of \(k_1k_3\) and \(k_2\) are obtained from App. A.3 using the maximum fiber strain and crack strain at the level of the top of the concrete core. The force is

\[
C_2' = k_1k_3 f_c' (b - b'')(k_r d - d' + \frac{D}{2})
\]

with a moment arm

\[
l_2' = d - d' + \frac{D}{2} - k_2 (k_r d - d' + \frac{D}{2})
\]
The values of \( k_1 k_3 \) and \( k_2 \) for the confined concrete force bounded by the stirrups are given in App. A.3 using the same values for outer fiber strain and crack strain used in the computation of \( C'_2 \). The confined concrete force is

\[
C'_1 = k_1 k_3 f_1 b'' \left( k_r d - d' + \frac{D}{2} \right)
\]

where \( f_1 \) is defined in App. A.2.

The moment arm of \( C'_1 \) with respect to the tension steel is

\[
l'_1 = d - d' + \frac{D}{2} - k_2 \left( k_r d - d' + \frac{D}{2} \right)
\]

The steel forces are determined exactly as in Sec. 4.3.2 in which the Bauschinger effect is considered.

The moment of the concrete forces and the compressive steel force about the tension steel is the moment on the cross-section.

\[
M'_r = C'_1 l'_1 + C'_2 l'_2 + C'_s l'_s
\]

The corresponding curvature is

\[
\phi'_r = \frac{e_c}{k_r d}
\]

End deflection and rotation of the beam are determined as in Sec. 4.3.2. Comparison between computed and measured quantities is discussed in Chapter 5.
CHAPTER 5 - COMPARISON OF MEASURED AND COMPUTED RESULTS

5.1 Introduction

The method of analysis proposed in Chapter 4 to predict load-deflection and load-rotation behavior was used to predict the response of the specimens tested in this investigation. The correlation between computed and measured results is discussed in this chapter. Differences between measured and computed results are examined. For comparison, the computed response is plotted on the measured load-deflection and load-rotation curves presented in Chapter 3. Since the analyses were of three types (one-directional loading, first load reversal, and subsequent load reversals), the following discussion will conform to the three classifications.

5.2 One-Directional Loading

As mentioned in Chapter 4, the analysis developed to predict the behavior of specimens under one-directional loading is valid for the response under monotonic loading, the upper envelope of the repeated loading tests, and the first cycle, primary direction of the reversed loading tests.

Comparison of the computed and measured load-deflection and load-rotation curves indicates the following trends:

1. In general, the measured load-deflection and load-rotation curves are in good agreement with computed values. The best correlation between measured and computed values is for specimens with 60 inch spans and 5 inch stirrup spacings.

2. In the RP type loading, the computed values of load exceed the measured values for large end deflections (Figs. 3.5, 3.7, 3.9, 3.11). The lower measured load carrying capacity of the specimen
is attributed primarily to bond deterioration in the beam and the anchorage zone.

3. For specimens with 2 in. stirrup spacing, the computed load is generally underestimated (Figs. 3.15, 3.16, 3.29, 3.30). The additional strength of the concrete resulting from confinement by the stirrups (as proposed in App. A.2) may be underestimated. The reduced stirrup spacing may also decrease the shear deformations.

4. The computed end rotation at the peak load of specimens under reversed loading generally exceeds the measured value (Figs. 3.25, 3.26, 3.39, 3.40, 3.43). In the analysis, the measured peak end deflection of each cycle was used to determine the corresponding load and end rotation. Hence the end rotation at the peak, unlike the end deflection, depends on the assumptions made in the analysis. The measured end deflection consists of both flexural and shear deformations. However, shear deformations are not considered in the analysis and therefore the rotations required to compute a given end deflection are overestimated. Thus the specimens with 30 in. spans showed the greatest error in computed end deflection because of the large shear deformations.

5. The analysis cannot be used to predict failures of the specimens. Failures observed in the test specimens were usually shear-compression-buckling failures. The axial load carrying capacity of reinforcing bars after buckling eventually decreases, however, the analysis does not incorporate post-buckling behavior (App. C.3). Secondly, the analysis does not include shear effects.
5.3 First Load Reversal

A separate analysis of the first load reversal is necessary because of the unique behavior exhibited by the specimens at this stage of loading. The concrete subjected to compression is intact but the flexural tensile cracks formed in the concrete during the primary direction of loading must close before the concrete carries compressive stresses.

In comparing measured results to computed results, the following trends are observed:

1. The general non-linear shapes of the curves (both load-deflection and load-rotation) are predicted.

2. The computed values of load consistently exceed the measured values. With larger end deflections, agreement between the measured and computed loads improves.

3. Concavities observed in the load-deflection curves of specimens with 30 in. shear spans are not predicted (Figs. 3.17, 3.18, 3.31, 3.32). It is realistic to assume that the concavities are a result of increased shear forces and large shear deformations. Although shear deformations are not included in the analysis for any of the test specimens, only specimens with 30 in. shear spans exhibited sufficient shear deformations to produce a noticeable deviation between computed and measured results at this stage of loading.

4. Computed end rotations are consistently higher than those measured for reasons cited previously.
5.4 Subsequent Load Reversals

After the first cycle of loading both the top and bottom concrete cover was spalled in the plastification zone. The measured load-deflection and load-rotation curves began to stabilize (i.e. reproduce the previous cycle) after the second cycle. With increased number of cycles a gradual reduction in load was observed in most specimens. Although the analysis was developed primarily to predict the behavior in the second cycle, the computed second cycle response is compared with the primary and secondary direction of loading for subsequent cycles since the observed behavior was reasonably stable. Only the primary direction of the second cycle is plotted in the figures.

Comparison of computed and measured results reveals the following trends:

1. The general shape of the second cycle load-deflection and load-rotation curves for the specimens with 60 in. spans and equal reinforcement, top and bottom, is predicted. With additional cycles the measured and computed curve began to deviate as shear deformations become more pronounced.

2. The concavities observed in the load-deflection curves for specimens with 30 in. shear spans are not predicted because of the large shear deformations (Figs. 3.17, 3.18, 3.31, 3.32).

3. A concavity is predicted in the load-deflection curves for the 86 specimens although it does not conform precisely to the observed results (Figs. 3.41, 3.42). The difference is attributed to the assumption that no force is carried by the concrete until residual cracks close and that cycling does not change the stress-strain relationship after the crack has closed.
4. The computed load in specimens with 2 inch stirrup spacing is slightly underestimated for the reasons cited in Sec. 5.2.

5. End rotations are overestimated because shear deformations are not considered in the analysis (Figs. 3.25, 3.26, 3.39, 3.40, 3.43).

5.5 Load and Energy Ratio

The ability to dissipate energy is a requirement for structures in seismic zones. The response of a structure to an exciting frequency equal to or near one of its natural frequencies is substantially reduced with increased damping provided by the energy absorbed in the hysteresis loops. Hence it is desirable that a structure continue to absorb energy through the duration of the earthquake. The forces which the structure is able to withstand as cycling continues will determine whether or not the structure will fail under the loads generated by the earthquake. The load and energy ratios presented in this section show how the variables considered in the test program affect the response of a member subjected to reversed cycling to failure.

As mentioned in the previous section, the analysis of the second cycle of loading is used to predict the behavior of the specimens in all subsequent cycles. However, observation of the measured load-deflection and load-rotation curves shows that both the maximum load and the energy absorption (hysteresis loop) decreases with cycling. To determine the response of a structure subjected to an earthquake, it is essential to know the load carrying capacity of its members and, as a measure of the damping, the energy absorbed with each cycle. In order to show quantitatively the decrease in load and energy absorption
with cycling, load and energy ratios were computed. The load ratio $P_m/P_c$ (Figs. 5.1-5.9) is defined as the ratio of the measured peak load in a given cycle to the computed load using the analysis for subsequent load reversals (Sec. 4.33). If the end-deflection between zero load and peak load were constant, $P_c$ would be constant, and the load ratio would be directly proportional to measured peak loads. However, because the end-deflection at zero load and peak load vary, $P_c$ is not constant with each cycle.

The energy ratio $E_m/E_c$ (Figs. 5.1-5.9) is defined as the ratio of the area under the measured load-deflection curve for any given half-cycle of loading to the area under the computed curve using the analysis for subsequent load reversals. The areas were obtained by using a planimeter to integrate the measured load-deflection curves and by integrating the computed load-deflection curves using the trapezoidal rule. The computed energy $E_c$ would always be constant if end deflection were constant between zero load and peak load as cited in the previous paragraph.

The purpose of the load ratio and energy ratio curves is to show the effect of cycling on the behavior of the specimens. Since the response in the first cycle is not duplicated in subsequent cycles, the abscissa begins with the second cycle. Ideally, for the second cycle both ratios should be equal to unity; the deviation from unity is a measure of the accuracy of the proposed analysis. The rate of decay of both curves is a measure of the effect of cycling on the load carrying and energy absorbing capacity of the specimen. The points which are plotted in Fig. 5.1-5.9 are obtained from selected cycles when the specimen demonstrated a deviation in behavior from
the previous cycle. The straight lines connecting the chosen points are a reasonable approximation to the ratios at intermediate cycles which were omitted.

Inspection of the load ratio and energy ratio curves reveals the following:

1. In specimens with #8 bars (Figs. 5.5-5.8) decay occurs at a faster rate than in specimens with #6 bars (Figs. 5.1-5.4).

2. After 10 cycles of RV5 loading, in all 60 in. span specimens both ratios were in excess of 80% of the cycle 2 ratios. The discontinuous behavior (increase from cycle 10 to 11) of the curves observed in the tenth cycle are a result of increasing the end-deflection from $5\Delta_y$ to $7\Delta_y$ as explained in Sec. 3.4.1.

3. Only specimens with 2 in. stirrup spacings were able to withstand 10 or more cycles of RV10 loading (Fig. 5.3, 5.7). For these specimens, both ratios were in excess of 75% of those of cycles 2 through 10 cycles.

4. Specimens with 30 in. shear spans exhibited a more rapid decay of both curves than those with 60 in. spans under comparable load histories (Figs. 5.4, 5.8). Only specimen 88-34-RV5-30 (Fig. 5.8a) was able to withstand more than 10 cycles of loading and showed substantially more decay in both curves than did the 60 in. span specimens. For the specimens with 30 in. spans, the decay of the energy ratio is more severe than the load ratio.

Observation of the load-deflection curves of the specimen with 30 in. spans (Chap. 3) indicates that the concave shape of the curve reduces the energy absorbed whereas the load is eventually regained at the peak of the cycle.
5. For specimens with 2 in. stirrup spacing under R\(V5\) loading, both ratios were at about 75% of that in the second cycle after 20 cycles of loading (Figs. 5.2, 5.6). Closely spaced stirrups substantially increased the number of cycles through which the specimens maintained a large portion of its original load carrying and energy absorbing capacity.

6. Specimens with unequal reinforcement (86) decayed at a faster rate in the direction of loading producing tension in the #8 bar (primary direction, Fig. 5.9). The forces on the specimen in the primary direction were greater than those in the secondary direction, producing more rapid concrete deterioration.

From the preceding observations based on the curves shown in Figs. 5.1-5.9, the number of cycles to failure and, consequently, the energy absorbing capacity of the members is increased if (a) the stirrup spacing is decreased, (b) the moment-shear ratio is increased, (c) the percentage of reinforcement is decreased.

In view of the limited number of tests performed in this investigation, a specific stirrup spacing cannot be recommended. However, it is clear that spacing stirrups (at least in the plastification zone) closer than required for shear considerations only will substantially improve the life of the structure under load reversal. Although the moment-shear ratio often cannot be controlled by the designer, he should recognize that large shear forces will reduce the life of a member under load reversals. Finally the life of the member is improved if the percentage of reinforcement (both p and p')
can be kept to a minimum. In some cases this may be possible by increasing the cross-section of the beam.

5.6 Concluding Remarks

In summary the method used in predicting the behavior of the specimens gives reasonably good correlation between measured and computed values of the load-deflection and load-rotation curves. The most important limitations of the proposed method are the inability to compute shear deformations, to predict failure, and to predict the effect of repetitions of reversed cycling of loads after the second cycle. The method does predict the behavior of the first two cycles reasonably well and appears to be a sound basis on which to build a more refined model which can overcome these limitations.

The load ratio and energy ratio curves indicate the effect of steel percentage, moment-shear ratio, and stirrup spacing on the ability of the specimen to maintain load carrying and energy absorbing capacity.
CHAPTER 6 - SUMMARY AND CONCLUSIONS

6.1 Object and Scope

The purpose of this investigation was to determine experimentally the response of reinforced concrete cantilever beams to cyclic load histories and to develop a method to predict the response under the prescribed load histories.

The test specimen was a cantilever beam cast monolithically with an enlarged end block into which the longitudinal reinforcement was anchored. The cross-sectional dimensions of the beam portion of the specimen were 6 in. x 12 in., the enlarged portion, 10 in. x 18 in. The influence of the following factors on the response of the test specimen under load reversal were studied: percentage of top and bottom reinforcement, stirrup spacing, and shear span. Three different arrangements of longitudinal reinforcement were used: #8 bars top and bottom, #6 bars top and bottom, and #8 bars top and #6 bars bottom. In 16 specimens the stirrup spacing was 4 in. or 5 in. as required for shear capacity, and in 4 specimens the spacing was reduced to 2 in. to provide additional concrete confinement. For 16 specimens the span of the cantilever beam was 60 in., and in 4 specimens the span was reduced to 30 in. Twenty specimens were tested, 2 under monotonic loading, 4 under loads repeated in one direction, and 14 under cyclic reversed loads. A description of the load histories is as follows:

Monotonic-loading in one direction to failure

Repeated-loading in one direction with an increment of deflection equal to either one or four times the end deflection at yield added in each cycle.
Reversed-cycles of loading to a constant end deflection in each
direction of either five or ten times the yield deflection.

Instrumentation was designed to measure load, end deflection, end
rotation, and the distribution of rotation along the beam and at the
face of the end block.

In order to develop a method to predict the response of the test
specimens under the prescribed load histories (Chap. 4), the results of
tests performed by other researchers on both steel and concrete under
cyclic loads were used to develop the response of the beam cross-section
to cyclic loads. Other factors such as the length of the plastic zone
and the unloading stiffness of the composite section were adjusted to
produce good correlation with test results. The contribution of
deformations of the anchored bars to that of the specimen were measured
and a set of empirical equations developed to include this effect in
the analysis.

6.2 Summary of Test Results

Monotonic and Repeated Loading

The cycling of load in one direction produced little if any loss of
ductility compared with monotonically loaded specimens. The monotonic
load-deflection curve was found to be a good approximation to the envelope
of the load-deflection curve under repeated loading. In those specimens
in which failure occurred, deformations in the anchorage zone were less
than in the specimens which did not fail. The beam portion of the
specimen was subjected to greater deformations in order to achieve a
given end deflection. Therefore, the ductility of the anchorage zone
was an important factor in the overall ductility of the specimen. The
mode of failure (in cases where failure occurred) was shear compression.
Reversed Loading

The behavior of the test specimens after load reversal differed significantly from that of specimens loaded in one direction. Both the load-deflection and load-rotation relationships were non-linear because of the Bauschinger effect (App. C.2), shear deformations (App. D), closing of residual concrete cracks (App. A.3), and non-linear load-slip relationships in the anchorage zone (App. B.2).

Of the two reversed load histories used, the type with larger deflection limits (RV10, Sec. 2.2.1) produced failure with fewer cycles than did the RV5 load history on specimens with the same section geometry (Fig. 3.27 vs. 3.28).

A larger percentage of reinforcement (p or p') reduced the number of cycles to failure apparently by producing greater compressive and shear forces in the concrete (Fig. 3.14 vs. 3.28).

Closely spaced stirrups significantly increased the number of cycles to failure by increasing the shear capacity of the specimens (Fig. 3.30 vs. 3.28).

A short shear span reduced the number of cycles to failure in every case because of the increased shear forces (Fig. 3.32 vs. 3.28). The shapes of both the load-deflection and load-rotation curves for the shorter spans were altered resulting in reduced energy absorption in each cycle.

The mode of failure of all specimens with equal top and bottom reinforcement appeared to be in shear across planes of weakness which were the result of load reversals and would not otherwise be expected under one directional loading. Shear failure was followed by buckling of the compression bars usually between adjacent stirrups. Failure was
initiated by buckling of the compression steel (smaller bars) only in specimens with unequal top and bottom reinforcing steel.

6.3 Prediction of Response Under Applied Loads

The method developed to predict the behavior of the specimens gave reasonably good correlation between measured and computed values of the load-deflection and load-rotation curves. The computed values obtained for the monotonically loaded specimens were a good approximation to the specimens under repeated loading. The general non-linear shape and magnitude of the load-deflection and load-rotation curves of specimens subjected to load reversals was predicted for the first two cycles. The effect of subsequent cycles was considered quantitatively by the rate of decay with cycling of the load and energy absorbing capacity of the specimens. These measured loads and energy absorbing capacities in a given cycle were normalized using the computed load and energy absorbed in the second cycle and presented in a nondimensional form. The resulting load and energy ratio curves (Sec. 5.5) indicated that the number of cycles of reversed loading to failure was increased if the stirrup spacing decreased, the moment-shear ratio increased, or the percentage of top or bottom reinforcement decreased. The change in both the load and energy ratio provided a measure of the decay in load and energy absorption with number of cycles.

The most important limitations of the proposed method were the inability to predict shear deformations or to predict failure.

6.4 Conclusions

The most significant conclusions based on the results of this investigation are:

1. Cycling of load in one direction has little effect on the load or
deformation capacity of the specimens. The upper envelope of the load-deflection and load-rotation curves of specimens under one directional cyclic loading may be approximated by those obtained for monotonic loading.

2. The behavior of test specimens under load reversal is markedly non-linear due to a combination of the Bauschinger effect in the steel, shear deformations, closing of residual concrete cracks, and non-linear load-slip behavior of the anchored steel.

3. The ability of the test specimen to maintain load and energy absorbing capacity under cyclic load reversals is significantly improved if the stirrup spacing is reduced, the moment-shear ratio is increased, or the percentage of either top or bottom steel is reduced. All of these factors appear to be related to the demands made of the concrete in shear.

4. Deformations of the steel in the anchorage zone contribute significantly to the total deformation of the test specimen. Hence the ductility of the structure is affected by the ability of the joint to deform in a ductile manner.

5. The load-deflection and load-rotation curves can be predicted with reasonable accuracy for the first two cycles of load reversal. The load and energy ratio curves (Sec. 5.5) give a quantitative indication of the effect of subsequent cycles.

For designing structures in seismic zones, the following general guides are indicated by this investigation:

The percentage of reinforcing (both top and bottom) should be kept as low as possible.
The moment-shear ratio (where some adjustment is possible) should be as large as possible.

Provision of stirrups at a spacing smaller than required for shear alone significantly improves the performance of a structure under load reversal at a relatively small cost.

The results of this test program give an indication of the effect of stirrup spacing, moment-shear ratio, and percentage of reinforcement. More tests should be performed using a wider range of values of the variables considered to provide quantitative relationships between the variables and the load history. In addition, the effect of axial load should be included in future research.
LIST OF REFERENCES


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* * denotes compression steel properties (bottom bars)

L = beam span, inches  
\( s = \) stirrup spacing, inches  
\( f_y = \) yield stress, ksi  
\( f_u = \) ultimate stress, ksi  
\( A_s = \) steel area, \text{in}^2  
\( \epsilon_{sh} = \) strain hardening strain; \text{in/in} \times 10^3  
\( f_c^* = \) concrete cylinder strength  
\( E_s = 29,000 \) ksi, Steel elastic modulus  
\( \epsilon_u = 0.02, \) steel fracture strain
**FIG. 2.3 - LOAD HISTORIES**

**a) Monotonic loading**
- Load: $\Delta y$
- Deflection

**b) Repeated loading**
- Load: $\Delta y$
- Deflection
- $\text{RPI - A} = \Delta y$
- $\text{RP4 - A} = 4\Delta y$

**c) Repeated Reversed Loading**
- Load: $\Delta y$
- Deflection
- $\text{RV5 - B} = 5\Delta y$
- $\text{RV10 - B} = 10\Delta y$
- Yield

Fig. 3.4 - Load vs. End Rotation, 88-35-M
Fig. 3.7 - Load vs. End Deflection, 66-35-RP1
Fig. 3.8 - Rotation Distribution, 66-35-RP1
Fig. 3.10 - Rotation Distribution, 88-35-RP4
Fig. 3.17 - Load vs. End Deflection, 66-35-RV5-30
Fig. 3. - Load vs. End Deflection, 66-35-RV10-30

- Measured
- × Computed, Cycle 1
- △ Computed, Cycle 2
Fig. 3.19 - Rotation Distribution at Cycle Peaks, 66-35-RV5
Fig. 3.22 - Rotation Distribution at Cycle Peaks, 66-32-RV10
Fig. 3.25 - Load vs. End Rotation, 65-35-1W10
Fig. 3.27 - Load vs. End Deflection, 88-35-RV5

- Measured
- Computed, Cycle 1
- Computed, Cycle 2
Fig. 3.28 0 Load vs. End Deflection, 88-35-RV10
Fig. 3.31 - Load vs. End Deflection, 88-34-RV5-30

- Measured
- × Computed, Cycle 1
- Δ Computed, Cycle 2
Fig. 3.32 - Load vs. End Deflection, 88-34-RV10-30
Fig. 3.34 - Rotation Distribution at Cycle Peaks, 88-35-RV10
Fig. 3.38 - Rotation Distribution at Cycle Peaks, 88-24-RV10-30
Fig. 2.35 - Rotation Distribution at Cycle Peaks, 86-35-RV10
Fig. 3.47 - Failure
(a) Equilibrium at Yield

(b) Curvature Distribution at Yield

Fig. 4.1 - Conditions at Yield
Fig. 4.2 - Stress and Strain for Monotonic Post-Yield Conditions
Fig. 4.3 - Moment and Curvature Distribution, Monotonic Post-Yield
Fig. 4.4 - Unloading Stiffness vs. End Deflection

\[ \frac{k_r}{k_y} = \left( \frac{d_y}{d} \right)^{0.2} \]
Fig. 4.6 - Curvature Distribution, Reversed Loading
Fig. 5.1 - Load and Energy Ratio, 66-35-RV5, 66-35-RV10
Fig. 5.5 - Load and Energy Ratio vs. Cycle, 88-35-RV5, 88-35-RV10
Fig. 5.9 - Load and Energy Ratio vs Cycle, 86-35-RV5, 86-35-RV10
APPENDIX A - CONCRETE BEHAVIOR

A.1 Unconfined Concrete

The stress-strain relationship for unconfined concrete subjected to a strain gradient was studied by Karsan\textsuperscript{1} using plain concrete specimens. He proposed the following expressions for stress-strain with the non-dimensionalized equations

\[ F = 0.85 S e^{(1-S)} \quad 0 \leq S \leq 1.1 \quad \text{A.1} \]

\[ F = 0.931 - 0.077S \quad S > 1.1 \quad \text{A.2} \]

where

\[ F = \frac{f_c}{f_1} \]

\[ f_1 = \text{maximum concrete strength} \]

\[ S = \frac{\varepsilon_c}{\varepsilon_o} \]

\[ \varepsilon_o = \text{strain at maximum concrete stress taken as } 0.0017 \text{ in this study} \]

Equations A.1 and A.2 are shown graphically in Fig. A.1*. The abscissa of Fig. A.1 is in units of strain which can easily be obtained by the expression

\[ \varepsilon_c = S \varepsilon_o \quad \text{A.3} \]

In the case of unconfined concrete, \( f_1 = f'_c \).

If the area under the non-dimensionalized stress-strain curve is divided by \( S \), the average stress \( f_{cu} \) over the concrete compressive zone is obtained. The ratio of the average concrete stress to the maximum

* The definition of \( \varepsilon_{cr} \) in Fig. A.1 is given in App. A.3 and is equal to zero for the stress-strain curves discussed in App. A.1 and A.2.
concrete strength is defined as follows:

\[ k_1 k_3 = \frac{f_{cu}}{f_1} = \frac{\int_0^S F \, dS}{S} \quad \text{A.4} \]

Substituting Eqs. A.1 and A.2 into Eq. A.4, the following expressions are obtained:

\[ k_1 k_3 = \frac{0.85 e}{S} \left[ 1 - \frac{S + \frac{1}{e}}{S} \right], \quad 0 \leq S \leq 1.1 \quad \text{A.5} \]

\[ k_1 k_3 = \frac{-0.283 + 0.931 S - 0.0385 S^2}{S}, \quad S > 1.1 \quad \text{A.6} \]

The resultant concrete force for a given external fiber strain is determined by applying the average stress over the area of compressed concrete.

\[ C = f_{cu} \times \text{Area} = k_1 k_3 f_1 \times \text{Area} \quad \text{A.7} \]

The location of the centroid \( S_c \) of the compressive force with respect to the point of zero strain may be obtained by area-moment as follows:

\[ S_c = \frac{\int_0^S F \times S \, dS}{\int_0^S F \, dS} \quad \text{A.8} \]

If \( k_2 \) is defined as the centroid of the compressive force with respect to the point of maximum non-dimensionalized strain \( S \) divided by \( S \), the following equation is obtained.

\[ k_2 = 1 - \frac{S_c}{S} \quad \text{A.9} \]

By substituting Eqs. A.1, A.2, and A.8 into Eq. A.9, the following expressions for \( k_2 \) are determined.

\[ k_2 = 1 - \frac{2}{S} + \frac{0.85 e}{k_1 k_3} \frac{1-S}{S}, \quad 0 \leq S \leq 1.1 \quad \text{A.10} \]
\[ k_2 = \frac{-0.07 + 0.4655S^2 - 0.0257S^3}{k_1k_3S^2}, \quad S > 1.1 \quad \text{A.11} \]

Values of \( k_1k_3 \) and \( k_2 \) are plotted against the external fiber strain in Fig. A.2. It was assumed that the concrete stress was zero beyond a 1\% strain. Observation of the test specimens indicated that the concrete began to spall when the maximum compressive strain was approximately 1\%. At an end deflection of about four times the yield deflection, the entire concrete cover was not intact due both to concrete crushing failure and to splitting of the concrete at the level of the compressive steel. The splitting is thought to be induced by slip between the steel and the concrete which creates tension in the concrete through a wedge effect.

\[ \text{A.2 Confined Concrete} \]

The strength and ductility of concrete subjected to a strain gradient is increased when confined by stirrups and compressive steel. An approximation to the stress-strain curve as a function of the stirrup spacing has been proposed by Yamishiro\(^2\) on the basis of tests performed at the University of Illinois. Yamishiro's approximation is shown in Fig. A.1 for 3 in. and 6 in. spacing of \#3 stirrups. Additional curves are extrapolated from those presented by Yamishiro to approximate the effect of the stirrup spacings used in this investigation.

Another effect of the confinement provided by the stirrup is to increase the concrete strength. This increase is a function of the geometry of the confined concrete cross-section as well as the stirrup spacing and does not consider the confinement provided by the compressive steel. Yamishiro introduced the following expression for the increase
in concrete strength:

\[ \Delta f_c = \frac{6000}{1 + \frac{2300}{f_2}} \text{ PSI} \quad \text{A.12} \]

where \( f_2 \) is a measure of the confining stress given by the following equation:

\[ f_2 = \frac{A_s f''}{s} \left( \frac{1}{2c} + \frac{1}{b''} \right) \quad \text{A.13} \]

where

\( s = \) stirrup spacing, in.

\( A_s = \) cross-sectional area of the stirrup, in.\(^2\)

\( f'' = \) yield stress of the stirrup, psi

\( c = \) depth of confined concrete taken as the distance from the neutral axis to the bottom of the stirrup.

\( b'' = \) width of confined concrete enclosed by the stirrup.

The maximum concrete stress \( f_1 \) used in Fig. A.1 for confined concrete is

\[ f_1 = f'_c + \Delta f_c \quad \text{A.14} \]

The numerical value of \( c \) used in Eq. A.13 for a cross-section with substantial compressive reinforcement can be quite small thus producing an unreasonably large increase in the computed concrete strength. Since the beams in this test program had large \( p'/p \) ratios, the value of \( c \) was limited to no less than the diameter of the compressive reinforcement. With this limit of \( c \) the increase in concrete strength for \#3 stirrups at different spacings is approximately
\[ s = 5 \text{ in.} \quad \Delta f_c = 0.25 f'_c \]
\[ s = 4 \text{ in.} \quad \Delta f_c = 0.30 f'_c \]
\[ s = 2 \text{ in.} \quad \Delta f_c = 0.40 f'_c \]

In view of the apparent limitation of Eq. A.12 and A.13, the values of \( \Delta f_c \) in Eq. A.15 are used.

With the values of \( k_1 k_2 \) and \( k_2 \) determined as discussed in App. A.1 (plotted in Fig. A.2), the magnitude and location of the resultant concrete force can be determined as a function of the outer fiber strain, the neutral axis, and the specimen geometry.

### A.3 Cracked Concrete

The only modification necessary to alter the stress-strain relationship for concrete with a residual tensile crack is to introduce a shift equal to the crack width \( \varepsilon_{cr} \) as shown in Fig. A.1. The effect of \( \varepsilon_{cr} \) on \( k_1 k_2 \) and \( k_2 \) shown in Fig. A.2 is to shift these relationships by an amount equal to \( \varepsilon_{cr} \).
Fig. A.2 - Concrete Effective Stress and Location of Centroid
APPENDIX B - BOND CHARACTERISTICS IN THE ANCHORAGE ZONE

B.1 Monotonic Loading

Deformations in the anchorage zone of the test specimen contribute significantly to the rotation and end deflection of the beam. The load-slip characteristics of the tension reinforcement at the interface between the beam and the enlarged anchorage zone are shown in Fig. B.1 through B.3. The ordinate T is an approximation of the tensile force (both bars) based on the assumption that the compressive force acts at the centroid of the compressive steel. Both theoretical analysis and transducer data indicate that this assumption is realistic because of the large compressive steel area. Hence

\[ T = \frac{M}{(d-d')} \]

The manner in which slip \( \Delta_{FE} \) is measured is discussed in detail by Ismail.\(^8\)

Prediction of the contribution of deformations in the anchorage zone to rotation and end deflection of the beam is simplified if the load-slip relation for all the specimens can be formulated. Inspection of the load-slip curves indicates a nearly bilinear behavior. The original slope corresponds to pre-yield conditions which are not of interest in this study but are discussed in greater detail by Ismail. Hence only the second slope corresponding to post yield behavior is of interest. Inspection of the slope of the post-yield load-slip curves indicates a noticeable difference between the 88-35 series of specimens and all other specimens with #8 bars in tension. The difference may be attributed to variations in the normal forces provided in the anchorage

- B.1 -
zone. In the 88-35 series, the normal force was not measured, whereas in all subsequent tests it was held constant at about 1000 psi.

Construction of the best straight line through the post yield data points for each specimen led to the conclusion that all of the specimens may be classified into three groups without sacrificing significant accuracy. The post-yield load-slip curve takes the form of the following equation for a straight line

$$ T = m \Delta_{PE} + T_0 $$

where

- $m$ = slope
- $T_0$ = intercept of the straight line on the $T$ axis

The bond stress capacity permitted in the ACI Building Code\textsuperscript{10} is proportional to $\sqrt{f_c'}/D$. If Eq. B.2 is normalized by this ratio, it takes the form

$$ \Psi = m' \Delta_{PE} + b' $$

where

- $\Psi = \frac{T D}{\sqrt{f_c'}}$
- $m'$ = modified slope of load-slip curve produced by normalization
- $b' = \frac{T_0 D}{\sqrt{f_c'}}$
- $f_c'$ = concrete strength, ksi
- $D$ = diameter of reinforcing bar

The normalization reflects at least some of the properties of the system and should produce better correlation between specimens with different bar sizes and concrete strength.

-B.2-
A good estimate of $b'$ is

$$b' = \frac{A_s f_y D}{\sqrt{f_{c}^*}}$$  \hspace{1cm} \text{B.4}$$

which implies that $T_0$ is the yield force. If Eq. B.3 is solved for $\Delta_{FE}$ and combined with Eq. B.4, the elongation in the anchorage zone becomes

$$\Delta_{FE} = \frac{(T - A_s f_y) D}{m' \sqrt{f_{c}^*}}$$  \hspace{1cm} \text{B.5}$$

The values of $m'$ for the three groups of specimens are

- #8 bars, 5 in. stirrup spacing, \hspace{0.5cm} m' = 25.6
- all other #8 bars \hspace{0.5cm} m' = 45.1
- all #6 bars \hspace{0.5cm} m' = 22.4

The rotation at the interface of the beam and the enlarged anchorage zone may be established from the value of $\Delta_{FE}$ given by Eq. B.5 and the expression

$$\theta_{FE} = \frac{\Delta_{FE}}{d - d'}$$  \hspace{1cm} \text{B.6}$$

which is based on the assumption that the neutral axis is located at the centroid of the compression steel.

The contribution of $\theta_{FE}$ to the end deflection is

$$\Delta = \theta_{FE} L$$  \hspace{1cm} \text{B.7}$$

\textbf{B.2 Repeated Loading}

In the repeated loading history used the end deflection was always increased over the previous cycle. The envelope of the load-slip
relationships for beams under repeated loads was very close to that observed for monotonic loading, and therefore no distinction is made in the load-slip behavior for repeated or monotonic loads. This assumption does not apply to the unloading and reloading conditions but only to points on the envelope of the specimens with repeated loads. Bond characteristics under cyclic loads are discussed in more detail by Ruiz and Winter.\(^4\) Their work indicates that the behavior observed in this investigation would not be true if cycling were performed without significantly increasing the end deflection with each cycle or if flexural cracks significantly affect bond properties.

B.3 Reversed Loading

The load-slip characteristics for the second cycle were measured as discussed in section B.1 and are plotted in Fig. B.4 - B.6. These curves exhibit a non-linear load-slip relationship for all specimens after load reversal. Again a difference in stiffness appears between the specimens with #8 bars with 5 in. stirrup spacing and the other specimens with #8 bars. A parabolic curve through the data points produces the relationships between load and slip as follows:

- **#8 bars, 5 in. stirrup spacing**
  \[ \Delta_{FE} = 0.0005T + 0.000044T^2 \]  
  \[ \text{B.8} \]

- **All other #8 bars**
  \[ \Delta_{FE} = 0.0013T + 0.00001T^2 \]  
  \[ \text{B.9} \]

- **All #6 bars**
  \[ \Delta_{FE} = -0.0002T + 0.00008T^2 \]  
  \[ \text{B.10} \]

Normalization by the factor \( \sqrt{f'_C/D} \) as in App. B.1 did not produce better correlation between specimens than that of Eq. B.8, B.9, and B.10.
The elongation $\Delta_{PE}$ may be obtained from these equations to determine the rotation (Eq. B.6) and end deflection (Eq. B.7) of the beam produced by deformations in the anchorage zone under load reversal.
Fig. B.2 - Load Slip, 8 Bars, 2" and 4" Stirrup Spacing
Fig. B. 4 - Load-Slip, Cycle 2, Primary Direction
Fig. B.5 - Load-Slip, Cycle 2, Primary Direction
Fig. B. 6 - Load vs. Slip, Cycle 2 Primary Direction
APPENDIX C - REINFORCING STEEL

C.1 Virgin Stress-Strain Curve.

The reinforcing steel used in this investigation demonstrated a well defined yield point and a flat yield plateau until the onset of strain hardening. A generalized stress-strain relationship for the tension and compression steel is shown in Fig. C.1. The curve is assumed to be perfectly elasto-plastic until strain hardening begins. A formulation for the strain hardening portion of the curve was proposed by Burns\textsuperscript{3} as

\[
    f_s = f_y \left[ \frac{112 \varepsilon' + 2}{60 \varepsilon' + 2} + \frac{\varepsilon'}{\varepsilon_u - \varepsilon_{sh}} \left( \frac{f_u}{f_y} - 1.7 \right) \right]
\]

where

- \( f_s \) = steel stress at any point beyond strain hardening
- \( f_y \) = yield stress of the steel bar
- \( \varepsilon' \) = strain beyond first strain hardening
- \( \varepsilon_u \) = strain at fracture
- \( \varepsilon_{sh} \) = strain hardening strain
- \( f_u \) = ultimate stress

Thus the stress-strain relationship may be completely defined by \( E_s \), \( f_y \), \( \varepsilon_u \), \( \varepsilon_{sh} \), and \( f_u \). Table 2.1 lists these properties for the reinforcement used in this test program.

C.2 Bauschinger Effect on Stress-Strain Curve.

The loading history significantly influences the stress-strain characteristics of reinforcing steel. Tests on reinforcing steel\textsuperscript{5} indicate that elastic-perfectly plastic behavior is no longer exhibited
upon reversal of loading if the strain in the first direction of loading exceeds yield as shown in Fig. C.2(a). The shape of this nonlinear compressive stress-strain curve is highly dependent upon the residual strain when the loading is reversed. The residual strain will be defined as the absolute value of the strain between sequential points of zero stress and denoted as initial plastic strain, IPS. If segment 2-3 of Fig. C.2(a) is inverted and plotted from the origin as a function of IPS, the family of curves shown in Fig. C.2(b) is obtained. The authors of Reference 5 proposed following the expression for the stress-strain curve of the first stress reversal.

\[ \sigma = 64.5 - 52.7 (0.838)^{1000} \epsilon \]  

C.2

where \( \epsilon \) is measured from the strain at zero stress. The analytical expression which is plotted in Fig. C.3(b) is based upon an IPS of 0.004 and a yield stress of about 53 ksi. Since in this test program the IPS in some cycles is significantly higher than 0.004, Eq. C.2 was modified to include both IPS and yield stress as variables. Before developing this expression, the following terms will be defined.

\[ F_s = f_s/f_y = \text{ratio of steel stress at any point to the virgin yield stress.} \]

\[ \epsilon'_{sh} = \text{strain at which the steel first exceeds the virgin yield stress after reversal.} \]

\[ S_s = \epsilon_s/\epsilon'_{sh} = \text{ratio of steel strain at any point to the effective strain hardening strain.} \]

A physical interpretation of \( \epsilon'_{sh} \) is the strain at which the reversed loading curves of Fig. C.2(b) reach the yield stress. Since
\( \varepsilon_{sh}' \), depends upon the IPS, these values obtained from Ref. 5 are plotted and an empirical relationship derived (Fig. C.3).

\[
\varepsilon_{sh}' = \varepsilon_{sh} \frac{\ln(\text{IPS}/\varepsilon_y)}{1.38}
\]

C.3

Using the data reported in Ref. 5 (Fig. C.2-b) the following empirical expression for the non-dimensionalized stress-strain curve for stresses below yield was derived

\[
F_s = 1 - e^{-2.05S_s} + 0.129S_s
\]

C.4

and is shown in Fig. C.4.

The authors of Ref. 5 recommend the use of the original strain hardening curve when yield is exceeded. Therefore, when the strain in the steel exceeds \( \varepsilon_{sh}' \), the strain hardening curve given by Eq. C.1 is assumed to apply. In this case \( \varepsilon' \) of Eq. C.1 is measured from \( \varepsilon_{sh}' \) rather than from \( \varepsilon_{sh} \), and the value of \( \varepsilon_{sh} \) is replaced by \( \varepsilon_{sh}' \). If the computed value of \( \varepsilon_{sh}' \) is less than \( \varepsilon_{sh} \), Eq. C.3 will apply until the yield stress is achieved after which the virgin stress-strain curve is assumed to govern the behavior (see Fig. C.5 for IPS = 0.004). Some typical stress-strain curves computed from this proposed method are presented in Figs. C.5 and C.6. The unloading curve is assumed to have the same slope as the original loading curve.

In summary, the stress-strain relationship for reinforcing steel subjected to stress reversal may be approximated by

\[
F_s = 1 - e^{-2.05S_s} + 0.129S_s \quad 0 \leq f_s \leq f_y
\]

C.4

where

\[
F_s = \frac{f_s}{f_y}
\]

-C.3-
\[ S_s = \varepsilon_s / \varepsilon_{sh} \]
\[ \varepsilon_{sh} = \frac{\varepsilon_{sh} \ln(\text{IPS} / \varepsilon_y)}{1.38} \]

and

\[ f_s = f_y \left[ \frac{112 \varepsilon' + 2}{60 \varepsilon' + 2} + \frac{\varepsilon'}{\varepsilon_u - \varepsilon_{sh}} \left( \frac{f_u}{f_y} - 1.7 \right) \right], \quad f_s > f_y \quad \text{C.1} \]

where \( \varepsilon' = \varepsilon_s - \varepsilon_{sh} \)

C.3 Buckling of Compression Reinforcement

During the course of this investigation it was observed that after one complete cycle of reversed loading the concrete cover spalls.

Therefore, it is possible that the compressive reinforcement may buckle in the direction unrestrained by concrete. In order to predict the buckling load, the following assumptions are made.

1. The stress-strain relationship for the compressive reinforcement is defined as in App. C.2.
2. Buckling occurs according to the tangent modulus theory.
3. Any buckling occurring will be in the first mode between adjacent stirrups.
4. The compressive reinforcement is restrained against rotation at the stirrups.
5. After buckling is initiated, the compressive force in the steel remains constant.

The third assumption is based on the relative stiffness of the compressive steel and the stirrups. If buckling should occur in higher modes, the stirrups must deflect with the compressive steel. However, analysis shows that the relative stiffness of the stirrup to that of the
compression member is too high to permit buckling in higher modes. An analysis of a similar problem by Ruiz and Winter indicates that a much more flexible stirrup would be required if a higher mode is to control.

The fourth assumption is based upon the restraint provided by the concrete in the core between the adjacent stirrups.

To rotate at A, the bar must deflect downward in span AB. However, the concrete in this region is still intact and resists such deformations.

The post-buckling behavior of a compressive member is dependent on the initial eccentricity of the axial force. The eccentricity is the result of both shear deflection and curvature, the first of which could not be determined in this experimental program. For this reason, the post buckling behavior could not be predicted and assumption 5 was made. Observation of the test specimens at failure showed that in specimens with equal top and bottom reinforcement buckling did not initiate failure but was secondary to a shear failure. Hence, assumption 5 should not affect the prediction of failure loads. If the specimen properties are such that the buckling load is substantially lower than those in this investigation, a more rigorous post buckling analysis would be necessary.

The tangent modulus theory states that a compression member will buckle according to the expression

\[ P = \frac{\pi^2 E_t I}{(kL)^2} \]  

C.5
where

\[ P = \text{buckling force} \]
\[ E_t = \text{slope of the stress strain curve at the buckling stress} \]
\[ I = \text{Moment of inertia about the axis of buckling given by} \]
\[ I = \frac{\pi d^4}{64} \quad \text{C.6} \]
\[ kL = \text{equivalent length of compression member} \]

Assumption 3 and 4 indicate that \( kL \) is one-half the stirrup spacing.

\[ kL = \frac{s}{2} \quad \text{C.7} \]

Dividing Eq. C.5 by the cross-sectional area and combining with Eqs. C.7 and C.8 yields the buckling stress

\[ \sigma = \frac{E_t}{4} \left( \frac{\pi d}{s} \right)^2 \quad \text{C.8} \]

The tangent modulus in the strain hardening zone may be obtained by differentiation of Eq. C.1 with respect to \( \varepsilon' \).

\[ E_t = f_y \left[ \frac{(60\varepsilon' + 2) 112 - (112\varepsilon' + 2) 60}{(60\varepsilon' + 2)^2} + \frac{f_u - 1.7}{e_u - \varepsilon_{sh}} \right] \quad \text{C.9} \]

It is possible in some cases that the bar will buckle at a stress lower than yield in which case Eq. C.4 must be used to determine \( E_t \). However, in this test program buckling occurs at a stress in excess of yield and Eq. C.9 controls. When the stress computed by Eq. C.8 is less than that of the stress-strain diagram (Eq. C.1), buckling has occurred and the stress is assumed to remain constant for increasing strains. A graphical solution of Eq. C.8 and C.1 is shown in Fig. C.7 for 5 in. stirrup spacing and both #6 and #8 compression steel bars. In the specimens with 4 in. and 2 in. stirrup spacing, buckling would be predicted at strains in excess of those which would occur in the specimen.
Fig. C.1 - Generalized Steel Stress-Strain Curve
Fig. C.3 - IPS vs. $\varepsilon'_{sh}$

$\varepsilon'_{sh} = \varepsilon_{sh} \ln \frac{\text{IPS}}{\frac{\varepsilon_y}{1.38}}$

Data Points (Ref. 5)
Fig. C.4 - Proposed Stress-Strain Relationship, Reversed Loading

F = 1 - e^{-2.05S_S + 1.29S_S}

\frac{\lambda}{S} = \rho

S_S = \frac{\varepsilon_S}{\varepsilon_{sh}}
Fig. C.5 - Effect of IPS on Reversed Stress-Strain Curve
Strain-hardening portion of virgin stress-strain curve

\[ f_y = 45 \text{ ksi} \]
\[ s = 5 \text{ inches} \] (s=stirrup spacing)

Fig. C.7 - Steel Stress vs. Strain, Buckling Solution
APPENDIX D - SHEAR

In the case of monotonic or repeated loading, the shear capacity of a member may be determined using the free-body diagram shown in Fig. D.1(a). The diagonal crack conforms to lines of principal tensile stress and intersects properly spaced stirrups. When the load is reversed, however, it is possible that the flexural tensile cracks produced by loading in both directions will intersect. If this occurs a free-body diagram for the resulting shear capacity is shown in Fig. D.1(b). The crack intersects no stirrups requiring all the shear force to be transferred by the dowel forces \( V_d \) and by the shear force in the concrete at the cracked section.

When the beam has been loaded upward and then unloaded to zero load, cracks along the bottom of the beam remain open even at zero load. If the load is reversed, the beam must undergo sufficient rotations to close the cracks before the shear force \( V_c \) becomes fully mobilized. Until the crack is closed the shear must be transferred by dowel forces (Fig. D.1-b) and by aggregate interlock. A simplified model of the shear transfer mechanism is shown in Fig. D.2. Under the shear deformation shown, the elastic stiffness of each bar is \( 968.1 \text{ kN} / \text{m}^3 \) which suggests that the shear deflection between the adjacent concrete surfaces is increased with a larger stirrup spacing because of the greater displacement between the surfaces. The abrasion of these surfaces after numerous load reversals substantially reduces the shear capacity of the concrete, \( V_c \), because the surfaces become smoother, and hence the aggregate interlock is steadily reduced. By decreasing the stirrup spacing, the dowel is stiffer and the abrasion at the crack is reduced. Observation of the test specimens indicates
that the number of flexural cracks increases as the stirrup spacing is reduced. Although the dowel is stiffer, with small stirrup spacing more cracks are formed and therefore the total shear deflection of the beam is made up of a large number of small shear deformations along the beam. No attempt is made to predict the shear deflection, but rather to show qualitatively the effect of stirrup spacing on shear capacity after load reversals.
Fig. D.1 - Shear Capacity

a) Monotonic Loading

b) Reversed Loading
Fig. D.2 - Effect of Stirrup Spacing on Dowel Force

\[ \delta = \frac{s^3}{96 \text{ FT}} \]

\( s \) and \( \delta \) are the stirrup spacing and deformation, respectively.