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ANALYSIS OF THREE FLUID, CROSS FLOW
HEAT EXCHANGERS

by

Noel Charles Willis, Jr.

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INTRODUCTION

Considerable effort has been expended in previous investigations to define performance characteristics of heat exchangers involving energy transfer between two fluids. Now that industrial processes have been developed which require simultaneous heat exchange between more than two fluids, analytical techniques are needed to describe the performance of multifluid heat exchangers. One example of such a process is the large scale production of oxygen in an air separation plant which requires heat exchange between oxygen, nitrogen, and air at very low temperatures. There are also possibilities for combining several separate two fluid heat exchanging operations more economically in a single multifluid arrangement.

Several investigators (refs. 1 through 5) have attacked the problem of multifluid heat exchangers in parallel or counter flow wherein only one physical dimension of the exchanger is considered. The purpose of this investigation is the detailed study of three fluid heat exchangers in cross flow, a completely two dimensional problem.

In this study the performance of three fluid cross flow heat exchangers is determined and presented graphically in terms of the temperature effectiveness of two of the fluids. The effectiveness is determined as a function of heat exchanger size for sets of fixed operating conditions. The introduction of non-dimensional operating variables reduces the volume of data required to represent a practical range of operating conditions. The number of boundary conditions for
the temperatures is reduced from three to one by the introduction of a non-dimensional inlet temperature parameter.

An expression for overall effectiveness is derived which compares the performance of a heat exchanger to that of an infinitely large exchanger operating at the same conditions. A study of the two-dimensional temperature distributions reveals circumstances for which the overall effectiveness may be greater than unity. This interesting result implies that in a three fluid cross flow heat exchanger, total heat transfer is not always maximized by increasing the size of the exchanger.

Effectiveness factors are determined for a wide range of operating parameters for single pass, three fluid, heat exchangers. Performance of multipass three fluid heat exchangers for both cocurrent and countercurrent flow is studied for selected operating conditions.

Sample problems are used to illustrate the application of the effectiveness curves to heat exchanger design problems. Since some of the performance data can be explained only in terms of the two-dimensional variation of the temperatures of each fluid, these problem solutions are also used to provide insight into the detailed behavior of the fluids within the heat exchangers.

The basic differential equations for the spatial distribution of the temperatures of the three fluids were solved numerically using a digital computer. A very flexible program is available for both single pass and multiple pass calculations. An automatic integration step size control was developed through the consideration of overall conservation of energy so that multiple cases may be run continuously with the optimum step size used for each individual case. Output op-
tions are available for a detailed study of spatial temperature dis-
tribution within the exchanger or for the determination of the overall
performance using only the average exit temperatures and effectiveness
values.
SYMBOLS

English Letters

$A = \frac{u_{12}x_{0}y_{0}}{\dot{m}_{1}c_{p_{1}}}$

$B = \frac{u_{23}x_{0}y_{0}}{\dot{m}_{3}c_{p_{3}}}$

$c_{p_{j}}$ specific heat of fluid $j$ (BTU/lb°F)

$C = \frac{u_{12}x_{0}y_{0}}{\dot{m}_{2}c_{p_{2}}}$

$D = \frac{u_{23}x_{0}y_{0}}{\dot{m}_{2}c_{p_{2}}}$

$E$ overall effectiveness $= \frac{Q_{2}}{Q_{3}}$

$K_{1} = \frac{\dot{m}_{1}c_{p_{1}}}{\dot{m}_{2}c_{p_{2}}}$ capacity rate ratio for fluid 1

$K_{3} = \frac{\dot{m}_{3}c_{p_{3}}}{\dot{m}_{2}c_{p_{2}}}$ capacity rate ratio for fluid 3

$\dot{m}_{j}$ mass flow rate of fluid $j$ (lb/hr)

$M$ largest value of the set (A, B, C, D)

$NTU_{1}$ number of transfer units of the heat exchanges referred to fluid 1, $= \frac{u_{12}x_{0}y_{0}}{\dot{m}_{1}c_{p_{1}}} = A$

$q_{j}$ heat transferred to fluid $j$ per unit time (BTU/sec)
\( Q_a \) total heat transfer rate in a heat exchanger of finite size (BTU/sec)

\( Q_\infty \) total heat transfer rate in a heat exchanger of infinite size (BTU/sec)

\( \bar{Q} \) non-dimensional heat transfer rate \( \frac{Q}{m_2 c_p (t_{1i} - t_{s1})} \)

\( t_j \) local temperature of fluid j (°F)

\( t_{ji} \) inlet temperature of fluid j (°F)

\( t_{je} \) exit temperature of fluid j (°F)

\( t_{jem} \) average exit temperature of fluid j (°F)

\( t_{1smix} \) inlet mixing temperature of fluids 1 and 3 = \( \frac{m_1 c_p t_{1i} + m_3 c_p t_{3i}}{m_1 c_p + m_3 c_p} \)

\( t_{2eo} \) exit temperature of fluid 2 for an infinitely large heat exchanger

\( t_{2max} (x=0) \) maximum value of \( t_2 \) along the y axis for an infinitely large heat exchanger = \( \frac{U t_{1i} + t_{ai}}{U + 1} \)

\( T \) non-dimensional temperature \( \frac{t - t_{s1}}{t_{1i} - t_{s1}} \) all above subscripts for \( t \) are applied to \( T \) also

\( u_{12} \) overall conductance for the surface separating fluids 1 and 2 \( \frac{\text{BTU}}{\text{hr-ft}^2 \circ F} \)
\( u_{23} \) overall conductance for the surface separating fluids 2 and 3 \( \left( \frac{\text{BTU}}{\text{hr-ft}^\circ\text{F}} \right) \)

\[ U \text{ conductance ratio} = \frac{u_{13}}{u_{23}} \]

\( x \) coordinate of heat exchanger surface (ft.)

\( y \) coordinate of heat exchanger surface (ft.)

\( x_o \) \( x \) dimension of heat exchanger (ft.)

\( y_o \) \( y \) dimension of heat exchanger (ft.)

\( X \) non-dimensional coordinate of heat exchanger surface \( = \frac{x}{x_o} \)

\( Y \) non-dimensional coordinate of heat exchanger surface \( = \frac{y}{y_o} \)

**Greek Letters**

\( \Delta t_i \) inlet temperature parameter \( = \frac{t_{1i} - t_{2i}}{t_{3i} - t_{2i}} \)

\( \theta_i \) temperature effectiveness of fluid 1 \( = \frac{t_{1i} - t_{1em}}{t_{1i} - t_{2i}} \)

\( \theta_3 \) temperature effectiveness of fluid 3 \( = \frac{t_{3i} - t_{3em}}{t_{3i} - t_{2i}} \)
PROBLEM FORMULATION

Derivation of the Governing Equations for Three Fluid Cross Flow Heat Exchangers

Figure 1 is a schematic representation of a single pass, three fluid, cross flow heat exchanger. Heat is transferred between the center fluid, 2, and each outer fluid, 1 and 3; however, there is no heat directly transferred between the two outer fluids. The immediate objective is to determine the temperature distributions of the three fluids in the heat exchanger for a given size and operating condition. Once the temperature distributions are known, the heat transferred to each fluid may be calculated from average exit temperatures, and subsequently the performance of the heat exchanger may be evaluated.

In this investigation certain simplifying assumptions have been made to reduce the complexity of the equations. These assumptions are:

1. The heat exchanger is considered to be adiabatic; that is, there is no heat loss to the surroundings.

2. Steady flow exists for all three fluids.

3. Fluid properties are constant.

4. For a particular surface, the local conductance is constant and equal to the overall conductance.

5. There are no temperature gradients normal to the heat transfer surfaces.

6. There is no lateral mixing in any fluid.
Under the above assumptions, the governing equations for the temperatures distributions in three fluid cross flow heat exchangers will be derived. Consider Figure 2 which represents the heat transfer surface between fluids 1 and 2. For a properly designed exchanger the two outer fluids, 1 and 3, will be either both hotter or both colder than the center fluid, 2. For purposes of discussion during this derivation the center fluid will be arbitrarily assumed colder that the two outer fluids, however, the resulting equations are independent of the assumed temperature levels.

Referring to Figure 2, the heat transferred per unit time from fluid 1 into fluid 2 across the elemental area $dx \, dy$ is

$$ dq_1 = u_{12} (t_2 - t_1) \, dx \, dy $$

where $t_1$ and $t_2$ are both functions of $x$ and $y$.

This expression may be equated to the energy decrease per unit time of the element of fluid 1 between $y$ and $y + dy$ as it moves from $x$ to $x + dx$ which is

$$ dq_1 = \dot{m}_1 c_{p1} \frac{(dy)}{y_0} \frac{\partial t_1}{\partial x} \, dx. $$

Introducing nondimensional coordinates $X = \frac{x}{x_0}$ and $Y = \frac{y}{y_0}$ and equating the above expressions for $dq_1$, the resulting differential equation is

$$ \frac{\partial t_1}{\partial X} = \frac{u_{12} x_0 y_0}{\dot{m}_1 c_{p1}} (t_2 - t_1). $$
Following a similar procedure for fluid 3 yields

\[
\frac{\partial t_3}{\partial x} = \frac{u_{23}x'y_o}{m_3c_{p_3}} (t_2 - t_3) .
\]

A differential volume element of fluid 2 is bounded by two surfaces and is in thermal communication with both fluids 1 and 3. The energy transferred to this element of fluid 2 from the outer fluids is

\[
dq_2 = \left[ u_{12} (t_1 - t_2) + u_{23} (t_3 - t_2) \right] dx \, dy .
\]

As the elemental volume of fluid 2 between \( x \) and \( x + dx \) moves from \( y \) to \( y + dy \), its thermal energy increase may be expressed as

\[
dq_2 = \dot{m}_2 c_{p_2} \left( \frac{dx}{x_0} \right) \frac{\partial t_2}{\partial y} dy .
\]

Equating these two expressions and introducing the nondimensional coordinates yields

\[
\frac{\partial t_2}{\partial y} = \frac{u_{12}x'y_o}{m_3c_{p_3}} (t_1 - t_2) + \frac{u_{23}x'y_o}{m_2c_{p_2}} (t_3 - t_2) .
\]

Three simultaneous partial differential equations have been derived which define the temperatures of each of the three fluids as functions of both space coordinates, \( x \) and \( y \). The resulting equations are:

\[
\frac{\partial t_1}{\partial x} = A (t_3 - t_1) ,
\]
\[ \frac{\partial t_2}{\partial y} = C(t_1 - t_2) + D(t_3 - t_2), \]
\[ \frac{\partial t_3}{\partial x} = B(t_2 - t_3), \]

where

\[ A = \frac{u_{12} x_0 y_0}{m_1 c_{p_1}}, \quad B = \frac{u_{23} x_0 y_0}{m_3 c_{p_3}}, \quad C = \frac{u_{12} x_0 y_0}{m_2 c_{p_2}}, \quad D = \frac{u_{23} x_0 y_0}{m_2 c_{p_2}}, \]

and

\[ t_j = t_j(x, y). \]

Under the assumptions of the problem, the nondimensional terms A, B, C, and D are constants which depend upon the heat capacity rates of the fluids, the heat exchanger dimensions and the values of conductance at the two heat transfer surfaces. When A, B, C, and D are specified along with the inlet temperatures of the three fluids, the equations may be solved for \( t_1, t_2 \) and \( t_3 \) as functions of position throughout the heat exchanger.

**Reduction of the Number of Boundary Conditions**

It is convenient to nondimensionalize the basic equations with respect to temperature in the following manner. All temperatures will be referred to the inlet temperature of the center fluid, \( t_{2_1} \), and divided the difference between the inlet temperatures of fluids 1 and 2, or \( (t_{1_1} - t_{2_1}) \). The resulting equations are:

\[ \frac{\partial t_1}{\partial x} = A (T_2 - T_1), \]
\[ \frac{\partial T_2}{\partial y} = C(T_1 - T_2) + D(T_3 - T_2), \]

\[ \frac{\partial T_3}{\partial x} = B(T_2 - T_3), \]

where

\[ T_1 = \frac{t_1 - t_{21}}{t_{1i} - t_{21}}, \]

\[ T_2 = \frac{t_2 - t_{21}}{t_{1i} - t_{21}}, \]

\[ T_3 = \frac{t_3 - t_{21}}{t_{1i} - t_{21}}. \]

The advantage of the above formulation is particularly apparent when the boundary conditions are examined. They become

\[ T_1 (X=0) = 1, \]

\[ T_2 (Y=0) = 0, \]

\[ T_3 (X=0) = \left( \frac{t_3 - t_{21}}{t_{1i} - t_{21}} \right). \]

The boundary conditions for any problem may be specified now by a single quantity called the inlet temperature parameter. This parameter is defined as

\[ \Delta t_i = \frac{t_{1i} - t_{21}}{t_{3i} - t_{21}}, \]

and is the ratio of the temperature levels of the two outer fluids referred to the temperature of the center fluid. The third boundary
condition becomes \( T_{3_1} = \frac{1}{\Delta t_1} \). The outer fluids may be "named" so that \( \Delta t_1 \) varies between zero and unity. For example, in the case of both outer fluids being hotter than the center fluid, fluid 1 is always the colder of the two hot fluids. When \( \Delta t_1 \) is unity, \( t_{1_1} \) equals \( t_{3_1} \) and when \( \Delta t_1 \) is very small, \( t_{3_1} \) is considerably greater than \( t_{1_1} \). For example, consider two sets of inlet temperatures:

\[
\begin{align*}
  t_{1_1} &= 300^\circ F \\
  t_{2_1} &= 100^\circ F \\
  t_{3_1} &= 500^\circ F
\end{align*}
\]

\[
\begin{align*}
  t_{1_1} &= 50^\circ F \\
  t_{2_1} &= 0^\circ F \\
  t_{3_1} &= 100^\circ F
\end{align*}
\]

In both cases \( \Delta t_1 = 0.5 \) and the problems are equivalent in the foregoing nondimensional formulation.

**Discussion of Nondimensional Independent Variables**

Now that the specification of the inlet temperatures has been reduced to a single parameter, any problem may be completely defined by five quantities, \( A, B, C, D \) and \( \Delta t_1 \).

In order to specify a particular problem in terms of quantities which are more useful to a designer, the quantities \( A, B, C, \) and \( D \) may be combined into a new set of parameters which are more amenable to physical interpretation.

The new parameters are:

\[
NTU_1 = A = \frac{u_{12}x_{0y_0}}{m_1c_{p_1}}
\]
\[ U = \frac{C}{D} = \frac{u_{12}}{z_3}, \]

\[ K_1 = \frac{A}{C} = \frac{\dot{m}_1 c_p_1}{\dot{m}_2 c_p_2}, \]

and

\[ K_3 = \frac{B}{D} = \frac{\dot{m}_3 c_p_3}{\dot{m}_2 c_p_2}. \]

The constant \( A \) is retained as the basic size parameter and called \( NTU_1 \), or the "number of transfer units" of the heat exchanger referred to fluid 1. This parameter represents the ability of the heat exchanger to change the temperature of fluid 1. A large value of \( NTU_1 \) can result from a large physical size \( (x_0 y_0) \), a high conductance for the surface separating fluids 1 and 2 \( (u_{12}) \) and a small capacity rate for fluid 1 \( (\dot{m}_1 c_p_1) \). All of these factors would make fluid 1 relatively easy to heat or cool. The nondimensional input parameter, \( NTU_1 \), is therefore, a good representation of the size of the exchanger. Since a similar parameter based on either fluid 2 or 3 could also have been defined, the choice of fluid 1 as a reference for size is arbitrary.

The parameters \( K_1 \) and \( K_3 \) are nondimensional heat capacity flow rates of the outer fluids (1 and 3) referred to the center fluid (2) and will be called "capacity rate ratios". For a well designed heat exchanger, the combined heat capacity flow rates of the outer two fluids should not be significantly different from the capacity flow rate of the center fluid. This implies that \( (K_1 + K_3) \) should be near unity for proper design.
The parameter \( U \) is called the "conductance ratio" and indicates the relative ability of the two separating surfaces to transfer heat.

Problems may be specified now by the five independent parameters \( \text{NTU}_1, K, K_2, U, \) and \( \Delta t_1 \).

Discussion of the Numerical Solution of the Basic Equations

The equations have been solved numerically by a first order predictor-corrector integration scheme. A very flexible FORTRAN program has been developed for use with the IBM-7094 or Univac 1107/1108 computers. The program can handle single pass and multiple pass calculations. An automatic step size control, governed by the values of certain input parameters, was developed so that a large number of cases could be run continuously with the optimum step size used for each individual case. Output options are available which allow the user to study either detailed temperature distributions or overall performance characteristics. The details of the numerical procedure are described in Appendix A, while the computer program is discussed in Appendix B.

Discussion of Nondimensional Dependent Variables

The solution of the basic equations provides two dimensional distributions of the temperatures of all three fluids throughout the heat exchangers. Some of the phenomena which occur in three fluid heat exchangers can be explained only by a study of these detailed distributions. Particular examples will be discussed in
later sections. The designer, however, is interested mainly in the overall performance characteristics of the heat exchanger, for example, the average exit temperatures of the fluids.

The dependent variables chosen to represent the performance of three fluid heat exchangers are the temperature effectiveness of the two outer fluids. These variables are defined in the following manner:

\[
\theta_1 = \left( \frac{t_{1i} - t_{1em}}{t_{1i} - t_{2i}} \right) \times 100\% ,
\]

\[
\theta_3 = \left( \frac{t_{3i} - t_{3em}}{t_{3i} - t_{2i}} \right) \times 100\% .
\]

The quantities \( t_{1em} \) and \( t_{3em} \) are the average exit temperatures of fluids 1 and 3. They are obtained by averaging the exact values for the exit temperatures obtained from the two-dimensional numerical integration of the basic equations.

The variables \( \theta_1 \), and \( \theta_3 \) represent the degree to which the temperatures of the outer fluids have approached the inlet temperature of the center fluid when they leave the heat exchanger. The effectiveness, \( \theta_1 \) or \( \theta_3 \), will be 100 percent when the average exit temperature of fluids 1 or 3 equals the inlet temperature of fluid 2. The value will be zero when there is no change in temperature. There are circumstances when one of the temperature effectivenesses can actually be negative. It has been assumed that for proper design the center fluid, 2, will either heat or cool both outer fluids. Consider the case for which \( t_{1i} \) and \( t_{3i} \) are
both greater than $t_{21}$ and the function of the heat exchanger is to cool fluids 1 and 3. If the inlet temperature and heat capacity rate of fluid 3 are considerably greater than those of fluid 1, heat will be transferred through fluid 2 to fluid 1. Fluid 1 will leave the heat exchanger at a temperature above its inlet value, just opposite of the desired effect. Therefore, a negative temperature effectiveness indicates that a fluid intended to be cooled was actually heated or vice versa. A designer must use $\theta_1$ and $\theta_3$ to determine the effect of the heat exchanger on both fluids.

An auxiliary dependent variable, the overall effectiveness, has been defined to compare the performance of a particular heat exchanger to one of infinite size.

The overall effectiveness is $E = \frac{Q_a}{Q_\infty}$ where $Q_a$ is the total heat transferred by an exchanger under fixed operating conditions and $Q_\infty$ is the heat that would be transferred by an infinitely large heat exchanger operating at the same conditions. The overall effectiveness has some interesting properties that can be discussed better in the next section following the presentation of results.
PRESENTATION AND DISCUSSION OF RESULTS

SINGLE PASS HEAT EXCHANGERS

Method of Presentation

One of the major problems in studying three fluid heat exchangers is the presentation of results in a manner which will be useful to designers. The performance of a two fluid heat exchanger may be expressed by a single dependent variable which is a function of two independent variables. For example, the overall effectiveness may be expressed as a function of the heat exchanger's number of transfer units (NTU) and the capacity rate ratios of the two fluids. An investigation of three fluid heat exchangers requires consideration of two dependent variables and five independent variables.

The performance of three fluid heat exchangers has been expressed in terms of two dependent dimensionless variables, $\theta_1$, and $\theta_3$, the temperature effectivenesses of the two outer fluids;

$$\theta_1 = \left( \frac{t_1 - t_{1em}}{t_{1i} - t_{2i}} \right) \times 100\%,$$

$$\theta_3 = \left( \frac{t_3 - t_{3em}}{t_{3i} - t_{2i}} \right) \times 100\%,$$

where $t_1$ and $t_3$ are the average exit temperatures of fluids 1 and $t_{1em}$ and $t_{3em}$ are functions of the five independent dimensionless exchanger parameters $K_1$, $K_3$, NTU, $U$, and $\Delta t_1$. Another dependent parameter, $E$, the overall effectiveness, provides additional insight into exchanger performance. The overall
effectiveness is defined as

\[ E = \frac{Q_a}{Q_\infty} \]

where \( Q_a \) is the total heat exchanged for a particular size exchanger under fixed operating conditions and \( Q_\infty \) is the heat that would be exchanged for an infinitely large heat exchanger operating at the same conditions. Results are presented in Figures 3 through 22 by plotting \( \theta_1, \theta_3 \) and \( E \) as functions of the size parameter, NTU, for fixed values of \( K_1, K_3 \), and \( U \) with \( \Delta t_1 \) as a parameter.

A range of values for the independent variables has been chosen to cover a realistic spectrum of operating conditions. The variation of the inlet temperature parameter may be confined to the range 0 to 1 by appropriately numbering the fluids. For example, in the case for which \( t_1 \) and \( t_3 \) are greater than \( t_2 \), \( \Delta t_1 \) will always be 1 or less, if the colder of the two outer fluids is designated as fluid 1. The conductance ratio, \( U \), may vary from 0 to \( \infty \), however, the selected values of 0.5, 1.0, and 2.0 should be sufficient to cover the range of practical interest. The heat capacity rate ratios, \( K_1 \) and \( K_3 \), may also vary from 0 to \( \infty \), however, \( (K_1 + K_3) \) must be reasonably close to unity for a balanced heat exchanger. Therefore, the values for \( K_1 \) and \( K_3 \) of 0.25, 0.5 and 1.0 should adequately cover the range of interest.

The variation of NTU from 0 to 7.5 is also sufficient to cover the range of practical sizes. The multiple run option of the computer program was used to determine performance factors for heat exchangers represented by all possible combinations of the following set of independent parameters:
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</tbody>
</table>

This set of parameters required 756 separate calculations of the two dimensional variations of all three fluid temperatures. The printout option was used which restricted the output to the overall performance factors $\theta_1$, $\theta_3$ and $E$. Automatic step size control was used to obtain the required accuracy and minimize computation time. The details of the numerical technique including the automatic step size control are contained in Appendix A. The computer program is discussed in Appendix B.

In the above table there are three values each for $K_1$, $K_3$ and $U$, therefore, there are 27 resulting performance charts. Even with this many charts only discreet values of the independent parameters are represented. The section Application of Performance Curves for Design presents an interpolation technique for investigating problems defined by intermediate values of these parameters.
Overall Effectiveness

It is useful to define a parameter which compares the performance of a particular heat exchanger to a heat exchanger of infinite heat transfer area operating at the same conditions. The overall effectiveness has been previously defined as

\[ E = \frac{Q_a}{Q_\infty} \]

where \( Q_a \) is the heat transferred by a particular exchanger and \( Q_\infty \) is the heat transferred by an infinitely large counterflow exchanger operating at the same conditions. The heat transfer, \( Q_a \), is obtained from the solution of the basic equations and may be expressed as either

\[ Q_a = \dot{m}_2 c_p (t_{2i} - t_{2e}) \]

or

\[ Q_a = \dot{m}_1 c_p (t_{1i} - t_{1e}) + \dot{m}_3 c_p (t_{3i} - t_{3e}). \]

In non-dimensional form, the expression becomes

\[ \overline{Q_a} = \frac{Q_a}{m_2 c_p (t_{2i} - t_{2e})} = \frac{T_{2e}}{T_{1e}} = K_1 \left( 1 - T_{1e} \right) + K_3 \left( \frac{1}{h_{t_1}} - T_{3e} \right). \]

Since there is some numerical inaccuracy in the computer program, \( Q_a \) is equated to the average of these two quantities.

In deriving an expression for \( Q_\infty \) for a three fluid heat exchanger, it is instructive to consider a similar problem for a two fluid heat exchanger as illustrated in Figure 30. In the two fluid, counterflow,
case the exit temperature of the fluid with the smaller capacity rate
will approach the inlet temperature of the fluid with the larger ca-
pacity rate as the heat transfer area becomes infinite. If we denote
the hot fluid by the subscript, \( h \), and the cold fluid by the subscript,
\( c \), then for \( \dot{m}_h c_p h \cdot t_{h e} = t_{c i} \), and for \( \dot{m}_h c_p h \cdot t_{c e} = t_{h i} \).

Therefore, the heat transfer for an exchanger of infinite area
is

\[
Q_\infty = (\dot{m} c_p)_{\text{minimum}} (t_{h i} - t_{c i}).
\]

Figure 31 depicts the analogous situation for three fluid heat ex-
changers. In the case for which the capacity rate of the center fluid
is greater than the sum of the capacity rates of the outer fluids, the
exit temperatures of the outer fluids both approach the inlet tempera-
ture of the center fluid.

For \( (\dot{m}_1 c_p_1 + \dot{m}_3 c_p_3) < \dot{m}_2 c_p_2 \), \( t_{1 e}\infty = t_{3 e}\infty = t_{2 i} \)
and \( Q_\infty = \dot{m}_1 c_p_1 (t_{1 i} - t_{2 i}) + \dot{m}_3 c_p_3 (t_{3 i} - t_{2 i}) \).

The equivalent non-dimensional quantities for \( (K_1 + K_3) < 1 \) are

\[
T_{1 e}\infty = T_{3 e}\infty = T_{2 i} = 0
\]

and

\[
\overline{Q}_\infty = \frac{Q_\infty}{\dot{m}_2 c_p_2 (t_{1 i} - t_{2 i})} = K_1 + \frac{K_3}{4t_{1 i}}.
\]

For the situation in which \( (\dot{m}_1 c_p_1 + \dot{m}_3 c_p_3) > \dot{m}_2 c_p_2 \), \( t_{e e} \) approaches
a limiting value, \( t_{2e} \), which lies somewhere between \( t_{1i} \) and \( t_{3i} \).

This limiting value corresponds to a single, effective inlet temperature of fluids 1 and 3. This effective inlet temperature is the mixing temperature of fluids 1 and 3 defined as

\[
t_{2e} = t_{13}^{\text{mix}} = \frac{\dot{m}_1 c_{p1} t_{1i} + \dot{m}_3 c_{p3} t_{3i}}{\dot{m}_1 c_{p1} + \dot{m}_3 c_{p3}}
\]

so that

\[
Q_{e} = \dot{m}_2 c_{p2} \left( t_{2e} - t_{2i} \right).
\]

The equivalent non-dimensional expressions are, for \((K_1 + K_3) > 1\),

\[
T_{2e} = \frac{K_1 T_{1i} + K_3 T_{3i}}{K_1 + K_3} = \frac{K_1 + K_3 / \Delta t_i}{K_1 + K_3}
\]

and

\[
\tilde{Q}_{e} = \frac{Q_{e}}{\dot{m}_2 c_{p2} (t_{1i} - t_{2i})} = \frac{T_{2e}}{t_{2e}} = \frac{K_1 / K_3 + 1 / \Delta t_i}{K_1 / K_3 + 1}.
\]

It should be noted that \( T_{2e} \) depends only on the ratio, \((K_1/K_3)\), and \( \Delta t_i \) and is independent of \( U \). For two fluid heat exchangers the effectiveness can never be greater than 100 percent; however, using the above definition, there are isolated cases corresponding to poor design practice for which the overall effectiveness of a three fluid heat exchanger in cross flow can actually exceed 100 percent. A specific sample will be used to explain the behavior of the fluids when \( E \) is greater than 100 percent and to show why this somewhat anomalous result corresponds to poor design practice. The case under consideration will be one of using the center fluid to cool the two outer fluids.
Values of effectiveness greater than 100 percent occur when 

$$(K_1 + K_3) > 1$$

and the configuration is such that $T_{e_m}^2$ is greater 

than $T_{e_m}^3 = T_{13}^{e_m}$. Figure 32 represents the performance of a three 
fluid cross flow heat exchanger in the terms of $\theta_1$, $\theta_3$ and $E$ for $K_1 = 2.0$, $K_3 = 0.5$ and $U = 2.0$. The value of $E$ reaches a maximum of 

101.5% at $\text{NTU}_1 = 4$ for $\Delta t_1 = .25$ and begins to decrease toward 100 percent as $\text{NTU}_1$ continues to increase. To understand why it is possible for $T_2$ to be greater than $T_{e_m}$, consider the distribution of $T_2$ 

along $X = 0$ from $Y = 0$ to $Y = 1.0$. Along this line $T_1$ and $T_3$ are 

constant at their inlet values $T_{i_1}$ and $T_{i_3}$. Along the narrow strip 

at $X = 0$ fluids 1 and 3 act as infinite sources between which fluid 2 must flow. If the heat exchanger becomes infinitely large, there is 

a maximum temperature which fluid 2 may reach along the $Y$ axis. Consider the case for $T_{i_3} > T_{i_1} > T_{i_2}$.

The maximum value of $T_2$ along the $Y$ axis, $T_{2 \text{max}}(X = 0)$, is 

reached when the heat flow rate into fluid 2 from fluid 3 is equal 

to heat flow rate from fluid 2 into fluid 1. This condition is expressed by the equation

$$u_{12} (T_{2 \text{max}} - T_{i_1}) = u_{23} (T_{3 i} - T_{2 \text{max}}).$$

Solving for $T_{2 \text{max}}$ gives

$$T_{2 \text{max}}(X=0) = \frac{U T_{i_1} + T_{3 i}}{U + 1}$$

or since $T_{i_1} = 1$ and $T_{3 i} = \frac{1}{\Delta t_1}$
\[ T_{2_{\text{max}}} (X=0) = \frac{U + \frac{1}{\Delta t_1}}{U + 1} \]

\( T_{2_{\text{max}}} \) is a function of \( \Delta t_1 \) and \( U \) and is independent of \( K_1 \) and \( K_3 \), therefore, independent of \( T_{2e\infty} \). If the value of \( T_{2_{\text{max}}} \) is greater than \( T_{2e\infty} \) (\( T_{13} \) mix), the effectiveness, \( E \), may be greater than 100 percent for sufficiently large values of \( NTU_1 \), the size parameter.

Figure 33 illustrates the distribution of \( T_1, T_2, \) and \( T_3 \) along the \( Y \) axis for \( NTU_1 = 7.5 \). \( T_1 \) and \( T_3 \) are constant and \( T_2 \) asymptotically approaches 2. For \( \Delta t_1 = 0.25 \) and \( U = 2 \),

\[ T_{2_{\text{max}}} (X=0) = \frac{U + \frac{1}{\Delta t_1}}{U + 1} = \frac{2 + \frac{4}{3}}{3} = 2.0 \]

Figures 34 and 37 illustrate the temperature distributions as functions of \( Y \) for \( X = 0.1, 0.25, 0.5 \) and 1.0. As \( X \) becomes larger all fluids approach a nondimensional temperature of 1.6 at \( Y = 1.0 \). This is more clearly illustrated in Figure 38 in which all temperatures are plotted vs \( X \) at \( Y = 1.0 \). For these conditions

\[ T_{2e\infty} = \frac{1}{K_1/K_3 + \frac{1}{\Delta t_1}} = \frac{2/0.5 + 0.25}{2/0.5 + 1} = 1.6 \]

If the heat exchanger size were made infinite, the exit temperature of fluid 2 would be \( T_{2e\infty} \), as defined above, since the amount of fluid at a temperature greater than \( T_{2e\infty} \) would be negligible. There, for \( T_{2_{\text{max}}} \) (X=0) greater than \( T_{2e\infty} \), the
overall effectiveness, \( E \), will rise as \( \text{NTU}_1 \) increases, reach a maximum value and asymptotically decrease to 100 percent as \( \text{NTU}_1 \) becomes infinite. If \( T_2^* (X=0) \) is less than \( T_{2e\infty}^\text{max} \), the average exit temperature of fluid 2 approaches \( T_{2e\infty}^\text{max} \) as a maximum and \( E \) increases monotonically to 100 percent as \( \text{NTU}_1 \) becomes infinite.

Upon examining the behavior of \( \theta_1, \theta_3, T_1 \) and \( T_3 \) for this case, one can see that it corresponds to poor design. The average temperature of fluid 1 is not affected and actually exceeds its inlet value at many points in the heat exchanger. Also it can be seen that very little heat transfer is accomplished after \( X = 0.5 \), indicating a highly oversized exchanger.

This example should help emphasize the fact that while the overall effectiveness, \( E \), is a very useful parameter, a designer should direct his attention to \( \theta_1 \) and \( \theta_3 \) to analyze the actual performance of the exchanger effectively.

The following table will allow the designer to identify quickly the cases for which \( T_2^* (X=0) \) is greater than \( T_{2e\infty}^\text{max} \).
TABLE II

\[ T_{2 \text{ max}} (x=0) = \frac{U + \frac{1}{\Delta t_i}}{U + 1} \]

<table>
<thead>
<tr>
<th>U</th>
<th>( T_{2 \text{ max}} ) ( \Delta t_i = .25 )</th>
<th>( T_{2 \text{ max}} ) ( \Delta t_i = .50 )</th>
<th>( T_{2 \text{ max}} ) ( \Delta t_i = .75 )</th>
<th>( T_{2 \text{ max}} ) ( \Delta t_i = 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3.00</td>
<td>1.667</td>
<td>1.22</td>
<td>1.00</td>
</tr>
<tr>
<td>1.0</td>
<td>2.50</td>
<td>1.50</td>
<td>1.167</td>
<td>1.00</td>
</tr>
<tr>
<td>2.0</td>
<td>2.00</td>
<td>1.33</td>
<td>1.11</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\[ T_{2e} = \frac{K_1/K_3 + \frac{1}{\Delta t_i}}{K_1/K_3 + 1} \]

<table>
<thead>
<tr>
<th>( K_1/K_3 )</th>
<th>( T_{2e} ) ( \Delta t_i = .25 )</th>
<th>( T_{2e} ) ( \Delta t_i = .50 )</th>
<th>( T_{2e} ) ( \Delta t_i = .75 )</th>
<th>( T_{2e} ) ( \Delta t_i = 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.25</td>
<td>3.40</td>
<td>1.80</td>
<td>1.267</td>
<td>1.00</td>
</tr>
<tr>
<td>1.0</td>
<td>2.50</td>
<td>1.50</td>
<td>1.167</td>
<td>1.00</td>
</tr>
<tr>
<td>2.0</td>
<td>2.00</td>
<td>1.33</td>
<td>1.111</td>
<td>1.00</td>
</tr>
<tr>
<td>4.0</td>
<td>1.60</td>
<td>1.20</td>
<td>1.067</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Since \( T_{\text{max}} \) depends only on \( U \) and \( \Delta t_1 \), and \( T_{2e} \) depends only on the ratio \( (K_1/K_3) \) and \( \Delta t_1 \), the above table provides an easy means of checking the possibility of \( E \) being greater than 100% for a given problem. For the case previously discussed, \( K_1 = 2.0 \) and \( K_3 = 0.5 \) therefore, \( K_1/K_3 = 4.0 \). For \( \Delta t_1 = .25 \) the table indicates \( T_{2e} = 1.6 \). Since \( U = 2.0 \), the tables indicate \( T_{\text{max}}(X=0) = 2.0 \). Therefore, the possibility exists for an overall effectiveness greater than 100%, and there will eventually be a decrease in \( E \) for an increase in size.
APPLICATION OF PERFORMANCE CURVES FOR DESIGN

Because of the large number of independent heat exchanger variables, a complete graphical presentation of performance data is not practical. The approach used in this study is to obtain performance data for selected values of the variables $K_1$, $K_3$, $U$, and $\Delta t_i$ which bracket the range of practical interest and to develop interpolation techniques for intermediate values. While the data presented by the curves are limited, considerable insight into the behavior of three fluid, cross flow heat exchangers may be obtained from them.

Three sample problems are solved below to demonstrate the application of the performance curves for design and to illustrate the physical significance of certain trends in the performance data which contributes to an understanding of the behavior of three fluid cross flow heat exchangers.

**Problem 1**

This problem will illustrate the use of the performance curves when no interpolation is required. It is desired to predict the outlet temperatures of three fluids for a heat exchanger operating at the following conditions:

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$\dot{m}$ (lb/hr)</th>
<th>$C_p \text{ BTU/lb}^\circ \text{F}$</th>
<th>$t_i$ ($^\circ \text{F}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>0.5</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>0.5</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>1.0</td>
<td>500</td>
</tr>
</tbody>
</table>
The surface conductances are $u_{12} = 50$ BTU/hr*ft$^2\cdot F$ and $u_{23} = 25$ BTU/hr*ft$^2\cdot F$ while the area, $x_0 y_0$, is 5 ft$^2$. The resulting nondimensional independent variables are

$$K_1 = \frac{\dot{m}_{1} cp_1}{m_2 cp_2} = 0.5, \quad K_3 = \frac{\dot{m}_{3} cp_3}{m_2 cp_2} = 1.0, \quad \Delta t_1 = \frac{t_{11} - t_{21}}{t_{31} - t_{21}} = 0.5,$$

$$U = \frac{u_{12}}{u_{23}} = 2.0, \quad NTU_1 = \frac{u_{12} x_0 y_0}{m_1 cp_1} = 2.0.$$

Referring to Figure 20, the resulting nondimensional dependent variables are:

$$\theta_1 = 48\%, \quad \theta_3 = 30\%, \quad \text{and} \quad E = 51\%.$$

The average outlet temperatures may be obtained in the following manner:

$$\theta_1 = \frac{t_{11} - t_{11}_{em}}{t_{11} - t_{21}} \times 100\%$$

For $\theta_1 = 49\%$, $t_{11}_{em} = 204°F$

$$\theta_3 = \frac{t_{31} - t_{31}_{em}}{t_{11} - t_{21}} \times 100\%$$

For $\theta_3 = 30\%$, $t_{31}_{em} = 380°F$

The average exit value of $t_2$ may be obtained from an energy balance.

$$\dot{m}_1 cp_1 (t_{11} - t_{11}_{em}) + \dot{m}_3 cp_3 (t_{31} - t_{31}_{em}) = \dot{m}_2 cp_2 (t_{21} - t_{21}_{em}),$$

$$125(300 - 204) + 250(500 - 380) = 250(t_{21} - 100)$$

$$t_{21}_{em} = 268°F.$$
It is interesting to note the effect of increasing the size of the heat exchanger to $x_0 y_0 = 10 \text{ ft.}^2$ or $NTU_1 = 4.0$. From Figure 20 it is seen that this change in area results in the following effectiveness factors:

$$\theta_1 = 46\%, \quad \theta_3 = 44\%, \quad \text{and} \quad E = 66\%.$$  

Both $E$ and $\theta_3$ have increased, however $\theta_1$ has decreased by 2%. This decrease occurs because the increased size of the heat exchanger allows the hottest fluid, 3, to heat the center fluid, 2, to a temperature greater than that of fluid 1 in some parts of the heat exchanger which have been added.

The dotted region in the above sketch indicates that part of the heat exchanger in which fluid 2 is hotter than fluid 1. Since fluid 1 is being heated in the dotted region, an increase in size actually
decreased the effectiveness of fluid 1. Fluid 1 is cooled for a smaller value of the coordinate Y. However, as fluid 2 moves through the exchanger, it is heated by both fluids 1 and 3. In the dotted region, heat transferred from the hotter fluid 3 has increased the temperature of fluid 2 above that of fluid 1. This effect is most pronounced for a small $\Delta t_1$ (large $t_{3_1}$), a large capacity rate for fluid 3 (large $K_3$), and a small conductance ratio (large $u_{23}$). All of these factors contribute to a high heat transfer rate from fluid 3 to fluid 2, with the possible result that fluid 1 is reheated in some part of the heat exchanger. In some cases, for example, $K_1 = 1.0$, $K_3 = 1.0$, $\Delta t_1 = 0.25$ (Figures 27-29), the effect is so pronounced that fluid 1 is actually heated and $\theta_1$ becomes negative as NTU increases. The above discussion has assumed an original intent to cool fluids 1 and 3 with fluid 2.

Problem 2

The conditions of this problem are chosen to illustrate how the performance curves may be used when the independent variables are not equal to those chosen for preparing the curves, namely the combinations resulting from the values listed in Table I.

It is desired to determine the temperature effectiveness, $\theta_1$ and $\theta_3$, for a heat exchanger operating under the following conditions:

$$K_1 = 0.40, \quad K_3 = 0.75, \quad U = 1.3, \quad \Delta t_1 = 0.85, \quad \text{NTU}_1 = 2.0$$
Since these values of the independent variables do not correspond to those for which the performance curves have been prepared, some interpolation scheme must be employed to determine temperature effectiveness for this case. A straightforward graphical technique has been used. Figure 39 presents the temperature effectiveness \((\theta_1, \theta_3)\) as a function of the conductance ratio \(U\), for \(NTU_1 = 2.0\) and \(\Delta t_1 = 0.85\). Nine curves are required to cover all possible combinations of \(K_1\) and \(K_3\). The data were obtained from the performance charts using visual interpolation for \(\Delta t_1 = 0.85\). Points were obtained for the three values of \(U\) used in the performance curves, namely, 0.5, 1.0 and 2.0. Figure 39 was used to determine \(\theta_1\) and \(\theta_3\) as functions of \(K_3\) for \(U = 1.3\) and \(K_1 = 0.25, 0.50,\) and 1.0. The results are plotted in Figures 40 a, b, and c. These curves were used to determine \(\theta_1\) and \(\theta_3\) as functions of \(K_1\) for \(U = 1.3\) and \(K_3 = 0.75\). The resulting Figure 40d may be used to determine \(\theta_1\) and \(\theta_3\) for \(K_1 = 0.40, K_3 = 0.75, U = 1.3\), and \(\Delta t_1 = 0.85\). Entering Figure 40d at \(K_1 = 0.40\) yields:

\[
\theta_1 = 58\% \quad \text{and} \quad \theta_3 = 41\%.
\]

The overall effectiveness \(E\), could have been obtained in a similar manner, however, it is easier to return to the basic definition and calculate it.

\[
\overline{Q_a} = K_1(1 - T_{1_{\text{em}}}) + K_3 \left[ \frac{1}{\Delta t_1} \right] - T_{3_{\text{em}}}
\]
or
\[
\bar{Q}_a = K_1 \theta_1 + K_3 \Delta t_i \theta_3 = .40(.58) + .75(.85)(.41) = .492.
\]

Since \( K_1 + K_3 > 1 \),
\[
\bar{Q}_\infty = \left( \frac{1}{K_1 + K_3} \right) \left( K_1 + \frac{K_3}{\Delta t_i} \right) = \frac{1.28}{1.15} = 1.11.
\]

Therefore \( E = \frac{\bar{Q}_a}{\bar{Q}_\infty} = 44\% \).

It is evident from examining Figures 39 and 40 that linear interpolation is not adequate, hence, the need for at least three points on each curve. An additional point is available on curves which present \( \theta_1 \) and \( \theta_3 \) as a function of either \( K_1 \) or \( K_3 \) (Figure 40). Whenever \( K_j \) approaches zero, \( \theta_j \) approaches unity. Physically, this means that as the flow rate of a fluid becomes infinitely small, it can be cooled or heated very easily. It should also be noted that as \( U \) becomes very large, \( \theta_3 \) will approach zero in Figure 39. This trend reflects the physical consequence of \( u_{23} \) approaching zero; that is, if fluid 3 is insulated from fluid 2, it obviously will not change temperature. At first, it may seem that \( \theta_1 \) should approach zero as \( U \) becomes small, however, if \( U \) approaches zero, \( NTU_1 = \frac{x_o y_o u_{12}}{m_1 c_{P_1}} \), could no longer be 2.0 as presumed in the problem. Therefore, it is impossible to consider \( U \) approaching zero for a finite, constant value of \( NTU_1 \).

When solving a problem where conditions do not correspond to those used in the performance curves, a set of figures similar to Figures 39 and 40 must be prepared for every set of values for \( NTU_1 \) and \( \Delta t_i \).
Problem 3

Problems 1 and 2 involved predicting output conditions for a given heat exchanger operating at specific input conditions. Problem 3 is one more frequently encountered by a designer, namely, given inlet conditions and capacity rates of the two outer fluids (1 and 3), determine the size of the exchanger and the mass flow rate of the center fluid (2) required to produce specified outlet conditions for fluids 1 and 3.

Consider fluids 1 and 3 entering the exchanger at the following conditions:

\[ \dot{m}_1 = 250 \text{ lb/hr} \quad c_{p1} = 1.0 \frac{\text{BTU}}{\text{lb} \cdot ^\circ F} \quad t_{i1} = 300^\circ F \]

\[ \dot{m}_3 = 250 \text{ lb/hr} \quad c_{p3} = 0.5 \frac{\text{BTU}}{\text{lb} \cdot ^\circ F} \quad t_{i3} = 500^\circ F \]

Coolant fluid (2) is available at \( t_{a2} = 100^\circ F \) with \( c_{p2} = 0.5 \frac{\text{BTU}}{\text{lb} \cdot ^\circ F} \).

Determine the NTU and \( \dot{m}_2 \) required to cool both fluids to 220°F. These temperature changes correspond to the following values of effectiveness:

\[ \theta_1 = \frac{t_{i1} - t_{e1}}{t_{i1} - t_{a2}} = \frac{300 - 220}{300 - 100} = 40\% \]

and

\[ \theta_3 = \frac{t_{i3} - t_{e3}}{t_{i3} - t_{a2}} = \frac{500 - 220}{500 - 100} = 70\% . \]

Possible solutions may be found by studying the design curves for which \( K_1/K_3 = 2.0 \). This condition is satisfied for two sets of curves; Fig-
ures 13, 14 and 15 for which $K_1 = 0.5$ and $K_3 = 0.25$ and Figures 24, 25 and 26 for which $K_1 = 1.0$ and $K_3 = 0.5$. For each of these sets of curves a table may be prepared to investigate possible solutions.

First the effectiveness, $\theta_1$, is fixed at 40% and the number of transfer units ($NTU_1$) is determined for each value of the conductance ratio, $U$. The values of $\theta_3$ at this value of $NTU_1$ are tabulated. A similar procedure is followed holding $\theta_3$ fixed at 70% and determining $\theta_1$. The object is to find a combination of $U$ and $NTU_1$ for which $\theta_1$ and $\theta_3$ are as close to the desired values as possible. The following table results for this problem:

<table>
<thead>
<tr>
<th>$K_1$ = 1.0</th>
<th>$K_3$ = 0.5</th>
<th>(Figures 24, 25, and 26)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\theta_1}{NTU_1}$</td>
<td>$\theta_3$</td>
<td>$\frac{\theta_3}{NTU_1}$</td>
</tr>
<tr>
<td>0.5</td>
<td>2.5 79</td>
<td>1.0 30</td>
</tr>
<tr>
<td>1.0</td>
<td>2.5 75</td>
<td>1.0 30</td>
</tr>
<tr>
<td>2.0</td>
<td>2.5 66</td>
<td>3.5 43</td>
</tr>
</tbody>
</table>

It can be seen that for $U = 2.0$ a value of $NTU_1$ between 2.5 and 3.5 should be acceptable. From Figure 26 we obtain for $U = 2.0$ and $NTU_1 = 3.0$, $\theta_1 = 42\%$, $\theta_3 = 68\%$ which is certainly close enough for design purposes. For this condition the overall effectiveness, $E$, is 83%.

The required value of $\dot{m}_2$, $c_{p_2}$ is $\dot{m}_2 = \frac{\dot{m}_1}{K_1} c_{p_1} = \frac{250 \text{ BTU}}{\text{hr} \cdot \text{F}}$. Since $c_{p_2} = 0.5 \frac{\text{ BTU}}{\text{lb} \cdot \text{F}}$, the required $\dot{m}_2$ is 500 lb/hr.

It is still necessary to consider the case of $K_1 = 0.5$, $K_3 = 0.25$. Preparing a similar table:
\[ K_1 = 0.5 \quad K_3 = 0.25 \quad \text{(Figures 12, 13 and 14)} \]

<table>
<thead>
<tr>
<th>U</th>
<th>(\frac{\theta_1}{\theta_3} )</th>
<th>(\frac{\theta_3}{\theta_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.85 85</td>
<td>0.40 23</td>
</tr>
<tr>
<td>1.0</td>
<td>0.80 70</td>
<td>0.80 40</td>
</tr>
<tr>
<td>2.0</td>
<td>0.75 45</td>
<td>1.80 57</td>
</tr>
</tbody>
</table>

In this case the values for \( U = 1.0 \) give the desired result exactly for \( \text{NTU}_1 = 0.80 \). From Figure 13 the overall effectiveness, \( E \), is 55%. The required value of \( \dot{m}_2 \) is 1000 lb/hr.

We now have two sets of conditions which satisfy the objectives of the problem:

\[
\begin{align*}
\text{NTU}_1 &= 3.0 \quad \text{NTU}_1 = 0.80 \\
\dot{m}_2 &= 500 \text{ lb/hr} \quad \dot{m}_2 = 1000 \text{ lb/hr} \\
U &= 2.0 \quad U = 1.0 \\
E &= 83\% \quad E = 55\%
\end{align*}
\]

The choice facing the designer is between a large physical size with low flow rate and high effectiveness or small size, larger flow rate and lower effectiveness. The ultimate choice must be based on factors such as construction, cost, space available, volume of coolant fluid available, etc.

Not all problems which are approached in this manner will have an adequate solution. Consider a case which has the same conditions as the previous problem except that \( K_1/K_3 \) has the value 0.50 instead of 2.0. That is, assume the hotter fluid has the higher capacity rate.
Performance curves must now be considered for which \( K_1 = 0.25 \), \( K_3 = 0.50 \) (Figures 6, 7, and 8) and \( K_1 = 0.50 \), \( K_3 = 1.0 \) (Figures 18, 19, and 20). Preparing a chart as before

<table>
<thead>
<tr>
<th>( K_1 = .25 )</th>
<th>( K_3 = .50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 = 40% )</td>
<td>( \theta_3 = 70% )</td>
</tr>
<tr>
<td>( \frac{\theta_1}{NTU_1} )</td>
<td>( \frac{\theta_3}{NTU_3} )</td>
</tr>
<tr>
<td>( 0.5 )</td>
<td>2.0</td>
</tr>
<tr>
<td>.85</td>
<td>53</td>
</tr>
<tr>
<td>.70</td>
<td>67</td>
</tr>
<tr>
<td>2.0</td>
<td>( \theta_1 ) always above ( \theta_3 ) for ( U = 2.0 )</td>
</tr>
</tbody>
</table>

The trends indicate that there will be no satisfactory solution. For \( K_1 = 0.5 \) and \( K_3 = 1.0 \), \( \theta_3 \) never even reaches 70\%. In this case the performance curves indicate the objectives of the problem are impossible under the imposed restrictions.
MULTIPLE PASS THREE FLUID CROSS FLOW HEAT EXCHANGERS

The use of multiple passes has long been recognized as a possible method of improving heat exchanger performance. Previous investigations reported in References 7 and 8, have attacked the problem of multipass two fluid crossflow heat exchangers.

The basic analysis and method of presentation of results used in this study for single pass, three fluid, crossflow heat exchangers will now be extended to multiple passes. The configuration for a two-pass, three fluid crossflow heat exchanger is illustrated in Figure 41. This particular arrangement will be referred to as countercurrent. A cocurrent arrangement is obtained by reversing the indicated direction of fluid 2. The flow detail for each pass is the same as that illustrated in Figure 1.

The solution of the cocurrent case is straightforward in that the outlet temperatures of the first pass become inlet temperatures of the second pass and so on for all passes. The solution in any pass is independent of the solutions of all subsequent passes. The temperature distribution in any pass is found in the same manner as in the single pass exchanger with the exception that the initial temperature will not, in general, be constant along the inlet boundaries after the first pass.

The problem is somewhat more difficult for the countercurrent exchanger. Figure 42 represents the mathematical configuration for this case. The difficulty arises because the initial temperature distributions are not completely known for either pass. Using the terminology of Reference 7, there are "outer" boundaries where the
initial conditions for the whole exchanger are given and "inner" boundaries which are effectively common to the two passes. The inlet temperature for fluid 2 in pass 1 along an "inner" boundary is dependent upon the solution in pass 2. Since it is evident that the solution in pass 2 is dependent in turn on pass 1, some iterative scheme is suggested using assumed distributions along "inner" boundaries.

Following the basic scheme outlined in Reference 7, an inlet value of $T_2$ is assumed for pass 1 and the resulting exit values of $T_1$ and $T_3$ are used as input for pass 2. The exit value of $T_2$ for pass 2 is used then in the second calculation for pass 1. This iterative procedure is continued until the average exit values of $T_2$ for pass 2 for consecutive iterations differ by less than a present convergence criterion. For the calculations in this study, the convergence criterion was agreement to within 1% of the average value of $T_2$. Depending upon the particular case, the number of iterations required for convergence was from three to five.

For more than two passes, values of $T_2$ would have to be assumed on all inner boundaries where needed, the number of such boundaries being one less than the number of passes. Numerical calculations in this investigation were confined to two pass heat exchangers.

For both the cocurrent and countercurrent cases there are several possibilities for the behavior of the fluids in the elbow sections of multiple pass exchangers. A fluid may be completely mixed such that it enters one pass at a constant temperature which is the average of the exit distribution from the other pass. The fluid may be completely unmixed in the elbow and approach the next pass with the identical
temperature distribution with which it left the previous pass. Another possible condition would be no mixing in the elbow with a flow arrangement to invert the fluid prior to entering the next pass. For each fluid we can consider the following possibilities for behavior in the elbow:

1. mixed,
2. unmixed, identical order,
3. unmixed, inverted order.

All possible combinations would give 27 different cases to consider, for each set of input variables $K_1$, $K_3$, and $U$. In order to restrict the presentation of performance data to a reasonable amount and still obtain an insight into the performance of three fluid multipass cross flow heat exchangers, numerical results have been obtained for a countercurrent exchanger under the following conditions: $K_1 = 0.5$, $K_3 = 0.5$, $U = 0.5$, 1.0, 2.0 with all fluids mixed in the elbows. To provide comparisons for some of the other possibilities, the following cases have been considered for $K_1 = 0.5$, $K_3 = 0.5$, $U = 1.0$:

1. Cocomitant mixed,
2. Countercurrent, unmixed identical,
3. Countercurrent, unmixed inverted.

Figures 43 through 48 are the performance curves for the six cases above.

The size parameter, $NTU_1$, for multiple pass cases is NA where $N$ is the number of passes and $A$, as previously defined is $\frac{u_{12}X_Y}{m_1 c_p}$.
In order to compare the performance of two pass countercurrent heat exchangers with single pass exchangers operating at the same conditions, one can compare Figures 15, 16 and 17 with Figures 43, 44, and 45 respectively. Although all of the effectiveness terms are increased in the two pass case, the most significant increases occur in $\Theta_1$, for the smaller values of $\Delta t_1$, particularly 0.25.

Previously, it was noted in the case of the single pass heat exchanger that for small $\Delta t_1$, the possibility existed for fluid 1 to be heated in some parts of the exchanger. This effect was the result of fluid 2 being heated by fluid 3 to a level that exceeded the local temperature of fluid 1. This tendency is decreased by the use of multiple passes in a countercurrent arrangement. Fluid 1 may still be heated in pass 1, but in pass 2, fluid 3 has been cooled sufficiently so that it no longer heats fluid 2 above the level of fluid 1. The results is that fluid 1 is well cooled in pass 2, therefore, a significant increase in $\Theta_1$ is obtained for small $\Delta t_1$. Again the discussion has assumed an intent to cool fluids 1 and 3. Similar reasoning applies to the case of heating the outer fluids.

Figure 46 presents the performance factors for a two pass co- current arrangement for $K_1 = 0.50$, $K_3 = 0.50$, and $U = 1.0$ with mixed flow in the elbows. This configuration is considerably less effective than even the single pass exchanger for similar conditions (Figure 16). The effect of the second pass was to reheat fluids 1 and 3 after they had been cooled in pass 1. The average exit temperature of fluid 2 was actually higher than the exit temperatures of fluids 1 and 3. The effectiveness even decreases with size after an $NU_1$ of 2. These re-
sults for a specific case should not categorically condemn the multiple pass cocurrent arrangement, but they definitely illustrate the potential problem of heating (or recooling) associated with this configuration.

Figure 47 represents the performance factors for a two pass countercurrent arrangement for $K_1 = 0.50$, $K_3 = 0.50$, and $U = 1.0$ for unmixed, identical order flow in the elbows. Figure 48 represents a similar case for inverted flow, and Figure 44 for mixed flow. Inspection of the curves indicates that all performance factors are the highest for inverted order and lowest for identical, with mixed being slightly above identical. The differences are most pronounced for $\theta_i$, when $\Delta t_i = 0.25$. Differences in overall effectiveness are slight, for example for NTU$_1 = 4$, $E_{\text{inv.}} = 78\%$, $E_{\text{mix}} = 76\%$, and $E_{\text{ident}} = 75\%$. However, for $\Delta t_i = .25$, $\theta_{1\text{inv.}} = 45\%$, $\theta_{1\text{mix}} = 39\%$, and $\theta_{1\text{ident}} = 37\%$ for NTU$_1 = 4$. Again, general conclusions may not be drawn from this specific case. It does indicate that the differences between the three possibilities for flow in the elbow are worth considering in design and may significantly affect some of the performance factors of the exchanger.

Figure 49 is an illustration of the convergence of $T_{21}$ to pass 1 for the case of countercurrent flow with inverted order, $K_1 = 0.5$, $K_3 = 0.5$, $U = 1.0$, NTU$_1 = 3$ and $\Delta t_i = 0.25$. Four iterations were required. The initial estimate for $T_{21}$ to pass 1 was automated for the computation of multiple cases. The initial value was assumed to be half of the mixing temperature of fluids 1 and 3 for each case.
Summary and Conclusions

Performance characteristics have been determined for a wide range of operating parameters for single pass, three-fluid, cross flow heat exchangers. Performance of two pass heat exchangers for both concurrent and countercurrent flow has been studied for selected operating conditions. Results have been presented in terms of the temperature effectiveness of the two outer fluids as functions of heat exchanger size for sets of fixed operating conditions.

Because of the infinite possibilities for combinations of operating conditions, selected values have been chosen to bracket the range of practical interest. Interpolation techniques have been used to obtain performance data for intermediate values. Sample problems are included to illustrate the use of the performance curves and the interpolation techniques.

An expression for overall effectiveness has been derived which compares the heat transferred by a particular exchanger to that transferred by one of infinite size. Isolated cases, corresponding to poor design, are cited for which the overall effectiveness may be greater than unity. This indicates the importance of using the temperature effectiveness of the two outer fluids as the primary design variables, and the overall effectiveness as an auxiliary parameter.

While data is necessarily limited to fixed sets of operating conditions, considerable insight into the behavior of three fluid cross flow heat exchangers may be obtained from the performance curves.

A very flexible computer program has been developed for the study
of both single and multiple pass heat exchangers. Output options are available for detailed studies of temperature distributions within a particular exchanger as well as for generation of performance data for a large number of heat exchangers.
REFERENCES


Figure 1. - Schematic representation of a single pass three fluid heat exchanger in cross flow.
Figure 2. - Schematic representation of the heat transfer surface between fluids 1 and 2.
Figure 3 - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .25$, $K_3 = .25$, $U = .5$.
Figure 4.- Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .25$, $K_2 = .25$, $U = 1.0$
Figure 5.- Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .25$, $K_3 = .25$, $U = 2.0$
Figure 6. Effectiveness factors for a single pass three fluid cross flow heat exchanger. $k_1 = 0.25, k_3 = 0.50, U = 0.5$.
Figure 7. - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .25$, $K_2 = .75$, $U = 1.0$.
Figure 8.- Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .25$, $K_3 = .50$, $U = 2.0$. 

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<table>
<thead>
<tr>
<th>$\Delta t_i$</th>
<th>$\theta_1$</th>
<th>$\theta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>$\Delta t_i$</th>
<th>Overall effectiveness, $E$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>.75</td>
<td></td>
</tr>
<tr>
<td>.50</td>
<td></td>
</tr>
<tr>
<td>.25</td>
<td></td>
</tr>
</tbody>
</table>
Figure 9 - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 25$, $K_3 = 1.0$, $U = .5$.
Figure 10. - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .25$, $K_2 = 1.0$, $U = 1.0$
Figure 11. - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .25, K_3 = 1.0, U = 2.0$
Figure 12 - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 0.50, K_3 = 0.25, U = 0.5$.
Figure 13. Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .50, K_3 = .25, U = 1.0$
Figure 14. - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 0.50$, $K_3 = 0.25$, $U = 2.0$
Figure 15. - Effectiveness factors for a single pass-three fluid cross flow heat exchanger. $K_1 = 0.50, K_2 = 0.75, U = 0.5$.
Figure 16. - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 0.50$, $K_3 = 0.50$, $U = 1.0$
Figure 17. - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .50$, $K_3 = .50$, $U = 2.0$
Figure 18. - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = .50$, $K_2 = 1.0$, $U = .5$
Figure 19 - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 0.50, K_3 = 1.0, \Delta U = 1.0$.
Figure 20. - Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 0.50$, $K_3 = 1.0$, $U = 2.0$
Figure 21: Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 1.0$, $K_2 = 0.25$, $U = 5$. 
Figure 22.- Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 1.0, K_3 = 0.25, U = 1.0$
Figure 23.- Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 1.0$, $K_3 = .25$, $U = 2.0$
Figure 24.- Effectiveness factors for a single pass three fluid cross flow heat exchanger. \( K_1 = 1.0, K_3 = 0.50, U = 0.5 \).
Figure 25: Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 1.0$, $K_3 = 0.50$, $U = 1.0$.
Figure 27.- Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 1.0, K_3 = 1.0, U = .5$
Figure 28. Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 1.0$, $K_3 = 1.0$, $U = 1.0$
Figure 29.- Effectiveness factors for a single pass three fluid cross flow heat exchanger. $K_1 = 1.0$, $K_3 = 1.0$, $U = 2.0$
Figure 30: Temperature distributions in two-fluid counter-flow exchangers with finite and infinite heat transfer area.
Figure 31. - Temperature distributions in three-fluid counter-flow exchangers with finite and infinite heat transfer area.
Figure 32.- Effectiveness factors for a simple pass three fluid cross flow heat exchanger. $K_1 = 2.0$, $K_3 = .50$, $U = 2.0$
Figure 33: Temperature distributions as functions of $Y$ for $X = 0$.

$K_1 = 2.0$, $K_3 = 0.5$
$U = 2.0$, $\Delta t_i = .25$
$NTU_1 = 7.5$
Figure 34.- Temperature distributions as functions of $Y$ for $X = .10$. 

$K_1 = 2.0 \quad K_3 = 0.5$

$U = 2.0 \quad \Delta t_i = .25$

$NTU_1 = 7.5$
Figure 35. - Temperature distributions as functions of Y for X = .25.
Figure 36. - Temperature distributions as functions of $Y$ for $X = .50$. 

$K_1 = 2.0 \quad K_3 = 0.5$

$U = 2.0 \quad \Delta t_i = .25$

$NTU_1 = 7.5$
Figure 37. - Temperature distributions as functions of Y for X = 1.0.
Figure 38.- Temperature distributions as functions of $X$ for $Y = 1.0$. 

$K_1 = 2.0 \quad K_3 = 0.5$

$U = 2.0 \quad \Delta t_i = 0.25$

$NTU_1 = 7.5$
Figure 39. Temperature effectiveness as a function of conductance ratio for $NTU_1 = 2.0$, $\Delta t_i = 0.85$. 
Figure 40. - Temperature effectiveness as a function of capacity rate ratio for $u = 1.3$, $NTU_1 = 2.0$, $\Delta t_1 = 0.85$. 
Figure 41. - Schematic representation of a countercurrent two pass three fluid cross flow heat exchanger.
Figure 42. - Mathematical configuration of a countercurrent two pass three fluid cross flow heat exchanger.
Figure 43 - Effectiveness factors for a two pass countercurrent three fluid cross flow heat exchanger with mixed flow in elbows.

\[ K_1 = .50, K_3 = .50, U = .5 \]
Figure 44. - Effectiveness factors for a two pass countercurrent three fluid cross flow heat exchanger with mixed flow in elbows. $K_1 = .50, K_3 = .50, U = 1$
Figure 45. Effectiveness factors for a two pass countercurrent three fluid cross flow heat exchanger with mixed flow in elbows. \(K_1 = 0.50, K_3 = 0.50, \Delta t = 2\)
Figure 46.- Effectiveness factors for a two pass cocurrent three fluid cross flow heat exchanger with mixed flow in elbows. $K_1 = .50, K_3 = .50, U = 1.0.$
Figure 47. Effectiveness factors for a two pass countercurrent three fluid cross flow heat exchanger with identical flow order in elbows. $K_1 = .50$, $K_3 = .50$, $U = 1$
Figure 48. - Effectiveness factors for a two pass countercurrent three fluid cross flow heat exchanger with inverted flow order in elbows. $K_1 = .50$, $K_3 = .50$, $U = 1$
Figure 49. - Convergence of $T_{2_i}$ for pass 1 in a two pass countercurrent heat exchanger with inverted flow in the elbows.
APPENDIX A - Numerical Procedure

Basic Logic

To determine the temperature distributions of each fluid in a three fluid cross flow heat exchanger, the partial differential equations which must be solved simultaneously are:

\[
\frac{\partial T_1}{\partial x} = A(T_2 - T_1) \tag{1}
\]

\[
\frac{\partial T_2}{\partial y} = C(T_1 - T_2) + D(T_3 - T_2) \tag{2}
\]

\[
\frac{\partial T_3}{\partial x} = B(T_2 - T_3) \tag{3}
\]

The region of solution of these non-dimensionalized equations is the portion of the \(X-Y\) plane bounded by \(X = 0, X = 1, Y = 0, Y = 1\).

The boundary conditions are

At \(X = 0\)
\[
T_1 = G(Y) \\
T_3 = H(Y)
\]

At \(Y = 0\)
\[
T_2 = J(X)
\]
The basic logic for solution will be outlined before the detailed numerical scheme is discussed.

(1) One can see that equations (1) and (3) could be integrated in the X-direction if $T_2$ were known along the Y-axis.

(2) To obtain $T_2 (0,Y)$ equation (2) is solved numerically.

(3) Using the initial values of $T_1$ and $T_3$ and the calculated values of $T_2$ at $X = 0$, $T_1$ and $T_3$ may be calculated at $X = \Delta X$ using equations (1) and (3).

(4) Now at $X = \Delta X$ the same situation exists as before, i.e., $T_1$ and $T_3$ are known and $T_2$ is to be calculated from equation (2).

(5) The above procedure is repeated at each increment $\Delta X$ until the solution is obtained over the entire region.

Summarizing with the aid of a sketch

At $X_n$, $T_1$ and $T_3$ are known

At $Y = 0$, $T_2$ is known

(1) Starting at $X = X_n$, $Y = 0$ integrate equations (2), for $T_2$ along $X = X_n$ from $Y = 0$ to $Y = 1$. 
(2) Evaluate $\frac{\partial T_1}{\partial X}$ and $\frac{\partial T_3}{\partial X}$ at $X_n$ from equations (1) and (3).
(3) Calculate $T_1$ and $T_3$ at $X_{n+1} = X_n + \Delta X = (n + 1)(\Delta X)$.
(4) Return to step (1) and repeat for $X = X_{n+1}$.

Integraction Scheme

The numerical technique used is a first order predictor-corrector integration scheme. The solution of equation (2) for $T_2(Y)$ at $X = X_1$ will be used to illustrate the procedure. Assume $T_1$, $T_2$ and $T_3$ are known at $X = X_1$, $Y = Y_1$ and the value of $T_2$ is desired at $Y = Y_1 + \Delta Y$. Recall from the outline of the basic logic that $T_1$ and $T_3$ are known along the line $X = X_1$ from $Y = 0$ to $Y = 1$. Equation (2) is used to evaluate $\left(\frac{\partial T_2}{\partial Y}\right) Y = Y_1$. A predicted value for $T_2$ at $Y = Y_1 + \Delta Y$ is calculated from

$$T^p_2(Y_1 + \Delta Y) = T_2(Y_1) + \Delta Y \left(\frac{\partial T_2}{\partial Y}\right) Y = Y_1 .$$

This value of $T^p_2$ is then used in equation (2) along with $T_1$ and $T_3$ at $Y = Y_1 + \Delta Y$ to evaluate $\left(\frac{\partial T_2}{\partial Y}\right) Y = Y_1 + \Delta Y$. A corrected value of $T_2 = T^c_2$ is calculated from

$$T^c_2 = T_2(Y_1) + \frac{1}{2}\left\{\left(\frac{\partial T_2}{\partial Y}\right) Y = Y_1 + \left(\frac{\partial T_2}{\partial Y}\right) Y = Y_1 + \Delta Y\right\} .$$

This procedure is equivalent to using the average value of the derivative over the interval in place of the value at the beginning of the interval for the forward integration.

A similar procedure is used to solve equations 1 and 3 in the $X$ direction. In the solution of these equations $T_2$ is assumed
constant over the interval $\Delta X$ between $X_1$ and $X_1 + \Delta X_1$ at its value $T_2(X_1)$.

Accuracy Check

The accuracy of the computation may be checked at any $x$ coordinate during the integration by comparing the energy gained (lost) by fluid 2 with that lost (gained) by fluids 1 and 3.

The calculation of the energy balance proceeds in the following manner:

At any station $X_1$, $t_1$ and $t_3$ are averaged from $y = 0$ to $y = y_o$ while $t_2$ is averaged from $x = 0$ to $x = x_1$.

Conservation of energy requires:

$$(\dot{m}_1Cp_1)[t_{11} - t_{1m}(x_1)] + (\dot{m}_3Cp_3)[t_{31} - t_{3m}(x_1)] = (\dot{m}_2Cp_2)(\frac{x_1}{x_o})[t_{2m}(x_1) - t_{2i}]$$

(5)

The coordinates $\frac{x_1}{x_o}$ may be replaced by $X_1$.

Dividing (5) by $(\dot{m}_2Cp_2)$, the resulting equation is:

$$\frac{\dot{m}_1Cp_1}{\dot{m}_2Cp_2}[t_{11} - t_{1m}(X_1)] + \frac{\dot{m}_3Cp_3}{\dot{m}_2Cp_2}[t_{31} - t_{3m}(X_1)] = X_1[t_{2m}(X_1) - t_{2i}]$$

(6)
Since \( \frac{\dot{m}_1Cp_1}{\dot{m}_2Cp_2} = K_1 \) and \( \frac{\dot{m}_2Cp_3}{\dot{m}_2Cp_2} = K_3 \), equation (2) may be written:

\[
K_1[t_{1i} - t_{1m}(X_1)] + K_3[t_{3i} - t_{3m}(X_1)] = X_1[t_{2m}(X_1) - t_{21}]
\]  
(7)

Dividing now by \( t_{1i} - t_{21} \) yields

\[
K_1[T_{1i} - T_{1m}(X_1)] + K_3[T_{3i} - T_{3m}(X_1)] = X_1[T_{2m}(X_1)]
\]  
(8)

Since the boundary conditions are

\[
T_{1i} = 1; \quad T_{3i} = \frac{1}{\Delta t_i}
\]

the accuracy of the overall computation may be checked at any station \( X_1 \) by comparing the quantities

\[
K_1[1 - T_{1m}(X_1)] + K_3[\frac{1}{\Delta t_i} - T_{3m}(X_1)]
\]

and

\[
X_1[T_{2m}(X_1)].
\]

This comparison was used to determine the appropriate step size for the different calculations.

Upon examining the basic equations

\[
\frac{\partial T_1}{\partial x} = A(T_2 - T_1),
\]

(1)

\[
\frac{\partial T_2}{\partial y} = C(T_1 - T_2) + D(T_3 - T_2),
\]

(2)
and
\[
\frac{\partial T_3}{\partial x} = B(T_2 - T_3)
\]  
(3)

one can see the direct influence of A, B, C, and D on the calculation.

For large A, B, C, or D the temperature gradients will be large and a smaller step size will be required to maintain acceptable accuracy. Since any of these constants is a non-dimensional representation of the size of the heat exchanger, it follows that larger exchangers will require more calculational steps.

It was arbitrarily decided that an acceptable limit for accuracy would be that the two overall energy balance terms would not differ from each other by more than 2% of their average value.

Because of the large number of cases which were needed to generate the performance curves, an automatic step size control was used in the computer program. For any calculation, the largest value of the set (A, B, C, D) is denoted as M. The following criteria for step size were established.

<table>
<thead>
<tr>
<th>M</th>
<th>Δx, Δy</th>
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<tbody>
<tr>
<td>M &gt; 20</td>
<td>0.002</td>
</tr>
<tr>
<td>7 ≥ M ≤ 20</td>
<td>0.005</td>
</tr>
<tr>
<td>4 ≤ M &lt; 7</td>
<td>0.01</td>
</tr>
<tr>
<td>2 ≤ M &lt; 4</td>
<td>0.02</td>
</tr>
<tr>
<td>1 ≤ M ≤ 2</td>
<td>0.05</td>
</tr>
<tr>
<td>M &lt; 1</td>
<td>0.10</td>
</tr>
</tbody>
</table>
In all cases this set of criteria was sufficient to insure agreement of the energy balance within 2\% and in the majority of cases the agreement was considerably better.
APPENDIX B - Computer Program

A computer program has been developed to solve the basic differential equations using the numerical procedure described in Appendix A. The program is capable of handling calculations for both single pass and two pass heat exchangers.

Since the program is written in FORTRAN source language, it may be run on the IBM-7094 or UNIVAC 1107/1108 Computers.

The following pages contain a listing of the complete program.
PROGRAM MAIN

DIMENSION T1(1001), T2(1001), T3(1001), XAVE(14), XII(1001), YII(1001),
I1, BCDX(12), BCDY(12), TIP(1001), TIT(10), T1P(301), T3P(301)
COMMON /AB/ A, B, C, D, DT1
REAL K1, K3

KCOU = 0

READ(5,16) (XAVE(KJ), KJ=1,14)
DO 40 JO=1,14
40 XAVE(JO) = XAVE(JO) -.001
IF( XAVE(1) .LT. 0.00001 .AND. XAVE(2) .LT. 0.00001) XAVE(1) = 1.0
IZERO = 0
CALL RESET
CONTINUE

READ(5,995) IPASS, ITYPE, IMIX, IDENT, IPRINT, IALL

CONTINUE
3722 READ(5,16) K1, K3, DTI, A, U
C = A + K1
B = A + (K1/K3) * (1.0/U)
D = (K1/U) * A
WRITE(6,1000)
IF(ITYPE .EQ. 1) WRITE(6,1001)
IF(ITYPE .EQ. 2) WRITE(6,1010) IPASS
IF(ITYPE .EQ. 3) WRITE(6,1011) IPASS
IF(ITYPE .EQ. 4) GO TO 101
IF(IMIX .EQ. 1) WRITE(6,1012)
IF(IMIX .EQ. 1) WRITE(6,1013)
IF(IDENT .EQ. 1) WRITE(6,1014)
IF(IDENT .EQ. 1) WRITE(6,1015)

101 CONTINUE
ASCOD = A
IF(ASCOD .LT. 0) ASCOD = B
IF(ASCOD .LT. 0) ASCOD = C
IF(ASCOD .LT. 0) ASCOD = D
NX = 500
IF(ASCOD .LE. 20.0 .AND. ASCOD .GE. 7.0) NX = 200
IF(ASCOD .LT. 7.0 .AND. ASCOD .GE. 4.0) NX = 100
IF(ASCOD .LT. 4.0 .AND. ASCOD .GE. 2.0) NX = 50
IF(ASCOD .LE. 2.0 .AND. ASCOD .GE. 1.0) NX = 20
IF(ASCOD .LT. 1.0) NX = 10
MY = NX
NN = NY + 1
MP1 = NX + 1
DELX = 1.0 / FLOAT(NX)
DELY = 1.0 / FLOAT(NY)
WRITE(6,1000) A, K1, K3, U, DTI
WRITE(6,1008) B, C, D
WRITE(6,1010) DELX, DELY

306 CONTINUE
T11 = 1.0
T22 = 0.0
T33 = 1.0 / DTI
T2GUES = 0.5*(K1+K3/DTI)/(K1+K3)
WRITE(6,1000) T33
IF(I1YPE .EQ. 3) WRITE(6,28) T2GUES
IF(I1YPE .EQ. 3) T22 = T2GUES
IF(I1PLOT .EQ. 0) GO TO 304
XII(1) = 0.0
YII(1) = 0.0
DO 41 IK=1,MP1
XII(IK+1) = XII(IK) + DELX
YII(IK+1) = YII(IK) + DELY
41 CONTINUE
304 CONTINUE
IAVE = 1
IF( IPASS .EQ. 1) IAVE = 0
WRITE(6,17)
INV = 0
DO 42 I66=1,10
I77 = 1
42 CONTINUE
IF(INV .EQ. 1) GO TO 200
DO 1 I=1,NN
T1(I) = T1
T2P(I) = T22
1 T3(I) = T33
200 X = 0.0
INV = 0
KK = 1
T2SUM = 0.0
IMO = MOD(I77,2)
IF(IPASS .EQ. 1) GO TO 3
WRITE(6,29) I66
IF(IMO .NE. 0) WRITE(6,21)
IF(IMO .EQ. 0) WRITE(6,22)
3 DO 4 J=1,MP1
CALL AB1 (T1(J), T2(J), T3, DELY, NY, T2P(J))
T2P(J) = T2(NN)
T2SUM = T2SUM + T2(NN)
KCOU = KCOU + 1
IF(KCOU .NE. IPRINT) GO TO 202
KCOU = 0
WRITE(6,11) X
WRITE(6,12)
WRITE(6,10) (T1(K),K=1,NN)
WRITE(6,13)
WRITE(6,10) (T2(K),K=1,NN)
WRITE(6,14)
WRITE(6,10) (T3(K),K=1,NN)
202 CONTINUE
IF(XAVE(KK) .LT. 0.1) GO TO 371
IF(XAVE(KK) .LT. 0.1E-03) GO TO 18
IF(XAVE(KK) .LT. X1) GO TO 18
IF(IALL .EQ. 0) GO TO 310
371 CONTINUE
WRITE(6,11) X
WRITE(6,12)
WRITE(6,10) (T1(K), K=1,NN)
WRITE(6,15)
WRITE(6,10) (T2(K), K=1,NN)
WRITE(6,14)
WRITE(6,10) (T3(K), K=1,NN)

310  KK = KK + 1
      T2S = T2SUM / FLOAT(J)
      T1SUM = T1SUM + T1(1K)
      T3SUM = T3SUM + T3(1K)
      T1SUM = T1SUM / FLOAT(NN)
      T3SUM = T3SUM / FLOAT(NN)
      IF (IPILOT .LT. 1) GO TO 302
      CALL QUIKML (-1, 0.0, 1.0, 0.0, 5.0, 1H1, BCDX, BCDY, NN, Y11, T1)
      CALL QUIKML (0, 0.0, 1.0, 0.0, 5.0, 1H2, BCDX, BCDY, NN, Y11, T2)
      CALL QUIKML (0, 0.0, 1.0, 0.0, 5.0, 1H3, BCDX, BCDY, NN, Y11, T3)

302  CONTINUE
      WRITE(6,1021)
      WRITE(6,1020) T1SUM, T2S, T3SUM
      COM = K1 * (1.0 - T1SUM) + K3 * (1.0/DTI - T3SUM)
      XX22 = 0.0
      IF (ITYPE .EQ. 0) XX22 = T2S
      COM1 = X * (T2S - XX22)
      IF (IHO .LT. 0) COM1 = K1 * (T11 - T1SUM) + K3 * (T3S - 1) T3SUM
      ACC = ABS(COM-COM1) / (COM+COM1) * 100.0
      IF (IHO .EQ. 2) WRITE(6,1004) COM, COM1, ACC

18  CONTINUE
      X = X + DELX
      T1P(J) = T1(NN)
      T3P(J) = T3(NN)
      DO 2 I=1,NN
      DEL = DELX * A * (T2(I) - T1(I))
      T1P1 = T1(I) + DEL
      DEL1 = DELX * A * (T2(I) - T1P1)
      T1P1 = T1(I) + DEL
      DEL = DELX * B * (T2(I) - T3(I))
      T3P1 = T3(I) + DEL
      DEL1 = DELX * B * (T2(I) - T1P1)
      2  T3(I) = T3(I) + (DEL+DEL1) * 0.5

4  CONTINUE
      CALL QUIKML (-1.0, 0.0, 1.0, 0.0, 5.0, 1H1, BCDX, BCDY, NN, Y11, T1)
      CALL QUIKML (0.0, 0.0, 1.0, 0.0, 5.0, 1H2, BCDX, BCDY, NN, Y11, T2)
      CALL QUIKML (0.0, 0.0, 1.0, 0.0, 5.0, 1H3, BCDX, BCDY, NN, Y11, T3)
      X = 1.0

311  CONTINUE
      THE1 = 1.0 - T1SUM
      THE2 = DTI + T2S
      THE3 = 1.0 - DTI*T3SUM
      CK = K1 + K3
      GAUE = 0.5 * (COM+COM1)
!HAX = (K1 + K3*UT1) & (1.0 / GX)
IF(GX .LT. 1.0) HMAX = K1 + K3/DT1
E = GAVE / HMAX
WRITE(6,1005)
3000 CONTINUE
IF(IND .NE. 0) T2SSS = T2S
IF(IVAVE .EQ. 0) GO TO 100
I77 = I77 + 1
IF(I77 .GT. 2) GO TO 44
IF(IMIX .NE. 0) GO TO 46
T1I = T1SUM
T2S = 0.0
T3S = T3SUM
IF(ITYPE .EQ. 2) T2S = T2S
GO TO 203
45 CONTINUE
T2S = T2S
T1I = 1.0
T3S = 1.0 / DT1
IF(IMIX .EQ. 1) IINV = 1
IF(IDENT .EQ. 0) GO TO 52
IINV = 1
DO 9 I55 = 1,NN
T1(I55) = T1I
T3(I55) = T3S
THPPP(I55) = T2P(I55)
DO 91 I55 = 1,NN
I551 = NN - I55 + 1
91 T2P(I55) = THPPP(I551)
GO TO 42
46 CONTINUE
IF(IDENT .EQ. 0) GO TO 47
DO 49 I55 = 1,NN
49 THPPP(I55) = T1(I55)
DO 5 I55 = 1,NN
I551 = NN - I55 + 1
5 T1(I55) = THPPP(I551)
DO 6 I55 = 1,NN
THPPP(I55) = T3(I55)
DO 7 I55 = 1,NN
I551 = NN - I55 + 1
7 T3(I55) = THPPP(I551)
DO 6 I55 = 1,NN
8 T2P(I55) = 0.0
T1I = T1SUM
T2S = T2S
T3S = T3SUM
IF(ITYPE .EQ. 2) GO TO 56
GO TO 200
56 DO 57 I55 = 1,NN
I551 = NN - I55 + 1
57 T2P(I55) = T2(I551)
GO TO 200
52 DO 53 KKO = 1,NN
T1(KKO) = T1I
53 T3(KKO) = T33
   IF ( I1IX .LE. 1 ) INV = 1
   GO TO 42
47 CONTINUE
   IF ( I1YPE .EQ. 2 ) GO TO 58
   DO 48 I55=1,NN
   T2P(I55) = 0.0
   GO TO 200
48 DO 59 KKO=1,NN
59 T2P(KKO) = T2(KKO)
   GO TO 200
44 CONTINUE
   IF ( I1YPE .LE. 2 ) GO TO 103
   T1T(I66) = T2S
   IF ( 166 .EQ. 1 ) GO TO 45
   IF ( ABS(T1T(I66-1) - T1T(I66))/T1T(I66-1) .LT. 0.01 ) GO TO 103
   T11 = 1.0
   T22 = T2S
   T33 = 1.0 / DTI
   IF ( I1DENT .EQ. 0 ) GO TO 52
   INV = 1
   DO 102 I55=1,NN
   T1(I55) = 1.0
   T3(I55) = T33
102 T1PPP(I55) = T2P(I55)
   DO 92 I55=1,NN
   I551 = NN - I55 + 1
92 T2P(I55) = T1PPP(I551)
42 CONTINUE
   WRITE(6,23)
103 CONTINUE
   WRITE(6,1022)
   THE10 = 1.0 - T1SUM
   THE30 = 1.0 - DTI * T3SUM
   Q20 = T2SSS
   IF ( I1YPE .EQ. 2 ) Q20 = T2S
   Q130 = K1 * (1.0-T1SUM) + K3 * (1.0/DTI - T3SUM)
   EO = (Q130 + Q20) * 0.5 / QMAX
   ACC = ABS(O20-Q130) / (Q130+Q20) * 100.0
   QAVEO = (Q20+Q130) * 0.5
   WRITE(6,1004) Q130, Q20, ACC
   IF (QX .LT. 1.0) WRITE(6,1003)
   IF (OK .GE. 1.0) WRITE(6,1002)
   AA2 = A + A
   WRITE(6,1024) QAVEO, QMAX, AA2, EO, THE10, THE30
   CALL TIME (ITIME)
   IZERO = ITIME - IZERO
   WRITE(6,1029) IZERO, ITIME
   IZERO = ITIME
1029 FORMA(1,10X,15HTIME FOR THIS CASE,15,13H MICROSECONDS, //
   1 11X,11HTOTAL TIME,16,13H MICROSECONDS )
   WRITE(6,1005)
   CALL DMPBUF
   GO TO 100
10 FORMAT (6,14.7)
11 FORMAT(1, I , 5X, 4H X = ,F5.3)
12 FORMAT(1, I , 5X, 16H T1(I) = , I = 1,N )
13 FORMAT(1, I , 5X, 16H T2(I) = , I = 1,N )
14 FORMAT(1, I , 5X, 16H T3(I) = , I = 1,N )
15 FORMAT(12E14. 6I1)
16 FORMAT(1F10.2)
17 FORMAT(1, I , 55X, 14H OUTPUT )
20 FORMAT(1H0, 4D0, 26H THE AVERAGE VALUE OF T1 = ,F8.3, / 41X,
1 26H THE AVERAGE VALUE OF T2 = ,F8.3, / 41X,
2 26H THE AVERAGE VALUE OF T3 = ,F8.3 )
21 FORMAT(1H0, 52X, 8H OCHAN I )
22 FORMAT(1H0, 52X, 8H OCHAN II )
23 FORMAT(1H0, 52X, 8H OCHAN III )
20 FORMAT(1H0, 46X, 24H T GUESS FOR OCHAN I = ,F5.2)
29 FORMAT(1H0, 56X, 10I, 16H ITERATION , I2 )
55 FORMAT(12A6/12A6)
955 FORMAT(7211 )
1000 FORMAT(1H1 , 58X, 13H INPUT )
1001 FORMAT(1H0, 59X, 11H SINGLE PASS )
1002 FORMAT(1H0, 60X, 10H T > 1.0 )
1003 FORMAT(1H0, 60X, 10H T < 1.0 )
1004 FORMAT(1H0, 35X, 4H HPOR Acuracy Check on Energy Balance Compare//
1 29X, 6H 013 = ,F8.3,13H WITH 02 = ,F8.3,5X,3HOR ,F7.4,
2 16H PER CENT ACCURACY )
1005 FORMAT(1H0, 128(1H0 )
1007 FORMAT(1H0, 40X, 23H Boundary Conditions )
1 43X ,
1 9HT1 = 1.00, 5X, 8HT2 = 0.0, 5X, 5HT3 = ,F4.2)
1008 FORMAT(1H0, 59X, 12H INPUT VALUES / 30X, 4HA = ,F4.2, 5X, 5HK1 = ,
1 F4.2, 5X, 5HK2 = ,F4.2, 5X, 4HU = ,F4.2, 5X, 11H DELTA T1 = ,F4.2)
1009 FORMAT(1H0, 40X, 30H THE RESULTING VALUES OF B, C, AND D ARE /
1 43X, 4HB = ,F5.2, 5X, 4HC = ,F5.2, 5X, 4HD = ,F5.2)
1010 FORMAT(1H0, 51X, 15H MULTIPLE PASS (12, 18H PARALLEL FLOW)
1011 FORMAT(1H0, 51X, 15H MULTIPLE PASS (12, 17H COUNTER FLOW)
1012 FORMAT(1H0, 63X, 5H MIXED)
1013 FORMAT(1H0, 61X, 5H MIXED)
1014 FORMAT(1H0, 57X, 5H MIXED)
1015 FORMAT(1H0, 57X, 5H MIXED)
1016 FORMAT(1H0, 50X, 10H DELTA X = ,F5.3, 51X, 10H DELTA Y = ,F5.3)
1017 FORMAT(1H0, 40X, 4H DATA IS PRINTED OUT AT EACH CALCULATED POINT. )
1018 FORMAT(1H0, 40X, 4H DATA IS PRINTED OUT AT THE FOLLOWING VALUES OF X
1 1)
1019 FORMAT( 51X, 2XH ,12, 4N ) = ,F5.3)
1020 FORMAT(1H0, 40X, 4H SAVE = ,F8.3, 10X, 4H MAX = ,F8.3 // 50X,
1 4HA = ,F8.3, 63X, 4HE = ,F8.3, 10X, 4HNTHETA1 = ,F8.3, 10X
2 9NTHETA3 = ,F8.3)
1021 FORMAT(1H0, 34X, 3H THE AVERAGE EXIT TEMPERATURES ARE )
1022 FORMAT(1H0, 41X, 32H THE OVERALL OUTPUT VARIABLES ARE)
1023 FORMAT(1H0, 35X, 44H PESR THE Acuracy Check on Energy Balance Compare
1 20X, 7H DIO = ,F8.3, 14N WITH 020 = ,F8.3, 5X, 3HOR ,F7.4,
2 16H PER CENT ACCURACY )
1024 FORMAT(1H0, 39X, 6H AVE = ,F8.3, 10X, 4H MAX = ,F8.3 // 48X,
17H T1U = ,F8.3//24X, 5DE = ,F8.3, 10X, 10HTHETA10 = ,F8.3, 10X,
2 10NTHETA30 = ,F8.3)
END
SUBROUTINE AB1 (T1, T2, T3, DELY, NX, T26)
COMMON /AB/ A, B, C, D, DT1
DIMENSION T1(1001), T2(1001), T3(1001)
T2(1) = T26
DO 1 N=1,NX
  DEL = DELY * (C*T1(N)) + D*T3(N) = (C*D) * T2(N)
  TEMPT2 = T2(N) + DEL
  DEL1 = DELY * (C*T1(N+1)) + D*T3(N+1) = (C*D) * TEMPT2
  T2(N+1) = T2(N) + (DEL+DEL1) * 0.5
1 CONTINUE
RETURN
END