LU, Sun, 1938–
STUDY OF THE POLARIZATION OF THE STIMULATED EMISSION FROM \( \text{Nd}^{3+} \) DOPED GLASS AND CaWO\(_4\) LASERS.

Rice University, Ph.D., 1966
Engineering, electrical

University Microfilms, Inc., Ann Arbor, Michigan
RICE UNIVERSITY

Study of the Polarization of the Stimulated Emission from Nd$^{3+}$ Doped Glass and CaWO$_4$ Lasers

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
Doctor of Philosophy

Thesis Director's Signature:

Henry C. Barrie

Houston, Texas
May, 1966
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I Introduction

The three characteristics of radiation are intensity, spectrum and polarization. For every radiating system, the spectrum and polarization of its radiation characterize the intrinsic properties of the system. For a long time, the analysis of spectrum has been used to identify the chemical composition of the light source. It has also been a leading contributor to the development of quantum theory of atomic physics up to the present stage. In the classical theory of radiation, polarization was regarded as resulting from the anisotropic nature of the radiation fields established by elementary radiators such as dipole, quadrupole etc. Every elementary radiator emits light in a definite polarization state and by measuring its polarization, we can better understand the nature of the radiation process of the light source. In quantum theory, emission and absorption of light can be considered as interactions between two stationary states through an interaction operator $\bar{F}$. The polarization of its radiation comes from the spatial anisotropy of the transition matrix elements defined as

$$P_{kn} = \langle \psi_k | \bar{F} | \psi_n \rangle$$ (1-1)

For a given interaction $\bar{F}$, certain selection rules for allowable transitions and the polarizations of the associated radiations can be stated. These rules are determined by the
nature of interaction and the quantum numbers which characterize the energy states of the radiating system. Therefore, a close connection between the polarization of the radiation and the properties of the microscopic system emitting the radiation exists. Practically, polarization studies have been used to obtain information about the structure of a radiating system in an isolated state or during its interaction with the surrounding medium when an external field is applied such as the Zeeman effect and crystal field splitting, etc.

According to both the classical theory and the quantum theory, the unpolarized nature of ordinary light is a result of an aggregation of infinitely many microscopic radiating systems radiating in a perfectly random manner. Therefore, the light from a source without macroscopic anisotropy is expected to be unpolarized. If the light source has an anisotropic structure or is placed in an anisotropic surrounding medium such as a magnetic field, its radiating field is expected to be at least partially polarized.

The invention of the laser has greatly changed our concepts about light sources. For the first time, due to the coherent properties of stimulated emission, electromagnetic waves in optical range can have the same desirable coherent properties of radio frequency waves that have long been used in communication. Very shortly after the successful operation of a ruby
laser by Malman in 1960, Nelson and Collins (1) reported that the light from a 90° cut ruby (a crystal in which the C-axis or principal axis of symmetry of ruby is at 90° to the rod axis) was linearly polarized with its electric vector perpendicular to the plane containing the principal axis of symmetry, while the output from a 0° cut ruby was unpolarized at threshold and above. Each individual spike was also unpolarized. Similar conclusion was reported by Abella and Cummins (2). The light output from large angle cut ruby crystals was also investigated. A theory based on the double refraction of the light in ruby crystals has been developed by Miyagi et al (3) to explain the polarization of light from their 67° cut ruby rod. It also clarified the unpolarized nature of light from 0° rubies, but unfortunately the theory was not applicable to 90° cut ruby rods. A later report by Tamai and Achiwa (4) about a 90° cut ruby mentioned that about 0.5% of the total radiation from the laser rod was observed to be polarized with its \( \vec{E} \) vector parallel to the C-axis instead of perpendicular to it. Nelson and Remeika (5) reported that with a nearly perfect flux-grown ruby cut at 0°, regular pulses similar to a decay oscillation appeared when the laser was just operated slightly above threshold (around 0.5% above threshold) and all these spikes were linearly polarized in the same direction. For higher pumping energy input (about 1% above threshold), there were two trains
of "decay oscillations". The first train of oscillation was found to be linearly polarized with $\mathbf{E}$ parallel to the long lateral dimension of the cavity. The second train was linearly polarized in a perpendicular direction. With higher energy input, irregular spikes appeared in the output of the laser and the light was unpolarized. More recently, Brunton reported on his polarization study about seven fine quality ruby rods of different orientations with the following conclusions (6):

1. Light from the $0^\circ$ cut ruby was polarized.
2. Light from different parts of the rod exhibited different polarizations. Only a small part of the transmission face of the crystal emitted completely polarized light, the remaining area emitted unpolarized light.
3. Light from the $90^\circ$ cut ruby was completely polarized at the beginning of a run but unpolarized light was observed at the end of the run.
4. In all the crystals the amount of polarized light decreased with the increasing of pumping energy.

By examining these experimental results, one can see that there is still some uncertainties associated with the polarization of the stimulated emission from rubies and a general theoretical explanation is also lacking.

In 1958 Sugano and Tanabe (7) published their well known work about the absorption spectra of $\text{Cr}^{3+}$ ion in $\text{Al}_2\text{O}_3$ (ruby) which greatly extended the understanding about the properties
of the spectral lines of ruby. The polarization and the Zeeman patterns of the particular important \( R_1 \) and \( R_2 \) lines were worked out in detail. Their theory was used to explain the polarization of stimulated emission from 90° cut ruby lasers. But if the existing experimental results about the polarization of ruby lasers are correct, it would seem that the polarization of the light from a laser, although basically determined by the polarization of its fluorescence, would also depend upon some other factors, which are closely associated with the oscillation process that give rise to the spiky nature of the laser output. In order to get a general picture of how the polarization is related to the fluorescence of the active ion in the laser and to the properties of the oscillation cavity, it is necessary to carry out extensive experiments not only on light from ruby but also light from other laser materials. By comparing these results, a clearer picture about polarization of laser output can be obtained. To this end, several experiments about Nd\(^{3+}\) doped lasers were carried out. The Nd\(^{3+}\) ion was chosen as the main subject because it can be doped in various kind of crystals and laser action can take place at a low threshold energy input. It also can be doped in different kinds of glasses to form lasers. Without crystalline structure, a glass laser is expected to have no preferred direction. A Nd\(^{3+}\) doped CaWO\(_4\) laser was also studied. The CaWO\(_4\) crystals have tetragonal symmetry and their principal symmetry axes
are mutually perpendicular. This gives us the advantage of testing the validity of the theoretical predictions about the relationship between polarization of fluorescence and the polarization of the laser output which comes from CaWO₄ in a crystal symmetry completely different from the uniaxial structure of ruby crystals.
II METHODS

2-1 Construction of the Laser

The pulsed laser used in the experiments is pumped by arranging a linear flash lamp at one focus of an elliptical reflector. With the laser rod at another focus, all the light from the flash lamp is concentrated on the laser rod, thus the efficiency of this pumping configuration is high.

The flash tube used is a xenon-filled flash tube with an active length about 3 inches. The maximum allowable energy input to the flash tube is 400 joules. The reflector is made of 4 1/2 " OD aluminum tubing with the ellipse cut by a cutter which is inclined at an angle of 13.5° with respect to the axis of the tubing. Two plane reflectors are placed on the two micarta end plates to increase the pumping efficiency. The end plates support the reflector, flash tube, and the laser rod and form a cover for the strong white light from the flash lamp.

The trigger of the flash tube is provided by discharging a small capacitor (0.5 uf) through an EG & G TR-35 pulse transformer with a thyatron switch. At the same time, a pulse is formed by differentiating the voltage across the 0.5 μf capacitor. This pulse is used as a synchronizing pulse for triggering oscilloscopes and other electronics devices. Both the Nd³⁺ doped glass and CaWO₄ lasers are operated at room temperature. Because of their low thresh-
olds, a small amount of energy is enough to achieve laser action. The laser rod is cooled by blowing nitrogen gas at room temperature through the laser.

Different power supplies are used for glass and CaWO$_4$ lasers. The power supply for the glass laser has an energy storage capacitance of 120 μf. The one for CaWO$_4$ has 50 μf capacitance. Both power supplies have series inductances combined with the flashtube to form approximately critically damped series RLC circuits with the flash periods approximately equal to 1 ms.

The distance between the laser rod and the flash tube is one inch. The peak current passing through the flash lamp during its flash is approximately 1000 amperes. This current can produce a magnetic field of several hundred gauss in the vicinity of the laser rod. The effect of this field on the polarization of the light output from the laser is unknown, but an external constant field of 7,000 gauss is applied to the glass laser and the 90° cut ruby laser. Within the accuracy of the measurement, there is no noticable definite effect on the polarization. Therefore this small field due to the flash tube is completely neglected in the explanation of the experimental results.

2-2 The Theory of Partial Polarization (8, 9)

The stimulated emission of a laser has a very narrow frequency range; it is much narrower than the line width of
the spontaneous emission between the levels at which lasing action takes place. In electronics terminology, a laser is essentially an optical amplifier driven by the internal "noise" due to spontaneous emission. In solid state lasers, multi-mode oscillations and multi-level transitions are almost inevitably involved in the lasing action if no particular precaution is taken to eliminate them. The spectrum of the output light of a solid state laser is composed of many sharply defined lines and in some cases extends to a fairly broad frequency range. For example, the stimulated emission of a Nd\textsuperscript{3+} doped glass laser centered at 1.06 micron is composed of tremendously large numbers of sharply defined lines which cover wavelengths ranging around 100 Å. This spread in wavelength of the stimulated emission has a maximum deviation about \( \pm 0.5\% \) from the central wavelength. If \( \bar{\lambda} \) is the mean wavelength of the stimulated emission and \( \Delta\lambda \) is the wavelength spread, then for the stimulated emission of the Nd\textsuperscript{3+} doped glass laser

\[
\frac{\Delta\lambda}{\bar{\lambda}} = 0.01 \ll 1
\]

This value is about the upper limit for solid state lasers.

A light wave satisfying the relation \( \frac{\Delta\lambda}{\bar{\lambda}} \ll 1 \) is called quasimonochromatic light and this ratio shows how closely the light wave may be treated by the approximation of the theory of quasimonochromatic light.
The small ratio of $\Delta \lambda / \bar{\lambda}$ for the stimulated emission of lasers allows the theory of quasimonochromatic light to be applied accurately in the calculation of polarization, coherency, etc., of the output light from lasers.

The Coherency Matrix

Consider a quasimonochromatic light wave with mean frequency $\bar{\nu}$ propagating along the $z$ direction. The $x$ and $y$-components of the electric field strength at certain point in space at time $t$ are

$$E_x(t) = a_1(t) e^{i(\phi_1(t) - 2\pi \bar{\nu} t)}$$
$$E_y(t) = a_2(t) e^{i(\phi_2(t) - 2\pi \bar{\nu} t)}$$

(2-1)

Where $\phi_1(t)$ and $\phi_2(t)$ are the phase angles of the $x$ and $y$-components of electric field strength respectively. If the light wave is represented by a vector

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

(2-2)

then after it passes through an optical device such as a compensator to introduce a phase delay between the $x$ and $y$-components of the electric field strength or a linear polarizer, attenuator, etc., it becomes

$$\vec{E}' = \vec{L} \vec{E}$$

(2-3)

Where $\vec{L}$ is the instrument operator which characterizes the property of the optical device. For example, the instrument operator for a non-isotropic attenuator is
\[ \widetilde{\mathbf{A}} = \begin{bmatrix} e^{-\gamma_x} & 0 \\ 0 & e^{-\gamma_y} \end{bmatrix} \quad (2-4) \]

For a compensator which introduces a delay \(\varepsilon\) of \(E_y\) with respect to \(E_x\), the instrument operator is

\[ \widetilde{C}(\varepsilon) = \begin{bmatrix} e^{i\varepsilon/2} & 0 \\ 0 & e^{-i\varepsilon/2} \end{bmatrix} \quad (2-5) \]

and for a linear polarizer with transmission axis at an angle \(\theta\) with respect to the x-axis, the instrument operator is

\[ \widetilde{P}(\theta) = \begin{bmatrix} \cos^2\theta & \sin\theta \cos\theta \\ \sin\theta \cos\theta & \sin^2\theta \end{bmatrix} \quad (2-6) \]

For \(N\) optical devices in cascade, the combined operator \(\widetilde{\mathcal{L}}\) will be the product of the \(N\) operators in the sequence.

\[ \widetilde{\mathcal{L}} = \widetilde{L}_N \widetilde{L}_{N-1} \widetilde{L}_{N-2} \ldots \widetilde{L}_2 \widetilde{L}_1 \quad (2-7) \]

If \(\overline{E}^\dagger\) is the Hermitian conjugate of \(\overline{E}\), then the coherency matrix of a light wave \(\overline{E}\) is defined as:

\[ J = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = \langle \overline{E} \overline{E}^\dagger \rangle = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} \]

\[ = \begin{bmatrix} a_1^2(t) & a_1(t) a_2(t) e^{i(\phi_1(t) - \phi_2(t))} \\ a_1(t) a_2(t) e^{-i(\phi_1(t) - \phi_2(t))} & a_2^2(t) \end{bmatrix} \quad (2-8) \]

Where \(\langle \ldots \rangle\) represents time average and the "X" signifies the direct product of two matrices.

The coherency matrix \(J\) is Hermitian; it has the properties that
\[ J_{xx} \geq 0 \quad J_{yy} \geq 0 \quad J_{xy} = J_{yx}^* \quad \text{and} \quad \text{Det}(J) = J_{xx} J_{yy} - J_{xy} J_{yx} \geq 0 \quad (2-9) \]

The last relation of (2-9) follows from Schwarz inequality. \( J_{xx} \) is the time average of \(|E_x|^2\), hence it is the light intensity of the component at \( x \) direction and \( J_{yy} \) is the intensity for \( y \)-component. The trace of the \( J \)-matrix represents the intensity of the light wave.

\[ I = \text{Tr}(J) = J_{xx} + J_{yy} = \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle \quad (2-10) \]

For a natural or unpolarized monochromatic wave, the electric vector of the radiation field changes its direction rapidly and randomly such that it is equally likely to be anywhere in the \( x \ y \)-plane, thus

\[ \langle E_x \rangle = \langle E_y \rangle = 0 \quad (2-11) \]
\[ \langle E_x E_x^* \rangle = \langle E_y E_y^* \rangle = I/2 \quad (2-12) \]

where \( I \) is the intensity of the light wave. Furthermore, there is no correlation between \( E_x \) and \( E_y \), hence

\[ \langle E_x E_y^* \rangle = \langle E_y E_x^* \rangle = 0 \quad (2-13) \]

Therefore, the coherency matrix of a natural light wave is

\[ J = I/2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2-14) \]

For completely polarized (linear, circular, and elliptical) light waves, the amplitudes of \( E_x \) and \( E_y \) are not functions of time and their phase difference is a constant; that is:

\[ a_1(t) = \text{constant} = a_1 \]
\[ a_2(t) = \text{constant} = a_2 \]
\[ \varphi_1(t) - \varphi_2(t) = \alpha = \text{constant} \]

The coherency matrix takes the form

\[
J = \begin{bmatrix}
  a_1^2 & a_1 a_2 e^{i\alpha} \\
  a_1 a_2 e^{-i\alpha} & a_2^2
\end{bmatrix}
\]

and

\[ \text{Det}(J) = a_1^2 a_2^2 - a_1^2 a_2^2 = 0 \] (2.16)

Thus for a completely polarized wave, the determinant of
its coherency matrix is always zero.

From equation (2.15), we obtain the coherency matrices for

(1) Linearly polarized wave:

When the phase difference between \( E_x \) and \( E_y \) is a multiple
of \( \pi \), the resultant electric field vector vibrates in a
definite direction and the light wave is linearly polarized.

In this case

\[ \alpha = m\pi \quad (m = 0, \pm 1, \pm 2, \cdots) \]

hence

\[
J = \begin{bmatrix}
  a_1^2 & (-1)^m a_1 a_2 \\
  (-1)^m a_1 a_2 & a_2^2
\end{bmatrix}
\] (2.17)

Therefore, all the matrix elements of the coherency matrix
of a linearly polarized wave are real.

(2) Circularly polarized wave:

When \( a_1 = a_2 \) and the phase angle difference \( \alpha = (2m + 1)\frac{\pi}{2} \)
\((m = 0, \pm 1, \pm 2, \cdots)\), the electric vector of the light wave
has a constant magnitude and rotates in space with a constant
angular speed and the light is circularly polarized.
In this case
\[
J = \begin{pmatrix}
    a_1^2 e^{-i(2m+1)\pi/2} & a_1^2 e^{i(2m+1)\pi/2} \\
    a_1^2 e^{-i(2m+1)\pi/2} & a_1^2
\end{pmatrix}
\]
\[
= I/2 \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}
\]  \hspace{1cm} (2-18)

The upper sign represents a right handed circularly polarized wave and the lower sign for the left handed circularly polarized wave.

(3) Elliptically polarized wave:

For all other coherency matrices with \( \text{Det}(J) \) equal to zero, the light wave is elliptically polarized. This is the most general form for completely polarized light.

When a beam of light with coherency matrix \( J \) passes through an optical device with instrument operator \( \tilde{L} \), we have
\[
\tilde{E}' = \tilde{L} \tilde{E}
\]
Since \( (\tilde{E}')^\dagger = (\tilde{L} \tilde{E})^\dagger = \tilde{E}^\dagger \tilde{L}^\dagger \), thus
\[
J' = \langle \tilde{E} \times \tilde{E}' \rangle = \langle \tilde{L} \tilde{E} \times \tilde{E}^\dagger \tilde{L}^\dagger \rangle
= \tilde{L} \langle \tilde{E} \times \tilde{E}^\dagger \rangle \tilde{L}^\dagger = \tilde{L} J \tilde{L}^\dagger
\]  \hspace{1cm} (2-19)

This is a "transformation law" for the coherency matrix in the quasimonochromatic approximation.

It is interesting to find the coherency matrix \( J' \) of a linearly polarized light wave by equation (2-19). Consider natural light passing through a linear polarizer with its transmission axis inclined at an angle \( \theta \) with respect to \( x \)-axis. Since
\[ \mathbf{L} = \mathbf{p}(\theta) = \begin{pmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} = \mathbf{L}^+ \text{ and} \]
\[ \mathbf{J} = \mathbf{I}/2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ therefore, we have} \]
\[ \mathbf{J}' = \mathbf{L} \mathbf{J} \mathbf{L}^+ = \mathbf{I}/2 \begin{pmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]
\[ \begin{pmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} = \mathbf{I}/2 \begin{pmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} \]
(2-20)

which is essentially the same coherency matrix for a linearly polarized wave given in equation (2-17). Furthermore, the polarizer only transmits half (I/2) of the original intensity since it absorbs those light component which are polarized in a direction perpendicular to the transmission axis of the polarizer.

The Decomposition of the Coherency Matrix

Consider that several quasimonochromatic light waves coexisting in space. The resultant electric field vector is the sum of the individual electric field vectors.
\[ \mathbf{E} = \sum_i \mathbf{E}_i \]  
(2-21)

If these light waves are independent or incoherent, then there is no correlation between any two of them, i.e.
\[ \langle \mathbf{E}_i \times \mathbf{E}_j^\dagger \rangle = 0 \quad i \neq j \]  
(2-22)

Therefore, the coherency matrix of the resultant light wave is the sum of individual coherency matrices:

- 15 -
\[ J = \langle \mathbf{E} \times \mathbf{E}^\dagger \rangle = \sum_i \langle \mathbf{E}_i \times \mathbf{E}_i^\dagger \rangle = \sum_i J_i \quad (2-23) \]

For coherent light waves, equation (2-23) is not applicable since in that case there is some correlation between these light waves and the cross terms are not zero.

Any coherency matrix \( J = \begin{pmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{pmatrix} \) may be decomposed into two parts such as:

\[
J = J_1 + J_2 = \frac{I_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} P & J_{xy} \\ J_{yx} & Q \end{pmatrix} \quad (2-24)
\]

with the conditions:

\[
I_0 \geq 0, \quad P \geq 0, \quad Q \geq 0, \quad \text{and} \\
PQ - \frac{1}{4} |J_{xy}|^2 = 0 \quad (2-25)
\]

and this decomposition is unique.*

It is easy to solve \( I_0, P, \) and \( Q \) in terms of elements of the \( J \)-matrix.†

\[
I_0 = (J_{xx} + J_{yy}) - \sqrt{(J_{xx} + J_{yy})^2 - 4 \det(J)} \\
P = (J_{xx} - J_{yy})/2 + \sqrt{(J_{xx} + J_{yy})^2 - 4 \det(J)} \quad (2-27)
\]

\[
Q = (J_{yy} - J_{xx})/2 + \sqrt{(J_{xx} + J_{yy})^2 - 4 \det(J)}
\]

The matrix \( J_1 = I_0/2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \) represents a natural or unpolarized light where the matrix \( J_2 = \begin{pmatrix} P & J_{xy} \\ J_{yx} & Q \end{pmatrix} \)

* For a proof of this statement, refer to Born and Wolf *Principle of Optics* Sec. 10.8.2.

† Born and Wolf *Principle of Optics* Sec. 10.8.2.
represents a totally polarized light as it satisfies equation (2-26).

The decomposition of an arbitrary coherency matrix $J$ by equation (2-24) has the physical meaning that any partially polarized quasimonochromatic light wave (represented by an arbitrary coherency matrix $J$) may be thought of as an incoherent superposition of a completely unpolarized light wave and a fully polarized light wave. The matrix $J_1$ represents the unpolarized component and $J_2$ represents the completely polarized component. Starting in this way, we find that the light intensity of the unpolarized component is

$$\text{Tr} \ (J_1) = I_0$$

and the intensity of polarized component is

$$I_1 = \text{Tr} \ (J_2) = P + Q$$

Therefore, the degree of polarization of the original quasimonochromatic wave is defined as

$$p = \frac{\text{Intensity of polarized component}}{\text{Total intensity of the light}} = \frac{P+Q}{I_0+P+Q}$$

By this definition, it is obvious that

$$0 \leq p \leq 1$$

When $p = 0$, the light is completely unpolarized. When $p = 1$, the light is completely polarized and the determinant of the coherency matrix vanishes.

The incoherency between the unpolarized component and the completely polarized component of a light wave makes
possible the decomposition given in equation (2-24). However, a similar decomposition of a completely polarized coherency matrix $J_2$ into a matrix which represents linearly polarized component and a matrix which represents circularly polarized component does not exist in general. We see that the imaginary part of $J_{xy}$ is closely related to the circular polarization because it never appears in those coherency matrices which represent unpolarized wave and linearly polarized wave. Therefore, when the imaginary part of $J_{xy}$ vanishes, the light essentially consists of an unpolarized component and a linearly polarized component. However, for completely polarized light, in general we cannot split the polarization into linear and circular components.

**The Stokes Parameters**

The Stokes parameters are another expression of polarization for a partially polarized light wave. From the properties of the coherency matrix, one can see that four real quantities are necessary and sufficient to completely characterize the polarization state of a quasmonochromatic light wave. In the coherency matrix representation, these four quantities are $J_{xx}$, $J_{yy}$, $\text{Re}(J_{xy})$, and $\text{Im}(J_{xy})$, where "Re" means the real part and "Im" means the imaginary part. Let us form a column matrix:

$$
\mathbf{J} = \begin{bmatrix} J_{xx} \\ J_{xy} \\ J_{yx} \\ J_{yy} \end{bmatrix}
$$

(2-32)
and make a unitary transformation to a column matrix \( S \) in such a way that:

\[
\begin{bmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 1 \\
1 & 0 & 0 & -1 \\
0 & 1 & 1 & 0 \\
0 & -1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
J_{xx} \\
J_{xy} \\
J_{yx} \\
J_{yy}
\end{bmatrix}
\]  

(2-33)

The new elements so defined are called Stokes parameters and are all real. The intensity of the light is represented by \( S_0 \):

\[ S_0 = J_{xx} + J_{yy} = I \]

The quantity \( S_3 = 2 \text{ Im } (J_{xy}) \) is related to circular polarization.

2-3 The Experimental Setup for Measuring the Polarization of the Stimulated Emission of Pulsed Lasers

The usual method of measuring the polarization of a continuous and steady light with a linear polarizer and quarter wave plate encounters difficulties when the measurement of polarization of pulsed light (such as the stimulated emission of a laser) is attempted. If the polarization of the stimulated emission does not change with time, such as the output light from a 90° cut ruby rod, very good results can be obtained in the laboratory by this conventional method of polarization measurement if a second photodetector is used to monitor the total output of the laser. In the actual setup, the output from the laser is directed to pass through a linear analyser and sometimes a quarter wave plate when circular polarization is measured. Then it is detected by a photo-
detector. The real output is also detected by another photo-
tube. The laser is fired successively with different settings
of the transmission axis of the analyser. The total output
changed irregularly from firing to firing; the real monitor
served as a correlation among successive data. The results
obtained by this simple setup for measuring the polarization
of the stimulated emission from a 90° cut ruby rod is very
satisfactory. The variation of light intensity with respect
to the rotation of the analyser follows the law of Malus
very closely (10).

If the polarization of the laser output changes from
firing to firing or even changes with time very rapidly in
the output of one flash, the simple way described above is
no longer applicable and other well developed techniques
have to be used instead. Brunton (11) and Pavliscoak (12)
have measured the polarization of light from ruby lasers by
using a biprism designed by J. S. Courtney-Pratt. The biprism
consists of two identical wedges of quartz of opposite rota-
tions cemented together by an optically inert cement (Fig. 2).
A linearly polarized light beam traversing the center of the
prism will have its plane of polarization rotated first to
the left; then on entering the second wedge, to the right
with an equal amount of rotation. Thus for a central beam
the plane of polarization is not changed on passing through
the prism. Beams to the left or right of the center will have
their planes of polarization rotated to the left or right by angles which depend on the thicknesses of the left-handed and right-handed wedges traversed. If the light beam is spread over the prism face, passed through the prism and viewed through a Polaroid analyzer set for extinction, a dark extinction band will appear at the center of the prism and near each edge where the rotation is 180°. In the actual experiment, a picture was taken. From the positions of the dark bands and the relative exposure of the photographic plate, the plane of polarization and the degree of polarization of the light was determined. High-speed photography on the
order of 500,000 frames/sec was used in order to obtain the high time resolution required.

The above method for the measurement of rapidly changing polarization has a relatively poor accuracy. By using a microdensitometer, curves which represented the polarization of the laser output were obtained from those photographic plates. However, the curves were so distorted that a quantitative measurements of the degree of polarization and the plane of polarization could hardly be made. The time resolution of the measurement was restricted by the speed of the camera system. Microsecond resolution is about the fastest limit of present techniques. This time resolution is found to be slightly insufficient to measure the polarization of a single spikes (which usually has a width around one microsecond.)

As previously stated, four real quantities are necessary and sufficient to completely characterize the polarization state of a light wave. Thus in dealing with a light wave which changes its polarization in time rapidly, if one can make four suitably chosen measurements simultaneously, the polarization of the light wave at any instant can be calculated by the data obtained. Instead of designing a complicated optical system and using an expensive high speed camera system, we simply split the light beam into six components and made six measurements simultaneously. By taking six measurements
instead of four, we had checks on the apparatus. In analysing
the data, this redundancy also permitted an error minimization
 technique to be used and improved the accuracy of the measure-
 ment.

In order to get six separate components for simultaneous
measurements, the light beam was directed to beam splitting
mirrors aligned at 45° to the laser beam. Both the transmitted
and reflected portions of the beam were used. Because the
polarization of the light will be changed by the reflection
from or passage through a layer of thin metal film (13), some
compensating technique should be used such that each of the
six components of the beam preserves the original polarization
state of the laser output. In the measuring setup, this was
done by using an arrangement as shown in figure 3. One can see
that before a component of the beam is detected by a phototube
it always undergoes an even number of reflections (transmis-
sions) and the plane of incidence in half the cases is perpen-
dicular to the plane of incidence for the other half.

To understand the process more easily, let us trace the
light beam as it passes through the optical system. The beam
(a) is incident on mirror (1) at 45° and splits into a trans-
mitted component (b) and a reflected component (d). The trans-
mitted component (b) passes through mirror (2) with a plane
of incidence perpendicular to the first plane of incidence
and goes through an analyser to a photodetector. The reflected
Fig. 3. Diagram showing splitting of a light beam without change of polarization
beam (d) is incident on another mirror (3) which is arranged so that its plane of incidence is perpendicular to the plane of incidence of beam (a) on mirror (1). The reflected beam (e) from mirror (3) then goes through another analyser and is detected by another phototube.

The polarization of a light wave is changed by reflection from or passage through a mirror at an oblique incident angle because the coefficient of reflection (transmission) for the wave for which the electric vector is in the plane of incidence is not equal to the coefficient of reflection (transmission) for the wave for which the electric vector is perpendicular to the plane of incidence. (figure 4)

![Diagram](image)

Electric vector in the plane of incidence  Electric vector perpendicular to the plane of incidence

Fig. 4

After being reflected or passed through a mirror, the electric field components change their relative amplitudes and the polarization of the original wave is altered.

Let us resolve the incident beam (a) into two perpendicular
components $E_{\alpha x}$ and $E_{\alpha y}$ (figure 3). When beam (a) hits mirror (1), $E_{\alpha x}$ is perpendicular to and $E_{\alpha y}$ is in the plane of incidence, thus

$$\begin{bmatrix} E_{bx} \\ E_{by} \end{bmatrix} = \begin{bmatrix} t_\perp & 0 \\ 0 & t_\parallel \end{bmatrix} \begin{bmatrix} E_{\alpha x} \\ E_{\alpha y} \end{bmatrix}$$

and

$$\begin{bmatrix} E_{dx} \\ E_{dy} \end{bmatrix} = \begin{bmatrix} r_\perp & 0 \\ 0 & r_\parallel \end{bmatrix} \begin{bmatrix} E_{\alpha x} \\ E_{\alpha y} \end{bmatrix}$$

Beam (b) hits mirror (2) with $E_{bx}$ in and $E_{by}$ perpendicular to the plane of incidence. Thus

$$\begin{bmatrix} E_{cx} \\ E_{cy} \end{bmatrix} = \begin{bmatrix} t_\parallel & 0 \\ 0 & t_\perp \end{bmatrix} \begin{bmatrix} E_{bx} \\ E_{by} \end{bmatrix} = \begin{bmatrix} t_\parallel & 0 \\ 0 & t_\perp \end{bmatrix} \begin{bmatrix} t_\parallel & 0 \\ 0 & t_\perp \end{bmatrix} \begin{bmatrix} E_{\alpha x} \\ E_{\alpha y} \end{bmatrix}$$

$$= t_\perp \cdot t_\parallel \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} E_{\alpha x} \\ E_{\alpha y} \end{bmatrix}$$

The electric field components $E_{cx}$ and $E_{cy}$ have exactly the same relativistic amplitudes as that of $E_{\alpha x}$ and $E_{\alpha y}$; therefore the original polarization is preserved. A similar argument applies to beam (e).

By this simple method, it is very easy to extract six components of the light beam with every component preserving the original polarization of the laser output. One way to achieve this purpose is shown in figure 5. This is the way that our experimental apparatus was built. Looking over the mirror system, it seems to be unnecessarily complicated when
compared to the one illustrated in figure 3. It is of practical consideration to use such a mirror system; by this design we could mount all the phototubes on one single base plate; otherwise a complicated three dimensional structure must be constructed.

The six components of the beam after they were extracted without altering the polarization state were directed to pass through Polaroid type HR linear polarizers. These plastic laminated polarizers are particularly designed for near infrared radiation between 8,000 A to 27,000 A. Two of the six components of the beam were also directed to pass through some compensators; one of which is a quarter wave plate at 10,829 A, and another one has a delay corresponding to 59° at 10,600 A. Actually this 59° delay device is a quarter wave plate at 6,943 A. It was used because it was readily available. The delay of this device has been measured by using the emission of a helium lamp which has a sharp and strong line at 10,829 A. The experimental value of delay is about 57° at 10,829 A and corresponds a delay of 58° at 10,600 A. Since the index of refractions of usual materials change very little from 6,943 A to 10,600 A, the delay of this device at 10,600 A was decided by a simple proportion

$$\frac{\delta}{90°} = \frac{6,943}{10,600}$$

This gives $$\delta = 59° = 0.328\pi$$, which is quite close to the value measured by using the helium lamp.
After the beams passed through delay devices and linear polarizers, they were detected by RCA type 7102 photomultiplier tubes with S-1 type spectral response. Silicon filters about 1 mm thick were placed in front of the photomultiplier tubes (abbreviated as PM tubes in the following). The spectral response of the combination had a peak sensitivity somewhere in the infrared near 10,000 Å. This was suitable to measure the polarization of radiation from Nd³⁺ doped lasers. The laser output is very much stronger than the light intensity necessary to saturate the photomultipliers; hence, heavy attenuators were placed in front of each tube. The sensitivities of the PM tubes were separately variable in a large range by varying the anode voltage on the tube (which was around 1,000 v.). The load resistance of each PM tube was 2,700 ohms. The output signals were coupled to 93 ohm RG 62/U coaxial cables through emitter followers and were displayed on oscilloscopes. The time resolution of the PM tube was better than 10 n.s. and this was very much faster than the rise time of the emitter followers as well as some of the vertical amplifier plug-in units of the oscilloscopes. The PM tubes were shielded with high μ metal tubes, this could prevent decrease of sensitivity due to some weak magnetic field present in the vicinity of the PM tubes. However, the shielding was insufficient when a strong magnetic field of several kilo-gauss was applied to the laser (about 1 meter
from the PM tubes). In this case, the whole apparatus should be housed in a heavy magnetic shielding box. Figure 6 shows the circuit diagram for supplying DC voltage to the PM tubes and figure 7 is a photograph of the whole apparatus with the laser shown in front of it.

2-4 Experimental Procedures, Analysis of Data, and Accuracy of the Measurement

Consider a quasimonochromatic light with coherency matrix

\[
J = \begin{bmatrix}
J_{xx} & J_{xy} \\
J_{yx} & J_{yy}
\end{bmatrix}
\]

passing through a compensator which introduces a delay \( \varepsilon \) of \( E_y \) with respect to \( E_x \). After that it passes through an analyser with its transmission axis set at an angle \( \theta \) with respect to the \( x \)-axis. The resultant instrument operator from (2-5) and (2-6) will be

\[
\hat{\mathcal{I}}(\theta, \varepsilon) = C(\varepsilon) P(\theta) =
\begin{bmatrix}
e^{i\varepsilon/2} \cos^2 \theta & e^{i\varepsilon/2} \sin \theta \cos \theta \\
e^{-i\varepsilon/2} \sin \theta \cos \theta & e^{-i\varepsilon/2} \sin^2 \theta
\end{bmatrix}
\]  \hspace{1cm} (2-34)

The coherency matrix of the light after passing through the delay device and linear analyser is

\[
J'(\theta, \varepsilon) = \hat{\mathcal{I}}(\theta, \varepsilon) \quad J \quad \hat{\mathcal{I}}(\theta, \varepsilon)
\]  \hspace{1cm} (2-35)

and the intensity of the light is \( I(0, \varepsilon) = \text{Tr}(J'(\theta, \varepsilon)) \).

Which gives
\[ I(\theta, \varepsilon) = J_{xx} \cos \theta + J_{yy} \sin \theta + \sin(2\theta) \Re(J_{xy} e^{-i\varepsilon}) \]  \hspace{1cm} (2-36)

When the light wave is detected by a PM tube, the output of the tube is proportional to this intensity. The proportional constant is actually the resultant sensitivity (including absorption by the compensator, analyser, and the attenuators) of the photodetector.

By rotating the analysers and inserting compensating devices in the experimental setup, we could measure light intensities of six components simultaneously with any six chosen combinations of \( \theta \) and \( \varepsilon \) values. For sake of convenience, they were chosen to be

- **PM 1** registered \( I(0, 0) \)
- **PM 2** \( I(45, 0) \)
- **PM 3** \( I(90, 0) \)
- **PM 4** \( I(45, \pi/2) \)
- **PM 5** \( I(45, 0.328\pi) \)
- **PM 6** had no analyser in front of it and registered the total intensity of light.

The output of the PM tubes were proportional to the light intensities they registered. Since we were only interested in the polarization state of the laser beam which was independent of the absolute intensity of the light, only the relative sensitivities of the photodetectors should be known. Hence a calibration of these PM tubes was necessary. In the experi-
ment, the sensitivities were calibrated to be the same. This was done by rotating all the analysers before the PM tubes to $0^\circ$ (or $90^\circ$), since $I(0^\circ, \varepsilon) = J_{xx}$ and $I(90^\circ, \varepsilon) = J_{yy}$, therefore, in both cases, all the PM tubes detected the same light intensity regardless of the type of compensator placed before them; their outputs should be the same if they were calibrated. By using a Tektronics type Z difference amplifier plug-in unit with type 585 oscilloscope, the outputs from the PM tubes were compared two at a time. The output of PM 3 or PM 2 was usually adjusted to a suitable magnitude for oscilloscope display and served as a standard output. The signal from other PM tubes were compared with this standard. The sensitivities of the tubes were adjusted until their outputs were exactly the same magnitude as the standard signal. When this was the case, the difference amplifier had a zero output and a straight line appeared on the screen. In practical cases, perfect cancellation of two signals rarely occurred. Some defects of the coupling circuits between the PM tubes and connecting cables set a limitations on this calibration. A typical trace on the screen during calibration is shown in figure 8 as compared with one of the signals. Since the absolute intensity of the laser beam did not take parts in polarization calculation, the outputs of the PM tubes, after they were calibrated, were assigned to be the intensities of the components they detected. Hence from equation (2-36):
Fig. 8. Typical traces during calibration. Upper trace, B - C 0.05 v/div. Lower trace, calibrated A 0.25 v/div. 10 μsec/div. sweep. The calibration between B and C is better than 6% in this picture.
The output of

\[ PM \ 1 = A = I(0^\circ,0) = J_{xx} \]  
\[ PM \ 2 = B = I(45^\circ,0) = (J_{xx}+J_{yy})/2 + \text{Re}(J_{xy}) \]  
\[ PM \ 3 = C = I(90^\circ,0) = J_{yy} \]  
\[ PM \ 4 = D = I(45^\circ,\pi/2) = (J_{xx}+J_{yy})/2 + \text{Im}(J_{xy}) \]  
\[ PM \ 5 = E = I(45^\circ,0.328\pi) = (J_{xx}+J_{yy})/2 + \text{Re}(J_{xy} e^{-1.328\pi}) \]  
\[ PM \ 6 = F = I = J_{xx} + J_{yy} \]  

(2-37-1)  
(2-37-2)  
(2-37-3)  
(2-37-4)  
(2-37-5)  
(2-37-6)

As there are only four quantities to be determined, namely \( J_{xx}, J_{yy}, \text{Re}(J_{xy}), \) and \( \text{Im}(J_{xy}) \), one can see that from A, B, C, and D it is easy to solve for these parameters. If the polarimeter is used in this way, E and F only serve as checks of the measurement.

The outputs of the PM tubes were displayed on several oscilloscopes and recorded by photographs. Among those oscilloscopes, one of them was a dual beam scope; therefore, five pictures were taken simultaneously. Thus the six outputs from PM tubes could be recorded for later analysis. In taking these pictures, the PM tubes were first calibrated with all analysers set at 0\(^0\) or 90\(^0\). After the calibration, analysers were set back to their proper angles, with all the shutters of cameras open (T shutters were used), then the laser was fired. The synchronizing pulse from the triggering circuit

* Note that PM 6 was not calibrated because of technical difficulties. Hence F just served as a check of the apparatus. sometime PM 6 was even removed and only 5 measurements were made.
triggered a delayed pulse generator, and this delayed pulse triggered all the oscilloscopes and made the exposure of the Polaroid films. Several sets of such pictures are shown in figures 15, 16, 17, and 18 in Chapter IV. The connection of the whole system is shown in figure 9.

Although the matrix elements of the J-matrix can be determined by making four measurements such as A, B, C, and D only, but in actual case an error minimization technique was used to improve accuracy. Assume from the data a coherency matrix J is evaluated by certain method. From this coherency matrix, we can calculate what should be the outputs of those PM tubes. Let us call them A', B', C', D', E', and F' respectively. Of course, they can be expressed in terms of the matrix elements of J exactly as equation (2-37) with the "'" added on A, B, C, D, E, and F. From these values, an error can be defined as follows:

\[
\Delta^2 = (A' - A)^2 + (B' - B)^2 + (C' - C)^2 + (D' - D)^2 + (E' - E)^2
\]

\[
= [J_{xx} - A]^2 + (\frac{1}{2} + \text{Re}(J_{xy}) - B)^2 + (J_{yy} - C)^2
\]

\[
+ (\frac{1}{2} + \text{Im}(J_{xy}) - D)^2 + (\frac{1}{2} + \text{Re}(J_{xy}e^{-i0.328\pi}) - E)^2
\]  \hspace{1cm} (2-38)

If the measurements are perfect, \( \Delta \) has to be zero. In actual cases, it is not zero and its value depends on the choice of the J-matrix.

We solved for the matrix elements \( J_{xx}, J_{yy}, \text{Re}(J_{xy}), \) and \( \text{Im}(J_{xy}) \) by minimizing the error defined above. Thus they were determined from the set of equations:
\[ \frac{\partial \Delta^2}{\partial J_{xx}} = 0 \]
\[ \frac{\partial \Delta^2}{\partial J_{yy}} = 0 \]
\[ \frac{\partial \Delta^2}{\partial (Re \ J_{xy})} = 0 \]
\[ \frac{\partial \Delta^2}{\partial (Im \ J_{xy})} = 0 \]

This minimized the \( \Delta^2 \) and thus \( \Delta \), and we called this minimum value \( \Delta_{\text{min}} \). The intensity of the light is \( F' = I = J_{xx} + J_{yy} \). This value was compared with \( F \) and a ratio \( R \) was defined as:

\[ R = \frac{I}{F} \]  \hspace{1cm} (2-40)

If our apparatus worked properly and in the ideal case, \( R \) should be a constant (the variation range of \( R \) could be considered as a measure of the accuracy). Besides the \( R \), a normalized error \( \delta \) was defined as:

\[ \delta = \frac{\Delta_{\text{min}}}{(J_{xx} + J_{yy})} \]  \hspace{1cm} (2-41)

This had been used much more frequently as a measure of accuracy since sometimes for simplifying the experiment, PM 6 was not used.

This method of measurement with the error minimization technique is by no means the best way to use the polarimeter. on the contrary, we did find a lot of discrepancies in using the error minimization technique described above. In spite of
these discrepancies, it gave us satisfactory results with simple mathematical calculations. We believe that, for most spikes analysed, the error was mainly due to the experimental apparatus rather than due to the technique used in analysing the data.

With this method, about 2,000 spikes of 100 firing of the Nd$^{3+}$ glass laser were analysed. The error $S$ was less than 5% for about 78% of the spikes, it was less than 10% for nearly all spikes (99%). The remaining spikes (1%) had error $S$ greater than 10%, and were usually spikes with very small amplitudes. In that case, the human factors in reading the pictures were responsible for the larger errors.

For the light output from CaWO$_4$ laser, the situation was almost the same except when the spikes were 100% linearly polarized at an angle near 90° or 0°. Let us assume a spike to be linearly polarized at an angle near 90°, then we have $C \gg A \approx 0$. From equation (2-38) and (2-39) one can see that equation (2-39) selects a set of $J_{xx}$ and $J_{yy}$ by equalizing their errors, that is

$$|(J_{xx} - A)| = |(J_{yy} - C)|$$

Both of these values are less than or at most equal to $S(J_{xx} + J_{yy})$, since $S$ is near 5%, we have

$$|(J_{xx} - A)| \leq 0.05 (J_{xx} + J_{yy})$$

but it can be greater than $A$ since $A \approx 0$. Sometimes the selected
\( J_{xx} \) may be several times greater or smaller than the true value so that \( \text{Det}(J) = J_{xx} J_{yy} - |J_{xy}|^2 \) may have an error far exceed 100%! This error introduces serious inaccuracy in calculating \( I_o \) and the degree of polarization (equation 2-27). Sometimes the selected \( J_{xx} \) and \( J_{yy} \) may even give a negative value of \( \text{Det}(J) \) and thus a negative value of \( I_o \) and a degree of polarization exceeds 100%! For this reason, when either \( J_{xx} \gg J_{yy} \equiv 0 \) or \( J_{yy} \gg J_{xx} \equiv 0 \), or equivalently when the spike is almost linearly polarized at 0° or 90°, particular care has been taken to minimize the value of \( \Delta^2 \) under the condition that \( \text{Det}(J) \equiv 0 \). This makes the problem much complicated and can only be solved numerically.

It has been mentioned that the \( \delta \) value is smaller than 5% for most spikes recorded. The contributors of this error were found to be

1. The change of polarization due to reflections and transmissions of mirrors are not perfectly compensated. The reason comes partly from slightly misalignment of the whole apparatus and partly from the divergence of the laser beam.

Kodak IR phosphor was used to find the position of the laser beam. The phosphor can emit orange light when it is irradiated by strong infrared radiation. Alignment of the polarimeter was done by adjusting the whole apparatus until the laser beam hit several points which were premarked particularly for alignment. With a pulsed laser light at infrared region, it is difficult
to align the polarimeter very accurately. The divergence of the beam which is quite large for CaWO₄ laser may be also responsible for this slight alteration of polarization when the beam was split into six components.

(2) Human error in reading pictures and measuring the heights of the spikes. This error dominates when spikes of very small amplitudes are analysed.

(3) The coupling circuits between the phototubes and connecting cable can distort the shape of a spike such that accurate calibration of the sensitivities of photodetectors become impossible. This put a limitation to the time resolution of the polarimeter. Fast time response circuits for this purpose are not difficult to design.

The polarimeter has a time resolution only restricted by the rise time of the external electronic circuits. Good coupling circuits are required to have fast rise time and to be able to deliver a fairly large (say 10 volts) output to the 93 ohm cable without loading the PM tubes to their non-linear region. With slight improvement of external electronic circuits, the time resolution can be extended to less than 100 n.s and the variation of polarization within a single spike would be able to be observed.

It is interesting to test the accuracy of the polarimeter by detecting a light with known polarization. The output of the laser was passed through a linear polarizer such that it was
linearly polarized at $45^\circ$ before it entered the polarimeter. The results of this measurement showed that errors less than $\pm 4^\circ$ for the direction of polarization and $\pm 5\%$ for the percentage polarization were observed. No accuracy test was done by detecting a known partially polarized light because of the difficulties in producing such kind of light sources.
III Properties of Nd$^{3+}$ Doped Lasers

3-1 Spectroscopic Properties of Rare Earth Ions in Crystalline Solids (14, 15)

The progressive filling of the 4f and 5f electron shells of atoms forms the elements generally known as rare earths. The filling of 4f electrons starting with the element cerium ($Z = 58$) and ending with the element lutecium ($Z = 71$) forms the group of elements called lanthanides, while the filling of 5f electrons form the group of actinides. Besides the 4f electrons, the electronic configuration of lanthanides possesses the common feature of a xenon core ($1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6$) and two or three outer electrons ($6s^2$ or $5d6s^2$). It is well known that, starting from cerium, the 4f-eigenfunction contracts so that its maximum lies well inside the $5s^2 5p^6$ closed shell of the xenon structure (16, 17). As a result of this contraction (called lanthanide contraction), the 4f electrons are well shielded by the $5s^2 5p^6$ closed shell such that the interactions between these 4f electrons and the outside environment are relatively weak. Consequently, these elements have similar chemical properties as well as similar spectroscopic properties.

The common structure of xenon core, which is inert either chemically or spectroscopically, shields partially the positive charge of the nucleus. Therefore, an effective nuclear charge $Z'$ can be assumed. The Hamiltonian of the atom containing N
electrons outside the core, including the relativistic effect, can be written as

$$\hat{H} = \sum_{i=1}^{N} \left( \frac{p_i^2}{2m} - \frac{Z_i e^2}{r_i} \right) + \sum_{i>j} \frac{\mathbf{e}_{i,j}^2}{r_{i,j}} + \sum_{i} \mathbf{\zeta}(r_i) \mathbf{\bar{S}}_i \cdot \mathbf{\bar{L}}_i$$  

(3-1)

Where $r_i$ is the distance of electron $i$ from the nucleus and $r_{i,j}$ is its distance from electron $j$.

By the theory of central field approximation, the first summation defines a configuration of the atom which is characterized by the principal and azimuthal quantum numbers of the individual electron orbits $(n_1 l_1, n_2 l_2, \cdots, n_N l_N)$. The second term is the electrostatic interaction between electrons causes the electrostatic splitting of configuration into many terms. Because of this interaction, the orbit and spin angular momenta of individual electrons are no longer good quantum numbers but the total angular and spin momenta defined as

$$\mathbf{\bar{L}} = \sum_{i} \mathbf{l}_i$$
$$\mathbf{\bar{S}} = \sum_{i} \mathbf{s}_i$$  

(3-2)

become good quantum numbers to label a term. Besides the electrostatic interactions, there are still other interactions acting in a free atom (ion). The most important one for the present interest is the spin-orbital interaction due to the relativistic effect. It causes the multiplet splitting of a term and therefore, in a strict sense, the $L$ and $S$ quantum numbers cease to be good quantum numbers and lose their meaning.
It is the total angular momentum $\mathbf{J} = \mathbf{S} + \mathbf{L}$ which becomes a good quantum number and can be used to label the states.

For rare earths, the separation between terms (by electrostatic interaction) is about several thousands of cm$^{-1}$. The spin-orbital interaction, which causes multiplet splitting of a term is usually even weaker, particularly for deep lying terms of high multiplicity. However, it is still comparable to the electrostatic interaction and plays an considerably important role. Only for deep lying terms, a good approximation of the normal coupling could be expected. When this is the case, the quantum numbers $L$ and $S$ approximately preserve their meaning and a level of the free atom or ion can be labeled by specifying its $\gamma SIJ$ quantum numbers ($\gamma$ are any other quantum numbers necessary to specify the level).

**States of an Atom (ion) in a Crystal with Definite Symmetry**

The states of an atom (ion) which forms part of a crystal lattice are different from the states of a free atom (ion). Strictly speaking, the actual states are states of the entire lattice as a whole. However, if the atom (ion) forming the lattice interacts comparatively weakly with respect to surrounding atoms or ions, the influence of this surrounding can be treated as an external interaction (external electric and magnetic field) on the free atom (ion) considered.

The actual fields acting on the ion of the rare earths in crystals are mainly electrical. Magnetic field plays only a
minor role. The magnitude of energy level splitting due to this crystal magnetic field, even in the case of shape lines, does not exceed the order of the width of the lines. Hence it usually can be neglected completely.

Consider the free atom (ion) as a whole, it possesses spherical symmetry, and the level with a given quantum number \( J \) has the degeneracy of multiplicity \( 2J + 1 \). That is, for a free atom (ion), the energy of the states does not depend on the orientation of the total angular momentum vector \( \vec{J} \). When the atom is in an external field of the crystal, it possesses spherical symmetry no more, but the symmetry of the crystal field. The energies of states do depend on the orientation of \( \vec{J} \) vector and the degeneracy of the state is partially removed. For atom (ion) with an odd number of electrons, it can be shown that all the levels are at least doubly degenerate even if the atom or ion is situated in a crystal field without symmetry. This kind of degeneracy is known as Kramers' degeneracy.

The symmetry of the field acting on the ion in the crystalline solids depends on the symmetry of the crystal and the position of the ion in it. Therefore, the contribution to the Hamiltonian due to the field will have a symmetry which either coincides with the symmetry of the point group for a given crystal or lower. For the ion of interest, it can be written as

\[
\tilde{H}' = -e \sum_i^N V(r_1, \theta_1, \phi_1)
\]

(3-3)
where the sum extends over all the electrons of the ion. It is usual to expand this into a series of spherical harmonics,

$$ H' = \sum_1^N \sum_k B_{k q}^i r_1^k Y_{k q}(\theta_1, \phi_1) $$  \hspace{1cm} (3-4) 

In dealing with equivalent electrons, the radial part can be averaged over the radial part of the wavefunctions when the matrix elements of $\tilde{H}'$ are calculated. Hence, by defining

$$ B_k^q = \left( \frac{2k+1}{4} \right)^{1/2} B_k^q \langle r^k \rangle, \text{ where} $$

$$ \langle r^k \rangle = \int_0^\infty R_{n l}(r) r^k R_{n l}(r) \, dr $$

$\tilde{H}'$ can be written as (15)

$$ \tilde{H}' = \sum_{1 q k} B_k^q \left( c_q^{(k)} \right)_1 $$  \hspace{1cm} (3-5) 

where $c_q^{(k)} = \left( \frac{4}{2k+1} \right)^{1/2} Y_{k q}(\theta, \phi)$ is a tensor operator of rank $k$.

The spherical harmonics under the inversion operator $\tilde{I}$ has the symmetry properties

$$ \tilde{I} Y_{k q}(\theta, \phi) = (-1)^k Y_{k q}(\theta, \phi) $$

Therefore, the terms in (3-5) with odd $k$, have odd parity and contribute nothing to the matrix elements of $\tilde{H}'$ between states of same configuration (which implies same parity). Hence the crystal field splitting of a given level is mainly determined by the even terms (with even $k$) in (3-5).*

*The odd terms can cause coupling between different configurations hence states of opposite parity, this contributes a small part of the crystal field splitting and also makes the intra-$f^N$ electric dipole transitions possible.

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If only $f$ electrons are involved, the terms in (3-5) with $k > 6$ contribute nothing. Hence

$$\langle f^{N} r S L J J_z | \widetilde{H}' | f^{N} r' S' L' J' J'_z \rangle = \sum_{k \leq 6} B_{q}^{k} S_{(S, S')}$$

$$\langle f^{N} r S L J J_z | \sum_{i=1}^{N} (C_{q}^{k})_{i}^{(k)} | f^{N} r' S' L' J' J'_z \rangle \quad (3-7)$$

The details of this calculation are very complicated and will not be included here. The general method of finding these matrix elements can be found in articles dealing with spectroscopic properties of rare earths (15, 18, 19, and 20).

The magnitude of the energy splitting due to the crystal field is determined to a first approximation by the diagonal terms of the matrix elements of $\tilde{H}'$. In the practical case, the coefficients $B_{q}^{k}$ are treated as parameters to fit the experimental data. Theoretically they could be calculated by setting a crystal field model and using the lattice constants determined from X-ray crystallographic studies. This kind of calculation has been performed by several people (21, 22). Their results sometimes have $B_{q}^{k}$ value about ten times bigger than the experimental values.

It has already been mentioned that $\tilde{H}'$ has the symmetry of the crystal structure around the ion of interest. Since the Hamiltonian of a free ion has spherical symmetry, the symmetry operator $\tilde{S}$ which leaves $\tilde{H}'$ invariant will leave the total Hamiltonian of the ion in the crystal invariant too. That is

$$\tilde{S} \tilde{H} = \tilde{H} \quad (3-8)$$
It can be proved that $\tilde{S}$ commutes with $\tilde{H}$ and the eigenfunctions of $\tilde{H}$ are simultaneous eigenfunctions of the operator $\tilde{S}$. Thus, the eigenvalues of $\tilde{S}$ will be good quantum numbers to characterize the states of the ion in the given crystal. By introducing these quantum numbers, one can divide the states of the ion into classes such that all the states in a given class interact only with states belonging to the same class and have the same energy. The number of states in a class or the degeneracy of the corresponding level does not exceed four. Therefore, the electronic levels of the ion can have the following degeneracy (multiplicity)

\[
\begin{align*}
    r = 1 & \quad \text{nondegenerate level (rep. by } A) \\
    r = 2 & \quad \text{doubly degenerate level (rep. by } E) \\
    r = 3 & \quad \text{triply degenerate level (rep. by } F) \\
    r = 4 & \quad \text{quadruply degenerate level (rep. by } G) \\
\end{align*}
\]

The most important symmetry elements of the crystal field are the presence of an $n$-fold rotational or inversional axis and the presence of a center of symmetry. The former classifies the states by the crystal quantum number $\mu$ and the latter divides the states into even and odd.

It can be proved that in the presence of a rotational axis $\tilde{C}_n$, the states of the free ion with quantum numbers

\[
M = \mu, \mu \pm n, \mu \pm 2n, \mu \pm 3n, \ldots
\]

will give the states of the ion in the field of given symmetry, characterized by the same crystal quantum number $\mu$ and have same energy. Therefore, one can write (3-9) as
\[ M = \mu \quad (\text{mod. } n) \quad (3-10) \]

This is very similar to the case of an electron in a crystal-
line solid, that is, the presence of a periodic potential makes
the states of the free electron with wave vectors \( \vec{p} = \vec{K} + \vec{k} \),
where \( \vec{K} \) is a vector of the reciprocal lattice, correspond to
the same degenerate state characterized by the wave vector \( \vec{k} \).

The presence of a center of symmetry to a group of the
type \( \overset{\sim}{C}_n \) doubles the order of the group. Each state will be
even or odd and the number of types of states will be simply
doubled. The even state is denoted by an index \( g \) and the odd
will be denoted by an index \( u \).

For the presence of an inversional axis such as \( \overset{\sim}{S}_4 \) (\( \overset{\sim}{C}_4 \)),
the situation is quite similar but with a slight difference as
compared with the case of \( \overset{\sim}{C}_n \) axis. A crystal quantum number \( \mu \)
denoted in equation (3-10) still can be introduced to label
an energy level, but the absence of a separate center of sym-
metry (\( \overset{\sim}{I} \)) leads a small mixture of the states with opposite
parity. A precise characterization of the states requires an
quantum number \( \mu_I \) introduced by Hellewege (24).

Although the absense of a center of symmetry (\( \overset{\sim}{I} \)) leads to
a small mixture of the states of opposite parity, the charac-
terization of levels as even or odd with definite value of \( \mu \)
still approximately maintains its meaning and can be used with-
out introducing the quantum number \( \mu_I \). In order to distinguish
it from the case of a true rotational axis \( \overset{\sim}{C}_n \), the \( u \) and \( g \)
indexes are enclosed in parentheses.
In the presence of a perpendicular rotational axis of second order \( \tilde{C}_2 \) or a plane of symmetry \( \sigma_v \) which passes through the principle symmetry axis, it is easy to prove that the eigenvalues of \( \tilde{C}_2 \) and \( \sigma_v \) are \( \hat{+}1 \) for states with integer \( J \) values and \( \hat{-}1 \) for half integer \( J \) values. Therefore, we designate the state by indexes "\( + \)" and "\( - \)" (positive and negative) for the presence of a \( \tilde{C}_2 \) axis. For the \( \sigma_v \) plane, the indexes "\( s \)" and "\( a \)" (symmetry and antisymmetry) will be used. For example, a state lebeled as \( A_{1g}^+ \) means that it is non-degenerate, \( \mu = 1 \), even, and positive (eigenvalue is \( +1 \) under the rotation of \( C_2 \)).

In the case when both directions of rotation about the principal symmetry axis are physically equivalent, the states \( A_{\pm \mu} \) and \( A_{-\mu} \) should have the same energy because they correspond to the states \( +M \) and \( -M \) which differ only in sign of the projection of the angular momentum on the principal symmetry axis. We shall denote such a doubly degenerate level by \( E' |\mu| \).

Complete tables for designating the states of a free ion under the influence of a crystal field have been given by El'yashevich in his book *Spectra of Rare Earth* (14). The tables include all crystallographic point groups with integer \( J \) values from 0 to 12 and half integer \( J \) values from 1/2 to 25/2.

To illustrate how many sublevels that a given level of a free ion can have, let us take the levels \( ^4F_{3/2} \) and \( ^4I_{11/2} \) of free \( \text{Nd}^{3+} \) ion in a crystal field with \( \tilde{S}_4 \) symmetry. The electronics configuration of the \( \text{Nd}^{3+} \) ion consists of a Xe structure plus three \( 4f \) electrons, hence all the levels have odd
parity. In this example, equation (3-10) can be written as

\[ M = \mu \, (\text{mod. } 4) \]. Hence we have, for the level \( ^{4}F_{3/2} \, (J = 3/2) \),

\[ \mu = \pm 3/2, \pm 1/2 \]. Since both directions of rotation of the \( \hat{S}_4 \) axis

are equivalent, the \( ^{4}F_{3/2} \) level will be split into two doubly
degenerate levels as following

\[
^{4}F_{3/2} \begin{array}{l}
\uparrow \ E'_{1/2} \ (u) \\
\downarrow \ E'_{3/2} \ (u)
\end{array}
\]

Denoting the original level by \( D_{J} \), we have

\[
^{3}D_{3/2} = E'_{1/2} \ (u) + E'_{3/2} \ (u)
\]  

Similarly, for the level \( ^{4}I_{11/2} \), we have

\[
^{3}D_{11/2} = E'_{1/2} \ (u) + E'_{1/2} \ (u) + E'_{1/2} \ (u) +
E'_{3/2} \ (u) + E'_{3/2} \ (u) + E'_{3/2} \ (u)
\]

\[
= 3 \ E'_{1/2} \ (u) + 3 \ E'_{3/2} \ (u)
\]

That is, it splits into six sublevels.

Transitions between Energy Levels of Ion in a Crystal

For an ion in a crystal, both permitted and forbidden
transitions are possible. It is well known, that the electric
dipole transition is possible only between levels with different
parity and magnetic dipole and electric quadrupole transitions
are possible only between levels with same parity. For free
rare earth atoms or ions, the intra-\( f^N \) electric dipole transi-
tions are forbidden because all levels of a same electronic
configuration have the same parity. This is not true when the
ion is a part of the crystal. Electric dipole transitions
between levels of the 4f shell not only can be observed but also form a major part of the spectra of rare earth ions in crystals and other solids. This kind of transition will be called the "forced electric dipole transition" (14).

The intensity of the spectra due to the forced dipole transition is much weaker than the normal electric dipole transition but it is stronger than the normal magnetic dipole and electric quadrupole transitions.

The prohibition of dipole transitions between levels with same parity is related to the presence of the inversion symmetry about the nucleus of the free atom or ion. The presence of a crystal field usually destroys this inversion symmetry and makes forced dipole transitions possible. Sometimes the crystal fields do have an inversion symmetry such as the field with point groups O_h, T_h, D_{6h}, D_{4h}, D_{3h}, D_{2h}, C_{6h}, C_{4h}, C_{31}, C_{2h}, and C_{1}, but forced dipole transitions still can be observed which is related to the oscillations of the ion.

For normal dipole transitions between levels a and b, the transition probability is (25)

\[ A = \frac{64}{3\hbar c} \frac{4}{3} P_0^2 \]  
(3-13)

For non-degenerate levels \( P_0^2 = |P_{ab}|^2 = |(P_x)_{ab}|^2 + |(P_y)_{ab}|^2 \)

\[ + |(P_z)_{ab}|^2 := |\langle \Psi_a | \vec{P} | \Psi_b \rangle|^2 \]  
(3-14)

and for degenerate levels

\[ P_0^2 = \frac{1}{\varepsilon_a} \sum_{ab} |P_{ab}|^2 \]  
(3-15)
\( g_a \) is the multiplicity of the initial level. It is more convenient to write \(|P_{ab}|^2\) in another form

\[ |P_{ab}|^2 = |(P_+)_ab|^2 + |(P_-)_ab|^2 + |(P_Z)_ab|^2 \]  \hspace{1cm} (3-16)

where \( P_+ = \frac{1}{\sqrt{2}} (P_x + iP_y) \) and \( P_- = \frac{1}{\sqrt{2}} (P_x - iP_y) \) \hspace{1cm} (3-17)

The components \( P_+ \) and \( P_- \) correspond to rotation dipoles with positive and negative directions of rotation, respectively, while \( P_Z \) is a linear dipole along \( z \) direction. When the radiation is observed at a direction perpendicular to \( z \) axis, the radiation due to \( P_Z \) is linearly polarized in \( z \) direction and those due to \( P_+ \) and \( P_- \) are polarized at a direction perpendicular to \( z \) axis. The former is called \( \pi \)-component and the latter is called the \( \sigma' \)-component.

![Diagram of \( \pi \)-component and \( \sigma' \)-component](image-url)

**Fig. 10.**

The selection rules for normal electric dipole transition are listed and derived in many books (26). Besides the strictly true selection rule of parity, the selection rule for quantum numbers \( J \) and \( M \) are
\[ \Delta J = 0, \pm 1 \]  

and

\[ \Delta M = 0 \quad (P_z)_{ab} \neq 0 \quad \pi\text{-component} \]

\[ \Delta M = 1 \quad (P_+)_{ab} \neq 0 \quad \sigma'\text{-component} \]  

\[ \Delta M = -1 \quad (P_-)_{ab} \neq 0 \]

Under the influence of a crystal field, the crystal quantum number \( \mu \) instead of \( M \) is used, by equation \( (3-10) \) \( \bar{M} = \mu \) (mod. \( n \)). Consequently, the selection rules of quantum number \( \mu \) for normal dipole transition are as following:

\[ \Delta \mu = 0 \quad (P_z)_{ab} \neq 0 \quad \pi\text{-component} \]

\[ \Delta \mu = 1 \quad (P_+)_{ab} \neq 0 \]

\[ \Delta \mu = -1 \quad (P_-)_{ab} \neq 0 \quad \sigma'\text{-component} \]  

(3-20)

For forced dipole transitions, the selection rules are much more complicated. The wave functions of level \( a \) and \( b \) actually have no definite parity since the inversion symmetry has to be destroyed either by the crystal field or the vibration of the ion. If the wave function of the states of the free ion (which have definite parity) are represented by \( \psi^{(o)}_n \), to the first order perturbation

\[ \psi_a = \psi^{(o)}_a + \sum_d C_{ad} \psi^{(o)}_d \]

\[ \psi_b = \psi^{(o)}_b + \sum_d C_{bd} \psi^{(o)}_d \]  

(3-21)

where \( \psi^{(o)}_d \) is a wavefunction of opposite parity in comparison with \( \psi^{(o)}_a \) and \( \psi^{(o)}_b \), while the coefficients \( C_{ab} \) are
\[ o_{ad} = \frac{\langle \psi_{d}^{(o)} | \tilde{H}_{\text{odd}} | \psi_{a}^{(o)} \rangle}{E_{a} - E_{d}} \]
\[ o_{bd} = \frac{\langle \psi_{d}^{(o)} | \tilde{H}_{\text{odd}}' | \psi_{b}^{(o)} \rangle}{E_{b} - E_{d}} \] (3-22)

where \( \tilde{H}_{\text{odd}}' \) is the perturbation Hamiltonian which causes this admixture of states with opposite parity. As mentioned earlier, \( \tilde{H}_{\text{odd}}' \) consists of two parts, a static term due to the crystal field and a dynamic term due to the oscillations of the ion. The lack of an inversion symmetry of the crystal field is represented by the odd terms of the crystal field expansion in equations (3-4) and (3-5). Hence representing these odd terms by \( \tilde{U}_{\text{odd}} \) and those terms due to oscillation by \( \tilde{U}_{\text{osc}} \), we have

\[ \tilde{H}_{\text{odd}}' = \tilde{U}_{\text{odd}} + \tilde{U}_{\text{osc}} \] (3-23)

The transition probability of the forced electric dipole transition is given by the matrix elements \( (P_{\alpha})_{ab} = \langle \psi_{a} | P_{\alpha} | \psi_{b} \rangle \), where \( \alpha \) stands for z, +, and -. Using the expressions for \( \psi_{a} \) and \( \psi_{b} \) and remembering that \( \psi_{a}^{(o)} \) and \( \psi_{b}^{(o)} \) have same parity and all \( \psi_{d}^{(o)} \)'s have opposite parity with respect to \( \psi_{a}^{(o)} \) and \( \psi_{b}^{(o)} \), we have

\[ (P_{\alpha})_{ab} = \sum_{d} \left[ C_{ad}^{*} \langle \psi_{d}^{(o)} | \tilde{P}_{\alpha} | \psi_{b}^{(o)} \rangle + C_{bd} \langle \psi_{d}^{(o)} | \tilde{P}_{\alpha} | \psi_{a}^{(o)} \rangle \right] \] (3-24)

By inserting the expressions for \( c_{ad} \) and \( c_{bd} \), one can find that the selection rules for forced dipole emission will be determined by the difference from zero of the products of matrix elements of the type

\[ \langle \psi_{d m}' | \tilde{H}_{\text{odd}}' | \psi_{d m}^{(o)} \rangle \langle \psi_{d m}^{(o)} | \tilde{P}_{\alpha} | \psi_{b m}^{(o)} \rangle \] (3-25)
where \( m \) stands for quantum number \( M \) or \( u \).

The second factor in (3-25) is different from zero when \( m = m'' \) if \( P_\alpha = P_Z \), and when \( m = m'' \pm 1 \) if \( P_\alpha = P_\pm \). The first factor can be decomposed further into two matrix elements

\[
\langle \psi_{am'}^{(o)} | \tilde{U}_{\text{odd}} | \psi_{dm}^{(o)} \rangle + \langle \psi_{am'}^{(o)} | \tilde{U}_{\text{osc}} | \psi_{dm}^{(o)} \rangle
\]

(3-26)

Let us first consider the first matrix element. If there is a term \( C_q^{(k)} \) in the odd parts of the crystal field expansion (3-5), then this matrix element is non zero provided \( m' - m = q \).

Therefore equation (3-25) will be non zero if

\[
m' - m'' = q \quad (P_Z \neq 0)
\]

\[
m' - m'' = q \pm 1 \quad (P_\pm \neq 0)
\]

As an example, let us consider that the crystal field around the ion has \( \widetilde{S}_4 \) symmetry. The odd terms \( (k = \text{odd}) \) in (3-5) are invariant under the operation of \( \widetilde{S}_4 \). In other word

\[
\widetilde{S}_4 \ Y_{kq}(\theta, \phi) = Y_{kq}(\theta, \phi).
\]

However

\[
\widetilde{S}_4 \ Y_{kq}(\theta, \phi) = \tilde{\Pi} \ C_4 \ Y_{kq}(\theta, \phi) = [\tilde{\Pi} \ Y_{kq}(\theta, \phi)] \ e^{i q \pi/2}
\]

\[
= (-1)^k \ e^{i q \pi/2} \ Y_{kq}(\theta, \phi)
\]

Therefore, it must have \(-e^{i q \pi/2} = 1\), which gives \( q = \pm 2, \pm 6 \).

The selection rule will be

\[
\Delta m = \pm 2, \pm 6 \quad \text{if} \ (P_Z)_{ab} = 0 \quad \text{and}
\]

\[
\Delta m = \pm 1, \pm 5, \pm 7 \quad \text{if} \ (P_\pm)_{ab} = 0
\]

For \( \widetilde{S}_n \) symmetry of the crystal field, \(-n/2 \leq \mu \leq n/2\) and since both ends of the \( \widetilde{S}_n \) axis are physically equivalent, the
two states $A_+\mu$ and $A_-\mu$ form a doubly degenerate state $E_{1|\mu|}$, hence the possible crystal quantum number will be $0 \leq \mu \leq n/2$. For $\tilde{S}_4$ symmetry, $\mu = 0, 1, 2$ for ions with even number of electrons and $\mu = 1/2, 3/2$ for ions with odd number of electrons. Under this condition, the selection rule and the polarization of the radiation for forced dipole transitions from the ion subjected to crystal field with $\tilde{S}_4$ symmetry will be

\[ \Delta \mu = 0, \pm 2 \quad \text{for } (P_z)_{ab} = 0 \quad \pi \text{-component} \]
\[ \Delta \mu = \pm 1 \quad \text{for } (P_\pm)_{ab} = 0 \quad \sigma' \text{-component} \quad (3-27) \]

The second term in equation (3-26) is associated with the vibration of the lattice. For small oscillations, $\tilde{U}_{osc}$ can be expanded into a Taylor series about the equilibrium position and can be treated in a similar manner. When the crystal field does have a center of symmetry ($\tilde{U}_{odd} = 0$), $\tilde{U}_{osc}$ become the major component which makes dipole transitions between levels with same parity possible.

**Spectroscopic Properties of Nd$^{3+}$ doped CaWO$_4$ Lasers**

Calcium tungstate forms crystals which have tetragonal symmetry with space group $O_{4h}$. The symmetry of the ions Ca and W is $\tilde{S}_4$ (the $\tilde{S}_4$ axis is the c-axis) and those of O is $\tilde{C}_1$. Each unit cell has four molecules. When the Nd$^{3+}$ ion is doped into the calcium tungstate crystal, it replaces divalent calcium ion and the local charge neutrality could not be maintained unless some charge compensation technique is used (29; 30).
\[
\begin{align*}
\text{Ca}^{++} \quad \text{WO}_4^{-} & \quad \text{Nd}^{+++} \quad \text{WO}_4^{-} \\
\text{WO}_4^{-} & \quad \text{Na}^{+} \quad \text{WO}_4^{-} \quad \text{Ca}^{++} \\
\text{Ca}^{++} \quad \text{WO}_4^{-} & \quad \text{Ca}^{++} \quad \text{WO}_4^{-} \\
\end{align*}
\]

\text{Na}^{+} \text{ compensated } \text{CaWO}_4

\[
\begin{align*}
\text{Ca}^{++} \quad \text{WO}_4^{-} & \quad \text{Ca}^{++} \quad \text{WO}_4^{-} \\
\text{WO}_4^{-} & \quad \text{Nd}^{+++} \quad \text{NbO}_4^{-} \quad \text{Ca}^{++} \\
\text{Ca}^{++} \quad \text{WO}_4^{-} & \quad \text{Ca}^{++} \quad \text{WO}_4^{-} \\
\end{align*}
\]

\text{Nb}^{5+} \text{ compensated } \text{CaWO}_4

\[
\begin{align*}
\text{Ca}^{++} \quad \text{WO}_4^{-} & \quad ( ) \quad \text{WO}_4^{-} \\
\text{WO}_4^{-} & \quad \text{Nd}^{+++} \quad \text{WO}_4^{-} \quad \text{Nd}^{+++} \\
\text{Ca}^{++} \quad \text{WO}_4^{-} & \quad \text{Ca}^{++} \quad \text{WO}_4^{-} \\
\end{align*}
\]

\text{Uncompensated } \text{CaWO}_4

\textbf{Fig. 11.} Three kinds of charge compensation of \text{Nd}^{+++} \text{ ion in calcium tungstate crystals.}
Three different kinds of compensation have been used frequently and these are shown in figure 11. All these charge compensations can introduce distortions of the \( \tilde{S}_4 \) symmetry of the crystal field acting on Nd\(^{3+}\) ion. The amount of distortion depends on how these charge compensating ions (vacancies) are distributed in the structure.

Paramagnetic resonance experiment on uncompensated Ce\(^{3+}\) doped CaWO\(_4\) was performed by Klein and Mins (31). Later, similar kind of experiment of Ce\(^{3+}\) doped, Na\(^+\) compensated CaWO\(_4\) was performed by Nassau and Loiarono (32), their result showed that the Na\(^+\) ion was more or less randomly distributed over all Ca\(^{2+}\) sites, most of the time, was far enough away from the Ce\(^{3+}\) ion such that its influence is limited to minor lines observed at very low intensities. Similar behavior is expected for Nd\(^{3+}\) doped crystals. We can assume that the field acting on Nd\(^{3+}\) ion is approximately unchanged by the charge compensation process and has the original \( \tilde{S}_4 \) symmetry.

The energy level diagram of Nd\(^{3+}\) ion in CaWO\(_4\) is shown in Fig. 12. Laser actions have been observed at 1.06 \( \mu \), 1.34 \( \mu \), and 0.91 \( \mu \) which correspond to transitions from multiplet \(^4F_{3/2}\) to multiplets \(^4I_{11/2}\), \(^4I_{13/2}\), \(^4I_{9/2}\) respectively. The 1.06 \( \mu \) transition is the strongest among these three. This transition is intra-\( f^3 \) electric dipole in nature and obeys the selection rules given in (3-27).

It has already been shown that for a crystal field with \( \tilde{S}_4 \) symmetry and for the multiplet \(^4F_{3/2}\)
Fig. 12. Energy levels of Nd$^{3+}$ in CaWO$_4$. 
\[ D_{3/2} = E'_{1/2} (u) + E'_{3/2} (u), \] hence it splits into 2 levels. For \( ^4I_{11/2} \), six sublevels are obtained, i.e.

\[ D_{11/2} = 3 E'_{1/2} (u) + 3 E'_{3/2} (u). \]

The energies of these sublevels can be determined experimentally and can be found in literatures. For example, the energy levels of these two multiplets for Na\(^+\) compensated crystal as given by Johnson (33) is shown in the following diagram

\[ ^4P_{3/2} \]

\[ \text{11469 cm}^{-1} \]

\[ 10,652 \text{ A} \]

\[ 10,582 \text{ A} \]

\[ ^4I_{11/2} \]

\[ 2227 \]

\[ 2189 \]

\[ 2057 \]

\[ 2016 \]

\[ 1977 \]

\[ \sigma \]

\[ \pi \]

Fig. 13. Energy level diagram of \(^4P_{3/2}\) and \(^4I_{11/2}\). (After L.F. Johnson)

The even part of the crystal field with \( \bar{S}_4 \) symmetry is

\[ \tilde{V}_{\text{even}} = B_0^2 c_0^{(2)} + B_0^4 c_0^{(4)} + B_4^4 c_4^{(4)} (c_{4}^{(4)} + c_{-4}^{(4)}) + B_6^6 c_0^{(6)} + B_4^6 (c_4^{(6)} + c_{-4}^{(6)}) \]  (3-28)

There are only five \( B_q^k \) coefficients to be determined. From the data reported by Johnson, it is possible to make a least square
error fitting and determine all these coefficients. Therefore, to each sublevel, a proper crystal quantum number can be assigned. Combining this with the selection rule given above, the polarization of the radiation due to each transition can be determined.

At room temperature, not all of the lines due to these transitions can be observed in the output light of the laser but only two lines are involved. They correspond to transitions shown in Fig. 13. The line of 10,652 Å is a $\sigma$-component and the other of 10,582 Å is a $\pi$-component.

If the axis of the laser rod coincides with the $a$-axis of the crystal, the $\sigma$-component will be linearly polarized with $\overline{E}$ vector along $b$-axis, while the $\pi$-component is linearly polarized with $\overline{E}$ vector along $c$-axis (Fig. 10. $c$-axis coincides with $z$-axis). The output light of the laser will involve two frequency components with perpendicular directions of polarization.

3-3 Properties of Nd$^{3+}$ doped Glass Lasers

Due to the contraction of the $4f$ wavefunction of rare earth ions, the energy levels of the Nd$^{3+}$ in glass is approximately the same as those in CaWO$_4$. Laser emissions have been observed at 1.06 μ and 0.918 μ (34, 35, 36) which are due to intra-$f^3$ dipole transitions from $4F_{3/2}$ to $4I_{11/2}$ and $4F_{3/2}$ to $4I_{9/2}$ respectively. The fluorescence spectra of these transitions are almost spikeless but are still quite sharp, although considerab-
ly broader than in crystals. This is in consistence with the random nature of the crystal field inside the glass matrix.

The broad and spikeless spectra of the fluorescence give little information about the energy levels and the transitions involved. When the temperature goes down to liquid nitrogen temperature (77°C), the spectra show two peaks clearly resolved, but the broadness of the lines are still about the same. It has been shown that the upper level $^4F_{3/2}$ is split into two groups with an energy spacing about 120 cm$^{-1}$ such that the upper group is partially filled at 300°C but is empty at 77°C (35). Hence the doublet structure of these lines is only possible if each terminal level is also split into two groups. The energy level diagram thus derived was published by Maurer (37) and is reproduced in figure 14. The splitting of the ground level which depends on the composition of the glass matrix has been found to be important to the pumping of the laser at room temperature (38).

The strongest laser emission occurs at 1.06 μ which bears the common feature of randomly shaped spikes of solid state lasers. The spectral analysis gave the results that this emission involved a large number of sharply defined lines which, in an ordinary pumping, can cover a band about 100 A width. Lowering the pumping can decrease the width of the band but a large number of sharp lines still have been observed at low pumping energy such that the laser output has only several spikes. Multimode phenomena are apparently inevitable due to
Fig. 14. Level scheme of Nd$^{3+}$ in soda lime glass. Arrow shows the strongest transition.
(After R. D. Maurer)
this wide frequency band of the stimulated emission. Similarly, each spike can be the superposition of several lines with different frequencies.

The internal electric field produced by the glass matrix apparently have random directions and magnitudes due to the amorphous nature of glass. It lacks a definite symmetry and makes a theoretical calculation of the polarization of the light from the laser almost impossible. However, we think that the situation is just quite similar to the case of CaWO₄ lasers but only much more complicated. We are dealing with more frequency components which have definite polarization randomly orientated. Each frequency component could be linearly, circularly, or more probably, elliptically polarized. These have been proved by the polarization measurement described in Chapter IV.
IV Experimental Results

3-1 Polarization Measurement of the Light from Nd\textsuperscript{3+} doped Glass Lasers

During the experiment, two samples of Nd\textsuperscript{3+} glass laser rods were used. The first one was a 1/4" D by 17/8" rod with 2\% (in atom) trivalent Nd ions doping. The rod was coated on both sides with multilayer dielectric coatings which form a high efficiency oscillating cavity. The threshold energy input was measured with our pumping configuration to be around 65 joules. Another sample was a 1/4" D by 21/2" rod with 2\% Nd\textsuperscript{3+} doping. This rod has a relatively higher threshold and was not frequently used.

The stimulated emission of both rods are centered at 1.06 \mu m, which is due to the transition from the multiplet \( ^4\text{F}_{3/2} \) to \( ^4\text{I}_{11/2} \). The terminal level for this transition (\( ^4\text{I}_{11/2} \)) is about 2,000 cm\(^{-1}\) above ground level, therefore, it is unoccupied at ordinary temperature. This makes the Nd\textsuperscript{3+} glass laser a low threshold four level laser and it can be operated easily at room temperature.

The stimulated emission of both Nd-glass lasers have the familiar randomly spiky nature of pulsed laser output. Generally, light from Nd-glass laser has a wide wavelength spread which depends on how much energy above threshold has been supplied to the flashtube. The wavelength of the output light covers a relatively narrow band when it is pumped just slightly
above threshold but a much wider wavelength interval is covered at higher energy input to the flashtube. For normal pumping, a 100 A wide band is a typical value that the stimulated emission at 1.06 μ can cover. In this wavelength band, a great number of sharp lines can be found for ordinary random spiky output but continuous bands were also observed with some clad glass rod that showed limit cycle behaviour (38).

In the following section, two polarization measurements with the light from Nd-glass lasers will be described in detail as well as some qualitative explanation of the experimental results to which we think the truth of the explanation is highly probable.

We first measured the polarization of individual spike. After a long time of observation, it was found that the spikes in a single firing of the laser did not have the same polarization. A spike usually has a width around 1 μ sec. In order to measure the polarization of a single spike, the high time resolution polarimeter was constructed and used. Several typical pictures of this measurement are shown in figures 15, 16, and 17.

With the 1/4" D × 17/8" rod as the sample, about 2,000 spikes from about 100 firing of the laser were analysed and only statistical results could be drawn. For each spike, its polarization state was calculated from the data recorded on the pictures and represented in coherency matrix form. From
Fig. 15. Typical traces of the Nd$^{3+}$-glass laser output at 72 joules pumping. From top to bottom: $I(0^\circ,0)$, $I(45^\circ,0)$, $I(90^\circ,0)$, $I(45^\circ,\pi/2)$, $I(45^\circ,0.328\pi)$. Three firings of the laser have been recorded on these pictures.

Amplitude 0.25 v/div. for B, C
0.2 v/div. for A, D, E
20 μsec/div. horizontal sweep
Fig. 16. Typical traces of the Na\(^{3+}\)-glass laser output at 80 joule pumping. Two fires of the laser have been recorded in these pictures. From top to bottom I(0°, 0), I(45°, 0), I(90°, 0), I(45°, π/2), I (45°, 0.328π).

Amplitude 0.2 V/div. for A, B, D
0.25 V/div. for C, E
20 μsec/div. horizontal sweep
Fig. 17. Typical traces of the Nd³⁺-glass laser output at 400 joule pumping. From top to bottom, I(0°, 0), I(45°, 0), I(90°, 0), I(45°, π/2), I(45°, 0.328π).

Amplitude A, D, E 0.2 v/div.
B, C 0.25 v/div.
10 μsec/div. sweep
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<tr>
<th>Pumping energy (J)</th>
<th>Spike order</th>
<th>J( _{xx} )</th>
<th>J( _{yy} )</th>
<th>Re(J( _{xy} ))</th>
<th>Im(J( _{xy} ))</th>
<th>I( _0 )</th>
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<th>Degree of polarization</th>
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Note: The table continues with more rows and columns, but they are not shown here.
the coherency matrix, the degree of polarization and type of polarization were determined. A tabulation of the measured results for several firing of the laser at different pumping energies is given in table 1. The spikes are tabulated in chronological order and their polarization states are given in coherency matrix form. The unpolarized component \( I_0 \) in table 1 and the intensities of the polarized component measured at \( x \) and \( y \) directions \((P \text{ and } Q \text{ in table 1})\) are also tabulated. Finally the degree of polarization and the normalized error are listed as a criticism of the measurement.

From these data, several general conclusions could be drawn.

1. The light from Nd\(^{3+}\) glass laser is polarized, the polarization being changed from spike to spike.

2. Each spike is in general partially polarized. Unpolarized component can be found in the output of the laser. The completely polarized component of the output light is mostly elliptically polarized, the degree of polarization and the type of polarization being changed from spike to spike.

3. The average degree of polarization per spike increases as the pumping energy is lowered. Table 2 shows how the degree of polarization per spike changes with pumping energy (Fig.18). From the figure, one can see that if the pumping energy is lowered to the threshold, the average degree of polarization per spike tends to 100\%, that is, the spikes are completely polarized. As the pumping energy increases, more and more unpolarized
<table>
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<th>Pumping Energy (joules)</th>
<th>72</th>
<th>80</th>
<th>140</th>
<th>240</th>
<th>400</th>
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<td>Total No. of Spikes registered</td>
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<td>608</td>
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<td>293</td>
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<td>0.68</td>
<td>0.44</td>
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<td>Threshold Energy Input = 65 joules</td>
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</table>

Table 2

components are involved in each spike.

(4) In spite of the average degree of polarization changes as shown in figure 18, the degree of polarization of each individual spike changes quite randomly in a single fire of the laser. For example, at 80 joules pumping, some spikes have degree of polarization around 20%, although most of them have degree of polarization ranges from 50% to 80%. Similarly, at 400 joule pumping, completely polarized spikes are also observed but most of them have degree of polarization around 30%.

(5) In a single firing of the laser, the spikes can have a certain preferred kind of polarization. For example, sometimes the \( J_{xx} \) value can be always greater than \( J_{yy} \) for all spikes in a single fire or \( \text{Im}(J_{xy}) \) can be always negative for all spikes in a single fire of the laser. This phenomenon is more obvious when the pumping energy is near threshold. For determining whether there is a preferred kind of polarization
for the output light from Nd$^{3+}$ doped glass lasers, some further experiments were done. This was the polarization measurement of the total radiant energy output.

If no compensating devices (1/4 λ plate etc.) are used in the polarization measurement, one can not detect circular polarization. Hence in equation (2-36), by letting $\xi = 0$, $I(\theta, 0)$ takes the form

$$I(\theta, 0) = J_{xx} \cos^2 \theta + J_{yy} \sin^2 \theta + \sin2\theta \Re(J_{xy})$$

$$= I_0/2 + I_1 \cos^2(\theta - \vartheta) \quad (4-1)$$

where

$$I_1 = \sqrt{(J_{xx} - J_{yy})^2 + 4 \left( \Re(J_{xy}) \right)^2} \quad (4-2-1)$$

$$I_0 = (J_{xx} + J_{yy}) - I_1 \quad (4-2-2)$$

$$\tan 2\vartheta = 2 \Re(J_{xy})/(J_{xx} - J_{yy}) \quad (4-2-3)$$

From equation (4-1), one can see that $I(\theta, 0)$ represents a partially polarized wave with unpolarized component $I_0$ and linearly polarized component $I_1$ which polarized at an angle $\vartheta$ with respect to the x-axis.

The experiment performed for checking whether there was prefered kind of polarization was to detect $I(\theta, 0)$ at various $\theta$ values and integrated the output signals from the PM tubes such that the polarization of the total light energy output of a single firing of the laser could be measured. In this experiment, 5 PM tubes with linear analysers were used. The analysers were set at 0°, 36°, 72°, 108°, and 144° respectively. The outputs of the PM tubes were integrated and then displayed
on oscilloscopes. The sensitivities of the PM tubes had been calibrated by a similar method described in Sec. 2-4. Therefore, from the heights of the traces on the screens, the polarization of the total energy output in a single firing of the laser could be calculated. This experiment has been performed at various pumping energies. Some of the traces are shown in figures 19 and 20. Many curves have been plotted according to these data and several of them are presented in figures 21 and 22. For the two rods studied, the experimental results have shown that

1. The total energy output in a single firing of the laser is usually polarized. With low pumping energy, more frequently an output with a high degree of polarization (up to 35%) would be observed. When the pumping increases, the degree of polarization was usually around 10% to 20%. Nearly unpolarized output have also been observed at various pumping energies.

2. The direction of the polarization of the polarized component is changed from firing to firing quite randomly but not randomly enough to completely rule out the possibility of the existence of a certain kind of preferred polarization at various pumping energies.

3. There is no obvious effect when a transverse magnetic field up to 7,000 gauss is applied at the vicinity of the laser rod.

The results of these measurements are consistent with the random nature of the local fields present in a glass matrix.
Fig. 19. Integrated output of the $\text{Nd}^{3+}$ glass laser at 80 joule pumping.
(1)=I(0°,0); (2)=I(360°,0); (3)=I(720°,0);
(4)=I(1080°,0); (5)=I(1440°,0).
Amplitude 20 mv/div. for (1), (2), (3), (4)
50 mv/div. for (5)
100 μsec/div. sweep.
(1) and (3) have been horizontally shifted.
Very large degree of polarization can be seen on the picture.
Fig. 20. Integrated output of the Nd$^{3+}$ glass laser at 140 joule pumping. Amplitude 50 mV/div. for I(144°,0) 20 mV/div. for others 100 µsec/div. sweep. The degree of polarization of the output light is quite low in this picture.
Fig. 21. Polarization of the total output of Nd$^{3+}$ doped glass laser at 80 joule pumping.

Fig. 22. Polarization of the total output of Nd$^{3+}$ doped glass laser at 140 joule pumping.
The orientation of this local field is completely random in space. The magnitude of this field, however, is not so randomly distributed as its direction. This is due to the fact that at the very close neighborhood of an imurity ion such as $\text{Nd}^{3+}$ in our case, the glass actually has a crystalline structure.

Any single mode of oscillation of the laser must possess a definite polarization. Due to the amorphous structure of glass and its random local field, it is obvious that there are no definite modes of oscillation, which have such low threshold as compared with other modes that they can predominate over any other modes of oscillation and become the only possible modes that can be observed at normal pumping. The situation is quite different in other kinds of lasers, for example, the mode of oscillation which is linearly polarized at c-axis of a $90^\circ$ cut ruby laser rod has such a low threshold as compared with other modes that it becomes the only mode which can oscillate at moderate pumping. Later, the experimental results about $\text{Nd}^{3+}$ doped $\text{CaWO}_4$ laser show two different modes of oscillation which have comparably low thresholds that predominate over other possible modes and become the only transitions (modes) that can occur at room temperature. In glass lasers, the spectroscopic analysis of their output light shows that a great number of sharply defined lines exist even there are only several spikes in the output light. Therefore, we expect that the glass laser can oscillate at a great number of different modes under normal pumping condition. In other words, these different modes
have comparable thresholds to oscillate. At low pumping energies very close to threshold, only the modes with lowest thresholds can oscillate, and each spike consists essentially of one mode of oscillation and possesses a definite polarization. This is the reason why the spikes approach 100% polarization near the threshold. As the pumping energy increases, each spike will be the result of an incoherent (may be not completely incoherent) superposition of several different modes of oscillation and the degree of polarization is decreased. In the experiments with Nd$^{3+}$ doped CaWO$_4$ lasers, we have some proof that a spike can consists two different modes of oscillation and can have a large portion of unpolarized component. This could be the result of several different areas emitting simultaneously but with different modes; however, if this is true, certain coupling mechanism should exist in order to have these different areas emit light synchronously. Otherwise they just emerge together by chance. To this problem, more experiments should be done before a definite conclusion can be achieved. One thing we can do with the high time resolution polarimeter is to analyse the change of polarization within a single spike. The situation is particularly simple when such measurement is performed on the light from a Nd$^{3+}$ doped CaWO$_4$ laser because it only composes of two modes of oscillation. The results of the measurement will give us some information about the mechanism of how a spike is formed.
Polarization Measurement of the Light from Nd$^{3+}$ doped CaWO$_4$ Laser

In this experiment, the sample used was a Na$^+$ compensated crystal with 2% Nd$^{3+}$ (in atom) doping. The rod is 1/4" D by 2" with multi-dielectric coatings on both end surfaces. The threshold energy input to the flashtube for laser action to start was measured to be around 16 joules with our pumping configuration and this value changed sensitively with respect to ambient temperature.

CaWO$_4$ forms crystals with tetragonal symmetry. The sample we used was cut with the rod axis coincident with the a-axis of the crystal. The b and c-axes are perpendicular to the a-axis. The stimulated emission is still due to the transition between the multiplets $^4F_{3/2}$ and $^4I_{11/2}$. The energy level diagrams are shown in figures 12 and 13. At room temperature, the output light from the sample involves two spectroscopic lines at 10,578 A and 10,648 A. These lines were measured by using a Jarrell Ash monochromator.

Two experiments have been performed with this CaWO$_4$ laser. The first experiment was the polarization measurement of individual spikes. The experimental results were checked by using the monochromator. Precisely the same results have been obtained.

The output light from the Nd$^{3+}$ doped CaWO$_4$ laser still have the familiar spiky nature but when the pumping is high enough, a DC component appears during most of the lasing period, which
shows that the crystal is lasing continuously in the period.

Similar procedures were used to measure the polarization of each spike with the high time resolution polarimeter. Several sets of typical traces are shown in figures 23, 24, and 25. From these pictures, one can see that there are more distortions among signal outputs from the PM tubes for CaWO$_4$ laser than those for glass lasers. Sometimes, it is even difficult to identify corresponding spikes. This shows that the spikes are highly polarized. Some of the results of this measurement are shown in table 3.

It is interesting to note that at the beginning of the lasing period, there are usually several spikes bearing quite regular shapes (figure 24). They are almost equally spaced and appear like a "decayed oscillation". At very low pumping energy, for example, 10% above threshold, only spikes with regular shapes occur in the laser output (figure 23). All these "regular shaped" spikes are polarized linearly at 70° with respect to the arbitrarily chosen 0° axis of the crystal. It was found that this 70° direction was actually the direction of the b-axis which was equivalent to a-axis for CaWO$_4$. The c-axis is at -20° with respect to the arbitrarily selected reference. At higher pumping energy (Fig. 24), besides the several regularly shaped spikes at the front of the lasing period, all other spikes are more or less randomly shaped and partially polarized. The imaginary part of J$_{xy}$ for these spikes are usually not zero but stayed very small, which indicated that the polariza-
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<sup>δ</sup> Negative value of $I_o$ and a degree of polarization greater than 1.00 are due to the error minimization technique used in the analysis of the data, see discussion on page 34.
Fig. 23. Traces of the Nd$^{3+}$ CaWO$_4$ laser output. 10% above threshold pumping. The spikes polarized at 70°.
Amplitude 0.1 v/div. for all traces
30 μsec/div. sweep
Fig. 24. Traces of the Nd$^{3+}$ CaWO$_4$ laser output. 50% above threshold pumping. Regular shaped spikes appear at the beginning of lasing period, they polarize at 70° and has 10,578 Å wavelength. The rest of the spikes have random shape.
Amplitude A and B 0.2 v/div.; C 0.125 v/div.; D and E 0.1 v/div.
20 μsec/div. sweep
Fig. 25. Traces of the Na$^{+}$ doped CaWO$_4$ laser output at 3.7 x threshold. Amplitude A, D, and E 0.2 V/div. B and C 0.1 V/div.

I(45°, 0.0)
I(90°, 0.0)
I(45°, 0.0)
I(0°, 0)}
tion of the completely polarized component is almost linear. At quite high pumping (Fig. 25), the spikes are much bigger and wider, the distortions among the signal outputs from the PM tubes are reduced and the identification of spikes becomes easier. In the meantime, the degree of polarization of these spikes is reduced to a value around 65%.

A very interesting feature of the measurement results is found when one ignores the imaginary part of $J_{xy}$ and represents the polarization by $I_0$, $I_1$, and $\phi$ as shown in equations (4-1) and (4-2). The results are amazing. Nearly all the spikes are partially polarized either at $-20^\circ$ (c-axis) or $70^\circ$ (b-axis). When the pumping energy is low, more spikes are polarized at the direction of b-axis. However, when the pumping energy is high enough, the number of spikes polarized at the direction of c-axis increases rapidly and finally dominates the most radiant energy output of the laser.

The results of these measurements suggest that two different trains of spikes are involved in the output light of the laser. At very low pumping energy, only the regularly shaped spikes (completely polarized at $70^\circ$) appear and their regular shape and uniform polarization show the possibility that the laser is operated in a single mode of oscillation and the shape of these spikes can be described by the equation of Statz and deMars (39). As the pumping energy is increased, irregular shaped spikes appear (partilly polarized, most of them are polarized at $-20^\circ$) which indicates that another mode of oscilla-
tion occurs and the unpolarized component of these spikes are the results of incoherent superposition of these two independent modes of oscillations. At very high pumping, say, 100% above threshold, this latter mode of oscillation dominates the output of the laser.

For further check of this conclusion, a monochromator was used to separate these two components. The light from the laser was incident on the slit of the monochromator after passing through a polarizer. There was a 7102 PM tube on the monochromator to detect the light. The slit could be easily adjusted to get a resolution around 1 A. The output light from the Nd$^{3+}$ doped CaWO$_4$ laser has been measured and the results showed that it involved two spectroscopic lines at 10,578 A and 10,648 A. Figure 26 shows the relative magnitudes of these two lines at various pumping energies. The time constant of the PM tube on the monochromator is relatively long as compared with the width of spikes. The traces in Fig. 26 are the integrations of the waveforms of the laser output. From the pictures, one can see that at sufficiently low pumping only the 10,578 A line appears and at higher pumping, the 10,648 A line is much stronger. It is also obvious that the 10,578 A line appears about 50 μsec earlier than the 10,648 A line (Fig. 26). This means that the former has a lower threshold and is easier to lase. It has been mentioned before that at the beginning of the lasing period, there were some regularly shaped spikes which were polarized at 70°. Therefore, these spikes compose only the
Fig. 26
Relative magnitudes of 10,578 A and 10,648 A lines in the output of Na\textsuperscript{+} CaWO\textsubscript{4} laser.
upper trace 10,648 A line
lower trace 10,578 A line
100 μsec/div. sweep
Fig. 27
Polarization of 10.578 A component in Na$^{3+}$ doped CaWO$_4$ laser

From upper to lower, analyser set at $70^\circ$, $40^\circ$, $10^\circ$, $-20^\circ$
100 $\mu$sec/div. sweep
Fig. 28
Polarization of 10,648 A component in Nd$^{3+}$ doped CaWO$_4$ laser

From upper to lower, analyser set at
-20°, 10°, 40°, 70°
100 μsec/div. sweep
10,578 Å line. From this coincidence, it is decided that the 10,578 Å line is polarized at 70° (b-axis) and the 10,648 Å line is polarized at -20° (c-axis). This result have been confirmed and the actual measurement of the polarization of these lines are shown in figures 27 and 28.

The conclusions about the polarization measurement of light from Nd³⁺ doped CaWO₄ laser can be drawn as follows:

(1) Two spectroscopic lines are involved in the light output of the laser when it is operated at room temperature. The wavelengths and the polarization of these two lines are listed in table 4

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<th>wavelength</th>
<th>polarization</th>
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<tr>
<td>10,578 Å</td>
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<td>10,648 Å</td>
<td>linearly polarized at -20° (c-axis)</td>
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Table 4

(2) When the pumping is sufficiently weak, only the 10,578 Å line can occur. The laser is in a single mode of oscillation and quite regular shaped spikes like a decayed oscillation are observed.

(3) As the pumping increases to sufficiently high value, another mode of oscillation occurs; which is the 10,648 Å line. Due to some incoherency between these two components, the spikes which are the superposition of light intensities of these two lines have random shapes and large amount of unpolarized
components.

(4) The imaginary value of $J_{xy}$, which represents circular polarization is usually small but it is appreciable for certain spikes. This value will increase with increasing pumping energy.

Johnson (33) has measured the polarization behavior of absorption and fluorescence spectra of Na$^3+$ in CaWO$_4$ (Na$^+$ compensated) and revealed that the transitions involved were all electric dipole (40). The polarization of the emission due to the transition between $^4F_{3/2}$ and $^4I_{11/2}$ is listed for Na$^+$ compensated crystal in table 5.

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Table 5

There were no detailed descriptions about polarization study and its relationship to the spiky nature of its output in his paper. The wavelength of the laser lines varies from crystal to crystal depending on their quality and treatment. The wavelengths of the lines of our sample are slightly different from those reported by Johnson (33) but the polarization measurement is consistent with the published results.
V Conclusions

Before making measurement of the polarization of light from Nd\(^{3+}\) doped glass laser, we proposed that two possible cases could occur. First, the light could be unpolarized because of the noncrystalline structure of glass. Second, a spike could be triggered by the spontaneous emission from a Nd\(^{3+}\) ion with the result that the whole spike would have the same polarization. But the spikes, triggered by different sources, would have different polarization in a perfectly random distribution such that the total radiation of the laser was unpolarized. The actual measurement showed that the polarization of the stimulated emission of the laser was much more complicated than these two oversimplified cases.

Both of the experimental results show that multimode oscillations are involved in the glass lasers. Each spike of the laser output can be the superposition of several different modes of oscillations (with different frequencies). Each mode of oscillation has a definite polarization determined by the polarization of the transition which triggered the oscillation. Each line in the spectrum of the stimulated emission is completely polarized, but each spike is partially polarized because it is formed by a lot of different lines. This multimode model of oscillation can explain all the experimental results obtained in the polarization measurement.

If a host material like a crystal is used, the polarization
of the transitions can be calculated by a pure crystal field theory. The degree of accuracy is high when the theory is applied to rare earth ions of the first group (lanthanides). When the host material is a glass, it is difficult to make such a calculation because of the random nature of the local field inside the material. However, each transition will still have a definite polarization, although it is randomly oriented.

In this measurement, there is no spatial resolution in detecting the changes of polarization across the end surface of the laser rod. It is very interesting to make such kind of a measurement. It is thought that several different areas can emit light more or less independently with different polarizations. There is strong evidence that the zero degree ruby does emit light from different emitting areas with different polarizations. The Nd$^{3+}$ doped glass laser is expected to behave similarly, but the actual behavior will have to be determined by further experiment since no such kind of experiment has been done so far.
ACKNOWLEDGMENT

The author is deeply grateful to Dr. T. A. Rabson for suggesting this problem and his continuous guidance and encouragement throughout this research. The support of the project by the National Aeronautics and Space Administration is also highly appreciated. The author also wish to thank the many members at Electrical Engineering Department of Rice University for their contributions in many aspects in making this work possible.
REFERENCES


(13) Born and Wolf, Principles of Optics, Sec. 13.4.1.


(24) K. Hellwege, Ann. der Phys. 4, 95 (1948).


(26) The selection rule and polarization of electric dipole, magnetic dipole, and electric quadrupole transitions for both free atoms (ions) and ions in crystal were treated in an elementary way by P. P. Feofilov in his book "The Physical Basis of Polarized Emission" (1959). English translation was published by Consultants Bureau Enterprises, Inc., New York (1961).

(27) P. P. Ewald und C. Hermann, Strukturbericht, Band I, 1913-1918.


