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A comparison of the magnetospheric specification model, the Hardy et al. model, and satellite observations for precipitating auroral electron energy fluxes

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Rice University, 1992
RICE UNIVERSITY

A COMPARISON OF THE MAGNETOSPHERIC SPECIFICATION MODEL,
THE HARDY ET AL. MODEL, AND SATELLITE OBSERVATIONS
FOR PRECIPITATING AURORAL ELECTRON ENERGY FLUXES

by

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Abstract

A semi-quantitative comparison has been made of the observed and calculated precipitating electron energy fluxes for the April 1988 magnetic storm. Electron energy fluxes were calculated by the Rice Magnetospheric Specification Model (MSM), a comprehensive model of the inner magnetospheric environment, and by the Hardy et al. model, a statistical model of electron precipitation in the auroral zone.

The MSM correlates better with the observed fluxes than does the Hardy et al. model in terms of auroral boundaries, latitudinal profile and extent, and the actual magnitude of the energy flux. The sources of error in the MSM are probably: 1.) Artificial flux dropouts created near the ionospheric projection of the model outer boundary, 2.) an overestimate of the convection electric field, and 3.) errors in locating the polar cap boundary.
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I. Introduction

Since the dawn of civilization, the ethereal radiance that we now call the *aurora borealis* has aroused the curiosity of many scientists and philosophers. People such as Aristotle, Seneca, Galileo, Descartes, Halley, Gassendi, and Benjamin Franklin, have all made some contribution to auroral research, associating the phenomena with ice crystals, luminous atmospheric constituents, reflected light, atmospheric circulation patterns, and other exotic sources. Poets often personified the aurora, for example as "fierce, fiery warriors [fighting] upon the clouds,...drizz[ing] blood upon the Capitol (Shakespeare, *Julius Caesar* II, 2,19)." Most of the populous, however, believed that the aurorae were divine manifestations, precursors to pestilence and disaster, light from distant conflagrations, or precursors to the birth of important personages, such as emperors.

After the birth of electromagnetic theory in the late 19th century, more credible theories of the aurora began to emerge. Around 1900, Birkeland's wonderful experiments helped form the hypothesis that the aurorae were the result of the interaction of charged particles from outer space with the atmosphere. Shortly afterwards, Carl Störmer proposed that charged particles can become trapped in a dipole magnetic field, bouncing back and forth between hemispheres as they spiral along the field lines. This theory taken alongside the observations of the aurora made by spectroscopists helped solidify the opinion we hold today: the auroral forms are the result of electrons and ions raining down upon the Earth's atmosphere, guided by the terrestrial magnetic field. These particles cause ionization and excitation of atmospheric constituents such as atomic oxygen, and both atomic and molecular nitrogen, and thus produce the enthralling and colorful displays familiar to most people.

The space age brought about a change in the way scientists viewed the aurora.
Early space flights confirmed the theory advanced by Chapman and Ferraro, that the terrestrial magnetic field 'carves out' a cavity in the solar wind. This region where the supersonic solar wind interacts with the Earth's field is called the magnetosphere, and is depicted in Figure I.1. The solar wind is thermalized at the bow shock, and is diverted around the magnetopause, the boundary between the terrestrial and interplanetary magnetic fields denoted by a sharp change in field magnitude and/or direction (Wolf [1988]). However, the magnetopause is not an impenetrable barrier, and it is believed that solar wind plasma enters the magnetosphere at the entry layer.
and the mantle. The dawn-dusk electric field observed to exist across the polar cap, when mapped out into the tail, interacts with the mantle plasma and causes it to flow into the tail lobes. This is a possible mechanism for populating the plasma sheet, a region which consists of nearly isotropic, slow-flowing kilovolt plasma, which is mainly of solar wind origin during quiet geomagnetic conditions, but which has a considerable ion component originating from the ionosphere during disturbed conditions (Wolf [1988]). Loss mechanisms operate to keep the plasma-sheet plasma near equilibrium, one of which is the scattering of particles into the equatorial loss cone so that they precipitate into the auroral ionosphere. Thus the aurora may be viewed as a dissipation of magnetospheric energy derived from the solar wind interaction.

The efficiency of the energy transfer between the solar wind and the magnetosphere is highly variable, and thus, so are the dissipation mechanisms. Sometimes, the conditions are right for a substantial increase of magnetospheric energy over a short period of time. This is usually due to an increase in the reconnection rate of interplanetary magnetic field lines and terrestrial field lines on the magnetopause, thus transferring flux into the magnetotail as the newly reconnected lines are swept back by the solar wind. Of course, Nature prefers a quieter state and this energy is released rather explosively in a phenomena called a magnetospheric substorm where the precipitation mechanisms are greatly enhanced and the energy flux into the auroral ionosphere increases dramatically. Of course, the visual aurora becomes spectacular at these times.

It is this most disturbed magnetospheric state that receives our attention. In addition to being an interesting and challenging physical problem, we choose to model the substorm because of its detrimental effects on space systems and high latitude power systems. The intense charged particle fluxes and currents that occur can cause satellite surface charging, direct penetration of its systems by energetic particles, and
electrical discharges, which can result in temporary or permanent damage to satellite software and hardware. Also the intense currents flowing into and through the auroral ionosphere tend to disrupt power grids by inducing their own currents in the system. In this day and age, everyone is heavily dependent on both technologies for communication, defense, and navigation, and a significant disturbance affects many individuals. A description of the effects of the great magnetic storm of March 1989 is contained in Allen et al. [1990].

The Magnetospheric Specification Model was designed to help diagnose and predict magnetospheric conditions hazardous to spacecraft. It also outputs energy fluxes and mean energies of auroral electrons which can be used to estimate currents flowing out of the ionosphere and as input for ionospheric specification models which provide estimates of ionospheric conductivity and electron density (Tascione et al. [1988]).

The next section of this paper describes the morphology and dynamics of the auroral precipitation in more detail. Section III describes the July 1990 version of the MSM, which is still under development, and its objectives and operation. Section IV is a short overview of the Hardy, et al. average model of electron precipitation. It is used as a default model in the MSM. Section V compares the auroral electron energy fluxes output by the MSM with those observed by the Defense Meteorological Satellite Program F8 and F9 satellites during the April 1988 magnetic storm. There is also a comparison between the MSM output and the output of the Hardy et al. default model. Lastly, there are conclusions and recommendations for future work.
II. Auroral Morphology and Dynamics

Perhaps one of the more challenging physical problems existing today is the modelling of the aurora. It is an extremely dynamic phenomenon, with both large and small spatial variations occurring on vastly different time scales ranging from a fraction of a second to many hours. However, the aurora also exhibits certain large-scale regularities in its structure and dynamics which have been described by simple models. Using concepts from some of these models, the MSM represents an attempt at comprehensive modelling of auroral characteristics over the entire polar region. The following is a description of those characteristics during quiet and disturbed conditions. It should provide an idea of how flexible any model such as the MSM must be in order to accurately describe the characteristics of auroral precipitation.

II.A Morphology of the Aurora

If one were able to look down from far above the high latitude regions of the north pole, it would be evident that on the largest scale, the auroral configuration is an oval. This was conclusively established by Feldstein in 1963 from a statistical study of all-sky camera photographs. At any instant in time, the auroral forms are seen to be confined to an oval roughly centered on the geomagnetic pole, but eccentric, with the oval distended further equatorward along the nightside of the Earth, than along the dayside (Akasofu [1968]). Another observer situated over the south pole would report that the auroral configuration is almost the same there as over the north pole; the aurora is a conjugate phenomena, with particles bombarding the earth in both hemispheres, roughly symmetric about the magnetic dipole equator. An example of the oval structure is provided in Figure II.1.
Figure II.1: UV Photo of Auroral Oval During Quiet and Disturbed Conditions. 
(Hill and Dessler [1991], Allen et al. [1989])

Upon closer examination of the oval, one would see that the forms vary both in latitude and longitude. The forms fall into two broad classifications, which are termed the diffuse and discrete aurora. Starting from the low latitude form, the diffuse aurora, also known as the continuous aurora, or veil aurora, is a diffuse glow that appears devoid of small scale structure. It appears as a faint glow, usually without noticeable coloration, which covers a very large area of the sky. The discrete aurora, on the other hand, usually consists of bright arcs, with brilliant red and/or green coloration, which span many degrees of longitude but have very limited latitudinal extent. However, the visual appearance of the aurora is not relevant to this work. A very good description of the visual characteristics is given by Eather [1980].

A more comprehensive picture of the aurora is obtained by studying the morphology and dynamics of the particles which precipitate into the atmosphere and generate the
visual signatures. This is accomplished using particle detectors which are usually carried on board balloons, rockets and satellites. Balloon flights are necessarily low-altitude and therefore best suited to observing the high energy 'by-products' of the precipitation such as bremsstrahlung radiation and x-rays. Rocket flights, on the other hand, can make in situ observations of the precipitation, but are hampered by limited observation time and spatial coverage. Satellites can obtain information on the particles closer to their magnetospheric source regions, but also suffer from limited spatial coverage. However, several polar orbiting satellites can quickly provide huge amounts of information on the global characteristics of the precipitation. For example, the work of Hardy et al. [1985, 1987] suggests that the majority of the energy flux into the auroral oval is carried by low energy electrons (30 eV-30 keV), with ions making a contribution that is typically lower by a factor of about 10 over most of the oval, but it can be higher than the electron contribution in localized regions. For this reason, and the fact that the MSM does not calculate the ion precipitation, the remainder of this section will be mainly concerned with describing the characteristics of auroral electrons.

The electron precipitation exists over a much larger spatial area than just the visual auroral oval. On the nightside of the Earth, the latitudinal morphology of the electrons precipitating into the auroral ionosphere can be described in this manner (please consult Figure II.2). Consider that we have an electron detector able to measure several different energies on board a polar orbiting satellite. As it moves up through the magnetic equatorial plane headed for the north magnetic pole, we first detect an energetic electron population with a trapped pitch angle distribution (i.e., very few particles inside the equatorial loss cone). Note there is no pitch angle information in Figure II.2. The precipitation at this low latitude is not very intense, but is quite energetic (E > 10 keV). These observations combined with a mapping of the
Figure II.2: Representative DMSP F8 pass. Each point is a one second average of the number flux or energy flux over all 20 channels (30 eV-30 keV) of the SSJ/4 curved plate electrostatic analyzer. The average energy is energy flux/number flux. CGMLAT refers to corrected geomagnetic latitude at 110 km. MLT is magnetic local time.

Precipitation along the field lines to the equatorial plane suggest that these electrons originate in the Van Allen radiation belts (Winningham et al. [1975], Gussenhoven et al. [1983]).

As the satellite moves to higher magnetic latitudes, the observed flux begins to increase. The number flux of precipitating electrons becomes very high, and the electrons become increasingly more energetic within the range of 1-10 keV. The precipitation is fairly uniform, and the pitch angle distribution is generally isotropic (Lui et al. [1977]). The energy spectrum is characteristically monotonic, with the flux
decreasing like a power law spectrum at low energies, but changing to a near-Maxwellian shape as energy increases. This region of uniform precipitation is co-located with the diffuse auroral forms, and it has been suggested that the source region for these electrons is the central plasma sheet (Lui et al. [1973], Winningham et al. [1975], Lui et al. [1977]). This was tested by Lui and co-workers, who compared the electron spectrum described above to the electron spectra previously measured in the plasma sheet. They found that the spectra are reasonably similar during quiet and disturbed geomagnetic conditions. Some recent studies utilize satellite conjunctions, i.e., instances where satellites located in the plasma sheet and at low altitude were believed to lie on the same magnetic field line at their respective altitudes. This allows measurement of the spectrum outside the loss cone in the plasma sheet and the spectrum inside the loss cone at low altitude. It is then assumed that the low altitude spectrum is comparable to the high altitude spectrum inside the equatorial loss cone. Good agreement has been found between the two sets of spectra by Meng et al. [1979], and Schumaker et al. [1989]. This suggests that the auroral precipitation observed at low altitude does indeed originate, unaccelerated, from the plasma sheet.

In view of these results, many researchers have hypothesized that the inner edge of the plasma-sheet electrons, which is observed to be rather sharp and well defined (Wolf [1988]), maps to the equatorward edge of the diffuse auroral precipitation above the ionosphere, which is also observed to be well defined (Winningham et al. [1975], Lui et al. [1977], Kamide and Winningham [1977], Meng et al. [1979], Hardy et al. [1981], Gussenhoven et al. [1981, 1983], Nakai et al. [1986], Schumaker et al. [1989]). The most convincing evidence supporting this hypothesis is that there is an observed energy dispersion in the precipitating electrons with the less energetic electrons lying farthest equatorward (Gussenhoven et al. [1981], see Figure II.2).
This dispersion compares very well with the observed energy dispersion at the inner edge of the plasma sheet where the inner edge of the lower energy particles lies at a smaller geocentric distance than the inner edge of the higher energy particles (for example, see Frank [1971]). It also compares well with the theory of the Alfvén layer. Electrons convecting earthward from the plasma sheet are eventually diverted around the earth due to the increasing importance of gradient/curvature drift. The distance from the Earth where gradient/curvature drift dominates the plasma motion is energy dependent, with lower energy particles penetrating closer to the Earth before being diverted. When this structure is projected along the field lines into the ionosphere, it reproduces the observed energy dispersion. It is important to note, however, that this hypothesis is not without its critics. Feldstein and Galperin [1985] insist that the equatorward edge of the auroral oval, as defined by Feldstein in 1963, maps to the inner edge of the plasma sheet. The equatorward edge defined by Feldstein is essentially the edge of the oval of discrete auroral arcs. This puts the equatorward edge of the diffuse aurora inside the outer radiation belt. The majority of the observations do not support this theory at all, and the assumption that the equatorward edge of the diffuse auroral precipitation maps to the inner edge of the plasma sheet is used in the MSM and throughout the remainder of this work.

Further poleward of the diffuse auroral precipitation lies a region of highly structured electron precipitation where the number and energy fluxes are localized and extremely variable from one orbit to the next. The energy spectra observed are similar to the spectra observed in the diffuse region except that they also show a prominent 'monoenergetic' peak at higher energies. The peak is narrower than a Maxwellian and its existence suggests that the electrons undergo acceleration before they are detected. This will be discussed in more detail later. The region is co-located with the region of visual discrete auroral forms and inverted-V type structures; a structure so
named because as the spacecraft passes over it, the energy of the electrons rises to a maximum and then declines to a lower level (Lui et al. [1977], Swift [1981]). Figure II.2 shows several discrete auroral arcs with inverted-V structure.

The mapping of these arcs is usually inferred by considering that the diffuse precipitation maps to the central plasma sheet and, therefore, the discrete arcs must map further out into the magnetotail. This has led to suggestions that the discrete forms map to the boundary layer of the plasma sheet. As noted above, Feldstein and Galperin have challenged this mapping insisting that the discrete forms map into the plasma sheet. More recent work indicates that the oval of discrete forms maps to a wide region in the GSM Z direction at 15 RE down the tail (Reiff et al. [1991]). However, the mapping of the discrete forms is as variable as the phenomena itself, and the question still remains unanswered.

As the detector crosses the poleward boundary of the discrete precipitation, it enters a region of fairly quiet and uniform electron precipitation over the polar cap. This 'polar rain' consists of a soft spectrum (E ≈ 100 eV), resembling that of the electrons detected in the magnetosheath (Winningham and Heikkila [1974]). The field lines threading through this region are generally believed to be connected to the interplanetary magnetic field, especially during disturbed conditions. This implies direct entry of the solar wind electrons over the polar cap, a theory supported by the soft spectrum and hemispheric asymmetries in the intensity of the precipitation (see, for example, Fairfield and Scudder [1985]). It should be emphasized that the polar cap precipitation is occasionally more energetic and localized, forming sun-aligned discrete arcs and trans-polar arcs (theta aurorae, Meng and Akasofu [1976], Frank et al. [1986]). These events usually occur when the IMF Z component is northward; i.e., when the magnetosphere and the auroral oval are fairly quiet (Hardy et al. [1986]). They have been associated with field aligned potential drops accelerating the
electrons, or the strahl component of the solar wind (a field aligned component of solar wind electrons) entering the polar cap (Winningham and Heikkila [1974], Newell and Meng [1990] and references therein). Energetic polar rain can make identification of polar cap precipitation boundaries difficult because the precipitation does not drop to a background level like that shown in Figure II.2.

This latitudinal morphology does extend in local time away from the midnight meridian in both directions remaining relatively unchanged as far as the dawn and dusk meridians. However, the vastly different configuration of the dayside magnetosphere (see Figure I.1) greatly alters the precipitation morphology on this side of the planet. As one moves from the dawn meridian to the noon meridian the spectra become less like those detected in the diffuse region (i.e., plasma sheet spectra), but instead resemble those usually detected in the magnetosheath (sheath spectra). Centered near noon in this region of softer precipitation is a localized region of very soft electron precipitation with sheath-like spectra. This area extends only 3-4° in latitude and 1-3 hours in local time, and is co-located with an observed gap in some of the visual discrete auroral forms. This region is most likely the ionospheric footprint of the polar cusp, while the slightly more energetic precipitation surrounding it probably maps to the low latitude boundary layer (Meng [1981], Hardy et al. [1985], Newell and Meng [1988]). The average characteristics of these regions will be discussed in a later section. Figure II.3 shows a polar schematic of the morphology that has been described.

It has been mentioned above that the electrons forming the diffuse and discrete regions of precipitation are scattered into the loss cone and precipitated from their respective magnetospheric source regions. It is useful to discuss the mechanisms that may cause this pitch angle scattering. The diffuse electron precipitation is
observed to be unaccelerated, therefore, the mechanism that scatters plasma sheet electrons into the loss cone should not change the energy of the electron. The most widely accepted mechanism for doing this is wave-particle interactions, with two different wave modes suggested. First is the whistler mode, an electromagnetic, right-hand circularly polarized wave which is capable of resonating with the gyrating electrons when its' frequency is equal to the doppler shifted gyrofrequency of the electron,

\[ \omega = \omega_{ce} - kv_R, \]
where \( \omega_{ce} \) is the electron gyrofrequency, and \( v_R \) is the resonant velocity. This interaction can cause the perpendicular and parallel energy of the electron to be exchanged (see Cornwall [1964]). When the whistler is not propagating along the field line, as assumed above, it can resonate with higher harmonics of the gyrofrequency, but these resonances are weaker than the primary one above.

This mechanism seems to be effective for higher energy electrons which are capable of moving fast enough to fulfil the resonance condition. However, as mentioned earlier, most of the observed diffuse precipitation is relatively low energy (1-10 keV) and it has been suggested that electrostatic electron cyclotron harmonic waves with frequencies near odd half-harmonics of the gyrofrequency are capable of interacting with these slower electrons (Schumaker et al. [1989] and references therein). However, recent studies refute the claim made by many that the ECH waves have amplitudes large enough to be responsible for the observed precipitation (Belmont et al. [1983], Roeder and Koons [1989]). An excellent 'review' of the topic is given by Roeder and Koons. At this point in time, the pitch angle scattering mechanism of the lowest energy electrons is still controversial.

Recall, however, that the spectra of the discrete precipitation suggests that the electrons undergo acceleration at some point along their journey from the magnetosphere. The number of theories that exist to explain the acceleration and precipitation of these electrons is very large and diverse in nature. The MSM does not attempt to model the discrete precipitation because of its dynamic nature and therefore, a discussion of the theories behind it is beyond the scope of this work. Good reviews of both the discrete and diffuse precipitation mechanisms are those of Mozer et al. [1980], and Swift [1981].
II.B Dynamics of the Aurora

The previous part of this section provides a snapshot of the auroral precipitation during a period of average geomagnetic activity. However, as mentioned earlier, the auroral forms are very dynamic on a number of spatial and temporal scales. For example, the auroral oval undergoes large scale variation driven by the changes in the geometry of the magnetic field due to the change in solar wind incidence angle. This occurs because the geomagnetic pole is offset by about 11° from the geographic pole. This dipole tilt causes the auroral oval to undergo diurnal variations and also seasonal variations due to the changing angle between the Earth's rotation axis and the ecliptic plane. The variation of incidence angle also disrupts the conjugacy between the northern and southern auroral zones. Meng and Akasofu [1968] noted that the visual auroral forms are shifted further equatorward in the summer hemisphere than in the winter hemisphere. Kamide and Winningham [1977] also obtained similar results from a study of ISIS II electron spectrograms.

In addition to the variations caused by the magnetic field geometry, there are also large scale variations in the oval due to the outside influence of the interplanetary magnetic field. Dungey suggested that a southward (-Z GSM direction) IMF might result in an increase in the reconnection rate of interplanetary and geomagnetic field lines, thereby increasing the amount of energy transferred from the solar wind to the magnetosphere. It has also been suggested that the magnitude and direction of IMF $B_Z$ has a large effect on the size of the auroral oval (Akasofu et al. [1973]), with the oval extended further equatorward during times of negative $B_Z$ than during times of positive $B_Z$. In fact, a linear correlation between the magnitude of negative IMF $B_Z$ and the latitude of the equatorward edge of the diffuse auroral precipitation has been found by several researchers (Kamide and Winningham [1977], Hardy et al. [1981], Nakai et al. [1983]). The correlation coefficients are not extremely high ($r \approx 0.65$), but
the polarity of $B_Z$ is believed to play a dominant role in the equatorward motion of the equatorward edge. During times of high magnetic activity, $B_Z$ is usually observed to be negative (Hirschberg and Colburn [1969]), and the inner edge of the plasma sheet moves earthward (Freeman [1974]) which corresponds to equatorward motion of the equatorward edge.

This in itself implies a correlation between oval size and geomagnetic activity, Kp (The Kp index is a three hour index of worldwide magnetic activity which is computed from multiple ground stations and is proportional to the logarithm of the amplitude of time variations in the horizontal magnetic field component). Indeed, such a linear correlation exists with high correlation coefficients ($r = 0.80$) in most local time zones (Gussenhoven et al. [1981, 1983]). Both relations are of the form,

$$\Lambda_i = \Lambda_{oi} + \alpha_i X,$$

where $i$ refers to a local time bin and $X$ can be either Kp or $B_Z$. Of course, the intercepts and slopes are different in the different relations.

It is important to note that the effects of Kp and $B_Z$ on the auroral oval cannot be entirely separated from the most dynamic and interesting of all auroral processes, the magnetospheric substorm. The systematic auroral phenomena associated with the substorm were first noted by Akasofu [1964]. The substorm is best defined by quoting the work of Rostoker et al. [1980]: "[The substorm is] a transient process initiated on the nightside of the Earth in which a significant amount of energy derived from the solar wind-magnetosphere interaction is deposited in the auroral ionosphere and in the magnetosphere." The substorm can be divided into several phases, the growth phase, expansion phase, and recovery phase. A southward turning of the IMF usually occurs and initiates the growth phase. It is believed that the reconnected
magnetic flux is transferred from the dayside magnetosphere to the tail lobes, storing magnetic energy and thinning the plasma sheet. It is during this phase that the oval expands equatorward, usually quite rapidly. Suddenly, one of the equatorward-most discrete arcs will undergo extensive brightening and begin expanding poleward and westward, signalling the onset of the expansion phase. The expanding discrete auroral arcs are known as the westward travelling surge (WTS). Intense field aligned currents are associated with the bright arcs, and the WTS. These events also result in an intensification of the auroral electrojet, the current flowing horizontally in the ionosphere which closes the upward and downward field aligned currents. In addition to the expansion of the WTS, the entire oval expands poleward somewhat, forming a bulge. All of these features are shown in Figure II.1. In addition, an observation-based cartoon of the disturbed auroral oval is presented in Figure II.4. These effects are believed to be associated with the disruption of the cross tail currents and resulting dipolarization of the magnetic field in the tail. Fresh plasma is injected earthward and precipitation is enhanced, which results in an overall brightening of both discrete and diffuse forms. After some time, the activity dies off and the oval contracts, usually over a period of several hours (Nakai et al. [1986]), to a more quiet configuration such as that shown in Figure II.3. Note that there can be many onsets before the recovery occurs. Further information on the effects of substorms in the magnetosphere can be found in Kan [1990] and Lopez [1990].

As mentioned in the Introduction, it is these times of high magnetic activity, and dynamic auroral activity that we wish to model. Unfortunately, many auroral processes are poorly understood and strongly coupled to a much larger system. The precipitation model in the MSM represents a 'best guess' as to what is actually happening in the magnetosphere to produce the auroral precipitation and it will be
Figure II.4: Disturbed Auroral Morphology (Akasofu [1977])

shown that, considering the dynamic nature of what we are trying to represent, we have made a good guess.
III. The Magnetospheric Specification Model

The Magnetospheric Specification Model was designed for the Air Weather Service branch of the U. S. Air Force with the objective of preventing and/or diagnosing environmentally related spacecraft damage. It is an operational model designed to provide comprehensive real-time monitoring of the inner magnetospheric environment. Specifically, the model is designed to:

1.) Specify fluxes in the L=2-10 region of electrons and ions with energies between 100 eV and 100 keV;
2.) Specify the energy flux and average energy of precipitating electrons and ions over the auroral zones with the intention of using this output as input for ionospheric and thermospheric specification models.

The MSM accomplishes these objectives by using a stream of ground-based and satellite data as input to semi-empirical electric and magnetic field sub-models. Then, utilizing an empirically specified initial condition, the model computes drift paths for test particles with a number of different energies by using the concept of conservation of invariant density, $\eta$, along a drift path except for loss. This concept is discussed in Harel et al. [1981]. Note that this computation is carried out by the third major sub-model of the MSM: the particle tracer. Each of these important sub-models will receive concise treatment below. For a full description of the July 1990 version of the MSM, consult Freeman et al. [1990], Hausman [1989], Hilmer [1989], and Nagai [1991].

The structure and operation of the MSM can be described with the help of Figure III.1 which presents a simplified flowchart. Input data, which are described in Table III.1, are read into the model and manipulated to place it in the correct format. Because the MSM requires the input at every model timestep (set at 15 minutes for
this run), this manipulation consists of interpolation of input from the values surrounding the timesteps. However, if the gap in the input data is too large, proxy data values are derived from a set of front-end models, which use the available input to estimate values for the missing input. These models are described in Freeman et al. [1990]. They were implemented because a design constraint requires that the MSM operate and produce useful output in the event of the loss of some or all of the input data, except Kp. Thus, the front end models and the default models, which provide geosynchronous and precipitating fluxes in the event of input data loss, are Kp driven.

The input data are then fed into the E and B-field models, and initial and boundary conditions are set up. This information goes to the particle tracer, which computes invariant densities for each timestep, energy species, and mass species, and corrects them for loss. Finally, a high-energy particle distribution is calculated and the fluxes in the equatorial plane as well as the auroral energy flux and mean energy are output.

Table III.1: MSM Input Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kp (Used for Initial and Boundary Fluxes)</td>
<td>None</td>
</tr>
<tr>
<td>Dst (Determines ring current strength in B-field model)</td>
<td>Nanotesla</td>
</tr>
<tr>
<td>Equatorward Edge of Diffuse Aurora (Determines auroral boundary in E-field model and constrains B-field mapping)</td>
<td>Degrees</td>
</tr>
<tr>
<td>Polar Cap Potential (Input for E-field model)</td>
<td>Kilovolts</td>
</tr>
<tr>
<td>Polar Cap Potential Pattern (Input for E-field Model)</td>
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</tr>
<tr>
<td>Solar Wind Velocity (Determines standoff distance for B-field model)</td>
<td>Km/sec</td>
</tr>
<tr>
<td>Solar Wind Density (Determines standoff distance for B-field model)</td>
<td>Protons/cm^3</td>
</tr>
<tr>
<td>GEO Electron Flux (Checks model predictions)</td>
<td>Elec/cm^2·sec·sr·keV</td>
</tr>
<tr>
<td>GEO Ion Flux (Checks model predictions)</td>
<td>Ions/cm^2·sec·sr·keV</td>
</tr>
</tbody>
</table>
MSM Simplified Flowchart

Figure III.1.
III.A The MSM Coordinate Grid

A key component of the MSM is the polar, ionospheric grid which is used mainly to govern the motion of plasma within the modelling region. This grid is set up at the geocentric distance of the ionosphere and its coordinates are magnetic colatitude and magnetic local time. Note, however, that at this time, we do not include the tilt of the Earth's main field away from the rotation axis, therefore, geographic coordinates, invariant coordinates, and magnetic coordinates all coincide.

The grid is assumed to rotate with the earth and has uniform spacing of grid points in local time corresponding to 0.5 hours of local time per grid point. There is also a wrap around of 3 grid points in local time for a total dimension of 51 grid points. The latitudinal grid spacing is non-uniform. It was designed by Wolf to have a closer spacing of the grid points in the region corresponding to the auroral zone, and wider spacing over the polar cap and lower latitudes (see Freeman et al. [1990]). An illustration of the latitudinal spacing of the grid points is presented in Figure III.2. The grid extends from \( \theta = 1^\circ \) to \( \theta = 49.1^\circ \), where \( \theta \) is magnetic colatitude. There are 62 grid points in colatitude.

![Figure III.2: Grid Colatitude vs. Grid Index I](image-url)
III.B The Magnetic Field Model

The semi-empirical magnetic field model developed for use in the MSM is fully described in Hilmer [1989] and Freeman et al. [1990]. Therefore, only an overview of its structure and operation will be presented here.

The magnetospheric magnetic field configuration is described as a superposition of the magnetic fields produced by four main current systems:

1.) The main terrestrial magnetic field, $B_d$, which is assumed to be a dipole field;

2.) The field due to the Chapman-Ferraro currents flowing on the magnetopause, $B_{cf}$;

3.) The field produced by the ring current, $B_{rc}$;

4.) The field produced by the cross-tail current, $B_{tail}$.

The model employs a pre-determined magnetopause that is not computed from pressure balance, and there is no shielding of the ring or tail currents at the magnetopause. There is also no explicit representation of the Birkeland currents.

The three-dimensional magnetic field configuration is computed under the assumptions that $\nabla \cdot B = 0$, and since only static configurations are considered, the currents then follow from $\nabla \times B = \mu_0 J$. Also, $(B_{cf} + B_{tail}) \cdot n = 0$, where $n$ is a unit vector normal to the magnetopause.

The model is incorporated into the MSM in the form of pre-computed matrices specifying the equatorial mapping points of each ionospheric grid point, the equatorial field strength at each of these points, and the flux tube volume raised to $-2/3$ power,

$$VM = \left( \int \frac{d\gamma}{B} \right)^{-2/3},$$
at each grid point for a large number of magnetic field configurations (the term \( ds \) is a line element along the magnetic field). This is necessary due to the limitations of present computing facilities which do not allow us to compute the field configuration in real-time.

A grid of different field configurations has been computed and each matrix is characterized by 3 parameters:

1.) Magnetopause standoff distance (6, 8, 10, 12, and 14 \( R_E \));
2.) \( Dst \) (-400, -300, -200, -150, -100, -50, and 50 nT);
3.) \( \Lambda_{meb} \), the equatorward edge of the diffuse aurora at midnight (16 latitudes ranging from 49.47° to 69.30°).

The inner edge of the cross tail current is constrained to map to \( \Lambda_{meb} \) along a field line. This is basically equivalent to the assumption that the inner edge of the plasma sheet maps to the equatorward edge of the diffuse aurora at that point.

During a run, the MSM uses this grid as a 'look-up' table of magnetic field configurations. Eight matrices which bracket the input parameters are selected for each time step, and the mapping points, field strengths, and flux tube volumes are interpolated from the chosen matrices. This process is shown schematically in Figure III.3.

The representation of the time dependence of the magnetic field by time-dependant mapping of the ionospheric grid points to the magnetic equatorial plane is important because it effectively simulates the effects of induction and co-rotation electric fields. Thus, these fields do not have to be explicitly represented elsewhere.
III.C The Electric Field Model

The ionospheric electric field model developed for the MSM is an input data driven model which calculates the potential at each grid point. It was developed by Wolf and Spiro and is described fully in Freeman et al. [1990]. The model divides the northern ionosphere into 4 regions which are shown in Figure III.4. Region 0 corresponds to the polar cap. Region 1 is the location of the electric field reversal or roughly the location of the Region-1 field aligned currents. Region 2 is the auroral sunward flow which is roughly coincident with the diffuse auroral zone. Region 3 is the sub-auroral zone and extends to the magnetic equator.

The regions are bounded by three ellipses which are defined by the relation:
Figure III.4: A Cartoon of the E-Field Model Regions and Boundaries

\[ \frac{(x-\text{DX})^2}{A^2} + \frac{(y-\text{DY})^2}{B^2} = 1 \]

where $\theta$ and $\phi$ are the colatitude and magnetic local time coordinates of the flat polar grid. So, each of the three ellipses is defined by the four parameters $A$, $B$, $\text{DX}$, and $\text{DY}$. The parameters $\text{DX}$ and $\text{DY}$ represent displacements of the ellipse from the geomagnetic pole.

**Placement of Ellipse B**

When the MSM is run, this ellipse is placed first. In the magnetic equatorial plane, we associate ellipse $B$ with the outer boundary of our main modelling region. This boundary is placed at:
\[ R_{\text{noon}} = 0.95(R_{\text{standoff}}), \]
\[ R_{\text{dusk}} = R_{\text{dawn}} = 1.4(R_{\text{standoff}}), \]
\[ R_{\text{midnight}} = 2.0(R_{\text{standoff}}). \]

We then use the magnetic field model to map the outer boundary onto the ionospheric grid. We derive the ellipse parameters based on the mapping location. This mapping from the equatorial plane to the ionosphere eliminates the possibility of mapping ellipse B from the ionosphere to a unphysically large distance in the magnetosphere should the magnetic field model produce a highly inaccurate field configuration.

**Placement of Ellipse C**

To place ellipse C, we recognize its close proximity to the equatorward edge of the diffuse aurora. Thus, at midnight, we use the equatorward edge derived from the DMSP satellites and assume:

\[ \Lambda_{\text{mid}} = \Lambda_{\text{MEB}}. \]

At dawn and dusk, we assume that:

\[ \Lambda_{\text{dawn}} = \Lambda_{\text{dusk}} = \Lambda_{\text{MEB}} + 1^\circ. \]

Finally, the location at local noon is determined by using the regression results of Gussenhoven et al. [1983] to derive:

\[ \Lambda_{\text{noon}} = 0.5147(\Lambda_{\text{MEB}})+35.29^\circ. \]

Once this is done, we again assume that the boundary is elliptical in shape and calculate the ellipse parameters.

It is important to note that the placement of ellipses B and C is driven by the input data and because of this we must guard against the unphysical situation of ellipse C being placed poleward of ellipse B. Therefore, a minimum thickness of region 2 is enforced:

\[ \Delta \Lambda_{\text{noon}} = \Delta \Lambda_{\text{midnight}} = 3^\circ \]
\[ \Delta A_{\text{dusk}} = \Delta A_{\text{dawn}} = \frac{4.5 \times 10^{-6} (\text{PCP})}{E_{\text{max}}}, \]

where PCP is the observed polar cap potential drop and \( E_{\text{max}} \) is the maximum allowable downward electric field and is set at 0.1 V/m.

**Placement of Ellipse A**

In order to place ellipse A, digitized versions of the Heppner-Maynard-Rich (HMR) polar cap potential patterns (Heppner and Maynard [1987], Maynard and Rich [1989]) were used. Note that these are also input patterns for the MSM (IPATT). Ellipses which correspond to the MSM's ellipse A and ellipse B were drawn on the HMR patterns and ellipse parameters (A, B, DX, and DY) were derived for the pattern. Then, assuming that ellipse B in the HMR pattern and the calculated ellipse B coincide, we directly scale the MSM ellipse parameters using the following expressions:

\[ A_{\text{MSM}(1)} = A_{\text{MSM}(2)} \frac{A_{\text{HMR}(1,\text{IPATT})}}{A_{\text{HMR}(2,\text{IPATT})}}, \]

\[ B_{\text{MSM}(1)} = B_{\text{MSM}(2)} \frac{B_{\text{HMR}(1,\text{IPATT})}}{B_{\text{HMR}(2,\text{IPATT})}}, \]

\[ DX_{\text{MSM}(1)} = DX_{\text{MSM}(2)} + \frac{A_{\text{MSM}(2)}}{A_{\text{HMR}(2,\text{IPATT})}} [DX_{\text{HMR}(1,\text{IPATT})} - DX_{\text{HMR}(2,\text{IPATT})}] , \]

\[ DY_{\text{MSM}(1)} = DY_{\text{MSM}(2)} + \frac{B_{\text{MSM}(2)}}{B_{\text{HMR}(2,\text{IPATT})}} [DY_{\text{HMR}(1,\text{IPATT})} - DY_{\text{HMR}(2,\text{IPATT})}] , \]
where 1 stands for ellipse A, 2 stands for ellipse B, and IPATT is the HMR pattern type.

**Computation of the Region 0 Potential**

The next step is to calculate the potentials in the four ionospheric regions. Recall that above we drew ellipse A on the digitized HMR polar cap potential patterns. This ellipse satisfies the relation:

\[
\frac{(x_{HM} - DXHM)^2}{A_{HM}^2} + \frac{(y_{HM} - DYHM)^2}{B_{HM}^2} = 1.
\]

Our ellipse A, however, will generally satisfy a slightly different relation:

\[
\frac{(x - DXA)^2}{A_A^2} + \frac{(y - DY_A)^2}{B_A^2} = 1.
\]

In order to calculate the potential for a given x and y, we use a 'rule of corresponding points' and assume:

\[
\frac{(x_{HM} - DXHM)^2}{A_{HM}^2} = \frac{(x_{HM} - DXA)^2}{A_A^2},
\]

\[
\frac{(y_{HM} - DYHM)^2}{B_{HM}^2} = \frac{(y_{HM} - DY_A)^2}{B_A^2},
\]

to calculate the values of x\(_{HM}\) and y\(_{HM}\) using a specific HMR polar cap pattern. Using these values, we then calculate the potential, \(V_{HM}\), at (x\(_{HM}\), y\(_{HM}\)) using an algorithm provided by Fred Rich and described in Rich and Maynard [1989]. This
potential is then scaled using the polar cap potential drop observed by the DMSP satellite:

\[
V_{MSM} = V_{HM} \left[ \frac{PCP}{V_{MAXHM(IPATT)} - V_{MINHM(IPATT)}} \right].
\]

Using this procedure, the HMR pattern is scaled to fit within the MSM's polar cap.

**Computation of the Region 3 Potential**

The expression for the sub-auroral potential evolved from experiments conducted with the Rice Convection Model for the SUNDIAL 1984 event (see Spiro et al. [1988]). The results of these runs suggested a low latitude potential of the form,

\[
V_{low}(\theta,\phi) = -2.75E_{west,max}^{m_{Vm}} \left[ \frac{\sin(\theta_{shield})}{\sin(25^\circ)} \right] \frac{\sin(\theta_{shield})^p}{\sin(\theta)} \times \\
[0.6103\sin(\phi) - 0.0154\sin(2\phi) - 0.0210\sin(3\phi) - 0.1092\cos(\phi) \\
+ 0.1676\cos(2\phi) - 0.0314\cos(3\phi)] + V_{BAR},
\]

where \(\theta_{shield} = A(3)-DX(3)\), and \(p\) is set equal to 1.38.

To estimate \(E_{west}(\theta_{shield})\), we assume that the equatorward edge of the aurora is the ionospheric footprint of the inner edge of the plasma sheet and that this is a convection boundary; that is, that the latitudinal motion of the equatorward edge is due primarily to the \(E \times B\) convection of plasma sheet electrons earthward from the magnetotail. Thus we write:
\[ R_I \frac{d \Lambda}{dt} = - \frac{E_{\text{west}}}{B}, \]

where \( R_I \) is the geocentric distance of the ionosphere and \( B \) is the magnetic field strength at the ionosphere: \( B = 0.5 \times 10^{-4} \) Tesla. Then,

\[ E_{\text{west}} = -1.576 \frac{d \Lambda^o}{dt_{\text{hrs}}}. \]

The term \( V_{\text{BAR}} \) is the average potential at low latitudes and we set it equal to:

\[ V_{\text{BAR}} = 0.6V_{\text{max}} + 0.4V_{\text{min}} - 0.22V_{\text{penet}}, \]

where \( V_{\text{penet}} \) is the penetration potential given by:

\[ V_{\text{penet}} = -13.1 \frac{d \Lambda^o}{dt_{\text{hrs}}} \sin(\theta_{\text{shield}}). \]

**Computation of the Region 2 Potential**

In this region a smooth extrapolation of the region 3 potential is added to an auroral zone potential. The extrapolation of the region 3 potential is a Taylor expansion about \( \theta_c \), the colatitude of the shielding layer:

\[ V_{\text{low, x}}(\theta, \phi) = V_{\text{low}}(\theta_c(\phi), \phi) + (\theta - \theta_c) \frac{\partial V_{\text{low}}(\theta_c, \phi)}{\partial \theta_c}. \]

The main auroral zone potential is represented by:
\[
V_{az}(\theta, \phi) = V_{b,az}(\phi) \left[ \frac{1 - \left(1 + \frac{(\theta_c - \theta)^2}{\Delta \theta^2} \right)^{-1}}{1 - \left(1 + \frac{(\theta_c - \theta_b)^2}{\Delta \theta^2} \right)^{-1}} \right] + F_{corr}(\theta, \phi),
\]

\[
\Delta \theta = (\theta_c - \theta_b)[1.25 - 0.75\cos(\phi - 3\pi/4)],
\]

where \( V_{b,az} \) is the contribution to the potential at boundary B from the auroral zone, and \( r=1 \). The term \( F_{corr} \) represents the distortion of the equipotentials around the Harang discontinuity:

\[
F_{corr}(\theta, \phi) = -6.75 \frac{dV_{b,az}}{d\phi} \phi' \Delta \phi_{amp} \frac{(\theta_c - \theta)^2(\theta - \theta_b)}{(\theta_c - \theta_b)^3},
\]

\[
\phi' = \cos \left( \frac{\pi(\phi - \pi)}{\Delta \phi_{width}} \right) \quad \text{if} \quad \left| \frac{2(\pi - \phi)}{\Delta \phi_{width}} \right| < 1,
\]

\[
\phi' = 0 \quad \text{otherwise}.
\]

Also, we set \( \Delta \phi_{width} = 1 \), and \( \Delta \phi_{amp} = 2 \).

**Computation of the Region 1 Potential**

Region 1 is treated as a transition region between the polar cap and the sunward-flow region. The potential in this region is designed to fit smoothly between these two regions. To ensure this we:

1.) Determine the colatitudes of boundaries A and B for some \( \phi: \theta_a(\phi), \theta_b(\phi) \),

2.) Find these quantities:
\[ v_0 = V_0[\theta_a(\phi), \phi]; \quad v'_0 = \frac{\partial V_0[\theta_a, \phi]}{\partial \theta_a}, \]

\[ v_2 = V_2[\theta_b(\phi), \phi]; \quad v'_2 = \frac{\partial V_2[\theta_b, \phi]}{\partial \theta_b}. \]

3.) Using these values, we set the potential equal to:

\[
V(\theta, \phi) = v_0 + v'_0(\theta - \theta_a) + [3(v_2 - v_0) - (2v'_0 + v'_2)\delta \theta] \frac{(\theta - \theta_a)^2}{\delta \theta^2}
\]

\[
+ [2(v_0 - v_2) - (v'_0 + v'_2)\delta \theta] \frac{(\theta - \theta_a)^3}{\delta \theta^3},
\]

where \( \delta \theta = \theta_b(\phi) - \theta_a(\phi). \)

### III.D The Particle Tracer

The particle trace algorithm uses the input from the electric and magnetic field models to compute the drift paths of test particles perpendicular to \( \mathbf{B} \) for a number of different energy species and mass species using a 4th-order Runge-Kutta scheme with a variable timestep. The particle drift is assumed to be due to \( \mathbf{E} \times \mathbf{B} \) and gradient/curvature drift. Assuming that the particles have an isotropic pitch angle distribution, we use a bounce-averaged drift law:

\[
v_{IE} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\lambda \mathbf{B} \times \mathbf{V}}{B^2} \left[ \int \frac{dS}{B} \right]^{-2/3},
\]
where $\lambda$ is an energy invariant for an isotropic particle distribution which is convecting adiabatically (Harel et al. [1981]). There are also further assumptions that the plasma is collisionless and that $\lambda$ is conserved, which implies strong pitch angle scattering without changes in energy. Note that under these assumptions, we may express the particle energy as:

$$E = \lambda \left[ \int \frac{ds}{B} \right]^{-2/3},$$

where the quantity in brackets is the flux tube volume. This form is the same as that for the energy of an ideal gas undergoing adiabatic expansion $E = \text{(constant)} V^{-2/3}$, where $V$ is volume. Using these assumptions, we can say that the invariant density, the number of particles per unit magnetic flux, is conserved along a drift path, except for loss. The invariant density is written:

$$\eta = n \int \frac{ds}{B},$$

where $n$ is the particle density.

If we know the plasma distribution at all of the grid points and for all invariant energy species at time $t = t_0$, we find the distribution at $t = t_0 + \Delta t$ in the following manner (at this time, $\Delta t$ is set equal to 15 minutes). Please reference Figure III.5. The magnetic and electric field configurations derived by the sub-models are used to trace a test particle of each energy and mass species backwards in time from $t_0 + \Delta t$ to $t_0$ and determine the location of the particle on the grid at time $t_0$. This is done for all grid
points. Then, we interpolate the invariant density from the four grid points surrounding the test particle's location at $t_0$. Since $\eta$ is conserved along the drift path, this interpolated $\eta$ becomes the new $\eta$ for the starting grid point at time $t_0 + \Delta t$, once it is corrected for loss. This correction takes the form:

$$\eta(i,j,t_0+\Delta t) = \eta(i',j',t_0)\exp[-\Delta t/\tau],$$

where $\tau$ is the loss lifetime. The loss calculation will be explained in detail later. Using this procedure, the plasma distribution will evolve forward in time. These results are then mapped to the equatorial plane along the magnetic field lines.

Note that the particle tracer in its present configuration requires the electric and magnetic field configurations to be computed for the entire run period before it will operate. This means that at this time, the MSM cannot operate in a real-time monitoring mode; it is a tool for retrospective investigation and diagnosis.

**III.E The Electron Precipitation Algorithm**

Above, a very simple description of the loss calculation was presented. However, the loss algorithm employed by the MSM is a very robust combination of empirical and theoretical loss relations for multiple electron energies. A separate algorithm exists for the modelling region and the region poleward of the model boundary (i.e., ellipse B). In the modelling region, the loss is actually calculated using various assumptions about the type and strength of the scattering mechanism. Poleward of
the boundary, we simply scale statistical fluxes from the Hardy, et al. model to fit into our Region 1. Although implied earlier, it should be explicitly stated that in our mapping of the invariant densities from the ionospheric grid to the equatorial plane we assume no field aligned potential drops exist ($E \cdot B = 0$). It should be evident that this assumption is not easily justifiable when modelling auroral precipitation.

**Precipitation in the Main Modelling Region**

Here, we assume that two basic types of loss are occurring: 1.) strong loss due to strong pitch angle scattering or, 2.) weak loss due to some type of background noise. Strong pitch angle scattering basically assumes that over a bounce period, frequent wave-particle interaction causes the electron to be scattered by an amount large
compared to the equatorial loss cone. Thus the pitch angle distribution is isotropic, and the loss cone is always full. Weak scattering also assumes wave-particle interactions causing scattering, but not at such a large rate. Thus, we write the basic loss equation in this form,

\[
\frac{D\eta_{\text{IE}}}{Dt} = -\text{Loss} = -\text{Max} \left[ \frac{\text{Max}[0, (\eta - \eta_{\text{eq}})]}{\tau_{\text{weak}}} , \frac{\text{Max}[0, (\eta - \eta_{\text{thr}})]}{\tau_{\text{strong}}} \right]_{\text{IE}},
\]

where \( \eta_{\text{thr}} \) is a threshold density for the onset of strong pitch angle scattering, \( \eta_{\text{eq}} \) is a quantity which corresponds to a flux minimum which has been set in the MSM to prevent \( \eta \) from dropping to an unphysically low value, \( \tau_{\text{weak}} \) is the weak loss lifetime, \( \tau_{\text{strong}} \) is the strong scattering loss lifetime, and IE represents a specific energy channel.

The loss equation is integrated in this manner. Consider the decrease in \( \eta \) during the MSM timestep \( (t+\Delta t) \) to \( t \) from 'Runge-Kutta' times \( t_{n-1} \) to \( t_n \), that is, consider the loss during one timestep of the particle traceback. We calculate and store the weak and strong loss lifetimes at \( t_{n-1} \) and \( t_n \). The algorithm also stores the values of \( \eta_{\text{eq}} \) and \( \eta_{\text{thr}} \) at time \( t+\Delta t \). We then determine which term in the above loss equation is larger by assuming the first term may be written,

\[
\frac{\text{Max}[0, (\eta_{n-1} - \eta_{\text{eq}}(t+\Delta t))]}{0.5[\tau_{\text{weak}}(n-1) + \tau_{\text{weak}}(n)]},
\]

and the second is written as

\[
\frac{\text{Max}[0, (\eta_{n-1} - \eta_{\text{thr}}(t+\Delta t))]}{0.5[\tau_{\text{strong}}(n-1) + \tau_{\text{strong}}(n)]}.
\]

Therefore, if one of these terms is positive, the equation takes the form,
\[
\frac{D\eta}{Dt} = \frac{(\eta_{i,t+\Delta t} - \eta_{i,t+\Delta t})}{\tau_i},
\]

where \(\eta_{i,t+\Delta t}\) is either \(\eta_{eq}(t+\Delta t)\) or \(\eta_{thr}(t+\Delta t)\), and \(\tau_i\) is the appropriate denominator from above. Both of these parameters are assumed to be constant over the time step so that the equation immediately integrates to,

\[
\eta(n) - \eta_i(t+\Delta t) = [\eta(n-1) - \eta_i(t+\Delta t)]\exp\left(-\frac{(t_n-t_{n-1})}{\tau_i}\right),
\]

with this adjustment being made at each grid point for every energy channel. Then, a net loss rate is defined, \(\text{Rate} = \text{Loss}/\eta\), where the term \(\text{Loss}\) is defined above. The number flux of precipitating electrons is assumed to be,

\[
\Phi_N = \frac{\text{Rate} \times \eta \times B_{ir}}{2 \times 10^{13}},
\]

where \(B_{ir}\) is the magnetic field strength at the ionosphere. The energy flux is then simply,

\[
\Phi_E = (1.6 \times 10^{-12}) \times \Phi_N \times \lambda \times VM.
\]

The constant converts the energy flux to ergs from eV. We then sum over energy species to get the total number and energy fluxes at each grid point. Division of the energy flux by the number flux gives the average energy for the grid point. Only the total energy flux and average energy are given as output by the MSM.
The real flexibility in the model lies in the calculation of the parameters $\eta_{\text{thr}}$, $\tau_{\text{weak}}$ and $\tau_{\text{strong}}$. The methods are detailed in Freeman, et al. [1990], and instead of repeating the large number of equations, a discussion of the physical assumptions involved will be substituted. Recall that $\eta_{\text{thr}}$ is a threshold for the onset of strong pitch angle scattering. The idea that such a threshold might exist was put forward by Kennel and Petschek in 1966. They argued that the whistler mode, which interacts readily with electrons, is susceptible to unstable growth if the electron pitch angle distribution is sufficiently anisotropic. It was also suggested that the wave growth rate only depends on the number of resonant particles and that the particle energy and wave energy are conserved together. Thus the wave growth rate is limited by the rate at which wave energy is propagated away, that is, if the growth rate becomes very large, the build up in wave energy must be offset by a loss of resonant particles. Thus the growth rate is reduced and the system stabilizes itself. The flux level at which this instability occurs is known as the Kennel and Petschek limiting flux, and it is proportional to $L^{-4}$. Recent observations by Baker et al. [1979], indicate that the $E > 30$ keV particle fluxes at geosynchronous orbit seem to reach a saturation flux level of about $5 \times 10^7$ cm$^{-2}$-sec$^{-1}$-sr$^{-1}$, which is interpreted as the approach to a strong diffusion limit. So, for the high energy electrons ($E \geq 40$ keV) we write,

$$j_{\text{thr}}(r, E) = (5 \times 10^7 \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}) E_R(r, E) \left(\frac{6.6}{r}\right)^4,$$

where $E_R$ is essentially the ratio of the differential number flux of electrons at some energy $E$ and the total number flux of electrons at energies above 40 keV. It has units of eV$^{-1}$, and thus $j_{\text{thr}}$ is a differential flux that must be converted to $\eta_{\text{thr}}$.

The work of Schumaker et al. [1989] seems to indicate that the threshold flux for the lower energy electrons ($E \leq 20$ keV) is much lower because they observe
isotropic distributions inside the loss cone and relatively low equatorial electron fluxes. Therefore, in the MSM we set $\eta_{\text{thr}}$ equal to $\eta_{\text{eq}}$ for these electrons and interpolate between the two results for the intermediate energy range ($20 \text{ keV} \leq E \leq 40 \text{ keV}$).

The strong pitch angle scattering loss lifetime can be written as (Wolf [1983]),

$$\tau_{\text{full}} = \frac{2B_{\text{ir}}}{v} \int \frac{ds}{B},$$

where $B_{\text{ir}}$ is the field strength at the ionosphere and $v$ is the particle velocity. This is a lower limit on the loss lifetime. The work of Schumaker et al. [1989] suggests that the efficiency of strong pitch angle scattering in the plasma sheet depends on $K_p$, being least efficient for low $K_p$ and most efficient for high $K_p$, but never quite reaching the full limit described above. Thus we multiply $\tau_{\text{full}}$ by a factor $f_0(K_p)$ to obtain $\tau_{\text{strong}}$. Figure III.6 illustrates the strong loss lifetime for several energies.

The weak loss lifetime is much higher than the strong pitch angle scattering lifetime, and our estimation of it depends on whether the grid point is inside or outside the plasmapause. Inside the plasmapause we assume that the loss of electrons is due to pitch angle scattering from plasmaspheric whistlers and directly use the results of the Lyons et al. [1972] model. Approximations to their lifetime curves are coded into the MSM and displayed in Figure III.7. Outside the plasmapause, there is very little information on a weak loss rate, therefore we use the observations of Frank, et al. [1964] to estimate that the peak daytime precipitation rate is roughly 10 times the nightside rate (see their Figure 9) for high energy electrons ($E > 50 \text{ keV}$). We also know that it takes these electrons about an hour to traverse the dayside of the
Figure III.6: Loss Lifetimes for Strong Pitch Angle Scattering (Wolf [1983]).

Figure III.7: Electron Loss Lifetimes from Lyons et al. [1972].
magnetosphere. So, at synchronous orbit, we set the peak rate at roughly 0.5 hours and construct a function that varies smoothly in local time,

\[
\tau_{\text{weak}}(6.6R_E) = \text{Min}\left\{ \frac{E}{20 \text{ keV}} - 1, 1 \right\} \int_1^{1 + \left(\frac{9}{11}\right) \cos(\phi)} 1\,d\phi
\]

However, for energies below 20 keV, it was noted that the strong scattering threshold is low, and we set this quantity equal to zero. For distances between synchronous orbit and the plasmapause, we perform a linear interpolation between Lyons et al.'s result and our empirical formula. Outside synchronous orbit, we simply assume that the rate is equal to the weak rate at synchronous orbit.

**Poleward of the Main Modelling Region**

In this region, we perform a scaling of the power in precipitating electrons calculated from the Hardy et al. [1985] model. It is assumed that power is a cubic function of the latitudinal grid index,

\[
P''(i,J) = a[(i-i_J) + b(i-i_J)^2 + c(i-i_J)^3]
\]

where \(i_J\) is the floating point location of the intersection of ellipse A with a grid line J. Above this point, we set \(P'' = 0\), that is, there is no polar cap precipitation in the MSM. The coefficients are determined by requiring that:

1.) The formula must agree with the computed results for the first two latitudinal grid points in the main modelling region for all local times;

2.) It must give a designated amount of power per unit grid line poleward of the main modeling region,
\[
P_T = \int_{l_0}^{l_{MIN}} P^*(I,J) \, dl = \frac{d\text{Power}}{dJ}_{\text{Hardy Model}},
\]

where

\[
F = 0.5 + 0.3\sin(\phi),
\]

and IMIN is the latitudinal grid location of the modeling boundary (i.e., ellipse B) at local time \( \phi \). Finally, the output is smoothed by integrating the power given by the cubic equation at each latitudinal grid point over the range of I-0.5 to I+0.5. These power values are converted to energy fluxes by using a simple relation (\( \text{Power} = \text{Energy Flux} \times \text{Area} \)) and given as output.

So, in summary, the MSM precipitation algorithm provides an extremely flexible calculation of the diffuse auroral precipitation but only estimates the precipitation in the region of discrete auroras. We do not try to model the discrete precipitation because the MSM space and time resolution is too low to bother, there is no consensus on the mapping of discrete auroral forms or acceleration mechanisms (but field-aligned potential drops are important), and the complex calculations of electron acceleration would undoubtedly increase the amount of time necessary to complete each traceback. This is unacceptable for a model designed to be an on-line monitor and forecasting tool.

**III.F The MSM Initial Plasma Distribution**

The initial distribution of electrons in the magnetospheric equatorial plane are set up by calculating a reference matrix of fluxes based on statistical studies and various observational data. The matrix has three dimensions: \( \text{Kp} \) (from 0 to 6), energy (29 separate energies from 10 eV to \( 10^8 \) eV in steps of \( 10^{0.25} \)), and radial distance (3, 4, 6.6, and 13 \( R_E \)).
Observations of electron fluxes in the plasma sheet indicate that the distribution function is not Maxwellian, but has a high energy tail. Therefore, we model the energy dependance of the flux by a kappa distribution,

\[ j(E) = (1.68 \times 10^8 \text{ keV}^{-1}\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}) F(\kappa) \left[ \frac{1 \text{ keV}}{E_0} \right]^{1/2} \left[ \frac{N}{1 \text{ cm}^{-3}} \right] \frac{(E/E_0)}{(1 + E/\kappa E_0)^{\kappa+1}}, \]

where \( E_0 \) is the most probable energy, \( N \) is the number density, and \( F(\kappa) \) is a slowly varying function of \( \kappa \). As \( \kappa \) approaches infinity, the distribution becomes Maxwellian (Vasyliunas [1968]).

For \( r=13 \text{ R}_E \), we assume that this is the plasma sheet and use the statistical study of Huang and Frank [1986], and the observations of Baumjoehann et al. [1989], to estimate \( N \) and \( E_0 \), which are then placed into the kappa distribution.

For \( 6.6 \text{ R}_E \), two \( \kappa=6 \) distributions are summed, one for lower energy electrons and one for higher energy electrons, as defined by Garrett and Gaudet [1989]. The number density and \( E_0 \) are estimated from this same study. However, we feel that their results for \( K_p = 0 \) are contaminated by substorm effects, and the values for the high energy (\( E > 30 \text{ keV} \)) electrons are overridden. The new values are based on observational data as described in Freeman et al. [1990].

For \( 3 \text{ R}_E \) (the electron slot region) and \( 4 \text{ R}_E \) (the outer radiation belt), we fit fluxes from figures 5-45 and 5-46 of Spjeldvik and Rothwell [1985], which correspond to the NASA AE7-HI and AE7-LO radiation belt models respectively. The fluxes were modeled by a kappa distribution for four energy channels. However, the AE7 models do not contain any information about \( K_p \) dependance of these fluxes. Therefore, we assumed that the average fluxes corresponded to average \( K_p \) (about 2), and designed a \( K_p \) dependance using other observational data (see Freeman et al. [1990]).
Once the matrix is calculated, an initial flux must be found for each grid point. A three dimensional interpolation is used, noting that:

1.) The interpolation in $K_p$ is linear, using the logarithm of the flux. If $K_p > 6$, then $K_p = 6$ is used.

2.) Interpolation in $r$ is linear, using the logarithm of the flux. For $r > 13 \, R_E$, a reasonable $1/r$ dependance is assumed.

3.) The interpolation in energy is linear using the logarithms of both the flux and the energy.

These fluxes are then converted to invariant densities. Note that the MSM does allow for an admixture of fluxes calculated using the method described above, and fluxes calculated from a previous run. However, this feature is not currently implemented in the code.
IV. The Hardy et al. Average Model

In the last section, I briefly mentioned the default models built into the MSM. The Hardy et al. average model of auroral electron precipitation is the default model for the electron energy fluxes and mean energies. In the event of the loss of all input data, except Kp or some failure in the MSM precipitation algorithm, the Hardy et al. model fluxes and mean energies are given as output by the MSM. This model was not chosen solely because of its Kp dependence (although it was the deciding factor); quite a bit of work went into testing the model against the models of Spiro, et al. [1982], and Evans, et al. [1985] before the final selection was made. This work is detailed in Shade [1989].

The model is fully described in Hardy et al. [1985], and Hardy et al. [1987], but it is useful to summarize it here. The goal of the model was to determine the average characteristics of auroral electron precipitation as a function of magnetic latitude and local time as well as Kp. The data used for this study came from the SSJ/3 detectors carried on board the DMSP F2 and F4 satellites, and the P78-1 satellite. These detectors each consist of 2 curved plate electrostatic analyzers, one covering an energy range between 50 eV and 1 keV in eight logarithmically spaced channels, and the other covering the energy range between 1 keV and 20 keV also in eight channels. The detectors were swept together so that a complete 16 point spectrum was produced each second. The data base consisted of roughly 14.1 million spectra, sampled over all seasons.

The orbital coverage of the three satellites is shown in Figure IV.1. The DMSP satellites are both in sun-synchronous orbits at altitudes of about 840 km and inclinations of 98°. The F2 satellite was launched into the 1000-2200 local time meridian but progressed slowly to the 0800-2000 meridian over 2.5 years, while the
F4 satellite orbited in the dawn-dusk meridian. These satellites are 3-axis stabilized, and the detectors point towards local zenith. The P78-1 satellite was launched into a sun-synchronous orbit at 600 km altitude in the noon-midnight meridian. It is a spin-stabilized satellite, with the spin plane in the orbital plane, and the detectors mounted at right angles, both in the spin plane. The coverage of the auroral region is quite good, except for two small sectors.

The data were binned in magnetic local time and corrected geomagnetic latitude, with a total of 48 equally spaced local time bins, five latitude bins between 80° and 90°, five bins between 50° and 60°, and 20 bins between 60° and 80°. After this, the data were divided into seven Kp groupings, one for each value between 0 and 5, and one for all Kp ≥ 6-. Number fluxes, energy fluxes, and mean energies were calculated for each spectrum and placed in the appropriate bin. Some normalization to reduce bias due to the different detectors was performed before determining the average values. These average spectra were then smoothed to reduce residual noise. The
authors then integrated the spectra to produce color coded polar maps of the average fluxes and energies. An example is presented in Figure IV.2.

![Figure IV.2: Global Average Energy Map for Kp = 0 (Hardy et al. [1985])](image)

In their discussion of these global maps, the authors decided to divide the precipitating electrons into a hot ($E_{\text{avg}} \geq 600$ eV) population and a cold ($E_{\text{avg}} < 600$ eV) population. The hotter electrons tend to be confined to an approximately annular region whose equatorward edge is the equatorward edge of the diffuse aurora. This region is discontinuous, however, with a 'gap' in the higher energy electrons located at about 1800 local time for Kp = 0, and moving to earlier local times for increasing Kp. The authors explain this as a change in the location of the electron 'forbidden zone' (inner edge of the Alfvén layer) with increasing Kp. This is consistent with the results of Gussenhoven et al. [1981]. They also note that this hotter population tends to be a small fraction of the total number flux of precipitating electrons but does carry
the majority of the energy flux into the oval. The energy and number flux is seen to increase with Kp on the nightside, but on the dayside, the number flux is observed to increase from Kp = 0 to 2 and then decrease, with a corresponding decrease in the average energy in the region while the energy flux remains relatively constant at higher Kp. They also report that the maximum local time of electron penetration (i.e., the local time that freshly injected electrons from the tail drift around to before being substantially affected by loss mechanisms) was typically past noon, but decreased with increasing activity. This led the authors to postulate that the pitch angle scattering mechanism believed to produce the diffuse aurora is less efficient in times of lower magnetic activity.

The colder electron population is concentrated in a crescent shaped region centered at about noon and extending a few degrees in latitude and many hours in local time east and west of noon. A minimum in the mean energy lies at about the center of this region, and the value of the mean energy here decreases with increasing Kp. The crescent itself persists for all Kp and the number flux remains relatively constant with Kp, but the total flux into the region increases with Kp. Also, the mean energy of the electrons observed in this region gradually increases with local time as one moves away from noon, and also with latitude poleward or equatorward of the minimum. These facts and the fact that the spectrum of the electrons in this region is observed to be soft and have characteristics of magnetosheath electrons lead the authors to postulate that the localized energy minimum probably corresponds to the location of the polar cusp, where solar wind electrons penetrate directly into the ionosphere, and that the extended low energy precipitation region probably maps to the low-latitude boundary layer. Further evidence supporting this identification is the observed spatial expansion of the region and the equatorward motion of the region from 81° to 78° with increasing Kp which is in good agreement with previous work. Lastly, on the
nightside of the auroral oval, the cool electron population tends to have slightly higher mean energies and lower number fluxes, which suggests that the averaging of cool precipitation and inverted-V type structures of the boundary plasma sheet region (Winningham et al. [1975], Lui et al. [1977]) is probably the cause.

It is immediately evident that the global maps produced by the Air Force group are very useful because they display average latitude and local time characteristics of the electron precipitation with good statistics. To make the model more portable, the group fit the number flux and energy flux maps to simple functional forms (Hardy et al. [1987]). The variation in latitude was found to be best represented by an Epstein transition function of the form:

\[
e(h) = r + S_1(h-h_0) + (S_2-S_1)\ln\left(\frac{1 - (S_1/S_2)e^{(h-h_0)}}{1 - (S_1/S_2)}\right),
\]

where \(e\) is either the number or energy flux, \(h\) is the corrected geomagnetic latitude, \(r\) is the maximum of the quantity \(e\) being fit, \(h_0\) is the latitude for \(r\) at a specific local time, \(S_1\) is the slope of the curve for \(h < h_0\), and \(S_2\) is the slope of the curve for \(h > h_0\). This functional representation has the advantage of being able to fit asymmetric latitudinal flux profiles better than other functions (Hardy et al. [1987]).

The local time variations of the Epstein function coefficients were found to be quite smooth, and therefore, to reduce the number of coefficients needed to reproduce each global map, the MLT dependance was expanded in a Fourier series of the form:

\[
\alpha(t) = \sum_{n=0}^{6} c_n^\alpha \cos\left(\frac{n\pi t}{12}\right) + s_n^\alpha \sin\left(\frac{n\pi t}{12}\right)
\]
where \( \alpha \) is the coefficient being fit, \( C_\alpha \) and \( S_\alpha \) are the Fourier coefficients, and \( t \) is local time in hours. Each of the four coefficients of the Epstein function are thus represented by 13 Fourier coefficients, resulting in a total of 52 coefficients which are necessary for each level of Kp to reconstruct the global flux profile.

In the MSM, the model operates by calling up the Fourier coefficients for the latitude and local time of a specific grid point to determine the values of \( r \), \( h_0 \), \( S_1 \), and \( S_2 \). These values are then fed into the Epstein function along with the latitude of the grid point. This procedure is repeated for all of the grid points for both the energy and number flux. The energy flux and mean energy are then output by the MSM.

From a modelling standpoint, the Hardy et al. model has the advantages of being a very portable model with good spatial resolution and a large statistical data base. An additional advantage from our point of view is that the model is Kp based. However, the very nature of the model renders it incapable of reproducing small scale structure so commonly observed in the aurora and it also tends to blur the sharp boundary associated with the equatorward edge. This will be demonstrated in the next section. Another drawback is that the model obviously has poorer statistics for times of extremely high activity (e.g., Kp > 6), and will be unable to reproduce the fluxes associated with such events.
V. Testing the Precipitation Algorithm for the April 1988 Event

V.A General Description of the Event

In order to determine the accuracy of the MSM electron precipitation algorithm and compare it to the default precipitation model, the model output was compared with precipitating electron data supplied by the Air Force Geophysics Laboratory from their DMSP F8 and F9 satellites for the period of 1800 UT April 21 through 0600 UT April 22, 1988. During the period of April 21-23, there occurred a relatively large magnetic storm whose character is described in Figures V.1 to V.8. In these figures, April 21st is day 112, April 22nd is day 113, et cetera. Note that except for IMF BZ, these are all input parameters necessary to run the MSM (see Table III.1). Figure V.1 shows the global magnetic activity index Kp, Figure V.2 shows the index Dst, which is the average change of the horizontal magnetic field strength measured on the ground at low latitudes. A large negative value for Dst indicates the build up of a storm-time ring current. Figure V.3 shows the equatorward edge of the diffuse auroral precipitation mapped to local midnight (Gussenhoven et al. [1983]), which is obtained from the DMSP F9 satellite. Figure V.4 shows the polar cap potential drop, obtained from DMSP F8 satellite. It is a measure of the strength of convection in the magnetosphere. Figure V.5 shows the solar wind velocity, Figure V.6 shows the solar wind density, and Figure V.7 shows IMF BZ, all obtained from the IMP-8 satellite. Lastly, Figure V.8 shows 40 keV geosynchronous electron fluxes from Air Force satellite #1.

The sequence of events can be traced in this manner:

1.) Around day 112.6, there is a positive excursion in Dst. This indicates a compression of the magnetosphere by an increase in solar wind ram pressure, which is evident in the figures as an elevated solar wind density.
2.) Kp begins to rise, and the auroral oval begins to expand equatorward around the end of day 112. Also, IMF BZ begins to turn southward at this time, indicating increased energy transfer to the magnetosphere. Also note the beginning of a flux dropout at geosynchronous orbit. This is a classic pre-storm dropout.

3.) At about day 113.0, we note that the geosynchronous fluxes increase dramatically, and the DMSP F9 satellite observes an intense discrete arc near midnight (see Figure V.11). These denote plasma injection and intensification of precipitation; both are classic signatures of substorm onset (see Section II).

4.) Dst attains a value of about -120 nanoteslas at day 113.4, indicating that injections have built up the ring current. The equatorward edge is at its lowest value, and the polar cap potential is at its highest value.

5.) Some recovery begins to occur after day 113.5, as evidenced by the Dst index, but around day 113.8, there appears to be another, smaller, substorm onset.

6.) Recovery begins after day 114.0.

Thus, there should be an increase in the precipitating energy flux at the beginning of day 113 as the substorm begins.

V.B Comparison with Satellite Observations

The precipitating electron energy fluxes computed by the MSM and the Hardy et al. model are compared with the observational data from the Air Force DMSP F8 and F9 spacecraft, kindly provided by W. Denig of the Geophysics Lab. Each spacecraft is three-axis stabilized and in a sun-synchronous polar orbit at an altitude of about 840 kilometers. The F8 spacecraft is located in the dawn-dusk meridian with an orbital period of 101 minutes. It provided 13 polar passes for study within the time frame mentioned earlier. The F9 spacecraft orbits roughly in the 1000-2200 local time meridian, also with a period of 101 minutes. It provided 14 polar passes for study.
Figure V.1: Magnetic Activity Index Kp for April 1988 Event.

Figure V.2: Magnetic Activity Index Dst.
Figure V.3: Equatorward Edge of Diffuse Precipitation at Local Midnight.

Figure V.4: Polar Cap Potential Drop (Kilovolts).
Figure V.5: Solar Wind Velocity (km/sec).

Figure V.6: Solar Wind Density (cm\(^{-3}\)).
Figure V.7: Interplanetary Magnetic Field Z-Component.

Figure V.8: Electron Flux at Geosynchronous Orbit. Satellite Local Time is LT = UT - 2.3 hours. Dark Line is MSM Output; Squares are Data.
Both spacecraft carry the SSJ/4 electron detector, which is similar to the SSJ/3 mentioned earlier. However, the SSJ/4 has 20 logarithmically spaced energy channels which cover the energy range from 30 eV to 30 keV. Each detector is swept once per second to produce a 20-point electron spectrum. The total energy flux is calculated by performing a weighted sum over all channels, where the energy flux in each channel is given by,

\[ J_{E_i}(E_i) = \frac{E_i \cdot C(E_i)}{\tau \cdot [GF]_i} \]

where \( i \) represents a specific channel, \( E_i \) is the mean energy of the channel, \( C(E_i) \) are the counts in the channel, \( \tau \) is the sampling time, and \( [GF]_i \) is the geometric factor of the channel which takes the detector geometric factor and energy resolution of the channel into account. The data was provided in this counts-per-channel format and were converted to 15 second averages of the total energy flux for comparison. Also, the units were converted from keV/cm^2-s-sr to ergs/cm^2-sec.

The MSM output was linearly interpolated along the spacecraft track for comparison. Since the output comes in 15 minute timesteps, it was necessary to locate the spacecraft on the modelling grid for two MSM timesteps surrounding the spacecraft time. Then, the fluxes from the four surrounding grid points were interpolated in time and then in space. This was done at variable time resolution to try and pick up as much detail as possible over the auroral zone. The Hardy model fluxes were also calculated along the spacecraft track using a modified version of the subroutine imbedded in the MSM.

**Detailed Comparison**
Representative comparisons of the model output and satellite data are presented in Figures V.9 through V.14, and in Appendix A. The dashed line with small squares denotes the MSM output, the dashed line with small triangles denotes the Hardy model output, and the solid line indicates the satellite data. These passes will be studied in detail to provide familiarity with the presentation format and the physical processes producing the disagreement between the MSM fluxes and the observed fluxes.

Case 1

Figure V.9 shows a pass over the north pole by the DMSP F8 spacecraft along the dawn-dusk meridian beginning at about 1900 UT on day 112 (April 21), well before the substorm onset. The vertical solid lines indicate the diffuse equatorward edge chosen using the criteria of Gussenhoven et al. [1983]. The vertical dashed line indicates the MSM equatorward diffuse edge, placed arbitrarily where the energy flux decreases to observed equatorward background levels.

Several things are noteworthy about this pass. First of all, a glance at the ephemeris shows that the spacecraft did not reach high magnetic latitudes (< 80°) and, therefore, probably did not have a chance to penetrate very far into the polar cap region and sample the characteristically low energy fluxes in that region. It was determined by examining the spacecraft ephemeris that due to the diurnal variation of the geomagnetic pole about the geographic pole, neither spacecraft was able to reach high values of magnetic latitude during the later portion of the day (about 1300 UT - 2030 UT). This type of pass will be termed a grazing incidence pass. Note, however, that the MSM did predict entry into the polar cap region for this pass, indicating that the MSM's polar cap (E-field algorithm ellipse A) was further equatorward than the actual polar cap boundary.
Also, the MSM's equatorward edge of the diffuse precipitation lies much further equatorward than the observed equatorward edge for both auroral crossings. Considering that we assume that the equatorward edge maps to the inner edge of the plasma sheet, this could indicate that the MSM's inner edge could lie further earthward than the observed inner edge. However, note the plateau-like feature in the energy flux labeled 'shoulder' in the figure. It was determined that the poleward portion of this feature (i.e., where there is a sudden change in the slope of the energy flux) almost exactly coincides with the position of ellipse C, the equatorward edge of the shielding layer. The invariant densities inside this region are not much affected by the convection electric field (roughly, they are shielded from the convection field by an oppositely directed polarization field), and thus just die off over time due to loss. We are seeing precipitation from the initial condition invariant densities which is obstructing the diffuse equatorward edge.

The MSM energy fluxes also have a local minimum where the observed fluxes tend to be high. The ionospheric projection of the outer boundary of the modelling region lies inside this minimum. The reason the calculated flux reaches a peak and then declines as latitude decreases is due to the assumption of adiabatic convection in the model. When the flux tubes begin convecting earthward from the boundary (i.e., equatorward in my figures), they try to conserve their invariant content (PV) and the precipitated flux is low. However, the pressure in the flux tube builds up quickly as it moves earthward, which corresponds to an increase in the precipitated flux. As earthward convection and precipitation continues, the flux tube will become depleted, and the precipitating flux will decrease (Spiro and Wolf, private communication). When the two precipitation algorithms are merged at the ionospheric projection of the modelling boundary, the 'bite-out' occurs. However, for the majority of this pass, the MSM overestimated the fluxes in the auroral zone by a factor of 2-10.
Figure V.9: Comparison of MSM, Hardy, and DMSP Energy Fluxes, and Cross-Correlation Coefficients of the Auroral Peaks (see text).
The Hardy model does predict a grazing pass, but overestimated the polar cap fluxes (this is characteristic of the Hardy model because they averaged together a number of different polar cap states for each Kp level). Note also that the MSM did a much better job of reproducing the equatorward edges and the observed latitudinal profile of the precipitation, especially on the dayside.

Finally, at the bottom of Figure V.9 are the cross-correlation coefficients for the auroral peaks. The solid line represents the correlation between the MSM energy flux and the DMSP energy flux, while the dotted line represents the correlation between the Hardy energy flux and the DMSP energy flux. The cross-correlation procedure is described in appendix B. The size of the coefficient indicates the strength of a linear relationship between the two profiles. The coefficient is not normalized to one, however, in order to gain information about the relative magnitude of the energy flux. It is plotted versus $i$, the latitudinal offset of the model output with respect to the observed data. For example, the coefficients for the dawnside peaks indicate that the MSM output would best fit the observations if the peak were shifted by $3^\circ$ to the left. Then the large peak in the MSM energy flux would overlap the large peak in the observed energy flux. Note however, that there is a weaker correlation if the output were shifted slightly to the right. Notice that this shift is the one which 'looks proper' to the eye. It does not correlate as well because when that shift is made, the majority of the MSM energy flux is of a lower magnitude than that which is observed.

On the dusk side, we see that if the MSM peak were shifted to the right by about $1.5^\circ$, then the correlation would be very large, indicating that the MSM flux is much higher than that observed. However, the MSM peak does not fit the observed peak very well no matter which way it is shifted. We also see that the Hardy peaks are not placed well either. This cross-correlation is a way to quantify the 'goodness of fit' suggested by visual examination, but requires careful interpretation.
Case 2

Figure V.10 shows a pass over the south pole by the DMSP F9 spacecraft in the pre-noon to pre-midnight meridian beginning at about 1930 UT on day 112, still well before the onset of the first large substorm. It is immediately evident that the spacecraft passed below the auroral oval and detected no significant increase in the energy flux due to diffuse precipitation. The increase near local dusk is probably due to radiation belt electrons contaminating the detector considering the low magnetic latitude at which it occurs. Note that even though the pass was observed to be subauroral, the MSM seems to predict a grazing incidence pass, in which case the diffuse aurora predicted by the MSM would lie at lower latitudes than what was observed. However, the spacecraft never ventured poleward of ellipse C during this pass, which indicates that the MSM fluxes shown are the decay of the initial condition inside the shielding layer, not precipitation of plasma-sheet electrons. Also note that the Hardy model and the MSM produce similar profiles, although the MSM fluxes are higher.

The cross-correlation did not have to be split into dayside and nightside correlations due to the character of the precipitation. Here, of course, the MSM output is predicted to correlate best with the increase due to the Van Allen electrons. Since the MSM does not try to model the precipitation of Van Allen electrons, it must be concluded that the cross-correlation gives no new information here.

Case 3

Figure V.11 shows a south-bound pass over the nightside northern auroral zone by DMSP F9 starting at about 0 UT on day 113. This is really the first case that occurs during or after the substorm onset. The spacecraft observed a very energetic discrete arc at high latitudes. It should also be noted that energy flux is well over 1 erg/cm²-sec in the diffuse aurora, and there is no longer a 'shoulder' on the precipitation profile.
Figure V.10: Same format as Figure V.9.
Figure V.11: Same format as Figure V.9.
The MSM does a very good job of reproducing the magnitude and equatorward edge of the precipitation; however, it greatly under-estimates the poleward extent. This seems to indicate that the polar cap (i.e., ellipse A) is placed too far equatorward in this local time sector for substorm expansion phase conditions.

The Hardy model was unable to reproduce the localized character of the precipitation, and under-estimated the magnitude of the energy flux. The correlation coefficients indicate that a good fit was obtained (only 0.75° shift to the right indicated), but the value of the coefficient is extremely low, probably due to the fact that neither model is capable of reproducing the flux profile of such an intense discrete arc.

Case 4

Figure V.12 shows a pass over the north pole by the DMSP F8 spacecraft along the dawn-dusk meridian, beginning at about 0007 UT on day 113. The most notable features here are the slightly oversized MSM polar cap, and the equatorward shift of the dusk precipitation peak, indicating that the electron inner edge may be too far earthward in this local time sector. The magnitude of the fluxes near local dawn is beginning to increase, as the electrons drift around from midnight. Again, the Hardy model is unable to reproduce the localized character of the precipitation.

The cross-correlation coefficients indicate that a small poleward shift of both peaks would make the fit better. These facts again indicate that the MSM plasma sheet may be further earthward than the actual plasma sheet.

Case 5

Figure V.13 presents a pass over the north pole from the DMSP F9 satellite which begins at about 0300 UT on day 113. There is an intensification in the observed
Figure V.12: Same format as Figure V.9.
Figure V.13: Same format as Figure V.9.
Figure V.14: Same format as Figure V.9.
nightside precipitation. The MSM energy flux agrees remarkably with the observed profile on the dayside, but disagrees on the nightside. Again, the MSM polar cap appears to be too far equatorward on the nightside of the Earth, thus it predicts no precipitation where very intense fluxes were observed. Also, the latitudinal extent of the observed diffuse precipitation is large on the nightside for this pass, which the MSM does not show. The correlation coefficient is very low on the nightside for the same reason as in Case 3. On the dayside, the correlation coefficient indicates a near perfect fit, but with low fluxes. The Hardy model fails to predict the observed flux profile.

Case 6

This last pass, shown in Figure V.14, is a south pole pass by the DMSP F8 spacecraft beginning about 0420 UT on day 113. This is about 4 hours after the first substorm onset. There is much structure near local dawn, indicating passage of the spacecraft over discrete arcs. The dawn precipitation region has also grown very wide, which is characteristic after an injection of electrons near local midnight. Some of the electrons manage to drift around to the dayside of the Earth before being lost.

The MSM did a good job predicting the flux profile in this case except for the fact that it under-estimated the poleward extent of the dawn side precipitation, due to the same problem with ellipse A mentioned previously. It did a good job of predicting the magnitude of the energy flux in both peaks, even though the cross-correlation indicates otherwise on the dawn side (recall, the MSM completely missed the large region of discrete arcs, which will affect the correlation coefficient). The Hardy model did an admirable job of reproducing the dayside peak although it is featureless. The dusk peak is too broad, and shifted poleward.
From the examination of these passes and the others in appendix A, we infer these general characteristics of the MSM output:

1.) The MSM is capable of reproducing the enhanced energy fluxes over the auroral zones. The latitudinal profile, extent, and equatorward edge of the precipitation are reproduced better than by the Hardy model in most cases.

2.) The Hardy model tends to produce one or two very broad and featureless flux enhancements, which over-estimate the energy flux in the early part of the event, and under-estimate it after the first substorm onset.

3.) The MSM has trouble reproducing the flux profile of a grazing incidence pass or sub-auroral pass. It also sometimes displaced the profile from the observed peak, usually equatorward. This will be discussed shortly.

4.) The increase in the magnitude of the auroral energy fluxes observed at the end of day 112 is reproduced by the MSM. However, the model seems to have been a little enthusiastic, predicting high energy fluxes in the dawn-dusk meridian, where the energy flux was observed to be low (Appendix A).

5.) The 'shoulder' appearing in the MSM energy flux for the early part of the run is observed to decrease in intensity as the run progresses (Appendix A). It is gone for the most part by the onset of the substorm.

6.) The equatorward edge of the diffuse precipitation predicted by the MSM usually lies at lower latitudes than the edge chosen from the SSJ/4 data in all local time sectors examined (see Figures V.15).

7.) Figures V.16 show the optimum shift in latitude for the MSM energy fluxes given by the cross-correlation algorithm, versus time, for four different local time sectors over each pole. Due to the fact that the method does not place the MSM precipitation relative to the observed precipitation in an aesthetically pleasing manner, but instead just finds the maximum correlation, not much quantitative information is
Figure V.15: Equatorward Edge of Diffuse Precipitation in four Local Time Sectors. Criteria for selection of boundaries is described in Section V.B. Dotted line is the MSM boundary.

given here. Note, however, that there seems to be a slight tendency for the peaks to be placed poleward during the early part of the event (they require an equatorward shift for highest correlation), but this shifts to a tendency for the peaks to be placed equatorward after the onset of activity. There is an exception with the dusk sector over the south pole. Note also that this information contradicts that given in Figures
Figure V.16: Optimum Offset vs. Time for four Local Time Sectors over North Pole. This is simply the offset which gave the highest correlation coefficient for each pass.
Figure V.16 (continued): Optimum Offset vs. Time for four Local Time Sectors over South Pole.
V.15, that the auroral precipitation is placed equatorward of the observed precipitation during the early part of the event.

It would seem that there are two major causes for the discrepancies between the observed energy fluxes and the MSM energy fluxes. First is the inaccuracy of the electric field sub-model. Consider the observation that the equatorward boundary of the MSM's precipitation lies at lower latitudes than the observed boundary, indicating that the inner edge of the plasma sheet may be too far earthward in the MSM. This can be accounted for if the electric field model were overestimating the strength of the convection electric field across the tail of the magnetosphere. To demonstrate this, I decided to determine where the Alfvén layer lies for cold electrons under the influence of different convection electric fields. The model of Kavanagh et al. [1968], and Chen [1970] was used, and their expression for the electric potential is,

\[ V(r, \phi) = -E_0 r \sin(\phi) - \frac{\omega_E B_0 R_E^2}{r} + \frac{\mu B_0 R_E^2}{qr^3}, \]

where \( \omega_E \) is the angular frequency of the Earth, \( B_0 \) is the magnetic field strength at the surface of the Earth (0.31 Gauss), \( R_E \) is the radius of the Earth, \( \mu \) is the electron's magnetic moment, and \( E_0 \) is the convection electric field strength. The first term represents the convection electric field, the second is the co-rotation field, and the third represents the potential from gradient drift. For cold electrons, \( \mu = 0 \). The resulting equation is quadratic in \( r \),

\[ r(\phi) = \frac{-V_s - \sqrt{V_s^2 - 4E_0 \omega_E B_0 R_E^3 \sin(\phi)}}{2E_0 \sin(\phi)}. \]
\[ V_s = -E_0 r_s - \frac{\omega_E B_0 R_E^2}{r_s}, \quad r_s = \sqrt{\frac{\omega_E B_0 R_E^3}{E_0}}. \]

Figure V.17 shows the results for two different values of \( E_0 \), the outer boundary with \( E_0 = 0.3 \) mV/m, and the inner boundary with \( E_0 = 0.6 \) mV/m. Considering that the Alfvén layer is roughly coincident with the inner edge of the electron plasma sheet, this demonstrates that strong convection electric fields will push the electrons further earthward than weaker fields, which would result in the diffuse equatorward boundary being further equatorward for the stronger field when mapped to the ionosphere. This would seem to indicate that the MSM over-estimated the convection electric field.

This explanation serves well for those cases slightly before the substorm onset, and all cases afterwards. However, in the early portion of the run, the 'shoulders' observed on the energy flux profiles tended to obscure the equatorward edge. In my opinion, the disagreement between the two sets of equatorward edges for the early part of the run is caused by enthusiastic precipitation inside the shielding layer, possibly due to inaccurate initial plasma distributions, or precipitation mechanisms.

A second discrepancy may also be related to the electric field model. I commented that in many cases, the MSM appears to place the polar cap boundary at a lower latitude than that observed. This is related to our rather subjective choice of where
Figure V.17: Alfvén Layer for Cold Electrons Under Influence of Convection Electric Field of 0.3 mV/m (outer) and 0.6 mV/m (inner). Middle axis is labeled in Earth Radii.

Ellipses should lie on the digitized Heppner-Maynard-Rich ionospheric potential patterns (see section III), and to our requirement that the model boundary and the equatorward edge of the electric field reversal region (ellipse B) coincide. This tends to place the edge of region 1 at too low a latitude, and our simple scaling procedure for ellipse A reproduces the same error. As of this writing, an attempt has been made to correct this by allowing the edge of region 1 to be decoupled from the ionospheric projection of the outer modeling boundary. This has also reduced the strength of the convection electric field. Time constraint precludes a discussion of the results of this effort in this thesis, but some improvement was achieved. Optimizing the placement of these ellipses is an ongoing process.
Additional Comments

It would be desirable to be able to compare our precipitation *mechanisms* to reality and thus determine if those assumptions are a source of error. Unfortunately, the MSM only gives the total precipitating energy flux and average energy at each grid point. In order to examine the precipitation mechanisms, it would be necessary to compare the energy fluxes in each energy channel of the MSM to a similar energy channel of the SSJ/4 detector and thus help determine the dominant contributors to the total energy flux. We could also compare MSM electron energy spectra to satellite energy spectra as well.

Some of this information may be retrievable by looking at the average electron energy output and comparing it to observations. For example, a high average energy may indicate that the MSM precipitated too many high energy electrons compared to reality. However, this would only be useful if the MSM's energy channels were set up to mimic the SSJ/4's, or if interpolation were carried out. For example, Figure V.18 displays average energy output from the MSM (dashed line with little squares) and one second average energy values from the F9 spacecraft. The high values at low latitudes are due to a drop in the invariant density inside the shielding layer (magnetospheric projection of ellipse C), and should be ignored. It can be seen that on the nightside we reproduce the observed energy dispersion quite well (although offset in latitude), in addition to reproducing the correct energy values. However, it should be noted that the MSM does not predict low average energies anywhere in the auroral zone, even though they are observed. This is characteristic of the entire period studied and indeed, before the level of activity increased on day 113, the MSM overestimated the average energy. This is due to the fact that the lowest energy channel for this run was 1 keV electrons (see Appendix C); the MSM could not predict low
average energies because none of its electrons were low energy, and no information could be returned on the precipitation mechanisms.

Also, it has already been noted how well the MSM compares to the Hardy et al. model, and it is expected that it would compare just as favorably with the other average models of Spiro et al. (Spiro et al. [1982], Simons, et al. [1985]) and Evans [1985]. Both models produce the broad, featureless energy fluxes observed with the Hardy model. In addition, the Spiro et al. model suffers from a small data base and low spatial resolution (compared to the Hardy et al model), as well as low resolution in magnetic activity. The NOAA model, on the other hand, has very fine spatial resolution and good resolution in activity, using 10 activity bins. However, the activity binning is based on the NOAA auroral power index, P, a line integral of the energy flux along the spacecraft track instead of a more conventional activity index.

In addition to the average models mentioned above, one theoretical model of diffuse auroral precipitation has been put foreword by Fontaine and Blanc [1983]. Due to the
fact that the authors assume strong pitch angle scattering exists everywhere for electrons in the magnetosphere, I would expect the MSM to compare well against this model too. Strong pitch angle scattering is an upper limit on the precipitation, a fact which the authors repeatedly mention, and thus their model tends to precipitate the electrons before they can move very far in the earthward direction, causing their equatorward edge to lie further poleward than observed.
VI. Conclusions and Recommendations for Future Work

A quantitative comparison has been made of the precipitating electron energy fluxes calculated by the Magnetospheric Specification Model, the Hardy et al. average model, and observed fluxes from the DMSP spacecraft. It was found that for most passes, the MSM produced a much better prediction of the energy fluxes than the Hardy et al model. The MSM's predictions of the boundary of diffuse auroral precipitation, the latitudinal profile and extent of the precipitation, and the magnitude of the energy flux, were all much better than the Hardy et al. model. However, the MSM did have trouble reproducing the flux profiles of passes which have grazing incidence upon the auroral zone, or pass below the auroral zone.

A method was examined for quantifying the results, namely a cross-correlation of the calculated fluxes with the observed fluxes. The correlation provides some information on whether or not the fluxes were placed in the right position and whether or not the magnitude is correct. However, due to the variable nature of the observed and calculated energy fluxes, the cross-correlations must be examined carefully to eliminate any spurious information.

Both the cross-correlation and visual examination suggest that the MSM tended to overestimate the convection electric field. The main evidence is that the MSM's equatorward edge of the diffuse precipitation, which maps to the inner edge of the plasma sheet, lies equatorward of the observed boundary for most passes studied. However, some of this error may be due to an inaccurate initial plasma distribution at low L values, or inaccurate loss mechanisms in that region. The MSM initial condition has not been compared to spacecraft observations, perhaps the recent CRRES mission, which will measure lower energy electrons (100 eV - 20 keV) and ions at low L values, will provide such an opportunity for improvement.
Another error which greatly affected the storm-time precipitation was the placement of the polar cap boundary at lower latitudes than observed. This caused underestimation of the latitudinal extent and magnitude of the precipitation. This can be traced to the requirement that the equatorward edge of the electric field reversal region in the E-field sub-model is required to coincide with the ionospheric projection of the model boundary. This often places ellipses A and B at lower latitudes than observed. A more flexible method of placing the boundary of the polar cap should be found. This will allow installation of a polar cap precipitation model, and possibly allow establishment of a single parameter to evaluate the accuracy of the precipitating fluxes as is done with the geosynchronous fluxes.

Also, the large flux drop out associated with the adiabatic convection assumption and the merging of the high and low latitude precipitation models, was another source of error.

Due to the manner in which output is obtained from the MSM, the precipitation mechanisms could not be evaluated quantitatively. In the future, it might be interesting to produce the energy flux in each energy channel to allow investigation of these important assumptions.

It was found after this study was completed that the time index of the polar cap potential drop (Figure V.4) was marked at the time when the DMSP F8 spacecraft crossed the equatorial plane headed poleward, which is a shift of about 25 minutes from the appropriate time index which should be when the spacecraft is over the pole. This probably affected plasma motion, and thus the precipitation. At this time, new evaluations correcting this problem, and the problem of the strong convection field are underway. The correction for the convection field strength involves a decoupling of the ionospheric projection of the modelling boundary and ellipse B, which essentially places ellipse B a little poleward of the modelling boundary.
Future work is likely to involve post-correction of the precipitated fluxes and auroral boundaries using additional input data from the DMSP satellites. This may include an algorithm to correct the discrepancy in the low-latitude boundary by adding small penetration electric fields in order to move the inner edge of the MSM plasma sheet and thus improve agreement between the observed and calculated boundary. Also, an algorithm for correcting the precipitation profiles by using the latitude integral of the DMSP electron energy flux in 15-second intervals is under consideration. Basically, a correction factor would be determined from comparison of the MSM output and the data and a conservative extrapolation of the factor in local time would be performed.
References


Evans, D. S., Global Statistical Patterns of Auroral Phenomena, Space Environment Laboratory, Environmental Research Laboratories, National Oceanographic and Atmospheric Administration, Boulder, Colorado, 80303.


Hausman, B. A., Particle Fluxes at Geosynchronous Orbit and in the Plasma Sheet During a Substorm: Implications for Adiabatic Convection, M. S. Thesis, Rice University, 1989.


Appendix A: Comparison Plots for the April 1988 Event

The following pages contain the remaining 21 passes that were not presented in detail in Section V.B. The format is the same as described in that section. The passes are arranged in chronological order (using the time that the pass begins). At the bottom of each page are the dayside and nightside cross-correlation coefficients (dayside being defined as magnetic local times from 0600 to 1800). The procedure for generating these coefficients is described in Appendix B.
ELECTRON ENERGY FLUX: DMSP F8

UT(SEC)  | 74700. | 74880. | 75060. | 75240. | 75420. | 75600. | 75780. | 75960.
COMLAT(GEO) | 52.0   | 61.4   | 69.9   | 75.6   | 75.8   | 70.3   | 62.6   | 53.5   
MLT(HRS)   | 4.35   | 4.83   | 3.85   | 1.95   | 23.2   | 21.3   | 20.3   | 19.6   
DAY       | 112    |        |        |        |        |        |        |        

DMSP F8   |        |        |        |        |        |        |        |        
MSM       |        |        |        |        |        |        |        |        
Hardy     |        |        |        |        |        |        |        |        

Cross-Correlation of Electron Energy Fluxes

Cross-Correlation Coefficient

Cross-Correlation Coefficient

N (36018 Neoplices) i (+1: 0.25 degrees)
ELECTRON ENERGY FLUX: DMSP F8

<table>
<thead>
<tr>
<th>UT (SEC)</th>
<th>77600</th>
<th>77840</th>
<th>78120</th>
<th>78300</th>
<th>78480</th>
<th>78660</th>
<th>78840</th>
<th>79020</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMLAT (DEG)</td>
<td>-45.1</td>
<td>-54.0</td>
<td>-63.2</td>
<td>-72.3</td>
<td>-79.9</td>
<td>-80.2</td>
<td>-72.4</td>
<td>-62.8</td>
</tr>
<tr>
<td>MLT (HRS)</td>
<td>17.4</td>
<td>17.2</td>
<td>16.9</td>
<td>16.3</td>
<td>14.4</td>
<td>10.1</td>
<td>8.02</td>
<td>7.25</td>
</tr>
</tbody>
</table>

DAY 112

---

Cross-Correlation of Electron Energy Fluxes

Cross-Correlation Coefficient

Cross-Correlation Coefficient

<table>
<thead>
<tr>
<th>1.6</th>
<th>1.4</th>
<th>1.2</th>
<th>1.0</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
<th>0.2</th>
<th>0.0</th>
</tr>
</thead>
</table>

[Graphs showing cross-correlation of electron energy fluxes]
ELECTRON ENERGY FLUX: DMSP F9

UT(SEC) 85800. 85920. 86040. 86160. 86280.
COMLAT(DEC) 50.5 57.0 63.2 68.9 73.5
MLT(HRS) 8.82 8.52 8.09 7.41 6.35
DAY 112

Cross-Correlation of Electron Energy Fluxes

Cross-Correlation Coefficient

Dayside
Appendix B: Cross Correlation of the Precipitating Energy Fluxes

In order to make the analysis more quantitative, it was decided to perform a cross-correlation of the MSM output with the DMSP data for each pass, and also to correlate the Hardy fluxes for comparison. This is essentially a convolution of the two data sets, and a good reference on the theory is Cartwright [1990]. The cross-correlation function takes the form,

$$
\rho(\lambda) = \frac{\int J_{DMSP}(\Lambda)J_{MSM}(\Lambda-\lambda)d\Lambda}{\int [J_{DMSP}(\Lambda)]^2 d\Lambda},
$$

where $\Lambda$ is corrected magnetic latitude, and $\lambda$ is the latitudinal offset which maximizes the correlation. This is what we wish to determine. This correlation function is not normalized and thus it will give information on the relative fluxes at the optimum latitudinal offset, with a value higher than one indicating that the model fluxes are higher than the observed fluxes.

The SSJ/4 sampling rate is constant in time but not latitude, therefore, some interpolation of the spacecraft ephemeris was necessary to obtain a constant sampling rate. Note that the fluxes were not interpolated, instead, at the time values determined for each latitude, the raw one second average total energy flux was used. Comparison of the raw flux profiles with the 15 second averages presented in this work show marked disagreement only in the polar cap. Since the MSM does not produce these fluxes, this was deemed unimportant.

The correlation coefficient was calculated for many values of $\lambda$ for each pass, and the coefficients shown in section V are essentially plots of $\rho(\lambda)$ vs. $\lambda$. For a perfect
linear correlation between the observed and calculated fluxes, there should be a maximum at zero and the coefficient should be symmetric about zero. A displacement of the maximum from zero indicates that the calculated profile is offset by some amount from the observed profile. Also, a non-symmetric peak implies that the latitudinal extent of the calculated peak does not agree with the observation. However, for multi-peaked phenomena such as the energy fluxes, spurious information is easily produced by this type of analysis. For example, most passes were split into nightside and dayside for correlation because the correlation of the calculated dayside peak with the observed nightside peak produces useless information. Most grazing passes did not have to be split in that manner. Careful interpretation is required.
Appendix C: MSM Energy Channels

For the run discussed in this thesis, the electron energies traced by the MSM are given in the following table. Note that these are electron energies at geosynchronous orbit.

Table C.I: MSM Energy Channels

<table>
<thead>
<tr>
<th>Channel Number</th>
<th>Energy at 6.6 R_E (eV)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1000.00</td>
</tr>
<tr>
<td>2</td>
<td>1778.28</td>
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<tr>
<td>3</td>
<td>3162.28</td>
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<td>4</td>
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<td>5</td>
<td>10000.00</td>
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<tr>
<td>6</td>
<td>17782.79</td>
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<tr>
<td>7</td>
<td>37000.00</td>
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<tr>
<td>8</td>
<td>54000.00</td>
</tr>
<tr>
<td>9</td>
<td>80000.00</td>
</tr>
</tbody>
</table>