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Response of earth dams in semi-elliptical canyons to oblique SH waves

Hsu, Ching-Heng, M.S.

Rice University, 1993
RICE UNIVERSITY

RESPONSE OF EARTH DAMS IN SEMI-ELLiptICAL CANYONS TO OBLIQUE SH WAVES

by

CHING-HENG HSU

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE MASTER OF SCIENCE

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ABSTRACT

RESPONSE OF EARTH DAMS IN SEMI-ELLIPTICAL CANYONS
TO OBLIQUE SH WAVES

by

Ching-Heng Hsu

Two analytical closed-form solutions are developed for steady-state lateral response of earth and rockfill dams built in semi-elliptical canyons. In the first model, the canyon is assumed to be rigid, while the dam is idealized as a two-dimensional linearly-hysteretic elastic body deforming only in shear (shear beam). Both free and base-induced oscillations are studied for various canyon geometries. In the second model, the canyon is assumed to consist of flexible elastic rock, subjected to asynchronous excitation consisting of obliquely incident harmonic SH waves. The solution accounts in a rigorous way for the complex wave reflection, transmission and diffraction phenomena associated with the dam-filled canyon. The study focuses on the effects of: (a) the angle of incidence, (b) the impedance ratio and (c) the canyon narrowness. It is shown that the effects of radiation damping and ground motion spatial variability are very important.
ACKNOWLEDGMENTS

This research was directed by my thesis advisor, Dr. Panos Dakoulas. His patient guidance and invaluable suggestions throughout this study are heartily and sincerely appreciated. The precious instruction and time he gave me made this study possible.

I also would like to thank the thesis committee members, Dr. Anestis S. Veletsos and Joel P. Conte, for reviewing and offering useful suggestions and thank Department of Civil Engineering, Rice University, for its financial assistance from September 1991 to December 1992.

Special thanks to my cousin, Wayne Ju, and his family, Gregory and Donna Rose, Yong-Ji Chen and Shiah-Sen Wang for their help, encouragement, friendship and helpful suggestions.

I wish to express my gratitude to Shingyuan Yu for his encouragement and many helpful suggestions.

I dedicate this thesis to my family for their love, encouragement as well as spiritual and financial support.
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List of Symbols

\( a_{nj} \) dimensionless frequencies for the dam of mode \( n \) and \( j \)

\( a_o \) dimensionless frequency

AF amplification function

B dam width

\( c_c \) dimensionless frequency of the canyon in the elliptical coordinate

\( c_d \) dimensionless frequency of the dam in the elliptical coordinate

\( c_{dnj} \) undamped natural frequencies of the dam in the elliptical coordinate of mode \( n \) and \( j \)

\( d \) half of the interfocal distance in the elliptical coordinate

\( G_c \) shear modulus for the canyon rock

\( G_d \) shear modulus for the dam soil

\( G_d^* \) complex shear modulus for the dam soil

H dam height

IR dam to canyon material impedance ratio

\( j_r \) spherical-Bessel function of the first kind with order \( r \)

\( J_r \) Bessel function of the first kind with order \( r \)

\( k_c \) wave number in the elastic canyon

\( k_d \) wave number in the dam

L dam length

\( P_{nj} \) participation factors of mode \( n \) and \( j \)

\( P_r \) Legendre polynomial of degree \( r \)

\( P_m^* \) associated Legendre function of the first kind with order \( m \) and degree \( n \)

\( r^* \) radial polar coordinate

\( R_{mn}^{(1)} \) radial prolate spheroidal wave function of the first kind with order \( m \) and degree \( n \)
\( R_{mn}^{(2)} \) radial prolate spheroidal wave function of the second kind with order \( m \) and degree \( n \)

\( R_{mn}^{(3)} \) radial prolate spheroidal wave function of the third kind with order \( m \) and degree \( n \)

\( R_{mn}^{(4)} \) radial prolate spheroidal wave function of the fourth kind with order \( m \) and degree \( n \)

\( S_{mn}^{(1)} \) angular prolate spheroidal wave function of the first kind with order \( m \) and degree \( n \)

\( S_{mn}^{(2)} \) angular prolate spheroidal wave function of the second kind with order \( m \) and degree \( n \)

\( T_{nj} \) natural period for the dam of mode \( n \) and \( j \)

\( U_1 \) scalar amplitude of incident harmonic motion in the elastic canyon

\( U_b \) complex amplitude of harmonic motion at the dam base

\( U_c \) complex amplitude of harmonic motion in the elastic canyon

\( U_d \) complex amplitude of harmonic motion in the dam

\( U_i \) complex amplitude of incident harmonic motion in the elastic canyon

\( U_r \) complex amplitude of reflected harmonic motion at the free field

\( U_{cr} \) complex amplitude of diffracted harmonic motion in the elastic canyon

\( \bar{U}_{nj} \) normalized modal displacement shapes of mode \( n \) and \( j \)

\( V_c \) shear wave velocity of the canyon rock

\( V_d \) shear wave velocity of the dam

\( V_d^* \) complex shear wave velocity of the dam

\( V_x, V_z \) shear wave velocities for the canyon rock in the \( x \) and \( z \) directions

\( y_r \) spherical-Bessel function of the second kind with order \( r \)

\( Y_r \) Bessel function of the second kind with order \( r \)

\( \alpha \) angle of the incident waves in the polar coordinate

\( \beta_d \) hysteretic material damping ratio of the dam
\( \phi \)  phase angle of the amplification function AF
\( \gamma \)  shear strain
\( \eta \)  angular elliptical coordinate
\( \lambda_{mn} \)  separation constants (eigenvalues) of order \( m \) and degree \( n \)
\( \mu \)  ratio of shear modulus for the canyon rock to complex shear modulus for the dam soil
\( \theta \)  angle of the position vector from \( x \) axis
\( \theta_o \)  angle of the incident waves from \( x \) axis
\( \theta^* \)  angular polar coordinate
\( \rho_c \)  mass density for the canyon rock
\( \rho_d \)  mass density for the dam soil
\( \tau \)  shear stress
\( \omega \)  circular frequency
\( \omega_{nj} \)  circular natural frequencies of mode \( n \) and \( j \)
\( \xi \)  radial elliptical coordinate
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1. INTRODUCTION

1.1 Objectives

Earthquakes may affect the safety of thousands of earthfill and rockfill dams, endanger lives and cause vast property damage in seismic regions around the world. Reliable assessment and better understanding of the seismic response characteristics of earth and rockfill dams can prevent such catastrophic failures by allowing remedial measures in unsafe existing dams and by improving the design of future dams.

Significant progress has been made in understanding the dynamic characteristics and seismic behavior of earth and rockfill dams during strong earthquakes in the last fifteen years. The studies made focused mainly on: the 3-D effects of the canyon geometry; the nonlinear and inelastic behavior of the material; the inhomogeneity of the material; the relative flexibility between the dam and the canyon or foundation soil; and the composition and spatial variability of excitation. Related developments have been reviewed in two state-of-the-art papers by Gazetas (1987) and Gazetas and Dakoulas (1991). Despite this significant progress, still, the effects of some factors influencing significantly the behavior of earth dams are yet to be fully understood. Furthermore, even less understood are the interrelationship and the combined effects of these factors. This is because, with rare exceptions, the studies that have been made so far have focused only on certain aspects of the problem, due to the lack of appropriate and efficient models for seismic analysis of dam-canyon systems.

The objective of the present work is to develop a new analytical model that will allow a comprehensive study of the individual and combined effects of these factors on the seismic response of dam-canyon systems. The developed model is used in extensive parametric studies focusing in particular on the effects of canyon geometry, canyon flexibility and variability of the ground motion on the response.
1.2 Scope Of Work

This study investigates the effects of canyon geometry, canyon flexibility and variability of the ground motion on the response of earth and rockfill dams to steady-state harmonic and transient excitations in the lateral (upstream-downstream) direction. To this end, the study utilizes a simplified analytical model for the dam, which is idealized as a linearly-hysteretic elastic body deforming only in shear. This generalized “shear beam” model extends in the vertical and longitudinal directions and assumes uniform response values for the upstream-downstream direction. The canyon has a semi-cylindrical shape of elliptical cross-section.

The first part of the study investigates the effect of canyon geometry for a dam on a rigid canyon, subjected to lateral synchronous identical motion along the dam-canyon interface. Thus, in this case, there is no dam canyon interaction and radiation damping effects. Rigorous analytical closed-form solutions are presented for the steady state and harmonic response based on the generalized shear beam model.

The second part of the study investigates the response of a dam built in a canyon consisting of linearly hysteric elastic (flexible) rock. The dam is subjected to obliquely-incident harmonic and transient SH waves, traveling along the longitudinal direction of the dam. Analytical closed-form solutions are presented that account in a rigorous way for the dam-canyon interaction phenomena, including wave reflection and diffraction phenomena caused by the presence of the dam-canyon system. A parametric investigation elucidates the effects of the dam-to-canyon material impedance ratio, of the angle of incidence, and of the frequency of excitation on the response along the crest of the dam. Similarities and differences are noted and explained with the response of the dam to “rigid-base”-type synchronous excitation, as well as with the response of an alluvial semi-cylindrical valley to oblique SH waves.
2. LATERAL RESPONSE OF DAMS IN SEMI-ELLIPTIICAL RIGID CANYONS

2.1 Introduction

For earth dams built in narrow valleys, the presence of relatively rigid abutments creates a three-dimensional (3-D) stiffening effect, in which natural periods decrease and near crest accelerations increase sharply as the canyon becomes narrower. Several studies considering various canyon geometries have demonstrated that the 3-D canyon geometry effects on the seismic response characteristics of dams are very important.

Due to the complexity of 3-D modeling and analysis of the dam-canyon system, it is usually assumed for simplicity that the supporting canyon vibrates as a rigid body resulting to "synchronous" identical excitation along the dam-canyon interface. Based on this assumption, numerical results have been published for several idealized canyon shapes, such as rectangular, semi-cylindrical, trapezoidal and triangular, as well as for some actual canyon geometries. Most of these analyses are linear, although a few 3-D inelastic solutions have also been reported. In the following, a brief account of the most representative studies performed and the methods used is given.

Analytical closed-form solutions are particularly valuable even if the canyon shapes are highly idealized. Hatanaka (1952) and Ambraseys (1960) presented closed-form solutions for the lateral response of dams in rectangular canyons, while Dakoulas and Gazetas (1986) derived a simple closed-form solution for the lateral response of a homogeneous earth dam in a semi-cylindrical canyon. These solutions at the shear-beam concept, in which only lateral displacements and shear deformations are allowed and assumed to be uniformly distributed across the lateral (upstream-downstream) direction of the dam. With these assumptions, the solutions are exact and no other approximation is introduced. The results are in the form of especially simple algebraic expressions for
natural periods, modal shapes, steady-state transfer functions, and participation factors for transient seismic excitation.

Martinez & Bielak (1980) have developed an efficient numerical procedure for dams having a plane of symmetry perpendicular to the longitudinal axis, by neglecting the longitudinal deformation, discretizing in finite elements only the dam midsection, and using Fourier series to compute the displacement distribution along the longitudinal direction. Results on natural frequencies and modal shapes for rectangular, trapezoidal and triangular canyons indicate significant differences due to canyon geometry.

By dividing the dam body through vertical closely spaced transverse planes into super-elements behaving as shear-beams, Ohmachi et al. (1982) have developed an approximate efficient formulation in which no symmetry is required. The solution is obtained by using a linear interpolation function for the displacement shape in the longitudinal direction and by enforcing compatibility of deformation between super-elements. Results on natural frequencies and modal shapes for rectangular, trapezoidal and triangular canyons confirm the conclusions drawn by Martinez and Bielak.

Using the Rayleigh-Ritz method, Abdel-Ghaffar and Koh (1982) have presented a semi-analytical solution for dams built in canyons having a plane of symmetry. This solution utilizes the shear-beam modal shapes or sinusoids as basis functions, and involves a transformation of the dam geometry into a cuboid. The method is versatile and could be used for an approximate solution of the nonlinear problem.

Makdisi et al. (1982) have developed a special 3-D dynamic finite-element formulation by replacing the 2-D plane-strain isoparametric elements of the computer code LUSH with prismatic longitudinal elements having six faces and eight nodal points. To reduce computer storage and time requirements, the longitudinal displacements are ignored, and only shear waves propagating vertically and horizontally in the embankment are
considered. Results have been presented for steady-state and transient response of homogeneous dams in triangular canyons. In similar studies, Mejia et al. (1983) used a 3-D finite-element formulation but without restricting the longitudinal deformations.

Finally Prevost et al. (1985) used a kinematic multi-surface plasticity constitutive model for soil in 3-D finite-element modeling of dams in arbitrarily-shaped canyons. The dam body is discretized in eight-node isoparametric “brick” elements. However, the finite-element mesh used in such 3-D analyses seems to be coarse, due to the very substantial computational requirements of the method. It is quite likely that high frequency components are artificially filtered out or at least reduced as they propagate through a coarse mesh, affecting adversely the computation of accelerations near the crest zone.

More details about the above developments may be found in the given references and in two state-of-the-art papers by Gazetas (1987) and Gazetas and Dakoulas (1991).

The above studies have provided valuable insight and help improve our understanding of the effects of the canyon geometry on the response of dams. Despite the substantial progress, much still has to be learned regarding the behavior of such structures built in narrow canyons. As stated above, analytical closed-form solutions are particularly valuable as they lead to results in the form of especially simple algebraic expressions for natural periods, modal shapes, and response to steady-state harmonic and transient seismic excitation. Such solutions allow extensive parametric studies offering considerable insight on the effects of the various key parameters influencing the response and provide a means of checking more complex numerical solutions, as well as a valuable tool for preliminary design computations.

However, for a large number of canyon shapes, the developed closed-form solutions for rectangular and semi-cylindrical shapes are highly idealized. Although, in these cases, an “equivalent” rectangular or semi-cylindrical model can be considered, such
approximations may not always be satisfactory as the actual canyon shape may induce
coupling of the vibrational modes in the longitudinal and vertical directions, that cannot
be expressed with the aforementioned models. A canyon with a semi-elliptical shape
offers considerable geometric flexibility, as it can approximate the geometry of several
actual canyons.

The objective of this study is to develop a closed-formed analytical solution for the
dynamic lateral response of earth dams built in semi-elliptical canyons. The models will
serve two purposes:

1. The results will be used to gain more insight on the effects of canyon
narrowness on the seismic response characteristics, by using canyons the
length to height ratios ranging from 2 (semi-cylindrical canyon) to infinity
(plane strain conditions) and comparing with results from other canyon
shapes.

2. By offering considerable insight on the response characteristics for a rigid
canyon base, the solution will serve as the natural first step in solving and
understanding the response characteristics of the more realistic problem of a
flexible canyon base, addressed in Chapter 3. As opposed to the rigid canyon
base, the flexible canyon model takes into account the effects of the dynamic
dam-canyon interaction, including the effects of the radiation damping and the
spatial variability of the ground motion.
2.2 Simplifying Assumptions

Figure 2.1 portrays a 3-D perspective view of the dam in a semi-elliptical canyon. The dam has a triangular cross-section, consisting of homogeneous and linearly hysteretic soil with a constant mass density \( \rho_d \), a constant shear modulus \( G_d \), and a material hysteretic damping ratio \( \beta_d \). The canyon consists of rigid rock and vibrates exclusively in the lateral (upstream-downstream) direction. Therefore, all points along the dam-canyon interface experience identical and synchronous (in-phase) oscillations. No slippage is allowed at the dam base.

The response of the dam to the seismic excitation, applied along the semi-elliptical boundary with the canyon, is assumed to be only in horizontal lateral shear deformation with the upstream-downstream displacements, \( u_d \), uniformly distributed across the width of the dam. In other words, the dam is idealized as a generalized “shear beam”, which extends in the vertical and longitudinal directions. The response values are assumed either uniform or average for the upstream-downstream direction. The uniformity of displacements, \( u_d \), across the width of the dam has been confirmed as a reasonable approximation by a series of seismic analyses of earth dams with finite-element and shear-beam models (Dakoulas and Gazetas 1985, 1986, Gazetas 1987). Indeed, comparisons of acceleration, displacement and shear stress time histories between shear-beam and numerical analyses at various points within the dam showed excellent agreement. Figure 2.2c shows an infinitesimal element \( b \Delta x \Delta z \) and the corresponding average (across the width \( b \)) shear stresses \( \tau_{yz} \) and \( \tau_{yx} \) applied on the horizontal and vertical sides, respectively. In reality, the distributions of the \( \tau_{yz} \) and \( \tau_{yx} \) are almost uniform for most of the width \( b \), except near the two slopes of the dam where they must vanish, due to decrease of the confining pressure. Nevertheless, by considering the average shear stresses and the average shear modulus across the width, no assumption regarding their exact distribution is required.
Figure 2.1. Three-dimensional view of a dam in a semi-elliptical canyon.
Figure 2.2. Dam in semi-elliptical canyon: (a) longitudinal section; (b) maximum cross-section; and (c) infinitesimal element with shear stresses acting at its faces.
The response of the dam to longitudinal and vertical direction of ground motion is not considered in this study. Some longitudinal and vertical response would indeed take place, but it is neglected for simplicity since they are much less than the lateral response. Finally, hydrodynamic effects are not taken into account, because they are of secondary importance for the lateral response of the earth dams (Hall and Chopra 1982).
2.3 Free Vibration: Analysis And Results

Consider the dynamic stresses acting on an infinitesimal horizontal element of the dam of volume \( b \, dx \, dz = (B / H) \, z \, dx \, dz \) (Figure 2.2c). The net shearing forces induced by earthquake shaking on the horizontal and vertical faces of the element are

\[
- \frac{B}{H} \frac{\partial}{\partial z} \left[ z \, \tau_y (x, z; t) \right] \, dx \, dz
\]  
(2.1)

and

\[
- \frac{B}{H} \frac{\partial}{\partial x} \tau_y (x, z; t) \, z \, dx \, dz
\]  
(2.2)

The total inertia force on the element equals

\[
\rho_d \ddot{u} (x, z; t) \frac{B}{H} \, z \, dx \, dz
\]  
(2.3)

where \( u = u(x, z; t) \) is the lateral displacement relative to the boundaries and \( \ddot{u} = \partial^2 u / \partial t^2 \). Considering the dynamic equilibrium of the above three forces while accounting for the stress-displacement relations

\[
\tau_y = G_d \frac{\partial u}{\partial x}, \quad \tau_z = G_d \frac{\partial u}{\partial z}
\]  
(2.4)

leads to the following equation of motion

\[
G_d \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{z} \frac{\partial u}{\partial z} \right) = \rho_d \ddot{u}
\]  
(2.5)

To take the dissipation of energy due to inelastic soil behavior into account, a linear hysteretic damping is introduced in equation (2.5) by replacing \( G_d \) with the complex valued modulus \( G_d^* = G_d (1 + 2 \, i \beta_d) \), where \( \beta_d \) is the damping ratio and \( i = \sqrt{-1} \).

For harmonic steady-state vibration of frequency \( \omega \), the total displacement in the dam, \( u_d \), may be written as
\[ u_d = U_d(x, z) \exp(i\omega t) \]  

(2.6)

Substituting equation (2.6) into equation (2.5) leads to

\[ \frac{\partial^2 U_d}{\partial z^2} + \frac{1}{z} \frac{\partial U_d}{\partial z} + \frac{\partial^2 U_d}{\partial x^2} + k_d^2 U_d = 0 \]  

(2.7)

where

\[ k_d = \frac{\omega}{V_d} \]  

(2.8)

and

\[ V_d^* = \sqrt{\frac{G_d(1+2i\beta_d)}{\rho_d}} = V_d \sqrt{1+2i\beta_d} \]  

(2.9)

in which \( V_d \) is the shear wave velocity of the dam material. Due to the geometry of the canyon, it is convenient to convert to elliptical coordinates (Figure 2.3), using the relationship

\[
\begin{align*} 
x &= d \eta \xi \\
z &= d \sqrt{(1-\eta^2)(\xi^2-1)} \end{align*}
\]

\[ 1 \leq \xi < \xi_b, \quad -1 \leq \eta \leq 1 \]  

(2.10)

where \( \xi \) is the “radial” coordinate, \( \eta \) is the “angular” coordinate, \( d \) is half of the interfocal distance and \( \xi_b \) is the value of \( \xi \) at the dam base, given by

\[ \xi_b = \frac{L}{\sqrt{L^2 - 4H^2}} \]  

(2.11)

The new coordinate system is formed by two families of confocal ellipses and hyperbolas, as shown in Figure 2.3. After some mathematical operations, equation (2.7) transforms into

\[ \frac{\partial}{\partial \xi} \left[ (\xi^2 - 1) \frac{\partial U_d}{\partial \xi} \right] + \frac{\partial}{\partial \eta} \left[ (1-\eta^2) \frac{\partial U_d}{\partial \eta} \right] + c_d^2 (\xi^2 - \eta^2) U_d = 0 \]  

(2.12)
Figure 2.3. Dam in semi-elliptical canyon and elliptical coordinate system.
where

\[ c_d = k_d \cdot d \]  

(2.13)

For free vibrations, the solution of equation (2.12) must satisfy the boundary condition of zero displacement at the elliptical base (no slippage, valley stationary)

\[ U_d(\xi_b, \eta) = 0 \quad -1 \leq \eta \leq 1 \]  

(2.14)

Moreover, it should yield zero shear stresses at the crest

\[ G_d \frac{\partial U_d}{\partial \eta}(\xi, 1) = G_d \frac{\partial U_d}{\partial \eta}(\xi, -1) = 0 \quad \text{for} \quad 1 < \xi \leq \xi_b \]  

(2.15a)

\[ G_d \frac{\partial U_d}{\partial \xi}(1, \eta) = 0 \quad \text{for} \quad -1 < \eta < 1 \]  

(2.15b)

Using variable separation, \( U_d \) may be written in the form of the Lamé products

\[ U_d = R_{mn}(c_d, \xi) S_{mn}(c_d, \eta) \]  

(2.16)

The “radial” solution \( R_{mn}(c_d, \xi) \) and the “angular” solution \( S_{mn}(c_d, \eta) \) must satisfy the ordinary differential equations

\[ \frac{d}{d\xi} \left[ (\xi^2 - 1) \frac{d}{d\xi} R_{mn}(c_d, \xi) \right] - \left( \lambda_{mn} - c_d^2 \xi^2 \right) R_{mn}(c_d, \xi) = 0 \]  

(2.17)

\[ \frac{d}{d\eta} \left[ (1 - \eta^2) \frac{d}{d\eta} S_{mn}(c_d, \eta) \right] + \left( \lambda_{mn} - c_d^2 \eta^2 \right) S_{mn}(c_d, \eta) = 0 \]  

(2.18)

in which the associated eigenfunctions \( R_{mn}(c_d, \xi) \) and \( S_{mn}(c_d, \eta) \) are, respectively, prolate spheroidal radial and angular functions of order \( m \) and degree \( n \) (Abramowitz and Stegun 1970; Flammer 1957). The separation constants \( \lambda_{mn} \) are to be determined so that \( R_{mn}(c_d, \xi) \) and \( S_{mn}(c_d, \eta) \) are finite at \( \xi = 1 \) and \( \eta = \pm 1 \). The only acceptable solutions for the dam response are the prolate spheroidal radial and angular functions of the first kind, denoted by \( R_{mn}^{(1)}(c_d, \xi) \) and \( S_{mn}^{(1)}(c_d, \eta) \), while the second kind functions,
$R_{mn}(c_d, \xi)$ and $S_{mn}(c_d, \eta)$, are rejected due to their singularity at the dam crest. The solution for the dam response is derived as a special case of the solution of the wave differential equation in prolate spheroidal coordinates, in which the second separation constant $m$ is equal to zero.

For $m = 0$, the angular function of the first kind is given by an infinite sum of the form

$$S_{on}^{(1)}(c_d, \eta) = \sum_{r=0,2,\ldots}^{\infty} d_r^{on}(c_d) P_r(\eta)$$

and the radial function of the first kind is an infinite sum of the form

$$R_{on}^{(1)}(c_d, \xi) = \frac{1}{\sum_{r=0,2,\ldots}^{\infty} d_r^{on}(c_d)} \sum_{r=0,2,\ldots}^{\infty} i^{r-n} d_r^{on}(c_d) j_r(c_d \xi)$$

where $P_r = \text{Legendre polynomial of order } r$; $j_r = \text{spherical Bessel function of the first kind and order } r$, defined by

$$j_r(z) = \sqrt{\frac{\pi}{2z}} J_{r+\frac{1}{2}}(z)$$

in which $J_{r+\frac{1}{2}}(z)$ is the Bessel function of the first kind and order $r+1/2$; the coefficients $d_r^{on}(c_d)$ are determined through a recursion formula; and $i = \sqrt{-1}$. The computation of $R_{mn}(c_d, \xi)$ and $S_{mn}(c_d, \eta)$ is very involved and has been the subject of studies in problems of acoustic and electromagnetic waves. A brief description of the basic computational procedure is given in APPENDIX A. APPENDIX B demonstrates that $U_d$ satisfies the boundary conditions (2.15).

Enforcing equation (2.14) along the elliptical canyon base, the undamped natural frequencies of the dam for each mode $(n, j)$ are computed from the roots
\[ c_{dnj} = \frac{\omega_{nj} d}{V_d} = \frac{\omega_{nj} H}{V_d \sqrt{(\xi_b^2 - 1)}} \]  

(2.22)

of the equation \( R^{(1)}_{bn}(c_d, \xi_b) = 0 \). The circular natural frequencies \( \omega_{nj} \) (in rad/s) are given by

\[ \omega_{nj} = \frac{\alpha_{nj} V_d}{H}, \quad n = 0, 2, 4, \ldots \quad j = 1, 2, 3, \ldots \]  

(2.23)

where the dimensionless natural frequencies

\[ \alpha_{nj} = c_{dnj} \sqrt{\left(\frac{\xi_b^2}{\xi_b^2 - 1}\right)} \]  

(2.24)

for \( n = 0, 2, 4 \) and \( j = 1 \sim 6 \) and for various aspect ratios \( L/H \) are given in Tables 2.1-2.3. The corresponding natural periods \( T_{nj} \) of the dam are

\[ T_{nj} = \frac{2\pi H}{\alpha_{nj} V_d} \]  

(2.25)

Notice in Table 2.1 that for \( L/H = 2 \) the frequencies \( \alpha_{nj} \) are identical to those derived independently for dams in semi-circular canyons (Dakoulas and Gazetas 1986), while as \( L/H \rightarrow \infty \) the results approach those of the independent plane-strain shear beam solution. The values of the dimensionless frequency \( \alpha_{nj} \) are also plotted versus the aspect ratio \( L/H \) in Figure 2.4 for \( j = 1 \sim 6 \). The results confirm the increase of natural frequencies of the dam as canyon narrowness increases (stiffening effect). For aspect ratios \( L/H > 5 \), the natural frequencies of the dam approach those for dams under plane-strain conditions. For both \( L/H = 2 \) and \( \infty \), the dam response depends only on the term \( R^{(1)}_{oo}(c_d, \xi) S^{(1)}_{oo}(c_d, \eta) \), i.e. \( n = 0 \), while terms with \( n > 0 \) do not contribute to the response. Thus, the corresponding roots \( \alpha_{nj} \) for \( n = 2 \) and 4 do not appear in Tables 2.2 and 2.3.

The modal displacement shapes, normalized to a unit value at the midcrest are given by
<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>Roots $a_{nj}$ for $n = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/H</td>
<td>j = 1</td>
</tr>
<tr>
<td>2</td>
<td>$\pi$</td>
</tr>
<tr>
<td>2.5</td>
<td>2.944</td>
</tr>
<tr>
<td>3</td>
<td>2.828</td>
</tr>
<tr>
<td>3.5</td>
<td>2.752</td>
</tr>
<tr>
<td>4</td>
<td>2.699</td>
</tr>
<tr>
<td>4.5</td>
<td>2.660</td>
</tr>
<tr>
<td>5</td>
<td>2.630</td>
</tr>
<tr>
<td>6</td>
<td>2.588</td>
</tr>
<tr>
<td>7</td>
<td>2.559</td>
</tr>
<tr>
<td>8</td>
<td>2.538</td>
</tr>
<tr>
<td>9</td>
<td>2.523</td>
</tr>
<tr>
<td>10</td>
<td>2.510</td>
</tr>
<tr>
<td>$\infty$</td>
<td>2.405</td>
</tr>
</tbody>
</table>

Table 2.1 Roots $a_{nj}$ for the first six modes and various dam length to height ratios.
<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>Roots $a_{nj}$ for n = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/H</td>
<td>j = 1</td>
</tr>
<tr>
<td>2.5</td>
<td>5.035</td>
</tr>
<tr>
<td>7</td>
<td>3.190</td>
</tr>
<tr>
<td>10</td>
<td>2.938</td>
</tr>
</tbody>
</table>

Table 2.2 Roots $a_{2j}$ for the first six modes and various dam length to height ratios.
<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>Roots $a_{nj}$ for n = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>j = 1</td>
</tr>
<tr>
<td>L/H</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>5.682</td>
</tr>
<tr>
<td>4.5</td>
<td>4.821</td>
</tr>
</tbody>
</table>

Table 2.3 Roots $a_{nj}$ for the first six modes and various dam length to height ratios.
Figure 2.4. Dimensionless natural frequencies versus length to height ratio for the first six modes and n=0.
\[
U_{nj} = \frac{U_d(c_{dnj} \xi, \eta)}{U_d(c_{dnj}^*, 1, 0)} = \frac{R_{oo}^{(1)}(c_{dnj}, \xi)}{R_{oo}^{(1)}(c_{dnj}^*, 1)} \frac{S_{oo}^{(1)}(c_{dnj}, \eta)}{S_{oo}^{(1)}(c_{dnj}^*, 0)}
\]

(2.26)

Figures 2.5-2.7 plot the normalized displacement shapes for vibrational modes with \(n = 0, 2, 4\) and \(j = 1 \sim 4\) and for aspect ratios \(L/H = 2, 2.5, 3\) and \(10\). The displacement shapes are plotted along the height of the dam at maximum cross-section as well as along the dam crest. Notice that the displacement shapes along the height of the dam are practically independent of the aspect ratio \(L/H\) in the first mode \((n=0, j=1)\). This is also true for modes corresponding to \(n=0\) and \(j>1\) for \(L/H \geq 3\), whereas as \(L/H \to 2\) there is a sharper attenuation of displacement shapes with depth. For \(L/H = 2\), the modal shapes from equation (2.26) are identical to those obtained from the independent solution by Dakoulas and Gazetas (1986), in which the modal displacement shapes are given by

\[
\bar{U}_j = \sin(j \pi \frac{r^*}{H}) \quad j = 1, 2, 3, \ldots
\]

(2.27)

where \(r^*\) is the radial polar coordinate and \(H\) is the dam height. The modal displacement shapes for \(L/H = 2\) corresponding to \(n = 2\) and \(4\) do not appear in Figures 2.6 and 2.7, as in this case the dam response depends only on modes with \(n = 0\).
Figure 2.5. Displacement shapes for dam in semi-elliptical canyons with \( L/H = 2, 2.5, 3 \) and 10: (a) Mode \( n=0, j=1 \) (continued).
Figure 2.5. Displacement shapes for dam in semi-elliptical canyons with $L/H = 2, 2.5, 3$ and 10: (b) Mode $n=0, j=2$ (continued).
Figure 2.5. Displacement shapes for dam in semi-elliptical canyons with $L/H = 2, 2.5, 3$ and 10: (c) Mode $n=0, j=3$ (continued).
(d) Mode \( n = 0, \ j = 4 \)

Figure 2.5. Displacement shapes for dam in semi-elliptical canyons with \( L/H = 2, 2.5, 3 \) and 10: (d) Mode \( n=0, j=4 \).
Figure 2.6. Displacement shapes for dam in semi-elliptical canyons with $L/H = 2.5$, 3 and 10: (a) Mode $n=2$, $j=1$ (continued).
(b) Mode $n = 2, \quad j = 2$

Figure 2.6. Displacement shapes for dam in semi-elliptical canyons with $L/H = 2.5, \ 3$ and 10: (b) Mode $n=2, \ j=2$ (continued).
Figure 2.6. Displacement shapes for dam in semi-elliptical canyons with $L/H = 2.5$, 3 and 10: (c) Mode $n=2, j=3$ (continued).
(d) Mode n = 2, j = 4

Figure 2.6. Displacement shapes for dam in semi-elliptical canyons with L/H = 2.5, 3 and 10: (d) Mode n=2, j=4.
Figure 2.7. Displacement shapes for dam in semi-elliptical canyons with $L/H = 2.5$, $3$ and $10$: (a) Mode $n=4$, $j=1$ (continued).
Figure 2.7. Displacement shapes for dam in semi-elliptical canyons with $L/H = 2.5$, $3$ and $10$: (b) Mode $n=4$, $j=2$ (continued).
(c) Mode n = 4, j = 3

Figure 2.7. Displacement shapes for dam in semi-elliptical canyons with $L/H = 2.5$, 3 and 10: (c) Mode n=4, j=3 (continued).
Figure 2.7. Displacement shapes for dam in semi-elliptical canyons with $L/H = 2.5, 3$ and 10: (d) Mode $n=4, j=4$. 
2.4 Response To Base Excitation

When the canyon is oscillating as a rigid base with an acceleration \( u_b = \ddot{u}_b(t) \) in the \( y \) direction, the displacement \( u = u(\xi, \eta, t) \) of the dam relative to the base is governed by

\[
\frac{\partial}{\partial \xi} \left[ (\xi^2 - 1) \frac{\partial u}{\partial \xi} \right] + \frac{\partial}{\partial \eta} \left[ (1 - \eta^2) \frac{\partial u}{\partial \eta} \right] = \frac{d^2}{V_d^2} (\xi^2 - \eta^2)(\ddot{u} + \ddot{u}_b) \tag{2.28}
\]

For an arbitrary seismic base excitation \( \ddot{u}_b(t) \), equation (2.28) is solved by mode superposition

\[
u(\xi, \eta, t) = \sum_{n=0,2,\ldots}^{\infty} \sum_{j=1,2,\ldots}^{\infty} \bar{U}_{nj}(\xi, \eta) P_{nj} D_{nj}(t) \tag{2.29}
\]

in which \( \bar{U}_{nj}(\xi, \eta) \) is the \( j \)th mode displacement shape given in equation (2.26); \( P_{nj} \) is the participation factor of the \( j \)th mode

\[
P_{nj} = \frac{\int_1^{\xi^2 -1} \int_1^{\eta^2 -1} \bar{U}_{nj}(\xi, \eta) \rho_d B d^3(\xi^2 - \eta^2) \, d\xi \, d\eta}{\int_1^{\xi^2 -1} \int_1^{\eta^2 -1} \bar{U}_{nj}^2(\xi, \eta) \rho_d B d^3(\xi^2 - \eta^2) \, d\xi \, d\eta} \tag{2.30}
\]

\[
= \frac{1}{c_{dnj}^2} \frac{(\xi_b^2 - 1) \, R_{on}^{0,0} (c_{dnj} \xi_b) \, R_{on}^{1,1} (c_{dnj} \xi_b) \, S_{on}^{1,0} (c_{dnj} \xi)}{\int_{-1}^{1} \int_{-1}^{1} \bar{U}_{on}^2 (c_{dnj} \xi, \eta) \, \eta^2 \, d\xi \, d\eta - \sum_{r=0,2,\ldots}^{\infty} \frac{(d_{on}^r)^2}{(2r + 1)} \int_{-1}^{1} \bar{U}_{on}^{1,2} (c_{dnj} \xi) \, \xi^2 \, d\xi}
\]

and \( D_{nj}(t) \) is the Duhamel integral. For \( n = 0, 2, 4 \) and \( j = 1 \sim 4 \) and for various aspect ratios \( L/H \) are given in Tables 2.4-2.6. Notice in Table 2.4 that for \( L/H = 2 \) the participation factors \( P_{nj} \) are identical to those derived independently for dams in semi-circular canyons (Dakoulas and Gazetas 1986). From equation (2.29), the absolute acceleration \( \ddot{u}_a \) is directly recovered

\[
\ddot{u}_a(\xi, \eta, t) = \ddot{u}_b(t) + \sum_{n=0,2,\ldots}^{\infty} \sum_{j=1,2,\ldots}^{\infty} \bar{U}_{nj}(\xi, \eta) P_{nj} \ddot{D}_{nj}(t) \tag{2.31}
\]

Finally, the expression for the maximum shear strain is given by
\[ \gamma_{\max}(\xi, \eta, t) = \gamma_{\xi \eta}(\xi, \eta, t) = \sum_{n=0,2,\ldots}^{\infty} \sum_{j=1,2,\ldots}^{\infty} \frac{\partial U_{nj}(\xi, \eta)}{\partial \xi} P_{nj} D_{nj}(t) \]  

(2.32)

while \[ \gamma_{\xi \eta} = \gamma_{\eta \eta} = 0. \]
<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>Participation Factors $P_{nj}$ for $n = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>j=1</td>
</tr>
<tr>
<td>L/ H</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2.5</td>
<td>2.006</td>
</tr>
<tr>
<td>3</td>
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Table 2.4 Participation factors $P_{nj}$ for the first four modes and various dam length to height ratios.
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<thead>
<tr>
<th>Aspect Ratio</th>
<th>Participation Factors $P_{nj}$ for n = 2</th>
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<tr>
<td>L/ H</td>
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<tr>
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Table 2.5 Participation factors $P_{2j}$ for the first four modes and various dam length to height ratios.
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<td>1.388</td>
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Table 2.6 Participation factors $P_{nj}$ for the first four modes and various dam length to height ratios.
2.5 Steady-State Harmonic Response

It is of interest to obtain the steady-state response to a harmonic base excitation 
\[ \ddot{u}_b = \dot{U}_b(\omega, \xi_b, \eta) e^{i\omega t}. \] The total motion inside the dam, \( U_d \), expressed as

\[ U_d = \sum_{n=0,2,\cdots}^{\infty} A_n R_{on}^{(1)}(c_d, \xi) S_{on}^{(1)}(c_d, \eta) \]  
(2.33)

satisfies equation (2.28) and the boundary condition (2.15) of zero stress at the dam crest. The coefficients \( A_n \) can be derived by enforcing the second boundary condition of total displacement \( U_d(c_d, \xi_b) = U_b \) at the dam base, where \( U_b \), for a rigid canyon, is equal to the motion at the free field. Thus, at \( \xi = \xi_b \)

\[ U_d(c_d, \xi_b) = \sum_{n=0,2,\cdots}^{\infty} A_n R_{on}^{(1)}(c_d, \xi_b) S_{on}^{(1)}(c_d, \eta) = U_b, \quad -1 \leq \eta \leq 1 \]  
(2.34)

Multiplying by \( S_{ol}^{(1)}(c_d, \eta) \) and integrating on the interval (-1, 1) both sides of the equality, equation (2.34) becomes

\[ \sum_{n=0,2,\cdots}^{\infty} \int_{-1}^{1} A_n R_{on}^{(1)}(c_d, \xi_b) S_{ol}^{(1)}(c_d, \eta) S_{on}^{(1)}(c_d, \eta) \, d\eta = U_b \int_{-1}^{1} S_{ol}^{(1)}(c_d, \eta) \, d\eta \]  
(2.35)

The integral on the left side of equation (2.35) is evaluated by using the orthogonality condition of the angular function

\[ \int_{-1}^{1} S_{mn}^{(1)}(c_d, \eta) S_{ml}^{(1)}(c_d, \eta) \, d\eta = N_{mn} \quad l = n \]
\[ = 0 \quad l \neq n \]  
(2.36)

where

\[ N_{mn} = 2 \sum_{r=0,1}^{\infty} \frac{(r + 2m)! (a_r^{mn})^2}{(2r + 2m + 1) r!} \]  
(2.37)

whereas the integral on the right side is computed from the identity
\[
R_{mn}^{(1)}(c_d, \xi) = \frac{i^{m-n}}{2 S_{mn}^{(1)}(c_d, 0)} \int_{-1}^{1} J_m[c_d \left(1 - \eta^2\right)^{1/2} \left(\xi^2 - 1\right)^{1/2}] s_{mn}^{(1)}(c_d, \eta) \, d\eta
\] (2.38)

which for \(m = 0\) and \(\xi = 1\) yields

\[
\int_{-1}^{1} S_{on}^{(1)}(c_d, \eta) \, d\eta = 2 \, i^n \, R_{on}^{(1)}(c_d, 1) S_{on}^{(1)}(c_d, 0)
\] (2.39)

Making use of equations (2.35), (2.36) and (2.39), the coefficients \(A_n\) are fully recovered

\[
A_n = \frac{2 \, i^n \, R_{on}^{(1)}(c_d, 1) S_{on}^{(1)}(c_d, 0)}{R_{on}^{(1)}(c_d, \xi_b) N_{on}} \, U_b
\] (2.40)

The results of the steady-state response are presented in the form of an amplification function, \(AF = AF(\omega, \xi, \eta)\), defined as the ratio of the total motion of the dam over its base motion, i.e.

\[
AF = \frac{U_d(c_d, \xi, \eta)}{U_b} = 2 \sum_{n=0, 2, \ldots}^{\infty} \frac{i^n \, R_{on}^{(1)}(c_d, 1) S_{on}^{(1)}(c_d, 0)}{R_{on}^{(1)}(c_d, \xi_b) N_{on}} \, R_{on}^{(1)}(c_d, \xi) S_{on}^{(1)}(c_d, \eta)
\] (2.41)

Figure 2.8 plots the amplification, \(AF\), at the midcrest (\(\xi = 1, \eta = 0\)) of five dams built in semi-elliptical rigid canyons, having length to height ratio \(L/H = 2, 2.5, 3\) and \(5\) versus a dimensionless frequency \(a_o\) given by

\[
a_o = \frac{\omega H}{V_d}
\] (2.42)

The amplification of a dam under plane-strain conditions is also plotted for comparison in Figure 2.8. All amplifications are given for material hysteretic damping ratio \(\beta_d = 10\%\). The amplification for \(L/H = 2\) computed from equation (2.41) is identical to that derived independently for dams in semi-circular canyons using the simpler expression
Figure 2.8. Midcrest amplification for dams in semi-elliptical canyons with $L/H = 2, 2.5, 3, 5$ and $\infty$. 
\[ AF = \frac{\sin(a \frac{r^*}{H})}{\left( \frac{r^*}{H} \right) \sin(a)} \]  
(2.43)

where

\[ a = \frac{\omega H}{V_d^*} = \frac{a_o}{\sqrt{1 + 2i\beta_d}} \]  
(2.44)

\( r^* \) = radial polar coordinate and \( H = \) dam height (Dakoulas and Gazetas 1986). Indeed, Figure 2.9 shows that the midcrest amplification from equation (2.43), represented by a solid curve, and from equation (2.41) for \( L/H = 2.01 \), represented by a dashed curve, are practically identical. Note that, as \( L/H \to 2 \), the parameter \( c_d \to 0 \), in which case \( R_{on}^{(l)}(c_d, 1) = 0 \) for \( n \neq 0 \). Consequently, for \( L/H \to 2 \) only the term \( n = 0 \) in equation (2.41) is contributing to the response. Moreover, as \( L/H \to \infty \), the midcrest amplification approaches the independent plane-strain shear beam solution, given by

\[ AF = \frac{J_0(a \frac{z}{H})}{J_0(a)} \]  
(2.45)

where \( z = \) depth from the crest (see Figure 2.8).

The results in Figure 2.8 show that for narrow canyons with \( L/H \leq 5 \) the midcrest amplification at first resonance is about 10, whereas as \( L/H \to \infty \), it reduces to 8. This is in agreement with earlier results for triangular canyons (Makdisi et al. 1982) and rectangular canyons. The amplification at higher frequencies is much larger for dams in narrow canyons than for dams under plane-strain conditions. For example, the amplification \(|AF|\) at the third resonance peak is about 8.7 for \( L/H = 2 \), and reduces to 4.7 for \( L/H = 3 \) and to 3.8 as \( L/H \to \infty \). This substantially higher amplification for dams in canyons with \( L/H = 2 \) is due to two factors: (a) the expected stiffening effect of the canyon narrowness and (b) wave focusing phenomena at midcrest further intensified
Figure 2.9. Comparison of midcrest amplification for dams in semi-elliptical canyons with $L/H = 2$ and 2.01.
by the idealized semi-circular shape of the canyon. As \( L/H \to \infty \), the effect of these two factors diminishes and the high midcrest amplifications reduce dramatically. For dams with \( 2 < L/H < 5 \) the amplification curves show more irregularity compared to that in dams under plane-strain conditions due to the presence of more peaks and valleys caused by the participation of a larger number of vibrational modes.

Figure 2.10 plots the contribution of the first term \((n = 0)\) to the midcrest amplification for the same dams shown in Figure 2.8. Comparison between the results of Figures 2.8 and 2.10 demonstrates that the first term \((n = 0)\) in equation (2.41) is very important.

Figure 2.8 demonstrates the effect of only the length to height ratio, \( L/H \), on the steady-state midcrest amplification of dams built in canyons with semi-elliptical shapes. In addition to the effect of \( L/H \), it is also of interest to examine the effect of the canyon shape on the response, by considering the amplifications of dams built in canyons with semi-elliptical and rectangular shapes of the same \( L/H \) ratio. The amplification \( AF \) of a dam in a rectangular rigid canyon is given by

\[
AF = 1 + \frac{4}{\pi} \sum_{j=1}^{\infty} \sum_{n=1}^{\infty} \frac{(1 - (-1)^n) a_0^2 J_0(\mu_j \frac{z}{H}) \sin(n \frac{x'}{L})}{n \mu_j J_1(\mu_j) \left( \left( \frac{n \pi H}{L} \right)^2 - a_0^2 + \mu_j^2 \right)}
\]

(2.46)

where \( \mu_j \) is the \( j \)th root of the equation \( J_0(\mu_j) = 0 \). (The origin of the \( x' - z \) coordinate is at the upper-left corner of the rectangular canyon cross-section).

Figures 2.11-13 plot the amplifications for the two canyon shapes with \( L/H \) ratios equal to 2, 3, and 5, respectively, using equations (2.41) and (2.46). Notice again the substantially higher midcrest amplification values of the dam in the semi-circular canyon \((L/H=2)\), caused by the higher boundary proximity, as well as the wave focusing phenomena, not present in the response of the dam in the rectangular canyon (Figure 2.11). For \( L/H=3 \), the significant differences in the response shown in Figure 2.12, for
Figure 2.10. First term \( n = 0 \) of midcrest amplification for dams in semi-elliptical canyons with \( L/H = 2, 2.5, 3, 5 \) and \( \infty \).
Figure 2.11. Midcrest amplification for two dams built in semi-circular and rectangular canyons with L/H = 2.
Figure 2.12. Midcrest amplification for two dams built in semi-circular and rectangular canyons with L/H = 3.
Figure 2.13. Midcrest amplification for two dams built in semi-circular and rectangular canyons with $L/H = 5$. 

$\beta_d = 10\%$
frequencies $a_o$ ranging from about 3 to 8, are due to the difference between the two
canyon shapes, with the semi-elliptical canyon being "narrower" than the rectangular one.
Finally, for $L/H=5$ (Figure 2.13) or higher, the effect of the canyon shape is less
significant, and the midcrest amplifications for both dams approach that of a dam under
plane strain conditions, given by equation (2.45).

Figures 2.14-17 plot the distribution of the amplifications along the crest and along the
depth at the mid-section for dams built in canyons with semi-elliptical and rectangular
shapes of $L/H$ ratios equal to 2, 3, 5 and 10, respectively, evaluated at the first resonance
of the dam. For narrow canyons the distribution of $|AF|$ is not very different for the two
canyon shapes. For long dams with $L/H=10$, the steady state response of a dam in the
rectangular canyon is very close to that of a dam a under plane-strain conditions for about
the middle 80% of the dam length. By contrast, the steady-state response of the dam in
the semi-elliptical canyon is close to that of a dam under plane-strain conditions only for
about the middle 25% of the dam length, while most of the rest of the dam moves a rigid
body, with $AF \approx 1$. More substantial differences between the response of the dams in
semi-elliptical and rectangular canyons occur for higher frequencies and $L/H < 5$.

It is of interest to examine also the distribution of the shear strain developed within the
dam body, as the level of shear strains controls the amount of stiffness degradation and
material hysteretic damping, associated with the level of the nonlinearity of the soil
response induced during a seismic shaking. Figure 2.18 plots the distribution of the
normalized shear strain $\frac{\gamma_{yz} H}{U_b}$ evaluated at the mid-section versus the normalized depth
$z/H$ for six dams built in semi-elliptical canyons with aspect ratios $L/H=2, 2.5, 3, 5, 10$ and $\infty$ (dam under plane strain conditions). The shear strain is computed for 10% material hysteretic damping for all dams. The results suggest consistently that of the
canyon geometry has little effect of the magnitude of the shear strain $\gamma_{yz}$ at the
midsection. Indeed, for depths $z/H < 0.7$, the shear strain curves show very consistent trends and similar values for all aspect ratios $L/H$, with $\gamma_{yz}$ decreasing slightly as $L/H$ increases. For depths $z/H > 0.7$, the same trend is observed for $2.5 \leq L/H \leq \infty$ but as $L/H \to 2$ the shear strain near the base approaches again that of the infinitely long dam. It should be noted that the shear strain $\gamma_{yz}$ at the dam midsection is zero due to symmetry. Finally, Figure 2.19 plots the normalized base shear strain $\frac{\gamma_{yz}^b H}{U_b}$ versus the dimensionless frequency $\omega = \frac{\omega H}{V_d}$ for five dams built in semi-elliptical canyons with aspect ratios $L/H = 2, 2.5, 3, 5$ and $\infty$. The results show that the amplitude of the normalized base shear strain $\frac{\gamma_{yz}^b H}{U_b}$ increases with frequency $\omega$. However, it should be noted that, the relationship of the normalized shear strain with $\omega$ depends on the normalizing factor. For example, if instead of the above normalization, the quantity $\frac{\gamma_{yz}^b V_d^2}{H \dot{U}_b}$ is plotted versus $\omega$, where $\dot{U}_b = -\omega^2 U_b$ is the amplitude of the base acceleration, the normalized shear strain will decrease with increasing frequency $\omega$. 
Figure 2.14. Distribution of amplification at first resonance along the depth (at mid-section) and along the crest for two dams built in semi-circular and rectangular canyons with $L/H = 2$. 

$L/H = 2$

$\beta_d = 10\%$
Figure 2.15. Distribution of amplification at first resonance along the depth (at mid-section) and along the crest for two dams built in semi-circular and rectangular canyons with $L/H = 3$. 

$L/H = 3$

$\beta_d = 10\%$
Figure 2.16. Distribution of amplification at first resonance along the depth (at mid-section) and along the crest for two dams built in semi-circular and rectangular canyons with $L/H=5$. 

$L/H = 5$

$\beta_d = 10\%$
Figure 2.14. Distribution of amplification at first resonance along the depth (at midsection) and along the crest for two dams built in semi-elliptical and rectangular canyons with $L/H=10$. 

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Figure 2.18. Normalized shear strain at first resonance at midsection versus depth for six dams in semi-elliptical canyons with aspect ratios $L/H = 2, 2.5, 3, 5, 10$ and $\infty$. 
Figure 2.19. Normalized shear strain at the base of the midsection versus a dimensionless frequency for five dams in semi-elliptical canyons with aspect ratios $L/H = 2, 2.5, 3, 5$ and $\infty$. 
2.6 Summary And Conclusions

A closed-form analytical solution has been developed for the dynamic lateral shear response of embankment dams supported by rigid semi-elliptical canyons. The results are used to gain more insight on the effects of the narrowness and the shape of the canyon on the seismic response characteristics of the dam. For this purpose, the semi-elliptical canyon shape is particularly useful as it allows significant flexibility in modeling a wide range of degrees of canyon narrowness. The solution for the dam response is given in terms of prolate spheroidal radial and angular functions of the first kind, and of zero order. Results have been presented for natural frequencies, modal displacement shapes, participation factors, and response to transient and steady-state harmonic base excitation for various dam length to height ratios. Parametric studies have been performed to examine the effect on the response of the length to height ratio and of the canyon shape, for dams built in semi-elliptical and rectangular canyons.

The conclusions drawn from this study are summarized as follows:

1. The developed model for the lateral response of dams in semi-elliptical canyons is a generalization of the shear beam model, and includes as special cases two earlier solutions for dams in semi-circular canyons and dams under plane-strain conditions.

2. The results confirm the increase of the natural frequencies of the dam as canyon narrowness increases (stiffening effect). For aspect ratios \( L/H > 5 \), the natural frequencies of the dam are almost identical with those for dams under plane-strain conditions.

3. The displacement shapes along the height of the dam in the first mode are practically independent of the aspect ratio \( L/H \). This is also true for modes
corresponding to \( n=0, \ j>1 \) for \( L/H \geq 3 \), whereas as \( L/H \rightarrow 2 \) there is a sharper attenuation of displacement shapes with depth.

4. Results on steady-state response to harmonic excitation suggest that the high midcrest amplifications observed for high frequency motion in dams with semi-circular canyon shape are the combined result of canyon narrowness and intense wave focusing phenomena. These high-frequency amplifications are reduced as \( L/H \) increases.

5. For \( L/H < 5 \), the midcrest amplification is affected significantly by both the \( L/H \) ratio and the particular canyon shape. For \( L/H > 5 \) the high-frequency midcrest amplification approaches that of a dam under plane-strain conditions.

6. For long dams in semi-elliptical canyons, the steady-state response is close to that of a dam under plane-strain conditions only along a small region near the midsection. For example, for \( L/H = 10 \), the steady-state response is close to that of a dam under plane-strain conditions only for about the middle 25% of the dam length, while most of the rest of the dam moves a rigid body. The response of equally long dams built in rectangular canyons is close to that under plane-strain conditions for about the middle 80% of the dam length.

7. The results suggest consistently that the canyon geometry shape has little effect of the magnitude of the shear strain \( \gamma_{yz} \) at the midsection.

8. The presented closed-form solution for the lateral response of dams in rigid semi-elliptical canyons can be a valuable tool for parametric studies and preliminary design computations, as the semi-elliptical canyon shape allows significant flexibility in modeling the 3-D geometry of several actual dams.
3. RESPONSE OF DAMS IN SEMI-ELLIPTICAL FLEXIBLE CANYONS SUBJECTED TO OBLIQUE SH WAVES

3.1 Introduction

The first part of this study examined the response of earth dams in semi-elliptical perfectly rigid canyons, subjected to synchronous, identical base motion along the dam-canyon interface. The results from this study demonstrated that for dams built in narrow valleys, the proximity of relatively rigid abutments creates a three-dimensional (3-D) stiffening effect, which manifests itself by higher natural frequencies and sharply higher near crest accelerations. This is in agreement with results from earlier studies for rectangular, semi-circular and triangular canyon shapes, which have demonstrated that the 3-D canyon geometry effects on the response characteristics of dams are very important.

In the second part of this work, the flexibility of the canyon and the ground motion spatial variability along the dam-canyon interface are introduced. Although the effects of the these two factors appear to be quite important, due to the complexity of 3-D analysis of the dam-canyon system, in engineering practice they are usually ignored and the supporting canyon is assumed to vibrate as a rigid body with a "synchronous" identical excitation along the dam base. In reality, of course, the seismic motion along the dam-canyon interface is rarely synchronous and identical. Indeed, recorded accelerations along various locations at the abutment and canyon base rock close to the dam demonstrate very frequently substantial differences in the amplitude, phase difference and frequency content (e.g. CSMIP 1989; Lai and Seed 1980). This is because the seismic shaking is the result of a multitude of body and surface waves that travel through different paths, encountering various materials, and strike the dam at various angles, creating complex reflection and diffraction phenomena.
There are only few studies investigating the combined effects of the canyon/foundation flexibility and the spatial variability of ground excitation. Dibaj and Penzien used the finite-element method to investigate the response of long earth dams to laterally traveling waves, ignoring the flexibility of the foundation material and therefore the energy radiated back to the halfspace. Haroun and Abdel-Hafiz (1987) presented another finite-element formulation for the response of a dam in a rectangular canyon, by idealizing the dam as a two-dimensional shear beam and examining various forms of excitation at the dam base to reflect the amplitude and phase variation of the ground motion.

A more complete formulation has been used by Nahhas (1987) who applied the Boundary Integral Equation Method, however, after introducing some substantial simplifications of the dam-canyon geometry, to analyze the dynamic behavior of earth dams subjected to P, SV and SH waves.

Using a generalized shear-beam model, Dakoulas and Hashmi (1992) developed an approximate analytical closed-form solution for a dam built in a rectangular canyon subjected to obliquely incident SH waves. In this study, the dam is idealized as a linearly-hysteretic elastic body deforming only in shear, while the supporting canyon is modeled as an elastic medium, representing a deformable rock half-space. Recently, Dakoulas (1993) developed an analytical solution for steady-state lateral response of earth dams built in semi-cylindrical flexible canyons subjected to obliquely-incident harmonic SH waves. The solution accounts in a rigorous way for the complex wave reflection, transmission and diffraction phenomena associated with the presence of the dam-filled canyon.

The results of the above studies on the response of very long dams and dams in rectangular and semi-cylindrical valleys, indicate clearly that the effects of the canyon/foundation flexibility and wave passage are very important. The wave passage effects in particular ignoring them may lead to a very unconservative analysis and unsafe design.
Valuable insight in the phenomena controlling the response of the dam-canyon system may also be gained from the results of analytical and numerical studies of the response of alluvial valleys with semi-circular, semi-elliptical, cosine and nearly rectangular shapes (Trifunac, 1971; Wong and Trifunac, 1974; Bard and Bouchon, 1980). These studies showed the response of the alluvium depends on the interference of the transmitted waves with their reflections on the alluvium boundaries, which results in the formation of Love waves, propagating back and forth within the two edges of alluvium.

By accounting for the spatial variability of the seismic excitation, this study aims at improving our understanding of the behavior of the earth and rockfill dams and providing more insight for the design of new dams in seismic regions.
3.2 Simplifying Assumptions

Figure 2.1 portrays a 3-D perspective view of the dam in a semi-elliptical canyon. The dam has a triangular cross-section, consisting of homogeneous and linearly hysteretic soil with a constant mass density $\rho_d$, a constant shear modulus $G_d$, and a material hysteretic damping ratio $\beta_d$. The canyon is a homogeneous linearly hysteretic solid, representing elastic rock, with mass density $\rho_c$, and shear modulus $G_c$.

The incident excitation consists exclusively of a steady-state train of harmonic SH waves of constant amplitude, $U_1$, and frequency, $\omega$, traveling from the left to the right along the dam in a upward direction forming an angle $\alpha$ to the vertical (see Figure 3.1a). The resulting displacement, $u_i$, is only in the upstream-downstream (or y) direction, having the form

$$u_i = U_1 \exp \left[ i \omega \left( t - \frac{x}{V_x} + \frac{z}{V_z} \right) \right] \quad (3.1)$$

in which $V_x$ and $V_z$ are the phase velocities along the x and z directions given by

$$V_x = \frac{V_c}{\sin \alpha} \quad (3.2)$$

$$V_z = \frac{V_c}{\cos \alpha} \quad (3.3)$$

where $V_c$ is the shear wave velocity expressed as

$$V_c = \sqrt{\frac{G_c}{\rho_c}} \quad (3.4)$$

The response of the dam to the plane SH excitation is assumed to be only in horizontal lateral shear deformation with the upstream-downstream displacements, $u_d$, uniformly distributed across the width of the dam. In other words, the dam is idealized as a "shear
Figure 3.1. Dam in semi-elliptical canyon: (a) longitudinal section (b) maximum cross-section (c) infinitesimal element with shear stresses acting at its faces.
beam”, which extends in the vertical and longitudinal directions and assumes uniform (or average) response values for the upstream-downstream direction.

3.3 Analysis Of Steady State Response

For the case of an elastic half-space (without the canyon), or at the free field far from the canyon, the total motion is the superposition of incident waves, \( u_i \), and reflected waves, \( u_r \), expressed as

\[
\begin{align*}
    u_i + u_r &= (U_i + U_r) \exp(i \omega t) \\
    &= U_1 \exp \left[ i \omega \left( t - \frac{x}{V_x} + \frac{z}{V_z} \right) \right] + U_1 \exp \left[ i \omega \left( t - \frac{x}{V_x} - \frac{z}{V_z} \right) \right] \\
    &= 2 U_1 \exp \left[ i \omega \left( t - \frac{x}{V_x} \right) \right] \cos \left( \frac{\omega z}{V_z} \right) \\
\end{align*}
\]

(3.5)

where \( U_i + U_r \) is their complex amplitude. This expression for the total motion satisfies the differential equation for the elastic half-space and the boundary conditions of zero shear stress at the free surface.

In the case of the semi-elliptical canyon filled with the dam, the total motion of the canyon may be written as

\[
    u_c = U_c(x, z) \exp(i \omega t) \\
\]

(3.6)

where \( U_c(x, z) \) is initially unknown and has to be determined by considering the dam-canyon interaction. The motion, \( u_c \), is the superposition of: (a) the incident and the reflected waves in the half-space without the canyon, \( u_i + u_r \) and (b) the reflected and diffracted waves caused by the presence of the semi-elliptical canyon and the dam, \( u_{cr} \). Thus

\[
    u_c = u_i + u_r + u_{cr} \\
\]

(3.7)
and, by eliminating the time factor,

$$U_c = U_i + U_r + U_{cr}$$  \hspace{1cm} (3.8)

in which $u_{cr} = U_{cr} \exp(i\omega t)$ represents outgoing waves.

Similarly, the response of the dam is denoted by

$$u_d = U_d(x, z) \exp(i\omega t)$$  \hspace{1cm} (3.9)

In the following, the expressions for $U_c$ and $U_d$ are derived by considering the dam-canyon interaction.

By considering the dynamic equilibrium of an infinitesimal element of the dam body (Figure 3.1c), the equation of motion is expressed as

$$\frac{\partial^2 U_d}{\partial z^2} + \frac{1}{z} \frac{\partial U_d}{\partial z} + \frac{\partial^2 U_d}{\partial x^2} + k_d^2 U_d = 0$$  \hspace{1cm} (3.10)

where

$$k_d = \frac{\omega}{V_d^*}$$  \hspace{1cm} (3.11)

and

$$V_d^* = \sqrt{\frac{G_d(1 + 2i\beta_d)}{\rho_d}} = V_d \sqrt{1 + 2i\beta_d}$$  \hspace{1cm} (3.12)

in which $V_d$ is the shear wave velocity of the dam material. Equation (3.10) expressed in elliptical coordinates becomes

$$\frac{\partial}{\partial \xi}[(\xi^2 - 1) \frac{\partial U_d}{\partial \xi}] + \frac{\partial}{\partial \eta}[(1 - \eta^2) \frac{\partial U_d}{\partial \eta}] + c_d^2 (\xi^2 - \eta^2) U_d = 0$$  \hspace{1cm} (3.13)

where
\[
x = d \frac{\xi}{\eta} \\
z = d \sqrt{\frac{\xi^2}{\eta^2} - 1} \sqrt{1 - \eta^2} \quad \text{for} \quad 1 \leq \xi < \xi_b, \quad -1 \leq \eta \leq 1
\]

(3.14)

where

\[
\xi_b = \frac{L}{\sqrt{L^2 - 4H^2}}
\]

(3.15)

\[
c_d = k_d \ d
\]

(3.16)

and \( H \) = dam height and \( 2d = \) interfocal distance (Figure 3.1b).

The solution of equation (3.13) must satisfy the continuity of displacements and shear stresses along the semi-elliptical dam-canyon interface. Thus,

\[
U_d(\xi_b, \eta) = U_c(\xi_b, \eta) \quad \text{for} \quad -1 \leq \eta \leq 1
\]

(3.17)

and

\[
G^*_d \frac{\partial U_d(\xi_b, \eta)}{\partial \xi} = G^*_c \frac{\partial U_c(\xi_b, \eta)}{\partial \xi} \quad \text{for} \quad -1 \leq \eta \leq 1
\]

(3.18)

Moreover, it should yield zero shear stresses at the dam crest

\[
G^*_d \frac{\partial U_d(\xi, 1)}{\partial \eta} = G^*_d \frac{\partial U_d(\xi, -1)}{\partial \eta} = 0 \quad \text{for} \quad 1 < \xi \leq \xi_b
\]

(3.19)

\[
G^*_d \frac{\partial U_d(1, \eta)}{\partial \xi} = 0 \quad \text{for} \quad -1 < \eta < 1
\]

(3.20)

Using separation of variables, \( U_d \) may be written in the form of the Lamé products

\[
U_d = R_{mn}(c_d, \xi) S_{mn}(c_d, \eta)
\]

(3.21)

The "radial solution" \( R_{mn}(c_d, \xi) \) and the "angular solution" \( S_{mn}(c_d, \eta) \) must satisfy the ordinary differential equations.
\[
\frac{d}{d\xi}[(\xi^2 - 1) \frac{d}{d\xi} R_{mn}(c_d, \xi)] - (\lambda_{mn} - c_d^2 \xi^2) R_{mn}(c_d, \xi) = 0
\] (3.22)

\[
\frac{d}{d\eta}[(1 - \eta^2) \frac{d}{d\eta} S_{mn}(c_d, \eta)] + (\lambda_{mn} - c_d^2 \eta^2) S_{mn}(c_d, \eta) = 0
\] (3.23)

in which the associated eigenfunctions \( R_{mn}(c_d, \xi) \) and \( S_{mn}(c_d, \eta) \) are, respectively, prolate spheroidal radial and angular functions of order \( m \) and degree \( n \) (Abramowitz and Stegun 1970; Flammer 1957). The separation constants \( \lambda_{mn} \) are to be determined so that \( R_{mn}(c_d, \xi) \) and \( S_{mn}(c_d, \eta) \) are finite at \( \xi = 1 \) and \( \eta = \pm 1 \). The only acceptable solutions for the dam response are the prolate spheroidal radial and angular functions of the first kind, denoted by \( R_{mn}^{(1)}(c_d, \xi) \) and \( S_{mn}^{(1)}(c_d, \eta) \), while the second kind functions \( (R_{mn}^{(2)}(c_d, \xi) \) and \( S_{mn}^{(2)}(c_d, \eta)) \) are rejected due to their singularity at the dam crest.

The angular function of the first kind is given by an infinite sum of the form

\[
S_{mn}^{(1)}(c_d, \eta) = \sum_{r=0,1}^{\infty} d_r^{mn}(c_d) P_m^{m+r}(\eta)
\] (3.24)

in which the prime over the summation sign, \( \sum' \), indicates that the summation is only over the even values of \( r \) when \( n - m \) is even, and over only odd values of \( r \) when \( n - m \) is odd; \( P_m^{m+r} \) is the associated Legendre function of the first kind defined according to Ferrer as

\[
P_n^m(z) = (1 - z^2)^{\frac{m}{2}} \frac{d^m P_n(z)}{d z_m} \quad \text{for} \quad -1 \leq z \leq 1
\] (3.25)

and \( d_r^{mn}(c_d) \) are the coefficients determined by a recursion formula (see APPENDIX A).

Similarly, the radial function of the first kind is an infinite sum of the form
\[ R_{mn}^{(1)}(c_d, \xi) = \frac{1}{\sum_{r=0}^{\infty} \sum_{r=0}^{\infty} a_r^{mn}(c_d) \frac{(2m+r)!}{r!} \frac{(\xi^2 - 1)^{m}}{\xi^{2r}} \frac{i^{r+m-n}}{r!} (2m+r)!} \frac{d_r^{mn}(c_d) J_{m+r}(c_d, \xi)}{r!} \]  

(3.26)

where \( j_r(z) \) is the spherical-Bessel function of the first kind given by

\[ j_r(z) = \sqrt{\frac{\pi}{2z}} J_{r+\frac{1}{2}}(z) \]  

(3.27)

in which \( J_{r+\frac{1}{2}}(z) \) is the Bessel function of the first kind and order \( r+\frac{1}{2} \).

The prolate spheroidal functions that satisfy the differential equation (3.13) for the dam response must be of zero order \( (m = 0) \). For flexible canyons, in which the excitation varies along the semi-elliptical dam-canyon interface, the general solution is given as a superposition of terms \( R_{mn}^{(1)}(c_d, \xi) S_{on}^{(1)}(c_d, \eta) \), where \( n = 0, 1, 2, \ldots \). Thus, the motion within the dam is expressed as

\[ U_d = \sum_{n=0}^{\infty} A_n R_{on}^{(1)}(c_d, \xi) S_{on}^{(1)}(c_d, \eta) \]  

(3.28)

where the constants \( A_n \) are determined in the sequel using the boundary conditions.

Let us now proceed with the formulation of the expression for the reflected and diffracted outgoing waves, \( U_{cr} = U_{cr}(\xi, \eta) \), caused by the presence of the semi-elliptical canyon and the dam. It is convenient to consider the governing equation of motion for the elastic-rock half-space in polar coordinates \( (r^*, \theta^*) \)

\[ r^*2 \frac{\partial^2 U_c}{\partial r^*2} + r^* \frac{\partial U_c}{\partial r^*} + \frac{\partial^2 U_c}{\partial \theta^*2} + r^* k_c^2 U_c = 0 \]  

(3.29)

where
\[ k_c = \frac{\omega}{V_c} \]  
(3.30)

and

\[
V_c = \sqrt{\frac{G_c}{\rho_c}}
\]  
(3.31)

is the shear wave velocity of the canyon rock.

The solution of equation (3.29) for the case of elastic-half-space, \( U_i + U_r \), is written in polar coordinate as

\[
U_i + U_r = \exp \left[ -ik_c r^* (\sin \theta^* \sin \alpha - \cos \theta^* \cos \alpha) \right] + \exp \left[ -ik_c r^* (\sin \theta^* \sin \alpha + \cos \theta^* \cos \alpha) \right]
\]

\[ = \exp \left[ i k_c r^* \cos(\theta^* + \alpha) \right] + \exp \left[ -i k_c r^* \cos(\theta^* - \alpha) \right] \]  
(3.32)

where \( \alpha \) is the angle of incidence, and \( r^* \) and \( \theta^* \) are defined in Figure 3.2. Introducing the angles \( \theta \) and \( \theta_o \) defined in Figure 3.2, the expressions for \( \theta^* + \alpha \) and \( \theta^* - \alpha \) are rewritten as

\[
\theta^* + \alpha = \theta_o - \theta
\]  
(3.33)

and

\[
\theta^* - \alpha = \pi - (\theta_o + \theta)
\]  
(3.34)

Now, utilizing the identity

\[
U_i = \exp \left[ i k_c r^* \cos(\theta_o - \theta) \right]
\]

\[ = 2 \sum_{m=0}^{\infty} \sum_{n=m,m+1,...}^{\infty} i^n \frac{(2 - \delta_{om})}{N_{mn}} S_{mn}^{(1)}(c_c, \cos \theta_o) S_{mn}^{(1)}(c_c, \eta) R_{mn}^{(1)}(c_c, \xi) \]

where

\[
\delta_{om} = 1 \quad m = 0
\]

\[ = 0 \quad m \neq 0
\]  
(3.35)
Figure 3.2. Definition of polar, elliptical and Cartesian coordinate systems used.
and

\[ N_{mn} = 2 \sum_{r=0}^{\infty} \frac{(r + 2m)!}{(2r + 2m + 1)!} r! \left( \frac{a_r^{mn}}{\sin \theta_o} \right)^2 \]  

(3.37)

(Flammer 1957), the incident waves are expressed in terms of prolate spheroidal functions similar to those encountered in the solution for the response of the dam. Furthermore, it can be shown (see Appendix D) that the reflected waves can also be expressed as

\[ U_r = \exp \left[ i \frac{k_c}{r} \cos(\theta_o + \theta) \right] = 2 \sum_{m=0}^{\infty} \sum_{n=m+1}^{\infty} i^n (-1)^m \frac{(2 - \delta_{om})}{N_{mn}} S_{mn}^{(1)}(c, \cos \theta_o) S_{mn}^{(1)}(c, \eta) R_{mn}^{(1)}(c, \xi) \]  

(3.38)

From equations (3.35) and (3.38), the total motion \( U_i + U_r \) becomes

\[ U_i + U_r = 2 \sum_{m=0}^{\infty} \sum_{n=m+1}^{\infty} i^n \left[ 1 + (-1)^m \right] \frac{(2 - \delta_{om})}{N_{mn}} S_{mn}^{(1)}(c, \cos \theta_o) S_{mn}^{(1)}(c, \eta) R_{mn}^{(1)}(c, \xi) \]

\[ = \sum_{m=0}^{\infty} \sum_{n=m+1}^{\infty} C_{mn} S_{mn}^{(1)}(c, \eta) R_{mn}^{(1)}(c, \xi) \]  

(3.39)

where

\[ C_{mn} = \frac{4}{N_{mn}} i^n S_{mn}^{(1)}(c, \cos \theta_o) \quad \text{for} \quad m = 0, 2, 4, \ldots \]

\[ C_{mn} = 0 \quad \text{for} \quad m = 1, 3, 5, \ldots \]  

(3.40)

The diffracted-wave amplitude \( U_{cr} \) must satisfy equation (3.29), the continuity of displacements and shear stresses along the dam-canyon interface (equations (3.17-18)), and zero stress at the surface of the half-space outside the canyon (equations (3.19-20)).

The expression for the reflected and diffracted outgoing waves, \( U_{cr} = U_{cr}(\xi, \eta) \), is written as a series expansion of the form
\[ U_{cr} = \sum_{n=0}^{\infty} B_n R_{rn}^{(4)}(c, \xi) S_{on}^{(1)}(c, \eta) + \sum_{m=2,4,6} \sum_{n=m,m+1,\ldots} D_{mn} R_{mn}^{(4)}(c, \xi) S_{mn}^{(1)}(c, \eta) \] (3.41)

where \( B_n \) and \( D_{mn} \) will be determined in the sequel using the boundary conditions. The expression \( R_{rn}^{(4)}(c, \xi) \) is the spheroidal radial function of the fourth kind

\[ R_{rn}^{(4)}(c, \xi) = R_{rn}^{(1)}(c, \xi) - i R_{rn}^{(2)}(c, \xi) \] (3.42)

where \( R_{rn}^{(2)}(c, \xi) \) is the spheroidal radial function of the second kind. The latter is obtained by replacing the spherical-Bessel function of the first kind, \( j_r(z) \), in \( R_{rn}^{(1)}(c, \xi) \) by the spherical-Bessel function of the second kind, \( y_r(z) \), which is defined by

\[ y_r(z) = \sqrt{\frac{\pi}{2z}} Y_{rt + \frac{1}{2}}(z) \] (3.43)

in which \( Y_{rt + \frac{1}{2}}(z) \) is the Bessel function of the second kind.

Note that the selection of the spheroidal radial function of the fourth kind, \( R_{rn}^{(4)}(c, \xi) \), was made based on the requirement that the scattered wavefield must be outward propagating. This can be easily shown by taking the asymptotic expansion of \( R_{rn}^{(4)}(c, \xi) \) given by

\[ R_{rn}^{(4)}(c, \xi) \xrightarrow{c, \xi \to \infty} \frac{1}{c_0 \xi} \exp \left \{ -i \left [ c_0 \xi - \frac{1}{2} (n + 1) \pi \right ] \right \} \] (3.44)

which, for time dependence in the form \( \exp(i\omega t) \), represents an outgoing wave and satisfies the Sommerfeld radiation condition. By contrast, the spheroidal radial function of the third kind, \( R_{rn}^{(3)}(c, \xi) \), given by

\[ R_{rn}^{(3)}(c, \xi) = R_{rn}^{(1)}(c, \xi) + i R_{rn}^{(2)}(c, \xi) \] (3.45)

represents an incoming wave and does not satisfy the radiation condition.
Notice that in equation (3.41) the expression for $U_{cr} = U_{cr}(\xi, \eta)$ is written as the sum of two series with corresponding unknown coefficients $B_n$ and $D_{mn}$. The first series corresponds to terms with $m = 0$ and the second series to terms with $m = 2, 4, 6, \ldots$

The derivation of coefficients $A_n$ and $B_n$ by enforcing the boundary conditions (3.17) and (3.18) along the semi-elliptical dam-canyon interface ($\xi = \xi_p$) encounters some special difficulty: the angular functions $S_{on}^{(1)}(c_c, \eta)$, for the canyon, and $S_{on}^{(1)}(c_d, \eta)$, for the dam, are not orthogonal, as they correspond to different material parameters ($c_c \neq c_d$). In order to overcome this problem, $S_{on}^{(1)}(c_c, \eta)$ and $S_{on}^{(1)}(c_d, \eta)$ are expressed as series expansions of the associated Legendre function $P_r^m(\eta)$, which is independent of the material parameters, using equation (3.24). Furthermore, since $U_d$ is expressed in terms of spheroidal functions of zero order ($m = 0$), the enforcement of the boundary conditions (3.17) and (3.18) along the semi-elliptical dam-canyon interface is separated into systems of equations corresponding to $m = 0$ and $m = 2, 4, 6, \ldots$, respectively. Thus, the expressions for the response of the dam and the canyon are rewritten as

$$U_d = \sum_{r=0}^{\infty} P_r(\eta) \sum_{n=0}^{\infty} A_n R_{on}^{(1)}(c_d, \xi) d_r^{on}(c_d)$$

and

$$U_i + U_r + U_{cr} =$$

$$\sum_{r=0}^{\infty} P_r(\eta) \sum_{n=0}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi) d_r^{on}(c_c) + \sum_{m=2,4,\ldots}^{\infty} \sum_{n=m,m+1,\ldots}^{\infty} C_{mn} S_{mn}^{(1)}(c_c, \eta) R_{mn}^{(1)}(c_c, \xi) +$$

$$\sum_{r=0}^{\infty} P_r(\eta) \sum_{n=0}^{\infty} B_n R_{on}^{(4)}(c_c, \xi) d_r^{on}(c_c) + \sum_{m=2,4,\ldots}^{\infty} \sum_{n=m,m+1,\ldots}^{\infty} D_{mn} R_{on}^{(4)}(c_c, \xi) S_{on}^{(1)}(c_c, \eta)$$

(3.47)

By substituting the above equations in the boundary conditions (3.17) and (3.18), for the terms having $m = 0$ and every term $r$ and after canceling the $P_r(\eta)$ terms from all equations, the following system is obtained:
\[ \sum_{n=0}^{\infty} A_n R_{on}^{(1)}(c_d, \xi_b) d_r^{on}(c_d) = \sum_{n=0}^{\infty} \left[ C_{on} R_{on}^{(1)}(c_c, \xi_b) + B_n R_{on}^{(4)}(c_c, \xi_b) \right] d_r^{on}(c_c) \]  

(3.48)

\[ \sum_{n=0}^{\infty} A_n R_{on}^{(1)}(c_d, \xi_b) d_r^{on}(c_d) = \mu \sum_{n=0}^{\infty} \left[ C_{on} R_{on}^{(1)}(c_c, \xi_b) + B_n R_{on}^{(4)}(c_c, \xi_b) \right] d_r^{on}(c_c) \]  

(3.49)

where

\[ \mu = \frac{G_i}{G_d} \]  

(3.50)

By solving the above system of equations separately for even and odd values of \( n \), the coefficients \( A_n \) and \( B_n \) can be recovered. Of course, it is numerically impossible to solve the system of infinite terms of \( A_n \) and \( B_n \). However, a sufficiently accurate solution can be obtained by using a finite number of terms, which depends on the frequency of the waves and the canyon geometry. By using the first ten terms in the series expansion, excellent results were obtained for all cases examined.

The terms corresponding to \( m = 2, 4, 6, \ldots \) do not affect the response of the dam, but are needed for the computation of the total motion in the canyon. These terms are included only \( U_r + U_o \) and \( U_{cr} \) and should have zero displacement and stress contribution at \( \xi = \xi_b \). Thus, for each term \( m = 2, 4, 6, \ldots \) and \( n = m, m+1, \ldots \), the coefficients \( D_{mn} \) are given by

\[ D_{mn} = \frac{C_{mn} R_{mn}^{(1)}(c_c, \xi_b)}{R_{mn}^{(4)}(c_c, \xi_b)} \]  

(3.51)

More details about the derivation of the coefficients \( A_n \), \( B_n \) and \( D_{mn} \) are given in APPENDIX E.
3.4 Parametric Study And Discussion

In the sequel, the solutions (3.46) and (3.47) are used to investigate the influence of the angle of incidence, $\alpha$, of the dam to canyon material impedance ratio, defined as

$$\text{IR} = \frac{V_c \rho_c}{V_d \rho_d}$$

(3.52)

and of the aspect ratios $L/H$ on the steady-state response of a dam in semi-elliptical canyon subjected to harmonic SH waves. The results are presented in the form of amplification functions, $AF$, of the motion with reference to the free field surface motion. The latter has amplitude equal to two times the amplitude $U_1$ of the incident waves. We study two such amplification functions:

$$|AF| = \left| \frac{U_d}{2U_1} \right|$$

(3.53)

for the dam motion and

$$|AF| = \left| \frac{U_e}{2U_1} \right|$$

(3.54)

for the motion in the supporting elastic canyon.

The phase angles, $\phi$, are also studied herein:

$$\phi = \arctan \left[ \frac{\text{Im}[AF]}{\text{Re}[AF]} \right]$$

(3.55)
3.4.1 Effect of impedance ratio and canyon geometry

To examine the effects of impedance ratio and canyon geometry, the crest amplifications of twenty five dams in semi-elliptical canyons having various aspect ratios, \(L/H\), and impedance ratios, \(IR\), are considered.

Figure 3.3 plots the magnitude of the midcrest amplification \(|AF|\) versus the dimensionless frequency \(a_o = \frac{\omega H}{V_d}\) for five dams built in semi-circular canyons \((L/H = 2)\) and corresponding to five impedance ratios \(IR = 2.5, 5, 10, 15\) and \(\infty\) (rigid canyon). All dams have the same mass density ratio, \(\rho = \frac{\rho_c}{\rho_d} = 1.25\), material hysteretic damping ratio \(\beta_d = 10\%\) and are subjected to vertically incident SH waves \((\alpha = 0^\circ)\). The \(|AF|\) values in Figure 3.3, computed from equation (3.46), are identical to those obtained independently from a different expression derived for the response of dams in semi-circular flexible canyons by Dakoulas (1993). (The agreement between the two solutions is illustrated in Figure 3.8, which compares the midcrest \(|AF|\) from the two solutions for a dam with impedance ratio \(IR=10\)). Similarly, Figures 3.4-3.7 plot \(|AF|\) at mid-crest versus \(a_o\) for dams with length to height ratios \(L/H = 2.5, 3, 5\) and \(\infty\), respectively, and for the same values of impedance ratios \(IR = 2.5, 5, 10, 15\) and \(\infty\) and vertically incident SH waves. The results in Figure 3.7 for \(L/H = \infty\) (dam under plane strain conditions) are obtained directly from the 1-D shear beam solution (Dakoulas 1985), since for \(L/H > 10\) the accuracy of the numerical computations of \(|AF|\) using equation (3.46) decreases for high frequency motion.

As discussed in the case of rigid semi-elliptical canyons, dams with \(2 < L/H < \infty\) have a double infinite series of natural frequencies, corresponding respectively to vibrational modal shapes in the elliptical “radial” and “angular” directions. For the special cases of \(L/H = 2\) and \(L/H = \infty\), the number of natural frequencies is reduced to a single series, corresponding to vibrational modes in the radial and vertical directions, respectively. The
presence of more natural frequencies for dams with \(2 < L/H < \infty\) is clearly demonstrated by the additional smaller peaks and the higher irregularity of the midcrest amplification \(|AF|\) versus \(a_o\) for \(L/H = 2.5, 3,\) and \(5\), compared to that for \(L/H = 2\) and \(L/H = \infty\). Finally, it should be noted that, as \(IR \to \infty\) (rigid base), the results obtained from equation (3.46) and plotted in Figure 3.3 to 3.6 with a thick solid curve, are identical with those derived independently for the case of a rigid canyon in Chapter 2, given by the simpler equation (2.41).

The results in Figures 3.3-3.7 indicate consistently a dramatic effect of the impedance ratio on the response for the entire frequency range and for all \(L/H\) ratios. At first resonance, the effect of \(IR\) on \(|AF|\) is almost independent of \(L/H\). For dams with \(2 \leq L/H \leq 5\), \(\beta_d = 10\%\), and for the examined range of \(IR\), the amplification \(|AF|\) at first resonance varies from about 2.7 to 10, i.e. by a factor of 3.7. As \(L/H \to \infty\), \(|AF|\) varies from 3 to 6.2, i.e. by a factor of 2.8.

The effect of \(IR\) on \(|AF|\) at higher frequencies is even more dramatic, as illustrated in Figures 3.3-3.7 by the significant reduction in the amplification as \(IR\) decreases from \(\infty\) to 2.5. This reduction of \(|AF|\) is particularly sharp for narrow canyons, as shown by comparing the results for \(L/H = 2\) (Figure 3.3) with those for \(L/H \to \infty\) (Figure 3.7). Thus, for \(L/H = 2\), \(|AF|\) at the third natural frequency is reduced from 9, for \(IR = \infty\), to 1.2, for \(IR = 2.5\), i.e. by a factor of 7.5. The corresponding reduction of \(|AF|\) for \(L/H \to \infty\) is only by a factor of 1.5.

The effect of the canyon geometry on the response amplification seems to be almost as dramatic as that of the impedance ratio. In fact, these two factors should be examined together, as they both affect considerably the spatial variability of the ground motion and the amount of radiation damping. As shown in Figures 3.3-3.7, for narrow rigid canyons the amplification becomes maximum. In this case there is no spatial variability of the
ground motion along the dam base and no energy is radiated back to the canyon. Instead, the proximity of the rigid boundaries leads to a stiffening effect that manifests itself with the higher midcrust amplification values for the “3-D” dam, higher natural frequencies, and sharper attenuation of the displacements shapes with depth.

As the impedance ratio decreases, however, the spatial variability of the ground motion increases. For vertically incident SH waves, the ground motion spatial variability increases along the right and left abutments of the dam, while the motion along the midsection base remains more or less uniform. Thus, very long dams subjected to vertical SH waves experience synchronous identical base motion, while dams in narrow canyons experience almost synchronous motion at the horizontal base but asynchronous motion along the left and right abutments. Of course, as canyon narrowness increases, the effect of the spatial variability of the ground motion along the left and right abutments on the dam response becomes more important.

Moreover, as the canyon-rock S-wave velocity and, therefore, the impedance ratio decrease, more energy is carried away from the dam through higher-amplitude outgoing waves; i.e. the amount of radiation damping also increases. Very long dams subjected to vertical SH waves radiate energy back to the half-space only along the vertical direction, while dams in narrower canyons radiate energy back to the half-space along the entire dam-canyon interface, including the abutments. This suggests that, for low impedance ratios, which allow more radiation of energy, dams in narrower canyons will experience more “geometric damping”, because the energy is radiated in two dimensions, as opposed to the 1-D radiation for infinitely long dams.

The combined effect of the ground motion spatial variability and radiation damping, associated with the canyon geometry and impedance ratio, leads to the substantial reduction of the amplification shown in Figures 3.3-3.6. As discussed above, the smaller amplification reduction observed in Figure 3.7 for a dam under plane strain conditions, is
due to (a) the fact that the base motion, in this case, is always synchronous and (b) the radiation of energy is only in the vertical direction.

For high-frequency excitation \( a_o > 5 \) the combined effects of radiation damping and of the spatial variability of the ground motion are even more dramatic for dams in narrow canyons. This is explained by the fact that, high-frequency motion results to a higher degree of destructive interference associated with the short wavelength SH waves. Of course, the latter applies only on the "3-D" dams built in (narrow) canyons, as for very long dams and vertically incident waves the motion is synchronous.

Finally, for large wavelengths, the dam-canyon system appears as a small detail in the half-space and is practically ignored by the propagating waves. In that case, the dam-canyon system tends to vibrate like a half-space excited by SH waves, showing little variation of response along the crest of the dam.
Figure 3.3. Midcrest amplification for dams in semi-cylindrical canyons ($L/H = 2$) and impedance ratios $IR = 2.5, 5, 10, 15$ and $\infty$. 
Figure 3.4. Midcrest amplification for dams in semi-elliptical canyons with L/H = 2.5 and impedance ratios IR = 2.5, 5, 10, 15 and ∞.
Figure 3.5. Midcrest amplification for dams in semi-elliptical canyons with $L/H = 3$ and impedance ratios $IR = 2.5, 5, 10, 15$ and $\infty$. 

$\alpha = 0^\circ$  
$\beta_d = 10\%$  
$L/H = 3$
Figure 3.6. Midcrest amplification for dams in semi-elliptical canyons with $L/H = 5$ and impedance ratios $IR = 2.5, 5, 10, 15$ and $\infty$. 
Figure 3.7. Midcrest amplification for dams under plane strain conditions ($L/H = \infty$) with impedance ratios $IR = 2.5, 5, 10, 15$ and $\infty$. 

\[ \alpha = 0^\circ \]
\[ \beta_d = 10\% \]

(Plane Strain)

\[ L/H = \infty \]
Figure 3.8. Midcrest Amplification for dams in semi-elliptical canyons with $L/H = 2$ (Dakoulas 1993) and $L/H = 2.01$ from this study (IR = 10).
3.4.2 Effect of angle of incidence

The effect of the angle of incidence on the response of a dam-canyon system is studied through a parametric study using four different angles of incidence $\alpha = 0^\circ, 30^\circ, 60^\circ$ and $90^\circ$. (Although it is rather unrealistic to expect values of $\alpha$ as high as $90^\circ$, this value is used here as an extreme case). The parametric study is performed for two impedance ratios, $IR=5$ and $10$, four length to height ratios, $L/\ H=2$, $2.5$, $3$ and $5$, and a number of different excitation frequencies $a_o$.

Figure 3.9 plots the amplification function $|AF|$ and the phase angle $\phi$ at the crest of the dam and the surface of the half-space near the dam versus a dimensionless distance $2 \ x/L$, for a dam with $L/\ H=2$ and $IR=5$ at a frequency $a_o = 3$, corresponding to the first resonance. The response of the dam corresponds to $-1 < 2 \ x/L < 1$, while the response of the half-space surface to $2 \ x/L < -1$ and $2 \ x/L > 1$. For $L/\ H=2$, the response at midcrest is identical for all values of $\alpha$ and equal to $|AF| \approx 6.55$. Notice that the effect of the angle of incidence on the amplification function at first resonance peak is not significant. For waves traveling from the left to the right, as $\alpha$ increases the peak amplification moves slightly to the right of the midcrest.

Similarly, Figure 3.10 plots the amplification function $|AF|$ and the phase angle $\phi$ of the same dam for a frequency $a_o = 4.2$. Notice that the results in Figure 3.10 demonstrate a substantial effect of the angle of incidence $\alpha$ on the dam response, which increases significantly as $\alpha$ increases. As shown in the figure, the maximum amplification is $|AF| \approx 2.8$ for $\alpha = 0^\circ$ (at midcrest), $4.4$ for $\alpha = 30^\circ$ (at $2 \ x/L = 0.3$), $6.2$ for $\alpha = 60^\circ$ (at $2 \ x/L = 0.4$) and $6.8$ for $\alpha = 90^\circ$ (at $2 \ x/L = 0.43$). Higher values of maximum AF with increasing $\alpha$ are obtained not only in the part of the dam which is at the right of midcrest, but on both sides, although at the right part, the obtained $|AF|$ values are even higher. It is interesting to note that even a relatively small inclination angle of $\alpha = 30^\circ$,
induces an amplification about 57% higher compared to that of vertically propagating SH waves. This substantial difference between vertically and obliquely incident SH waves is also observed in the surface response of semi-cylindrical and semi-elliptical alluvial valleys subjected to incident SH waves at various angles \( \alpha \) (Trifunac 1971; Wong and Trifunac 1974), although perhaps to a slightly less extend due to the differences between the dynamic characteristics of the alluvial valley and the dam.

The above results demonstrate that the spatial variability of the ground motion, associated with obliquely incident SH waves, for certain excitation frequencies, leads to response values much higher than those obtained for vertically incident waves. Such higher amplification values are attributed to the excitation of antisymmetric vibrational modes, which are not present in the response to vertically incident waves. The importance of the presence of these antisymmetric terms in the response of the dam has been demonstrated in detail by Dakoulas (1993) for dams in semi-circular flexible canyons.

Figure 3.11 plots the amplification \( |A_F| \) and the phase angle \( \phi \) for a frequency \( a_o = 6 \), which corresponds to the second natural frequency of the dam (see also Figure 3.3). Although there are significant differences in the response for various incident angles \( \alpha \), these differences are much smaller than those shown in Figure 3.10. For \( \alpha = 60^\circ \), Figure 3.11 shows some peculiarity with respect to the phase angle \( \phi \), which at the right side of the dam decreases at a faster rate than for the other values of \( \alpha \) examined. Normally, the phase angle \( \phi \) along the right side of the halfspace for \( \alpha = 60^\circ \) would lie between \( \alpha = 30^\circ \) and \( 90^\circ \), in contrast with Figure 3.11, where \( \phi \) for \( \alpha = 60^\circ \) is translated by \( 2\pi \). (Of course, the response along the right side of the halfspace, given in terms of trigonometric functions of \( \phi \), is not affected by this translation).

Figures 3.12-3.14 plot the amplification function \( |A_F| \) and the phase angle \( \phi \) at the crest of the dam and the surface of the half-space, for a dam with \( L/H = 2.5 \) and \( IR = 5 \) at
frequencies \( a_o = 2.9 \) (natural frequency), 4.2 and 6, respectively. The conclusions from these results are very similar to those drawn above. At the first natural frequency (\( a_o = 2.9 \)) the effect of \( \alpha \) on the \(|AF|\) is small, while at the higher frequencies \( a_o = 4.2 \) and 6 the effect is significant. For high frequency motion, the response at midcrest is no longer identical for all values of \( \alpha \) (see Figure 3.14) and its distribution along the crest is more irregular due to the higher contribution of high order vibrational modes.

Figures 3.15-3.18 plot the amplification function \(|AF|\) and the phase angle \( \phi \) at the crest of the dam and the surface of the half-space, for a dam with \( L/H = 3 \) and \( IR = 5 \) at frequencies \( a_o = 2.7 \) (natural frequency), 4.2, 5 and 6, respectively. Notice that for waves traveling from the left to the right at \( a_o = 2.7 \) and 4.2, as \( \alpha \) increases the peak amplification moves slightly to the right of the midcrest (Figures 3.15-3.16). At \( a_o = 5 \) the response for vertically incident waves is maximum near the two symmetric mid-quarter points of the dam crest (Figure 3.17). As the angle \( \alpha \) increases, in contrast with the results in Figures 3.15-3.16, the response becomes maximum at the left side of the longitudinal section of the dam. This trend is reversed again for \( a_o = 6 \) (Figure 3.18).

Figures 3.19-3.21 plot the amplification function \(|AF|\) and the phase angle \( \phi \) at the crest of the dam and the surface of the half-space, for a dam with \( L/H = 5 \) and \( IR = 5 \) at frequencies \( a_o = 2.5 \) (natural frequency), 4.2 and 6, respectively. The trends observed and the conclusions drawn from these results are similar to those found above.

Finally, Figures 3.22-3.33 plot the amplification function \(|AF|\) and the phase angle \( \phi \) at the crest of the dam and the surface of the half-space, for \( IR = 10 \). For this higher impedance ratio, the spatial variability of the ground motion is smaller than that corresponding to \( IR = 5 \), examined above. The results from Figures 3.22-3.33 lead to qualitatively similar conclusions, except that the effects of the spatial variability of the ground motion are
smaller, and, therefore, the differences in the response corresponding to various angles of incidence are quite smaller.

The above results from all analyses performed demonstrate consistently that the spatial variability of the ground motion, associated with obliquely incident SH waves, for certain excitation frequencies, leads to response values much higher than those obtained for vertically incident waves. The variations of amplification suggested by the presented model are indeed observed in actual dams during earthquakes. One example is the response of the Long Valley Dam during the Mammoth Lakes, California, earthquake series of May 25 to 27, 1980 (Lai and Seed 1980). A plan view of the dam is shown in Figure 3.34. This dam has a length to height ratio $L/H \approx 3$. It is interesting to compare the recorded response of the dam in a clearly qualitative manner with the above findings. Thus, for the May 27 shaking, the maximum base acceleration of the dam was 0.18 g, while the near mid-crest acceleration was 0.44 g (Channel 6, see Figure 3.34). Notice that the corresponding accelerations at channels 4 and 14 were respectively 0.27 g and 0.48 g. A possible explanation for this significant difference between the recorded maximum acceleration at the two channels is that the wave excitation was propagating at an angle to the vertical.

Finally, Figure 3.35 demonstrates the contribution of each term $n = 0, 2, 4, \text{ and } 6$ in equation (3.46) for a dam with $L/H=3$ and $IR=10$. Notice in Figure 3.35a, which plots the amplitude of each term, that the most important contribution is due to the first term, while terms corresponding to $n>6$ can be safely neglected for the frequency range of interest.

3.4.3 Effect of Nonlinearity

The effects of the dam-canyon interaction depend significantly on the value of the impedance ratio IR. The latter is bound to change during an earthquake excitation,
depending on the intensity level of the ground motion and the resulting amplitude of average seismic shear strain. For large seismic strain amplitudes, the reduction of the shear wave velocity will tend to increase the value of IR. Consequently, during a strong shaking, although the hysteretic behavior of soil will induce substantial material damping, the radiation damping in the dam-canyon system will tend to decrease.
Figure 3.9. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2$, $IR = 5$, $a_o = 3$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.10. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2$, $IR = 5$, $a_o = 4.2$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.11. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2$, $IR = 5$, $\alpha_0 = 6$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.12. (a) Amplitude and (b) Phase angle of amplification $|AF|$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2.5$, $IR = 5$, $a_o = 2.9$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.13. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2.5$, $IR = 5$, $a_o = 4.2$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.14. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for \( L/H = 2.5 \), \( IR = 5 \), \( a_o = 6 \) and four angles of incidence \( \alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ \).
Figure 3.15. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 3$, $IR = 5$, $a_o = 2.7$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.16. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 3$, IR = 5, $a_o = 4.2$ and four angles of incidence $\alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ$. 
Figure 3.17. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 3$, $IR = 5$, $a_o = 5$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.18. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 3$, $IR = 5$, $a_o = 6$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.19. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 5$, $IR = 5$, $a_o = 2.5$ and four angles of incidence $\alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ$. 

- $L/H = 5$
- $IR = 5$
- $\beta_d = 10\%$
- $a_o = 2.5$
Figure 3.20. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for \(\frac{L}{H} = 5\), IR = 5, \(a_o = 4.2\) and four angles of incidence \(\alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ\).
Figure 3.21. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 5$, $IR = 5$, $a_o = 6$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.22. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2$, $IR = 10$, $a_o = 3$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.23. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2$, $IR = 10$, $a_o = 4.2$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 

\[ L/H = 2 \]
\[ IR = 10 \]
\[ \beta_d = 10\% \]
\[ a_o = 4.2 \]
Figure 3.24. (a) Amplitude and (b) Phase angle of amplification \( AF \) along the crest of the dam and along the surface of the half-space near the dam for \( L/H = 2 \), \( IR = 10 \), \( \alpha_0 = 6 \) and four angles of incidence \( \alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ \).
Figure 3.25. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2.5$, $IR = 10$, $a_o = 2.9$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 

L/H = 2.5
IR = 10
$\beta_d = 10\%$
a_o = 2.9

α

0°
30°
60°
90°
Figure 3.26. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2.5$, IR = 10, $a_o = 4.2$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.27. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 2.5$, $IR = 10$, $\alpha_o = 6$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 

Parameters:
- $L/H = 2.5$
- $IR = 10$
- $\beta_d = 10\%$
- $\alpha_o = 6$
Figure 3.28. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 3$, $IR = 10$, $a_o = 2.8$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.29. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 3$, $IR = 10$, $a_o = 4.2$ and four angles of incidence $\alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ$. 
Figure 3.30. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 3$, $IR = 10$, $a_o = 6$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Figure 3.31. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 5$, $IR = 10$, $a_o = 2.6$ and four angles of incidence $\alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ$. 
Figure 3.32. (a) Amplitude and (b) Phase angle of amplification AF along the crest of the dam and along the surface of the half-space near the dam for $L/H = 5$, $IR = 10$, $a_o = 4.2$ and four angles of incidence $\alpha = 0^\circ, 30^\circ, 60^\circ, 90^\circ$. 
Figure 3.33. (a) Amplitude and (b) Phase angle of amplification $AF$ along the crest of the dam and along the surface of the half-space near the dam for $L/H = 5$, $IR = 10$, $a_o = 6$ and four angles of incidence $\alpha = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$. 
Plan View of Long Valley Dam:

Location of Recorded Peak Accelerations.

Figure 3.34. Plan view of Long Valley Dam and location of recorded peak acceleration.
Figure 3.35. (a) Amplitude of the first four terms \( (n = 0, 2, 4, 6) \) of the midcrest amplification for a dam in semi-elliptical canyon with \( L/H = 3 \) and \( IR = 10 \) (continued).
Figure 3.35. (b) Real part and (c) Imaginary part of the first four terms ($n = 0, 2, 4, 6$) of the midcrest amplification for a dam in semi-elliptical canyon with $L/H = 3$ and $IR = 10$. 

\[ \alpha = 0^\circ \]
\[ \beta_d = 10\% \]
3.5 Summary and Conclusions

The effects of dam-canyon interaction on the response of earth and rockfill dams built in narrow flexible canyons are considered by using an analytical closed-form solution for a dam in a semi-elliptical canyon, subjected to obliquely-incident harmonic SH waves. The model accounts rigorously for the wave reflection, transmission and diffraction phenomena of the dam-canyon system. The conclusions from this study are summarized as follows:

1. The presented solution for the lateral response of dams in semi-elliptical flexible canyons, for the special case of semi-circular canyon, leads to results which are identical to those obtained independently for the response of dams in semi-circular flexible canyons (Dakoulas 1993). Moreover, as $IR \to \infty$ (rigid base), the results of equation (3.46) obtained for a flexible canyon are identical with those derived independently for the case of a rigid canyon, given by equation (2.41).

2. The results indicate consistently a dramatic effect of the impedance ratio on the response for the entire frequency range, and even more intensely for the high-frequency excitation. This is due to the fact that the impedance ratio affects both the amount of energy radiated back to the canyon and the spatial variability of the ground motion along the dam canyon interface. For a hysteretic damping ratio of 10% for the dam material, the presence of a flexible base rock may reduce the amplification by factor of 2 to 10 compared to the amplification in the case of a rigid base.

3. The effect of the canyon geometry on the response amplification seems to be almost as dramatic as that of the impedance ratio. For vertically incident SH waves, the ground motion spatial variability increases along the right and left abutments of the dam. Very long dams subjected to vertical SH waves
experience synchronous identical base motion, while dams in narrow canyons may experience almost synchronous motion at their horizontal base but asynchronous motion along the left and right abutments. Moreover, as the impedance ratio decreases, more energy is carried away from the dam through higher-amplitude outgoing waves. Very long dams subjected to vertical SH waves radiate energy back to the half-space only along the vertical direction, while dams in narrower canyons experience more "geometric damping", because the energy is radiated in two dimensions.

4. For high-frequency excitation the combined effects of radiation damping and of the spatial variability of the ground motion are even more dramatic for dams in narrow canyons. This is because high-frequency motion results to a higher degree of destructive interference associated with the short wavelength SH waves.

5. When obliquely incident waves impinge on the dam-canyon interface, they induce both symmetric and antisymmetric response components. Even at a small incidence angle of the SH wave, such as 30°, the antisymmetric components may lead to substantially higher response in the dam than the response caused by vertically propagating waves. The observed differences in the response are both in the amplitude and its distribution along the dam crest. For obliquely incident waves traveling from the left to the right, as the angle of incidence increases, a gradual shift of the location of the peak response is observed from the mid-crest to the sides of the dam, as well as higher irregularity of its amplitude along the crest of the dam.

6. For low-frequency SH excitation, the response of the dam shows little variation along the crest and approaches the response of the elastic half-space. In this case, the dimensions of the dam are small compared to the wavelength of the
incident motion and, therefore, the presence of the dam has only a small effect on the site response.

7. The response depends only on the first 4-6 terms of the solution.

8. The presented model allows substantial flexibility in modeling the geometry and the material properties of the dam-canyon system, and, therefore, can be used effectively to predict the response of several actual dams to transient earthquake excitations using frequency response analysis.
REFERENCES


Aki, K., (1988) "Local Site Effects on Strong Ground Motion", *Proceedings of a Conf. on Earthquake Engineering and Soil Dynamics II: Recent Advances in Ground Motion Evaluation.*, Editor T. L. Von Thun, Park City, Utah.


"CSMIP Strong Motion Records from the Santa Cruz Mountains (Loma Prieta), California earthquake of October 17, 1989" (1989), Cal. Dept. of Conserv., Div. of Mines and Geology, Office of Strong Motion Studies, Report OSMS 89-06.


APPENDIX A

In this appendix the derivation of the eigenvalues $\lambda_{mn}$, the associated coefficients $d_r^{mn}$, the angular function $S_{mn}(c, \eta)$ as well as the radial function $R_{mn}(c, \xi)$ is presented.

The recursion formula for the coefficients $d_r^{mn}$ is found by substituting into the differential equation (2.18) (or (3.23)), the infinite sum of the angular function of the first kind, $S_{mn}^{(1)}(c, \eta)$, given by

$$S_{mn}^{(1)}(c, \eta) = \sum_{r=0}^{\infty} d_r^{mn}(c) P_{m+r}^m(\eta)$$  \hspace{1cm} (A.1)

where the prime over the summation sign indicates that the summation is over only even values of $r$ when $n - m$ is even, and over only odd values of $r$ when $n - m$ is odd, and by making use of the associated Legendre differential equation

$$P_n^m(z) = (1 - z^2)^{\frac{m}{2}} \frac{d^m}{dz^m} P_n(z) \hspace{1cm} -1 \leq z \leq 1$$  \hspace{1cm} (A.2)

The derived recursion formula for the coefficients $d_r^{mn}$ is

$$\alpha_r \cdot d_{r+2}^{mn} + (\beta_r - \lambda_{mn}) d_r^{mn} + \gamma_r \cdot d_{r-2}^{mn} = 0 \hspace{1cm} r \geq 0$$  \hspace{1cm} (A.3)

(Flammer 1957) where $\alpha_r$, $\beta_r$ and $\gamma_r$ are respectively

$$\alpha_r = \frac{(2m + r + 2)(2m + r + 1)c^2}{(2m + 2r + 3)(2m + 2r + 5)}$$  \hspace{1cm} (A.4)

$$\beta_r = (m + r)(m + r + 1) + \frac{2(m + r)(m + r + 1) - 2m^2 - 1}{(2m + 2r - 1)(2m + 2r + 3)} c^2$$  \hspace{1cm} (A.5)

$$\gamma_r = \frac{r(r-1)c^2}{(2m + 2r - 3)(2m + 2r - 1)}$$  \hspace{1cm} (A.6)
To obtain a converging solution, \( d_r^{mn} / d_{r-2}^{mn} \) has to decrease as \(-c^2 / (4r^2)\) as \( r \) tends to infinity. This condition that \( d_r^{mn} / d_{r-2}^{mn} \) approaches zero as \( r \) tends to infinity leads to a transcendental equation for \( \lambda_{mn} \) and \( c^2 \). To form this equation, we first define the following quantities

\[
\gamma_r^m = (m + r) (m + r + 1) + \frac{1}{2} c^2 \left[ 1 - \frac{4m^2 - 1}{(2m + 2r - 1)(2m + 2r + 3)} \right] \quad r \geq 0 \quad (A.7)
\]

\[
\beta_r^m = \frac{r (r - 1)(2m + r)(2m + r - 1) c^4}{(2m + 2r - 1)^2 (2m + 2r - 3)(2m + 2r + 1)} \quad r \geq 2 \quad (A.8)
\]

\[
N_r^m = \frac{(2m + r)(2m + r - 1)c^2}{(2m + 2r - 1)(2m + 2r + 1)} \frac{d_r^{mn}}{d_{r-2}^{mn}} \quad r \geq 2 \quad (A.9)
\]

By using the recursion formula equation (A.3), we obtain

\[
N_r^m = \frac{\beta_r^m}{\gamma_r^m - \lambda_{mn} - N_{r+2}^m} \quad r \geq 2 \quad (A.10)
\]

\[
N_{r+2}^m = \gamma_r^m - \lambda_{mn} - \frac{\beta_r^m}{N_r^m} \quad r \geq 2 \quad (A.11)
\]

\[
N_2^m = \gamma_0^m - \lambda_{mn}, \quad N_3^m = \gamma_1^m - \lambda_{mn} \quad (A.12)
\]

From the condition that \( N_r^m \) goes to zero as \( r \) approaches infinity, we obtain the convergent infinite continued fraction by iterating equation (A.10)

\[
N_{r+2}^m = \frac{\beta_{r+2}^m}{\gamma_{r+2}^m - \lambda_{mn} - \frac{\beta_{r+4}^m}{\gamma_{r+4}^m - \lambda_{mn} - \frac{\beta_{r+6}^m}{\gamma_{r+6}^m - \lambda_{mn} - \cdots}} \quad (A.13)
\]

Similarly, from the condition that \( d_r^{mn} = 0 \) for \( r < 0 \), we also get the finite continued fraction by iterating equation (A.11)
\[ N_r^m = \gamma^m_r - \lambda^m_{mn} - \frac{\beta^m_r}{\gamma^m_{r-2} - \lambda^m_{mn} - \frac{\beta^m_{r-2}}{\gamma^m_{r-4} - \lambda^m_{mn} - \frac{\beta^m_{r-4}}{\gamma^m_{r-6} - \lambda^m_{mn} - \ldots \ldots}}} \]  

(A.14)

where the last partial denominator is \( \gamma^m_0 - \lambda^m_{mn} \) for even \( r \) and \( \gamma^m_1 - \lambda^m_{mn} \) for odd \( r \).

By equating equations (A.13) and (A.14) for a certain \( c \), the eigenvalues \( \lambda^m_{mn} \) can be recovered. After computing \( \lambda^m_{mn} \), \( N_r^m \) are obtained by equations (A.7), (A.8) and (A.13) (or (A.14)), and the ratios \( \frac{d^m_{mn}}{d^m_{r-2}} \) are determined from (A.9). Any ratios \( \frac{d^m_{0}}{d^m_{2r}} \) and \( \frac{d^m_{1}}{d^m_{2r+1}} \) may be calculated then by

\[
\frac{d^m_{2r}}{d^m_{2r}} = \left( \frac{d^m_{2r}}{d^m_{2r}} \right) \left( \frac{d^m_{2r}}{d^m_{4r}} \right) \ldots \left( \frac{d^m_{2r}}{d^m_{2r}} \right) \]  

(A.15)

\[
\frac{d^m_{2r+1}}{d^m_{2r+1}} = \left( \frac{d^m_{2r+1}}{d^m_{2r+1}} \right) \left( \frac{d^m_{2r+1}}{d^m_{2r+1}} \right) \ldots \left( \frac{d^m_{2r+1}}{d^m_{2r+1}} \right) \]  

(A.16)

The coefficients \( d^m_{r} \) are determined in terms of an arbitrary coefficient \( d^m_{0} \) for even \( r \) and \( d^m_{1} \) for odd \( r \). The coefficients \( d^m_{0} \) and \( d^m_{1} \) can be defined by using a suitable normalization. In the following, we adopt the Flammer normalization, which requires

\[
S^{(1)}_{mn}(c,0) = P^m_n(0) = \frac{(-1)^{n-m}}{2^n \left( \frac{n-m}{2} \right)! \left( \frac{n+m}{2} \right)!} \quad \text{for } n-m \text{ even} \]  

(A.17)

\[
S^{(1)}_{mn}(c,0) = P^m_n'(0) = \frac{(-1)^{n-m-1}}{2^n \left( \frac{n-m-1}{2} \right)! \left( \frac{n+m+1}{2} \right)!} \quad \text{for } n-m \text{ odd} \]  

(A.18)

where the prime in the above equations indicates derivative. The above equations lead to the following normalizing relations.
\[
\sum_{r=0,2,\ldots}^{\infty} \frac{(-1)^{\frac{r}{2}}(r+2m)!}{2^r(r+2m)!} d_r^{mn} = \frac{(-1)^{\frac{n-m}{2}}(n+m)!}{2^{n-m}(\frac{n-m}{2})!(\frac{n-m}{2})!} \quad \text{for } n-m \text{ even} \quad (A.19)
\]
\[
\sum_{r=1,3,\ldots}^{\infty} \frac{(-1)^{\frac{r}{2}}(r+2m+1)!}{2^r(r+2m+1)!} d_r^{mn} = \frac{(-1)^{\frac{n-m-1}{2}}(n+m+1)!}{2^{n-m}(\frac{n-m-1}{2})!(\frac{n-m-1}{2})!} \quad \text{for } n-m \text{ odd} \quad (A.20)
\]

which are used to obtain the coefficient \(d_0^{mn}\) for even \(r\) and \(d_1^{mn}\) for odd \(r\). Utilizing equation (A.14) and (A.15), the expansion coefficients \(d_r^{mn}\) are completely defined.

Consequently, we can obtain the angle function \(S^{(1)}_{mn}(\xi, \eta)\) by (A.1), whereas the radial function of the first kind, \(R^{(1)}_{mn}(c, \xi)\) is given by

\[
R^{(1)}_{mn}(c, \xi) = \sum_{r=0,1,\ldots}^{\infty} \frac{1}{d_r^{mn}(c)} \frac{(\xi^2-1)^{\frac{1}{2}}}{(2m+r)!} \sum_{r=0,1,\ldots}^{\infty} i^{r+m-n} \frac{d_r^{mn}(c)}{r!} \frac{(2m+r)!}{r!} j_{m+r}(c, \xi) \quad (A.21)
\]

and the second kind, \(R^{(2)}_{mn}(c, \xi)\) is given by

\[
R^{(2)}_{mn}(c, \xi) = \sum_{r=0,1,\ldots}^{\infty} \frac{1}{d_r^{mn}(c)} \frac{(\xi^2-1)^{\frac{1}{2}}}{(2m+r)!} \sum_{r=0,1,\ldots}^{\infty} i^{r+m-n} \frac{d_r^{mn}(c)}{r!} \frac{(2m+r)!}{r!} y_{m+r}(c, \xi) \quad (A.22)
\]

where \(j_r\) and \(y_r\) are the spherical-Bessel function of the first kind and the second kind respectively, given by

\[
j_r(z) = \sqrt{\frac{\pi}{2z}} J_{r+\frac{1}{2}}(z) \quad (A.23)
\]
\[ y_r(z) = \sqrt{\frac{\pi}{2z}} Y_{r+\frac{1}{2}}(z) \]  

(A.24)

in which \( J_{r+\frac{1}{2}}(z) \) and \( Y_{r+\frac{1}{2}}(z) \) are the Bessel function of the first kind and the second kind of order \( r+1/2 \).

The radial function of the third kind, \( R_{mn}^{(3)}(c, \xi) \), and the fourth kind, \( R_{mn}^{(4)}(c, \xi) \), respectively are

\[ R_{mn}^{(3)}(c, \xi) = R_{mn}^{(1)}(c, \xi) + i R_{mn}^{(2)}(c, \xi) \]  

(A.25)

\[ R_{mn}^{(4)}(c, \xi) = R_{mn}^{(1)}(c, \xi) - i R_{mn}^{(2)}(c, \xi) \]  

(A.26)

An alternative expression for the radial function of the first kind \( R_{mn}^{(1)}(c, \xi) \) is given by

\[ R_{mn}^{(1)}(c, \xi) = \frac{1}{S_{mn}^{(1)}(c, 0)} \sum_{r=0,2,\ldots}^{\infty} i^{r+m-n} d_{r}^{mn}(c) P_{m+r}^{m}(0) j_{m+r}[c (\xi^2 - 1)^{\frac{1}{2}}] \]  

(A.27)

for \( n - m \) even, and

\[ R_{mn}^{(1)}(c, \xi) = \frac{\xi}{\sqrt{\xi^2 - 1}} \sum_{r=1,3,\ldots}^{\infty} i^{r+m-n} d_{r}^{mn}(c) P_{m+r}^{m}(0) j_{m+r}[c (\xi^2 - 1)^{\frac{1}{2}}] \]  

(A.28)

for \( n - m \) odd, where \( S_{mn}^{(1)}(c, 0) \), \( P_{m+r}^{m}(0) \), \( S_{mn}^{(1)}(c, 0) \) and \( P_{m+r}^{m}(0) \) may be obtained by equations (A.17) and (A.18) respectively. Equations (A.27) and (A.28) lead to more accurate results than those computed by equations (A.21) and (A.22), when \( c \xi \) is large.
APPENDIX B

This part proves in detail the boundary condition given in equation (2.15) (or equations
(3.17-18)) are satisfied. In polar coordinates, the boundary condition of zero shear stress
$\tau_{yz}$ at the dam crest is given by

$$\tau_{yz}(r, \frac{\pi}{2}) = \frac{G_d}{r} \frac{\partial U_d}{\partial \theta} = 0 \quad \text{(B.1)}$$

$$\tau_{yz}(r, -\frac{\pi}{2}) = \frac{G_d}{r} \frac{\partial U_d}{\partial \theta} = 0 \quad \text{(B.2)}$$

The relationship between the Cartesian, polar and elliptical coordinate systems is

$$x = r \sin \theta = d \eta \xi \quad \text{(B.3)}$$

$$z = r \cos \theta = d \sqrt{(1 - \eta^2)(\xi^2 - 1)} \quad \text{(B.4)}$$

By letting $\xi = \cosh \xi^* \quad \text{and} \quad \eta = \cos \eta^*$, equations (B.3) and (B.4) can be rewritten as

$$x = r \sin \theta = d \cosh \xi^* \cos \eta^* \quad \text{(B.5)}$$

$$z = r \cos \theta = d \sinh \xi^* \sin \eta^* \quad \text{(B.6)}$$

The above relationships to transform the boundary conditions (B.1) and (B.2) into
elliptical coordinates in terms of $\xi^*$ and $\eta^*$.

First, we can express $r^*$ as

$$r^* = d \sqrt{\cosh^2 \xi^* - \sin^2 \eta^*} \quad \text{(B.7)}$$

and express $\frac{\partial U_d}{\partial \theta^*}$ as

$$\frac{\partial U_d}{\partial \theta^*} = \frac{\partial U_d}{\partial \xi^*} \frac{\partial \xi^*}{\partial \theta^*} + \frac{\partial U_d}{\partial \eta^*} \frac{\partial \eta^*}{\partial \theta^*} \quad \text{(B.8)}$$
In view of the above equation, we need to obtain \( \frac{\partial \xi^*}{\partial \theta^*} \) and \( \frac{\partial \eta^*}{\partial \theta^*} \) in terms of \( \xi^* \) and \( \eta^* \).

Differentiating equations (B.5) and (B.6) respect to \( \theta^* \), they become

\[
\begin{align*}
    r^* \cos \theta^* &= d \sinh \xi^* \cos \eta^* \frac{\partial \xi^*}{\partial \theta^*} - d \cosh \xi^* \sin \eta^* \frac{\partial \eta^*}{\partial \theta^*} \\
    -r^* \sin \theta^* &= d \cosh \xi^* \sin \eta^* \frac{\partial \xi^*}{\partial \theta^*} + d \sinh \xi^* \cos \eta^* \frac{\partial \eta^*}{\partial \theta^*}
\end{align*}
\]  

Equations (B.9)

Substituting from equations (B.5) and (B.6) the expressions

\[
\begin{align*}
    r^* \cos \theta^* &= d \sinh \xi^* \sin \eta^* \\
    -r^* \sin \theta^* &= -d \cosh \xi^* \cos \eta^*
\end{align*}
\]  

Equations (B.11) and (B.12)

into equations (B.9) and (B.10), the latter become

\[
\begin{align*}
    \frac{\cos \eta^* \partial \xi^*}{\sin \eta^* \partial \theta^*} - \frac{\cosh \xi^* \partial \eta^*}{\sinh \xi^* \partial \theta^*} &= 1 \\
    \frac{\sin \eta^* \partial \xi^*}{\cos \eta^* \partial \theta^*} + \frac{\sinh \xi^* \partial \eta^*}{\cosh \xi^* \partial \theta^*} &= -1
\end{align*}
\]  

Equations (B.13) and (B.14)

By solving the above equations, \( \frac{\partial \xi^*}{\partial \theta^*} \) and \( \frac{\partial \eta^*}{\partial \theta^*} \) are recovered

\[
\begin{align*}
    \frac{\partial \xi^*}{\partial \theta^*} &= -\frac{\sin \eta^* \cos \eta^*}{\cosh^2 \xi^* - \cos^2 \eta^*} \\
    \frac{\partial \eta^*}{\partial \theta^*} &= -\frac{\sinh \xi^* \cosh \xi^*}{\cosh^2 \xi^* - \cos^2 \eta^*}
\end{align*}
\]  

Equations (B.15) and (B.16)

Finally, combining equations (B.1), (B.2), (B.7), (B.8), (B.15) and (B.16) leads to

\[
\frac{G_d}{r^*} \frac{\partial U_d}{\partial \theta^*} = A (\sin \eta^* \cos \eta^* \frac{\partial U_d}{\partial \xi^*} + \sinh \xi^* \cosh \xi^* \frac{\partial U_d}{\partial \eta^*})
\]  

Equation (B.17)
where

\[
A = \frac{-G_d}{d \sqrt{\cosh^2 \xi^* - \sin^2 \eta^* (\cosh^2 \xi^* - \cos^2 \eta^*)}} \quad (B.18)
\]

Now we check the new boundary condition (B.17) to see if it is satisfied. At the dam crest:

(a) Between the two foci, where \( \xi^* = 0 \).

\[
\frac{G_d^*}{r} \frac{\partial U_d}{\partial \theta^*} = \frac{-G_d^*}{d \sin \eta} \frac{\partial U_d}{\partial \xi^*} \quad (B.19)
\]

By letting \( \xi = \cosh \xi^* \), equation (B.19) can be expressed as

\[
\frac{G_d^*}{r} \frac{\partial U_d}{\partial \theta^*} = \frac{-G_d^*}{d \sin \eta} \frac{\partial U_d}{\partial \xi} \frac{\partial \xi}{\partial \xi^*} = \frac{-G_d^*}{d \sin \eta} \frac{\partial U_d}{\partial \xi} \sinh \xi^* \quad (B.20)
\]

\[
= \frac{-G_d^*}{d \sin \eta} \frac{\partial U_d}{\partial \xi} \sinh 0 = 0
\]

(b) Outside the two foci, where \( \eta^* = 0 \) or \( \pi \).

\[
\frac{G_d^*}{r} \frac{\partial U_d}{\partial \theta^*} = \frac{-G_d^*}{d \sinh \xi^*} \frac{\partial U_d}{\partial \eta^*} \quad (B.21)
\]

Similarly, by letting \( \eta = \cos \eta^* \), equation (B.21) can be expressed as

\[
\frac{G_d^*}{r} \frac{\partial U_d}{\partial \theta^*} = \frac{-G_d^*}{d \sinh \xi^*} \frac{\partial U_d}{\partial \eta} \frac{\partial \eta}{\partial \eta^*} = \frac{G_d^*}{d \sinh \xi^*} \frac{\partial U_d}{\partial \eta} \sin \eta^* = 0 \quad (B.22)
\]

Therefore the boundary condition (2.15) are satisfied.
APPENDIX C

The relationships between the Cartesian, polar and elliptical coordinate systems are expressed in detail below (also refer to Figure 3.2). The Cartesian coordinate system is denoted by \((x,z)\), the polar coordinate by \((r^*, \theta^*)\), the elliptical coordinate by \((\xi, \eta)\) and \(d\) is half of the interfocal distance in elliptical coordinate system.

(a) From Cartesian coordinates to polar and elliptical coordinates

\[ x = r^* \sin \theta^* = d \eta \xi \]  \hspace{1cm} (C.1)
\[ z = r^* \cos \theta^* = d \sqrt{1 - \eta^2} \left( \frac{\xi^2}{\xi^2 - 1} \right) \]  \hspace{1cm} (C.2)

(b) From polar coordinates to Cartesian and elliptical coordinates

\[ r^* = \sqrt{x^2 + z^2} = d \sqrt{\xi^2 + \eta^2 - 1} \]  \hspace{1cm} (C.3)
\[ \theta^* = \tan^{-1} \frac{x}{z} = \tan^{-1} \frac{\eta \xi}{\sqrt{1 - \eta^2} (\xi^2 - 1)} \]  \hspace{1cm} (C.4)

(c) From elliptical coordinates to polar and Cartesian coordinates

\[ \xi = \frac{\sqrt{\frac{r^*}{d^2} + \frac{2r^*}{d} \sin \theta^* + 1} + \sqrt{\frac{r^*}{d^2} - \frac{2r^*}{d} \sin \theta^* + 1}}{2} \]  \hspace{1cm} (C.5)
\[ = \frac{\sqrt{(x+d)^2 + z^2} + \sqrt{(x-d)^2 + z^2}}{2d} \]

\[ \eta = \frac{\sqrt{\frac{r^*}{d^2} + \frac{2r^*}{d} \sin \theta^* + 1} - \sqrt{\frac{r^*}{d^2} - \frac{2r^*}{d} \sin \theta^* + 1}}{2} \]  \hspace{1cm} (C.6)
\[ = \frac{\sqrt{(x+d)^2 + z^2} - \sqrt{(x-d)^2 + z^2}}{2d} \]
APPENDIX D

The expression for the amplitude of the incident and the reflected waves, $U_i + U_r$, in a half-space without the canyon is derived below in terms of elliptical coordinates. As shown in equation (3.32), in polar coordinates

\[ U_i = \exp\left[ i k_c r^* \cos(\theta^* + \alpha) \right] \]  
\[ U_r = \exp\left[ -i k_c r^* \cos(\theta^* - \alpha) \right] \]  

Referring to Figure 3.2, we may express $\theta^* + \alpha$ and $\theta^* - \alpha$ as

\[ \theta^* + \alpha = \theta_o - \theta \quad \text{for the incident waves} \]  
\[ \theta^* - \alpha = \pi - (\theta_o + \theta) \quad \text{for the reflected waves} \]  

The expression for the incident waves has been originally derived for a prolate spheroidal coordinate system, which is formed by rotating the two-dimensional elliptical coordinate system about the $x$ axis forming an angle $\varphi$ with the $x-z$ plane.

Introducing the identity (Flammer 1957)

\[ \exp\left[ i k_c r^* \cos\Theta \right] = \]  
\[ 2 \sum_{m=0}^\infty \sum_{n=n,m, m+1,...} \frac{(2 - \delta_{om})}{N_{mn}} S^{(1)}_{mn}(c_c, \cos \theta_o) S^{(1)}_{mn}(c_c, \eta) R^{(1)}_{mn}(c_c, \xi) \cos[m(\varphi - \varphi_o)] \]  

where

\[ \cos\Theta = \cos \theta \cos \theta_o + \sin \theta \sin \theta_o \cos (\varphi - \varphi_o) \]  

D.5

D.6
in which $\theta_o$ and $\phi_o$ are the spherical coordinates of the positive direction of propagation of the plane waves and $\Theta$ is the angle between the position vector and the propagation vector.

By selecting special values of $\phi$ and $\phi_o$, equation (D.5) can be used to derive the expression for the incident and reflected waves in an elliptical coordinate system. By letting $\phi - \phi_o = 0$, we can obtain the following equation for the incident waves in elliptical coordinates

$$\exp \left[ i k_c r^* \cos(\theta_o - \theta) \right] = 2 \sum_{m=0}^{\infty} \sum_{n=0, n \neq m+1}^{\infty} i^n \frac{2-\delta_{om}}{N_{mn}} S^{(1)}_{mn}(c_c, \cos \theta_o) S^{(1)}_{nn}(c_c, \eta) R^{(1)}_{nn}(c_c, \xi) \tag{D.7}$$

Similarly, by letting $\phi - \phi_o = \pi$, we obtain the expression for the reflected waves in elliptical coordinates

$$\exp \left[ i k_c r^* \cos(\theta_o + \theta) \right] = \sum_{m=0}^{\infty} \sum_{n=0, n \neq m+1}^{\infty} i^n (-1)^n \frac{2-\delta_{om}}{N_{mn}} S^{(1)}_{mn}(c_c, \cos \theta_o) S^{(1)}_{nn}(c_c, \eta) R^{(1)}_{nn}(c_c, \xi) \tag{D.8}$$

By making use of equation (D.7) and (D.8), $U_i$ is written as

$$U_i = \exp \left[ ik_c r^* \cos(\theta^* + \alpha) \right] = \exp \left[ ik_c r^* \cos(\theta_o - \theta) \right] \tag{D.9}$$

$$= 2 \sum_{m=0}^{\infty} \sum_{n=0, n \neq m+1}^{\infty} i^n \frac{2-\delta_{om}}{N_{mn}} S^{(1)}_{mn}(c_c, \cos \theta_o) S^{(1)}_{nn}(c_c, \eta) R^{(1)}_{nn}(c_c, \xi)$$

and $U_r$ is written as
\[ U_r = \exp[-i k_c r^* \cos(\theta^* - \alpha)] \]
\[ = \exp[-i k_c r^* \cos(x - (\theta_o + \theta))] \]
\[ = \exp[i k_c r^* \cos(\theta_o + \theta)] \]
\[ = 2 \sum_{m=0}^{\infty} \sum_{n=m,m+1,\ldots} i^n (-1)^m \frac{(2 - \delta_{om})}{N_{mn}} S_{mn}(c_c, \cos \theta_o) S_{mn}(c_c, \eta) R_{mn}(c_c, \xi) \] (D.10)
APPENDIX E

This part describes in detail the derivation of $A_n$, $B_n$ and $D_{mn}$ given in equations (3.28) and (3.41) based on equations (3.28), (3.39), (3.41) and the boundary conditions (3.17-18). Recall that the motion within the dam can be expressed as

$$U_d = \sum_{n=0}^{\infty} A_n R_{on}^{(1)}(c_d, \xi) S_{on}^{(1)}(c_d, \eta)$$  \hspace{1cm} (E.1)

and the motion of the incident and the reflected waves

$$U_i + U_r = \sum_{m=0}^{\infty} \sum_{n=m, m+1, \ldots}^{\infty} C_{mn} S_{mn}^{(1)}(c, \eta) R_{mn}^{(1)}(c, \xi)$$  \hspace{1cm} (E.2)

where

$$C_{mn} = \frac{4}{N_{mn}} i^n S_{mn}^{(1)}(c, \cos \theta_o) \quad \text{for} \quad m = 0, 2, 4, \ldots$$  \hspace{1cm} (E.3)

$$C_{mn} = 0 \quad \text{for} \quad m = 1, 3, 5, \ldots$$

Moreover, the motion of the reflected and diffracted waves caused by interference of the semi-elliptical canyon and the dam is

$$U_{cr} = \sum_{n=0}^{\infty} B_n R_{on}^{(4)}(c, \xi) S_{on}^{(1)}(c, \eta) + \sum_{m=2, 4, \ldots}^{\infty} \sum_{n=m, m+1, \ldots}^{\infty} D_{mn} R_{mn}^{(4)}(c, \xi) S_{mn}^{(1)}(c, \eta)$$  \hspace{1cm} (E.4)

The above equations must satisfy the boundary conditions of continuity of displacements and shear stresses at the dam base

$$U_d(\xi_b, \eta) = U_c(\xi_b, \eta) \quad \text{for} \quad -1 \leq \eta \leq 1$$  \hspace{1cm} (E.5)

and
\[ G_d \frac{\partial U_d}{\partial \xi_b}(\xi_b, \eta) = G_e \frac{\partial U_e}{\partial \xi_b}(\xi_b, \eta) \quad \text{for} \quad -1 \leq \eta \leq 1 \] (E.6)

where

\[ U_c = U_i + U_r + U_{cr} \] (E.7)

Equations (E.1), (E.2) and (E.4) must satisfy the above equations (E.5) and (E.6) along the dam-canyon interface, where \( \xi_b = \frac{L}{\sqrt{L^2 - 4H^2}} \), \(-1 \leq \eta \leq 1\), the order \( m = 0 \), and all the terms of \( m \neq 0 \) must vanish. Therefore at the dam base (E.2) and (E.4) may be rewritten as

\[ U_i + U_r = \sum_{n=0}^{\infty} C_{on} R^{(1)}_{on}(c_c, \xi_b) S^{(1)}_{on}(c_c, \eta) \] (E.8)

\[ U_{cr} = \sum_{n=0}^{\infty} B_n R^{(4)}_{on}(c_c, \xi_b) S^{(1)}_{on}(c_c, \eta) \] (E.9)

\( A_n \) and \( B_n \) must be independent of \( \eta \) along the boundary. After reviewing equations (E.1), (E.8) and (E.9), we may express \( S^{(1)}_{on}(c_c, \eta) \) as

\[ S^{(1)}_{on}(c_c, \eta) = \sum_{r=0,1,...}^{\infty} d_r^{on}(c_c) P_r(\eta) \] (E.10)

and substitute it into equation (E.1) (with \( \xi = \xi_b \)), (E.8) and (E.9), we obtain

\[ U_d = \sum_{r=0}^{\infty} P_r(\eta) \sum_{n=0}^{\infty} A_n R^{(1)}_{on}(c_d, \xi_b) d_r^{on}(c_d) \] (E.11)

\[ U_i + U_r = \sum_{r=0}^{\infty} P_r(\eta) \sum_{n=0}^{\infty} C_{on} R^{(1)}_{on}(c_c, \xi_b) d_r^{on}(c_c) \] (E.12)

\[ U_{cr} = \sum_{r=0}^{\infty} P_r(\eta) \sum_{n=0}^{\infty} B_n R^{(4)}_{on}(c_c, \xi_b) d_r^{on}(c_c) \] (E.13)
Now we apply boundary conditions (E.5) and (E.6) for every term \( r \)

\[
(1) \sum_{n=0}^{\infty} A_n R_n^{(1)}(c_d, \xi_b) d_r^{\alpha n}(c_d) = \sum_{n=0}^{\infty} (C_n R_n^{(1)}(c_c, \xi_b) + B_n R_n^{(4)}(c_c, \xi_b)) d_r^{\alpha n}(c_c) \quad (E.14)
\]

\[
(2) \sum_{n=0}^{\infty} A_n R_n^{(1)}(c_d, \xi_b) d_r^{\alpha n}(c_d) = \mu \sum_{n=0}^{\infty} (C_n R_n^{(1)}(c_c, \xi_b) + B_n R_n^{(4)}(c_c, \xi_b)) d_r^{\alpha n}(c_c) \quad (E.15)
\]

where \( \mu = \frac{G_c}{G_d} \). \( P_r(\eta) \) is cancelled in both sides of the equality signs, and thus the condition that \( A_n \) and \( B_n \) must be independent of \( \eta \) along the dam-canyon interface is satisfied. Equation (E.14) may be rewritten as

\[
r = 0
\]

\[
A_0 R_0^{(1)}(c_d, \xi_b) d_0^{\alpha 0}(c_d) + A_2 R_2^{(1)}(c_d, \xi_b) d_2^{\alpha 2}(c_d) + A_4 R_4^{(1)}(c_d, \xi_b) d_4^{\alpha 4}(c_d) + \cdots
-B_0 R_0^{(4)}(c_c, \xi_b) d_0^{\alpha 0}(c_c) - B_2 R_2^{(4)}(c_c, \xi_b) d_2^{\alpha 2}(c_c) - B_4 R_4^{(4)}(c_c, \xi_b) d_4^{\alpha 4}(c_c) - \cdots
= \sum_{n=0,2,4,\ldots} C_n R_n^{(1)}(c_c, \xi_b) d_0^{\alpha n}(c_c)
\]

\[
r = 2
\]

\[
A_0 R_0^{(1)}(c_d, \xi_b) d_2^{\alpha 0}(c_d) + A_2 R_2^{(1)}(c_d, \xi_b) d_2^{\alpha 2}(c_d) + A_4 R_4^{(1)}(c_d, \xi_b) d_4^{\alpha 4}(c_d) + \cdots
-B_0 R_0^{(4)}(c_c, \xi_b) d_2^{\alpha 0}(c_c) - B_2 R_2^{(4)}(c_c, \xi_b) d_2^{\alpha 2}(c_c) - B_4 R_4^{(4)}(c_c, \xi_b) d_2^{\alpha 4}(c_c) - \cdots
= \sum_{n=0,2,4,\ldots} C_n R_n^{(1)}(c_c, \xi_b) d_2^{\alpha n}(c_c)
\]

\[
r = 4
\]

\[
A_0 R_0^{(1)}(c_d, \xi_b) d_4^{\alpha 0}(c_d) + A_2 R_2^{(1)}(c_d, \xi_b) d_4^{\alpha 2}(c_d) + A_4 R_4^{(1)}(c_d, \xi_b) d_4^{\alpha 4}(c_d) + \cdots
-B_0 R_0^{(4)}(c_c, \xi_b) d_4^{\alpha 0}(c_c) - B_2 R_2^{(4)}(c_c, \xi_b) d_4^{\alpha 2}(c_c) - B_4 R_4^{(4)}(c_c, \xi_b) d_4^{\alpha 4}(c_c) - \cdots
= \sum_{n=0,2,4,\ldots} C_n R_n^{(1)}(c_c, \xi_b) d_4^{\alpha n}(c_c)
\]
\[ r = r \]

\[ A_0 R_{00}^{(1)}(c_d, \xi_b) d_r^{00}(c_d) + A_2 R_{02}^{(1)}(c_d, \xi_b) d_r^{02}(c_d) + A_4 R_{04}^{(1)}(c_d, \xi_b) d_r^{04}(c_d) + \cdots - B_0 R_{00}^{(3)}(c_c, \xi_b) d_r^{00}(c_c) - B_2 R_{02}^{(3)}(c_c, \xi_b) d_r^{02}(c_c) - B_4 R_{04}^{(3)}(c_c, \xi_b) d_r^{04}(c_c) - \cdots \]

\[ = \sum_{n=0,2,\ldots}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi_b) d_r^{on}(c_c) \]

for \( n \) and \( r \) even, and

\[ r = 1 \]

\[ A_1 R_{01}^{(1)}(c_d, \xi_b) d_1^{01}(c_d) + A_3 R_{03}^{(1)}(c_d, \xi_b) d_1^{03}(c_d) + A_5 R_{05}^{(1)}(c_d, \xi_b) d_1^{05}(c_d) + \cdots - B_1 R_{01}^{(3)}(c_c, \xi_b) d_1^{01}(c_c) - B_3 R_{03}^{(3)}(c_c, \xi_b) d_1^{03}(c_c) - B_5 R_{05}^{(3)}(c_c, \xi_b) d_1^{05}(c_c) - \cdots \]

\[ = \sum_{n=1,3,\ldots}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi_b) d_1^{on}(c_c) \]

\[ r = 3 \]

\[ A_1 R_{01}^{(3)}(c_d, \xi_b) d_3^{01}(c_d) + A_3 R_{03}^{(3)}(c_d, \xi_b) d_3^{03}(c_d) + A_5 R_{05}^{(3)}(c_d, \xi_b) d_3^{05}(c_d) + \cdots - B_1 R_{01}^{(5)}(c_c, \xi_b) d_3^{01}(c_c) - B_3 R_{03}^{(5)}(c_c, \xi_b) d_3^{03}(c_c) - B_5 R_{05}^{(5)}(c_c, \xi_b) d_3^{05}(c_c) - \cdots \]

\[ = \sum_{n=1,3,\ldots}^{\infty} C_{on} R_{on}^{(3)}(c_c, \xi_b) d_3^{on}(c_c) \]

\[ r = 5 \]

\[ A_1 R_{01}^{(5)}(c_d, \xi_b) d_5^{01}(c_d) + A_3 R_{03}^{(5)}(c_d, \xi_b) d_5^{03}(c_d) + A_5 R_{05}^{(5)}(c_d, \xi_b) d_5^{05}(c_d) + \cdots - B_1 R_{01}^{(7)}(c_c, \xi_b) d_5^{01}(c_c) - B_3 R_{03}^{(7)}(c_c, \xi_b) d_5^{03}(c_c) - B_5 R_{05}^{(7)}(c_c, \xi_b) d_5^{05}(c_c) - \cdots \]

\[ = \sum_{n=1,3,\ldots}^{\infty} C_{on} R_{on}^{(5)}(c_c, \xi_b) d_5^{on}(c_c) \]
\[ r = r \]

\[ A_1 R_0^{(1)}(c_d, \xi_b) d_r^{01}(c_d) + A_2 R_0^{(2)}(c_d, \xi_b) d_r^{02}(c_d) + A_5 R_0^{(5)}(c_d, \xi_b) d_r^{05}(c_d) + \cdots \]
\[ -B_1 R_0^{(4)}(c_c, \xi_b) d_r^{01}(c_c) - B_2 R_0^{(2)}(c_c, \xi_b) d_r^{02}(c_c) - B_5 R_0^{(4)}(c_c, \xi_b) d_r^{05}(c_c) - \cdots \]
\[ = \sum_{n=1,3,\ldots}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi_b) d_r^{on}(c_c) \]

for \( n \) and \( r \) odd.

Similarly, equation (E.15) can also be rewritten as

\[ r = 0 \]

\[ A_0 R_0^{(1)}(c_d, \xi_b) d_0^{00}(c_d) + A_2 R_0^{(2)}(c_d, \xi_b) d_0^{02}(c_d) + \cdots \]
\[ -\mu (B_0 R_0^{(4)}(c_c, \xi_b) d_0^{00}(c_c) - B_2 R_0^{(2)}(c_c, \xi_b) d_0^{02}(c_c) - \cdots) \]
\[ = \mu \sum_{n=0,2,\ldots}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi_b) d_0^{on}(c_c) \]

\[ r = 2 \]

\[ A_0 R_0^{(1)}(c_d, \xi_b) d_2^{00}(c_d) + A_2 R_0^{(2)}(c_d, \xi_b) d_2^{02}(c_d) + \cdots \]
\[ -\mu (B_0 R_0^{(4)}(c_c, \xi_b) d_2^{00}(c_c) - B_2 R_0^{(2)}(c_c, \xi_b) d_2^{02}(c_c) - \cdots) \]
\[ = \mu \sum_{n=0,2,\ldots}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi_b) d_2^{on}(c_c) \]
\[ r = r \]
\[ A_0 R_{00}^{(1)}(c_d, \xi_b) d_r^{00}(c_d) + A_2 R_{02}^{(1)}(c_d, \xi_b) d_r^{02}(c_d) + \cdots \]
\[ - \mu (B_0 R_{00}^{(4)}(c_c, \xi_b) d_r^{00}(c_c) - B_2 R_{02}^{(4)}(c_c, \xi_b) d_r^{02}(c_c) - \cdots) \]
\[ = \mu \sum_{n=0}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi_b) d_r^{on}(c_c) \]

for \( n \) and \( r \) even, and

\[ r = 1 \]
\[ A_1 R_{01}^{(1)}(c_d, \xi_b) d_1^{01}(c_d) + A_3 R_{03}^{(1)}(c_d, \xi_b) d_1^{03}(c_d) + \cdots \]
\[ - \mu (B_1 R_{01}^{(4)}(c_c, \xi_b) d_1^{01}(c_c) - B_3 R_{03}^{(4)}(c_c, \xi_b) d_1^{03}(c_c) - \cdots) \]
\[ = \mu \sum_{n=1}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi_b) d_1^{on}(c_c) \]

\[ r = 3 \]
\[ A_1 R_{01}^{(1)}(c_d, \xi_b) d_3^{01}(c_d) + A_3 R_{03}^{(1)}(c_d, \xi_b) d_3^{03}(c_d) + \cdots \]
\[ - \mu (B_1 R_{01}^{(4)}(c_c, \xi_b) d_3^{01}(c_c) - B_3 R_{03}^{(4)}(c_c, \xi_b) d_3^{03}(c_c) - \cdots) \]
\[ = \mu \sum_{n=1}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi_b) d_3^{on}(c_c) \]

\[ \vdots \]

\[ r = r \]
\[ A_1 R_{01}^{(1)}(c_d, \xi_b) d_r^{01}(c_d) + A_3 R_{03}^{(1)}(c_d, \xi_b) d_r^{03}(c_d) + \cdots \]
\[ - \mu (B_1 R_{01}^{(4)}(c_c, \xi_b) d_r^{01}(c_c) - B_3 R_{03}^{(4)}(c_c, \xi_b) d_r^{03}(c_c) - \cdots) \]
\[ = \mu \sum_{n=1}^{\infty} C_{on} R_{on}^{(1)}(c_c, \xi_b) d_r^{on}(c_c) \]

for \( n \) and \( r \) odd.
By solving the above linear equation system separately for even and odd values of \( n \), the coefficients \( A_n \) and \( B_n \) can be recovered.

As mentioned earlier, along the dam-canyon interface, the total motion in the canyon for all the terms of \( m \neq 0 \) is zero, and since \( C_{mn} = 0 \) for \( m = 1, 3, 5, \ldots \), we only need to determine the terms corresponding to \( m = 2, 4, 6, \ldots \).

\[
\sum_{m=2,4,\ldots}^{\infty} \sum_{n=m,m+1,\ldots}^{\infty} C_{mn} S_n^{(1)}(c_c, \eta) R_m^{(1)}(c_c, \xi_b) + \\
\sum_{m=2,4,\ldots}^{\infty} \sum_{n=m,m+1,\ldots}^{\infty} D_{mn} R_n^{(4)}(c_c, \xi_b) S_m^{(1)}(c_c, \eta) = 0
\]  

(E.16)

Solving (E.16) for each \( m \) and \( n \), the coefficients \( D_{mn} \) are found to be

\[
D_{mn} = \frac{C_{mn} R_m^{(1)}(c_c, \xi_b)}{R_n^{(4)}(c_c, \xi_b)}
\]  

(E.17)

It is, of course, numerically impossible to solve the system of infinite terms of \( A_n \), \( B_n \) and \( D_{mn} \). However, since the response of the system depends on the first few terms of the series. Therefore, it is possible to approximate the solution by a finite number of terms. The number of terms needed depends on the frequency of the waves and the canyon geometry.