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THE AVERAGE ENERGY EXPENDED PER IONIZED ELECTRON-HOLE PAIR
IN SEMICONDUCTORS

by

FRANK EDWARD EMERY

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Thesis Director's signature:

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I. INTRODUCTION

A. Historical Notes

The discovery of the fundamental principles of many present day nuclear radiation detector methods date back to the beginning of the twentieth century. Becquerel discovered radioactivity by exposing a covered photographic plate with radiation from uranium salts in 1896. Within a few years Rutherford found that the arrival of an individual particle of radiation could be detected by observing scintillations as the particles impinged upon a fluorescent screen. Rutherford and Geiger used a detector consisting of a chamber filled with rarified gas. In the chamber a high electric field was applied to a pair of electrodes such that when a particle of radiation passed through the gas it would break down. These methods of radiation detection have been much improved and are in use today.

The direct detection of the ionization produced in solids by ionizing radiation is a comparatively new field. The first report of successful solid counters was by Van Heerden in 1946 (66). Several years of work on semiconductors and insulators followed with little success. These counters were conductivity ionization chambers, i. e., two electrodes applied to a block of material with an electric field applied to collect the ionized charge. To keep the dark current sufficiently small, insulators and high resistivity semiconductors were employed. The chief difficulty of these type of counters lay in effects of charge trapping or capture by empty electron levels within the forbidden gap. This permitted only very small detectors.
B. Semiconductor Diode Detectors

In 1950, McKay of Bell Telephone Laboratories successfully utilized a germanium p-n junction as a solid counter (41). In this type of detector the p-n junction is reverse biased to limit the current; the active region of the detector is the depletion zone formed about the junction only. Since McKay's detector was quite small also, not much came of his discovery at first. However during the early 1950's, a group at Purdue was engaged in learning how to make large area germanium photo cells under the guidance of Karl Lark-Horowitz. Using some of the Purdue techniques, Walters of Oak Ridge made several successful germanium barrier detectors starting in 1958. These detectors had to be cooled to very low temperatures (liquid nitrogen) to achieve adequately low dark currents. To overcome this difficulty, work was started on the use of silicon for a detector material. The availability of high grade, extremely pure germanium and silicon single crystal material from the burgeoning transistor industry was a prime impetus to these developments. By late 1960, enough work had been done on semiconductor diode type detectors to warrant two special conferences (16, 57). Since 1960, the major development which has occurred enables the manufacture of large semiconductor detectors. This development involves the application of the lithium ion drift technique of semiconductor compensation originated by Pell (47) to silicon and germanium detectors. This technique has enabled detector volumes of $8\text{cm}^3$ to be fabricated (31). The fabrication of large volume germanium detectors has essentially changed the nature of gamma-ray spectroscopy.

The basic understanding of the mechanisms present in semiconductor
detectors was fairly well outlined by McKay in 1951 (41). With the increasing interest in this work several review papers and two books have been written which more than adequately describe the basic operation theory of these devices (19, 21, 43, 64, 59, 15). Since these are available in the open literature in massive detail, little time will be wasted in a discussion of the working principles for the detectors in this thesis. In fact the amount of literature available on semiconductor detectors is overwhelming.
C. The Average Energy Expended per Ionized Electron-Hole Pair

Shockley introduced a phenomenological theory which had great success in predicting, $c$, the average energy per hole-electron pair created by the high energy charged particles (53). A table of experimental values for this quantity is found in Czulius et al. Two facts are striking concerning this table:

1) After the appearance of Shockley's theoretical value, the data converge to his value;

2) The data for electrons is consistently higher than the data for alphas,

Czaja has been successful in extending Shockley's theory for various materials (14) besides silicon.

In the past year, new data have come from a group working with E. Gatti which show that $c$ for electrons is different from $c$ for alphas (12). A temperature dependence was noted, also. Various experimenters have noted a temperature dependence of the pulse height obtained from semiconductor detectors as the temperature is lowered into the cryogenic region. Prior to the appearance of these data, this author had tried extending Shockley's empirical model to predict the temperature dependence of the detector response. As it began to appear worthwhile to investigate this phenomena more fully; experiments concerning detector behavior at temperatures from $20^\circ$ K to room temperature were performed. This thesis is the result of these experiments. The primary results of the thesis are data about $c$ as a function of temperature for silicon and germanium using natural alpha and electron sources.
D. Chapter Description

First the models for $\mathcal{C}$ will be presented, followed by the methods used for detector manufacture. A thorough discussion of the experimental procedure follows. After this, chapters concerning the data results and conclusions are presented. A few derivations which were mainly gleaned from the literature are presented in the appendices.
II. FACTORS DETERMINING THE AVERAGE ENERGY PER PAIR, $\varepsilon$

A. Heuristic Discussion

We discuss below factors which will affect $\varepsilon$ from the viewpoint of the processes within the counter itself. Let us examine heuristically what might occur when a high energy charged particle enters a counter as shown in figure 1.

The ionization of hole-electron pairs by the incoming particle itself will occur along the path of the particle through the solid. These primary ionizations may involve electrons deep within the atoms (a low energy shell) as well as the valence band electrons. The process would undoubtedly be complicated as some very high energy electrons and vacant states would be created. By a combination of various processes (photon and phonon radiation and absorption, secondary impact ionization and Auger recombination) we might suspect that these very energetic particles will descend to the valence and conduction bands with still relatively high energy with respect to the band edges. During this complicated process we suspect that secondary pairs are generated by the primary pairs. Solving this problem analytically would be difficult and so far as is known has not been attempted. However, we intuitively feel that successive ionization processes will occur until the remaining carriers have such small kinetic energies that they cannot ionize new pairs. The existence of a threshold level for pair ionization is well established in the literature (42). It appears that the number of pairs generated is not dependent on the type of the entering particle.(16).
Figure 1.

Schematic diagram of a charged particle entering a detector, necessary bias arrangement, and signal output.
It seems that very energetic electrons (or holes) would create new pairs rather than lose their energy by other processes. Photon radiation may be ruled out as a lossy process, since such a photon would be quickly absorbed with a new indistinguishable pair taking the place of the old. Thus we consider phonon collisions as the main process competing with impact ionization to slow down the energetic electrons and holes. This process has been a subject of investigations for many years. A fairly simple model which explains several phenomena has been developed by Shockley (53) and has led to new understanding of the phenomena. A brief summary of Shockley's model as it pertains to charged particle detectors follows.
B. The Four-parameter Model of Shockley

The model which Shockley proposed is admittedly a very crude and simple one. The main justification for it is primarily empirical; he had been able to use it to predict six experimental relationships in four independent experiments. It is highly probable that the various parameters do not have the simple significance assigned to them, but are instead averages of a variety of processes.

It is well established that carriers capable of secondary ionizations will be more likely to be scattered by phonons of the optical branch rather than those of the acoustic branch. The most likely phonons are those of the highest frequency. This is the so-called "Raman vibration" where the two atoms of the unit cell are vibrating out of phase with one another. The energy $\hbar \omega$ associated with this vibration is called the Raman energy, $E_R$. Generally, the entire optical branch is fairly flat and thus small error is made by assuming that the energy of the phonons is constant at $E_R$. All the phonon collisions are assumed to be lossy since at normal temperatures the optical modes are not excited.

The other three parameters of the model are: $E_i$, the threshold energy measured from the band edge above which a carrier may produce a new hole-electron pair; $L_R$, the mean-free-path between scattering by Raman modes; and $r = L_i / L_R$ where $L_i$ is the mean-free-path between ionizations for a carrier with energies greater than $E_i$. Clearly, an electron with energy greater than $E_i$ will generate $r$ phonons for each pair producing ionization on the average.

Shockley applies his model to the problem of determining the
average energy per hole-electron pair produced by high energy charged particles (16). He accounts for the incident energy as follows. Each pair-producing ionization consumes an energy \( E_i \) because of the ionization and \( rE_R \) is consumed in phonon collisions. After the produced particles no longer ionize, they will retain a final energy less than \( E_i \) from the band edge denoted \( E_f \). A particle with this final energy \( E_f < E_i \) will be incapable of ionizing new pairs and thus will lose its excess kinetic energy as heat. Since the hole and electron are treated equally the final energy for the pair will be \( 2E_f \). From the above it is apparent that

\[
E_f = E_i + 2E_f + rE_R. \tag{e-1}
\]

Assuming the carriers are equally likely to lie anywhere in the Brillouin zone and spherical \( E(K) \) surfaces, Shockley estimates

\[
E_f = \frac{3}{5} E_i. \tag{e-2}
\]

A better calculation of \( E_f \) would involve a more detailed picture of the ionization process in order to predict the sharing of the kinetic energy by the particles after an ionization occurs.

It seems reasonable to assume \( E_i \) is approximately the energy of the forbidden gap, \( E_g \). The energy \( E_R \) may be independently determined by neutron diffraction experiments and is given by Shockley for silicon and germanium. The constant \( r \) may be evaluated from photo conductivity experiments or from data concerning avalanche multiplication in diodes. Table I gives a summary of Shockley's results.
<table>
<thead>
<tr>
<th>Mat.</th>
<th>$E_{\text{gap}}$(ev)</th>
<th>$E_R$(ev)</th>
<th>$r$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>1.10</td>
<td>0.063</td>
<td>18</td>
<td>3.5ev</td>
</tr>
<tr>
<td>Ge</td>
<td>0.66</td>
<td>0.037</td>
<td>54</td>
<td>3.5ev</td>
</tr>
</tbody>
</table>
C. An Extension of Shockley's Model

About the same time as Shockley's paper appeared, Baldinger and Czaja determined new values for $\varepsilon$ for silicon and germanium ($3,2$) empirically indicating

$$\frac{\varepsilon_{\text{Si}}}{\varepsilon_{\text{Ge}}} = 1.5 \approx \frac{E_{\text{gap Si}}}{E_{\text{gap Ge}}} \quad (e-3)$$

Thus, they postulated:

$$\varepsilon = \text{const. } E_{\text{gap}} \quad (e-4)$$

Later Czaja (14) extended Shockley's theory for group IV semiconductors as follows. He assumes that $rE_R = r^' k^{\Theta_D}$. Where $\Theta_D$ is the Debye temperature. He then justifies a relation, determined empirically, that

$$\Theta_D = x + y E_{\text{gap}} \quad (e-5)$$

for $\alpha$-tin, silicon, germanium and diamond. From this Czaja then postulates

$$\varepsilon = C_1 + C_2 E_{\text{gap}} \quad (e-6)$$

a relation similar to the average energy per pair for gases with $E_{\text{gap}}$ replacing the ionization energy $I$.

While observing the logic of Baldinger et al one is struck that for germanium they used the data of Walters (67) et al to determine $\varepsilon$ for germanium while measuring $\varepsilon$ for silicon themselves. They carefully state that the temperature is $300^\circ K$ for the silicon measurement. Walters carefully states that his germanium data is at liquid nitrogen temperature. Since the original Shockley model is based on an interaction between the impact ionization process and optical phonon creation, one might suspect some temperature dependence. A great deal of temperature dependence could invalidate some of Czaja's conclusions.
D. Temperature Dependence Model

Let us consider Shockley's model further with regard for temperature dependence. First of all the gap energy \( E_{\text{gap}} \) varies slightly with temperature.

silicon

\[
E_{\text{gap}} = 1.21 - 3.6 \times 10^{-4} T \quad T \leq 700^\circ\text{K} \quad (e-7a)
\]

Germanium

\[
E_{\text{gap}} = 0.785 - 3.5 \times 10^{-4} T \quad T \leq 500^\circ\text{K} \quad (e-7b)
\]

The above equations are given by Valdes (65), while Smith (55) presents graphical data indicating a parabolic shape as the gap energy increases less at low temperatures than the linearized formulas show.

Since \( E_R \) is a function mainly of the lattice type and spacing, one does not expect that it should be a function of temperature. We may well expect the competition process between phonon and ionization collisions to be temperature sensitive.

In Shockley's paper of 1951 (52), concerning hot electrons and the electron mobility in germanium, the interactions between electrons and both acoustical and optical mode phonons are considered. The essential nature of the calculation has been summarized in appendix A. The interesting result which Shockley derived is that electron-phonon transition probability is independent of temperature, to the first approximation, for optical mode phonons. Thus mean free time between optical mode collisions is independent of temperature.

The Auger recombination-impact ionization process in semiconductors has been analyzed by Beattie and Landsford (6). A summary of their analysis is also included in appendix A. In this case the transition probability is temperature dependent and leads to the following temperature
dependent expression for the mean free time.

\[ \tau_i \propto E_g^{3/2}(T) \exp\left( \frac{1 + 2\mu}{1 + \mu} \frac{E_g(T)}{kT} \right) \]  \hspace{1cm} (e-8)

where \( \mu = \frac{m_e^*}{m_h^*} \), the ratio of the effective masses of the electron and the hole.

We assume that the hot carrier with energy \( E \) has a velocity given by the classical approximation

\[ v = \left( \frac{2|E - E_{edge}|}{m^*} \right)^{1/2} \]  \hspace{1cm} (e-9)

and is the same for either process. Therefore we may take

\[ r = \frac{\tau_i}{L_i} = \frac{v \tau_i}{L_i} = \frac{\tau_i}{\tau_R} \]  \hspace{1cm} (e-10)

Using the models of Shockley and Beattie and Landsford we find that the parameter \( r \) in the \( \mathcal{E} \) model has a temperature dependence.

\[ r \propto E_g^{3/2}(T) \exp\left( \frac{1 + 2\mu}{1 + \mu} \frac{E_g(T)}{kT} \right) \]  \hspace{1cm} (e-11)

Combining equations 1, 2, and 11 we see that the temperature dependence of \( \mathcal{E} \) may be predicted by

\[ \mathcal{E} = E_g(T) \left( 2.2 + C_1 E_g^{3/2}(T) E_R \exp\left( \frac{1 + 2\mu}{1 + \mu} \frac{E_g(T)}{kT} \right) \right) \]  \hspace{1cm} (e-12)

The constant \( C_1 \) is the constant of proportionality for \( r \) and is difficult, if not impossible, to calculate.

Data generally available in the literature is of the form shown in figure 2 for low temperatures \((22, 23, 27)\). At temperatures not too far from room temperature Fabri et al \((24, 3)\) have measured \( \mathcal{E} \) as a function of temperature with electrons and alphas for silicon; their data is presented in figure 3. We may write equation 12

\[ \mathcal{E} = E_g(T) \left( 2.2 + C \frac{E_g(T)}{\tau} B \frac{E_g(T)}{T} \right) \]  \hspace{1cm} (e-13)
Figure 2.

Pulse amplitude versus temperature for a lithium-drift silicon detector using $^{207}$Bi 1 Mev conversion electrons.

Detector bias 100 volts. (Figure 6 of Gregoire et al (27)).
Figure 3.

Literature data curves showing $\varepsilon_\infty$ as a function of the gap energy, and thus of temperature. (Bussolati et al (12)).

(Energy gap versus temperature from Smith (55)).
to a first approximation. Using the high temperature data we may evaluate \( A \) and \( B \). The accuracy of the evaluation is limited because of the small range of the data on the logarithmic scale. Figure 4 shows hypothetical relative pulse height curves (RPH \( \propto 1/C \)) for different values of \( B \) and \( C \). The constant \( B \) may not be determined from cyclotron resonance data since the effective masses obtained from these experiments are band edge effective masses. The effective mass to be used in the \( \epsilon \) case should be an average effective mass considering the entire energy band. Notice that in figure 4 the relative pulse height stays almost constant for all easily obtainable temperature levels, dropping to zero rapidly as the temperature decreases below the liquid nitrogen level. This implies the assumptions made by Czaja in his energy gap dependence theory for various materials are correct.

Figure 5 shows the agreement of equation 13 with the data of Fabri et al for \( \alpha \) particles, with \( C = 0.897 \) and \( B = 19.5 \). The remarkable thing here is the flatness of the curve in the temperature region of their data. But where they predict a straight line variation as \( T \) decreases, equation 13 shows a marked increase in \( \epsilon \). It is clear that further experimentation is necessary to determine the correctness of equation 13. It seems remarkably successful on first glance.

An objection to the limiting behavior of equation 12 for \( \epsilon \) may be voiced. It is clear that it predicts \( \epsilon \) grows infinite as \( T \) approaches zero. Since the basic process of carrier ionization as outlined by Bohr and Bethe does not have any temperature dependence, some number of ion pairs must be produced by the impinging energetic particle even at the lowest temperatures. It is also likely that
Figure 4.

Relative pulse height (v 1/E) versus temperature from equation e-13. Curve a for B = 53, C = 0.794; curve b for B = 19.5, C = 0.897.
Figure 5.

Data of Fabri, Gatti and Svelto compared with equation e-13. $B = 19.5, C = 0.9$. 
\[ \epsilon = E_g \left( 2.12 + 0.9 \frac{E_g(T)}{T} \right) \]

\[ \epsilon = \frac{19.5 E_g(T)}{T} \]

DATA OF FABRI, GATTI, & SVELTO
these "primary pairs" will have very large energies and will produce secondary pairs in a manner analogous to incident energetic electrons. This leads us to split the total number of ion pairs as follows

\[ N_t = N_1 + N_2 \]  

(e-14)

Where \( N_t \) represents the total number of pairs created; \( N_1 \), the number of pairs created by "primary" processes; and \( N_2 \), the number of pairs produced by low energy Auger excitation between the conduction and valence bands. It seems intuitively clear from the preceding passages that \( N_2 \) must be proportional to \( N_1 \), i.e. the number of pairs created by the Auger process is proportional to the number of primary pairs which will eventually fall to the conduction band through the primary ionization processes. The simplest assumption we can make is that

\[ N_2 = XN_1 . \]  

(e-15)

Since Shockley's ideas concerning \( \xi \) are fairly successful we must conclude that \( X \gg 1 \) or that most of the pairs are created by Auger excitation between the conduction and valence band. Thus we may make the approximation

\[ \xi = \frac{E_g}{N_1(1 + X)} \approx \frac{qE}{XN_1} = \frac{2.2E_g + rE_R}{XN_1} \]  

(e-16)

Since no other criterion exists we will let

\[ \frac{qE}{N_1} = \mu \xi^2 \text{ a constant.} \]  

(e-17)

Thus

\[ X = \frac{\mu \xi^2}{2.2E_g + rE_R} \]  

(e-18)

and we may express \( \xi \) as follows
\[ \mathcal{E} = \frac{2.2E_g + rE_R}{1 + \frac{2.2E_g + rE_R}{\mathcal{Q}}} \]  

As we have introduced it, \( \mathcal{Q} \) is the "average energy per primary pair" with the term primary pair meaning pairs produced by processes analogous to the original pairs produced by the impinging particle. This author has seen no work which would enable this quantity to be deduced. It may be that \( \mathcal{Q} \) is sensitive to the ionization density created and thus would be different for different particle types. If such a conjecture is true it would explain the difference between \( \mathcal{E} \) for \( \alpha \) particles and \( \mathcal{E} \) for \( \beta \) particles. At any rate the definition of such a quantity will allow the temperature dependence expression to saturate at very low temperatures.
E. Particle Differences in $\mathcal{E}$

The general theory of the energy loss by charged particles in matter has been developed by N. Bohr (10) and Bethe (7). The theory is essentially the same for particles heavier than protons. The theory for electrons must account for the larger scattering angles that the incoming electron undergoes because of its smaller mass. The electron theory is quite similar to the heavier particle theory and may be considered approximately the same as far as energy losses are concerned. Range equations are a different matter.

According to Bohr, an energetic charged particle of energy $E$, mass $M_1$, and charge $Z_1q$ may lose its energy in matter in accordance with the following formula.

$$-\frac{dE}{dx} = \frac{4\pi N (Z_1^*)^2 q^4 L_e}{\mu v^2} + \frac{4\pi N Z_1^2 Z_2^2 q^4}{M_2 v^2} L_U$$  \hspace{1cm} (e-20)

Where

$N$ = density of atoms in the stopping material

$Z_1^*q$ = effective charge of the entering particle

$Z_1q$ = nucleous charge of the entering particle

$v$ = velocity of the entering particle

$Z_2q$ = nucleous charge of stopping material

$M_2$ = nucleous mass of the stopping material

$L_e = Z_2 \ln \frac{2 \mu v^2}{I}$ for non-relativistic particles

$L_U = \ln \frac{T_m}{T_{ia}}$

$\mu$ = reduced mass of an electron and $M_1$

$I$ = ionization energy of the stopping material

$T_m = \frac{4M_1M_2}{(M_1+M_2)^2} E$, the maximum energy transfer by classical collision
\[ T_a = T_m \sin^2 \theta_a, \text{ the minimum energy transfer under Rutherford scattering.} \]

The first term in the equation is due to electron excitation; the second, nuclear collisions. Bohr states that the two logarithmic terms are comparable, showing that as long as \( Z_1^* \) is about the same as \( Z_1 \) the first term will dominate and almost all of the energy lost will be transferred to excited electrons in the material.

The difference between \( Z_1^* \) and \( Z_1 \) is brought about by the slowing down of the incident energetic particle. As this particle slows down it attracts electrons effectively shielding its charge. An early estimate of \( Z_1^* \) is

\[ Z_1^* = Z_1^{1/3} \frac{v}{v_0} \quad (e-21) \]

where \( v_0 = q^2/\hbar \). This approximation for \( Z_1^* \) is fairly crude but experimental data for better estimates is difficult to get so that it has survived. At any rate it seems that eventually the particle will slow down to a point where the energy loss mechanism becomes dominated by nuclear collisions rather than electronic excitation.

Under these conditions it seems likely to postulate a critical energy which a particle must have before appreciable electronic excitation takes place. That is to say, the charge generated in the detector will be proportional to the difference between the original energy \( E \) and the critical energy \( E_c \).

\[ Q_{\text{gen}} = \frac{(E - E_c) q}{\xi} \quad (e-22) \]

An analysis of this problem by Czaia (8) and Flicker (25) postulate that the critical energy should be of the form

\[ E_c = \frac{M_1}{\xi} \xi \quad (e-23) \]
Another estimate of the critical energy may be made using equation 20, equating the two terms and substituting for \( Z_1^* \). We then approximate \( A = 2Z \) for the size particles of interest and can derive a critical energy which is of the form

\[
E_C \propto Z_1^{7/3}
\]

(H. W. Schmidt (50) of Oak Ridge has performed experiments with heavy ions of bromine and iodine incident upon surface barrier detectors. After corrections for window loss, his data is shown in figure 6. We suspect that the critical energy for \( \alpha \) particles is so small that it is undetectable so that if we compare the pulse heights for the heavy ions to that of the alpha particles, we obtain an expression as follows

\[
\frac{H_{hp}}{H_{\alpha}} = \gamma \left( 1 - \frac{E_C}{E} \right)
\]

where the \( \gamma \) represents a factor present because the charge collection efficiency may differ for the heavy particles compared with the \( \alpha \) particles. The range of the heavy particle in the detector is much shorter than that of an \( \alpha \) of the same energy. Thus the ion pair density is much higher in the case of heavy particles; this should increase the recombination of pairs prior to collection (11). Figure 7 shows Schmidt's data presented in this fashion. From these data we obtain \( E_C = 4.71 \text{ Mev} \) for bromine (\( Z = 35 \)) and \( E_C = 7.0 \text{ Mev} \) for iodine (\( Z = 53 \)). From the data of Bilger, Baldinger, and Czaja (8), we obtain \( E_C = 0.18 \text{ Mev} \) for silicon ions (\( Z = 14 \)). As can be seen in figure 8, only the crudest of fits to the data is obtained.

Lindhard has presented a theory for obtaining the average energy lost by energetic particles to nuclear collisions (38). From his
Figure 6.

Schmidt's data from spectra of energetic Br\textsuperscript{n+} and I\textsuperscript{n+} ions incident upon a surface barrier detector.
Figure 7.

Schmidt's data portrayed in the manner of equation e-25.

Curve 1: \[ \frac{H}{H_0} = 9.5 \left(1 - \frac{4.7}{E}\right) \]

Curve 2: \[ \frac{H}{H_\infty} = 9.4 \left(1 - \frac{7.0}{E}\right) \]
Figure 8.

Atomic number versus critical energy data, and two slopes for $E_c \propto Z$ and $E_c \propto Z^{7/3}$. (Presented on a plot of $Z$ versus $E_c$ because of the log-log scales available.)
theory he expects the critical energy of a 6 Mev \( \alpha \) particle to be 12 Kev (37). Attempts to extend the Lindhard theory to estimate the heavy particle critical energy have not proved worthwhile thus far.

One problem that is encountered with respect to semiconductor detectors concerns the so-called pulse-height defect in the response from heavy particles. Recently data has been presented to the effect that \( \varepsilon \) is not the same for electrons and \( \alpha \) particles (12). A short perusal of the literature will lead one to the conclusion that not enough is known concerning the interaction of charged particles and matter to resolve these questions at the present time. It can only be said that the pulse height defect comes in part from the nuclear collisions and in part from the increased ionization density in the case of the heavier particle. It is possible although not likely, the response from electrons and alphas can be traced to similar effects. The idea of a critical energy may be converted into a model concerning \( \varepsilon \) as follows.

The charge detected by the system is \( Q_c \), the charge collected by the detector "plates". This can be expressed in terms of the energy lost by the incoming particle in electronic collisions only.

\[
Q_c = \int \frac{(E - E_c) \, q}{\varepsilon_T} \quad (e-26)
\]

where \( \varepsilon_T \) is the true value for \( \varepsilon \).

Defining an apparent \( \varepsilon \) by the following

\[
Q_c = \frac{qE}{\varepsilon_A} \quad , \quad (e-27)
\]

and equating the two collected charges, we arrive at \( \varepsilon_A \) as a function of \( \varepsilon_T \) and \( E \).
\[ \varepsilon_A = \frac{\varepsilon_T}{(1 - \frac{E_C}{E})} \]  

(e-28)

Equation 28 may be used to partially explain the pulse height ratio curves of figure 7.
F. Some Concluding Remarks

It is evident from the above remarks that the theories concerning the nature of the average energy per pair in semiconducting materials are at best categorized phenomenological at the present time. This of course is the way the balance between theory and experiment happens. The rationale of the experimentalist is then to track down categorically as many facts as possible pointing to agreement or disagreement as he sees it. This hopefully leads eventually to better theories. The rationale for the engineering experimentalist is to catalog data as necessary so that useful design predictions or change may be made. The failure of semiconductor detectors in connection with possible bubble chamber use has been given. Data and possible reasons for behavior in extreme environmental areas are clues to design criteria and better products even for ordinary environmental use. Thus a study of the temperature dependence of ε seems warranted for the following reasons.

1) To determine if the poor response exhibited by semiconductor detectors lies in lack of generation (ε temperature dependence) or in charge collection (trapping of the generated charge).

2) To test the extension of Shockley's simple model and hence Shockley's simple model itself.
III. DETECTOR DESIGN AND FABRICATION

A. Introduction

Preliminary expectations for this project indicated need for detecting both $\alpha$ and $\beta$ particles available from the natural sources which were on hand. These sources included the ThC' $\alpha$'s and Bi$^{207}$ conversion electrons; also considered were Am$^{241}$ $\alpha$'s, Cs$^{137}$ conversion electrons. These requirements indicate a thin window (hopefully negligible) for $\alpha$ particles coupled with the deep depletion layer requirement for the 1.06 Mev electrons available from the Bi$^{207}$ source. Temperature cycling seemed to rule out any type of detector with epoxy casting; therefore a pressure contact was required. In view of the poor success which has been exhibited in this laboratory with non-epoxied surface barrier detectors, the lithium-drift-surface-barrier type were chosen as suitable for the $\xi$ experiments. The p-i-n detectors of this type have wide depletion regions at low voltages, compared with voltages necessary for a junction or surface barrier type with the equivalent depletion depth. Hopefully this helps the leakage problem although most likely accentuates the charge collection problem.

The source and the detector may be placed in very close proximity, allowing a small area detector to be used. However, a detector which will not allow the escape of any electrons from the sides is also desired. Fairly easily handling characteristics, number of possible detectors from available material are also to be considered in the size decision. Weighing all of these factors led to the decision of a detector approximately 9 mm by 9 mm by the necessary 2 1/2 mm depletion depth. This size was to be the same for both silicon and germanium detectors.
B. Lithium Drifting

The characteristics of lithium in silicon and germanium have been extensively studied. Lithium dissolves interstitially in both germanium and silicon and is ionized as a donor at room temperature. The ionization energy of lithium is on the same order as the commonly used donor "dopants" in both silicon and germanium. Hence it may be used to fabricate the n region of a p-n junction. Lithium diffuses quite rapidly in both elements, however, making shallow junctions difficult to achieve; the diffusion constants are given in Hannay (29) as

\[
25 \times 10^{-4} \quad e^{-11,800/RT} \quad \text{for germanium}
\]

\[
23 \times 10^{-4} \quad e^{-15,200/RT} \quad \text{for silicon.}
\]

These diffusion constants are $10^5$ times greater than those for the group III or V elements at a given temperature.

Coupled with the high diffusion constants lithium has a relatively low solubility in both silicon and germanium and may precipitate from solid solution causing loss of a device junction. This effect does not seem to occur in silicon at room temperature but has been noticed in germanium (18,30). Carter and Swalin show that the lithium precipitation may be slowed by pre-doping germanium with copper (13). It might be thought that copper doping would increase the trapping, but this may be avoided. Another difficulty with lithium as a dopant is the presence of oxygen in the crystal. Lithium will combine quite readily with the oxygen and will thus be out of the solution. It is imperative that the crystal oxygen content be kept as low as possible for good junctions, although statements have been made concerning an optimum oxygen content (63). Little or no quantitative data exists in the open literature concern-
ing many of these effects. Many recipes for Li-drift detectors allude to them qualitatively. As most makers of detectors buy their semiconductor crystals, a period of trial and error is rather expensive and could be solved by proper specifications being published. In the case of germanium, only zone leveled germanium from the Sylvania Electric Company has been found suitable for detectors (61). It was thought that Texas Instruments Lopex Silicon would prove suitable, but it has not.

Aside from these various drawbacks as a donor material, lithium possesses possible unique characteristics which allow large regions of closely compensated material to be fabricated in a particularly useful form for nuclear detectors. This makes possible the manufacture of large p-i-n junction diodes. As can be seen from the analysis summarized in appendix B, a p-i-n diode with a large i region can have a large depletion depth. The manufacturing process of compensation is termed ion-drift and has been adequately covered in the literature by several authors (39,36,47). The original work by Pell showed that a p-n junction with lithium as the donor could be used to create a large intrinsic region by drifting the Li⁺ ions in a large reverse bias electric field applied at slightly elevated temperature. Lehrer and Reiss made approximate calculations which predict a remarkable degree of compensation to be achieved by the drift process. Mayer's work is particularly applicable to the manufacture of nuclear particle detectors.

As has been previously mentioned, several recipes for the manufacture of Li-drift particle detectors are available in the literature (40,9,44,1,20,26,62,31). A wide diversity of methods is presented. In all cases some source of controlled heat is applied to a reverse-biased p-n junction formed by diffusing Li into a suitable p type material.
Generally the original material resistivity is about 100 ohm-cm in the case of silicon and 20 -- 40 ohm-cm for germanium. Lithium is diffused from an oil suspension or a film deposited in a vacuum evaporator. Assuming that one has ready access to the equipment necessary for basic semiconductor work, the only special equipment necessary for Li-drifting is the control apparatus for the drift. The necessity for some type of control is readily seen upon consideration of the temperature stability problem encountered in heated, reverse-biased junctions. The next section will discuss two basic types of drift control and some of the heating methods.
C. Drift Control

The diode under drift may do so at constant voltage and temperature or constant power with varying temperature. The constant voltage-temperature method implies a heat sink that is quite large compared to the diode so that the self heat is carried rapidly away from the diode. This allows some variation of the diode current without thermal run-away. Since a component of the diode leakage current is increasing with increasing intrinsic layer width (the recombination current), a thickness may be reached where run-away occurs. In practice this thickness may be made quite large so that several mm thick detectors may be produced. The main advantage to this system of drift is its simplicity.

The constant power, variable temperature method of drifting is inherently stable with respect to thermal run-away. The disadvantages of this system lie in equipment complexity and a lengthened drift time. The fact that the temperature decreases as the drift proceeds lowers the rate of drift. A variation of the constant power scheme is to drift at constant current with a constant diode voltage allowing the temperature to decrease as the drift progresses. Since the major component of the diode leakage is the diffusion current at the elevated temperatures encountered under drift, the temperature does not vary too much as the drift proceeds until near the end of the process. Drift times which are comparable for thin diodes may be achieved with this method coupled with the constant power advantages for thick diodes. A discussion involving time comparison of the two processes may be found in Stiff's thesis (58). Since the apparatus for drifting was also to be
used for large volume $^{29}$Si neutron detector manufacture, the constant power method was favored.

The drift controller developed by Goulding and Hansen (26) was taken as the model from which to start. The basic circuit was modified for available components and fewer power supplies and tried. The drifter hot plates were calculated to have a heat leak to a copper sink cooled by running tap water. The plates for silicon diodes were to have a maximum temperature of 175°C with about 50 watts of heat applied. The plates for germanium diodes were designed for a temperature of 75°C with 50 watts of applied power. The heat capacity of the plates was kept at a minimum to provide good control. Heat is furnished by dissipating power in a low cost, high power, silicon transistor. Although the design called for the heater transistor to burn out at the design temperature of the hot plates, the allowed temperature would be much lower giving a safety margin. Sketches of the preliminary control circuit and heater design are found in figures 9 and 10 respectively. The circuit works as follows.

The drift current, $I_D$, flows through resistor $R_1$ which may be adjusted for a given amount of drift current. When $I$ is less than the set amount of drift current the transistor base of $Q_1$ will be reverse-biased. The base of $Q_2$ keeps the emitters of $Q_1$ and $Q_2$ above a voltage level of -1 volt. As the drift current increases $Q_1$ turns on gradually causing $Q_2$ to turn off. When $Q_2$ is off so is $Q_3$ and therefore the heater transistor $Q_4$ also. As the diode under drift (DUD) cools, $Q_1$ turns off allowing the other transistors to turn on. The 2N334 transistors were chosen only because they were available as a gift from T. I. and were the only n-p-n transistor available in sufficient quantity. It
Figure 9.

*Initial circuit of drifter control.*
Figure 10.

Sketch of drifter plate assembly.

Legend:

1. Brass compression top contact
2. Teflon insulating post
3. Gold-plated brass drifter hot plate
4. Asbestos insulating plate
5. Buchner funnel, top cover
6. Lucite sealing ring
7. "O" ring
8. Diode under drift
9. Collector resistor
10. Heat leak support legs
11. Mica washer covered with Dow-Corning 340 compound
12. Heater transistor
13. Copper cold plate
14. Nylon screw
15. Copper tubing for refrigerant.
was found that the control power supplies need to be fairly well regulated to maintain a constant drift current according to the setting when several controls operate from the same supply.

The requirements of the 50 volt high current supply and the high voltage supplies were not critical. These supplies were designed and built from available "scrap" parts. No regulation is supplied and a capacitor input filter is used. It was found more convenient for each drifting diode to have its own high voltage supply. This latter is necessary for the germanium units because of the power level required.

Upon putting the design into service, several of the original design assumptions were found to be in error. These troubles were gradually corrected. The cold water was found to have a temperature of 85° F in midsummer, forcing the use of a small Freon refrigerator unit to cool the sink. This copper plate is now kept at 0° F. Difficulty in adequately coupling the diodes to the drifter plates caused the controls to stick in a half-on half-off position. To make the heat sink coupling less critical, the first stage of the control circuit was made into a Schmidt trigger for more positive on-off control. The hysteresis of the circuit was minimized to be less than 5% of the drift current setting.

The addition of the refrigerator forced the use of relatively airtight seals over the drifter plates to avoid frost build up when the heaters are off. This turned out to be a blessing in disguise when it was found that pure dry nitrogen was a necessary atmosphere for the germanium diodes. The large copper cold plate was encased in styrofoam and sealed from the atmosphere with caulking compound.
It was also found that not enough safety margin had been allowed for some heater transistors. It is felt that the junction-to-case thermal resistance of the transistor varies from unit to unit by a large amount. To improve the dissipation problem a series resistor was added in the collector of the heater transistor. By proper choice of resistor style, the heat from the collector resistor may also be fed to the hot plate allowing the temperature limits to be kept. The control circuit was also modified to prevent thermal runaway of the heater transistor by the addition of an 18 volt Zener diode. The modified control circuit is shown in figure 11.

Figure 12 is a photograph of the drifter plates, control, and power supplies as they appear in the laboratory. Switching arrangements allow the various drift voltages, currents, and temperatures to be read with a single set of meters. Small inexpensive voltmeters are used to indicate the power drawn by the heater transistors.
Figure 11.

Final version of drifter control circuit including 1000 volts, 100 ma schematic.
Figure 12.

Photograph of drifter plates and control rack.
D. Detector Recipes

While the process of detector manufacture is an organic one, a brief catalog of the present recipes in use at the time of writing may prove useful. Some amount of laboratory skill must be acquired in any case to provide adequate yield of detectors. Several of the process steps in the manufacture of nuclear detectors are still in the "black magic" stage and are not well understood. This statement may be applied in general to the entire semiconductor industry as this author sees it*. If one has faith that certain of his laboratory techniques are good, other steps in the process are altered to make the over-all process work. A perusal of the literature will quickly show the diversity of recipes available. Before a given process may be duplicated, it is likely that all of the existing initial conditions must be met, i.e. atmospheric contaminants of the laboratory, type of water and chemicals used, jigging and fixtures, phase of the moon, position of the planets, and other incantations. "Modern alchemy" is not far from a good description of the detector manufaturing processes.

1. Silicon Detectors*

T. I. Lopez Silicon, which is boron doped with resistivities of 100 -- 2000 ohm-cm, has been used. The lifetime of this material is quite long exceeding $10^{-3}$ sec for the higher resistivity material and $10^{-4}$ sec for the 100 ohm-cm material. The etch pit count was on the order of 500/cm$^2$. The material was purchased in 1/8 inch thick slices approximately one inch in diameter. Lopez Silicon is a T. I.

*P. J. Holmes provides some quantification of laboratory techniques but it is only a first step.
trade name for low oxygen content silicon; they claim that the oxygen content is below the detectable amount.

The slices are lapped on both sides with 600 mesh lapping powder and cleaned by ultrasonic agitation in a detergent solution to remove all the lapping particles. An ultrasonic rinse in high purity deionized water follows. Thirty to sixty second soakings in electronic grade propanol, trichloroethylene, propanol, and water complete the rinsing process. The slice is checked to see if it is hydrophilic, if not the cleaning process is repeated with longer solvent rinses. Acetone and methanol have been successfully used as the intermediate rinse. From the final water rinse the slice is transferred to a plastic beaker containing 125 ml of CP-4A (33) and etched for 3 minutes. The etch is quenched by alternately flooding and decanting with high resistivity deionized water.

The etch polished slice is then placed in a vacuum evaporator and a 2 μm thick coating of lithium evaporated upon one face. Stainless steel jiggling is used to facilitate cleaning of the masks by a weak nitric acid rinse. Lithium evaporation is done with a tantalum filament. Skill is required to prevent the charge from leaping from the boat before it wets the filament surface. The bell jar is lifted with dry nitrogen and the slice is quickly transferred to an adjacent argon diffusion furnace preheated to 500°C (58). With some practice this latter step may be performed as quickly as 10 sec.

Some experimentation yielded a 500°C diffusion for 150 sec to be an optimum choice. Surface concentration of $10^{18}$/cm² with a 300 μ to 500 μ junction depth are obtained. A $\frac{1}{2}$" graphite block serves as
a wafer holder providing a pad for the thermal shocks encountered in the diffusion process. The slice and block are cooled on a water cooled brass block in the room air. By providing a super abundant amount of lithium and having minimum exposure to air the oxygen problem is essentially swamped. The lithium-silicon interface has $10^5$ more lithium atoms than are necessary for the desired surface concentration.

After cooling the excess lithium is washed off in propanol and slowly added water for minimum surface pitting. The slice is then given a 10 sec rinse in CP-4A quenching the etch as before. At least a 24 hour storage in a dust tight container in the room air follows. After a day in the room air metallic film contacts are evaporated to the slice --- aluminum to the lithium side, gold to the other side. Care should be taken with the alignment of the masks and the lithium area. After contact evaporation the diode characteristics are checked. Dark leakage currents on the order of $1\mu\text{A/cm}^2$ are easily achievable at 200 volt reverse voltage. The diode may now be stored until a drifter is available.

The diodes are drifted until a capacitance check shows an intrinsic region greater than $2\frac{1}{2}$ mm (39). A few samples were stained using the copper staining technique to verify the accuracy of the capacitance measurements. Ten to fifteen mils were then removed from the back face by lapping with 600 mesh abrasive as before. The diode is then cleaned and etched as previously. After a day in a dry air atmosphere a thin gold film is applied to the face opposite the lithium rich side and the aluminum contact is renewed. This is supposed to form a surface barrier contact for the p region of the p-i-n diode. It has been found difficult to do this without encapsulating
the diode in epoxy. The few wafers which have been fair diodes have been noisy detectors. Surface problems leading to poor inversion layer formation seem to offer the most likely explanation. To qualitatively investigate this possibility, a few drifted wafers were potted in epoxy and surface barriers were made in the normal fashion. All of these potted wafers formed better diodes than the unpotted type. In one case where an unpotted semi-diode was potted, a 10 to 1 improvement in leakage was found.

The "low" leakage potted silicon detectors were also found to be poor detectors. Little or no peak was seen in the case of electron sources; $\alpha$ resolution was poor, also. It has been found that the spectra initially improve with increasing bias and then deteriorate as the diode leakage increases. This fact indicates that the detectors have poor charge collection efficiency. The fact that $\alpha$ particles may be observed correlates with the capacitance and stain measurements to show that the lithium has drifted through to the back. The initial lifetime of the material may of course be drastically altered by the heating steps in the process.

The leakage current points to a value of lifetime which would indicate good collection. A trapping phenomena caused by unwanted impurities or vacancies is a possible explanation. Figures 13 and 14 show data from a typical potted silicon detector.

After these experiments correspondence with Geoff Dearnaley revealed that others had had difficulty with T. I. Lopex Silicon and a decision was made to concentrate more fully on germanium detectors and purchase a silicon detector.
Figure 13.

Capacitance and leakage current data for detector LSi13C2 with an area of 0.75 cm².
Figure 14.

Am$^{241}$ spectrum of detector LSil3C2 at a bias of 250 volts.
2. Germanium Detectors

Originally it was hoped that germanium surface barrier detectors could be fabricated. All attempts to do this have been failures to date thus the process to be described is for lithium-drifted p-i-n diodes. Intrinsic region widths of 5½ mm have been achieved in this laboratory, using the similar processes Hansen (31) has achieved 11 mm thick intrinsic regions.

Gallium doped, zone-leveled germanium with resistivity around 40 ohm-cm is obtained from the Sylvania Electric Company. The ingots are sliced to form bread shaped pieces of the desired thickness, 4 inch in our case. Care must be used in cutting to avoid thermal shock. These bread loaf wafers are then diced into 6, 1 x 1 cm square pieces. After cleaning the germanium pieces are allowed to stand for 30 minutes in a 2% solution of cupric nitrate with two drops of HF added per liter of solution. The pieces in the solution are exposed to a strong incandescent lamp for 15 minutes on a side. This process forms a thin copper layer on the diode surface. The wafers are rinsed in deionized water, blown dry, and heated in the argon furnace for 5 minutes at 500°C. The wafers are then lapped all over with 600 mesh SiC powder until the copper is removed and all saw marks have disappeared. The graphite block is used to prevent thermal shock to the germanium. The wafers are then stored until a drifter is available.

Lithium is evaporated and diffused as was done with the silicon material, using a diffusion cycle of 5 minutes at 525°C. Surface resistivities of less than ½ ohm are desirable to get a good diode. Care must be taken not toOverheat the material as the eutectic point is
close to this temperature. If alloying occurs all of the lithium may be removed in the etch steps which follow.

After the lithium diffused slice has cooled so that the graphite pad may be handled, the excess lithium is washed off with a propanol and water solution. The sides of the slice are trimmed up to the lithium diffusion using a wire saw and 600 mesh abrasive slurry. The slice is cleaned as in the post lapping steps and then etched in White etch (33) for 3 minutes at room temperature. After rinsing in the usual manner, the surface resistivity is checked. If \( \rho \) is sufficiently low, the slice is ready to be mounted on the drifter plate; the value of surface resistivity will also locate the lithium side of the slice.

A eutectic alloy of Ga-In is used to provide electrical and thermal contacts to both sides of the germanium. The diode is placed lithium side up on the drifter plate and clean dry nitrogen is blown into the chamber at a rapid flow. Drift voltage is applied slowly; if the leakage is more than 2 ma/cm\(^2\) at 100 volts and room temperature the diode is removed from the drifter plate, its contacts removed, and re-etched. Usually a second etch is sufficient to make a good diode. With a good diode the heater power is turned to full and the drift voltage gradually brought to 500 volts with the necessary drift current to achieve a hot plate temperature about 50\(^\circ\) C. Nitrogen flow is reduced to 1 l/m for each drifter. Drift times have been correlated to Hansen's (31) with the use of the square root of the ratio of the drift voltage to 500 volts as an adjustment. It has been found necessary to be able to remove a diode before it drifts completely to the back side in order to get a good detector.
The test chamber which was designed for the temperature experiment was found to be good for testing the germanium diodes. A sketch of the chamber is shown in figure 15. Figure 16 shows a gamma ray spectrum. The photo peak resolution is largely limited by the electronic system noise.
Figure 15.

Sketch of germanium detector test chamber.
Figure 16.

$^{137}\gamma$ spectrum of germanium-lithium-drifted detector at liquid nitrogen temperature. Most of the 15 kev noise comes from the electronics.
IV. EXPERIMENTAL PROCEDURE

A. Summary of Apparatus

1. Electronic Apparatus

The electronic apparatus used in this experiment is similar to the usual electronic chain used in nuclear experimentation. The signal from the detector is fed to an integrating preamplifier located as close to the detector as possible to minimize the cable capacity; this maximizes the "integrator gain" thus improving the signal to noise ratio. The output stage of the preamplifier (a modified Tennelec 100B) is a White follower which enables several feet of cable to be driven to connect to the main amplifier. In this case the main amplifier is an A-61, an integral part of the RCL 11209 256 Channel Pulse Height Analyzer. The input time constant of the A-61 was modified to be $1.6 \mu s$ in accordance with the requirements of the Tennelec preamplifier.

Incorporated into the A to D converter of the 256 Channel Analyzer is a lower level control which allows a spectrum to be spread out over the 256 channels in the range of interest. This acts as an offset on the channel number versus energy curve. Use of the lower level control with the 0.1% channel width enables the experimenter to achieve high accuracy if calibration means are available. Calibration may be easily obtained by inserting pulses through a series capacitor to the input of the preamplifier. If two pulse heights are available, these may be used to straddle the spectrum channel thus correctly determining the channel number versus charge curve. In this way only the differential linearity of the A to D converter is important if the
calibration pulses are sufficiently close to the spectrum channel.

A block diagram of the apparatus is shown in figure 17. Radiation from a known source is incident on the detector. The ionized charge collected by the "plates" of the detector is then sent to the pulse height analyzer through the amplifier chain. Test pulses are fed through $C_T$ to the preamplifier input from a fast pulser and can be used for calibration, linearity checks and a determination of the system noise. A later section describes the calibration system in detail.

2. Temperature Apparatus

The detector and source are placed in an Air Products and Chemicals, Inc. "Cryotip." The detector mounted to the heat exchanger with an OFHC copper heat sink. This unit is capable of refrigeration to 150 K. It is a new miniature, two-fluid, open cycle Joule-Thompson refrigerator designed to provide a convenient and inexpensive source of cryogenic temperatures for various applications. The schematic for a typical Cryotip system is shown in figure 18.

The temperature of the copper block is measured by means of a silicon diode thermometer. The diode is operated under forward bias at a constant current. The forward voltage is measured and found to be a strong and reproducible function of temperature. A detailed discussion of this apparatus, its calibration and its accuracy is delegated to a later section.
Figure 17.

Block diagram of electronic apparatus used in the experiment.
Figure 18.

Schematic diagram of the "cryotip" and control panel.
B. **Calibration Procedure for Determining the Average Energy per Pair**

1. **Source Energies**

   Sources of radioactive energy included \(^{241}\text{Am}\) and \(^{223}\text{ThC'}\) for \(\alpha\)'s, \(^{137}\text{Cs}\) and \(^{207}\text{Bi}\) for conversion electron spectra, \(^{137}\text{Cs}\) and \(^{22}\text{Na}\) for \(\gamma\)-ray spectra. All sources were commercially obtained with the exception of the \(\text{ThC'}\) sources. Specially constructed source rods were made for collecting the \(\text{ThC'}\). The tips of the rods were demountable for use in the test apparatus. These tips only were exposed to the gas chamber, and they were conically shaped, mechanically and electrochemically polished bronze (56).

   The various energies for the various sources were obtained from the data of Strominger, Hollander and Seaborg (60). Since these energies were shown to four significant figures their accuracy was taken to be exact for the purpose of this experiment.

2. **Calibration Procedure for Determining the Collected Charge**

   a. **Test Pulser.** The requirements for the test pulser are that it be capable of delivering a fast rise time step function of variable, determinable amplitude. It would be convenient to have two determinable "steps" at different times so that time may be saved during a run. This feature was not too difficult to achieve accurately. The quality of the step function for long times need not be great since the pulses are differentiated several times in the preamp-amplifier chain. The important factor becomes the initial value of the voltage pulse fed to the test capacitor.

   Consider an ideal operational amplifier as shown in figure 19. Let the detector be represented by a current source in parallel with
Figure 18.

Ideal charge sensitive preamplifier input stage showing detector model and calibration model.

Figure 19.

Idealized detector current.
the total capacitance, $C'$. The calibration circuit is represented by a series capacitor $C_T$ and a step function input $V_u(t)$. It is tacitly understood that the two signals are not occurring simultaneously and thus may be considered as separate inputs and are not summed. If the operational amplifier is ideal the summing point voltage, $V_s$, may be taken as zero hence all the detector current must flow through the feedback capacitor $C_f$. Transferring to the Laplace Transform domain then we have, using the notation of figure 19

$$I_f(s) = -I_d(s) \quad \text{(P.B-1)}$$

substituting

$$sC_fV_o(s) = -\frac{I_d}{s}(1 - e^{-sT_c}) \quad \text{(P.B-1a)}$$

where $T_c$ is the collection time (length of current block). Therefore, transferring to the time domain:

$$v_o(t) = -\frac{I_d t u(t)}{C_f} + \frac{I_d (t - T_c) u(t - T_c)}{C_f} \quad \text{(P.B-2)}$$

If $v_o(t)$ is differentiated with a time constant much longer than $T_c$, the resulting pulse will clearly have a height proportional to $-I_d T_c/C_f$ or $-Q_c/C_f$. Where $Q_c$ is the collected charge. In practice a large resistor is placed in parallel with $C_f$; however if the resulting time constant $T_f$ is long compared with $T_c$, there is no essential difference in the pulse height output. The result is more general than the derivation, which is shown for convenience only.

Consider now the test input, with the capacitors initially uncharged.

$$I_f(s) = -I_T(s) \quad \text{(P.B-3)}$$

$$sC_fV_o(s) = -sC_T V_T(s) \quad \text{(P.B-3a)}$$

Therefore:

$$v_o(t) = \frac{-C_T V_T(t)}{C_f} = \frac{-Q_T}{C_f} \quad \text{(P.B-4)}$$
If \( V_T(t) = V_1 u(t) \) then \( Q_T = V_1 C_T \) and if \( V_1 \) and \( C_T \) can be determined quite accurately \( Q_c \) may be determined to the linearity and stability of the electronics involved.

Since the signals are differentiated later in the preamp-amplifier chain \( v_1(t) \) need not be a step function but may be an exponentially decaying one

\[
v_1(t) = V_1 e^{-t/T_p} u(t) \tag{P.B-5}
\]

if \( T_p \) is relatively long. As we shall see later a correction factor may need to be added to account for the interaction of the various RC networks. Otherwise a consistent error may occur which will lead to an incorrect determination of \( Q_c \).

A simple method of building a fast rise time exponentially decaying pulse is to charge a capacitor and then dump its charge through a resistor. Use of mercury wetted contact relays eliminate contact bounce and allow a repetition rate up to 100 cps. Such a pulser using a chopper relay (as shown in figure 21) is a common testing device for a nuclear electronics chain. The height of the voltage peak may be determined quite accurately if the transit time of the switch is very short compared with the dielectric relaxation time of the capacitor. For ease of calculation it is convenient if the charging cycle is something like 10 time constants.

A double pulse height pulser may be derived from the simple pulser by using two relays and a voltage divider on the discharge side. This scheme is advantageous in that the highly accurate potentiometer in the charge side is quite expensive (10 relays). It is disadvantageous in that the discharge time constant must be reduced
to the order of the smallest time constant in the preamp-amplifier chain. The schematic for the double pulse height device is shown in figure 22.

Not unimportant in the construction of such a pulser is the matching of the transmission line impedance with the source and terminating resistors. This bears on the discharge time constant and the ratio of the larger to smaller pulse. It was experimentally shown that reflections and bounces of the signal in the first 50 ns of the pulse were of little consequence; see figure 23. However, the reflections from the sending end of the receiving end mismatch must drive the small pulse higher and the large pulse lower. This was the cause of many hours of recalculation to find the 2\% error in large pulse to small pulse ratio which appeared because of these reflections. Excellent matching at the receiving end was obtained by using time domain reflectometry (46). The sending end must necessarily be mismatched for the double height pulser to work.

The cable between the pulser and the preamplifier is miniature 72 ohm cable. Although the length of this cable is short, cable losses were investigated as a mechanism which could lead to inaccurate results. This was tested by shorting the receiving end of the cable, applying a step function input of a known size and measuring the remainder of the step function after two electrical lengths of the line have passed. A type Z plug-in for a Tektronix 545 oscilloscope was used, allowing a comparison measurement of high accuracy. The loss increment was found to be 0.02351 ± 0.5\% for a single traverse of the line.

At the same time the assumption that the capacitor did not
Figure 22.

Circuit diagram of the double height pulser.
Figure 23.

Experiment to determine effects of termination reflections.

Both photos: top trace is preamplifier output at 0.1 volt/cm; bottom trace is $V(t)$ at 0.2 volt/cm; time scale is 50 ns/cm. Photo (b) has a 12 foot coaxial stub added in parallel with the test input. In both cases the analyzer put the pulse into channel 202.
discharge during the transit time of the relay swinger was checked. To avoid loading the capacitor with the oscilloscope, the signal was taken from the voltage source side of the relay and the capacitor terminal. In this manner the impedance of the scope should tend to hold the charge on the capacitor. Again the type Z plug-in was used for high accuracy and it was found that \( v(0) \) should be corrected by a factor, \( E_D \), of \( 0.97704 \pm 0.04\% \).

The cable has a total capacity measured at the receiving end of the line of 279 pf. Precise accuracy requirements demand an accounting of this. Therefore an analysis was made of \( V_T(s) \), the transform of the test voltage. We assume the operational amplifier is ideal to account for the impedance to the right of \( V_T \) node (figure 20). The resulting transform of \( v_T(t) \) is

\[
V_T(s) = \frac{v(0)}{R_1 C_W} \frac{1}{(s + 1/R_s C_p) (s + 1/R_H C_W)}
\]

(P.B-6)

where \( C_W \) is the sum of all the strays and the test capacitance; \( R_s = R_1 + R_2; R_H = R_1 R_2/(R_1 + R_2) \). \( v(0) \) is the capacitor voltage at the time of contact with the discharge circuit. It may be taken as the open circuit voltage of the potentiometer divider and adjusted for the loss described above with a later calculation.

b. Preamp-Amplifier Chain and Multi-Channel Analyzer. The most obvious sources of error in the electronics chain are non-linearity and instability. Repeated tests have found these instruments to be quite linear and stable over a time period of 10 minutes. Tests were made on the entire system. Linearity was found to be \( \pm \frac{1}{2} \) channel from channel 30 to channel 220. Stability was found to be up and down 2 channels at channel 128 over the period of an hour. Readings
were taken every 10 minutes; less than 1 channel drift was noted in that time.

A less obvious and multiplicative error in the test charge circuit is found to be the interaction of the differentiating circuits with the exponential decay of the test pulse. A reasonably complete analysis was made of the preamp-amplifier chain with the following assumptions. Except for the RC networks explicitly analyzed, the net contribution of the other RC networks is negligible. The phase shifts of various sub-amplifier stages is 0° or 180° so that the gains are \( A_i \) or \(-A_i\). The White followers used in the preamplifier have negligible output impedance. Reverse biased grids on triodes have infinite input impedance. A block diagram of the model analyzed is shown in figure 24.

If the amplifier gains are assumed unity for ease of calculation using the transform domain notation of figure, it is clear that

\[
\frac{V_A(s)}{V_T(s)} = \frac{\left( \frac{C_T}{C_f} \right) T_f T_0 T_1 s^3}{(T_f s + 1) (T_0 s + 1) (T_1 s + 1)}.
\]

(P.B-7)

Substituting equation P.B-6

\[
V_A(s) = \frac{R_2}{R_1 + R_2} v(0) \frac{C_T}{C_f} \frac{T_p T_f T_0 T_1 s^3}{(T_f s + 1)(T_0 s + 1)(T_1 s + 1)}
\]

(P.B-7a)

where \( C_f' \) is the modified feed back capacitance

\[
C_f' = \left( \frac{A + 1}{A} \right) C_f + \frac{C_T + C_{W2}}{A}
\]

and the \( T_i \) are the appropriate time constants.

We note that \( T_f > T_0 > T_p > T_1 > T_w \) and see that to first approximation we may consider \( T_f s \ll 1, T_0 s \ll 1 \). The resulting transform is

\[
V_A(s) = \frac{Q_T}{C_f} \frac{T_p T_1 s}{(T_f s + 1) (T_0 s + 1) (T_0 s + 1)}
\]

(P.B-7b)
Figure 24.

Block diagram of the model of electronic apparatus analyzed for effect of pulse decay on calibration.
\( T_f \approx 250 \, \mu s \)
\( T_0 \approx 50 \, \mu s \)
\( T_1 \approx 1.6 \, \mu s \)
\( R_0 C_0 = T_0 \)
\( R_1 C_1 = T_1 \)
with the resulting time function
\[ V_A(t) = \frac{Q_T'}{C_F} F(t) \]
\[ = \frac{Q_T'}{C_F} \left( 1 - \frac{T_W}{T_1} \right) e^{-\frac{t}{T_P}} - (1 - \frac{T_P}{T_1}) e^{-\frac{t}{T_W}} - \left( \frac{T_P}{T_1} - \frac{T_W}{T_1} \right) e^{-\frac{t}{T_1}} \frac{1}{(1 - \frac{T_W}{T_P})(1 - \frac{T_W}{T_1})(1 - \frac{T_P}{T_1})}. \] (P.B-8)

Since the maximum is of interest
\[ V_{A_{\text{max}}} = \frac{Q_T'}{C_F} F(t_m). \] (P.B-9)

If we assume \( t_m/T_P \) and \( t_m/T_1 \) are very small, then we determine
\[ e^{-\frac{t_m}{T_W}} = \frac{T_W}{T_P} \frac{T_1 + T_p}{T_P T_1}. \] (P.B-10)

Substituting the above for \( F(t_m) \) we obtain
\[ F(t_m) = \frac{1 - \frac{T_W}{T_1} \frac{T_P + T_1}{T_P T_1}}{1 - \frac{T_W}{T_1}}. \] (P.B-11)

Since \( T_W \) and \( T_P \) are slightly different for the two pulses, it is necessary to determine \( F(t_m) \) for each pulse. Then
\[ Q_T(LP) = \frac{R_2}{R_1 + R_2} v(0) C_T F(t_m;LP) F_L F_D \] (P.B-12)
\[ Q_T(SP) = \frac{R_2}{R_1 + R_2} v(0) C_T F(t_m;SP) F_L F_D \]

Table II lists the pertinent data. Note that in the present case a constant error of slightly greater than 0.1% would have been present if the correction factor \( F(t_m) \) had been ignored.
TABLE II.

<table>
<thead>
<tr>
<th></th>
<th>Small Pulse</th>
<th>Large Pulse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor Ratio</td>
<td>0.43776</td>
<td>0.50198</td>
</tr>
<tr>
<td>Test Capacitor</td>
<td>1.1372pf</td>
<td>1.1372pf</td>
</tr>
<tr>
<td>$F(t_m)$</td>
<td>0.99882</td>
<td>0.99867</td>
</tr>
<tr>
<td>$F_L$</td>
<td>0.97649</td>
<td>0.97649</td>
</tr>
<tr>
<td>$F_D$</td>
<td>0.97704</td>
<td>0.97704</td>
</tr>
</tbody>
</table>

Calculated Pulse Ratio = 1.1466

Average of 5 measurements of pulse ratio on the 256 = 1.147
R₂/Rₛ for the pulsers is measured by a potentiometric measurement with a 0.005% error. Fₜₘ is known to 0.01% even though the Tₜ are not so well known because the Tₜ₁/T₟ factor is so small. Fₕ is known to 0.01%, Fₜ to 0.04%. Cₜ was measured to 0.2% accuracy with a General Radio 1620-A capacitance bridge. Repeated measurements have been made on Cₜ with no discernible variation with temperature. V(0) is determined by measuring Vₑₐ with a L & N K-3 potentiometer and using the reading on a 0.05% linearity ESI Dek-a-Pot. Repeated checks on the ratio of the larger pulse to the smaller pulse have been made by using the analyzer. The average value of several such readings is 1.147 with a spread of ± 0.2%. This latter is less than the analyzer linearity. The calculated ratio of the two pulses is 1.1466. The agreement is well within the accuracy of the analyzer so that the pulse ratio is taken to be exact. This implies the chief source of pulser error lies in the determination of Cₜ.
3. Charge Collection Efficiency Effects

a. Bias Dependence of the Charge Collection Efficiency. The charge collected on the electrodes of a detector is related to the charge generated in the detector by the charge collection efficiency factor.

\[ Q_c = \eta Q_g \]  \hspace{1cm} (P.B-13)

Since \( Q_g \) is needed to calculate \( \epsilon \), a method of achieving \( \eta = 1 \) would be useful.

It is shown in appendix C that the charge collection efficiency is of the form

\[ \eta = \frac{\mu \epsilon E}{W} \left( 1 - e^{-\frac{W}{\mu \epsilon E}} \right) \]  \hspace{1cm} (P.B-14)

where \( \mu \) is the mobility of the carriers, \( \epsilon \) is the carrier lifetime, \( E \) is the electric field in the detector, \( W \) is the spacing of the parallel electrodes of the detector. The quantity \( \mu \epsilon E \) is known as the "Schubweg." If \( W/\mu \epsilon E \) is sufficiently small, the exponential term in P.B-14 may be expanded as a McClaurin series. The factor \( \mu \epsilon E/W \) may be multiplied through with the resulting series approximated as

\[ \eta = \frac{1}{1 + \frac{1}{2} \frac{L}{\mu \epsilon E}} \]  \hspace{1cm} (P.B-15)

For a given experimental configuration \( L/\mu \epsilon \) will be fixed and only \( E \) may be varied easily. In this event, we may say

\[ \eta = \frac{1}{1 + \frac{\text{const}}{E}} \]  \hspace{1cm} (P.B-16)

which is in agreement with the analysis of Baldinger et al. (4).

In the case of the p-i-n diode \( E \) is proportional to the applied bias voltage in the reverse direction for sufficiently high bias. This is the case of interest for lithium ion drift detectors. Thus we may take
\[ J = \frac{1}{1 + \frac{\text{const}}{V_D + V_0}} \]  

(P.B-17)

where \( V_0 \) represents contact potential of the diode, and \( V_D \) represents the voltage measured across the detector itself. \( V_0 \) may be measured easily from the forward bias characteristic of the particular diode by extrapolation of the straight portion to the zero current line (68).

b. **Determining \( \varepsilon_\infty \).** It is clear from the preceding development that if infinite bias could be obtained the collected charge would be the generated charge. In practice infinite bias is unrealizable and therefore data must be taken at sufficiently high bias to allow extrapolation to infinite bias. This is made clear by the following.

\[ \varepsilon_{AP} = \frac{qE}{Q_c} = \frac{qE}{\gamma Q_E} = \varepsilon_\infty \left( \frac{1 + \frac{\text{const}}{V_D + V_0}}{V_D + V_0} \right) \]  

(P.B-18)

If points are taken for several values of \( V_D \) and then plotted against \( \frac{1}{V_D + V_0} \) the resulting curve may be extrapolated to the intercept with the \( \varepsilon \) axis. This intercept will be \( \varepsilon_\infty \). \( V_D \) must be sufficiently high to allow the approximations outlined in the preceding section.

In practice if the bias level is high only three points need be taken and a computer may be used to determine the intercept with high accuracy. In fact the computer may be used without the straight line approximation if sufficiently high bias can not be obtained.

c. **Determining Detector Bias.** Since the detector operation necessitates a large bias resistor, provisions must be made to determine the detector bias. Measuring the detector bias is not difficult; however, it is important to be able to measure it accurately enough.
Such analysis is rather straightforward and only the results need be summarized. Referring to figure 17

\[ V_D = V_B - I_B (R_{L1} + R_{L2}) = V_B - I_B R_B \] (P.B-19)

for an acceptable effect on the determination of \( \mathcal{E}_{\infty} \); so that the overall errors due to the pulser, analyzer, and bias is less than 0.4\%, the slope of the \( \mathcal{E}_{\text{AP}} \) versus 1/V line must have less than 2\% error. This in turn implies that \( V_D \) be known to better than ½\%.

The laboratory equipment for determining \( V_D \) consists of a commercial detector bias supply. Provisions are made for determination of \( V_B \) within ½\% and \( I_B \) within 2\% if no extra calibration is made. Calibration of the \( I_B \) meter was made to bring it to within 0.1\% for detector currents less than 10 µA. \( R_B \) is a very large resistor and difficult to measure accurately. However, if \( I_B R_B \) are less than \( V_B / 10 \) the best they need be known is 3\%. If \( I_B R_B \) rise to \( V_B / 2 \) their product must be known to better than 1\%. 
C. Temperature Measurements

In the light of the wide range of temperatures envisioned for this experiment a simple means was sought to use a one device thermometer over the entire range. Since the true detector temperature could not adequately be measured (but only the temperature of the copper heat sink), extreme accuracy was not felt warranted for the thermometer. Several methods were considered: various resistance thermometers, thermocouple, and silicon diode thermometer. The silicon diode thermometer was chosen because it was inexpensive, convenient, and presented increasing signal level as the temperature decreased. The chief disadvantage is the necessity for calibration; this is true of almost every scheme considered.

An excellent article is available covering the theory of the diode thermometer (54). Thus only the major points in this design need to be covered here. An epoxy encapsulated silicon diode was chosen for its ruggedness. The leads of this diode are especially designed by the manufacturer to carry heat away from the diode; they are No. 14 silver wire formed like a common nail. The diode carries a forward bias current of approximately \( \frac{1}{2} \) ma. This current is sufficiently low to prevent local heating of the junction. The forward voltage of the diode is compared through a zero center 20 \( \mu \)a meter to a 25 turn precision potentiometer. If care is taken in setting the potentiometer in the same direction each time, less than \( \frac{1}{2} \) a small division reproducibility may be achieved. The thermometer circuit is shown in figure 25a.

The diode thermometer was calibrated against a copper resistance thermometer by inserting a calibrating assembly in various cold dewars.
The calibration assembly and calibration curves are shown in figures 25b and 26 respectively. Care was taken during the winding of the copper coil to prevent straining the copper wire. The copper resistance temperature scale of Dauphinee and Preston-Thomas (17) was adjusted at the helium point for the residual resistance of the copper wire. The accuracy of the calibration is within 1° K at the low temperatures.
Figure 25.

a) Circuit diagram of silicon diode thermometer.

b) Schematic drawing of calibration assembly.
(a)  

(b)  

#30 Teflon Insulated Wire for Leads

Thin Wall Stainless Tube

Epoxy

GE "Glyptol"

#40 Formvar Insulated Magnet Wire 100 Ohms @ Room Temp.

Pressure Sensitive Teflon Tape

Dummy Lead Wire

Copper Mandrel for Copper Coil Thermometer

Diode to be Calibrated
Figure 26.

Calibrated temperature curve of silicon diode thermometer. Calibrated for potentiometer dial setting of the particular circuit shown.
Silicon Diode Thermometer

Room Temperature

Ice Water

Dr. Ice & Propane

A Brief Article

A Brief Report

A Brief Note

A Brief Study

A Brief Review
Silicon Diode Thermometer

- Room Temperature
- Ice Water
- Dry Ice & Propanal
- Liquid Nitrogen
- Liquid Helium

Calibrated Against Liquid Nitrogen (Atmospheric Pressure)
0 Calibrated Against Copper Resistance Scale
V. EXPERIMENTAL RESULTS

A. Specific Procedure

Prior to the complete assembly of the cryostat the silicon detector was checked using a small room temperature vacuum chamber. After a few measurements on the calibration system the results for ∆ agreed quite well with the results published by Gatti's group. After this temperature runs were made down to 20° K in the following manner.

A source, usually Am$^{241}$ or Cs$^{137}$, was mounted inside one of the 4 ports which are on the cryostat dewar. Surrounding the detector and heat sink was a copper radiation shield. This shield has a small hole to allow the particles to impinge directly upon the detector. A high vacuum was necessary for the operation of the dewar, thus no energy loss should be suffered by the particles on their way to the detector. The radiation shield could also be used to "turn off" the source by rotation of the dewar. Figure 27 schematically shows the arrangement.

After a room temperature check was made to see if all the counting apparatus was operating correctly, the cooling process began. Spectrums were monitored qualitatively as the refrigerator cooled the detector. However, since the low temperatures were maintainable only with high pressure in the bottles, no data was taken until the detector reached the hydrogen temperature. In the case of the germanium detectors, the bias was not applied until the detector had cooled to less than 100° K.

At the extremely low temperatures, good temperature control
Figure 27.

Arrangement of source and detector in the cryostat.
was achieved by increasing the pressure in the hydrogen outlet line. For temperatures above 330 K, temperature is controlled by use of more or less pre-cooling of the hydrogen gas by the nitrogen circuit. It was found that after an hour or so at the low temperatures a single set of cylinders did not have enough pressure left to always establish good temperature control. However, the heating of the detector could always be considerably reduced. While the collection time for a given spectrum was small enough to be at a fairly constant temperature, the calibration and print out process took enough time to enable a sometimes considerable temperature change. This limited the number of different biases that could be used at a given temperature. If the temperature changed much over the time of spectrum collection, this fact was easily noticed on the display tube of the analyzer with the appearance of an unusually noisy spectrum. Generally this was judged unacceptable and cleared. Live times of 100 seconds were usually sufficient for enough counts. Spectrums taken at 100 volts detector bias with Cs\textsuperscript{137} source are shown in figures 35 through 40; the calibration peaks have been omitted.
B. \( \mathcal{E} \) Data Analysis

The print-out of the various spectra were then analyzed for the mean of the source peak channels and the channels corresponding calibration peaks. To sharpen the calibration peaks the detector was unplugged while the calibration was taking place. It was discovered that this step changed the gain of the system by 1%. These data were analyzed on the IBM 1620 along with the bias voltage and current. A \( V_o(T) \) function was empirically determined and placed in the computer program. In this manner the apparent \( \mathcal{E} \) at a particular temperature and electric field strength were determined.

The apparent \( \mathcal{E} \) data were plotted versus the inverse of the detector voltage and \( V_o \) sum. A sample of these plots is shown in figure 28. The straight lines were extended to the ordinate to determine the true value of \( \mathcal{E} \), or \( \mathcal{E}_o \). This technique, as discussed in the previous chapter, eliminated the trapping and recombination effects. Baldinger, Gutman, and Matile (5) also discuss this technique in connection with the varying series resistance of the detector and show that the same type of relationship exists in this case. If all the points for a given temperature lie on a line, the accuracy of \( \mathcal{E} \) is as good as that of the various apparent \( \mathcal{E} \), \( \pm 0.01 \) ev. These points are generally at the temperature extremes where good temperature control was available. Where the points did not lie on a line, an extreme variation of \( \pm 0.03 \) ev could be made by various line fits.

The deterioration of the pulse height for a given bias may be observed by taking the inverse of the apparent \( \mathcal{E} \) along a vertical line, as shown in figure 29. Thus it can easily be seen that the decrease
Figure 28.

Plot of apparent $\xi$ versus analogous inverse of electric field for various temperatures. Data from $\chi$ runs with silicon detector.
Figure 29

Relative pulse height as a function of temperature and detector bias for the silicon detector using the Am$^{241}$ source.
of pulse height of a detector is a combination of two effects; that due to the changes in $\varepsilon$ and that due to changes in the collection efficiency of the detector. The changes in the collection efficiency are likely the results of changes in the occupation of various trapping levels. However, good data on the mobility of the carriers as a function of temperature is necessary before even qualitative results could be made. Another necessary experiment would be to make a distinction (if possible) between the two types of carriers.

The $\varepsilon$ versus temperature data are presented in figures 30 through 32. The temperature error bars include the extremes for the various levels obtained and the thermometer uncertainty. This was not felt too critical in the middle range because of the slow variation of $\varepsilon$ with temperature. Also shown in figures 30 through 32 are the best fits of equation e-13 of chapter II to the data. It is readily seen that the curves fit well in a qualitative manner. It is also noted that the data agree quite well with that of Bussolati et al. (12) for the silicon detector in the overlapping temperature range. The germanium detector was not good for $\alpha$ particles and these data are omitted.

A disturbing factor is the size of the exponent B. According to equation e-12, B should be in the range of $1/k$, which is about $10^5$ $^0$K/ev, the units used. However we see that the values of B for the best fitting curves are on the order of 1 to 10. To see if any other temperature dependence existed, the data were analyzed by subtracting $2.2 \ E_g$, taking the natural logarithm of the remainder and the product of the various powers of T and $E_g$. The logarithmic
Figure 30.

\[ \xi \text{ versus temperature for } 5.477 \text{ MeV } \alpha \text{ particles} \]

incident upon silicon detector. Solid line equation given by

\[ \xi = 2.2 \ E_g(T) + 0.96 \ E_g^{3/2}(T) \ \exp \left( 0.75 \ E_g(T) / T \right). \]

\( E_g(T) \) data is from Smith.
Figure 31.

$\xi$ versus temperature for conversion electrons incident upon silicon detector. Solid line equation given by

$$\xi = 2.2 E_g(T) + 1.08 E_g^{3/2}(T) \exp (11.2 E_g(T)/T).$$

$E_g(T)$ data is from Smith.
SILICON DETECTOR
ELECTRON SOURCE

△ RUN 0319
○ RUN 0329
□ ITALIAN DATA


\[ \epsilon_{EV/PR} \]

T (DEG K)
Figure 32.

$\epsilon$ versus temperature for Cs$^{137}$ gamma rays incident upon germanium detectors. Solid line equation given by

$$\mathcal{G} = 2.2 \ E_g(T) + 1.99 \ E_g^{3/2}(T) \ \text{exp} \ (4.75 \ E_g(T)/T).$$

$E_g(T)$ data is from Smith.
GERMANIUM DETECTOR

○ RUN 0401
△ RUN 0406

$\varepsilon$ (EV/PR)

$T$ (DEG K)
result was then multiplied by $T$

$$\text{Term (x, y)} = T \ln \left( \frac{E - 2.2E_g}{T x E_g y} \right).$$  \hspace{1cm} (r-1)$$

A comparison of equations r-1 and e-13 show that the proper value of $x$ and $y$ will lead to an equation of the form

$$\text{Term} = k_1 T + k_2$$  \hspace{1cm} (r-2)$$

neglecting the small temperature dependence of $E_g$. The results of this program are shown in figure 33. Although the results of this analysis are not completely conclusive, they show the essential temperature dependence to be that which was expected.

A family of $\xi$ versus $T$ curves for various values of $\Omega$ are shown in figure 34 for the data obtained from the silicon detector. The exponent $B$ used for this family lay between the values determined--as is shown by the steeper climb of the $\beta$ curve and lesser climb for the $\alpha$ curve. The $\alpha$ data may be fit closely but the $\beta$ data may not. This may imply that the difference in the exponent $B$ may be a function of ionization density.
Figure 33.

Term (x, y)--equation r-l--versus temperature for various values of x and y. Primed lettered curves for germanium detectors; unprimed for silicon.

Legend:

a, a'  \( x = -1\frac{1}{2} \)  \( y = 0 \)
b, b'  \( x = -1 \)  \( y = \frac{1}{2} \)
c, c'  \( x = 0 \)  \( y = 0 \)
d, d'  \( x = 0 \)  \( y = 1\frac{1}{2} \)
Figure 34.

Silicon data presented with $\sigma(\mathcal{L}, T)$ results.
\[ \epsilon = \frac{\epsilon_s}{1 + \epsilon_s/\Omega} \]
C. Resolution and Temperature

The data shown in figures 35 to 40 are the spectra for the Cs\textsuperscript{137} source for various temperatures at 100 volts bias. Looking over these spectra qualitatively will show how the pulse height decreases with the temperature. All spectra are adjusted to the same channel number versus charge line. Another fact that stands out is that the resolution of the detectors is a fluctuating function with respect to temperature. Other authors have noted various temperatures where the resolution of the detector is best. The best spectra here is at 131° Kelvin where the FWHM for the $E_K$ peak decreases to 15 kev. This is likely to be mostly the noise in the electronic chain since the FWHM resolution of the gamma ray for the germanium detector is also 15 kev. The pulser usually has a 4 to 8 channel spread when the detector is attached to the preamplifier with the gain adjusted for maximum. This is the reason the detector was unplugged when the calibration pulses were applied to the system.
Figures 35-39.

Spectra from silicon detector at 100 volts bias and Cs\textsuperscript{137} conversion electrons incident for various temperatures.
Figure 40.

Spectra from germanium detector with 100 volts bias
and Cs$^{137}$ gamma source incident.
VI. CONCLUSIONS

It is evident from the experimental results shown in the previous section that there is a temperature dependence of $\varepsilon$ which is more than that for the energy gap. This fact may be seen more clearly on figures 41 and 42 which show $\varepsilon$ versus the width of the energy gap. It is also clear from figures 28 and 29 that only part of the decrease in pulse height with temperature is caused by changes in $\varepsilon$.

It can only be said that the extension presented in this thesis of the Shockley model is qualitatively successful. There are three possibilities for explaining this. The first is that the basic model is in error. Secondly, either or both of the lifetime calculations are in error. Thirdly, new processes enter the picture at the lower temperatures which are not present at higher temperatures. It seems highly likely that even a mixture of these three reasons is a good possibility. Certainly there is quite a bit of room for theoretical development in this area.

The fact that the silicon detector used in this experiment had spectrum peaks even at $20^\circ$ K while the detector used by Gregoire's group did not is probably a function of the method of manufacture. The author strongly suspects that the electrical contacts are a large part of the trouble. It is also suspected that the contacts are responsible for the erratic behavior of the detector resolution with temperature. If detector performance at these extremely low temperatures is necessary, a thorough development program on the detector contacts should be made. Mechanical strain relief may not be enough to guarantee consistent electrical behavior.
Figure 41.

6 data for silicon detector portrayed against gap energy. \( E_g(T) \) data is from Smith.
SILICON DETECTOR

\[ \epsilon \quad \text{(EV/Pr)} \]

GAP ENERGY (EV)

\[ \beta \]

LINE WITH SLOPE 2.2

\[ \alpha \]
Figure 42.

G data of germanium detectors portrayed against gap energy. $E_g(T)$ data is from Smith.
GERMANIUM
DETECTOR

$\epsilon \ (\text{eV/pr})$

$\epsilon$

3.3

3.0

2.5

0.73

0.74

0.75

GAP ENERGY \ (\text{eV})
The difference between $\epsilon$ for electrons and $\epsilon$ for alpha particles for silicon detectors is borne out by the agreement of this experimental data with that previously published. Since the detector used had a depletion depth of roughly twice the range of the electrons used, it seems unlikely that many electrons escaped the detector without losing all of their energy. It now becomes imperative for a germanium detector which will detect both $\alpha$'s and electrons to be constructed. Another experiment which might bear on this question is the response of the detectors to protons and other particles which are unavailable with the use of natural sources.

The apparatus used for achieving detector temperature variation is convenient for adaptation to scattering chamber experiments. Thus in conjunction with a machine such as the Van der Graeff accelerator many experiments could be performed to test the temperature and particle response of the detectors. Parenthetically it may be said that the cryostat could be used to achieve optimal resolution for a particular detector in a critical nuclear experiment.

It is clear that the response of semiconductor detectors is a discernible function of temperature. The mechanisms underlying the performance of these detectors are understood only to the extent of qualitative prediction of the temperature response. If more detailed information is required, a more thorough understanding of the interaction of radiation and matter will have to occur.
VII. APPENDICES

A. Calculation of Mean-Free-Times

1. Equivalence of Classical Statistics and Statistics from Quantum Mechanical Models

The energy bands of the semiconductor may be approximated by spherical constant energy surfaces; with the $E$ versus $K$ equations taken as

$$ E = \frac{\hbar^2 K^2}{2m_e} \quad (a-1) $$

in the conduction band, and

$$ E = \Delta E - \frac{\hbar^2 K^2}{2m_h} \quad (a-2) $$

in the valence band. Consider a thin shell in $\bar{K}$ space bounded by two closely neighboring surfaces of constant energy. If the energy difference between the shells is small, then we may treat the shell as a surface. We then may talk about the number of states per unit area of this energy surface.

Let us take an ensemble of $N$ systems which are in state $\bar{K}_o$ at $t = 0$ and consider that at a later time $t$ that we have the following arrangement of the systems. The systems have been acted upon by a disturbing situation in the interval $t$.

\( \mathcal{N} = \) number of systems still in state $\bar{K}_o$

\( \mathcal{P}_j = \) number of systems per unit area around state $\bar{K}_j$

which have been scattered once

\( \mathcal{Y}_i = \) number of systems per unit area around state $\bar{K}_i$

which have been scattered twice

and so forth.
Then
\[ \beta_j \frac{d\alpha_j}{\beta_j} = \text{number of systems scattered once in the range } \alpha_j, \]
and \[ \beta_j \frac{d\alpha_j}{\beta_j} = \text{the number of systems for which scattering occurs once.} \]
Let \( W_0 \) be the probability per unit time that a system leaves state \( K_a \) under the influence of a scattering event.

Then
\[ \frac{d\alpha_j}{dt} = -W_0 \alpha_j \]  \hspace{1cm} (a-3)

Thus
\[ \alpha_j \sim e^{-W_0 t} = e^{-t/\gamma_0} \] \hspace{1cm} (a-3a)

where
\[ \gamma_0 = \frac{1}{W_0} \] \hspace{1cm} (a-4)

is called the mean free time. Rates of changes for more than single events may be similarly calculated but are of no interest to us here.

We concern ourselves now with the calculation of \( W_0 \), the probability that the system leaves state \( K_a \). This quantity may be calculated from a knowledge of the wave-functions which describe the unscattered events and the interaction Hamiltonian which describes the scattering event. Time-dependent-perturbation theory is used. Following the notation of Schiff (49), we see that the transition probability per unit time from a state \( m \) to a group of states around \( K \) with density \( \rho(K) \) is
\[ W_{mk} = \frac{2 \pi \hbar}{\hbar} | \rho(K) | \text{ } H_{km} |^2. \] \hspace{1cm} (a-5)

Where \( \rho(K) \) represents the density of possible end states \( K \) and \( H_{km} \) represents the matrix element.
\[ H_{km} = \int u_k^* \mathcal{H}' u_n \, \mathrm{d}t \]  
(a-6)

with the \( u \)'s representing the unperturbed wave functions and \( \mathcal{H}' \) the interaction Hamiltonian. We identify \( W_{km} \) with \( W_0 \) after Shockley (51). Thus:

\[ \frac{1}{\tau_w} = W_0 \rightarrow W_m = \int W_{mk} \, \mathrm{d}t \langle 2(k) \rangle \]  
(a-7)

2. Electron Scattering by Optical Phonons

In his work, *Electrons and Holes in Semiconductors*, Shockley gives a thorough treatment of the interactions of electrons or holes with the accoustical branch phonons. From this we abstract the following results. The electron-phonon interaction must conserve both energy and wave vectors. This last requirement is tantamount to conservation of momentum, leading to the fact that the number of phonons, \( n \), with wave vector \( \mathbf{k} \) increases or decreases by one

\[ k_{\alpha} = k_2 - k_1 \]

\[ n \rightarrow n \pm 1 \]

depending on emission or absorption of the phonon. The matrix element for the electron-phonon interaction in the case of accoustical phonons is given by

\[ \left| H_{km} \right|^2 = \frac{E_1^2 \kappa T}{V_c c_{ll}} \]  
(a-8)

Where \( E_1 \) is the slope of the band edge with respect to change of the lattice constant; \( V \) is the volume of the crystal; \( c_{ll} \) is the elastic constant of longitudinal elastic waves; \( k \) is Boltzmann's constant; and \( T \) the absolute temperature (51).

The optical phonons in silicon and germanium will have a frequency
on the order of $10^{13}$ cps, or an energy corresponding to $520^\circ$ K. Thus we can see that the collisions with optical phonons will involve loss for the electron energy. Shockley has also considered this case and his arguments are somewhat approximate. The chief reasons for the approximate methods lie in the fact that the matrix elements for the optical phonon collisions vanish for low wave number phonons using the spherical energy band approximation. However, it is known that the actual bands are not spherical. Shockley (52) feels that a correct analytic treatment is unobtainable because of great complexity and so he estimates the optical scattering matrix element as

$$\langle H_{\text{opt}} \rangle^2 = \frac{h\nu}{kT} \langle H_{\text{opt}}^2 \rangle^2 .$$

(a-8)

If we take the energy $h\nu$ of the optical phonon branch as a relative constant we may express

$$H_{\text{opt}}^2 = \frac{E_{ER}^2}{V_c}$$

(a-9)

To perform the integration we need to express the density of states $\tilde{\rho}(k)$, and the elemental area of the constant energy surface $d\Omega k = k^2_0 \sin \Theta d\Theta d\Phi$. To obtain the equation for $W_{mk}$, an integration with respect to energy was performed. Thus what is normally thought of as the density of states in $k$ space must be modified to account for this. Thus

$$\tilde{\rho}(k) = \frac{V_{m^*}}{8 \pi^3 \hbar k}$$

(a-10)

since the spin does not change.

For the optical transitions the constant energy surface is taken to be the final state energy surface and assumed spherical; thus
\[
\frac{1}{C_{op}} = \int \frac{2}{k_{2}(k)} \frac{V m^{*}}{8\pi^{3} h k_{2}} \frac{E_{F1}^{2}}{c} k_{2}^{2} \sin \theta d\theta d\phi
\]

\[
= \frac{m^{*} k_{2}}{\pi h^{2}} \frac{E_{F1}}{c} \mu
\]

\[
= \frac{E_{F1}}{h^{3} c} \frac{m^{*}}{\mu} \left( \frac{h k_{2}}{m^{*}} \right)^{2}
\]

\[
= \frac{E_{F1}^{2}}{h^{3} c} \frac{m^{*}}{\mu} v_{2}
\]

(a-11)

where \( v_{2} \) is the "velocity" of an electron in state \( k_{2} \).

A fairly obvious assumption which Shockley makes throughout the work is that all end states for a given transition are available and thus the density of states need not be modified by the statistics of end state occupation.

3. Impact Ionization in Semiconductors

We follow here an analysis of the Auger effect in semiconductors by Beattie and Landsford (6). The following assumptions are made. The electrons in the conduction band and holes in the valence band are assumed to be in internal equilibrium and the statistics of each may be represented by the use of quasi-Fermi levels. The electrons are described as in the previous discussion with Bloch wave functions. The energy bands are assumed to be spherical. Figure A1 shows a schematic representation of the process possibilities. The perturbing Hamiltonian is taken to be a screened Coulomb potential, \( \frac{q^{2}e^{-\lambda r}}{r} \) with \( 1/\lambda \) taken as the screening radius. Standard time dependent perturbation theory is used.

The expression for the matrix element is given as

\[
|H_{if}| = \frac{4 e^{2}}{\xi V} \left[ F_{1} F_{2} \frac{\Delta 21}{\lambda^{2} + g^{2}} - F_{3} F_{4} \frac{\Delta 12}{\lambda^{2} + h^{2}} \right] \left\{ \frac{k_{1} - k_{2} - k_{1}' - k_{2}'}{\lambda} \right\}
\]

(a-12)
\[ g \equiv \vec{k}_1 - \vec{k}_1', \quad h \equiv \vec{k}_2 - \vec{k}_1' \] (a-12a)

\[ \Delta_{21} = \Delta_{12} = 1 \quad \text{when the electrons 1 and 2 have like spins} \]
\[ \Delta_{21} = 1, \Delta_{12} = 0 \quad \text{when the electrons 1 and 2 have unlike spins} \]
\[ \text{and their spins are preserved} \]
\[ \Delta_{21} = 0, \Delta_{12} = 1 \quad \text{when the electrons 1 and 2 have unlike spins} \]
\[ \text{and their spins are changed.} \]

\[ F_1 \equiv \int \psi_c^* (\vec{k}_1, \vec{r}) \ u_v (\vec{k}_1', \vec{r}) \ d\vec{r} \] (a-13)
\[ F_2 \equiv \int \psi_c^* (\vec{k}_2, \vec{r}) \ u_c (\vec{k}_2', \vec{r}) \ d\vec{r} \]
\[ F_3 \equiv \int \psi_c^* (\vec{k}_1, \vec{r}) \ u_c (\vec{k}_2', \vec{r}) \ d\vec{r} \]
\[ F_4 \equiv \int \psi_c^* (\vec{k}_2, \vec{r}) \ u_v (\vec{k}_1', \vec{r}) \ d\vec{r} \]

The evaluation of the matrix element was carried out with the help of the usual reciprocal lattice sums yielding the selection rules for the process. Neglected in the final result are the Umklapp type process as they are taken to be far from the band edges and thus will have small Fermi probabilities.

In the development of the case of optical phonon scattering, it was assumed that the electrons of interest had sufficient energy such that it was not necessary to adjust the density of states for the occupation statistics. All end states were assumed essentially empty and available for the scattered electron. For the present case of impact ionization and Auger recombination all levels of electron energy are of interest. In this event the density of states \( \mathcal{f}(k) \) in equation a-5 is modified by the statistics that state \( k \) will be empty.

Thus the density of states is modified by the probability that states 1 and 2 are filled, \( 1' \) and \( 2' \) are empty for the process shown schematically in figure Al-a and the corresponding probabilities for the
Figure A1.

Auger effects leading to construction or destruction of an electron-hole pair in a semiconductor. Unprimed numbers refer to initial states; primed numbers to final states.

(a) Destruction of hole-electron pair by e-e collision
(b) Creation of hole-electron pair by e-e collision
(c) Destruction of pair by h-h collision
(d) Creation of pair by h-h collision.
inverse process figure A1-b. The net probability of an e-e process may then be calculated with a knowledge of the occupation statistics for the semiconductor. In this case these statistics are taken to be Fermi statistics with the non-equilibrium taken care of by the use of the quasi-Fermi levels.

The bulk of the Beattie and Landsford paper is then given over to the evaluation of the probability integrals, the results of which are an expression for the lifetime of the Auger process. The dominant temperature dependence of this process is given by

\[ \left( E_g(T) \right)^{3/2} \exp \frac{1 + 2\mu E_g(T)}{1 + \mu} \frac{E_g(T)}{kT} \]  

(a-14)

in the case of a non-degenerate and intrinsic semiconductor. Here \( \mu = m_e^*/m_h^* \) if \( m_e^* < m_h^* \). The temperature dependence of the probability integrals comes entirely from the inclusion of the statistics in the density of states function. The temperature to use in equation a-14 would not necessarily be the lattice temperature in the event of a strong non-equilibrium situation. Just moving the Fermi level may not account for the occupation of high energy states especially if an inversion is present. It would be well to keep these points in mind when applying equation a-14 to processes which occur with strong non-equilibrium conditions.
B. DC Field Distribution in a Reverse Biased p-i-n Diode

In this appendix the relations between the bias voltage and the electric field in a p-i-n diode will be discussed. In particular, the bias necessary to achieve an approximately linear field in the *i* region will be determined and the value of this field in terms of the external bias voltage will be determined.

Prim has solved the problem of the reverse biased p-i-n planar diode for the case of negligible current densities. The geometry which he considers is shown in figure A2. A qualitative picture of the potential distribution throughout the structure is shown in figure A3. A detailed description of the solution need not be delved into here; if more detail is needed, one is referred to Prim's article (48).

Unfortunately, the solution for the electrostatic potential \( \phi \) as a function of distance \( x \) throughout the planar structure turns out to be a very messy function. Therefore, a graphic presentation is chosen for clarity. Figure A4 shows \( \phi(x) \) for three values of applied bias. Since \( E = -\nabla \phi \) is the electric field it can be readily seen that for sufficiently high bias the electric field will be uniform in the intrinsic region.

When a p-i-n diode is operating as a particle or \( \gamma \)-ray detector, it is desirable to operate a detector at the maximum electric field in the depletion layer consistent with the lowest possible leakage current generated noise. Thus we can see that the electric field in the intrinsic region of a p-i-n detector will usually be uniform. In order to present a quantitative picture of the field penetration into the interior of the intrinsic region, Prim introduces a field
Figure A-2

Doping schematic for a p-i-n planar diode.

Figure A-3

Sketch of the electrostatic potential in a reverse-biased p-i-n diode showing the nomenclature used in the various regions.
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Figure A4.

Graphical representation of Prim's intrinsic region
solution for $W = 11 \mu$, $N = P = 10^{18}$ cm$^{-3}$, $T = 300^\circ$
for silicon. $V_{INT}$ is the potential across the intrinsic
region only.
penetration parameter $\zeta$ defined by

$$\zeta = \frac{\text{minimum field intensity}}{\text{average field intensity}}$$

where the average field intensity is given by

$$E_{av} = \frac{V_{int}}{W}.$$  

$V_{int}$ is the potential across the intrinsic region of the diode. Figure A5 shows a normalized plot of one half of the voltage drop across the intrinsic region as a function of the normalized intrinsic layer length. The normalization parameters are as follows.

$$\hat{V} = \text{normalized voltage} = \frac{qV}{kT}$$

$$\hat{L} = \text{normalized length} = \frac{W}{L}$$

$$\mathcal{L}^2 = \frac{kT}{2q\sqrt{NF}}: \text{the "mean Debye length"}$$

$N$ and $P$ are the donor and acceptor concentrations in the $n$ and $p$ regions respectively. For the case of lithium drifted detectors $N \sim 10^{18}$ cm$^{-3}$ $P \sim 10^{14}$ cm$^{-3}$ for either silicon or germanium. Therefore we have:

$$\mathcal{L}_{si} \sim 2.92 \times 10^{-6} \sqrt{\frac{T}{300}},$$

$$\mathcal{L}_{ge} \sim 3.37 \times 10^{-6} \sqrt{\frac{T}{300}}.$$  

For the purpose of getting an idea of how much voltage is necessary for a value of $\zeta = 0.95$ as a function of temperature, take an average value of $\mathcal{L} = 3.13 \times 10^{-6} \sqrt{\frac{T}{300}}$. For practical detectors $\mathcal{L}$ will lie in the range of $10^4$ to $5 \times 10^6$. Figure A6 shows the necessary $V_{int}$ to achieve $\zeta = 0.95$ as a function of temperature.

The true voltage across the detector is the difference between
Figure A5.

Normalized voltage across the intrinsic region of a p-i-n diode as a function of the normalized width and the field penetration parameter $\xi$. 
Figure A6.

Necessary bias to achieve a field penetration parameter of 0.95 as a function of intrinsic layer width and temperature.
the two Fermi potential represented in figure A3 as \( \phi_n \) and \( \phi_p \).

Referring to figure A3, we see that to determine \( V_{\text{int}} \) and thus the electric field, we must determine \( \phi(0) \) and \( \phi(W) \) in terms of \( \phi_n \) and \( \phi_p \). Valdes calculates a value for \( \phi(W) - \phi_p \) assuming near \( x = W \) the majority carriers are holes (65). The calculation is the same for \( \phi_n - \phi(0) \).

Thus we may state with Valdes

\[
\phi_n - \phi(0) = \phi(W) - \phi_p = 0.57 \text{ kT/q.} \quad (a-15)
\]

Referring to figure A3:

\[
\begin{align*}
\phi_n - \phi_p &= \phi(0) - \phi(W) + 1.14 \text{ kT/q} \\
&= V_{\text{int}} + 1.14 \text{ kT/q} \quad (a-16)
\end{align*}
\]

\[
\begin{align*}
\phi_n - \phi_p &= \phi_n - (\phi_N - \phi) - (\phi_p - (\phi - \phi_p)) \\
&= \phi_n - \phi_p + (\phi_N - \phi_p) \\
&= V_{\text{bias}} + V_{\text{int}} + 1.14 \text{ kT/q} \quad (a-17)
\end{align*}
\]

where reverse bias has been taken positive for convenience.

Equating equations a-16 and a-17 we see that

\[
V_{\text{int}} = V_{\text{bias}} + V_o \quad (a-18)
\]

where \( V_o \) is essentially the "contact" potential of the diode. The electric field of a highly reverse biased p-i-n diode may then be taken as

\[
|E| \sim V_{\text{bias}} + V_o. \quad (a-19)
\]

To evaluate the built in potential, \( V_o \), consider the ideal diode equation

\[
I = -I_o (e^{-\frac{V_{\text{bias}} + V_o}{\Delta}} - 1) \quad (a-20)
\]

where our previous sign convention has been kept. If the diode current is zero then
\[ e^{-\alpha(V_{bias} + V_0)} = 1 \quad \text{(a-21)} \]

or

\[ V_{bias} = -V_0 \quad \text{(a-22)} \]

In practice if the diode is forward biased and the current curve extrapolated to the \( I = 0 \) axis in a straight line, a close approximation to \( V_0 \) may be measured, as shown in figure A7 (68).
Figure A7.

A sketch showing means for determining the contact potential of a diode.
Figure A-7.

Sketch showing the scheme for measuring $V_0$. 
C. Charge Collection Efficiency

There exist two approaches to determining the charge collection efficiency of a detector in the literature. One stems from the work of Baldinger et al. (3) who use an analysis of the diffusion equation by Jaffe (34). The other stems from the early work with crystal counters and was originally developed by Hecht (32), the same concepts are presented in English by Mott and Gurney (45). Both presentations appear equally valid to this author although the Hecht form seems a little more general in scope than Jaffe's original derivation. In the high electric field limit, the two can be shown to be equivalent. Since at high electric fields the charge collection efficiency tends to unity, the high field limit is the most applicable in efficient nuclear detector work. The arguments behind Hecht's derivation (with suitable modifications) will be presented.

From the work of C. K. Jen (35), we obtain an expression for the current to an electrode induced by a charge moving in a space change region under the influence of an external electric field.

For the special case of parallel geometry, two plates of spacing W, with one electrode grounded the current induced in the lead to the second electrode by a single moving charge q', is

\[
i = \frac{q'}{W} \frac{dx}{dt}.
\]

(a-23)

It is understood in the above expression that the electric field is perpendicular to the two plates and the charge moves in the direction parallel to the field, taken as the x direction. In the case of nuclear detectors and a charge sensitive preamplifier circuit equation, one may be expressed in terms of the charge sensed.
\[ dQ = \frac{q'}{w} \, dx \] (a-24)

Consider now a charged particle entering a depletion layer of a slab geometry diode as shown in figure A8. The \( x \) axis is chosen to be co-planar with the track of the particle. It is assumed that the depletion layer boundary planes extend to infinity. Since in most cases \( R \cos \phi \) is much less than the lateral dimensions of the detector, this model is reasonable. Let \( n(r) \) be the density of electron-hole pairs along the track of the particle. Therefore

\[ \int_{0}^{R} n(r) \, dr = N: \text{ the total number of pairs generated.} \]

The arguments of Mott and Gurney show for \( n \) electrons originally at a point \( x_1 \) under the influence of an electric field in the \( x \) direction and having a lifetime \( \tau_e \)

\[ n(x) = n(x_1) \, e^{-\frac{x}{\mu_e \tau_e E}} \] (a-25)

where \( \mu_e \) is the electron mobility for the material under consideration.

The charge detected due to the electrons generated between \( r \) and \( r + dr \) moving under the influence of the electric field \( \overline{E} \) is

\[ Q_e = \frac{q}{w} n(x, r) \, dr \, x \] (a-26)

where \( n(x, r) \) means the element of charge which was originally at \( x_1 = r \sin \phi \) and has moved, rather its center has moved, and the dispersion is assumed small. Then

\[ n(x, r) = n(r) \, e^{-\frac{x}{\mu_e \tau_e E}} \] (a-27)

and the total charge detected from the electrons located originally at \( r \) along the particle track is

\[ dQ_e = \frac{qn(r) \, dr}{w} \int_{0}^{x} e^{-\frac{x}{\mu_e \tau_e E}} \, dx \] (a-28)
Figure A8.

Particle entering detector and creating a plasma of holes and electrons.
\[ dQ_e = \frac{q \cdot n(r)}{W} \cdot \frac{dr}{\mu_e C_e} \left[ 1 - e^{-r} \sin \varphi / \mu_e C_e E \right] \]  

(a-28)

Similar arguments lead to the charge detected from the current induced by the moving holes. The positive holes move with the field and the negative electrons move against it; thus the current induced by both holes and electrons is in the same direction. The total charge detected from the pairs generated between \( r \) and \( r + dr \) is therefore

\[ dQ_t = \frac{q \cdot n(r)}{W} \cdot \frac{dr}{\mu_e C_e} \left[ \mu_e C_e \left( 1 - e^{-r} \sin \varphi / \mu_e C_e E \right) \right. \\
+ \left. \mu_h C_h \left( e^{-r} \sin \varphi / \mu_h C_h E \right) - e^{-W/\mu_h C_h E} \right] \]  

(a-29)

In silicon and germanium the electron and hole mobilities differ at most by a factor of 5. Not much is known concerning the lifetimes of excess carriers in the depletion region itself. It is felt highly likely that the electron and hole magnitudes are of the same order of magnitude. With the preceding as justification, we make the approximation

\[ \mu_e C_e = \mu_h C_h = \mu C \]  

(a-30)

Thus

\[ dQ_t = \frac{q \cdot n(r)}{W} \cdot \frac{dr}{\mu C} \left[ 1 - e^{-W/\mu C E} \right]. \]  

(a-31)

Integrating over the range of the particle, we obtain

\[ Q_t = \frac{q \cdot N \cdot \mu C E}{W} \left( 1 - e^{-W/\mu C E} \right). \]

The defining relation for the charge collection efficiency is

\[ Q_c = \gamma Q_g \]  

(a-32)

where \( Q_c \) is the detected charge, \( \gamma Q_g \) is the generated charge, and \( \gamma \) is the charge collection efficiency. We see therefore under the assumptions made above that the charge collection efficiency is given by the following expression.
\eta \frac{Q_e}{qW} = \frac{\mu C E}{W} \left(1 - e^{-W/\mu C E}\right). \tag{a-33}

Where \mu C is the mobility lifetime product for either holes or electrons. A more detailed analysis may be made without the assumption of equation (a-30), however, not much new information is gained. The essential differences between this derivation and the ones of Hecht and Mott--Gurney are that both positive and negative generated charges are considered and the range need not extend through the active region of the detector. The assumption of equal lifetime--mobility products makes it immaterial from the collection efficiency point of view whether the fast or slow particles travel the longest distance.
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