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A Complex Phase Shift Analysis of
Elastically-Scattered Protons from Carbon

by

Jerry B. Swint

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I. INTRODUCTION

The elastic scattering of protons by C^{12} has both experimental and theoretical interest and has been extensively studied. The available experimental data below 13 MeV has been previously summarized. These previous data are, in general, of poor energy resolution and have comparatively poor accuracy. In the present experiment precise measurements of the absolute differential cross section for the elastic scattering of protons by C^{12}, and for some reactions which occur, were made to enable the deduction of the scattering phase shifts as a function of energy and to provide information of the nuclear structure of N^{13}.

In quantum mechanics the phase shifts of the partial waves completely determine the scattering. In the case of carbon the nuclear radius equals approximately 4.8 fermi. For proton energies of the order of 10 MeV the de Broglie wavelength $\lambda \approx 1.5$ fermi. Therefore, partial waves through $l = 3$ are expected to take part in the nuclear scattering. In the C^{12} + p scattering process, reactions may occur as low as 5 MeV which result in the phase shifts' becoming complex and the number of parameters' being approximately doubled. A complete analysis of the C^{12} + p scattering experiment has been made by finding the complex phase shifts as a function of energy and then relating them to the eigenstates of the compound nucleus N^{13}.
Elastic- and inelastic-scattering cross sections were measured at fixed angles with the energy increasing in small steps, so that these data may be referred to as "cross sections as a continuous function of energy". Elastic-scattering and reaction angular distributions at nineteen energies were also measured. These data will be referred to as "cross sections at discrete energies". The complex phase shifts which fit the elastic-scattering angular distributions and the total inelastic cross sections at the discrete energies were found. At some of the energies, polarization data were included among the data to be parameterized by the phase shifts. Smooth curves were drawn through the phase shifts at the "off-resonance" discrete energies and were used to approximate the continuous energy-dependence of the slowly-varying part of the collision matrix diagonal element. The part of the matrix element which gives sharp resonances was approximated by a single-level form, except in one case where the overlap of states of the same spin and parity was very strong. It was found that up to $E_p = 8.5\text{ MeV}$ a reasonable fit could be obtained, as a continuous function of energy, to the elastic differential cross sections, the total reaction cross sections, and the polarization at $\Theta_{\text{lab}} = 50^\circ$. Above $E_p = 8.5\text{ MeV}$ the inclusion of only one of the states near $E_p = 9.14\text{ MeV}$ caused the calculated results to differ significantly from the experimental data.
When the complex structure of the 9.14 MeV anomaly was first noticed, it was presumed that two levels were involved, the $J^\pi = 7/2^-$ state previously associated with this anomaly and the $J^\pi = 9/2^+$ state predicted by shell model calculations but not seen experimentally. Extensive efforts were made to fit the data with these assumptions, and other positive parity assignments to the second state, but without success. Recently Terrell and Bernstein\(^2\) showed that the second state had $J^\pi = 5/2^-$. The complex phase shift analysis over the 9.14 MeV anomaly has confirmed a $5/2^-$, $7/2^-$ assignment. These assignments have reproduced the features of the elastic-scattering cross sections as a function of energy over the doublet. Since the weaker state does not appear in conventional p-shell calculations\(^3\), it is presumed due to two-nucleon excitation into higher shells.
II. MEASUREMENT OF THE EXPERIMENTAL DATA

A type EN tandem Van de Graaff accelerator was used to accelerate protons which entered a differentially-pumped gas scattering chamber constructed and described in detail by Jones\textsuperscript{4).} The beam energy value was known to better than 0.1\%\textsuperscript{5).} The gaseous target was 99\% pure methane. The scattering chamber has four detector ports, and during this experiment ten detectors (eight silicon surface barrier detectors, one silicon diffused junction detector, and a CsI(Tl) scintillator) were used at various times. The detected particles were analyzed by a Technical Measurement Corporation (TMC) 400-channel pulse height analyzer.

A. Scattering Chamber\textsuperscript{4)}

The precision small-volume gas scattering chamber was constructed primarily in order to provide an accurate and conveniently-operable instrument for the study of scattering and reactions from gaseous targets. The chamber consists mainly of three cylindrical pieces: the center piece, held fixed in space; the upper and lower pieces, free to rotate about an axis perpendicular to the incident beam. The center piece contains four ports: an entrance port; a Faraday cup port; a gas handling port; and a viewing port. The upper and lower pieces each contain two detector ports 90° apart in a plane perpendicular to their axis of rotation. The detector ports house the rectangular slit systems, and the detectors are mounted externally. The upper piece contains a mid-range
(27° to 153°) and a forward (20° to 27°) detector; the lower piece has a mid-range and a backward (153° to 160°) detector. The limitation on the angular range of the detectors results from the changing relative orientation of the beam and the slit long axis. The angular measurements are made by two precision transit azimuth circles. The separation of the accelerator vacuum system and the experimental target is accomplished by an entrance foil whenever a rare gaseous target is used, and a differential pumping system whenever an abundant gaseous target such as methane is used. Cross section measurements are estimated to be in errors of no greater than 0.1% due to geometrical inaccuracies in the scattering chamber.

B. Counter Slit System

The product of the solid angle and the target length is given by the G and f factors. The G factor is defined by the dimensions of the rectangular detector slit system: width of front slit; width and height of rear slit; distance between front and rear slits; and distance from rear slit to center of scattering chamber. The measurements of the dimensions were made with a traveling microscope and micrometer. The RMS error in G resulting from the inaccuracies of the above measurements was estimated to be less than 0.3%. The angle-dependent factor f which accounts for the change in target length was calculated by Harris with a stated accuracy of 0.1%. Harris tabulated the four detector laboratory angles.
and corresponding f factors for 0.1° intervals of the upper
and lower azimuth circle readings from 1° to 359°.

G. Detectors.

The eight detectors used between May 1962 to August
1963 have been previously discussed1). Only two additional
detectors have been used since August 1963, both being used
at the forward detector port. Both detectors were Nuclear
Diode silicon surface barrier types (#309 and #802) with maxi-
mum stopping energies of 12 MeV protons and sensitive areas of
20 mm². Tennelec preamplifiers (model 100B) were used with the
above two detectors with external and internal bias voltage
supplies.

D. Scattering Centers.

The number of target nuclei per cm³ was determined by
measuring the temperature and pressure of the gas. The tem-
perature was measured by inserting a thermometer in a hole in
the scattering chamber and assuming that the gaseous target
was in thermal equilibrium with the aluminum chamber. The
chamber pressure was determined by means of a manometer filled
with butyl phthalate pump oil. The oil level difference was
measured with a cathetometer. The error in the temperature
and pressure measurements is estimated to be +0.8% and +0.1%,
respectively.

The molecular composition of the gas was known from a
mass spectrometric analysis performed by Petroleum Analytical
Research Corporation, Houston, Texas, so that various effective
pressures could be obtained. For instance, if scattering from $^{12}C$ was resolved from scattering from all other nuclides, the effective pressure used was for $^{12}C$ alone. If scattering from $^{13}C$ was unresolved, the effective pressure was higher by the factor (total carbon)/$^{12}C$, and so on. The length of the target volume was known from the chamber geometry so that the number of scattering centers per cm$^2$ of target was obtained. The target thickness depends on the gas temperature and pressure, beam energy, and angle of observation. For 5 MeV protons, at the chamber pressure used, the target thicknesses at various detector ports were 1.3 keV at 30°, 6.8 keV at 159°, 0.9 keV at 90°, and 1.3 keV at 26°.

E. Incident Proton Flux

The total number of incident protons was determined from the integrated charge $q$ which was measured by a current integrator. The capacitor of the current integrator is charged to a known voltage $V_1$, then discharged to a second known voltage $V_2$ by the positive current of the incident protons. When the second known voltage $V_2$ is reached, the data-taking apparatus is remotely stopped. The integrated charge is given by $q = C(V_1 - V_2)$, where $C$ is the capacitance of the capacitor. The capacitance is determined by measuring the time required to discharge the capacitor through a 10 megohm resistor from $V_1$ to a final known voltage $V_f$. During this experiment the maximum deviation of the capacitance measurements was ±0.4%. The error in the charging voltage is estimated to be less than
0.1%. Therefore, the estimated accuracy of the number of incident protons is ±0.5%.

F. Beam Energy Determination

The energy loss of protons in traversing the differential pumping system and first half of the scattering chamber cannot be accurately calculated since the pressure distribution along the proton path is not well known. An empirical formula obtained by Jones gave the energy loss at proton energy \( E_p = 5 \text{ MeV} \) as 22 keV for the chamber pressure used. Extreme assumptions concerning the pressure distribution in the system change the energy loss by ±10 keV, so that ±30% seems a reasonable estimate of the accuracy of the energy loss. At \( E_p = 5 \text{ MeV} \) the beam energy uncertainty arising from the uncertainty in the 90° bending magnet calibration is about ±5 keV. These two systematic errors compound to approximately ±12 keV. There is also a random error in the beam energy due to possible different paths through the 90° magnet with different lens and deflector settings. Since the energy of sharp resonances was usually repeatable to within a few keV, this is estimated as the random energy uncertainty. The energy spread of the beam is also estimated as a few keV. At higher energies the beam energy uncertainty increases, but the energy loss before the target decreases, leaving the total systematic uncertainty approximately constant. Thus this uncertainty should be less than ±15 keV at all energies used in this experiment. The peak cross section of the 4.8 MeV anomaly in the elastic
scattering cross sections appeared at 4.803 MeV at \( \Theta = 161.14^\circ \) and 4.809 MeV at \( \Theta_{c.m.} = 140.76^\circ \) in this experiment. An absolute measurement\(^7\) at 180\(^\circ\) has placed the peak at \((4.806 \pm 0.005)\) MeV.

G. Analysis of Detected Particles

Output signals from the detector preamplifiers were fed into a TMC 400-channel pulse height analyzer. During the excitation function measurements the 400-channel analyzer was used in the 4 x 100 channel mode. When the angular distributions were measured, the analyzer was operated in the 2 x 200 channel mode to obtain better energy resolution. Two of the detector outputs were also fed through linear amplifiers, five-channel integral pulse height analyzers (discriminators), and fast scalers, giving two five-channel integral bias curves. Reliable counting rates over a particular part of the spectrum were obtained from the integral bias curves since the dead time correction of this system was of the order of a few microseconds per event, making the counting losses small at the counting rates used. The five-channel integral discriminators were adjusted at 35, 40, 45, 50, and 55 volts. To maintain an acceptable bias curve (sharp drop in last channel) the linear amplifier gains were adjusted occasionally during excitation functions and for each data point during angular distributions. The dead time of the TMC 400-channel analyzer was 40 to 120 microseconds per pulse; hence, an appreciable counting loss resulted. This counting loss made it necessary
to make a correction to the number of counts recorded by the analyzer. This correction was made from the comparison of counting rates from the 100-channel spectrum and the five-channel integral bias curves. The number of counts above the 55 volt discriminator was compared with the 100-channel spectrum to determine the channel number corresponding to the 55 volt level. Then the corresponding channel numbers of 50, 45, 40, and 35 volt levels were determined from the location of the 55 volt level. The comparison of counts in a certain group of channels in the spectrum and corresponding voltage levels of the bias curve gave a reliable correction for the counting losses. The corrections for the two detectors were averaged unless one had an unreasonable value. In nearly all cases the corrections for the two detectors agreed quite well.

The largest counting loss correction made on the excitation function and angular distribution data was 7.7% and 3.3%, respectively, while the average corrections were 2.3% and 1.3%, respectively.

The 100- and 200-channel spectra were examined to identify those proton groups due to scattering from $C^{12}$, as well as groups due to scattering from the impurities, to determine the upper and lower channel limits on the groups of interest, and to make background corrections.

In the reduction of the elastic data, small corrections were made for the elastic scattering from the $C^{13}$, $N^{14}$, and $O^{16}$ impurities. The elastically-scattered protons from the
three impurities are close in energy to protons elastically-scattered from C\textsuperscript{12}. When the impurity peaks were unresolved, it was assumed for the purpose of making these small corrections that the proton scattering cross sections of the impurities and C\textsuperscript{12} were equal. This assumption should introduce negligible errors into the cross sections. At the more backward angles some of the impurity peaks were well resolved and could be ignored in the data reduction.

The groups from inelastic scattering were summed between upper and lower channel limits chosen in a consistent manner. Since there were no impurity groups close in energy to the inelastic groups, the only correction necessary was for the "flat" spectral background. For inelastic scattering to the first excited state, the number of counts per channel above and below the peak of interest was typically less than 1\% of the peak counts, so that any change in the choice of upper and lower limits would have had only a small effect on the total number of counts. Since the second inelastic group was located on the upper side of low energy noise, a plot of counts/channel versus channel number for each spectrum was drawn in order to obtain the best corrected peak count.
III. EXPERIMENTAL RESULTS

A. Cross Sections as a Continuous Function of Energy

Absolute differential cross sections for elastic scattering of protons by $^{12}\text{C}$ and for the inelastic scattering reaction $^{12}\text{C}(p,p')^{12}\text{C}^*\ (Q = -4.43 \text{ MeV})$ were measured at six angles in the energy ranges $4.7$ to $12.8 \text{ MeV}$ and $6.0$ to $12.8 \text{ MeV}$, respectively. The inelastic scattering reaction $^{12}\text{C}(p,p'')^{12}\text{C}^*\ (Q = -7.66 \text{ MeV})$ was observed at five angles from $10.2$ to $12.8 \text{ MeV}$. Differential cross sections for the reaction $^{12}\text{C}(p,\alpha)^9\text{B}\ (Q = -7.56 \text{ MeV})$ were measured at $\theta_{\text{lab}} = 85.22^\circ$ from $11.8$ to $12.8 \text{ MeV}$. The data at $\theta_{\text{lab}} = 85.22^\circ$, $121.16^\circ$, and $159.45^\circ$ were taken at one time and the data at $\theta_{\text{lab}} = 25.54^\circ$, $105.23^\circ$, and $137.52^\circ$ at a later time. From the close agreement between the positions of the $4.8 \text{ MeV}$ anomaly in the elastic-scattering cross sections at $\theta_{\text{c.m.}} = 161.14^\circ$ and $140.76^\circ$, the two energy scales are in satisfactory agreement. The existence of the $9.14 \text{ MeV}$ double anomaly was confirmed by repeating the measurements of the elastic- and inelastic-scattering cross sections at $\theta_{\text{c.m.}} = 90^\circ$ and $140.76^\circ$ from $9.11$ to $9.22 \text{ MeV}$ (fig. 5). The elastic-scattering and reaction cross sections as a continuous function of energy are shown in figs. 1-5. The energy steps were generally $6$ to $9 \text{ keV}$, except that over narrow resonances $2.5$ to $4 \text{ keV}$ steps were taken, and above $12.3 \text{ MeV}$ approximately $20 \text{ keV}$ steps were taken. Counting statistics, which are the largest source of error, and overall RMS errors of the excitation functions (figs. 1-3) are given in table 1.
Figures 1a-d: Excitation functions for $p + C^{12}$ elastic scattering. The dots represent the experimental data points, and the dashed lines are cross sections for scattering from a hard charged sphere of radius 4.8 fm. The experimental points represented by triangular points (fig. 1a) at 161.14° are plotted multiplied by one-fifth. The angular distribution points are plotted as + (ref. 7), O (ref. 8), and X (present work). A small correction should be made to the energy scale: the points are plotted at energies too low by 0.25%.
Figure 1a
Figure 10

\[ C^{12}(p,p)C^{12} \]

\[ \theta_{OM} = 27.61^\circ \]

\[ \theta_{OM} = 109.88^\circ \]

\[ \theta_{OM} = 140.76^\circ \]
Figure 2: Excitation functions for the inelastic scattering reaction $^{12}_C(p,p')^{12*}_C$ ($Q = -4.43$ MeV). The dots represent the experimental points. Only every second point was plotted except over narrow resonances. Angular distribution points of the present work are plotted as x. A small correction should be made to the energy scale: the points are plotted at energies too low by 0.25%. The top three excitation functions continue uneventfully to 12.8 MeV.
Figure 3: Excitation functions for inelastic-scattering reaction $\text{Cl}^{12}(p,p')\text{Cl}^{12*}$ ($Q = -7.66 \text{ MeV}$). The dots represent the experimental points. The energy scale is correctly shown.
Figure 3
Table 1

Errors in the Excitation Functions

<table>
<thead>
<tr>
<th>Laboratory Angle</th>
<th>Average counting statistics (± %)</th>
<th>RMS overall error (± %)</th>
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<tr>
<td></td>
<td>elastic</td>
<td>first inelastic</td>
</tr>
<tr>
<td>25.54°</td>
<td>0.9(1.4)</td>
<td>3.6(7.9)</td>
</tr>
<tr>
<td>85.22°</td>
<td>1.8(4.0)</td>
<td>3.7(6.3)</td>
</tr>
<tr>
<td>105.23°</td>
<td>1.6(3.0)</td>
<td>3.3(6.3)</td>
</tr>
<tr>
<td>121.16°</td>
<td>1.8(4.1)</td>
<td>2.9(8.3)</td>
</tr>
<tr>
<td>137.52°</td>
<td>1.2(4.1)</td>
<td>2.7(5.0)</td>
</tr>
<tr>
<td>159.45°</td>
<td>0.8(5.8)</td>
<td>1.6(5.3)</td>
</tr>
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</table>

Extreme values of the counting statistics are given in the parentheses.
Figure 4: Absolute differential cross sections for the reaction $^{12}\text{C}(p,\alpha)\text{B}^9$ ($Q = -7.56$ MeV) as a function of energy. The dots represent the experimental points. Errors indicate statistical uncertainties only.
Figure 5: Excitation functions for p + C^{12} elastic scattering and for inelastic-scattering reaction C^{12}(p,p')C^{12*} (Q = -4.43 MeV). The dots represent the experimental points. The arrow indicates the energy position (9.15 MeV) of the first angular distribution taken at the 9.14 MeV anomaly.
$^{12}_C(p, p')^{12}_C$

$\theta_{cm} = 90^\circ$

$\theta_{cm} = 140.76^\circ$

$^{12}_C(p, p')^{12}_C$

$\theta_{LAB} = 85.22^\circ$

$\theta_{LAB} = 137.52^\circ$

**Figure 5**

**Differential Cross Section - mb/sr (cm)**

**Incident Proton Energy - MeV (LAB)**
The average value of the counting statistics for the \( \text{C}^{12}(p, \alpha_0)\text{B}^9 \) reaction was 4.5% with an extreme value of 7%. Contributions to the RMS error from sources other than statistics have been previously tabulated\(^1\). Experimental points from the angular distributions of Reich et al.\(^7\), Moss and Haeberli\(^8\), and the present work are plotted in figs. 1 (a-d) to demonstrate the consistency of the measurements.

B. Cross Sections as a Function of Angle

Absolute differential cross sections as a function of angle for the elastic scattering of protons by \( \text{C}^{12} \), and for the reactions which occur, were measured at nineteen incident proton energies: "off-resonance" energies 2.39, 2.97, 3.97, 4.99, 5.66, 6.18, 6.65, 6.77, 8.07, 8.39, 8.96, 9.39, 9.81, and 11.6 MeV and "on-resonance" energies 8.20, 9.10, 9.13, 9.15, and 9.16 MeV. The differential cross sections at the above discrete energies for the elastically-scattered protons are shown in figs. 6-7. The cross sections for the inelastic scattering reaction \( \text{C}^{12}(p,p')\text{C}^{12*} \) \((Q = -4.43 \text{ MeV})\) were measured at the thirteen highest energies, with the exception of 6.77 MeV, and are shown in figs. 7-8. The worst counting statistics were \( \pm 1\% \) for the elastic angular distributions and \( \pm 6\% \) for the inelastic angular distributions so that the overall accuracy is estimated as \( \pm 4.4\% \) for the elastic and \( \pm 6.3\% \) for the inelastic cross sections.

When the \( E = 11.60 \text{ MeV} \) angular distribution was being measured, some cross sections for the inelastic scattering
reaction \( ^{12}\text{C}(p,p')^{12}\text{C}^\ast \) \( (Q = -7.66 \text{ MeV}) \) and the \( ^{12}\text{C}(p,\alpha_0)^9\text{B} \) reaction were obtained and are shown in fig. 9. The inelastic-scattering angular distributions have been fitted to a series of Legendre polynomials, the necessary coefficients being shown in table 2. The fits were carried out for 5, 7, and 9 polynomials. The number of polynomials was judged adequate if adding two more improved the fit only marginally. The Legendre series taken to represent the data are plotted as solid lines in figs. 7-9. The total cross section was evaluated from \( \sigma_T = 4\pi A_0 \), where \( A_0 \) is the coefficient of the zero-order polynomial. Except at \( E_p = 11.60 \text{ MeV} \), this was also taken to be the total reaction cross section, since no other reaction groups were observed. At 11.60 MeV the inelastic scattering \( (Q = -7.66 \text{ MeV}) \) total cross section and an estimate of the \( ^{12}\text{C}(p,\alpha_0)^9\text{B} \) total cross section were included in the total reaction cross section. The total cross sections at the discrete energies are given in table 2.

The total cross sections for inelastic scattering were estimated as a continuous function of energy in the range \( 6.7 \leq E_p \leq 11.5 \text{ MeV} \) for \( Q = -4.43 \text{ MeV} \) and \( 10.2 \leq E_p \leq 11.1 \text{ MeV} \) for \( Q = -7.66 \text{ MeV} \), using the data shown in figs. 2 and 3, respectively. The results are shown in fig. 10, together with the total cross sections derived from the angular distribution data at the discrete energies of the present work.
Figures 6a-d: Absolute differential cross sections for the elastic scattering of protons by $^{12}$C as a function of angle at the energies shown. The dots are the experimental data, and the solid and dashed curves were calculated from the derived phase shifts (table 3). Note that the cross sections have been displaced by the amount given in the parentheses.
Figure 6a
$C^{12}(p,p)C^{12}$

**Figure 6b**

- Differential cross section (b/sr)
- Centre-of-mass angle (degrees)
- Data points for different energies:
  - 6.77 MeV (+0.3 b/sr)
  - 6.65 MeV (+0.2 b/sr)
  - 6.18 MeV (+0.1 b/sr)
  - 5.66 MeV
$^{12}\text{C}(p, p)^{12}\text{C}$

**Figure 6e**

- $11.60 \text{ MeV} \pm 0.6 \text{ b/sr}$
- $9.81 \text{ MeV} \pm 0.5 \text{ b/sr}$
- $9.39 \text{ MeV} \pm 0.3 \text{ b/sr}$
- $8.96 \text{ MeV} \pm 0.2 \text{ b/sr}$
- $8.39 \text{ MeV} \pm 0.1 \text{ b/sr}$

**Differential Cross Section (b/sr)**

**Centre-of-Mass Angle (degrees)**
Figure 6d
Figure 7: Absolute differential cross sections for the elastic scattering of protons by $^{12}\text{C}$ and the inelastic scattering reaction $^{12}\text{C}(p,p')^{12}\text{C}^*$ ($Q = -4.43 \text{ MeV}$) as a function of angle at 8.20 MeV are shown. The dots are the experimental data. The dashed curve was calculated from the derived phase shifts in table 3, and the solid curve was calculated from the Legendre series in table 2. Note that the cross sections have been displaced by the amount given in the parentheses.
$E_p = 8.20$ MeV

$C_{12}^{12}(p,p) C_{12}^{12}$

$C_{12}^{12}(p,p') C_{12}^{12*}$

$x10 \text{ b/sr}$

Figure 7
Figures 8a-b: Absolute differential cross sections for the inelastic-scattering reaction $^{12}\text{Cl}(p,p')^{12}\text{Cl}^*$ $(Q = -4.43 \text{ MeV})$, as a function of angle at the energies shown. The dots are the experimental data, and the solid curves were calculated from the Legendre series in table 2. Note that the cross sections have been displaced by the amount given in the parentheses.
$C^{12}(p,p')C^{12*}$

Differential Cross Section (b/sr)

Centre-of-Mass Angle (degrees)

11.60 MeV
(+0.05 b/sr)

9.81 MeV
(+0.04 b/sr)

9.39 MeV
(+0.03 b/sr)

8.96 MeV
(+0.02 b/sr)

8.39 MeV
(+0.01 b/sr)

8.07 MeV

6.65 MeV

Figure 8a
Figure 8b
Figure 9: Absolute differential cross sections as a function of angle for the $^{12}_{C}(p,p'')^{12}_{C}$ ($Q = -7.66$ MeV) and $^{12}_{C}(p,\alpha)_{B^9}$ reactions at $E_p = 11.60$ MeV. The dots are the experimental data, and the crosses are repeated points. Errors indicate statistical uncertainties only. The solid curve was calculated from the Legendre series in table 2.
Figure 9
Table 2
Coefficients of Legendre Series Which Represent the Reaction Data \((p,p')\), \((p,p'')\), and the Resulting Total Reaction Cross Sections.

\[ C^{12}(p,p')C^{12}^n \]

<table>
<thead>
<tr>
<th>( E_p ) (MeV)</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
<th>( a_6 )</th>
<th>( a_7 )</th>
<th>( a_8 )</th>
<th>( \chi^2 )</th>
<th>( \sigma_p ) (mb)</th>
</tr>
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<tr>
<td>6.65</td>
<td>0.0047</td>
<td>-0.000004</td>
<td>0.0004</td>
<td>-0.0007</td>
<td>-0.0007</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0051</td>
<td>59</td>
</tr>
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\[ C^{12}(p,p'')C^{12}^n \]

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Table 2 (continuation)
Figure 10: Estimates of total cross sections for the $^{12}_C(p,p')C^{12*} (Q = -4.43 \text{ MeV})$ and $^{12}_C(p,p'')C^{12*} (Q = -7.66 \text{ MeV})$ reactions as a function of incident proton energy, based on the differential cross sections at six angles presented in figs. 2 and 3 are plotted as dots. The crosses and open circle are the total cross sections from table 2, based on the angular distributions. The solid curve is the total reaction cross section as a continuous function of energy, calculated as described in the text.
IV. ANALYSIS OF THE EXPERIMENTAL RESULTS

The methods of phase shift analysis and single-level dispersion theory were used to extract information concerning the nuclear states of $N^{13}$. Complex phase shifts were found which represent the elastic-scattering and total inelastic cross sections and available polarization data. These phase shift values were used to represent the slowly-varying contribution to the elastic-scattering cross sections as a continuous function of energy while the rapidly-varying contributions (due to narrow resonances) were represented by a single-level form.

A. Phase Shift Analysis

For the scattering of spin-1/2 particles from spin-0 nuclei the elastic-scattering cross section and the polarization are

$$
\sigma(\theta) = \frac{1}{k^2} \left( |f_c(\theta)|^2 + |f_i(\theta)|^2 \right)
$$

and

$$
|\hat{P}(\theta)| = \frac{2 \text{ Im} (f_c f_i^*)}{k^2 \sigma(\theta)}
$$

where $f_c(\theta)$ and $f_i(\theta)$ are the coherent and incoherent scattering amplitudes, respectively; $k$ is the wave number of the incident beam, and $\theta$ is the center-of-mass scattering angle. The coherent and incoherent scattering amplitudes are defined in terms of the scattering phase shifts as
\[ f_c(\theta) = -\frac{1}{2} \csc^2(\theta/2) e^{i\gamma \ln \csc^2(\theta/2)} \]

\[ + \frac{i}{2} \sum_{l=0}^{\infty} e^{2i\omega_l} P_l(\cos \theta) \left[ (2l+1)-(l+1) e^{2i\delta_l^+} - l e^{2i\delta_l^-} \right] \]

and

\[ f_{l}(\theta) = \sum_{l=1}^{\infty} \frac{i e^{2i\omega_l}}{2} \sin \theta P_l(\cos \theta) \left[ e^{2i\delta_l^+} - e^{2i\delta_l^-} \right] \]

where \( \delta_l \) are the phase shifts of the \( l \)th partial wave of total angular momentum \( J = l \pm 1/2 \), \( \omega_l \) is the \( l \)th Coulomb phase shift, and \( \gamma = \frac{\mu Z_1 Z_2 e^2}{h^2} \). All symbols used have their usual significance and are defined in ref. 7. The expressions (3,4) for \( f_c \) and \( f_l \) are valid when only elastic scattering is energetically possible.

In the present experiment the inelastic-scattering channel opens at approximately 5 MeV resulting in the phase shifts \( \delta \) becoming complex that is

\[ e^{21\delta} \rightarrow e^{21(\delta+i\gamma)} = e^{-2\gamma} e^{21\delta} = \alpha e^{21\delta} . \]

The complex phase shifts are expressed in terms of a real phase shift \( \delta \) and a damping parameter \( \alpha \) which describes the absorption of the incident partial wave into all open reaction channels.

The total reaction cross section can be written as (Appendix A)
\[ \sigma_{\ell} = \frac{\pi}{k^2} \sum_{\ell'} \left[ (2\ell' + 1) - (\ell + 1)(\alpha^+_{2\ell})^2 - \ell (\alpha^-_{2\ell})^2 \right]. \] (5)

The expressions for differential elastic-scattering cross section, total reaction cross section, and polarization have been programmed for use on IBM 1401 and 7040 computers. The method used to deduce the complex phase shifts \( (\delta^J_\ell, \alpha^J_\ell) \) from the experimental results is given in Appendix B. The complex phase shift values necessary to represent the differential elastic-scattering cross sections at discrete energies, total inelastic cross sections where measured, and available polarization data are shown in fig. 11 and tabulated in table 3. The quantity \( \chi^2 \) is defined in Appendix B.

B. Calculation of Cross Sections and Polarization

as a Continuous Function of Energy

The expression for the cross section is now written in terms of the collision matrix \( U^J_\ell \) of the \( \ell \)th partial wave and total angular momentum \( J \)

\[ f_c(\theta) = -\frac{n}{2} \text{csc}^2(\theta/2) e^{i2 \ln \text{csc}^2(\theta/2)} \]
\[ + \sum_{\ell=0}^{\infty} i \left( \frac{\pi}{2\ell' + 1} \right)^{1/2} Y^0_\ell(\theta) \left[ (2\ell' + 1) e^{2i\phi} - (\ell + 1) U^+_{\ell'} - \ell U^-_{\ell} \right] \] (6)

\[ f_i(\theta) = i n^{1/2} \sum_{\ell=1}^{\infty} \left[ \frac{A(\ell + 1)}{2\ell' + 1} \right]^{1/2} Y^1_\ell(\theta) \left[ U^-_{\ell} - U^+_{\ell} \right]. \] (7)
Figures 11a-b: The phase shifts ($\delta^+_\ell$) and damping parameters ($\alpha^+_\ell$) determined from angular distributions at the discrete energies are shown as dots (fig. 11 (a)) and dots and open circles (fig. 11 (b)). The solid curves were used for the calculation described in the text. The dashed curves at $\delta^-_1$ and $\delta^-_3$ are hard sphere scattering phase shifts and at $\delta^-_0, \delta^+_1$, and $\delta^+_2$ are dispersion theory calculations including resonance effects. The dashed curve fitted to $\delta^-_2$ between 4.5 and 6.2 MeV represents a dispersion theory calculation using a two-level formula for the resonant phase shift and the parameters of the 6.35 MeV level. Resonance energies are indicated by arrows.
Figure 11a
Table 3
Complex Phase Shifts which Represent the Data.

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<td>0.72</td>
<td>0.93</td>
<td>0.35</td>
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</table>
To take account of a sharp resonance in data otherwise varying slowly with energy, the R-matrix may be split into a single-level part and a "background" part. The collision matrix splits accordingly: \( U = U^0 + U^1 \). For the \( p + C^{12} \) problem the background part \( U^0 \) must be nondiagonal to account for the slowly-varying absorption. Only the diagonal elements of \( U^0 \) enter the expression for the elastic-scattering cross sections, and these elements were approximated by \( U^0_J(E) = \alpha^J e^{2i\phi^J} \). In the resonant part \( U^1 \) the approximation \( R^0 L^0 = 0 \) was used (although this choice cannot be entirely correct since it implies that \( U^0 \) is diagonal). Then the diagonal element is approximately

\[
U_{cc}(E) = U^J_J(E) = e^{2i\omega_J} (\alpha_J e^{2i\phi_J}) [1 + \alpha_J^2 (e^{2i\beta_J} - 1)].
\]

Provided that \( \alpha \leq 1 \) and \( \alpha \leq 1 \), this form guarantees that \( U_{cc}^* U_{cc} = 1 \) so that the unitarity condition cannot be violated by the diagonal element alone. The parameter \( \alpha \) is the fractional elastic width of the resonance. Some other forms which did not make this guarantee were discarded.

The resonance phase shifts \( \beta \) were approximated by \( \beta = \arg \tan \frac{\Gamma}{E_0 - E} \) where \( \Gamma \) is real and independent of energy (i.e., the level shift and the energy-dependence of the width were neglected for the narrow levels) and \( E_0 \) is the resonance energy.

In expression (8) for \( U_{cc} \) the term outside the square parentheses describes the slowly-varying processes and the
inside term describes the rapidly-varying processes due to a single level in the compound nucleus. The definitions of "slowly-" and "rapidly-varying" are obviously somewhat arbitrary and, correspondingly, some effects could be included in either term. Terms of the outside type can simply be regarded as representing the effects of those levels not represented by terms of the inside type.

In the present calculation \( 5.0 \leq E_p \leq 9.5 \text{ MeV} \) the levels whose effects were represented by the outside terms were (a) the known levels below the energy range of the calculation; (b) the known and unknown levels above the energy range of the calculation; (c) the strongly overlapping \( J^m = 3/2^+ \) levels at \( E_p = 5.3 \) and 6.6 MeV, because of the possible importance of the interference terms between them, and because of the large width of the upper state. Thus the outside terms were taken from the complex phase shift values of the solid curves in fig. 11.

To take account of the rapidly-varying contributions to the total reaction cross section a "total" damping parameter \( A^J_2 \) which includes both slowly- and rapidly-varying contributions has been derived from expression (8) (Appendix A). Thus

\[
A^J_2 = \alpha^J_2 \sqrt{1 - 2a^J_2 (1 - a^J_2)(1 - \cos 2\beta^J_2)}
\]

replaces \( \alpha^J_2 \) in expression (5).

The differential elastic and total inelastic cross sections calculated as a continuous function of energy in the range \( 5.0 \leq E_p \leq 9.5 \text{ MeV} \) are shown as solid curves on figs. 12
and 10, respectively. The parameters used in these calculations are the complex phase shifts shown as solid curves in Fig. 11 and the level parameters indicated in table 4. Note that only one of the levels of the 9.14 MeV doublet was included in the calculations, resulting in rather poor fits above $E_p = 8.5$ MeV. The calculated polarization as a continuous function of energy at $\theta_{lab} = 50^\circ$ shown as a solid curve in fig. 13 is compared to the experimental data of ref. 8. Calculated polarization (table 3 for parameters used) as a function of angle at discrete energies shown as dashed curves in fig. 14 is compared to available experimental polarization data at energies close to those used. Polarization calculated from the complex phase shifts of the solid curves in fig. 11 is compared (fig. 15) to available experimental results in the energy range of interest. Below $E_p = 8.5$ MeV good agreement between the calculated and experimental polarization has been obtained, so a contour map of spin polarization versus energy and angle based on the solid curves of fig. 11 has been calculated (fig. 16).

C. Revised Level Parameters

Obtaining the level parameters for the wide $J^\pi = 3/2^+$ state at $E_p = 6.35$ MeV is complicated by the presence of the $E_p = 5.3$ MeV state of the same spin and parity, which is overlapped by the wider state. The values of the $\delta_2^-$ phase shift between $E_p = 4.5$ and 4.9 MeV and between $E_p = 5.6$ and 7.8 MeV (that is, excluding the region of the 5.3 MeV state
Figure 12: Absolute differential cross sections for the elastic scattering of protons by $^{12}$C as a function of incident proton energy at the angles shown. The dots are a condensation of fig. 1. The solid curves were calculated as described in the text. The break in the curves around 5.35 MeV is to indicate the uncertainty of the resonating $\delta_2$ phase shift in this region.
Figure 12

Differential Cross Section — mb/sr (cm)

Incident Proton Energy — MeV (Lab)
Figure 13: Polarization at $\theta_{\text{lab}} = 50^\circ$ as a continuous function of energy. The dots are the experimental values from ref. 8, and the solid curve was calculated using the solid curves in fig. 11. Errors indicate statistical uncertainties only.
Figure 14: Polarization data as a function of angle at discrete energies close to some of those used in the present experiment. The experimental data are taken from the following sources: the solid curve at 5.66 and 6.18 MeV from ref. 10, the dots at 6.18, 6.77, 7.99, and 8.66 MeV from ref. 8, and the dots at 6.60, 8.60, and 11.70 MeV from ref. 11. The dashed curves were calculated from the complex phase shifts in table 3.
Figure 14
Figure 15: Polarization data as a function of angle and energy in the energy range of the present experiment.

The experimental data represented by dots are taken from the following sources: (a) function of angle: 4.4 MeV from ref. 12; 4.66, 5.04, 7.21, and 7.55 MeV from ref. 8; and 5.4 and 7.4 MeV from ref. 11 (b) function of energy: $\theta_{\text{lab}} = 60.8^\circ$ from ref. 13 and $\theta_{\text{lab}} = 51.5^\circ$ from ref. 14. The solid curves were calculated from complex phase shifts taken from the solid curves of fig. 11.
Figure 15
Figure 16: Polarization Map for $C^{12}(p,p)C^{12}$. Lines of equal spin polarization are shown as dashed (zero polarization) and solid curves. The polarization was calculated from the complex phase shifts represented by solid curves in fig. 11.
### Table 4

Level Parameters in $^{13}N$

<table>
<thead>
<tr>
<th>$E_p$ (MeV)</th>
<th>$E_x$ (MeV)</th>
<th>$J^\pi$</th>
<th>$r$ (keV)</th>
<th>$a_{lj}$</th>
<th>$E_\lambda$ (MeV)</th>
<th>$\gamma_\lambda$ (MeV)</th>
<th>$\gamma_\lambda/(3n^2/2\mu R^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.80$^c$</td>
<td>6.37$^c$</td>
<td>5/2$^+$</td>
<td>12$^c$</td>
<td>1.0$^c$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.30</td>
<td>6.83</td>
<td>3/2$^+$</td>
<td>74</td>
<td></td>
<td>6.87</td>
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<td>0.014</td>
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<tr>
<td>5.88$^{+0.018}$</td>
<td>7.37</td>
<td>5/2$^-$</td>
<td>70</td>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.35</td>
<td>7.79</td>
<td>3/2$^+$</td>
<td>1720</td>
<td>1.00</td>
<td>8.86</td>
<td>0.9</td>
<td>0.31</td>
</tr>
<tr>
<td>7.53$^{+0.025}$ $^{-0.018}$</td>
<td>8.89</td>
<td>1/2$^-$</td>
<td>250</td>
<td>0.66</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.16$^{+0.018}$</td>
<td>9.47</td>
<td>3/2$^-$</td>
<td>30</td>
<td>0.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.13</td>
<td>10.37</td>
<td>7/2$^-$</td>
<td>84</td>
<td>0.90</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Energies in the laboratory system  
(b) Relative to ground state of $^{13}N$  
(c) From ref. 3)
but including the rest of the 6.35 MeV state) were fitted to
an expression of the form
\[ \delta_1^+ = \phi_1^+ + \arctan \left( \frac{\gamma_{1}^{p} f_{1}^p}{E_{l}^2 + \Delta_{l}^2 - \Delta} \right) \]
to determine some of the parameters of the wider state. A
radius \( R = 4.8 \text{ fm} \) was used. Since the \( \alpha_2^- \) parameter shows no
dip corresponding to the state at 6.35 MeV, the partial re-
action width for this state was taken to be zero.

The parameters for the 6.35 MeV state, and estimates of
the parameters of the 5.30 MeV state, were then used to calcu-
late the resonance phase shift from 4.5 to 6.2 MeV. A simple
two-level formula was used for the resonant part of the phase
shift:
\[ \delta_2^- = \arctan \left( \frac{\Gamma/2}{E_0 - E} + \frac{\gamma_{2}^{p} f_{2}^p}{E_{l}^2 + \Delta_{2}^2 - \Delta} \right) . \]
The \( \delta_2^- \) phase shifts calculated are shown as a dashed curve
in fig. 11a.

The level parameters used for these calculations are
given in table 4 with the other states.

D. The 9.14 MeV Double Anomaly

The four angular distributions over the 9.14 MeV anomaly
were taken at two different times. The energy location of
the first angular distribution was determined (when the
presence of the double anomaly was unknown) by inelastic
yield curves (\( Q = -4.43 \text{ MeV} \)) at \( \theta_{lab} = 85.22^\circ \) and 121.16°.
In fig. 5 an arrow indicates the energy position chosen
(\( E_p = 9.15 \text{ MeV} \)) which is between the two anomalies. After
the double anomaly was found three angular distributions were taken at energy positions determined by elastic-scattering yield curves at $\theta_{\text{c.m.}} = 90^\circ$ and $140.76^\circ$. The energy positions of the latter three angular distributions ($E_p = 9.10$, 9.13, and 9.16 MeV) and the first one are indicated by arrows in fig. 17.

The complex phase shifts which represent the experimental results of the angular distributions over the doublet are given in table 3. The calculated elastic angular distributions are shown as solid and dashed curves on fig. 6d. The elastic cross sections as a continuous function of energy were calculated ($9.0 \leq E_p \leq 9.24$ MeV) as previously described and are shown in fig. 17. The level parameters and the complex phase shifts used in this calculation are given in table 5.
Figure 17: Absolute differential cross sections for the elastic scattering of protons by $^1\text{C}^{12}$ as a function of energy. The dots are experimental data from fig. 1. The solid curves were calculated as described in the text. The arrows indicate the energy position (9.10, 9.13, 9.15, and 9.16 MeV) of the angular distributions over the 9.14 MeV anomaly.
Figure 17
Table 5
Level Parameters and Complex Phase Shifts which Represent the Experimental Data over the 9.14 MeV Doublet

<table>
<thead>
<tr>
<th>$E_p$ (MeV)</th>
<th>$J^π$</th>
<th>$Γ^{(a)}$ (keV)</th>
<th>$a_{\ell j}$</th>
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<tr>
<td>9.145</td>
<td>5/2−</td>
<td>12</td>
<td>0.28</td>
</tr>
<tr>
<td>9.152</td>
<td>7/2−</td>
<td>90</td>
<td>0.75</td>
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</table>

$δ_0$ $δ_1^-$ $δ_1^+$ $δ_2^-$ $δ_2^+$ $δ_3^-$ $δ_3^+$
$α_0$ $α_1^-$ $α_1^+$ $α_2^-$ $α_2^+$ $α_3^-$ $α_3^+$

<table>
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<tr>
<th>72.50</th>
<th>-35.00</th>
<th>-14.50</th>
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<th>-15.00</th>
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<td>0.85</td>
<td>0.80</td>
<td>0.98</td>
<td>0.70</td>
<td>0.77</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

(a) Energies in the laboratory system.
V. DISCUSSION

The level structure of $^{13}$N is quite well known, and most of the anomalies in the elastic-scattering cross sections have the shapes and positions expected. Level parameters have been assigned (tables 4 and 5) to all states observed in the present experiment in the range $4.8 \leq E_p \leq 9.5$ MeV. No evidence was observed in the elastic-scattering data to indicate a previously-reported\textsuperscript{15} weak and narrow resonance at $E_p = 5.68$ MeV which is below the energy where good inelastic data were obtained. However, inelastic scattering yield curves indicate the presence of this narrow resonance. Of considerable interest is the previously-unreported splitting of the $9.14$ MeV anomaly, indicating a new state very close to the known $7/2^-$ state at an excitation energy $E_x(N^{13}) = 10.4$ MeV. In the region 10 to 11 MeV four resonances were observed. In the inelastic scattering data (fig. 2) there is evidence of a level seen previously only by Adams et al.\textsuperscript{16} at $E_p = 10.74$ MeV.

The phase shifts that would be expected on the basis of the known level scheme of $^{13}$N are shown as dashed curves in fig. 11a. Where no known levels of a given spin and parity are nearby, hard sphere phase shifts are plotted. Dispersion theory calculations are shown for cases where the effects of known levels would be expected to be appreciable. It is seen that the phase shifts necessary to fit the experimental data differ significantly from the dashed curves, so that the
scattering of protons by $^{12}\text{C}$ is not properly explained by the known $^\text{13}\text{N}$ levels in the energy range of the experiment. As previously discussed, the outside terms of the collision matrix are represented by the complex phase shifts shown as solid curves in fig. 11, and the inside terms are represented by a single-level form. The break in the solid curve (fig. 11a) at $E_p = 5.30$ MeV indicates the uncertainty of the phase shift $\delta_2$ in this region. The elastic-scattering cross sections at the resonance position (5.30 MeV) cannot be represented by letting the phase shift $\delta_2$ go through 90° as would be expected. The phase shift $\delta_2$ at $E_p = 5.34$ MeV jumps from 50° to 193° at $E_p = 5.36$ MeV, and no values can be found between which fit the experimental data.

The calculated results of the elastic-scattering cross sections, total inelastic cross sections, and polarization from the complex phase shifts (solid curves fig. 11) are shown in fig. 12 and, in general, are quite good. The single-level form $[1-\delta_2 J^e(\frac{2\pi J^e}{\lambda^2} -1)]$ is used to represent the five levels: 4.80, 5.88, 7.55, 8.17, and 9.14 MeV. The $J^e = 3/2^+$ levels at $E_p = 5.30$ and 6.35 MeV were represented by the outside terms. Inclusion of only one state ($J^e = 7/2^-$) at the 9.14 MeV doublet accounts for the significant difference in the experimental and calculated results above $E_p = 8.5$ MeV.

The complex phase shifts over the 9.14 MeV doublet indicate that the $5/2^-$ state is narrow and lies below the $7/2^-$
state in energy. The nonresonant phase shifts in this region agree quite well with those at nearby energies. Although the lower state of the doublet is a weak one overlapped by a stronger state, the assignment \( J^m = 5/2^- \) to the weaker state is now viewed with confidence. The angular distribution for the inelastic-scattering reaction was measured first only at 9.15 MeV, and the striking lack of symmetry about \( \theta = 90^\circ \) was at first taken to indicate that the two states of the doublet were of opposite parity. This agreed with the expectation of a \( J^m = 9/2^+ \) state in this energy region, but it was not found possible to fit the elastic-scattering data with a \( 7/2^- , 9/2^+ \) assignment for the doublet. Further, the dominant term in the inelastic angular distribution is proportional to the Legendre polynomial \( P_3 \), which is not characteristic of \( 7/2^- , 9/2^+ \) interference. On the other hand \( 7/2^- , 3/2^+ \) interference gives a pure \( P_3 \)-term and efforts were made to fit the elastic-scattering data with this assignment. These and other positive-parity assignments were unsuccessful.

The conclusions changed considerably when the three later angular distributions over the doublet were considered. It can be seen from fig. 8b that the inelastic angular distributions do not change dramatically over the doublet, and in particular that the sign of the coefficient of \( P_3 \) remains the same. Thus this term is due to interference between a resonant state and a nonresonant background of opposite parity. The argument that the two states of the doublet must have
opposite parity fails, and the $5/2^-$, $7/2^-$ assignment is found to fit the elastic-scattering data. The strong $P_3$ term in the inelastic-scattering angular distribution is presumed due to interference between the $7/2^-$ state and the tail of the very wide $3/2^+$ state at 6.35 MeV. Further, if the wide $3/2^+$ state interferes with the $7/2^-$ state it might also be expected to interfere with the $5/2^-$ state. Such interference is characterized by a $P_1$ term and it can be seen from table 1 that this term is indeed anomalous close to the $5/2^-$ state. Thus both the elastic- and inelastic-scattering data are consistent with the $5/2^-$ assignment. The polarization data and cross sections of Terrell and Bernstein are similarly consistent with this assignment.

In Kurath's calculations of the negative-parity states of $N^{13}$, all possible states for a given number of nucleons in the $1-p$ shell are formed. No $J^\pi = 5/2^-$ state appears with the excitation energy in the 10 MeV region. However, Kurath notes that excitation of two nucleons from the $1-p$ shell to the $1-d$ would give higher-lying negative-parity states. The $J^\pi = 5/2^-$ is assumed to be one of these.
APPENDIX A

In this section the expressions used for the elastic differential cross section and the total reaction cross section are shown to follow the development of nuclear reactions by Lane and Thomas\(^9\). The differential cross section for the processes \(\alpha \rightarrow \alpha'\) is given as

\[
d\sigma_{\alpha\alpha'} = \left(\frac{2I_1+1}{2I_2+1}\right) \sum_{\nu \nu'} |A_{\alpha,\nu;\alpha',\nu'}(\Omega_{\alpha'})|^2 d\Omega_{\alpha'},
\]

where \(I_1\) and \(I_2\) are the spins of the particles of the pair \(\alpha\) (channel index). The notation follows that of Lane and Thomas. The scattering amplitude in the \([\alpha sLM]\) scheme may be written as

\[
A_{\alpha',\nu';\alpha,\nu}(\Omega_{\alpha'}) = -\frac{\eta_{\alpha}}{2\lambda_{\alpha}} \csc(\theta_{\alpha}/2) \mathcal{C}(\lambda_{\alpha}) \mathcal{A}(\lambda_{\alpha}) \chi_{\nu'}
\]

\[
+ \frac{i\pi\eta_{\alpha}}{\lambda_{\alpha}} \sum_{JMM'LM'} (2L+1)^{1/2} (s\nu;JM')(s'\nu';JM) x_{\nu'}(\Omega_{\alpha'}) (e^{i\lambda_{\alpha} J} \delta_{\nu\nu'} - U_{\nu\nu'}^{J}) \chi_{\nu'}.
\]

For elastic scattering of spin-1/2 particles on spin-0 nuclei, the conservation of angular momentum and parity require that \(\alpha = \alpha', \ s = s'\), and \(J = J'\). Now let the scattering amplitude be defined as

\[
A_{\alpha',\nu';\alpha,\nu} \equiv A_{\nu'} = \frac{A_{\nu s}}{A_{\nu s} - \eta_{\alpha}}
\]
It may be shown that \( A_{1/2 \rightarrow 1/2} = A_{1/2 \rightarrow -1/2} \) so the elastic differential cross section is given as

\[
\frac{d\sigma_{\text{el}}}{d\Omega} = \sigma_{\text{el}}(\phi) = \left| A_{1/2 \rightarrow 1/2} \right|^2
\]

where

\[
A_{1/2 \rightarrow 1/2} = -\frac{n}{2 \pi} \cos^2(\phi/2) e^{i\gamma \cos^2(\phi/2)} \chi_{1/2}^{1/2}
\]

\[
+ \frac{i \pi}{\hbar} \sum_{l=1}^{l_{\text{max}}} (2l+1)^{1/2} \begin{bmatrix} J_{1/2} & l_{1/2} \end{bmatrix} \begin{bmatrix} J_{1/2} \end{bmatrix} \chi_{1/2}^{1/2} \left( e^{i\gamma \cos^2(\phi/2)} - \mathbf{U}_l^{1/2} \right) \chi_{1/2}^{1/2}
\]

\[
A_{1/2 \rightarrow -1/2} = -\frac{n}{2 \pi} \cos^2(\phi/2) e^{i\gamma \ln \cos^2(\phi/2)} \chi_{1/2}^{1/2}
\]

\[
+ \frac{i \pi}{\hbar} \left\{ \sum_{l=1}^{l_{\text{max}}} (2l+1)^{1/2} \begin{bmatrix} l^{1/2} & 0 \end{bmatrix} \begin{bmatrix} l^{1/2} \end{bmatrix} \chi_{1/2}^{1/2} \left( e^{i\gamma \cos^2(\phi/2)} - \mathbf{U}_l^{1/2} \right) \chi_{1/2}^{1/2}
\]

\[
+ \sum_{l} (2l+1)^{1/2} \begin{bmatrix} l^{1/2} & 0 \end{bmatrix} \begin{bmatrix} l^{1/2} \end{bmatrix} \chi_{1/2}^{1/2} \left( e^{i\gamma \cos^2(\phi/2)} - \mathbf{U}_l^{1/2} \right) \chi_{1/2}^{1/2}
\]

\[
+ \sum_{l} (2l+1)^{1/2} \begin{bmatrix} l^{1/2} & 0 \end{bmatrix} \begin{bmatrix} l^{1/2} \end{bmatrix} \chi_{1/2}^{1/2} \left( e^{i\gamma \cos^2(\phi/2)} - \mathbf{U}_l^{1/2} \right) \chi_{1/2}^{1/2}
\]

\[
+ \sum_{l} (2l+1)^{1/2} \begin{bmatrix} l^{1/2} & 0 \end{bmatrix} \begin{bmatrix} l^{1/2} \end{bmatrix} \chi_{1/2}^{1/2} \left( e^{i\gamma \cos^2(\phi/2)} - \mathbf{U}_l^{1/2} \right) \chi_{1/2}^{1/2}
\]
The spin wave functions designated by $\chi_{1/2}^{1/2}$ and $\chi_{1/2}^{-1/2}$ are associated with the coherent and incoherent scattering amplitudes, respectively. The coherent scattering amplitude $f_c(\theta)$ represents those protons whose spins do not change direction in the scattering process, while the incoherent scattering amplitude $f_i(\theta)$ represents those protons whose spins have reversed during scattering.

From the orthogonality of the spin functions and the values of the Clebsch-Gordon coefficients, the elastic differential cross section is written as

$$
\sigma_{el}(\theta) = \frac{1}{k^2} \left( |f_c(\theta)|^2 + |f_i(\theta)|^2 \right)
$$

where

$$
f_c(\theta) = -\frac{n}{2} \csc^2(\phi/2) e^{i\pi} 2^n \csc^2(\phi/2)
$$

$$
+ \frac{1}{2} \frac{(-1)^L}{\sqrt{2}} \gamma_i(\phi) \left[ \frac{J}{2L+1} \right] \sum_L \left( 2L+1 \right)^{1/2} \gamma_i(\phi) \sqrt{\frac{2L+1}{2L+1}} \left[ \frac{U_2^{L-1/2}}{U_2^{L+1/2}} \right]
$$

and

$$
f_i(\theta) = \frac{1}{2} \frac{(-1)^L}{\sqrt{2}} \gamma_i(\phi) \sqrt{\frac{2L+1}{2L+1}} \left[ \frac{U_2^{L-1/2}}{U_2^{L+1/2}} \right].
$$

The total cross section for the reactions $\alpha s \rightarrow \alpha's'$ is given as

$$
\sigma_{\alpha\alpha'} = \frac{4\pi}{k^2} \sum_{J, L, M} \frac{2J+1}{(2J+1)(2I_2+1)} |T_{\alpha\alpha', \ell s l'}|^2
$$

where
\[ T_{\alpha_s' \ell', \alpha_{sl}}^{J} = e^{2i\omega \tau} \delta_{\alpha_s' \ell', \alpha_{sl}} - U_{\alpha_s' \ell', \alpha_{sl}}^{J} . \]

Hence, for the problem under consideration \((I_1 = 1/2, I_2 = 0)\)

\[ \sigma_{\alpha \alpha'} = \sigma_T = \frac{\pi}{2\alpha^2} \sum_{J,I,s,s'} (2J+1)(2s+1) |U_{\alpha_s' \ell, \alpha_{sl}}^{J}|^2 . \]

The diagonal elements of the collision matrix have been represented by either

\[ \alpha_e^J e^{2i\phi} \quad \text{or} \quad \alpha_e^J e^{2i\phi} \left[ -a_e^J (e^{2i\phi} - 1) \right] . \]

When the former form of the collision matrix is considered, the total reaction cross section may be written as

\[ \sigma_T = \frac{\pi}{2\alpha^2} \sum_{J,I} (2J+1)(2\ell+1)(\alpha_e^J)^2 \left( 1 - |\alpha_e^J e^{2i\phi}|^2 \right) \]

\[ \sigma_T = \frac{\pi}{2\alpha^2} \sum_{J,I} \left[ \left\{ 2(\ell+1)+1 \right\} - \left\{ 2(\ell+\frac{1}{2})+1 \right\} (\alpha_e^J)^2 \right. \]

\[ 
\left. + \left\{ 2(\ell-\frac{1}{2})+1 \right\} - \left\{ 2(\ell-1)+1 \right\} (\alpha_e^J)^2 \right] \]

\[ \sigma_T = \frac{\pi}{2\alpha^2} \sum_{J,I} \left[ (2\ell+1) - (\ell+1)(\alpha_e^J)^2 - \ell (\alpha_e^-)^2 \right] \]

where

\[ \alpha_e^J = \alpha_e^{\ell+\frac{1}{2}} - \alpha_e^\ell \]
The latter form of the collision matrix may be shown to give the same \( \sigma_T^J \).

A "total" damping parameter \( A^J \) is derived to take account of the rapidly-varying contributions to the total reaction cross section. The diagonal element of the collision matrix

\[
U^J e^{i \omega e^{i \theta}} \left[ 1 + A^J \left( e^{2i \theta} - 1 \right) \right]
\]

is simply reduced to a form \( A^J e^{i \Delta^J} \) where \( A^J \) and \( \Delta^J \) are considered the "total" damping parameter and phase shift

\[
e^{2i \omega e^{i \theta}} \left[ 1 + A^J \left( e^{2i \theta} - 1 \right) \right]
\]

\[
= A^J e^{2i \Delta^J}
\]

where the quantities inside the square parentheses represent \( A^J \) and \( \Delta^J \).
APPENDIX B

The expressions for the differential elastic and total reaction cross sections and polarization in terms of the complex phase shifts for the charged particle scattering problem of spin-1/2 on spin-0 are previously given. The complex phase shifts cannot be expressed in terms of the cross sections due to the complicated nature of the expression, so the phase shifts are usually found by numerical methods. In the present analysis two computers (IBM 1401 and 7040) were used to carry out the phase shift analysis, and the first attempts were based on values of Reich et al. \(^7\) and Moss and Haeberli \(^8\).

1401 Program

The differential and total reaction cross sections were calculated from the estimated complex phase shifts, and the differences between the calculated and experimental cross sections ("difference" function) were printed out. Also, changes in the calculated cross sections ("change" function) caused by specified changes in the real phase shifts \(\delta\) were evaluated and printed out. Then the "difference" function and the "change" functions were plotted and an attempt was made to generate a curve like the "difference" function by linear combinations of the "change" functions. The suggested changes were made and the procedure repeated, until the differences were comparable with the errors in the experimental
cross sections. The damping parameters $\alpha$ were varied in order to obtain good agreement between the experimental total inelastic cross sections and the calculated total reaction cross sections and to improve the fits to the differential cross sections. The quantity used to determine the quality of the fit is written as

$$\sum_{1}^{M} \left[ \frac{\sigma_{\exp}(\theta) - \sigma_{\text{cal}}(\theta)}{\sigma_{\exp}(\theta)} \right]^2 \right]^{1/2}$$

where $M$ is the maximum number of angles. When complex phase shifts were not necessary, the damping parameters $\alpha$ were set to unity, resulting in a conventional phase shift analysis.

7040 Program

If the complex phase shifts are expressed in terms of $(\delta, \alpha)$ then the cross section at the $1^{\text{st}}$ angle may be written in terms of $2N$ parameters

$$\sigma_{1}^{\exp} = f_1(\delta_1 \delta_2 \ldots \delta_N, \alpha_1 \alpha_2 \ldots \alpha_N).$$

The present analysis used partial waves through $l = 3$, $N = 7$ so that cross section data at $14$ angles would give $14$ equations like that above. The equations are nonlinear and cannot be solved directly. However, if a set of trial parameters $\delta_1^{(0)} \ldots \delta_N^{(0)}$

$\alpha_1^{(0)} \ldots \alpha_N^{(0)}$ can be found with a corresponding cross section $\sigma_{1}^{(0)}$ then the original problem can be replaced by the problem of reducing the quantity $e_1 = \sigma_{1}^{\exp} - \sigma_{1}^{(0)}$ to zero for all $i$. Small changes $\Delta\delta$ and $\Delta\alpha$ in the parameters produce a change $\Delta\sigma_{1}^{(0)}$ in $\sigma_{1}^{(0)}$ which change can be
approximated by a Taylor series in which only linear terms are retained. Then if $\Delta \delta$ and $\Delta \alpha$ could be ideally chosen

$$e_1 = \left( \frac{\delta f_1}{\delta \delta_1} \right) \delta_1^{(o)} \Delta \delta_1 + \left( \frac{\delta f_1}{\delta \delta_2} \right) \delta_2^{(o)} \Delta \delta_2 + \ldots \left( \frac{\delta f_1}{\delta \alpha_1} \right) \alpha_1^{(o)} \Delta \alpha_1 + \ldots$$

(E1)

If perfect measurements of $e_1$ at $2N$ angles were available, the simultaneous linear equations (E1) could be solved for the $2N$ parameters ($\Delta \delta$, $\Delta \alpha$). Thus the non-linear problem has been linearized with the aid of the trial set of parameters. Although the symbol $\sigma$ is used above, the $2N$ pieces of experimental data could be any mixture of cross section and polarization data.

In practice, more than $2N$ pieces of imperfect data are available and the problem is changed slightly. If $e$ is regarded as a dependent variable and the partial derivatives as independent variables, the problem is to find the values of the ($\Delta \delta$, $\Delta \alpha$) such that an equation of the form E1 is a best fit to the experimental $e_1$. Using the least-squares criterion the object is to minimize a quantity such as

$$\sum_{i} (e_1 \text{ exp} - e_1 \text{ calc})^2.$$

The quantity actually used was

$$\chi^2 = \sum_{i} \left( \frac{\sigma_i \text{ exp} - \sigma_i \text{ calc}}{\sigma_i \text{ exp}} \right)^2 + \sum_{j} \left( \frac{P_j \text{ exp} - P_j \text{ calc}}{3P_j \text{ exp}} \right)^2.$$
where the factor 3 was inserted because the polarization data were approximately three times less accurate than the cross section data.
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