McASHAN, Michael Sanford, 1939—
A MECHANICAL STUDY OF THE FORCES ON
MOVING INTERMEDIATE–STATE REGIONS IN
A TYPE I SUPERCONDUCTOR.

Rice University, Ph.D., 1966
Physics, solid state

University Microfilms, Inc., Ann Arbor, Michigan
RICE UNIVERSITY

A Mechanical Study of the Forces on Moving Intermediate-State Regions in a Type I Superconductor

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Thesis Director's signature: [Signature]

Houston, Texas
May 1966
The author wishes to acknowledge with thanks the guidance of Dr. Houston, under whose supervision this work was carried out, and the help of several illuminating discussions with Mr. Michael Melich, of this department.

This research has been supported in part by grants from the National Aeronautics and Space Administration.
ABSTRACT

This paper describes experiments performed on a system consisting of a superconducting tin disc suspended to oscillate about its axis in a torsion pendulum. This is acted upon by a pair of stationary coils which apply two local regions of magnetic field at points diametrically opposite on the disc and directed normal to its surface. As the system oscillates, the intermediate-state regions produced by the coils are moved through the superconductor, and the resulting forces are determined from the motion.

The results can be understood phenomenologically in terms of the same ideas that have been applied to flux motion in type II superconductors. For small velocities, there is an exponential dependence of the velocity of the flux motion upon the applied force, indicating that flux creep is occurring. For larger forces and velocities, the dependence is linear, showing flux flow.

Because of a dependence of the behavior of the system on history, however, no very firm conclusions have been reached concerning the dependence of the parameters of the phenomenological theory on the properties of the superconductor or on the magnetic field. The exception to this is in the measurements of the flow viscosity. These show a reasonably well-defined, although unexplained, field dependence and a probable dependence on the first power of the normal-state conductivity.
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I. Introduction

A. General Properties of Superconductors

Considering only the gross magnetic properties, superconductors may be classified into two types usually designated by the roman I and II. The differences are illustrated in the idealized magnetization curves in figure I-1.

![Figure I-1: Ideal Magnetization Curves](image)

The magnetization of the type I superconductor is equal to $-\frac{1}{4} \pi H$ so that $B = 0$ inside the material for $H$ less than the thermodynamic critical field $H_c$. At $H_c$ the magnetization drops to zero, and the transition to the normal state occurs. In the type II superconductor, however, the flux of $B$ is excluded only up to $H_{c1}$, the lower critical field, where the magnetization drops abruptly. It thereafter decreases slowly with increasing $H$, disappearing at the upper critical field $H_{c2}$. 
Central to the understanding of these phenomena is the interphase surface energy. It was early realized\textsuperscript{1)} that the $B = 0$ state of the type I superconductor cannot be the lowest energy state unless a positive surface energy is postulated, and the most generally successful phenomenological theory of superconductivity, that of Ginzberg and Landau\textsuperscript{2)}, was originally formulated to include this feature. The solutions of Landau's equations show that the superconducting phase presents a coherence effect in that there is a characteristic distance $\xi$ over which the order parameter characterizing the superconducting state is found to vary. Although strictly speaking both the penetration depth $\lambda$ and this coherence length in the Landau theory are temperature-dependent, it is usual to speak of the coherence length as being given by $\xi \sim \lambda / \kappa$, where $\lambda$ is the empirically determined value and $\kappa$ is a temperature-independent parameter.

To see that coherence results in an interphase surface energy, consider the unit area of boundary illustrated schematically in figure I-2.

\begin{itemize}
  \item 2. Ginzberg and Landau, \textit{JETP} \textbf{20}, 1064 (1950). This theory has undergone great development in recent years. The reader is referred to a general work such as \textit{Superconductivity} by E. A. Lynton (Methuen & Co., London, 1964) for details.
\end{itemize}
In order to create this boundary, a volume of the superconductor approximately equal to \( \xi + z_o \) must be raised into the normal state, which process requires free energy in amount \( H_c^2 / 8\pi (\xi + z_o) \). By admitting magnetic field to the volume \( (\lambda + z_o) \) however, about \( H_c^2 / 8\pi (\lambda + z_o) \) of free
energy is recovered, so the surface energy to be associated with the presence of the boundary is about \( Hc^2/8\pi \) \( (\frac{3}{2} - \lambda) \).

The conclusion from this argument is that the surface energy should be positive for values of \( \kappa \) less than one and negative for values greater than one. Careful solutions of the Ginzberg-Landau equations show that the zero of surface energy is at \( \kappa = 1/\sqrt{2} \) rather than one, and they confirm that the \( \kappa < 1/\sqrt{2} \) case corresponds to type I superconductivity. The solutions also show that the \( \kappa > 1/\sqrt{2} \) case is to be associated with the ideal type II behavior pictured in figure I-1.

Because of the negative surface energy, the flux penetration of a type II sample in the region between \( H_{c1} \) and \( H_{c2} \) would be expected to occur on as small a scale as is permitted by flux quantization. According to the Ginzberg-Landau theory, the minimum-energy configuration is one in which the flux passes through a regular array of normal filaments of negligible thickness which are arranged parallel to the external field. Along the filaments, the order parameter is zero and rises as quickly as possible, that is, over a distance of about \( \frac{3}{2} \), with increasing distance from the center. The flux passing along the filament extends over a radius of \( \lambda \), and the associated supercurrent forms a vortex line along the field direction. Since the vortices interact appreciably only when at a distance of \( \lambda \) or less, at \( H_{c1} \), where vortex
formation becomes energetically favorable, the superconducting body becomes filled with an array of vortex lines about \( \lambda \) apart, and the magnetization drops suddenly. As the field increases from \( H_c \), the magnetization drops more slowly since the interaction energy of the vortices increases with their density.

In most type II superconductors these ideal features are rarely observed due to flux pinning by crystal defects. Type II superconductors as a class are somewhat difficult to produce in the form of perfect, single crystals. The various defects create localized regions in the lattice in which, due to shortening of the electron mean free path, \( \lambda \) is decreased and \( \lambda \) increased, resulting in a small region of lowered surface energy. Thus the flaws attract and hold vortices, distorting the magnetization curve and obscuring the predicted behavior. It is important to notice that the interaction energy of the vortex lines is fairly large so that local density fluctuations in the vortex structure are energetically unfavorable. Thus though a flaw may interact with a single line, the resultant pinning force is distributed to the whole surrounding structure.

The magnetization curves in figure I-1 have been idealized also in that they are drawn for bodies of zero demagnetization factor. The properties of a more general superconducting body are illustrated by an ellipsoid of
demagnetization factor D in a field Ho applied parallel to its axis of symmetry. If Ho is increased from zero it is easy to show that the magnetization appearing in the body to exclude the flux is \(-Ho/4\pi(1-D)\), and that the field at its equator is \(Ho/(1-D)\). Thus when \(Ho/(1-D)\) becomes equal to the critical field, the magnetization is \(-Ho/4\pi\) and flux must enter the body. In a type II superconductor, a vortex structure is formed at this point, but in a type I material, the positive surface energy prevents the flux entering on any such small scale. Instead, as was first suggested by London\(^4\), the body becomes divided into laminar normal and superconducting regions the size and arrangement of which are dependent on the size and shape of the body. As the field applied to this structure increases, the normal lamirae widen and the magnetization decreases until the bulk critical field is reached, at which point the body is entirely normal. This state of flux penetration in a type I superconductor is called the intermediate state.

There have been some theoretical studies of the intermediate state and a good deal of experimental work\(^5\), and

3. See, for example, Lynton, op. cit., p. 24.
5. See Lynton, op. cit. or Shoenberg, D., Superconductivity (Cambridge University Press, 1960), for the details. It is unfortunate that a good deal of this interesting work is in untranslated Russian journals.
the two are in order of magnitude agreement. The experiments show clearly the effects of flux pinning centers. The laminar boundaries are attracted to flaws just as are vortex lines, and the result is that in the observed intermediate-state structures, the laminae have a wrinkled and folded appearance. The reader is referred to the excellent powder photographs in Lynton 6). These distortions do not seem to greatly affect the characteristic width of the laminae, however, which for macroscopic bodies varies between $10^{-2}$ and $10^{-1}$ cm. The theory indicates that this width should decrease as the size of the body decreases, a situation which is to be contrasted with flux penetration in type II superconductors. In that case the scale is on the order of $10^{-5}$ cm, and the size of the vortices is independent of the size of the body. It is interesting to note with regard to this size dependence, that Tinkham 7) has recently shown that in thin films of type I oriented normal to a field, a vortex structure similar to that of type II is to be expected. Presumably at some thickness, there is a transition between the laminar and the vortex structures.

B. Flux Motion in Type II Superconductors

The present understanding of the technologically im-

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Important field of hard superconductors is based upon their identification as superconductors of the second kind with a very high density of pinning centers. In a type II superconductor, transport currents exert a Lorentz force on the vortex line structure, which, if it becomes large enough to overcome the pinning, can set the whole structure into motion. The importance of this lies in the fact that such motion is inherently a dissipative process and results in the magnetic field dependent resistance or resistive state observed in hard superconductors.

To see that viscous flux flow under the Lorentz force gives the proper type of resistive behavior, consider the force on a unit length of a flux line carrying a flux \( \Phi_0 \). This is

\[
F = \frac{1}{c} \int (\mathbf{j} \times \mathbf{B}) \, dV = \frac{1}{c} \mathbf{j} \Phi_0
\]

Assume that when this force exceeds the pinning force \( F_0 \), viscous flow defined by the equation

\[
\eta \mathbf{v} = F - F_0 \quad \text{(flux flow)}
\]

occurs. Here \( \eta \) is the viscosity and \( \mathbf{v} \) is the resultant velocity of the flux line. The power dissipated by \( n \) such lines per unit volume under these conditions may be set equal to the power input of the system \( \mathbf{J} \cdot \mathbf{E} \) where \( \mathbf{E} \) is the electric field developed across the sample. Thus
\[ J \cdot E = n \left( \frac{\phi_0/c - F_0}{\eta} \right) \phi_0/c. \]

\[ E = (\phi_0/c - F_0) B/\eta c \quad \text{and} \quad \zeta = dE/dJ = B_0/\eta c^2 \]

where \( n \phi_0 \) has been set equal to \( B \), the field applied to the sample. These expressions have been well verified in the work of Kim, Hempstead and Strnad. These authors also find an additional effect, however. When \( E \) and \( J \) are small enough, the relationship between them becomes exponential rather than linear. This dependence has been explained by Anderson in his flux creep theory. He assumes a pinning center which is of linear dimension \( \delta \) and binding energy \( P_0 \). The force on a line trapped at this site is \( F_0 \delta \) and the energy associated with this force is about \( F_0^2 \). Thus the total binding energy of the line is \( P_0 - F_0^2 \), and there is a probability of its escaping from this flaw and hopping to the next. Anderson writes this as


\[ V = mC e^{-\frac{(P_0 - B^2 F)}{kT}} \]  

(flux creep)

where \( \delta^2 \) has been replaced by a parameter \( \beta \) and \( C \) is a constant. This is the expression for flux creep, and the experiments verify this form.

There is another class of experiments which are directed toward proving that moving flux can generate voltages in superconductors. This is the work of Giaever \(^{11} \) and of Solomon \(^{12} \) in which two thin films of superconductor are separated by an insulating layer thin compared to the size of a vortex line. A field is applied normal to the surface and a current is passed through one of the films. This one enters the resistive state, and the flux begins to move across it, but the other film is so close that the vortex lines in the two are coupled. Thus the lines are moved across the second film and a voltage can be measured in it also. Giaever used a type II superconductor, but Solomon used a type I.

Similar to these is the experiment of Pearl \(^{13} \) in which the flux is moved along a thin strip of type II material.

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mechanically, by a moving magnet, and the voltage developed across the ends is measured. All three of these experiments give good evidence for the correctness of the flux flow model of the resistive state.
II. Apparatus

A. Purpose and Method of the Measurements

At the time that this experimental work was undertaken, the ideas of flux creep and flow were current, but there was a lack of any very direct experimental evidence that flux lines actually move in this way or are responsible for the resistive states of type II superconductors. The recently reported work of Giaever, of Solomon, and of Pearl, which were described briefly above, fill this gap to some degree. This present work is a mechanical study of the forces involved in moving flux through a superconductor and so provides a different approach to the same problems. However since a type I superconductor is used here, the results cannot be directly compared to the previous work.

The method used in making these measurements is as follows: a disc-shaped sample is suspended to oscillate about its axis in a torsion pendulum, and a localized magnetic field is applied normal to its surface in two spots by means of a pair of stationary coils. The field passes through intermediate-state regions in the disc, which as the pendulum oscillates, are moved back and forth. The forces due to this flux motion are determined by observing their effects on the oscillations.
B. Apparatus

a) The Pendulum Systems

Figure II-1 shows the coil and disc arrangement, and Figure II-2 gives a more general view of the apparatus. Also included are two photographs. The first of these, figure II-3, shows the coils with the disc in position between them, together with their glass container separated at the ground joint. The surface of the disc is quite shiny, and the reflections are somewhat confusing, particularly because the polish extends only part of the way up on the central post. However, reference to figure II-1 helps to clarify this. When this part of the apparatus is assembled for operation, the coil support rests in a holder which can be seen in the bottom of the container on the right, and the glass tube of the pendulum passes through the tube extending from the left of the picture. Figure II-4 is a view of the disassembled apparatus showing on the left the helium dewar, in the center the pendulum container, and on the right a pendulum system.

Two discs in five suspension systems were used in the measurements. Table II-1 below gives their properties.
Figure 11-1

Pendulum System

- Fiber
- Mirror
- Glass Tube
- Coil
- Tin Disc

Side View
Top View
Figure II- 2
View of the Apparatus
(not to scale)
Table II-1

<table>
<thead>
<tr>
<th>System number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disc</td>
<td>A</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>$I (\text{cm}^2)$</td>
<td>225</td>
<td>253</td>
<td>10900</td>
<td>10900</td>
<td>3390</td>
</tr>
<tr>
<td>$K (\text{dyne-cm./rad.})$</td>
<td>4.92</td>
<td>4.92</td>
<td>751</td>
<td>279</td>
<td>43.1</td>
</tr>
<tr>
<td>$T (\text{sec})$</td>
<td>42.52</td>
<td>45.05</td>
<td>23.92</td>
<td>39.27</td>
<td>55.70</td>
</tr>
<tr>
<td>$P/I (\text{sec}^{-1})$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>at $4.2^\circ \text{K.}$</td>
<td>$1.95 \times 10^{-4}$</td>
<td>$1.78 \times 10^{-4}$</td>
<td>$2.0 \times 10^{-4}$</td>
<td>$2.35 \times 10^{-4}$</td>
<td>$6.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>at $3.455^\circ \text{K.}$</td>
<td>$1.38 \times 10^{-4}$</td>
<td>$1.33 \times 10^{-4}$</td>
<td>$2.1 \times 10^{-4}$</td>
<td>$2.35 \times 10^{-4}$</td>
<td>$8.1 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Critical damping current (A) at $4.2^\circ \text{K.}$: 1.48 0.365 3.28 3.28 1.18

Disc A is made of "Baker Analyzed" tin, not annealed, but electropolished. Disc B is made of Vulcan "high purity grade" tin (total impurity content not to exceed .002%), annealed for 12 days in vacuo at $210^\circ \text{C.}$ and then electropolished. Both discs are 1/16 inch thick and 2-5/8 inches in diameter. They were first cast in vacuo, and then carefully machined to size. Each has a post in the center about 3/8 inch in diameter and 5/8" high with a hole drilled to receive the glass rod of the pendulum. The other quantities in this table are parameters in the free pendulum equation of motion.
\[ I \ddot{\theta} + P \dot{\theta} + K(\theta - \theta_0) = 0, \]

and the critical damping current is that current in the coils required to critically damp the system in the normal state.

Each disc has permanently attached by means of epoxy glue a 3mm diameter pyrex tube 61 cm. in length with a 1/2 inch diameter galvanometer mirror fixed near the top and a fitting with a setscrew above that. Systems 1 and 2 were used without auxiliary moments of inertia, so the torsion fiber was attached directly to the fitting at the top of the pendulum. In systems 3-5, however, small brass tubes were attached to the top of the pendulum and these, through a stainless steel bellows, to brass moments of inertia, and the fibers were fixed to the tops of these.

The fibers used in these systems are of tungsten wire of 3, 5, 7.7 and 10 mil diameter and about 70 cm. length. Their attachment is by means of epoxy into 16 mil holes in brass end-pieces. These are in turn fixed by means of screws to the pendulum at one end and a rod in the top of the apparatus at the other. This rod can be adjusted slightly in the vertical direction.

b) The Coils

The coils are toroidal in shape and wound with two layers of 10 mil diameter niobium-zirconium wire. The diameter of the pole faces is 3/8 inch, and the gap is 0.18
inch. These are arranged on a stand with 1-1/2" between the centers of the pole faces. The maximum field produced in the gap is 34 gauss per ampere, which was measured with a Hall-effect gaussmeter manufactured by F. W. Bell, Inc. The average field is 28 gauss per ampere, which was measured with a search coil and ballistic galvanometer. The current supply for these coils is a current-regulated power supply manufactured by Princeton Applied Research, Inc. This instrument has an integral potentiometer with which the current was measured.

c) Cryogenic Apparatus

The pendulum system is entirely isolated from the helium bath by a glass container with a ground glass joint sealed with silicone stopcock grease. Although the pressure in this pendulum chamber can be maintained at any value, it was found that if this pressure was lowered much below that of the helium bath, severe thermal oscillations resulted. It was therefore customary while making measurements to leave a valve connecting the two regions open a crack. An air-leak manostat was used to regulate the pressure above the helium bath, which was measured with a mercury manometer.

C. Data Recording Equipment

The optical lever used with this apparatus is 109.5 inches long, and the scale is 125 inches long. This scale is read either by means of a telescope, which was used to
record end points of the motion, or by a 16 mm camera. The camera has no shutter and is controlled by a synchronous motor which advances the film once a second and flashes four xenon flash tubes on the scale to make the picture. The shadow of a cross-hair immediately in front of the film provides a fiducial line for the reading, and a flash tube and counter, also operated by the motor, with a stationary galvanometer mirror records a number on each picture.

The error to be associated with readings made by either method is about plus or minus .01 scale inch or $4.6 \times 10^{-5}$ radians. This is also the accuracy with which the position of the top of the fiber can be reset by means of a second telescope and mirror after exciting the pendulum.

D. Data Analysis

The data with which one deals are observations of the displacement of the pendulum, either end points, or from the camera measurements, readings at one second intervals. The quantity which one wishes to determine from these measurements is the force on the superconductor due to the magnetic field. It was expected originally that the deviations of the motion from damped harmonic would be small, and that the oscillator equation would provide the basis for calculation of the force. Although things have proven a bit more complicated than this, casting the results in a form that shows these deviations from harmonic behavior is
a good starting point for analysis.

This is easily done for the end-point measurements as follows: First consider the maximum displacements $\Theta_n$ of a damped harmonic oscillation. These are given by

$$-\frac{P}{2I} \left( t_0 + \frac{nT}{2} \right)$$

$$\Theta_{(n+1)} - \Theta_0 = Ae^{(-1)^n}$$

which is the solution of the equation

$$I \ddot{\Theta} + P \dot{\Theta} + K(\Theta - \Theta_0) = 0$$

for times $t = nT = 2\pi n(K/I-(P/2I)^2)^{-1/2}$. The equilibrium position $\Theta_0$ is inconvenient to measure independently of the $\Theta_n$, so it is best removed from the equation by taking the difference of successive end points.

$$\Theta_{(n+1)} - \Theta_{(n+2)} = A(-1)^n e^{-P/2I(\tau + T/2)} (1 + e^{-P T/4I})$$

Therefore

$$ln(\Theta_{(n+1)} - \Theta_{(n+2)}) = -P T/4I + c (\text{independent of } n),$$

and a plot of $-ln(\Theta_{(n+1)} - \Theta_{(n+2)})$ against even values of $n$ should be a straight line of slope $P T/2I$.

All of the end point data have been treated in this way, and in the cases in which the curve is straight, $P/I$ has been determined by the method of least squares. Often, however, this plot shows a curved line, and the period is
not constant for the successive swings. The end-point data
presented below are in the form of either $-\ln(\theta_{n+1}\theta_{n+2})$
plots, which quantity is called $\Delta \theta$, or plots of a quantity
called $P/I$ (magnetic), which is the difference between the
measured $P/I$ and the residual $P/I$ in zero field, against the
current in the coils, whichever is appropriate.

The analysis of the camera data presents a somewhat
larger problem than the end-point data. There are usually
several hundred measurements of displacement, again called
$\theta_n$, in each series, and these are treated in the following
way: From the $\theta_n$, values of $\theta$ and $\dot{\theta}$ are calculated by fit-
ting a second order polynomial to each three points, and
then calculating the derivatives at the middle. The result
is

$$\dot{\theta}_n = \frac{1}{2} \left( \theta_{n+1} - \theta_{n-1} \right)$$

$$\ddot{\theta}_n = \left( \theta_{n+1} + \theta_{n-1} - 2 \theta_n \right)$$

For each pair of $n$ values, one could calculate values for
$X_n$ and $Q_n$ in the equation

$$\ddot{\theta}_n + X_n \dot{\theta}_n + Q_n (\theta_n - \theta) = 0$$

In practice, however, it proves better to do a bit of curve
smoothing at this point in the analysis by making a weighted
least squares fit of $X_n$ and $Q_n$ to the fifteen sets of values
between \((n-7)\) and \((n+7)\). The value of \(\theta_0\) for a series is extrapolated from the last few end points after the oscillation has been allowed to die down. The weighting factor was chosen to have the form

\[ W(k) = 1 - (\frac{k-n}{\delta})^2 \]

where \(k\) is an index running from \((n-7)\) to \((n+7)\).

The result of this analysis is a plot of \(X_n\) and \(O_n\) as functions of \(n\) or time. Since the second derivative of the displacement is proportional to the force on the pendulum, calculation of \(X\) and \(O\) in this way is a resolution of this force into a damping and a restoring component respectively, and in the case of a harmonic oscillation, \(X\) and \(O\) should be constants in time.

The camera data in the following section are all presented in the form of plots of \(X\) and \(O\), and it will be seen that each has a characteristic departure from the straight line.
III. Results

A. Behavior of the System in the Normal State

Measurements were made with all five systems at 4.2°C K. in order to observe normal eddy current damping. These were end-point measurements, and in all cases, plots of ln ΔΘ were perfectly straight lines, indicating that the system in the normal state behaves very exactly as a damped harmonic oscillator. Under these circumstances it is possible to obtain values of K/I and P/I from the measurements.

The values of K/I were found to be independent of the current in the magnet coils.

The results for P/I (magnetic) are summarized in figure III-1 which is a log-log plot of the current against this quantity, and the lines drawn through the points have a slope of 2 as is expected for eddy-current damping. The x on the line for system 5 is a point measured by the camera, and figure III-8 shows the X-0 plot for these data.

B. Behavior of the System in the Superconducting State

All of the measurements were made at a temperature of 3.455 ± 0.005°C K. at which the critical field is 39 gauss or approximately 1.4 amperes in the coils. Two kinds of states were studied, either the disc was cooled with zero current or with 2 amperes. The former will be called zero-field transitions and the latter, 2 ampere transitions, and the various measurements described below will be arranged according to system number and the kind of transition.
Figure III-1
Normal Damping
a) Data from System 1

1) Zero field transition: Figure III-2 shows ln $\Delta \theta$ plots for a number of different fields. These measurements were made in the order of increasing field, and they show a characteristic progression of shapes. A list of the average periods together with some guesses as to the values of the first swing periods also appears.

ii) 2A. transitions: There were no useful measurement of this kind made with this system.

b) Data from system 2:

1) Zero field transition: Plots of ln $\Delta \theta$ are shown in figure III-3. One can see that the damping is generally much greater than for system 1, and at 300 ma., the system is almost critically damped. The measurements of the time of the first swing were made with a strip-chart recorder.

ii) 2A. transitions: It was found that this system would not oscillate until the current was reduced below 100 ma.

c) Data from Systems 3 and 4

The ln $\Delta \theta$ plots from these systems always appeared straight. It was therefore possible to reduce the data to P/I (magnetic) values, plots of which for both kinds of transitions are shown in figures III-4 and III-5.
Figure III-2  Zero-field Transitions for System I

<table>
<thead>
<tr>
<th>Current</th>
<th>T (ave)</th>
<th>T (1st swing, estimated)</th>
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</thead>
<tbody>
<tr>
<td>0.0</td>
<td>41.52</td>
<td>42.52</td>
</tr>
<tr>
<td>0.05</td>
<td>41.29</td>
<td>41</td>
</tr>
<tr>
<td>0.10a</td>
<td>38.65</td>
<td>38</td>
</tr>
<tr>
<td>0.10b</td>
<td>38.75</td>
<td>38</td>
</tr>
<tr>
<td>0.15</td>
<td>31.00</td>
<td>37</td>
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<tr>
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<td>25.25</td>
<td>36</td>
</tr>
<tr>
<td>0.30</td>
<td>19.62</td>
<td>35</td>
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<td>0.90</td>
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<td>1.78</td>
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</tr>
</tbody>
</table>
Figure III-4
Damping in System 3

P/I (magnetic)

2 Amp. Transition
Zero-field Transition
Normal-state Behavior

Current (amp.)
Figure III-5
Damping in System 4

P/I (magnetic)

Current (amp.)

2 Amp. Transition
Zero-field Transition
Normal-state Behavior
The periods measured for these systems varied also.

Plots of a quantity, $\Delta K/I$, which is the measured value of $K/I$ for a particular field minus $K/I$ for the free system, are shown in figure III-6 for the two kinds of transitions in system 4.

d) Data for System 5

1) Zero-field transitions: Figure III-7 gives $\ln(\Delta \theta)$ plots for some end-point data. In addition, there are good camera data for the currents 0.2, 0.3, 0.4, 0.6, and 2.0 amperes. Because all of these sets of data present more or less the same features, only three of the $X-0$ plots are reproduced here, those for 0.3, 0.6, and 2 amperes in figures III-8, 9 and 10. These figures also show the amplitudes of the pendulum motion and the times at which $\theta = \theta_0$. Figure III-11 shows plots for two successive measurements under the same conditions.

ii) 2A. transitions: All of these measurements were made by the camera. Because the system was overdamped, the readings were made by moving the top of the torsion fiber through some angle and then recording the subsequent motion. Figure 12 shows displacement against time for the system at 2A. The units of the ordinate are inches on the scale, so that this is a plot of the data on the film. Measurements were also made at 1.5, 1.0, and 0.6 amperes, all of which have the same appearance as figure III-12. When the current
Figure III - 6

Period Changes in System 4

$\Delta K/K = 0.39$
Figure III-7

Zero-field Transitions for System 5

<table>
<thead>
<tr>
<th>Current</th>
<th>T (ave)</th>
<th>T (1st swing)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>55.68</td>
<td>55.68</td>
</tr>
<tr>
<td>0.10</td>
<td>53.90</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>50.60</td>
<td>54.5</td>
</tr>
<tr>
<td>0.30</td>
<td>50.60</td>
<td>54.3</td>
</tr>
<tr>
<td>0.40</td>
<td>49.23</td>
<td>54.3</td>
</tr>
<tr>
<td>0.60</td>
<td>51.32</td>
<td>54.3</td>
</tr>
<tr>
<td>0.80</td>
<td>50.68</td>
<td>54.3</td>
</tr>
<tr>
<td>1.20</td>
<td>52.33</td>
<td>54.3</td>
</tr>
</tbody>
</table>
Displacement for 2A Transition in System 5

Figure 111-12
was reduced to 0.4 amperes, the pendulum showed a slight
tendency to oscillate. This is shown in figure III-13. As
the current was reduced further, the oscillations became
more pronounced superimposed upon a monotonic decay until
at .15 amperes the motion appeared to be more or less
purely oscillatory. More complete details of these data
are to be found in Table IV-1 below.
Figure III-13
Displacement Against Time at 0.4A.
IV. Discussion

A. The Mechanical Force on the Flux

At first sight, the results presented above may seem a bit confusing, and indeed the system does not behave in the way that was expected. However, it appears that this behavior can be understood in terms of the ideas of flux creep and flow in the superconductor.

To see that this is so, one must first discover what quantity in this system is the analogue of the $J \times B$ force that enters in the flux-creep theory. Assume for the moment that the pendulum is turned slightly between the magnets. The pattern of normal laminae of the intermediate state, which was initially in equilibrium between the poles of the magnets, will be moved to one side causing the field lines to be bent through a small angle $\delta$ proportional to the amount that the disc is rotated. This situation is illustrated schematically in figure IV-1.
Figure IV-1

Moving Flux Passing Through a Normal Lamina

Considering this in detail, one can see that the bending of the field lines causes the flux distribution in the normal laminae to alter, increasing the field slightly in some parts and decreasing it in others. This throws the normal-superconducting boundaries out of equilibrium, and they move until the field uniformity is re-established.
For the present purposes, however, one wishes not to consider the detailed motion of the phase boundaries, but to look at the process as an overall motion produced by a force. Therefore drawing a black box around the surface of the disc, the force transmitted to the enclosed volume may be found by integrating the stress tensor $T$ in the expression

$$\vec{F} = \oint \vec{T} \cdot \vec{dS}$$

where $\vec{dS}$ is the outward directed normal. Assuming that the field at the surface may be written $H_x = -H \cos \delta$, $H_y = \pm \sin \delta$, where the sign is negative if one is on the upper and positive if on the lower surface, and ignoring any $z$ component, $T$ is given by

$$\hat{\Sigma}_T = \frac{H^2}{8\pi} \begin{pmatrix} \cos 2\delta & \mp \sin 2\delta & 0 \\ \pm \sin 2\delta & -\cos 2\delta & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The normal is $\hat{T}$, so that the force is

$$\vec{F} = F_y = \frac{1}{4\pi} \sin 2\delta \int H^2 \, ds \tag{1}$$
In the case that the field applied by the coils is less than the bulk critical field, the field on the superconducting surface is zero and the field on the normal laminae is \( H_c \), so that this force can be written

\[
F_y = \frac{1}{4\pi} \sin 2\delta \bar{H}_c \bar{\phi}
\]

where \( \bar{\phi} \) is the flux passing through the sample. The sine of the small angle can be approximated as \( 2\delta \), and the flux is approximately given by the area of the magnet pole faces times the average field. Combining all of these constants, one has for the particular case under consideration here,

\[
F = 290 i \delta \text{ (dynes)} \quad i \leq i_c
\]

Here \( i \) is the current in the coils.

In the case in which the applied field is greater than \( H_c \), one would expect to find regions of the surface of the disc on which \( H > H_c \). This makes it difficult to evaluate the integral in equation (1), but as a first guess it might be taken to be equal to the square of the average field of the coils times the area of their pole faces. This yields

\[
F = 208 i^2 \delta \text{ (dynes)} \quad i \gg i_c
\]

which at least has the virtue of being equal to the above expression when \( i = i_c \).
It is interesting to note that the same expression as (1) comes from considering the forces due to a current in the disc necessary to produce the distorting field \( H_y = \mp H \sin \delta \). By Ampere's law this is

\[
4\pi/c \int \gamma dy = 2 H \sin \delta dy \hat{y}
\]

where \( \gamma \) is the thickness of the disc, and the resultant force is

\[
\vec{F} = 1/c \int 3x\vec{B} \, dV = 1/2\pi \sin \delta \int 1/\gamma \ H^2 \ \gamma \, dy \, dz \hat{y}
\]

\[
= 1/2\pi \sin \delta \int \frac{H^2}{\gamma} \, ds \hat{y}
\]

which for small \( \delta \) is the same as (1).

B. The Equations of Motion

Having a functional form for the "force" upon the normal regions in the superconductor, and at least an order of magnitude estimate of its size, one can derive equations of motion for the pendulum system. A variable to describe the position of some average center of the intermediate-state structure will be needed.
Illustration of the Variables Used in the Equations of Motion

Call this $\phi$ and let it be measured to the vector $P$ which is fixed in the disc. The observable in these experiments is the direction of $P$ which is measured by $\theta$, so with the equilibrium position of $P$ ($\theta = \theta_0$) defined to lie on the
radius of the magnets, the angle through which the center of the normal regions is displaced from the magnet pole will be given by \((\theta - \theta_0 - \phi)\). Thus

\[
(\theta - \theta_0 - \phi)^{1/2}(s-r) = s
\]

for small \(s\), where \(r\) is the radius from the center of the disc to the center of the pole face, \(s\) is the separation of the pole faces, and as before, \(\gamma\) is the thickness of the disc. This geometrical factor has a value of roughly 14, so

\[
f = c(\theta - \theta_0 - \phi) \text{ dyne-cm.}
\]

where \(c = 4000\) for \(1 < ic\)

\[
c = 2800\text{ }i^2 \text{ for } i > ic
\]

Here another factor of \(\gamma\) has been included to convert the forces on the disc to torques.

With this coupling force, the equation of motion for \(\theta\) is

\[
I\ddot{\theta} + P\dot{\theta} + K(\theta - \theta_0) + c(\theta - \theta_0 - \phi) = 0
\]

This \(I\) is the moment of inertia of the system, \(K\) is its torsion constant, and \(P\) is close to the residual damping since the losses due to flux motion enter through the last term.
The equation for $\varphi$ has two forms depending on whether flux flow or flux creep are taking place. In the case of the former, the equation is that of viscous-damped flow

$$\dot{\varphi} = c(\Theta - \Theta_0 - \varphi)^+ F_0^+ \quad \text{(flow)}$$

This equation is, of course, valid only for $c(\Theta - \Theta_0 - \varphi) > F_0$, and the sign of $F_0$ is chosen to have the opposite sign from $c(\Theta - \Theta_0 - \varphi)$. In the case of flux creep, one might immediately write

$$\dot{\varphi} = C e^{\frac{(P_0 - g c(\Theta - \Theta_0 - \Phi))}{\kappa T}}$$

however, this is unsatisfactory in that $\varphi$ is not zero when $(\Theta - \Theta_0 - \varphi)$ goes to zero. In order to remedy this, one must take into account the increasing probability of flux motion against the direction of the force as the force approaches zero. This is done thus

$$\dot{\varphi} = C \left[ e^{\frac{(P_0 - g c(\Theta - \Theta_0 - \Phi))}{\kappa T}} - e^{\frac{(P_0 + g c(\Theta - \Theta_0 - \Phi))}{\kappa T}} \right]$$

Since all of the measurements under discussion here were made at a single temperature, and since it is unlikely that $1/T$ correctly represents the temperature dependence of these exponents anyway, it seems proper to lump all of the undeter-
mined constants, giving the result

\[ \dot{\phi} = \alpha \sinh (\beta (\theta - \theta_0 - \phi)) \]  \hspace{1cm} \text{(creep)} \hspace{1cm} (5)

where \( \alpha \) and \( \beta \) are assumed to vary with temperature and with magnetic field and probably with the history of the sample also.

There will be some transition region between the ranges of validity of equations (4) and (5) in which both creep and flow occur, so that the overall variation of \( \dot{\phi} \) with the force should be as indicated in figure IV-3.

\[ (\theta - \theta_0 - \phi) \]

**Figure IV-3**

Relationship of Velocity and Force in the Equation of Motion
In drawing this picture, it has been assumed that $\alpha \beta < 1/\eta$, as it must be if equations (4) and (5) are to be consistent.

C. Discussion of the Data

a) The X-0 Plot Data

The constant $c$ appearing in these equations of motion is large, much larger than $k$, so that $(\theta - \theta_0 - \phi)$ will always be small. This means that when $\dot{\theta}$ is large, $\dot{\phi}$ will also be large, and it seems reasonable to make the approximation that $\ddot{\theta} = \ddot{\phi}$ under these conditions. Therefore for oscillating solutions of large enough amplitude for $\dot{\phi}$ to be well up into the flux-flow region, equations 3 and 4 may be combined, assuming $\dot{\phi} = \ddot{\theta}$, to yield

$$I\dddot{\theta} + (p + \eta) \ddot{\theta} + k(\theta - \theta_0) + F_0 = 0$$  \hspace{1cm} (6)

where now the sign of $F_0$ is to be the same as that of $\ddot{\theta}$. This equation should be valid for the regions in the X-0 plots, figures III-8-11, for which $\ddot{\theta}$ is large. One can see that in these regions, the values of $X$, which is the damping torque on the system, lie on relatively constant plateaus, and this suggests that these values of $X$ (plateau) be identified with $\frac{p + \eta}{I}$ or, since $p$ is small, simply with $\eta/I$.

Under this interpretation, the value of $\eta$ may be seen to increase slightly from one plateau to the next. Figure III-11 shows that this increase does not continue indefinitely, and one would guess that it is probably due to rearrangement
in the structure of the intermediate state, which finally reaches an equilibrium configuration for the field value of the measurement. Therefore the proper equilibrium value of $\eta/I$ for a given set of data is most closely given by the largest plateau value of $X$.

In the regions of the $X-O$ plots in which $\theta$ is small, that is to say the regions near the end-points of the swings and to the right of the graph where the amplitude is small, system is entering the flux creep region, and equations 3 and 5 should be used to describe the motion. No simple solution of these equations presents itself, but consider the motion of the system if equation (5) were replaced by

$$\dot{\theta} = 0 \quad \text{for} \quad |\theta - \theta_o - \phi| \leq F_o,$$

which is a fairly good approximation. As the system enters this state, the equations for $\phi$ and $\theta$ uncouple, and the motion is described by the equation

$$\ddot{\theta} + P/I \dot{\theta} + K/I (\theta - \theta_o) + \frac{c}{I} (\theta - \theta_o - \phi) = 0$$

($\phi$ constant)

Thus the apparent effect is that the damping torque decreases and the restoring torque increases. With regard to this last, it should be noted that although $c$ is much greater than $K$ in this equation, $(\theta - \theta_o)$ is much greater than
\( (\theta - \theta_o - \phi) \), so one would not expect any catastrophic change in the restoring force. The X-0 plots show these effects clearly. Between the plateaus the value of X drops, however the corresponding rise in 0 does not show up, and as the amplitude decreases below a certain value, X begins to decrease and 0 to increase.

Pursuing this interpretation, it seems reasonable to identify this critical amplitude with the limit of validity of equation (6) where the maximum value of \( K(\theta - \theta_o) \) is becoming comparable to \( F_o \). Efforts were made to determine \( F_o \) by including it as a third parameter in the X-0 analysis, but this introduced so much scatter into the results that it had to be left out of account. One will therefore have to be satisfied with estimates of \( F_o \) given by the critical values of \( K(\theta - \theta_o) \), which, to give an example, are .7 and .3 dyne-cm. for the .3 and the .6 ampere measurements respectively.

Tests were made on the .3 ampere data to determine the effect of this \( F_o \) on the X-0 plots. The value of \( \theta_o \) was changed by an amount equivalent to an \( F_o \) of about 1 dyne-cm. but no changes were discernible in the part of the plot for the large amplitudes. Although this is not really a fair test because \( F_o \) must change sign with \( \theta \), it is felt that \( F_o \) is sufficiently small that ignoring it does not affect any of the arguments presented above.
One more thing might be mentioned with regard to the applicability of the equations 3, 4 and 5. The approximate set of equations 3, 4 and 7 were solved by means of an analog computer, and good fits were obtained by eye to several sets of end-point data. Such fits are, of course, unreliable for any quantitative argument, but they do show that the behavior of the solutions of these equations is at least similar to that of the pendulum system.

b) Overdamped-motion Data

The velocity of the pendulum in the overdamped state is small, so the equations that should apply are (3) and (5) describing flux creep. Although it is very difficult to study the oscillating solutions to these equations, for this overdamped motion, the problem can be considerably simplified by dropping the first and second derivatives of $\Theta$ from equation (3), leaving

$$k(\Theta - \Theta_o) + c(\Theta - \Theta_o - \phi) = 0 \quad (3a)$$

$$\dot{\phi} = \alpha \sinh \beta c(\Theta - \Theta_o - \phi) = 0 \quad (5)$$

Dropping the term $I\Theta$ is justified by the small, monotonic velocity of the expected solution and dropping $P\dot{\Theta}$ by the smallness of $P$ as well as of $\dot{\Theta}$.

These equations can be easily solved. First $\phi$ is eliminated between them, giving for $\Theta$
\[ \theta = -\frac{c}{(K+c)} \alpha \sinh \beta K(\theta - \theta_0), \]

which can be integrated with the result

\[ (\theta - \theta_0) = \frac{2}{\beta K} \tanh^{-1} e^{-\frac{c\alpha \beta K}{(K+c)} (t + t_0)} \]

About 30 points were taken from each set of data, and a least-squares fit of the function

\[ \theta = B_1 \tanh^{-1} e^{-B_2(t+B_3)} + B_4 \tag{8} \]

was made. Table IV-1 gives the results of this and values of \( \alpha \) and \( \beta \) calculated assuming \( c \gg K \).

<table>
<thead>
<tr>
<th>Current</th>
<th>2A</th>
<th>2'A</th>
<th>1.5A</th>
<th>1A</th>
<th>0.6A</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta ) head</td>
<td>-.0456</td>
<td>.0913</td>
<td>-.0456</td>
<td>-.0456</td>
<td>.0913</td>
</tr>
<tr>
<td>( B_1 ) (sec(^{-1}))</td>
<td>.047</td>
<td>-.122</td>
<td>.0395</td>
<td>.0295</td>
<td>-.046</td>
</tr>
<tr>
<td>( B_2 ) (sec(^{-1}))</td>
<td>.00453</td>
<td>.00667</td>
<td>.00623</td>
<td>.00999</td>
<td>.0161</td>
</tr>
<tr>
<td>( \beta ) (sec(^{-1}))</td>
<td>1.09x10(^{-4})</td>
<td>4.07x10(^{-4})</td>
<td>1.23x10(^{-4})</td>
<td>1.47x10(^{-4})</td>
<td>3.27x10(^{-4})</td>
</tr>
<tr>
<td>( \beta ) (dyne-cm.)</td>
<td>.969</td>
<td>3.80</td>
<td>1.17</td>
<td>1.57</td>
<td>1.14</td>
</tr>
<tr>
<td>( Q ) (in.)</td>
<td>1x10(^{-3})</td>
<td>5x10(^{-3})</td>
<td>5x10(^{-3})</td>
<td>8x10(^{-3})</td>
<td>2x10(^{-2})</td>
</tr>
</tbody>
</table>
The quantity $\Delta\text{head}$ is the angle through which the top of the torsion fiber was turned to initiate the motion, and the quantity $Q$ is the root-mean-square difference per point between the measured and the calculated values at the points used in the fitting.

This last is included as a guide to the quality of the fits, which for all but the .6A measurement are excellent. The best is the first 2A measurement pictured in figure III-12. If the reader will look closely, he will see that the curve in this figure consists of points on a line. The line fairly represents the function equation (8) above since the maximum difference between a computed and a measured point among those used for the fit is .01 inch, which is about the size of the points on the line and about the error associated with reading the data on the film. This maximum error remains below .02 inch for the rest of the data except for the .6A readings for which the maximum is about .1 inch. The distribution of the errors on this curve suggests that the largest differences are due to the onset of the sort of behavior pictured in figure III-13, which can be associated with the increase in size of the neglected $1\theta$ term in equation (3).

One is now in a position to test the consistency of the model described in the above paragraphs by constructing a graph similar to figure IV-3. It is clear that the damping
factor $\eta/I$ in equation (6) can be identified with the $P/I$ (magnetic) in figures II-4 and 5, so by extrapolating the descending curve for the 2A transitions in these figures, approximate values of $\eta$ can be determined under about the same conditions as prevailed for the $\alpha$ and $\beta$ measurements. These three parameters are all that are necessary to make a plot of $\dot{\phi}$ against $c(\theta - \Theta_o - \varphi)$ from equations (4) and (5) if the boundary condition that the slopes of the two functions match at their common point is assumed. Figure IV-4 shows such a plot for the .6A data, and Table IV-2 gives the relevant information for the 2A and 1A data as well.

<table>
<thead>
<tr>
<th>Table IV-2</th>
<th>2A</th>
<th>1A</th>
<th>.6A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\eta$ (dyne-cm. sec.)$^{-1}$</td>
<td>$2.10^{-4}$</td>
<td>$5.7\times10^{-4}$</td>
<td>$1.3\times10^{-3}$</td>
</tr>
<tr>
<td>$\alpha\beta$ (dyne-cm. sec.)$^{-1}$</td>
<td>$1\times10^{-4}$</td>
<td>$2.3\times10^{-4}$</td>
<td>$3.7\times10^{-4}$</td>
</tr>
<tr>
<td>$\phi_c$ (sec.$^{-1}$)</td>
<td>$1.9\times10^{-4}$</td>
<td>$3.4\times10^{-4}$</td>
<td>$1.1\times10^{-3}$</td>
</tr>
<tr>
<td>$c(\theta-\Theta_o-\varphi)$ (dyne-cm.)</td>
<td>1.36</td>
<td>1.00</td>
<td>1.70</td>
</tr>
<tr>
<td>$F_o$ (dyne-cm.)</td>
<td>.4</td>
<td>.4</td>
<td>.85</td>
</tr>
<tr>
<td>$\phi_{max}$ (sec.$^{-1}$)</td>
<td>$2.10^{-4}$</td>
<td>$3.5\times10^{-4}$</td>
<td>$1.1\times10^{-3}$</td>
</tr>
</tbody>
</table>
Figure IV-4

Velocity against Force at .6A.

\(10^3 \dot{\phi}\) (sec\(^{-1}\))

- Slope \(1/\eta\)
- Slope \(\alpha\beta\)
- Point \(c(\theta-\theta_0-\phi)_c\)
- Force \(F_0\)

(dyne-cm.)
The quantities \( \dot{\phi}_c \) and \( c(\Theta - \Theta_0 - \phi)_c \) give the position of the boundary point between the creep and flow states, and so \( \dot{\phi}_c \) is in this picture the critical velocity for flux creep. The velocity \( \dot{\phi}_{\text{max}} \) is the largest observed velocity in each set of measurements. This comes uncomfortably close to \( \dot{\phi}_c \) in these three examples, but it is consistent to treat the motion as creep. The assumptions made about flux flow and about \( F_0 \) in the X-0 analysis also appear to be consistent since the velocities observed in the oscillating system are typically greater than \( 6 \times 10^{-3} \) sec.\(^{-1} \), and the values of \( F_0 \) quoted above are about the same size as the estimates made from the X-0 plots.

There is not enough data here to tell much about the variation of \( \alpha \) and \( \beta \) with field. They do not seem to be strong functions, however, so it is possible that at some field \( \alpha \beta \) will equal \( 1/\gamma \). At this point, \( \dot{\phi}_c \) and \( F_0 \) will go to zero and the system should exhibit only flux flow. Certainly the X-0 data do show that \( \dot{\phi} \) decreases rapidly with increasing field. However, the model does not take into account that there may be flux pinning centers of many strengths present in the superconductor. This would tend to blur the transition between creep and flow, and it seems probable that even for large fields, there will be a certain amount of creep.
D. The Damping Constants

It has been argued that the maximum plateau value of \( x \) in the \( X-0 \) plot should be associated with \( \gamma / I \) for the system. In the case of end-point data for which the \( \ln \Delta \theta \) plots are straight lines, it is clear that the derived value of \( P/I \) is the same as this \( X \) value. Analogously for the curved plots, an estimate of the maximum \( X \) value can be made by using the steepest slope of the \( \ln \Delta \theta \) line and the period associated with that swing of the pendulum. Thus almost all of the data can be made to yield a value of \( \gamma \), although sometimes a rather crude one. These values have been collected together in figure IV-5 together with the normal-damping curves from figure III-1 multiplied by their respective values of \( I \).

There are two principal things to notice about this rather jumbled-looking data. First in the spread of points for the second pendulum system, that is systems 2-5, the more recently measured values of \( \gamma \) tend to lie highest on the graph. The measurements with system 2 were made first, followed by the rest in order over about an 8 month period. This suggests that this spread of values should be attributed, in part at least, to temperature cycling or annealing or perhaps, most probably, with some migration of impurities in the sample, since the normal conductivity changed very little during this period of time.
Figure IV-5  Values of $\eta$

Normal Damping in Systems 2 - 5

- Normal Damping in System 1

$\eta$ (Dyne-cm./sec.)

100

10

Current (A)

0.1

1

System 1

System 2

System 3

System 4

System 5 (end-point)

System 5 (camera)
The second thing to notice is that the curves for
the two samples bear more or less the same relationship to
their respective normal damping lines. Since the discs had
almost the same thickness, the difference in the normal
properties must be due to a difference in conductivity.
One therefore concludes that \( \eta \) has almost the same dependence
on conductivity as the normal eddy-current damping.

To see what this dependence is, first consider the
skin depth

\[
\delta = \frac{c}{\sqrt{2 \pi \sigma \omega}} \quad \text{(gaussian units)}
\]

in the material. For tin at room temperature, the handbook
gives a resistivity of \( 13 \times 10^8 \text{ ohm-meters} \), which converts to
a conductivity of \( 7 \times 10^{15} \text{ sec}^{-1} \) in gaussian units. Thus \( \delta \)
is for this low frequency oscillation \( (\omega = .113 \text{ sec}^{-1}) \)

\[
\delta = 4.3 \times 10^2 \text{ cm ,}
\]

so even if one assumes a resistivity ratio of \( 10^4 \), \( \delta \) at \( 4.2^\circ \text{K.} \)
is still much greater than the thickness of the disc.

Therefore the magnetic field in the material can be
assumed to be the same as the field applied by the coils,
and the electric field in the disc, if it is moving with
velocity \( \hat{v} \), is given by

\[
\hat{E} = \frac{1}{c} (\hat{v} \times \hat{B})
\]
to first order in \(V/c\). Thus the current density is, again to first order,

\[
\mathbf{j} = \mathbf{E} \sigma = \frac{e}{c} (\mathbf{V} \times \mathbf{B})
\]

and the resultant force is

\[
F = \frac{1}{c} \int \mathbf{j} \times \mathbf{B} dV = \frac{\sigma}{c^2} \int (\mathbf{V} \times \mathbf{B}) \times \mathbf{B} dV
\]

\[
= - \frac{\sigma}{c^2} \mathbf{V} \int \mathbf{B}^2 dV.
\]

The conclusion from all of this is that \(\gamma\) varies as the first power of the conductivity of the normal tin.

In addition to this dependence, there is a characteristic variation of \(\gamma\) with the field. This surely calls for an explanation, but it is not at all clear on what sort of model of the intermediate state to begin a calculation, and no satisfactory results have been obtained.

E. The Restoring Force

To this point, most of the discussion has been directed toward damping torques on the system, and there are few loose ends left over from this which need a little attention. These are concerned with the problem, which was mentioned above, that the model being used does not include the possibility that pinning centers of different strengths may be present in the superconductor. Due to the large value of the constant \(c\), the pinning of a small amount of flux could have
a noticeable effect on the torsion constant of the system while changing the damping very little. This sort of change in the torsion constant is to be seen in a number of places in the data, indicating the presence of some strong pinning centers.

The most obvious occurrence of this is to be seen in figure III-6. On all but the descending branch of the $2A$ transition curve, the value of $\Delta K$ rises sharply to a value of about 10 dyne-cm. at $0.2A$ and then decreases after that to about 1 dyne-cm. at $2A$. Very much the same thing is seen in the $X-0$ data. The minimum values of 01 for the largest amplitudes are larger than $K$ by about 5 dyne-cm. at $0.2A$ and 2 dyne-cm. at $2A$. These $\Delta K$ would not be expected to be the same since the end-point measurements give values of $K/I$ averaged over time, but they are close. The decrease in $\Delta K$ with increasing field after the initial rise is probably due to the engulfing of the strongest trapping centers by the expanding normal laminae, which would find it energetically favorable to pass through regions of low surface energy.

The effect of a distribution of pinning strengths is probably also responsible for part of the rise in the $O$ values on the right in the $X-0$ plots. Certainly some of the increase is associated with the bulk of the flux motion entering the creep state and the consequent decrease in $X$, but $O$ begins to rise appreciably before $X$ falls. In this increase, one is
seeing the effect of the pinning centers on the torsion
constant before it shows up in the damping constant because
of the large value of c. This behavior is responsible also
for the very large values of $\Delta K$ observed in the 2A tran-
sition curve in figure III-6. A notation in the data book
reminds that the period for these oscillations was strongly
dependent on amplitude, which suggests that the X-0 plots
for this motion would show a large rise in 0 as the motion
died out and that the correct values of $\Delta K$ are actually much
lower than those measured.

Changes in the restoring force can also be seen in the
behavior of the overdamped system. In the 2A measurement in
Table IV-1, the top of the torsion fiber was rotated through
an angle of $-0.0456$, which is $-10$ scale inches, and the
initial rest position was 67.40 inches. However, the final
rest position as indicated by the value of $B_4$ in equation (8)
was 58.46 inches. This leaves about an inch unaccounted for
except by the assumption that trapped flux was opposing the
force of the torsion fiber and lowering the effective value
of K. Therefore the $\alpha$ and $\beta$ calculated using the fiber
value of K seem a bit low, and the values for the 2'A
measurement, which was made when the top of the fiber was
turned through +20 scale inches from its position after the
2A measurement, are high.
Enough has probably been said on this matter, but one more curious feature presents itself and should be mentioned. In figure III-11 there is a jump in the 0 values over as short a time as the X-0 analysis permits, occurring in both plots at about the same place with regard to the amplitude of the motion. This certainly looks like flux being caught by some strong pinning center.
V. Conclusion

The most important conclusion to be drawn from the experimental work is that the results are consistent with the phenomenological theory of Anderson. Both flux creep and flux flow states are seen, and there appears to be a rather broad range of velocities for which a mixture of the two occur, giving evidence that the pinning centers in the material have varying strengths. In short, it seems that qualitatively considered, the motion of the intermediate state of this type I superconductor is like that observed in the mixed state of type II superconductors.

No very firm conclusions can be reached, however, concerning the dependence of the parameters of this phenomenological theory upon the properties of the superconductor or the value of the magnetic field. The exception to this is the data for the viscosity $\eta$. This shows a well-defined if unexplained field dependence and the probability that depends upon the first power of the normal-state conductivity, but a strong dependence of the behavior of the system on history obscures most of the rest of the details.

The major disadvantage of the method of measurement used in all of this work lies in the problem of dealing with an oscillating system. The second time derivative in the equations of motion makes it difficult to infer the forces from
the motion of the pendulum in all but the simplest cases, and the back-and-forth motion of the normal laminae prevents any study of flux trapping and other possible hysteresis effects.

These difficulties could be avoided by modifying the apparatus so that the coils could be rotated about the axis of the disc. The torsion pendulum then could be used statically to determine the forces, and curves such as figure IV-4 could be measured directly and in detail. This type of study as a function of temperature and disc thickness for several superconductors, both type I and II, would be useful in giving an overall picture of flux pinning mechanisms and viscosity forces. It would also be interesting to observe the effect upon a system like this of passing a current across the disc as the coils were allowed to make a complete revolution. This should show the effects of the lorentz force very clearly.