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THE HYDRODYNAMIC FORCES ACTING ON A CYLINDER IN AN OSCILLATING FLOW

by

CHARLES MITCHELL McBRIDE

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NOMENCLATURE

A	Cross-sectional area of wake
A_O	Body cross-sectional area
C_D	Steady state drag coefficient
C_M	Coefficient of virtual mass
d	Diameter of body
F	Total force ; total force per unit length
h	Double-Amplitude of sinusoidal motion
I	Inertia force ; inertia force per unit length
T	Period of oscillation
t	Time ; time lapse between encounters
U_m	Maximum body velocity in sinusoidal motion
u_1	Centerline wake velocity ; velocity of particle
U_∞	Body velocity ; body velocity relative to a fluid particle at encounter
V	Relative velocity between fluid and body
x	Location of a fluid particle ; position of body in Fugure 4
x_1	Distance of particle from body
β	Empirical constant = 0.18
ρ	Density of fluid
φ	Location of body in cycle relative to a point of zero velocity

φ_1	Location of body at time of first encounter
φ_2	Location of body at time of second encounter

1. INTRODUCTION

Hydrodynamic forces are examined which are acting on a body when executing an oscillatory motion, that is, motions are considered in which the body reverses its direction of motion. A body traveling through a fluid imparts momentum to the fluid, thereby establishing a wake. If the body then reverses its direction of motion, the force it will experience will be influenced by the existence of the wake.

Stokes (1)* and Basset (2) investigated motions of this type, but restricted themselves to small amplitude oscillations of the body which involve high accelerations and low velocities. This investigation considers cases in which inertia forces do not dominate over drag forces. For sinusoidal motions this implies a low frequency of oscillation. The bodies considered are cylinders with axes perpendicular to the direction of motion. The body motion is restricted to a straight line and the wake is assumed to be turbulent.

The previously mentioned conditions which are imposed on this problem correspond to situations in which a body is submerged in a fluid which is oscillating back and forth. Such situations occur near the floor of the ocean. Here the fluid motion is oscillatory due to the action of surface waves, yet the motion is restricted to almost a straight line due to the presence of the floor.

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* Numbers in parentheses denote references.

DESCRIPTION OF THE WAKE IN THE STEADY STATE

2.1 The Wake at Large Distances from the Body

H. Schlichting (3), (4), first investigated two-dimensional, steady state wakes. His analysis is based on Prandtl's mixing length hypothesis (3) and on the similarity of the velocity profile at large distances downstream from the body. Schlichting's result for the velocity along the centerline of the wake is

$$\frac{u_1}{U_\infty} = \frac{\sqrt{10}}{18\beta}\left(\frac{C_D d}{X_1}\right)^{1/2} , \qquad (2\text{-}1)$$

where U_∞ is the velocity of the body, u_1 is the velocity of the fluid in the center of the wake, X_1 is the distance downstream from the body, d is the diameter of the body, C_D is the drag coefficient of the body, and β is the constant of proportionality. Experimental evaluation of this constant gives β = 0.18 for cylinders. If this value is used for β, then $\sqrt{10}/(18\beta) \cong 1$, and equation (2-1) can then be written approximately as

$$\frac{u_1}{U_\infty} = \left(\frac{C_D d}{X_1}\right)^{1/2} . \qquad (2\text{-}2)$$

Equation (2-2) will be assumed to represent the wake velocity at locations far behind the body.

2.2 Model of the Wake at Locations Near the Body

Consider a steady state, one-dimensional momentum equation.

$$\frac{1}{2} C_D A_o \rho U_\infty^2 = \rho A u_1 (U_\infty - u_1) \quad , \tag{2-3}$$

where A_o is the cross-sectional area of the body, A can be considered to be the cross-sectional area of the wake of constant velocity and ρ is the fluid density. In equation (2-3) it is assumed that the static pressure is ambient. Equation (2-3) can be rearranged:

$$\frac{1}{2} C_D \frac{A_o}{A} = \frac{u_1}{U_\infty} \left(1 - \frac{u_1}{U_\infty}\right) \quad . \tag{2-4}$$

A maximum value for the area ration A_o/A exists when

$$\frac{1}{2} C_D \frac{d(A_o/A)}{d(u_1/U_\infty)} = 1 - 2\frac{u_1}{U_\infty} = 0 \quad ,$$

or when

$$\frac{u_1}{U_\infty} = \frac{1}{2} \quad . \tag{2-5}$$

For this value of the wake velocity u_1 ; the wake area has the value

$$A = 2 C_D A_o \quad . \tag{2-6}$$

Equation (2-6) represents the minimum possible value for the wake area in the absence of static pressure variations. For locations near the body, equations (2-5) and (2-6) will be assumed to represent the wake.

There are now two descriptions of the wake, one which is assumed to apply near the body and the Schlichting model which is restricted to regions far downstream of the body. In order to describe the wake in its entirety, these two models must be joined together, at the proper distance from the body.

To extend the Schlichting model of the wake to its minimum cross-section we require that the velocity be continuous. This requirement is met by equating the velocities for the two models as follows,

$$\frac{1}{2} U_\infty = U_\infty \left(\frac{C_D d}{X_1} \right)^{1/2} ,$$

or

$$X_1 = 4 C_D d . \qquad (2\text{-}7)$$

Thus, in this description of the wake, the Schlichting model may be used for distances which are more than $4C_D d$ from the body.

Summarizing, the entire length of the wake is assumed to be completely described by the following two equations:

$$u_1 = \frac{1}{2} U_\infty , \quad 0 \leq x_1 \leq 4C_D d , \qquad (2\text{-}8)$$

$$u_1 = U_\infty \left(\frac{C_D d}{X_1} \right)^{1/2} , \quad x_1 \geq 4C_D d . \qquad (2\text{-}9)$$

2.3 The Motion of a Fluid Particle

We now consider the problem of finding the displacement of a fluid particle from its initial location after the body sets it in motion. If the present model for the wake is used, equations (2-8) and (2-9) describe the motion of a particle of fluid. These equations are

$$\frac{dx}{dt} = \frac{1}{2} U_\infty \quad , \qquad 0 \leq x_1 \leq 4C_D d, \tag{2-8a}$$

$$\frac{dx}{dt} \pm \frac{U_\infty \sqrt{C_D d}}{x_1^{1/2}} \quad , \qquad x_1 \geq 4C_D d, \tag{2-9a}$$

where x_1 is the distance of the particle downstream of the body, U_∞ is the velocity of the body relative to the fluid particle at encounter, and x is the location of the fluid particle in an inertia frame in which the particle is initially at rest.

If x is measured from the initial location of the particle and t is measured from the instant the body passes the particle, then the solution to equation (2-8a) is

$$x = \frac{1}{2} U_\infty t \quad , \qquad x \leq 4C_D d. \tag{2-10}$$

With x and t measured in the above manner it follows that

$$x_1 = U_\infty t - x$$

and equation (2-9a) can be written as

$$\frac{dx}{dt} = \frac{U_\infty \sqrt{C_D d}}{(U_\infty t - x)^{1/2}} \; , U_\infty t - x \geq 4C_D d. \tag{2-11}$$

The solution to this differential equation (2-11) is obtained by a transformation $\eta = (U_\infty t - x)^{1/2}$, with which equation (2-11) may be written as

$$\frac{dx}{d\eta} = 2\sqrt{C_D d} \left(\frac{\eta}{\eta - \sqrt{C_D d}} \right) . \tag{2-12}$$

Equation (2-12) has the solution

$$x - x_0 = 2\sqrt{C_D d}\left[\eta + \sqrt{C_D d}\ \ln\left(\eta - \sqrt{C_D d}\right)\right]_{\eta_0}^{\eta} \qquad (2\text{-}13)$$

The initial condition for this equation is equation (2-10).

Using this initial condition, equation (2-13) becomes

$$X = 2\sqrt{C_D d}\left[\sqrt{U_\infty t - X} + \right. \qquad (2\text{-}14)$$

$$\left. + \sqrt{C_D d}\ \ln\left(\sqrt{\frac{U_\infty t - X}{C_D d}} - 1\right)\right], \quad \frac{U_\infty t - X}{C_D d} \geq 4.$$

3. DESCRIPTION OF THE UNSTEADY WAKE

3.1. Assumptions

Analytical prediction of forces on bodies oscillating in a fluid has been successful only for inertia forces or when treating small oscillating velocities. Quantitatively different results must be expected when these oscillating velocities are large.

In view of the difficulties arising in the analytical treatment, the approach taken here will be to apply the steady state parameters to the unsteady problem. This approach should not be excessively in error since the motions to be considered are those in which accelerations are not the prime consideration, that is, motions in which the drag force dominates over the inertia force are considered. In applying the steady state result as an input to the unsteady case, the variation of the velocity of the body must be accounted for insofar as it affects the relative velocity between the body and the fluid.

3.2. MAXIMUM FORCES

3.2.1. General Remarks

Consider a body which is submerged in a fluid which is at rest. If the body begins an oscillatory motion, then, according to the wake flow model which has been adopted, the maximum forces will occur in the second half cycle of the body motion, half-cycles being measured from points of reversal of body motion. The reason for this is:

During any half-cycle, the body velocity varies between zero and some maximum value. Consider the body at a position in the first half-cycle where the body velocity has a value of U_∞ . According to the model being used for the wake, the fluid particle encountered by the body at this location will be set in motion with a velocity of $1/2\ U_\infty$ since the particle had an initial velocity of zero. Now consider the body at a position in any other half-cycle where the body has a velocity of U_∞ . The body will encounter a particle having some velocity, u. After the encounter with the body, this particle will have a velocity of $1/2\ (U_\infty + u)$. The velocity u is in the opposite direction of U_∞ and is thus subtractive to U_∞ in the preceding value for the velocity of the particle after the encounter. Therefore the wake velocity after any other half-cycle of motion is less than the wake velocity established in the first half-cycle, and the relative velocity between the body and the fluid is largest during the second half-cycle. Since the drag force is proportional to the square of the relative velocity, the maximum forces will occur during the second half-cycle of the body motion.

Because the maximum forces occur in the second half-cycle, it is desirable to calculate the relative velocity between the body and the fluid during this entire half-cycle. Once the relative velocity is known, all forces can be calculated.

3.2.2 Sinusoidal Motion

To obtain numerical results, a specific body motion must be considered. For this purpose, and because sinusoidal motions occur in nature, a sinusoidal body motion is considered first.

The equations to be solved in this care are equations (2-10) and (2-14) along with two equations arising from the geometry of sinusoidal motion (see Figure #1). These two equations are:

$$U_m t = \frac{h}{2}\left(\varphi_2 - \varphi_1\right) , \quad (3\text{-}1)$$

and

$$x = \frac{h}{2}\left(\cos\varphi_1 - \cos\varphi_2\right) , \quad (3\text{-}2)$$

where U_m is the maximum velocity of the body during the cycle, h is the double-amplitude of the motion, φ is the location of the body in the cycle relative to the position of zero velocity, t is the time lapse from the instant the body encounters a fluid particle until the instant the body re-encounters that same fluid particle, x is the distance traveled by the particle in the time t, and the subscripts $_1$ and $_2$ denote the first and second encounters, respectively.

For amplitudes of motion that are small enough that the wake does not decay ($x \leq 4 C_D d$), equation (2-10) must be satisfied. Substituting equations (3-1) and (3-2) into equation (2-10), and noting $U_\infty = U_m \sin\varphi_1$, leads to the equation

$$\frac{\varphi_1}{2}\sin\varphi_1 + \cos\varphi_1 = \frac{\varphi_2}{2}\sin\varphi_1 + \cos\varphi_2 \quad , \qquad (3\text{-}3)$$

The solution to this equation is presented in Figure #2.
The restriction on this equation is that $\frac{X}{C_D d} = \frac{h}{d2C_D}(\cos\varphi_1 - \cos\varphi_2) \leq 4$.
The maximum value of $X/(\frac{h}{2})$ corresponding to the solution to equation (3-3) is 1.24. This non-decaying model for the wake then holds for values $0 \leq h/(2C_D d) \leq 3.22$.
To calculate the drag on the body, the relative velocity, V, is needed. Equation (2-8a) gives the particle velocity for this case, and the relative velocity can be written as

$$\frac{V}{U_m} = \sin\varphi_2 - \frac{u_1}{U_m} = \sin\varphi_2 - \frac{\sin\varphi_1}{2} \quad . \qquad (3\text{-}4)$$

Consideration is now turned to equation (2-14). Ignoring terms of order $\left[X/(U_\infty t)\right]^2$, this equation can be approximated by

$$\frac{X}{C_D d} = \frac{\ln\left(\frac{U_\infty t}{C_D d}\right) + 2\sqrt{\frac{U_\infty t}{C_D d}} - 2\sqrt{\frac{C_D d}{U_\infty t}}}{1 + \sqrt{\frac{C_D d}{U_\infty t}} + \frac{C_D d}{U_\infty t} + \left(\frac{C_D d}{U_\infty t}\right)^{3/2}} \quad . \qquad (3\text{-}5)$$

Substituting equations (3-1) and (3-2) into equation (3-5) leads to

$$h(\cos\varphi_1 - \cos\varphi_2)/(2C_D d) =$$

$$= \frac{\ln\left[\frac{h\sin\varphi_1}{2C_D d}(\varphi_2-\varphi_1)\right] + 2\left[\frac{h\sin\varphi_1}{2C_D d}(\varphi_2-\varphi_1)\right]^{1/2} - 2\left[\frac{2C_D d}{h\sin\varphi_1(\varphi_2-\varphi_1)}\right]^{1/2}}{1+\left[\frac{2C_D d}{h\sin\varphi_1(\varphi_2-\varphi_1)}\right]^{1/2} + \left[\frac{2C_D d}{h\sin\varphi_1(\varphi_2-\varphi_1)}\right] + \left[\frac{2C_D d}{h\sin\varphi_1(\varphi_2-\varphi_1)}\right]^{3/2}} .$$

The use of trigonometric identities in the above equation yields

$$2H\sin\frac{\Delta}{2}\left(\cos\frac{\Delta}{2} + \cot\varphi_1\sin\frac{\Delta}{2}\right) = \frac{\ln(H\Delta) + 2(H\Delta)^{1/2} - \frac{2}{(H\Delta)^{1/2}}}{1+\left(\frac{1}{H\Delta}\right)^{1/2} + \left(\frac{1}{H\Delta}\right) + \left(\frac{1}{H\Delta}\right)^{3/2}} ,$$

where $H = h\sin\varphi_1/(2C_D d)$ and $\Delta = \varphi_2 - \varphi_1$.

The preceding equation can be written as

$$\varphi_1 = \cot^{-1}\left\{\frac{1}{(1-\cos\Delta)}\left[\frac{\ln(H\Delta) + 2(H\Delta)^{1/2} - \frac{2}{(H\Delta)^{1/2}}}{H\left\{1+\left(\frac{1}{H\Delta}\right)^{1/2} + \left(\frac{1}{H\Delta}\right) + \left(\frac{1}{H\Delta}\right)^{3/2}\right\}} - \sin\Delta\right]\right\} . \tag{3-6}$$

Values for φ_1 were calculated from equation (3-6) by assigning values to H and Δ . Then φ_2 and the parameter $h/(2C_D d)$ were calculated. From this information, plots of φ_1 vs. φ_2 for constant values of the parameter $h/(2C_D d)$

were obtained. These results are presented in Figure #3.

The relative velocity for the case in which the wake decays may be calculated using equations (2-11), (3-1), and (3-2). Use of these equations leads to the following expression for the relative velocity, V:

$$\frac{V}{U_m} = \sin\varphi_2 - \frac{\sin\varphi_1 \left(\frac{2C_D d}{h}\right)^{1/2}}{\left[\sin\varphi_1(\varphi_2-\varphi_1)+\cos\varphi_2-\cos\varphi_1\right]^{1/2}} . \quad (3\text{-}7)$$

The results for equation (3-7) for various values of the parameter $h/(2C_Dd)$, along with the result for equation (3-4), are plotted in Figure #4. The relative velocities presented in this figure determine the maximum force that the body will experience when the motion is sinusoidal.

It is seen from Figure 4 that the relative velocity for any value of the parameter $h/2C_Dd$ is greater than the body velocity. This will result in a drag force acting on the body which is greater than the drag force as calculated using the body velocity. For instance, in the case that $h/2C_Dd = 10$, Figure 4 shows the maximum value of the relative velocity to be 1.15 times the maximum body velocity. Since the drag force is proportional to the square of the relative velocity, this means that the body will experience a maximum drag force which is $(1.15)^2 = 1.32$ times the maximum drag predicted when the effect of the wake is ignored.

3.3 DIRECT METHOD

3.3.1 Approximation

The procedure presented in the previous section is indirect and, as a result, is too cumbersome to use to extend the solution beyond the second half-cycle of motion. A more useful method is needed. This method should be relatively simple so that it can be carried through many cycles until the steady state is reached.

Since the dependent and independent variables cannot be separated in equation (2-14), we seek an approximation to the original differential equation (2-11), with the expectation of obtaining an explicit expression for the displacement of a fluid particle in terms of the body location. The velocity of a fluid particle in the model adopted for the wake at locations not near the body depends on the distance between the particle and the body. This distance might be approximated by replacing the particle location with an extrapolation of equation (2-10), which is the expression for the location of a fluid particle near the body. If this approximation is made, then

$$x_1 = U_\infty t - x \cong U_\infty t - \frac{1}{2} U_\infty t = \frac{1}{2} U_\infty t \quad ,$$

where x_1 is the distance between the body and the particle.

Using the above approximation, the expression for the velocity, equation (2-11), may be written as

$$\frac{dx}{dt} = \frac{U_\infty \sqrt{C_D\, d}}{\left(\frac{1}{2} U_\infty t\right)^{1/2}} \quad , \quad U_\infty t \geq 8\, C_D d \,. \tag{3-8}$$

This approximation to the velocity is good in the region in which the particle velocity is only beginning to decay, since in this region the true value for x differs only slightly from the value of $\frac{1}{2}\, U_\infty\, t$.

Equation (3-8) may be integrated to yield

$$x - x_o = \sqrt{8 C_D d}\left[(U_\infty t)^{1/2} - (U_\infty t_o)^{1/2}\right] \,. \tag{3-9}$$

The initial conditions for the above equation are taken from equation (2-10). These initial conditions, when substituted into equation (3-9), lead to the expression

$$x = \sqrt{8 C_D d}\,(U_\infty t)^{1/2} - 4 C_D d \quad , \quad U_\infty t \geq 8\, C_D d \,. \tag{3-10}$$

For locations far from the body, equation (3-10) will be assumed to describe the motion of a fluid particle as a function of time (or of the body location). Equation (2-10) is again assumed to hold when the particle is near the body.

3.3.2 Graphical Procedure.

There is no explicit solution to equations (2-10) and (3-10), (3-1), and (3-2). There is, however, a simple graphical procedure which applies to any oscillatory body motion. This graphical method is explained, using Figure 5, as follows:

The location of the body is plotted as a function of time. The curve AO in Figure 5 is the path of a particular fluid particle previous to its encounter with the body at the point O. The straight line OB represents the instantaneous particle velocity at the instant of encounter. This line OB locates the reference coordinate system in which equations (2-10) and (3-10) apply for the particular particle being considered, that is, OB locates the inertia frame in which the fluid particle would appear to be stationary at the instant the particle encounters the body. The line OC represents the instantaneous body velocity at encounter. At any time the distance between OC and OB is the value of $U_\infty t$, which appears in equations (2-10) and (3-10); this distance is represented by the line GI in Figure 5. The curve OD represents the path the particle would have taken had it experienced no encounter with the body. This curve OD accounts for the deceleration ($\partial u/\partial t$) of the surrounding fluid into which the body is traveling. The difference between the distances travelled by a particle of the surrounding fluid when the particle's deceleration is taken into account and when its velocity is assumed to be constant is the length GH in Figure 5.

Equations (2-10) and (3-10) were derived using the assumption that the body is traveling in a surrounding fluid which is moving at a constant velocity. If this surrounding fluid is losing momentum, then the decay of the wake which is being generated by the body will be reduced with time. With time a fluid particle will conserve more momentum in a wake for which the surrounding fluid has a decaying velocity than it will for the case in which the surrounding fluid has a constant velocity. The difference in momentum for the particle in the wake in these two cases is now assumed to be equal to the difference in momentum for a particle of the surrounding fluid for the same two cases. Under this assumption the location of the fluid particle relative to the line OB as calculated from either equation (2-10) or equation (3-10) should be displaced by a distance equal to the length GH. Therefore equation (2-10) or (3-10) gives the location of the fluid particle relative to the curve OD. This location of the fluid particle is represented by the curve OE in Figure 5.

3.3.2.1. Simplification of the Procedure.

In the previous section the corrections to the location of a fluid particle due to deceleration of the surrounding fluid are usually small compared to the magnitude of the distance of the fluid particle from the reference coordinate system. If the decay of the velocity of the surrounding fluid is ignored, then a simpler graphical procedure results. Figure 6 is used in the explanation of this graphical method. The procedure is as follows:

The location of the body is plotted as a function of time. The curve AO is the path of a particular fluid particle before encounter with body at the point O. The straight lines OB and OC represent the instantaneous velocity of the particle and of the body, respectively, at the instant of encounter. The value of $U_\infty t$ is the distance between OC and OB and is represented by the line HG in Figure 6. The straight line OD passing through the point of encounter and the mid-point of HG is then a plot of equation (2-10), which holds for values of $U_\infty t \leq 8 C_D d$. For values of $U_\infty t \geq 8 C_D d$, the displacement of the fluid particle from the line OB may be calculated by equation (3-10). This calculated displacement is represented by the curve DF in Figure 6.

Ignoring the deceleration of the surrounding fluid may lead to situations in which the velocity in the wake decays to zero or even reverses its direction. If in the procedure described above the particle velocity decays to zero, then it must be forced to remain at a value of zero until the body encounters the particle again.

4. RESULTS AND DISCUSSION

In order to check the accuracy of the model adopted for the wake, a comparison between theory and experiment was made. Experimental results were taken from Keulegan and Carpenter (5) and from tests currently being performed in the Ryon Wave Tank at Rice University.

Keulegan and Carpenter (5) present the complete time history of the force acting on a circulat cylinder in an oscillating fluid for only one case in the range being considered. For this case the motion is sinusoidal and has the following characteristics: a maximum velocity of 41.0 cm/sec and a period of 2.075 seconds. The cylinder diameter is 0.75 inches. The graphical procedure and its simplified counterpart were carried out to determine the relative velocity between the body and the fluid for these conditions. The steady state drag coefficient used in the graphical procedure was taken to have a value of 1.2, which is the drag coefficient for a cylinder at the Reynolds number based on the maximum velocity of the body's oscillation for this case. Once the relative velocity between the body and the fluid was determined, the force acting on the body was calculated as the sum of the drag and the inertia forces. The inertia force per unit body length, I, was obtained from

$$I = C_M \rho \frac{\pi d^2}{4} \frac{dU_\infty}{dt} , \tag{4-1}$$

where ρ is the density of the fluid, d is the body diameter, t is time, U_∞ is the body velocity, and C_M is the coefficient of mass and is taken to have a value of 2.0 for a circular cylinder. The total force per unit length, F, acting on the body is then

$$F = C_D \frac{1}{2} \rho d V^2 + I , \tag{4-2}$$

where C_D is the steady state drag coefficient for the body (having a value of 1.2 for the case under consideration), V is the relative velocity between the body and the fluid, and I is defined by equation (4-1).

Figure 7 presents a comparison of the time history of the force acting on the body predicted from the adopted wake model with the experimental result of Keulegan and Carpenter (5). As can be seen, there is agreement between the predicted and the experimental results. The predicted result does, however, begin to depart noticeably from the experimental result in the regions in which the body reverses its direction of motion and inertia (acceleration) forces are important. In Figure 7 the points at which the body reverses its direction of motion are t/T = 0.25 and t/T = 0.75, where T is the period of the sinusoidal motion.

Figure 8 presents a comparison between the result of the simplified graphical procedure and the experimental result of Keulegan and Carpenter (5). There is again agreement between the predicted and the experimental results. The result predicted using the simplified graphical procedure differs very little from the result obtained from the more complicated graphical method.

An experimental program to measure the forces acting on a body oscillating in a fluid is currently underway at Rice University. Data from this program were selected to make another comparison with the present theory. The particular experimental data selected for this comparison were for a cylinder with a steady state drag coefficient of $C_D = 0.85$ in the range of the Reynolds number which existed during the test. The double-amplitude of the cylinder's oscillation in the test was about twenty feet. The body location as a function of time is shown in Figure 9)b). It can be seen from this figure that the acceleration of the body is very small except near locations where the direction of the body's motion reverses.

The graphical procedure was carried out for the test conditions, and the relative velocity between the fluid and the body was determined. The theoretical force acting on the body was then calculated as if it were totally a drag force (C_D = 0.85), that is, inertia forces were neglected in calculating the force acting on the body. Because of the nature of the body motion, this is a reasonable approximation to the total force acting on the body except in the neighborhood of the points at which the body reverses its direction of motion. Figure 9(a) is a comparison of the time history of the predicted and the experimentally determined force acting on the body. The correlation between the two forces is good, although the predicted force does differ significantly from the experimental result in the regions of appreciable acceleration of the body. This is to be expected since inertia forces were neglected in the calculation of the theoretical force. Figure 10 compares the experimentally determined force to the force predicted using the simplified graphical procedure to determine the relative velocity between the body and the fluid. There is again agreement between theory and experiment. The results of the two graphical procedures differ very little in this case, and in comparison of theory to experiment, the same comments may be made about the result of the simplified procedure as were made about the result of the complex procedure.

Figure 11(a) shows a comparison of the theoretical and the experimental values of the force acting on the body divided by the factor $\frac{1}{2}\rho A_o U_\infty^2$, where ρ is the fluid density, A_o is the cross-sectional area of the body, and U_∞ is the body velocity. This quantity $F/(\frac{1}{2}\rho A_o U_\infty^2)$ represents a drag coefficient needed to calculate the force acting on the body when the effect of the body's previous motion on the flow field is ignored, that is, it is a pseudo-drag coefficient which correctly predicts the force acting on the body when only the velocity of the body is considered. This pseudo-drag coefficient is shown only for the parts of the motion in which effects due to the acceleration of the body are unimportant. The experimental and the predicted values of this pseudo-drag coefficient are in agreement except in the second half-cycle of the body motion, in which instance the predicted value is high. Figure 11(b) gives the body location as a function of time. The purpose of this figure is to illustrate the regions of the motion in which the pseudo-drag coefficient of Figure 11(a) is shown.

In Figures 9-11 the predicted drag force has been compared to the measured total force acting on the body. A closer comparison of theory with experiment could be made if the drag force in the experiment were known. Only the total force can be measured in an experiment, and, since inertia forces are present, the drag force cannot be measured directly. To obtain the experimental drag force, the inertia force must be subtracted from the measured total force acting on the body. To calculate the drag force in this manner requires that the inertia force be approximated. The acceleration of the test model was determined by graphically estimating the second time derivative of the body location curve. The inertia force to be subtracted from the measured force acting on the test cylinder was then calculated.

Figure 12 compares the predicted drag force with the experimental drag force. The predicted force is slightly out of phase with the experimental drag force and has a slightly higher maximum value. In Figure 12 the discrepancy between the two forces in the regions of high acceleration of the body is much less than the discrepancy between the two forces shown in Fig. 9(a), in which inertia forces were not subtracted from the experimental force.

In summary, the results given by the wake model which was used matched the available experimental data very well. The theoretical result was slightly out of phase with the experimental data in both cases, but this was a small deviation as compared to the period of oscillation. The magnitude of the predicted forces corresponded to the measured forces in both cases.

REFERENCES

1. Stokes, G. G., "On the effect of the internal friction of fluids on the motion of pendulums", Trans. Cambridge Phil. Soc. 9, 8, 1851.

2. Basset, A. B., A Treatise on Hydrodynamics - Vol. 2, Dover, 1961

3. Schlichting, H., Boundary Layer Theory, McGraw - Hill, 1960

4. Abramovich, G. N., The Theory of Turbulent Jets, The M. I. T. Press, Cambridge, Mass., 1963.

5. Keulegan, G. H., and Carpenter, L. H., "Forces on Cylinders and Plates in an Oscillating Fluid", Jour. Res. Nat'l. Bureau Standards, Vol. 60, No. 5, May, 1958, Research Paper 2857.

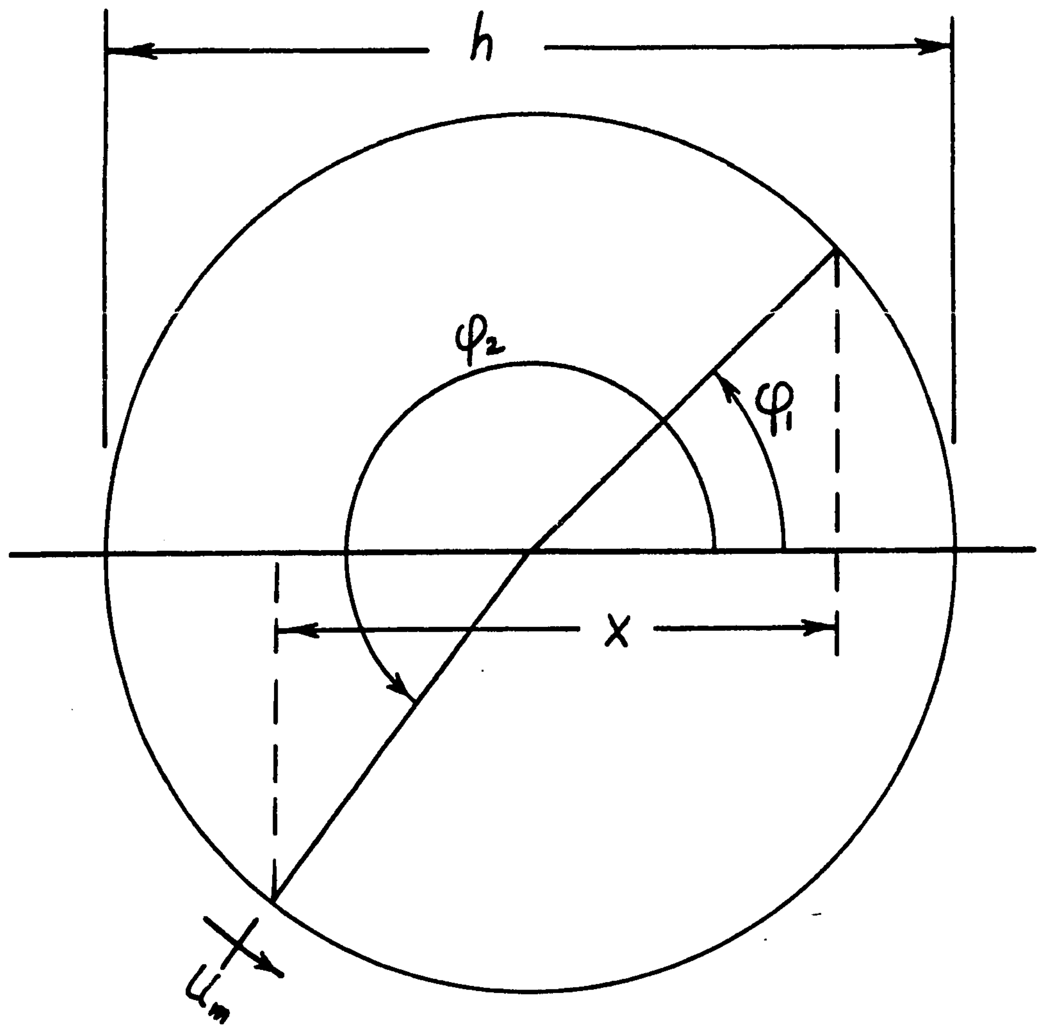


Figure 1 Illustration of the nomenclature used to locate

a body undergoing sinusoidal motion

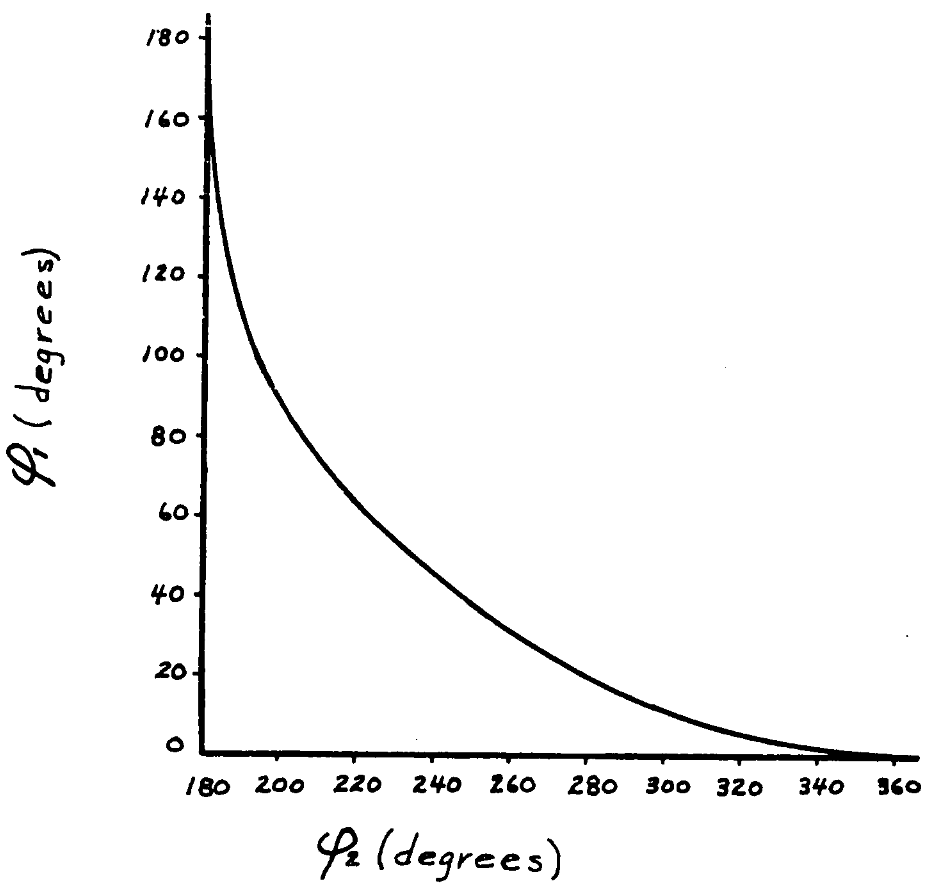


Figure 2 Solution to equation (3 - 3)

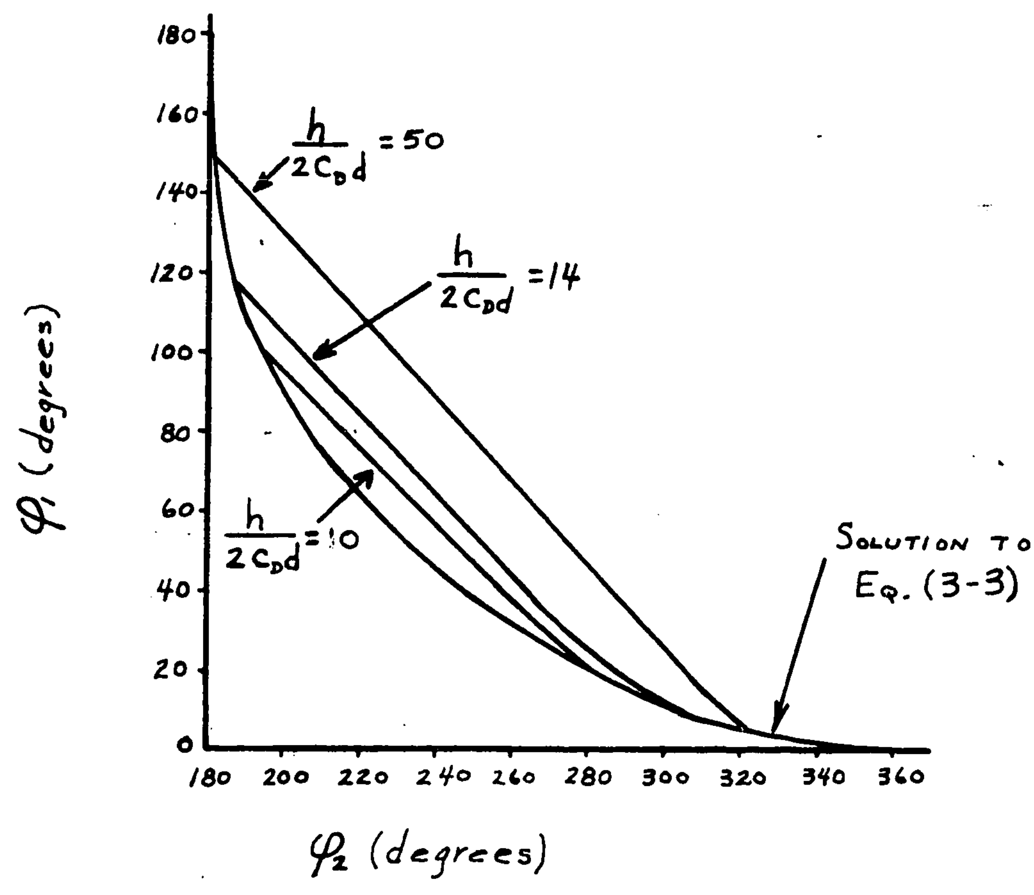


Figure 3 Solution to equation (3 - 6)

Figure 4 Relative velocity as a function of body location

for various values of h/ ($2C_D$ d)

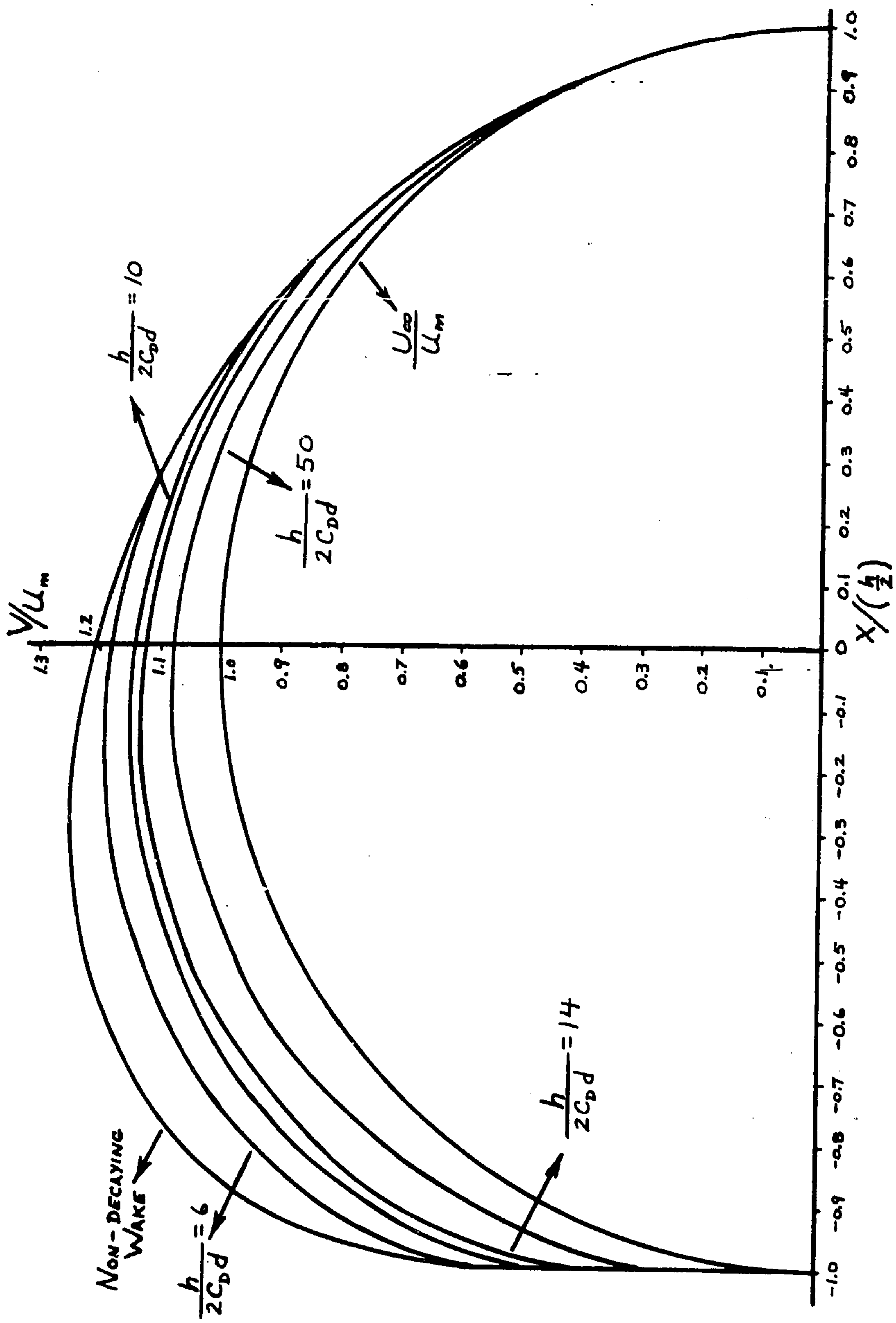


Figure 5 Illustration of graphical construction

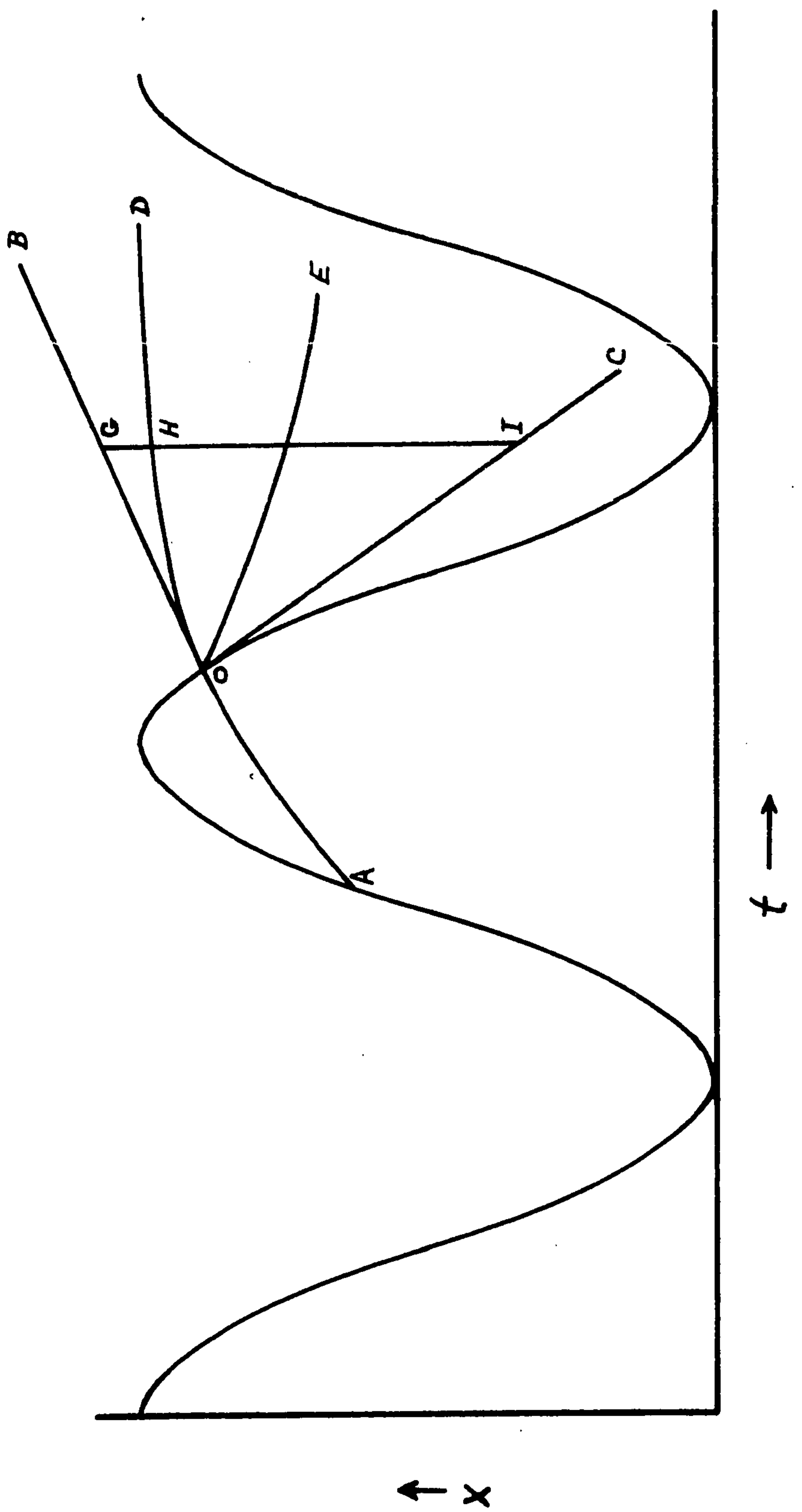
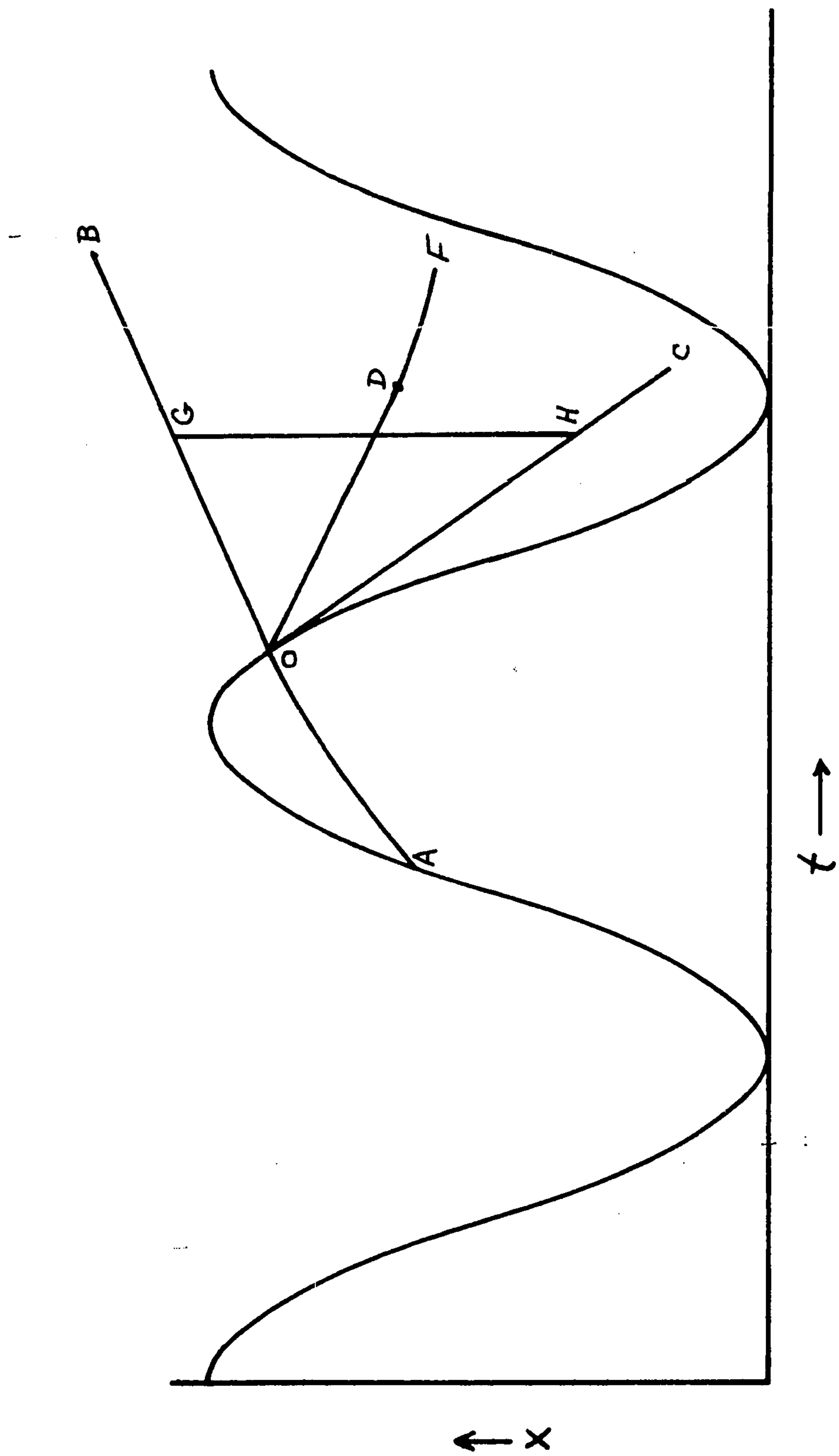


Figure 6 Simplified graphical procedure



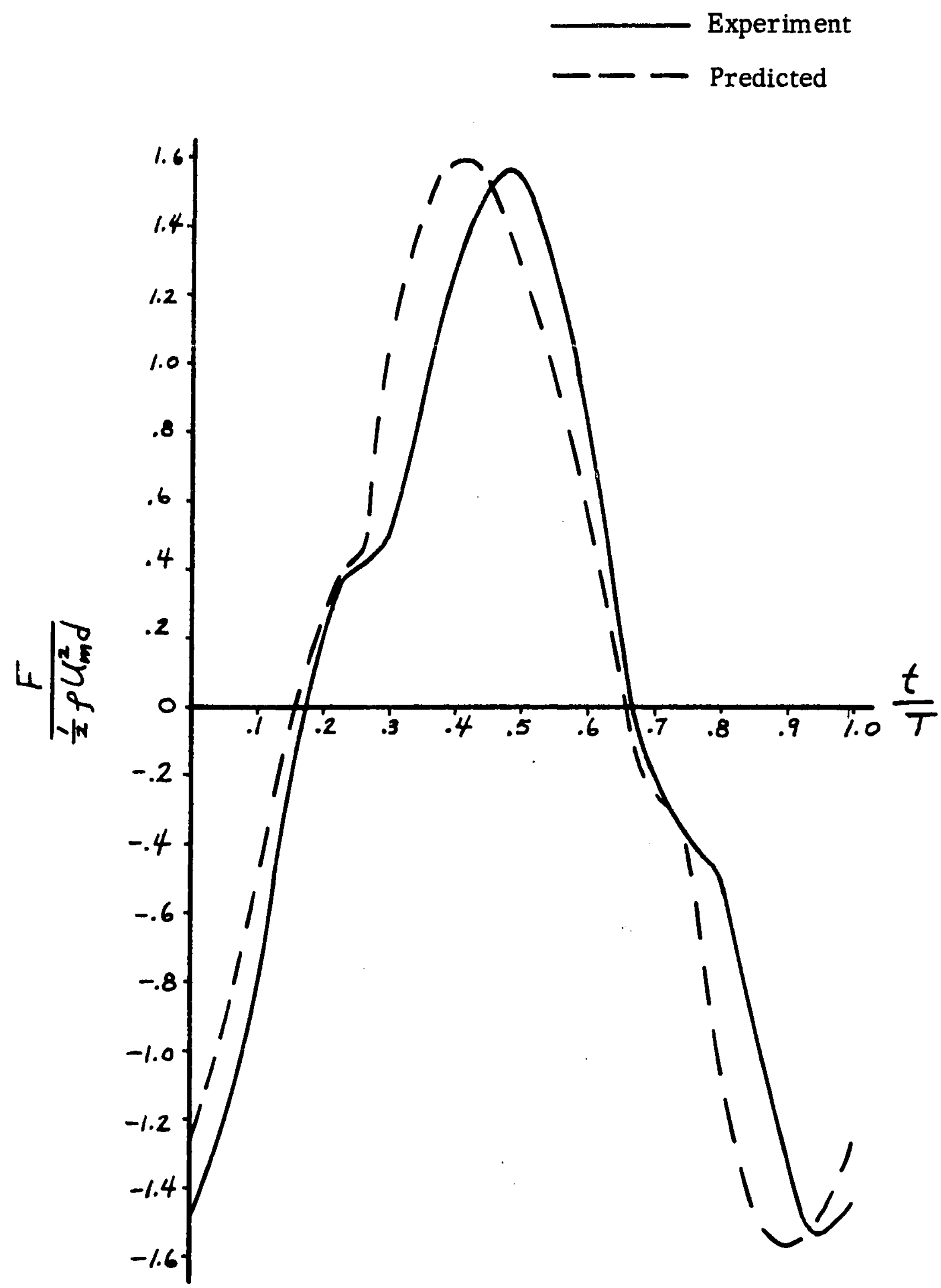


Figure 7 Comparison of the predicted force to

the data of Keulegan and Carpenter

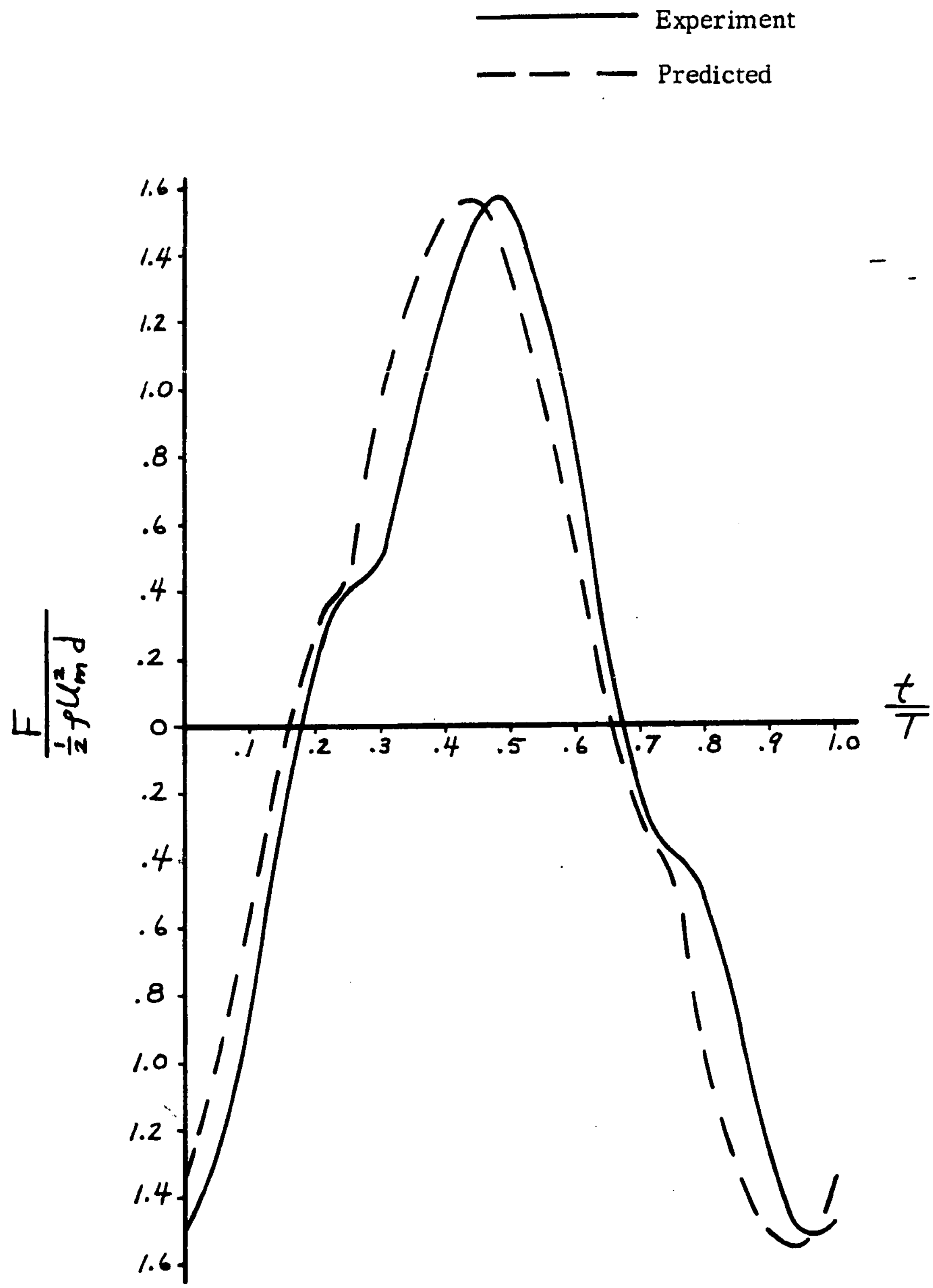


Figure 8 Comparison of the predicted force based on the simplified procedure with the data of Keulegan and Carpenter

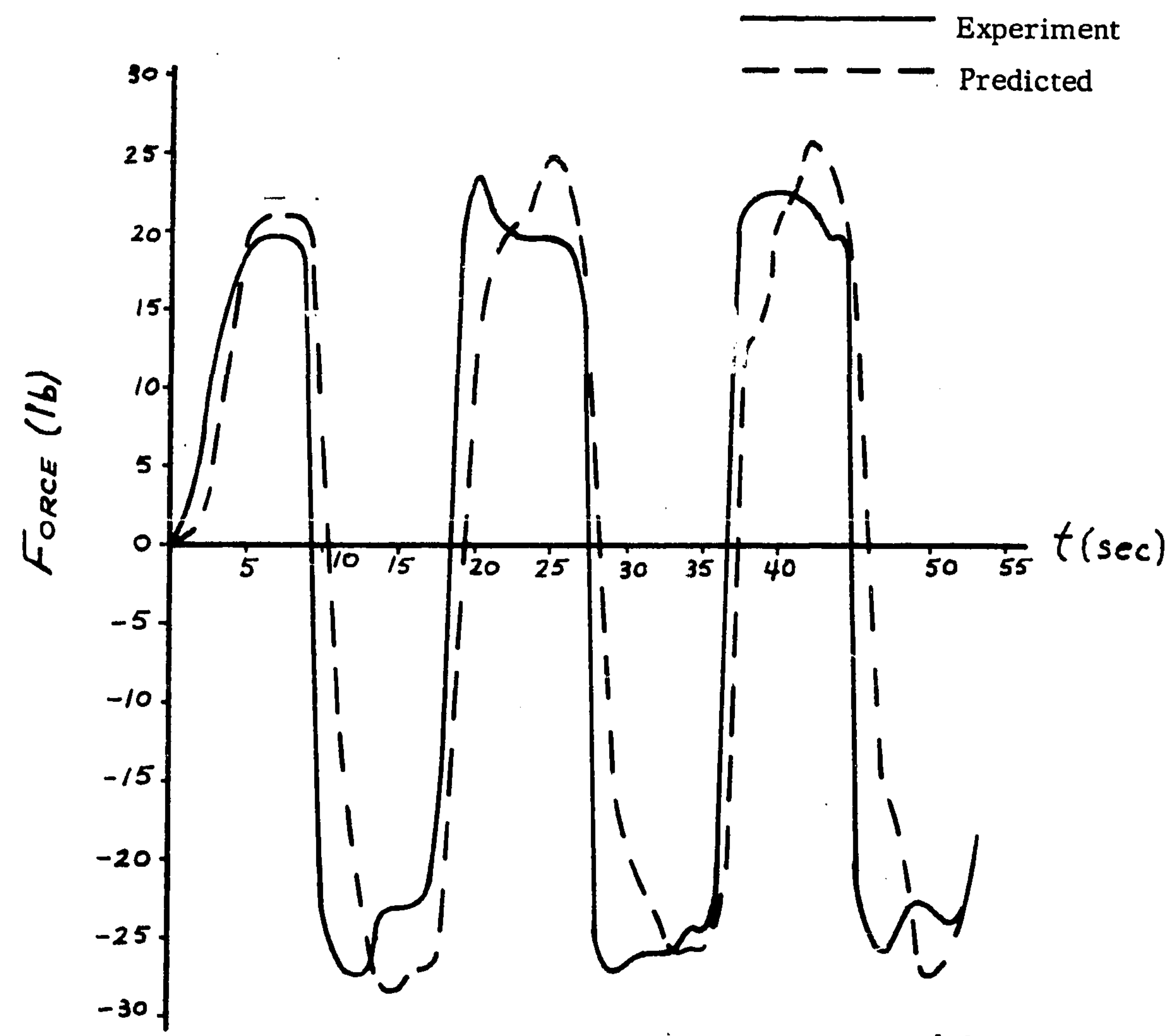


Figure 9 (a) Time history of the force

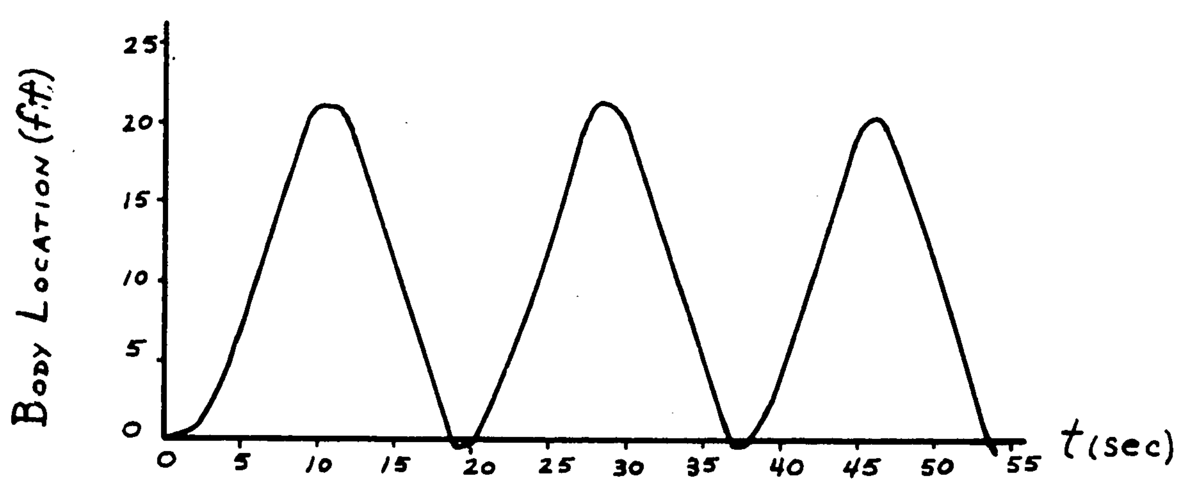


Figure 9 (b) Body location vs. time

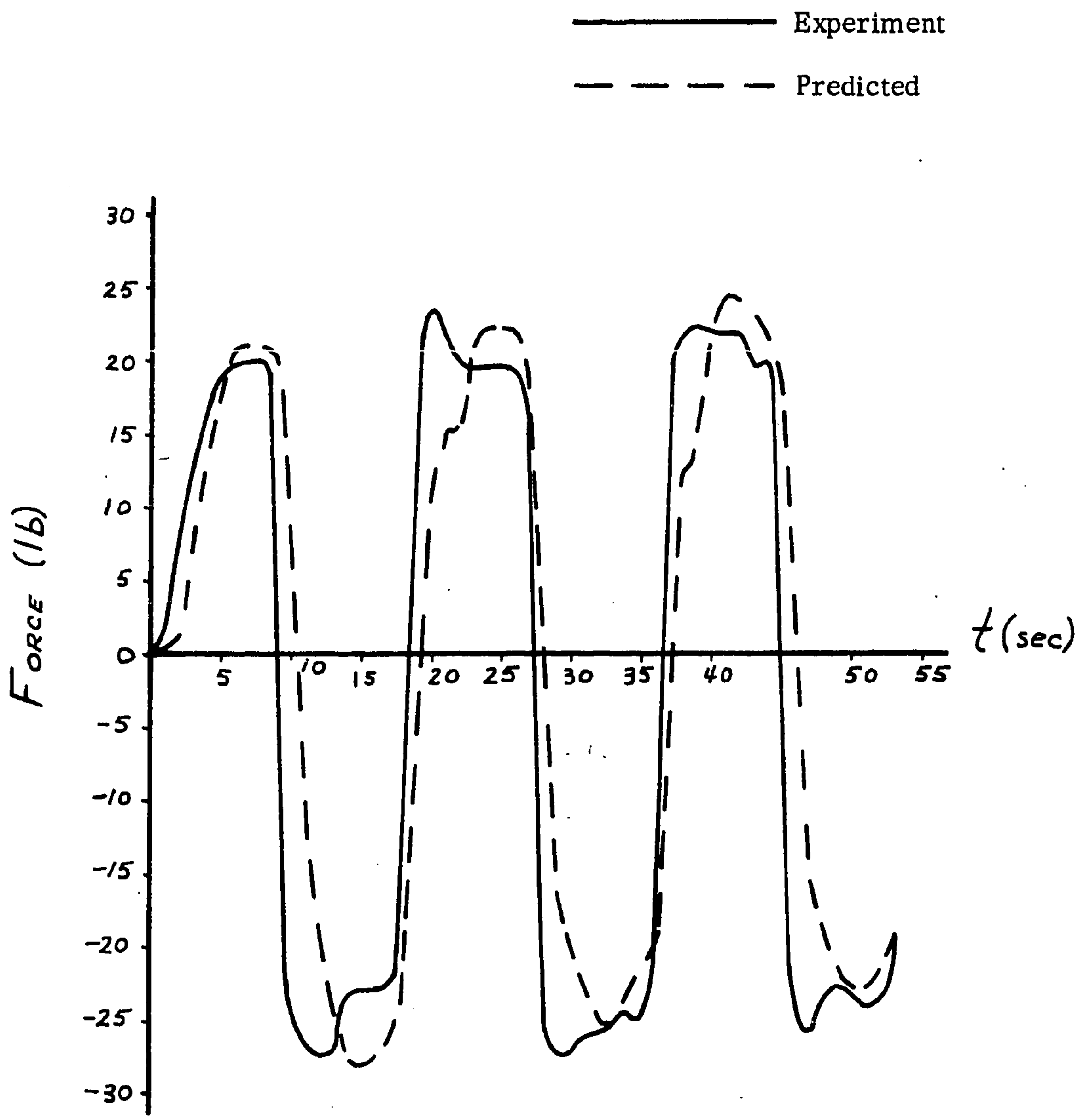


Figure 10 Time history of the force - simplified theory compared to experiment

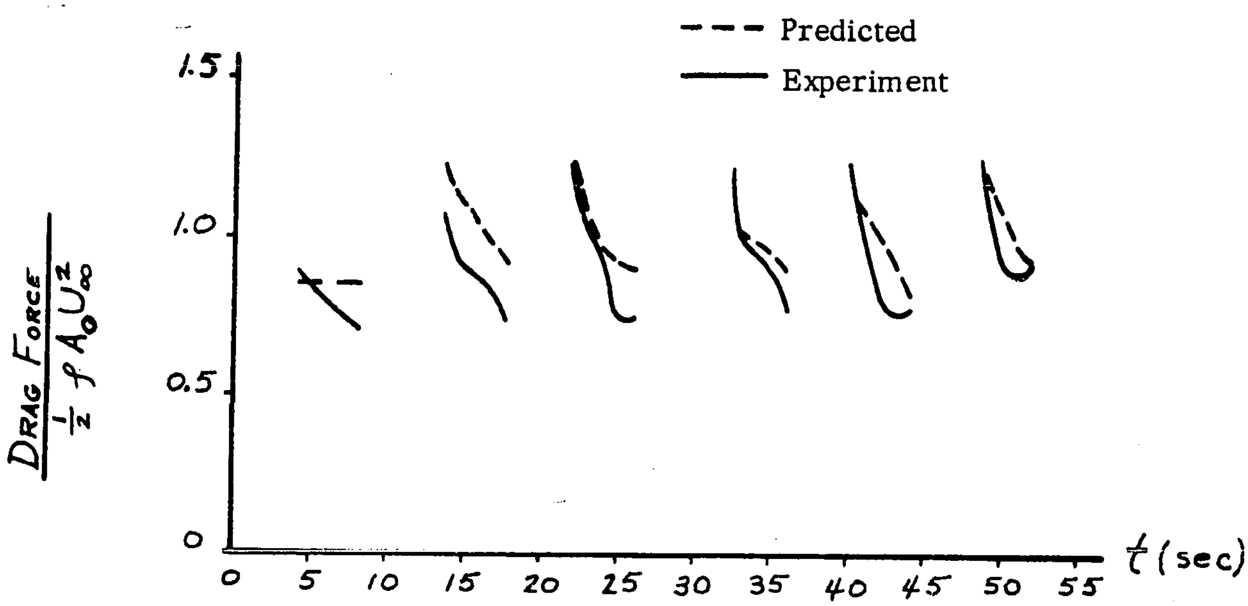


Figure 11 (a) Pseudo-drag coefficient vs. time

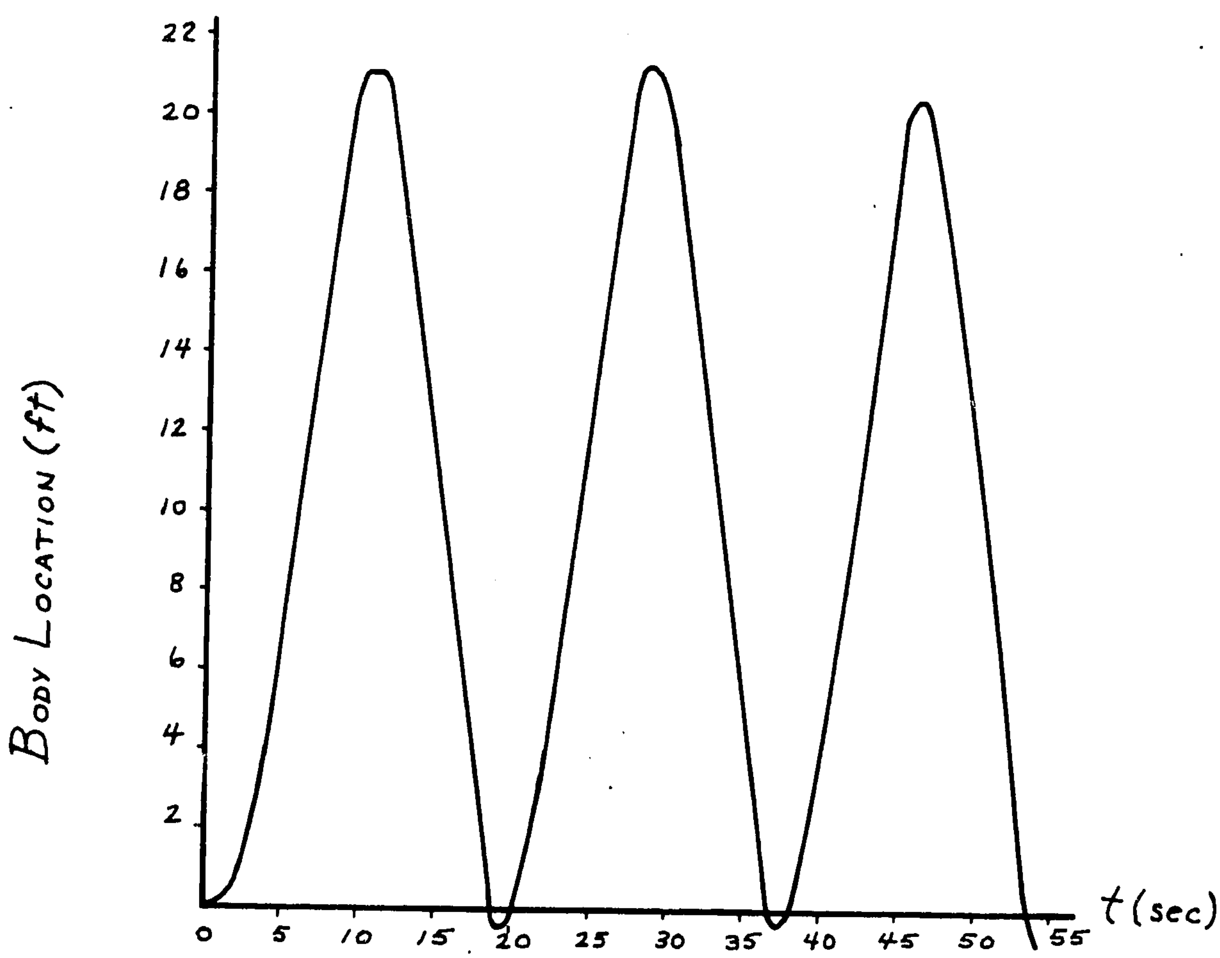


Figure 11 (b) Body location vs. time

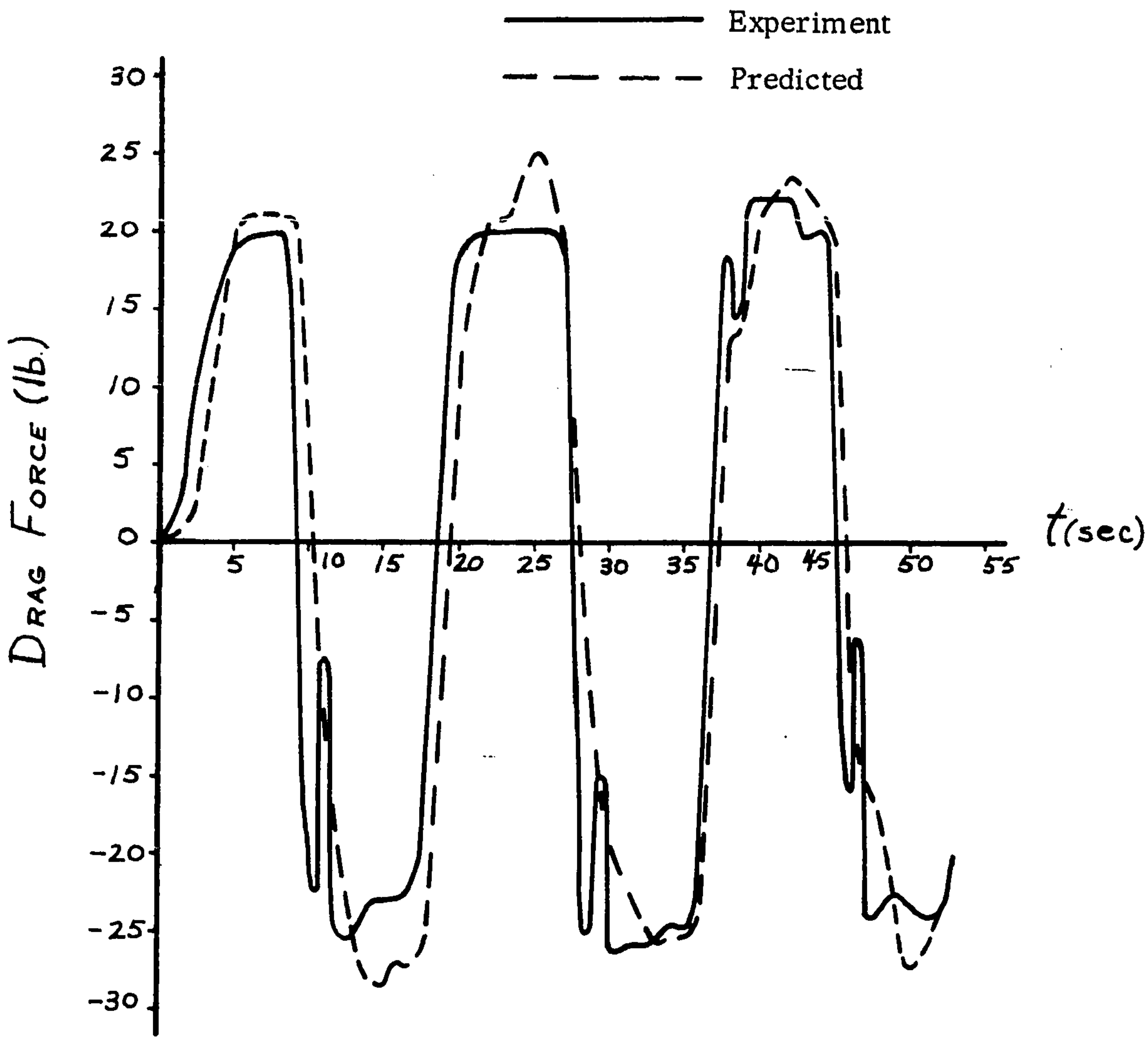


Figure 12 Comparison of predicted and

experimental drag forces