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The vaporization of an electrically conducting droplet through the use of an external alternating magnetic field

Stickel, Victor George, Jr., M.S.
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THE VAPORIZATION OF AN ELECTRICALLY CONDUCTING DROPLET THROUGH THE USE OF AN EXTERNAL ALTERNATING MAGNETIC FIELD

by

VICTOR G. STICKEL, JR.

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APPROVED, THESIS COMMITTEE

Y. Bayazitoglu, Professor, Director
Mechanical Engineering and Materials Science

A. J. Chapman, Professor
Mechanical Engineering and Materials Science

A. J. Meade, Assistant Professor
Mechanical Engineering and Materials Science

Houston, Texas
April, 1994
Abstract

The Vaporization of an Electrically Conducting Droplet through the Use of an External Alternating Magnetic Field

by

Victor G. Stickel, Jr.

A nondimensionalized model is developed for a falling, electrically conducting, spherical droplet undergoing equilibrium vaporization. Vaporization is caused by the presence of an external, alternating magnetic field. This model incorporates the influence of magnetic field parameters, phase change, convective heat transfer and radiative heat transfer. The model is used to predict the behavior of a stationary droplet under the same conditions. An experimental electromagnetic vaporization device is designed and constructed. Experimental vaporization rates obtained from the device are found to be in good agreement with numerical results from the stationary model for low power levels. A discrepancy between experimental data and the model for higher power levels is attributed to the formation of a thin metal film in the experimental device.
Acknowledgments

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Nomenclature

$a$  coil radius
$Al$  Alfven number
$B$  magnetic induction
$Bi$  Biot number
$c$  specific heat
$f$  frequency
$F$  force
$g$  gravitational acceleration
$G(x)$  skin depth function
$h$  coefficient of heat transfer
$H(x)$  skin depth function
$Ha$  Hartman number
$I$  electric current
$k$  thermal conductivity
$L$  coil length
$L$  latent heat
$L_y$  Lykoudis number
$m$  mass
$N$  turns
$N_g$  gravitational interaction parameter
$p$  pressure
$P$  power
$R$  droplet radius
$Re$  Reynold’s number
$\bar{s}$  position vector
$Ste$  Stefan number
\( t \) time
\( T \) temperature
\( v \) velocity magnitude
\( \mathbf{v} \) velocity vector
\( V \) Voltage
\( x \) dimensionless parameter
\( z \) position
\( \beta \) thermal expansion coefficient
\( \beta \) dimensionless parameter
\( \delta \) skin depth
\( \zeta \) dimensionless parameter
\( \varepsilon \) emissivity
\( \sigma \) electrical conductivity
\( \sigma_r \) Stefan Boltzmann constant
\( \nabla \) Gradient Operator
\( \nu \) dynamic viscosity
\( \rho \) density
\( \mu \) magnetic permeability
\( \omega \) angular frequency

**Subscripts**

- \( a \) atmospheric
- \( b \) buoyancy
- \( c \) chamber
- \( D \) drag
- \( d \) droplet
- \( i \) initial
- \( f \) final
- \( L \) Lorentz
- \( L \) liquid
- \( v \) vaporization
- \( s \) sublimation

**Superscripts**

- \( * \) dimensionless
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Chapter I

Introduction

The demand for metal powders and particles has grown significantly over the past decade. In the 1990's, metal powders are increasing their market share while other semiproducts, such as, steel sheet, strip, and bar, iron castings, and zinc die castings decline. Engineers are striving to decrease weight, increase quality, trim component cost and improve recyclability in their designs [1]. Metal powders and components fabricated from metal powders help designers meet these criteria through the attributes of corrosion resistance, high temperature susceptibility, strong mechanical properties and near-net shape forming. Also of particular interest recently is powder metallurgy's advantage over competing technologies as being environmentally friendly [2]. The applications of powder metals are manifold with automotive applications having the lion's share of the market followed by aerospace, domestic equipment, instruments, mechanical devices, business machines and hand tools [3]. Specific parts formed from powders include, automotive and aerospace structural components, pulleys, sprockets, hubs, connecting rods, gears, rotors, etc. [4]. In addition to this, powders are finding increasing uses as additives in other media such as, paint, plastic, magnetic tapes, conducting films and food [5].

Traditional methods of powder production include (i) atomization, (ii) mechanical machining and crushing, (iii) chemical reduction, and (iv) electrolysis [6]. In the atomization process a stream of molten metal is
disintegrated by an impinging gas jet. This jet is usually an inert gas in order to keep impurities to a minimum [7]. Careful consideration of the atomization chamber size is required in this process so that newly formed particles do not adhere to or touch chamber walls. Atomization provides the majority of all powders produced today. However, production yields are often poor due to the fact that it is difficult to control particle size. Powders are also produced by machining (e.g. milling) and crushing. This technique is usually only useful for producing particles of irregular shape made of brittle materials. During this procedure, contamination is often a problem due to the source material being contacted by a mechanical element. Nearly all metals can be reduced to powders by chemical reactions. The classical method of oxide reduction is achieved by thermochemical reactions involving reducing gases such as carbon monoxide or hydrogen. This process results in a sponge powder which requires further grinding to reduce the powder. Finally, electrolysis can be used to precipitate a powder at the cathode of an electrolytic cell. The process begins with dissolution of the anode under an applied voltage. Transport through the electrolyte is used to purify the powder. Final powder particles are often dendritic in shape and the process is regularly limited to elemental powders [8]. These traditional powder production methods, for the various reasons outlined, are incapable of simultaneously meeting today's powder metal manufacturing requirements of (1) fine particle size, (2) spherical particle shape, (3) high purity and (4) high production yield [5].

Concurrent with the increased demand for powder metals there has surfaced a growing interest in new methods of metal powder manufacture. One new method of metal powder manufacture called Electromagnetic
Vaporization (EV) was first proposed by Bayazitoglu in 1991 [9]. It is suggested as a means of producing ultrafine metal powders through the use of alternating magnetic fields. This process may begin either with molten metal droplets or with solid powder droplets designated for further reduction in size. The droplets are exposed to an alternating magnetic field. Due to the electrical conductivity of the metal droplets, the field causes Joule heating of the particles. Sustained Joule heating produces subsequent vaporization or sublimation of the droplets. After the inner core of the droplets has reached desired diameter (i.e. after a certain corresponding time period) the particles are cooled and collected. A device using this principle, known as an Electromagnetic Vaporization Device (EVD), was also described by Bayazitoglu in 1991 [9]. Using this device, droplets of molten metal are permitted to fall under the influence of gravity through a vaporization chamber. The vaporization chamber houses a solenoid which subsequently heats and vaporizes the particles. Finally, the particles and vapor are cooled, separated, and collected (see Figure I.A). In this paper, we propose a simplified model of this EVD.

In general, when an electrically conducting body is placed in an alternating magnetic field, sinusoidally alternating currents, all of the same frequency are induced in the body (see Figure I.B) [10]. These currents, known as the eddy currents, can cause Joule heating of the body.

If the heating is sufficient to overcome energy losses of the body, melting and even vaporization may occur. In addition to the heating effect, the eddy currents react with the external field to produce Lorentz forces on the body. These forces can be used to levitate the body as suggested by Okress [11]. The combined effects of levitation and heating are sometimes used in the process of levitation melting [12]. In the EV process we are
Figure I.A
interested in utilizing Joule heating to vaporize the conducting particles. We are also interested in the effect the associated Lorentz force has on the particle’s motion and thus, its residence time in the vaporization chamber. The electromagnetic vaporization of multiple conducting droplets in motion is considered a magnetohydrodynamics problem. In general, it

\[ \vec{B}(\vec{s}) = \vec{B}_0(\vec{s}) e^{j\omega t} \]

Figure 1.B

requires the solution of the Maxwell equations for electrodynamics along with the Navier Stokes equations for fluid flow coupled through Ohm's law for moving media. In addition to this, equations of phase change, free surface geometry, and motion for the falling bodies, must be formulated. Due to extremely rapid heating rates, interface velocities may be high enough to consider nonequilibrium vaporization [13]. Also, if the surrounding medium is deemed to be participating, solution of the
Radiative Transfer Equation is required. Our model will use several simplifying assumptions in order to make this a more tractable problem.

The problem of fluid flow in a conducting liquid of given surface shape under the influence of an alternating magnetic field has been studied by Moffat [14] and Sneyd [15, 16]. Metsel [17] analyzed liquid metal levitating in an alternating magnetic field to determine the free surface geometry and velocity field. There has also been a great deal of work done in the area of absorbed power and Lorentz forces on conducting bodies exposed to an alternating magnetic field. Rony [10] determined the power absorption and the levitation force for a conducting sphere exposed to an homogeneous alternating magnetic field. Fromm and Jehn, in a more detailed model using a series of stacked current carrying loops to model the field, derived the power absorption and levitation forces for a conducting sphere [12]. Lohofer [18] derived power absorption for a levitating body exposed to an arbitrary but sinusoidally varying magnetic field. Vaporization and sublimation phase change and energy transport considerations, however, have received little attention. Vutsens [19] studied vaporization of a solid sphere being levitated assuming a constant temperature of vaporization while being subjected to an alternating magnetic field. Recently, Bayazitoglu and Cerny looked at nonequilibrium vaporization [13], and also at a distributed temperature [20] within a molten liquid sphere.

In this paper, we derive a model for a single, electrically conducting sphere falling through a set of coaxial, current carrying loops. Depending on the external parameters, the droplet undergoes equilibrium vaporization or sublimation phase change followed by cooling.
Analysis

Magnetic Fields

In our model we look at a single sphere falling through an homogeneous alternating magnetic field $\vec{B}(\vec{s}) = B_o(\vec{s}) e^{j\omega t} \hat{z}$ where $\vec{s} = \rho \hat{\rho} + \theta \hat{\theta} + z \hat{z}$ (see Figure I.C)

![Figure I.C](image)

For a homogenous sphere of permittivity, $\varepsilon$, permeability, $\mu$, and conductivity, $\sigma$, exposed to such a field and surrounded by a nonconducting medium of permittivity, $\varepsilon_o$, permeability, $\mu_o$, the Maxwell equations may be expressed as:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
\[ \nabla \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t} \]
\[ \nabla \cdot \bar{B} = 0 \]
\[ \nabla \cdot \bar{D} = \rho' \]

along with the constitutive relations;
\[ \bar{D} = \varepsilon \bar{E} \]
\[ \bar{B} = \mu \bar{H} \] inside the droplet
\[ \bar{J} = \sigma \bar{E} \]

and;
\[ \bar{D} = \varepsilon_0 \bar{E} \]
\[ \bar{B} = \mu_0 \bar{H} \] outside the droplet

where we have assumed that the inner and surrounding media are linear and the surrounding medium is also non-conducting (an assumption which we will return to later). Since frequencies are small enough for the displacement current to be negligible, we make the quasi-steady assumption. Rony [10] has shown that for a sphere in a quasi-steady state field the Lorentz force acting on the center of mass of the sphere may be expressed as:

\[ \bar{F}_L = -\frac{2\pi R^3}{\mu} G(x)(\bar{B} \cdot \nabla)\bar{B} \]  \hspace{1cm} (1)

where;

\[ G(x) = 1 - \frac{3}{2x} \frac{\sinh 2x - \sin 2x}{\cosh 2x - \cos 2x} \]  \hspace{1cm} (2)
\[ x = \frac{R}{\delta} \]  \hspace{1cm} (3)
\[ \delta = \sqrt{\frac{2}{\omega \sigma \mu}} \]  \hspace{1cm} (4)
and the power absorbed in the sphere due to Joule heating, \( P \), may be expressed as;

\[
P = 3\pi \frac{RH(x)}{\sigma \mu^2} (\bar{B} \cdot \bar{B})
\]

where;

\[
H(x) = \frac{x(\sinh 2x + \sin 2x)}{\cosh 2x - \cos 2x} - 1
\]

Equation (1) has been validated for brass spheres of up to 1.0 in diameter by Okress [11]. Thus, we will assume our particles to be small i.e. \( R_O < 0.5 \text{ inches} \) and spherical in geometry. It should be noted that the skin-depth functions \( G(x) \) and \( H(x) \) which contain the frequency terms have been separated from the spatial characteristics of the applied magnetic field. We are thus able to compute the terms \( (\bar{B} \cdot \bar{V})\bar{B} \) and \( \bar{B} \cdot \bar{B} \) from magnetostatic calculations (see Appendix A). This brings us back to an earlier assumption of a uniform alternating magnetic field.

In our calculations of \( \bar{F}_L \) and \( P \) we have assumed that \( \bar{B}(\bar{s}) = \text{B}_0(\bar{s})e^{j\omega t} \) only. However, in order for this to be valid we must first determine \( \text{B}_0(\bar{s}) \) due to the given coil geometry using magnetostatic analysis. The alternating magnetic field is generated by a current carrying coil of \( N \) turns. A typical EVD as suggested by Bayazitoglu will utilize a solenoid of \( N/L \equiv 200(\text{turns/m}) \) [9]. It has been shown by Sathuvalli and Bayazitoglu that for \( N/L \geq 200 \) the magnetic field due to a current passing through such a coil can be accurately modeled as a series of \( N \) stacked loops [21]. Sathuvalli and Bayazitoglu also show that the field is fairly uniform in the region \( \rho/a \leq 0.3 \) where \( a \) is the coil radius. Thus, we designate the field due to the \( i^{th} \) loop as \( \text{B}_i(\bar{s}_i) \) where \( \bar{s}_i(\rho, \theta, z) \) is the relative coordinate system to the \( i^{th} \) loop. The field due to a single loop of
radius, \( a \), with current, \( I \), is easily found using the Biot-Savart Law (see Smythe [22]);
\[
\vec{B}_o(\vec{s}) = \frac{\mu I}{4\pi} \oint_\vec{s} \frac{d\vec{r} \times \vec{s}_{\perp}}{s^3}.
\]

and may be expressed as;
\[
\vec{B}_o(\vec{s}) = B_{o\rho}(s)\hat{i}_\rho + B_{o\theta}(s)\hat{i}_\theta.
\]

where;
\[
B_{o\rho} = \frac{\mu I}{2\pi \rho} \frac{z}{[(a+\rho)^2 + z^2]^{1/2}} \left[-K(k) + \frac{a^2 + \rho^2 + z^2}{(a-\rho)^2 + z^2} E(k)\right] \tag{7}
\]
\[
B_{o\theta} = \frac{\mu I}{2\pi} \frac{1}{[(a+\rho)^2 + z^2]^{1/2}} \left[K(k) + \frac{a^2 - \rho^2 - z^2}{(a-\rho)^2 + z^2} E(k)\right] \tag{8}
\]

and;
\[
K(k) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}, \quad E(k) = \int_0^{\pi/2} \frac{1}{\sqrt{1 - k^2 \sin^2 \phi}} d\phi
\]
\[
k = \sqrt{\frac{4a\rho}{(\rho + a)^2 + z^2}}
\]

We note here that, using a stacked loop model, the \( \theta \)-component of the field is zero due to symmetry. Consequently, using superposition of the contributions due to each loop, we find that, for a coil of \( N \) turns;
\[
\vec{B}_o(\vec{s}) = \sum_{i=1}^{N} \vec{B}_o(\vec{s}_i).
\]  \tag{9}

Plots of the dimensionless magnetic field, Lorentz force, and skin depth functions may be found in the Results and Discussion section of Chapter I.

Kinetic Theory/Vaporization

As mentioned before we assume that the surrounding medium is non conducting, thus, we pose some conditions on geometry and the vaporization process itself. We first assume that the vaporization chamber
volume $V_c \gg V_d$ and that there is fast removal of the metal vapor and heated ambient gas. We also assume that interface velocities are such that vaporization takes place in local thermodynamic equilibrium [13]. Thus, during the vaporization process $T = T_v$. In addition to this, all metal vapor particles leaving the sphere have the same mass, $m_i$, and velocity, $v_i$.

Mass, Momentum and Energy

In our model we consider a small conducting droplet of initial radius, $R_0$, at temperature $T_\alpha$ at time $t = 0$. The droplet is released from an initial position $z_0$ at time $t > 0$ and permitted to fall through a current carrying coil of length, $L$, and radius, $a$. The drop is surrounded by a medium at temperature, $T_\alpha$, and convection heat transfer exists between the droplet and the medium. As the drop falls through the coil, the magnetic field generated by the coil $B_\alpha(s)$ changes relative to the drop. Thus, Joule heating varies as a function of time. As the Joule heating increases and becomes greater than the convective and radiative losses of the droplet, the drop temperature reaches the vaporization temperature, $T_v$ at a time, $t_{v,t}$. At this point, the temperature remains constant and the power absorbed is sufficient to overcome the enthalpy of vaporization. The outer shells of the drop become vapor and the radius $R(t)$ begins to decrease. Since the metal vapor has been assumed to be removed very quickly, radiation is modeled as nonparticipating. When the Joule heating is no longer sufficient to overcome energy losses the drop begins to cool for time $t > t_{v,t}$. Thus, we express our model in terms of three time regimes (i) the heating phase $0 < t < t_{v,t}$, (ii) the vaporization phase $t_{v,i} \leq t \leq t_{v,f}$ and (iii) the cooling phase where $t > t_{v,f}$. In addition to this,
our droplet is small (i.e. $R \leq 10^{-3} m$), and $v < 10 m/s$ $T < 10^3 K$, thus $h < 10^3 W/m^2 K$ and typical Biot numbers for our model are $Bi = hR/k_a < 10^{-3}$. Since $Bi < 0.1$, we assume an infinite conductivity model and use a lumped analysis to formulate our equations [23]. During the time the drop is being heated (phase (i)) we may express the volume heat balance as:

$$ mc_v \frac{dT}{dt} = P - S[h(T - T_a) + \sigma_r \varepsilon (T^4 - T_a^4)] \quad (10) $$

and for a sphere:

$$ m = \rho_d V = \frac{4}{3} \pi \rho_d R^3 \quad S = 4 \pi R^2 $$

Substituting for $P$ (eq. (5)), $S$, and $m$ and noting that during phase (i) $R(t) = R_o = \text{constant}$, we write equation (10) as:

$$ \frac{dT}{dt} = \frac{9}{4} \frac{H(x) (\bar{B} \cdot \bar{B})}{\rho d c_L \sigma u^2 R_o^2} - \frac{3 \sigma_r \varepsilon}{\rho d c_L R_o} (T^4 - T_a^4) - \frac{3 h (T - T_a)}{\rho d c_L R_o} \quad (11) $$

where

$$ H(x) = H(R_o/\delta) $$

and we express the change in drop radius as:

$$ \frac{dR}{dt} = 0 \quad (12) $$

for

$$ 0 < t < t_{v,i} $$

When the drop reaches the vaporization temperature, $R = R(t)$ and phase change occurs. Thus, the energy equation now becomes:

$$ \rho_d L_v \frac{dV}{dt} = -P + S[h(T_v - T_a) + \sigma_r \varepsilon (T_v^4 - T_a^4)] \quad (13) $$

Noting for a sphere:

$$ \frac{d(V)}{dt} = \frac{d}{dt} \left( \frac{4}{3} \pi \rho_d R^3 \right) = 4 \pi \rho_d R^2 \frac{dR}{dt} $$

Substitution for $\frac{dV}{dt}$, $S$, and $P$ into equation (13) and rearranging yields;
\[
\frac{dR}{dt} = \frac{h(T_v - T_d)}{\rho_d L_v} + \frac{\sigma_T^2 (T_v^4 - T_d^4)}{4 \rho_d L_v} + \frac{3}{4} H(x)(\vec{B} \cdot \vec{B})
\]

where;

\[
H(x) = H(R(t)/\delta)
\]

During phase (ii) \( T(t) = T_v \) = constant implying;

\[
\frac{dT}{dt} = 0 \quad \text{for} \quad t_{v,i} \leq t \leq t_{v,f}
\]

When the drop begins to cool (phase (iii)), equations (11) and (12) again result for the droplet temperature and radius for the time regime, \( t > t_{v,f} \).

As mentioned before, the general solution of this problem would require the solution of the Navier-Stokes equations (in the reference frame of the moving drop) in order to determine the mass transport within the drop. As a simplifying approximation again since the drop is small we assume no mass transport within the drop.[24]. Thus, we are left with the Newtonian equations of motion to describe the path of the drop. The forces acting on the drop are the Lorentz force, \( \vec{F}_L \), the drag force, \( \vec{F}_D \), the buoyancy force, \( \vec{F}_B \), and the gravitational force, \( \vec{F}_g \), where;

\[
\vec{F}_g = -mg
\]

\[
\vec{F}_B = \rho_a \beta_a (T - T_a) \vec{V} \vec{g}
\]

\[
\vec{F}_D = \frac{1}{2} \rho_a A_a C_D (\vec{V} \cdot \vec{V})
\]

and for a sphere \( A_s = \pi R^2 \). Using the general relation for momentum of a body with changing mass [25];

\[
\vec{F}_{ext} = m \frac{\partial \vec{V}}{\partial t} - \vec{V} \frac{\partial m}{\partial t}
\]
where \( m \), represents the mass of the body, \( \bar{v} \), represents the velocity of the center of mass of the body, and \( \bar{V} \), represents the velocity of the mass leaving or attaching to the body relative to the body, we see for our case;

\[
\bar{F}_{ext} = m \frac{\partial \bar{v}}{\partial t} - \frac{\partial m}{\partial t} \sum_{i=1}^{n} \bar{V}_i
\]

The last term on the right hand side of this equation represents the vectorial sum of the relative velocities of \( n \) vapor particles leaving the sphere. Since we have made the assumption of equilibrium vaporization, this implies;

\[
\sum_{i=1}^{n} \bar{V}_i = 0 \Rightarrow \bar{F}_{ext} = m \frac{\partial \bar{v}}{\partial t}
\]

Thus, we may consider the momentum of the drop only. Writing the external force balance on the drop gives;

\[
m \frac{d\bar{v}}{dt} = -\frac{2\pi R^3}{\mu} G(x)(\bar{B} \cdot \bar{v}) \bar{B} + \frac{1}{2} \rho_a A_d C_b (\bar{v} \cdot \bar{v}) + \rho \beta_a (T - T_a) Vg - mg
\]

(16)

Using a cylindrical coordinate system as shown in Figure I.C we note for a stacked loop model the Lorentz force in the \( \theta \)-direction is zero (see Appendix A). If we consider a drop falling along the axis of the coil, a further simplification results, namely, the Lorentz force in the \( \rho \)-direction vanishes for \( \rho = 0 \). Thus, the velocity is only in the \( z \)-direction and \( \bar{v} = v \). Using this simplification and substituting for \( m \), \( V \), and \( A_d \) into equation (16) gives;

\[
\frac{dv}{dt} = -\frac{3}{2} \frac{G(x)}{\rho_d \mu} (\bar{B} \cdot \bar{v}) \bar{B} + \frac{3 \rho_a C_D}{8 \rho_d R_o} v^2 + \frac{g \rho_a \beta_a (T - T_a)}{\rho_d} - g
\]

(17)

During vaporization \( T = T_v \) and equation (17) becomes;

\[
\frac{dv}{dt} = -\frac{3}{2} \frac{G(x)}{\rho_d \mu} (\bar{B} \cdot \bar{v}) \bar{B} + \frac{3 \rho_a C_D}{8 \rho_d R} v^2 + \frac{g \rho_a \beta_a (T_v - T_a)}{\rho_d} - g
\]

(18)
where \( \frac{dR}{dt} \) is given by equation (14). Again during phase (iii) \( R = R_f \) and we may use equation (17) for \( \nu(t) \). In addition to this, during all phases we use the relation

\[
\frac{dz}{dt} = \nu \tag{19}
\]

to determine the drop position \( z(t) \). We note that in order to obtain an accurate description for \( C_d \) we ran order of magnitude analyses and determined the assumption of Stoke’s flow to be inaccurate. Thus, we use the following empirical relationships for \( C_d \) and \( h \) [24, 26];

\[
C_d = \frac{24}{Re} (1 + 0.15 Re^{0.887})
\]

\[
h = \frac{k_r}{2R} \left\{ 2 + 0.493 Re^{0.12} \left[ 1 + (0.759 \Omega)^{3.5} \right]^{0.5} \right\}
\]

where;

\[
Re = \frac{2 \nu R}{\nu_a}
\]

\[
\Omega = (\frac{Gr}{Re^2})^{1/4}
\]

\[
Gr = 8g \beta_a (T - T_a) R^3 / \nu_a^2
\]

and \( h \) is valid for \( 3.5 < Re < 5.9 \times 10^3 \), \( 1 < Gr < 10^5 \) and \( C_d \) is valid for \( 3.5 < Re < 5.9 \times 10^3 \).

**Initial Conditions**

In this model we assume the initial temperature of the droplet to be the melting temperature, \( T_m \). Referencing Figure I.C and allowing the drop to be released from rest at the top of the coil, we may write the following initial conditions;

\[
T(0) = T_m, \quad R(0) = R_o, \quad \nu(0) = 0, \quad z(0) = 0 \tag{20}
\]
Thus, due to the occurrence of the phase change in our problem we are left with three sets of equations to describe the problem depending on the time interval. For \(0 < t < t_{v,i}\) the drop radius \(R\) is known \((R = R_0)\) and equations (11),(12),(17) and (19) may be used with initial conditions (20) to determine the four unknowns \(T, R, v, z\). When the temperature, \(T\), has reached the vaporization temperature, \(T_v\), equilibrium vaporization occurs for \(t_{v,i} \leq t \leq t_{v,f}\) and \(T = T_v = \text{constant}\). Here, \(R = R_f (t)\) and equations (14),(15),(18) and (19) along with initial conditions (20) are needed to determine the unknowns \(T, R, v, z\). Finally, during \(t > t_{v,f}\) the drop begins to cool and equations (11),(12),(17) and (19) may be used again with \(R_0\) replaced by \(R_f\), the final drop radius, in initial conditions (20).

**Nondimensionalization**

In order to provide meaningful descriptions of basic trends we nondimensionalized our problem. Introducing the dimensionless variables:

\[
T^* = \frac{T}{T_m}, \quad t^* = \left( \frac{B_0^2}{\sigma \mu^2} \right) t, \quad R^* = \frac{R}{R_o}, \quad v^* = \frac{v}{(c_L R_o T_m \sigma \mu)}, \quad z^* = \frac{z}{R_o}, \quad \bar{B}^* = \frac{\bar{B}}{B_o}, \quad H^* = H(R^*), \quad G^* = G(R^*), \quad h^* = h(T^*, R^*, v^*)/h_o, \quad C^*_D = C_D(R^*, v^*), \quad 
\]

where;

\[
B_o = \mu l/2 \pi R_o \quad \text{and} \quad h_o = k_a/2R_o
\]

Substituting these variables into equations (11),(12),(17) and (18), respectively, gives for phase (i) and phase (iii):

\[
\frac{1}{3} \frac{dT^*}{dt^*} = \frac{3}{4} H^* \left( \bar{B}^* \cdot \bar{B}^* \right) - \zeta \left( T^{*4} - \left( \frac{T_a}{T_m} \right)^4 \right) - \beta h^* \left( T^* - \frac{T_a}{T_m} \right) \quad (21)
\]
\[
\frac{dR^*}{dt^*} = 0
\]

\[
\frac{dv^*}{dt^*} = -\frac{3}{2} \frac{G^* (\bar{B}^* \cdot \bar{V}^*) \bar{B}^* + \frac{3}{8} C_{p} v^* v^*}{Ha^2} + \frac{1}{Ly} \left( \frac{T^* - T_a}{T_m} \right) - \frac{1}{N_g}
\]

\[
\frac{dz^*}{dt^*} = \frac{1}{Al} v^*
\]

where;

\[
\zeta = \frac{\varepsilon \sigma_{r} T_m^4 R_o}{B^2/\sigma \mu^2}
\]

\[
\beta = \frac{h_o T_m R_o}{B_o^2/\sigma \mu^2}
\]

\[
Ha^2 = \frac{B_o^2/\mu}{\rho_a (c_L R_o T_m \sigma \mu)^2}
\]

\[
Ly = \frac{B_o^2/\mu}{\varepsilon \rho_a \beta_a T_m R_o}
\]

\[
N_g = \frac{B_o^2/\mu}{\varepsilon \rho_d R_o}
\]

\[
Al = \frac{B_o^2/\mu}{\rho_d (c_L R_o T_m \sigma \mu)^2}
\]

Substituting the dimensionless variables into equations (14),(15),(18) and (19), respectively, gives for phase (ii) \( t_{v_i}/t_o < t/t_o < t_{v_f}/t_o \)

\[
\frac{dT^*}{dt^*} = 0
\]

\[
\frac{1}{Ste} \frac{dR^*}{dt^*} = \beta \left( \frac{T_v}{T_m} - \frac{T_a}{T_m} \right) + \zeta \left[ \left( \frac{T_v}{T_m} \right)^4 - \left( \frac{T_a}{T_m} \right)^4 \right] - \frac{3}{4} \frac{H^* (\bar{B}^* \cdot \bar{B}^*)}{R^*}
\]

\[
\frac{dv^*}{dt^*} = -\frac{3}{2} \frac{G^* (\bar{B}^* \cdot \bar{V}^*) \bar{B}^* + \frac{3}{8} C_{p} v^* v^*}{Ha^2} + \frac{1}{Ly} \left( \frac{T_v}{T_m} - \frac{T_a}{T_m} \right) - \frac{1}{N_g}
\]

\[
\frac{dz^*}{dt^*} = \frac{1}{Al} v^*
\]

where;

\[
Ste = \frac{c_L T_m}{L_v}
\]

The initial conditions now become;

\[
T^*(0) = 1, \quad R^*(0) = 1, \quad v^*(0) = 0, \quad z^*(0) = 0
\]

Equations (21)-(29) represent a system of four equations in four unknowns for three different time (i.e. temperature) domains. The system
is nonlinear and coupled through the magnetic field. This is due to the fact that since the drop is falling the field is changing in time relative to the drop.

Nondimensionalization of the governing equations results in the ten dimensionless parameters; \( \zeta, \beta, Ste, T_v/T_m, T_a/T_m, x, , Ha, Ly, N_g, Al \). During phase (i) the drop is heating and it is clear that the dimensionless coefficients, \( \zeta \) and \( \beta \) represent the energy lost to radiation and convection, respectively, over the energy absorbed due to the magnetic field. During the vaporization phase, the coefficients, \( \zeta \) and \( \beta \) are also found along with a parameter of phase change, \( Ste \), and the relative temperature coefficients \( T_v/T_m \) and \( T_a/T_m \). The Stefan number relates the internal energy to the enthalpy of phase change which in this case is the enthalpy of vaporization. The coefficients \( T_v/T_m \) and \( T_a/T_m \), are a parameter both of the material being considered and the surrounding environment. The parameter \( x \), a function of drop radius and skin depth, is constant for phases (i) and (iii) if the material properties, \( \sigma \) and \( \mu \), and the frequency, \( f \), are treated as constants. This skin depth function will definitely vary with time in phase (ii), however, since \( R=R(t) \) in this phase. The remaining parameters, \( Ha, Ly, N_g, Al \), are normally associated with magnetohydrodynamics problems, and in particular, problems involving internal flows. In this problem, however, they provide useful interpretations to underlying phenomena and their definitions are essentially the same except for the characteristic velocity variables used above. The square of the Hartman number, \( Ha \), represents the force due to the magnetic field over the viscous force on the body. The ratio of the the force due to the magnetic field to buoyancy force induced by natural convection is known as the Lykoudis number, \( Ly \). \( N_g \), the gravitational
interaction parameter, relates the magnetic field force to the gravitational force. Finally, the Alfven number, $Al$, is a measure of the kinetic energy due to the field over the kinetic energy due to inertia. It should be noted that any of these coefficients may also be functions of time depending on the model of the physical parameters (e.g. $\epsilon, \rho_d, c_L, \sigma$, etc.) which may themselves be functions of temperature.

**Numerical Solution**

In order to solve the nondimensionalized equations, a Fortran code along with a *Matlab* code and a corresponding gateway program were developed. The main program was a *Matlab* program which used a fourth/fifth order Runge-Kutta ODE solver with a variable time step to solve the nonlinear ODE's. In order to more efficiently evaluate values of $\vec{B}(\vec{s})$ and $\vec{F}_L(\vec{s})$, for large numbers of coil turns ($N$), a Fortran program was written which employed Simpson's Rule to numerically integrate the elliptic integrals in the field and Lorentz force terms. In addition to this, a gateway program was developed to allow the main *Matlab* program to communicate with the Fortran program. Using this scheme, the position $\vec{s}(t)$, which was determined by the *Matlab* routine at a particular time step, was input to the Fortran routine which calculated the corresponding $\vec{B}(\vec{s})$ and $\vec{F}_L(\vec{s})$ values and output them to the main program. Tolerance in the ODE solver was set at $10^{-6}$. 
Results and Discussion

It is clear from the nondimensionalization of this problem that a great many parameters are influencing the values of $T$, $R$, $v$, and $z$. In order to temper our theoretical results with real engineering numbers as opposed to purely hypothetical cases, we used dimensionless parameters based on known physical constants and external parameters. In addition to this, an order of magnitude analysis was performed. Calculations show convection to be the dominant mode of heat loss with typical Boltzman numbers of $10^2$. Looking at the forces on the drop we find the buoyancy force to be quite small with ratios of gravitational force to buoyancy force on the order of $10^4$. Thus, we are mainly interested in the parameters involving convection, Lorentz force, viscous force and gravitational force. In our calculations we use a 0.1 (mm) radius zinc droplet in an ambient argon atmosphere as a baseline. The baseline field strength was derived from using a coil with $N =100$ turns, $I =300$ (A), $a=0.01$ (m) and $N/L =200$. This results in the following values of our dimensionless parameters;

$\zeta = 7.6e^{-7}$ \hspace{1cm} $\beta = 6.6e^{-5}$ \hspace{1cm} $T_r/T_m = 1.7$ \hspace{1cm} $T_a/T_m = 0.43$

$Ste = 0.19$ \hspace{1cm} $x/R_o/\delta = 1.03$ \hspace{1cm} $Ha^2 = 19$ \hspace{1cm} $N_g = 4.4e4$

$Ly = 9.58e7$ \hspace{1cm} $Al = 0.0034$

In all of our results these parametric values are used unless indicated otherwise. In some instances, in place of the dimensionless variable $x$, the results are stated in terms of the frequency, $f$, for the sake of clarity.

Figure 1.D shows dimensionless drop temperature as a function of time for varying values of the magnetic field frequency. The temperature increases as the drop falls through the coil, reaches the
vaporization temperature (if heating is sufficient) and then is cooled as it falls from the coil. For lower frequencies, there is a slow temperature rise but the droplet never reaches the vaporization temperature before passing through the coil. This is due to the fact that the skin depth, $\delta$, is inversely proportional to the square of the frequency. At lower frequencies the skin depth is quite large compared to the drop radius and the power from the field is distributed over a large volume. At frequencies of $10^7$ (Hz) the temperature increase is more rapid and vaporization takes place since the heating effect is concentrated in a small outer shell of the sphere. During
the heating phase, the power absorbed is proportional to $H(x)/x^2$. Looking at Figure I.E it is clear that this function will increase for increasing frequencies ($x$ values). However, at value of $x \approx 2.41$ the heating rate will reach a maximum. As frequencies are increased, the vaporization time is at first lengthened since the droplet is still being vaporized at the time it exits the coil. Gradually, the frequency becomes high enough for the droplet to reach a final drop radius, $R_f$. At this point the power absorbed by the droplet has decreased to the point where it is equivalent to the heat losses from convection and radiation. During the cooling phase, higher frequencies result in faster cooling rates. This is due to the result that droplets which have been exposed to higher frequencies reach a smaller
final radius as indicated in Figure I.F.

During the vaporization phase, the change in radius is proportional to $H(x)/x$ as opposed to $H(x)/x^2$. The function $H(x)/x$ is shown in Figure I.G. As vaporization continues, since the skin depth is fixed, the value of $x$ decreases due to a decreasing droplet radius. As $x$ decreases the power absorbed balances the energy lost and $x$ reaches a final value. Again, higher frequencies indicate a higher initial value of $x$ which also yields a higher value of $H(x)/x$. The onset of vaporization is related directly to the heating rates found in Figure I.D. Thus, we see that one optimum parameter for vaporization would be to start at an initial $x$ value
of $x \approx 2.41$ to maximize heating rates and then to increase the frequency during the vaporization phase.

In Figure I.H we have shown the radius of the drop as a function of the droplet position. The position used is the relative position within the coil with 0.0 being the coil entrance and 1.0 being the exit. In all cases the droplet reaches the final radius before it exits the coil. At higher frequencies, the droplet has reached its final radius within the first 1/10 of the coil length. At first glance it would seem that the remaining portion of the coil is of little value. However, the power absorbed is proportional to
the square of the number of turns, $N$. Thus, the remaining $9/10$ of coil length is serving to build the total strength of the field available for vaporizing the droplet.

As seen in the Analysis section, the nondimensionalization of the energy equation results in the dimensionless parameters $\beta$ and $\zeta$. For the scales of the problem considered, the radiative losses amounted to approximately $2\%$ of the total heat losses. Typical Reynolds numbers for the problem were on the order of $10^1$ resulting in appreciable convective effects in the cooling of the droplet. However, our value of $\beta = 6.6e-5$ resulted in only a minimal heat loss effect during the heating and
vaporization phases. This is apparent in Figure I.J where we see that the temperature curve for $\beta = 0$ is not much different than a curve with our value during these phases. However, after vaporization has been completed, the influence of convection is immediately apparent in the cooling phase. It is clear that a $\beta$ value on the order of $10^{-4}$ would be more desirable than our value. This would increase the overall efficiency of the process with a decrease in the final radius of only 15% as can be seen in Figure I.K. This figure also shows that for high enough $\beta$ vaporization is completely suppressed. It is not likely that $\beta$ values much larger than $10^{-4}$ would be
used in such a process. Impurities are to be kept at a minimum and an inert gas atmosphere is envisioned for the process.

In considering the forces acting on the falling drop, we find the gravitational force to dominate as may be expected. A more interesting result, however, is that the Lorentz force is found to be quite negligible except at the ends of the coil. Typical ratios of gravitational force to Lorentz force are on the order of $10^5$ except at the coil ends where the ratio is decreased to $10^2$. This variation in magnitude can be explained clearly with reference to Figures I.L and I.M which show dimensionless
magnetic field strength and dimensionless Lorentz force, respectively, as functions of relative coil position. In our model we have assumed that the droplet falls along the axis of the coil. This results in a Lorentz force per unit volume proportional to \(-B_Z \frac{\partial B_z}{\partial z}\). From Figure I.L we can see that B_Z is negligible just above the coil entrance. As we enter the coil however, there is a rapid increase in B_Z which then quickly becomes constant again. This produces a net upward spike in the Lorentz force. After passing this spike the Lorentz force becomes negligible in the center of the coil since B_Z is virtually constant. At the bottom of the coil we see another spike in the Lorentz force but in the downward direction (Figure I.M).
Figure I.M

Figure I.N shows the dimensionless velocity of the droplet as a function of the relative droplet position for varying frequencies. It appears that as the frequency is increased the Lorentz force becomes appreciable and retards the motion of the droplet. This is not the case, however. In actuality, as the frequency is increased the vaporization rate is increased and the droplet reaches a smaller final radius. This smaller radius leads to a smaller ratio of gravitational to viscous force and the droplet reaches a lower terminal velocity. In the case of a frequency of $10^8$ (Hz) the droplet has decreased in radius by an order of magnitude from 1e-4 to 1e-5 (m) and has almost reached terminal velocity by the time it exits the coil.
Figure I.N

At the bottom of the coil, we expect to see some influence on droplet motion by the Lorentz force for a droplet which is near terminal velocity. This is unlikely though because the Lorentz force is also proportional to the skin depth function $G(x)$. This function decays rapidly as $x$ decreases. Therefore, by the time the droplet has reached its final size, $G(x)$ is negligible (see Figure I.G). In figure I.N we have also plotted the velocity profile of a drop falling through argon without the influence of a magnetic field. It is clear that frequencies below which vaporization occurs (i.e. below $1e7$ (Hz)) will have very little influence on particle motion.
As mentioned before, typical Reynolds numbers for the falling droplet are on the order of $10^1$, but change with time due to the varying droplet radius and velocity. Thus, we find that viscous forces are important and on the order of 1/10 of the gravitational force. Using the dimensionless parameter $Ha^2$, representing the ratio of magnetic to viscous forces, we examine the influence of the viscous force on the droplet velocity and radius, respectively, in Figures I.P and I.Q. As $Ha^2$ increases, the viscous force is decreased and the residence time of the droplet within the coil is decreased. The limit of $Ha^2 \to \infty$ corresponds to that of a free falling droplet through vacuum. At smaller values of $Ha^2$
Figure I.Q

the droplet reaches terminal velocity more rapidly and its residence time is increased. Since the residence time is increased, the droplet is able to reach a smaller final radius do to longer vaporization times. Thus, again we may conclude that a vacuum atmosphere would be undesirable for this process due to high values of $Ha^2$.

The last parameter we study is the interaction parameter, $N_g$. Variations in this parameters indicate the influence of the gravitational force with respect to the magnetic force. This parameter is particularly useful in comparing the behavior of droplets of varying densities. Our value of $N_g = 4.4e4$, corresponded to zinc. In other practical applications
for materials such as copper and titanium, we expect $N_g$ to decrease for a fixed field strength with values likely between $5e4$ and $1e4$. In Figure I.R we see that for decreasing $N_g$, droplet accelerations are increased as expected. As a consequence of this, the final droplet radius is decreased due to shorter vaporization times. We conclude, that materials such as aluminum may achieve a larger final radius in the process than zinc. However, aluminum would have a smaller Stefan number and thus lower vaporization rates.

In addition to the parameters we have examined we also note that similar plots could be made for other values of $T_v/T_m$, $T_a/T_m$. The
effect of the variation in a similar temperature coefficient and in $Ste$ is discussed for our stationary droplet problem in Chapter II (Results and Discussion). The same conclusions, however, are applicable to the falling droplet. Namely, the final droplet radius will become larger as $T_a/T_m$ and Stefan number is decreased and smaller as $T_v/T_m$ is increased. Since our value of $Ste$ is quite high (being for zinc), it is expected that other prospective materials such as copper and titanium will require higher field strengths to achieve a similar size reduction. The same is true for other materials with high $T_v/T_m$ and low $T_a/T_m$ values.
Chapter II

Introduction

In recent years there has been a resurgence of interest in the field of magnetohydrodynamics. This is particularly apparent in the area of materials processing. Within the realm of materials processing, scientists have been particularly interested in examining metals in the solid or liquid phase or metals undergoing phase change between the solid and liquid states in the presence of a magnetic field. The majority of the research has been concentrated in two areas; (i) the influence of an external magnetic field on convection in an electrically conducting molten liquid and (ii) levitation and/or melting (heating) of an electrically conducting body through the use of an applied alternating magnetic field.

It is well known that convection heat transfer in a conducting fluid may be influenced by an external magnetic field. This effect is of particular interest to materials scientists as a means of controlling crystal growth in molten metals during the cooling phase. In the 1950’s, Mori gave a series solution to the problem of free convection of an electrically conducting fluid on a vertical plate in a transverse magnetic field [1]. Lykoudis was later able to achieve a similarity solution to a similar problem [2]. The next group of researchers, Seki et. al., Michiyoshi et. al., and Ozoe et. al., were concerned with the fluid behavior under the combined effects of gravitational and magnetic fields [3-5]. More recent efforts have focused attention on the dependence of convection suppression on the direction of the applied field. Maekawa and Tanasawa considered
two-dimensional natural convection in a fluid heated from below and subjected to a field of arbitrary angle [6]. Ozoe and Okada have undertaken both experimental and analytical studies of heat transfer rates of natural convection of molten gallium suppressed under an external magnetic field in either the x, y, or z-directions [7, 8].

It is also well known that a conducting body may be both heated and levitated through the use of an alternating magnetic field. This is a phenomenon which scientists have utilized to perform containerless melting of metals and measurement of various physical properties of the metals e.g., surface tension, emissivity, density, etc. In the early 1950’s Okress et. al. experimented with the levitation of small conducting spheres [9]. Rony [10] and Fromm and Jehn [11] calculated expressions for both the Lorentz force and power absorption due to joule heating in an electrically conducting sphere in the presence of an homogeneous alternating magnetic field. Moffat and Sneyd examined the fluid flow within a conducting fluid of given surface shape [12-14]. Mestel analyzed liquid metal levitating in an alternating magnetic field to determine is free surface geometry and velocity field [15]. More recently, Zong et. al. have described a volume integral method for calculating electromagnetic forces and power absorptions generated in spherical and deformed bodies of rotational symmetry [16].

Thus, we see that the majority of this research concerns electrically conducting fluids subjected to a magnetic field which are heating or cooling in the liquid state or passing between the liquid and solid phases. Of primary interest to the authors is the use of alternating magnetic fields to vaporize or sublime a conducting body. In particular, we are interested in the generation of sufficient joule heating within a solid or molten body
to vaporize its outer shells. In 1964 Van Audenhove gave a description and results for an experimental technique of vacuum evaporation of metals by high frequency levitation heating [17]. Vutsens provided both an analytical expression and experimental data for a levitating drop of aluminum in a coil of unspecified geometry [18]. More recent studies by Bayazitoglu and Cerny consider an electrically conducting drop of liquid metal of finite conductivity undergoing nonequilibrium vaporization [19]. Bayazitoglu and Cerny have also considered an infinite conductivity model undergoing nonequilibrium vaporization [20].

In the current chapter, we examine both analytically and experimentally the heating and subsequent sublimation of a small electrically conducting body caused by the Joule heating effect associated with an alternating magnetic field. The analysis is based on a limiting case of the falling droplet considered in Chapter I. Namely, the body is stationary, sublimating in vacuum and undergoing radiative heat loss. Equations are developed for energy conservation and phase change and careful attention is paid to the modeling of the alternating magnetic field so as to incorporate important engineering design parameters (e.g. $I$, $N$, $N/L$, $a$, etc.). An experiment is designed and built to test the validity of the results. Data are developed for the nondimensional radius as a function of nondimensional time.
Analysis

In this model of the vaporization process, we assume that a stationary, electrically conducting sphere of initial radius, \( R_0 \), and temperature, \( T_0 \), is subjected to an external alternating magnetic field at time \( t > 0 \) (see Figure II.A).

![Figure II.A](image)

The magnetic field is generated by a current carrying coil modeled by a series of stacked current carrying loops (see Chapter I Analysis). Due to the field, induced eddy currents generated within the drop produce subsequent joule heating. This heating takes place until the drop temperature, \( T(t) \), reaches the vaporization temperature \( T_v \) at time \( t = t_v \). When the drop temperature reaches \( T_v \), the outer shell begins to vaporize and the drop radius \( R(t) \) starts to decrease. It is assumed that the volume surrounding the drop is much larger than the drop volume i.e. \( V_c \gg V_d \) and equilibrium vaporization is present (see Chapter I Analysis). The
atmosphere surrounding the drop is vacuum and heat is lost by surface radiation. In addition to this, since the drop remains stationary and the ambient surrounding is vacuum, both free and forced convection are assumed negligible. With these assumptions in mind, we see that this model is a special case of the more general model formulated in Chapter I for a falling drop. In particular, this drop is stationary and there is no convective heat transfer. In this case we shall write the equations for the model in terms of two time regimes; (i) \(0 < t < t_c\) (heating phase) and (ii) \(t \geq t_c\) (sublimation phase). Referring to Chapter I - Analysis equation (10) we may thus write the volume heat balance for this drop during phase (i) as:

\[
mc \frac{dT}{dt} = P - S[\sigma, \epsilon(T^4 - T_a^4)] \quad (1)
\]

where

\[
P = 3\pi \frac{RH(x)}{\sigma \mu^2} (\vec{B} \cdot \vec{B}) \quad (2)
\]

\[
H(x) = \frac{x(\sinh 2x + \sin 2x)}{\cosh 2x - \cos 2x} - 1 \quad (3)
\]

\[
x = \frac{R}{\delta} \quad (4)
\]

\[
\delta = \sqrt{\frac{2}{\omega \sigma \mu}} \quad (5)
\]

now for a sphere;

\[
m = \rho_v V = \frac{4}{3} \pi \rho_v R^3 \quad (6)
\]

\[
S = 4\pi R^2 \quad (7)
\]

Rearranging terms and recognizing that for phase (i) \(R(t) = R_o = \text{constant}\), we write equation (1) as;

\[
\frac{dT}{dt} = \frac{9}{4} \frac{H(x)(\vec{B} \cdot \vec{B})}{\rho_d c_L \sigma \mu^2 R_o^2} - \frac{3\sigma \epsilon}{\rho_d c_L R_o} (T^4 - T_a^4) \quad (8)
\]
where
\[ H(x) = H(R_o/\delta) \]

and we express the change in drop radius as;
\[ \frac{dR}{dt} = 0 \] (9)

for
\[ 0 < t < t_v \]

Thus, equations (8) and (9) describe the behavior of our drop during the heating phase. Referring to Chapter I - Analysis equation (13) we may write the phase change relation during phase (ii) as;
\[ \rho_d L_v \frac{dV}{dt} = S \sigma_e (T_v^4 - T_a^4) - P \] (10)

Again noting for a sphere with constant density;
\[ \frac{d(V)}{dt} = \frac{d}{dt} \left( \frac{4}{3} \pi \rho_d R^3 \right) = 4 \pi \rho_d R^2 \frac{dR}{dt} \] (11)

Substitution for \( \frac{dV}{dt}, S, \text{ and } P \) into equation (11) and rearranging yields;
\[ \frac{dR}{dt} = \frac{\sigma_e (T_v^4 - T_a^4)}{\rho_d L_v} - \frac{3}{4} \frac{H(x)(\bar{B} \cdot \bar{B})}{\rho_d L_v \sigma \mu^2 R} \] (12)

where;
\[ H(x) = H(R/\delta) \]

During phase (ii) \( T(t) = T_v = \text{constant implying} \)
\[ \frac{dT}{dt} = 0 \] (13)

for \( t > t_v \)

Thus, equations (12) and (13) describe the change in drop radius and temperature during phase (ii).

The initial conditions for this problem may be stated as the starting drop radius and temperature at time \( t = 0 \) are \( R_o \) and the ambient temperature, \( T_a \), respectively, i.e.;
\[ R(0) = R_0, \quad T(0) = T_a \]

The system of equations (8), (9), (12) and (13) along with initial conditions (14) represent a nonlinear set of ordinary differential equations. These equations are no longer coupled, however, as they were in Chapter I. This is due to the fact that the drop is now stationary and thus the magnetic field is constant relative to the drop. The field strength can be determined separately since the drop position is always known.

**Nondimensionalization**

Introduction of the dimensionless variables
\[ T^* = \frac{T}{T_a}, \quad R^* = \frac{R}{R_0}, \quad t^* = \frac{t(B^2/\sigma\mu^2)}{(\rho_d c_L R_0^2 T_a)}, \]
\[ H^* = H(R^*) \]

into equations (8), (9) and (14) results in the following equations for phase (i);
\[ \frac{1}{3} \frac{dT^*}{dt^*} = -\zeta(T^{*4} - 1) + \frac{3}{4} H^* \quad (15) \]
\[ \frac{dR^*}{dt^*} = 0 \quad (16) \]

along with initial conditions
\[ T^*(0) = 1, \quad R^*(0) = 1 \quad (17) \]

where;
\[ \zeta = \frac{\varepsilon \sigma_r T_a^4 R_0}{B^2/\sigma\mu^2} \]

and for the stacked loop model;
\[ B^2 = B \cdot \bar{B} = \sqrt{B_{\rho}^2 + B_z^2} \]

Expressions for \( B_{\rho} \) and \( B_z \) for a stacked loop model are given in Appendix A. For phase (ii) we get
\[
\frac{1}{Ste} \frac{dR^*}{dt^*} = \zeta \left[ \left( \frac{T_v}{T_a} \right)^4 - 1 \right] - \frac{3}{4} \frac{H^*}{R^*} \tag{18}
\]
\[
\frac{dT^*}{dt^*} = 0 \tag{19}
\]

where:
\[
Ste = \frac{c_L T_a}{L_v}
\]

Nondimensionalization of the governing equations results in the four dimensionless parameters; \( \zeta \), \( Ste \), \( T_v/T_a \), and \( x \). During phase (i) the drop is heating and it is clear that the dimensionless coefficient, \( \zeta \), represents the energy lost to radiation over the energy absorbed due to the magnetic field. During the vaporization phase, the coefficient \( \zeta \), is also found along with a parameter of phase change, \( Ste \), and the relative temperature coefficient \( T_v/T_a \). The Stefan number relates the internal energy to the enthalpy of phase change which in this case is the enthalpy of sublimation. The coefficient \( T_v/T_a \) is a parameter both of the material being considered and the surrounding environment. The parameter, \( x \), a function of skin depth, is constant for phase (i) if the material properties, \( \sigma \) and \( \mu \), and the frequency, \( f \), are treated as constants. The function \( x \) will definitely vary with time in phase (ii), however, since it is also a function of the sphere radius, \( R(t) \). It should be noted that the coefficients \( \zeta \) and \( Ste \) may also be functions of time depending on the model of the physical parameters (e.g. \( \varepsilon, \rho_\sigma, c_L, \sigma \), etc.) which may themselves be functions of temperature.

The system was solved using a similar scheme to that in Chapter I (see Numerical Solution) except in this case the magnetic field strength is only calculated once based on the location of the drop within the coil.
Special Case for $R \geq 2\delta$

If we make the assumption of constant physical parameters in this problem (e.g., $\varepsilon, \rho, c, \sigma$ are not functions of temperature $T(t)$ or drop radius $R(t)$), we may derive a special result for the range in which $x = \frac{R}{\delta} \geq 2$. Noting expression (3) for $H(x)$ and Figure II.B we can clearly see that for $x \geq 2$;

$$H(x) \equiv x - 1$$

Substituting this expression back into equation (12) we find that we are able to solve equations (9), (12) and (14) exactly to obtain the following piecewise continuous transcendental equation for $R(t)$;

$$R(t) = R_o \quad \text{for } 0 < t < t_v \quad (20)$$

$$\left( R(t) - R_o \right) \frac{\alpha_2}{\alpha_1} \ln \left( \frac{\left( R(t) + \frac{\alpha_2}{\alpha_1} \right)}{\left( R_o + \frac{\alpha_2}{\alpha_1} \right)} \right) = \alpha_1 (t - t_v) \quad \text{for } t \geq t_v \quad (21)$$

where

$$\alpha_1 = \frac{4\sigma \mu^2 \delta \sigma e (T_v^4 - T_s^4) - 3B^2}{4\rho L \sigma \mu^2 \delta}$$

$$\alpha_2 = \frac{3B^2}{4\rho L \sigma \mu^2}$$

In the same manner we note here that equations (8) and (13) along with initial condition (14) can be solved exactly resulting in the following piecewise continuous transcendental equation for $T(t)$;

$$\ln \left( \frac{(T(t) + k)(T_s - k)}{(T(t) - k)(T_s + k)} \right) + \tan^{-1} \left( \frac{T(t)}{k} \right) - \tan^{-1} \left( \frac{T_s}{k} \right) = \frac{2\beta_1}{k} t \quad \text{for } 0 < t < t_v \quad (22)$$

$$T(t) = T_v \quad \text{for } t \geq t_v \quad (23)$$

where
\[ k = \sqrt{\frac{\beta_1}{\beta_2}} \]

\[ \beta_1 = \frac{9H(R_0/\delta)B^2 + 12\sigma_c\epsilon T_a^4 \sigma \mu^2 R_0}{4\rho_d c_l \sigma \mu^2 R_0^2} \]

\[ \beta_2 = \frac{-3\sigma_c \epsilon}{\rho_d c_l R_0} \]

**Figure II.B**
Experimentation

Apparatus

In order to validate analytical predictions for a single droplet an experimental electromagnetic vaporization device (EVD) was designed and constructed at Rice University by the investigating team. Figure II.C is a schematic of the EVD.

Its critical features are the induction coil and custom vacuum chamber which are designed to achieve efficient coupling between the coil and the droplet that is being heated. In addition, the design permits a choice of the ambient atmosphere into which the droplet evaporates. Depending on experimental requirements, the droplet may be exposed to an inert gas atmosphere or vacuum. The chamber uses a specially designed airlock/manifold at the top. The airlock/manifold is made of quartz and has two inlets, two outlets, two greaseless needle valves, and two glass vacuum valves. The top inlet permits insertion of the droplets without depressurizing the chamber. The other inlet permits the introduction of an inert gas atmosphere and the outlets provide for vacuum pressure monitoring and entrance into the main chamber. A quartz tube (5' length, 1/2" diameter) serves as the main chamber. The induction coil is mounted outside the quartz tube. The coil is specifically designed to transfer the maximum amount of energy from the frequency generator to the droplet. The coil is made of 1/4(in.) x 1/2 (in.) square copper tubing with wall thickness of 0.062 (in.). It has 6 turns, an inside diameter of 1/2", a length of 22.38", and a calculated inductance of 0.175 \( \mu H \) [21]. The bottom of
the chamber consists of an aluminum "catch basin" which houses a ceramic "catcher" and provides a connection for the vacuum pump. The custom vacuum chamber is fully adjustable to accommodate other coil geometries and droplet sizes. The quartz tube can be easily replaced if coils with smaller/larger diameters and lengths are used. The bottom portion of the chamber rides along Plexiglas rods and its position can be adjusted to accommodate shorter/longer quartz tube/coil combinations. The main support structure of the chamber is made of Plexiglas rods and mounted into aluminum base and top plates. This EVD was used in conjunction with other laboratory equipment to generate data for the stationary droplet, but will also be used for the verification of the predicted behavior of a falling droplet. Figure II.D. is a schematic of the experimental setup.

The equipment used consists of the EVD, a Lepel radio frequency (RF) generator capable of supplying 30 kW at 400 kHz [21], a Welch Scientific Co. mechanical vacuum pump, two Ingersoll-Rand hydraulic pumps, and a 4-to-1 step-down transformer. The electrical circuit consists of the RF generator, the step-down transformer, and the induction coil in the EVD. The transformer was needed to match the impedance of the RF generator. The leads to the coil are made of copper tubing of varying outside diameter (from 1/8 (in.) to 3/8 (in.)). Since there is significant heating due to the large currents that flow in the electrical circuit, the circuit is cooled by two separate hydraulic circuits. One circuit cools the RF generator while the other circuit is dedicated to cooling the transformer and the EVD coil. Each circuit uses a 100 psi pump with water as the cooling medium. The temperature of the cooling water flowing through the induction coil is critical and is monitored at the coil outlet. This is accomplished with chromel-alumel thermocouple coupled with an Omega
Figure II.D
digital thermometer. The ambient temperature is recorded using a standard mercury thermometer.

**Procedure**

A zinc shot of approximately 3\(mm\) diameter (99.999\% purity [22]) is weighed on a *Sartorius* laboratory electrical balance to an accuracy of +/- 0.0001\(g\). Using a thin tungsten wire (0.01 (mm) diameter) the shot is suspended from the top of the vacuum chamber such that it is positioned in the middle of the coil. Room temperature is recorded. The hydraulic circuits are then energized and a flow rate of (20 \(mL/s\)) is established through the coil. The vacuum chamber is pumped down to 40 \(\text{mTorr}\). The frequency generator is then turned on providing a fixed high frequency current depending on the coil in use. The current magnitude is adjusted via controls on the RF generator. Heating/vaporization times are then recorded using a stopwatch. The vacuum pressure is monitored using a *Varian 801 MilliTorr* vacuum gage. During the experiment, cooling water temperature at the coil outlet is continuously monitored. After a given vaporization time has been reached, the RF current is turned off and the vacuum chamber pressure is returned to ambient conditions. The zinc shot is then removed from the chamber and weighed again. In this manner, various data points along a single curve were obtained by varying the vaporization time for a given fixed current, vacuum pressure, ambient temperature, coil geometry and initial droplet mass.
Results and Discussion

In our computations as well as our experiment we used zinc as our subject material. Zinc is commonly used in the metallurgy industry and has a relatively low enthalpy of sublimation at rough vacuum pressures [23]. Material properties for zinc may be found in Appendix C.

The problem of Chapter II is less involved than the problem considered in Chapter I due to the droplet being stationary. After nondimensionalization, we are left with the four parameters; $\zeta$, $Ste$, $T_v/T_a$, and $x$. In order to determine the influence of these parameters on the vaporization process we develop dimensionless plots for varying values of the parameters. Again, we express changes in the starting value of $x$ as differing values of the frequency, $f$. We then plot our experimental results against the predicted behavior of our model.

Dimensionless curves are plotted for $\zeta = 5.83e-4$, $T_v/T_a = 2.2$, $x_o = 4.43$, and $Ste = 0.0235$. These values correspond to our experimental parameters for zinc at a vacuum pressure, $p_v = 40 mTorr$, ambient temperature, $T_a = 296(K)$ vaporization temperature, $T_v = 658(K)$ and a frequency, $f = 245.31 kHz$. The field strength, $B$, was derived from a coil with $N = 6$ turns, $I = 300 \text{ A}$, $a = 0.0063(m)$ and $N/L = 200$. The calculated dimensionless magnetic flux density for a coil with this geometry (using our stacked loop model) can be seen in Figure II.E. The field is not constant along the axis, as seen for the model in Chapter I., due to the smaller number of loops. However, this will only effect us in the calculation of the error in the positioning of the droplet. This is discussed in the Uncertainty Analysis in Appendix D.
Figure II.E

In Figure II.F we have plotted dimensionless temperature versus dimensionless time for various values of the energy parameter, $\zeta$. The sloped portion of the curves represents the heating phase while the flattened portion is indicative of our equilibrium vaporization assumption. We note that for lower values of $\zeta$ the temperature increases linearly with time. This is due to the fact that power absorption is the dominant term in equations (15) and (18) and is independent of temperature. For higher values of $\zeta$ radiation becomes appreciable and eventually radiation loss is sufficient to prevent the droplet from vaporizing. Thus, an equilibrium temperature is reached below the vaporization temperature. Our experimental values of $\zeta = 1.52 \times 10^{-3}$ and $\zeta = 5.83 \times 10^{-4}$ are quite close to
the limit of $\zeta = 0$ corresponding to heating without any radiation losses. This leads us to conclude that radiation is not a dominant factor in our experiment. We would point out however, that at higher values of $\zeta$ an order of magnitude in a radiation parameter, such as emissivity, will produce a substantial change in the heating rate.

![Graph showing dimensionless temperature vs. dimensionless time]

**Figure II.F**

Figure II.G shows dimensionless droplet radius as a function of dimensionless time for changing values of $\zeta$. Again we have used the corresponding parameters of $T_v/T_a = 2.2$, $f = 245.31 kHz$, and $Ste = 0.065$ as seen in our experiment. All of these curves begin with a small horizontal portion corresponding to the time needed to reach the vaporization temperature from the ambient temperature. In addition to
this, we see a linear change in drop radius for larger $\zeta$ gradually changing to a nonlinear change for smaller $\zeta$. Clearly for large $\zeta$, as before, the radiation loss per unit area is appreciable and this is a not a function of the droplet radius. For smaller $\zeta$, when power absorption dominates the change in radius behaves like $H(x)/x$ (see figure I.G). That is, as $x$ decreases $H(x)/x$ becomes smaller and a limit is reached where radiative losses are equivalent to power absorbed. Since our model uses a constant skin depth, as $x$ decreases the radius is becoming smaller relative to the skin depth. Thus, the heating effect is no longer concentrated in a thin outer shell of the sphere. It is also apparent that even for $\zeta = 0$ a limiting
droplet size will be reached. This phenomena is more apparent for low values of $\zeta$ in Figure II.G.

In order to analyze the results for different materials we have varied the phase change parameter, $Ste$. In Figure II.H we have plotted dimensionless radius versus dimensionless time for $\zeta = 5.83e-4$ and various Stefan numbers. Our experimental value of $Ste = 0.065$ is quite favorable for the vaporization process. For other materials at the same atmospheric conditions, such as copper and titanium, we would expect much lower Stefan numbers. As $Ste$ increases it is clear that droplet radius will reduce more rapidly due to the need to overcome a relatively smaller enthalpy of phase change.

![Figure II.H](image-url)
Figure II.J shows dimensionless drop temperature as a function of time for varying values of the magnetic field frequency. This result is very similar to the heating and vaporization curves in Chapter I for $\beta = 0$ (i.e. no convection heat transfer). In this case, however, the frequencies needed to vaporize the droplet are much lower. This is a result of the longer time scales for the stationary droplet in addition to the absence of convection. For lower frequencies there is a slow temperature rise but the droplet gradually reaches the vaporization temperature. As before, this is due to the fact that the skin depth, $\delta$, is inversely proportional to the square of the frequency. At lower frequencies the skin depth is quite large.

Figure II.J
compared to the drop radius and the power from the field is distributed over a large volume. At frequencies of $10^5$ (Hz) the temperature increase is more rapid and vaporization takes place more quickly since the heating effect is concentrated in a small outer shell of the sphere. During the heating phase, the power absorbed is proportional to $H(x)/x^2$. Looking at Figure I.E it is clear that this function will increase for increasing frequencies ($x$ values). However, at value of $x \equiv 2.41$ the heating rate will reach a maximum. As frequencies are increased, the vaporization time is lengthened. Gradually, the frequency becomes high enough for the droplet

During the vaporization phase, the change in radius is proportional

![Figure II.K](image)

Figure II.K

to reach a final drop radius, $R_f$. At this point the power absorbed by the droplet has decreased to the point where it is equivalent to the heat loss
from radiation. Droplets which have been exposed to higher frequencies reach a smaller final radius as indicated in Figure II.K.

to \( H(x)/x \) as opposed to \( H(x)/x^2 \). The function \( H(x)/x \) is shown in Figure I.G. As vaporization continues, since the skin depth is fixed, the value of \( x \) decreases due to a decreasing droplet radius. As \( x \) decreases the power absorbed balances the energy lost and \( x \) reaches a final value. Again, higher frequencies indicate a higher initial value of \( x \) which also yields a higher value of \( H(x)/x \). The onset of vaporization is related directly to the heating rates found in Figure II.G. Thus, we see that one optimum parameter for vaporization would be to start at an initial \( x \) value of \( x \approx 2.41 \) to maximize heating rates and then to increase the frequency during the vaporization phase.

Our experimental data can be seen along with the theoretical predictions in Figures II.L and II.M. These plots show dimensionless droplet radius versus actual time in seconds. This will give the reader a "feel" for the actual process we are talking about. In these figures we have indicated the uncertainty in the measurement of the drop radius and time via error bars. The error in these measured values was quite small due to the high accuracy of our scale and the long time durations. The uncertainty of the other measured parameters, i.e. coil geometry, droplet position relative to the coil, current, and vacuum pressure have been represented as a single uncertainty in the parameter \( \zeta \). The reader may consult Appendix D for more detail. Thus, these figures also indicate the variation in the theoretical curves based on the uncertainty in \( \zeta \). The experimental data for Experiment #1 is shown in figure II.L. This data is for a measured current level of 81 (A) corresponding to
\( \zeta = 1.52 \times 10^{-3} \pm 6.0\% \). It is apparent that the data exhibits a downward trend in the direction of the predicted values. However, the vaporization rate predicted by the model is slightly slower than the observed values.

![Graph showing dimensionless radius change with time for different conditions.](image)

**Figure II.L. (Experiment #1)**

It is felt that part of this error may be attributed to the uncertainty in values of the material parameters. Our calculation of error does not take the variation of reported values of emissivity, specific heat, enthalpy of sublimation, etc., into account. The author particularly found discrepancies in the literature with regard to emissivity values. Other values found for \( \varepsilon \), however, were greater than the value used. This would mean an increase in the value of \( \zeta \) rather than a decrease. On the other hand, in order to obtain a noticeable reduction in vaporization rates the emissivity would have to be increased by an order of magnitude to 0.5 This is five times higher than any data the authors observed. We conclude
that since our experimental $\zeta$ values are so small vaporization is in a regime where it is unlikely that uncertainty in emissivity warrants the discrepancy between predicted and observed values. There is also conflicting information on the values of the specific heat at elevated temperatures. The specific heat of a solid at elevated temperature was used for our predictions since we assumed the droplet to be sublimating. Specific heats of molten zinc at the same temperature are found to be higher. In some experiments the droplet was observed to deform slightly suggesting appreciable creep. We believe that this may warrant a higher value of specific heat than was used. This would mean a greater Stefan number and thus, greater vaporization rates as indicated in Figure II.F. The experimental Stefan number lies in a regime where small changes in its
value could produce noticeable changes in vaporization rates as shown in Figure II.H. In Experiment #2, a higher current level of 131 (A) was used. This corresponds to an increased power level and $\zeta = 5.83 \times 10^{-4} \pm 4.5\%$. The data recorded for this experiment (Figure II.M) also shows a trend in the downward direction as predicted. In this case, however, the model predicts a vaporization rate greater than the observed data. Again the material parameters could be questioned. A higher emissivity in this case would decrease the predicted vaporization rates and more closely match the data. However, a higher Stefan number would increase the vaporization rate and cause greater disparity between predicted and observed values. In

![Graph showing dimensionless radius vs dimensionless time](image)

**Figure II.N**

In some instances at higher power levels, the vacuum pressure was observed to increase slightly during vaporization from 40 mTorr to 50 mTorr.
Thus, vaporization will occur at a higher droplet temperature. This increase in pressure corresponds to a change in $T_v/T_a$ from 2.22 to 2.24. Figure II. N shows that this will produce a negligible change in the vaporization curve. This figure indicates the influence of vaporization temperature on the vaporization rate. Since this pressure measurement was made at a relatively far distance from the droplet, it is possible that a vapor cloud of hot zinc vapor may be surrounding the droplet. This would increase the near field ambient pressure even higher.

During vaporization in Experiment #2, another phenomenon was observed which could provide an explanation for the disparity. When generating the data for longer time periods, a thin zinc film began to form on the inner surface of the quartz vacuum chamber. Measurements of this film show its thickness, $\Delta$, to be approximately 8.9e-5 (m). Using a shielding model to predict the effect of this film on the magnetic field shows that the field inside the film (i.e. the field heating the droplet) should be approximately 36% of the external field for a thickness of $\Delta$ and 60% for $\Delta/2$ [24]. This corresponds to an increase in the time scale, $t_0$. In Figure II.M, we have plotted the predicted vaporization curves with a correction for the shielding effect. The curve for $\Delta/2$ was plotted as an approximation since we do not know the growth of the film thickness in time. It is clear that the $\Delta/2$ curve more closely approximates the trend of the data. Our data now falls between the predicted behavior of the model and the $\Delta/2$ curve. Since vaporization is apparently nonlinear, increasing rapidly at first and then decreasing slowly, it is felt that $\Delta(t)$ may also be nonlinear. Thus, we conclude that the film produced a shielding effect
which was nonlinear as a function of time. A more accurate model of $\Delta(t)$ may provide an improved correlation with the data recorded.
Conclusions

In conclusion we may summarize our findings as follows;

(1) We have developed a general model for an electrically conducting droplet falling through an external alternating magnetic field and undergoing equilibrium vaporization. The model is nondimensionalized and takes into consideration the geometry of the coil generating the electromagnetic field. Within the reference frame of powdered metal production, we have determined that the process is feasible only at extremely high frequencies of the magnetic field (on the order of 10 (MHz)). The reduction in droplet radius is expected to be one order of magnitude for a starting radius of 0.1 (mm). Radiation is negligible for a falling droplet and convection is the dominant mode of heat loss. Increasing convection may improve the efficiency of the process to a certain degree by providing faster cooling rates with only a small increase in final droplet radius. The Lorentz force in such a device is negligible as well as the buoyancy forces. The dominant forces are the gravitational force and the viscous force. Vaporization generally occurs within a small portion of the device i.e. within the top 1/10 of its length. Vaporization may result in a reduction in size sufficient to result in terminal velocity of the particles. For this reason, as well as the efficiency of a convecting atmosphere, it is preferable to perform the vaporization in an inert gas atmosphere as opposed to vacuum. The process will work for high density high enthalpy of vaporization materials with only a moderate decrease in efficiency.
(2) Using our general model for the falling droplet we have derived a model for the limiting case of a stationary droplet. This model considers a stationary electrically conducting droplet in a vacuum atmosphere. In order to validate the stationary model an experimental device is designed and built and data are taken. Observed data indicate the model to be accurate at lower power levels but liberal in estimation of vaporization rates for higher power levels. Part of the discrepancy may be attributed to uncertainty in physical parameters at elevated temperatures. Other possible explanations include an elevated pressure in the near field of the drop, and thus a higher vaporization temperature, when using higher power levels through the coil. The most likely cause for the discrepancy is the shielding effect of the field produced by the formation of a thin film of condensed zinc vapor.
References - Chapter I


[5] Personal communication from Mr. J. B. Huddleston, President, Metallurgical Technologies, Inc., 14435 Max Road, Route 4, Box 4752, Pearland, TX 77581.


References - Chapter II


[21] Lepel Corporation, 50 Heartland Boulevard, Edgewood, NY 11717. Model # T-30-3-KC-SW

[22] Johnson Matthey Electronics, 30 Bond Street, Ward Hill, MA 01835.


References - Appendix


Appendix A
Calculation of $\bar{B} \cdot \bar{B}$ and $(\bar{B} \cdot \bar{\nabla})\bar{B}$
Smythe [1] has given the magnetic field due to a single current loop of radius \( a \) in cylindrical coordinates \( \rho, \theta, z \), as;

\[
\vec{B}(\vec{s}) = B_\rho(\vec{s}) \hat{\rho} + B_z(\vec{s}) \hat{z}
\]

where;

\[
\vec{s} = \rho \hat{\rho} + \theta \hat{\theta} + z \hat{z}
\]

\[
B_\rho = \frac{\mu I}{2\pi \rho} \frac{z}{\sqrt{(a+\rho)^2 + z^2}} \left(-K + \frac{a^2 + \rho^2 + z^2}{(a-\rho)^2 + z^2}E\right)
\]

\[
B_z = \frac{\mu I}{2\pi \sqrt{(a+\rho)^2 + z^2}} \left(K + \frac{a^2 - \rho^2 - z^2}{(a-\rho)^2 + z^2}E\right)
\]

and;

\[
K(k) = \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}
\]

\[
E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2 \phi} d\phi
\]

\[
k = \sqrt{\frac{4a\rho}{(\rho + a)^2 + z^2}}
\]

Here, we note that as expected \( \vec{B}(\vec{s}) \) is independent of \( \theta \). Defining the del operator, \( \vec{\nabla} \), in cylindrical coordinates \( \rho, \theta, z \), as;

\[
\vec{\nabla} = \hat{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho} \hat{\theta} \frac{\partial}{\partial \theta} + \hat{z} \frac{\partial}{\partial z}
\]

we see;

\[
(\vec{B} \cdot \vec{\nabla})\vec{B} = B_\rho \frac{\partial}{\partial \rho} \left(B_\rho \hat{\rho} + B_z \hat{z}\right) + B_z \frac{\partial}{\partial z} \left(B_\rho \hat{\rho} + B_z \hat{z}\right)
\]

performing the differentiation and collecting terms we are left with;

\[
(\vec{B} \cdot \vec{\nabla})\vec{B} = \hat{\rho} \left(B_\rho \frac{\partial B_\rho}{\partial \rho} + B_z \frac{\partial B_\rho}{\partial z}\right) + \hat{z} \left(B_\rho \frac{\partial B_z}{\partial \rho} + B_z \frac{\partial B_z}{\partial z}\right)
\]

letting;

\[
B_\rho = \alpha \beta \gamma
\]
where;
\[
\alpha = \frac{\mu l}{2\pi z}
\]
\[
\beta = \frac{z}{\rho [(a + \rho)^2 + z^2]^{\frac{3}{2}}}
\]

and
\[
\gamma = -K + \frac{a^2 + \rho^2 + z^2}{(a - \rho)^2 + z^2} E
\]

Using the chain rule and differentiating \( B_\rho \) with respect to \( \rho \) gives;
\[
\frac{\partial B_\rho}{\partial \rho} = \alpha \gamma \frac{\partial \beta}{\partial \rho} + \alpha \beta \frac{\partial \gamma}{\partial \rho}
\]

where
\[
\frac{\partial \beta}{\partial \rho} = -z \left[ \frac{z^2 + (a + 2\rho)(a + \rho)}{(a + \rho)^2 + z^2} \right]
\]
\[
\frac{\partial \gamma}{\partial \rho} = -\frac{\partial K}{\partial k} \frac{\partial k}{\partial \rho} + \frac{2a(a^2 - \rho^2 + z^2)}{[(a - \rho)^2 + z^2]^{3/2}} E + \frac{a^2 + \rho^2 + z^2}{(a - \rho)^2 + z^2} \frac{\partial E}{\partial k} \frac{\partial k}{\partial \rho}
\]
\[
\frac{\partial K}{\partial k} = \frac{E}{k(1 - k^2)} - \frac{K}{k}
\]
\[
\frac{\partial E}{\partial k} = \frac{E - K}{k}
\]
\[
\frac{\partial k}{\partial \rho} = \frac{1}{2\rho} - \frac{k^3}{4a}
\]

differentiating \( B_\rho \) with respect to \( z \) gives;
\[
\frac{\partial B_\rho}{\partial z} = \alpha \gamma \frac{\partial \beta}{\partial z} + \alpha \beta \frac{\partial \gamma}{\partial z}
\]

where;
\[
\frac{\partial \beta}{\partial z} = \frac{(a + \rho)^2}{\rho [(a + \rho)^2 + z^2]^{3/2}}
\]
\[
\frac{\partial \gamma}{\partial z} = -\frac{\partial K}{\partial k} \frac{\partial k}{\partial z} - \frac{2a\rho z}{[(a - \rho)^2 + z^2]^{3/2}} E + \frac{a^2 + \rho^2 + z^2}{(a - \rho)^2 + z^2} \frac{\partial E}{\partial k} \frac{\partial k}{\partial z}
\]
\[
\frac{\partial k}{\partial z} = -\frac{zk^3}{4a\rho}
\]
Now letting;

\[ B_z = \alpha \xi \psi \]

where;

\[ \xi = \frac{1}{\left[ (a + \rho)^2 + z^2 \right]^{\frac{3}{2}}} \]

and

\[ \psi = K + \frac{a^2 - \rho^2 - z^2}{(a - \rho)^2 + z^2} E \]

Using the chain rule and differentiating \( B_z \) with respect to \( \rho \) gives;

\[ \frac{\partial B_z}{\partial \rho} = \alpha \psi \frac{\partial \xi}{\partial \rho} + \alpha \xi \frac{\partial \psi}{\partial \rho} \]

where

\[ \frac{\partial \xi}{\partial \rho} = \frac{-(a + \rho)}{\left[ (a + \rho)^2 + z^2 \right]^{\frac{3}{2}}} \]

\[ \frac{\partial \psi}{\partial \rho} = \frac{\partial K}{\partial \rho} \frac{\partial k}{\partial \rho} + \frac{2a[(a - \rho)^2 - z^2]}{\left[ (a - \rho)^2 + z^2 \right]^2} E + \frac{a^2 - \rho^2 - z^2}{(a - \rho)^2 + z^2} \frac{\partial E}{\partial \rho} \frac{\partial k}{\partial \rho} \]

differentiating \( B_z \) with respect to \( z \) gives;

\[ \frac{\partial B_z}{\partial z} = \alpha \psi \frac{\partial \xi}{\partial z} + \alpha \xi \frac{\partial \psi}{\partial z} \]

where;

\[ \frac{\partial \xi}{\partial z} = \frac{-z}{\left[ (a + \rho)^2 + z^2 \right]^{\frac{3}{2}}} \]

\[ \frac{\partial \psi}{\partial z} = \frac{\partial K}{\partial z} \frac{\partial k}{\partial z} - \frac{4az(a - \rho)}{\left[ (a - \rho)^2 + z^2 \right]^2} E + \frac{a^2 - \rho^2 - z^2}{(a - \rho)^2 + z^2} \frac{\partial E}{\partial z} \frac{\partial k}{\partial z} \]

**Special Case at \( \rho = 0 \)**

at \( \rho = 0 \);

\[ B_\rho \to 0, \quad \frac{\partial B_\rho}{\partial \rho} \to 0, \quad \frac{\partial B_\rho}{\partial z} \to 0, \quad \frac{\partial B_z}{\partial \rho} \to 0 \]

Thus,
\[(\vec{B} \cdot \nabla) \vec{B} = \hat{i} \left( B_z \frac{\partial B_z}{\partial z} \right) \]

Now at \( \rho = 0; \)

\[B_z = \frac{1}{2} \frac{\mu a^2 I}{[a^2 + z^2]^{3/2}} \]

and

\[\frac{\partial B_z}{\partial z} = -\frac{3}{2} \frac{\mu a^2 I z}{(a^2 + z^2)^{5/2}} \]
Appendix B

Nondimensionalization
Nondimensionalization of $k$

Given:

$$k = \frac{4a\rho}{\sqrt{(\rho + a)^2 + z^2}}$$  \hspace{1cm} (1)

We introduce the dimensionless variables:

$$\rho^* = \rho/R_o$$
$$z^* = z/R_o$$

substituting into equation (1) gives:

$$k = \frac{4\rho^*(a/R_o)}{\sqrt{\left((a/R_o) + \rho^*\right)^2 + z^{*2}}}$$

Thus, let $k^* = k$.

Nondimensionalization of Derivatives of $k$

Given:

$$\frac{\partial k}{\partial \rho} = \frac{k}{2\rho} - \frac{k^3}{4\rho} - \frac{k^3}{4a}$$  \hspace{1cm} (2)
$$\frac{dk}{dz} = \frac{zk^3}{4\rho}$$  \hspace{1cm} (3)

We introduce the dimensionless variables:

$$\rho^* = \rho/R_o$$
$$z^* = z/R_o$$
$$k^* = k$$

substituting into equations (2) and (3) gives:

$$\frac{\partial k}{\partial \rho} = \left(\frac{k^*}{2\rho^*} - \frac{k^{*3}}{4\rho^*} - \frac{k^{*3}}{4(a/R_o)}\right)$$
\[
\frac{\partial k}{\partial z} = -\frac{z^* k^*^3}{4\rho^*}
\]

Thus, we define:

\[
\frac{\partial k^*}{\partial \rho} = \left(\frac{k^* - k^* third}{2\rho^*} - \frac{k^* third}{4\rho^*} \frac{k^* third}{4(a/R_o)}\right)
\]

\[
\frac{\partial k^*}{\partial z} = -\frac{z^* k^*^3}{4\rho^*}
\]

**Nondimensionalization of \(B\)**

Given that;

\[
B = \sqrt{B_p^2 + B_z^2}
\]

\[
B_p = \frac{\mu l}{2\pi} \frac{z}{\rho[(a + \rho)^2 + z^2]^{1/2}} \left[-K(k) + \frac{a^2 + \rho^2 + z^2}{(a - \rho)^2 + z^2} E(k)\right]
\]

(4)

and

\[
B_z = \frac{\mu l}{2\pi} \frac{1}{[(a + \rho)^2 + z^2]^{1/2}} \left[K(k) + \frac{a^2 - \rho^2 - z^2}{(a - \rho)^2 + z^2} E(k)\right]
\]

(5)

We introduce the dimensionless variables;

\[
\rho^* = \rho/R_o
\]

\[
z^* = z/R_o
\]

\[
k^* = k
\]

substituting into equations (4) and (5) gives;
\[ B_{\rho} = \frac{\mu I}{2\pi a} \frac{\rho^*}{\rho^*[(1+\rho^*)^2 + z'^*]^1/2} \left[ -K(k^*) + \frac{1+\rho'^* + z'^*}{(1-\rho^*)^2 + z'^*} E(k^*) \right] \]

\[ B_{z} = \frac{\mu I}{2\pi a} \frac{1}{[(1+\rho^*)^2 + z'^*]^1/2} \left[ K(k^*) + \frac{1-\rho'^* - z'^*}{(1-\rho^*)^2 + z'^*} E(k^*) \right] \]

defining;

\[ B_o = \frac{\mu I}{2\pi R_o} \]

and

\[ B'_{\rho} = B_{\rho}/B_o \]

\[ B'_{z} = B_{z}/B_o \]

\[ B' = B/B_o \]

we derive the nondimensionalized field components as;

\[ B'_{\rho} = \frac{\rho^*}{\rho^*[(1+\rho^*)^2 + z'^*]^1/2} \left[ -K(k^*) + \frac{1+\rho'^* + z'^*}{(1-\rho^*)^2 + z'^*} E(k^*) \right] \]

\[ B'_{z} = \frac{1}{[(1+\rho^*)^2 + z'^*]^1/2} \left[ K(k^*) + \frac{1-\rho'^* - z'^*}{(1-\rho^*)^2 + z'^*} E(k^*) \right] \]

and the expression for the nondimensionalized field, \( B' \), as;

\[ B' = \sqrt{B'^2_{\rho} + B'^2_{z}} \]

**Nondimensionalization of Governing Equations**
Given:
\[
\frac{dT}{dt} = \frac{9}{4} \frac{H(x)(\bar{B} \cdot \bar{B})}{\rho_d c_L \sigma \mu^2 R_o^2} - 3 \frac{\sigma_r e \varepsilon}{\rho_d c_L R_o} (T^4 - T_a^4) - \frac{3h(T - T_a)}{\rho_d c_L R_o}
\]

let
\[T^* = \frac{T}{T_m}, \quad H^* = H, \quad \bar{B}^* = \frac{\bar{B}}{B_o}, \quad t^* = \left(\frac{B_o^2}{\sigma \mu^2}\right)t/\left(\rho_d c_L R_o^2 T_a\right)\]

then
\[
\frac{dT^*}{dt^*} = \frac{9}{4} H^* (B^* \cdot B^*) - 3 \frac{\sigma_r e T_m R_o}{B_o^2 / \sigma \mu^2} \left(\frac{T^* - T_a}{T_m}\right) - 3 \frac{\sigma_r T_m R_o}{B_o^2 / \sigma \mu^2} \left(T^* - \left(\frac{T_a}{T_m}\right)^4\right)
\]

and
\[
\frac{1}{3} \frac{dT^*}{dt^*} = \frac{3}{4} H^* (B^* \cdot B^*) - \beta h^* \left(\frac{T^* - T_a}{T_m}\right) - \zeta \left(T^* - \left(\frac{T_a}{T_m}\right)^4\right)
\]

where;
\[\beta = \frac{\sigma_r T_m R_o}{B_o^2 / \sigma \mu^2}, \quad \zeta = \frac{\sigma_r T_m R_o}{B_o^2 / \sigma \mu^2}\]

Given;
\[
\frac{dR}{dt} = \frac{h(T_v - T_a)}{\rho_d L_v} + \frac{\sigma_r e (T_v^4 - T_a^4)}{\rho_d L_v} - \frac{3}{4} \frac{(\bar{B} \cdot \bar{B}) H(x)}{\rho_d L_v \sigma \mu^2 R}
\]

let
\[R^* = \frac{R}{R_o}, \quad H^* = H, \quad \bar{B}^* = \frac{\bar{B}}{B_o}, \quad t^* = \left(\frac{B_o^2}{\sigma \mu^2}\right)t/\left(\rho_d c_L R_o^2 T_a\right)\]

thus;
\[
\frac{dR^*}{dt^*} = \frac{c_L T_m}{L_v} \frac{h T_m R_o}{B_o^2 / \sigma \mu^2} \left[\left(\frac{T_v}{T_m}\right) - \frac{T_a}{T_m}\right] + \frac{c_L T_m}{L_v} \frac{\sigma_r e T_a R_o}{B_o^2 / \sigma \mu^2} \left[\left(\frac{T_m}{T_a}\right)^4 - \left(\frac{T_a}{T_m}\right)^4\right] - \frac{3}{4} \frac{H^* (\bar{B}^* \cdot \bar{B}^*)}{R^*}
\]

and
\[
\frac{1}{Ste} \frac{dR^*}{dt^*} = \beta \left[\left(\frac{T_v}{T_m}\right) - \frac{T_a}{T_m}\right] + \zeta \left[\left(\frac{T_v}{T_m}\right)^4 - \left(\frac{T_a}{T_m}\right)^4\right] - \frac{3}{4} \frac{H^* (\bar{B}^* \cdot \bar{B}^*)}{R^*}
\]

where;
\[Ste = \frac{c_L T_m}{L_v}, \quad \zeta = \frac{\sigma_r e T_a^4 R_o}{B_o^2 / \sigma \mu^2}, \quad \beta = \frac{h T_m R_o}{B_o^2 / \sigma \mu^2}\]
Given;

\[
\frac{dv}{dt} = -\frac{3}{2} \frac{G(B \cdot \vec{v})B}{\rho_d \mu} \frac{1}{8} \frac{\rho_a c_D v^2 + g \rho_a \beta (T_v - T_a)}{\rho_d} - g
\]

let

\[
T^* = T/T_m, t^* = \left( \frac{B_o^2}{\sigma \mu^2} \right) t/\left( \rho_d c_L R_o^2 T_a \right), R^* = R/R_o,
\]

\[
v^* = v/\left( c_L R_o T_a \sigma \mu \right), \quad B^* = B/B_o, \quad G^* = G, \quad c_D^* = c_D, \quad \vec{v}^* = R_o \vec{v}
\]

then

\[
\frac{B_o^2/\mu \cdot dv^*}{\rho_d R_o \cdot dt^*} = -\frac{3}{2} \frac{B_o^2/\mu}{\rho_d R_o} G^* (B^* \cdot \vec{v}^*) B^* + \frac{3 \rho_a v_o^2}{8 \rho_d R_o \cdot R^* c_D^*} + \frac{g \rho_a \beta_a T_m \left( T^* - T_a \right)}{\rho_d} - g
\]

and

\[
\frac{dv^*}{dt^*} = -\frac{3}{2} \frac{G^* (B^* \cdot \vec{v}^*) B^*}{8 H a^2 R^* c_D^*} + \frac{1}{8} \frac{v^2}{H a^2 R^* c_D^*} + \frac{1}{L_y} \left( T^* - T_a \right) - \frac{1}{N}
\]

where;

\[
H a^2 = \left( \frac{B_o^2/\mu}{\rho_d (c_L R_o T_m \sigma \mu)^2} \right)^{1/2}, L_y = \frac{B_o^2/\mu}{g \rho_a \beta_a T_a R_o}, N = \frac{B_o^2/\mu}{g \rho_a R_o}
\]

Given;

\[
\frac{dz}{dt} = v
\]

\[
v^* = v/\left( c_L R_o T_a \sigma \mu \right), \quad t^* = \left( \frac{B_o^2}{\sigma \mu^2} \right) t/\left( \rho_d c_L R_o^2 T_a \right), \quad R^* = \frac{z}{R_o}
\]

then

\[
\frac{dz^*}{dt^*} = \rho_d \left( \frac{c_L R_o T_m \sigma \mu}{B_o^2/\mu} \right) v^* = \frac{1}{Al} v^*
\]

where;

\[
Al = \frac{B_o^2/\mu}{\rho_d (c_L R_o T_a \sigma \mu)^2}
\]
Appendix C

Material Properties
<table>
<thead>
<tr>
<th>Material Parameter</th>
<th>Zinc</th>
<th>Argon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$-specific heat (solid) (J/kgK)</td>
<td>444</td>
<td></td>
</tr>
<tr>
<td>$c$-specific heat (liquid) (J/kgK)</td>
<td>481</td>
<td></td>
</tr>
<tr>
<td>$\nu_a$-kinematic viscosity (m$^2$/s)</td>
<td></td>
<td>1.418e-5</td>
</tr>
<tr>
<td>$\rho$-density (kg/m$^3$)</td>
<td>6660</td>
<td>1.202</td>
</tr>
<tr>
<td>$L_v$-latent heat of sublimation (J/kg)</td>
<td>2.016e6</td>
<td></td>
</tr>
<tr>
<td>$L_v$-latent heat of vaporization (J/kg)</td>
<td>1.76e6</td>
<td></td>
</tr>
<tr>
<td>$T_a$-ambient temperature (K)</td>
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<td>300</td>
</tr>
<tr>
<td>$T_m$-melting temperature (K)</td>
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<td></td>
</tr>
<tr>
<td>$T_v$-vaporization temperature (K)</td>
<td>1184</td>
<td></td>
</tr>
<tr>
<td>$T_v$-sublimation temperature* (K)</td>
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<td></td>
</tr>
<tr>
<td>$\beta_a$-expansion coefficient (1/K)</td>
<td></td>
<td>3.66e-3</td>
</tr>
<tr>
<td>$\sigma$-electrical conductivity(1/Ωm)</td>
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</tr>
<tr>
<td>$k$-thermal conductivity (W/mK)</td>
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<td>1.63e-2</td>
</tr>
<tr>
<td>$\mu$-magnetic permeability (H/m)</td>
<td>4$\pi$e-7</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$-emissivity</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

* at 40 mTorr vacuum pressure.
Appendix D

Uncertainty Analysis
The nature of our problem is such that it involves many parameters. In an effort to accurately model the uncertainty analysis the method as outlined by Moffat was utilized [3]. Moffat suggests to express the experimental uncertainty in terms of the uncertainty in the governing dimensionless parameters of the problem. Given a result, \( R \), calculated from the input data \( (x_1, x_2, x_3, \ldots , x_n) \), the uncertainty, \( dR \), may be expressed as [5]:

\[
\delta R = \left[ \left( \frac{\partial R}{\partial x_1} \delta x_1 \right)^2 + \left( \frac{\partial R}{\partial x_2} \delta x_2 \right)^2 + \ldots + \left( \frac{\partial R}{\partial x_n} \delta x_n \right)^2 \right]^{1/2}
\]

In two cases, where we felt this method to be unnecessarily cumbersome, we used a conservative uncertainty in the input parameters and directly calculated the maximum deviation due to this effect. This technique was employed to calculate (i) the uncertainty in the magnetic field due to uncertainties in droplet position and coil geometry \( (B'_i) \) and (ii) the uncertainty in the vaporization temperature \( (T_V) \) due to uncertainty in the vacuum pressure. In all other instances the standard method as outlined above was employed.

\[ R_0 \]

Using:

\[
m = \rho_s V = \frac{4}{3} \pi \rho_s R_o^3
\]

\[
\Rightarrow R = \left( \frac{3m}{4 \pi \rho_s} \right)^{1/3}
\]

now,
\[ \delta R = \left( \frac{2}{\omega \sigma \mu} \right)^{\frac{1}{2}} \]

and,

\[ \frac{\partial R}{\partial m} = \left( \frac{1}{4\pi \rho_d} \right)^{\frac{1}{2}} \left( \frac{1}{3m} \right)^{\frac{3}{2}} \]

Typical calculation:

\[ m = 0.1700 \pm 0.0001(\text{g}) \]
\[ \rho_d = 6660(\text{kg/m}^3) \]
\[ \Rightarrow R = 0.0018265(m) \]
\[ \Rightarrow \delta R = 0.35815 \times 10^{-6}(m) \]

Thus,

\[ R_s = 1.8265 \times 10^{-3} \pm 0.0164\%(m) \]

\[ \delta \]

Using;

\[ \delta = \sqrt{\frac{2}{\omega \sigma \mu}} \]

with,

\[ \omega = 2\pi f \Rightarrow \delta = \frac{1}{\sqrt{\pi f \sigma \mu}} \]

now,

\[ \delta \delta = \left( \frac{\partial \delta}{\partial f} \frac{\delta f}{\delta} \right)^{\frac{1}{2}} \]

and,

\[ \frac{\partial \delta}{\partial f} = -\frac{1}{2} \left( \frac{1}{\pi \sigma \mu} \right)^{\frac{1}{2}} \left( \frac{1}{f} \right)^{\frac{3}{2}} \]
Typical calculation:
\[ \sigma = 6.06 \times 10^6 (1/\Omega \cdot m) \]
\[ \mu = 4\pi \times 10^{-7} (H/m) \]
\[ f = 245.31 \pm 0.01 (kHz) \]
\[ \Rightarrow \delta = 4.1278 \times 10^{-4} (m) \]
\[ \Rightarrow \delta \delta = 8.4136 \times 10^{-9} (m) \]

Thus,
\[ \delta = 4.1278 \times 10^{-4} \pm 0.00204\% (m) \]

**H(R/\delta)**

Using;
\[ H(R/\delta) = \frac{R}{\delta} \left[ \frac{\sinh(2R/\delta) + \sin(2R/\delta)}{\cosh(2R/\delta) - \cos(2R/\delta)} \right] - 1 \]

now,
\[ \delta H = \left[ \left( \frac{\partial H}{\partial R} \delta H \right)^2 + \left( \frac{\partial H}{\partial \delta} \delta \delta \right)^2 \right]^{1/2} \]

and,
\[ \frac{\partial H}{\partial R} = \frac{B + A}{\delta (D - C)} + \frac{2R(D + C)}{\delta^2 (D - C)} - \frac{2R(B + A)^2}{\delta^3 (D - C)^2} \]
\[ \frac{\partial H}{\partial \delta} = \frac{R(B + A)}{\delta^2 (D - C)} - \frac{2R^2(D + C)}{\delta^3 (D - C)} + \frac{2R^2(B + A)^2}{\delta^4 (D - C)^2} \]

where;
\[ A = \sin(2R/\delta) \]
\[ B = \sinh(2R/\delta) \]
\[ C = \cos(2R/\delta) \]
\[ D = \cosh(2R/\delta) \]
Typical calculation:

\[ R_0 = (1.8265 \pm 0.0004) \times 10^{-3} (m) \]

\[ \delta = (4.1278 \pm 0.00008) \times 10^{-4} (m) \]

\[ \Rightarrow H = 3.4245 \]

\[ \Rightarrow \delta H = 0.0009705 \]

Thus,

\[ H = 3.4245 \pm 0.0283\% \]

The waveform measured from the RF generator was observed to have a complex structure due to the superposition of both an RF signal and a low frequency signal due to power supply ripple. The voltage magnitude due to the RF signal, \( V_{RF} \), was calculated from the equation:

\[ V_{RF} = V_a - V_b / 2 \]  \hspace{1cm} (1)

where \( V_a \) was the amplitude of the signal and \( V_b \) was a peak to peak measure of the power supply ripple. \( V_r \cdot V_{RF} \) was at a frequency \( f_{RF} = 245 \text{ kHz} \), while \( V_r \) was at a frequency \( f_r = 344 \text{ Hz} \). In order to determine the magnitude of the effect on heating due to the low frequency signal we look at the ratio of \( P_r / P_{RF} \) (power absorbed due to the RF signal/power absorbed due to the ripple) Using:

\[ P = 3\pi \frac{RH(x)}{\sigma \mu^2} \left( \bar{B} \cdot \bar{B} \right) \]

and recognizing for our model \( \left( \bar{B} \cdot \bar{B} \right) = B_z^2 = B_0^2 B_z^{2} \)

We may express this as:

\[ \frac{P_r}{P_{RF}} = \frac{H(x_r)B_z^{2}}{H(x_{RF})B_{0r}^{2}} \]
Now
\[ B_o = \frac{\mu I}{2\pi R_o} \propto \frac{\mu V}{2\pi R_o} \]
thus;
\[ \frac{P_r}{P_{RF}} = \frac{H(x_r)V_r^2}{H(x_{RF})V_{RF}^2} \]  \hspace{1cm} (2)

Looking for a moment at \( V_r \) we see that for a frequency \( f_r = 344(\text{Hz}) \) and \( R \equiv R_o = 1.827(\text{mm}) \) we get an \( R/\delta_r = x_r \equiv 0.1657 \). Rony has shown that for small values of \( x \) (i.e. \( x \leq 1 \)), \( H(x) \equiv 0.091x^4 \) [3]. For a frequency \( f_{RF} = 245(\text{kHz}) \) we get an \( R/\delta_{RF} = x_{RF} \equiv 4.425 \). Rony has also shown that for large values of \( x \) (i.e. \( x \geq 2 \)), \( H(x) \equiv x - 1 \) [3]. Substitution into equation (2) gives:
\[ \frac{P_r}{P_{RF}} \equiv \frac{0.09x^4V_r^2}{(x_{RF} - 1)V_{RF}^2} \]
Using the values of \( V_{RF} = 81(\text{mV}), V_r = 13(\text{mV}) \) gives \( \frac{P_r}{P_{RF}} \equiv 5.16 \times 10^{-7} \). Thus, Joule heating due to this low frequency voltage will be negligible and we use \( V_{RF} \) to calculate the current through our coil.

The voltages \( V_a \) and \( V_b \) were measured with the aid of a current transformer having negligible parasitic losses through the ferrite core. Based on the construction of the current transformer the current through the coil can be expressed as:
\[ I = \frac{N_2}{N_1} \frac{V_{RF}}{R_{eq}} \]
Substituting equation (1) yields the following expression in terms of measured quantities:
\[ I = \frac{N_2}{N_1} \frac{(V_a - V_b/2)}{R_{eq}} \]
Now
$$
\delta I = \left[ \left( \frac{\partial I}{\partial V_a} \delta V_a \right)^2 + \left( \frac{\partial I}{\partial V_b} \delta V_b \right)^2 + \left( \frac{\partial I}{\partial R_{eq}} \delta R_{eq} \right)^2 \right]^{1/2}
$$

and,

$$
\frac{\partial I}{\partial V_a} = \frac{N_2}{N_1 R_{eq}}
$$
$$
\frac{\partial I}{\partial V_b} = -\frac{1}{2} \frac{N_2}{N_1 R_{eq}}
$$
$$
\frac{\partial I}{\partial R_{eq}} = -\frac{N_2 (V_a - V_b/2)}{N_1 R_{eq}^2}
$$

Typical calculation:

- $N_2 = 1000\ (\text{turn})$
- $N_1 = 1\ (\text{turn})$
- $R = 1.000 \pm 0.004\ (\Omega)$
- $V_a = 94 \pm 2\ (mV)$
- $V_{a_1} = 144 \pm 2\ (mV)$
- $V_h = 26 \pm 2\ (mV)$
- $V_{h_1} = 26 \pm 2\ (mV)$

\[ \Rightarrow I_1 = 81.0\ (A) \]
\[ \Rightarrow \delta I_1 = 2.3\ (A) \]
\[ \Rightarrow I_2 = 131.0\ (A) \]
\[ \Rightarrow \delta I_2 = 2.3\ (A) \]

Thus,

- $I_1 = 81 \pm 2.8\%\ (A)$
- $I_2 = 131 \pm 1.8\%\ (A)$
Using our stacked loop analysis we may express the field magnitude, \( B \), as:

\[
B = \sqrt{B^2 + B^2_z}
\]

(see Appendix A for a derivation of the field). In our analytical model we assume that \( \rho = 0 \) and thus, \( B_\rho \to 0 \). We also make this assumption in the uncertainty analysis of our experiment by approximating the uncertainty in the \( \rho \)-direction as zero. This is due to the fact that the drop is suspended from a thin wire and free to move in the \( \rho \)-direction. Thus, the drop will move to the position in which the \( \rho \)-component of the Lorentz force is a minimum, i.e. the center of the coil.

Thus,

\[
B = B_z
\]

and we may write this as (see Appendix B);

\[
B_z = \frac{\mu I}{2\pi R_o} B_z^*
\]

Now

\[
\delta B = \left[ \left( \frac{\partial B}{\partial I} \delta I \right)^2 + \left( \frac{\partial B}{\partial B_z} \delta B_z^* \right)^2 + \left( \frac{\partial B}{\partial R_o} \delta R_o \right)^2 \right]^{\frac{1}{2}}
\]

and,

\[
\frac{\partial B}{\partial I} = \frac{\mu B_z^*}{2\pi R_o}
\]

\[
\frac{\partial B}{\partial B_z} = \frac{\mu I}{2\pi R_o}
\]

\[
\frac{\partial B}{\partial R_o} = -\frac{\mu I}{2\pi R_o^2} B_z^*
\]
In order to estimate $\delta B_z^*$, we need only estimate the uncertainty due to an uncertainty in the $z$-position of the drop since $B_z^* = B_z(z)$ only for $\rho = 0$. Here again in order to avoid unnecessary complications we use the maximum deviations in $z$ to calculate the corresponding uncertainty in $B_z^*$.

Calculation for $B_z^*$;

$$z = 0.000 \pm 0.001 (m)$$

at $z = 0.001 (m)$; $B_z^* = 1.7890$

at $z = 0.000 (m)$; $B_z^* = 1.7808$

at $z = -0.001 (m)$; $B_z^* = 1.7890$

Thus,

$$B_z^* = 1.7808 \pm 0.0082 = 1.7808 \pm 0.458\%$$

Typical calculation:

$$\mu = 4\pi \times 10^{-7} (H/m)$$

$I_1 = 81 \pm 2 (A)$

$I_2 = 131 \pm 2 (A)$

$B_z^* = 1.781 \pm 0.008$

$R_b = (1.8265 \pm 0.0004) \times 10^{-3} (m)$

$\Rightarrow B_1 = 0.0158$

$\Rightarrow \delta B_1 = 0.000396$

$\Rightarrow B_2 = 0.02554$

$\Rightarrow \delta B_2 = 0.000407$

Thus,

$$B_1 = 0.0158 \pm 2.53\%$$

$$B_2 = 0.0255 \pm 1.57\%$$

$T_v$
Using;

\[ \log_{10} p = -\frac{A}{T_v} + B + C \log_{10} T_v + D \]  \hspace{1cm} (3)

where for zinc [4],

\[ A = 6883 \]
\[ B = 9.418 \]
\[ C = -0.0503 \]
\[ D = -0.33 \times 10^{-3} \]

we can iterate using the known pressure, \( p \) (in mmHg), to get the vaporization temperature, \( T_v \). Since equation (3) is a transcendental equation and not solvable for \( T_v \) in terms of \( p \), we calculate the maximum uncertainty in \( T_v \) based on the maximum uncertainty in the measured value of \( p \).

Typical calculation:

\[ p = 40 \pm 5 \text{(mTorr)} \]

at \( p = 45 \text{ (mTorr) } \); \( T_v = 661.53 \text{ (K) } \)

at \( p = 40 \text{ (mTorr) } \); \( T_v = 658.22 \text{ (K) } \)

at \( p = 45 \text{ (mTorr) } \); \( T_v = 654.51 \text{ (K) } \)

Thus,

\[ T_v = 658 \pm 4 = 658 \pm 0.608\% \text{(K)} \]

\[ \zeta \]

Using:

\[ \zeta = \frac{\sigma T^4 a R_o}{B^2 / \sigma \mu^2} \]

now,
\[
\delta\zeta = \left[ \left( \frac{\partial \zeta}{\partial B} \delta B \right)^2 + \left( \frac{\partial \zeta}{\partial R_o} \delta R_o \right)^2 + \left( \frac{\partial \zeta}{\partial T_a} \delta T_a \right)^2 \right]^{1/2}
\]

and,

\[
\begin{align*}
\frac{\partial \zeta}{\partial B} &= -2 \frac{R_o \sigma_r e T_a^4}{B^3 / \sigma \mu^2} \\
\frac{\partial \zeta}{\partial R_o} &= \frac{\sigma_r e T_a^4}{B^2 / \sigma \mu^2} \\
\frac{\partial \zeta}{\partial T_a} &= 4 \frac{R_o \sigma_r e T_a^3}{B^2 / \sigma \mu^2}
\end{align*}
\]

Typical calculation:

\[
\begin{align*}
B_1 &= 0.0158 \pm 0.0004 \\
B_2 &= 0.0255 \pm 0.0004 \\
\sigma &= 6.06 \times 10^6 (1/\Omega \cdot m) \\
\mu &= 4\pi \times 10^{-7} (H/m) \\
R_o &= (1.8265 \pm 0.0004) \times 10^{-3} (m) \\
\sigma_r &= 5.6696 \times 10^{-8} (W/m^2 K^4) \\
\varepsilon &= 0.05 \\
T_a &= 296 \pm 1 (K)
\end{align*}
\]

\[
\Rightarrow \zeta_1 = 1.524 \times 10^{-3}
\]

\[
\Rightarrow \delta \zeta_1 = 7.985 \times 10^{-5}
\]

\[
\Rightarrow \zeta_2 = 5.834 \times 10^{-4}
\]

\[
\Rightarrow \delta \zeta_2 = 1.998 \times 10^{-5}
\]

Thus,

\[
\begin{align*}
\zeta_1 &= 1.524 \times 10^{-3} \pm 5.24\% \\
\zeta_2 &= 5.834 \times 10^{-4} \pm 3.42\%
\end{align*}
\]
Using:

$$Ste = \frac{c_L T_a}{L_v}$$

now,

$$\delta Ste = \left[ \left( \frac{\partial Ste}{\partial T_a} \delta T_a \right)^2 \right]^{1/2}$$

and,

$$\frac{\partial Ste}{\partial T_a} = \frac{c_L}{L_v}$$

Typical calculation:

$$c_L = 444 (J/kg \cdot K)$$

$$L_v = 2.02 \times 10^6 (J/kg)$$

$$T_a = 296 \pm 1 (K)$$

$$\Rightarrow Ste = 0.0651$$

$$\Rightarrow \delta Ste = 2.198 \times 10^{-4}$$

Thus,

$$Ste = 0.0651 \pm 0.338\%$$

In order to account for the error introduced by the Stefan number, frequency and temperature terms, we introduce the artificial parameter $\zeta'$. We calculate the error in $\zeta'$ and then use this as a maximum error in the parameter $\zeta$ in the plots for our experimental data.
\( \zeta' \)

Using:

\[
\zeta' = \zeta \frac{\left(\frac{T_v}{T_a}\right)^4 - 1}{H \left(\frac{R_o R^*}{\delta}\right)} \text{Ste}
\]

now,

\[
\delta\zeta' = \left[ \left( \frac{\partial \zeta}{\partial \zeta} \delta\zeta \right)^2 + \left( \frac{\partial \zeta}{\partial T_v} \delta T_v \right)^2 + \left( \frac{\partial \zeta}{\partial T_a} \delta T_a \right)^2 + \left( \frac{\partial \zeta}{\partial H} \delta H \right)^2 + \left( \frac{\partial \zeta}{\partial \text{Ste}} \delta\text{Ste} \right)^2 \right]^{1/2}
\]

and,

\[
\frac{\partial \zeta'}{\partial \zeta} = \frac{\left(\frac{T_v}{T_a}\right)^4 - 1}{H} \text{Ste}
\]

\[
\frac{\partial \zeta'}{\partial T_v} = \frac{4 \zeta T_v^3}{T_a^4 H} \text{Ste}
\]

\[
\frac{\partial \zeta'}{\partial T_a} = -\frac{4 \zeta T_v^4}{T_a^5 H} \text{Ste}
\]

\[
\frac{\partial \zeta'}{\partial H} = -\frac{\zeta \left[\left(\frac{T_v}{T_a}\right)^4 - 1\right]}{H^2} \text{Ste}
\]

\[
\frac{\partial \zeta'}{\partial \text{Ste}} = \frac{\zeta \left[\left(\frac{T_v}{T_a}\right)^4 - 1\right]}{H}
\]

Typical calculation:
\[ \zeta_1 = 1.524 \pm 0.079 \times 10^{-3} \]
\[ \zeta_2 = 5.834 \pm 0.199 \times 10^{-3} \]
\[ T_v = 658 \pm 4 (K) \]
\[ T_a = 296 \pm 1 (K) \]
\[ H = 3.4245 \pm 0.00097 \]
\[ Ste = 0.0651 \pm 0.0002198 \]
\[ \Rightarrow \zeta'_1 = 6.784 \times 10^{-4} \]
\[ \Rightarrow \delta \zeta'_1 = 4.069 \times 10^{-5} \]
\[ \Rightarrow \zeta'_2 = 2.597 \times 10^{-4} \]
\[ \Rightarrow \delta \zeta'_2 = 1.169 \times 10^{-5} \]

Thus,
\[ \zeta'_1 = 6.784 \times 10^{-4} \pm 5.99\% \]
\[ \zeta'_2 = 2.597 \times 10^{-4} \pm 4.50\% \]

and we use these total percentage errors for our variation of zeta in experiments #1 and #2, respectively.
Appendix E
Experimental Data
\[ a = 0.00630 \pm 0.0002 (m) \]
\[ L = 0.03363 \pm 0.0002 (m) \]
\[ N = 6 (\text{turns}) \]
\[ f = 245.31 \pm 0.01 (kHz) \]
\[ V_a = 94 (mV) \]
\[ V_b = 144 (V) \]
\[ N_1 = 1 (\text{turn}) \]
\[ N_2 = 1000 (\text{turns}) \]
\[ P_v = 40 \pm 5 (mTorr) \]
\[ T_a = 296 \pm 1 (K) \]
\[ R = 1.000 \pm 0.004 (\Omega) \]
\[ \rho = 0.0 \]
\[ z = 0.000 \pm 0.001 (m) \]

<table>
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<th>Experiment #2</th>
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<td>time (sec)</td>
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