INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI
University Microfilms International
A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
313/761-4700 800/521-0600
Investigation of a gamma guidance scheme for flight in windshear

Aizawa, Takeshi, M.S.

Rice University, 1992
RICE UNIVERSITY

Investigation of a Gamma Guidance Scheme for Flight in Windshear

by

Takeshi Aizawa

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE Master of Science

APPROVED THESIS COMMITTEE:

[Signatures and names of committee members]

Houston, Texas
May, 1992
ABSTRACT

Investigation of a Gamma Guidance Scheme for Flight in Windshear

by

Takeshi Aizawa

This thesis refers to windshear recovery systems, designed to increase the survival capability of an aircraft. With reference to the take-off problem, it is known that optimal trajectories are difficult to implement because of the lack of global information on the wind flow field and the lack of enough time and computing capability onboard. Consequently, one is forced to employ local/prior information on the windshear and the downdraft.

Within the above context, an investigation of the gamma guidance scheme is presented with reference to the take-off problem. Attention is focused on the feedback control form of the gamma guidance law. This law is modified so that not only the aircraft can react to unfavorable shears, but can take advantage of favorable shears. Indeed, every unfavorable shear (central core of the downburst) is both preceded and followed by favorable shears.

The windshear efficiency of the gamma guidance law depends on the proper choice of several parameters. Therefore, a systematic investigation of the effect of these quantities on survival capability is presented for two windshear models. The results indicate that, through a proper choice of the parameters, the windshear efficiency of the gamma guidance law is within 5% of that of an optimal trajectory.
ACKNOWLEDGMENTS

The author expresses sincere gratitude to his advisor, Dr. A. Miele, for suggesting the topic of this thesis and for providing guidance throughout the ensuing research. Thanks are also due Dr. T. Wang for many helpful discussions and consultations.

The contributions of the other thesis committee members, Dr. R. Cohen and Dr. D. Dyson, are also gratefully acknowledged.

This research was supported by Nikkiso Company, Tokyo, Japan and by Texas Advanced Technology Program, Grant No. TATP-003604020.

DEDICATION

This thesis is dedicated to Mr. Keijiro Oto, Chairman of the Board, Nikkiso Group, and Mr. Masahiko Hatano, President, Nikkiso Co., Ltd., for their understanding and giving me the opportunity to study abroad.

This thesis is also dedicated to Dr. Yukihiro Nosé, Professor, Baylor College of Medicine, to his wife Ako, and to Dr. Setsuo Takatani, Adjunct Professor, Baylor College of Medicine, for their continued support.

Finally I also dedicate this thesis to my wife Eika for her loving support and selfless backing from the start and all throughout the pursuit of this degree.
Table of Contents

Abstract
Acknowledgments
List of Tables
List of Figures
Notations

1. Introduction
2. System Description
   2.1. Equations of Motion
   2.2. Approximations for the Forces
   2.3. Shear/Downdraft Factor
3. Aircraft Data
4. Wind Models
   4.1. Wind Model 1
   4.2. Wind Model 2
5. Gamma Guidance
   5.1. Guidance Law
   5.2. Feedback Control
   5.3. Nominal Angle of Attack
   5.4. Guidance Parameters and Constants
6. Numerical Results
7. Summary and Conclusions

References
Tables
Figures
List of Tables

Table 1A. Spline nodal points, horizontal wind, WM1.
Table 1B. Spline lengths, horizontal wind, WM 1.
Table 1C. Spline coefficients, horizontal wind, WM 1.
Table 1D. Summary results, horizontal wind, WM 1, \( \lambda = 1 \).
Table 2A. Spline nodal points, vertical wind, WM 1.
Table 2B. Spline lengths, vertical wind, WM 1.
Table 2C. Spline coefficients, vertical wind, WM 1.
Table 2D. Summary results, vertical wind, WM 1, \( \lambda = 1, h = h_* \).
Table 3. Wind components and their derivatives, WM 2, \( \lambda = 1, h = h_0 \).
Table 4A. Survival capability \( \Delta W_{xc}(fps) \), gamma guidance,
\( C_2 = 4.0 \), Wind Model 1, \( x_0 = -2300 \) ft.
Table 4B. Survival capability \( \Delta W_{xc}(fps) \), gamma guidance,
\( C_2 = 5.0 \), Wind Model 1, \( x_0 = -2300 \) ft.
Table 4C. Survival capability \( \Delta W_{xc}(fps) \), gamma guidance,
\( C_1 = 1.0 \), Wind Model 1, \( x_0 = -2300 \) ft.
Table 5A. Survival capability \( \Delta W_{xc}(fps) \), gamma guidance,
\( C_2 = 4.0 \), Wind Model 2, \( x_0 = -6000 \) ft.
Table 5B. Survival capability \( \Delta W_{xc}(fps) \), gamma guidance,
\( C_2 = 6.5 \), Wind Model 2, \( x_0 = -6000 \) ft.
Table 5C. Survival capability \( \Delta W_{xc}(fps) \), gamma guidance,
\( C_1 = 1.0 \), Wind Model 2, \( x_0 = -6000 \) ft.
Table 6. Comparison of the survival capability \( \Delta W_{xc}(fps) \) for different gamma guidance schemes.
List of Figures

Fig. 1. Cross section of a microburst.

Fig. 2. Coordinate system and force diagram.

Fig. 3A. Nodal points and splines, horizontal wind, WM 1.

Fig. 3B. Nodal points and splines, vertical wind, WM 1.

Fig. 4A. Horizontal wind velocity component, WM 1, \( \lambda = 1 \).

Fig. 4B. Vertical wind velocity component, WM 1, \( \lambda = 1 \).

Fig. 5. Vortex ring pair.

Fig. 6A. Horizontal wind velocity difference vs altitude, WM 2, \( \lambda = 1 \).

Fig. 6B. Vertical wind velocity difference vs altitude, WM 2, \( \lambda = 1 \).

Fig. 6C. Horizontal wind velocity component, WM 2, \( \lambda = 1 \).

Fig. 6D. Vertical wind velocity component, WM 2, \( \lambda = 1 \).

Fig. 7A. Typical optimal trajectories:
   altitude vs time.

Fig. 7B. Typical optimal trajectories:
   absolute path inclination vs time.

Fig. 8. Nominal angle of attack vs relative velocity.

Fig. 9A. Gamma guidance trajectories, WM 1, \( x_0 = -2300 \text{ ft} \),
   altitude vs time.

Fig. 9B. Gamma guidance trajectories, WM 1, \( x_0 = -2300 \text{ ft} \),
   wind velocity components vs time.

Fig. 9C. Gamma guidance trajectories, WM 1, \( x_0 = -2300 \text{ ft} \),
   relative velocity vs time.

Fig. 9D. Gamma guidance trajectories, WM 1, \( x_0 = -2300 \text{ ft} \),
   shear/downdraft factor vs time.

Fig. 10A. Gamma guidance trajectories, WM 1, \( x_0 = -2300 \text{ ft} \),
   altitude vs time.
Fig. 10B. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, wind velocity components vs time.

Fig. 10C. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, relative velocity vs time.

Fig. 10D. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, shear/downdraft factor vs time.

Fig. 11A. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, altitude vs time.

Fig. 11B. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, wind velocity components vs time.

Fig. 11C. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, relative velocity vs time.

Fig. 11D. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, shear/downdraft factor vs time.

Fig. 12A. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, altitude vs time.

Fig. 12B. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, wind velocity components vs time.

Fig. 12C. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, relative velocity vs time.

Fig. 12D. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, shear/downdraft factor vs time.
Notations

Wind Models

\( h \) = altitude, ft ;

\( H \) = height of vortex ring, ft ;

\( r \) = radial distance, ft ;

\( R \) = radius of vortex ring, ft ;

\( W_h \) = wind velocity component in the h-direction, ft sec \( ^{-1} \) ;

\( W_r \) = wind velocity component in the radial direction, ft sec \( ^{-1} \) ;

\( W_x \) = wind velocity component in the x-direction, ft sec \( ^{-1} \) ;

\( W_y \) = wind velocity component in the y-direction, ft sec \( ^{-1} \) ;

\( x \) = horizontal distance, ft ;

\( y \) = lateral distance, ft ;

\( \Gamma \) = circulation strength, ft \( ^2 \) sec \( ^{-1} \);  

\( \mu \) = argument of a complete elliptic integral ;

\( \psi \) = stream function .

Aircraft Trajectories

\( ARL \) = aircraft reference line ;

\( C_D \) = drag coefficient ;

\( C_L \) = lift coefficient ;

\( D \) = drag force, lb ;

\( g \) = acceleration of gravity, ft sec \( ^{-2} \) ;

\( h \) = altitude, ft ;

\( K \) = gain coefficient ;

\( L \) = lift force, lb ;

\( m \) = mass, lb ft \( ^{-1} \) sec\( ^2 \) ;

\( S \) = reference surface area, ft\( ^2 \) ;

\( T \) = thrust force, lb ;
\[ V = \text{relative velocity, } \text{ft sec}^{-1} ; \]
\[ V_e = \text{absolute velocity, } \text{ft sec}^{-1} ; \]
\[ W = mg = \text{weight, } \text{lb} ; \]
\[ \alpha = \text{relative angle of attack, } \text{rad} ; \]
\[ \alpha_e = \text{absolute angle of attack, } \text{rad} ; \]
\[ \beta = \text{engine power setting} ; \]
\[ \gamma = \text{relative path inclination, } \text{rad} ; \]
\[ \gamma_e = \text{absolute path inclination, } \text{rad} ; \]
\[ \delta = \text{thrust inclination, } \text{rad} ; \]
\[ \theta = \text{pitch altitude angle, } \text{rad} ; \]
\[ \rho = \text{air density, } \text{lb ft}^{-4} \text{sec}^2 . \]

**Superscripts**

\[ \cdot = \text{derivative with respect to time} ; \]
\[ \sim = \text{nominal condition} . \]

**Subscripts**

\[ 1 = \text{primary ring} ; \]
\[ 2 = \text{image ring} . \]
1. Introduction

Development of effective guidance schemes for aircraft flight in windshear has been a demanding task for the past decade. Survey records covering a 20-year period include some 30 airplane accidents in take-off or landing which can be attributed to windshear (Ref. 1). Especially two recent aircraft accidents, involving considerable loss of life, have focused the attention of the engineering and scientific community on the windshear problem: one occurred at New Orleans International Airport (Pan Am Flight 759 on July 9, 1982) and involved a Boeing B-727 in takeoff; the other occurred at Dallas-Fort Worth International Airport (Delta Airlines Flight 191 on August 2, 1985) and involved a Lockheed L-1011 in landing (Refs. 2-3).

The cause of the above accidents is a particular meteorological condition, called the downburst by Fujita (Ref. 4-5). In a downburst, there is a descending column of air, which then spreads horizontally in the neighborhood of the ground. Hence, there is a shear coupled with downdraft and updraft (Fig. 1). In this situation, an aircraft in take-off or landing encounters a headwind coupled with a downdraft, followed by a tailwind coupled with a downdraft. Associated with the headwind-to-tailwind transition is a transport acceleration, hence a windshear inertia force, which in some cases can be as large as the drag of the aircraft or the thrust of the engines. Under these conditions, the controllability of the aircraft is diminished, occasionally resulting in a crash.

Considerable work has been done at Rice University over the past eight years on the optimization and guidance of flight trajectories in windshear. For the take-off problem, optimal trajectories were investigated in Refs. 6-8 under the assumption that global information on the wind flow field is known in advance. It was concluded that: (i) for weak-to-moderate windshears, the optimal trajectories are characterized by a monotonic climb; and (ii) for severe windshears, the optimal trajectories are characterized by an initial climb, followed by nearly-horizontal flight, followed by renewed climbing after the aircraft
has passed through the shear region.

In practice, an optimal trajectory is difficult to implement for two reasons: global information on the wind flow field might not be available; even if it were available, there might not be enough time and computing capability onboard to process it adequately. As a consequence, the significance of an optimum trajectory is that it constitutes an ideal benchmark for developing guidance laws as well as assessing the relative merits of guidance schemes.

Since global information on the wind flow field is not available, one is forced to employ local/prior information on the windshear and downdraft. It is in this context that guidance schemes approximating the properties of the optimal trajectories were developed at Rice University, specifically the gamma guidance and acceleration guidance (Refs. 9-11).

This thesis continues the work of Refs. 9-11. With reference to the take-off problem, it focuses attention on the feedback control form of the gamma guidance law. This law is modified in such a way that not only the aircraft can react to unfavorable shears, but can take advantage of favorable shears. Indeed, it is known that every unfavorable shear (central core of the downburst) is both preceded and followed by favorable shears.

Two windshear models are used in this thesis. One is the idealized wind model used in Refs. 9-11, and the other is the wind model produced by a pair of vortex rings, the primary ring and the image ring, symmetrically located with respect to the ground plane and having circulation strengths identical in modulus, but opposite in sign.

From fluid mechanics, it is known that the induced velocity at any point of the flow field of a vortex line can be computed using the Biot-Savart law or the properties of the stream function (Refs. 12-14). Both methods produce exactly the same flow field. In this thesis, the method employing the properties of the stream function is used.
2. System Description

In this thesis, we make use of the relative wind-axes system (Fig. 2) in connection with the following assumptions:

(i) the aircraft is a particle of constant mass;
(ii) flight takes place in a vertical plane containing the axis of the microburst.
(iii) Newton's law is valid in an Earth-fixed system;
(iv) the wind flow field is steady.

2.1. Equations of Motion

With the above premises, the equation of motion include the kinematic equations (Ref. 6)

\[ \dot{x} = V \cos \gamma + W_x, \]  \hspace{1cm} (1a)
\[ \dot{h} = V \sin \gamma + W_h, \]  \hspace{1cm} (1b)

and the dynamic equations (Ref. 6)

\[ V = \frac{T}{m} \cos (\alpha + \delta) - \frac{D}{m} - g \sin \gamma - (\dot{W}_x \cos \gamma + \dot{W}_h \sin \gamma), \] \hspace{1cm} (2a)
\[ \dot{\gamma} = \frac{T}{mV} \sin (\alpha + \delta) + \frac{L}{mV} - (g/V) \cos \gamma + (1/V) (\dot{W}_x \sin \gamma - \dot{W}_h \cos \gamma). \] \hspace{1cm} (2b)

Because of the assumption (iv), the total derivatives of the wind velocity components and the corresponding partial derivatives satisfy the relations

\[ W_x = \frac{\partial W_x}{\partial x} (V \cos \gamma + W_x) + \frac{\partial W_x}{\partial h} (V \sin \gamma + W_h), \] \hspace{1cm} (3a)
\[ \dot{W}_h = \frac{\partial W_h}{\partial x} (V \cos \gamma + W_x) + \frac{\partial W_h}{\partial h} (V \sin \gamma + W_h). \] \hspace{1cm} (3b)

These equations must be supplemented by the functional relations
\[ T = T(h, V, \beta), \]  
\[ D = D(h, V, \alpha), \quad L = L(h, V, \alpha), \]  
\[ W_x = W_x(x, h), \quad W_h = W_h(x, h), \]

and the analytical relations

\[ V_e = \sqrt{(V \cos \gamma + W_x)^2 + (V \sin \gamma + W_h)^2}, \]  
\[ \gamma_e = \arctan \frac{V \sin \gamma + W_h}{V \cos \gamma + W_x}, \]  
\[ \theta = \alpha + \gamma = \alpha_e + \gamma_e. \]  

For a given value of the thrust inclination \( \delta \), the differential system (1)-(5) involves four state variables [the horizontal distance \( x(t) \), the altitude \( h(t) \), the velocity \( V(t) \), and the relative path inclination \( \gamma(t) \)] and two control variables [the angle of attack \( \alpha(t) \) and the power setting \( \beta(t) \)]. However, the number of control variables reduces to one [the angle of attack \( \alpha(t) \)] if the power setting \( \beta(t) \) is specified in advance. The quantities defined by the analytical relations (5) can be computed a posteriori once the values of \( x, h, V, \gamma, \alpha, \beta \) are known.

The angle of attack \( \alpha \) and its time derivative \( \dot{\alpha} \) are subject to the inequalities

\[ \alpha \leq \alpha_*, \]  
\[ -\dot{\alpha}_* \leq \dot{\alpha} \leq \dot{\alpha}_*, \]

where \( \alpha_* \) is a prescribed upper bound and \( \dot{\alpha}_* \) is a prescribed positive constant.

2.2. Approximations for the Forces

In this section, we discuss the approximations employed for the thrust, the drag, the lift, and the weight. Because the trajectories under investigation involve relatively minor variations of the altitude, the air density is assumed to be constant.

Thrust. The thrust \( T \) is approximated with the quadratic function
\[ T = A_0 + A_1 V + A_2 V^2, \]  

(7)

where \( V \) is the relative velocity. The coefficients \( A_0, A_1, A_2 \) depend on the altitude of runway, the ambient temperature, and the engine power setting. They can be determined via a least-square fit of manufacturer-supplied data over a given velocity range.

**Drag.** The drag \( D \) is written in the form

\[ D = (1/2) C_D \rho S V^2, \]  

(8a)

\[ C_D = B_0 + B_1 \alpha + B_2 \alpha^2, \quad \alpha \leq \alpha_*, \]  

(8b)

where \( \rho \) is the air density, \( S \) is a reference surface, \( V \) is the relative velocity, and \( C_D \) is the drag coefficient. The coefficients \( B_0, B_1, B_2 \) depend on the flap setting and the undercarriage position (gear up or gear down). They can be determined via a least-square fit of manufacturer-supplied data over a given angle-of-attack interval.

**Lift.** The lift \( L \) is written in the form

\[ L = (1/2) C_L \rho S V^2, \]  

(9a)

\[ C_L = C_0 + C_1 \alpha, \quad \alpha \leq \alpha_{**}, \]  

(9b)

\[ C_L = C_0 + C_1 \alpha + C_2 (\alpha - \alpha_{**}), \quad \alpha_{**} \leq \alpha \leq \alpha_*, \]  

(9c)

where \( \rho \) is the air density, \( S \) is a reference surface, \( V \) is the relative velocity, and \( C_L \) is the lift coefficient. The coefficients \( C_0, C_1, C_2 \) depend on the flap setting and the undercarriage position (gear up or gear down). They can be determined via a least-square fit of manufacturer supplied data over a given angle-of-attack range.

**Weight.** The mass \( m \) is regarded as constant. Hence, the weight \( W = mg \) is also regarded as constant.
2.3. Shear/Downdraft Factor

Let $E$ denote the aircraft energy per unit weight,

$$E = h + V^2/2 \, g.$$  \quad (10)

Upon taking the time derivative of both sides of Eq. (10), we obtain the relation

$$\dot{E} = h + V \dot{V} / g,$$  \quad (11)

which in light of (1b) and (2a) can be rewritten as

$$\dot{E} = \left[ (T/W) \cos (\alpha + \delta) - D/W - F \right] V,$$  \quad (12)

where $F$ is the shear/downdraft factor (Ref. 15),

$$F = \left( \dot{W}_x / g \right) \cos \gamma + \left( \dot{W}_h / g \right) \sin \gamma - \dot{W}_h / V,$$  \quad (13a)

which simplifies to

$$F = \dot{W}_x / g - \dot{W}_h / V,$$  \quad (13b)

if $\gamma$ is small and if $\dot{W}_h$ is at most of the same order as $\dot{W}_x$. Clearly, the shear/downdraft factor combines the effect of the shear and downdraft into a single entity. Note that $F > 0$ for an unfavorable shear, $F < 0$ for a favorable shear, and $F = 0$ if there is no shear nor downdraft.

In the absence of shear and downdraft, $F = 0$ and Eq. (12) simplifies to

$$\dot{E}_0 = \left[ (T/W) \cos (\alpha + \delta) - D/W \right] V.$$  \quad (14)

On the other hand, in the presence of shear and downdraft, $F \neq 0$ and Eq. (12) can be rewritten as

$$\dot{E} = \dot{E}_0 - F \, V.$$  \quad (15)
Therefore, the product $FV$ represents the degradation of the energy time rate due to the combined effect of the shear and downdraft. This is why the quantity described by either of Eqs. (13) has considerable importance in guidance.
3. Aircraft Data

The aircraft under consideration is a Boeing B-727, powered by three JT8D-17 turbofan engines. The following assumptions are employed.

(i) the aircraft flies in a vertical plane containing the axis of symmetry of the downburst;

(ii) the aircraft becomes airborne from a runway located at sea level altitude;

(iii) the engines are operating at maximum power setting,
\[ \beta = 1, \quad \dot{\beta} = 0; \]  

(iv) the gear is up and flap setting is \( \delta_F = 15.0 \) deg;

(v) the inequality constraints (6) on the angle of attack and its time derivative are enforced with
\[ \alpha_{**} = 12.0 \text{ deg}, \quad \alpha_* = 16.0 \text{ deg}, \]  
\[ \dot{\alpha}_* = 3.0 \text{ deg/sec}; \]  

(vi) the ambient temperature is \( T_a = 100 \) deg Fahrenheit.

Thrust. At low altitudes, the dependence of the thrust on the altitude is disregarded, and the thrust is assumed to depend on the velocity only. At \( t = 0 \), the thrust function is represented by Eqs.(7), with

\[ A_0 = 0.4456 \times 10^5 \text{ lb}, \]  
\[ A_1 = -0.2398 \times 10^2 \text{ lb ft} \cdot \text{sec}, \]  
\[ A_2 = 0.1442 \times 10^1 \text{ lb ft} \cdot \text{sec}^2. \]  

The thrust inclination with respect to the aircraft reference line is assumed to be

\[ \delta = 0.2000 \times 10^1 \text{ deg}. \]  

Drag. The dependence of the density on the altitude is disregarded, and the drag is assumed to be depend on the velocity and the angle of attack only. The drag function is
represented by Eqs. (8), with

\[ \rho = 0.2203 \times 10^{-2} \text{ lb ft}^{-3} \text{ sec}^2, \]  
\[ S = 0.1560 \times 10^4 \text{ ft}^2, \]  
\[ B_0 = 0.7351 \times 10^{-1}, \]  
\[ B_1 = -0.8617 \times 10^{-1}, \]  
\[ B_2 = 0.1996 \times 10^1. \]  

Lift. Like the drag, the lift is assumed to depend on the velocity and the angle of attack only. The lift function is represented by Eqs. (9), with \( \rho, S \) given by Eqs. (20) and

\[ C_0 = 0.1667, \]  
\[ C_1 = 0.6231 \times 10^1, \]  
\[ C_2 = -0.2165 \times 10^2. \]  

Weight. Disregarding the fuel consumed, the weight of the aircraft is assumed to be constant and its value is

\[ W = 0.1800 \times 10^6 \text{ lb}. \]  

Initial Conditions. At \( t = 0 \), the aircraft is assumed to be in quasi-steady climbing flight. The initial conditions are as follows:

\[ x(0) = -2300 \text{ ft}, \text{ WM 1}, \]  
\[ x(0) = -6000 \text{ ft}, \text{ WM 2}, \]  
\[ h(0) = 50 \text{ ft}, \]  
\[ V(0) = 276.8 \text{ fps}, \]  
\[ \gamma(0) = 6.989 \text{ deg} = 0.1220 \text{ rad}, \]  
\[ \alpha(0) = 10.36 \text{ deg} = 0.1808 \text{ rad}. \]
Final Conditions. The final time is assumed to be

\[ \tau = 40 \text{ sec}, \text{ WM 1}, \]  

\[ \tau = 60 \text{ sec}, \text{ WM 2}. \]
4. Wind Models

A downburst (in particular, a microburst) consists of a descending column of air, which then spreads horizontally in the neighborhood of the ground (Fig. 1). Although no two microbursts are exactly alike, two basic phenomena always exist: shear and downdraft. Therefore, it is important that these essential characteristics be present in the wind models employed in optimization and guidance studies. These characteristics are present in the two wind models employed in this thesis.

4.1. Wind Model 1

This is an approximate wind model having the following properties:

(a) there is a transition from a uniform headwind to a uniform tailwind, with nearly constant shear in the core of the downburst;

(b) the downdraft achieves maximum negative value at the center of the downburst;

(c) the downdraft vanishes on the ground;

(d) the wind velocity components satisfy approximately the continuity equation and the irrotationality condition in the core of the downburst;

(e) random effects due to free-stream turbulence are neglected.

Wind Model 1 is represented by the relations

\[ W_x = \lambda A(x), \quad W_h = \lambda \left(\frac{h}{h_0}\right) B(x), \]  

(26)

and

\[ \Delta W_x = \lambda \Delta W_{x_0}, \quad \Delta W_h = \lambda \left(\frac{h}{h_0}\right) \Delta W_{h_0}. \]  

(27)

Here, \( W_x \) is the horizontal wind, \( W_h \) is the vertical wind, \( \Delta W_x \) is the horizontal wind velocity difference (maximum tailwind minus maximum headwind at constant altitude), and \( \Delta W_h \) is the vertical wind velocity difference (maximum updraft minus maximum downdraft
at constant altitude. Also, $A(x)$ is a shape function for the horizontal wind, $B(x)$ is a shape function for the vertical wind, $h_0 = 1000$ ft is a reference altitude, $\Delta W_{x_0} = 100$ fps is a reference value for the horizontal wind velocity difference, and $\Delta W_{h_0} = 50$ fps is a reference value for the vertical wind velocity difference. Finally, $\lambda$ is a parameter characterizing the intensity of the shear/downdraft combination.

Decreasing values of $\lambda$ (hence, smaller values of $\Delta W_x$ and $\Delta W_h$) correspond to milder windshears; conversely, increasing values of $\lambda$ (hence, larger values of $\Delta W_x$ and $\Delta W_h$) correspond to stronger windshears. Therefore, by changing the value of $\lambda$, one can generate shear/downdraft combinations ranging from extremely mild to extremely severe.

For the horizontal wind, the shape function $A(x)$ is represented via seven cubic splines involving the nodal points $P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7$ (Fig. 3A). Each spline is represented by the relations

$$A = C \left( A_0 + A_1 y + A_2 y^2 + A_3 y^3 \right), \quad (28a)$$

$$y = \frac{x - x_L}{x_R - x_L}, \quad (28b)$$

$$C = (1/2) \Delta W_{x_0} = 50 \text{ fps}, \quad (28c)$$

in which the dimensionless distance $y$ is normalized in such a way that $y_L = 0$ and $y_R = 1$. At the nodal points, the continuity of $A$, $dA/dx$, $d^2A/dx^2$ is enforced. This leads to the results summarized in Table 1.

Table 1A gives the position of the nodal points. Table 1B gives the length of each spline together with the positions of the left and right boundaries of each spline. Table 1C gives the coefficients $A_0, A_1, A_2, A_3$ for each spline. Table 1D supplies summary results for $W_x, \partial W_x/\partial x, \partial W_x/\partial h$ under the assumption that the wind intensity parameter is $\lambda = 1$, corresponding to $\Delta W_x = 100$ fps. See also Fig. 4A.
For the vertical wind, the shape function $B(x)$ is represented via seven cubic splines involving the nodal points $P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7$ (Fig. 3B). Each spline is represented by the relations

$$B = C (B_0 + B_1 y + B_2 y^2 + B_3 y^3), \quad (29a)$$

$$y = (x - x_L)/(x_R - x_L), \quad (29b)$$

$$C = \Delta W_{h*} = 50 \text{ fps}, \quad (29c)$$

in which the dimensionless distance $y$ is normalized in such a way that $y_L = 0$ and $y_R = 1$. At the nodal points, the continuity of $B$, $dB / dx$, $d^2B / dx^2$ is enforced. This leads to the results summarized in Table 2.

Table 2A gives the position of the nodal points. Table 2B gives the length of each spline together with the positions of the left and right boundaries of each spline. Table 2C gives the coefficients $B_0, B_1, B_2, B_3$ for each spline. Table 2D supplies summary results for $W_h, \partial W_h / \partial x, \partial W_h / \partial h$ under the assumption that the wind intensity parameter is $\lambda = 1$ and the altitude is $h = h*$, corresponding to $\Delta W_h = 50 \text{ fps}$. See also Fig. 4B.

4.2. Wind Model 2

This wind model retains properties (a) through (e) of Wind Model 1, with two exceptions: concerning (a), there is no assumption of uniformity of the wind before and after the central core of the downburst; concerning (d), the wind velocity components satisfy "exactly" the continuity equation and the irrotationality condition.

Assuming that the downburst is an axisymmetric phenomenon and that the axis of symmetry is vertical, the flow field can be modelled by postulating the presence of two horizontal vortex rings of radius $R$ (Refs.12-14) : the primary ring, having circulation $+\Gamma$, is located at altitude $+H$ above ground; the image ring, having circulation $-\Gamma$, is located at
altitude - H below ground (Fig. 5). If one assumes steady flow and uses either the Biot-Savart law or the properties of the stream function, Wind Model 2 is represented by the relations

\[ W_r = \lambda A(r,h), \quad W_h = \lambda B(r,h), \quad (30) \]

and

\[ \Delta W_r = \lambda F(h), \quad \Delta W_h = \lambda G(h). \quad (31) \]

Here, \( W_r \) is the horizontal wind, \( W_h \) is the vertical wind, \( \Delta W_r \) is the horizontal wind velocity difference (maximum tailwind minus maximum headwind at constant altitude), and \( \Delta W_h \) is the vertical wind velocity difference (maximum updraft minus maximum downdraft at constant altitude). Also, \( A(r,h) \) is a shape function for the horizontal wind, \( B(r,h) \) is a shape function for the vertical wind, \( F(h) \) is a shape function for the horizontal wind velocity difference, and \( G(h) \) is a shape function for the vertical wind velocity difference. The parameter \( \lambda \) characterizes the intensity of the shear/downdraft and is proportional to the circulation strength \( \Gamma \). While the shape functions \( A(r, h) \), \( B(r,h) \), \( F(h) \), \( G(h) \) depend on \( H \) and \( R \), they are independent of \( \lambda \).

**Stream Function.** For a flow field which is incompressible, irrotational, and axisymmetric, the wind velocity components can be written in terms of the stream function using either cylindrical coordinates \((r, \theta, h)\) or Cartesian coordinates \((x, y, h)\). Specifically, in cylindrical coordinates, we have

\[ W_r = \frac{1}{r} \frac{\partial \psi}{\partial h}, \quad (32a) \]

\[ W_\theta = 0, \quad (32b) \]
\[ W_h = -\frac{1}{r} \frac{\partial \psi}{\partial r}; \]  
\hspace{10em} (32c)

Alternatively, in Cartesian coordinates, we have

\[ W_x = \frac{x}{r} W_r, \]  
\hspace{10em} (33a)

\[ W_y = \frac{y}{r} W_r, \]  
\hspace{10em} (33b)

\[ W_h = \frac{1}{r} \frac{\partial \psi}{\partial r}. \]  
\hspace{10em} (33c)

Therefore, if the stream function \( \psi = \psi(r, h) \) is known, the wind velocity components can be computed with either (32) or (33).

Let \( \psi \) denote the stream function produced by the vortex ring pair; let \( \psi_1 \) and \( \psi_2 \) denote the primary and image contributions, respectively. Because of the linearity of the problem, the following relation holds:

\[ \psi = \psi_1 + \psi_2. \]  
\hspace{10em} (34)

For given values of \( x, y, h \) (hence, for given point \( P \)), let \( a_1, b_1 \) denotes the minimum and maximum values of the distance \( PP_1 \) with respect to \( \theta \); let \( a_2, b_2 \) denote the minimum and maximum values of the distance \( PP_2 \) with respect to \( \theta \). Therefore, by definition,

\[ a_1 = \min |p_1 - \rho| = \sqrt{(r - R)^2 + (h - H)^2}, \]  
\hspace{10em} (35a)

\[ b_1 = \max |p_1 - \rho| = \sqrt{(r + R)^2 + (h - H)^2}, \]  
\hspace{10em} (35b)

and

\[ a_2 = \min |p_2 - \rho| = \sqrt{(r - R)^2 + (h + H)^2}, \]  
\hspace{10em} (36a)
\[ b_2 = \max |p_2 - p| = \sqrt{(r + R)^2 + (h + H)^2} , \]  
\[ \text{where} \]
\[ r = \sqrt{x^2 + y^2} . \]  

Let \( \mu_1, \mu_2 \) denote the ratios

\[ \mu_1 = \frac{b_1 - a_1}{b_1 + a_1} , \]  
\[ \mu_2 = \frac{b_2 - a_2}{b_2 + a_2} . \]  

With this understanding, the components of the stream function can be written as

\[ \psi_1 = + \frac{\Gamma}{2} (a_1 + b_1) \frac{\mu_1^2}{1 + 3 \sqrt{1 - \mu_1^2}} , \]  
\[ \psi_2 = - \frac{\Gamma}{2} (a_2 + b_2) \frac{\mu_2^2}{1 + 3 \sqrt{1 - \mu_2^2}} , \]  

so that the stream function (34) becomes

\[ \psi = + \frac{\Gamma}{2} (a_1 + b_1) \frac{\mu_1^2}{1 + 3 \sqrt{1 - \mu_1^2}} - \frac{\Gamma}{2} (a_2 + b_2) \frac{\mu_2^2}{1 + 3 \sqrt{1 - \mu_2^2}} . \]  

With reference to Eqs. (32)-(40), the following comments are in order:

(i) With the stream function known, the wind velocity components (32) or (33) can be computed, once the partial derivatives appearing in (32) are computed.

For example, if a central difference technique is used, we have

\[ \frac{\partial \psi}{\partial r} = \frac{\psi(r + \Delta r, h) - \psi(r - \Delta r, h)}{2\Delta r} , \]  

(41a)
\[
\frac{\partial \psi}{\partial h} = \frac{\psi(r, h + \Delta h) - \psi(r, h - \Delta h)}{2\Delta h}.
\]

(ii) Equations (32) or (33) can be used everywhere in the axisymmetric flow field, except on the axis of symmetry. On the axis, a singularity occurs, but can be removed via application of the l'Hospital rule. This leads to the relations

\[
W_r = 0,
\]

\[
W_h = -\frac{\Gamma}{2R} \left[ 1 + \left(\frac{h - H}{R}\right)^2 \right]^{-1.5} + \frac{\Gamma}{2R} \left[ 1 + \left(\frac{h + H}{R}\right)^2 \right]^{-1.5}.
\]

(iii) At the center of the primary ring, \( h = H \); hence, Eq. (42b) yields

\[
W_h = -\frac{\Gamma}{2R} + \frac{\Gamma}{2R} \left[ 1 + \left(\frac{2H}{R}\right)^2 \right]^{-1.5}.
\]

(iv) On the ground, \( h = 0 \); hence, Eq. (42b) yields

\[
W_h = 0.
\]

(v) At low altitudes, where \( h / R \) is small, a Taylor expansion of Eq. (42b) leads to

\[
W_h = -\frac{3\Gamma HR^2}{(R^2 + H^2)^{2.5}} h.
\]

Clearly, the vertical wind velocity component vanishes on the ground and varies linearly with the altitude above the ground.

**Numerical Data.** Let the ring vortex pair be characterized by

\[
R = 2000 \text{ ft},
\]
\[ H = 4000 \text{ ft.} \quad (43b) \]

By changing the values of the circulation strength \( \Gamma \), a one parameter family of winds is generated. In particular, the value \( \Gamma = 0.2471 \times 10^7 \text{ ft}^2/\text{sec} \) yields \( \Delta W_x = 100 \text{ fps at } h = 0 \).

Therefore, this value of \( \Gamma \) is assumed to correspond to \( \lambda = 1 \).

The computation of the velocity components is done by using the stream function in conjunction with (32) and (41) for increments

\[ \Delta r = 0.1 \text{ ft}, \quad (44a) \]
\[ \Delta h = 0.1 \text{ ft}. \quad (44b) \]

For \( \Gamma = 0.2471 \times 10^7 \text{ ft}^2/\text{sec} \), corresponding to \( \lambda = 1 \), the results are shown in Table 3 and Fig. 6.

Table 3 supplies summary results for the wind velocity components and their first derivatives (Wind Model 2, \( \lambda = 1 \), \( h = h_0 \)).

Fig. 6A shows the horizontal wind velocity difference versus the altitude. In the low altitude region \( (h \leq h_*) \), \( \Delta W_r \) is nearly independent of the altitude. This is consistent with the analyses of Refs. 6-11.

Fig. 6B shows the vertical wind velocity difference versus the altitude. In the low altitude region \( (h \leq h_*) \), \( \Delta W_h \) is nearly linear with the altitude. Again, this is consistent with the analyses of Refs. 6-11.

Figs. 6C and 6D show the wind velocity components \( W_r, W_h \) for several values of the altitude, namely, \( h = 50, 500, 1000 \text{ ft} \). It appears that the profiles of the horizontal and vertical wind components resemble those of sinusoidal and cosinusoidal functions, respectively.

**Remark.** In the following sections, we assume that the aircraft flies in a vertical plane containing the axis of symmetry of the microburst. Also, we assume that the coordinate system \((x, y, h)\) is chosen so that
\[ y = 0. \quad (45) \]

This implies that
\[ r = x, \quad (46) \]

so that
\[ W_r = W_x. \quad (47) \]

An analogous remark holds for the partial derivatives of (47) with respect to \( x \) and \( h \), respectively.
5. Gamma Guidance

Optimal take-off trajectories can be determined by minimizing the maximum deviation of the absolute path inclination $\gamma_e$ from the initial value $\gamma_{e0}$ (Ref. 7). From the analysis of these trajectories, the following basic properties can be established:

(i) for weak-to-moderate windshears, the optimal trajectories are characterized by a monotonic climb (Fig. 7A);

(ii) for severe windshears, the optimal trajectories are characterized by an initial climb, followed by nearly-horizontal flight, followed by renewed climbing after the aircraft has passed through the shear region (Fig. 7A);

(iii) initially, the absolute path inclination $\gamma_e$ must be decreased until a certain critical value is reached; the critical value of $\gamma_e$ is nearly maintained for a relatively long time interval; after passing through the shear region, the value of $\gamma_e$ is gradually increased to $\gamma_e = \gamma_{e0}$ (Fig. 7B);

(iv) the critical value of $\gamma_e$ depends on the intensity of the shear/downdraft factor $F$ [see Eqs. (13)]; it decreases as the intensity of the shear/downdraft factor increases (Fig. 7B).

5.1. Guidance Law

The above properties of the optimal trajectories lead to the following gamma guidance law:

$$\gamma_e = \bar{\gamma}_e (F),$$

$$\bar{\gamma}_e = \gamma_{e0} C_1 (1 - C_2 F),$$

$$F = W_x / g - W_h / V,$$

$$\gamma_{e1} \leq \gamma_e \leq \gamma_{e2}.$$
In (48), $\gamma_e$ is the absolute path inclination, $\tilde{\gamma}_e$ is the nominal absolute inclination, and $F$ is the shear/downdraft factor. The remaining quantities are parameters and constants to be suitably determined.

5.2. Feedback Control

In feedback control form, the guidance law (48) is implemented as follows:

$$\alpha - \tilde{\alpha}(V) = -K[\gamma_e - \tilde{\gamma}_e(F)],$$

(49a)

$$\alpha \leq \alpha_*,$$

(49b)

$$-\dot{\alpha}_* \leq \dot{\alpha} \leq \dot{\alpha}_*.$$  

(49c)

Here, $K$ is the gain coefficient, $\alpha$ is the angle of attack, and $\tilde{\alpha}$ is the nominal angle of attack.

5.3. Nominal Angle of Attack

Assume that, along a large portion of a trajectory,

$$\cos \gamma \approx 1, \quad \sin \gamma \approx \gamma,$$  

(50a)

$$|\dot{W}_X/g| \ll 1, \quad |\dot{W}_h/g| \ll 1, \quad |V\dot{\gamma}/g| \ll 1.$$  

(50b)

Under these conditions, Eq. (2b) yields the following nondifferential equation:

$$(T/W) \sin (\alpha + \delta) + L/W - 1 = 0,$$  

(51)

which supplies implicitly the function $\tilde{\alpha}(V)$. Next, we employ the representation (7) for the thrust and the representation (9) for the lift, and we expand the trigonometric term $\sin (\alpha + \delta)$ in Taylor series as follows:
\[ \sin (\alpha + \delta) = (\alpha + \delta). \] (52)

Therefore, from Eq. (51), we obtain the following algebraic equations:

\[ \begin{align*}
D_0 + D_1 \alpha &= 0, \quad \alpha \leq \alpha^{**}, \\
E_0 + E_1 (\alpha - \alpha^{**}) + E_2 (\alpha - \alpha^{**})^2 &= 0, \quad \alpha^{**} \leq \alpha \leq \alpha^{*},
\end{align*} \] (53a-b)

which admit the solutions

\[ \begin{align*}
\tilde{\alpha} &= -\frac{D_0}{D_1}, \quad \tilde{\alpha} \leq \alpha^{**}, \\
\bar{\alpha} &= \alpha^{**} + \left( \frac{1}{2E_2} \right) \left[ -E_1 + \sqrt{E_1^2 - 4E_0E_2} \right], \quad \alpha^{**} \leq \bar{\alpha} \leq \alpha^{*}.
\end{align*} \] (54a-b)

The coefficients \( D_0, D_1, E_0, E_1, E_2 \) depend on the velocity and are given by

\[ \begin{align*}
D_0 &= -1 + (\delta \text{ / mg}) (A_0 + A_1 V + A_2 V^2) + (C_0 \rho S / 2mg) V^2, \\
D_1 &= (1 \text{ / mg}) (A_0 + A_1 V + A_2 V^2) + (C_1 \rho S / 2mg) V^2,
\end{align*} \] (54c-d)

and

\[ \begin{align*}
E_0 &= D_0 + D_1 \alpha^{**}, \\
E_1 &= D_1, \\
E_2 &= (C_2 \rho S / 2mg) V^2.
\end{align*} \] (54e-f-g)

Equations (54) supply explicitly the function \( \bar{\alpha}(V) \); this function is shown in Fig. 8 with reference to the Boeing B-727 aircraft.

5.4. Guidance Parameters and Constants

To sum up, the feedback control form of the gamma guidance law is represented by the relations (48), (49), (54). Present in these relations are the parameters \( C_1, C_2, K \), which must be determined so as to ensure the survival capability of the aircraft in a severe
windshear. While the systematic investigation of the effect of these parameters on survival capability is the main objective of this thesis, we remark that typical values are

\[ C_1 = 1.0, \quad C_2 = 4.0, \quad K = 5.0. \]  \hspace{1cm} (55)

Concerning the bounds for the nominal absolute path inclination, typical values are

\[ \gamma_{e1} = \varepsilon \gamma_{e0}, \quad \gamma_{e2} = \gamma_{e0}. \]  \hspace{1cm} (56)

where \( \varepsilon \) is either zero or a small positive number.
6. Numerical Results

Using the equations of motion of Section 2, the aircraft data of Section 3, the wind models of Section 4, and the guidance scheme of Section 5, gamma guidance trajectories were computed. The angle of attack was subjected to Ineq. (49b) with \( \alpha_\ast = 16.0 \) deg; the angle of attack rate was subjected to Ineq. (49c) with \( \dot{\alpha}_\ast = 3.0 \) deg/sec; the nominal absolute path inclination was subjected to Ineq. (48d) with \( \gamma_e = 0 \) and \( \gamma_{e_o} = \gamma_{e_o} \). Because the guidance scheme (48), (49), (54) depends on the triplet \( (C_1, C_2, K) \), the parameters of the triplet were systematically varied. An analogous remark refers to the intensity parameter of the wind model.

For Wind Model 1, a one-parameter family of winds can be generated by varying the wind intensity parameter \( \lambda \) in Eqs. (26)-(27). Note that \( \lambda = 1 \) corresponds to \( \Delta W_x = 100 \) fps. As \( \lambda \) increases, more intense shear/downdraft combinations are generated until a critical value \( \lambda_c \) is found (hence, a critical wind velocity difference \( \Delta W_{xc} \) is obtained) such that the aircraft scrapes the ground. The critical value \( \Delta W_{xc} \) is called the survival capability of the aircraft and constitutes a measure of the goodness of a guidance scheme.

For Wind Model 2, a one-parameter family of winds can be generated by varying either the parameter \( \lambda \) in Eqs. (30)-(31) or the circulation strength \( \Gamma \) in Eqs. (39)-(40). Note that \( \lambda = 1 \), corresponding to \( \Gamma = 0.2471 \times 10^7 \) ft \(^2\)/sec, yields \( \Delta W_x = 100 \) fps at \( h = 0 \). For Wind Model 2, the survival capability \( \Delta W_{xc} \) is defined in the same way as for Wind Model 1.

The results of the numerical experiments are shown in Tables 4-6 and Figs. 9-12.

Table 4, pertaining to Wind Model 1 and \( x_0 = 2300 \) ft, shows the survival capability \( \Delta W_x(C_1, C_2, K) \) by freezing one of the parameters and varying systematically the other two. Specifically, Table 4A refers to \( C_2 = 4.0 \), Table 4B refers to \( C_2 = 5.0 \), and Table 4C refers to \( C_1 = 1.0 \).
Table 5, pertaining to Wind Model 2 and $x_0 = -6000$ ft, shows the survival capability $\Delta W_x(C_1, C_2, K)$ by freezing one of the parameters and varying either the other two. Specifically, Table 5A refers to $C_2 = 4.0$, Table 4B refers to $C_2 = 6.5$, and Table 4C refers to $C_1 = 1.0$.

For particular choices of the triplet $(C_1, C_2, K)$, Table 6 shows the survival capability $\Delta W_{xc}$ in both Wind Model 1 and Wind Model 2.

Figures 9-10, pertaining to Wind Model 1 and $x_0 = -2300$ ft, refer to particular choices of the parameter triplet, specifically $(1.0, 4.0, 5.0)$ in Fig. 9 and $(1.0, 5.0, 4.0)$ in Fig. 10.

Figures 11-12, pertaining to Wind Model 2 and $x_0 = -6000$ ft, refer to particular choices of the parameter triplet, specifically $(1.0, 4.0, 5.0)$ in Fig. 11 and $(1.0, 6.5, 14.0)$ in Fig. 12.

Each of the above figures includes four parts. Consider for example Fig. 9. Figure 9A shows the altitude profile $h(t)$ of the family of gamma guidance trajectories parametrized in terms of the wind velocity difference $\Delta W_x$. The lowest member of the family is the limiting trajectory generated by the critical wind velocity difference $\Delta W_{xc}$. For the above limiting trajectory, Figs. 9B, 9C, 9D show the time histories of the wind components, the relative velocity, and the shear/downdraft factor.

From the inspection of the results, the following comments arise.

(i) There are many parameter triplets generating gamma guidance trajectories that perform nearly as well as an optimal trajectory. For instance, consider Wind Model 1. For these conditions, the survival capability of an optimal trajectory is $\Delta W_{xc} = 119.5$ fps. Table 4 shows that there are 49 parameter triplets (13 in Table 4A, 6 in Table 4B, and 30 in Table 4C) yielding a survival capability above 113.5 fps, namely, within 5% of that of the optimal trajectory.

(ii) Comparing Tables 4 and 5, we see that the survival capability of a gamma
guidance trajectory in Wind Model 2 is considerably above (in some cases, 30-50% above) that in Wind Model 1. The reason is that, in Wind Model 1, the shear/downdraft combination is either unfavorable to the aircraft or neutral; on the other hand, in Wind Model 2, the shear/downdraft combination is partly unfavorable (central core of the downburst) and partly favorable to the aircraft (peripheral regions of the downburst). The increase in survival capability is due to the ability of the gamma guidance scheme to take advantage of the regions where the effects of the shear/downdraft combination are favorable to the aircraft.

(iii) The parameter triplets that are best in Wind Model 1 are not necessarily the same that are best in Wind Model 2. However, some particular parameter triplets yield gamma guidance trajectories performing well in both wind models (Table 6).

(iv) For a given parameter triplet, Figs. 9A to 12A show the trajectory deterioration associated with the increasing values of the wind velocity difference. The gamma guidance trajectory climbs monotonically for mild-to-moderate windshears, becomes nearly-horizontal for severe windshears, ultimately scraping the ground for extremely severe windshears.

(v) For a given parameter triplet, Figs. 9C to 12C and Figs. 9D to 12D show a clear correlation between the relative velocity and the shear/downdraft factor. Specifically, the relative velocity decreases whenever $F > 0$ (shear/downdraft combination unfavorable to the aircraft) and increases whenever $F < 0$ (shear/downdraft combination favorable to the aircraft).
7. Conclusions

This thesis presents an investigation of the gamma guidance scheme with reference to the take-off problem. Attention is focused on the feedback control form of the gamma guidance law. This law is modified in such a way that not only the aircraft can react to unfavorable shears, but can take advantage of favorable shears.

The windshear efficiency of the gamma guidance law depends on the proper choice of a parameter triplet. A systematic investigation of the effect of the triplet on survival capability is presented for two windshear models. In Wind Model 1, the shear/downdraft combination is either unfavorable to the aircraft or neutral; in Wind Model 2, the shear/downdraft combination is partly unfavorable (central core of the downburst) and partly favorable to the aircraft (peripheral regions of the downburst). The major conclusions are as follows:

(i) There are many parameter triplets generating gamma guidance trajectories that perform nearly as well as an optimal trajectory, namely, yield a survival capability within 5% of that of the optimal trajectory.

(ii) The survival capability of a gamma guidance trajectory in Wind Model 2 is considerably above that in Wind Model 1. The increase in survival capability is due to the ability of the gamma guidance scheme to take advantage of the regions where the effects of the shear/downdraft combination are favorable to the aircraft.

(iii) Some particular parameter triplets yield gamma guidance trajectories performing well in both Wind Model 1 and Wind Model 2.

(iv) For a given parameter triplet, there is trajectory deterioration associated with increasing values of the wind velocity difference. The gamma guidance trajectory climbs monotonically for mild-to-moderate windshears, becomes nearly-horizontal for severe windshears, ultimately scraping the ground for extremely severe windshears.

(v) For a given parameter triplet, there is a clear correlation between the relative
velocity and the shear/downdraft factor. Specifically, the relative velocity decreases whenever the shear/downdraft combination is unfavorable to the aircraft and increases whenever the shear/downdraft combination is favorable to the aircraft.
References


Table 1A. Spline nodal points, horizontal wind, WM 1.

<table>
<thead>
<tr>
<th>Point</th>
<th>x (kft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₀</td>
<td>-2.3</td>
</tr>
<tr>
<td>P₁</td>
<td>-2.1</td>
</tr>
<tr>
<td>P₂</td>
<td>-1.9</td>
</tr>
<tr>
<td>P₃</td>
<td>-1.7</td>
</tr>
<tr>
<td>P₄</td>
<td>1.7</td>
</tr>
<tr>
<td>P₅</td>
<td>1.9</td>
</tr>
<tr>
<td>P₆</td>
<td>2.1</td>
</tr>
<tr>
<td>P₇</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Table 1B. Spline lengths, horizontal wind, WM 1.

<table>
<thead>
<tr>
<th>Spline</th>
<th>xₐₗ (kft)</th>
<th>xᵣᵢₗ (kft)</th>
<th>xᵣᵢₗ - xₐₗ (kft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₀P₁</td>
<td>-2.3</td>
<td>-2.1</td>
<td>0.2</td>
</tr>
<tr>
<td>P₁P₂</td>
<td>-2.1</td>
<td>-1.9</td>
<td>0.2</td>
</tr>
<tr>
<td>P₂P₃</td>
<td>-1.9</td>
<td>-1.7</td>
<td>0.2</td>
</tr>
<tr>
<td>P₃P₄</td>
<td>-1.7</td>
<td>1.7</td>
<td>3.4</td>
</tr>
<tr>
<td>P₄P₅</td>
<td>1.7</td>
<td>1.9</td>
<td>0.2</td>
</tr>
<tr>
<td>P₅P₆</td>
<td>1.9</td>
<td>2.1</td>
<td>0.2</td>
</tr>
<tr>
<td>P₆P₇</td>
<td>2.1</td>
<td>2.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Table 1C. Spline coefficients, horizontal wind, WM 1.

<table>
<thead>
<tr>
<th>Spline</th>
<th>120A0</th>
<th>120A1</th>
<th>120A2</th>
<th>120A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0P1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>P1P2</td>
<td>-119</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>P2P3</td>
<td>-113</td>
<td>9</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>P3P4</td>
<td>-102</td>
<td>204</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P4P5</td>
<td>102</td>
<td>12</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>P5P6</td>
<td>113</td>
<td>9</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>P6P7</td>
<td>119</td>
<td>3</td>
<td>-3</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ W_x = \lambda A(x), \ A = C (A_0 + A_1y + A_2y^2 + A_3y^3), \ y = (x - x_L)/(x_R - x_L), \ C = 50 \text{ fps}. \]

Table 1D. Summary results, horizontal wind, WM 1, \( \lambda = 1 \).

<table>
<thead>
<tr>
<th>Point</th>
<th>x (kft)</th>
<th>( W_x ) (fps)</th>
<th>( \partial W_x / \partial x ) (1/sec)</th>
<th>( \partial W_x / \partial h ) (1/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>-2.3</td>
<td>-0.50000E+02</td>
<td>0.00000E+00</td>
<td>0.00000E+00</td>
</tr>
<tr>
<td>P1</td>
<td>-2.1</td>
<td>-0.49583E+02</td>
<td>0.62500E-02</td>
<td>0.00000E+00</td>
</tr>
<tr>
<td>P2</td>
<td>-1.9</td>
<td>-0.47083E+02</td>
<td>0.18750E-01</td>
<td>0.00000E+00</td>
</tr>
<tr>
<td>P3</td>
<td>-1.7</td>
<td>-0.42500E+02</td>
<td>0.25000E-01</td>
<td>0.00000E+00</td>
</tr>
<tr>
<td>P4</td>
<td>1.7</td>
<td>0.42500E+02</td>
<td>0.25000E-01</td>
<td>0.00000E+00</td>
</tr>
<tr>
<td>P5</td>
<td>1.9</td>
<td>0.47083E+02</td>
<td>0.18750E-01</td>
<td>0.00000E+00</td>
</tr>
<tr>
<td>P6</td>
<td>2.1</td>
<td>0.49583E+02</td>
<td>0.62500E-02</td>
<td>0.00000E+00</td>
</tr>
<tr>
<td>P7</td>
<td>2.3</td>
<td>0.50000E+02</td>
<td>0.00000E+00</td>
<td>0.00000E+00</td>
</tr>
</tbody>
</table>
Table 2A. Spline nodal points, vertical wind, WM 1.

<table>
<thead>
<tr>
<th>Point</th>
<th>x (kft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₀</td>
<td>- 2.30000</td>
</tr>
<tr>
<td>P₁</td>
<td>- 1.63333</td>
</tr>
<tr>
<td>P₂</td>
<td>- 0.96666</td>
</tr>
<tr>
<td>P₃</td>
<td>- 0.30000</td>
</tr>
<tr>
<td>P₄</td>
<td>0.30000</td>
</tr>
<tr>
<td>P₅</td>
<td>0.96666</td>
</tr>
<tr>
<td>P₆</td>
<td>1.63333</td>
</tr>
<tr>
<td>P₇</td>
<td>2.30000</td>
</tr>
</tbody>
</table>

Table 2B. Spline lengths, vertical wind, WM 1.

<table>
<thead>
<tr>
<th>Spline</th>
<th>xₜ (kft)</th>
<th>xᵣ (kft)</th>
<th>xᵣ - xₜ (kft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₀P₁</td>
<td>- 2.30000</td>
<td>- 1.63333</td>
<td>0.66666</td>
</tr>
<tr>
<td>P₁P₂</td>
<td>- 1.63333</td>
<td>- 0.96666</td>
<td>0.66666</td>
</tr>
<tr>
<td>P₂P₃</td>
<td>- 0.96666</td>
<td>- 0.30000</td>
<td>0.66666</td>
</tr>
<tr>
<td>P₃P₄</td>
<td>- 0.30000</td>
<td>0.30000</td>
<td>0.60000</td>
</tr>
<tr>
<td>P₄P₅</td>
<td>0.30000</td>
<td>0.96666</td>
<td>0.66666</td>
</tr>
<tr>
<td>P₅P₆</td>
<td>0.96666</td>
<td>1.63333</td>
<td>0.66666</td>
</tr>
<tr>
<td>P₆P₇</td>
<td>1.63333</td>
<td>2.30000</td>
<td>0.66666</td>
</tr>
</tbody>
</table>
Table 2C. Spline coefficients, vertical wind, WM 1.

<table>
<thead>
<tr>
<th>Spline</th>
<th>6B₀</th>
<th>6B₁</th>
<th>6B₂</th>
<th>6B₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₀P₁</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>P₁P₂</td>
<td>-1</td>
<td>-3</td>
<td>-3</td>
<td>2</td>
</tr>
<tr>
<td>P₂P₃</td>
<td>-5</td>
<td>-3</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>P₃P₄</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>P₄P₅</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>P₅P₆</td>
<td>-5</td>
<td>3</td>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>P₆P₇</td>
<td>-1</td>
<td>3</td>
<td>-3</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ W_h = \lambda(h/h_*)B(x), \quad B = C(B_0 + B_1y + B_2y^2 + B_3y^3), \quad y = (x - x_L)/(x_R - x_L), \quad C = 50 \text{ fps}. \]

Table 2D. Summary results, vertical wind, WM 1, \( \lambda = 1, h = h_* \).

<table>
<thead>
<tr>
<th>Point</th>
<th>x</th>
<th>( W_h )</th>
<th>( \partial W_h / \partial x )</th>
<th>( \partial W_h / \partial h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₀</td>
<td>-2.30000</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
</tr>
<tr>
<td>P₁</td>
<td>-1.63333</td>
<td>-0.83333E+01</td>
<td>0.375000E-01</td>
<td>-0.83333E-02</td>
</tr>
<tr>
<td>P₂</td>
<td>-0.96666</td>
<td>-0.41666E+02</td>
<td>0.375000E-01</td>
<td>-0.41666E-01</td>
</tr>
<tr>
<td>P₃</td>
<td>-0.30000</td>
<td>-0.50000E+00</td>
<td>0.000000E+00</td>
<td>-0.50000E-03</td>
</tr>
<tr>
<td>P₄</td>
<td>0.30000</td>
<td>-0.50000E+00</td>
<td>0.000000E+00</td>
<td>-0.50000E-03</td>
</tr>
<tr>
<td>P₅</td>
<td>0.96666</td>
<td>-0.41666E+02</td>
<td>0.375000E+00</td>
<td>-0.41666E-01</td>
</tr>
<tr>
<td>P₆</td>
<td>1.63333</td>
<td>-0.83333E+01</td>
<td>0.375000E-01</td>
<td>-0.83333E-02</td>
</tr>
<tr>
<td>P₇</td>
<td>2.30000</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
</tr>
</tbody>
</table>
Table 3. Wind components and their derivatives, WM 2, $\lambda = 1$, $h = h_0$.

<table>
<thead>
<tr>
<th>x (ft)</th>
<th>$W_x$ (fps)</th>
<th>$W_h$ (fps)</th>
<th>$\partial W_x / \partial x$ (1/sec)</th>
<th>$\partial W_x / \partial h$ (1/sec)</th>
<th>$\partial W_h / \partial x$ (1/sec)</th>
<th>$\partial W_h / \partial h$ (1/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>-18.9</td>
<td>0.2</td>
<td>-0.007447</td>
<td>0.000050</td>
<td>0.000050</td>
<td>0.004284</td>
</tr>
<tr>
<td>5500</td>
<td>-23.0</td>
<td>0.2</td>
<td>-0.008868</td>
<td>0.000024</td>
<td>0.000024</td>
<td>0.004676</td>
</tr>
<tr>
<td>5000</td>
<td>-27.8</td>
<td>0.2</td>
<td>-0.010233</td>
<td>-0.000034</td>
<td>-0.000034</td>
<td>0.004665</td>
</tr>
<tr>
<td>4500</td>
<td>-33.2</td>
<td>0.1</td>
<td>-0.011221</td>
<td>-0.000142</td>
<td>-0.000142</td>
<td>0.003837</td>
</tr>
<tr>
<td>4000</td>
<td>-38.9</td>
<td>0.0</td>
<td>-0.011320</td>
<td>-0.000320</td>
<td>-0.000320</td>
<td>0.001592</td>
</tr>
<tr>
<td>3500</td>
<td>-44.2</td>
<td>0.1</td>
<td>-0.009854</td>
<td>-0.000570</td>
<td>-0.000570</td>
<td>0.002799</td>
</tr>
<tr>
<td>3000</td>
<td>-48.3</td>
<td>0.4</td>
<td>-0.006163</td>
<td>-0.000867</td>
<td>-0.000867</td>
<td>0.009968</td>
</tr>
<tr>
<td>2500</td>
<td>-50.0</td>
<td>1.0</td>
<td>0.000051</td>
<td>-0.001139</td>
<td>-0.001139</td>
<td>0.020060</td>
</tr>
<tr>
<td>2000</td>
<td>-47.9</td>
<td>1.6</td>
<td>0.008349</td>
<td>-0.001288</td>
<td>-0.001288</td>
<td>0.032344</td>
</tr>
<tr>
<td>1500</td>
<td>-41.5</td>
<td>2.2</td>
<td>0.017466</td>
<td>-0.001235</td>
<td>-0.001235</td>
<td>0.045157</td>
</tr>
<tr>
<td>1000</td>
<td>-30.6</td>
<td>2.8</td>
<td>0.025657</td>
<td>-0.000965</td>
<td>-0.000965</td>
<td>0.056337</td>
</tr>
<tr>
<td>500</td>
<td>-16.3</td>
<td>3.1</td>
<td>0.031290</td>
<td>-0.000526</td>
<td>-0.000526</td>
<td>0.063908</td>
</tr>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.033289</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.066358</td>
</tr>
<tr>
<td>500</td>
<td>16.3</td>
<td>3.1</td>
<td>0.031290</td>
<td>0.000526</td>
<td>0.000526</td>
<td>0.063908</td>
</tr>
<tr>
<td>1000</td>
<td>30.6</td>
<td>2.8</td>
<td>0.025657</td>
<td>0.000965</td>
<td>0.000965</td>
<td>0.056337</td>
</tr>
<tr>
<td>1500</td>
<td>41.5</td>
<td>2.2</td>
<td>0.017466</td>
<td>0.001235</td>
<td>0.001235</td>
<td>0.045157</td>
</tr>
<tr>
<td>2000</td>
<td>47.9</td>
<td>1.6</td>
<td>0.008349</td>
<td>0.001288</td>
<td>0.001288</td>
<td>0.032344</td>
</tr>
<tr>
<td>2500</td>
<td>50.0</td>
<td>1.0</td>
<td>0.000051</td>
<td>0.001139</td>
<td>0.001139</td>
<td>0.020060</td>
</tr>
<tr>
<td>3000</td>
<td>48.3</td>
<td>0.4</td>
<td>-0.006163</td>
<td>0.000867</td>
<td>0.000867</td>
<td>0.009968</td>
</tr>
<tr>
<td>3500</td>
<td>44.2</td>
<td>0.1</td>
<td>-0.009854</td>
<td>0.000570</td>
<td>0.000570</td>
<td>0.002799</td>
</tr>
<tr>
<td>4000</td>
<td>38.9</td>
<td>0.0</td>
<td>-0.011320</td>
<td>0.000320</td>
<td>0.000320</td>
<td>0.001592</td>
</tr>
<tr>
<td>4500</td>
<td>33.2</td>
<td>0.1</td>
<td>-0.011221</td>
<td>0.000142</td>
<td>0.000142</td>
<td>0.003837</td>
</tr>
<tr>
<td>5000</td>
<td>27.8</td>
<td>0.2</td>
<td>-0.010233</td>
<td>0.000034</td>
<td>0.000034</td>
<td>0.004665</td>
</tr>
<tr>
<td>5500</td>
<td>23.0</td>
<td>0.2</td>
<td>-0.008868</td>
<td>-0.000024</td>
<td>-0.000024</td>
<td>0.004676</td>
</tr>
<tr>
<td>6000</td>
<td>18.9</td>
<td>0.2</td>
<td>-0.007447</td>
<td>-0.000050</td>
<td>-0.000050</td>
<td>0.004284</td>
</tr>
</tbody>
</table>
Table 4A. Survival capability $\Delta W_{xe}$ (fps), gamma guidance,
$C_2 = 4.0$, Wind Model 1, $x_0 = -2300$ ft.

<table>
<thead>
<tr>
<th>K</th>
<th>$C_1 = 0.0$</th>
<th>$C_1 = 0.5$</th>
<th>$C_1 = 1.0$</th>
<th>$C_1 = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>112.3</td>
<td>112.2</td>
<td>109.1</td>
<td>105.4</td>
</tr>
<tr>
<td>5.0</td>
<td>119.1</td>
<td>118.9</td>
<td>116.5</td>
<td>111.2</td>
</tr>
<tr>
<td>10.0</td>
<td>99.4</td>
<td>118.9</td>
<td>118.1</td>
<td>113.9</td>
</tr>
<tr>
<td>15.0</td>
<td>65.6</td>
<td>118.5</td>
<td>118.0</td>
<td>115.5</td>
</tr>
<tr>
<td>20.0</td>
<td>65.6</td>
<td>113.5</td>
<td>117.6</td>
<td>115.7</td>
</tr>
</tbody>
</table>

Table 4B. Survival capability $\Delta W_{xe}$ (fps), gamma guidance,
$C_2 = 5.0$, Wind Model 1, $x_0 = -2300$ ft.

<table>
<thead>
<tr>
<th>K</th>
<th>$C_1 = 0.0$</th>
<th>$C_1 = 0.5$</th>
<th>$C_1 = 1.0$</th>
<th>$C_1 = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>112.3</td>
<td>113.5</td>
<td>112.3</td>
<td>109.9</td>
</tr>
<tr>
<td>5.0</td>
<td>119.1</td>
<td>119.2</td>
<td>118.7</td>
<td>118.5</td>
</tr>
<tr>
<td>10.0</td>
<td>99.4</td>
<td>99.4</td>
<td>108.7</td>
<td>116.5</td>
</tr>
<tr>
<td>15.0</td>
<td>65.6</td>
<td>99.4</td>
<td>108.7</td>
<td>106.5</td>
</tr>
<tr>
<td>20.0</td>
<td>65.6</td>
<td>99.3</td>
<td>105.1</td>
<td>104.0</td>
</tr>
</tbody>
</table>
Table 4C. Survival capability $\Delta W_{\infty}$ (fps), gamma guidance,
$C_1 = 1.0$, Wind Model 1, $x_0 = -2300$ ft.

<table>
<thead>
<tr>
<th>$C_2$</th>
<th>$K = 1.0$</th>
<th>$C_2 = 5.0$</th>
<th>$C_2 = 6.5$</th>
<th>$C_2 = 10.0$</th>
<th>$C_2 = 15.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>109.1</td>
<td>112.3</td>
<td>112.9</td>
<td>113.2</td>
<td>113.3</td>
</tr>
<tr>
<td>5.0</td>
<td>111.4</td>
<td>116.6</td>
<td>117.1</td>
<td>116.7</td>
<td>116.8</td>
</tr>
<tr>
<td>6.0</td>
<td>113.6</td>
<td>118.7</td>
<td>118.4</td>
<td>118.8</td>
<td>119.0</td>
</tr>
<tr>
<td>7.0</td>
<td>115.0</td>
<td>119.3</td>
<td>119.4</td>
<td>119.5</td>
<td>119.5</td>
</tr>
<tr>
<td>8.0</td>
<td>116.5</td>
<td>118.7</td>
<td>118.4</td>
<td>119.5</td>
<td>119.3</td>
</tr>
<tr>
<td>9.0</td>
<td>116.7</td>
<td>118.6</td>
<td>118.0</td>
<td>113.2</td>
<td>119.2</td>
</tr>
<tr>
<td>10.0</td>
<td>118.1</td>
<td>109.8</td>
<td>84.2</td>
<td>103.1</td>
<td>99.4</td>
</tr>
</tbody>
</table>
Table 5A. Survival capability $\Delta W_{xc}$ (fps), gamma guidance, $C_2 = 4.0$, Wind Model 2, $x_0 = -6000$ ft.

<table>
<thead>
<tr>
<th></th>
<th>$C_1 = 0.0$</th>
<th>$C_1 = 0.5$</th>
<th>$C_1 = 1.0$</th>
<th>$C_1 = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>K = 1.0</td>
<td>281.2</td>
<td>176.9</td>
<td>147.9</td>
<td>144.6</td>
</tr>
<tr>
<td>K = 5.0</td>
<td>288.9</td>
<td>176.4</td>
<td>147.4</td>
<td>152.5</td>
</tr>
<tr>
<td>K = 10.0</td>
<td>287.5</td>
<td>176.0</td>
<td>147.4</td>
<td>159.0</td>
</tr>
<tr>
<td>K = 15.0</td>
<td>289.6</td>
<td>175.1</td>
<td>147.5</td>
<td>159.8</td>
</tr>
<tr>
<td>K = 20.0</td>
<td>289.7</td>
<td>175.1</td>
<td>148.6</td>
<td>160.4</td>
</tr>
<tr>
<td>K = 25.0</td>
<td>289.5</td>
<td>175.0</td>
<td>148.6</td>
<td>160.4</td>
</tr>
</tbody>
</table>

Table 5B. Survival capability $\Delta W_{xc}$ (fps), gamma guidance, $C_2 = 6.5$, Wind Model 2, $x_0 = -6000$ ft.

<table>
<thead>
<tr>
<th></th>
<th>$C_1 = 0.0$</th>
<th>$C_1 = 0.5$</th>
<th>$C_1 = 1.0$</th>
<th>$C_1 = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>K = 1.0</td>
<td>281.2</td>
<td>165.8</td>
<td>151.1</td>
<td>148.3</td>
</tr>
<tr>
<td>K = 5.0</td>
<td>288.9</td>
<td>165.4</td>
<td>158.0</td>
<td>153.6</td>
</tr>
<tr>
<td>K = 10.0</td>
<td>287.5</td>
<td>165.2</td>
<td>162.3</td>
<td>156.4</td>
</tr>
<tr>
<td>K = 15.0</td>
<td>289.6</td>
<td>165.3</td>
<td>164.2</td>
<td>156.3</td>
</tr>
<tr>
<td>K = 20.0</td>
<td>289.7</td>
<td>164.7</td>
<td>164.1</td>
<td>156.9</td>
</tr>
<tr>
<td>K = 25.0</td>
<td>289.5</td>
<td>164.6</td>
<td>164.1</td>
<td>156.9</td>
</tr>
</tbody>
</table>
Table 5C. Survival capability $\Delta W_{x_0}(\text{fps})$, gamma guidance, 
$C_1 = 1.0$, Wind Model 2, $x_0 = -6000$ ft.

<table>
<thead>
<tr>
<th></th>
<th>$C_2 = 4.0$</th>
<th>$C_2 = 6.5$</th>
<th>$C_2 = 10.0$</th>
<th>$C_2 = 15.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K = 10.0$</td>
<td>147.4</td>
<td>162.3</td>
<td>162.3</td>
<td>162.3</td>
</tr>
<tr>
<td>$K = 12.0$</td>
<td>147.3</td>
<td>162.2</td>
<td>162.2</td>
<td>162.2</td>
</tr>
<tr>
<td>$K = 14.0$</td>
<td>147.5</td>
<td>164.3</td>
<td>164.3</td>
<td>164.3</td>
</tr>
<tr>
<td>$K = 15.0$</td>
<td>147.5</td>
<td>164.2</td>
<td>164.2</td>
<td>164.2</td>
</tr>
<tr>
<td>$K = 16.0$</td>
<td>147.4</td>
<td>164.3</td>
<td>164.3</td>
<td>164.3</td>
</tr>
<tr>
<td>$K = 18.0$</td>
<td>147.5</td>
<td>164.3</td>
<td>164.3</td>
<td>164.3</td>
</tr>
<tr>
<td>$K = 20.0$</td>
<td>148.6</td>
<td>164.1</td>
<td>164.1</td>
<td>164.1</td>
</tr>
</tbody>
</table>
Table 6. Comparison of the survival capability $\Delta W_{xc}$ (fps) for different gamma guidance schemes.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$K$</th>
<th>WM 1, $x_0 = -2300$ ft</th>
<th>WM 2, $x_0 = -6000$ ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>116.5</td>
<td>147.4</td>
</tr>
<tr>
<td>S2</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>119.3</td>
<td>153.0</td>
</tr>
<tr>
<td>S3</td>
<td>1</td>
<td>8</td>
<td>4</td>
<td>119.4</td>
<td>155.7</td>
</tr>
<tr>
<td>S4</td>
<td>1</td>
<td>8</td>
<td>5</td>
<td>119.0</td>
<td>158.0</td>
</tr>
</tbody>
</table>
Fig. 1. Cross section of a microburst.
Fig. 2. Coordinate system and force diagram.
Fig. 3A. Nodal points and splines, horizontal wind, WM 1.

Fig. 3B. Nodal points and splines, vertical wind, WM 1.
Fig. 4A. Horizontal wind velocity component, WM 1, $\lambda = 1$.

Fig. 4B. Vertical wind velocity component, WM 1, $\lambda = 1$. 
Fig. 5. Vortex ring pair.
Fig. 6A. Horizontal wind velocity difference vs altitude, WM 2, $\lambda = 1$.

Fig. 6B. Vertical wind velocity difference vs altitude, WM 2, $\lambda = 1$. 
Fig. 6C. Horizontal wind velocity component, WM 2, $\lambda = 1$.

Fig. 6D. Vertical wind velocity component, WM 2, $\lambda = 1$. 
Fig. 7A. Typical optimal trajectories: altitude vs time.

Fig. 7B. Typical optimal trajectories: absolute path inclination vs time.
Fig. 8. Nominal angle of attack vs relative velocity.
Fig. 9A. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, altitude vs time.

Fig. 9B. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, wind velocity components vs time.
Fig. 9C. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, relative velocity vs time.

$C_1 = 1.0$
$C_2 = 4.0$
$K = 5.0$
$\Delta W_x = 116.5$ fps

Fig. 9D. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, shear/downdraft factor vs time.

$C_1 = 1.0$
$C_2 = 4.0$
$K = 5.0$
$\Delta W_x = 116.5$ fps
Fig. 10A. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, altitude vs time.

$c_1 = 1.0$
$c_2 = 5.0$
$K = 4.0$

$\Delta W_x = 119.3$ fps

Fig. 10B. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, wind velocity components vs time.

$W_x, W_h$ (fps)
Fig. 10C. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, relative velocity vs time.

$C_1 = 1.0$
$C_2 = 5.0$
$K = 4.0$
$\Delta W_x = 119.3$ fps

Fig. 10D. Gamma guidance trajectories, WM 1, $x_0 = -2300$ ft, shear/downdraft factor vs time.

$C_1 = 1.0$
$C_2 = 5.0$
$K = 4.0$
$\Delta W_x = 119.3$ fps
Fig. 11C. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, relative velocity vs time.

$C_1 = 1.0$
$C_2 = 4.0$
$K = 5.0$
$\Delta W_x = 147.4$ fps

Fig. 11D. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, shear/downdraft factor vs time.

$C_1 = 1.0$
$C_2 = 4.0$
$K = 5.0$
$\Delta W_x = 147.4$ fps
Fig. 11A. Gamma guidance trajectories: WM 2, $x_0 = -6000$ ft, altitude vs time.

Fig. 11B. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, wind velocity components vs time.
Fig. 12A. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, altitude vs time.

Fig. 12B. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, wind velocity components vs time.
Fig. 12C. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, relative velocity vs time.

$C_1 = 1.0$
$C_2 = 6.5$
$K = 14.0$
$\Delta W_x = 164.3$ fps

Fig. 12D. Gamma guidance trajectories, WM 2, $x_0 = -6000$ ft, shear/downdraft factor vs time.

$C_1 = 1.0$
$C_2 = 6.5$
$K = 14.0$
$\Delta W_x = 164.3$ fps