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Three dimensional localised slant stacks and their application to the analysis of synthetic reverse vertical seismic profiles

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Three dimensional localised slant stacks and their application to the analysis of synthetic reverse vertical seismic profiles

by

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ABSTRACT

Three dimensional localised slant stacks and their application to the analysis of synthetic reverse vertical seismic profiles

by

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Three-dimensional slant stacks simulate the response of an elastic medium for both an incident (ray parameter q) and a scattered (ray parameter p) plane wave and are implemented by following a slant stack of common shot gathers by a second slant stack of common ray parameter gathers. Three-dimensional localised slant stacks are developed, and applied to analyse reverse vertical seismic profiles. The application of slant stacks to borehole data differs from the surface seismic case in that the velocities measured from plane wave decomposition are horizontal and vertical phase velocities. Velocity and dip estimates for a model with iso-velocity layers and plane interfaces with any dip are obtained using the slant stacks without explicitly utilizing traveltimes. The estimation scheme is based on a layer stripping approach. The direct arrivals are used to determine the velocity and the reflections are used to determine the dip of the reflector.
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1 INTRODUCTION

1.1 Overview

Vertical Seismic Profiling (VSP) is a measurement procedure in which a signal generated at the surface of the earth is recorded by geophones secured at various depths to the wall of a drilled well. The direction that geophones are deployed during a VSP survey thus differs by 90 degrees relative to the lateral geophone placement used when recording surface seismic data. Because the geophone is located far below the earth's surface when recording VSP data, it responds to both upgoing and downgoing seismic events. This type of geophone response is an important difference between VSP data and surface recorded reflection data because downward traveling events cannot be identified in data recorded by geophones positioned on the earth's surface.

Reverse Vertical Seismic Profiling (RVSP) method is a newly emerging seismic imaging technique which differs from the conventional VSP technique with regard to the source-receiver configuration and has several advantages over it. In conventional VSP a seismic receiver is positioned in a well bore close to and above the target horizon. A seismic source on the earth's surface is used to produce a single trace of data. Significant gains in vertical and horizontal resolution of subsurface images are possible, though at great expense, as each shot is recorded by a single geophone station or a limited number of stations. A common VSP procedure is to record with a receiver clamped at a fixed depth while successive shots are placed along a radial surface line that intersects the well. This is known as "walk-away-shooting". Small faults, pinchouts, and missed production areas have been successfully mapped using VSP methods (Dillon 1984). An alternative to the VSP methods is to place the seismic source in the borehole while an unlimited number of receivers record the seismic response on the surface of the earth and is called the Reverse
VSP method. The principle of seismic reciprocity between source and receiver asserts that a complete "walk-away shooting VSP" that might have taken many hours or several days to shoot can be recorded with a single shot in a well. When the shot generated energy at the receiver falls below the random ambient surface noise level, such as for very deep shots or at very large geophone offsets, records from several shots at the same depth can be summed for coherent signal enhancement. The RVSP method produces seismic sections similar to those of conventional VSP. The difficulty with RVSP technology is devising a downhole source with sufficient energy to transmit through thousands of feet of earth without damaging the borehole. RVSP is becoming an attractive alternative to conventional VSP surveying as more downhole seismic sources are being developed.

In RVSP sections, upgoing and downgoing wave fields are not the same as the upgoing and downgoing wavefields in conventional VSP sections because the source and receiver arrangement is reversed. Upgoing and downgoing waves are referenced to the source in the well, whereas in VSP sections upgoing and downgoing waves are referenced to the receiver in the well. Reflected P waves are downgoing wavefields, while all the upgoing wavefields except the direct P waves are unwanted arrivals.

1.2 The Controlled Directional Reception method

In a constant velocity medium, the velocity of a wave does not change with depth. In a stratified medium, \( \frac{dt}{dx} \) does not change with depth. In a stratified medium the wavefronts are curved, and they are horizontal translations of each other. Figure (1.1) illustrates the differential geometry of the wave. The diagram shows that
Figure 1.1: Geometry of plane waves in a constant velocity medium.

\[
\frac{dt}{dx} = \frac{\sin(\theta)}{v} \quad \text{and} \quad \frac{dt}{dz} = \frac{\cos(\theta)}{v}
\]

These two equations define two slownesses. The first is the inverse of the horizontal speed measured along the earth's surface, called the horizontal phase velocity. The second is a vertical speed measurable in a borehole (as in a VSP), called the vertical phase velocity. Both these speeds are greater than the velocity $v$ of wave propagation in the medium. The projection of wave fronts onto coordinate axes gives speeds larger than $v$, whereas projection of rays onto coordinate axes gives speeds smaller than $v$.

The Controlled Directional Reception (CDR) method is based on the automated picking of pre-stack reflection surface seismic data (Riabinkin et al., 1962). The goal of this picking is to obtain the parameters of reflected waves. One of the picked parameters is travelt ime (the time it takes for a wave to travel from the source to the receiver). Another is ray parameter, defined as the change in travelt ime as the position of the shot, receiver or both varies. The ray parameters required for velocity estimation are the shot ray parameter
(the change in traveltime as the shot position is varied down the hole, while the geophone position is held constant), and the geophone ray parameter (the change in traveltime as the geophone position is varied, while the shot position is held constant).

In the CDR method, the parameters are determined by selecting a gather of nearby traces and slant stacking the gather from x-t to tau-p space where p is the ray parameter. The Tau-p transform is the sum of the samples from each trace along a given slope. That is for a given slope (or slant) of \( p = \frac{dt}{dx} \), a trace is formed by summing all the amplitudes along that slope. In Tau-p space, the vertical axis "Tau" represents two-way reflection time at zero offset distance and the horizontal axis is a set of traces, each trace corresponding to a particular "p" or ray parameter. The slant-stack that is being used for parameter estimation is modified so that the intercept-time axis is located at an offset where the ray parameter is to be measured rather than at zero-offset. This slant stack is known as localized slant stacking. On such a tau-p gather, the point of maximum amplitude is picked and the position of this maximum gives the ray parameter (p) and traveltime information directly provided the wavelet was zero-phase.

The method of CDR has been used for simultaneous estimation of structure and velocity for a layered earth from the pre-stack surface seismic reflection data (Gray, W.C. and Golden, J.E., 1983 and Inderweisen, P.L., 1986,1987). The methods of Gray and Inderweisen essentially consists of two steps - a downward continuation step and an imaging step. The downward continuation step involves tracing rays from the source and receiver locations downward through the previously defined velocity-depth structure to the layer boundary just above the unknown layer. Snell's law is used to determine ray refractions at layer boundaries. The imaging step consists of guessing a velocity for the second layer and Snell's law is used to continue the raypaths into the unknown layer to their intersection point. If the computed traveltime for the completed raypath is not in agreement with the observed traveltime, then the velocity guess is assumed incorrect and a
new trial velocity used. The velocity which makes the computed and observed traveltimes match to an agreeable limit is taken as the correct velocity for the second layer.

**Approaches to ray parameter estimation**

Different approaches have been used to pick the events in the slant stack domain. Sword, C. H. Jr. (1987) uses the 'maxima trace concept'. In this method the picking process begins with finding the maximum amplitude at each time on each slant stack section. A trace is obtained each for the shot ray parameter and the geophone ray parameter panels, this trace is called the 'maximum trace'. The length of the trace is the duration of the time on the tau-p section. The two "maximum traces" one each for the shot \( S_s \) and receiver gather \( S_g \) are combined to give a correlated-maximum trace. Such a trace should have large amplitudes wherever the two traces have a good correlation and low amplitude wherever they do not. This is because a given event will manifest itself on both the traces at the same travelt ime but with different ray parameter. One measure of the correlation is the quantity \( S_s(t) - S_g(t) \). When this quantity is near zero, the traces are correlated; otherwise they are not (this quantity is not a good measure if the traces differ by only a multiplicative constant). The correlation criteria that has been used by Sword (Sword, C.H.Jr., 1987) is essentially a Gaussian function which is unity when the amplitude difference between the two maxima traces is low and falls exponentially as the square of the distance, the sharpness of the exponential fall depends on a user assigned value. Once the correlated maxima trace \( q(t) \) has been obtained, it has to be picked for determining the ray parameters of the correlated events. The maximum value \( q_{\text{max}} \), and the corresponding traveltime \( t_{\text{max}} \) are found on \( q(t) \). Then the slant-stack sections are examined to find the respective points of maximum amplitude at time \( t_{\text{max}} \). The positions of these maxima give the corresponding ray parameters for the shot and receiver gathers which will be used to trace rays from the
shot and receiver. Inderweisen (1984) picks the event with the best coherency on the tau-p and tau-q panels and then correlates them to get the parameters corresponding to an event. In the three-dimensional localised slant stacks that I have developed there is no need for the correlation of the events on the tau-p panel and the tau-q panel, since it is equivalent to summing the data along planes.
2 THREE DIMENSIONAL SLANT STACKS

2.1 Physical meaning of the three-dimensional slant stack

Stacking along the shot direction simulates a generalised downgoing wave at non-vertical incidence. Figure 2.1 shows a schematic of the plane-wave simulation. When receiver gathers are summed along the slope $q = dt/ds$ it is essentially equivalent to firing shots at an inverse rate of $dt/ds = S/v$. When data is summed over the geophone axis instead of the shot axis, the result is point-source experiments recorded by receiver antennas that have been tuned to receive upcoming waves at different angles to the vertical.

Figure 2.1: Three-Dimensional slant stacks: shows the physical basis of the double slant stacks. The line receiver $p$ is generated by summing along geophone axis. The line source $q$ is generated by summing along shot axis.

Summing along both the axes leads to plane waves generated by a line-source being received by a line-receiver. The parameters of the three-dimensional slant stacks $p$ and $q$.
quantify the emergent angle at the receiver and the exit angle at the source respectively. The 3-D localised slant stack is a method for determining the ray angles from the shot and geophone simultaneously. The 3-D localised slant stack has the advantage that it is more resistant to noise, since more traces are summed than in the 2-D case. Figure 2.2 shows the stacking chart for a 2-D and 3-D localised slant stack.

![Diagram](image)

**Figure 2.2** (a) shows the gathers that are to be slant stacked to determine p and q, using single slant stacks. (b) shows the same data but now a hatched diamond shape shows all the gathers to be included in the 3-D slant stack. The 3-D slant stack is more resistant to noise, since more data are summed.

Localised slant stack differs from the global slant stack, in that the intercept time axis is modified so that the intercept - time axis is located at the receiver offset where the ray parameter is to be measured rather than at zero-offset. Only the traces within an aperture around the receiver position at which the ray parameter is being estimated are summed, whereas in a global slant stack all the traces in the section are summed. Localised slant
stacks allow a laterally adaptable extraction of locally linear events. It is possible to extract weak signal from behind strong Gaussian noise. Continuous events on the gathers can be specified as a summation of short, tapered line segments of all dips using localised slant stacks. Global slant stacks require signal to be easily expressed as a sum of lines extending across the section - an assumption that produces artefacts when the data do not agree. The great advantage of localised slant stack lies in its resistance to artefacts. Its lateral adaptability leads it to parametrize the events well, without oversimplifying focused events, and by not straightening or extending events.

2.2 Implementation of the 3-D localised slant stack.

3-D Localised slant stack

The 3-D localised slant stack algorithm simulates the response of the 2-D earth for both an extended line source (ray parameter p) and an extended line receiver (ray parameter q). A double (τ, p) transform in the continuous form can be defined as \( \Psi(\tau, p, q) \) (Equation 2.2.1). This transform would lead to traces in the three dimensional (τ, p, q) space. Each point \( \Psi(\tau, p, q) \) corresponds to the stack of all data in a plane defined by

\[ \tau = \tau + pg + qs. \]

\( \Psi(\tau, 0, 0) \) would be the horizontal stack of the complete wavefield.

\[
\Psi(\tau, p, q) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(s, g, \tau + pg + qs) ds dg 
\]  

(2.2.1)

Two-way localised slant stack is defined by the equation 2.2.2. (Harlan and Burridge 1983). It generates a cube of data \((p, q, \tau)\) where \(p\) is the geophone ray parameter, \(q\) is the shot ray parameter and \(\tau\) is the traveltime. This method circumvents the cross correlation between two separate slant-stack panels (the shot ray parameter panel and the geophone
receiver panel), it allows the determination of $p$ and $q$ simultaneously. It is essentially a two-pass, 2-D slant stacking which produces a three dimensional function. The 3-D slant stack data are computed in the time domain as,

$$r_{ij}(m\Delta p, n\Delta q, t) = \sum_{k=-mwz}^{mwz} \sum_{l=-mwx}^{mwx} d(i+k,j+l,(t+nk\Delta q\Delta s+ml\Delta p\Delta g)),$$  \hspace{1cm} (2.2.2)

where $i$ = shot index, $j$ = geophone index, $\Delta s$ = the shot spacing, $\Delta g$ = the geophone spacing, $\Delta p$ = the geophone ray parameter sampling, $\Delta q$ = the shot ray parameter sampling, $n$ = shot ray parameter index, $m$ = geophone ray parameter index, $mwz = (nz-1)/2$, $mwx = (nx-1)/2$ where $nz$ and $nx$ are the number of traces added together in the shot and geophone direction respectively.

*Time domain implementation*

For each shot-receiver pair the double transform is obtained by the summing the recorded wavefield $d(s,g,t)$ along planes defined by $p$ and $q$. The number of traces summed along the plane is $nx \times nz$, whereas in the 2-D transform it is $nx$ if it is stacked along $g$ and $nz$ if stacked along $s$. This is essentially carried out in two steps:

(i) For each shot index, add the traces in the shot gather for a range of offsets within the window defined for the localised slant-stack, along straight lines with slope $p$.

(ii) For each $p$, collect the slant stack trace for consecutive shot points and construct a slant stack by summing along the shot direction with straight lines of slope $q$.

Each of the steps require data interpolation because the slanting line does not necessarily intersect with the traces at the grid points.
**Frequency domain implementation**

The algorithm is implemented in the frequency domain by doing phase shift and summing the traces horizontally. The method essentially consists of doing two single slant stacks in order to obtain the double slant stacked data. Whereas single slant stacks are obtained by summing along slant lines, double slant stacks can be visualized as data obtained by summing along planes, where the plane is defined by lines with slopes $p$ and $q$ along the receiver and shot direction respectively. The steps are:

(i) Extract the traces within a cylindrical window about the shot and receiver position at which the stack is to be done. The number of traces in the window along the shot and geophone directions could be different.

(ii) Transform the traces within the window (which is rectangular) into frequency domain using an FFT routine.

\[
D(it,ig,is) \rightarrow D(F,g,s) \quad \text{FFT} \quad (2.2.3)
\]

(iii) Carry out slant stack along the geophone direction by phase shift and summation.

\[
G(ip,if,is) = M(ip,ix) * D(ig,is,if) \quad (2.2.4)
\]

where $M(ip,ix) = \left[ e^{i2\pi F_i px} \right]$ is the matrix containing the phase shift elements for a frequency $F_i$.

The elements of the matrix $M$ need not be computed at every frequency since the elements at a frequency $F_{i+1}$ can be computed from that at frequency $F_i$ using the relation

\[
\left[ e^{i2\pi F_{i+1} px} \right] = \left[ e^{i2\pi (F_i + dF) px} \right] = \left[ e^{i2\pi F_i px} \right] \cdot \left[ e^{i2\pi dF px} \right] \quad (2.2.5)
\]
where $dF$ is the frequency increment. Since $F_i$ goes from $F_0$ to $F$ with increment $dF$, we only need to generate the matrix at zero frequency ($F_0$), which is an identity matrix and the matrix $\left[ e^{i2\pi dF p} \right]$. The phase shift matrix at all other frequencies can be obtained using the equation (3.3). At this stage we have all the tau-p sections for the different shots within the window in the frequency domain. The next step is the slant stacking along the shot direction.

(iv) This step can be written in the frequency domain as

$$ R(\omega,p,q) = \sum_{m,w,z} G(\omega,p,z) e^{i\omega q z} \quad (2.2.6) $$

where $G$ is the data obtained after the slant stack along the geophone direction. The summation is carried out along the shot axis over the traces $z_i - mwz$ to $z_i + mwz$ where $z_i$ is the shot index at which the stack is done and $mwz$ is half the number of traces in the window. This can be written in matrix notation as,

$$ R(ip,if,iq) = N(ip,iq) * G(ip,is,if) \quad (2.2.7) $$

where $N = \left[ e^{i2\pi F_i q z} \right]$ is the matrix containing the phase shift elements for the frequency $F_i$. The elements of the matrix $N$ need not be computed at every frequency since the elements at $F_i + 1$ can be computed from that at frequency $F_i$ using a similar relation as (2.2.5).

(v) The double slant stacked data are then transformed back to the time domain using an inverse FFT to obtain the data cube $S(p,q,t)$, where $p$ and $q$ are the geophone and shot ray parameters respectively.

The double slant stack program was implemented in the Fourier domain. This essentially consists of a 2-pass scheme, first the data is summed along slants in the geophone direction and then the tau-p sections are summed along the shot direction. The computation is carried out using the two-pass method. The inputs to the program are the number of traces to be summed along the shot direction and geophone direction, the
minimum and maximum geophone and shot ray parameters, the shot and receiver spacing and the number of geophone and shot ray parameter samples. For a given position of the shot and receiver a window of traces is created from the input shot gathers and the window is summed along planes defined by p and q. The summation is carried out in the frequency domain first along the geophone direction for all shot positions in the window surrounding the shot position about which the stack is to be done, this summation gives us the data(p,s,f), next the data are summed along the shot direction along slopes defined by 'q' for all the p. This gives us a cube of data which has peaks at times corresponding to the arrival of an event and the corresponding p and q.

**Aliasing in the ray-parameter domain**

If we consider a plane wave propagating at an angle \( \theta \) in a homogeneous medium, the parameter p is defined as \( p = \frac{\sin \theta}{v} \). In transforming from x-t into \( \tau-p \), we need to consider (a) the maximum p, \( p_{\text{max}} \) and (b) the sampling of p, \( \Delta p \). Let us assume that \( p_{\text{max}} = \frac{1}{v} \), where v is the velocity considered, so that no data are lost in the tau-p transform. It is well known that to avoid aliasing in the \( \omega-k \) domain, the condition \( \Delta k_x < \frac{1}{x_{\text{max}}} \) must be satisfied, where \( x_{\text{max}} \) is the maximum length of the record in space and \( \Delta k_x \) is the wavenumber sampling interval. Thus for a particular frequency \( \omega \), if no aliasing is to occur, then the sampling interval \( \Delta p \) should satisfy \( \omega \Delta p = \Delta k_x < 1/x_{\text{max}} \). That is \( \Delta p < \frac{\omega x_{\text{max}}}{\omega_{\text{max}} x_{\text{max}}} \). This has to be satisfied for all \( \omega < \omega_{\text{max}} \), the maximum frequency occurring in the data. Hence to avoid aliasing, \( \Delta p \) must be smaller than or equal to \( \frac{1}{\omega_{\text{max}} x_{\text{max}}} \).

**Event picking**

Once the 3-D slant-stack cube is generated for a given source-receiver pair, then the next step is to scan the cube along the time axis for determining the travel time of an event.
The time spent on scanning the time-slices has been reduced by giving apriori traveltimes information for a given event and a given source-receiver position by picking the traveltimes from the receiver gathers for these events. The program only scans the timeslices within a window around the given rough traveltimes to determine the timeslice which has the highest envelope amplitude. Once the timeslice has been sorted next we scan along p and q axes to determine the other two coordinates (p&q) for the discrete maximum value. Once the coordinates for the discrete maximum are found, the coordinates for the actual peak is found by interpolating in three dimensions to get the coordinates (pmax,qmax,amax). The interpolation is carried out by using a three dimensional adaptation of the Lagrangian form of interpolating polynomials. Holding one variable constant, a series of Lagrangian polynomials are written for interpolation at the given value of the other variable, and then combine these in a final Lagrange form. The net result is a Lagrangian polynomial in which the function factors are replaced by Lagrangian polynomials. The traveltimes which is obtained from the stacks is a much refined and accurate estimate over the rough picks done from the shot gathers. Figure 2.3 shows the event picking process done for the first arrival for the model shown in Figure 3.6. The figure shows the traveltime profiles for every other shot, starting with shot index 3. There are 23 traveltime profiles for the direct arrival corresponding to the 23 shots.

Window width

The choice of the aperture width for localised slant stack is not very obvious. If the aperture is too wide, the desired x-localisation is lost, and with it, resolution in the x-direction. For the double slant stacks, the p,q resolution is dependent on the aperture width along the shot as well as the geophone direction. If the aperture is too narrow, p and q resolution is lost because discrimination between p, q values relies on having an adequate baseline over which to measure slopes. If one considers the limit of an aperture that is only
Figure 2.3: Event picking: Traveltime profiles for the direct arrival for different shots obtained from the time sample corresponding to the maxima envelope in the tau-p-q domain.
one trace wide it is clear that all p values will have the same amplitude and will be equal to the largest amplitude in the trace. It takes a finite aperture to measure an apparent velocity. The optimum width is that distance over which constructive interference is obtained for the p under consideration. This has previously been referred to as the Fresnel zone (McMechan & Ottolini 1980). The spatial wavenumber is given by \( k_x = \omega p \), so the optimum aperture width changes with position and with local temporal frequency \( \omega \) in the data. The effect of aperture shape is investigated. A simple triangular window is used to decrease linearly the weighting of each trace lying within the aperture from 1.0 at the center to 0.0 at the edges.

\[
W(x-x_i) = 1.0 - \frac{|x_i - x|}{\delta} \quad (2.2.8)
\]

where \( \delta \) is the half width of the aperture expressed in the same units as \( x \). This shape satisfies the requirements that the contributions in each localised slant stack come predominantly from traces near the center of the aperture and that edge effects related to the finiteness of the aperture are reduced, but has no justification other than satisfactory performance.

2.3 Ray parameter estimation schemes

Two versions of the ray parameters estimations were implemented, the first one requires human interaction, the schematic flow chart is shown in Figure 2.4(a). In this version, the events are picked from the tau-p and tau-q panels from the screen. The picks made from the two panels are collected together and sorted, redundant picks are deleted and a window is added around the sorted picks to take into account any errors in picking and also the phase characteristics of the wavelet. Once the picks are sorted, the next step is to run the program which stores the timeslices corresponding to the picks and clusters the picks into events. For each event, the peak coordinates for all the timeslices around the
event traveltime are determined. The maximum among the peaks is determined for each event group. The coordinates corresponding to the maximum are the parameters for the event at that particular source-receiver coordinates. This is then repeated for the next source-receiver pair and so on.

The second type of implementation is automatic, the picks of the events are made on the shot gathers and the program stores the timeslices around the approximate picks to handle errors in picking and then does the peak envelope sorting process. The schematic flow chart is shown in Figure 2.4(b). This version determines the parameters for all the events which were picked from the shot gathers.
Figure 2.4: Implementation of the 3-D slant stack: Flow chart showing, (a) the interactive version, where the events are picked in the ray parameter domain and (b) the automatic version, where the picks are made in the shot domain.
3 APPLICATION OF SLANT STACKS TO RVSP DATA

3.1 Characteristics of the double transform for RVSP geometry.

The geophone ray parameter is always positive for the direct as well as the reflection event because the event arrival time increases with the distance from the bore for both the reflection as well as the transmission events. The shot parameter 'q' is positive for the transmission event since the arrival time increases with the depth of the shot whereas for the reflection events q is negative since the reflection arrival times decrease as the shot moves closer to the reflecting horizon. The 3-D localised slant stacks were computed for the data from the models as described below. Then the relationships between the shot and geophone ray parameters are used to develop a velocity and dip estimation scheme.

(i) model 1: flat layer in a constant velocity medium.

The travelt ime equation for the reflection from a horizontal layer in a constant velocity medium is given by,

\[ v^2 t^2 = g^2 + (2zr - s)^2 \]  \hspace{1cm} (3.1.1)

where g is the distance to the geophone from the well-bore, zr - is the depth of the reflector, s - is the shot depth in the well. Figure 3.1 shows the geometry for the flat layer model.

Differentiating equation with respect to \( g \) we get,

\[ v^2 t \frac{dt}{dg} = g \]
Figure 3.1: Flat layer model. The interface is at a depth $z_r$, $v$ is the velocity of the layer, $s$ is the shot depth along the well and $g$ is the geophone offset from the well.

$$v^2 \cdot t \cdot p = g \quad (3.1.2)$$

Differentiating equation with respect to $s$ we get,

$$v^2 \cdot t \cdot \frac{dt}{ds} = s - 2z_r$$

$$v^2 \cdot t \cdot q = s - 2z_r \quad (3.1.3)$$

Eliminating $g$ and $s$ from the traveltime equation by substituting the above two equations we get,

$$p^2 + q^2 = \frac{1}{v^2} \quad (3.1.4)$$

The direct arrival event and the reflection from a horizontal interface in a constant velocity medium satisfy the relation 3.1.4., where $v$ is the velocity of the medium. Therefore the $(p,q)$ pair corresponding to each shot-receiver $(s,g)$ pair lie on a circle for a reflection from
a flat interface in a constant velocity medium. For a subsurface with horizontal planar interfaces, the interval velocities can be determined by using equation 3.1.4. This is due to the fact that the geophone ray parameter $p$ is constant. Therefore, all the $(p,q)$ pairs will satisfy the equation 3.1.4.

(ii) model 2: *dipping layer in a constant velocity medium*.

The traveltime equation for the reflection from a dipping interface for borehole geometry is given by,

$$v^2 t^2 = \frac{g^2}{s^2} + 4(zr - s)\cos\beta(zr \cos\beta - g \sin\beta)$$

(3.1.5)

where $\beta$ is the dip of the layer, $z r$ is the depth to the point where the dipping layer intercepts

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**Figure 3.2**: Dipping layer model: The dip of the interface is $\beta$, $z r$ is the depth to the intercept of the interface with the well.
the well, and s and g mean the same as in model 1. Figure 3.2 shows the geometry for the
dipping layer.

The (p,q) profile for the reflection from the dipping interface satisfies the relation (see the
Appendix A.)

\[ p^2 + q^2 - 2pq \sin(2\beta) = \frac{\cos^2(2\beta)}{v^2} \] (3.1.6)

\[ \frac{(p + q)^2}{(\sin\beta + \cos\beta)^2} + \frac{(p - q)^2}{(\sin\beta - \cos\beta)^2} = \frac{2}{v^2} \] (3.1.7)

where \( \beta \) is the dip of the layer. In this case the (p,q) corresponding to each shot-receiver
pair (s,g) lie on an ellipse whose major axis is at 45 degrees to the p axis. The length of the
major axis decreases with increase in the velocity. When the dip is zero \( (\beta = 0) \) then the
ellipse becomes a circle. The vertices of the p+q - p-q ellipse is given by

\[ \frac{\sqrt{2}(\sin\beta + \cos\beta)}{v} \] along the horizontal and \[ \frac{\sqrt{2}(\sin\beta - \cos\beta)}{v} \] along the vertical axes.

(iii) model 3: Point scatterer in a constant velocity medium. The geometry of the diffractor
model is shown in the Figure 3.3.

A point scatterer at the point \((A,B)\) satisfies the following relations in the p-q domain.

\[ vt = \sqrt{A^2 + (B - y)^2} + \sqrt{(A - x)^2 + B^2} \] (3.1.8)

\[ \frac{v^2 p^2}{1 - v^2 p^2} = \frac{(A-x)^2}{B^2} \] (3.1.9)
Figure 3.3: Point scatterer model: (A,B) are the coordinates of the diffractor.

\[
\frac{v^2 q^2}{1 - v^2 q^2} = \frac{(B-y)^2}{A^2} \tag{3.1.10}
\]

Eliminating \(x\) and \(y\) we can relate the traveltime to the location of the point scatterer and the two ray-parameters \(p\) and \(q\).

\[
v_t = \frac{A}{\sqrt{1 - v^2 q^2}} + \frac{B}{\sqrt{1 - v^2 p^2}} \tag{3.1.11}
\]

For a given shot, the \(p-q\) profile for all receivers is a straight line, parallel to the \(p\) axis. This is due the fact that for a given diffractor and shot, the angle of incidence of the waves from the shot to the diffractor is constant, therefore \(q\) is constant for a given shot.

(iv) model 4: *Layer with a constant velocity gradient*. The geometry of the model is shown in Figure C in the appendix. The velocity profile is defined by the equation 3.1.12. See appendix B for the details of the derivation. The traveltime relation is given by,
\[ t = \frac{1}{a} \cosh^{-1} \left[ 1 + \frac{a^2(x^2 + z^2)}{2v_0(v_0 + az)} \right]; \]  

Figure 3.4: Velocity gradient model: \( z \) is the depth of the shot, \( v(z) \) is defined by 3.1.13.

\[ v(z) = v_0 + az \]  

\[ p^2 + q^2 = \frac{1}{v^2(z)} \]  

where \( v(z) \) is the velocity at the depth of the shot for the RVSP geometry for the transmission event, \( a \) is the gradient of velocity and \( v(z) \) is the velocity at the depth of the reflector for the reflection event.

(v) Model 5: *Dipping layer in a constant velocity medium with the receiver line inclined to the horizontal.* The geometry of the model is shown in Figure C in the appendix C. The \( p-q \) relation for the direct arrival from a source in a vertical well to the geophone along an inclined surface making an angle \( \alpha \) with the horizontal is given by

\[ \frac{(p+q)^2}{1 + \sin\alpha} + \frac{(p-q)^2}{1 - \sin\alpha} = \frac{2}{v^2} \]  

(3.1.15)
The p-q profile for the direct arrival is an ellipse, with the major axis making 45 degrees with the horizontal. The p-q profile for the reflection from a dipping interface as measured from an inclined surface is also an ellipse. The relation is given by

\[ p^2 + q^2 + 2pq\sin(\alpha - 2\beta) = \frac{\cos^2(\alpha - 2\beta)}{v^2} \]  
(3.1.16)

\[ \frac{(p + q)^2}{1 - \sin(\alpha - 2\beta)} + \frac{(p - q)^2}{1 + \sin(\alpha - 2\beta)} = \frac{2}{v^2} \]  
(3.1.17)

These are the basic relations for the reflection from a dipping interface, which are used in the layer stripping approach described in the next section.

3.2 Scheme for estimation of velocity and dips.

A method for the estimation of velocities and dips of interfaces from RVSP data was developed using the shot and geophone ray parameter relationships. The subsurface is assumed to consist of iso-velocity layers separated by interfaces which can take on any dip. For each source-receiver pair and each event we can determine a velocity, using the shot ray parameter q and the geophone ray parameter p for that event measured for a given shot and receiver. The method utilizes a layer-stripping approach. The velocity and the dip of the first layer are determined using the direct arrival from the source to the receiver for shots within the first medium. The transmission event is used for determining the velocity. This is determined using the equation 3.1.15. Once the velocity is obtained from the transmission event, this velocity is utilized along with the ray parameters of the reflection event to determine the dip of the interface. Knowing the position and the dip of the first interface, the geophone ray parameters corresponding to the shots in the second medium are downward continued, so that they correspond to the ray parameters that would be measured if the geophones were at the interface rather than at the surface. For plane
horizontal interfaces the geophone ray parameter measured at the surface will be the same as at the interface because of Snell's law. For a dipping interface, the ray parameter as would be measured at the interface, is determined from the ray parameter \( p_1 \) measured at the surface using the relation,

\[
p_1 = \frac{\sin(\theta + \alpha)}{v_1} = p \cos \alpha + \frac{\sqrt{1 - v_1^2 p^2}}{v_1} \sin \alpha
\]

where \( p_1 \) is the geophone ray parameter that would be measured if the geophones were at the interface rather than at the surface, for a shot below the dipping interface. \( v_1 \) is the velocity above the interface. The dip of the interface is \( \alpha \).

Using the \( p \) for the transmission events for shots in the second medium we determine the velocity in the second medium using the \( p-q \) relation for the transmission event for inclined measuring surface. In this case, the measuring surface is the interface. This relation holds for an interface with any dip. Using the velocity determined from the transmission event, dip of the second interface is determined from the reflection information. The intercept of the bed with the well is used to position the interface in space, relative to the surface.

### 3.3 Modeling and Processing

**Acoustic Finite Difference Modeling**

Finite-difference modeling of seismic data has long been used as an interpretation tool for enhancing our knowledge of wave propagation and interaction in complex media. Numerical solutions to both acoustic and elastic wave equations have been obtained for borehole data (Hu, McMechan and Harris, 1988). In this study, the reverse VSP data is modeled using the 2-D acoustic finite difference scheme for a unique velocity model.

The reverse VSP geometry places a source in the drill hole and recorders at the free surface (Figure 3.2.1). The velocity model, \( C \), can be spatially varying. The acoustic wave equation is given by:
\[ U_{tt} = c^2(U_{xx} + U_{zz}) + S(x,z,t), \quad (3.3.1) \]

where \( U \) is the acoustic pressure at spatial location \((x,z)\), and time \( t \). Subscripts denote partial differentiation of the subscripted quantity with respect to the subscript. The function \( S(x,z,t) \) is a line source with both spatial and temporal dependencies. Its temporal form can be any smooth function such as 1) derivative of the Gaussian function to represent an impulsive explosion or 2) a sine wave for harmonic vibrations. Its spatial form can have directivity such as horizontal dipole or a uniform compression. The free surface condition is given by,
\[ U_z = 0 \quad (3.3.2) \]

For a source applied at the free surface the source term is excluded from wave equation (3.3.1) and added to the free surface

\[ U_z = S(x,z,t) \text{ for } |x,z| \leq a, \quad (3.3.3) \]

where \( a \) is the source radius. The absorbing boundary condition of Clayton and Enquist (1977) is used to reduce edge reflections generated from numerical boundaries. The acoustic wave equation is combined with appropriate initial and boundary conditions to simulate seismic responses.

Three-Layer Model

The survey geometry consists of a three-layer velocity model containing a point diffractor (Figure 3.5). A grid of 401 x 401 nodes serves as the finite difference model. A total of 51 shot records at 40 meters depth interval, each consisting of 51 surface traces were computed. Two representative data slices are shown in Figures (3.6(a) and (b)). The shot
(S1) is located at a depth of 0.65 km, the surface receiver is offset 0.6 km from the well. Receiver spacing is four gridpoints or 40 meters. Dominant source frequency is 10 Hz, corresponding to a pulse width of 20 grid points (200 m). The shot gather in panel (a) contains several significant events: (i) direct arrival, (ii) primary reflection from the flat interface (iv) primary reflection from the dipping interface (vi) diffraction from diffractor D. In the field data, free surface reflections are usually seen. The free surface reflections are similar to surface ghosts commonly observed in the dynamite shooting. The separation between the three primary events is seen to increase on the common receiver gather shown in panel (b). The two traces shown by arrow heads are identical.

If source-receiver reciprocity is applied without considering the radiation pattern and the influence of the source well, each common receiver gather would have the appearance of a single offset VSP (Figure 3.6(b)). Thus conventional processing methods for VSP data can be applied directly to RVSP data. The most important benefit for RVSP surveys is obtaining multi-offset VSP's from single source efforts.

**Separation of Upgoing and Downgoing Wavefields**

Wavefield separation has to be carried out for the RVSP type data to enable analysis using localised slant stacks. The separation removes the interference of the up and downgoing events and therefore avoids anomalous high amplitudes in the ray parameter
Figure 3.5: Geometry of the three-layer model: shows the velocities in the different media. The geophones are placed on the surface from 500 to 2500 m, and the shots run down the well from 100 m down to 2100m. There are 51 shots and 51 receivers. The shot and receiver spacing is 40m. Every alternate geophone and shot are displayed.
Figure 3.6: Data from three-layer model. (a) Shot gather for the shot at depth of 660m. (b) Receiver gather for the receiver at 900m. The 11th trace in the shot gather and the 15th trace on the receiver gather are identical. They are marked by arrow heads. Four distinct events are seen on the gathers.
domain which might be caused if there is interference between the events. The separation leads the events to be well parametrised in the ray parameter domain. The separation is carried out using F-K filtering.

F-K filtering is an established processing procedure for filtering unwanted energy from seismic data. It is based on the phase velocities of different wave modes which differ either in the magnitude of the velocities or in the direction of propagation or both. Upgoing VSP energy modes exhibit negative velocities and the 2-D Fourier Transform therefore places all upgoing modes in the negative wavenumber half-plane, the transform expresses downgoing wave modes in terms of positive wavenumbers. Whereas, upgoing and downgoing events always criss-cross each other in the space time (Z-T) domain, they do not overlap in the F-K domain. This separation of VSP wave-modes in F-K space provides a convenient way by which downgoing events can be attenuated without suppressing upgoing events. The separation is carried out in the following manner:

(i) Transform the data into F-K domain using 2D FFT transform.
(ii) Multiply every data point in the positive wavenumber half-plane by a value which is much less than unity. (0.001 = 60 decibels.)
(iii) Performing an inverse 2D FFT transform on the modified data generates a new set of VSP data, in time and space in which the downgoing modes are 60 dB weaker than the downgoing modes in the original data set.

In RVSP sections, the upgoing and downgoing wavefields are not the same as in the conventional VSP sections because the source and receiver configuration is reversed. Upgoing and downgoing waves in RVSP section are referenced to the source in the well, whereas in the VSP sections they are referenced to the receiver in the well. Therefore the nomenclature for up and down are reversed when comparing VSP to RVSP literature. In RVSP sections the primary reflected arrivals are the downgoing wavefields, whereas all the upgoing wavefields except for the direct arrivals are unwanted arrivals. The downgoing
wave fields along with the direct arrivals are used for velocity and structure determination. Figures 3.7 - 3.8 show F-K filtering applied to the data for the three layer model obtained using finite difference modeling. Figure 3.7 shows F-K spectrum for a shot gather, we see that the direct arrival (high energy) and the reflections (low energy and upgoing are well separated in the F-K domain). Figure 3.8 (a) shows the direct arrival event after wavefield separation (b) shows the primary reflections after wavefield separation.
Figure 3.7: F-K plot: showing the up and downgoing wavefields present in the data. The direct arrivals from the shots to the receiver, plot in the positive wavenumber and the primary reflection events plot in the negative wavenumber domain. The F-K spectrum shown is for the receiver gather with the receiver at 500m.
Figure 3.8: Wavefield Separation: Receiver gather showing, (a) the transmission event and (b) the primary reflections after wavefield separation. The receiver gathers are for the receiver at 500m.
4 RESULTS AND APPLICATIONS

Tests were made to evaluate the method of 3-D localised slant stacks as a viable method for ray angle estimation and the possible advantages of the ray-parameter domain. Synthetic data was generated for a RVSP type geometry for each of the models studied in chapter 3 using traveltime relations. Data were generated using a symmetrical wavelet with onset time at the peak of the wavelet.

4.1 Flat layer model

Figure 4.1(a) shows the shot gathers for a flat layer model in a medium with velocity 2500m/s, the shot spacing is 40m, the geophone spacing is 40m and sample rate is 4 ms. The geophones are on either side of the borehole. The ensembles show the direct arrivals from the source to the receivers as well as the reflection event. As the shot is lowered down the well the reflections arrive faster. The apparent velocity of the reflected waves along the axis of the well is negative, therefore the reflections plot in the lower half of the p-q plot. The apparent velocity of the direct arrival event along the borehole is positive since the arrival time increases with depth. Therefore the transmission event plots in the upper half of the p-q plot show in Figure (4.4). For receivers to the left of the borehole the apparent velocity of the direct arrival along the geophone axis is negative since the closer the geophone is to the well-head lower will be the traveltime. Both the transmission and the reflected events to the left of the well plot on the left half of the p-q plot, and the (p,q) pairs for the receivers on the right of the borehole plot on the positive half of the p-q plot. As has been shown in equation 3.1.4, the (p,q) pairs plot along a circle for a homogeneous medium. The transmission event plots in the upper semi-circle, and the reflection event plots in the lower semi-circle. Therefore, the circle is well constrained even
Figure 4.1: Data for flat layer model. (a) Shot gather for flat layer model with shot at 180 m depth, (b) receiver gather for a receiver at 580m. The well head is at 1500m.
Figure 4.2: Slices of the tau-p-q cube for a flat layer for the direct event. The slant stack is done at the shot and receiver indices (3,3) respectively. (a) tau-p section. (b) tau-q section. The trace with the maximum amplitude are marked with vertical arrow heads. The marked traces on the two sections are identical to each other.
Figure 4.2: (c) Time-slice of the tau-p-q cube at the direct arrival time as measured from maximum envelope amplitude.
Figure 4.3: Slices of the tau-p-q cube for a flat layer for the reflection event (a) tau-p section. (b) tau-q section. The trace with the maximum amplitude are marked with vertical arrow heads. The marked traces on the two sections are identical to each other.
Figure 4.3: (c) Time-slice of the cube at the arrival time of the reflection event as measured from maximum envelope amplitude.
Figure 4.4: p-q plot for the flat layer. The (p,q) pairs corresponding to the maxima for each source-receiver pair for the transmission and the reflection events are plotted. The transmission event plots on the upper half space and the reflection event plots in the lower half space. They fall on a circle whose radius is equal to the inverse of the velocity of the medium. The points are plotted for every shot and every other geophone.
though we do not get points on the complete circle. The distribution of the points depends on the ray parameter sampling, which in turn is dependent on the spacing of the shots and the geophones. The \((p,q)\) pairs for the transmission event are more spread apart than the ones for the reflection event which are clustered closer this is because of the fact that the transmission event has a wider aperture than the reflected ray paths, put in other words the range of the variation in reflected ray paths is lesser than the range in the transmission path. Figure 4.2(a), (b) and (c) show the slices of \(p-q\) cube for the direct arrival. (a) shows the \(p\) section (b) the \(q\) section and (c) the time-slice corresponding to the arrival time as obtained from the peak envelope. Figure 4.3(a) shows the \(p\) slice corresponding to the peak of the reflection event (b) shows the corresponding \(q\) slice and (c) shows the time slice corresponding to the peak envelope amplitude for the reflection event for the source at 180m and receiver at 580m. The traveltime obtained from the timeslice is accurate to 1 ms after interpolation. The resolution of the ray parameter sample is 5.74 degrees for a velocity of 2500 m/s. Ray parameter range is \(-5e-4\) to \(5e-4\) s/m. The traveltime obtained from the \(p-q\) domain is 0.3749 s and 0.8123 s for the direct and the reflection event respectively. The times calculated from geometry are 0.3742 s and 0.8122 s respectively.

### 4.2 Dipping Layer model

Eleven shot gathers with 51 traces, with shot spacing 30m and geophone spacing 30 m, for a dipping layer model with velocity 3000 m/s and dip 30 degrees. Figure 4.5(a) shows the shot gather corresponding to the index at which the stack is done, (b) shows the receiver gather corresponding to the index at which the stack is done. Updip direction is to the right of the borehole. The \((p,q)\) pairs for the RVSP geometry plot into an ellipse whose major axis is inclined at 45 degrees angle to the \(p\) axis. The vertices of the \(p-q\) ellipse give the reciprocal of the velocity. When \(p+q\) vs \(p-q\) is plotted then we get an ellipse with its axes parallel to the coordinate axes. Higher the velocity smaller is the major axis of the \(pq\)
Figure 4.5: Data for dipping layer model. There are 11 shots and 51 geophones (a) Shot gather for shot index 6. (b) Receiver gather for receiver index 21. The traces with the vertical arrows placed on them are identical. The slant stack for the figures shown in Figure 4.6 are done at shot index 6 and receiver index 21.
Figure 4.6: p-q plot for the dipping layer. The (p,q) pairs for the reflection from a dipping layer plot onto an ellipse. The major axis of the ellipse is at 45 degrees to the horizontal. The velocity in the layer is 3000 m/s and the dip is 30 degrees, updip to the right of the borehole. The (p+q, p-q) points plot on an ellipse with its major axis horizontal.
Figure 4.7: p-q plot for dipping layer for different dips. (a) velocity of the medium is 4500 m/s and dip is 15 degrees (b) velocity of the medium is 4500 m/s and dip is 30 degrees.
Figure 4.8: Dip estimates for a dipping layer: velocity of the medium is 3000m/s and true dip is 15 degrees.
ellipse. The minor axis of the pq ellipse decreases with the angle of the dip. The pq ellipse is defined by the equation 3.1.6 and the p+q - p-q ellipse is defined by the equation (3.1.7). Figures 4.6 shows the (p,q) pairs, and the corresponding pq and p+q - p-q ellipses on which the points lie. The velocity in the layer is 3000 m/s and the dip of the bed is 30 degrees. Figure 4.7 shows the pq plots for dipping bed for different dips. Plot (a) is for a layer with dip 15 degrees and velocity 4500 m/s (b) Layer with dip 15 degrees and velocity 4500 m/s. Figure 4.8 shows the dip estimates made for the layer using the equation 3.1.7. The true dip is 15 degrees.

4.3 Diffractor model

Figure 4.9(a) shows the shot gather for a diffractor model, for a diffractor at (1800,600). (b) shows the corresponding receiver gather. Figure 4.10 shows the p-q plot for all the source-receiver positions, we see that the the p-q plot is a straight line parallel to the p-axis, this could be explained by the fact that the incident wave for a given shot is parametrised by a constant q, that is the incident wave to the diffractor travels in a single direction therefore for a given shot the q is constant for the p-q profile. The scattered waves travel in all directions and therefore the p varies with the geophone position. For each shot we get one straight line in the p-q domain.

4.4 Layer with a constant vertical velocity gradient

Figure 4.11(a) shows the shot gather showing the direct arrival event within the layer. Data were generated for 11 shots with shot spacing 25 m and 41 geophones with trace spacing 25 m. The sampling rate is 4 ms. The receivers are on either side of the borehole. The velocity is 2500 m/s at the surface, a = 0.5 s/m. The (p,q) points plot into arcs of circles, the (p,q) points corresponding to a given shot plot along an arc of a circle whose radius is equal to the inverse of the velocity at the depth of the shot in the well. The
(p,q) pairs for the receivers to the left of the borehole plot on the upper left quadrant and the (p,q) pairs for the receivers to the right plot on the upper right quadrant, as shown in Figure 4.12.

Figure 4.9: Data for diffractor model. (a) Shot gather for shot index 5. It has 61 traces with a spacing of 25m. (b) Receiver gather for shot index 23. It has 20 traces with spacing of 25m. The diffractor is at (1800,600). Shots start from 300m and geophones from 700m.
Figure 4.10: p-q plot for diffractor model. The p-q pairs plot onto straight lines. For each shot we get a unique straight line. This is because of the fact that for a given shot the angle from the source to the diffractor is constant, but the diffracted waves travel in all directions to arrive at the different geophones. There are sixteen shots.
Figure 4.11: Data for a model with a vertical velocity gradient: (a) shot gather showing the transmission event for shot index 3. (b) Receiver gather for the receiver index 3. The velocity at the surface is 2500 m/s and increases at the rate of 0.5 m/s per unit depth downwards.
Figure 4.12: p-q plot for layer with vertical velocity gradient. The (p,q) pairs for the transmission event plot onto arcs of circles whose radius decreases with shot depth, since the velocity increases with depth. The velocity at the surface is 2500 m/s. The velocity at the first shot plotted is 2650 m/s.
4.5 Dipping layer with inclined measuring surface

Figure 4.13 shows the p-q plot for the direct event from the source in a medium with velocity 6000 m/s to receivers on the surface which is inclined at 30 degrees to the horizontal. The points plot on an ellipse as defined by equation C6 where $\alpha'$ here is 120 degrees (the angle from the vertical). The p+q - p-q ellipse is defined by the equation 3.1.15. The p-q ellipse makes an angle of 45 degrees with the horizontal. The vertices of the p+q - p-q ellipse are shown in the figure and they satisfy the equation 3.1.15. Figure 4.14(a) shows the p-q plot for reflection from a dipping interface as measured from an inclined surface. The velocity of the medium is 2500 m/s. The dip of the bed is 15 degrees and the measuring surface is inclined at an angle of 30 degrees to the horizontal. The p - q and the p+q - p-q points plot into circles, this is due to the fact that, $\alpha = 2\beta$ where $\alpha$ is the dip of the measuring surface and $\beta$ is the dip of the layer. Figure 4.14(b) shows the plot for bed with dip 30 degrees, and dip of the inclined surface also is 30 degrees. In this case the (p,q) points plot into ellipse defined by equation 3.1.17.

4.6 Three-layer model with a diffractor

The data for the upgoing wavefields and the downgoing wavefields are separated before analysis using double slant stacks. The separation is done using F-K filtering as described in chapter 3.

Upgoing events: Only the direct arrival event is present in the data, since absorbing boundary conditions are used. The automatic version of the 3-D localised slant stack program was used to determine the p, q and t for each source -receiver pair corresponding to the peak amplitude due to this event. Figure 4.15 shows the $p^2 + q^2$ relationship for the p, q determined from the slant stacks. We see that there are three main groups, the first one is at a velocity of 2500 m/s which is the velocity of the first medium. The second group
Figure 4.13: p-q plot for transmission for an inclined measuring surface: The surface is inclined at an angle of 30 degrees to the horizontal. The well is vertical. The velocity of the medium is 6000 m/s.
Figure 4.14: p-q plot for reflection from dipping bed for an inclined measuring surface: (a) The surface is inclined at an angle of 30 degrees to the horizontal. The well is vertical. The dip of the bed is 15 degrees. It is updip to the right of the borehole. The velocity of the medium is 2500 m/s. (b) The inclination of the surface is 30 degrees. The bed dips at 30 degrees. The velocity is 2500 m/s.
plots at a velocity of 3300 m/s which is the velocity of the second medium. The third group shows a sloping line with center of the group plotting at 4000 m/s which is the velocity of the third medium. The sloping trend in the velocity is due to the fact that the second interface is dipping updip to the right and the velocity also shows the same updip trend. Figure 4.15(b) shows the same data in Figure (a) but now with the horizontal axis representing the shot axis, which in the RVSP case is the depth axis. We can see the three velocities present in the model. There is scatter below the second interface because this interface is dipping. Figure 4.16 (a) shows the geophone parameter profiles for different shots. The horizontal axis is the geophone index, the geophone ray parameter p, changes rapidly from geophone to geophone when the shot is shallower. Deeper the shot lesser is the variation of p from shot to shot. The geophone ray parameter is zero over the well head, because at this point waves travel vertically upward, $\theta = 0$. The profile for the shallow shot approaches $4x10^{-4}$ s/m for geophones farthest from the well on either side. Figure 4.16(b) shows the q profiles for each of the shots. The profiles cluster into three different groups corresponding to the three different velocities present in the medium. When the shots are in medium 1 of layer velocity 2500 m/s the q parameter for the geophone over the well bore goes to $\frac{1}{2500} = 4e-4$ s/m, when the shots are in the second medium $q = \frac{1}{3300} = 3e-4$ s/m and in the third medium $q = \frac{1}{4000} = 2.5e-4$ s/m. We can clearly see the three different groups corresponding to the velocities in three media of the model. Within the first layer the q profiles for the receivers on the left and the right of the borehole are the same since it is a flat interface and therefore the raypaths will be symmetrical for geophones equidistant from the well bore on either side of the well. It is also true for shots within the second medium. For the shots below the dipping interface q profiles for the receivers on either side of the well will be different as seen in the shots with shot index greater
Figure 4.15: $p^2 + q^2$ profiles: velocity obtained using the relation 3.1.4 obtained from direct arrival for the two-layer model. (a) plotted against the receiver axis. (b) plotted against the shot axis.
Figure 4.16: Ray parameter profiles for the direct event for different shots. (a) Geophone ray parameter profiles. (b) Shot ray parameter profiles.
than 18. The receiver over the well shows the reciprocal of the velocity of the third medium $q = \frac{1}{4000} = 2.5 \times 10^{-4} \text{ s/m}$.

The downgoing events: Figure 4.17(a) and (b) show the plot of the downgoing events in the ray parameter domain along different slices of the p-q-tau cube. The slices correspond to the reflection from the first interface. Figure(a) shows the tau-p gather and (b) shows the corresponding tau-q gather which show the three distinct events. (c) shows the corresponding timeslice.

4.7 Velocity and dip estimation for the three-layer model

The following parameters were used for the scheme. 61 shot and geophone ray parameter samples were used. The ray parameter range was $-5 \times 10^{-4} \text{ s/m}$ to $5 \times 10^{-4} \text{ s/m}$ along both the directions. 5*5 traces were summed in the localised slant stack.

The first step is to use the transmission information to get a velocity profile in the subsurface. If the interfaces are horizontal, we can read out the interval velocities directly from the velocity profiles for different shots.

For the shots in the first medium, the (p,q) pairs were determined for the reflection event, for every other shot and every other geophone. The (p,q) pairs for the reflection event as well as the transmission event are plotted in Figure 4.18. The points seem to plot on a circle. The dip of the interface is determined using the relation given by equation 3.1.17. Using the average the dip obtained from the dip profiles (0 degrees in this case), The geophone ray parameters for the shots below the first interface are determined using the equation 3.2.1. Knowing the geophone ray parameters at the first interface for the shots in the second medium, The (p,q) pairs can be used to determine the velocity using the transmission event, using the equation 3.1.15 for layers with any dip. Figure 4.19 shows the dip estimates for the first interface using equation 3.1.15. The velocity profiles
Figure 4.17: Slices of the tau-p-q cube for the reflection from the first interface: The slices are shown at the peak envelope amplitude. The slant stack is done for shot index 15 and receiver index 11. (a) tau-p section (b) tau-q section. The traces marked with vertical arrow heads on the two sections are identical.
Figure 4.17(c) Time slice at the time sample corresponding to the arrival time of the reflection from the horizontal interface.
obtained for the second medium is shown in Figure 4.20. Knowing the velocity in the second medium, the dip of the second interface is determined using the relation 3.1.19. The Figure 4.21 shows the dip estimates for the different source-receiver positions. The estimates of the velocity of the medium below the second interface is shown in Figure 4.22.
Figure 4.18: p-q plot for the transmission and reflection from the first layer. The points are for eight shots and twenty four geophones. A circle with radius equal to the reciprocal of the velocity estimated as 2480 m/s passes through the points.
Figure 4.19: Dip estimates of the first interface for the three-layer model. The interface is horizontal.
velocity profiles for shots below first interface

geophone index, spacing = 80m, index 1 = 580m

Figure 4.20: Velocity estimates for the second medium for the three-layer model.
Figure 4.21: Dip estimates of the second interface for the three-layer model. The profiles are over the geophones to the right of the borehole.
Figure 4.22: Velocity estimates for the medium below the second interface for the three-layer model.
SUMMARY AND CONCLUSIONS

The relationship between the shot ray parameter q and the geophone ray parameter p were derived for simple models. The 3-D slant stacks were run with simple models in order to understand the parameters of the model in the ray parameter domain. A single layer model with horizontal interface showed that the (p,q) pairs plot on a circle. The velocity obtained had an accuracy of 0.4% - 0.7%. The angles from the source and receiver were accurate to 0.6 degrees on either side of the true value. The (p,q) pairs for a dipping layer model plot onto an ellipse. The major and the minor axes of the ellipse are dependent on the dip and the velocity of the medium. The dip estimates for the layer were obtained using the velocity obtained from the direct arrival. The dip estimates were accurate to 0.5 degrees. The 3-D slant stacks were run for a diffractor model. The (p,q) pairs plot into straight lines. This model could be used as the basis for any generalized model since any model can be generated by using point diffractors. Transmission in a medium with a vertical velocity gradient was studied. The velocity obtained using the ray parameters is a function of the depth. The velocity obtained is equal to the velocity at the depth of the shot. The next step was to obtain some generalized p and q relationships for the transmission and the reflection from a dipping interface as measured from an inclined surface to enable layer stripping approach to velocity and dip determination. The transmission event plots onto an ellipse. This event can be used to determine the velocity in the medium. The reflection event also plots onto an ellipse and was used for determining the dip of the interface. The parameters of the ellipse are dependent on the dip of the bed, the dip of the inclined surface and the velocity in the medium.

A scheme for velocity and dip estimation was developed and run on a three layer model. The model was generated using acoustic finite difference modeling. The characteristics of the geophone and the shot ray parameter profiles were studied for the model. The data was transformed into the ray-parameter domain only once. Using the
transmission data for shots in the first medium the velocity is determined from the relation between \( p \) and \( q \). Once the velocity is determined is used along with \( p \) and \( q \) for the reflection event to determine the dip of the interface. Knowing the dip of the interface, the geophone ray parameter values for the shots below the first interface as measured on the surface are downward continued to the first interface. This continuation is done such that they resemble the values that would have been measured were the receivers at the first interface. The transmission for the shots in the second medium are used to determine the velocity in the second medium and the reflection is used to determine the dip of the interface. This process is continued downwards. For the three-layer model, the velocity estimates had an error of 1\%. The dip estimates had an error range 0 to 1.5 degrees, the larger being for the geophones directly over the well head for the flat interface. The average velocity obtained for the first layer is 2480 m/s. The average velocity for the second layer is about 3285 m/s. The velocity obtained for the third layer is 3990 m/s.

3-D localised slant stacks enable the simultaneous determination of the geophone ray parameter \( p \) and the shot ray parameter \( q \) without the need for any type of correlation measures between the tau-\( p \) panels and tau-\( q \) panels which is the drawback of two-dimensional localised slant stacks.

The determined \( p \) and \( q \) parameters can be used for velocity and depth estimations using the \( p \) and \( q \) relationships that hold for the generic elements present in the model, namely diffractor, dipping interface etc.

Dips of the interfaces can be obtained for walkaway VSP's too, since a suite of shot gathers will be available to do the 3-D slant stack.
Future Research and suggestions

The scheme can be used for data collected areally on the earth's surface, for shots in the well. A tomographic scheme could be used for velocity estimation, using the p,q parameters as apriori information would lead to better convergence of the inversion. For layers with arbitrary curvature, we have to include the traveltime information into the reflector parameter estimation process in order to determine the subsurface position of the reflector segment. The localised slant stack method can be used even for data with Gaussian noise. In this domain the signal can be extracted from behind strong noise because of the inherent summation involved in slant stacking.

The most generalised approach would be to assume the subsurface to be made up of diffractors and to plot dip segments at each subsurface locations obtained using the five reciprocal parameters - the geophone ray parameter, the shot ray parameter, the location of the shot, the location of the geophone and the traveltime.
Appendix A

Dipping layer in a constant velocity medium.

The geometry for a single interface dipping at an angle $\beta$ is shown in Figure (A1). The bed intercepts the well at $O'$. In this figure, $x$ is the well-to-geophone offset, $y$ is the offset of the image point $I$ along the line joining the image point $I$ and $O'$ as measured from the point $I'$ and $I'$ is the intercept of the line $IO'$ with the earth's surface.

Figure A1: Schematic for dipping layer illustrating the geometry of the different parameters.
The traveltime relation for the reflection event is given by
\[ v^2t^2 = (x - y\sin 2\beta)^2 + (y\cos 2\beta)^2 \]  \hspace{1cm} (A1)

Differentiating A1 with respect to \(x\), we get
\[ v^2t \frac{dp}{dt} = x - y\sin 2\beta \]  \hspace{1cm} (A2)

Differentiating A1 with respect to \(y\), we get
\[ v^2t \frac{dq}{dt} = y - x\sin 2\beta \]  \hspace{1cm} (A3)

where \(p = \frac{dt}{dx}\) and \(q = \frac{dt}{dy}\).

\(p\) and \(q\) are the geophone and shot ray parameters respectively. The next step is to eliminate \(x\) and \(y\) from equations A1, A2 and A3.

Multiplying equation A2 with \(\sin 2\beta\) and adding with equation A3, we get
\[ v^2t [p\sin 2\beta + q] = y\cos 2\beta \]  \hspace{1cm} (A4)

Multiplying equation A3 with \(\cos 2\beta\) and adding with equation A2, we get
\[ v^2t [q\sin 2\beta + p] = x\cos 2\beta \]  \hspace{1cm} (A5)

Substituting \(x\) and \(y\) obtained from equations A4 and A5 into the traveltime equation A1 and also taking into account that \(\frac{dt}{dz} = -\frac{dt}{dy}\), we obtain
\[ p^2 + q^2 - 2pqs\sin 2\beta = \frac{\cos 2\beta}{v^2} \]  \hspace{1cm} (A6)

which can be cast in the form of the equation for an ellipse as
\[ \frac{(p + q)^2}{(\sin \beta + \cos \beta)^2} + \frac{(p - q)^2}{(\sin \beta - \cos \beta)^2} = \frac{2}{v^2} \]  \hspace{1cm} (A7)
Appendix B

Horizontal Layer with constant velocity gradient

For a layer with a constant vertical velocity gradient the velocity can be represented by

\[ v = v(z) = v_0 + az \]

where \( v_0 = v(z=0) \), 'a' is a constant and is called the vertical velocity gradient, and \( z \) is the depth measured from the surface.

For the borehole geometry (Fig 3.1.4) the travelt ime for the direct arrival from the source to the receiver in such a medium is given by

\[ t = \frac{1}{a} \cosh^{-1} \left[ 1 + \frac{a^2(x^2+z^2)}{2v_0(v_0+az)} \right] \]  \hspace{1cm} (B1)

differentiating B1 with respect to \( z \), the source coordinate and rearranging terms, we get

\[ \left( \frac{2v_0}{a} \right) \sinh(at)q = \frac{2z}{v_0+az} - \frac{a(x^2+z^2)}{(v_0+az)^2} \]  \hspace{1cm} (B2)

\[ \left( \frac{2v_0}{a} \right) \sinh(at)p = \frac{2x}{v_0+az} \]  \hspace{1cm} (B3)

where \( p = \frac{dt}{dx} \) and \( q = \frac{dt}{dz} \) are the geophone ray parameter and the shot ray parameter respectively.

Squaring and adding equations (B2) and (B3) and using the trigonometric identity \( \cosh^2(at) - \sinh^2(at) = 1 \) we obtain the relation between \( p \) and \( q \) for the direct arrival event as

\[ p^2 + q^2 = \frac{1}{(v_0 + az)^2} = \frac{1}{v_z^2} \]  \hspace{1cm} (B4)

where \( v_z \) is the velocity in the medium at the depth of the shot in the well about which the localised slant stack is done.
Appendix C

Dipping layer in a constant velocity medium with an inclined measuring surface.

(a) Transmission event:

The traveltime for the direct arrival event from the source to the receiver for a vertical well, and receivers on an inclined surface is a function of the angle between the receiver line and the well. Using the triangle formula we can write the traveltime relation as

\[ v^2t^2 = x^2 + y^2 - 2xy \cos \alpha' \]  \hspace{1cm} (C1)

where \( \alpha' \) is the angle between the well and the geophone line. Figure C shows the geometry of the vertical well.

Differentiating equation C1 with respect to \( x \), we get

\[ v^2tp = x - y \cos \alpha' \]  \hspace{1cm} (C2)

Differentiating equation C2 with respect to \( y \), we get

\[ v^2tq = y - x \cos \alpha' \]  \hspace{1cm} (C3)

where \( p \) and \( q \) are the geophone and shot ray parameters respectively.

Multiplying equation C2 with \( \cos \alpha' \) and adding with equation C3, we get

\[ v^2t (p \cos \alpha' + q) = y \sin^2 \alpha' \]  \hspace{1cm} (C4)

Multiplying equation C3 with \( \cos \alpha' \) and adding with equation C2, we get

\[ v^2t (p + q \cos \alpha') = x \sin^2 \alpha' \]  \hspace{1cm} (C5)

Substituting the \( x \) and \( y \) obtained from C4 and C5 into the traveltime equation C1 we obtain a relation between \( p \), \( q \), \( \alpha' \) and the velocity \( v \):

\[ p^2 + q^2 + 2pq \cos \alpha' = \frac{\sin \alpha'}{v^2} \]  \hspace{1cm} (C6)

if the angle of the geophone line is measured from the horizontal then \( \alpha' = 90 + \alpha \) and we can write the equation C6 in terms of \( \alpha \) cast in the equation of an ellipse as
\[\frac{(p+q)^2}{1 + \sin \alpha} + \frac{(p-q)^2}{1 - \sin \alpha} = \frac{2}{\nu^2}\] (C7)

(b) Reflection Event

The geometry for the reflection from a layer dipping at an angle \(\beta\) as measured along an inclined surface is shown in Figure C. The bed intercepts the well at \(O'\). In this figure \(g\) is the well to geophone offset. \(y\) is the offset of the image point \(I\) along the line joining the image point \(I\) and \(O'\), as measured from the point \(I'\). \(I'\) is the intercept of the line \(IO'\) with the earth's surface.

Figure A3: Schematic illustrating the geometry of the different parameters.

The traveltme relation for the reflection event is given by,
\[ v^2 t^2 = (y \sin 2\beta - g \cos \alpha)^2 + (y \cos 2\beta + g \sin \alpha)^2 \quad (C8) \]

Differentiating (C8) with respect to \( g \), we get,

\[ v^2 t_p = y \sin (\alpha - 2\beta) + g \quad (C9) \]

Differentiating (C8) with respect to \( y \), we get,

\[ v^2 t_q = y + g \sin (\alpha - 2\beta) \quad (C10) \]

Using the relations (C7), (C8) and (C9) and eliminating \( y, g \) and \( t \) between them we obtain the relation,

\[ p^2 + q^2 + 2pq \sin (\alpha - 2\beta) = \frac{\cos^2 (\alpha - 2\beta)}{v^2} \quad (C11) \]

which can be cast in the form of the equation for an ellipse as,

\[ \frac{(p + q)^2}{1 - \sin (\alpha - 2\beta)} + \frac{(p - q)^2}{1 + \sin (\alpha - 2\beta)} = \frac{2}{v^2} \quad (C12) \]

which reduces to the equation (A7) when \( \alpha = 0 \).
REFERENCES


