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A STUDY OF SHORT PERIOD SEISMIC NOISE

by

Changsheng Wu

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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Statistical treatments have been performed on short period (0.5 to 5.0 cps) seismic noise observed near Houston, Texas with a vertical array, a triaxial array, and a L-array of buried seismometers at depths from 3 feet to 150 feet.

The spectra are quite similar in structure over a horizontal distance of 900 feet or less, and show prominent peak at about 2.0 cps. The coherence of different components of the same array is generally high for frequencies smaller than 2.5 cps. It is possible to follow wavelet packets from one seismometer to another in an organized manner. The ground velocity decreases approximately exponentially both with depth of burial and with frequency. From cross-correlation computations, the velocity of propagation of the seismic noise in the near surface section is about 1150 to 1250 ft/sec in nearly horizontal directions of propagation regardless of the depth of burial.

These observations suggest that, in this area, much of the seismic noise near 2 cps is due to air-coupled Rayleigh waves generated by infrasonic waves of about 2 dynes/cm² in the atmosphere. Such infrasonic waves are known from the literature. The S wave velocity in the weathering layer is less than the sound wave velocity in air thus providing the necessary condition for air coupling. The frequency is determined by the phase velocity curve and corresponds fairly well to the depth and velocity of the weathering layer. A minimum in the group velocity near that frequency may contribute to the predominance of these waves.
It might be possible to take advantage of this phenomenon substantially
to increase the signal-to-noise ratio of teleseismic signals.
ACKNOWLEDGEMENTS

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1. STATEMENT OF THE PROBLEM

This thesis presents the results of studies of the structure of short period seismic noise in the frequency range of 0.5 to 5.0 cps in the solid earth within 150 feet of the surface. The incentive for conducting this study is of theoretical and practical interest. It is sufficient to indicate that an understanding of the spatial and temporal seismic noise structure is closely related to the realization of optimum detection of weak teleseismic signals amongst ever-present ambient noise. Teleseismic signals are defined as either earthquake or explosion signals of a seismic magnitude 5.0 or less that occurred at least 2000 km from the recording station. Short period seismic noise is defined as the organized disturbances appearing on the seismograms in the frequency range of 0.5 to 5.0 cps excluding teleseismic signals and instrumental noises.

The basic concept of interpretation of the experiments is that the chronologically recorded short period seismic noise at discrete time intervals, i.e., the data time series, may be thought of as portions of zero mean, normal, weakly stationary, stochastic processes during some arbitrary time interval. Studying such time series in the time as well as in the frequency domain may yield the characterizations of such processes and shed light on both the spatial and temporal properties and on the structure of short period seismic noise. From such results, possible generating mechanisms of such processes may be deduced.
2. SPECTRAL ANALYSES OF SHORT PERIOD SEISMIC NOISE

A stochastic process \( \{ x(t), t \in T \} \) is a family of random variables indexed on a continuous parameter \( t \) which represents time and takes on all values in the set \( T \). More rigorously, if the set \( T \) comprises all points on the real line, one may describe the stochastic process as \( \{ x(t), -\infty \leq t \leq \infty \} \). Because the behavior of a random variable is governed by a probability law, the \( n \)-dimensional joint probability distribution function of \( \{ x(t) \} \) at points \( t_1, t_2, \ldots t_n \) governs a stochastic process. Knowledge of the probability distribution function for any \( t_i, i = 1, 2, \ldots \leq n \) implies a knowledge of how \( \{ x(t) \} \) behaves for any time interval \( (t_i - t_j, i \neq j) \). However, this multivariate probability distribution function is seldom known, therefore one is forced to use the linear dependence of \( \{ x(t) \} \) on its past to infer information about the process. For this class of stochastic processes, this requires that the second moments, \( E[ x(t), x(t+\tau) ] \), be finite and be functions only of the reference time of \( \tau \).

The symbol \( E \) denotes mathematical expectation. The second moments of zero mean stochastic process are equivalent to its covariance function, \( C(\tau) \), and depend only on relative displacements of the time function in time and not on a fixed time origin. Therefore this assumption implies a time invariance in the mechanism which generates the stochastic process. However, this time invariance does not necessarily hold for all ensemble averages as it does in the more stringent requirements of strict stationarity. Stochastic processes with these properties
are called weakly stationary, synonymous with second order, wide-sense, or covariance stationary.

Let \( \{ x(t) \} \) be real valued, weakly stationary, with zero mean. Its moments show the relationships between the present and the past as a function of probability distribution, \( P [ x(t), x(t+\tau) ] \), and are commonly referred to as ensemble, probability or phase averages of the observations. Thus

\[
E [ x(t), x(t+\tau) ] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t) x(t+\tau) \, dP [ x(t+\tau) ]
\]

\[
= \int_{-\infty}^{\infty} x(t) \, dP [ x(t) ] \int_{-\infty}^{\infty} x(t+\tau) \, dP [ x(t+\tau) \mid x(t) ].
\]  

In order to apply the theory of stochastic processes to physical systems, it is necessary to be able to compute or measure from the limited available observations of such a process \( \{ x(t), 0 \leq t \leq T \} \) useful quantities such as the ensemble average, and the probability distribution function. Since one only has observed a single finite record of a discrete parameter stochastic process, the conditions (if they exist) under which the time average (sample average) can be identified ultimately with the corresponding ensemble average (population average) are desired. Physical systems obeying such conditions are called ergodic (Lee, p. 108, 1960).

Thus if the sample averages, given by

\[
\overline{x(t)} = \frac{1}{T} \sum_{t=1}^{T} x(t),
\]  

(2)
have variances which tend to zero as the sample size $T$ increases to infinity, then the sample average is approximately equal to the ensemble average, i.e.,

$$\bar{x}(t) = E[ x(t) ]$$

for almost all possible sample functions of the stochastic process that could have been recorded. Thus by invoking the ergodic hypothesis, one may consider that each record of the ensemble is statistically equivalent to every other record, and therefore the ensemble average, which is only obtainable over the ensemble at a fixed time, now can be replaced identically by the corresponding time average on a single sample record of the ensemble. However, one must be aware that a necessary condition for the ergodic property is that the stochastic process be stationary, although this is not a sufficient condition (Parzen, p. 74, 1962). This point shall not be pursued further.

Many aspects of a zero mean, normal, ergodic, weakly stationary time series are best understood in terms of their spectral density functions. For this purpose, the Fourier integral and Wiener approach is used with modifications. The recorded time function is considered as an aperiodic integrable function of equally-spaced discrete time samples with the result that the spectral density function is also an aperiodic integrable function of discrete frequencies. Since, in practice, only a finite portion of a time function is available, only estimations of the true spectral density function can be computed. Theoretical and practical considerations on such estimations and related problems, such as sampling theory, smoothing of sample spectral density, and stability of the estimation, are discussed
by Blackman and Tukey (1959), Lee (1960), Doob (1953), Parzen (1962), and Blackman (1965), and shall not be repeated in detail. But a short review shall be given here in order to insure continuity on discussion of the techniques of data analyses.

2.1 Empirical Analysis of Single Time Series

The sample autocovariance function $C_s(\tau)$ is defined by

$$C_s(\tau) = \frac{1}{T - |\tau|} \sum_{t=1}^{T-|\tau|} x(t) x(t + \tau); \quad \tau = 0, \pm 1, \pm 2, \ldots, \pm (T-1),$$

$$= 0 \quad \text{otherwise},$$

where $T$ is the total sample size in unit of time and $\tau$ is the time lag in units of sampling interval $\Delta t$. It is noted that $C_s(\tau)$ is an unbiased estimate by this definition. The corresponding two-sided sample spectral density function $P_s(f)$ is the cosine transform of $C_s(\tau)$:

$$P_s(f) = \sum_{\tau=-T}^{T} C_s(\tau) \cos(2\pi f \tau).$$

(4)

It can be shown that $P_s(f)$ fluctuates widely about the true spectrum $P(f)$ even though the sample size is taken quite large. This fluctuation, however, may be decreased by multiplying $C_s(\tau)$ with a suitable weighting or tapering function $W(\tau)$, which is an even function such that

$$W(0) = 1$$

$$W(\tau) < 1 \text{ for } |\tau| < \tau_m$$

$$W(\tau) = 0 \text{ for } |\tau| \geq \tau_m$$
where \( \tau_m \) is the maximum time lag. The \( W(\tau) \), which is also known as a lag window in the time domain, may be considered as a window of variable transmission which smooths or smears the values \( C_s(\tau) \) differently for different values of \( \tau \). The resulting modified or smoothed sample autocovariance function \( C_m(\tau) \) is obtained as

\[
C_m(\tau) = W(\tau) \cdot C_s(\tau).
\]  

(5)

If the ergodic hypothesis is applied, it can be shown that

\[
P_m(f) = 2 \sum_{\tau=0}^{T} C_m(\tau) \cos(2\pi f \tau),
\]

(6)

where \( P_m(f) \) may be regarded as the collection of the weighted spectral density of the true spectral density \( P(f) \) as seen through a window of variable transmission \( Y(f - f_1) \), which is the Fourier transform of \( W(\tau) \), in the frequency domain.

The \( P_m(f) \) is not an estimate of \( P(f) \) itself in the true sense, but a smoothing of \( P(f) \) over frequencies near \( f \) with weights proportional to \( Y(f - f_1) \). Thus one can obtain only a smoothed spectral density function when a finite length observation of a stochastic process is available.

It is convenient to measure the stability of any positive estimated spectral density function by the number of degrees of freedom \( k \) associated with a multiple of a chi-square variate. If the \( \tau_m \) used in the computation of \( C_s(\tau) \) is less than 10% of the sample size \( T \), then \( k \) is approximately equal to \( \frac{2T}{\tau_m} \), (Blackman and Tukey, p.112, 1959.) Therefore the distribution of deviations of the calculated approximate spectrum values from the true values may be thought to follow a chi-
square distribution, and the related confidence limit may be calculated.

The frequency resolution, \( r \), of an estimated spectral density is defined as a measure of the concentration of a spectral density in frequency (cps units), and may be defined as \( r = 1/\tau_m \) (Blackman and Tukey, p. 147, 1959).

2.2 Empirical Analysis of Multiple Time Series

Given finite samples of \( n \) real time series \( \{ x_i(T), t=1, 2, \ldots T \} \), \( \ldots, \{ x_n(t), t=1, 2, \ldots T \} \), the following definitions, similar to the ones used in the single time series analysis, may be used: The sample cross-covariance function \( \hat{C}_{hj}(\tau) \), of lag \( \tau \), between \( x_h(t) \) and \( x_j(t) \) is defined to be

\[
\hat{C}_{hj}(\tau) = \frac{1}{T-|\tau|} \sum_{t=1}^{T-|\tau|} x_h(t) x_j(t+\tau); \quad \tau = 0, \pm 1, \pm 2, \ldots \pm (T-1),
\]

\[
= 0 \quad \text{otherwise},
\]

and one notes that \( \hat{C}_{hj}(\tau) = \hat{C}_{jh}(-\tau) \) showing \( \hat{C}_{hj}(\tau) \) is not an even function.

The sample cross-spectral density function between \( x_h(t) \) and \( x_j(t) \) is defined by

\[
\hat{P}_{hj}(f) = \sum_{\tau = -T}^{T} \hat{C}_{hj}(\tau) \exp(-i2\pi f \tau).
\]

The sample cross-correlation function \( \hat{R}_{hj}(\tau) \) is defined by

\[
\hat{R}_{hj}(\tau) = \hat{C}_{hj}(\tau) \sqrt{\hat{C}_{hh}(0) \cdot \hat{C}_{jj}(0)}^{\frac{1}{2}},
\]

and its Fourier transform is the normalized sample cross-spectral density function \( \hat{Q}_{hj} \).
This function $R_{h_1}(\tau)$ has the property that $|R_{h_1}(\tau)| \leq 1$ and provides a measure of the similarity of series $h$ and $j$: if $R_{h_1}(\tau') = 1$, then series $h$ and $j$ are identical but with a time shift $\tau'$.

It is well known that if one is seeking to estimate the spectral density functions of weakly stationary time series, one must eliminate the truncation effect by using windowed sample covariance as discussed in the section 2.1.

Therefore the modified sample cross-covariance function $C_{h_1;m}$ is obtained by

$$C_{h_1;m}(\tau) = C_{h_1}(\tau) \cdot W(\tau), \quad (10)$$

where $W(\tau)$ is a suitable lag window. The Fourier transforms of $C_{h_1;m}(\tau)$ is then the desired estimate of the cross-spectral density function $P_{h_1;m}(f)$. This is generally complex. Therefore $P_{h_1;m}(f)$ may be written as

$$P_{h_1;m}(f) = \Re P_{h_1;m}(f) + \Im P_{h_1;m}(f)$$

$$= |P_{h_1;m}(f)| \exp \{ i \theta_{h_1;m}(f) \}, \quad (11)$$

where

$$\theta_{h_1;m}(f) = \arctan \frac{\Im P_{h_1;m}(f)}{\Re P_{h_1;m}(f)}.$$

$\Re P_{h_1;m}(f)$ and $\Im P_{h_1;m}(f)$ are known as the smoothed co-spectral density function and the smoothed quadrature spectral density function respectively.

Generally speaking, if the time series are random functions, the cross spectra provide information at each frequency about (1) the cross amplitude,
\[ |P_{hj;m}(f)|, \text{ or the product of the amplitudes of the random waves at a given frequency in the two series and (2) the phase difference between them.} \]

A quantity of great use in empirical multiple time series analysis is the coherence function which is defined by (Jenkins, 1963; Tick, 1963)

\[ B_{hj}(f) = \frac{|P_{hj;m}(f)|}{[P_{hh;m}(f) \cdot P_{jj;m}(f)]^{1/2}}. \]  

(12)

By the well-known Schwarz inequality, it can be shown that

\[ |P_{hj;m}(f)|^2 \leq P_{hh;m}(f) \cdot P_{jj;m}(f) \], which implies \( 0 \leq B_{hj} \leq 1 \). If the frequency responses of the systems used to record \( x_h(t) \) and \( x_j(t) \) are identical and each system noise is negligible, the coherence function may be thought of as a measure of the similarity of the two time series in the sense that \( B_{hj}(f) \) attains a theoretical maximum of unity if the two series are identical for any given frequency. If the coherence function is less than unity, at any frequency, it indicates the lack of complete similarity at that frequency of the two series due to some unknown factors that are not taken into consideration, e.g., noise.
3. PROCEDURES OF DATA COLLECTION AND ANALYSIS.

The research undertaken under this thesis is to measure and analyze the spatial and temporal variation of ambient seismic noise and teleseismic signals observed in shallow boreholes at a given geological and geophysical environment. The study was concerned with the supposedly unorganized short-period seismic noise and also organized signals centering around frequencies near one cycle per second at depths of 150 feet below the ground surface or shallower. It had been demonstrated earlier by Phillips and Wu, (1964, and 1965) that the amplitudes of a large portion of the undesired seismic noise, which may propagate horizontally in the form of surface wave modes or body waves, may be attenuated by the earth section with depth. Since the almost vertically propagating teleseismic signals suffer less attenuation with depth, the signal-to-noise ratio may be improved by this technique. However, the lack of multi-channel recording equipment had prevented quantitative measurement of the outputs of seismometers at different depths of burial. Therefore the equipment assembled for the present research project was designed to remedy those deficiencies, and to provide for detailed study of the spatial, temporal, and spectral properties of the ambient noise and of teleseismic signals.

3.1 Equipment

The complete field recording system is summarized in the block diagram
FIELD EQUIPMENT SYSTEM BLOCK DIAGRAM

FIGURE 1
of Figure 1. Seven Hall Sears HS-10-2 cps seismometers with
associated amplifiers served as seven inputs for simultaneous recording while the
summed output of the systems (arbitrarily divided by a factor of seven) served
as the 8th input. A single, vertical, short-period Benioff instrument with its
photo-tube amplifier served as another independent input for comparison purpose.
Ambient noise and signals were recorded continuously by (a) a Helicorder on paper for
two channels only, and (b) a magnetic tape FM system for all channels. Timing was
accomplished by a precise timing system capable of being compared to WWV
timing signals. Photographic records, with proper timing lines, could be obtained
at any time for visual study, either from the magnetic tape system or from the
amplified seismometer signals for visual inspection.

A set of low pass filters has been provided in the solid-state SPA-10
amplifiers for optimizing frequency filtering. Using its 0-7 cycles low pass filter
and -27 dB output, an overall system magnification curve and velocity sensitivity
curve were obtained and are shown in Figure 2. A 70% critical damping
HS-10-2 type seismometer was used for this test. It is noted that the maximum
gain of the system which occurs at about 3.5 cps is $3 \times 10^4 \times 22.4 = 6.72 \times 10^5$
where 22.4 is gain factor corresponding to 27 dB. Under these conditions of
maximum magnification the signal/noise ratio was 40 db at the normal recording
level.

In-place calibration of the seismometers (HS-10-2) was provided to allow
Figure 2. Overall System Transfer Function of HS-10-2 Seismometers and SPA-10 Amplifier With 0-7 cps Low Pass Filter. -27db Output Was Used.
the sensitivity of each buried seismometer to be compared after installation in borehole.

3.2 Seismometer Arrangements

The field site selected for the field station was near Houston. This site was located at latitude 29° 46.9’ N, longitude 95° 37’ W, about 12 miles west of the city limits of Houston in a ranch field with the nearest highway and railway about two miles away. The station site was selected in such a way that it was as far away from man-made noise sources as local conditions permitted. In this way, typical seismic noise and teleseismic signal data may be collected comparatively free of cultural noise.

In order to study the effect of burial on the attenuation of the seismic noise of a local origin, a vertical array was set up at the field site by planting seismometers firmly at the bottoms of uncased boreholes of different depths which were drilled within a circle of 5 feet in radius. The horizontal spatial effect on characteristics of the simultaneously recorded data are expected to be negligible and the data have been treated as if they were recorded in one borehole but at different depths. The purpose of using different boreholes was to insure better seismometer-ground coupling than could be obtained in a single hole containing many seismometers.

Assuming that seismic noise propagates as plane waves in the near surface layers, a triaxial seismometer array was also used at each site to study the direction
and speed of propagation of such waves. The triaxial array was actually a Cartesian coordinate system with a seismometer at the origin and two seismometers equally spaced along each axis. The x and y axes were on the surface of the ground with z axis extending into the ground. The distance between adjacent seismometers was 100 feet which was limited by the available depth of the borehole in the z axis direction. Therefore the maximum distance between two seismometers along each axial direction was 200 feet.

A L-shape seismometer array was also used at the Houston Site. Three seismometers were placed at equal distance (300 feet) along each line with one seismometer at the apex, and all seismometers buried at 150 foot depth. This array gave better resolution in the frequency-wave number space for the desired measurements but was still small enough to insure the uniformity of geologic effects.

The output of each seismometer was continuously recorded on magnetic tape with appropriate gain and filter settings. Measurements were made over a period of at least two weeks. The response of the systems and the relative sensitivity of the buried seismometers were checked twice daily using the calibration unit. At all times, the output of at least one buried seismometer was recorded with Helicorder for visual inspection. These Helicorder records were visually edited to find the portions of typical seismic noise and teleseismic signals for further study for use of the magnetic tapes. The portions were chosen to have small amounts of
visible interferences such as train noise, weather front induced noises, automobile traffic on nearby roads, tail-ends of other teleseismic events, etc. This method of editing may appear rather subjective and biased but it was hoped that such selections would be representative purely seismic noise. Such selected portions were found from the corresponding magnetic tape reels by interrupting the daily recording routine and playing back the magnetic tapes on Helicorder recording papers and comparing them with the master records. Then such chosen portions of magnetic tape were cut out for later digitizations. This process was laborious, inefficient, and possibly degraded the quality of magnetic recording due to the collection of dust on the tapes and the recording-playback heads during this cut-off-and-splice process. However, this method was the only possible procedure with the available equipment.

3.3 Techniques of Analyses

Data obtained in the field were analyzed in three ways. First, the Helicorder paper records were scanned in an attempt to recognize different typical seismic noise conditions, to identify interesting teleseismic events, and to locate other phenomena which might yield diagnostic information. Daily seismometer calibration signals and teleseismic signal amplitudes were also measured on these records. Any signs of instrument malfunction were also searched visually. The times of occurrence of desired portions of data were recorded for future use.
The second type of analysis involved the playback of the previously selected portion of data from magnetic tape to make photographic records for further visual study. This type of playback record had timing lines of 0.1 second interval on a six inch width paper and was used to isolate noise segments which appeared to be correlated on all traces and to check the performance of the instruments, including the magnetic tape recorder.

Finally, the spliced [as discussed in section 3.2] magnetically recorded information was transcribed to another data tape reel with proper identification markers at a speed (3-3/4", per second) which was four times faster than the original recording speed (15/16" per second). This process was designed to save computer time during the analog-to-digital conversion process, which was carried out using the analog to digital conversion facilities of the Rice Computer. Since the converter had only three channels available, and there were nine channels from the magnetic tape (seven seismometers, sum of the seven, and Benioff) to be digitized, one set of data had to be fed into the converter five times since a trigger pulse was recorded on a channel and was used on each digitization run. The computer was programmed to begin the digitization of each run at the same height of the trigger pulse thus this procedure gave almost simultaneous digitization of all time series in a set of data even they were digitized in five runs. This scheme was checked by using identical data in each channel. Results showed that no relative time shift between traces was introduced. The
finite time interval between samples of adjacent channels was 10 micro-sec; therefore, no correction was made for this shift. The linearity of the converter quantization was checked with known inputs; the result is shown in Figure 3. The converter has a signal to noise ratio of approximately 50 db and a range of 1.0 volt rms. It produced a word length of 7 bits plus sign at each sample point of each trace. Six consecutive data words were packed together to form a machine word (54 bits). These words were then written on magnetic tape in blocks of 2000 machine words to each block. A specially designed, two channel d.c. operational amplifier, VICTOR-1000, with filters was used at appropriate gains in order to ensure better resolution on quantization of information. Filters were used to eliminate noise above 7 cps (as recorded in the field). The filter amplitude response was quite flat for frequencies recorded at 5 cps and below; the filter output was down 3 db at 7 cps and decreased about 48 db per octave for higher frequencies.

A sampling rate of 250 data points per second was used, equivalent to 62.5 points per second at the original recording speed. The corresponding folding frequency is 31.25 cps. However, later during the computation, a decimation by 2 was used, thus the folding frequency becomes 15.625 cps, which is still well above the highest frequency of interest, 5.0 cps. Therefore no desired information was lost by this process which saved substantial computation time. Because of the filters used in recording and playback, aliasing (Blackman and Tukey, p. 117,
Figure 3. Linearity of A/D Converter of the Rice University
1959) does not present a problem in this case.

The digital analysis of the field data were carried out using the Rice Computer of the Rice University, Houston, Texas. The noise analysis programs performed the following operations:

1. A time interval and starting time was first decided upon. The noise record of this interval of any seismometer was then read into the computer from the data tape reel. These digital data were reconverted to analog form and plotted by a strip chart recorder connected to the computer to verify the selection and gross accuracy of the digitization process before each data analysis. The length of the records varied from 12 to 96 sec.

2. Deaveraging and detrending computations were performed on all data by subtracting the least squares straight line from each sample point.

3. The autocovariance function of the sample, $C_s(\tau)$, was computed using equation (3) of section 2.1. The maximum time lag used was, unless otherwise noted, 4% of the sample length; this gave a degree of freedom $k \approx 50$, which may be thought as a compromise designed to keep the random covariance rather low and positive covariance reasonably high.

4. The sample autocovariance function was multiplied with a tapering function $W(\tau)$, where $W(\tau) = \left[1 - \left(\frac{\tau}{T_m}\right)^2\right]^2$, as discussed by DeBremaecker (1964), thus obtaining the smoothed or modified autocovariance $C_m(\tau)$. Of course, since $C_m(\tau)$ is an even function, one only needs to compute $C_m(\tau)$ for values
of $0 \leq \tau \leq \tau_m$.

5. The cosine transforms of $C_m(\tau)$ were computed at the interval of 0.05 cps from 0 to 5 cps yielding the desired smoothed spectral density function, $P_m(f)$. For all the spectra presented in this thesis, no correction was made for the instrument effects and was plotted using relative power in linear scale vs. frequency. One may correct the spectra for the instrument effects by means of the curves on Figure 2. It is also known that sensitivity of the overall system in volts rms at the A/D converter input per millimicron ($\text{m} \mu$) of ground displacement near 1 cps may be measured and such measurement may be used to normalize the spectral densities with units of $(\text{rms m} \mu)^2$ per cps at 1.0 cps.

The 80% confidence limits of the estimated spectral densities were computed on the assumption that the distribution of deviations of the calculated values from the true, long-run values follows a chi-square distribution as discussed by Blackman and Tukey (1959, p. 22). However, it is noted that the end points of any confidence interval depend on the values of the estimated spectral density values, therefore the 80% confidence limits as shown are strictly correct for only that value of estimated spectral density. This limit defines a band about the individual peculiarities of the sample at that frequency, within which there is a reasonable hope that the spectral density function at that frequency appropriate to the related stochastic process is to be found. However, in order to avoid cluttering up the graph, only one confidence limits for one particular frequency is plotted to guide
such inferences.

6. In order to facilitate the comparison of spectral densities from various seismometers of different depths, a program was written to compute the relative attenuation caused by the burial using the ratio of the spectral densities. This attenuation may be defined by $\text{db} = 10 \log_{10} \frac{P_x}{P_0}$, where $P_x$ is the power at depth $x$, and $P_0$ is the power at a reference depth which is usually the surface seismometer. Using this definition db values at a given frequency are the same for amplitude and power.

7. The sample cross-covariance function, $C_{hj}(\tau)$, and its tapered function $C_{hj;m}(\tau)$ of any two pairs of seismometer outputs, $h$ and $j$, were computed following equations (7) and (10) with $|\tau_m| = 0.04T$ for $-\tau_m \leq \tau \leq \tau_m$. It is important to note that a two-sided taper function $W(\tau)$ must be used since the $C_{hj}(\tau)$ is not an even function.

The cross-correlation function $R_{hj}(\tau)$ was computed using equation (9); it is also known as a cross-correlation coefficient. One of the purposes of computing $R_{hj}(\tau)$ was to provide a reasonable estimate of similarities of the waveforms in the two time series under comparison. If a single pure signal, composed of oscillations of different frequencies, is propagating as a plane wave in a homogeneous nondispersive, and linear medium, then the cross-correlation function of $x_h(t)$ and $x_j(t)$, where $x_h(t)$ and $x_j(t)$ are two time series recorded simultaneously at distance $\Delta x = x_h - x_j$ apart, shows a central maximum at a time shift of $\tau'$ from the zero line, and $R_{hj}(\tau')$ has a value of +1. The central maximum has a period
corresponding to the frequency component which possess the greatest spectral
density present in $x_h(t)$ and $x_i(t)$. Any successive maxima also show a similar
relation between their periods and their component spectral density functions but
in general do not have values as large as the central maximum. The value of $\tau'$
obtained from the position of the central maximum is a function of $\Delta x$ and the
apparent velocity for the waves. If $x_h(t)$ and $x_i(t)$ are contaminated with random
noise, the value of $R_{h_i}(\tau')$ is less than 1, indicating the lack of similarity of the
two time series. Nevertheless when the central maximum approaches unity it
may be taken as an indication of the strong similarity of the two signals and the
value of $\tau'$ deduced may be used to obtain the apparent velocity. If three
signals from instruments defining a plane are compared, the true propagation
velocity and direction of propagation of the coherent noise can be deduced.

A computer program was written to search for two local maxima near the
zero line of $R_{h_i}(\tau)$ and to compute the time shift by determining the first moment
of 5 points adjacent to the maximum along the time axis using the formula:

$$\tau' = \sum_{k=1}^{5} \left[ R_{h_i}(\tau) \right]_k \tau_k / \sum_{k=1}^{5} \left[ R_{h_i}(\tau) \right]_k .$$

(13)

This method allows the time shift to be evaluated more accurately than is possible
visually. For the triaxial array defined in section 3.2, the propagation velocity
$|\vec{v}|$ is given by
\[ |\vec{v}| = \frac{d}{\left( \sum_{i=1}^{3} \Delta t_i \right)^{\frac{1}{2}}} , \quad i = 1, 2, 3; \quad \Delta t = \text{sampling time} \]  

where \( d \) is the distance between seismometers in feet, \( \Delta t_i \) is the time shift of the maximum in seconds along the \( X_i \) axis, and \( i = 1, 2, 3 \) represent the \( x, y, z \) axis respectively. The direction cosines of the normal to the wave front are given by

\[ \text{Cosine } X_i = \frac{V_i}{\left( V_1^2 + V_2^2 + V_3^2 \right)^{\frac{1}{2}}} , \quad i = 1, 2, 3 , \]  

where \( V_i \) is the apparent velocity along the \( X_i \) axis.

8. Fourier transforms of the tapered cross-covariance functions were also computed so that comparisons of the seismic noise involved could be studied as a function of frequency. The resulting magnitude of the cross spectral density function \(|P_{hj;m}(f)|\), and the phase angle of this function, \(\theta_{hj;m}(f)\), were calculated.

9. Coherence relations were computed between the noise measured simultaneously at various seismometers in the array using equation (12). Such relations provide a measure of the similarity of noise power measured at two seismometers which is due to some common cause. However, the computed coherence depends on the sample size, maximum time lag used, and the form of the spectral
window by virtue of the assumption that the spectral densities are flat over the
width of the spectral window. The coherence is equal to one at any frequency
only if that frequency has suffered a pure time shift. For completely uncorrelated
waveforms the function $B_{h1}(f)$ can range from one to zero with an rms value for
random waveforms of the order of $\sqrt{\frac{\tau_m}{T}}$ (Seriff et al, 1964). Thus for
analyses given in this thesis, the $|B_{h1}|$ of random waveforms is about 0.2.
Therefore when $|B_{h1}|$ is significantly larger than 0.2 it may be inferred that
the signals are at least partly coherent.

Based on computations prepared by Amos and Koopmans (1963), 80% confidence limits bars were plotted on computed graphs of coherence estimates.
Since the end points of any confidence interval depends on the values of the
estimated coherence values, the 80% confidence limits as shown are strictly
correct for only that value of estimated coherence. However, it is believed that
a better overall impression may be conveyed this way than a graph plotted as an
envelope of estimated values. Also, the variations of end points for coherence
$\geq 0.7$ are rather small for $k \approx 50$.

10. In this thesis, one is interested in the seismic noise in the band of
0.5 - 5.0 cps. Therefore it is desirable to remove microseismic noises of frequencies
lower than 0.5 cps during the analysis to facilitate the determination of velocities
of higher frequencies. For the data obtained at the Houston Site, the low
frequency microseismic noises had low power densities and were not filtered out
for the major part of the computation. But when it was needed, such undesirable
low frequencies were removed from the sample data by convolving the data with a 99 point, zero phase, digital bandpass filter, whose characteristics in the time domain and frequency domain are shown in the Figure 4.

3.4 Accuracies of Measurements and Computations

In this type of experiment, data obtained from the field site are a mixture of true seismic noise and noises generated in the recording and reproduction equipment. From the beginning care was taken to ensure that the rms system noise for every channel was well below the observed seismic noise levels. In general, this condition was successfully maintained, and the system noise was about -40db of the recorded seismic noise level.

The gain of each channel was calibrated and the instruments were set to match each other as closely as possible; the gain stability of the SPA-10 amplifier used was good. Despite these precautions it is expected that a certain amount of error was introduced into the field measurements, due to many factors such as temperature change of the instrument house, undetectable malfunctioning of electronic components, misreading of calibrations, etc. Therefore it is probably reasonable to assume that the deviation of gain of each channel is about ±0.5 db i.e., about ±5.6%.

The accuracy of phase comparisons and response similarity comparisons between channels has been investigated by planting seismometers (HS-10-2) within a 5 foot radius in a huddle arrangement and recording seismic noise for a
period of time. Visual inspection and analysis of the playback records showed no appreciable deviations. Later phase cross-spectrum and coherences of such huddle tests were computed and are shown in Figures 5 and 6. It is seen that the phase difference of any two channels is about 0.06 π or 10.8 degrees with a maximum of 22 degrees. The linear phase difference between the down-hole seismometer system and Benioff system (lagging) is 36 degrees between 1.0 to 1.8 cps; 16 degrees from 1.8 to 3.5 cps and becomes non-linear at frequencies beyond. This much greater phase difference is due to the difference of response between the Benioff seismometer and the other instruments. The coherence among channels is generally close to the theoretical value of unity and the deviations beyond 3.5 cps are probably caused by the slightly different frequency responses of the two-channel amplifiers used during the digitization process.

It is important to point out that in digital spectral analyses, one can occasionally obtain negative spectral estimates when some underlying assumptions are not completely fulfilled. The lag window used does have negative side lobes which can contribute negative power to the digital estimates. Furthermore, when data are convolved with a digital bandpass filter, side lobes effects can also cause the computed value of coherence to be larger than 1 or less than 0 at frequencies near the ends of the passband, even though in theory they are limited to the range 0 to 1. Such a result generally arises either where the spectral density is rapidly changing or where it is close to zero.
Figure 5. Phase Cross Spectral Densities of Ambient Noise, Obtained With Seismometers Planted Within 5 Foot Radius For Comparisons of Instruments.
Figure 6: Coherences of Ambient Noise Obtained With Seismometers Placed Within 5 Foot Radius For Comparisons of Instruments.
Finally, the statistical fluctuation of the results compiled in this report should be recognized. The confidence limits as shown on spectral density graphs are a measure showing the expected statistical fluctuations if the time series analyzed are assumed to be Gaussian.
4. RESULTS AND DISCUSSIONS

Geologically, the Houston Site is situated on the northern edge of the Gulf Coast Geosyncline where the underlying Cenozoic clastics are of 30,000 to 40,000 feet thick. This thick sequence of Cenozoic sediments presumably rests upon Mesozoic sediments which are in turn superimposed upon Paleozoic and pre-Cambrian rocks.

Typical lithology encountered from the ground surface to a depth of 200 feet is as follows: 0-2, top soil; 2-3, brownish gummy clay; 3-12, yellow gummy clay; 12-28, fine sand; 28-105, reddish clay; 105-115, gravel; 115-200, reddish clay. The borehole was uncased to avoid any unwanted noise sources which might be induced by casing. Circulation of drilling fluid in the hole was kept up for at least 30 minutes in order to clean the hole, and to give the side wall better strength against collapsing. The seismometer was firmly planted at the bottom of this 5½" diameter hole with a specially designed tool. Little trouble was encountered in retrieval of the seismometers; they were retrieved by jetting high pressure water through the bit-less drill pipes to loosen up the settled sediments. The probability of recovering seismometers that had been in the hole for an average period of two months was about 95 per cent.

No experiment was conducted at the Houston Site to determine the compressional wave velocity, shear wave velocity and other related elastic parameters in the near surface layers. However, from past experience on seismic exploration work done in this area and other unpublished data, it is reasonable to surmise
such quantities as follows:

<table>
<thead>
<tr>
<th>Thickness (ft.)</th>
<th>Longitudinal Velocity (ft/sec.)</th>
<th>Poisson's Ratio Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weathering layer</td>
<td>100-150</td>
<td>1500-2000</td>
</tr>
<tr>
<td>Substratum</td>
<td>-</td>
<td>6500-7000</td>
</tr>
</tbody>
</table>

These values are generally obtained by the so-called up-hole shooting technique as described by Dix (1952, p. 95) and are subject to considerable uncertainty. Nevertheless, this method generally give the more reliable results as other methods are subject to errors even more severe in nature.

The instruments were set at a magnification factor of 15K at 1 cps. After a study of the field records, it was determined that the average daytime ambient noise had a ground motion of about 150 millimicrons near 1 cps. By comparison with other areas as reported by Shopland and Stephens (1962), the Houston Site was noisy: the average ground motion at other sites was about 50 millimicrons near 1 cps. It was also noted that the average peak-to-peak amplitude of the noise recorded during the night time was about 1/3 of the daytime average.

Typical spectral density graphs (resolution = 0.26 cps) in Figure 7 show the general features of typical night-time and daytime noise at the Houston Site without correction for the instrument frequency response. All spectral density graphs presented in this thesis are plotted with the spectral density in linear scale. The long period microseismic noise, whose amplitude is decreased by the instrument response, is observed at 0.4 cps. The sharper peak at 2.0 cps is the dominant
Figure 7  Spectral Densities of Typical Night-time (top) and Daytime (bottom) Ambient Noise
Recorded at the Houston Site.
feature and has been observed throughout the United States as indicated by Shopland and Stephens (1962) and Frantti (1963). It is interesting to note that the 0.4 cps microseismic noise has more power during the daytime than night time while the 2.0 cps remains constant. Above 2.2 cps the noise falls off rapidly for the night observation in comparison with the day observation at an average ratio in amplitude about 1 to 3. There was also evidence indicating that noise recorded on holidays resembles night-time noise in spectral structure and power. Such observations suggest that at least a great portion of the ambient noise observed at the Houston Site probably was related to cultural or man-made noise; the noise near 2 cps is clearly of a different nature.

Some other types of noise were also observed. For instance, the noise of trains has a peak at about 3 cps. Cold fronts and distant tropical storms tend to produce a large increase in power near the low frequencies around 0.4 cps. The effects of local winds was also noted. The average noise level was about doubled when the wind velocity reached the value of 18 mph and maintained such a velocity for a period of hours. Such an occasion was encountered only on April 5, 1965.

4.1 Vertical Arrays

The noise attenuation vs depth graph of records obtained on this exceptional windy day as shown in Figure 8D indicates that the power of wind induced noise
Figure 8. Spectral Densities of Ambient Noise and Attenuations with Buried Seismometers Recording Simultaneously During Daytime at the Houston Site on April 5, 1965. Wind Velocity was 18 MPH.
was effectively reduced by burial. From the spectral density graphs of Figures 8A, 8B, and 8C (note the change in vertical scale), it is seen that the dominant noise lies between 1.4 to 3.1 cps, and the power in this band is reduced by an average of -4.8 db and of -9.2 db due to burial of seismometers at 100 foot depth and 200 foot depth respectively. The coherences also decrease as the depths of burial are increased as shown in Figure 9. It is interesting to note that the coherence of the outputs of the seismometer at the surface and the 200 foot depth approaches approximately the random level (0.2) at the frequency of about 2.4 cps, while the surface and the 100 foot depth coherence is maintained to about 3.9 cps.

The ever-present 2 cps noise is shifted to about 1.9 cps and its amplitude decreases with increase of depth of burial. The unusual dominant peak near 2.8 cps and the secondary peak near 3.0 cps, which are probably wind induced noises, are gradually attenuated by burial, and the latter almost disappears at the 200 foot depth.

Figures 10 to 14 show representative samples of attenuation of typical noise and signals due to burial at different depths as observed at the Houston Site. It is estimated that these data were obtained at an average wind velocity of about 5 mph.

It is possible that these curves could have been explained if a detailed velocity versus depth profile had been available, but such was not the case. Because the bulk of seismic noise is of Rayleigh waves (see detailed discussion in section 4.3), it should, moreover, be recognized that velocity logs only measure
Figure 9. Coherences of Ambient Noise with Buried Seismometers Recording Simultaneously During Daytime at the Houston Site on April 5, 1965.
Figure 10. Attenuations of the Noise as Shown by Buried Seismometers Recording Simultaneously with Respect to the Surface Seismometer. (Day Time)
Figure 11. Attenuations of the Noise as Shown by Buried Seismometers Recording Simultaneously with Respect to the 30 foot Depth Seismometer. (Day Time)
Figure 12. Attenuations of the Noise as Shown by Buried Seismometers Recording Simultaneously with Respect to the Surface Seismometer. (Night Time)
Figure 13. Attenuations of the Noise as Shown by Buried Seismometers Recording Simultaneously with Respect to the 30 foot Depth Seismometer. (Night Time)
Figure 14. Attenuations of Earthquake Signals as Shown by Buried Seismometers Recording Simultaneously with Respect to the Surface Seismometer.
the velocity of P waves, \( \alpha \), whereas the velocity and particle motion of Rayleigh waves are mostly governed by the velocity of S waves, \( \beta \). It is known that in sediments near the earth's surface \( \beta \) can be very low compared to \( \alpha \) (Poisson's ratio is much greater than 0.25), and that the ratio of \( \alpha/\beta \) can vary greatly. Consequently it would have been difficult to reach firm conclusions even if velocity logs had been available.

Figure 15 to 16 show typical field records of noise and earthquake obtained with the vertical array.

A remark should be made here: If the seismic noise is a mixture of body waves (\( \leq 1.0 \) cps) and Rayleigh modes, as reported by other investigators, [see Seriff, Velzeboer, and Haase (1965), Gupta (1965)], their presence in the near surface layer could be detected from measurements obtained with a vertical array. Body wave noise which travels more or less along a vertical path, will show a time difference between seismometers, while the horizontally traveling Rayleigh wave noise will not. Because of the limited vertical extent of our array such separation is impossible in the present case: the effective sampling interval is 0.032 sec while the expected \( \Delta t \) would be about 0.015 sec or less than half the sampling interval, taking \( d = 100 \) ft and \( c = 6500 \) ft/sec and the angle of incidence to be 90 degrees.

4.2 Triaxial Array and L Array

The triaxial array and the L array were installed in an attempt at
Figure 15. Ambient Noise Recording Simultaneously at Different Depths Showing Attenuations Due to Burial at the Houston Site. Benioff Gain was Twice as the Others.

Figure 16. Earthquake Signal Recording Simultaneously at Different Depths Showing Attenuations Due to Burial at the Houston Site. Benioff Gain was Twice as the Others.
measure the propagation velocities of the noise observed near the surface. Such velocity measurements were expected to provide information to help in identifying the type of waves propagating predominantly in the near surface layers. Two different field arrangements were used [see Section 3.2]: The distance between seismometers for the L-array was 300 feet, and 100 feet spacing was used for the triaxial array. The geometry of these arrays is shown in Figures 17 to 21. The outstanding appearances of wave packets formed by wavelets of about the same frequency should be noted on all these figures.

By visual inspection of the playback seismograms, which have timing lines every 0.1 sec, individual noise pulses and wave-packets were identified and their transit times across the array were measured. As shown in Figures 17 to 21, these noise pulses and wave-packets generally indicate a unique direction of propagation in the interval of the recording. Occasionally, the change of direction of propagation can be clearly identified on the record, as seen on Figure 19. The similarity in wavelet shapes and frequency content are obvious, indicating that there is a large degree of coherence. However, the most interesting observation is that the resultant horizontal velocities of propagation, after studying numerous records, are always in the range of 1150 to 1250 feet per second. The Δt between the two seismometers along the z-axis of the triaxial array is negligible, but the small vertical separation is insufficient to indicate whether the waves are propagated horizontally or not.
Figure 17. Ambient Noise Recorded Simultaneously with L-array Showing Direction of Propagation. Seismometers were Buried at 150 Foot Depth. Benioff was Buried at 3 Foot Depth. Note the Well-developed Wave Packets Showing on Each Trace.

Figure 18. Ambient Noise and Earthquake Signal Recorded Simultaneously with L-array Showing Direction of Propagation. Seismometers were Buried at 150 Foot Depth. Benioff was Buried at 3 Foot Depth. Note the Wave Packets Showing on Each Trace and the Cancellation Effect of Noise on the Sum Trace.
Figure 19. Ambient Noise Recorded Simultaneously with L-array Showing Directions of Propagation. Seismometers were Buried at 150 Foot Depth. Benioff was Buried at 3 Foot Depth. Note the Wave Packets Showing on Each Trace.
Figure 20. Ambient Noise Recording Simultaneously by Triaxial Array at the Houston Site Showing the Wave Packets and Direction of Propagation.

Figure 21. Ambient Noise Recording Simultaneously by Triaxial Array at the Houston Site Showing the Wave Packets and Direction of Propagation.
However, at this point two comments must be made. First, the number of seismometers and the array length between seismometers, are probably marginal for studies of the velocities in the expected range. That is to say the spacing of timing line may be too close for reliable resolution. Secondly, if waves proceed across the array simultaneously in two or more directions then this method of study of spatial and temporal coherence becomes much more complex and simple conclusions from visual inspection are probably impossible.

A second method for estimating the propagation velocity across the array involves computing the cross correlation functions, the cross-spectra and the coherences between various elements of the array. [See section 3.3, §. 7]. This is a much more extensive average than that resulting from visual measurements on the seismograms. However, in spite of this difficulty this method has the appeal, in addition to the computation speed, that it gives information about the extent of similarity of the two time series. Therefore about 20 data samples were processed for estimations of their cross-correlation coefficients, cross-power densities, and coherences. Sample plots of the obtained cross spectral density estimates, $|P_{h_i, m}(f)|$ and coherences, $B_{h_i}(f)$, are shown in Figures 22 to 24. In the present study, the coherence graphs should be interpreted in reference with the cross spectral density graphs because the high coherences are only significant at frequencies where the spectral densities are also of substantial magnitudes. The vertical bars show the 80% confidence limits of these estimations at frequencies as shown.

The general features of these graphs may be enumerated as follows: (1)
Figure 22. Cross spectral estimates and coherence estimates of seismic noise recording simultaneously with the triaxial array at the Houston Site.
Figure 23  Cross spectral estimates and coherence estimates of seismic noise recording simultaneously with the triaxial array at the Houston Site.
Figure 24  Cross spectral estimates and coherence estimates of seismic noise recording simultaneously with the L array at the Houston Site.
the indication of spatial coherence is high ($\geq 0.7$) for frequencies where the cross-
power spectra are high but the coherence decreases as the separation distance be-
tween seismometers is increased either in the horizontal direction or in the vertical
directions. (2) The magnitudes of cross-power spectra decrease as the separation
distance between seismometers was increased. (3) The coherence values at
frequencies where the cross-power densities are very low, have little relevance to
the main objective of this research which is to study the behavior of seismic noise
at frequencies with relative high spectral densities; the true significance of these
coherences remains to be investigated.

Here again, the most interesting result is that the propagation velocities of
the seismic noise at the dominant frequency are in the ranges of 1150–1250 ft/sec.
In addition, the average cross-correlation coefficient, $R_{hj} (\tau')$, has a value greater
or equal to 0.7. The directions of propagation are always very nearly horizontal
and unidirectional. The direction of propagation has no apparent relation with any
physical objects in the vicinity of the station, such as highway traffic, irrigation
pumps, trees. It is true that if there are two or more wave trains propagating at
the same frequency from different directions, the computation of such data as
described above would yield an ambiguous result. But the above results were
obtained after processing 20 or more data samples which were chosen in the
manner as described in Section 3.3, hence such coincidence seems unlikely.
Visual inspection of data samples as shown in Figures 17 to 21 also tends to rule
out such coincidence.
Therefore it is concluded that the seismic noise at the dominant frequency (around 2.0 cps) has a measured propagation velocity of 1150-1250 ft/sec with a single direction of propagation at the Houston Site. The length of seismograms used for such measurement varied from 12 seconds to 96 seconds. Seismograms were obtained either with seismometers buried at 3 feet or at 150 feet depth. The triaxial array and the L-array yielded the same general result.

4.3 Discussion

The following experimental properties of the seismic noise in the range of 0.5 to 5.0 cps as observed at the Houston Site will now be discussed:

1. Its power is attenuated vertically by the earth's section. In addition, at a given depth the power shows an approximate exponential decrease with frequency as shown in Figure 8, and 10 to 13.

2. Its horizontal velocity of propagation in the near surface section is about 1150-1250 ft/sec regardless of the depth of burial, which was in the range of 3 feet to 150 feet below the ground surface.

3. Its coherence indicates a high degree of similarity between outputs of different seismometers. This implies that a large portion of such seismic noise may be generated by a single mechanism or many such identical mechanisms at the same time.

4. Its power is centered around 2.0 cps, and such phenomenon has been
observed throughout the United States.

From such observations, one is immediately tempted to think that the source of such noise in the band of interest might be situated near or above the ground; such a source could be a pressure variation in the atmosphere of sufficient magnitude to produce the observed particle velocity in the ground. For an isotropic, homogeneous elastic half space, the elastic deformation of the earth's surface produced by a pressure variation may be estimated from the pressure radiation formula (for a plane wave) \( p = \rho \omega w \). Here \( p \) is the radiated pressure fluctuation received by the ground surface, \( \rho \) being the density of the ground layer, \( c \) the velocity of the pressure wave in the ground layer and \( \omega \) the velocity of displacement. From the field measurements obtained at the Houston Site, \( \omega \) is typically equal to \( 150 \times 10^{-7} \) cm/sec at 1 cps, \( \rho \) is estimated about 2.0 gm/cm\(^3\) and \( c \) is about 2000 \times 30.5 \) cm/sec. Therefore the required pressure fluctuation is 1.83 dynes/cm\(^2\) or about 2.0 dynes/cm\(^2\).

Cook (1962) and Cook and Young (1962) have reported that sounds of substantial intensity at infrasonic frequencies (less than 15 cps) are always present in the atmosphere. These sounds have many causes, including natural sources such as distant tornadoes, earthquakes, magnetic storms (Chrzanski, et al. 1961), volcanoes, and man-made sources such as factories and airplanes. The pressure variations due to the sound waves have been reported by Cook (1962) in the range of 0.1 dynes/cm\(^2\) to 50 dynes/cm\(^2\) so that the value of 2 dynes/cm\(^2\) appears as a reasonable requirement. Of particular interest is that such waves can be propagated
over a very long distance without substantial loss of energy. The attenuation coefficient, $a$, of infrasound in the atmosphere due to viscosity and heat conduction, may be defined by the spatial attenuation of $p$, $p(X) = p_0 \exp(-aX)$, and it is about $1.6 \times 10^{-4} / T^2B$ dB/m (Cook, 1962) where $T$ is the period in seconds and $B$ is the atmospheric pressure in dynes/cm² ($\approx 1.013 \times 10^6$ dynes/cm² at 0 meter).

For a plane sound wave of 1 dyne/cm² in the standard atmosphere with a period of $T = 1.0$ sec, the attenuation is less than $2 \times 10^{-8}$ dB/km. (This, of course, does not include the effect of the geometrical spreading.) Thus the loss of sound pressure in the band of interest due to attenuation is very small. These arguments suggest that the mechanism postulated above is physically plausible.

Pressure variations could also be caused by the turbulent passage of the wind over obstructions on the ground surface, such as buildings, trees, plants, and hills. The turbulent motion can cause significant localized pressure fluctuations. The magnitude of the pressure variation, $\Delta p$, can be estimated using the Bernoulli principle with the formula $\Delta p \approx \rho v \Delta v$, where $\rho$ is the atmospheric density in a wind moving with speed $v$ and fluctuation $\Delta v$. Therefore a wind whose speed varies irregularly from 5 to 6 mph has a corresponding random pressure fluctuation of about 11 dynes/cm². Although such turbulence-induced pressure fluctuations are much greater than pressures observed for sounds at infrasonic frequencies, the wind pressure fluctuation is localized and would not be expected to generate much seismic noise that is coherent over a great distance, on the other hand it
may generate a sound pressure wave which may propagate for great distances. Therefore, while the wind turbulence may not contribute directly to seismic noise, it may be a contributing factor for producing seismic noise, via its generation of sound waves in the air.

In this experiment, only the vertical movement of the ground-particle velocity was measured, it is thus impossible to identify the type of seismic noise waves from the orbital motion point of view. However, many investigations, for instance, Wilson (1953), Akamata (1961), Frantti (1963), Douze (1965) and Seriff, et al. (1963, 1964, 1965), have indicated that a substantial portion of ambient seismic noise originates at the earth's surface and has a near-surface propagation path, i.e., a larger portion of the seismic noise consists of surface waves.

Based upon the above argument, it is conjectured that at least a large portion of seismic noise whose frequency is in the range of 0.5 to 5.0 cps is due to sound waves in the air coupled into the ground layer. Lamb (1916a, 1916b), Press and Ewing (1951), Jardetzky and Press (1952), and Ewing, Jardetzky, and Press, (1957, p. 34, p. 67, p. 230, p. 288) have treated the sound wave in the atmosphere as a pressure pulse traveling with a constant velocity $a_0$ applied normal to the surface of a dispersive system. The result is that constructive interference is only possible for those waves whose phase velocity $c$ equals the speed of sound in air $a_0$. Ewing et al. have shown that the effect of air
air coupling is essentially to introduce an additional branch in the phase and
group velocity curves, but that the curves computed without taking the air
coupling into account are substantially unchanged. Therefore curves without
the air coupling terms were computed and sketched in the additional branches
due to air coupling terms.

The presence of a minimum in the group velocity at the frequency at which
constructive interference can occur, will contribute to making these waves more
prominent, because this so-called Airy phase from the fundamental mode
attenuates less rapidly with distance than other waves. This is the situation in
Figure 4-62 of Ewing, Jardetzky and Press (p. 235, 1957). This figure was first
published in 1952 by Jardetzky and Press.

It is noted that the Poisson's ratio \( \alpha_1 = \alpha_2 = 0.25 \) was used by Ewing et al.
However, it is known that the Poisson's ratio of the near surface layer is generally
greater than 0.25. Based on papers of Dorbin, Simmon and Lawrence (1952),
Barkan (1962), and McDonal et al. (1958), it is reasonable to assume that
Poisson's ratio has a value in the range of 0.39 - 0.43 for the near surface layers.
Therefore, the phase and group velocity curves versus the dimensionless factor, \( kH \)
as computed by Mooney and Bolt (1965, p. 168) (neglecting the air-coupling term
of the frequency equation) are plotted in Figure 25 together with Figure 4-62
of Ewing, Jardetzky and Press (1957). The pertinent models and parameters are
shown in the graph.
Figure 25. Group and Phase Velocity Curves of Rayleigh Waves.
For the Houston Site, additional phase and group velocity curves were computed for different values of $\alpha$ and $\beta$ but without the air coupling term. The computations were performed by a computer program which uses Haskell's method (Haskell, 1953). This program was checked by comparing its results with those of Mooney and Bolt (1965). Two representative samples of such phase and group velocity curves are shown in Figure 26 (Model A) and 27 (Model B) together with the elastic parameters.

Figures 25, 26, and 27 show that the minimum of the group velocity for the fundamental Rayleigh mode varies very little with the parameters of the model when $kH$ is used as abscissa. This fact was also noticed by Mooney and Bolt. It is remarkable that this minimum always occur fairly close to the frequency corresponding to the condition for constructive interference in these four cases. As remarked before, this condition will favor the dominance of these waves over waves of other frequencies.

For the case of air coupled Rayleigh waves, real values of $kH$ must correspond to the interval $0.9194 \leq \frac{c}{\beta_1} \leq \frac{\alpha_0}{\beta_1}$. For the models A and B, the upper limits for $\frac{c}{\beta_1}$ would be about 1.567 ($= 1100/702$) and 1.794 respectively if the air-coupling terms of the frequency equations were not neglected. Therefore the $kH$ values of constructive interference for air-coupled Rayleigh waves would be 1.95 and 1.6 respectively. Hence the required thickness would be about 171 ft for model A and 140 ft for model
Figure 26. Group and Phase Velocity Curves of Rayleigh Waves for Model A.
Figure 27. Group and Phase Velocity Curves of Rayleigh Waves for Model B.
B, if \( c = 1100 \text{ ft/sec} \) and \( f = 2.0 \text{ cps} \) are used. These values are in fair agreement with the possible thickness (100-150 ft) of the low-velocity weathering layer at the Houston Site.

The phase velocities of 1150 to 1250 ft/sec observed with the buried arrays have a slightly higher value than the speed of sound in the air (1100 ft/sec). This discrepancy may be attributed partly to the inaccuracy of the measurements, partly to the computation method, and partly to the complex structure of the noise.

It is thus reasonable to suggest that the presence of a low velocity weathering layer and the sound waves in the atmosphere may be the cause of the 2 cps seismic noise. This interpretation is in agreement with the two hypotheses advanced by Frantti (p. 560, 1963) in regarding the 2 cps spectral peak i.e., that “(1) This frequency corresponds to a minimum or maximum in the group velocity curve for the structure at the recording site; (2) There exist specific noise sources over extensive regions of the earth’s surface which generate surface waves of this frequency.”

It is also interesting to note that if the sound wave is one of the generating mechanisms of seismic noise, the observed wave packets as shown in Figure 17 to 21 may be explained as the result of superposition of two harmonic waves with the same velocity but slightly different frequencies, i.e., an interference phenomenon. Such phenomenon might well be expected due to the small variations in the thickness of the weathering layer at the vicinity of the recording site.
It should be mentioned that short period seismic noise data had also been obtained from other sites, for instance the Montana area near Miles City. The preliminary results from this site show that the observed phase velocity for 2 cps noise is much higher (about 2500 ft/sec) than the speed of sound in air. It is therefore uncertain whether this explanation is valid for the world-wide 2 cps noise.

It may be remarked that higher modes of Rayleigh waves may also be air-coupled. Clearly these will have higher frequencies, but the same phase velocity.
5. CONCLUSIONS

The short period (0.5 to 5.0 cps) seismic noise from a field station near Houston, Texas was recorded and collected with a vertical array, a triaxial array, and a L-array consisting of buried seismometers at various depths. The relative constancy of the estimated spectral density functions suggests that the time series is at least weakly stationary.

The estimated power density spectra are very similar in structure over a horizontal distance of 900 feet or less. There is a prominent peak at about 2.0 cps. The usual low frequency microseism is cutoff on the low end by the recording instruments. The power spectrum of the output of each seismometer is attenuated by the earth's section. At a given depth, the power also shows an approximate exponential decrease with frequency.

From cross-correlation computations, the velocity of propagation of the seismic noise in the near surface section is about 1150 to 1250 ft/sec. in nearly horizontal directions of propagation regardless of the depth of burial or direction of propagation. The seismometers were buried from 3 feet to 150 feet below the ground surface. The coherence of the seismic noise indicates a high degree of similarity between outputs of different seismometers, and wave packets may be followed from one seismometer to another. Therefore it is inferred that a large portion of such seismic noise may be generated by a single mechanism or many such identical mechanisms at the same time.

These observations suggest that at least a large portion of the seismic noise
at the Houston vicinity may be generated by the infrasonic waves in the atmosphere. The sound waves while traveling in air near the ground surface, exert a pressure front vertically on the surface of ground; this disturbance may be coupled to Rayleigh waves traveling in the ground. The necessary condition for such coupling is that there is a low velocity surface layer where the S wave velocity is less than the speed of sound in air. As a result of air coupling, an additional branch may be introduced to the dispersion curves, corresponding to a train of approximately constant frequency waves. The frequency is determined by the phase velocity curve and corresponds fairly well to the depth and velocity of the weathering layer. A minimum in the group velocity near that frequency may contribute to the predominance of these waves.

It would be interesting to perform additional experiments to measure the magnitudes and waveforms of sound waves in the atmosphere near the ground surface, and to evaluate the correlations between the waveforms of such sound waves and the observed seismic noise at various depths in the frequency band of interest. The implication is that if a good correlation is found, a simple subtraction of the sound waves from the seismic records containing 2 teleseismic events would significantly improve the signal-to-noise ratio without further data processing procedures. Throughout this thesis, the emphasis has been placed on the statistical approach on treatment of data. The cross correlation results on the array data certainly do not represent an exhaustive study. It is possible by using partial
coherences to determine the number of major noise components present and the minimum number of elements required to study the noise field. Summation of outputs from seismometers of each array with variable time lags would also be quite interesting. Multichannel prediction and filtering of data is another avenue that should be explored. However, if the direct subtraction of the sound waves from the seismic record is as advantageous as the preliminary test suggests it will then be by far the simplest method.
REFERENCES


20. Lamb, H., On wave patterns due to a traveling disturbance, with an application to waves in superposed fluids, Phil. Mag. [6], 13, pp. 386-399, 1916.


