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A comparison of the magnetospheric specification model, the Garrett model and satellite data for the geosynchronous electron fluxes

Nagai, Akira, M.S.
Rice University, 1991
RICE UNIVERSITY

A COMPARISON OF THE MAGNETOSPHERIC SPECIFICATION MODEL, THE GARRETT MODEL AND SATELLITE DATA FOR THE GEOSYNCHRONOUS ELECTRON FLUXES

by

AKIRA NAGAI

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE MASTER OF SCIENCE

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A Comparison of the Magnetospheric Specification Model, the Garrett Model and Satellite Data for the Geosynchronous Electron Fluxes

by Akira Nagai

Abstract

The Magnetospheric Specification Model (MSM) calculates electron and ion fluxes that may endanger spacecraft. This thesis is to evaluate the electron flux levels specified by the MSM by comparison with the Garrett model output and spacecraft observations for the large magnetic storm of April 1988. The MSM is a magnetospheric physics model which uses ground-based and satellite data as input. The Garrett model, on the other hand, is a statistical model based on average geosynchronous electron fluxes.

The MSM flux enhancement predictions are in better temporal agreement than the other model. The largest error of the MSM is associated with flux dropouts which are observed by the spacecraft but not predicted by the MSM. The other possible error sources are 1) the MSM does not properly represent extreme thinning of the plasma sheet, 2) the MSM tends to overestimate the convection electric field.
Acknowledgments

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I. INTRODUCTION

As a result of an extremely large geomagnetic storm on March 6-19, 1989, many spacecraft experienced problems involving temporary damage or permanent loss. These problems include the following: "a stable low-latitude satellite (500-600 km) in nearly circular orbit inclined at 60 degree began uncontrolled tumbling"; "a Japanese communications satellite suffered permanent loss of half of the communication bands."; "NASA satellite SMM dropped in altitude as if it hit a brick wall."; "the Japanese geostationary meteorological satellite GMS-3 suffered severe scintillations and lost data transmissions for about one hour."; "NORAD lost track of over one thousand satellites, temporarily."; etc. (private communication, Heinemann, 1990).

These examples represent a severe impact on spacecraft operations not only from a scientific standpoint but also from governmental, industrial, even economic points of view. Research related to predicting magnetic storms and protecting spacecraft from these phenomena is becoming more and more important.

If the electrons and ions inside the magnetosphere could be traced, we could compute particle fluxes around spacecraft and carry out appropriate action to protect them from potentially dangerous storms. However, the phenomena involve extreme complexities both in a physical and a computational sense. The interaction between the solar wind, the IMF (Interplanetary Magnetic Field) and the magnetosphere is not yet completely understood. Processes inside the magnetosphere are even more complicated. Even if we could parameterize the system with all our known variables, the computation would still be a problem, since we have to deal with a tremendous amount of data and huge numbers of numerical integrations. Therefore, in order to realize a storm prediction system, appropriate methodology must be developed. This
requires a very high speed, large capacity computer. Recent rapid progress in high speed computer development now enables us to simulate the magnetic storm phenomena within a reasonable computing time. Further, an important method was developed by Jaggi and Wolf [1973], which utilizes the adiabatic invariant quantity $\eta$, defined to be the number of particles per unit magnetic flux. Instead of tracing particles, or number densities, we now trace $\eta$ using the adiabatic drift law. The MSM uses this method to provide higher computation accuracy with less computation time, thus we can compare the Magnetospheric Specification Model (MSM) output with actual satellite observations. The first run of the fully automated MSM was performed and the geosynchronous electron flux levels were also determined using the Garrett model for a large magnetic storm that occurred on April 21-23, 1988. In this thesis, the electron flux levels specified by both the MSM and the Garrett model are compared with spacecraft observations. Along with the comparison, a brief explanation of the two models is presented.

The geosynchronous orbit is one of the most important orbits both scientifically and economically due to presence of many spacecraft. Unfortunately, the plasma environment at geosynchronous orbit is also extremely complicated. It is probably one of the most complicated regions because of the following various phenomena: electrons and ions undergo a complex energy-dependent drift, the inner edge of the plasma sheet is usually located near the orbit and it may move in and out of the orbit during the substorm [Freeman, 1967, 1968], in addition, the plasma sheet may get extremely thin and the satellite may lie outside the center of the sheet. These phenomena were considered to be observed during the run and each will be discussed in relation with the MSM and the Garrett model output.
II. THE MAGNETOSPHERIC SPECIFICATION MODEL

The Magnetospheric Specification Model is fully described in Freeman et al. [1990], Hilmer [1989] and Hausman [1990]. Therefore, I will not present the details of the model here. Instead, I will explain how the model computes the magnetospheric parameters used internally in the MSM system. The current MSM consists of three major subsystems, which are the particle-tracer model, the electric-field model and the magnetic-field model. To compute the motion of the magnetospheric plasma the particle-tracer model traces the particle drift paths according to the adiabatic bounce-averaged drift law. For this trace, the particle tracer model requires the electric field and the magnetic field at every grid point. The E-field model and the B-field model compute these field values using the available ground based and satellite observations of magnetospheric parameters.

Another important subsystem of the MSM is the gateway machinery that governs the flow of parameters to the main calculation algorithms. It is called the front-end controller and it controls and processes the proper input for the MSM from available observations. The front-end controller produces the default parameters when desired data are not available. The following paragraphs describe these systems.

II.A The Particle Tracer Model

Figure 2.1 is a schematic diagram showing how the Particle Tracer Model advances the calculation of $\eta$ in time. This is one of the most important features in the MSM. This model computes plasma motions in the inner magnetosphere based on the electric and magnetic field supplied by the other submodels. The model assumes that
(1) particles convect adiabatically, (2) particle bounce times are far smaller than the drift time, and (3) strong pitch angle scattering results in isotropic pitch angle distributions.

The particle tracing algorithm works as follows. We assume an initial plasma configuration, i.e., assume that we know $\eta$ at all grid points at time $t=t_0$, where $\eta$ is the number of particles per unit magnetic flux. Harel et al. [1981] showed that $\eta$ is conserved along the drift path of a particle under lossless adiabatic conditions. Now, we want to calculate $\eta_P(t_1)$, $\eta$ on the grid point P in Figure 2.1 at a later time, $t=t_1>t_0$. This is done by tracing a particle of each energy invariant backward in time by following the bounce-averaged drift law

$$v_D = \frac{E \times B}{B^2} - \frac{\lambda}{q} \nabla \left( \frac{\dot{\mathcal{L}}/B}{B} \right)^{2/3} \times B$$

where $q$ is the charge of the particle and $\lambda$ is the energy invariant [Harel et al. 1981].

The particle energy in strong pitch-angle scattering then is expressed

$$W = \lambda \left( \frac{\dot{\mathcal{L}}/B}{B} \right)^{2/3},$$

where $\dot{\mathcal{L}}/B$ is the volume of a magnetic flux tube with one unit magnetic flux. The integral extends along a magnetospheric field line, from the southern ionosphere to the northern. The energy is proportional to the -2/3 power of the confining volume, as in an ordinary gas with adiabatic exponent $\gamma=5/3$.

Now, we have reached the traced point Q at time $t_0$. Since we know $\eta_A(t_0)$, $\eta_B(t_0)$, $\eta_C(t_0)$ and $\eta_D(t_0)$, we can obtain $\eta_Q(t_0)$ by a simple linear interpolation. Then, using the fact that $\eta$ is conserved along the path and including the possibility of the particle loss, we finally acquire

$$\eta_P(t_1) = \eta_Q(t_0) \exp(-\frac{t_1-t_0}{\tau})$$
where \( \tau \) is the loss lifetime. To update \( \eta \) from \( t_0 \) to \( t_1 \) for all grid points, we need to trace particles back from all grid points. Thus, we have obtained the new \( \eta \) for all grid points from the assumed initial \( \eta \) configuration.

The initial and boundary values of \( \eta \) at each grid point are empirically formulated by Wolf and Spiro (see Freeman et al. [1990]) from published plasma-sheet data (Huang and Frank [1986]), from the Garrett model (Garrett, private communication [1989]) and from the NASA radiation-belt model. The fluxes are first evaluated in the equatorial plane using these formulations and converted into \( \eta \) to be used as the initial and boundary conditions.

Figure 2.1 The schematic of the Particle Tracer Model, shown in the equatorial plane of the magnetosphere.
II.B Model Grid System

The MSM has a grid coordinate system on the ionosphere at the height defined by the variable $R_I$. The geocentric distance $R_I$ was set to 6475 km for this run. Sixty-two circles of constant latitude are used. The latitudinal grid spacing is non-uniform and ranges from 1.0 to 49.1 degrees in magnetic colatitude. This non-linear grid system was developed by Wolf so that the model could have more grid points in more important region of the magnetosphere, near the auroral zone (see Freeman, et al. [1990]). Figure 2.2 shows the relationship between the grid index, I, and geomagnetic colatitude. The local time is divided into 48 grid spaces, which is equivalent to a 30 minute separation.

The magnetic-field model is used to trace the ionospheric grid points to the equatorial plane. Although the grid is fixed in the ionosphere, its mapping to the equatorial plane is time-dependent. By means of this time-dependent mapping, the effects of the "corotation electric field" and of the "induction electric field" that is associated with the time-dependence of the magnetic-field model are automatically built in, without any additional calculation.

![Graph](image)

Figure 2.2 The MSM grid colatitude vs. grid index I.
Figure 2.3 shows the relationship between the ionospheric grid system used by the MSM and the GSM coordinate system, which is, the x-axis points sunward from the center of the Earth's dipole, xz-plane includes the earth's magnetic dipole, z-axis is perpendicular to the x-axis, and the y-axis is perpendicular to both axes in the right-hand manner. The figure is a dawn-dusk meridian cut, $R_E$ is the radius of the earth and $R_I$ is the modeling geocentric distance of the ionosphere. The particle-tracer model actually calculates the particle drift velocity relative to this rotating grid. This is performed as follows:

$$
\frac{di}{dt} = \frac{1}{d_{lat} d_{loc} B_{ir}} \frac{\partial V_{eff}}{\partial j}
$$

$$
\frac{dj}{dt} = -\frac{1}{d_{lat} d_{loc} B_{ir}} \frac{\partial V_{eff}}{\partial i}
$$

where $d_{lat}$ and $d_{loc}$ represent the distances between latitudinal grid points and between local-time grid points respectively, $B_{ir}$ is the radial component of geomagnetic flux at the ionospheric height, and $i$ and $j$ are floating point values of the indices $I$ and $J$. The $E\times B$ drift and gradient-curvature drift velocities were integrated into the form

$$
V_{ID} = \frac{B \times \nabla V_{eff}}{B^2}
$$
\[ V_{\text{eff}} = V + \frac{\lambda}{q} \left[ \int \frac{ds}{B} \right]^{-2/3} \]

and

\[ B_{ir} = -|B_{ir}| \hat{k} \]

II.C The Electric-Field Model

The electric field model was developed and formulated by Wolf. This model supplies the electric field at each grid point to the Particle Tracer Model and is driven by the DMSP satellite real-time observations. The model divides the northern ionosphere into four major regions using a simplified flat polar coordinate system. These regions are shown in the Figure 2.4. In the figure, region-0 is the polar cap, region-1 is the electric field reversal region, region-2 is the diffuse aurora area and region-3 is mid- and low-latitude area. They are separated by the boundaries a, b and c. The Electric Field Model first computes locations and shapes of these boundaries in the following order:

Figure 2.4. Regions and boundaries of the Electric Field Model
Determination of boundary-b

In the equatorial plane, the model first chooses the following four points on the dawn, dusk, noon and midnight meridians as shown in Figure 2.5;

\[ r_{\text{noon}} = 0.95 \, r_{\text{standoff}} \]
\[ r_{\text{dawn}} = r_{\text{dusk}} = 1.40 \, r_{\text{standoff}} \]
\[ r_{\text{midnight}} = 2.00 \, r_{\text{standoff}} \]

where \( r_{\text{standoff}} \) is the distance between the center of earth and the magnetopause at the sub-solar point. Then, the magnetic field model is used to map these four points back onto the ionospheric grid system. Points mapped from the dawn (dusk) equator are not generally on the dawn (dusk) meridian in the ionosphere because magnetic field lines tend to be swept anti-sunward from their ionospheric foot. Using these four ionosphere grid points, we obtain four parameters of the elliptic boundary which satisfies

\[ \left( \frac{x - DX_1}{A_1} \right)^2 + \left( \frac{y - DY_1}{B_1} \right)^2 = 1 \]

where \( x = \theta \cos \phi \) and \( y = \theta \sin \phi \), and where \( \theta \) is the invariant colatitude and \( \phi \) is the magnetic local time (0 at noon, \( \pi/2 \) at dusk, ...). Boundary b is an ellipse on a flat-polar ionospheric grid, but maps to a more complicated shape in the equatorial plane.
Determination of boundary-c

The equatorward edge of the diffuse aurora at local midnight, $\Lambda_{\text{midnight}}$ (in latitude), as obtained from the DMSP satellite observations, is used to determine the boundary-c. At local dawn and dusk, we choose

$$\Lambda_{\text{dawn}} = \Lambda_{\text{dusk}} = \Lambda_{\text{midnight}} + 1 \text{ degree}$$

as shown in Figure 2.6. Results of a study by Gussenhoven et al. [1983] are applied to obtain $\Lambda_{\text{noon}}$ such that

$$\Lambda_{\text{noon}} = \Lambda_{\text{midnight}} + \frac{(66.95 - \Lambda_{\text{midnight}}) \times 7.725 + (\Lambda_{\text{midnight}} - 56.8) \times 2.80}{10.15}.$$

The minimum width of region-2, the region between boundaries b and c, must be maintained as follows

$$\Delta \Lambda_{\text{noon}} = \Delta \Lambda_{\text{midnight}} = 3 \text{ degrees}$$

$$\Delta \Lambda_{\text{dawn}} = \Delta \Lambda_{\text{dusk}} = \frac{(4.5 \times 10^{-6} \text{ PCP})}{E_{\text{max}}}$$

where $E_{\text{max}} = 0.1 \text{ V/m}$ and PCP is the polar cap potential in volts.
If the width is less than the limit, the model expands the outer boundary-\( c \) so that the region-2 can keep the minimum width. The boundary is again assumed to be an ellipse and

\[
\left(\frac{x - DX_3}{A_3}\right)^2 + \left(\frac{y - DY_3}{B_3}\right)^2 = 1.
\]

![Diagram](image)

Figure 2.6 The mid- and low- latitude boundary \( c \).

**Determination of the boundary-\( a \)**

In order to determine boundary \( a \), we use the digitized Heppner-Maynard-Rich (HMR) empirical model of the ionospheric potential distribution. On the HMR empirical patterns, we defined ellipses \( a \) and \( b \), for each Heppner-Maynard potential pattern type (IPATT in the MSM code). By scaling HMR's ellipse-\( a \) so that the MSM's ellipse-\( b \) and HMR's ellipse-\( b \) coincide, we obtain the polar cap boundary-\( a \). The scaling relationships and the HMR ellipse parameters are shown next in fortran notation:
\[ A(1) = A(2) \times \frac{AHM(1, IPATT)}{AHM(2, IPATT)} \]
\[ B(1) = B(2) \times \frac{BHM(1, IPATT)}{BHM(2, IPATT)} \]
\[ DX(1) = DX(2) + \frac{A(2)}{AHM(2, IPATT)}[DXHM(1, IPATT) - DXHM(2, IPATT)] \]
\[ DY(1) = DY(2) + \frac{A(2)}{AHM(2, IPATT)}[DYHM(1, IPATT) - DYHM(2, IPATT)] \]

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### Computation of the Region-1 Potential

By relating the Heppner-Maynard-Rich ellipse-a and the MSM ellipse-a with
\[
\left( \frac{x_{HM} - DX_{HM}}{A_{HM}} \right)^2 + \left( \frac{y_{HM} - DY_{HM}}{B_{HM}} \right)^2 = 1,
\]
\[
\left( \frac{x - DX_1}{A_1} \right)^2 + \left( \frac{y - DY_1}{B_1} \right)^2 = 1
\]
and
\[
(x_{HM} - DX_{HM}) / A_{HM} = (x - DX_1) / A_1,
\]
\[
(y_{HM} - DY_{HM}) / B_{HM} = (y - DY_1) / B_1,
\]
we obtain \( V_1(x,y) \), the potential anywhere in the region-1 in the following manner:

1) Calculate \((x_{HM}, y_{HM})\) from given \((x,y)\) using above relationship.
2) Call a subroutine to obtain $V_{HM}(x_{HM}, y_{HM})$.

3) Compute $V_1(x, y) = V_{HM}(x_{HM}, y_{HM}) \times PCP / (V_{MAX}(IPATT) - V_{MIN}(IPATT))$

It should be noted that the subroutine that computes $V_{HM}$ was kindly supplied by F. J. Rich.

**Computation of the Region-3 Potential**

This describes the middle and low latitude electric field. We use studies and experimental results with the Rice Convection Model by Spiro et al. [1988] in this region. The RCM SUNDIAL run provided us with the following expression for potential at colatitude $\theta$ and local-time angle $\varphi$:

$$
V_{low}^{volt}(\theta, \varphi) = -2.75 \times 1000 \ E_{\text{west}}^{mV_{m}}(\theta_{\text{shield}}) \left[ \frac{\sin (\theta_{\text{shield}})}{\sin 25^\circ} \right]^b \left[ \frac{\sin (\theta_{\text{shield}})}{\sin \theta} \right] \\
\times [0.6103 \sin \varphi - 0.0154 \sin 2\varphi - 0.0210 \sin 3\varphi \\
- 0.1092 \cos \varphi + 0.1676 \cos 2\varphi - 0.0314 \cos 3\varphi ] + V_A
$$

with $p=1.38$ and $\theta_{\text{shield}} = A(3) - DX(3)$. $\varphi$ is measured from noon, i.e. $\varphi=\pi/2$ is dusk and $\varphi=3\pi/2$ is dawn. Then we assume that the latitudinal motion of the equatorward edge of the auroral zone is due predominantly to $E \times B$ drift as shown in Figure 2.7. Therefore,

$$
\frac{R_I d\Lambda}{dt} = -\frac{E_{\text{west}}}{B_{ir}}
$$

where $R_I$ is the geocentric distance of the ionopause, $\Lambda$ is the latitude of the boundary, $B_{ir}$ is the radial component of geomagnetic dipole on the ionopause, and $E_{\text{west}}$ is the westward electric field. Converting to units of degrees and hours and setting $B_{ir} = 0.5 \times 10^{-4}$ (T), we obtain
\[ E_{\text{west}}^{\text{Vhel}} = -1.576 \frac{d\Lambda^\circ}{dt_{\text{hour}}}. \]

\( V_A \) is the average potential at low latitude and is defined by

\[ V_A = 0.6 \ V_{\text{max}} + 0.4 \ V_{\text{min}} - 0.22 \ V_{\text{penet}} \]

where \( V_{\text{penet}} \) is the penetration potential. The total potential drop across the shielding layer, is given approximately by

\[ V_{\text{penet}}^{\text{volt}} = -13.1 \times 1000 \times g \frac{d\Lambda^\circ}{dt_{\text{hour}}} \sin(\theta_{\text{shield}}) \]

where \( g \) is a constant factor that is set to be 1.0 for this run.

---

**Figure 2.7** The motion of the equatorward edge of auroral zone and field line mapped \( E \times B \) drift motion on the equatorial plane.

---

**Computation of the Region-2 Potential**

The potential in this region, \( V_2(\theta, \varphi) \), is the sum of a smooth extrapolation of region-3 and an extra auroral-zone potential, namely

\[ V_2(\theta, \varphi) = V_{\text{low, } x}(\theta, \varphi) + V_{\text{az}}(\theta, \varphi) \]

where

\[ V_{\text{low, } x}(\theta, \varphi) = V_{\text{low}}(\theta_c(\varphi), \varphi) + (\theta - \theta_c) \frac{\partial V_{\text{low}}(\theta_c, \varphi)}{\partial \theta_c} \]
\[ V_{az}(\theta, \phi) = V_{b, az}(\phi) \left[ 1 - \frac{1}{\Delta \theta^2} \left( \frac{1 + (\theta_c - \theta)^2}{\Delta \theta^2} \right)^r \right] + F_{corr}(\theta, \phi) \]

and

\[ F_{corr}(\theta, \phi) = -6.75 \frac{dV_{b, az}(\phi)}{d\phi} \frac{\phi'}{\Delta \phi_{amp}} \frac{\Delta \phi_{amp}}{(\theta_c - \theta)^2 (\theta - \theta_b)} \]

with

\[ \phi' = \cos \left( \frac{\pi (\phi - \pi)}{\Delta \phi_{width}} \right) \quad \text{if} \quad \left| \frac{2(\pi - \phi)}{\Delta \phi_{width}} \right| < 1 \]

\[ \phi' = 0 \quad \text{otherwise} \]

\[ \Delta \phi_{width} = 1.0 \]

\[ \Delta \phi_{amp} = 2.0 \]

\[ r = 1 \]

\[ \Delta \theta = (\theta_c - \theta_b) \left[ 1.25 - 0.75 \cos \left( \frac{\phi - \frac{\pi}{4}}{\Delta \phi_{width}} \right) \right] \]

**Computation of the Region-1 Potential**

Region 1 is a transition region between regions 0 and 2. The following procedure provides the region-1 potential.

1) Find \( \theta_a(\phi) \) and \( \theta_b(\phi) \) for the \( \phi \)-value in question.

2) Find \( v_0 = V_0(\theta_a(\phi), \phi) \), \( v_0' = \partial V_0(\theta, \phi)/\partial \theta / \theta_a(\phi) \), \( v_2 = V_2(\theta_b(\phi), \phi) \) and \( v_2' = \partial V_2(\theta, \phi)/\partial \theta / \theta_b(\phi) \) using the analytic formulas described for regions 0 and 2.

3) Set

\[ V(\theta, \phi) = v_0 + v_0'(\theta - \theta_a) + [3(v_2 - v_0) - (2v_0' + v_2') \delta \theta] \frac{(\theta - \theta_a)^2}{\delta \theta^2} \]

\[ + [2(v_0 - v_2) + (v_0' - v_2') \delta \theta] \frac{(\theta - \theta_a)^3}{\delta \theta^3} \]

where \( \delta \theta \equiv \theta_b(\phi) - \theta_a(\phi) \). This formula makes both \( V \) and its derivative continuous at both \( \theta_a(\phi) \) and \( \theta_b(\phi) \).
II.D The Magnetic Field Model

Since the magnetic field model is fully described by Hilmer [1989], I will not go into the details. The model's objective is to provide a scale to map the MSM ionospheric grid points to the inner magnetosphere under various magnetospheric conditions and to compute the flux tube volumes required by the Particle Tracer Model. The B-field model is used to produce the supermatrix which is a look-up table of information connecting the ionospheric grid points with the magnetic equatorial plane for a large number of configurations. Since the present MSM code uses the look-up table, the B-field model is not called directly as a real-time procedure. The B-field model is a semi-empirical, 3 dimensional model which consists of following four components.

\[ \mathbf{B} = \mathbf{B}_d + \mathbf{B}_{roc} + \mathbf{B}_{tail} + \mathbf{B}_{cf} \]

where

- \( \mathbf{B}_d \) = Earth's main magnetic field (Dipole)
- \( \mathbf{B}_{roc} \) = Equatorial ring current field
- \( \mathbf{B}_{tail} \) = Cross-tail current field
- \( \mathbf{B}_{cf} \) = Chappman-Ferraro current field.

The model computes the field configuration so that the total magnetic field satisfies \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \) and \( \nabla \cdot \mathbf{B} = 0 \). There is no magnetopause shielding of either the ring or the cross-tail currents and no explicit Birkeland currents are included. Also \( (\mathbf{B}_d + \mathbf{B}_{cf}) \cdot \mathbf{n} = 0 \) where \( \mathbf{n} \) is a unit vector normal to the magnetopause.

Sets of B-field matrices are precomputed and specified by selecting the following indices (for the April '88 event run):
\( \Psi = \text{dipole tilt angle, which is the angle between the magnetic dipole axis and the } Z_{\text{GSM}} \text{ direction and is defined to be positive for northern hemisphere summer. This is currently set to be } 0^\circ \text{, i.e., the dipole is assumed to be aligned with the } Z_{\text{GSM}} \text{ axis. (In the MSM, the dipole axis is always aligned with the Earth's rotation axis.)} \)

\( r_s = \text{magnetopause stand-off distance. } r_s \in \{ 6, 8, 10, 12, 14 \text{ RE} \} \)

\( r_s \) is determined from the solar wind pressure and used to describe the size of the magnetosphere. It is also used to define the MSM calculation region, see Figure 2.5.

\[ \text{Dst} = \text{geomagnetic index (mid-latitude average of } \Delta H). \]
\[ \text{Dst} \in \{ -400, -300, -200, -150, -100, -50, 0, 50 \text{ nT } \}. \]

Magnetic field contributions from all components (\( B_d, B_{rc}, B_{\text{tail}} \) and \( B_{cf} \)) must produce a final equatorial field which is consistent with the \( \text{Dst} \) at the Earth's surface.

\[ \Lambda_0 = \text{latitude of the equatorward edge of diffuse aurora at local midnight.} \]
\[ \Lambda_0 \in \{ \text{16 latitudes from } 49.5 \text{ to } 69.3 \text{ degree } \}. \]

The midnight ionosphere point of this latitude must map, using the magnetic field, to a specified point in the equatorial plane at midnight where the inner edge of the cross-tail current is placed, see Hilmer [1989].
II.E Sequence of the MSM

The general sequence of the Magnetospheric Specification Model is approximately as follows.

START

Grid Initialization

Flux Initialization

Read input data and select B-field models

Compute the E-Field for the entire run period

Compute the boundary for the entire run period

Compute initial plasma distribution

for ( time=start to end ) {
    Trace particles
    Compute fluxes
    renew η matrix
}

Compute errors

Compute high energy fluxes

Output results

END

The MSM initializes the grid according to the given configuration, then initializes fluxes using Kp based statistical methods. The E-field and B-field model compute the electric potential matrix and selects the magnetic field configuration matrices for the entire run period. Then, the MSM sets the model boundary and computes the initial plasma distribution based on either Kp based statistics or previous run results. Now, all parameters required by the particle tracer are ready and the tracer is initiated and
continues to compute the flux and $\eta$ for desired steps. The MSM output is compared
with data from geosynchronous spacecraft, if these data are available, and summaries
of errors are printed out. A high energy flux computation based on the study by Koons
and Gorney, Aerospace Corporation, has been installed.

It should be remarked here that the electric field is precomputed at nearly the
beginning of the run and before the particle tracing. And the B-field matrix is also a
precomputed look up table. In other words, the electric and magnetic configuration are
not computed step by step in the particle tracing routine. The actual field values, flux
tube volumes, etc., are determined by interpolation using the values of the established
configuration matrix. The values at a particular time are computed by interpolation,
not by creation of new model configuration. This implies that the current MSM is not a
real-time process. Prior to the initiation of the tracer, the tracer must know all of the
parameters for the entire run period. In this mode, the MSM can be used for
retrospective analysis or diagnostic work a short time after the input data for a given
run are available.
III. THE GARRETT MODEL

Garrett et al. [1979, 1981a, 1981b] studied a large amount of plasma data from geostationary orbit and, on the basis of this, Garrett developed an empirical model to describe the electron and ion fluxes at geostationary orbit. Recently, Garrett and Gaudet revised Garrett's previous models [unpublished, 1989]. The revised model was provided to Rice University. This revised model, which we will use for the evaluation of the MSM, consists of a Fourier expansion of the local time dependence and a linear relationship to Ap, where Ap is a linear version of Kp. The model first computes four moments of the plasma distribution and then gives the Bi-Maxwellian distribution parameters. This enables us to obtain flux predictions at the synchronous orbit based on the statistical study. The model uses data from the geosynchronous spacecraft ATS-5 and ATS-6 and near-geosynchronous P78-2 SCATHA spacecraft. ATS-5 and ATS-6 were launched in August 1969 and May 1974 respectively and both of them have electrostatic analyzers (from the University of California at San Diego) which were calibrated before the launch. ATS-5 is a spin-stabilized cylinder, of length 1.8 meters and diameter 1.5 meters and the spin axis is parallel to the earth's rotation axis with a 0.79 second spin period. It has detectors parallel and perpendicular to magnetic field lines. The ATS-6, on the other hand, is a 3-axes stabilized large dish (10 meters in diameter) equipped with north-south and east-west particle detectors. (But, the east-west detector didn't work.)

Garrett et al. analyzed over 100 of days of data at 1 to 10 minute time resolution which had been binned according to Kp, or Ap, and to local time. A key assumption made was that there exists only weak-coupling between the plasma moments, Ap and local time.
The spacecraft plasma detector measures particle count rates in selected energy ranges. The UCSD 64-channel detector can observe from 51eV to 51keV for ATS-5 and 1eV to 80keV for ATS-6. The 10-minute averaged data was used to construct the distribution function \( f(E) \). The derivation of the distribution function from spacecraft data is discussed in Appendix B. By assuming an isotropic plasma distribution at geosynchronous orbit, the first 4 moments were computed as follows:

\[
\begin{align*}
ND_e &= 4\pi \int_0^\infty v^0 f_e(v) \, dv \\
NF_e &= \int_0^\infty v^1 f_e(v) \, dv \\
ED_e &= 4\pi \int_0^\infty \left(\frac{1}{2}mv^2\right) f_e(v) \, dv \\
EF_e &= \int_0^\infty \left(\frac{1}{2}mv^3\right) f_e(v) \, dv
\end{align*}
\]

where the subscript \( e \) stands for electrons. \( ND_e \) is number density, \( NF_e \) is number flux, \( ED_e \) is energy density and \( EF_e \) is energy flux. The model also includes an analysis for the ion distribution which is not discussed in this thesis. These 4 moments were then plotted in terms of \( Ap \) and local time independently (see Figure 3.1). Regression procedures were employed to obtain the following expressions:

\[
F_j(Ap) = C_{1j} + C_{2j} \cdot Ap
\]

\[
G_j(LT) = D_{1j} + D_{2j} \cdot \cos(t \cdot 2\pi/24) + D_{3j} \cdot \sin(t \cdot 2\pi/24) \\
+ D_{4j} \cdot \cos(t \cdot 4\pi/24) + D_{5j} \cdot \sin(t \cdot 4\pi/24)
\]

where \( C_1, C_2, \) and \( D_1 \) to \( D_5 \) are constants to be determined and \( j \) refers to the particular moment. \( Ap \) is directly calculated from \( Kp \) and the relationship is shown in
Table 3.1. To maintain compatibility with their earlier models, t is set to LT +6.5h. Then, Gj is normalized and multiplied by Fj to obtain Mj such as

\[ M_j(Ap, LT) = F_j(Ap) \cdot \text{G}'_j(t) \] \hspace{1cm} (3.7)

The orthogonality of Ap and local time is explicitly assumed. In the program code this is expressed in the form

\[ M_j(Ap, LT) = \left[ C_{1j} + C_{2j}\cos(t \, 2\pi/24) + C_{3j}\sin(t \, 2\pi/24) + C_{4j}\cos(t \, 4\pi/24) + C_{5j}\sin(t \, 4\pi/24) \right] \cdot Ap \] \hspace{1cm} (3.8)

The coefficients are shown in Table 3.2 with Fortran syntax.

Garrett and DeForest [1979] published a discussion of the validity of the Bi-Maxwellian, or 2-Maxwellian, distribution function for geosynchronous plasma, i.e.,

\[ f_0(E) = 27.2 \left[ n_{1e}T_{10}^{-3/2}e^{-E/T_{10}} + n_{2e}T_{20}^{-3/2}e^{-E/T_{20}} \right] \] \hspace{1cm} (3.9a)

\[ f_i(E) = 2.14 \times 10^6 \left[ n_{1i}T_{11}^{-3/2}e^{-E/T_{11}} + n_{2i}T_{21}^{-3/2}e^{-E/T_{21}} \right] \] \hspace{1cm} (3.9b)

where \( f = \) distribution functions [s\(^3\) km\(^{-6}\)], \( n = \) number densities [cm\(^{-3}\)], \( T = \) temperatures [keV], \( E = \) particle energies [keV]. Figure 3.2 shows the typical Bi-Maxwellian fit for geosynchronous electrons and ions taken from their paper. It can be seen that the 2-population Maxwellian distribution improved the fit to the spacecraft observation especially for ions. By substituting this distribution function (3.9) back into (3.1) to (3.4), we obtain the expression for Bi-Maxwellian parameters in terms of 4 moments. This was demonstrated by Garrett and Deforest [1979] and the result is:

\[ T_1^{1/2} = -B + \sqrt{B^2 + 4AC} \] \hspace{1cm} (3.10)

\[ T_2^{1/2} = -B + \sqrt{B^2 - 4AC} \] \hspace{1cm} (3.11)
\[ n_2 = \frac{C_2 - T_1^{1/2} C_1}{T_2^{1/2} - T_1^{1/2}} \quad (3.12) \]
\[ n_1 = n - n_2 \quad (3.13) \]

where the above relationships are obtained by solving the next quadratic equations in terms of \( T_1^{1/2} \):

\[
\begin{align*}
  n_1 + n_2 &= n = C_1 \\
  n_1 T_1^{1/2} + n_2 T_2^{1/2} &= NF \left( \frac{\pi m}{2k} \right)^{1/2} = C_2 \\
  n_1 T_1 + n_2 T_2 &= \frac{P}{k} = C_3 \\
  n_1 T_1^{3/2} + n_2 T_2^{3/2} &= \frac{B F (\pi m)^{3/2}}{2k} = C_4
\end{align*}
\]

And new variables are defined as \( A = C_2 - C_1 C_3 \), \( B = C_1 C_4 - C_2 C_3 \) and \( C = C_3^2 - C_2 C_4 \).

Thus, we obtain the Bi-Maxwellian distribution function as a function of \( Ap \), i.e. \( Kp \), and local time, enabling us to estimate geosynchronous electron fluxes. The dependence on \( Kp \) and local time of 4 moments observed by ATS-5 and ATS-6 and derived Bi-Maxwellian parameters are shown in Figure 3.1.

<table>
<thead>
<tr>
<th>( Kp )</th>
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<th>( Kp )</th>
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<td>176</td>
<td>7+</td>
<td>1232</td>
</tr>
<tr>
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<td>16</td>
<td>4</td>
<td>216</td>
<td>8-</td>
<td>1432</td>
</tr>
<tr>
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<td>24</td>
<td>4+</td>
<td>256</td>
<td>8</td>
<td>1656</td>
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<tr>
<td>1</td>
<td>32</td>
<td>5-</td>
<td>312</td>
<td>8+</td>
<td>1888</td>
</tr>
<tr>
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<td>40</td>
<td>5</td>
<td>384</td>
<td>9-</td>
<td>2400</td>
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<tr>
<td>2-</td>
<td>48</td>
<td>5+</td>
<td>448</td>
<td>9</td>
<td>3200</td>
</tr>
<tr>
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<td>536</td>
<td>3-</td>
<td>536</td>
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<td>752</td>
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<td>888</td>
<td>3+</td>
<td>1056</td>
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<tr>
<td>3+</td>
<td>144</td>
<td>7+</td>
<td>1056</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.2 The Garrett model coefficients

| DATA ( (C(I,J), I=1,10), J=1,8) /
| $0.75230B+02,-0.28078E+02, 0.24575E+02,-0.56359E+01, 0.10929E+01,
| $0.24140B+00,-0.90099E-01, 0.78857E-01,-0.18085E-01, 0.35069E-02,
| $0.90430B+02, 0.93146E+01,-0.16894E+01, 0.27812E+01, 0.21889E+01,
| $0.77620E-01, 0.79952E-02,-0.14501E-02, 0.23873E-02, 0.18788E-02,
| $0.17521E+02,-0.10863E+02, 0.74818E+01,-0.12412E+01, 0.21019E+01,
| $0.12618E+00,-0.78228E-01, 0.53878E-01,-0.89379E-02, 0.15136E-01,
| $0.10450E+03, 0.20327E+02, 0.61179E+01, 0.83670E+00, 0.11514E+01,
| $0.15994E+00, 0.31112E-01, 0.93636E-02, 0.12806E-02, 0.17622E-02,
| $0.14077E+03,-0.81727E+02, 0.49576E+02,-0.84721E+01, 0.14674E+02,
| $0.97372E+00,-0.56532E+00, 0.34292E+00,-0.58602E-01,-0.10150E+00,
| $0.33013E+02, 0.73802E+01,-0.31617E+01,-0.18114E+01, 0.15129E+01,
| $0.75400E-01, 0.16856E-01,-0.72214E-02,-0.41372E-02, 0.34556E-02,
| $0.93976E+02,-0.55286E+02, 0.47522E+02,-0.61020E+01,-0.93218E+01,
| $0.66958E+00,-0.39391E+00, 0.33800E+00,-0.43477E-01,-0.66418E-01,
| $0.69145E+01, 0.11542E+01, 0.82090E+00,-0.19881E-01,-0.53300E-02,
| $0.10519E-01, 0.17559E-02, 0.12488E-02,-0.30244E-04,-0.81084E-05/ |

where j=1,7,3,5 are for 4 moments of electrons (NDe, NFe, EDe, EFe) and j=2,8,4,6 are for ions (NDi, NFi, EDi, EFI). These moments can be retrieved with relationship (3.8). To obtain proper units for the moments, the following factors must be multiplied: 0.01 (for j=1 and 2), 1.0e-6 (for j=7 and 8), 1.5×6.24146e11×1.0e-10 (for j=3 and 4), 0.01×6.24146e11 (for j=5 and 6). Then the units will be ND[cm⁻³], NF[cm⁻²s⁻¹sr⁻¹] ED[eVcm⁻³] and EF[eVcm⁻²s⁻¹sr⁻¹].
Figure 3.1  Averaged Kp, local time dependence of 4 moments and Bi-Maxwellian parameters of electrons and ions observed by ATS-5 and ATS-6. From Garrett et al. [1981].
Figure 3.2  Electron and ion distribution functions observed by ATS-5. Maxwellian and Bi-Maxwellian fits are also shown. From Garrett and Deforest [1979].
IV. TESTING THE MODEL WITH THE APRIL 1988 EVENT.

IV.A General Description of the Event

In order to obtain statistics to determine the accuracy of the MSM, the model output was compared with electron flux data supplied by the Air force from three of their geostationary satellites. The event used for this evaluation was the large magnetic storm lasting from April 21 to 23, 1988. The magnetospheric parameters observed during the event are shown in Figures 4.1 to 4.8. In the figures, Kp is the planetary magnetic activity indices, Dst is the average change in magnetic-field strength measured on the ground at low latitudes (large negative Dst represents a storm-time ring current), SWDEN and SWVEL denote the solar wind density and velocity, and PCP, XIPATT and DLATAZ represent the polar cap potential drop, the polar cap potential pattern and the latitude of the equatorward edge of the diffuse aurora at local midnight, respectively.

For descriptions of geomagnetic indices see Rostoker [1972] and Mayaud [1980]. Also, for descriptions of typical substorm sequence see Rostoker et al. [1980], McPherron [1979], and Hones et al. [1977].

Several major features of the storm can be determined from these figures as follows:

1) There seems to have been a complicated succession of substorm-like events in the MSM run period, as can be seen from the DLATAZ and the Dst observations. The expansion phase of the first big substorm started near the end of day 112. There is some indication of a relatively well defined substorm starting at day 114.0.
2) The Dst index was positive just before the first substorm (from about day 112.6 to 112.9). The solar wind dynamic pressure was relatively high in the period. The solar wind pressure is calculated and plotted in Figure 4.9.

3) As seen from Figure 4.8, the IMF-Bz seems to be positive during the positive Dst period, but, the details can't be determined because of the data gap. In this period, the solar wind compressed the magnetosphere and caused the increase of the Dst index.

4) The IMF-Bz was mostly negative after day 112.8. (Again, it can't be determined clearly because of the fluctuation and the gaps in the spacecraft observational data of IMF.) A high probability of magnetic reconnection is expected at the magnetopause after day 112.8.

5) This was a large storm with the Dst index declining below -100 nT.

6) Some more substorm activities were observed during the recovery phase. Also, in the recovery phase, two big enhancements of the solar wind pressure were observed at about day 113.7 and 114.2 (these are not very precise due to data gaps).

The observed interplanetary magnetic field (IMF) is plotted in Figure 4.8. The solid lines are 1-hour averages. The data show large fluctuations and there seem to be no clear relationships between IMF-Bz and Dst. However, on average, the IMF-Bz was mostly southward during the large substorm period, which is consistent with generally accepted picture of a typical substorm.
Figure 4.1  The magnetospheric parameter of the April 1988 event: $K_p$.

Figure 4.2  The magnetospheric parameter of the April 1988 event: $Dst$. 
Figure 4.3  The magnetospheric parameter of the April 1988 event: SWDEN, the solar wind density.

Figure 4.4  The magnetospheric parameter of the April 1988 event: SWVEL, the solar wind velocity.
Figure 4.5  The magnetospheric parameter of the April 1988 event: PCP, the polar cap potential drop.

Figure 4.6  The magnetospheric parameter of the April 1988 event: XIPATT, the polar cap potential pattern type.
Figure 4.7  The magnetospheric parameter of the April 1988 event: DLATAZ, the latitude of the equatorward edge of the diffuse aurora at local midnight.
Figure 4.8  The magnetospheric parameter of the April 1988 event: IMF. The solid lines are one hour running averages.
Figure. 4.9 The solar wind pressure of the April 1988 event.

This figure was simply computed from the solar wind density and velocity shown in Figures 4.3 and 4.4. The geometric factor was not taken into account and $n_{SW}(m^{-3}) \cdot (V_{SW}(m \ s^{-1}))^2 \cdot 10^{-18}$ is plotted, which is sufficient for a qualitative discussion.
IV.B Use of the Input Parameters in the MSM

The parameters discussed in Chapter IV.A are the fundamental inputs to the Magnetospheric Specification Model. This section will explain the schematic flow of these magnetospheric parameters used for the evaluation of the April 1988 event.

Figure 4.10a shows the flow of the magnetic-field model input parameters. The inputs of the model are the magnetopause standoff distance, Dst, the latitude of the equatorward edge of diffuse aurora at local midnight, the magnetic field collapse indicator and the dipole tilt angle. The standoff distance is computed from the solar wind density (SWDEN) and the solar wind velocity (SWVEL). The standoff distance is derived from the pressure balance, at magnetopause subsolar point, between the plasma pressure exerted by the solar wind and the geomagnetic field pressure multiplied by a geometric factor. The formula used in this run is

\[
R_0 = \left[ \frac{1.3456 \times \frac{B_0^2}{2 \pi m_H n_{sw} v_{sw}^2}}{} \right]^{1/6}
\]

where \( B_0 \) = geomagnetic field strength at equator = 0.311829 [gauss], \( m_H \) = proton mass [gram], \( n_{sw} \) in [1/cm^3], \( v_{sw} \) in [cm/s], \( R_0 \) in [RE]. No Helium atoms were assumed to be in the solar wind.

The collapse parameter indicates whether or not the field lines are collapsed in the midnight region of the magnetotail; such collapse occurs in the expansion phase of a substorm. The dipole tilt angle is the angle between the direction to the dipole axis and the Z_GSM direction. For the evaluation of the April 1988 event described in this thesis, the collapse parameter is turned-off and the dipole tilt angle is 0°.

Figure 4.10b shows the schematic flow of the model parameters used by the electric-field model of the MSM. The boundary c, the equatorward edge of the shielding region, is estimated from the equatorward edge of diffuse aurora at local
midnight (DLATAZ) or if necessary from Kp using the study by Gussenhoven et al. [1983]. The boundary b, the equatorward edge of the electric-field reversal region, is computed from the magnetic field model.

The Heppner-Maynard potential pattern (XIPATT) is determined from the interplanetary magnetic field $B_y$ and $B_z$. This pattern determines the precomputed shapes of the Heppner-Maynard ellipses a and b. Then the boundary b is scaled so that it coincides with the former boundary b computed from the magnetic-field model. The boundary a is then scaled with the same factor. Thus, all boundaries are determined. With these and PCP, the polar cap potential drop, we can obtain the electric-field. All boundaries are assumed to be ellipses.

Kp is used by the Particle Tracer Model of the MSM in order to set the initial condition of the plasma distribution.

The planetary magnetic index Kp is assumed to be always available, even though other parameters may not always be available. The MSM must work even if some input parameters are not available at the MSM's input request. The Front-End Models of the MSM determine the input parameters to the MSM when the direct observational data are not available for longer than the maximum data gap periods. If there is a gap in the data stream, a linearly interpolated value is sent to the MSM. But, if the time gap happens to be very long, the interpolation loses accuracy. Therefore, we decided to employ statistical, empirical formulas to produce a proxy value for the large data gap. In order to switch from simple linear interpolation to statistical, empirical formula, we must specify the maximum data gap. An experiment was performed to obtain the maximum gap values. The determination of the maximum data gap for various input parameters is discussed in Appendix A. The formulas for the backup input parameters are shown in Table 4.1. Fortunately, the data were all
available for this evaluation; therefore, the default models for these formulas were not required. Table 4.2 shows the sources and additional information on the input parameters for this run.

![Diagram](image)

(a)

![Diagram](image)

(b)

Figure 4.10 The schematic flow of the B-Field and E-Field Model inputs.
### Table 4.1 Front-End Model: Kp based formula

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Formula</th>
<th>Source</th>
</tr>
</thead>
</table>
| Dst [nT]                    | 1) Dst=-355+5.25×DLATAZ  
                             | 2) Dst=-Kp×4 for Kp<4,  
                             |                         |                         |                         |
|                             | Dst=-20-(Kp×10-40)×2.6 for Kp>4                                      | Freeman                 |
|                             |                                                                         | (private comm.)         |
| Equatorward edge            | DLATAZ=66.95-2.03×Kp                                                  | Gussenhoven et al.      |
| of aurora at midnight [deg] |                                                                         | [1983]                  |
| Polar cap potential         | PCP=14.587+17×Kp                                                       | Reiff                   |
| [kV]                        |                                                                         | (private comm.)         |
| Polar cap pattern           | XIPATT=1                                                               |                         |                         |
| Standoff Distance           | Rs=11.7-Kp for Kp<5,                                                    | Freeman                 |
| [Re]                        | Rs=6 for Kp>9                                                          | (private comm.)         |

### Table 4.2 The summary of the MSM input parameters

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<thead>
<tr>
<th>Input</th>
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</tr>
<tr>
<td>Dst</td>
<td>Ground observation</td>
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<td>Solar wind density</td>
<td>Satellite-OMNI data set</td>
<td>NSSDC*</td>
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<tr>
<td>Solar wind velocity</td>
<td>Satellite-OMNI data set</td>
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<td>Satellite DMSP</td>
<td>GL/UTD*</td>
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<tr>
<td>Polar cap potential pattern</td>
<td>Satellite DMSP</td>
<td>GL/UTD*</td>
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<td>Equatorward edge of aurora at midnight</td>
<td>Satellite DMSP</td>
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</tbody>
</table>

*GL: Geophysical Laboratory  
*NSSDC: The National Space Science Data Center  
*UTD: University of Texas at Dallas
IV.C Calculation of Geosynchronous Fluxes

In order to compare the MSM output with the satellite observations, the MSM output fluxes must be determined at the three Air Force satellite locations. Since the output time intervals of the MSM and the satellite data times are generally different, and since the MSM grid points mapped onto the equatorial plane are not necessarily at the geosynchronous orbit, the model output fluxes had to be interpolated in time and space. This interpolation was carried out as follows: (1) First, the spatial interpolation was performed both for $t_1$ and $t_2$ where, $t_1 < t_{\text{satellite}} < t_2$ and $(t_1, t_2)$ are the times of the closest MSM output. A linear spatial interpolation was performed using the model output fluxes at the grid points as shown in Figure 4.11. (2) Then the two model fluxes $j(t_1)$ and $j(t_2)$ were linearly interpolated in time:

$$j(\text{at the satellite}) = j(t_1) \cdot \frac{(t_2 - t_{\text{satellite}})}{(t_2 - t_1)} + j(t_2) \cdot \frac{(t_{\text{satellite}} - t_1)}{(t_2 - t_1)}.$$  

The MSM uses the GSM coordinate system, with the $x$-axis pointing sunward from the center of the Earth's magnetic dipole, the $z$-axis is perpendicular to $x$-axis, and the $y$-axis is perpendicular to the $xz$-plane which contains the dipole (see Russel [1971]). In this evaluation, the MSM computes fluxes in the magnetic equator, while satellites measure fluxes in the geographic equator. The satellite data have 5 to 10 minute-interval on average, while the MSM output is updated every 15 minutes.

Before the interpolation, the model output must be converted to electron fluxes. The MSM keeps track of the adiabatic invariant $\eta$, the number of particles per unit magnetic flux, instead of particle fluxes. The derivation of the flux is as follows:

We have rigorously

$$\text{NF} = \int v f v^2 dv \int d\Omega$$  

(4.2)

where NF is the number flux and $f$ is the distribution function. Then, the differential flux $j(E)$ is
Figure 4.11 The interpolation of the MSM fluxes at the satellite location. 
\[ J_{\text{satellite}} = [1-(f_j-j)](f_i-I) + (1-(f_i-I))J_B \] + \[ [f_i-J](f_i-I) + (1-(f_i-I))J_C] \].

The I and J are the MSM grid index (integer) and the satellite location is expressed by \( (f_i, f_j) \), the floating point index in (I, J) coordinates. This is the equatorial plane and the MSM grid points were mapped by the B-Field Model from the ionospheric grid points. Therefore, the geosynchronous orbit is not always between the same lines of I or J since the magnetic field configuration is changing in time.

\[ j(E) = \frac{d^2(NF)}{dE \ d\Omega} = \frac{d}{dE} \int v^3 f \ dv \]  
(4.3)

With \( E=mv^2/2 \) and \( dE=mv \ dv \), we obtain
\[ j(E) = \frac{2}{m^2} \frac{d}{dE} \int f \ E \ dE = \frac{2}{m^2} E f \]  
(4.4)

Now, we need to obtain the expression of the distribution function \( f \) in terms of the model parameters. We use \( \eta_k \), the \( \eta \) for a given energy range,
\[ \eta_k = \int_{v_{\text{min}}}^{v_{\text{max}}} f v^2 \ dv \int d\Omega \int \frac{ds}{B} \]  
(4.5)

where the energy range is \( E_{\text{min}} < E < E_{\text{max}}, E_{\text{min}} = (E_{k-1} + E_k) / 2 \) and \( E_{\text{max}} = (E_k + E_{k+1}) / 2 \).

We assume an isotropic and constant distribution function inside the given energy range. Then, the distribution function \( f \) can be factored out of the integral and
\[
\eta_k = 4\pi f \int_{E_{\text{min}}}^{E_{\text{max}}} v^2 \, dv \int \frac{ds}{B} = 4\pi f \left[ \frac{2E}{m} \right]^{3/2}_{E_{\text{max}}} \int \frac{ds}{B} = f \frac{8\pi}{3} \sqrt{\frac{2}{m^3}} \left[ E_{\text{max}}^{3/2} - E_{\text{min}}^{3/2} \right] \int \frac{ds}{B}.
\]

(4.6)

Solving for \( f \) with the energy invariant \( \lambda \), where \( E = \lambda \left[ \int \frac{ds}{B} \right]^{-2/3} \), we obtain the distribution function:

\[
f(E_k) = \frac{3}{8\pi} \sqrt{\frac{m^3}{2}} \frac{\eta_k}{\lambda_{\text{max}}^{3/2} - \lambda_{\text{min}}^{3/2}}.
\]

(4.7)

Substituting (4.7) into (4.3), finally we obtain the differential flux in terms of the model parameters:

\[
j(E_k) = \frac{3}{4\pi} \sqrt{\frac{1}{2m}} \frac{\lambda_{k}}{\lambda_{\text{max}}^{3/2} - \lambda_{\text{min}}^{3/2}} \eta_k \left[ \int \frac{ds}{B} \right]^{-2/3}
\]

(4.8)

---

Note:

In the MSM code, (4.9) is used instead of (4.8). This was derived by assuming \( f v \) is constant. The difference between (4.8) and (4.9) is negligible (smaller than or of the order of a percent) in the MSM energy channel setup.

\[
j(E_k) = \frac{1}{2\pi} \sqrt{\frac{1}{2m}} \frac{\sqrt{\lambda_{k}}}{\lambda_{\text{max}} - \lambda_{\text{min}}} \eta_k \left[ \int \frac{ds}{B} \right]^{-2/3}
\]

(4.9)
IV.D Comparison with the Satellite Observation

The geosynchronous electron fluxes computed by the MSM and by the Garrett model are compared with the data observed by three Air Force geosynchronous spacecraft. These spacecraft are located at 322° east, 205° east and 70° east, which I would like to call satellite-1, -2 and -3, respectively, in this thesis. The local times of these satellites are UT - 2.5 hours, UT + 13.7 hours and UT + 4.7 hours, respectively, where UT is the universal time.

The two lowest energy channels, 30-44 keV and 44-64 keV, of the particle detectors on the Air Force spacecraft were selected for this evaluation from the following considerations and limitations; (1) the MSM was able to compute fluxes less than ~ 100 keV, (2) the high-energy submodel had not been installed in the MSM at the time of evaluation, (3) the Garrett model is valid for fluxes less than ~80 keV, and (4) the Air Force spacecraft were not equipped with a lower energy particle detector.

The differential channel passbands of the detectors are much larger than these widths and so the fluxes in adjacent channels were subtracted to form these smaller differential channels. In consultation with Tom Cayton of Los Alamos National Laboratory, a standard correction was added for detector dead-time. At the same time, the lowest channel flux was increased by a factor of 2 and the next lowest by 1.4 to achieve agreement with ATS data [Garrett, Private Communication].

The MSM and the Garrett Model Output Fluxes

These models' output fluxes for two energy channels and the corresponding satellite fluxes are plotted in Figures 4.12 to 4.17. In these figures, the thick solid lines denote the MSM output, while the solid lines with square markings are the
satellite measurements. The solid thin lines indicate the Garrett model output. Both model's fluxes are interpolated at the satellite local time and at the satellite data acquisition time.

The following results are obtained from these figures;

1) The MSM generally tends to overestimate fluxes.

2) The Garrett model, being a statistical model, showed flatter plots, as expected. The Garrett model outputs were too high for 40 keV electrons by a factor of approximately 10, except during times of very high fluxes. For 65 keV electrons, the Garrett model usually failed to predict the high fluxes.

3) The satellites frequently experienced large flux dropouts that were not predicted by the MSM. (For example, see decimal day 113.3, 113.9, 114.18 and 114.25 of the satellite-1 observation, shown in Figure 4.12.)

4) The MSM occasionally predicted very low fluxes when the satellites observed quiet fluxes. (see decimal day 114.1 of the satellite-2 observation shown in Figure 4.13.)

Over all, the MSM predicts electron fluxes reasonably well. As a matter of fact, the MSM never failed to predict high fluxes. However, the model has difficulty in predicting the flux dropouts. This discrepancy will be discussed later in Section IV.E-Detailed Analyses of the MSM Outputs.

Evaluation of Accuracy

For the evaluation of the accuracy of a model, it would seem to be desirable to have a single parameter or an accuracy index $\epsilon$, which can be taken as an indicator of the extent to which the model follows the observations. In fact, as we shall see, such an approach is risky when the goal is to build a model which accurately predicts
occurrences of worst-case or adverse storm conditions. Nonetheless, we have chosen for such an index the root-mean-square of the base-10-logarithm of the ratio of the model to the satellite fluxes averaged over the event period. It is essential to use the logarithm because of the large dynamic range of the fluxes, nearly four orders of magnitude.

\[ \varepsilon = \sqrt{\left\langle \log_{10} \left( \frac{J_{\text{model}}}{J_{\text{satellite}}} \right) \right\rangle^2} \]

This parameter was calculated for each of the two differential energy bands mentioned above and for all three satellites. The results are shown in Tables 4.3 and 4.4.

The MSM's predictions were very much better than the Garrett model for 40 keV electron fluxes. This is partly because the Garrett's average flux were too high for this energy range. However, for 65 keV fluxes, the MSM's predictions were worse than the Garrett model. But, the Garrett model fail to predict high fluxes. Overall, the MSM's output flux level was off approximately by a factor of 3 to 10 in linear scale, and 3 to 20 for the Garrett output.

<table>
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<td>Garrett</td>
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<table>
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<th>Table 4.4. The rms errors (65 keV)</th>
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<td>Garrett</td>
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Figure 4.12  The 40 keV electron differential fluxes of the MSM, the Garrett model and satellite-1.
Figure 4.13  The 40 keV electron differential fluxes of the MSM, the Garrett model and satellite-2.
Figure 4.14  The 40 keV electron differential fluxes of the MSM, the Garrett model and satellite-3.
Figure 4.15 the 65 keV electron differential fluxes of the MSM, the Garrett model and satellite-1.
Figure 4.16 The 65 keV electron differential fluxes of the MSM, the Garrett model and satellite-2.
Figure 4.17  The 65 keV electron differential fluxes of the MSM, the Garrett model and satellite-3.
IV.E Detailed Analyses of the MSM Outputs

The use of a single error-indicator can not adequately describe the accuracy of the system in most cases. In this section, six intervals selected from the event are studied. The selected cases were summarized in Table 4.5.

<table>
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<th>Case</th>
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<th>5</th>
<th>6</th>
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</thead>
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<td>113.1972</td>
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<td>114.0625</td>
<td>114.2188</td>
<td>113.6250</td>
</tr>
<tr>
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<td>113.3847</td>
<td>113.9785</td>
<td>114.1035</td>
<td>114.28135</td>
<td>114.1250</td>
</tr>
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<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<td>4.20</td>
<td>4.21</td>
<td>4.22</td>
<td>4.23</td>
</tr>
</tbody>
</table>

Case 1

This is a case where the MSM predicted a peak flux very well. Figure 4.18 shows the change of the 40 keV differential electron fluxes of the MSM outputs from the day 113, 0014UT to day 113, 0614UT. The electron fluxes were mapped onto the equatorial plane. As before, the magnetic and the geographic equatorial planes were assumed to be the same. In these figures, the Sun is to the left. The fluxes are color coded: the blue area corresponds to low flux and the red to high. The dotted area on white shows the highest flux. The geostationary orbit has been plotted and the location of the three Air Force satellites are also shown by the numbers 1, 2 and 3.

The outer boundary of the modeling region changes its shape due to the B-field mapping. However, the change of the boundary is not very significant here, since the geosynchronous orbit discussed in this thesis is far inside the boundary. At about 0444UT of day 113, the MSM flux reached the local maximum at satellite-1. This agrees well with the observation. Then, at 0459UT of day 113, we can see that the
region of highest model flux, the banana shaped white area in local midnight sector at geosynchronous orbit, was located at local midnight, which indicates the arrival of fresh electrons from the tail. This appearance of the white region probably represent the onset of the expansion phase. The thinning of the plasma sheet is expected to become enhanced. The process visualized by the MSM agrees with the theory and observations well.

**Case 2**

This is a case where the MSM was unable to predict a large flux dropout when satellite-1 was located near local dawn. This happened during the peak of the first large substorm when the ring current was strong and the polar cap potential drop was very high (120-140 kV); consequently a strong injection of plasma sheet electrons was expected, as was the thinning of the plasma sheet. It would seem possible that the plasma sheet on the dawn side was extremely thin and the satellite was outside the plasma sheet at this time. However, determining this would require further investigation.

**Case 3**

This is also a case where the MSM failed to predict a flux dropout. The observed flux dropped down to $10^2 \text{[cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{keV}^{-1}]$. The satellite was near the dusk meridian and probably far away from the center of the plasma sheet.

**Case 4**

This is an example opposite to the previous two cases. This time the MSM showed a flux dropout but satellite-2 did not. In this period, the polar cap potential
drop was high and strong sunward convection of plasma would be expected. The model may have overestimated the magnitude of the sunward motion and caused particle drift paths to move close to the day side magnetopause, leaving the satellite earthward of the newly injected plasma.

Case 5

The model output is higher by an order of magnitude than the spacecraft observations. It seems that this error may have been caused by the same phenomenon discussed in test case 4. In this case, the satellite is on the night side and so we see the opposite effect. The intense model electric field cause plasma injection too close to the earth to be consistent with observation.

Case 6

The model output was higher by an order of magnitude than the observations by satellite-3 from 1500UT to 1900UT on day 113. The satellite was between 8 pm and 0 am in local time. It appears again that the model predicted plasma sheet too close to the earth.

Judging from the large discrepancies of these examples, it would seem that there are two major different causes of prediction errors. One is probably the rapid change of the location of the inner drift paths of 40 keV electrons. In order to understand what is happening in the MSM, I made a simple electron-tracing program based on the study by Chen [1970], Kavanagh et al. [1968] and Freeman [1968]. Figure 4.24 shows the equatorial tracing plots for electrons of cases with the convection electric field $E_0=0.3$ and 0.6 [mV/m], and electron magnetic moments $\mu=0$ and 400 [eV/nT].
In the figures, the circle at the center is the Earth, the large circles represent the
geostationary orbit and the small circles on the +y axes are the stagnation points
where the electrons have zero drift velocity. The Sun is to the top. The
magnetopause-like boundaries are drawn for convenience, but they are not accurate.
Scales are relative and the distances can be measured by the tick marks on axes,
which are separated by two earth radii. The consecutive tracing points are separated
by 100 seconds for $\mu=0$ and 50 seconds for $\mu=400$ [eV/nT] electrons. A 400 [eV/nT]
electron can attain approximately 40keV at geostationary orbit. A simple dipole
magnetic field, uniform convection electric field and a rotation electric field are
assumed for use with this tracer model. The drift formula used here was
\[
\mathbf{v}_d = \frac{\mathbf{B} \times \nabla \Phi}{B^2}
\]
\[
\Phi = -\frac{K}{R} - E_0 R \sin \varphi + \frac{\mu M_E}{q R^3}
\]
where $K=14.5$ [mV/m], and $M_E=8.0 \times 10^{28}$ [gauss cm$^3$], $R$ is radius in [R$_E$].

It can be seen that the inner paths of $\mu=400$ [eV/nT] electrons is located near
the geosynchronous orbit and that newly injected 400 [eV/nT] electrons drift sunward
through the near-dawn region inside the geosynchronous orbit when the convection
electric field is strong.

If the MSM overestimates the enhancement of the convection electric field,
following consequences would be inferred;

1. At night side, the MSM places a storm-front of the 40 keV electrons much
closer to the Earth, more inside the geosynchronous orbit. Consequently, the
MSM overestimates the electron flux. This process can probably explain the
discrepancies in the cases 3, 5 and 6.

2. Similarly, in the dawn sector, the MSM locates the drift paths of the 40 keV
electrons much closer to the Earth. This also gives a higher flux output. This
may be one of the reasons why the model flux was extremely high from day 113.3 to 113.4 in the case 2.

(3) At the region from noon to dusk, the MSM places the 40 keV electrons much closer to the magnetopause, more distant from the Earth. As a result, the MSM gives a lower flux output. This process explains the error shown in the case 4 very well.

It seems probable that the overestimate of the electric field enhancements is a major source of error, however, there must be other phenomena at work in order to explain the large flux dropouts observed by the spacecraft but not predicted by the MSM.

Satellites-1 and -3 observed larger flux dropouts than satellite-2. Both experienced the dropouts when the MSM reported high or normal flux level. This phenomenon was not observed by satellite-2. April 21-23 is approximately one month after the spring equinox and the earth rotation axis is roughly perpendicular to the earth-sun line. Between the geophysical and geomagnetic coordinate systems, there are following relationships;

\[
\begin{align*}
\cos \Theta &= \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos (\varphi - \varphi_0) \\
\cos \Phi &= \cos \psi \cos (\varphi - \varphi_0) + \sin \psi \sin (\varphi - \varphi_0) \cos \Theta \\
\sin \psi &= -\sin \theta_0 \sin (\varphi - \varphi_0) / \sin \Theta
\end{align*}
\]

where \( \theta \) and \( \varphi \) are colatitude and longitude measured in geographical system, \( \Theta \) and \( \Phi \) are in geomagnetic colatitude and longitude, and \( \theta_0 \), and \( \varphi_0 \) are the position of the north pole. Also, \( \psi \) is the angle formed by geographic meridian and geomagnetic meridian planes both including the point \( P(\theta, \varphi) \). According to IGRF-1980, \( \theta_0=11.2^\circ \).
and \( \phi_0 = -70.8^\circ \). A rough estimate tells us that the geomagnetic colatitudes of these satellites are \( \Theta = 81^\circ, 89^\circ \) and \( 99^\circ \) for satellites 1, 2 and 3 respectively. Satellites 1 and 3 can then be approximately \( 1 \text{ RE} \) away from the magnetic equator, whereas satellite-2 can only be away about \( 0.1 \text{ RE} \). According to the study by Sergeev, et al. [1990], the plasma sheet thickness can be compressed down to \( 0.2 \text{ RE} \) at \( r \sim 9 \text{ RE} \). Spacecraft 1 and 3 may have been out of the plasma sheet.

Figure 4.25 shows the local time locations where the flux dropouts were observed during the run. Several things can be noticed. The dropouts are all observed from night to morning local time and are not exactly centered on the night side.

There are still some unexplained problems. The thinning of plasma sheet is usually reported near the local midnight sector. Is \( 1 \text{ RE} \), the distance from the magnetic equator, sufficient to be outside of the plasma sheet on the dawn or dusk side? Maybe the dropouts are caused by the combination of the plasma sheet thinning and another phenomenon such as the quick motion of the inner edge of the plasma sheet.
Figure 4.18  Detailed analysis of the MSM output: Case 1. 40 keV electron flux at satellite-1, from 113-0014UT(113.0097) to 113-0614UT(113.2597). Satellite local time = UT - 2.5 hours.
Figure 4.19  Detailed analysis of the MSM output: Case 2.  
40 keV electron flux at satellite-1, from 113-0744UT(113.1972) to 113-0914UT(113.3847). Satellite local time = UT - 2.5 hours.
Figure 4.20  Detailed analysis of the MSM output: Case 3.
40 keV electron flux at satellite-1, from 113-2029UT(113.8535) to 113-2329UT(113.9785). Satellite local time = UT - 2.5 hours.
Figure 4.21  Detailed analysis of the MSM output: Case 4.
40 keV electron flux at satellite-2, from 114-0130UT(114.0625) to 114-0229UT(114.1035). Satellite local time = UT + 13.7 hours.
Figure 4.22  Detailed analysis of the MSM output: Case 5. 40 keV electron flux at satellite-2, from 114-0515UT(114.2188) to 114-0645UT(114.2813). Satellite local time = UT + 13.7 hours.
Figure 4.23 Detailed analysis of the MSM output: Case 6. 40 keV electron flux at satellite-3, from 113-1500UT(113.6250) to 114-0300UT(114.1250). Satellite local time = UT + 4.7 hours.
Figure 4.24  The trajectory of electrons from the tail. Based on the study by Chen [1970], Kavanagh et al. [1968] and Freeman [1968]. Simple dipole field, uniform convection field and rotation field is assumed.
Figure 4.25  Local time locations where large flux dropouts were observed.
V. CONCLUSIONS

Throughout the run, the Magnetospheric Specification Model was able to predict storm related electron enhancements in reasonable time agreement with geosynchronous satellite data. The MSM's predictions were very much better than those of the Garrett model for 40 keV electron fluxes. And the Garrett's average fluxes were too high for this energy range. However, for the higher energy channel, the accuracy index showed that the MSM predictions were worse than those of the Garrett model. At the same time, the MSM followed enhancements in the fluxes far better than the Garrett model. The Garrett model fails completely in predicting the high fluxes, illustrating the inappropriateness of a single parameter index for the purpose of assessing model accuracy.

The MSM never failed to predict high fluxes when they were observed. However, it sometimes predicted high fluxes when they were not observed, and on one occasion, the MSM predicted a dropout that was not observed. These two errors were found to have a strong correlation to the MSM's overestimate of the electric field enhancements.

Also, the MSM usually failed to predict observed flux dropouts. Although predicting flux drop-outs was not a primary objective, it would be better if we could predict them. The error in predicting the flux dropouts may be caused by the phenomenon of plasma sheet thinning and by not taking into account the fact that the satellites are not actually at the geomagnetic equator. If the lower energy geosynchronous fluxes had been observed, the mechanism of the error could have been analyzed further.
It has been found by T. Cayton (private communication) that there seem to be seasonal, annual or longer fluctuation cycles in geosynchronous fluxes and that the average geosynchronous electron fluxes in April 1988 were unusually low. This could be the reason why the MSM's prestorm fluxes were higher than the satellite observations. There are still a lot of things remaining to be understood. The MSM should be tested with more different cases.

It has also been found that the time-index of polar cap potential data had been marked at the time when the DMSP-F8 satellite crosses the equatorial plane, which is about 25 minutes earlier than the appropriate time-index for the MSM. The polar cap potential should be indexed when the satellite is just above the pole. The difference, 25 minutes, is almost twice as long as the computing step-time of the MSM. This could have affected the electron injection time. The new evaluation for the April 1988 event with the corrected time-index of the polar cap potential is in progress. In the meantime, a new data analysis method, "cross-correlation", has been applied to analyze the significance of the error of time-index. This recent work is shown in Appendix C.
REFERENCES


APPENDIX A

The Determination of the Maximum Data Gap for the MSM Input Process
This section will deal with the methodology of the handling of input data. The data input to MSM is controlled by the Front-End Model named FECON. The index \( K_p \) is assumed to be available at anytime in the MSM run, but, satellite observations may not always be available at the MSM's input request times. In general, satellites can not always be located at the desired points. Or, even if they are at the right spots, the satellite observations usually have time intervals which do not coincide with the MSM's timing. Some satellites might be inoperative during a MSM run period. Therefore, we have to consider some methodology to interpolate or create an appropriate input data set to the MSM. A simple linear interpolation will probably work well, but if the data have a substantially large time gap, the interpolation will lose physical sense. It was suggested that we do a linear interpolation, but switch to a statistical empirical formula when there is a large time gap in the data. In order to switch from the interpolation to the empirical method, we must define the maximum data gap for various data types. Wolf proposed the following experiment to obtain the value of the maximum gap.

1) Select the source data set. \( X_{\text{source}}(i), \ i=1..N \)

2) Throw every other data point away. This makes the artificial gap \( IGAP=1 \).

3) Make the following two data series from gapped data in (2);
   \( X_{\text{interp}}(i) = \) use linear interpolation,
   \( X_{\text{formula}}(i) = \) use empirical formula
   to set the proxy values for the data points thrown away in (2).

4) Compute the root-mean-square of the errors

   \[
   \sigma_{\text{interp}} = \sqrt{\frac{1}{N} \sum (X_{\text{interp}}(i) - X_{\text{source}}(i))^2}
   \]

   \[
   \sigma_{\text{formula}} = \sqrt{\frac{1}{N} \sum (X_{\text{formula}}(i) - X_{\text{source}}(i))^2}
   \]

5) Compute the average time gap \( AGAP \).
6) Increase IGAP by 1.

7) Repeat the process 3 to 6 for appropriate loops.

In general, the interpolation method gives the better estimate of the real value if IGAP is small. Therefore, plots of $\sigma_{\text{interp}}$ vs. IGAP and $\sigma_{\text{formula}}$ vs. IGAP will cross at some IGAP. We define the averaged gap at this crossing point to be the maximum data gap.

The Kp based empirical formulas used are shown in Table A.1. The numerical result of the maximum gap (AGAP) is tabulated in Table A.2. Figures A.1 to A.4 are the plots of root-mean-squares of errors vs. IGAP, number of artificially inserted gaps, and average gap AGAP vs. IGAP. In the figures, SI and SF represent $\sigma_{\text{interp}}$ and $\sigma_{\text{formula}}$, respectively. Also, the comparison between the source data and Kp-based empirical value (or the default value for XIPATT) are plotted in Figures A.6 to A.8.

### Table A.1 The Empirical Formulas

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<th>Formula</th>
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### Table A.2 Maximum Data Gap

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</tr>
<tr>
<td>XIPATT</td>
<td>7</td>
<td>0.36</td>
</tr>
<tr>
<td>DLATAZ</td>
<td>6</td>
<td>0.31</td>
</tr>
</tbody>
</table>
Figure A.1. The Maximum data gap of Dst. SI and SF are the root-mean-square of errors $\sigma_{\text{interp}}$ and $\sigma_{\text{formula}}$. 
Figure A.2  The maximum data gap of PCP
Figure A.3  The maximum data gap of XIPATT
Figure A.4  The maximum data gap of DDATAZ
Figure A.5  Comparison of the Kp-based empirical formula. [ Dst ]

Figure A.6  Comparison of the Kp-based empirical formula. [ PCP ]
Figure A.7  Comparison with the default value. [ XIPATT ]

Figure A.8  Comparison of the Kp-based empirical formula. [ DLATAZ ]
APPENDIX B

Derivation of the distribution function from spacecraft observations.
The number of counts $C$ and the count rate $CR$ are as follows:

$$C = \int_0^T dt \int_{\Delta A} dA \int d^3v \ f(v) \left(v \hat{n}_v\right)$$

$$CR = \Delta A \int_{\Delta \Omega_v} d\Omega_v \int dv \ v^2 f(v) \left(v \hat{n}_v\right)$$

$$= \Delta A \ \Delta \Omega_v \int dv \ v^3 f(v)$$

With $E = mv^2/2$, and therefore $v^3 dv = 2E dE/m^2$,

$$CR = \Delta A \ \Delta \Omega_v \frac{2}{m^2} \int dE \ E \ f(E)$$

We assume that the energy bandpass $\Delta E$ is proportional to the selected energy $E_0$, i.e. $\Delta E = k E_0$, and that $f(E)$ is constant for a small integration bound $E_0 \pm \Delta E/2$, so that

$$CR = \Delta A \ \Delta \Omega_v \frac{2}{m^2} f(E_0) \int \frac{dE}{E} \ E^2$$

For small $\Delta E/E$,

$$CR = \Delta A \ \Delta \Omega_v \frac{2}{m^2} f(E_0) \frac{\Delta E}{E} \ E_0^3$$

By defining $H = \Delta A \ \Delta \Omega_v \ \Delta E/E$, we obtain the distribution function:

$$f(E_0) = CR(E_0) \ m^2 / (2 \ H \ E_0^2)$$

The conversion factor $H$ for ATS-6 detector is shown in Figure B.1.
Figure B-1. ATS-6 Conversion Factor  (from ATS-6 Handbook)
APPENDIX C

Cross Correlation Analysis.
A new analysis method was suggested by Hawthorn of Rice University, for evaluating model output. It is a "cross-correlation method", which is a convolution between two data series to be compared. \( H(\tau) \), in equation D.1, is the cross-correlation function used for this evaluation. To compare with the best fit pattern, the auto-correlation \( I(\tau) \), equation D.2, is computed. The cross-correlation function, in this evaluation, is a convolution of the MSM flux and the satellite flux, normalized by the satellite flux. The integration was performed over the run period which is from day 112.75 to 114.75.

\[
H(\tau) = \frac{\int j_{\text{satellite}}(t) j_{\text{MSM}}(t-\tau) \, dt}{\int [j_{\text{satellite}}(t)]^2 \, dt} \tag{D.1}
\]

\[
I(\tau) = \frac{\int j_{\text{satellite}}(t) j_{\text{satellite}}(t-\tau) \, dt}{\int [j_{\text{satellite}}(t)]^2 \, dt} \tag{D.2}
\]

Figure D.1 shows the result of the cross-correlation analysis for the 40 keV electron fluxes at three satellites. In the figure, \( H(\tau) \) and \( I(\tau) \) are plotted as a function of \( i \), not \( \tau \). The variable \( i \) is an integer index of spacecraft measurement time, which ranges from 1 to 400 approximately. Since there are many data gaps in the satellite measurement, \( i \) does not have equal time interval, however, for a simple test of the new analysis method, I think it is sufficient to use the \( i \) for computational convenience. On average, \( \Delta i = 1 \) corresponds to about 10 minutes. The auto-correlation, \( I(\tau) \), is
symmetric about i=0 by definition, therefore I(τ) can be easily distinguished from H(τ).

From Figure D.1(a), it is seen that if we shift the MSM output in negative time direction by Δi=8, Δt~80 minutes, the maximum convolution value is obtained. Typical interpretation is that the MSM output had a time delay of about 80 minutes. The satellite location information may have been incorrect. From Figure D.1(b), it can be seen that the MSM output flux had a fair time-agreement. But, there are two large peaks at i~40 and i~70. These two peaks of the convolution is probably due to the MSM's error in the dusk side region (see Figure 4.13, from day 113.2 to 113.4 and from day 114.0 to 114.4). Because satellite-3 had a large data gap (see Figure 4.14, from day 113.0 to 113.3), the cross-correlation function does not seem relevant.

In order to investigate further, I evaluated another function E(τ), cross RMS-error (equation D.3), in Figure D.2.

\[
E(\tau) = \frac{\int [j_{\text{satellite}}(t) - j_{\text{MSM}}(t-\tau)]^2 dt}{\int [j_{\text{satellite}}(t)]^2 dt}
\]  

(D.3)

This function E(τ) is basically an RMS-error evaluated with time-shifts. In the figure, the index i is used instead of τ, as before. From Figure D.2(a), it is clearly seen that the negative shift of the MSM output by ~80 minutes reduces the RMS-error, which is consistent with the previous cross-correlation analysis. Similar consistency can be seen for satellite-2. The result of satellite-3 does not show physical significance because of the data gap.
In order to discuss the time-shift precisely, the index i must be converted to a uniform time scale. This will be done by a simple interpolation and the algorithm will be applied to the next run of the MSM.

It has been found that the polar cap potential data used in this thesis had an inappropriate time index. The time was indexed when DMSP-F8 satellite crosses the equatorial plane. The proper time for the MSM input is the time when the spacecraft is just above the polar cap. There is about 25 minutes difference. The new MSM run with a corrected time index is in progress. The cross correlation analysis will be a powerful tool to clarify the systematic time-lag hidden in the input data stream and to investigate the sensitivity of the MSM over the input data.
Figure D1  Cross correlation analysis of the MSM output compared with the spacecraft observation.
Figure. D2   Cross RMS-error analysis of the MSM output compared with the spacecraft observation.