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PREDICTION OF THE PHASE SHIFT IN THE LOW FREQUENCY MOTION RESPONSE OF VESSELS TO IRREGULAR SEAS

by

Joseph Robert Mayersak

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DOCTOR OF PHILOSOPHY

Thesis Director's Signature:  

Herbert B. Mann

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**Nomenclature**

\[ A^2(w) = \text{spectral density as a function of frequency} \]

\[ A^2(f) = \text{spectral density as a function of cycle frequency} \]

\[ A^2(T) = \text{spectral density as a function of period} \]

\[ a = \text{wave amplitude} \]

\[ c = \text{wave celerity} \]

\[ c(w) = \text{response frequency operator} \]

\[ d = \text{ocean depth} \]

\[ E = \text{energy per unit area of the sea, or the lagged ensemble expectation value} \]

\[ f = \text{frequency in cycles/second} \]

\[ f_m = \text{frequency at maximum ordinate location in the energy spectrum} \]

\[ g = \text{acceleration of gravity} \]

\[ k = \text{wave number} \]

\[ KE = \text{kinetic energy} \]

\[ L = \text{distance between the hulls of a twin hull vessel} \]

\[ p = \text{pressure} \]

\[ PE = \text{potential energy} \]

\[ r(t) = \text{vessel response in roll} \]

\[ R^2(w) = \text{spectral vessel response in roll} \]

\[ s(w) = \text{response frequency operator} \]

\[ t = \text{time} \]

\[ t' = \text{lagged time intervals} \]

\[ T = \text{wave period} \]

\[ T^2(w) = \text{response amplitude operator} \]
\[ u \] = horizontal fluid particle velocity
\[ U \] = wave group velocity
\[ v \] = vertical fluid particle velocity
\[ w \] = frequency in radians/second
\[ x \] = abscissa distance
\[ y \] = ordinate distance, or free fluid surface elevation
\[ \lambda \] = wave length
\[ \rho \] = fluid density
\[ \psi \] = velocity potential
\[ \psi \] = stream function
I Introduction

This investigation develops a procedure which will predict the roll and pitch response of double hull vessels by considering the phase shift in the sea state over the distance between the hulls.

Linear motion response when extrapolated to spectral response in irregular seas for twin hull vessels is, to a certain extent, in discordance with the observed response for this type of vessel. Although it does predict the proper high frequency response it does not predict the downward frequency shift associated with the low frequency response. Most investigators consider the low frequency phenomenon to be associated with non-linear response effects which are assumed to dominate in the low frequency range \((1, 2)\). The shortcoming of this non-linear theory is that the response transfer functions involved are extremely complex in nature.

The linear theory \((3, 4)\), employing response amplitude operators, can not predict the low frequency response shift since it considers the individual sinusoidal harmonics seperately and disregards the possibility of interference effects. This applies for single hull ships as well as for twin hull vessels. The encounter

Numbers in parentheses denote references found at the end of the thesis
spectrum usually has very little, or no energy at all, in the frequency range where the motion response is experienced. The low frequency response of double hull vessels has to be coupled with those waves of the encounter spectrum which have short lives. The energy which is contained in the apparent short life waves is, in part, associated with the low frequency vessel response. Irregularities in the response spectrum of double hull vessels may be predicted by the application of the linear theory if a deviation in the analysis is undertaken. A pseudo-sea encounter spectrum can be found by coupling the distance between the hulls to the usual encounter spectrum. The deviation in the motion analysis is accomplished through superposition of certain pairs of harmonics of different frequency to generate short lived waves.

The procedure indicates that the non-linear interactions associated with the employed encounter spectrum can be treated as linear interactions. The usual linear response operator theory may be applied to obtain the regular response spectrum for twin hull vessels. The frequency shifts will be obtained by means of an additional pseudo-sea encounter spectrum.

The technique suggested may be used to predict
the roll and the pitch response of a twin hull vessel by considering the phase shift of the encounter sea at the two hull locations.
1.1 The Theory of an Irregular Sea

A sea state is composed of irregular waves. The compound wave train which describes the sea surface may be thought to be composed of an infinite number of simple single harmonics. Consider one such wave of the compound wave train. The motion can be assumed to be induced only by the action of forces acting normal to the free surface boundary of the fluid. Under this assumption the motion is irrotational. The single harmonic of the compound wave train may then be treated by potential theory.

The velocity, $\vec{V}$, of a fluid particle is given by,

$$\vec{V} = -\varphi_x \overrightarrow{e_x} - \varphi_y \overrightarrow{e_y}$$  \hspace{1cm} I.1)

where $\overrightarrow{e_x}$ and $\overrightarrow{e_y}$ are unit vectors in the x and y direction respectively, with

$$u = -\varphi_x = \psi_y$$  \hspace{1cm} I.2)

and

$$v = -\varphi_y = -\psi_x$$  \hspace{1cm} I.3)

where $\varphi$ is the velocity potential, $\psi$ is the stream function, and $u$ and $v$ are the horizontal component and the vertical component, respectively, of the velocity.

An additional relationship is available from
the non-steady form of the Bernoulli equation,
\[
y = \frac{1}{g} \varphi_t - \frac{1}{2g} \left[ u^2 + v^2 \right]
\]  
(I.4)

A first order approximation may be introduced if the
wave amplitude is small. If the wave amplitude is small:
equation I.4) may be rewritten as,
\[
y = \frac{1}{g} \varphi_t
\]  
(I.5)

Under the first order approximation the coordinate
system may be shifted from the free fluid surface
to a point on the still water level plane (Figure 1)).
If equation I.5) is differentiated with respect to time and
substituted into the free surface boundary condition the
condition becomes,
\[
\left[ g \varphi_y + \varphi_{tt} \right]_y = 0
\]
(I.6)

which is a modified surface boundary condition under
the first order approximation. The other available
boundary condition is,
\[
v \biggl|_{y = -\infty} = -\varphi \biggl|_{y = -\infty} = 0
\]  
(I.7)

The solution satisfying the continuity equation and
equations I.6) and I.7) is,
\[
\varphi = ac \exp(ky) \sin k(ct - x)
\]  
(I.8)

where
\[
a = \text{the amplitude of the wave}
\]
\[
k = \text{the wave number} = \frac{2\pi}{\lambda} = \frac{w^2}{g}
\]
\[ c = \text{the wave celerity} = \frac{\lambda}{T} \]

\[ \lambda = \text{the wave length} \]

\[ w = \text{the wave frequency in radians per second} \]

\[ g = \text{the acceleration of gravity} \]

\[ T = \text{the wave period in seconds} \]

The free fluid surface is described by,

\[ y = a \cos k(ct - x) \quad \text{I.9} \]

If the location of the origin of the coordinate system

is shifted by,

\[ x = x - \frac{\pi}{2k} \quad \text{I.10} \]

the free surface may be described by,

\[ y = a \sin k(ct - x) \quad \text{I.11} \]

The potential energy contained in a single traveling

surface wave over one wave length is given by,

\[ \text{PE} = \frac{\varphi g}{2} \int_0^\lambda y^2 \, dx \quad \text{I.12} \]

or,

\[ \text{PE} = \frac{\varphi g a^2}{2} \int_0^\lambda \cos^2 k(ct - x) \, dx \quad \text{I.13} \]

The kinetic energy over one wave length is (5),

\[ \text{KE} = \frac{\varphi g a^2}{2} \int_0^\lambda \sin^2 k(ct - x) \, dx \quad \text{I.14} \]

The total energy per wave length is given, then, as

\[ E = \frac{\varphi g a^2 \lambda}{2} \quad \text{I.15} \]

The energy per unit area of the sea is,

\[ E = \frac{\varphi g a^2}{2} \quad \text{I.16} \]
The rate at which energy is transferred in the direction of wave propagation can be determined by calculating the rate at which work is being done on the fluid contained to the right of a vertical control plane if the direction of propagation of the wave is to the right, as,

\[ E_t = \int_{-\infty}^{0} p u \, dy \quad \text{I.17) } \]

where \( p \) (6) is the pressure. Or,

\[ E_t = \frac{\varrho g a^2 c}{2} \cos^2 k(ct - x) \quad \text{I.18) } \]

The mean value of equation \text{I.18) } is,

\[ E_t \bigg|_{\text{mean}} = \frac{\varrho g a^2}{2} \cdot \frac{c}{2} \quad \text{I.19) } \]

The rate of energy transfer proceeds at a velocity which is half the wave celerity.

It has been observed that when a group of waves of approximately the same wave length is moving over deep water that the group velocity is less than the wave celerity. A single wave advances through the group attenuating as it approaches the front of the group while another single wave enters the rear of the group. Assume that a disturbance is introduced into a quiescent fluid surface which is capable of generating waves of different frequencies. These waves will tend to sort out into groups.
The wave length can be considered to be a function of time and position relative to the initial impulse. If the observer travels with the group and if \( U \) is the group velocity, then,

\[
\lambda_t + U \lambda_x = 0 \tag{I.20}
\]

If the observer travels with the waves composing the group rather than with the group, then,

\[
\lambda_t + c \lambda_x = \lambda_c x \tag{I.21}
\]

Combining equations I.20) and I.21) yields,

\[
U = \frac{c}{2} \tag{I.22}
\]

The group velocity is half the wave celerity.

It was shown in equation I.16) that the energy contained in a single surface wave is proportional to the amplitude of the wave squared. The sea state may be represented by an infinite sum of harmonics of various amplitude and of various frequency. The energy in the component waves would be proportional to their amplitudes squared. The spectral energy distribution is given by,

\[
\Delta a^2 = A^2(T) \Delta T \tag{I.23}
\]

where \( A^2(T) \) is the spectral height distribution, \( \Delta T \) is a spectral band width, and \( \Delta a^2 \) is associated with a band of wave amplitudes of approximately the same height (Figure 2)).
The predominate wave height over a spectral band of
\[ T - \frac{\Delta T}{2} < T < T + \frac{\Delta T}{2} \]
where \( T \) is the characterizing period associated with the spectral group of waves which possess a period of approximately \( T \), is,
\[ a = \sqrt{A^2(T)\Delta T} \quad \text{(I.24)} \]
In the limiting process where the sea state is presumed to be composed of an infinite number of constituent waves the wave amplitude of an individual harmonic is given by,
\[ a = \sqrt{A^2(T)dT} \quad \text{(I.25)} \]
Neumann (7) observed differing sea states over an extensive period of time. He correlated his observations with those of other investigators and postulated that the relation between wave height distribution and the associated period is,
\[ A(T) = 0.219 T^2 e^{-2.438T^2/U^2} \quad \text{(I.26)} \]
where \( A(T) \) is in meters, \( T \) is in seconds, and \( U \) is in meters per second. To determine the wave height spectral distribution \( A^2(T) \) is required and is given by,
\[ A^2(T) = C_1 T^4 e^{-4.876T^2/U^2} \quad \text{(I.27)} \]
The constituent wave amplitudes are given by,

\[ a^2 = C_1 T^4 e^{-4.876T^2/U^2} \text{ dT} \quad \text{I.28} \]

Substituting the wave frequency for its period yields,

\[ a^2 = C_2 f^{-6} e^{-4.876/f^2U^2} \text{ df} \quad \text{I.29} \]

or,

\[ a^2 = A^2(f) df \quad \text{I.30} \]

Differentiation of equation I.29) leads to (8),

\[ f_m = \frac{1}{0.785 U} \quad \text{I.31} \]

as the frequency at which the maximum ordinate value is realized, which may be considered as a positioning parameter of the spectrum.

I.2 Ship Response

The theory involved with the prediction of the behavior of a seakeeping variable is based on the assumption of the applicability of the linear superposition principle in that the response of the vessel to a compound wave train is assumed to be a summation of the responses to each individual harmonic. Vessel response prediction is predicated on a statistical approach associated with the sea encounter energy spectrum coupled with a directional encounter frequency which is a function of vessel heading (9, 10). Let a beam sea be unidirectional
and be propagating in the positive x direction towards a twin hull vessel at rest. In this case the encounter spectrum and the sea spectrum are identical. The sea surface is given by (4),

\[ y(x, t) = \int_0^\infty \sqrt{A^2(w)dw} \cos \left[ wt - \frac{w^2 x}{g} \right] \quad \text{I.32} \]

If sea state disturbances are measured at the center of gravity of the vessel the sea surface is given by,

\[ y(t) = \int_0^\infty \sqrt{A^2(w)dw} \cos wt \quad \text{I.33} \]

The integral of equation I.33) may be approximated by a finite sum,

\[ y(t) = \sum \sqrt{A^2(w) \Delta w} \cos wt \quad \text{I.34} \]

It is now proposed that the principle of linear superposition of the responses of the vessel to the wave harmonics is applicable. The response of the vessel, for example in roll, to an individual wave is,

\[ r(t) = a c(w) \cos wt + a s(w) \sin wt \quad \text{I.35} \]

The vessel response would be,

\[ r(t) = \sum \sqrt{A^2(w)\Delta w} \left[ c(w) \cos wt + s(w) \sin wt \right] \quad \text{I.36} \]

The terms,

\[ \sqrt{A^2(w)\Delta w} \quad c(w) \]

are associated with the twin hull vessel response in phase
with the forcing function, while the terms,
\[ \sqrt{A^2(w) \Delta w} \quad s(w) \]
are associated with the vessel response out of phase with the forcing function.

The ensemble expectation value of the time lagged product of, for instance, roll is given by,
\[
E \left[ r(t) r(t + t') \right] = \frac{1}{2} \sum A^2(w) \Delta w \left[ c^2(w) + s^2(w) \right] \cos wt'
\]  
(III.37)

In the limit this becomes,
\[
E \left[ r(t) r(t + t') \right] = \frac{1}{2} \int_{0}^{\infty} A^2(w) \left[ c^2(w) + s^2(w) \right] \cos wt' \; dw
\]  
(III.38)

A modified Fourier cosine transform yields the roll response spectrum as,
\[
R^2(w) = T^2(w) A^2(w)
\]  
(III.39)

where,
\[
T^2(w) = c^2(w) + s^2(w)
\]  
(III.40)

is a modified form of the usual response amplitude operator associated with roll.
II Solution Considerations

The Neumann spectrum may be employed to provide a representation of the encounter spectrum for a double hull vessel which is at rest and subjected to a unidirectional beam sea. At a position relative to the ship the compound wave train may be approximated by a modified Fourier series of the form,

\[ y = \sum \sqrt{A^2(w) \Delta w} \cos wt \]  \hspace{1cm} \text{II.1) }

or by a series of the form,

\[ y = \sum \sqrt{A^2(w) \Delta w} \sin wt \]  \hspace{1cm} \text{II.2) }

where \( y \) represents the elevation of the free sea surface at the location. The power spectrum analysis of the compound wave train reverts, in either case, to the original sea spectrum.

II.1 Test Data

Model tests on the twin hull Mohole drilling platform revealed oscillations in roll with a natural period of the order of 40 seconds when the platform was subjected to low sea states. This long natural period in roll can be associated with excessively weak restoring forces in connection with returning the vessel to a level position. Therefore, relatively small impulses can, with time, cause large excursions. During the tests
it was noted that the platform returned to a level position via many oscillations of the natural period, as shown in Figure 5) and in Figure 6).

A roll oscillation possessing a natural period of the order of 40 seconds corresponds to a response to the forcing function having a frequency of the order of 0.16 radians per second, as shown in Figure 7). The linear response amplitude operator theory would appear to be inadequate in this instance. The encounter spectrum as shown in Figure 8) is such that those waves in sea state 7 having a frequency less than 0.4 radians per second and those waves in sea state 5 having a frequency less than 0.7 radians per second possess, for all intensive purposes, insignificant energy. The linear response amplitude operator method apparently fails due to the fact that in that range where the low frequency response occurs,

$$A^2(w) = 0$$

Therefore, a modification in the motion response for twin hull vessels in an irregular sea as applied to spectral response is required. There are two possible modifications involved. One would involve a complete reconsideration of the response operator method as indicated by K. Hasselmann (1). However this non-linear approach is extremely complex and the calculations
involved in determining the compound transfer functions would be tedious and difficult. Rather than advocate this method it will be postulated that the low frequency response characteristic of twin hull vessels may be predicted from superposition considerations.

An insight to the linear response as applied to the low frequency response of a twin hull vessel may be obtained by considering a single simple surface wave. Low sea states usually contain waves of approximately the same wave length traveling in groups. The energy of such a group is constant with respect to time. For the sake of the argument it will be assumed that the energy is concentrated over one-half wave length and that the group consists of two waves. If the peak energy of a single wave is concentrated in a wave crest as it passes the first hull the first hull will experience a force upward causing an impulse to roll motion in the downwave direction. After the wave has traveled one wave length the peak energy will be in a wave trough in that the energy travels at a velocity which is one-half the wave celerity. When the second hull is in this trough it will experience a downward force causing an impulse to roll motion in the downwave direction. The total effect would tend to produce two roll motion pulses
in the same angular direction. If the restoring force associated with returning the vessel to a level position is weak, which it would be for a slightly damped twin hull vessel, the phenomenon could cause excessive excursions in roll. However the response to such a phenomenon can be superimposed and treated in the same fashion as a response in the linear response operator theory. The example utilized a "critical" wave with a wave length which was twice the distance between the hulls, or,

\[ \lambda_c = 2L \] \hspace{1cm} \text{II.3)}

The roll motion of the vessel may be considered to be tuned to this phenomenon if its natural period in roll is such that,

\[ \frac{w}{4} \text{vessel} = w \text{wave} \] \hspace{1cm} \text{II.4)}

The above reasoning, though not conclusive, would tend to give an explanation as to the cause of the low frequency response in beam seas of a twin hull vessel such as the Mohole drilling platform.

Hence, rather than considering the response of a twin hull vessel at low frequencies as a non-linear effect it will be postulated that the effect is the result of wave dispersion and can be predicted through the
application of the linear theory employing a pseudo-sea encounter spectrum generated from the characteristics inherent in the actual encounter spectrum.

II.2 Double Hull Vessel Encounter Spectrum

A compound unidirectional sea state may be generated from the Neumann spectrum. The difference in the sea surface elevation at the two hull locations, where the coordinate system is located in the vertical plane containing the center of gravity of the first hull is given by,

\[ y^* = \sum \sqrt{A^2(w) \Delta w} \left[ \sin wt - \sin \left( \frac{wt - w^2L}{g} \right) \right] \quad \text{II.5} \]

where \( L \) is the distance between the hulls of the vessel. This expression would represent the forcing function to which the vessel would respond in roll. The actual response of the vessel in roll would be a function of the characteristics of the vessel. Assuming the principle of linear superposition of the response of the vessel to individual harmonics of the sum of equation II.5) is applicable the roll would be given by,

\[ r(t) = \sum \sqrt{A^2(w) \Delta w} \left[ c(w) \left[ \sin wt - \sin \left( \frac{wt - w^2L}{g} \right) \right] \right. \\
\left. + s(w) \left[ \cos wt - \cos \left( \frac{wt - w^2L}{g} \right) \right] \right] \quad \text{II.6} \]
The spectrum which would result upon analysis of the forcing function in roll defined by equation II.5) would not possess the ability to predict a low frequency vessel response since it does not transfer energy into the low frequency range. \( A^2(w) \) still dominates the forcing function and for low frequencies,

\[ A^2(w) = 0 \]

A normalized comparison of the encounter spectrum and of the spectrum of the forcing function in roll is given in Figure 9).

On first consideration it would appear that neither of the two spectral distributions could be used in connection with the linear response amplitude operator method to predict the low frequency vessel response. However, the forcing function of equation II.5) possess a characteristic which is not apparent in the power spectrum analysis in that spectral considerations do not allow that different harmonics travel at different velocities.

The low frequency response is apparently coupled with those waves which have short lives. These short life waves must therefore be apparent waves which result from a compound effect of the predominate waves in the encounter spectrum. To approximate a sea state at
least three waves must be employed (7). If more
harmonics are used the representation will be more
accurate. It is possible to approximate the various sections
of the compound wave train by the sum of two equal
amplitude waves. From consideration of Figure 5) and of Figure 6) it can be seen that the low frequency
excursions of the tested platform are associated with
groups in the compound wave train which are characterized
by containing the most significant wave heights. A
typical such group may be closely approximated in
the time trace by,
\[ y = A \sin w_1 t + A \sin w_2 t \]  \hspace{1cm} \text{II.7) }

or by,
\[ y = A \cos w_3 t + A \cos w_4 t \]  \hspace{1cm} \text{II.8) }

For example, a group from the wave trace of sea state 5
may be approximated by,
\[ y = A \cos 1.130t + A \cos 0.960t \]  \hspace{1cm} \text{II.9) }

where \( A \) is the properly chosen constant to duplicate
the maximum amplitude associated with the group. A
normalized comparison of the sea state 5 group and the
function of equation II.9) is given in Figure 10). They
are in agreement. Consider a sea state group defined
by an equation of the form of equation II.7). The
forcing function associated with the roll takes on the form,

\[ y^* = A \sin w_1 t + A \sin w_2 t \]

\[ - A \sin \left[ \frac{w_1 t - 2\pi L}{\lambda_1} \right] \]

\[ - A \sin \left[ \frac{w_2 t - 2\pi L}{\lambda_2} \right] \]

Equation II.10) may be rearranged to yield,

\[ y^* = 2A \sin wt \cos \Delta wt \]

\[ -2A \sin \left[ \frac{wt - 2\pi L}{2} \left[ \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right] \right] \cdot \cos \left[ \Delta wt - \frac{2\pi L}{2} \left[ \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right] \right] \]

where,

\[ w = \frac{w_1 + w_2}{2} \]

II.12)

and

\[ \Delta w = \frac{w_1 - w_2}{2} \]

II.13)

It is now assumed that,

\[ \frac{2\pi L}{2} \left[ \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right] \]

II.14)

represents an insignificant phase shift with respect to the wave length associated with \( \Delta w \), and that the arithemetic mean and the log mean of,

\[ \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \]

II.15)
are equal. Equation II.11) may then be rearranged to yield the approximation,

\[ y^* = 2A \sin \omega t \cos \Delta \omega t \]

\[ - 2A \sin \left[ \omega t - \frac{2\pi L}{\lambda} \right] \cos \Delta \omega t \]

The maximum difference in the two sea surface elevations, or the maximum in the forcing function, is realized when,

\[ L = \frac{2N + 1}{2} \lambda \]

where \( N = 0, 1, 2, \ldots \).

Equation II.17) not only determines the maximum of equation II.16). It determines the values of the frequencies occurring in equation II.16) for a given distance between the hulls of a twin hull vessel. Equation II.16) may now be rewritten, employing equation II.17), as,

\[ y^* = 4A \sin \omega t \cos \Delta \omega t \]

This equation may be altered to yield,

\[ y^* = 4A \sin \omega t \cos \frac{\omega t}{n} \]

where \( n \) is any positive number. The forcing function as defined by equation II.19) possess a peculiar characteristic in that for even integer values of \( n \) there is a beat phenomenon present. For odd integer values of \( n \) the beat phenomenon does not manifest itself. The beat phenomenon refers to the fact that a particular node is preceded and followed by two crests, while the following subsequent node is preceded and followed
by two troughs. The behavior of equation II.19) for odd and even integer values of $n$ is shown in Figure 11) and in Figure 12) for various values of $n$.

It is postulated that the beat phenomenon in the forcing function causes the large excursions in roll associated with a twin hull vessel. The large departures from equilibrium are the consequence of the action of a group of waves in the compound wave train of the sea state possessing certain characteristics. Since the effect manifests itself in its strongest form for only even integer values of $n$ in equation II.19) the equation may be written to concern itself only with the beat phenomenon as,

$$y^* = 4A \sin wt \cos \frac{wt}{2n} \quad \text{II.20)}$$

where $n = 1, 2, 3, \ldots$.

It was noted that equation II.17) determines the frequencies involved to produce the effect. By considering equation II.17) in an alternate form the frequency is given as,

$$w = \sqrt{\frac{(2N + 1) \pi g}{L}} \quad \text{II.21)}$$

where $N = 0, 1, 2, \ldots$.

In addition it may be shown that equation II.17) further
requires that,

$$w_1 = \frac{2n + 1}{2n} w$$  \hspace{1cm} (II.22)$$

and that,

$$w_2 = \frac{2n - 1}{2n} w$$  \hspace{1cm} (II.23)$$

If a typical group from the sea state is represented by an approximation of the form of equation II.8) the approximation may be put into an alternate form of,

$$y^* = 4A \cos wt \cos \frac{wt}{n}$$  \hspace{1cm} (II.24)$$

where \( n = 1, 2, 3 \ldots \)

Equation II.24) is obtained by following a procedure similar to that employed to obtain equation II.19).

The forcing function as defined by equation II.24) also possess the characteristic of having a beat phenomenon associated with it. However here the beat phenomenon is such that a particular node is preceeded and followed by two crests or two troughs, while the following subsequent node is preceeded and followed by two crests or two troughs. The beat phenomenon is exactly the same at a particular node and the preceeding and following node.

In addition, as shown in Figure 13) the effect occurs for only odd integer values of \( n \). Equation II.24) may be written to concern itself only with the beat phenomenon as,

$$y^* = 4A \cos wt \cos \frac{wt}{2n + 1}$$  \hspace{1cm} (II.25)$$
where \( n = 1, 2, 3 \ldots \)

Equations II.21), II.22) and II.23) are still the governing equations in connection with the frequencies necessary for the phenomenon to occur.

The beat effect in either case tends to broaden the response spectrum and produce a skewness in the probability distribution. The interaction generates new low frequencies which are not obtainable from the total sea state consideration, and the phenomenon is predominate in the low frequency range.

Equation II.9), representing a sea state 5 group, may be rewritten in an alternate form as,

\[
y = A \cos \frac{2\pi t}{6} \cos \frac{2\pi t}{74} \tag{II.26}
\]

If the beat phenomenon manifests itself here the ratio of the two frequencies would have to be approximately an odd integer and equation II.21) would have to be satisfied. The distance between the centers of gravity of the hulls of the tested platform is 67 meters (12). In this instance the sea state 5 group would satisfy equation II.21) with,

\[ N = 1 \]

The low frequency phase shift effect would offset the low frequency band to a characterizing frequency equal to 0.17 radians per second. The associated period would
be 37 seconds. A plot of the actual response trace and the wave trace in time which precipitated the response is given in Figure 14). They are in agreement.

A similar form for a group from sea state 7 would be,

\[ y = A \sin \frac{2\pi t}{7.5} \cos \frac{2\pi t}{30} \]  \hspace{1cm} \text{(II.27)}

The sea state group is such that the low frequency beat effect occurs when the forcing function is considered, with

\[ N = 0 \]

in equation II.21). The low characterizing frequency of the band would be of the same order for the group of sea state 7 as for the group of sea state 5.

As previously mentioned, low sea states usually contain waves of approximately the same wave length traveling in groups in the compound wave train representing the sea state. The length of these groups is closely coupled to the band width of the sea spectrum. Any such group possessing the characteristics necessary for either of the two type of beat phenomena to occur in the forcing function would tend to precipitate a low frequency response in the roll of the vessel. Hence a summation of all such groups in the sea state possessing the characteristics
would be necessary. In general this summation would result in a spectral distribution in the low frequency range. If the low frequency spectral distribution is added to the normal spectral distribution of the encounter spectrum the pseudo-encounter spectrum thus generated can be employed in connection with the linear response amplitude operator method to predict the spectral response of the vessel. Hence the method of linear superposition is still applicable.
Conclusions

From this research the following conclusions may be drawn:

1. The response amplitude operator method for predicting the spectral distribution of roll and pitch for a twin hull vessel may be used if the encounter spectrum is replaced by a pseudo-encounter spectrum. The method of employing such a pseudo-encounter spectrum has been shown to be applicable. The spectrum is obtained from the actual sea state by considering wave groups of finite length. The method employed uses simple surface wave theory.

2. The phase shift in the low frequency motion responses in roll and pitch associated with twin hull vessels are shown to be a result of wave groups in the sea state of finite length.

3. The low frequency response of twin hull vessels to an irregular sea is not a non-linear phenomenon. The response of the vessel to the wave groups can be obtained through the method of linear superposition.
4. For all intensive purposes the low frequency response of a twin hull vessel will be situated such that its maximum will be banded about a frequency of the order of one-quarter the frequency associated with the distance between the hulls of the vessel.

5. The low frequency response phase shift is the result of a beat phenomenon associated with the forcing function related to roll and pitch. The conditions necessary for the beat phenomenon to manifest itself are coupled with wave groups found in the compound wave train of the sea state.

6. The Neumann spectrum may be decomposed into a finite number of waves in the form of a Fourier sum to generate the approximate sea state wave train to determine if the beat phenomenon will manifest itself.
Appendix
SINGLE TRAVELING SURFACE WAVE
COORDINATE SYSTEM

\[ a = \text{wave amplitude} \]
\[ \lambda = \text{wave length} \]
\[ u = \text{horizontal velocity component} \]
\[ v = \text{vertical velocity component} \]
\[ x = \text{abscissa distance} \]
\[ y = \text{ordinate distance of free fluid surface} \]

FIGURE 1)
SPECTRAL ENERGY DISTRIBUTION IN AN IRREGULAR SEA STATE

Neumann Spectrum

\[ T = \text{Spectral Band Characterizing Period} \]

\[ \frac{\Delta T}{2} \]

Period in secs

FIGURE 2)
APPROXIMATION TO THE SEA STATE
BY SIX SURFACE WAVES
NEUMANN SPECTRUM

Wave Amplitude given by,

\[ a = \sqrt{A^2(w) \Delta w} \]

**Figure 3**

Spectral Density \( A^2(w) \)

\( A^2(w)_1 \)

\( A^2(w)_2 \)

\( A^2(w)_3 \)

\( \Delta w \)

Frequency in radians/sec

\( w = \) spectral band characterizing frequency
COMPARISON BETWEEN THE NEUMANN SPECTRUM AND THE REGENERATED POWER SPECTRUM

Dashed curve represents the Neumann spectrum

**FIGURE 4**
Wave

Stevens Series III
(Sept. 1965)

Run No. 699

6.5 Draft ~ 90° Hdg.
SS #5
Fins Only

Roll Angle

6.75 Deg./Div.

Pitch Angle

0.75 Deg./Div.

Heave Displacement
WAVE - .533 FT/Div.

ROLL ANGLE - 300 DEG/Div.

PITCH ANGLE - .523 DEG/Div.

HEAVE DISPLACEMENT - .208 FT/Div.

65' DRAFT - 10' HAS
SS # 7
FINS ONLY
SPECTRAL DISTRIBUTION OF MODEL PLATFORM RESPONSE IN ROLL
65 ft Draft 90° HDG
SI Series VII
Run 698
Run 699
Horizontal Fins Only

Sea State 7

S. S. 7

S. S. 5

S. S. 5

Frequency in radians/sec

FIGURE 7)
WAVE SPECTRA SI SERIES VII TESTS
65 ft Draft 90° HDG

Run 698
Run 699
Horizontal Fins Only

FIGURE 8)
NORMALIZED COMPARISON BETWEEN THE NEUMANN SPECTRUM AND THE SPECTRAL DISTRIBUTION OF THE FORCING FUNCTION FOR ROLL FOR A TWIN HULL VESSEL IN A BEAM SEA

Lag = 0.00667 cycles/sec

Dashed curve represents the Neumann spectrum

Factor = 0.549
Factor = 2.2

FIGURE 9)
NORMALIZED COMPARISON OF A WAVE GROUP FROM SEA STATE 5
SI SERIES VII TEST No. 699 AND THE APPROXIMATION TO THE
GROUP

\[ y = A \cos 1.130t + A \cos 0.960t \]

Approximate wave group

Sea State 5 Wave Group

FIGURE 10)
PLOT OF $\sin \omega t \cos \frac{\omega t}{n}$ FOR EVEN INTEGER VALUES OF $\frac{n}{\omega}$

BASE FUNCTION = $\sin \omega t$

$n = 2$

$n = 4$

$n = 6$

FIGURE 11)
PLOT OF $\sin \omega t \cos \frac{\omega t}{n}$ FOR ODD INTEGER VALUES OF $n$

BASE FUNCTION = $\sin \omega t$

$n = 3$

$n = 5$

$n = 7$

FIGURE 12)
PLOT OF $\cos wt \cos \frac{wt}{n}$ FOR ODD INTEGER VALUES OF $n$

BASE FUNCTION = $\cos wt$

$n = 3$

$n = 5$

FIGURE 13)
NORMALIZED COMPARISON BETWEEN MODEL PLATFORM RESPONSE IN ROLL
AND THE APPROXIMATE WAVE GROUP

SEA STATE 5

\[ y = A \cos 1.130t + A \cos 0.960t \]

FIGURE 14)
References
References


