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3D AVO Analysis: A Modeling Approach

by

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3D AVO Analysis : A Modeling Approach

ABSTRACT

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3D AVO (Amplitude Variation with Offset) analysis has evolved as an important interpretation method of 3D seismic data, in the last decade. A new approach is proposed in the present work for quantitative AVO analysis of 3D seismic data.

The method is based on fitting the observed seismic data, in a least-squares approach, to a function of average angle (average of angle of incidence and angle of transmittance) or offset. The aim of the method is to obtain the estimates of changes in the elastic parameters (P-wave modulus, S-wave modulus and density) at a particular output location, using a combination of pre-stack Kirchhoff migration and matrix inversion. The method includes ways to do migration velocity analysis and estimation of dip.

As a further extension, the method is modified to find out the orientation of vertical fractures and the extent of anisotropy, for the case when an isotropic medium overlies a vertically fractured medium.
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<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title Page</td>
<td>i</td>
</tr>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Acknowledgment</td>
<td>iii</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>iv</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vi</td>
</tr>
<tr>
<td>Symbols</td>
<td>viii</td>
</tr>
<tr>
<td>1: Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2: Approximation of the P-wave Reflection Coefficient</td>
<td>6</td>
</tr>
<tr>
<td>2.1: Background</td>
<td>6</td>
</tr>
<tr>
<td>2.2: Approximation</td>
<td>8</td>
</tr>
<tr>
<td>2.3: Results</td>
<td>13</td>
</tr>
<tr>
<td>3: Inversion (Least squares approach):</td>
<td>16</td>
</tr>
<tr>
<td>Angle of incidence vs. Reflection coefficient data</td>
<td></td>
</tr>
<tr>
<td>3.1: Estimation of elastic parameters from angle of incidence vs. Reflection coefficient data</td>
<td>16</td>
</tr>
<tr>
<td>3.1.1: Result</td>
<td>18</td>
</tr>
<tr>
<td>4: Inversion (Least squares approach):</td>
<td>20</td>
</tr>
<tr>
<td>4.1: Regular data: Pre-migration weighting and extraction of elastic parameters</td>
<td>21</td>
</tr>
</tbody>
</table>
LIST OF FIGURES:

Figure 1: Partitioning of P-wave energy

Figure 2:
(a) Comparison of the performance of the approximate formula with angle of incidence and average angle: Shale-Salt Case
(b) Comparison of performance of the Approximate formula with the Zoeppritz equation: Shale-Salt case

Figure 3:
(a) Comparison of the performance of the approximate formula and the Zoeppritz equation for the cases with medium 1 at top and medium 2 at top, with angle of incidence on the x-axis
(b) Comparison of the performance of the approximate formula and the Zoeppritz equation for the cases with medium 1 at top and medium 2 at top, with average angle on the x-axis

Figure 4: Flowchart for pre-migration-weighting scheme

Figure 5: Scheme of the iteration

Figure 6: Regular 3D geometry: CMP(x)-CMP(y)-Offset domain

Figure 7: Convergence with iteration

Figure 8: Data and result: Shale-Salt Interface

Figure 9: Grid element and the trace: weighting for irregular data

Figure 10: Irregular 3D geometry: CMP(x)-CMP(y)-Offset domain

Figure 11: Flowchart for post-migration (pre-stack constant-offset migration) weighting and estimation of elastic parameters

Figure 12: Flowchart for generalized way of estimating elastic
parameters from multi-fold 3D data

Figure 13: Migration velocity analysis: Comparison of traces 46

Figure 14: Migration velocity analysis: Equality of phase-angles 48

Figure 15: Relation between Shot, Receiver and Output location 51

Figure 16: Correct normal, computed normal and deviation between them 52

Figure 17: An isotropic medium overlying a vertically fractured medium 55

Figure 18: Variation of the reflection coefficient parallel and perpendicular to the fracture orientation 57

Figure 19: Relation among half-offset, $h_x$ and $h_y$ 59
Symbols Used:

- $p$: Ray parameter
- $i_1$: Angle of incidence of P-wave
- $i_2$: Angle of transmittance of P-wave
- $j_1$: Angle of reflection of S-wave
- $j_2$: Angle of transmittance of S-wave
- $i$: Average of $i_1$ and $i_2$
- $\rho$: Density
- $M$: P-wave modulus
- $\mu$: S-wave modulus
- $V_P$: P-wave velocity
- $V_S$: S-wave velocity
- $R, R_{PP}$: P-wave reflection coefficient
- $\theta$: Dip of the interface
- $h$: Offset
- $\alpha, \beta, \gamma$: Elastic moduli of Anisotropic medium
- $\phi$: Azimuth
- $\delta$: Effective Anisotropic parameter
Chapter 1

INTRODUCTION

Efforts to relate rock-properties to amplitude information from seismogram have been made from the earliest times of reflection seismic technology of exploration geophysics. Zoeppritz (1919), defined the reflection coefficient for plane waves at a plane interface of two elastic half-spaces. Koefoed (1955) was one of the first to emphasize the importance of the effect of change in Poisson’s ratio on the reflection coefficient, with change in angle of incidence. Ostrander (1984) demonstrated that gas sand reflection coefficients vary in an anomalous fashion with increasing offset and showed that this can be utilized for direct hydrocarbon detection.

In seismic studies, one rarely deals with a single isolated interface. In fact, measured data are a superposition of events, arising from complicated geometry of layers, along with complications stemming from velocity-variation, tuning and many other problems. In spite of these difficulties, it is worthwhile to investigate how reflection coefficients change with the angle of incidence. Knott (1899) and Zoeppritz (1919) presented a set of equations to compute the reflection coefficients. But the complicated appearance of the expressions somehow makes it difficult to gain a physical insight into the phenomenon of anomalous behavior of the reflection coefficients. There have been a number of approximations offered to clarify Zoeppritz equations. Bortfeld (1961) linearized the Zoeppritz equations with the assumption of small changes in the layer-parameters. Aki and Richards (1980) followed a similar line to present an approximate expression for the reflection
coefficients in terms of density, P-wave velocity and S-wave velocity. Hilterman (1983) and Smith and Gidlow (1987) offered an approximation, comprising "shear" and "fluid" terms. Shuey (1985) modified the Aki and Richards (1980) approximation to present an approximate expression in which different terms correspond to different range of angular-coverage.

Since 1984, many studies have been made of different aspects of AVO. AVO analysis has been used to characterize carbonate reservoirs, to determine porosity, to monitor Enhanced Oil Recovery (EOR) projects etc. The basic idea of AVO analysis is that, when P-wave is incident on an interface, some fraction of its energy is converted to S-waves, reflecting changes in P-wave reflectivity with angle of incidence. AVO analysis exploits the observation of anomalous amplitude-information to get an estimate of lithology, porosity etc. In the beginning, AVO analysis started with the 2D seismic data. Gas detection stood out as the most promising application of AVO analysis. Ostrander (1984) first showed the usefulness of AVO analysis in the study of gas-sand reflection coefficients. Rutherford and Williams (1989) classified different gas-sand AVO behaviors. Successful application of AVO analysis was reported from different areas of the world, e.g., Sacramento Valley, the North Sea, the Gulf of Mexico, Alaska, Arabian Gulf etc. Soroka et al. (1990) described the advantages gained in moving from 2D AVO to 3D AVO analysis. They reported that positive 3D AVO response led to reliable identification of gas-bearing beds, whereas 2D AVO signatures were difficult to interpret. Skidmore (1992) described the necessities and differences in change-over from 2D AVO to 3D AVO analysis.
In the last decade, AVO analysis has been experimented with extensively in the hydrocarbon industry in different forms. But, most of the approaches have been in interpreting 2D reflection seismic data. Subsurface geological features of interest in hydrocarbon exploration being three-dimensional in nature, 3D seismic is coming to be used more widely in the recent times. The advent of 3D seismic methods has created a requirement for an appropriate approach of interpreting the amplitude-information. Also, 3D seismic surveys now comprising the larger percentage of the seismic survey, blending 3D data and AVO-analysis, promises to be a potential tool for the interpretation of the subsurface.

In spite of the apparent potential of 3D methods for improving the AVO (Amplitude Variation with Offset) method, it suffers from wide variety of complications and pitfalls, arising from problems like tuning, near-surface complications, structural complexity, geometrical spreading, undersampling, pre-AVO analysis processing effects and above all, the data quality, especially the amplitude of the data for AVO analysis.

In the present work, a quantitative approach to the analysis of 3D-AVO data is proposed. There have been a number of approximations, proposed for the Zoeppritz equations. In the present work, the validity of one approximation is analyzed and later used to fit a non-linear curve to the observed seismic amplitude in terms of change in elastic parameters of the involved media. We also propose a method of extracting the estimates of change in elastic parameters, by using a combination of pre-stack Kirchhoff migration and simple matrix inversion.

Gardner (Nov., 1993, GTRI, HARC) proposed a method for "Amplitude
Preserving Processing for Prestack Migration”. Following on a similar line, a method is proposed for quantitative analysis of 3D-AVO data. An approximation for the Zoeppritz equation is used, which is a rewritten form of Aki and Richards (1980) linearized approximation of the Zoeppritz equations (Chapter 2).

The basic principle of the method proposed in the present work utilizes the fact, that given a set of angle of incidence vs. reflection coefficient data and a suitable approximation of the expression of the reflection coefficient as a function of the angle of incidence, it is possible to estimate the changes in the elastic parameters, namely, P-wave modulus, S-wave modulus and density (Chapter 3).

For estimation of the changes in the elastic parameters from 3D dataset, the problem has a difference in that the amplitude at a particular subsurface point has contribution from the points surrounding it. So for this case, migration has to be used to take care of this problem.

The next phase is migrating each coverage of a multi-offset data (while preserving amplitude) and then fitting a three-parameter least-squares curve to the amplitude at each output location, the three parameters of the curve being related to the three physical quantities (Chapter 4.1). In section 4.2, the performance of the above-mentioned method is examined for irregular data, before and after using weights to the data (according to the extent of irregularity).

From the chapter 4.1, it becomes obvious that the method presented here is costly and time-consuming, because it utilizes re-computation of (3x3) matrix, all elements of which are angle of incidence, and three pre-stack migrations. So in chapter 4.3
and 4.4, two more generalized alternatives are presented to the approach, described in section 4.1.

For the analysis, described in section 4.1, 4.3 and 4.4, it is necessary to know the migration velocity and the dip of a reflecting surface. In section 4.5, a method is described, which is used to find the migration velocity, using the concept of equality of phase angles. In section 4.6, a method is described for finding the normal to the reflecting horizon.

In chapter 5, a similar method (as described in section 3.1) is described to find out the orientation of the vertical fractures and extent of anisotropy for the case, when an isotropic medium overlies a vertically fractured medium. The data was generated with the assumption of azimuthal transverse isotropy and Mohammed Al-Otaibi’s(1995) approximation for the P-wave reflection coefficient for Transverse Isotropic media.

**AVO analysis is an attempt to look at the amplitude variation of seismic data with offset, in order to have an idea of the subsurface rock properties.**

*Application of AVO technique for 3D seismic data appears to have good potential.*

The present work addresses the first few steps of a complex problem: **AVO analysis of 3D seismic data.**
Chapter 2

Approximation of the P-wave Reflection Coefficient

2.1: Background

The reflection coefficient for plane-wave at a plane interface (figure 1) for any combination of media can be obtained by the Zoeppritz equations (2.1), which are derived by solving for the boundary conditions for the continuity of the normal and the tangential displacements and the stresses at the interface.

\[
D = \begin{bmatrix}
\begin{pmatrix} -\alpha_1 \cdot p \\ \cos i_1 \end{pmatrix} & \begin{pmatrix} -\cos f_1 \\ -\beta_1 \cdot p \end{pmatrix} & \begin{pmatrix} \alpha_2 \cdot p \\ \cos f_2 \end{pmatrix} & \begin{pmatrix} \cos f_2 \\ -\beta_2 \cdot p \end{pmatrix} \\
\begin{pmatrix} 2 \cdot \rho_1 \cdot \beta_1^2 \cdot p \cdot \cos i_1 \end{pmatrix} & \begin{pmatrix} \rho_1 \cdot \beta_1 \cdot (1 - 2 \cdot \beta_1^2 \cdot p^2) \end{pmatrix} & \begin{pmatrix} 2 \cdot \rho_2 \cdot \beta_2^2 \cdot p \cdot \cos i_2 \end{pmatrix} & \begin{pmatrix} \rho_2 \cdot \beta_2 \cdot (1 - 2 \cdot \beta_2^2 \cdot p^2) \end{pmatrix} \\
-\rho_1 \cdot \alpha_1 \cdot (1 - 2 \cdot \beta_1^2 \cdot p^2) & \begin{pmatrix} 2 \cdot \rho_1 \cdot \beta_1^2 \cdot p \cdot \cos i_1 \end{pmatrix} & \begin{pmatrix} \rho_2 \cdot \alpha_2 \cdot (1 - 2 \cdot \beta_2^2 \cdot p^2) \end{pmatrix} & \begin{pmatrix} -2 \cdot \rho_2 \cdot \beta_2^2 \cdot p \cdot \cos i_2 \end{pmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
R_{PP} \\
R_{PS} \\
T_{PP} \\
T_{PS}
\end{bmatrix} = D^{-1} \cdot \begin{bmatrix}
-\alpha_1 \cdot p \\
\cos i_1 \\
2 \cdot \rho_1 \cdot \beta_1^2 \cdot p \cdot \cos i_1 \\
\rho_1 \cdot \alpha_1 \cdot (1 - 2 \cdot \beta_1^2 \cdot p^2)
\end{bmatrix} \quad \text{......... (2.1)}
\]

where,

\( R_{PP} \) = P-wave Reflection coefficient;
\[ R_{PS} = \text{S-wave reflection coefficient}; \]
\[ T_{PP} = \text{P-wave transmission coefficient}; \]
\[ T_{PS} = \text{S-wave transmission coefficient}. \]

[In each of the cases, Incident wavefront is compressional, hence the first letter in the subscript of each of the coefficients is P].

Expression of reflection coefficients from Zoeppritz equation being algebraically complicated, they are both difficult to grasp intuitively and difficult to utilize for inversion, due to computational reasons.

Aki and Richards (1980) proposed a linearized approximation of the Zoeppritz solution of P-wave reflection coefficient, as

\[ R = \frac{1}{2} \left( 1 - 4V_S^2 \frac{\Delta p}{p} \right) + \left( -\frac{1}{2\cos^2\theta} \right) \left( \frac{\Delta V_p}{V_p} \right) - 4V_S^2 \frac{\Delta p}{p} \left( \frac{\Delta V_s}{V_s} \right), \quad (2.2) \]

where

- \( R = \text{P-wave reflection coefficient}, \)
- \( p = \text{ray-parameter}; \)
- \( V_P = \text{average of P-wave velocities}; \)
- \( \Delta V_P = \text{difference in P-wave velocities}; \)
- \( V_S = \text{average of S-wave velocities}; \)
- \( \Delta V_S = \text{difference in S-wave velocities}; \)
- \( p = \text{average of densities}; \)
\[ \Delta \rho = \text{difference of densities}; \]
\[ i = \text{average of angles of incidence and transmittance}. \]

In the above approximation the assumption was that the two media have similar properties, meaning that changes in the elastic properties are small. So resulting reflection-coefficients will be much smaller compared to one and the transmission coefficients will be of the order of one. The other assumption involved in the approximation was that none of the angles involved is near 90°.

2.2: Approximation

Relations among P-wave modulus, S-wave modulus, density and corresponding velocities can be used to rewrite the above expression (equation 2.2) of approximating reflection coefficient, as discussed below.

P-wave velocity \( V_P \), P-wave modulus \( M \) and density \( \rho \) are related as:

\[ V_P = \sqrt{\frac{M}{\rho}} \] \hspace{1cm} (2.3)

from which we can write:

\[ V_P = \sqrt{\frac{M}{\rho}} \]

\[ \ln(V_P) = \left( \frac{1}{2} \right) \left[ \ln(M) - \ln(\rho) \right] \]

\[ \frac{\Delta V_P}{V_P} = \left( \frac{1}{2} \right) \left[ \left( \frac{\Delta M}{M} \right) - \left( \frac{\Delta \rho}{\rho} \right) \right] \] \hspace{1cm} (2.4)

Similarly, it can be shown that:

\[ \frac{\Delta V_S}{V_S} = \left( \frac{1}{2} \right) \left[ \left( \frac{\Delta \mu}{\mu} \right) - \left( \frac{\Delta \rho}{\rho} \right) \right] \] \hspace{1cm} (2.5)
from the relation between S-wave velocity \(V_S\), S-wave modulus \(\mu\) and density \(\rho\)

\[V_S = \sqrt{\frac{\mu}{\rho}} \quad \text{(2.6)}\]

From the equations (2.2), (2.4) and (2.5), we can write:

\[R \cos^2 i = \left( -\frac{1}{4} \right) \left( \frac{\Delta \rho}{\rho} \right) + \left( \frac{\Delta M}{M} \right) \left[ \left( \frac{1}{2} \right) \left( \frac{\Delta \rho}{\rho} \right) + 2 \left( \frac{\Delta M}{M} \right) \right] \sin^2 i - 2 \left( \frac{\Delta \mu}{M} \right) \sin^4 i \quad \text{(2.7)}\]

For \(\Delta M = 0, \Delta \mu = 0\):

\[R_\rho = \left( -\frac{1}{4} \right) \left( \frac{\Delta \rho}{\rho} \right) \left( 1 + \tan^2 i \right) \quad \text{(2.8)}\]

For \(\Delta \mu = 0, \Delta \rho = 0\):

\[R_M = \left( -\frac{1}{4} \right) \left( \frac{\Delta M}{M} \right) \left( 1 + \tan^2 i \right) \quad \text{(2.9)}\]

---

**Figure 1: Partitioning of P-wave energy**

For \(\Delta \rho = 0, \Delta M = 0\):
\[ R_\mu = 2\left(\frac{\Delta \mu}{M}\right)\sin^2 i \quad \ldots \ldots (2.10) \]

It can be shown that:

\[ R = R_\rho + R_\mu + R_M \quad \ldots \ldots (2.11) \]

We can rewrite the above equation in terms of P-wave modulus(M), S-wave modulus(\mu) and density(\rho), as:

\[ R = \left(-\frac{1}{2}\right)\left(\frac{\Delta \rho}{\rho}\right)\left(1 - \tan^2 i\right) + \left(-\frac{1}{2}\right)\left(\frac{\Delta M}{M}\right)\left(1 + \tan^2 i\right) + 2\left(\frac{\Delta \mu}{M}\right)\sin^2 i \quad \ldots \ldots (2.12) \]

Two points to be noted here:

Firstly, there are two assumptions in the above approximation:

(a) small angle of incidence (of the order of 40°),

(b) small change in elastic parameters.

Secondly, the angle ‘i’ in the equation is not the angle of incidence in the first medium, it is the average of the angles of incidence and transmittance.

The reason for rewriting the equation 2.2, the linearized approximation to the Zoeppritz equations offered by Aki and Richards, is that the elastic parameters are not clearly separated from the average angle ‘i’. The rewritten equation is suitable for using least-squares method of inverting the elastic parameters. Another advantage of this approximate equation (2.12) lies in the fact that this approximate equation can be utilized for sample-by-sample analysis of change in elastic parameters, whereas using the exact Zoeppritz equation, we cannot achieve that.

If we look at the expression of the approximate equation for the reflection
Figure 2(a) : Comparison of the results from using the approximate equation, with angle of incidence and average angle.

Figure 2(b) : Comparison of the results obtained from using the approximate equation and the exact Zoeppritz equation.
Figure 3(a) : Comparison of the performance of the approximate equation and the Zoeppritz equation for the cases with medium 1 overlying medium 2 and medium 2 overlying medium 1, with the angle of incidence on the X-axis.

Figure 3(b) : Comparison of the performance of the approximate equation and the Zoeppritz equation for the cases with medium 1 overlying medium 2 and medium 2 overlying medium 1, with the average angle on the X-axis.
coefficient, we can see that it is symmetric in "i", the average angle. That is to say, if we compute reflection coefficient vs. average angle of incidence, using equation 2.12, at the interface between medium A and medium B (A overlying B), we will get the same result for the case when B overlies A. But if we compare the results for the cases ‘A over B’ and ‘B over A’ from using the exact Zoeppritz equations; there will be a difference. If this difference is small, the approximation will be accurate.

2.3: Results

Figures 2 illustrates the use of equation(2.12) for a particular combination of elastic moduli (Shale over Salt).

(a) If ‘i’ is used instead of i₁, which equals (i₁+i₂)/2, in equation (2.12), the curves are substantially different;

(b) Using i=(i₁+i₂)/2, the result from the approximate equation compares well with the results from the exact Zoeppritz equations.

From each of the figure 2(a), we can see that the values of the computed reflection coefficient is very different; when calculated, using the angle of incidence in the first medium and when calculated using the average angle of incidence.

From each of figure 2(b), we can see that the approximation fares well compared to the actual Zoeppritz equation. But if the change in the elastic parameters had been large across the interface, (b), the results from the approximation and the Zoeppritz equation would have differed a lot.

In each of the case figure 2(b), we can see that with increase in angle, the
performance of the approximation deteriorates in comparison with the results from Zoeppritz equation (violation of the first assumption, concerning the angle of incidence).

The advantages of the approximate equation lie in the facts that the effects due to different elastic parameters are isolated in the three terms in the equation (2.12) and this can be used in the inversion scheme (discussed later). Usage of the elastic moduli, instead of the velocities, may help in extending the method for the case of anisotropy. It appears that, it is more logical to deal with the intrinsic parameters (elastic moduli) of a medium rather than the extrinsic parameters like P-wave- or S-wave-velocities.

We compare the performance of the Zoeppritz equation and the approximate formula for finding out the reflection coefficient for two combinations of two media; once with medium 1 overlying medium 2 and the other time with medium2 overlying medium 1.

The result is described in figure 3(a) and 3(b). In figure 3(a) we plot the angle of incidence vs. absolute value of the reflection coefficient for all the cases (because the switch of the media-combination changes the polarity of the reflection coefficient). In both the cases, the approximate formula gives a good match with the results from the Zoeppritz equation.

In figure 3(b), we plot the same result, but this time with average angle (i.e. average of the angle of incidence and the angle of transmittance) in the x-axis, we see that the magnitude of the reflection coefficient from the approximate formula is same for both the cases, upto a range of 40° and the curve lies in between the two curves from the results from the Zoeppritz equation for the two cases. From figure 3(b), we can see that the reflec-
tion coefficient-values computed with the Zoeppritz equation for the two combinations are very close, which gives rise to a very good result from the approximate equation.

The results suggest a test for finding out suitability of the approximation of the P-wave reflection coefficient, given in equation (2.12), when either the change in the rock-properties across an interface is large or the angle of incidence is too large for the equation (2.12) to be useful.

Zoeppritz equations give the exact P-wave reflection coefficient. Many approximations to Zoeppritz equations have been presented in the literature.

Aki and Richards (1980) approximation is rewritten in terms of P-wave modulus S-wave modulus, density and average of angle of incidence and angle of transmittance.
Chapter 3

INVERSION - LEAST SQUARES APPROACH

Angle of incidence vs. reflection coefficient data

3.1: Estimation of elastic parameters from
angle of incidence vs. reflection coefficient data

We propose a new approach to obtain the estimates of the physical quantities from the AVO data (reflection coefficients calculated for a range of angles of incidence, using the Zoeppritz equations), based on the approximation, derived in the last section. We utilize the concept of least-squared-error (LSE) approach, to obtain a best fit to the observed seismic amplitude. We use the following approximation, for the LSE derivation:

\[ R_{fit}(i) = A_1(1 - \tan^2 i) + A_2(1 + \tan^2 i) + A_3 \sin^2 i \] .......... (3.1)

This model is chosen to get an estimate of relative changes in density \((\Delta \rho / \rho)\) from \(A_1\), P-wave modulus \((\Delta M/M)\) from \(A_2\), S-wave modulus \((\Delta \mu/M)\) from \(A_3\),

where \(R(i)\) is the data, obtained from the seismogram and "i", is the average angle, \(R_{fit}(i)\) is the least-square fit.

In the least-squared-error approach, we try to get the fit by minimizing the sum of the square of errors:

\[ \epsilon^2 = \Sigma (R(i) - R_{fit}(i))^2 \] .......... (3.2)

The least-squares equations can be written in the matrix form, as given
below:

\[
\begin{bmatrix}
\Sigma \left( 1 - \tan^2 i \right)^2 & \Sigma \left( 1 - \tan^2 i \right) \left( 1 + \tan^2 i \right) \Sigma \left( 1 - \tan^2 i \right) \sin^2 i \\
\Sigma \left( 1 - \tan^2 i \right) \sin^2 i & \Sigma \left( 1 + \tan^2 i \right) \sin^2 i & \Sigma \left( 1 + \tan^2 i \right) \sin^2 i \\
\Sigma \left( 1 - \tan^2 i \right) \sin^2 i & \Sigma \left( 1 + \tan^2 i \right) \sin^2 i & \Sigma \sin^4 i
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2 \\
A_3
\end{bmatrix} =
\begin{bmatrix}
\Sigma R(i) \left( 1 - \tan^2 i \right) \\
\Sigma R(i) \left( 1 + \tan^2 i \right) \\
\Sigma R(i) \sin^2 i
\end{bmatrix}
\]

We have to keep in mind that the angle "i" in the above equation is not the angle of incidence, rather it is the average of the angle of incidence and the angle of transmittance. But we have to estimate the elastic parameters from the angle of incidence vs. the reflection coefficient data. So, angle of transmittance is another unknown, which is implicit in the equation (3.3). For finding out the angle of transmittance, we can start with an initial guess for the angle of transmittance (i_2) = i_1; meaning that for the first iteration, i=i_1. After the first iteration, we can find the first estimate of the (Δρ/ρ), (ΔM/M) and (Δμ/M). Then for the second iteration, remembering that (Δ(sin(i)/V_p))=0 across the interface, we can estimate Δi as Δi = (ΔV_p/V_p)*tan(i), or,

\[
Δi = (0.5) \cdot \left[ \left( \frac{ΔM}{M} \right) - \left( \frac{Δρ}{ρ} \right) \right] \cdot \tan i \ldots \ldots \ldots \ldots \ldots \ldots (3.4)
\]

In the next iteration, we can use the (3x3) and the (3x1) matrices in the equation (3.3), and re-estimate the values of (Δρ/ρ), (ΔM/M) and (Δμ/M). Once again, we can get a value for Δi, and keep on doing iterations, till we get a satisfactory result.
3.1.1: Result

\[ M_2 = 34.85 \]
\[ \mu_2 = 12.29 \]
\[ \rho_2 = 2.4 \]

Table 1: Actual values

<table>
<thead>
<tr>
<th></th>
<th>( \Delta \rho / \rho )</th>
<th>( \Delta M / M )</th>
<th>( \Delta \mu / M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0368</td>
<td>0.06457</td>
<td>0.0109</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Traces with reflection coefficients computed using Zoeppritz equation

<table>
<thead>
<tr>
<th>Trace #</th>
<th>Offset</th>
<th>Angle of Incidence</th>
<th>Maximum amplitude of the trace</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.0</td>
<td>4.7636</td>
<td>0.00697</td>
</tr>
<tr>
<td>2</td>
<td>190.0</td>
<td>8.9972</td>
<td>0.00705</td>
</tr>
<tr>
<td>3</td>
<td>280.0</td>
<td>13.1340</td>
<td>0.00721</td>
</tr>
<tr>
<td>4</td>
<td>370.0</td>
<td>17.1363</td>
<td>0.00749</td>
</tr>
<tr>
<td>5</td>
<td>460.0</td>
<td>20.9735</td>
<td>0.00793</td>
</tr>
<tr>
<td>6</td>
<td>550.0</td>
<td>24.6236</td>
<td>0.00859</td>
</tr>
<tr>
<td>7</td>
<td>640.0</td>
<td>28.0725</td>
<td>0.00951</td>
</tr>
<tr>
<td>8</td>
<td>730.0</td>
<td>31.3136</td>
<td>0.01073</td>
</tr>
</tbody>
</table>

Table 3: Result from inversion using the approximate equation

<table>
<thead>
<tr>
<th>Iter #</th>
<th>( \Delta \rho / \rho )</th>
<th>% error in ( \Delta \rho / \rho )</th>
<th>( \Delta M / M )</th>
<th>% error in ( \Delta M / M )</th>
<th>( \Delta \mu / M )</th>
<th>% error in ( \Delta \mu / M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0427</td>
<td>15.9857</td>
<td>0.0705</td>
<td>9.1682</td>
<td>0.0124</td>
<td>13.0343</td>
</tr>
<tr>
<td>2</td>
<td>-0.0351</td>
<td>-4.6268</td>
<td>0.0629</td>
<td>-2.6262</td>
<td>0.0105</td>
<td>-4.0427</td>
</tr>
<tr>
<td>3</td>
<td>-0.0359</td>
<td>-2.5127</td>
<td>0.0637</td>
<td>-1.4092</td>
<td>0.0107</td>
<td>-2.2806</td>
</tr>
</tbody>
</table>
Table 4: Traces with some error introduced

<table>
<thead>
<tr>
<th>Trace #</th>
<th>Offset</th>
<th>Angle of Incidence</th>
<th>Maximum amplitude of the trace</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.0</td>
<td>4.7636</td>
<td>0.006977</td>
</tr>
<tr>
<td>2</td>
<td>190.0</td>
<td>8.9972</td>
<td>0.00705</td>
</tr>
<tr>
<td>3</td>
<td>280.0</td>
<td>13.1340</td>
<td>0.00721</td>
</tr>
<tr>
<td>4</td>
<td>370.0</td>
<td>17.1363</td>
<td>0.00754</td>
</tr>
<tr>
<td>5</td>
<td>460.0</td>
<td>20.9735</td>
<td>0.00794</td>
</tr>
<tr>
<td>6</td>
<td>550.0</td>
<td>24.6236</td>
<td>0.00866</td>
</tr>
<tr>
<td>7</td>
<td>640.0</td>
<td>28.0725</td>
<td>0.00959</td>
</tr>
<tr>
<td>8</td>
<td>730.0</td>
<td>31.3136</td>
<td>0.01077</td>
</tr>
</tbody>
</table>

Table 5: Results from inversion using the approximate equation

<table>
<thead>
<tr>
<th>Iter#</th>
<th>$\Delta \rho / \rho$</th>
<th>% error in $\Delta \rho / \rho$</th>
<th>$\Delta M / M$</th>
<th>% error in $\Delta M / M$</th>
<th>$\Delta \mu / M$</th>
<th>% error in $\Delta \mu / M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0403</td>
<td>9.5242</td>
<td>0.0681</td>
<td>5.3933</td>
<td>0.0114</td>
<td>4.3761</td>
</tr>
<tr>
<td>2</td>
<td>-0.0333</td>
<td>-9.4302</td>
<td>0.0611</td>
<td>-5.4470</td>
<td>0.0097</td>
<td>-11.1041</td>
</tr>
<tr>
<td>3</td>
<td>-0.0340</td>
<td>-7.6063</td>
<td>0.0617</td>
<td>-4.4073</td>
<td>0.0099</td>
<td>-9.6509</td>
</tr>
</tbody>
</table>

In the above cases, we created the angle of incidence vs. reflection coefficient data using the Zoeppritz equation. Then using the equation (3.3), we computed the first estimate of $\Delta \rho / \rho$, $\Delta M / M$ and $\Delta \mu / M$. Using the expression (3.4), we found out the value of guess $\Delta i$ and went for the second iteration. Similarly, we did the third iteration. From comparison with the actual values of $\Delta \rho / \rho$, $\Delta M / M$ and $\Delta \mu / M$, we can see from the table 2 that at the end of the third iteration, we get a very reasonable estimate of the changes in the elastic parameters. In the next case, we introduced some random error to the computed values of the reflection coefficient. We use the same method and we get a reasonable estimate.
Chapter 4

INVERSION: LEAST SQUARES APPROACH

Synthetic 3D Data

Amplitude variation with offset analysis of 3D data is much different from that of 2D AVO analysis, because of the addition of one more dimension, the amount of data to be handled and other facts like consideration of amplitude contribution at a point from the surrounding points.

In this section, we try to approach the problem in stages of generalization, as discussed below.

We start with a 3D data volume and our aim is create three data volumes, containing changes in P-wave modulus, S-wave modulus and density. The method described, can be used at a particular output location and can be applied on a sample-to-sample basis. The result can be expressed both as numbers or as wiggle plots.

For the beginning, we start with the simple case of 3D data, generated on a regular grid, over a horizontal reflector. We use the inversion scheme, described in the previous section (a difference in the present case being that we use Kirchhoff migration for summation of the scaled data), with the assumption of knowledge of migration velocity and no dip. In the next few sub-sections, we try to address the problems of tackling multi-fold data, dipping reflector, irregularity in the data and estimating the migration velocity.
4.1: Regular 3D data: Pre-migration weighting and extraction of elastic parameters

For 3D data, where there are a number of lines in the survey area and a number of CMPs/line and a number of traces/CMP, the method of extracting the change in the elastic parameters is going to be different from the previous chapter, because of the fact that the amplitude at every subsurface point is going to have contributions from the points surrounding it. The aim of doing an AVO analysis is to get the estimate of the elastic parameters (or, the changes in them). So although the method remains almost the same, as discussed in the last section, we have to take care of the 3D nature of the data in order to utilize this volume-coverage to enhance the analysis at some point of the area. So, the computation of the (3x3) matrix in the equation (3.3) remains the same, given that the interface we are dealing with is flat.

In the previous chapter, while testing the method for 2D data, we did not employ any migration for summation, but for 3D data we use migration, as discussed below. This is because for 2D data case, the data was the reflection coefficients; but for the 3D data, we created the data, taking care of spherical divergence. So we need migration to compensate for that.

For obtaining the terms on the right-hand-side of the equation(3.3), we utilize the method of pre-stack Kirchhoff summation migration of the data, scaled by the corresponding weight for three times. This helps us in using the correct image and also helps in getting the summation of data for 3D data-volume. The algorithm used is a constant-velocity pre-stack Kirchhoff summation migration scheme, which involves two stages:
(a) Differentiation of the input trace with respect to time;

(b) Multiplication of the amplitude of the differentiated trace by an obliquuity factor and spreading out over an ellipsoid, with shot and receiver as the foci.

The expression, used for migration, given by Gardner (1993, GTRI 3D report), is given below:

\[
A (p, t) = \left( \frac{1}{\pi} \right)^{1/2} \cdot (dx) \cdot (dy) \cdot \sum_{j} \left( \frac{t_j}{T} \right) \left( \frac{1 + \sigma^2}{1 - \sigma^2} \right)^{1/2} \left( \frac{d}{dt} \right)_{data} \tau \ldots \ldots \quad (4.1)
\]

where,

\[A(p,t) = \text{amplitude;}\]
\[p=\text{output location;}\]
\[t=\text{output time;}\]
\[dx,dy=\text{midpoint grid-spacing;}\]
\[t_1=\text{time corresponding to the receiver location;}\]
\[t_2=\text{time corresponding to the shot location;}\]
\[T=t_1+t_2;\]
\[\sigma=(t_1-t_2)/(t_1+t_2).\]

Below, the algorithm (the flow-chart is given on the next page, figure 4) for the method, utilized for extraction of subsurface parameters is given:

(i) Compute the (3x3) matrix, given in the left side of the LSE-equation;

(ii) Migrate the scaled data to get the (3x1) matrix on the right side of the LSE equation;

(iii) Invert to obtain the parameters and the guess for the next stage of iteration;

(iv) Iterate to get the convergence and to obtain the estimate of the elastic parameters.
Scheme of Iteration: 1

The diagram below (figure 5) shows the scheme of iteration. In the algorithm to find out the estimates of the physical quantities, angle is also an unknown, because the angle supposed to be used being average angle, we need to find out the angle of transmittance. So we start with the initial guess of angle of transmittance = angle of incidence, and modify the guess at each stage. So in the beginning we start with the initial guess of average angle = angle of incidence. Then at the end of the first iteration, we get a value of Δi. Then, for the next iteration, we use the guess of average angle = angle of incidence+Δi. This method gives us convergence, but at a slower rate. We check the convergence, by checking on the percentage change in the estimates at each iteration.

Scheme of Iteration: 2

For a faster("forced") convergence, we use an approximate method: (i) for the case when the reflection-coefficient is positive, we consider a guess g=(g_{i1}+g_{i2})/2 in place of g_{i2}; and (ii) for the case, when the reflection coefficient is negative, we consider the guess g=(2 x g_{i2} - g_{i1})/2 in place of g_{i2}. For this case, we do the iteration only two times, to get a reasonable estimate.

For the above two cases, the inputs to the program are (i) depth of the interface, (ii) P-wave-modulus of the top-layer (M_1), (iii) S-wave modulus of the top layer (μ_1) and (iv) density of the top layer (ρ_1).

The results for the above two types of iteration are furnished in Case 1.
Figure 4: Flow chart for pre-migration weighting and estimation of the elastic parameters.
Scheme of Iteration: 3

There can be another way of iteration. For this one, the input needed is only the depth of the interface and the P-wave velocity in the top layer. The output will be the relative change in P-wave modulus ($\Delta M/M$), the relative change in S-wave modulus ($\Delta \mu/M$) and the relative change in density ($\Delta \rho/\rho$) across the interface.

For the first iteration, we can use $i_2 = i_1$, thereby using $i = i_1$. But for the second iteration, remembering that $(\Delta (\sin(i)/V_p)) = 0$ across the interface, we can estimate $\Delta i$ as, $\Delta i = (\Delta V_p/V_p) \cdot \tan(i)$ or,

$$\Delta i = (0.5) \cdot [(\Delta M/M) - ((\Delta \rho)/\rho)] \cdot \tan(i) \ldots \ldots \ldots (4.2)$$

We can estimate $(\Delta M/M)$ and $(\Delta \rho/\rho)$ from the first iteration. So for the second iteration, we can use $i_2 = i_1 + \Delta i$.

The results for the last type of iteration is given in Case2 and Case3.

With a closer observation, one can understand, that from the first scheme of iteration to the third scheme of iteration, we proceed to a more generalized approach to the problem. In the first scheme, we need all of P-wave modulus, S-wave modulus, and density of the top layer and in addition to that, four or five iterations are needed to get a good
convergence. For the second scheme of iteration, we need as many input values, but fewer number of iterations are required. In the third scheme of iteration, we need only the P-wave velocity in the top layer as the input.

Figure 5: Scheme of iteration
4.1.1: Results

Case 1: In the first case, we consider the interface between Shale and Salt, at a depth of 600 m. We use Shot-receiver geometry consisting of 10 lines (separated by 10m), with 20 CMP/line and 3 traces/CMP. Maximum offset used is 640m (figure 6).

Table 2 gives the result, when we do the iteration for five times without any forced convergence. In the figure 7, the convergence is described in terms of %error and the number of iteration. We can see that, after fifth iteration, we get a satisfactory match. Five iteration means that migration has to done five times, which is costly in terms of resources. Another point to consider is that, from the figure 7, we can see that for successive iteration, we overshoot the zero-error point and oscillate from a positive error to a negative error, slowly converging to a zero error.

So, from the observation of two successive iterations, we can try to obtain a forced convergence, as described in the second scheme of iteration. For the case, when we do a “forced” convergence, we get a satisfactory match at the second iteration (table 3).

In figure 8(a), a part of the data is shown along with the zero-offset model in figure 8(b). The result of the method can also be shown as wiggles (figure 8(c)), which give the nature and magnitude of the change in the elastic parameters. So from a 3D data volume, three separate data volumes can be created, each containing the change in a particular elastic parameter.
Figure 6: Regular 3D geometry: CMP(x)-CMP(y)-Offset domain

Figure 7: Convergence with iterations, for the case of Shale-Salt interface

- Change in successive M2-estimates: o---o
- Change in successive mu2-estimates: x---x
- Change in successive rho2-estimates: +---+

Change in parameters -->

Iteration # -->
Figure 8: Shale-Salt interface:
(a) Part of data: CMP gathers;
(b) Model: horizontal interface at a depth of 600 m.;
(c) Result: Traces containing the relative changes in the elastic parameters at the output location.
SHALE-SALT INTERFACE (REGULAR CASE)

INPUT PARAMETERS:

P-wave modulus of top-layer: 34.85  Number of traces: 600
S-wave modulus of top-layer: 12.29  Number of traces/CMP: 3
Density of top-layer: 2.4  Depth of the interface: 600.0
P-wave modulus of layer 2: 42.87  Minimum offset: 100
S-wave modulus of layer 2: 15.26  Maximum offset: 640
Density of layer below: 2.05

ACTUAL VALUES

<table>
<thead>
<tr>
<th>Δρ/ρ</th>
<th>ΔM/M</th>
<th>Δμ/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0368</td>
<td>0.06457</td>
<td>0.0109</td>
</tr>
</tbody>
</table>

Table 6: RESULTS FOR SHALE-SALT INTERFACE (WITHOUT FORCED CONVERGENCE)

<table>
<thead>
<tr>
<th>Iter #</th>
<th>Δρ/ρ</th>
<th>% Error in Δρ/ρ</th>
<th>ΔM/M</th>
<th>% Error in ΔM/M</th>
<th>Δμ/M</th>
<th>% Error in Δμ/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.248</td>
<td>57.96</td>
<td>0.296</td>
<td>43.68</td>
<td>0.105</td>
<td>38.15</td>
</tr>
<tr>
<td>2</td>
<td>-0.105</td>
<td>-33.12</td>
<td>0.156</td>
<td>-24.27</td>
<td>0.060</td>
<td>-21.05</td>
</tr>
<tr>
<td>3</td>
<td>-0.172</td>
<td>9.55</td>
<td>0.222</td>
<td>7.76</td>
<td>0.082</td>
<td>7.89</td>
</tr>
<tr>
<td>4</td>
<td>-0.143</td>
<td>-8.28</td>
<td>0.189</td>
<td>-8.25</td>
<td>0.071</td>
<td>-6.57</td>
</tr>
<tr>
<td>5</td>
<td>-0.157</td>
<td>0.00</td>
<td>0.205</td>
<td>-0.48</td>
<td>0.076</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 7: RESULTS FOR SHALE-SALT INTERFACE (WITH FORCED CONVERGENCE)

<table>
<thead>
<tr>
<th>Iter#</th>
<th>Δρ/ρ</th>
<th>% Error in Δρ/ρ</th>
<th>ΔM/M</th>
<th>% Error in ΔM/M</th>
<th>Δμ/M</th>
<th>% Error in Δμ/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.248</td>
<td>57.96</td>
<td>0.296</td>
<td>43.68</td>
<td>0.105</td>
<td>38.15</td>
</tr>
<tr>
<td>2</td>
<td>-0.157</td>
<td>0.00</td>
<td>0.211</td>
<td>2.42</td>
<td>0.078</td>
<td>2.63</td>
</tr>
</tbody>
</table>

CASE 1: 3 Traces/CMP, 600 Traces, SHALE_SALT Interface and Depth of 600 m.
In Case 2 and 3, we furnish the result for the third type of iteration, when the input to the program is only P-wave velocity in the top layer and the output are the relative changes in the elastic parameters.

CASE 2: SAND-SHALE INTERFACE:

In this case, the interface is between Sand and Shale. The change in the elastic parameters across the interface is small. The input to the program is the P-wave velocity in the first medium and the result obtained at the end of the third iteration is very good. We compare the result (estimates of change in P-wave modulus, S-wave modulus and density) and it matches with the actual values remarkably well, other than the S-wave modulus. In fact, the results from the second iteration and the third one do not differ much, meaning that in the second iteration itself we get reach good convergence with the correct values.

INPUT PARAMETERS:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P-wave modulus of top</td>
<td>32.67</td>
<td>P-wave modulus of layer 2:</td>
<td>34.85</td>
</tr>
<tr>
<td>layer</td>
<td></td>
<td>S-wave modulus of top</td>
<td>11.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>layer</td>
<td>12.29</td>
</tr>
<tr>
<td>S-wave modulus of top</td>
<td>11.92</td>
<td>density of top layer</td>
<td>2.47</td>
</tr>
<tr>
<td>layer</td>
<td></td>
<td>density of layer below</td>
<td>2.43</td>
</tr>
<tr>
<td>Depth of the interface</td>
<td>600.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Actual $\Delta \rho/\rho = -0.0368$ ; Actual $\Delta M/M = 0.0645$ ; $\Delta \mu/M = 0.0109$

Table 8: Results from inversion using the approximation

<table>
<thead>
<tr>
<th>Iter #</th>
<th>$\Delta \rho/\rho$</th>
<th>% Error in $\Delta \rho/\rho$</th>
<th>$\Delta M/M$</th>
<th>% Error in $\Delta M/M$</th>
<th>$\Delta \mu/M$</th>
<th>% Error in $\Delta \mu/M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0446</td>
<td>21.1985</td>
<td>0.0750</td>
<td>16.2011</td>
<td>0.0152</td>
<td>38.7389</td>
</tr>
<tr>
<td>2</td>
<td>-0.0365</td>
<td>-0.7199</td>
<td>0.0670</td>
<td>3.7067</td>
<td>0.0131</td>
<td>19.952</td>
</tr>
<tr>
<td>3</td>
<td>-0.0359</td>
<td>-2.5334</td>
<td>0.0663</td>
<td>2.6374</td>
<td>0.0129</td>
<td>17.2981</td>
</tr>
</tbody>
</table>
CASE 3: SHALE-SALT INTERFACE:

In this case, we consider a large change in M-value, the combination being Shale-Salt. The %error improves a lot from the first iteration to the second, and from the second to the third. The result after the third iteration is very good. When compared with the results from the first and the second scheme of iterations, we can see one large difference. We get a nice convergence within the first three iterations in the present scheme of iteration and this scheme needs only the P-wave velocity in the top layer as the input.

INPUT PARAMETERS:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-wave modulus of top layer</td>
<td>34.85</td>
</tr>
<tr>
<td>S-wave modulus of top layer</td>
<td>12.29</td>
</tr>
<tr>
<td>Density of top layer</td>
<td>2.4</td>
</tr>
<tr>
<td>Depth of the interface</td>
<td>600.0</td>
</tr>
</tbody>
</table>

P-wave modulus of layer 2: 42.87
S-wave modulus of layer 2: 15.26
Density of layer below: 2.06

Actual $\Delta \rho/\rho = -0.157303$; Actual $\Delta M/M = 0.206382$; $\Delta \mu/M = 0.0764282$.

Table 9: Result from inversion using the approximation

<table>
<thead>
<tr>
<th>Iter#</th>
<th>$\Delta \rho/\rho$</th>
<th>% Error in $\Delta \rho/\rho$</th>
<th>$\Delta M/M$</th>
<th>% Error in $\Delta M/M$</th>
<th>$\Delta \mu/M$</th>
<th>% Error in $\Delta \mu/M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.2443</td>
<td>55.3164</td>
<td>0.2959</td>
<td>43.3678</td>
<td>0.1053</td>
<td>37.8387</td>
</tr>
<tr>
<td>2</td>
<td>-0.1177</td>
<td>-25.1567</td>
<td>0.1691</td>
<td>-18.0531</td>
<td>0.0646</td>
<td>-15.5181</td>
</tr>
<tr>
<td>3</td>
<td>-0.1583</td>
<td>0.6411</td>
<td>0.2097</td>
<td>1.6270</td>
<td>0.0779</td>
<td>1.9836</td>
</tr>
</tbody>
</table>

Inversion scheme, discussed in chapter 3, is applied for 3D regular data for a horizontal reflector.

The summation of the scaled seismic data is obtained, using pre-stack Kirchhoff migration.

The method is first discussed with the assumption of the knowledge of elastic moduli in the top layer and then for the generalized case of assumption of the knowledge of only the P-wave velocity.
4.2: Irregular 3D data and weighting

A lot of time, we have to deal with seismic data which is not regular in CMP(x)-CMP(y)-Offset space. For doing the AVO analysis as described earlier, we need to regularize the data. If the data is not too irregular, then we can use the following method. by “not-too-irregular” we mean that, there is at least one trace per grid element of the regular grid we want to impose on the irregular data in CMP(x)-CMP(y)-Offset space. The following method will give a weight to the irregular data, so that we can deal with the data as if it is regular. A similar method has been discussed by Lu(1993) for 2D seismic data.

For a particular trace, we find out the nearest eight grid-points in the CMP(x)-CMP(y)-Offset space(figure 9) and then compute the relative volume of the space. This is done for all the eight grid-points. Then we compute a weight for a particular grid-point, inversely proportional to its distance from the trace. Then we multiply the trace by the weight
and put it at that grid-point. We explain below, how it is done: in the above diagram, the volume denotes the grid superposed on the cmp(x)-cmp(y)-h space.

\((x_t,y_t,h_t)\) denotes the co-ordinate of a trace in the grid-element.

\((x_1,y_1,h_1)\) is the co-ordinate of the grid-point, nearest to the trace.

dx: cmp(x) grid-spacing.

dy: cmp(y) grid-spacing.

In the following discussion:

\(r_{v1} = \text{relative volume of the i-th sub-element and} \)

\(w_i = \text{weight for the i-th grid-point.} \)

\[
rv_1 = w_7 = \frac{(x_1-xt) \cdot (y_1-yt) \cdot (h_1-h)}{dx \cdot dy \cdot dh} \quad (4.3a)
\]

\[
rv_2 = w_8 = \frac{(dx - (x_1-xt)) \cdot (y_1-yt) \cdot (h_1-h)}{dx \cdot dy \cdot dh} \quad (4.3b)
\]

\[
rv_3 = w_5 = \frac{(dx - (x_1-xt)) \cdot (dy - (y_1-yt)) \cdot (h_1-h)}{dx \cdot dy \cdot dh} \quad (4.3c)
\]

\[
rv_4 = w_6 = \frac{(x_1-xt) \cdot (dy - (y_1-yt)) \cdot (h_1-h)}{dx \cdot dy \cdot dh} \quad (4.3d)
\]

\[
r_{v5} = w_3 = \frac{(x_1-xt) \cdot (y_1-yt) \cdot (dh - (h_1-h))}{dx \cdot dy \cdot dh} \quad (4.3e)
\]

\[
r_{v6} = w_4 = \frac{(dx - (x_1-xt)) \cdot (y_1-yt) \cdot (dh - (h_1-h))}{dx \cdot dy \cdot dh} \quad (4.3f)
\]

\[
r_{v7} = w_1 = \frac{(dx - (x_1-xt)) \cdot (dy - (y_1-yt)) \cdot (dh - (h_1-h))}{dx \cdot dy \cdot dh} \quad (4.3g)
\]

\[
r_{v8} = w_2 = \frac{(x_1-xt) \cdot (y_1-yt) \cdot (dh - (h_1-h))}{dx \cdot dy \cdot dh} \quad (4.3h)
\]

In this method, we divide the seismic data (traces) in the CMP(x)-CMP(y)-Offset space into number of grids. Then each trace is divided (weighted by fractional weights) and put at the
eight nearest grid-points. Then after this operation is repeated for all the traces, we essentially deal with traces, mapped into regular grids.

This algorithm of weighting works only for that case, when the extent of irregularity is not too much, e.g. when there is at least a single trace per grid element. In following examples, the data we use, each of them satisfies the above condition.

4.2.1: Results

Case 4:

In this case, Shale-Salt combination of media across the interface is considered. But this time the data, dealt with, is irregular in nature, which is irregular in CMP(x)-CMP(y)-Offset space (figure 12). The method discussed in the previous section is used on the irregular data to check the performance. The result obtained is close to the correct value and the values obtained, when the data is regular. But this is due to the fact that, the data is not very irregular.

Case 5:

According to the algorithm given above (section 4.2), we computed the weights to regularize the data, by mapping the data into a regular grid-space. Then the data were weighted by these computed weights and the same method (discussed in the section 4.1) is applied to find the elastic parameters. As can be seen from the results obtained from both first and second kind of iteration schemes, the results are much better compared to the results obtained in the case 4 (for the irregular data) and the results obtained in the final iteration are much closer to the actual values.
Irregular 3D geometry: CMP(x)-CMP(y)-Offset domain
SHALE-SALT INTERFACE (IRREGULAR CASE):

INPUT PARAMETERS:

P-wave modulus of top-layer: 34.85
S-wave modulus of top-layer: 12.29
Density of top-layer: 2.4
P-wave modulus of layer 2: 42.87
S-wave modulus of layer 2: 15.26
Density of layer below: 2.05

Number of traces: 600
Traces/CMP: 3
Depth of the interface: 600.0
Minimum offset: 100
Maximum offset: 640.0

ACTUAL VALUES

<table>
<thead>
<tr>
<th>Δρ/ρ</th>
<th>ΔM/M</th>
<th>Δμ/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0368</td>
<td>0.06457</td>
<td>0.0109</td>
</tr>
</tbody>
</table>

Table 10: RESULTS FOR SHALE-SALT INTERFACE (WITHOUT FORCED CONVERGENCE)

<table>
<thead>
<tr>
<th>Iter #</th>
<th>Δρ/ρ</th>
<th>% Error in Δρ/ρ</th>
<th>ΔM/M</th>
<th>% Error in ΔM/M</th>
<th>Δμ/M</th>
<th>% Error in Δμ/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.227</td>
<td>44.82</td>
<td>0.276</td>
<td>33.99</td>
<td>0.092</td>
<td>30.50</td>
</tr>
<tr>
<td>2</td>
<td>-0.096</td>
<td>-38.80</td>
<td>0.145</td>
<td>-29.23</td>
<td>0.056</td>
<td>-25.44</td>
</tr>
<tr>
<td>3</td>
<td>-0.157</td>
<td>0.193</td>
<td>0.206</td>
<td>0.297</td>
<td>0.0769</td>
<td>1.22</td>
</tr>
<tr>
<td>4</td>
<td>-0.128</td>
<td>-18.08</td>
<td>0.176</td>
<td>-14.39</td>
<td>0.069</td>
<td>-11.86</td>
</tr>
<tr>
<td>5</td>
<td>-0.142</td>
<td>-9.00</td>
<td>0.191</td>
<td>-7.27</td>
<td>0.071</td>
<td>-5.40</td>
</tr>
</tbody>
</table>

Table 11: RESULTS FOR SHALE-SALT INTERFACE (WITH FORCED CONVERGENCE)

<table>
<thead>
<tr>
<th>Iter #</th>
<th>Δρ/ρ</th>
<th>% Error in Δρ/ρ</th>
<th>ΔM/M</th>
<th>% Error in ΔM/M</th>
<th>Δμ/M</th>
<th>% Error in Δμ/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.227</td>
<td>44.82</td>
<td>0.276</td>
<td>33.99</td>
<td>0.092</td>
<td>30.50</td>
</tr>
<tr>
<td>2</td>
<td>-0.133</td>
<td>-15.07</td>
<td>0.176</td>
<td>-14.51</td>
<td>0.065</td>
<td>-14.26</td>
</tr>
</tbody>
</table>

CASE 4: IRREGULAR GEOMETRY
SHALE-SALT INTERFACE (IRREGULAR_WEIGHTED):

INPUT PARAMETERS:

- P-wave modulus of top-layer: 34.85
- S-wave modulus of top-layer: 12.29
- Density of top-layer: 2.4
- P-wave modulus of layer 2: 42.87
- S-wave modulus of layer 2: 15.26
- Density of layer below: 2.05
- Number of traces: 600
- Traces/CMP: 3
- Depth of the interface: 600.0
- Minimum offset: 100
- Maximum offset: 640.0

ACTUAL VALUES

<table>
<thead>
<tr>
<th>Δρ/ρ</th>
<th>ΔM/M</th>
<th>Δμ/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0368</td>
<td>0.06457</td>
<td>0.0109</td>
</tr>
</tbody>
</table>

Table 12: RESULTS FOR SHALE-SALT INTERFACE (WITHOUT FORCED CONVERGENCE)

<table>
<thead>
<tr>
<th>Iter #</th>
<th>Δρ/ρ</th>
<th>% Error in Δρ/ρ</th>
<th>ΔM/M</th>
<th>% Error in ΔM/M</th>
<th>Δμ/M</th>
<th>% Error in Δμ/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.227</td>
<td>44.82</td>
<td>0.293</td>
<td>42.70</td>
<td>0.0102</td>
<td>34.94</td>
</tr>
<tr>
<td>2</td>
<td>-0.114</td>
<td>-27.04</td>
<td>0.151</td>
<td>-26.36</td>
<td>0.056</td>
<td>-25.67</td>
</tr>
<tr>
<td>3</td>
<td>-0.162</td>
<td>3.28</td>
<td>0.221</td>
<td>7.73</td>
<td>0.082</td>
<td>8.41</td>
</tr>
<tr>
<td>4</td>
<td>-0.128</td>
<td>-18.08</td>
<td>0.186</td>
<td>-9.55</td>
<td>0.066</td>
<td>-12.34</td>
</tr>
<tr>
<td>5</td>
<td>-0.152</td>
<td>-2.88</td>
<td>0.204</td>
<td>-0.71</td>
<td>0.075</td>
<td>-1.02</td>
</tr>
</tbody>
</table>

Table 13: RESULTS FOR SHALE-SALT INTERFACE (WITH FORCED CONVERGENCE)

<table>
<thead>
<tr>
<th>Iter #</th>
<th>Δρ/ρ</th>
<th>% Error in Δρ/ρ</th>
<th>ΔM/M</th>
<th>% Error in ΔM/M</th>
<th>Δμ/M</th>
<th>% Error in Δμ/Μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.227</td>
<td>44.82</td>
<td>0.293</td>
<td>42.70</td>
<td>0.0102</td>
<td>34.94</td>
</tr>
<tr>
<td>2</td>
<td>-0.157</td>
<td>0.19</td>
<td>0.207</td>
<td>0.85</td>
<td>0.075</td>
<td>-1.54</td>
</tr>
</tbody>
</table>

CASE 5: IRREGULAR DATA, MAPPED ONTO A REGULAR GRID
WITH WEIGHT COMPUTED.
4.3: Weighting after constant-offset prestack migration and
extraction of elastic parameters

As we went through the last section (4.1), finding the estimate of the changes
in the elastic parameter, one thing became obvious: it is not feasible to iterate for a number of
times, especially as each iteration causes recomputation of the (3x3) matrix and three pre-
stack migrations. Also to deal with the effect of dip of the interface, we have to do a migration
in order to have an idea of the dip.

In order to take care of these difficulties, there can be another way of approaching the problem. Say, we want to find the desired results at a particular location. We have con-
stant-fold coverage in the area around the location. We can then use a pre-stack constant-
offset migration scheme to create a migrated offset gather at the output location. From the
migration, we can find the dip of the interface. Now, once we have the offset gather and know
the dip-value, we can create the angle of incidence vs. the reflection coefficient data for the
output location. If we know the dip and the offset values, we can find the corresponding angle
of incidence for a particular offset value as:

\[
\sin i_1 = \frac{h \cdot \cos \theta}{vt} \quad \ldots \ldots \ldots \ (4.6)
\]

where, \(h\)=offset; \(\theta\)=dip, \(vt\) = travel-time from shot-to-receiver and \(i_1\)=angle of incidence.

Once we have the angle of incidence vs. the reflection coefficient data, we can just
use the methodology described in section 3.1 to find out the estimate the changes in the elastic
parameters. This is a step towards the generalization to the approach described in the section
4.1, in order to tackle the problems of dip and requirements of multiple migrations.
Figure 11: Flowchart for post-migration (pre-stack constant offset migration) weighting and estimation of elastic parameters.
4.3.1: Results

Case 1:

In the first case we consider the following actual values of relative changes in elastic parameters:

$$\Delta \rho/\rho = -0.157303; \quad \Delta M/M = 0.206382; \quad \Delta \mu/M = 0.0764282.$$

Below we compare the results (Table 14 and Table 15) from the approaches discussed in the sections 4.1 and 4.4. From the comparison, it becomes obvious that the second method gives much better result both in terms of execution time and the results obtained, if the result obtained at the end of third iteration is compared from both the approaches.

### Table 14: Weighting before migration:

<table>
<thead>
<tr>
<th>Iter#</th>
<th>$\Delta \rho/\rho$</th>
<th>% error in $\Delta \rho/\rho$</th>
<th>$\Delta M/M$</th>
<th>%error in $\Delta M/M$</th>
<th>$\Delta \mu/M$</th>
<th>% error in $\Delta \mu/M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.2398</td>
<td>52.4621</td>
<td>0.2912</td>
<td>41.0772</td>
<td>0.1038</td>
<td>35.7892</td>
</tr>
<tr>
<td>2</td>
<td>-0.0316</td>
<td>-79.9261</td>
<td>0.0816</td>
<td>-60.4855</td>
<td>0.0335</td>
<td>-56.2282</td>
</tr>
<tr>
<td>3</td>
<td>-0.1631</td>
<td>3.6693</td>
<td>0.2142</td>
<td>3.8051</td>
<td>0.0793</td>
<td>3.7976</td>
</tr>
</tbody>
</table>

TOTAL TIME ELAPSED: 114.230 sec

### Table 15: Weighting after migration

<table>
<thead>
<tr>
<th>Iter#</th>
<th>$\Delta \rho/\rho$</th>
<th>% error in $\Delta \rho/\rho$</th>
<th>$\Delta M/M$</th>
<th>%error in $\Delta M/M$</th>
<th>$\Delta \mu/M$</th>
<th>% error in $\Delta \mu/M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.2397</td>
<td>52.4087</td>
<td>0.2911</td>
<td>41.0360</td>
<td>0.1037</td>
<td>35.7168</td>
</tr>
<tr>
<td>2</td>
<td>-0.1037</td>
<td>-34.0662</td>
<td>0.1546</td>
<td>-25.0993</td>
<td>0.0595</td>
<td>-22.0988</td>
</tr>
<tr>
<td>3</td>
<td>-0.1538</td>
<td>-2.2293</td>
<td>0.2049</td>
<td>-0.7154</td>
<td>0.0763</td>
<td>-0.2187</td>
</tr>
</tbody>
</table>

TOTAL TIME ELAPSED: 57.7900 sec
Case 2:

In the second case, we consider much smaller values of relative changes in the elastic parameters across the interface:

\[ \Delta \rho/\rho = -0.064516; \quad \Delta M/M = 0.109061; \quad \Delta \mu/M = 0.0534455. \]

From the results obtained, the first thing that becomes clear is that, as with the Case 1, the time required is almost half of the time taken in using the method described in section 4.1. The results obtained using the present method are also much better compared to the results obtained using the previous method..

**Table 16: Weighting before migration** : TOTAL TIME ELAPSED: 114.080sec

<table>
<thead>
<tr>
<th>Iter#</th>
<th>( \Delta \rho/\rho )</th>
<th>% error in ( \Delta \rho/\rho )</th>
<th>( \Delta M/M )</th>
<th>% error in ( \Delta M/M )</th>
<th>( \Delta \mu/M )</th>
<th>% error in ( \Delta \mu/M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0955</td>
<td>47.9896</td>
<td>0.1417</td>
<td>29.8940</td>
<td>0.0648</td>
<td>21.2587</td>
</tr>
<tr>
<td>2</td>
<td>-0.0398</td>
<td>-38.3275</td>
<td>0.0858</td>
<td>-21.3505</td>
<td>0.0430</td>
<td>-19.5336</td>
</tr>
<tr>
<td>3</td>
<td>-0.0599</td>
<td>-7.2259</td>
<td>0.1059</td>
<td>-2.8705</td>
<td>0.0512</td>
<td>-4.2668</td>
</tr>
</tbody>
</table>

**Table 17: Weighting after migration** : TOTAL TIME ELAPSED: 63.3200sec

<table>
<thead>
<tr>
<th>Iter#</th>
<th>( \Delta \rho/\rho )</th>
<th>% error in ( \Delta \rho/\rho )</th>
<th>( \Delta M/M )</th>
<th>% error in ( \Delta M/M )</th>
<th>( \Delta \mu/M )</th>
<th>% error in ( \Delta \mu/M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0950</td>
<td>47.2671</td>
<td>0.1412</td>
<td>29.4649</td>
<td>0.0647</td>
<td>20.9678</td>
</tr>
<tr>
<td>2</td>
<td>-0.0633</td>
<td>-1.9387</td>
<td>0.1094</td>
<td>0.2851</td>
<td>0.0525</td>
<td>-1.7214</td>
</tr>
<tr>
<td>3</td>
<td>-0.0696</td>
<td>7.8515</td>
<td>0.1157</td>
<td>6.0954</td>
<td>0.0550</td>
<td>2.8984</td>
</tr>
</tbody>
</table>

In order to avoid computation due to the pre-stack migration at each stage of iteration as discussed in chapter 4.1, a method is proposed to apply pre-stack constant offset migration to create a gather at an output location and apply post-migration weighting and inversion to obtain the changes in the elastic parameters.
4.4: Generalized method of extracting elastic parameters

The method described in the previous section, takes care of the difficulty in applying the method described in section 3.2 of using three pre-stack migration at each stage of iteration, but there can be some difficulty even with the method described in the section 3.3, which may be due to the usage of constant-offset pre-stack migration. If there are a number of offset coverages, then it may be very costly. A new way of approaching the problem is proposed below.

In this method, we try to "pack" the amplitude value at a particular output location as a function of offset, using a similar method as in section 3.2, but this time we try to fit the data to a function of offset, as given below

\[
R(h) = A + B \cdot h^2 + C \cdot h^4 \quad \ldots \ldots \ldots \quad (4.7)
\]

We use least-square approach to fit the observed data to R(h), which results in three normal equations, given in the matrix form below

\[
\begin{bmatrix}
  n & \Sigma h^2 & \Sigma h^4 \\
  \Sigma h^2 & \Sigma h^4 & \Sigma h^6 \\
  \Sigma h^4 & \Sigma h^6 & \Sigma h^8
\end{bmatrix}
\begin{bmatrix}
  A \\
  B \\
  C
\end{bmatrix}
= 
\begin{bmatrix}
  \Sigma (R) \\
  \Sigma (R \cdot h^2) \\
  \Sigma (R \cdot h^4)
\end{bmatrix} \quad \ldots \ldots \quad (4.8)
\]

Here also, we migrate the data scaling the data with three weights: 1, h^2 and h^4, for a particular output location and from the knowledge of the offset values, we can compute the (3x3) matrix. So we can find the unknowns: A, B and C.

Among the three migrations, the first migration is a migration of the data itself.
Read in h-vs.-R(h) data

Find the biggest offset

Normalize the offset values w.r.t. the biggest offset values and Create the (3 x 3) matrix.

Weight the data with the weights: \( I, h^2 \) and \( h^4 \) and Migrate the data to an output location.

Invert the (3 x 3) matrix and find the least-square coefficients for the output location.

From the migrated section find the dip of the interface

Using the dip-information and the least-square coefficients create the angle of incidence vs. reflection coefficient data.

Create the (3 x 3) matrix, from the computed values of the angles of incidence

Weight the R(i) data by weights \( w_1, w_2 \) and \( w_3 \) and obtain the sums to create the (3 x 1) array.

Update the (3x3) and (3x1) matrices

Invert to obtain the estimates the relative changes in the elastic parameters and from them, obtain the angle of deviation \( \Delta i \), from the angle of incidence \( i \).

END

Figure 12: Flowchart for generalized way of estimating the elastic parameters from multifold 3D data.
So from it, we can find out the dip of the interface.

Now knowing the dip and the offset-values, we can find the angle of incidence. From the values of A, B and C, we can construct the amplitude values each of the offset or angle of incidence values. Once the angle of incidence vs. reflection coefficient data is constructed, we can use the method described in the section 3.1 to find the desired parameters.

**4.5.1 Results:**

Input Parameter: $\Delta \rho/\rho = -0.1573$; $\Delta M/M = 0.2063$; $\Delta \mu/M = 0.07642$

**Table 21: Case 1: Dip = 0 degree**

<table>
<thead>
<tr>
<th>Iter#</th>
<th>$\Delta \rho/\rho$</th>
<th>% error in $\Delta \rho/\rho$</th>
<th>$\Delta M/M$</th>
<th>% error in $\Delta M/M$</th>
<th>$\Delta \mu/M$</th>
<th>% error in $\Delta \mu/M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.1140</td>
<td>27.4893</td>
<td>0.1579</td>
<td>23.4763</td>
<td>0.0582</td>
<td>23.7880</td>
</tr>
<tr>
<td>2</td>
<td>-0.1203</td>
<td>23.5196</td>
<td>0.1642</td>
<td>20.4283</td>
<td>0.0603</td>
<td>21.0736</td>
</tr>
<tr>
<td>3</td>
<td>-0.1555</td>
<td>1.1095</td>
<td>0.1996</td>
<td>3.24537</td>
<td>0.0718</td>
<td>6.02380</td>
</tr>
</tbody>
</table>

**Table 22: Case 2: Dip = 10 degree**

<table>
<thead>
<tr>
<th>Iter#</th>
<th>$\Delta \rho/\rho$</th>
<th>% error in $\Delta \rho/\rho$</th>
<th>$\Delta M/M$</th>
<th>% error in $\Delta M/M$</th>
<th>$\Delta \mu/M$</th>
<th>% error in $\Delta \mu/M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.2136</td>
<td>-35.7917</td>
<td>0.2474</td>
<td>-19.8812</td>
<td>0.0837</td>
<td>-9.6177</td>
</tr>
<tr>
<td>2</td>
<td>-0.1387</td>
<td>11.7902</td>
<td>0.1715</td>
<td>16.9010</td>
<td>0.0594</td>
<td>22.2395</td>
</tr>
<tr>
<td>3</td>
<td>-0.1553</td>
<td>1.21142</td>
<td>0.1884</td>
<td>8.69055</td>
<td>0.0649</td>
<td>14.9770</td>
</tr>
</tbody>
</table>

**Table 23: Dip = 15 degree**

<table>
<thead>
<tr>
<th>Iter#</th>
<th>$\Delta \rho/\rho$</th>
<th>% error in $\Delta \rho/\rho$</th>
<th>$\Delta M/M$</th>
<th>% error in $\Delta M/M$</th>
<th>$\Delta \mu/M$</th>
<th>% error in $\Delta \mu/M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.2338</td>
<td>-48.6747</td>
<td>0.2698</td>
<td>-30.7376</td>
<td>0.0953</td>
<td>-24.7598</td>
</tr>
<tr>
<td>2</td>
<td>-0.1382</td>
<td>12.0853</td>
<td>0.1718</td>
<td>16.7312</td>
<td>0.0624</td>
<td>18.3476</td>
</tr>
<tr>
<td>3</td>
<td>-0.1643</td>
<td>-4.4997</td>
<td>0.1988</td>
<td>3.6716</td>
<td>0.0717</td>
<td>6.14313</td>
</tr>
</tbody>
</table>
4.5: Migration Velocity Analysis

The above method can also be utilized to do a migration velocity analysis. We have to start with a good guess of the velocity, from the previous velocity analysis done on the data. We can use this initial velocity to do the “packing” of the amplitude value and find the values of A, B and C, as in equation(4.7). We can plot the three migrated traces (resulting from the migration of the traces scaled respectively by 1, (normalized offset-value)^2 and (normalized offset-value)^4), which have their peak-amplitudes as A, B and C. If these three peaks line up (figure 13b), then we can come to the conclusion that the velocity used for the migration is correct. If the three peaks create “concave-upward” formation (figure 13a), then

![Figure 13: Migration velocity analysis](image)

(a) Migration with a smaller velocity;
(b) Migration with the correct velocity;
(c) Migration with a bigger velocity.
the velocity used for the migration is smaller than the correct velocity. If the three peaks create a “convex-upward” formation (figure 13c), then the velocity used for the migration is bigger than the correct velocity.

We can also do the migration velocity analysis, by checking on the phase-angles of the three migrated traces. We take the Hilbert Transform of the three migrated traces and compare the phase-angles of the same samples from the three migrated traces. If the phase-angles are very similar, then the velocity used for migration, is a good approximation to the actual velocity. In figure 14, we show the result of this scheme. Here the correct velocity is 3812.62 m/sec. We find out the sample-number corresponding to the highest amplitude of the first migrated trace. Let’s say, that is “isam”. Then, we plot the phase-angle corresponding to the samples “isam-2” to “isam+2” for each of the three migrated traces. In the figure 14, we can see that phase-angle values for the same sample number are very different for the three traces, for the cases when the velocity used for the migration is far from the correct velocity (4500 and 3000). The difference decreases, as the difference of the migration velocity from the correct velocity decreases (4000 and 3500). For the case, when we use a velocity of 3810 m/sec (which is almost the same value as the correct velocity), the difference between the phase angles is very small.

Using Hilbert Transform for each of the three migrated trace (migrated with traces weighted with 1.0, h^2 and h^4), we can get the corresponding phase angles. But in stead of finding the phase-angle, we compute the corresponding real and imaginary parts (say, a and b, where phase angle is atan(b/a)).
Figure 14: Migration velocity analysis: Comparison of the phase angles
(a) Much bigger velocity;
(b) Much smaller velocity;
(c) Bigger velocity (close to correct velocity);
(d) Smaller velocity (close to correct velocity);
(e) Approximately correct velocity.
For a particular velocity, \( v_1 \), we find the real and imaginary parts for each traces: For migrated trace, obtained with scaling the input data with 1.0, let the real and imaginary parts be \( a_1 \) and \( b_1 \). Then for the migrated trace, obtained with scaling the input data with \( h^2 \), let the real and imaginary parts be \( a_2 \) and \( b_2 \). Similarly, for the migrated trace, obtained with scaling the input data with \( h^4 \), let the real and the imaginary parts be \( a_3 \) and \( b_3 \).

Then, for the velocity \( v_1 \) we define a variable, say, \( d_{11} \) as:

\[
d_{11} = a_1 b_2 - a_2 b_1; \text{ and another variable } d_{12}, \text{ as } d_{12} = a_1 b_3 - a_3 b_1.
\]

Then, for another velocity \( v_2 \), we find the variables, \( d_{21} \) and \( d_{22} \), which are similar to \( d_{11} \) and \( d_{12} \) respectively.

From the knowledge of \( d_{11} \) and \( d_{21} \), we try to interpolate the value of the velocity, for which we will get a zero-value for \( d \). We can get the velocity as:

\[
v_{in1} = \frac{(v_1 \cdot d_{21}) - (v_2 \cdot d_{11})}{(d_{21} - d_{11})}
\]

Similarly from the, other values of \( d \) (namely, \( d_{12} \) and \( d_{22} \)), we can get another measure of velocity as:

\[
v_{in2} = \frac{(v_1 \cdot d_{22}) - (v_2 \cdot d_{12})}{(d_{22} - d_{12})}
\]

So, we can get a measure of correct velocity as:

\[
v = \frac{(v_{in1} + v_{in2})}{2}
\]

We used the interpolation of the velocities obtained from equality of the
phase-angles for the same case, described in the figure 14. The correct velocity is 3810.62.

We compute the phase-angles for the three traces for each of the two velocities (3500 and 4000), by using Hilbert Transform. Then for each of the two velocities, we find the difference in the phase-angle values at each sample, between trace#1 and trace#2 & trace#1 and trace#3. Then from these two differences, we arrive at the value of the velocity, for which the difference in the phase angle will be zero. So after the above-mentioned steps, we obtain two velocities (one from using the difference of phase-angle between trace#1 and trace#2 and the other from using trace#1 and trace#3).

In the following table, we present the result for seven sample points, the peak (the reflecting horizon) being at sample 73. We get a very good match between the predicted value 3737.06m/sec and the correct value 3810.62 m/sec.

Table 22: Velocity from interpolation of the results obtained from using 3500 m/s and 4000m/s for migration velocities

<table>
<thead>
<tr>
<th>Wavelet</th>
<th>Sample #</th>
<th>Velocity from Trace#1 and Trace#2</th>
<th>Velocity from Trace#1 and Trace#3</th>
<th>Average velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>70</td>
<td>3858.73</td>
<td>3847.32</td>
<td>3853.03</td>
</tr>
<tr>
<td></td>
<td>71</td>
<td>3825.89</td>
<td>3812.73</td>
<td>3819.31</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>3791.04</td>
<td>3775.87</td>
<td>3783.45</td>
</tr>
<tr>
<td></td>
<td>73</td>
<td>3746.04</td>
<td>3729.16</td>
<td>3737.06</td>
</tr>
<tr>
<td></td>
<td>74</td>
<td>3702.33</td>
<td>3681.99</td>
<td>3692.16</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>3664.94</td>
<td>3641.50</td>
<td>3653.22</td>
</tr>
<tr>
<td></td>
<td>76</td>
<td>3634.06</td>
<td>3610.73</td>
<td>3622.39</td>
</tr>
</tbody>
</table>
4.6: Finding the unit normal vector to the dipping plane

After finding the correct migration velocity; for a particular output location, we can find out the dip of the reflector, using migration.

![Diagram](image)

**Figure 15: Relation between Shot, Receiver and Output location**

In figure 15, S denotes the shot-location and R denotes the receiver-location for a particular trace. Let’s say, O is the output location.

We find out the unit vector $u_s$ between the shot and the output sample, which is given by:

$$u_s = i \left[ \frac{(x_o - x_s)}{s} \right] + j \left[ \frac{(y_o - y_s)}{s} \right] + k \left[ \frac{(z_o - z_s)}{s} \right]$$
and \( u_r \) between the receiver and the output sample, given by:

\[
u_r = \hat{i} \left[ \frac{(x_o - x_r)}{r} \right] + \hat{j} \left[ \frac{(y_o - y_r)}{r} \right] + \hat{k} \left[ \frac{(z_o - z_r)}{r} \right]
\]

So, we can find out the direction of the normal to the Shot-receiver line from the output sample (not necessarily going through the mid-point), as \( u_{sr} \):

\[
u_{sr} = u_s + u_r = \hat{i} \cdot a + \hat{j} \cdot b + \hat{k} \cdot c
\]

Then, for a particular output sample, for each trace (i.e. each shot-receiver combination), we will scale the amplitude at the input time sample for the trace by the weights a, b and c; and do the migration at that particular output sample. After repeating the same action for all the traces, we will get three migrated values at the output sample,

---

**Figure 16: Correct normal, computed normal and the deviation between them**

\[O'\] Surface

\[N\] Normal at the output location

\[\theta\] Deviation between \(N\) and \(n\)

\[\theta\] Normal at the output location, estimated using migration.
say \( n_1, n_2 \) and \( n_3 \). Then the vector \( \mathbf{n} = n_1 \mathbf{i} + n_2 \mathbf{j} + n_3 \mathbf{k} \), will give the estimated normal to the dipping plane.

The advantage of this method is that this method can be used for each of the output sample separately. Once the normal vector to the dipping plane is known, we can find out the dip and the azimuth of the dipping plane. The accuracy of the method can be checked by finding the deviation between the computed normal-vector and the correct normal-vector, as shown in the figure 16. In table 23, we present results for several cases. In the first three cases, we generated 2400 synthetic traces (20 lines, 40 CMPs/line, 3 traces/CMP) for three different combinations of dip and azimuth of the reflecting plane. For each of the cases, we get very good results, reflected from the deviation-values in the last column of the table. There is one point to remember that the requirement of the number of the traces depends on the dip and the depth of the reflector. So for the cases when we consider the reflectors, at the same depth, but with greater value of dip, e.g. the last case, we used 9000 traces (50 lines, 60 CMPs/line, 3 traces/CMP). For this case also we get a very good match between the computed normal direction and the correct normal direction to the reflecting interface.

Table 23: Results obtained in finding the orientation of the normal to a dipping plane

<table>
<thead>
<tr>
<th>Number of traces used</th>
<th>Azimuth of the dipping plane (in degrees)</th>
<th>Dip of the dipping plane (in degrees)</th>
<th>Deviation of the computed normal from the correct normal (in degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2400</td>
<td>10</td>
<td>0</td>
<td>0.693516</td>
</tr>
<tr>
<td>2400</td>
<td>10</td>
<td>5</td>
<td>0.811566</td>
</tr>
<tr>
<td>9000</td>
<td>20</td>
<td>10</td>
<td>0.919198</td>
</tr>
</tbody>
</table>
Chapter 5

TRANSVERSE ISOTROPY

AND

VERTICAL FRACTURES

Anisotropy can be defined as the variation of physical properties of a medium, depending on the orientation of measurement direction.

In exploration seismology, the medium for wave-propagation is usually assumed to be isotropic; though almost all the time it not quite so. This is mostly because wave-propagation in anisotropic media is a complicated subject and assumption of isotropy works well in most of the time. So, for the reasons of economy and speed, the subject of anisotropy was not used in the industry for a long time, until recently, when consideration of anisotropy in several cases pointed out the reasons previous exploration failures. Even then, full anisotropy is not considered at present, once again due to the reasons of economy and speed. So, approximations of anisotropy, e.g., transverse isotropy (azimuthal and polar) are assumed. Thomsen(1986, 1988) described the concept of weak elastic anisotropy and azimuthal isotropy. Banik(1987) defined an effective anisotropic parameter, coined after several approximations to the coefficients from weak elastic anisotropy. Blangy and Nur(1992) presented a similar but different expression for P-wave reflection coefficient in Transversely Isotropic media, with consideration different range of angular coverage. Kim et al.(1992) described the results on the effects of Transverse Isotropy on
gas-sand reflection coefficients.

In the context of seismic exploration, we are interested in a particular context of anisotropy, which is seismic anisotropy rather than intrinsic anisotropy. Seismic anisotropy occurs when an ordered arrangement of elements (e.g. fractures) are small compared to the wavelength employed, because the seismic waves cannot "see" the above-mentioned features, but can "feel" them, giving rise to seismic anisotropy. Explorationists are interested in large scale transverse isotropy. Transverse isotropy is defined as the state, when material properties do not change in two direction, but changes in the third direction. Horizontal thin layering (polar transverse isotropy) and vertical fractures (azimuthal transverse isotropy) are two examples.

For defining wave-propagation in an isotropic medium, one needs to define two elastic moduli: $\lambda$ and $\mu$, but for describing the wave-propagation in anisotropic media one needs to

![Figure 17: An isotropic medium overlying a vertically fractured medium](image-url)
define three more elastic moduli: \( \alpha \), \( \beta \) and \( \gamma \). For a transverse isotropic medium, Synge (1957) showed that the parameters can be expressed as:

\[
C_{pqrst} = \lambda \cdot \delta_{pq} \cdot \delta_{rs} + \mu \left( \delta_{pr} \cdot \delta_{qs} + \delta_{ps} \cdot \delta_{qr} \right) + \alpha \left( \delta_{pq} \cdot J_r \cdot J_s + \delta_{rs} \cdot J_p \cdot J_q \right) + \\
\beta \left( \delta_{pr} \cdot J_q \cdot J_s + \delta_{qr} \cdot J_p \cdot J_s + \delta_{ps} \cdot J_q \cdot J_r + \delta_{qs} \cdot J_p \cdot J_r \right) + \gamma \cdot J_{pq} \cdot J_q \cdot J_r \cdot J_s \ldots (5.1)
\]

\( C_{pqrst} \) are the 81 constants, required to describe a general elastic material. \( J_i \) are the direction vectors and \( \delta_{kl} \) are Kronecker delta. Otaibi et al (1995) has shown that the reflection coefficient from an interface above a vertically fractured medium (figure 17), can be approximated as:

\[
R = \left( -\frac{1}{4} \right) \left( \frac{\Delta \rho}{\rho} \right) \left( 1 - \tan^2 i \right) + \left( -\frac{1}{4} \right) \left( \frac{\Delta M}{M} \right) \left( 1 + \tan^2 i \right) + 2 \left( \frac{\Delta \mu}{M} \right) \left( \sin^2 i \right) + \\
\left[ 2 \left( \frac{\Delta \alpha}{M} \right) \tan i + 2 \left( \frac{\Delta \beta}{M} \right) \sin i \left( 1 - \tan^2 i \right) - \left( \frac{\Delta \gamma}{M} \sin^2 i \tan^2 i \right) \cos^2 \phi \right] (5.2)
\]

where

- \( R \) = P-wave reflection coefficient,
- \( \rho \) = ray-parameter,
- \( M \) = average of P-wave moduli,
- \( \Delta M \) = difference in P-wave moduli,
- \( \mu \) = average of S-wave moduli,
- \( \Delta \mu \) = difference in S-wave moduli,
- \( \rho \) = average of densities,
- \( \Delta \rho \) = difference of densities,
- \( i \) = average of angles of incidence and transmittance,
- \( \Delta \alpha, \Delta \beta, \Delta \gamma \) = changes in the elastic moduli of transverse isotropy;
Figure 18(a): Plot of angle of incidence vs. P-wave reflection coefficient for an interface between an isotropic medium and a vertically fractured medium (transverse isotropic medium), (parallel and perpendicular to the fracture orientation). [Taken from Otaibi et al.]

Figure 18(b): Traces parallel to the fracture

Figure 18(c): Traces perpendicular to the fracture
\( \alpha, \beta, \gamma \) = average of elastic moduli of transverse isotropy;

\( \phi \) = azimuth.

Figure 18 shows the variation of reflection coefficient parallel to and perpendicular to the fractures (using the above equation). The plots show that, that parallel to the fractures the reflection coefficients are same as the ones for the isotropic case, obtained by using the Zoeppritz equation.

A similar approach (as in AVO) can be utilized to find out the orientation of the vertical fractures for the case, when an isotropic medium overlies a vertically fractured medium.

Gardner (Nov, 1993) has proposed an idea. For clarity of what we are going to do, we are going to discuss the theory in brief.

As shown in figure 20, the energy reflected from a vertically fractured formation for a given angle of incidence has a maximum, when the waves travel perpendicular to the fracture-orientation.

The procedure to identify the orientation of vertical fractures is based on a similar procedure, as in section 3.1. But this time the weights being dependent on azimuth and offset, which means that we try obtain a fit of the reflection amplitude with azimuth and offset.

We try to obtain a fit to the migrated amplitude \( A_m \), with half-offset(h) and
azimuth(\(\phi\)), as:

\[
A_m = A + B \cdot hx \cdot hx + C \cdot hx \cdot hy + D \cdot hy \cdot hy \ldots \ldots (5.3)
\]

the relation between \(h\), \(h_x\) and \(h_y\), being given by the following diagram (figure 19)

![Figure 19: Relation among half-offset(h), \(h_x\) and \(h_y\)](image)

So, we can write : \(h_x = h \cos \phi\) and \(h_y = h \sin \phi\).

So we can write the previous equation as:

\[
A_m = A + (B \cos^2 \phi + C \cos \phi \sin \phi + D \sin^2 \phi) \ h^2, \quad (5.4)
\]

\[=> A_m = A + f(\phi) \ h^2 \quad (5.4a)\]

where \(\phi\) is the azimuth and \(h\) is the half-offset.

Following a similar argument as in section 3.1, we can arrive at matrix equation.

\[
\begin{bmatrix}
\Sigma (h^2 \cdot \cos^2 \phi) & \Sigma (h^2 \cdot \cos \phi \cdot \sin \phi) & \Sigma (h^2 \cdot \sin^2 \phi) \\
\Sigma (h^2 \cdot \cos^2 \phi) & \Sigma (h^2 \cdot \cos^2 \phi) & \Sigma (h^4 \cdot \cos^2 \phi \cdot \sin^2 \phi) \\
\Sigma (h^2 \cdot \cos \phi \cdot \sin \phi) & \Sigma (h^4 \cdot \cos^2 \phi \cdot \sin^2 \phi) & \Sigma (h^2 \cdot \cos^2 \phi \cdot \sin^2 \phi) \\
\Sigma (h^2 \cdot \sin^2 \phi) & \Sigma (h^4 \cdot \cos^2 \phi \cdot \sin^2 \phi) & \Sigma (h^4 \cdot \cos^2 \phi \cdot \sin^2 \phi) \\
\end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} \Sigma \Sigma M(h, \phi) \\ \Sigma (h^2 \cdot \cos^2 \phi) M(h, \phi) \\ \Sigma (h^2 \cdot \cos \phi \cdot \sin \phi) M(h, \phi) \\ \Sigma (h^2 \cdot \sin^2 \phi) M(h, \phi) \end{bmatrix}
\]

\(M(h, \phi)\) is the data from the seismogram.
From the above equation, we can find the values for A, B, C and D.

The expression (5.4) has a minimum or maximum value, when \( \frac{dA_m}{d\phi} = 0 \)

\[
\frac{d}{d\phi} A_m = 0 \ldots \ldots \text{(5.6)}
\]

\[
(-2B) \cdot \cos\phi \cdot \sin\phi + C \cdot \cos^2\phi + (-C) \cdot \sin^2\phi + (2D) \cdot \sin\phi \cdot \cos\phi = 0 \ldots \text{(5.7)}
\]

\[
\tan 2\phi = \frac{C}{(B-D)} \ldots \ldots \text{(5.8)}
\]

\( f(\phi) \) in expression (5.4a) has minimum and maximum magnitudes, given by:

\[
f(\phi)_{\text{min}} = \frac{B + D \pm \sqrt{C^2 + (B - D)^2}}{2} \ldots \ldots \text{(5.9)}
\]

Two possible directions of fracture orientation can now be indicated. These two direction are either parallel or perpendicular to the fracture-orientation.

Next we can use a similar approach as in section 3.1, to find the elastic parameters in these two directions, using the traces lying within a range of azimuth, from Banik’s approximation or Otaibi’s approximation.

In their discussion Thomsen (1986) and Banik (1987) have introduced the concept of an “effective anisotropic parameter” \( \delta \). Using this, we can describe the reflection coefficient for a transverse isotropic medium (azimuthal, i.e., vertical fractures) as:

\[
R_{app}(i) = R_{ipp}(i) + \left( \frac{\Delta \delta}{2} \right)^2 \cos^2 \phi \ldots \ldots \text{(5.10)}
\]
where,

\[ R_{app}(i) = \text{reflection coefficient for transverse isotropic medium}; \]

\[ R_{pp}(i) = \text{reflection coefficient in absence of anisotropy}; \]

\[ \delta = \text{coefficient of transverse isotropy}; \]

\[ \phi = \text{the angle between the normal to the fracture orientation and the trace-azimuth}; \]

\[ i = \text{average of angles of incidence and transmittance}. \]

Physical significance of \( \delta \) being that:

"(i) For vertical symmetry axis, \( \delta \) describes the square of the move-out velocity at short offset for Transverse Isotropic media;

(ii) \( \delta \) describes relative competitiveness between P-wave anisotropy and SV-wave anisotropy in affecting properties Transverse Isotropic media." (Banik, 1987).

For the present work, we used Otaibi et al’s approximation as the forward model to generate the data. In the earlier part we found out the fracture orientation. Now using a method combining migration and inversion (similar to that in the case of isotropic medium), utilizing Banik’s approximation, we should be able to have an estimate of anisotropy.

We follow a similar least-square approach as in section 3.1., the matrix equation being,

Here \( \phi \) is the angle between the fracture-orientation and the azimuth of the trace. We estimate \( p_2, M_2 \) and \( \mu_2 \) from \( A_1, A_2 \) and \( A_3 \) (as in section 3.1). The coefficient
\[
\begin{bmatrix}
\Sigma (1 - \tan^2 \gamma) \Sigma (1 - \tan^2 \gamma)(1 + \tan^2 \gamma) \Sigma (1 - \tan^2 \gamma) \sin^2 \gamma \\
\Sigma (1 - \tan^2 \gamma)(1 + \tan^2 \gamma) \Sigma (1 + \tan^2 \gamma) \sin^2 \gamma \\
\Sigma (1 - \tan^2 \gamma) \sin^2 \gamma \\
\Sigma (1 - \tan^2 \gamma)(\cos^2 \gamma \cdot \cos^2 \phi) \Sigma (1 + \tan^2 \gamma)(\cos^2 \gamma \cdot \cos^2 \phi)
\end{bmatrix}
\begin{bmatrix}
\Delta_1 \\
\Delta_2 \\
\Delta_3 \\
\Delta_4
\end{bmatrix}
= 
\begin{bmatrix}
\Sigma R(i)(1 - \tan^2 \gamma) \\
\Sigma R(i)(1 + \tan^2 \gamma) \\
\Sigma R(i) \sin^2 \gamma \\
\Sigma R(i)(\cos^2 \gamma \cdot \cos^2 \phi)
\end{bmatrix}
\]

$A_4$ gives us estimate of $\delta$, the effective anisotropic parameter, which gives an idea of the extent of anisotropy in the layer at the bottom.
5.1: Result:

5.1.1: Fracture Orientation detection:

We tried to check the performance for the following case, (we used Mohammed Al-Otaibi’s approximation for generating the reflection coefficients), the input data being:

- P-wave modulus of top-layer: 34.85
- S-wave modulus of top-layer: 12.29
- Density of top-layer: 2.4
- P-wave modulus of layer 2: 42.87
- S-wave modulus of layer 2: 15.26
- Density of layer below: 2.05
- Alpha: 0.001
- Beta: 1.0
- Gamma: 0.001
- Phi: 27.0
- Number of traces: 4000
- Depth (z0): 600.0

We used 5 traces/CMP and four azimuths 0°, 30°, 60°, and 90°.

The result we got is following:

\[ A = -6.95416E-03; \]
\[ B = -1.32635E-02; \]
\[ C = -6.43097E-03; \]
\[ D = -8.56949E-03. \]

Orientation of the fracture = 26.9371° or 116.9371°.

For the second case, we considered everything same, but interchanged the values of alpha and gamma; so, the input values were:
P-wave modulus of top-layer: 34.85
S-wave modulus of top-layer: 12.29
density of top-layer: 2.4
P-wave modulus of layer 2: 42.87
S-wave modulus of layer 2: 15.26
density of layer below: 2.05
alpha: 1.0
beta: 0.001
gamma: 0.001
phi: 27.0
number of traces: 4000
depth (z0): 600.0

We used 5 traces/CMP and four azimuths 0°, 30°, 60° and 90°.

The result we got is following:

\[ A = -7.20568E-03; \]
\[ B = -6.33664E-03; \]
\[ C = -2.81614E-02; \]
\[ D = 1.41453E-02. \]

Orientation of the fracture = 26.9858° or 116.9858°.

5.1.2: Estimate of anisotropic parameter (δ):

We generated the data, using Otaibi’s approximation. For the following four cases, we tried to find an estimate of “effective” anisotropic parameter(δ). The two media for each of the case are Shale and Salt. Interface is at a depth of 600m and we used 2400 traces (20CMPS/line, 10 lines, 3 traces/CMP, offsets: 100m, 370m and 640m, azimuths: 0°, 30°, 60°, 90°.)
Case 1:

\[ \alpha = 0.001; \quad \beta = 0.001; \quad \gamma = 0.001; \]
\[ \phi = 60^\circ. \]

Estimated \( \delta = -0.002. \)

Case 2:

\[ \alpha = 0.001; \quad \beta = 0.111; \quad \gamma = 0.001; \]
\[ \phi = 60^\circ. \]

Estimated \( \delta = 0.003. \)

Case 3:

\[ \alpha = 0.001; \quad \beta = 1.000; \quad \gamma = 0.001; \]
\[ \phi = 60^\circ. \]

Estimated \( \delta = 0.0245. \)

Case 4:

\[ \alpha = 1.000; \quad \beta = 1.000; \quad \gamma = 1.000; \]
\[ \phi = 60^\circ. \]

Estimated \( \delta = 0.057. \)

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Transverse isotropy is a special case of general anisotropy, arising from layering of thin layers (polar transverse isotropy) or vertical fracturing (azimuthal transverse isotropy).

The method migrating scaled 3D seismic data, followed by in version is used to find the orientation of vertical fractures for the case of azimuthal transverse isotropy.

A method, similar to the one discussed in chapter 4, is used to find the extent of “effective” anisotropy for the case of azimuthal transverse isotropy.
Chapter 6
CONCLUSIONS

In the present work, the subject of quantitative analysis of Amplitude Variation with Offset is approached with a method that combines Pre-stack migration and inversion. The basic algorithm and the examples are described for the simplest case, i.e., a single interface and constant velocity. But the results obtained from this approach are compelling enough to go for the experimentation with the more complicated case of velocity variation and multiple layer model.

We started with the derivation of an approximation of P-wave reflection coefficient from an earlier existing previous expression and compared the performance of the approximate with that of the exact Zoeppritz equation.(Chapter 2). The approximation performs very well in predicting the reflection coefficient for small change in elastic moduli and for angle of incidence, to a limit of 38° (Figure 2, 3, 4 and 5).

In chapter 3, we use the approximate reflection coefficient formula to extract the elastic parameters from reflection coefficient vs. angle of incidence data for a single gather. For a single gather, we get a very good result after three iterations (Table 1 and table 2). We introduced some error to the angle of incidence vs. reflection coefficient data and tried to check the performance of the approximation in inversion, and we get a very good result after three iterations (Table 3 and Table 4).

In chapter 4, we discussed the topic of extracting the elastic parameters from 3D synthetic data, with different stages of generalization.
In the first attempt to extract the elastic parameters from the 3D seismic data, we scale the input data by three different weights (functions of the average of angle of incidence angle of transmittance) and migrate them to an output location and invert to obtain the estimate of change in the elastic parameters. This methodology, understandably takes a long time and, more importantly, needs the angle of incidence.

In the second method, we present an alternative, in which we apply pre-stack constant-offset migration on the data to get a migrated gather at an output location. From the knowledge of the dip of the interface at the output location, we can construct angle of incidence vs. reflection coefficient data, from which we can attempt to extract the estimate of the elastic parameters. This approach, though an approximation to the first method, produces a very good result.

At last, an even more generalized approach is presented, in which we scale the data with three weights (much like the first method, but this time the weights are functions of normalized offset-values) and migrate to a particular output location, and then we can construct the reflection-coefficient vs. angle of incidence data, from which we can estimate the change in the elastic parameters.

As an extension, it has been shown that on the way to apply the generalized method, that an automated migration velocity analysis can be done, utilizing the equality of phase angle of the three migrated trace. Also, a method to identify the dip, while doing migration, has been discussed.

In chapter 5, we show two more usages of this method of weighting-migration-inversion combination for the case of transverse isotropy. In the first one, we scale the
data by function of normalized offset-value and azimuth and then migrate and invert to obtain the orientation of the vertical fractures for a azimuthally transverse isotropic media. In the second one, we utilize a similar method to obtain the “effective anisotropic parameter” for vertically fractured medium.

**Direction for future**

In the present work, all the methods are for simple cases, namely, constant velocity and simple single-layer geometry. This can be considered as an introduction to solving a more difficult problem, namely, attempt to solve problems with complicated geometry and velocity-variation.

This, as we mentioned earlier, is a preliminary step towards attempting the complicated problem of AVO analysis of real 3D seismic data. There are number of problems associated with AVO interpretation of tackling data, e.g., amplitude-balancing, vertical and lateral variation in the velocity field, complication in the subsurface geology, tuning problem, directionality of source and receiver, irregularity in the acquisition geometry etc. In the present work, simplistic approach has been used to take care of some of these problems.

In the present work, for all the sections, there is an attempt to generalize the approach as much as possible. For example, for the extraction of elastic parameters (chapter 4), we started with a specific attempt (section 4.1), where we need the migration velocity and we know the dip. But in the later two sections, 4.2 and 4.3, we tried to make the
method more generalized, by addressing the problems of unknown dip and irregularity in acquisition geometry. In section 4.4, we present a method, which is very generalized in approaching the aim of extracting the elastic parameters. The method, presented in this section, utilizes the results from the next two sections, namely, migration velocity analysis (section 4.5) and finding the normal to the dipping reflector (section 4.6). Both of these methods can be utilized in sample-by-sample basis for each output location.

The migration-velocity analysis can be done, in practical cases, by using any existing Kirchhoff migration code, followed by a code for finding the Hilbert transform. The method for finding the normal to the dipping surface can also be used for a general case. Because, all we need is the travel-time of the wave from the source-location and receiver location to the output sample, which can be computed using a ray-tracing code. We did not address the problem for the cases of vertically and laterally varying velocity fields.

In Chapter 5, methods for finding the direction of vertical fractures and measuring extent of anisotropy is used. The first one is very generalized in the sense that we start with the assumption of transverse isotropy, but after that we just utilized the azimuth and offset information to predict the orientation of vertical fractures. The second method, described in the chapter, also aims at finding out the extent of anisotropy, given that the nature of anisotropy is vertical transverse isotropic in nature.
REFERENCES


Gardner, G.H.F., 1994, Measuring Fracture Orientation by the AVO Response in Different Directions, 3D Seismic Workshop, GeoTechnology Research Institute (GTRI), Houston Advanced Research Center (HARC).


Appendix 1: Generation of the Synthetic Data

Below we discuss the way we created the 3D synthetic data to be used as the input to the various AVO programs discussed in the present work.

We started with a geometry-generation file, which creates the co-ordinates the shot and the receiver points. The input to the program is number of lines, number of CMPS/line and number of traces/CMP (the azimuth-range also can be specified, but in the present work, we did not use any azimuth-varying data). The geometry-generation file creates a database, which contains the shot-receiver co-ordinates.

In the next phase, the program used, calls two subroutines (one for trace generation and the other for computation of the reflection coefficient, using Zoeppritz equation or Otaibi’s approximate equation for the anisotropic case). The input to the modelling program is the geometry database and the model (the elastic parameters and the depth of the interface and the dip and the azimuth of the interface) and other important parameters for generating the data (e.g. the maximum frequency, the sampling rate etc.).

For each shot-receiver combination (i.e. for each trace), the angle of incidence is computed and then the Reflection-coefficient computing subroutine is called. In the next phase the trace-generation routine is called, in which a symmetric (zero-phase) wavelet is created, with the time of arrival at the center of the wavelet, with the reflection
coefficient computed previously, at the peak of the wavelet.

The output of the modelling program is the gather-data (binary), trace-headers (containing line number, shot number, receiver number, shot-coordinate, receiver coordinate) and the traces (ASCII) and a model (zero-offset data, created automatically).

The traces and the trace-header are used in the migration-program.
Appendix 2: Computation of Hilbert Transform

For the migration-velocity analysis, we used the Hilbert Transform for finding the phase-angle of the migrated trace. Below we discuss in brief about the Hilbert Transform and the method utilized for finding the Hilbert Transform in the present work.

Hilbert Transform addresses the problem of studying the relations between real function $\alpha(\omega)$ and $\beta(\omega)$, when $\phi(\omega) = \alpha(\omega) + i \beta(\omega)$ is the Fourier transform of a function $f(t)$. $f(t)$ should have zero values for $t<0$, and should be absolutely integrable on the interval:

$$0 \leq t < \infty$$

The relation between $f(t)$, $\alpha(\omega)$ and $\beta(\omega)$ is:

$$f(t) = \int_{-\infty}^{\infty} \left( [\alpha(w) + i \beta(\omega)] \cdot e^{i\omega t} \right) d\omega$$

Hilbert transforms are:

$$\beta(\omega) = \left( \frac{1}{\pi} \right) \cdot P_{\omega} \cdot \int_{-\infty}^{\infty} \frac{\alpha(u)}{u - \omega} du$$

$$\alpha(\omega) = \left( -\frac{1}{\pi} \right) \cdot P_{\omega} \cdot \int_{-\infty}^{\infty} \frac{\beta(u)}{u - \omega} du$$

where $P(\omega)$ refers to the discontinuity at $u=\omega$

$$\left( P_{\omega} \right) \cdot \int_{-\infty}^{\infty} \frac{\alpha(u)}{u - \omega} du = \lim_{\varepsilon \to 0} -\int_{-\infty}^{(\omega - \varepsilon)} \frac{\alpha(u)}{u - \omega} du + \int_{(\omega + \varepsilon)}^{\infty} \frac{\alpha(u)}{u - \omega} du$$
In the previous equations, \( \beta(\omega) \) is defined as the Hilbert transform of \( \alpha(\omega) \), or \( \alpha(\omega) \) is the Hilbert transform of \( -\beta(\omega) \). (Ref: Operational Methods for Linear Systems, Kaplan, Addison-Wesley).

Below we describe the methodology of finding the Hilbert transform:

1. **Read Real data**: \( x(n) \)
2. **Forward (real->complex)** FFT to obtain the Fourier Transform of \( x(n) : a(i) + i \cdot b(i) \)
3. **Identify**
   - zero, +ve, Nyquist and -ve frequencies
4. **Multiply**:
   - (i) Zero-freq element by 1.0;
   - (ii) +ve-freq element by 2.0;
   - (iii) Nyq-freq element by 1.0;
   - (iv) -ve -freq element by 0.0.
5. **Inverse FFT** the resulting sequence (complex -> complex) to obtain the sequence:
   \( p(i) + i \cdot q(i) \)
6. **Result**:
   - \( p(n) = x(n) \);
   - \( q(n) = \) Hilbert transform of \( x(n) \)
Appendix 3: The relation between angle of incidence and dip of the interface

In Section 4.5, a generalized approach of estimating elastic parameters from multi-fold 3D data, have been discussed. In that approach, after finding out the three least-square coefficients, using migration-inversion combination, a reflection coefficient vs. offset table is created, which is then modified to reflection coefficient vs. angle of incidence data, to extract the elastic parameters. For this, a formula is required to relate the angle of incidence and the dip. Below that useful formula is derived from geometry:

\( \theta \): Apparent dip  
\( i \): angle of incidence

\[ 2h \]

\[ \begin{align*} 
\text{SR} &= 2h; \text{Angle RSD} = \theta; & \text{Angle SBR} &= \text{Angle ERD} = i; \\
\text{SD} &= 2h \cos \theta; & \text{RD} &= 2h \sin \theta; & \text{RE} &= 2h \sin \theta / \cos i; \\
\text{DE} &= 2h \sin \theta \tan i; & \text{RB} &= vt. 
\end{align*} \]

\[ SE = SD - ED = 2h \cos \theta - 2h \sin \theta \tan i \quad BE = BR - ER = vt - 2h \sin \theta / \cos i \]

\[ \sin i = \frac{SE}{BE} = \frac{2h \cdot \cos \theta - (2h \cdot \sin \theta \cdot \tan i)}{vt - (2h \cdot \sin \theta) / (\cos i)} \]

\( \Rightarrow vt \sin i - 2h \sin \theta \tan i = 2h \cos \theta - 2h \sin \theta \tan i \Rightarrow i = \sin^{-1}(2h \cos \theta / vt). \)