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PARTIAL RING CURRENTS
AND SOME OF THEIR GEOPHYSICAL EFFECTS

by

Warren David Cummings

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IN PARTIAL FULFILLMENT OF THE
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Thesis Director's signature:

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INTRODUCTION

A. The Phenomenon of a Geomagnetic Storm

The fluctuations of the geomagnetic elements H, V, and D about their quiet day values is called a geomagnetic storm. (H is the horizontal component of the earth's field, V is the vertical component, and D is the declination of H from true north.) Sometimes the storm can be associated with an eruption of a solar flare that preceded the geomagnetic phenomenon by 1 or 2 days. The storm may last 2 or 3 days, and during this time the geomagnetic elements may deviate from their quiet day values by as much as ~ 1%. The high-latitude fluctuations are most erratic and exhibit large local-time variations, while the low-latitude variations are smoother and more likely to exhibit universal-time characteristics that are reproducible from storm to storm.

The details of the average characteristic of geomagnetic storms are described in Chapman and Bartels (1940). The morphology of the low-latitude fluctuations in H may be summarized as follows:

(a) Most storms are characterized by a sudden commencement in which H increases by typically 20-30 γ with a rise time of 1-6 minutes. (1 γ = 10^-5 gauss = 10^-9 webers/m^2 -- the earth's field at the equator is approximately 0.3 gauss.)
(b) H remains above its pre-storm value throughout the initial phase. The initial phase lasts some two to eight hours, after which H falls below its pre-storm value.

(c) The average value of H continues to decrease during the second or main phase. Average decreases of 50 to 100 γ are typical and the main phase usually lasts from 12 to 24 hours.

(d) The recovery phase follows and may last 1 or 2 days. The recovery phase is sometimes free of geomagnetic activity except for the steady return of H to its pre-storm value.

In addition to the above storm phases, which are seen in more or less the same form at low-latitude stations of all longitudes, one finds that the amplitude of the variations in H also depends on local-time. In particular, at low-latitude stations main phase decreases observed during the afternoon hours tend to be larger than main phase decreases observed during the morning hours. The local-time dependent part of the storm variation is called the disturbance-daily variation. An example of the phases of a storm as well as the local-time effect is shown in Figure 1.

The above morphological description of low-latitude magnetic storms is an inadequate description of the high-latitude storm. Large (typically 500 γ) and rapid fluctuations together with a pronounced local-time dependence overshadow the world-wide progression of the storm through its phases. At high latitudes there also occur large rapid fluctuations in V and D.
B. The Present Theoretical Explanation of Geomagnetic Storms

Geomagnetic storm theory has been evolving for more than one hundred years. Chapman and Bartels (1940) discuss the contributions of the early workers in the field to ~ 1940. With the advent of space probes in the late 1950's and early 1960's the nature of the interplanetary environment was precisely determined and several important advances in storm theory were made. A good review of the present theory of geomagnetic storms has been given by Parker (1962). In the brief review that follows, emphasis will be placed on that part of the storm theory with which this thesis is concerned, namely the theory of the main and recovery phases.

The present theory of the initial phase is based on the idea first examined by Chapman and Ferraro (1931, 1932), that the geomagnetic storm is the result of "solar corpuscular radiation" incident on the geomagnetic field. The "solar corpuscular radiation", now called "solar wind" after the work of Parker (1958, 1960) has been observed with detectors aboard space vehicles (Gringauz et al., 1960; Bridge et al., 1961).

Since the highly ionized solar wind behaves like a perfectly conducting fluid, the geomagnetic field is frozen out of the stream by currents induced on its surface. The geomagnetic field is thus compressed into an elongated cavity, the boundaries of the cavity being
determined by a balance between the solar wind pressure and the geomagnetic field pressure.

The initial phase is understood to be the result of an enhanced solar wind pressure following the eruption of a solar flare. The enhanced solar wind pressure causes a compression of the geomagnetic cavity, and therefore a world-wide increase in H as measured at low latitudes.

The idea that charge particle motion in the geomagnetic field might be the cause of a storm phenomena is apparently due to Birkeland (1908, 1913). Alfvén (1955) introduced the idea that currents could arise from the drift of charged particles in an inhomogeneous magnetic field, and Singer (1957) proposed that the main phase decrease could be explained in terms of the currents produced by particles trapped in the magnetosphere and drifting in the way Alfvén proposed. From this point of view the main phase decrease is understood as the net magnetic effect of three currents:

1) the drift current resulting from the gradient in the geomagnetic field,
2) the drift current resulting from the curvature in the geomagnetic field,
and 3) a magnetization current determined by the distribution of the hot plasma in the magnetosphere.

It is assumed that the hot plasma responsible for the main phase decrease forms a symmetrical ring about the earth. When this is the case, each of the above three
currents flows in a ring about the earth; hence the name 
"ring current" theory of the main phase.

The origin of the particles responsible for the 
main phase ring current is unknown. Presumably a signi-
ficant particle pressure builds up inside the magnetosphere 
during the early main phase. The particle pressure within 
the magnetosphere inflates the geomagnetic field and thus 
causes a decrease in H as observed at low latitudes. From 
this point of view, the main phase decrease is a result 
of stresses imposed on the geomagnetic field by a hot 
plasma temporarily trapped within the magnetosphere (Dessler 
and Parker, 1959). This hydromagnetic description of the 
main phase decrease is equivalent to the drift current 
explanation.

Smith (1963) has reviewed the experimental evidence 
for the ring-current plasma. At the time of Smith's 
article the evidence was inconclusive; recent satellite 
experiments (Cahill, 1966; McIlwain, 1966), however, 
have apparently made unambiguous measurements of the ring 
current's magnetic effects. The low energy particles that 
make up the ring current belt have not yet been directly 
detected.

The theory of the recovery phase is concerned with 
the removal of the ring-current-particle strain on the 
geomagnetic field. Parker (1962) has given a critical 
review of the relaxation processes currently under discussion:

1) scattering of the ring current particles 
into their pitch angle loss cone by 
hydromagnetic waves,
or 2) by coulomb scattering,

and 3) energy loss of the ring current belt by
means of a charge exchange between protons
and ambient neutral hydrogen.

The effectiveness of each of these relaxation
processes depends in large part on basic ring-current
parameters:

1) particle type,
2) particle energy distribution,
3) particle pitch angle distribution,
and 4) position in the magnetosphere.

Unfortunately, little is known about these basic parameters.
However, the available indirect experimental observations,
combined with a number of theoretical arguments, indicate
the following range of values for the above parameters:

1) protons,
2) \(1 < E_p < 100\) kev,
3) unknown,

and 4) \(\sim 2 < R_{RC} < \sim 6\), where \(R_{RC}\) is the geocentric
distance to the ring current belt in units
of \(R_E\) (earth radii).

With these parameters for the ring current belt, the
most probable relaxation mechanism is the charge exchange
process. \textit{Fite et al.} (1960) have measured the total cross
section for charge exchange, \(Q\), and found that the product
of \(Q\) and \(v\), the incident proton velocity, is approximately
constant for the energy range $0.4 < E_p < 40$ kev. Liemohn (1961) combined the cross-section measurements of Fite et al., (1960) with a model of the terrestrial atmospheric hydrogen distribution (Johnson and Fish, 1960) to derive a lifetime against charge exchange:

$$\tau = \left( n_H Q v \right)^{-1}$$

At $R \approx 4R_e$, the value of $\tau$ is $\sim 1$ day for protons, independent of energy in the range $1 < E_p < 100$ kev.

Chapman (1935) proposed an atmospheric current system that could give rise to the disturbance-daily variation part of the magnetic storm. This current system is concentrated in the polar regions, particularly in the auroral zone (Figure 2). The current system is supposed to remain fixed with respect to the earth-sun line, so that a given station experiences a daily variation as it rotates beneath.

Fejer (1961) and Axford and Hines (1961) have developed physical models for the Chapman current system. However, a study by Akasofu and Chapman (1964) suggests that the low-latitude disturbance-daily variation is not due to the equatorward extensions of polar ionospheric currents. Recent high-latitude magnetic observations also are in disagreement with the original Chapman current system (Akasofu et al., 1965). In short, the low-latitude disturbance-daily variation is unexplained.
C. Object of this Thesis

The uniform ring current model has been fairly successful in providing a first order explanation for the average characteristics of the main and recovery phases of a geomagnetic storm. However, there are many phenomena associated with geomagnetic storms that are apparently not related to this first-order model, e.g.,

1) ionospheric heating (Jacchia, 1961a,b, 1963),
2) redistribution of low energy magnetospheric plasma (Carpenter, 1963, 1966),
3) redistribution of high energy magnetospheric particles (Rothwell and McIlwain, 1960; Pizzella, McIlwain, and Van Allen, 1962; McIlwain, 1966).
4) disturbance-daily variation (Akasofu and Chapman, 1964),

and 5) auroral phenomena.

There have in the past been discussions of variations on the uniform ring current model aimed at explaining in more detail a particular facet of the geomagnetic storm or of some related phenomenon, e.g.,

1) the recovery phase (Akasofu, Chapman, and Venkatesan, 1963),

In this thesis a non-uniform ring current model is explored. The various effects of small scale energy density fluctuations in the ring current belt are discussed. In particular, we examine storm related atmospheric heating as a consequence of a non-uniform ring current belt.

We also discuss the magnetic effects of a large scale asymmetry in the ring current belt. We show that a simple asymmetric ring current model can explain most of the low-latitude disturbance-daily variation.
FIGURE 1: A sample magnetic storm. As observed at San Juan, the storm shows clearly the initial, main, and recovery phases. The main phase decrease at San Juan is more than 200 \( \gamma \), while at Honolulu, it is only about 100 \( \gamma \). This discrepancy is attributed to the disturbance-daily variation. Note that the main phase at San Juan is centered about the afternoon hours, which are the morning hours for Honolulu.

FIGURE 2: The idealized Chapman current system (from Chapman and Bartels, p. 302). Most of the current is concentrated in the polar regions where the disturbance daily variation is the dominant effect during a magnetic storm. The low latitude variation is caused by the equatorward extensions of the polar currents, according to this explanation.
FIGURE 1

SAN JUAN

100γ

HONOLULU

10/28/61

MAIN

RECOVERY

INITIAL

SAN JUAN TIME

HONOLULU TIME

4 8 12 16 20 24 4 8 12 16 20 24
FIG. A VIEW FROM THE SUN

FIG. B VIEW FROM ABOVE THE NORTH POLE

FIGURE 2
NON-UNIFORM RING CURRENTS

A. A Survey of the Problem

In this section we briefly survey the consequences of an energy density enhancement in the ring current belt. Each of the aspects of the problem that are introduced in this section will be developed in more detail in subsequent sections.

In studying uniform ring currents from the single particle orbit theory approach, one is concerned with three current densities:

\[ J_G = u_\perp \frac{\vec{B} \times \vec{v}_B}{B^3} \quad \text{(m.k.s. units) Gradient B current, 1)} \]

\[ J_R = 2u_\parallel \frac{R_C \times \vec{B}}{R_C^2 B^2} \quad \text{Curvature current, 2)} \]

\[ J_M = \text{curl } \vec{M} = \text{curl } \left( \frac{u_\perp \vec{B}}{B^2} \right) \quad \text{Magnetization current, 3)} \]

where:

- \( u_\perp \) = the particle kinetic energy density of motion perpendicular to the magnetic field,
\(-11-\)

\[ u_{\parallel} = \text{the particle kinetic energy density of motion parallel to the magnetic field,} \]

\[ \vec{B} = \text{the magnetic field vector,} \]

\[ \vec{R}_c = \text{the radius of curvature of the field lines,} \]

and

\[ \vec{M} = \text{the magnetization of the ring current plasma.} \]

The current densities \( \vec{J}_G \) and \( \vec{J}_R \) arise from drift motion of the center of rotation, i.e., the guiding center, of the ring current particles. A gradient in the magnetic field causes a drift of the guiding center with velocity:

\[ \vec{v}_G = w_1 \frac{\vec{B} \times \vec{V}B}{eB^3} \quad \text{Gradient drift velocity 4) } \]

and a curvature in the B lines causes a drift of the guiding center with the velocity:

\[ \vec{v}_R = 2w_{\parallel} \frac{\vec{R}_c \times \vec{B}}{eR_c B^2} \quad \text{Curvature drift velocity 5) } \]
where:

\[ w_\perp = \text{the particle kinetic energy due to motion perpendicular to } \vec{B}, \]

\[ w_\parallel = \text{the particle kinetic energy due to motion parallel to } \vec{B}, \]

and

\[ e = \text{the charge of the ring current particle}. \]

The circular motion of a charged particle in the magnetic field \( \vec{B} \) produces a magnetic moment given by:

\[ \vec{I}_1 = -\frac{w_\perp}{B^2} \vec{B}. \]

The magnetization of the plasma is just the magnetic moment per unit volume, or:

\[ \vec{M} = n\vec{I}_1 = -\frac{u_1 \vec{B}}{B^2}, \]

where:

\[ n = \text{number density of the ring current particles}. \]
If the magnetic field is assumed to have zero curl, i.e., if the perturbation of the geomagnetic field by the ring current plasma is assumed negligible, then we have:

\[
\text{div}(\vec{j}_G) = \frac{(\vec{B} \times \vec{v}B) \cdot \vec{v}u_\perp}{B^3}
\]

\[
\text{div}(\vec{j}_R) = \frac{2(\vec{R}_c \times \vec{B}) \cdot \vec{v}u_\parallel}{R_c^2 B^2}
\]

\[
\text{div}(\vec{j}_M) = 0
\]

A uniform ring current belt, in the curl-free field approximation, may be defined as one in which:

\[
(\vec{B} \times \vec{v}B) \cdot \vec{v}u_\perp = 0
\]

and

\[
(\vec{R}_c \times \vec{B}) \cdot \vec{v}u_\parallel = 0
\]

There is never any build up of charge within such a plasma, since:

\[
\frac{\partial \rho}{\partial t} = - \text{div}(\vec{j}) = 0
\]
A non-uniform ring current belt may be defined as one in which:

\[(\vec{B} \times \vec{vB}) \cdot \vec{v}_{\perp} \neq 0\]

and

\[(\vec{R}_C \times \vec{B}) \cdot \vec{v}_{\parallel} \neq 0\]

Since \(\text{div}(\vec{j}_G)\) and \(\text{div}(\vec{j}_R)\) are no longer necessarily zero, the charge density at any point within the belt may change with time. It is therefore possible to have time dependent electric fields in a non-uniform ring current belt.

The presence of ring current electric fields in a magnetosphere pervaded by a low temperature background plasma means that the behavior of the ring current belt will be influenced by the ionosphere. Because of the background plasma, the geomagnetic field lines are almost perfect conductors parallel to \(B\). The field lines may be regarded as equipotential lines, and therefore a potential difference developed between field lines in the magnetosphere will also appear in the ionosphere. The subsequent development of the magnetospheric potential difference, and hence of its effect on the ring current belt, will depend to some extent on the response of the ionosphere.

The presence of electric fields in the ring current belt and consequently in the ionosphere, makes it necessary to consider two more current densities:
\[
\mathbf{J}_p = \frac{\rho_m}{B^2} \left( \mathbf{E} \perp \mathbf{B} \right) \quad \text{(Polarization current) \quad 6)}
\]

and

\[
\mathbf{J}_I = \sigma_1 \mathbf{E} \quad \text{(Pederson current) \quad 7)}
\]

where:

\( \rho_m \) = the total plasma mass density,

\( \mathbf{E} \) = the electric field vector,

and

\( \sigma_1 \) = the Pederson, or reduced, conductivity.

The polarization current results from the pulling apart of positive and negative charges by a changing electric field. The quantity \( \frac{\rho_m}{B^2} \equiv \varepsilon \) may be regarded as the low frequency magnetic plasma permittivity. It is related to the hydromagnetic wave velocity by the expression:

\[
\nu_{hm} = \frac{B^2}{\sqrt{\mu_0 \rho_m}} = \frac{1}{\sqrt{\mu_0 \varepsilon}}.
\]

The Pederson current is just an Ohm's law current. The Pederson conductivity takes into account the effect of both particle collisions and a transverse magnetic field in retarding the current. For low frequency electric
fields, the Pederson conductivity is a sharply peaked function of altitude, having a maximum in the ionosphere (Figure 3).

In Figure 4 we have summarized the consequences of an energy density enhancement in the ring current belt. The effect of the drift currents, \( \mathbf{J}_G \) and \( \mathbf{J}_R \), is to build up charge at the boundaries of the enhancement, while the effect of the currents \( \mathbf{J}_P \) and \( \mathbf{J}_I \) is to diminish this charge. All of the currents flow perpendicularly to the magnetic field.

In addition to causing the polarization and Pederson currents, a ring current belt electric field gives rise to the electric drift velocity:

\[
\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \quad \text{Electric drift velocity.} \quad 8)
\]

This drift of the plasma does not give rise to a current. The drift is not confined to the particles that create the electric field; all particles, independent of energy, assume the electric drift velocity.

The electric field associated with an energy density enhancement is eastward as viewed from the earth. The electric drift velocity of the enhancement is therefore outward as viewed from the earth, i.e., energy density enhancements tend to be ejected from the ring current belt. The velocity of ejection will depend, among other things,
on the strength of the electric field, and hence on the magnitude of the ionospheric conductivity. For example, if the ionospheric Pederson conductivity is infinite the electric field associated with an enhancement will be "shorted out" and the ejection velocity will be zero.

The ionospheric conductivity need not be infinite to prevent the loss of an energy density enhancement from the ring current belt. The non-uniform ring current belt is stabilized if the conductivity is high enough so that typical electric drift velocities are less than the gradient and curvature drift velocities that characterize the belt.

If the ionospheric conductivity is not high enough to stabilize the ring current belt some of the particle energy lost from the belt may appear in the ionosphere. The ring current particle energy appears in the ionosphere as Joule heating from the stabilizing currents.
FIGURE 3: The Pederson conductivity as a function of altitude for four points in the sunspot and day-night cycles. These curves are from Hanson (1965).

FIGURE 4: A summary of the consequences of an energy density enhancement in the ring current belt. The enhancement is represented by the shaded region. The curvature and gradient drift currents tend to build up charge on the boundaries of the enhancement, and the polarization and Pederson currents tend to diminish this charge build up. The tube of flux associated with the enhancement assumes the electric drift velocity given in magnitude by $v_E = \frac{E}{B}$.
FIGURE 3
B. The Model Used for Calculations

In order to proceed beyond a qualitative discussion, it is necessary to make calculations based on some model of the magnetosphere and ring current belt. In this section we list the assumptions of the model used in this thesis and briefly discuss some of the consequences of these assumptions:

We assume:

1) the ring current belt energy content is sufficiently low to allow a curl-free (dipole) magnetic field approximation,

2) the background low temperature plasma is always able to maintain a given magnetic field line as an equipotential,

3) the magnetic-moment and bounce actions \( I_1 \) and \( I_2 \) of the ring current particles are invariant.

and 4) energy density enhancements in the ring current belt are in the shape of tubes of flux; the ring current particles have pitch angle and energy distributions such that:

\[
\int_{-\infty}^{\infty} w_n \exp(w, \alpha_{eq}) dw = \frac{u_{eq} \sin \delta \alpha_{eq}}{2\pi \int_{-\pi/2}^{\pi/2} \sin^{\delta+1} \alpha_{eq} d\alpha_{eq}},
\]
where:

\[ w = \text{particle energy,} \]

\[ \alpha_{eq} = \text{particle pitch angle in the equatorial plane,} \]

\[ n_{eq}(w, \alpha_{eq}) = \text{equatorial number density per unit solid angle and per unit energy,} \]

and

\[ \delta = \text{a parameter that characterizes the pitch angle distribution of the ring current particles.} \]

Since we are making a dipole approximation for the magnetic field, we can facilitate calculations by choosing a special dipole coordinate system. In addition to the ordinary spherical polar coordinate system we will occasionally use an orthogonal dipole coordinate system having the following properties: Any point in space is determined by three numbers \((P, R, \varphi)\), where \(P\) and \(R\) are related to the spherical polar coordinates \(r\) and \(\theta\) by the equations:

\[ r = P \sqrt{|\cos \theta|} \tag{9} \]

and

\[ r = R \sin^2 \theta \tag{10} \]
Curves of constant $P$ and constant $R$ are shown in Figure 5. The curves of constant $R$ are just the ordinary dipole lines, and in subsequent calculations they will be assumed identical with the earth's field lines. The curves of constant $P$ are the orthogonal trajectories of the dipole lines. The coordinate $\varphi$ in the dipole system is the same as $\varphi$ in the spherical polar coordinate system.

The element of length along a curve of constant $R$ is given by the expression:

$$dS_R = R \sin \theta (1 + 3 \cos^2 \theta)^{\frac{3}{2}} d\theta$$ \hspace{1cm} (11)$$

The element of length along a curve of constant $P$ is given by:

$$dS_P = \frac{-P(1 + 3 \cos^2 \theta)^{\frac{3}{2}} d\theta}{2 \sqrt{\left| \cos \theta \right|}}$$ \hspace{1cm} (12)$$

or

$$dS_P = \frac{|\sin^3 \theta| \, dR}{(1 + 3 \cos^2 \theta)^{\frac{3}{2}}}$$ \hspace{1cm} (13)$$
The element of area in a plane of constant $\phi$ is given by:

$$dA_\phi = dS_R \, dS_P = R \sin^4 \theta \, d\theta \, dR,$$  \hspace{1cm} (14)

and the element of area in a surface of constant $P$ is given by:

$$dA_P = r \sin \theta \, d\phi \, dS_P,$$  \hspace{1cm} (15)

or

$$dA_P = \frac{P^2}{2} (1 + 3 \cos^2 \theta)^{\frac{1}{3}} \sin \theta \, d\theta \, d\phi,$$  \hspace{1cm} (16)

or

$$dA_P = \frac{R \sin^6 \theta \, d\phi \, dR}{(1 + 3 \cos^2 \theta)^{\frac{1}{3}}}.$$  \hspace{1cm} (17)

The differential volume element is given by:

$$dv = r \sin \theta \, dS_R \, dS_P \, d\phi = R^2 \sin^7 \theta \, d\theta \, dR \, d\phi.$$  \hspace{1cm} (18)
For future reference, we also note that the variation of field strength along a line of constant $R$ is given by:

$$ B = B_{eq} \frac{(1 + 3 \cos^2 \theta)^{\frac{3}{2}}}{\sin^6 \theta} $$

where:

$B_{eq}$ = the equatorial magnetic field strength at a distance $R$ from the center of the earth.

The variation of the radius of curvature along a line of force is given by:

$$ R_c = \frac{R \sin \theta (1 + 3 \cos^2 \theta)^{\frac{3}{2}}}{3(1 + \cos^2 \theta)} $$

Since we are assuming the geomagnetic field lines are equipotentials, the potential difference, $\Delta \psi$, between two adjacent field lines with the same value of $R$ is independent of $\theta$. The (longitudinal) electric field associated with such a potential difference is a simple function of $\theta$. We have:
\[ \vec{E}(R, \theta, \phi) = \frac{\Delta \psi(R, \theta, \phi) \vec{e}_\phi}{R \sin^3 \theta \Delta \phi} \]

\[ = \frac{\Delta \psi(R, \theta', \phi) \vec{e}_\phi}{R \sin^3 \theta \Delta \phi} \]

\[ = \frac{\sin^3 \theta' \Delta \psi(R, \theta', \phi) \vec{e}_\phi}{\sin^3 \theta R \sin^3 \theta' \Delta \phi} \]

Hence:

\[ \vec{E}(R, \theta, \phi) = \frac{\sin^3 \theta'}{\sin^3 \theta} \vec{E}(R, \theta', \phi) \quad . \quad (21) \]

The charge density along a dipole field line is also simply related to the charge density at any other point along the same line. We have:
\[ \rho(R, \theta, \varphi) = \varepsilon_0 \text{div} \bar{E}(R, \theta, \varphi) \]

\[= \frac{\varepsilon_0}{R \sin^3 \theta} \frac{\partial E(R, \theta, \varphi)}{\partial \varphi} \]

\[= \frac{\varepsilon_0 \sin^3 \theta'}{R \sin^6 \theta} \frac{\partial E(R, \theta', \varphi)}{\partial \varphi} \]

\[= \frac{\varepsilon_0 \sin^6 \theta'}{\sin^6 \theta} \frac{1}{R \sin^3 \theta'} \frac{\partial E(R, \theta', \varphi)}{\partial \varphi} \]

\[= \frac{\varepsilon_0 \sin^6 \theta'}{\sin^6 \theta} \text{div} \bar{E}(R, \theta', \varphi) \]

\[\text{Hence:} \]

\[\rho(R, \theta, \varphi) = \frac{\sin^6 \theta'}{\sin^6 \theta} \rho(R, \theta', \varphi) \] \hspace{1cm} 22)
As we will see in subsequent sections, it will be necessary to integrate particle energy density along a tube of flux. The assumption that the pitch angle and energy distributions are separable (3), together with the assumption that \( I_1 \) and \( I_2 \) are invariant (4), simplifies these integrations. We now derive (following Longmire, p.58, 1963) expressions for \( u_\parallel \) and \( u_\perp \) as a function of position along a tube of flux.

Let \( n_\text{eq}^+ (w, I_1) dw \, dI_1 \) be the equatorial number density of particles moving along the tube toward the north magnetic pole and having total energy \( w \) in \( dw \) and magnetic moment \( I_1 \) in \( dI_1 \). The particles move along the tube of force with velocity \( v_\parallel \) given by:

\[
v_\parallel = \sqrt{\left(\frac{2}{m}\right)(w - I_1 B)} \quad ,
\]

where:

\[
m = \text{particle mass}.
\]

Since the cross-sectional area of the flux tube is proportional to \( \frac{1}{B} \), we must have for any point along the tube that is accessible to particles with magnetic moment \( I_1 \):

\[
n^+ (w, I_1) \frac{v_\parallel}{B} = \text{constant} = n_\text{eq}^+ (w, I_1) \frac{v_\parallel \text{eq}}{B \text{eq}} \quad ,
\]
or

\[ n^+(w, I_1) = n_{eq}^+(w, I_1) \frac{v_{\parallel eq}}{v_{\parallel}} \frac{B}{B_{eq}} \]

The part of \( u_\parallel \) that arises from particles with energy \( w \) and magnetic moment \( I_1 \) moving toward the north pole is given by:

\[ u^+_{\parallel}(w, I_1) = n^+(w, I_1)\left(\frac{1}{2} m v_{\parallel}^2\right) \]

\[ = \frac{n_{eq}^+(w, I_1)m v_{\parallel eq}}{2} v_{\parallel} \frac{B}{B_{eq}} \]

\[ = n_{eq}^+(w, I_1)w \frac{B \sqrt{(1 - \frac{I_1}{w} B_{eq})(1 - \frac{I_1}{w} B)}}{B_{eq}} \]

For the pitch angle distributions under consideration an equal contribution to \( u_\parallel \) is provided by particles moving toward the south pole. We therefore have:

\[ u_\parallel = \int_0^\infty \int_0^\infty n_{eq}^+(w, I_1)w \frac{B \sqrt{(1 - \frac{I_1}{w} B_{eq})(1 - \frac{I_1}{w} B)}}{B_{eq}} \, dw \, dI_1 \]
where:

\[ n_{eq}(w, I_1) = n^+_{eq}(w, I_1) + n^-_{eq}(w, I_1) \, . \]

Since:

\[ I_1 = \frac{w}{B_{eq}} \sin^2 \alpha_{eq} \, , \]

we can define \( n_{eq}(w, \alpha_{eq}) \) by:

\[ n_{eq}(w, \alpha_{eq}) 2\pi \sin \alpha_{eq} \, d\alpha_{eq} = n_{eq}(w, I_1) dI_1 \, , \]

and write:

\[ u_\parallel = \int_0^{\sin^{-1} \sqrt{\frac{B_{eq}}{B}}} \int_0^\infty \left[ n_{eq}(w, \alpha_{eq}) \right] w \frac{B}{B_{eq}} \cos \alpha_{eq} \, d\alpha_{eq} \, . \]

\[ \sqrt{1 - \frac{B}{B_{eq}} \sin^2 \alpha_{eq} (2\pi \sin \alpha_{eq})} \, dw \, d\alpha_{eq} \, . \]
or
\[
  u_\| = \frac{u_{eq} \left( \frac{B}{B_{eq}} \right)^{\frac{3}{2}}}{\left( \int_0^{\pi/2} \sin^{\delta+1} \alpha_{eq} \, d\alpha_{eq} \right)} \int_0^1 \sqrt{\frac{B_{eq}}{B} - \sin^2 \alpha_{eq}} \sin^{\delta+1} \alpha_{eq} \cos \alpha_{eq} \, d\alpha_{eq} ,
\]
or
\[
  u_\| = \frac{u_{eq} \left( \frac{B}{B_{eq}} \right)^{\frac{3}{2}}}{\left( \int_0^{\pi/2} \sin^{\delta+1} \alpha_{eq} \, d\alpha_{eq} \right)} \int_0^1 \sqrt{\frac{B_{eq}}{B} - x^2} x^{\delta+1} \, dx .22
\]

We can similarly derive an expression for \( u_\perp \). The part of \( u_\perp \) that arises from particles with energy \( w \) and magnetic moment \( I_\perp \) is given by:

\[
  u_\perp(w, I_\perp) = n_{eq}(w, I_\perp) \left( \frac{\hbar}{2m} \right)^2 \frac{v_{\parallel eq}}{v_\|} \frac{B}{B_{eq}}
\]

\[
  = n_{eq}(w, I_\perp) I_\perp \frac{B^2}{B_{eq}} \sqrt{\frac{I_\perp}{1 - \frac{I_\perp}{w} B}}
\]
Hence:

\[ u_\perp = \int_0^\infty \int_0^\infty n_{eq}(w, I_1) I_1 \frac{B^2}{B_{eq}} \sqrt{1 - \frac{I_1}{w} \frac{B_{eq}}{B}} \sqrt{1 - \frac{I_1}{w} \frac{B_{eq}}{B}} \, dw \, dI_1 \]

\[ = \int_0^\infty \int_0^\infty \left[ n_{eq}(w, \alpha_{eq}) w \frac{B^2}{B_{eq}} \sin^2 \alpha_{eq} \right] \cos \alpha_{eq} \left( \frac{2\pi \sin \alpha_{eq}}{\sqrt{1 - \frac{B}{B_{eq}} \sin^2 \alpha_{eq}}} \right) \frac{\sin^{-1} \left( \sqrt{\frac{B_{eq}}{B}} \right)}{\sqrt{1 - \frac{B}{B_{eq}} \sin^2 \alpha_{eq}}} \, dw \, d\alpha_{eq} \]

\[ = \frac{\sin^{-1} \left( \sqrt{\frac{B_{eq}}{B}} \right)}{\sin^{3+\delta} \alpha_{eq} \cos \alpha_{eq} \, d\alpha_{eq}} \left( \frac{u_{eq} \left( \frac{B}{B_{eq}} \right)^{3/2}}{\pi/2} \int_0^{\pi/2} \frac{\sin^{\delta+1} \alpha_{eq} \, d\alpha_{eq}}{\sin^{3+\delta} \alpha_{eq} \cos \alpha_{eq} \, d\alpha_{eq}} \right) \]
or

\[ u_{\perp} = \frac{u_{\text{eq}} \left( \frac{B}{B_{\text{eq}}} \right)^{3/2}}{\left( \int_{0}^{\pi/2} \sin^{\delta+1} \alpha_{\text{eq}} \, d\alpha_{\text{eq}} \right)} \int_{0}^{\sqrt{\frac{B_{\text{eq}}}{B}} - x^2} \sqrt{\frac{x^{3+\delta}}{\sqrt{\frac{B_{\text{eq}}}{B}}}} \, dx \]

Expressions for \( u_{\parallel} \) and \( u_{\perp} \), for \( \delta = 0, 1, 2, \) and 3, are given in Table 1.

Finally, we note that if \( I_1 \) and \( I_2 \) are conserved, the equatorial pitch angle and energy variations with \( R \) can be calculated. The relevant equations are:

\[ I_1 = \frac{w \sin^2 \alpha_{\text{eq}}}{B_{\text{eq}}} \tag{24} \]

and

\[ I_2 = m \int v \cos \alpha \, ds_R \tag{25} \]

where the integration is over a complete north-south oscillation.
Substituting:

\[ dS_R = R \sin \theta (1 + 3 \cos^2 \theta)^{\frac{5}{6}} \, d\theta \ , \quad (11) \]

\[ v = \sqrt{\frac{2w}{m}} \ , \]

and

\[ \cos \alpha = \sqrt{1 - \frac{B L_1}{w}} \]

\[ = \sqrt{1 - \sin^2 \alpha_{eq} \frac{(1 + 3 \cos^2 \theta)^{\frac{5}{6}}}{\sin^6 \theta}} \]

into eq. (25) yields:

\[ I_2 = \sqrt{w} \, RF(\alpha_{eq}) \ , \quad (26) \]

where:

\[ F(\alpha_{eq}) = 4 \sqrt{2m} \int_0^{\theta_m} \left\{ \left[ 1 - \frac{\sin^2 \alpha_{eq} (1 + 3 \cos^2 \theta)^{\frac{5}{6}}}{\sin^6 \theta} \right] \right\}^{\frac{5}{6}} \sin \theta \, d\theta \ , \]
and $\theta_m$ is given by:

$$1 - \frac{\sin^2 \alpha_{eq} (1 + 3 \cos^2 \theta_m)^{\frac{1}{3}}}{\sin^6 \theta_m} = 0 .$$

The changes in $\alpha_{eq}$ with $R$ can be computed from the equation:

$$\frac{I_1}{I_2} = \frac{\sin^2 \alpha_{eq}}{B_{eq} R^2 P^2 (\alpha_{eq})} ,$$

(27)

and once $\alpha_{eq}$ is known, $w$ can be computed from either of the action invariant equations.

Figures 6 and 7, taken from a paper by Nakada et al., (1965) illustrate the functional dependence of $\alpha_{eq}$ and $w$ on $R$.

The assumptions discussed in this section are the main ones used in the thesis. Others, such as the neglect of a latitude dependence in the ionospheric conductivity, are more restricted in their influence and will be discussed individually.
FIGURE 5: The dipole coordinate system. Curves of constant $R$ are dipole lines. Curves of constant $P$ are orthogonal trajectories to the dipole lines.

FIGURE 6: The variation of equatorial pitch angle with geocentric radial distance if $I_1$ and $I_2$ are conserved (from Nakada et al., 1965).

FIGURE 7: The variation of particle energy with geocentric radial distance. The curves are normalized to particle energy at $R = 7R_E$. Pitch angles characterizing each curve refers to the equatorial pitch angle at $R = 7R_E$ (from Nakada et al., 1965).
FIGURE 5

\[ r = R \sin^2 \theta \]

\[ r = P \sqrt{\cos \theta} \]
TABLE 1. The perpendicular and parallel components of the energy density for four values of the pitch angle parameter.

<table>
<thead>
<tr>
<th>δ</th>
<th>$u_\perp$</th>
<th>$u_\parallel$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{2}{3} , u_{eq}$</td>
<td>$\frac{1}{4} , u_{eq}$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{3}{4} , u_{eq} \left( \frac{B_{eq}}{B} \right)^{\frac{1}{2}}$</td>
<td>$\frac{1}{4} , u_{eq} \left( \frac{B_{eq}}{B} \right)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{4}{5} , u_{eq} \left( \frac{B_{eq}}{B} \right)^{\frac{1}{5}}$</td>
<td>$\frac{1}{5} , u_{eq} \left( \frac{B_{eq}}{B} \right)^{\frac{1}{5}}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{5}{6} , u_{eq} \left( \frac{B_{eq}}{B} \right)^{\frac{3}{6}}$</td>
<td>$\frac{1}{6} , u_{eq} \left( \frac{B_{eq}}{B} \right)^{\frac{3}{6}}$</td>
</tr>
</tbody>
</table>
C. Response of the Magneto-Ionospheric Plasma

In this section we show that the magneto-ionospheric plasma responds to the potential difference associated with a ring current energy density enhancement in the same way that a leaky capacitor responds to an applied potential difference. The time dependent polarization effects die away with a time constant that depends on the magneto-ionospheric plasma permittivity and on the ionospheric Pederson conductivity. The time constant for this decay varies with the sunspot cycle, with local time, and with the position of the enhancement in the magnetosphere.

We also show that the charge density that develops on the east and west boundaries of an energy density enhancement is much less than $n_{eq}$, where $n_{eq}$ is the equatorial number density of ring current particles within the enhancement.

The assumption that the magnetic field lines are equipotentials is equivalent to the assumption that the charges redistribute themselves "instantaneously" along the B lines. The polarization effects that give rise to a leaky capacitor time constant are due to motion of the plasma transverse to the field. In order to calculate the response of the magneto-ionospheric plasma we use the charge conservation equation appropriate to the equipotential field line approximation. That is, the equation of local conservation:

$$\frac{\partial \rho}{\partial t} + \text{div } j = 0 \quad \text{(locally)}$$
must be replaced by a volume integral:

\[ \int_{\text{vol}} \left( \frac{\partial \rho}{\partial t} + \text{div} \mathbf{j} \right) dv = 0 \quad , \]

where the volume of integration is bounded by two planes of longitude separated by a small angle \( \Delta \phi \), two dipole shells separated by a small equatorial distance \( \Delta R \), and the earth's surface. In terms of the distance \( S_R \) along the field line, \( R \), and \( \theta \), the volume integral is:

\[ \int_{\Delta R} \int_{\Delta \phi} \int_{\text{along field line}} \left( \frac{\partial \rho}{\partial t} + \text{div} \mathbf{j} \right) \frac{R \sin^6 \theta \, dS_R \, d\phi \, dR}{\sqrt{1 + 3 \cos^2 \theta}} \quad . \]

In order that the above integral be identically zero, we require the inner integral to be zero:

\[ \int_{\text{along field line}} \left( \frac{\partial \rho}{\partial t} + \text{div} \mathbf{j} \right) \frac{\sin^6 \theta \, dS_R}{(1 + 3 \cos^2 \theta)^{1/2}} = 0 \quad . \]
Substituting:

\[ \rho = \frac{\rho_{eq}}{\sin^6 \theta} \]

and

\[ \mathbf{j} = \mathbf{j}_G + \mathbf{j}_R + \mathbf{j}_P + \mathbf{j}_I \]

into eq.(29), we have:

\[
\frac{3}{\delta t} \left[ \rho_{eq} \int \frac{\mathrm{d}S_R}{(1 + 3 \cos^2 \theta)^3} \right] = - \int \frac{(\mathbf{B} \times \overline{\mathbf{v}} \mathbf{B})}{B^3} \cdot \frac{\overline{\mathbf{v}} \mathbf{u}_\parallel \sin^6 \theta \mathrm{d}S_R}{(1 + 3 \cos^2 \theta)^3} \]

\[
= -2 \int \frac{(\mathbf{R} \times \overline{\mathbf{B}} \cdot \overline{\mathbf{v}} \mathbf{u}_\parallel \sin^6 \theta \mathrm{d}S_R}{R_c^2 B^2 (1 + 3 \cos^2 \theta)^3} \]

\[
- \int \frac{\text{div}(\mathbf{v} \mathbf{E}) \sin^6 \theta \mathrm{d}S_R}{(1 + 3 \cos^2 \theta)^3} \]

\[
- \int \frac{\text{div}(\mathbf{c} \mathbf{E}) \sin^6 \theta \mathrm{d}S_R}{(1 + 3 \cos^2 \theta)^3} \]
Assuming:

\[ \vec{E} \cdot \vec{\nabla} \varepsilon = \vec{E} \cdot \vec{\nabla} \sigma_1 = 0 , \]

we have:

\[ \text{div}(\varepsilon \vec{E}) = \varepsilon \frac{\partial (\vec{\nabla} \cdot \vec{E})}{\partial t} = \frac{\varepsilon}{\epsilon_0} \frac{\partial \rho}{\partial t} = \frac{\varepsilon}{\epsilon_0 \sin^6 \theta} \frac{\partial \rho_{eq}}{\partial t} , \]

and

\[ \text{div}(\sigma_1 \vec{E}) = \sigma_1 (\vec{\nabla} \cdot \vec{E}) = \frac{\sigma_1 \rho}{\epsilon_0} = \frac{\sigma_1}{\epsilon_0 \sin^6 \theta} \rho_{eq} . \]

The charge conservation equation [eq.(32)] may now be expressed in terms of the charge density in the equatorial plane:

\[ \left[ \int \frac{dS_R}{(1 + 3 \cos^2 \theta)^{\frac{3}{2}}} + \frac{1}{\epsilon_0} \int \frac{\varepsilon dS_R}{(1 + 3 \cos^2 \theta)^{\frac{3}{2}}} \right] \frac{\partial \rho_{eq}}{\partial t} + \left[ \frac{1}{\epsilon_0} \int \frac{\sigma_1 dS_R}{(1 + 3 \cos^2 \theta)^{\frac{3}{2}}} \right] \rho_{eq} + \Lambda = 0 , \quad 33 \]
where:

\[ \Lambda = \Lambda_\perp + \Lambda_\parallel \]

\[ = \int \frac{\langle \bar{B} \times \bar{v}B \rangle \cdot \bar{v}u_\perp \sin^6 \theta \, ds_R}{B^3(1 + 3 \cos^2 \theta)^{\frac{3}{2}}} \]

\[ + 2 \int \frac{\langle \bar{R}_C \times \bar{B} \rangle \cdot \bar{v}u_\parallel \sin^6 \theta \, ds_R}{R_C^2B^2(1 + 3 \cos^2 \theta)^{\frac{3}{2}}} \]

Since \( \epsilon > \epsilon_0 \) throughout the magnetosphere (Figures 8 and 9), we have:

\[ \int \frac{ds_R}{(1 + 3 \cos^2 \theta)^{\frac{3}{2}}} \ll \frac{1}{\epsilon_0} \int \frac{\epsilon \, ds_R}{(1 + 3 \cos^2 \theta)^{\frac{3}{2}}} \]

Hence, writing:

\[ E = \int \frac{\epsilon \, ds_R}{(1 + 3 \cos^2 \theta)^{\frac{3}{2}}} \]

35)
and

\[ \Sigma' = \int \frac{c_{1} dS_{R}}{(1 + 3 \cos^{2} \theta)^{\frac{3}{2}}} \]

\[ \text{eq. (33) becomes:} \]

\[ \frac{\partial \rho_{eq}}{\partial t} + \frac{\Sigma'}{E} \rho_{eq} \left( \frac{\varepsilon_{0} \Lambda}{E} \right) = 0 \]

The solution to eq. (37) is:

\[ \rho_{eq}(t) = \rho_{eq} \left( e^{-t/\tau} - 1 \right) \]

where:

\[ \rho_{eq} = \frac{\varepsilon_{0} \Lambda}{\Sigma'} \]

\[ \tau = \frac{E}{\Sigma'} \]

and

\[ \rho_{eq}(t = 0) = 0 \quad \text{is assumed} . \]
\[ E \text{ has the units of capacitance (farads), and } \Sigma' \text{ has the units of conductance (mhos). The equation expressing the build up of charge density with time [eq.(38)] is the leaky capacitor equation.} \]

In order to evaluate \( \rho_{eq} \) and \( \tau \), it is necessary to estimate values for \( E \) and \( \Sigma' \). In view of the scarcity of relevant magneto-ionospheric data and the large fluctuations known to occur in the pertinent parameters during magnetic storms, estimates based on extrapolation from low-latitude, idealized conditions must be regarded as tentative. Such, however, are the conditions on which the following estimates are based.

Above an altitude of \( \approx 1500 \) km, hydrogen ions provide the main contribution to the magnetic plasma permittivity. The contribution of the hydrogen plasma to \( E \) may be calculated if one knows the distribution of the protons along a given field line. For example, we have plotted in Figure 8:

\[
\frac{\varepsilon_{H^+} R_E}{(1 + 3 \cos^2 \theta)^{1/2}} \text{ vs } S_4 \text{ (in units of } R_E)\]

for the \( R = 4R_E \) field line, assuming the background hydrogen plasma is in diffusive equilibrium at \( T_e = T_p = 1250^0K \) [Hanson (1965), p. 35]. Twice the area
under the curve in Figure 8 represents the contribution of the hydrogen plasma to $E$. In this case the contribution is $\approx 15$ farads.

Below an altitude of about 1500 km, helium and oxygen ions provide the main contribution to $E$. The distribution of these ions, especially at high latitudes is not well known. The contribution to $\varepsilon$ from these ions at a geomagnetic latitude of $\approx 30^\circ$ is given in Figure 9. Assuming the same distribution for these ions at the base of the $R = 4R_E$ field line, the contribution to $E$ ranges from $\approx 0.5$ farads at night sunspot minimum to $\approx 10$ farads at day sunspot maximum.

Since $\sigma_1$ is a sharply peaked function of altitude, we have:

$$
\Sigma' = \frac{2}{\sin I} \frac{\Sigma_1}{\left(1 + 3 \cos^2 \theta\right)^\frac{1}{3}} = \frac{2\Sigma_1}{\sin I \left(4 - 3 \frac{R_E}{R}\right)^\frac{1}{3}}
$$

where:

$$
\Sigma_1 = \int \sigma_1 \, dh = \text{the height integrated Pederson conductivity},
$$

and

$$
I = \text{the dip angle}.
$$
Since:

\[
\sin I = \frac{2 \cot \theta}{\sqrt{1 + 4 \cot^2 \theta}} = \frac{2 \sqrt{\frac{R}{R_E}} - 1}{\sqrt{\frac{R}{R_E}} \sqrt{4 - 3 \frac{R_E}{R}}} = \frac{2 \sqrt{1 - \frac{R_E}{R}}}{\sqrt{4 - 3 \frac{R_E}{R}}}
\]

we have:

\[
\Sigma' = \frac{\Sigma}{\sqrt{1 - \frac{R_E}{R}}}
\]

High-latitude values of \( \Sigma \), especially during magnetic storms, are poorly known. It is generally thought that the conductivity in the auroral regions is larger than the equatorial conductivity by about a factor of 5. The low-latitude quiet day values of \( \Sigma \) can be obtained from Figure 3. These values range from \( \approx 0.07 \) mhos for night sunspot-minimum to \( \approx 25 \) mhos for day sunspot-maximum. Adopting these values for \( \theta = 30^\circ \), we have \( \Sigma' \approx 0.1 \) mhos for night sunspot-minimum and \( \Sigma' \approx 30 \) mhos for day sunspot-maximum.

According to these estimates, average (over the sunspot and local time cycles) values for \( E \) and \( \Sigma' \) for \( R = 4R_E \) are:

\[
E \approx 30 \text{ farads}
\]

and

\[
\Sigma' \approx 15 \text{ mhos}.
\]
This gives an average value for the time constant of:

\[ \tau \approx 2 \text{ seconds.} \]

We now show that the charge density at the boundaries of an energy density enhancement is much less than \( n_{\text{eq}} q \), where \( n_{\text{eq}} \) is the equatorial number density of ring current particles. We make the cautious assumption that:

\[ |\vec{v}_\perp| = \frac{u_\perp}{a_c}, \quad 42 \]

and

\[ |\vec{v}_\parallel| = \frac{u_\parallel}{a_c}, \quad 43 \]

where:

\[ a_c = \sqrt{\frac{2m_p}{qB}} \]

is the cyclotron radius for a proton with energy \( w \) (here \( w \) is the average energy of the proton energy density enhancement).

Using the fact that:

\[ \left| \frac{\vec{v}_\perp B}{B} \right| = \frac{1}{R_c}, \quad 44 \]
we have:

\[
\Lambda_\perp = \int \frac{(\overline{B} \times \overline{vB}) \cdot \overline{v u}_\perp \sin^6 \theta \, dS_R}{B^3(1 + 3 \cos^2 \theta)^3} d\theta
\]

\[
= \int \frac{u_\perp \sin^6 \theta \, R \sin \theta \, d\theta}{R_c \bar{B} \bar{a}_c}
\]

\[
= \frac{3q}{\sqrt{2m_w \gamma^*}} \int \frac{u_\perp (1 + \cos^2 \theta) \sin^6 \theta \, d\theta}{(1 + 3 \cos^2 \theta)^{3/2}}
\]  

\[
. \quad 45)
\]

Similarly

\[
\Lambda_\parallel = \frac{6q}{\sqrt{2m_w \gamma^*}} \int \frac{u_\parallel (1 + \cos^2 \theta) \sin^6 \theta \, d\theta}{(1 + 3 \cos^2 \theta)^{3/2}}
\]  

\[
. \quad 46)
\]
For the isotropic model \((\delta = 0)\), we have:

\[
\Lambda = \Lambda_\parallel + \Lambda_\perp = \frac{4q \ u_{eq}}{\sqrt{2m_p w}} \int_0^{\pi/2} \frac{(1 + \cos^2 \theta) \sin^6 \theta \ d\theta}{(1 + 3 \cos^2 \theta)^{3/2}}
\]

\[
< \frac{8q \ u_{eq}}{\sqrt{2m_p w}} \int_0^{\pi/2} \frac{(1 + \cos^2 \theta) \sin^6 \theta \ d\theta}{(1 + 3 \cos^2 \theta)^{3/2}}
\]

\[
< \frac{8q \ u_{eq}}{\sqrt{2m_p w}} \int_0^{\pi/2} 2 \sin^6 \theta \ d\theta
\]

\[
= \frac{16q \ u_{eq}}{\sqrt{2m_p w}} \left( \frac{5\pi}{32} \right)
\]

\[
= \frac{5\pi \sqrt{w} \ n_{eq} \ q}{2 \sqrt{2m_p}}
\]

\[
= 1.7 \times 10^6 \sqrt{K'} n_{eq} \ q , \quad 47)
\]
where:

\[ K = \text{the average proton energy in units of kev.}, \]

and

\[ n_{eq} = \text{the equatorial proton number density}. \]

We therefore find that an upper limit for \( \rho_{eq} \) is:

\[ \rho_{eq} < \varepsilon_0 \frac{\Delta}{\Sigma'} < 1.5 \times 10^{-6} \frac{\sqrt{K}}{\Sigma'} n_{eq} q \quad (\delta = 0) \quad 48 \]

A similar calculation for \( \delta = 1, 2, \) and 3 shows that:

\[ \rho_{eq} < 1.2 \times 10^{-6} \frac{\sqrt{K}}{\Sigma'} n_{eq} q \quad , \quad (\delta = 1) \quad 49 \]

\[ \rho_{eq} < 9.6 \times 10^{-7} \frac{\sqrt{K}}{\Sigma'} n_{eq} q \quad , \quad (\delta = 2) \quad 50 \]

and

\[ \rho_{eq} < 4 \times 10^{-7} \frac{\sqrt{K}}{\Sigma'} n_{eq} q \quad , \quad (\delta = 3) \quad 51 \]

Since \( K \) and \( \Sigma' \) are of order 10, we conclude that

\[ \rho_{eq} \ll n_{eq} q . \]
FIGURE 8: A plot of the integrand in the expression for $E$. Twice the area under the curve represents the contribution to the field line integrated permittivity from hydrogen ions along the $\frac{R}{R_E} = 4$ field line. In deriving this curve diffusive equilibrium at $T_e = T_p = 1250^\circ K$ is assumed. The curve was derived from the data of Hanson (1965).

FIGURE 9: The contribution to $\varepsilon$ from helium and oxygen ions at a geomagnetic latitude of $\approx 30^\circ$. These curves were derived from data in The Satellite Environment Handbook (1965).
D. The Electric Drift Velocity of the Enhancement

As discussed briefly in the introduction, an energy density enhancement in the ring current belt tends to be ejected outward as viewed from the earth. In this section we show that all plasma along a given field line tends to move together. We derive an expression for the terminal electric field and electric drift velocity for a given enhancement. (By "terminal" we mean the electric field and drift velocity after the polarization effects have died away.)

The direction of the electric drift velocity is along curves of constant $P$, i.e., the orthogonal trajectories of the dipole field lines. The magnitude of the electric drift velocity at any point $(r, \theta)$ along a given dipole line is:

$$V_e = \frac{E_{eq} \sin^3 \theta}{B_{eq} \sqrt{1 + 3 \cos^2 \theta}}$$  \hspace{1cm} (52)$$

The time it takes for plasma initially at $(r_1, \theta_1)$ to go from the dipole field line characterized by $R_1$ to a dipole field line characterized by $R_2$ is:
\[
\Delta T_{1,2} = \int \frac{dS}{v_E} = - \frac{p}{2 E_{eq}} \int_{\theta_1}^{\theta_2} \frac{(1 + 3 \cos^2 \theta)}{\sin^3 \theta \cos \theta} d\theta \\
= \frac{p B_{eq}}{E_{eq}} \left[ \frac{\cos \theta_2}{\sin^2 \theta_2} - \frac{\cos \theta_1}{\sin^2 \theta_1} \right].
\]

But by eqs. (9) and (10):

\[
p \frac{\cos \frac{1}{2} \theta_2}{\sin^2 \theta_2} = \frac{r}{\sin^2 \theta_2} = R_2,
\]

and

\[
p \frac{\cos \frac{1}{2} \theta_1}{\sin^2 \theta_1} = R_1
\]

Hence:

\[
\Delta T_{1,2} = \frac{B_{eq}}{E_{eq}} (R_2 - R_1).
\]
which is independent of \( P \). Thus all plasma along a given field line tends to move together.

The statement "all plasma along a given field line tends to move together" is equivalent to the statement that the plasma is "tied" to the field lines. The tendency to move together is greater the lower the energy of the particles involved, since for high energy particles the electric drift velocity may not be the dominant drift velocity.

To calculate the electric field, and hence the electric drift velocity, associated with a given energy density enhancement, we use the fact that the charge density at the boundaries of an enhancement is always much smaller than \( q \) times the number density of ring current particles. Along a given line of magnetic force the sum of the currents associated with the current densities \( \vec{J}_G \), \( \vec{J}_R \), \( \vec{J}_P \), and \( \vec{J}_I \) is approximately zero:

\[
\int_{\text{field line}} (\vec{J}_G + \vec{J}_R + \vec{J}_P + \vec{J}_I) dA = 0 \tag{55}
\]

or

\[
\int \int (\vec{J}_G + \vec{J}_R + \vec{J}_P + \vec{J}_I) ds_R \ ds_P = 0 \tag{56}
\]
or by eq. (13):

\[
\int \int (\vec{j}_G + \vec{j}_R + \vec{j}_P + \vec{j}_I) \, ds_R \frac{\sin^3 \theta \, dR}{(1 + 3 \cos^2 \theta)^{\frac{3}{2}}} = 0. \quad 57)
\]

Again we require the inner integral to be zero.

Hence:

\[
\int (\vec{j}_G + \vec{j}_R + \vec{j}_P + \vec{j}_I) \frac{\sin^3 \theta \, ds_R}{\sqrt{1 + 3 \cos^2 \theta}} = 0 \quad 58)
\]

or by eqs. (1), (2), (6), and (7):

\[
\int \left( \frac{u_1}{B^3} \frac{\vec{B} \times \vec{vB}}{1 + 3 \cos^2 \theta} \right) \sin^3 \theta \, ds_R + \int \frac{2u_\parallel}{R_c} \frac{\vec{R_c} \times \vec{B} \sin^3 \theta \, ds_R}{R_c^2 B^2 (1 + 3 \cos^2 \theta)^{\frac{3}{2}}}
\]

\[
+ \int \left( \epsilon \frac{\vec{E} + \sigma_1 \vec{E}}{1 + 3 \cos^2 \theta} \right) \sin^3 \theta \, ds_R = 0. \quad 59)
\]

In terms of the electric field in the equatorial plane, we have by eqs. (21), (35), and (36):
\[ \frac{\partial \mathcal{E}_{eq}}{\partial t} + \frac{\Sigma'}{E} \mathcal{E}_{eq} + \frac{\bar{J}}{E} = 0 \quad \text{60)} \]

where:

\[ \bar{J} = \left[ \frac{u_\perp (\overline{B} \times \overline{vB}) \sin^3 \theta \, ds_R}{B^3(1 + 3 \cos^2 \theta)^{3/2}} \right] \quad \text{61)} \]

\[ \bar{J} = \left[ \frac{2u_\parallel \overline{R_c} \times \overline{B} \sin^3 \theta \, ds_R}{2R_c^2B^2(1 + 3 \cos^2 \theta)^{3/2}} \right] \quad \text{62)} \]

and

\[ \bar{J} = \bar{J}_\perp + \bar{J}_\parallel \quad \text{63)} \]

The solution for the equatorial electric field, assuming \( \mathcal{E}_{eq} = 0 \) at \( t = 0 \), is:

\[ \mathcal{E}_{eq}(t) = \frac{\bar{J}}{\Sigma'} \left( e^{-t/\tau} - 1 \right) = \mathcal{E}_{eq} \left( e^{-t/\tau} - 1 \right) \quad \text{64)} \]

where again the time dependent polarization effects die away with the time constant:

\[ \tau = \frac{E}{\Sigma'} \quad \text{40)} \]
The electric drift velocity is then given by:

\[
\tilde{v}_E(t) = \frac{E}{B} \cdot \tilde{B} = \frac{J \times \tilde{B}}{B^2} \left( e^{-t/\tau} - 1 \right) = \tilde{v}_E \left( e^{-t/\tau} - 1 \right).
\]

The drift velocity at any point along the field line in question is simply related to the equatorial drift velocity, as discussed earlier in this section.

In the above expressions for the currents \( \tilde{J}_\parallel \) and \( \tilde{J}_\perp \), the energy densities \( u_\parallel \) and \( u_\perp \) refer to the enhancement in the ring current belt, i.e., not including the energy density of the uniform ring current belt. The currents can also be written:

\[
\tilde{J}_\perp = \int \frac{\Delta u_\perp (\tilde{B} \times \tilde{\nabla}B) \sin^3 \theta \, dS}{B^3 \left( 1 + 3 \cos^2 \theta \right)^{1/2}},
\]

and

\[
\tilde{J}_\parallel = \int \frac{2\Delta u_\parallel (\tilde{R}_c \times \tilde{B}) \sin^3 \theta \, dS}{\tilde{R}_c^2 B^2 \left( 1 + 3 \cos^2 \theta \right)^{1/2}},
\]

where:

\( \Delta u_\perp \) and \( \Delta u_\parallel \) represent the difference in the energy density of the enhancement and the energy density on either side of the enhancement.
Substituting eqs. (11), (20), and (44) into eqs. (66) and (67), we have:

\[
J_\perp = \frac{6}{B_{eq}} \int_{\theta^*}^{\pi/2} \frac{\Delta u_\perp \sin^9 \theta \left(1 + \cos^2 \theta \right) \, d\theta}{(1 + 3 \cos^2 \theta)^2}, \tag{68}
\]

and

\[
J_\parallel = \frac{12}{B_{eq}} \int_{\theta^*}^{\pi/2} \frac{\Delta u_\parallel \sin^9 \theta \left(1 + \cos^2 \theta \right) \, d\theta}{(1 + 3 \cos^2 \theta)^2}, \tag{69}
\]

where:

\[\theta^* \]

is the polar coordinate latitude of the tip of the energy density enhancement.

(If the particles within the tube of force containing the energy density enhancement are assumed to extend down to the surface of the earth, then \[\theta^* = \theta_0\].

Atmospheric absorption would attenuate the energy density enhancement near the earth's surface, so \[\theta^* > \theta_0\].)
For the isotropic model we have:

\[ J = J_\perp + J_\parallel = \frac{\hbar \Delta u}{B_{eq}} \int_{\theta^*}^{\pi/2} \frac{\sin^9 \theta (1 + \cos^2 \theta) d\theta}{(1 + 3 \cos^2 \theta)^2} \].

The integral:

\[ \int_{\theta^*}^{\pi/2} \frac{\sin^9 \theta (1 + \cos^2 \theta) d\theta}{(1 + 3 \cos^2 \theta)^2} \]

\[ \sqrt{1 - \frac{R^*}{R}} \int_0^1 \frac{(1 - x^2)^4 (1 + x^2) dx}{(1 + 3x^2)^2} = G\left(\frac{R^*}{R}\right), \]

where:

\[ R^* = R \sin^2 \theta^* \]

is plotted in Figure 10. It has a constant value near 0.3 over a wide range of \( \frac{R^*}{R} \).
Substituting \( G(R^*) = 0.3 \) in the expression for \( J \), we have:

\[
J = \frac{2.4\Delta u}{B_{eq}} \text{ amps/meter, } \quad (\delta = 0) \quad 71)
\]

hence:

\[
E_{eq} = \frac{2.4\Delta u}{B_{eq}^{\Sigma}} \text{ volts/meter, } \quad (\delta = 0) \quad 72)
\]

and

\[
v_{eq} = \frac{2.4\Delta u}{B_{eq}^{2\Sigma}} \text{ meters/second. } \quad (\delta = 0) \quad 73)
\]

It is perhaps more useful to express the equatorial electric drift velocity in terms of \( \beta \), the ratio of the particle energy density to the magnetic field energy density.

\[
\beta_{eq} = \frac{u}{(B_{eq}^2/2\mu_0)} \quad 74)
\]

Hence:

\[
\Delta\beta_{eq} = \frac{\Delta u}{(B_{eq}^2/2\mu_0)} \quad 75)
\]
and

\[ v_{E_{eq}} = 9.6 \times 10^5 \frac{v \beta_{eq}}{\Sigma'} \]

\[ \approx \frac{v \beta_{eq} \times 10^6}{\Sigma'} \text{ meters/second. (} \delta = 0 \text{) 76) } \]

We note here that the equatorial electric drift velocity is not seriously affected by the pitch angle distribution. For example, the drift velocity for \( \delta = 2 \) is:

\[ v_{E_{eq}} = 7.3 \times 10^5 \frac{v \beta_{eq}}{\Sigma'} \text{ meters/second. (} \delta = 2 \text{) 77) } \]

For the same total excess energy in a given flux tube, the electric drift velocities for different pitch angle distributions are almost equal. For example:

\[ (v_{E_{eq}})_{\delta = 2} = 1.05 (v_{E_{eq}})_{\delta = 0} \quad 78 \)
FIGURE 10: A plot of:

\[ G \left( \frac{R^*}{R} \right) = \int_0^{\sqrt{1 - \frac{R^*}{R}}} \frac{(1 - x^2)^4}{(1 + 3x^2)^2} \frac{(1 + x^2)dx}{(1 + 3x^2)^2} \]

= \int_0^{\pi/2} \frac{\sin^9 \theta}{(1 + 3 \cos^2 \theta)^2} d\theta
E. STABILITY AND ENERGETICS

The results of Sections C and D show that a non-uniform ring current belt tends to be stabilized by the conducting ionosphere. The equatorial ejection velocity associated with an energy density enhancement is inversely proportional to the height integrated Pederson conductivity.

In this section, we will discuss another stabilizing effect, caused by a spread in energies of the ring current particles. We will also examine the energetics of the problem and show that when a tube of flux containing an energy density enhancement is ejected from the ring current belt, most of the available particle energy is dissipated in the ionosphere.

Since the gradient and curvature drifts are energy dependent, an energy density enhancement that is not mono-energetic will tend to smear itself out. A crude indication of whether or not the electric drift velocity is large enough to overcome the stabilizing effect of a spread in energies in the enhancement is the ratio:

\[
\frac{v_E}{v_D}
\]

where:

\(v_D\) is a characteristic longitudinal drift velocity for the enhancement.
If:

\[
\frac{v_E}{v_D} \geq 1, \tag{78}
\]

then the energy density enhancement may be considered unstable.

If we define:

\[
v_D = \frac{|j|}{nq}, \tag{79}
\]

then for the isotropic model, we have [eqs. (1), (2), (44); Table 1]:

\[
v_D = \frac{4}{3} \frac{u}{nq R_c B} = \frac{4}{3} \frac{K \times 10^3}{R_c B}. \tag{80}
\]

The ratio \(\frac{v_E}{v_D}\), evaluated in the equatorial plane is then [eqs. (77) and (80)]:

\[
\left(\frac{v_E}{v_D}\right)_{eq} \approx \frac{5 \times 10^4 \Delta \beta_{eq}}{\Sigma^' \kappa R_o^2}. \tag{81}
\]

By evaluating the above ratio and comparing it with unity, one can estimate the stability of a given enhancement.
A value of $\beta > 0.5$ is probably required to produce a main phase decrease of 100 $\gamma$. The exact value depends on the (presently unknown) distribution of the ring current particles in the magnetosphere. The quiet-time belt of low-energy protons ($E_p > 100$ kev) reported by Davis and Williamson (1963) has a maximum value of $\beta \approx 0.25$ at $R = 3.5 R_E$. These protons produce a magnetic field at the earth's equator of the order of 10 $\gamma$ (Akasofu, Cain, and Chapman, 1961; Hoffman and Bracken, 1965).

Assuming ten per cent fluctuations in a ring current belt with $\beta_{eq} = 0.5$, we have:

$$\Delta \beta_{eq} = 0.05$$

Substituting:

$$\Sigma' = 15 \text{ mhos}$$
$$K = 10 \text{ kev}$$

and

$$R_c = 4$$

into eq. (81) gives:

$$\left( \frac{v_E}{v_{D,eq}} \right) \approx 1$$
This value is derived from an extrapolation of the quiet-day low-latitude conductivity. However it indicates that for cautious estimates of $\Delta \beta_{eq}$, a non-uniform ring current belt of 10 kev protons is only marginally stable. For the same values of $\Delta \beta_{eq}$ and $R_e$ the stability of the ring current belt increases with the average particle energy. For the same values of $\Delta \beta_{eq}$ and $K$ the stability increases with decreasing geocentric radial distance to the ring current belt. We note also that since the night time values of the height integrated Pederson conductivity may be lower than the day values by as much as a factor of 50, the night side of a non-uniform ring current belt should be less stable than the day side.

Provided the ejection is slow enough, the magnetic-moment and bounce actions of the ring current particles are conserved. In this case, the energy of the particles within the enhancement decreases in a way that is subject to calculation (e.g., Nakada et al., 1965). In particular, for particles that mirror in the equatorial plane the particle energy decreases with the cube of the geocentric distance to the tube of flux. The decrease in particle energy for other pitch angles is less severe, but still considerable (see Figure 7).

The energy made available by the adiabatic ejection of a flux tube containing ring current particles may be channeled in two directions. It can appear as kinetic energy of electric drift motion or it may be dissipated in the ionosphere as Joule heating from the ionospheric currents.
In the limiting case of high ionospheric conductivity the
ejection velocity of a flux tube is low, and most of the
available ring current-particle energy is dissipated in
the ionosphere. For low ionospheric conductivities the
ejection velocity is high, and most of the available energy
appears as kinetic energy of electric drift motion.

The kinetic energy due to electric drift motion within
a given tube of flux is:

\[
\int \int \int \phi \rho v_E^2 dv = \int \int \int \frac{\phi \rho}{B^2} \frac{E^2 \sin^6 \theta R dS_R d\phi dR}{(1 + 3 \cos^2 \theta)^{3/2}}
\]

\[
= \int \int \int \frac{\phi e E_{eq}^2 dS_R d\phi R d\phi}{(1 + 3 \cos^2 \theta)^{3/2}}
\]

or,

\[
\int \int \int \phi \rho v_E^2 dv = \int \int \phi E^2 R d\phi dR . \quad 83)
\]
On the other hand the energy of the ring current particles themselves, i.e., the kinetic energy of circulatory motion, is given by:

\[
\int_{SR} \int_{\Delta R} \int_{\Delta \phi} u \, dv = \int_{SR} \int_{\Delta R} \int_{\Delta \phi} \frac{u \sin^6 \theta \, dS_R \, d\phi \, dR}{(1 + 3 \cos^2 \theta)^{3/2}}. \tag{84}
\]

For the isotropic model, \( u \) is constant along a line of force, hence:

\[
\int_{SR} \int_{\Delta R} \int_{\Delta \phi} u \, dv = \int_{SR} \int_{\Delta R} \int_{\Delta \phi} uR^2 \int_{0}^{\pi-\theta^*} \sin^7 \theta \, d\theta \, d\phi \, dR.
\]

The integral:

\[
\int_{0}^{\pi-\theta^*} \sin^7 \theta \, d\theta = H \left( \frac{R^*}{R} \right)
\]

is plotted in Figure 11. As can be seen from this figure, \( H \left( \frac{R^*}{R} \right) \) is fairly constant over a wide range of \( \frac{R^*}{R} \), having the value \( \approx 0.45 \).
Setting:

\[ H \left( \frac{R^*}{R} \right) = 0.45 , \]

we have:

\[
\int_{\Delta R} \int_{\Delta \varphi} u dv = \int_{\Delta R} \int_{\Delta \varphi} [0.45 uR] R \ d\varphi \ dR . \tag{85}
\]

We can estimate the relative importance of the two channels for the available ring current energy by comparing the integrands of eqs. (83) and (85):

\[
\frac{1}{2} \left( \mathbf{E} \cdot \mathbf{E} \right)_{\text{eq}}
\]

and

\[ 0.45 uR \approx \frac{1}{2} uR . \]

If:

\[ \mathbf{E} \cdot \mathbf{E} \gtrless \ uR , \tag{86} \]

then most of the ring current belt energy is dissipated in the ionosphere. On the other hand, if:

\[ \mathbf{E} \cdot \mathbf{E} \approx \ uR , \tag{87} \]
then most of the ring current belt energy is being converted into kinetic energy of electric drift motion. This interpretation becomes particularly clear when one considers that:

\[ E \quad \text{corresponds to} \quad R_c = \frac{R_0}{B^2} \]

hence:

\[ E E^2 \quad \text{corresponds to} \quad R_0 \frac{E^2}{B^2} = R_0 \nu_E \]

\[ = 2 R \left( \frac{1}{2} \rho v_E^2 \right) \]

Substituting:

\[ E = 30 \text{ farads} \]

and

\[ E_{eq}^2 = B_{eq}^2 v_{eq}^2 \quad \text{[eq. (52)]} \]

\[ \approx B_{eq}^2 v_{D_{eq}}^2 \quad \text{[eq. (82)]} \]

\[ \approx B_{eq}^2 \left( \frac{4}{3} \frac{K \times 10^3}{R_c B} \right)^2 \quad \text{[eq. (80), } \delta = 0] \]

\[ = \frac{K^2 \times 10^6}{R_E^2} \quad \text{[} R_c = 4 \]
we find for \( K = 10 \):

\[
E \, E_{eq}^2 \approx 8 \times 10^{-5} \quad [\delta = 0, \, R_o = 4, \, K = 10]
\]

For 10 kev protons with a number density of \( 10^7 / m^3 \)
at \( R_o = 4 \), we have:

\[
uR \approx 4 \times 10^{-1}
\]

Hence, these estimates indicate:

\[
E \, E_{eq}^2 \ll uR
\]

For night sunspot minimum conditions, the electric
drift velocity may be larger than we have assumed. However,
unless:

\[
\frac{v_E}{v_D} \approx 100
\]

we may assume that most of the available ring current energy
is dissipated in the ionosphere.

In this section we have argued that, in order for the
ejection process to be important, the energy density
enhancement must satisfy a local instability criterion:

\[
\frac{v_E}{v_D} \geq 1
\]
We use the word "local" because the ring current belt parameters (and the ionospheric conductivity) may change with geocentric distance (geomagnetic latitude) and thus change the value of $v_F/v_D$. For example, the background ring current belt energy density may not decrease sufficiently rapidly with geocentric distance to allow the instability criterion to be satisfied throughout the belt.

The reason for this is most easily seen from the hydromagnetic point of view rather than from the point of view of single-particle-orbit theory (c.f. Gold, 1959; Sonnerup and Laird, 1963). From this point of view, the instability is important if it is energetically possible for two tubes of flux, separated in radial distance, to interchange. Since the particle energy decreases roughly as $R^{-3}$ and the volume of a tube of flux increases as $R^4$, we see that unless the energy density of the ring current belt decreases faster than $\sim R^{-7}$, the interchange of flux tubes is energetically impossible.

From these considerations, we see that the instability is most likely to be important at the outer boundary of the ring current belt, where a sharp negative gradient in the energy density could be present. A sharp field-aligned gradient in the equatorial profile of both the electron (Carpenter, 1963) and positive-ion (Taylor et al., 1965) background plasma has been observed. It is not yet known if this "boundary" in the background plasma corresponds to a "boundary" in the ring current belt.
FIGURE 11: The integral:

\[ \int_{\theta^*}^{\pi-\theta^*} \sin^7 \theta \, d\theta = H \left( \frac{R^*}{R} \right) . \]
F. APPLICATION TO IONOSPHERIC HEATING

In Section E we have argued that the energy of the ejected ring current particles is likely to be dissipated in the ionosphere via Joule heating of the Pederson current. In this section we examine the possibility that the non-uniform ring current belt model could account for the magnetic storm associated phenomenon of ionospheric heating. We estimate the power input into the ionosphere caused by the Joule heating of the stabilizing currents associated with a non-uniform ring current belt, and compare this power input with that caused by EUV (Extreme Ultraviolet Light). We show that if the recovery of a magnetic storm is a result of the proposed transfer of ring current particle energy to the ionosphere, the power input is \( \sim 10^{11} \) watts. This value is about a factor of ten less than the estimated power input caused by absorption of solar EUV, \( \sim 10^{12} \) watts (Hunt and Van Zandt, 1961; Harris and Priester, 1963).

It has been shown that the temperature variations of the upper atmosphere are correlated with geomagnetic activity as measured by the \( a_p \) index (Jacchia, 1961a,b). Several mechanisms have been discussed that relate to this phenomenon, including heating due to the dissipation of hydromagnetic wave energy (Dessler, 1959; Karplus et al., 1962), magnetic storm related heating by internal atmospheric gravity waves (Hines, 1965), and Joule heating from ionospheric currents (Cole, 1961).
Cole (1961) calculated the ionospheric heating rate from estimates of the ionospheric currents and conductivities. If the non-uniform ring current belt is a valid model, it is possible to take a different approach in calculating the ionospheric heating rate. In the non-uniform ring current model the decrease in energy content of the ring current belt appears as heat energy in the ionosphere. The ionospheric heating rate is given by:

\[
\frac{dQ_I}{dt} = - \frac{dU_{RC}}{dt}
\]

where:

\[Q_I = \text{heat input into the ionosphere},\]

and

\[U_{RC} = \text{energy content of the ring current belt}.\]

But, as pointed out by Dessler and Parker (1959), the energy content of the ring current belt is related to the magnetic field decrease, \(-B_{RC}\), produced at the earth's equator by the expression:

\[
U_{RC} = -\frac{2}{3} \frac{B_{RC} U_m}{B_0}
\]
where:
\[ U_m = 8 \times 10^{17} \text{ joules is the total geomagnetic field energy external to the earth,} \]

and
\[ B_o = 3 \times 10^{-5} \text{ webers/m}^2 \text{ is the field strength at the earth's equator.} \]

Using this relation, the power input into the ionosphere may be calculated as a function of \( \frac{dBR_C}{dt} \), the rate of change of the ring current magnetic field as measured at the earth's equator:

\[ P_I = \frac{dQ_I}{dt} = 1.8 \times 10^{22} \frac{dBR_C}{dt} \text{ watts.} \tag{90} \]

For a main phase decrease of 100 \( \gamma \) that recovers in \( \sim 3.6 \times 10^4 \) seconds (10 hours), we have:

\[ \frac{dBR_C}{dt} \approx 2.8 \times 10^{-10} \text{ webers/m}^2 \text{ sec}, \]

hence:

\[ P_I \approx 0.5 \times 10^{11} \text{ watts.} \tag{91} \]
This power input is less than the estimates of the power input due to EUV, \( \sim 10^{12} \) watts, (Harris and Priester, 1963; Hunt and Van Zandt, 1961). In order to account for the temperature variations of the upper atmosphere during magnetic storms the power input must be somewhat above the average EUV value. We note however that the above estimate of the power input into the ionosphere is a lower bound to the actual Joule heating rate during a magnetic storm. The estimate is only appropriate for the recovery phase and cannot take into account Joule heating during the active part of the main phase.
ASYMMETRIC RING CURRENTS

A. THE LOW-LATITUDE DISTURBANCE-DAILY VARIATION

As discussed in the Introduction, the low-latitude disturbance-daily variation is presently unexplained. Some authors have argued that the local time dependence of the storm field implies that most of the source currents are in the ionosphere (Vestine, 1953; Sugiura, 1953). The Chapman $S_D$ current system (Chapman and Bartels, 1940, p. 302) or some variation of this current system, represents an attempt to explain the local time dependence in this way. However, Akasofu and Chapman (1964) have argued, using high- and low-latitude magnetic data, that the low-latitude disturbance-daily variation cannot be attributed to the equatorward extension of a polar ionospheric current system. By a detailed study of several large storms, Akasofu and Chapman showed that the main phase storm field can be asymmetric even when there are no significant ionospheric currents flowing in the polar regions.

In this part of the thesis an extension of the symmetric ring current model is described that could account for the low-latitude disturbance-daily variation. The proposed extension is simply that the main phase ring current belt is asymmetric, having a larger energy content on the evening side than on the morning side of the earth.

In order to check the proposed explanation for the low-latitude disturbance-daily variation, a mathematical model of an asymmetric ring current is developed. Asymmetric
ring current functions are derived that give the magnetic effects that would be observed by a low-latitude station as it rotates beneath an asymmetry in an otherwise uniform ring current belt. Allowance is made in the mathematical model for the drift of the asymmetry in local time and for the recovery in universal time of both the symmetric ring current and the asymmetry in the ring current belt.

Recovery phases for stations with various longitudes are derived and compared with measured recovery phases for a few magnetic storms. The comparisons show that even a very simple model of the asymmetric ring current can account for most of the low-latitude disturbance-daily variation.
B. THE ASYMMETRY IN THE STORM FIELD

The low-latitude disturbance-daily variation is a result of an asymmetry in the main phase storm field. The asymmetry is generally east-west, as can be seen from Figure 12. In each of the polar histograms shown in Figure 12, the frequency of occurrence of an event is plotted against local time. An "event" occurs whenever the hourly averaged value of $H$ falls below its monthly 5-quiet-day average by more than 100 $\gamma$. Five years of data (1957-1961) were used to make these histograms for Honolulu, San Juan, Tucson, and Fredericksburg. By defining zero for each hour of local time as the 5-quiet-day values, we have attempted to subtract the quiet-day daily variation from the disturbance-daily variation in $H$.

The first thing to note about Figure 12 is that the histograms are of the same form for Honolulu (geomagnetic latitude $\lambda_m = 21.1^\circ$), San Juan ($\lambda_m = 29.9^\circ$), and Tucson ($\lambda_m = 40.4^\circ$). The maximum frequency of occurrence of events ($|\Delta H| \geq 100 \, \gamma$) is centered about 1800 hours for each of these low latitude stations. There is a decrease in the total number of events with increasing geomagnetic latitude. If the storm field lines are roughly perpendicular to the equatorial plane, one expects this decrease since $H$ represents the projection of the storm field on the local north-south horizontal.

The second thing to notice about the histograms of Figure 12 is the sudden change in form at Fredericksburg ($\lambda_m = 49.6^\circ$). For Fredericksburg, this histogram
representation of the disturbance-daily variation is not very appropriate because it presents only part of the effect. At high latitude stations, large positive excursions of $H$ occur. A more appropriate representation of the high-latitude disturbance-daily variation is given by Chapman and Bartels (1940, p. 277 ff.). Chapman illustrated the disturbance-daily variation by deriving an average storm from some 40 storms and then comparing each individual storm with the average storm to derive a local time dependence.

Presumably, the local time dependence of the storm field at magnetic stations with $\lambda_m > 45^\circ$ is due primarily to ionospheric currents. Some of these ionospheric currents may be associated with the asymmetric ring current belt, as we will discuss briefly below.
FIGURE 12: A polar histogram showing as a function of local time the frequency with which \((H_s - H) \geq 100 \gamma\). This figure shows all data from Honolulu (magnetic latitude \(\lambda_m = 21.1^\circ\)), San Juan \((\lambda_m = 29.9^\circ)\), Tucson \((\lambda_m = 40.4^\circ)\), and Fredericksburg \((\lambda_m = 49.6^\circ)\), for the years 1957 through 1961.
Figure 12
C. THE PARTIAL RING CURRENT SYSTEM

An asymmetric ring current is equivalent to a symmetric ring current plus a superimposed partial ring current system. The current system for the partial ring current, proposed earlier by Fejer (1961), is shown in Figure 13. In this figure the shaded region represents the asymmetry in an otherwise uniform ring current belt. The current in the equatorial element of the partial ring current system is due to the gradient and curvature drift motion of the charged particles that make up the asymmetry. The current flowing along the earth's field lines is carried by the low energy background plasma of the magnetosphere.

Not shown in Figure 13 are the Hall currents that flow in the ionosphere around the points of entry and exit of the current carrying field lines in the partial ring current system (c.f. Fejer, 1961). The magnetic fields of the Hall currents associated with the partial ring current system are important for high latitude stations. However, in this thesis we restrict our attention to low-latitude stations and neglect the magnetic effects of the Hall currents.

As a station rotates beneath the asymmetry, the value of H decreases below the symmetric ring current value. Note that an equatorial station beneath the asymmetry is, in effect, midway between two current loops, the upper and lower portions of the partial ring current system. Each
current loop carries current I/2 circulating clockwise as viewed from above the north pole.

All charged particles in the magnetosphere tend to co-rotate with the earth, so the asymmetry in the ring current belt will not in general remain fixed in local time. Protons have a westward gradient and curvature drift velocity and electrons drift eastward. Hence, the net drift of the asymmetry in local time can either be eastward or westward depending on the particle type, energy distribution, and geocentric distance of the ring current particles. For example, 10 keV protons between $R = 3R_E$ and $R = 4R_E$ drift westward with a drift rate of approximately 0.3 hours in local time per hour of universal time. (Note the drift rate in a non-rotating magnetosphere would be 1.3 hours in local time per hour of universal time.) The eastward drift rate for 10 keV electrons at the same geocentric distance is approximately 2.3 hours in local time per hour of universal time.
FIGURE 13: The partial ring current system. The current flowing along the field lines is carried by the low energy background plasma of the magnetosphere. The current in the equatorial segment arises from the drift motion of the ring current particles. Note that an equatorial magnetic station is, in effect, midway between two current loops, each carrying current $I/2$ circulating in the clockwise direction as viewed from above the north pole. The current loops are parallel in the equatorial plane giving the current $I$ in the equatorial segment of the partial ring current system.
D. THE MATHEMATICAL MODEL

In this section we describe the mathematical model used to simulate the physical model of an asymmetric ring current.

In order to compute the partial ring current functions, a wire model of the current system was built to serve as an analog computer. In the model, sketched in Figure 14, the geocentric distance to the equatorial element was scaled to 4 earth radii. The dipole loops of the current model were connected in the "polar" region with copper braid so that the angle, \( \gamma \), between the two dipole loops could be easily varied.

Measurements of the model partial ring current field were made with a small flux gate magnetometer. The magnetometer was perpendicular to the equatorial plane of the scale model at a distance corresponding to one earth radius. The partial ring current functions were derived by sampling the magnetic field at various longitudes.

The derived partial ring current functions are shown in Figure 15. The angle characterizing each function corresponds to the separation angle between the two dipole arms of the current system. The abscissa for these functions is longitude expressed in hours (1 hour corresponds to 15°) from \( \phi_0 \), the local time about which the partial ring current is centered. The partial ring current is assumed fixed in local time and each function is normalized to -1.0 at \( \phi_0 \).
Note that at those local times opposite to \( \varphi_o \), the partial ring current actually provides a slight positive contribution to the storm field. The magnetic effect of the current along the dipole arms outweighs the effect of the equatorial segment for these hours.

In order to simulate the recovery of magnetic storms, we assume the partial ring current decays exponentially. That is, if \( \mathcal{P}_y(\varphi) \) denotes one of the partial ring current functions shown in Figure 15, then:

\[
\mathcal{P}_y(\varphi, t) = \mathcal{P}_y(\varphi) \exp(-t/\tau_p), \tag{92}
\]

where:

\( \tau_p = \) the time constant for decay of the partial ring current.

In addition to the partial ring current, we assume a symmetric ring current that also decays exponentially. We have:

\[
S(t) = -S \exp(-t/\tau_s), \tag{93}
\]

where:

\( \tau_s = \) the time constant for decay of the symmetric ring current.
and

\[ S = \text{the amplitude of the symmetric ring current, i.e., } -S \text{ is the symmetric part of the decrease in } H \text{ observed at the equator at the beginning of the recovery phase.} \]

We take account of the movement of the partial ring current in local time in the following way. Suppose the partial ring current is initially (i.e., at the beginning of the recovery phase) centered about some longitude \( \varphi_0 \), and that it subsequently moves in local time with a constant angular velocity \( \dot{\varphi} \). At a later time \( t \), the partial ring current is centered about the longitude:

\[ \varphi = \varphi_0 - \dot{\varphi} t \]

where \( \dot{\varphi} \) is positive if the partial ring current drifts westward in local time. The magnetic effect observed at an equatorial station with longitude \( \varphi_X \) is given by:

\[ \Delta H = A P_\psi(|\varphi - \varphi_X|, t) + S(t) \]
\[ \Delta H = A P_y(|\phi - \phi_x|, t) + S(t) \]
\[ = A P_y(|\phi_0 - \phi t - \phi_x|) \exp(-t/\tau_p) \]
\[ - S \exp(-t/\tau_S) \]
\[ = A P_y(|\phi_0 - \phi x_0 - (\phi + 1)t|) \exp(-t/\tau_p) \]
\[ - S \exp(-t/\tau_S) \]

where:

- \( A \) = the amplitude for the partial ring current, i.e., -A is the asymmetric part of the decrease in \( H \) observed at an equatorial station directly beneath the center of the partial ring current at the beginning of the recovery phase.

Finally, we try to account for the geomagnetic latitude, \( \lambda_x \), of station \( x \) by multiplying eq. (94) by \( \cos \lambda_x \). That is:

\[ \Delta H(\phi x_0, \lambda_x, t) = \cos \lambda_x \left\{ A P_y(|\phi_0 - \phi x_0 - (\phi + 1)t|) \cdot \exp(-t/\tau_p) - S \exp(-t/\tau_p) \right\} . \]
The physical model under consideration represents a simple, idealized extension of the uniform ring current hypothesis. We have assumed there is no structure in the uniform ring current belt except for a single uniform enhancement spanning a large angle.

The mathematical model we have developed only approximates the physical model under consideration. There is provision in the model for the size of the partial ring current and for its drift in local time. There is little or no provision, however, for:

1) the change in size with time, e.g., the smearing out of the partial ring current in time, although the time constant \( \tau_p \) may partially take account of this effect,

2) possible changes in local time drift rate,

and 3) the change in radial position of the partial ring current with time; in fact, the radial position of the partial ring current is fixed at \( R = 4R_E \) in the model. Again, the time constant \( \tau_p \) may partially account for the outward radial drift and consequent de-energization of the asymmetry.
FIGURE 14: A sketch of the wire model used as an analog computer to calculate the partial ring current functions.

FIGURE 15: Three partial ring current functions corresponding to separation angles between the dipole arms of the partial ring current system of 100°, 140°, and 180°.
FIGURE 15
E. COMPARISON WITH MEASURED RECOVERY PHASES

In this section recovery phases calculated by the asymmetric ring current model are compared with measured recovery phases as observed at low-latitude magnetic observatories that are well spaced in longitude. The geographic longitude and geomagnetic latitude of the stations used in this study are listed in Table 2.

Ten sudden commencement storms from the five-year period 1957-1961 were chosen for this study. An attempt was made to select storms that had a quiet recovery phase, since the model used only provides for the decay of an asymmetric ring current presumably established at the height of the main phase (i.e., the model is not valid for storms in which the ring current belt is reformed during the recovery phase).

Since it was expected from the statistical study (Figure 12) that the asymmetry in the ring current belt would be centered about 1800 hours L.T., several storms were chosen that had \( \approx 1800 \) L.T. for the hour of main phase decrease as observed at San Juan. If the asymmetry of the ring current is centered about 1800 hours, then the Honolulu and Guam magnetic observatories would be outside of the partial ring current system at the height of the main phase for these storms but would subsequently rotate under the asymmetry (provided the bulk of the energy of the ring current particles was in low energy protons). The differences in the recovery phases at San Juan, Honolulu and Guam should be most striking for this particular set of circumstances.
Calculated and measured recovery phases for the storm that began on July 15, 1959 are compared in Figure 16. The parameters that determine the asymmetric ring current for this storm are listed in Table 3, along with the parameters for the other storms analyzed. The initial position and size of the asymmetry, the values of $S$ and $A$, and the initial position of the magnetic stations for the storm of July 15 are also given schematically in the top left corner of Figure 16.

For each station in Figure 16 the bars represent the hourly averaged values of $H$, where the zero levels for each hour are the 5-quiet-day values. The solid curve is the theoretical fit to the data. The curves used to determine the parameters for the asymmetric ring current are marked $F$ (fitted) and the other curves are marked $T$ (tested). The abscissa for each curve is the local time of the magnetic station.

There are two important things to notice about the curves of Figure 16. First, note the differences in the form of the recovery phases for the various stations. For example, the initial recovery rate at San Juan is more rapid than the initial recovery rate at Honolulu and Guam. As mentioned above, the recovery at Honolulu and Guam is retarded because these stations are initially outside the partial ring current system and subsequently rotate beneath the asymmetry.

The second thing to note about Figure 16 is the difference in the main phase decreases between stations
initially beneath the partial ring current system and stations initially outside the system. For example, the difference between the main phase decreases (normalized to the equatorial plane) at San Juan and Tashkent is about 100 \( \gamma \). This difference is fairly well reproduced by the asymmetric ring current model.

Measured and computed recovery phases for the storm that began on April 30, 1960 are compared in Figure 17. Particularly striking in this case is the difference in the form of the recovery phases as recorded at Honolulu and Hermanus. These two stations are about 180° apart in longitude, and for this storm Hermanus was directly beneath the partial ring current at the beginning of the recovery phase.

Measured and computed recovery phases for the other storms analyzed are compared in Figures 18 through 25. Similar comments about the differences in the recovery phases and magnitudes of main phase decreases can be made for some of these curves. However, as these figures show, this simple asymmetric ring current model is only partially successful in fitting some storms.

The uncertainty in the zero level for \( \Delta H \) could be one reason for the apparent failure of the asymmetric ring current model to fit the recovery phases of weak storms. As mentioned in the previous section, the zero levels for the measured \( \Delta H \) are the 5-quiet-day values. By subtracting out the 5-quiet-day averages, we are attempting to eliminate the quiet day diurnal variation from the storm
field. There is, of course, some uncertainty in this process, since the quiet day variation changes slightly from day to day. Also, one is not sure that the ionospheric current patterns causing the quiet day variation remain the same during a magnetic storm.

To get an idea of the uncertainties involved in defining the zero level for the storm field, we have plotted in Figures 26 through 28 the storm of July 15, as observed at San Juan, Tucson and Honolulu, using three different definitions of zero. In the first set of data, the hourly average of the five quiet days in July is defined as zero. For the second set of data, we have used the hourly averages of the day preceding the storm. For the third set, we have not tried to subtract out the quiet day diurnal variation. The average over all hours for the ten quiet days in July is defined as zero.

These figures show that during the daylight hours the uncertainty in the zero level is greater than during the night hours.

A source of error that affects all the comparisons between measured and computed recovery phases is the neglect of the diamagnetic effect of the ring current particles that make up the asymmetry in the ring current belt. For the symmetric ring current, the magnetization of the ring current particles cancels about a third of the main phase decrease caused by the drift currents (Dessler and Parker, 1959). By assuming the same correction is necessary for the particles in the asymmetry, one
can derive partial ring current functions that take account of the diamagnetic effect. In Figure 29 a "corrected" partial ring current function ($\psi = 140^\circ$) is compared with the corresponding function used in this study. As can be seen from this figure, the two partial ring current functions differ only at local times opposite to $\varphi_0$. For these local times the magnetic field of the partial ring current is much less than the magnetic field of the symmetric current, so the form of the recovery phase would not be greatly changed by inclusion of the diamagnetic effect in the asymmetric ring current model.

In computing the partial ring current functions by the scale model of the current system, the contribution to $\Delta H$ from the "ionospheric" element was found to be much less important than the contribution from the current flowing along the field lines. Changes in the position of the copper braid of the scale model produced little or no effect on the magnetometer, but movements of the dipole arms of the current system were easily detected.

Because of the diamagnetic earth, the ionospheric element of the partial ring current system probably contributes very little to the main phase decrease observed at an equatorial station. Even discounting the diamagnetism of the earth, the contribution of the ionospheric element is only a fraction of the contribution of the equatorial element. The earth current below the ionospheric element further reduces this contribution, since the induced current is antiparallel with the ionospheric current. The
Ionospheric current flows at an altitude of \(\approx 100\) km, and
the earth current flows at a depth of less than \(\approx 260\) km
\cite{chapman1951}, so that the separation between the two
oppositely directed currents is less than \(\approx 360\) km. The
magnetic effect is negligible at an equatorial station
several thousand kilometers from the two currents.

The magnetic effect of earth currents induced in the
vicinity of the low latitude magnetic observatories are
lumped into the amplitudes \(S\) and \(A\).
TABLE 2. The geographic longitude and geomagnetic latitude of the magnetic observatories used in this thesis.

<table>
<thead>
<tr>
<th>MAGNETIC OBSERVATORY</th>
<th>GEOGRAPHIC LONGITUDE</th>
<th>GEOMEMAGNETIC LATITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>M'BOUR</td>
<td>17.0° W</td>
<td>21.3° N</td>
</tr>
<tr>
<td>SAN JUAN</td>
<td>66.1° W</td>
<td>29.9° N</td>
</tr>
<tr>
<td>TUCSON</td>
<td>110.8° W</td>
<td>40.4° N</td>
</tr>
<tr>
<td>HONOLULU</td>
<td>158.0° W</td>
<td>21.1° N</td>
</tr>
<tr>
<td>GUAM</td>
<td>144.9° E</td>
<td>4.0° N</td>
</tr>
<tr>
<td>TASHKENT</td>
<td>69.3° E</td>
<td>32.0° N</td>
</tr>
<tr>
<td>ADISS-ABABA</td>
<td>38.8° E</td>
<td>5.4° N</td>
</tr>
<tr>
<td>HERMANUS</td>
<td>19.2° E</td>
<td>33.3° S</td>
</tr>
</tbody>
</table>
FIGURE 16: Comparisons between measured and computed recovery phases for the storm that began on July 15, 1959. The solid bars represent the measured hourly averages, and the curves are the computed recovery phases. The zero level for each hour is defined by the monthly 5-quiet-day average values. The diagram in the top left corner shows the positions of the magnetic stations at the beginning of the recovery phase. The values of $S$ and $A$, and the size and initial position of the asymmetry in the ring current belt are also represented schematically by this diagram.

FIGURES 17-25: Comparisons between measured and computed recovery phases for the storms that began on:

17) April 30, 1960,
18) January 21, 1957,
19) September 29, 1957,
20) February 11, 1958,
21) July 27, 1961,
22) October 28, 1961,
23) December 3, 1958,
24) November 6, 1957,
and 25) September 13, 1957.
FIGURE 16
FIGURE 17
FIGURE 18
<table>
<thead>
<tr>
<th>Location</th>
<th>December 3, 1958</th>
<th>Guam</th>
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<tbody>
<tr>
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<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>-100</td>
</tr>
<tr>
<td>24</td>
<td></td>
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</tr>
<tr>
<td>SAN JUAN</td>
<td>ΔH(γ)</td>
<td>T</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>-100</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>-200</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TASHKENT</td>
<td>ΔH(γ)</td>
<td>T</td>
</tr>
<tr>
<td>12</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HONOLULU</td>
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<td>HERMANUS</td>
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</tr>
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</table>

**Figure 23**
TABLE 3. The asymmetric ring current parameters for the storms analyzed in this thesis.

<table>
<thead>
<tr>
<th>STORM DATE</th>
<th>A</th>
<th>S</th>
<th>( \tau_p ) (hrs.)</th>
<th>( \tau_s ) (hrs.)</th>
<th>( \nu ) (hrs.L.T./hrs.U.T.)</th>
<th>( \psi ) (deg.)</th>
<th>( \varphi_0 ) (deg.)</th>
</tr>
</thead>
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<tr>
<td>JULY 15, 1959</td>
<td>100</td>
<td>300</td>
<td>6</td>
<td>20</td>
<td>0.3</td>
<td>180</td>
<td>1630</td>
</tr>
<tr>
<td>APRIL 30, 1960</td>
<td>180</td>
<td>240</td>
<td>3</td>
<td>27</td>
<td>0.5</td>
<td>180</td>
<td>2000</td>
</tr>
<tr>
<td>JAN. 21, 1957</td>
<td>100</td>
<td>260</td>
<td>4</td>
<td>24</td>
<td>0.3</td>
<td>100</td>
<td>1900</td>
</tr>
<tr>
<td>SEPT. 29, 1957</td>
<td>150</td>
<td>200</td>
<td>4</td>
<td>24</td>
<td>0.4</td>
<td>180</td>
<td>1930</td>
</tr>
<tr>
<td>FEB. 11, 1958</td>
<td>200</td>
<td>460</td>
<td>6</td>
<td>15</td>
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<td>140</td>
<td>1830</td>
</tr>
<tr>
<td>JULY 27, 1961</td>
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<td>120</td>
<td>4</td>
<td>27</td>
<td>0.3</td>
<td>100</td>
<td>2100</td>
</tr>
<tr>
<td>OCT. 28, 1961</td>
<td>180</td>
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<td>24</td>
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<td>DEC. 3, 1958</td>
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<td>24</td>
<td>0.3</td>
<td>180</td>
<td>1900</td>
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<tr>
<td>NOV. 6, 1957</td>
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<td>0.6</td>
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<td>SEPT. 13, 1957</td>
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<td>2</td>
<td>15</td>
<td>0.6</td>
<td>180</td>
<td>2100</td>
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</table>
FIGURES 26-28: The storm of July 15, 1959, using three different zero definitions. The solid bars correspond to defining the July 5-quiet-day hourly averages as zero. The dashed bars correspond to defining the hourly averages of the preceding day as zero. For the x bars, the average over all hours for the ten quiet days in July is defined as zero.

FIGURE 29: The partial ring current function (ψ = 140°) that has been corrected for the diamagnetic effect of the ring current particles. The corrected function differs from the uncorrected function at local times opposite to ψo.