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AND $^9\text{Be}(^3\text{He}, \alpha)^2\alpha$.

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A STUDY OF THE REACTIONS

$^9$Be($^3$He, $^6$Li)$^6$Li AND $^9$Be($^3$He, $\alpha$)$_2$$\alpha$

by

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I. INTRODUCTION

The object of this set of experiments has been to investigate a pair of reactions in which nucleons are transferred from the target nucleus to the bombarding nucleus forming reaction products which are intermediate in mass between the target and beam particles. Such investigations may provide information about cluster configurations within the nuclear surface and nucleon correlations in the low density tails of the nuclear matter distribution. This area of nuclear physics is concerned more with the correlations between groups of nucleons than basic nucleon-nucleon forces.

The $^9$Be($^3$He, $^6$Li) $^6$Li study is one of a number of recent experiments $^{6-9}$ which imply that essentially any reaction which is not forbidden by basic selection rules is likely to occur to some extent. The yield from some of these reactions can be quite low, and only with electronic developments which have taken place within the past few years have studies of these reactions become feasible.

One of the first groups to undertake multinucleon transfer studies was Daehnick and Denes $^{8,9}$ at Pittsburgh. Their primary work has been with ($d$, $^6$Li) reactions on a variety of nuclei of A from 9 to 19, but they have also reported
a number of other reactions such as \((d, ^7\text{Li})\) and \((d, ^9\text{Be})\). The \((d, ^6\text{Li})\) reactions which they have reported show particularly strong yields from \(^{12}\text{C}\) and \(^{16}\text{O}\) which are described as resulting from the direct transfer of a preformed alpha particle from the target nucleus to the bombarding nucleus. This work is consistent with the popular concept that nuclei are formed of groups of nucleons clustered together which behave as a unit.\(^{10,11}\) The most favorable cluster group to be found in nuclei is the alpha cluster because of its stability. Thus, the presence of alpha clusters lends itself to the description of high yields from \((d, ^6\text{Li})\) on \(^{12}\text{C}\) and \(^{16}\text{O}\). The cluster model has been used extensively in describing the properties of light nuclei\(^{15}\) primarily on the basis of \(\alpha\) clusters and \(d\) clusters.

To describe the \(^9\text{Be}(^3\text{He}, ^6\text{Li})^6\text{Li}\) reaction as the direct pickup of a cluster would require \(^9\text{Be}\) to have a \((^6\text{Li} + t)\) cluster configuration and \(^6\text{Li}\) to have a \((^3\text{He} + t)\) configuration. There is a great deal of controversy over the existence of triton clusters. Some photo-dinintegration reactions\(^{16}\) such as \(^6\text{Li}(\gamma, t)^3\text{He}\) indicate that triton clusters are indeed present in \(^6\text{Li}\). However, others such as 150 MeV protons on \(^6\text{Li}\) yield 20 times smaller cross sections for \((^3\text{He} + t)\) breakup than for \((d + \alpha)\) breakup.\(^{20}\)
Because of this controversy no attempt was made to explain the data in terms of a triton pickup reaction. Furthermore, in the absence of an appropriate theory, no effort has been made to describe the results quantitatively. Only a discussion of the gross features of the data in terms of various possible models is presented.

It should be noted, however, that this reaction has never before been reported, and a positive identification of its existence is presented in the second chapter of this thesis. This in itself is of some interest in view of the lack of such justification in the past.

The $^9\text{Be}(^3\text{He},\alpha)^{2\alpha}$ reaction presents a slightly different situation. The observed data along with the results from experiments by Bronson $^2$ and others imply that three-alpha breakup takes place almost exclusively by a sequential decay through well defined states in $^8\text{Be}$. However, measurements on the first emitted particles indicate that the sequential process is initiated by the knockout of an alpha cluster rather than the more likely pickup of a neutron from ($\alpha + \alpha + \alpha$). This conclusion is based upon the fact that the spectrum of first emitted particles is peaked at larger angles than expected on the basis of neutron pickup. The results of these experiments
indicate that compound nucleus formation does not take place. In studying the \(^{11}\text{Be}(p,\alpha)2\alpha\) reaction, Bronson observed a final-state interference phenomenon involving the first excited state in \(^8\text{Be}\). Unfortunately, it is kinematically impossible to observe interference of this 2.9 MeV state in \(^8\text{Be}\) with itself via the \(^9\text{Be}(\text{He},\alpha)2\alpha\) reaction. However, the 19.9 MeV state in \(^8\text{Be}\) has a width (\(\sim 1.5\) MeV) of the same order as the first excited state, and it is kinematically possible to observe the presence of interference between these two states which are both \(2^+\) states. Data were taken at a number of angles where states were superimposed, but they failed to show any evidence of interference.

Angular correlations between the first emitted alpha particle and breakup \(\alpha\) particles from the 2.9 MeV and 16.9 MeV states were obtained and compared with results from other experiments\(^{17,18}\) and with the predictions of a generalized density of states calculation. A density of states function derived by Phillips, Griffy and Biedenharn\(^{21}\) is combined with the angular correlation treatment of Goldfarb and Devons\(^{28}\) to describe the laboratory coincidence yield. The density of states treatment which relies on experimental values for alpha-alpha phase shifts is valid when the breakup occurs via a sequential process.
II. IDENTIFICATION OF MASS 6 PARTICLES

Experimental Procedure

Most nuclei between $^4\text{He}$ and $^{12}\text{C}$ in mass have rather large mass defects. Thus, a study involving these nuclei as primary reaction products requires a beam of particles which also have large mass defects. For this reason the $^3\text{He}$ beam is most convenient for these studies. One such reaction is $^9\text{Be}(^{3}\text{He},^6\text{Li})^6\text{Li}$.

The charged particle spectrum at $20^\circ$ in the laboratory resulting from the bombardment of $^9\text{Be}$ with a beam of 8 MeV $^3\text{He}^{++}$ is shown in Figure 1. A number of peaks are clearly present. These large peaks are due to strong processes such as elastic scattering, $^9\text{Be}(^{3}\text{He},p)^{11}\text{B}$ and $^9\text{Be}(^{3}\text{He},d)^{10}\text{B}$. Some weaker peaks and a continuum from the breakup process $^9\text{Be}(^{3}\text{He},\alpha)^{2}\alpha$ are also present. There may also be events from other weaker reactions present in this single parameter display which are engulfed by the stronger processes. It might be said that data presented in this manner is degenerate. For the two-body final states this degeneracy can be removed by measuring an additional parameter. The measurement of the energy of the recoil particle or the time-of-flight of the primary particle would resolve all two-body events. This would still leave three-body events
Figure 1. Singles spectrum from 8 MeV $^3$He on $^9$Be measured at a laboratory angle of 20°.
in an unresolved state unless still an additional parameter was measured. However, masking by three-body events does not present a problem for this particular reaction because of the high "Q" value for three-alpha decay and very little evidence of other three-body processes obscuring the region of interest when time-of-flight was chosen as the other parameter.

The flight-time is not a unique property to work with, but it can be used to determine the mass of the primary particle (particle detected in counter 1) which is a unique property. This is accomplished by simply computing \( E_T^2 \) over a given flight-path. In the absence of a pulsed beam, the associated particle technique was used to determine the time-of-flight and subsequently the mass of the primary particle. This provides a means for identifying reactions which are not evident from the singles spectra.

The mass identification arrangement used in this experiment consists of two solid state detectors, one placed near the target (5 cm) and the other 70 cm away as illustrated in Figure 2. A detailed discussion of the development of this technique is given by Emerson\(^1\) et.al. The quantities actually measured are \( E_1 \) - the energy of the primary particle, \( E_2 \) - the energy of the recoil particle,
Figure 2. Schematic drawing of the arrangement of the detectors for mass identification studies.
and $T_1 - T_2$ the difference between the flight times of particles 1 and 2. A discussion of the scattering chamber with which this arrangement was used is given in Chapter IV.

Figure 3 shows a diagram of the electronics required to make these measurements. A fast timing pulse is obtained by inserting an ORTEC Model 260 time pickoff unit in series with the cathode of the third cascode amplifier stage in the Tennelec Model 100 charge sensitive preamp. This arrangement provides sufficient amplitude to trigger the pickoff units with incident particles of a few hundred keV. When the pickoff unit was connected directly to the detector, it had a threshold of about 3 MeV. The pickoff units provide a timing pulse of 0.5V with a rise time of less than 2 nanosec.

The signals from the pickoff units were fed into EG&G Model TR 104 fast trigger modules in order to present square-wave inputs of constant amplitude and width to the time-to-amplitude converter. The EG&G Model TH 200 time-to-amplitude converter used in this experiment was set for a full scale output corresponding to 300 nanosec difference between the start and stop signals. For most particles of interest the flight time over a 70 cm flight path is less than 100 nanosec. Hence, 100 nanosec of delay was added to
Figure 3. Block diagram of the electronic circuitry for mass identification studies. The 1000 x 1000 x 1000 channel analyzer is an IBM 1401 computer.
the stop pulse so that converter pulses would lie in the upper two-thirds of their range. The resulting TAC signal was fed into a Tennelec Model TC 200 linear amplifier, while the signals from the two charge sensitive preamplifiers were fed to Cosmic Model 900 linear amplifier. The prompt output signals of the three amplifiers were sent through proper delays into separate modules of a Cosmic Model 900 coincidence unit. A 1 microsec coincidence was required between all three signals to gate the computer analyzer. This triple coincidence requirement was very effective in reducing accidentals due to noise in the time pickoff circuitry. The delayed amplifier signals were recorded in the computer as three-parameter events.

In order to obtain an accurate calibration of the time-of-flight axis, it was necessary to make corrections for the energy dependence of the firing-time of the pickoff units and the flight-time of the start particle. It has been found (Emerson et.al.\(^1\)) that the time required for the pulses to rise to the time pickoff discrimination level is inversely proportional to the energy of the incident particle. Incorporation these effects into an expression for the measured time-of-flight gives

\[ T = T_1 - T_2 + t_1 - t_2 + T_0 \]  

(1)
where $t_1$ and $t_2$ are the times for the pulses to rise to the pickoff discriminator level and $T_0$ is approximately equal to the inserted electronic delay. When the energy dependence of the correction terms (the correction for the flight-time of the start particle varies as $E_2^{-\frac{1}{2}}$) is included, the relationship becomes

$$T_1 = T - T_0 + \frac{C_1}{E_2} - \frac{C_2}{E_1} + \frac{C_3}{E_2},$$

(2)

For equation (2) to be useful, it is necessary to determine the constants $T_0$, $C_1$, $C_2$ and $C_3$. By performing a scattering experiment in which $T$ is measured for a series of values of $T_1$, $E_1$ and $E_2$, it is possible to obtain a set of data which can be fit with equation (2) by the method of least squares, thus determining the required constants.

A series of calibration spectra were measured by detecting elastically scattered $^3\text{He}$ particles from $^3\text{He} + ^9\text{Be}$ in detector 1 and the recoil $^9\text{Be}$ in detector 2, then switching the position of detector 2 so that it detected the $^3\text{He}$ and detector 1 received the $^9\text{Be}$ recoils. This provided calibrations for two different masses in detector 1 whose values were half and twice the mass of $^6\text{Li}$. This convenience contributed a very good set of constants for use with $^6\text{Li}$ particles.
Data is collected in the form \((E_1, E_2, T)\) which can be used in conjunction with formula (2) to compute \(E_1 T_1^2\) for particle 1. The \(E T^2\) spectrum from the calibration data is shown in Figure 4. Assuming a linear scale, it is possible to assign a mass code to the \(E T^2\) axis from the mass 3 and mass 9 peaks. A computer program was written to edit data from the \((E_1, E_2, T)\) format to a \((E_1, E_2, E_1 T_1^2)\) format. This data could be displayed in one of several ways. By sorting the events on \(E_1 T_1^2\), a mass yield distribution such as the one in Figure 4 is obtained. For three-dimensional displays the data can be sorted versus \((E_1, E_2), (E_1, E_1 T_1^2)\) or \((E_2, E_1 T_1^2)\). For the purpose of searching for \(^6\)Li, \(^6\)Li events the second form is the most convenient.

**Results**

The three-parameter mass identification technique was first employed by Emerson in the study of \(^9\)Be\((P,d)2\alpha\) and \(^9\)Be\((P, \alpha \alpha \alpha), d\). It was decided that this technique would be very useful for investigating reactions resulting in two heavy particles in the final state. A search was made for heavy products from \(^3\)He induced reactions for a number of nuclei \((^{19}F(\gamma, ^6\text{Li})^{16}O, \, ^{19}F(\gamma, ^6\text{Li})^{12}C, \, ^9\text{Be}(\gamma, ^6\text{Li})^{16}\text{Li})\).

No yield was observed for the first two reactions. This was probably due to experimental difficulties such as thin
Figure 4. Calibration spectrum showing peaks in $E T^2$ due to elastically scattered $^3\text{He}$ and recoil $^9\text{Be}$. The data were taken at 0.5 MeV steps from 5.0 to 10.0 MeV and least squares fit to equation (2) to determine the calibration coefficients. The resulting equation for $T_1$ was then applied to the calibration data to obtain this spectrum.
\[ \theta_1 = 50^\circ \]
\[ \theta_2 = 57.6^\circ \text{ for } ^3\text{He Peak} \]
\[ \theta_2 = 62.5^\circ \text{ for } ^9\text{Be Peak} \]
targets. However, the third reaction showed surprising yield for mass 6 in the mass yield spectrum when the second detector was placed at the proper kinematic angle to receive the recoil $^6\text{Li}$ and no yield for mass 6 when the second detector was moved to a different angle. Figure 5 shows the mass spectrum resulting from setting the angles so as to observe two $^6\text{Li}$ particles. Notice that there is also a peak corresponding to mass 7 indicating that the three-body channel $p, \alpha, ^7\text{Li}$ is also present. There are also peaks corresponding to most other available masses up to masses 10 and 11.

The same spectrum is presented in a three-dimensional $(E_1, E_1T_1^2)$ or $(E_1, M_1)$ display in Figure 6. In this form of presentation, a two-body event appears as a peak corresponding to the energy and mass of particle 1 for a given exit channel while a three-body event appears as a band at a mass corresponding to particle 1. The data contains a peak at mass 3 from elastic $^3\text{He}$, a corresponding peak at mass 9 from the recoil $^9\text{Be}$ and a band at mass 4 corresponding to the three-alpha exit channel. Also, a strong peak corresponding to $^6\text{Li}$ is clearly present. This figure presents positive evidence that the $^9\text{Be}(^3\text{He}, ^6\text{Li})^6\text{Li}$ reaction does exist. Normalization of the yield to a
Figure 5. Mass spectrum determined by ET\(^2\) for detector 1 at 50\(^{\circ}\) and a \(^3\)He bombarding energy of 8 MeV. The mass scale is a linear scale obtained from the two peaks in Figure 4.
Figure 6. Two-parameter spectrum of number of counts versus the mass and energy of particle 1 for detector 1 at 50° and a $^3$He bombarding energy of 9 MeV. The data show peaks due to $^3$He, $^6$Li, $^9$Be and $^{11}$B. There is also a bank of counts resulting from three-alpha breakup located at mass 4.
$E_{\Delta \text{He}} = 9 \text{ MeV}$
$\theta = 50^\circ$
recorded elastic scattering peak indicates that the reaction cross section is reasonably large $\sim 0.4$ mb/ster at this point.

Conclusions

Because of its large mass defect, $^3\text{He}$ is a very useful probe for studying reactions in which the products also have large mass defects. This is particularly true for Van de Graaff accelerators which are limited in energy range. The reaction $^9\text{Be}(^3\text{He}, ^6\text{Li})^6\text{Li}$ has a $-1.89$ MeV "Q" value. For other projectiles, especially protons and alphas, "Q" values for reactions of this type are so small that their study is not possible with a Van de Graaff accelerator. Hence, more and higher energy $^3\text{He}$ Van de Graaff beams are needed. Furthermore, the experimental technique used for this three-parameter measurement is cumbersome and unreliable. This points up the need for a pulsed $^3\text{He}$ beam which would allow the study of this reaction with a single counter.
III. THE $^{9}\text{Be}(^{3}\text{He},^{6}\text{Li})^{6}\text{Li}$ REACTION

Experimental Procedure

The time-of-flight techniques employed for identification of the $^{6}\text{Li}$ particles are very limited as a means of routinely collecting data. This is because the long arm must be mounted on a scattering chamber with fixed ports. This reduces the number of points which can be measured on an angular distribution to about 7 for identical particles. Also, it makes measurements of excitation functions at constant angle in the center-of-mass system impossible. Finally the time-to-amplitude converter has a dead-time of the order of 10 microseconds which allows noise from the time pickoff units to play an important role in the overall efficiency of the system. Successive measurements of the same data point with this arrangement indicated fluctuations of as much as 50 percent.

A new scheme for data collection was devised in order to remove these limitations. The long arm was eliminated and a chamber with two movable detector mounts was employed. A diagram of this versatile chamber is shown in Figure 7. The chamber was originally designed by J. D. Bronson $^{2}$ for the study of reactions involving three particles in the final state. For that reason one of the detectors was mounted with
Figure 7. A simplified drawing of the charged-particle angular correlation chamber showing the beam-defining telescope, counter arms 1 and 2, the target holder, and the Faraday cup system with target for determination of energy calibration points using known neutron thresholds.

Legend:

A. Tantalum disc with .0995" aperture
B. Quartz disc with 3/16" aperture
C. Ledex Rotary Solenoid
D. Counter holder and telescope
E. Target holder
F. Counter arm 2
G. Faraday cup and neutron threshold system
H. Threshold target positioning rod
I. Neutron threshold target
J. Cold trap
K. Viewing port (covered when not used to read scales)
L. Graduated angular scale
M. Counter arm 1
N. Beam-defining telescope
O. Beam tube of Van de Graaff accelerator
P. Diffusion pump port for Faraday cup system
two degrees of freedom for taking data out of the plane. To insure that the two detectors were in the same plane their alignment was checked optically and then a $^3\text{He} + d$ elastic scattering measurement was performed with coincidence required between the two detectors.

To improve the electronic efficiency, the time pickoff units and the time-to-amplitude converter were removed from the circuitry. A diagram of the resulting electronic arrangement is presented in Figure 8. The only coincidence requirement is that the pulses from the two charge sensitive preamplifiers are within 100 nanoseconds of each other. The resulting two-parameter spectra are very clean due to kinematical selection as shown in Figure 9. This spectrum is for detector 1 at $50^\circ$ in the laboratory and a bombarding energy of 8 MeV. A peak corresponding to $^6\text{Li}$, $^6\text{Li}$ is clearly visible although there are some background counts along the edges of the spectrum due to coincidences between elastically scattered $^3\text{He}$ and the corresponding recoil $^9\text{Be}$. In general, these counts are far removed from the region of interest and do not create any difficulty. Also, for some combinations of angles and energies counts from $p, \alpha, ^7\text{Li}$ decay make the $^6\text{Li}$, $^6\text{Li}$ peak slightly difficult to resolve. Again, this difficulty rarely occurred.
Figure 8. Block diagram of the circuitry employed in measuring particle-particle coincidences without time-of-flight.
Figure 9. Photograph of computer scanned cathode-ray tube display of coincidence spectrum from 8 MeV $^3$He on $^9$Be with detector 1 at $50^\circ$ in the laboratory and detector 2 on the $^6$Li recoil axis ($\sim 60.1^\circ$). The peak in the center of the spectrum is due to $^6$Li, $^6$Li coincidences while the ones near the axes are due to elastic scattering.
This method provides a convenient means for performing detailed studies of the $^{9}\text{Be}(^{3}\text{He},^{6}\text{Li})^{6}\text{Li}$ reaction once the products have been identified. However, additional care must be taken to insure that a coincidence pulse is recorded for each $^{6}\text{Li}$ particle incident upon detector 1. Because energy loss and multiple scattering in the target can be quite significant for these heavier particles, the targets were limited to $\sim 50$ micrograms per cm$^2$ in thickness. This choice represents a compromise with the need for thicker targets for higher count rates. Another method of increasing the count rate is to use larger detector solid angles, but this advantage is quickly lost as the finite size of the target spot creates very unfavorable geometrical conditions. The following diagram illustrates the effect of a 1/16 inch target spot on the size of the cone of recoil particles when detector 1 subtends a solid angle of approximately 0.01 steradians and the laboratory solid angle factors for the two directions are equal.

It is evident that the solid angle of the second detector must be nearly an order of magnitude greater to insure that all events are observed. Data were taken with
detector 1 always at a greater laboratory angle than detector 2, thus assuring that the geometrical conditions were at least as good as the above illustrated arrangement requires. The solid angle of detector 2 was chosen so as to give a reasonable margin for error in the detector positions. In Figure 10 the number of coincidences versus the angle of detector 1 for detector 2 fixed is displayed. The solid line represents the calculated effective solid angle of the two detectors while the data points represent the number of coincident events for a fixed amount of collected charge. This figure indicates a margin for error of two degrees in the relative placement of the detectors. The count rate is shown to be flat over a sizable angular range.

Absolute cross sections were obtained by normalizing the data to the elastic scattering data of Earwaker \(^3\), and Miller and Duggan \(^4\). Figure 11 shows the elastic scattering data at 45° in the laboratory from these two references. The elastic spectrum was monitored frequently to insure that experimental conditions remained uniform throughout the run. A record was kept of the target position at all times in order to make corrections for the variation of the effective target thickness with target angle. Much of the data was taken while recording the free spectrum
Figure 10. Number of \(^6\text{Li}\), \(^6\text{Li}\) coincidences for a fixed angle of detector 2 as a function of the angle of detector 1. The solid curve represents the calculated effective solid angle of the two detectors.
number of counts vs angle of detector 1 for detector 2 fixed at 29.8°

$E_{^3\text{He}} = 8 \text{ MeV}$
Figure 11. Elastic scattering cross section for $^3\text{He} + ^9\text{Be}$ at a laboratory angle of 45°. This curve was used to obtain absolute cross sections for $^6\text{Li}, ^6\text{Li}$. 
\( \theta_{\text{Lab}} = 45^\circ \)
from the small solid angle detector on a separate Nuclear Data 1024 channel analyzer, thus providing a convenient means for normalizing the results.

**Excitation Functions**

The "Q" value for the reaction \( ^9\text{Be}(^3\text{He}, ^6\text{Li})^6\text{Li} \) is - 1.89 MeV, thus requiring a laboratory energy of 2.6 MeV for \(^3\text{He} \) to reach the threshold for this process. However, the Coulomb barrier for \(^6\text{Li}, ^6\text{Li} \) is of the order of 2.5 MeV which corresponds to about 5.7 MeV laboratory energy for the \(^3\text{He} \), and considerably more than 2.6 MeV is necessary for the reaction to proceed at an observable rate. The effective threshold for the reaction can be seen from the exciting functions in Figures 12-15 to be of the order of 4 MeV. These excitation curves show the conventional Coulomb barrier shape with very little structure, except for the 50° lab case. The fluctuations in the 50° lab data can be explained by the shift in center-of-mass angle peaks in the angular distribution as the bombarding energy is increased. This point will be discussed later.

**Angular Distributions**

Angular distributions of the \(^6\text{Li} \) particles were measured at bombarding energies of 6, 7, 8, 9 and 10 MeV.
Figure 12. The excitation function for $^{9}\text{Be} (^{3}\text{He}, ^{6}\text{Li}) ^{6}\text{Li}$ measured at a laboratory angle of 50 degrees.
Figures 13, 14, 15. Excitation functions for $^9\text{Be}(^3\text{He},^6\text{Li})^6\text{Li}$ measured at center-of-mass angles of 50, 70 and 90 degrees.
$^9\text{Be}(^3\text{He}, ^6\text{Li})^6\text{Li}$

Excitation Function

$\theta_{cm} = 50^\circ$

$\sigma(\theta_{cm})$ (mb/sr)

$E_{^3\text{He}}$ (MeV)

4  6  8  10
$^{9}$Be ($^{3}$He, $^{6}$Li) $^{6}$Li
Excitation Function
$\theta_{\text{cm}} = 70^\circ$
$^{9}\text{Be}(^{3}\text{He},^{6}\text{Li})^{6}\text{Li}$
Excitation Function
$\theta_{\text{cm}} = 90^\circ$

$\sigma(\theta_{\text{cm}})$ (mb/ster)

$E_{^{3}\text{He}}$ (MeV)
These data are presented in Figures 16-20 along with Legendre polynomial fits. These fits were obtained by extrapolating the angular distributions to zero degrees and calculating the coefficients $A_l$ with the orthogonality relationship

$$\int_{-1}^{1} P_l(\chi) \sigma(\theta) d\chi = \frac{2}{2l+1} A_l$$

(1)

where $\sigma(\theta) = \sum_k A_{l,k} P_k(\cos \theta)$.

Because only identical particles are present in the state, only even values of $l$ are involved and the angular distribution is symmetric about $90^0$ in the center-of-mass system. The resultant coefficients $A_l$ are plotted versus bombarding energy in Figure 21. A number of extrapolations were tried in order to produce the most consistent set of coefficients for the entire set of data. While the coefficients for $l > 0$ were strongly dependent upon the choice of extrapolation, the $A_0$ terms remained nearly constant for all extrapolations. For this reason, the total cross section computed as $4\pi A_0$ and shown in Figure 22 should be very accurate ($\sim 5\%$).

In order to know which values of angular momentum may be expected to contribute to the reaction, it is necessary to determine the barrier penetration factors for various
Figures 16, 17, 18, 19, 20. Angular distributions for the reaction $^9\text{Be}(^3\text{He}, ^6\text{Li})^6\text{Li}$ measured at bombarding energies of 6, 7, 8, 9 and 10 MeV. The solid curve represents a Legendre polynomial fit to the data.
$^9\text{Be} (^3\text{He}, ^6\text{Li}) ^6\text{Li}$

Angular Distribution

$E_{^3\text{He}} = 6.0 \text{ MeV}$

$\sigma (\theta) = 0.39 + 0.075 P_4 + 0.15 P_6$
$^{9}\text{Be (}^{3}\text{He, }^{6}\text{Li})^{6}\text{Li}$
Angular Distribution
$E_{^{3}\text{He}}=7.0\text{ MeV}$

\[\sigma (\theta) = 0.41 + 0.05P_2 + 0.186 + 0.125P_8\]
$^9_{\text{Be}}(^3_{\text{He}}, ^6_{\text{Li}}) ^6_{\text{Li}}$
Angular Distribution
$E_{^3_{\text{He}}} = 8.0$ MeV

\[ \sigma (\theta) = 0.425 + 0.12 P_2 + 0.1 P_6 + 0.13 P_8 \]
$^{9}\text{Be}(^{3}\text{He},^{6}\text{Li})^{6}\text{Li}$
Angular Distribution
$E_{^{3}\text{He}} = 9.0 \text{ MeV}$

\[
\sigma(\theta) = 0.435 + 0.002P_2 + 0.08P_4 + 0.05P_6 + 0.16P_8
\]
$^{9}\text{Be}(^{3}\text{He},^{6}\text{Li})^{6}\text{Li}$

Angular Distribution

$E_{^{3}\text{He}} = 10$ MeV

$\sigma(\theta) = 0.455 + 0.15P_4 + 0.22P_8$
Figure 21. The Legendre polynomial coefficients resulting from fits to the $^9\text{Be}(^3\text{He}, ^6\text{Li})^6\text{Li}$ data. For clarity only one error bar is given for each curve with $L>0$. 
Figure 22. The total cross section for the reaction $^9\text{Be}(^3\text{He}, ^6\text{Li})^6\text{Li}$ as determined from the Legendre polynomial fit to the angular distributions.
values of \( \lambda \). For charged particles, the effective potential outside the nuclear radius is given by

\[
V(r) = \frac{\alpha' e^2}{r} + \frac{\hbar^2 \lambda (\lambda+1)}{2 \mu r^2}
\]

(2)

where \( \mu \) is the reduced mass \( \frac{m_1 m_2}{m_1 + m_2} \).

Here the first term is the Coulomb potential and the second is the "centrifugal" potential which repels particles that bring in or emerge with angular momenta greater than zero. The penetration factor is defined as

\[
\mathcal{P}_\lambda \equiv \frac{R R}{F^2_\lambda (r) + G^2_\lambda (r)}
\]

(3)

where \( R \) is the nuclear radius and \( k \) is the wave number outside the nucleus, \( k = \sqrt{2mE/\hbar^2} \). The functions \( F_\lambda \) and \( G_\lambda \) are the regular and irregular solutions of the confluent hypergeometric equation. A linear combination of these functions is used to form the Coulomb wave function. For potentials involving charged particles the functions \( F_\lambda \) and \( G_\lambda \) are very difficult to evaluate. However, to the extent that shielding by the atomic electrons can be neglected \(^5\), their asymptotic behavior is

\[
F_\lambda (r) \approx \sin \left[ kr - \frac{1}{2} \ell \pi - \eta \ln (2 \pi r) + \phi_\lambda \right]
\]

(4)

\[
G_\lambda (r) \approx \cos \left[ kr - \frac{1}{2} \ell \pi - \eta \ln (2 \pi r) + \phi_\lambda \right]
\]
for $R >> \rho$. The parameter $\eta$, which determines the importance of Coulomb effects is given by

$$\eta = \frac{r_2' e^2}{\hbar \nu},$$

and $\rho_2$ is the Coulomb phase shift which determines the pure Rutherford scattering. $\nu$ is the asymptotic relative velocity of the two particles.

Values for the penetration factors for $^3$He + $^9$Be and $^6$Li + $^6$Li were calculated from equation (3) and are shown in Figure 23. Both sets of penetration factors are given in terms of $^3$He bombarding energy. The $^6$Li + $^6$Li bombarding energy corresponding to a specific $^3$He + $^9$Be bombarding energy is

$$E_0 = 2 \left[ \frac{9}{12} E_3 - 1.9 \right] \text{MeV}.$$  

The values for $R$ used in these calculations were the sum of the radii of the two interacting particles.

**Interpretation of Data**

Reactions such as the three-nucleon transfer reaction $^9$Be($^3$He, $^6$Li) $^6$Li have not been studied to any great extent. Hence, there is very little theoretical information available with which to describe the data from this experiment. However, the existing data do exhibit a couple of consistent qualitative features. 6-9)
Figure 23. The penetration factors for $^3\text{He} + ^9\text{Be}$ and $^6\text{Li} + ^6\text{Li}$ as a function of $^3\text{He}$ bombarding energy. The Coulomb radius in each case was assumed to be equal to the sum of the radii of the interacting particles.
$^6\text{Li} + ^6\text{Li}$
Penetration Factor
for $R = 5.3$ fm

$^3\text{He} + ^9\text{Be}$
Penetration Factor
for $R = 4.25$ fm

$p_{\text{B}}$ vs $E_{^3\text{He}}$ (MeV)
1. Angular distributions show a distinct peak in the forward direction and one or more additional peaks. Little can be said of the forward peak at this time because of imprecise measurements at forward angles. The position of the other peak or peaks depends upon the bombarding energy. With increase in energy the peaks move toward smaller angles.

2. The energy dependence of the total cross section has the following feature. For a transfer of one nucleon the cross section increases very sharply in the region of the Coulomb barrier. As the energy is further increased this variation becomes slower. For a multinucleon transfer the energy dependence of the cross section is less sharp compared to the case of the single nucleon transfer.

The data from the $^9\text{Be} (^3\text{He}, ^6\text{Li})^6\text{Li}$ reaction possesses the features described above, hence this can probably be classified as a transfer reaction. Thus, the peaks in the 50$^\circ$ laboratory excitation function can be explained in terms of the shifting of peaks with energy mentioned in statement 1. above. It must be emphasized, however, that most of the reactions in this category are of the types $^{16}\text{O} (^{14}\text{N}, ^{15}_0)^{15}\text{N}$ and $^{12}\text{C} (^{12}\text{C}, ^\alpha)^{20}\text{Ne}$.

Before continuing a discussion of the data in terms of transfer mechanisms it is worth considering the likelihood
of compound nucleus formation. $^3$He has an intrinsic spin and parity of $1/2^+$ while the $^9$Be ground state is $3/2^-$. These couple to give a channel spin of $1^-$ and $2^-$. In the outgoing channel are two $^6$Li particles with ground state spins of $1^+$. This implies that the angular momenta in the entrance and exit channels must have opposite parity. The dashed line in Figure 22 corresponds to the penetration factor for $\mathcal{Q} = 0$ is the entrance channel and $\mathcal{Q} = 1$ in the exit channel. This curve agrees well with the $A_0$ term. Thus, at low energies compound nucleus formation could play an important role. But, for higher $\mathcal{Q}$ values it is quite evident from Figure 23 that the penetration factors are quite small for the given radii. This in turn implies that larger radii are involved and that direct mechanisms are important. To have a significant contribution from $\mathcal{Q} = 4$ as implied by the $A_8$ curve in Figure 22, the minimum complexity of the incoming and outgoing radiation must each be at least $\mathcal{Q} = 4^{22,23)$. Figure 24 shows the effect of dividing the $A_8$ coefficient by the product of penetration factors $\mathcal{P}_5 \mathcal{P}_4$ for different values of the nuclear radius. There are two possible interpretations of this figure. First, the resulting curves indicate that the $A_8$ coefficient corresponds to a radius of about 7 fm. This value is much greater than the values
Figure 24. A comparison of the Legendre polynomial coefficient for $\ell = 4$ (A$\ell$) with the penetration factor $P_5^4$ ($\ell = 5$ incoming and $\ell = 4$ outgoing) for different values of the nuclear radius. The kR factor has been removed from the penetration factors so that they asymptotically approach 1.0 at high energies.
Ratio of $A_8$ to $P_{2,6}$

$P_5 P_8 = 1$

$R = 7.0 \text{ fm}$

$R = 5.5 \text{ fm}$

$E_{3_{\alpha}} \text{ (MeV)}$
obtained by simply summing the radii of the two interacting particles. Compound nucleus formation cannot very reason-
ably take place at this distance; this again implies that a more direct process must be important in this case. Converse-
ly, the peak in the 5.5 fm curve indicates that the system could be passing through a resonance at that point.

One simple mechanism for describing reactions in which nucleons are transferred from the target nucleus to the beam nucleus is the pickup mechanism. Generally, the incoming nucleus reacts with only a portion of the target nucleus and only two-body forces are involved. The simplest of these are (p,d) and (n,d) reactions in which only one nucleon is transferred. Another reaction of this type is the (d, α ) reaction in which two nucleons are transferred. Some recent studies 6,7 have shown that the (d, 6 Li) reaction is a very prevalent one in which an alpha particle is transferred. Explanations of these processes are based upon the existence of cluster configurations within the nucleus as proposed by Phillips 8,9 and others. 10) Thus, in a reaction such as 10 B(d, 6 Li) 6 Li, 10 B is assumed to have a cluster configuration of (α +α +α ). Then the deuteron simply picks up one of the alpha clusters. This is not such an unlikely process since the alpha particle is a very
tightly bound configuration. Measurements by Daehnick and Denes \(^8,9\) show that \((d, ^6\text{Li})\) occurs fairly strongly for a number of nuclei \((A = 10-19)\). On the other hand, multi neutron transfer data \(^7\) indicate that the cross section for the transfer of an equivalent number of uncorrelated neutrons is considerably lower.

A logical extension of the above reasoning is employed by Young \(^6\) et.al. to explain the results of measurements on the \(^{11}\text{B}(_{3}\text{He}, ^6\text{Li})^{8}\text{Be}\) reaction. \(^6\text{Li}\) is assumed to have a significant \(_{3}\text{He} + t\) cluster configuration, and the claim is made that \(^{11}\text{B}\) consists partially of the cluster configuration \(_{8}\text{Be} + t\). \(^6\) Thus, the reaction is described as the direct pickup of a triton from \(^{11}\text{B}\) and the data is fit with the optical model.

Young's experiment implies that the \(^{9}\text{Be}(_{3}\text{He}, ^6\text{Li})^{6}\text{Li}\) reaction should also be expected to proceed via a triton pickup. This would require \(^{9}\text{Be}\) to have a \((^6\text{Li}, t)\) cluster configuration. Such a configuration seems highly unlikely in competition with an \((\varphi + \varphi + \varphi)\) configuration, but measurement by Bliden \(^25\) indicate that the \(^{9}\text{Be}(p, \varphi) ^6\text{Li}\) reaction proceeds via the pickup of a triton. Satchler \(^12\) and Radvanyi \(^20\) suggest that \(^6\text{Li}\) shows almost no parentage for \(_{3}\text{He} + t\) in the ground state implying that triton
pickup by a $^3$He particle is highly unlikely. However, measurements of $^6$Li($\gamma$, t) $^3$He cross section $^{24}$ ($\sim$ 30% of total cross section) do indicate some ($^3$He + t) parentage for $^6$Li in excited states.

There is no substantial evidence that triton pickup is the mechanism through which this reaction proceeds. Moreover, in the light of controversial opinions on the subject, it is hardly worth while to apply a direct pickup fitting technique to the data from this experiment, especially since a good fit to the data would not be proof of the existence of the mechanism.

Scattering experiments involving medium weight ions $^{13,14}$ show evidence for a "quasi molecular" interaction between nuclei. This is a highly deformed weakly bound state that exists for a time which is long with respect to a direct reaction, but short compared to the lifetime of compound nucleus states during which a number of nucleons could be transferred via a rearrangement process. This model allows any combination of nucleons to be transferred from one nucleus to the other without requiring the pre-existence of any unlikely clusters.

A "quasi molecular" potential of the Lennard-Jones type $^{14}$ is employed to explain fluctuations in the elastic
scattering cross sections from $^{12}_C + ^{12}_C$ and $^{16}_0 + ^{16}_0$.

This potential has the following appearance:

The dip in the potential barrier at radius $A > R$ is postulated to be a result of surface interactions and the Pauli exclusion principle. This model is similar to the "onion skin" model of Phillips and Tombrello.\textsuperscript{11)}

Consider $^3$He as a $(d + p)$ cluster configuration and $^9$Be as $(\alpha + \alpha + m)$. If only two-body forces are important in the reaction, then the Hamiltonian of the system could have an interaction potential of the form

$$V = V_{d\alpha_1} + V_{d\alpha_2} + V_{pd_1} + V_{pd_2} + V_{dm} + V_{pm} \quad (5)$$

This would allow evaluation of the cross section for this
reaction
\[ \mathcal{P}_1 \mathcal{P}_2 \int \psi_i \nabla \psi_f^* \, d\mathbf{r} \]  

(6)

in terms of well understood two-body interaction potentials. Such a treatment removes the objections associated with direct pickup of certain clusters without involving compound nucleus formation. This idea could explain the observed high angular momentum states.

Conclusions

With the exception of investigations performed with moderately high energies, the study of heavy particle transfer reaction mechanisms is yet in its very early stages. Advances in experimental techniques have made heavy ion studies of a wide variety feasible at energies near the Coulomb barrier. Because of this lack of knowledge about such processes, no effort was made to identify the data from this experiment with a specific reaction mechanism. It is evident that heavy ion studies offer a useful tool for investigating the outer fringes of the nucleus, and will be the subject of many future investigations.
IV. THE $^9$Be($^3$He, $\alpha$) 2$\alpha$ REACTION

**Experimental Procedure**

The technique employed in collecting three-alpha data is similar to the one described in Chapter III for $^6$Li, $^6$Li data. Four silicon surface barrier detectors were mounted within a scattering chamber which is 15 inches in diameter and approximately 5 inches deep. The solid angle of each detector was determined by a colimating system with a 1/8 inch diameter tantalum defining slit placed 2 inches from the target. The use of this chamber arrangement did not allow for measurements out of the plane, so one of the four detectors was placed on the opposite side of the beam from the other three and in the same plane. Coincidence between this detector and each of the other three allowed for the collection of three simultaneous two-parameter spectra. It was not deemed necessary to record accidental spectra for background subtraction as the three alpha locus is separated from most of the region where accidentals may occur by its high "Q" value.

Figure 25 shows the experimental arrangement of the electronics for these measurements. When a coincidence is observed between detector 1 and any one of the other three detectors, it is recorded in a three-alpha spectrum corresponding to that particular pair of detectors. Because the
Figure 25. A block diagram of the electronic circuitry employed in measuring three-alpha spectra. Four detectors were used in order to record three separate spectra simultaneously.
alpha particle energies for this reaction vary over a wide range (0-20 MeV), it was necessary to minimize the variation in timing signals with the pulse height of the input to the Cosmic coincidence modules. To achieve a timing pulse with minimal "walk", double delay line clipping was employed in the ORTEC linear amplifiers. This allowed for the use of the zero cross-over network in the Cosmic modules to trigger coincidence signals. This method is much less sensitive to pulse amplitudes than triggering on the leading edge. It was necessary to adjust the delays of signals to each of the Cosmic coincidence modules for detectors 2-4 separately so that they were each within a 100 nanosecond coincidence with detector 1. A different gating signal was generated for each of the three possible spectra. This allowed for the events from all three spectra to be recorded concurrently for later separation by searching on a tag corresponding to each particular gate condition. Figures 26-28 show three typical spectra for a twelve hour measurement at 4 MeV.

Presentation of Data

The basic goal of this experiment was to search for final-state interference effects in the three-alpha breakup of $^{12}\text{C}$ as observed by Bronson$^2$ for $^{11}\text{B}(p,\alpha)2\alpha$. 
Figures 26, 27, 28. Typical three-alpha coincidence spectra and profiles for one twelve hour accelerator run at a bombarding energy of 4 MeV.
$^3\text{He} + ^9\text{Be} \rightarrow \alpha_1 + \alpha_2 + \alpha_3$

$E_{^3\text{He}} = 4.0\ \text{MeV}$

$\theta_1 = 40^\circ$

$\theta_2 = 132.5^\circ$
Bronson was able to see evidence for both constructive and destructive interference of the 2.9 MeV state in $^8$Be with itself. This effect is attributed to ambiguities which result from the sequential breakup into a system of identical particles. In order to know where to expect these effects to occur, it is necessary to examine the kinematics for the reaction. The kinematics for three-body breakup are derived in Appendix A for both simultaneous and sequential decay. If the process is simultaneous, the locus of events observed in a two-energy coincidence spectrum is a curve. However, when the decay is sequential, only limited portions of the curve will be populated depending upon the energies and widths of the states in the recoiling nucleus. Figure 29 shows a plot of the loci of states versus the energy of the particle in detector 1 and the angle of detector 2 for detector 1 fixed at an angle of $30^\circ$ and a bombarding energy of 6 MeV. There are three sets of curves corresponding to three different observations of the sequential process. The lines of constant $E_1$ correspond to detection of the first emitted particle in detector 1. The dashed curve corresponds to detection of the first emitted particle in detector 2. And the solid curve corresponds to the first emitted particle being undetected. The intersections of these loci correspond to overlaps of states from two of the three
Figure 29. A plot of the loci of states in $^8$Be corresponding to each of the three possible decay modes for a bombarding energy of 6 MeV. The points at which the loci intersect represent sources of possible interference effects.
possible $^8$Be recoil systems. If the overlapping states have the same quantum numbers $J \pi$, then it may be possible for the wave functions to interfere. Figure 30 is an isometric presentation of data taken at angles where favorable kinematic crossings occur in Figure 29. Other isometric presentations for data measured at 3 and 4 MeV are given in Figures 31-34. In addition to an investigation of interference phenomena, the data also provide information about the angular correlations between the first emitted alpha and a breakup alpha from the recoiling $^8$Be system.

Interference

The only evidence for interference in Bronson's$^{2}$ data occurs where the loci for the 2.9 MeV state in $^8$Be overlap. Unfortunately, for the $^{9}$Be($^3$He,$\alpha$)$2\alpha$ reaction it is not kinematically possible to interfere this state with itself because of the high "Q" value ($\sim 19.006$ MeV). However, it may be possible to see this effect for any two states of the same $J$ and $\pi$. The most intense state in the spectra is the 16.9 MeV state in $^8$Be which accounts for about half of the total cross section. However, its width is only about 84 kev making interference impossible to observe with the $\sim 200$ kev per channel resolution employed. A broader state which is much less intense is the $4^+$, 11.4
Figure 30. An isometric display of three-alpha profiles for a bombarding energy of 6 MeV and a fixed detector angle of 30 degrees. The angles of the second detector were chosen to correspond to intersections of loci for states in $^8$Be indicated in Figure 29.
Figure 31. An isometric display of fixed detector energy profiles for detector 1 at 40 degrees and a bombarding energy of 4 MeV.
\[ {\text{He}}^3 + {\text{Be}}^9 \rightarrow a_1 + a_2 + a_3 \]

\[ E_{3\text{He}} = 4.0 \text{ MeV} \]

\[ \theta_1 = 40^\circ \]

Projection on energy axes of alpha particle detected at \( \theta_1 \)
Figures 32, 33, 34. Isometric displays of fixed detector energy profiles for a bombarding energy of 3 MeV. The angles of the fixed detector are 40, 60 and 120 degrees respectively.
$^{3}\text{He} + ^{9}\text{Be} \rightarrow \alpha_1 + \alpha_2 + \alpha_3$

$E_{^3\text{He}} = 3 \text{ MeV}$

$\theta_1 = 40^\circ$

Projection on energy axes of alpha particle detected at $\theta_1$
\[ ^3\text{He} + ^9\text{Be} \rightarrow a_1 + a_2 + a_3 \]
\[ E_{^3\text{He}} = 3 \text{ MeV} \]
\[ \theta_1 = 120^\circ \]
Projection on energy axes of alpha particle detected at \( \theta_1 \)
MeV state which is \( \sim 7 \) MeV wide.\(^{35}\) There is some evidence for interference of this state in Figure 32 for \( \Theta_1 = 40^\circ \) and \( \Theta_2 = 70^\circ \). However, the yield is so low that it is statistically insufficient to provide conclusive evidence. This is the only broad state for which the loci are observed to cross. Other possible sources of interference are where the 2.9 and 19.9 states cross. There are a number of points on the isometric displays where this condition exists, but none exhibit any significant evidence for interference. Also, the 2.9 state can be seen to cross with the 22.5 MeV state in Figure 30. One possible reason why no interference is observed in these instances is the relative yields from the different states. The states are of the same width so that they would show a good interference peak. But, if one state is much stronger than the other, then any interference effects could be unobservable.

Another possible technique for studying interference with this reaction is to take a region where the 16.9 state crosses itself and enlarge it so that the channel width is of the order of 5 to 10 kev. With the high yield from this state it would then be possible to observe any effect which occurs. This was not done in the present experiment, but it might prove to be a useful future experiment.
Density of States

By treating three-body decay as a time sequence of two-body interactions, Phillips, Grifffy and Biedenharn \(^{21}\) have presented a description of the cross section for leaving the recoil metastable nucleus in a state \(E_B\) in terms of a density of states factor

\[
\rho = \frac{1}{\pi} \frac{d}{dE_B} \left[ \delta_k (E_B) + \phi_k (a, E_B) \right]. \tag{1}
\]

This treatment assumes that the second decay occurs after the first emitted particle is outside the range of nuclear forces. The reaction can be written in the following manner:

\[
\begin{align*}
\alpha + A & \longrightarrow D^* \longrightarrow b + B + Q_1 \quad \text{(2a)} \\
B & \longrightarrow c + C + Q_2 \quad \text{(2b)}
\end{align*}
\]

The advantages to this technique are that no assumptions are made about the life times of the metastable states \(B\). Also, the phase-space dependence is considered by this technique, and it is possible to include effects which results from the order of emission of the experimentally detected particles. In addition, the finite size of the three-particle system is considered.

If the process in equation 2a) is treated by perturbation theory, the resultant cross section is \(^{21}\)
\[ \sigma_{ab}^{(\Theta, \Phi, \kappa_b)} = \left( \frac{\alpha \beta^2 a^2}{4 \pi \hbar^2} \right) \langle b + b, E_b | H^1 | a + a, E_a \rangle^2 \]  

(3)

where \( |a + a, E_a \rangle \) represents the initial state vector, \( |b + b, E_b \rangle \) the final state vector, and \( H^1 \) the interaction Hamiltonian for the process (2a). For a system B which has only sharp, long lived states (3) can be rewritten as

\[ \sigma_{ab}^{(E_b)} = \left( \frac{\alpha \beta^2 a^2}{4 \pi \hbar^2} \right) \langle b + b, E_b | H^1 | a + a, E_a \rangle^2 \rho(E_b) \]  

(4)

where \( \rho(E_b) \) represents only a set of sharp states at energies \( E_n \),

\[ \rho(E_b) = \sum_m \delta(E - E_m). \]  

(5)

This function can be extended to include continuum states by arguing that the probability of emission of b to form a continuum state B is proportional to the probability that c and C be localized within a nuclear volume of radius \( a \) which includes the interaction volume of b with B. This probability can be calculated for states of c + C by considering the final state wave function

\[ \psi = N(E) [U_k(E, r)/r] \gamma^{\infty}(\Theta, \Phi) \]  

(6)

where \( \Theta \) and \( \Phi \) are the polar azimuthal angles of particle c in the recoil center-of-mass system with respect to the
recoil axis, and $\psi_{r}(E,r)/r$ is the radial wave function normalized in the volume of radius $a$. The factor $N(E)$ then permits the definition of the generalized density of states function in terms of the usual density of states function $\rho_0$:

$$\rho(E) = \rho_0(E) N^3(E)$$  \hspace{1cm} (7)

where $\rho_0 = M^* R / m \pi^2 K$. The wave function is normalized over a volume of radius $R$ defined by:

$$\int_{0}^{R} |\psi|^2 d\nu = 1.$$  

The result of this renormalized form of the density of states function is

$$\rho = \frac{M^*}{\pi^2 R K} \left[ \frac{d}{dK} (\delta_0 + \phi_K) - \frac{1}{2} \left( \frac{1}{K} - \frac{2 A_2}{A_2^2} \right) \sin^2 (\delta_0 + \phi_K) - \frac{1}{K} \left( A_2 \frac{\partial A_2}{\partial K} - \frac{A_2}{K} \frac{\partial^2 A_2}{\partial K^2} \sin^2 (\delta_0 + \phi_K) \right) \right]$$  \hspace{1cm} (8)

where $A_2 = (F_2 + G_2)^2$. The hard sphere on Coulomb phase shift $\phi_2$ is given by

$$\phi_2 = \tan^{-1} \left( F_2 / G_2 \right).$$

For the case where $U_2$ for $r \geq a$ is independent of the energy, (8) reduces to

$$\rho = \left[ 2 M^* a / m \pi^2 s^2 \right] \left[ \sin^2 (\delta_0 + \phi_K) / \rho_0 \right]$$  \hspace{1cm} (9)
where \( f = U_\lambda(E, a) \) is a constant, and \( P_\lambda = \frac{k_c a}{A_\lambda^2} \) is the penetrability for the \( \lambda \)th wave \( c + C \) scattering process.

The phase shift can also be written \(^{21}\)

\[
\rho_\lambda(E_B) = \frac{1}{\pi} \frac{d}{dE_B} \left( \delta_\lambda(E_B) + \phi_\lambda(E_B, a) \right)
\]

for states which are reasonably sharp. If further the decay process can be described by the single level dispersion theory, then

\[
\delta_\lambda = \beta_\lambda - \phi_\lambda
\]

where

\[
\beta_\lambda = \tan^{-1} \left[ \frac{\frac{1}{2} \Gamma_\lambda}{E_\lambda - E} \right]
\]

with

\[
\frac{1}{2} \Gamma_\lambda = k a \frac{\sigma_\lambda^2}{A_\lambda^2}
\]

Thus

\[
\rho(E_B) = \frac{\sin^2(\delta_\lambda + \phi_\lambda)}{\pi \sigma_\lambda^2 P_\lambda}
\]

where \( \sigma_\lambda^2 \) is the reduced width of level \( \lambda \).

**Angular Correlations**

Until about ten years ago, the normal calculation of angular correlations followed the conventional use of the scattering matrix. When applied to processes involving arbitrary angular momenta and measurements of spin and polarization, this treatment required an extensive summation over magnetic quantum numbers resulting in very tedious
computations. A more recent approach which makes use of angular momenta and parities of nuclear states and their symmetry properties under operations of spatial rotations and reflections and time-reversal is summarized by Goldfarb, Goldfarb and Devons, and Coester and Jauch. The following discussion gives a summary of the treatment as it applies to a three-body decay.

Consider a sequential reaction process in which an incoming particle with spin \( \vec{s}_1 \) impinges upon a target nucleus of spin \( \vec{s}_2 \) to form channel spin \( \vec{a} \) with relative angular momentum \( \vec{\lambda}_1 \). Denote the total angular momentum of the compound system \( \tilde{B} \) by \( \vec{\tilde{b}} \). The system decays with the emission of a particle of spin \( \vec{s}_1' \), with angular momentum \( \vec{\lambda}_2 \). The channel spin of this intermediate system is \( \vec{\tilde{J}} = \vec{s}_1' + \vec{c} \) where \( \vec{c} \) is the spin of the recoil system. This recoil subsequently breaks up into two particles of spins \( \vec{s}_2' \) and \( \vec{s}_3' \), and relative angular momentum \( \vec{\lambda}_3 \). The angular correlation between two successive radiations resulting from the decay of a state \( B \) with total angular momentum \( \vec{b} \) can be written as the product of a density matrix with a statistical matrix as follows:

\[
W = \sum_{\vec{b}' \vec{b}'} \langle \vec{b} \vec{b}' \vec{b} \vec{b}' \rangle \langle \vec{b}' \vec{b}' \vec{b}' \vec{b}' \rangle
\]

(13)
where $\beta$ is the projection of $b$ onto the $z$ axis and $B, B'$ represent the quantum numbers necessary to describe the initial and final states of the system plus radiation. The angular dependence of the above expression as extracted by Devon and Goldfarb$^{28}$ for the case of three particles in the final state is$^{30}$

$$W \sim \sum (-1)^{a' - b' + s_1 + l_3' - j - c' + s_3} \hat{a} \hat{b} \hat{a}', \hat{b}', \hat{l}, \hat{l}' \hat{l}_2 \hat{l}_2'$$

$$\hat{a}_3 \hat{a}_3' \hat{c}' (l_1, l_1', 0 | r_0) (l_2, l_2', 0 | r_2, 0) (l_3, l_3', 0 | r_3, 0)$$

$$W(\hat{a} \hat{c}'; \hat{a}_3 \hat{c}_3'; \hat{r}_3 \hat{s}_3) W(\hat{c} \hat{c}_3'; \hat{a}_3 \hat{e}_3'; \hat{r}_3 \hat{s}_3) W(\hat{l}, \hat{l}', \hat{b} \hat{b}'; \hat{r}_b \hat{a})$$

$$\left\{ \begin{array}{c} l, l' \hfill \\
    b, b' \hfill \\
    k, k' \hfill \\
\end{array} \right\} \sum_\eta (k_2 \eta, k_3 - \eta | k_0) Y_{k_2}^\eta (\theta_1, \phi_1) Y_{k_3}^{-\eta} (\theta_2, \phi_2)$$

where the quantities $\hat{r}$ represent $(2l+1)^{1/2}$. The primed quantities represent the values of the quantum numbers after the emission of any unobserved radiation. For this particular case the primed and unprimed quantities are equal since there is no intervening radiation.

By combining the density of states treatment of Phillips, Grify and Biedenharn$^{21}$ and the angular correlation treatment presented by Devons and Goldfarb,$^{28}$ it is
possible to obtain a relationship for predicting the yield from a three-body reaction as follows:

\[
\frac{d^2 \sigma}{d \Omega_2 d \Omega_3} = \frac{\lambda^2}{4 \sigma_1 \sigma_2} \sum (-1)^{a-b'+s_1+\epsilon_3'-i-c-c'+s_2} \hat{a}_b \hat{a}'_{b'} \hat{l}_1 \hat{l}_2 \hat{l}_3 \hat{l}_2' \hat{l}_3'
\]

\[
\hat{a}_b \hat{a}'_{b'} (l_1, \alpha_1, \lambda_1; k_8, 0; l_2, \alpha_2, 0; k_2, 0) (l_3, \alpha_3, 0; k_3, 0)
\]

\[
\mathcal{W}(i, s'; c, c'; k_i, s_i) \mathcal{W}(cc', l_3, l_2', k_3, s_3) \mathcal{W}(\lambda, \lambda', \beta, \beta'; k_6, \lambda)
\]

\[
\sum \left\{ \begin{array}{c} \ell_2 \ell_1 \\ \ell_2' \ell_1' \end{array} \right\} \sum (k_2 \eta k_i \eta' k_6) \eta_{k_2} (\Theta_i, \phi_i) \eta_{k_3} (\Theta_i', \phi_i') \times \langle b b' R | b' b \rangle \langle b' b' | R^+ | b b \rangle \rho(E_c).
\]

(15)

Aldridge has applied a similar treatment to the reaction \(^3\text{He}(^3\text{He}, p)p, \alpha\) to predict the resulting energy spectrum with fair success. His treatment includes a density of states function but is not satisfactory for predicting the yield from very broad states. Simpson has applied the density of states treatment to the three-nucleon system to predict the yield from the singlet nucleon-nucleon system. These two techniques are combined in equation (15) which will be employed to describe the relative yield from \(2^+\) states in \(^8\text{Be}\).

**Application to Data**

Treatment of the \(^9\text{Be}(^3\text{He}, \alpha)2\alpha\) reaction is greatly simplified by the presence of identical spinless particles
in the final state. By considering only the $2^+$ states in $^8\text{Be}$ at 2.9 and 16.9 MeV the quantum numbers of the recoiling nucleus are fixed as $c = 2, s_2 = 0, s_3 = 0, l_3 = 2$. In the incoming channel $\sigma_1 = \frac{1}{2}$ and $\sigma_2 = \frac{3}{2}$. Thus we have the following possible quantum numbers:

\[
\begin{align*}
\frac{3}{2}^+ & \quad \frac{3}{2}^- \\
\alpha & \quad \frac{1}{2}^- \\
\alpha & \quad \frac{3}{2}^+ \\
\alpha & \quad \alpha \\
\frac{1}{2}^+ & \quad \frac{1}{2}^+ \\
\lambda_3 & = 2
\end{align*}
\]

Data was actually taken with detector 1 fixed at $40^\circ$. This means that the angular correlation will depend only upon the relative angles between the recoil particle and the direction of the first emitted particle. Angular distributions of the first emitted particle measured by Dorenbusch\textsuperscript{19} indicate that the distribution at $40^\circ$ is similar to $\lambda_2 = 1$. Moreover, since the correlation is independent of the distribution of the first emitted particle, little generality will be lost in assuming that $\lambda_1 = 0$. This leads to
the following set of possible quantum numbers:

\[ l_1, l_1' = 0 \]
\[ \sigma_1, \sigma_1' = \frac{1}{2} \]
\[ \sigma_2, \sigma_2' = \frac{3}{2} \]
\[ a = b = b' = 1, 2 \]
\[ l_2, l_2' = 1 \]
\[ j = j' = 2 \]
\[ s_1 = 0 \]
\[ c = c' = 2 \]
\[ s_2 = 0 \]
\[ s_3 = 0 \]
\[ l_3 = l_3' = 2 \]

Again for simplicity, only the case of opposite spins (i.e. \( a=b=1 \)) will be considered since the angular dependence comes out the same in both cases. The choice of \( l_1 = 0 \) forces \( k_b = 0 \) through the Clebsch-Gordan coefficient

\[ \left( l_1 \ 0 \ l_1' \ 0 \mid k_b \ 0 \right). \]

This condition forces \( k_2 = k_3 \) in the \( j \)-\( j' \) symbol leading to a reduction in the spherical harmonics to a form

\[
\frac{\sum_{m} (k_2 m k_3 m - m 1 0 0)}{2} \left( k_2 \right) \left( k_3 \right) P_{l_2} \left( \cos \theta \right)
\]

where

\[
\cos \theta = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2)
\]

and \( \theta \) is the angle that the second emitted particle makes.
with the recoil axis. Evaluation of the coupling coefficients leads to the relationship.

\[
\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{3 \alpha^2}{128\pi} \left[ P_0(\cos\theta) - P_2(\cos\theta) \right] \langle bB'B''|B'B''|\rho(E_0) \\
= \frac{9 \alpha^2}{256\pi} \left[ 1 - \cos^2\theta \right] \langle bB'B''|B'B''|\rho(E_0)
\]

This relationship predicts a distribution which shows a minimum along the recoil direction.

Figure 35 shows a comparison of this equation with the observed laboratory cross section at \( \theta_1 = 40^\circ \) and a bombarding energy of 4 MeV for the 2.9 and 16.9 states in \(^8\)Be. The density of states function for the 2.9 MeV state was obtained by applying equation (9) to phase shifts reported for alpha-alpha scattering by Russell, Phillips and Reich\(^{34}\). The density of states function was obtained from a computer program written by S. T. Emerson\(^{32}\) which employed a curve fit to the experimental phase shifts to compute the density of states distribution. The 16.9 state was much easier to treat. The Breit-Wigner or dispersion theory form for the resonance was used leading to a relationship for the density of states as given in equation (12).
Figure 35. Angular correlations between the first emitted alpha particle and a subsequent breakup particle from $^8$Be for the 2.9 and 16.9 MeV states. The solid curves are the predictions on the basis of density of states for an $\ell = 1$ first emitted particle.
2. 1 State
θ₁ = 40°
E_{\text{He}} = 4.0 \text{ MeV}

Ground State + 1st State
θ₁ = 40°
E_{\text{He}} = 4.0 \text{ MeV}
The angles involved in the calculation are given in the recoil center-of-mass system, RCM. This requires that the transformations derived in Appendix B be used to translate the appropriate laboratory quantities to the RCM. To convert the resulting values for the yield back into the laboratory system requires the transformations described in Appendix C. These transformations include the effects due to phase space.

The resulting fits to the data shown in Figure 35 are quite remarkable. The sharp peak in the spectrum for the 16.9 state is due to inclusion of the 0\(^+\) ground state in the fit. The ground state which should show an isotropic distribution in the RCM system could not be resolved from the experimental data for the 16.9 MeV state. The dashed curve corresponds to the predicted yield excluding the ground state. By using the results of Figure 35 to normalize the two theoretical fits together, the predictions were compared to the energy distributions of the two states. These comparisons are shown in Figures 36 and 37. The goodness of the fit to states for which the first emitted particle was not detected by Counter 1 implies that the equations hold reasonably well for a wide range of angles.
Figures 36, 37. Density of states fits to energy profile spectra for a bombarding energy of 4 MeV.
Discussion

The failure of this experiment to show any positive evidence of interference can not be interpreted as a negative result largely due to the differences in intensities of the states being interfered. The yield from the $4^+$ state in $^8$Be at 11.4 MeV is insufficient to provide information about interference. Stotland$^{33)}$ has reported seeing interference of the state with 27 MeV $^3$He.

Good fits obtained by a density of states calculation with inclusion of the spin of the recoil nucleus provide strong support for interpreting the process as a pure sequential breakup. In support of this claim, the argument can be made that all events observed in the coincidence spectra which do not obviously belong to one of the longer lived states in $^8$Be can be attributed to the 11.4 MeV state. The time for a 10 MeV alpha particle leaving $^8$Be in this state to separate by more than one nuclear radius is of the order of $10^{-20}$ seconds while the state has a life time of less than $10^{-21}$ seconds, so some complicated distributions from this state might be anticipated.
V. COMPARISON OF CROSS-SECTIONS

A rough estimation of the total cross section for the $^{9}\text{Be}(^{3}\text{He}, \alpha)^{2}\alpha$ reaction can be obtained by summing the data from Figures 32-34 over all possible angles for detector 1 and detector 2 and normalizing the results to the elastic cross section data from Figure 11. This gives a value for the total cross section which is of the order of 25 mb. The data of Dorenbusch $^{19}$ indicates a total cross section of comparable value. The total cross section for the $^{9}\text{Be}(^{3}\text{He}, {}^{6}\text{Li}){}^{6}\text{Li}$ reaction is of the order of 5 mb which compares favorably with the three-alpha cross section. However, neither of these reactions compares favorably with the geometrical cross section for $^{3}\text{He} + {}^{9}\text{Be}$ which is approximately 700 mb. The three-alpha yield from this reaction is also considerably lower than that observed by Bronson $^{2}$ for the $^{11}\text{B}(\text{P}, \alpha)^{2}\alpha$ process.
APPENDIX A

General Three-Body Kinematics

In order to determine the locus of the energy $E_2$ of particles counted in detector 2 at $\Theta_2, \phi_2$ versus the energy $E_1$ of particles counted in detector 1 at $\Theta_1, \phi_1$, we proceed as follows:

Conservation of momentum and energy gives

\[
\vec{P}_0 = \sum_{i=1}^{3} \vec{P}_i
\]

or

\[
\sum_{i=1}^{3} P_i \cos \Theta_i = P_0, \quad (z \text{ direction}) \tag{A1}
\]
\[
\sum_{i=1}^{3} P_i \sin \Theta_i \cos \phi_i = 0, \quad (y \text{ direction}) \tag{A2}
\]
\[
\sum_{i=1}^{3} P_i \sin \Theta_i \sin \phi_i = 0, \quad (x \text{ direction}) \tag{A3}
\]

and

\[
\frac{P_0^2}{2m_0} + Q = \sum_{i=1}^{3} \frac{P_i^2}{2m_i}. \tag{A4}
\]

Now, in order to determine $E_2$ in terms of $E_1$ (or $P_2$ in terms of $P_1$), we need to eliminate $P_3$ from equation (A4). It is evident that this can be done by solving (A1), (A2) and (A3) for $P_3$, squaring and adding to get:

\[
P_3^2 = (P_0 - P_1 \cos \Theta_1 - P_2 \cos \Theta_2)^2
\]
\[
+ (P_1 \sin \Theta_1 \cos \phi_1 + P_2 \sin \Theta_2 \cos \phi_2)^2
\]
\[
+ (P_1 \sin \Theta_1 \sin \phi_1 + P_2 \sin \Theta_2 \sin \phi_2)^2
\]
This gives
\[ P_3^2 = P_0^2 + P_1^2 + P_2^2 - 2P_0(P_1 \cos \theta_1 + P_2 \cos \theta_2) 
+ 2P_1P_2 \cos \theta_1 \cos \theta_2 + 2P_1P_2 \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2). \]

Substituting into equation (A4) gives
\[ \frac{P_0^2}{2m_0} + \frac{Q}{2m_1} = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \frac{1}{2m_3} \left[ P_0^2 + P_1^2 + P_2^2 
- 2P_0(P_1 \cos \theta_1 + P_2 \cos \theta_2) + 2P_1P_2 (\cos \theta_1 \cos \theta_2 
+ \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2)) \right]. \]

This can be rearranged as
\[ \left[ \frac{1}{2m_2} + \frac{1}{2m_3} \right] P_2^2 + \left[ \frac{P_1}{2m_3} (\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2)) 
- \frac{P_0}{m_3} \cos \theta_2 \right] P_2 + \left[ P_1^2 \left( \frac{1}{2m_1} + \frac{1}{2m_3} \right) + P_0^2 \left( \frac{1}{2m_3} - \frac{1}{2m_0} \right) 
- \frac{1}{m_3} P_0P_1 \cos \theta_1 - Q \right] = 0. \]

which has the quadratic form
\[ A P_2^2 + B P_2 + C = 0 \]

where
\[ A = \left[ \frac{1}{2m_2} + \frac{1}{2m_3} \right], \]
\[ B = \frac{1}{m_3} \left[ P_1 \cos \theta_1 \cos \theta_2 + P_2 \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2) - \frac{P_0}{m_3} \cos \theta_2 \right], \]
\[ C = \frac{1}{m_3} \left[ P_1^2 \left( \frac{1}{2m_1} + \frac{1}{2m_3} \right) + P_0^2 \left( \frac{1}{2m_3} - \frac{1}{2m_0} \right) - \frac{1}{m_3} P_0P_1 \cos \theta_1 \right]. \]
\[ c = \left[ P_0^2 \left( \frac{1}{2m_a} - \frac{1}{2m_0} \right) + P_1^2 \left( \frac{1}{2m_1} + \frac{1}{2m_2} - \frac{1}{m_2} P_0 P_1 \cos \theta_1 \right) - Q \right]. \]

The solution \( P_2 \) is just
\[ P_2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \]

There are thus two possible solutions for \( P_2 \) in terms of \( P_1 \). Both solutions are real although a negative value does not correspond to a detectable event in counter 2.

**Sequential Breakup**

The kinematics for simultaneous and sequential decay into three particles are identical. However, the step-wise process must be considered in determining the internal energies of the intermediate two-body system.

Consider the following two-step process:

(a) \( A + B \rightarrow i + (jk) \)

(b) \( i + (jk) \rightarrow i + j + k \)

where \( i, j \) and \( k \) are any permutation of the particles 1, 2 and 3. Conservation of energy requires that

\[ E_0 + Q = E_i + E_{jk} + E_{jk} \]  \( \text{(A5)} \)

where \( E_{jk} \) is the internal energy of the recoil system \( (jk) \).
Now, conservation of momentum requires that

\[ P_i \cos \theta_i + P_{3R} \cos \theta_{3R} = P_0, \]
\[ P_i \sin \theta_i \sin \phi_i + P_{3R} \sin \theta_{3R} \sin \phi_{3R} = 0, \]
\[ P_i \sin \theta_i \cos \phi_i + P_{3R} \sin \theta_{3R} \cos \phi_{3R} = 0. \]

Both \( \phi_i \) and \( \phi_{3R} \) can be eliminated from equations (A7) and (A8) by squaring and adding to get

\[ P_{3R}^2 \sin^2 \theta_{3R} = P_i^2 \sin^2 \theta_i. \]

In a similar manner, \( \theta_{3R} \) can be eliminated from (A6) and (A7) giving

\[ P_{3R}^2 = P_i^2 \sin^2 \theta_i + (P_0 - P_i \cos \theta_i)^2. \]

Thus

\[ E_{3R} = \frac{1}{2m_{3R}} \left( P_0^2 + P_i^2 - 2P_0P_i \cos \theta_i \right), \]

and from (A5)

\[ E_{3R} = E_0 + q - E_i - \frac{1}{2m_{3R}} \left( P_0^2 + P_i^2 - 2P_0P_i \cos \theta_i \right). \]

When the first emitted particle is detected in counter 1, the internal energy of the recoil (23) system is:

\[ E_{23} = E_0 + q - E_i - \frac{1}{2m_{23}} \left( P_0^2 + P_i^2 - 2P_0P_i \cos \theta_i \right). \]
Thus $\mathcal{E}_{23}$ is single-valued in $E_1$ and when plotted versus $E_1$ will be a single-branched curve.

When the first emitted particle is detected in counter 2, the internal energy of the recoil (13) system is:

$$\mathcal{E}_{13} = E_0 + Q - E_2 - \frac{1}{2m_{13}} \left( p_0^2 + p_2^2 - 2p_0p_2\cos\theta_2 \right).$$

Thus $\mathcal{E}_{13}$ is a single-valued function of $E_2$, but as a function of $E_1$ it is double-valued since $E_2$ is a double-valued function of $E_1$. This means that $\mathcal{E}_{13}$ will have two branches of mathematically possible loci when plotted versus $E_1$.

When the first emitted particle is undetected (both breakup particles are counted), the internal energy of the recoil system is:

$$\mathcal{E}_{12} = E_0 + Q - E_3 - \frac{1}{2m_{12}} \left( p_0^2 + p_3^2 - 2p_0p_3\cos\theta_3 \right).$$

By applying the same approach that was employed to solve for $P_2$ in terms of $P_1$ in the first part of this appendix, it is possible to obtain a similar quadratic expression relating $P_3$ to $P_1$. Thus $\mathcal{E}_{12}$ is also a double-valued function of $E_1$.

If the metastable recoiling nucleus has well defined levels, then the values of $\mathcal{E}_{jk}$ will be restricted. This implies that only limited portions of the kinematic locus will be populated by a sequential process.
APPENDIX B

Special Kinematic Relations

Because of the sequential nature of most three-body processes, it is necessary to consider the kinematics as viewed from three different coordinate systems. Appendix A gives the kinematics in the laboratory coordinate system. However, for theoretical considerations these quantities must be transformed to the other systems. These are the system center-of-mass (SCM) and recoil center-of-mass (RCM) frames of reference. Since all measurements for this experiment were made with both detectors in the same plane, the azimuthal angles will not be considered in this treatment.

The SCM is moving with a velocity

$$v_{SCM} = v_0 \frac{m_0}{M}.$$  

The first emitted particle has a velocity in the SCM $v_i$ determined by the law of cosines as

$$v_i = (v_i^2 + v_{SCM}^2 - 2v_i v_{SCM} \cos \theta_i)^{1/2}.$$  

The SCM polar angle is given by

$$\cos \Theta_i' = (v_i \cos \Theta_i - v_{SCM}) / v_i.$$  

Thus the velocities of all three particles in the SCM can be obtained from observed laboratory quantities. These
values can be used to determine the relationship of the breakup particles to the recoil axis. Figure 38 shows a diagram of the SCM and RCM systems. Two additional necessary quantities are the velocity of the recoil nucleus

\[ u_{RCM}^{(i)} = \frac{m_i}{m_i + m_R} \cdot u_i \]

and the breakup velocity of a particle from the recoil cluster in the RCM

\[ u_j^{(i)} = \sqrt{\frac{2m_k E_{sR}}{m_j (m_j + m_R)}} \]

From Figure 38 we see that the angle which a breakup particle makes with the beam direction is given by:

\[ \cos \theta_j^{(i)} = \frac{(u_j \cos \theta_j' + u_{RCM} \cos \theta_i')}{u_j^{(i)}} \]

and

\[ \sin \theta_j^{(i)} = \frac{(u_j \sin \theta_j' - u_{RCM} \sin \theta_i')}{u_j^{(i)}} \]

The angle which this particle makes with the recoil axis is \( \Theta = \pi - \theta_j' - \theta_j^{(i)} \). Thus the cosine of this angle becomes

\[ \cos \Theta = -\cos \theta_i' \cos \theta_j^{(i)} + \sin \theta_i' \sin \theta_j^{(i)} \]

This is the important quantity for expressing the angular correlation of a breakup particle with the first emitted particle.
Figure 38. The definition of variables and relationships between variables in the SCM and RCM coordinate systems.
APPENDIX C

Jacobian Notation

Theoretical expressions for the breakup of a system of particles must, in general, be transformed from a system in which the center-of-mass of the particles is at rest to the laboratory system. A general discussion of the Jacobians necessary to perform these operations is given by Buck.\textsuperscript{26)}

In the recoil center-of-mass system (RCM), the cross section is defined as

\[
\sigma_{\text{RCM}}^{(i)} = \frac{d^3 \sigma}{d \varepsilon_{ik} d \Omega_{i}^{'} d \Omega_{j}^{(i)}} \tag{C1}
\]

where \(d \Omega_{i}^{'}\) is the solid angle of the first emitted particle in the system center-of-mass (SCM), \(d \Omega_{j}^{(i)}\) is the solid angle for one of the recoil particles in the RCM and \(\varepsilon_{ik}\) is the internal energy in this recoil system.

Application of the Jacobian \(J_i\) transforms equation (C1) to the SCM as follows:

\[
\sigma_{\text{SCM}} = \frac{d^3 \sigma}{d \varepsilon_i d \omega_i d \omega_2} = J_i \sigma_{\text{RCM}}^{(i)} \tag{C2}
\]

where the indices 1 and 2 refer to the SCM variables corresponding to the detected particles. A second operator, \(J_L\), transforms the SCM cross section into the laboratory giving

\[
\sigma_{\text{Lab}} = \frac{d^3 \sigma}{d \varepsilon_i d \Omega_i d \Omega_2} = J_L \sigma_{\text{SCM}} \cdot \tag{C3}
\]
If the first emitted particle is detected in counter 1 at an SCM angle \((\Theta'_1, \varphi'_1)\), then the recoil axis has the orientation \((\pi - \Theta'_1, \pi - \varphi'_1)\). The direction of emission of particle 2 with respect to the recoil axis is \((\Theta''_2, \varphi''_2)\). The transformation involves solid angles of the form:

\[
dw = \sin \Theta \, d\Theta \, d\varphi = -d(\cos \Theta) \, d\varphi.
\]

The transformation required to transform \(\Theta''_2\) to the SCM is:

\[
J_1 = \frac{\partial \left( E_{12}, \cos \Theta'_1, \varphi'_1, \cos \Theta''_2, \varphi''_2 \right)}{\partial \left( E'_1, \cos \Theta'_1, \varphi'_1, \cos \Theta'_2, \varphi'_2 \right)}.
\]

A similar relationship holds for the first emitted particle being detected in counter 2:

\[
J_2 = \frac{\partial \left( E_{13}, \cos \Theta'_2, \varphi'_2, \cos \Theta''_3, \varphi''_3 \right)}{\partial \left( E'_1, \cos \Theta'_1, \varphi'_1, \cos \Theta'_3, \varphi'_3 \right)}.
\]

When the first emitted particle is undetected the Jacobian becomes:

\[
J_3 = \frac{\partial \left( E_{12}, \cos \Theta'_3, \varphi'_3, \cos \Theta''_3, \varphi''_3 \right)}{\partial \left( E'_1, \cos \Theta'_1, \varphi'_1, \cos \Theta'_3, \varphi'_3 \right)}.
\]

The results from evaluation these three Jacobians are:

\[
J_1 = \left| \frac{m_2}{M} \frac{p_2'}{p_2''} \left[ \frac{1}{\Delta p + p'_1 \cos \Theta'_3} \right] \right|,
\]
\[ J_2 = \frac{p'_1 p'_2}{p'_1 p'_2} J_1, \]

\[ J_3 = \left| \frac{M}{m_2} \frac{p'_2}{p'_1} \frac{p'_2^2}{p'_1} \left[ \frac{1}{A p'_2 + p'_1 \cos \delta_{12}} \right] \right| \]

where \( M = m_1 + m_2 + m_3 \), \( A = (m_2 + m_3)/m_2 \), \( \delta_{12} \) is the angle between the directions of particles 1 and 2 in the SCM, \( p'_1 \) is the momentum of particle 1 in the SCM and \( p'_j \) is the momentum of particle \( j \) in the \((j,k)\) RCM. The use of \( J_1 \), \( J_2 \) and \( J_3 \) transforms the original calculations into the SCM, so that the transformation to the laboratory must next be made by applying \( J_L \):

\[ J_L = \frac{\partial}{\partial} \left( \varepsilon'_1, \cos \theta'_1, \phi'_1, \cos \theta'_2, \phi'_2 \right) \]

\[ J_L = \left| \frac{P_1 P_2^2}{P'_1 P'_2} \left[ \frac{A p'_2 + p'_1 \cos \delta_{12}}{A p'_2 + p'_1 \cos \theta_{12} - p_0 \cos \theta_{2}} \right] \right| \]

where \( P \) and \( \Theta \) represent laboratory quantities.

The resulting transformations from the individual RCM systems to the laboratory can be expressed as:

\[ J_1 J_L = \left( \frac{M}{m_1 m_2 m_3} \right) \frac{1}{p'_1 p'_2} N(\varepsilon_1, \Omega_1, \Omega_2) \]
\[ J_2 J_L = \left( \frac{M}{m_1 m_2 m_3} \right) \frac{1}{p_{L}^{0} p_{L}^{0}} N(E_1, \Omega_1, \Omega_2) \]

\[ J_3 J_L = \left( \frac{M}{m_1 m_2 m_3} \right) \frac{1}{p_{L}^{0} p_{L}^{0}} N(E_1, \Omega_1, \Omega_2) \]

where

\[ N(E_1, \Omega_1, \Omega_2) = \frac{m_1 m_3 p_1 p_2^2}{p_2 \left( \frac{m_1 + m_2}{m_2} \right) - p_0 \cos \Omega_2 + p_1 \cos \Omega_3} \]

is the differential element of available phase space.
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