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Axisymmetric fluid jet impingement of a rock half-space

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Rice University, 1990
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AXISYMMETRIC FLUID JET
IMPINGEMENT OF A ROCK HALF-SPACE

by

Michael F. Welsh

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

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ABSTRACT

The effects of an axisymmetric fluid jet impinging on a rock half-space are examined. A recently developed constitutive model for porous elastic materials, which explicitly provides for the compressibilities of the solid grains and fluid that comprise the material, is reconciled to the model developed by M. A. Biot for soils. An increase in pore fluid pressure is shown to reduce the compression in the rock matrix, displacing the Mohr's circle in the direction of the Griffith failure surface. Shear stress due to the subsequent radial flow of fluid, though small and neglected in all previous work, is shown to have significant effect. Shear stress increases the difference between the maximum and minimum effective principal stresses, enlarging the Mohr's circle toward the failure surface. Accurate predictions of threshold pressures to cause failure were achieved when the component compressibilities and shear stress were taken into account.
ACKNOWLEDGEMENTS

I would like to express my appreciation to the Hughes Tool Company and its management for the complete funding and support of this research.

I have benefitted greatly from the valuable guidance and advice provided by Dr. J. E. Akin, Dr. R. P. Nordgren, and Mr. L. W. Ledgerwood III.

I thank Georgia Andrews for her assistance in the preparation of this manuscript.

And I especially thank my wife, Stella, whose love and patience sustained me through our quest for my Masters degree.
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VARIABLE LIST

\[ a_i = \text{polynomial coefficients in Equation 50 (i = 0 to 7)} \]
\[ C_1 = \text{constant in impact pressure distribution, Equations 29 and 46} \]
\[ C_2 = \text{constant in stagnation pressure, Equations 32 and 34} \]
\[ C_3 = \text{constant in impact pressure distribution, Equation 33} \]
\[ c_f = \text{local coefficient of skin friction} \]
\[ d = \text{nozzle diameter} \]
\[ dV = \text{small volume of soil or rock} \]
\[ dV_o = \text{initial small volume of soil or rock} \]
\[ dV_g = \text{solid grain volume of } dV \]
\[ dV_w = \text{fluid volume of } dV \]
\[ dV_{wo} = \text{fluid volume of } dV_o \]
\[ E = \text{Young's modulus} \]
\[ e = \text{volumetric strain} \]
\[ e = \text{exponentiation, Equations 31 and 33} \]
\[ e_g = \text{volumetric strain of the solid grains} \]
\[ e_w = \text{volumetric strain of the fluid} \]
\[ H = \text{one of Biot's constitutive parameters} \]
\[ h = \text{height of nozzle above impinged surface} \]
\[ I = \text{identity matrix} \]
\[ J = \text{Jacobian} \]
\[ K = \text{effective bulk modulus of soil or rock} \]
\( K_g \) = bulk modulus of solid grains
\( K_w \) = bulk modulus of fluid
\( k_s \) = coefficient of surface roughness
\( n \) = porosity
\( n_o \) = initial porosity
\( Q \) = one of Biot's constitutive parameters
\( P \) = impact pressure, a function of \( r \)
\( P_s \) = stagnation pressure
\( p \) = pore fluid pressure
\( r \) = radial distance from fluid jet centerline
\( T_o \) = uniaxial tensile strength
\( U_o \) = nozzle exit velocity of fluid
\( U_r \) = radial velocity of fluid after impingement
\( u \) = displacement vector of soil or rock matrix
\( V \) = pore fluid velocity vector
\( \alpha \) = one of Biot's constitutive parameters
\( \epsilon \) = strain tensor
\( \epsilon' \) = effective strain tensor
\( \eta \) = fluid viscosity
\( \theta \) = one of Biot's constitutive parameters
\( \kappa \) = permeability
\( \lambda \) = Lame constant
$\mu = \text{Lame constant (shear modulus)}$

$\nu = \text{Poisson Ratio}$

$\pi = \Pi$

$\rho = \text{fluid density}$

$\sigma = \text{stress tensor}$

$\sigma' = \text{effective stress tensor}$

$\sigma_m = \text{mean normal stress}$

$\sigma'_m = \text{mean normal effective stress}$

$\sigma_{\text{max}} = \text{maximum principal stress}$

$\sigma'_{\text{max}} = \text{maximum principal effective stress}$

$\sigma_{\text{min}} = \text{minimum principal stress}$

$\sigma'_{\text{min}} = \text{minimum principal effective stress}$

$\tau = \text{shear stress}$

$\tau_w = \text{shear stress at impinged surface due to radial spread of fluid}$
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INTRODUCTION

The purpose of this work is to study the effects of an axisymmetric fluid jet impinging normally on a rock half-space. High pressure jets are used in the mining industry to cut rock. They are also an integral part of the earthboring tools used in the oil and gas industry to drill holes to subsurface reservoirs.

Fluid is circulated during drilling primarily to maintain hole stability, cool the drilling tool, and to clean the hole bottom of drilled debris. The debris is typically generated through the gouging and crushing action of cutters mounted on the tool. To maximize the cleaning of debris from the hole bottom the fluid is accelerated through a jet as it exits the tool. An increased understanding of the effects of the jet on the rock should lead to greater optimization of the drilling process.

In the arrangement modeled, fluid from the jet impacts the rock with a Gaussian pressure distribution. Some of the fluid enters the rock pore space while most of it is diverted at the surface, forming a wall jet which spreads radially. The rock matrix and pore fluid are simultaneously stressed and pressurized, respectively. The total stress on the rock, therefore, consists of the effective stress on the rock matrix plus the effect of the uniform, volumetric strain from the pressure of the pore fluid.

Previous studies have treated the rock as solid elastic material, a porous-elastic material, and as a rigid-porous material. The solid elastic model overpredicted the threshold pressure required to initiate failure. The addition of porosity and the effect of the pore fluid improved the prediction of failure initiation.
However, the predicted values were now below the measured threshold pressures. The rigid porous model predicted only the location of initial failure based on the direction of fluid flow through a rigid material.

The stress equations used in the porous material studies were derived for the behavior of soil, a material whose bulk compressibility is much greater than the compressibilities of the solid grains or fluid which comprise it. The use of these equations for a porous material like rock is inaccurate because the bulk compressibility of rock falls between that of the solid grains and the fluid, and all three compressibilities are typically within two orders of magnitude.

In this study the rock is treated as a porous-elastic material. The strains on the solid grains and fluid are used to derive the stresses. It is not possible to solve for the exact solution analytically. The commercial finite element analysis software ABAQUS is used to approximate the solution numerically. The plane Griffith criterion for tensile failure is used to predict the zone of fracture initiation.

It is assumed in this study that the rock is an isotropic, homogeneous material which obeys the elastic linear stress-strain laws and linear strain-displacement equations. The area affected by the jet is small compared to the total mass of the rock. The deformation of the rock is symmetric with respect to the nozzle axis. The impacting fluid is a jet of water diffusing in air, and the fluid flows through the rock pores according to Darcy's Law.

Three major points will be made during the course of this study. First it will be shown that previous studies did not adequately account for the solid grain or fluid
compressibilities. Second it will be shown that the shear stress distribution resulting from the radial flow of fluid away from the impact area, though small and neglected in previous studies, has significant effect. And third it will be shown that the predicted failure levels are more accurate if the solid grain and fluid compressibilities and the shear stress are taken into account.
PORO-ELASTICITY

Introduction

Soils and rocks are multi-phase materials. They are made up of grains of solid material and a volume of one or more fluids. It is assumed in this study that the rock is saturated with only one fluid — water.

A small volume of a rock, \( dV \), is made up of a volume of solid grains, \( dV_g \), and a volume of fluid, \( dV_w \).

\[
dV = dV_g + dV_w \tag{1}
\]

The porosity of the rock, \( n \), is the ratio of the fluid volume to the total volume.

\[
n = \frac{dV_w}{dV} \tag{2a}
\]

\[
1 - n = \frac{dV_g}{dV} \tag{2b}
\]

A Continuum of Theory Poro-Elasticity

A continuum theory of poro-elasticity treats the material as a homogeneous solid traversed by a network of interconnected pores. Biot formulated the initial
equations in order to study problems in soil mechanics. He assumed the following basic properties of soil:

1. the material is isotropic
2. the solid matrix is linearly elastic
3. strains are small
4. the fluid flows through the pores according to Darcy's law
5. the fluid is incompressible

Using the first three assumptions he derived the following constitutive laws:

\[ \sigma = 2\mu \varepsilon + \lambda \text{tr}(\varepsilon) I - \alpha p I \]  \[ \theta = \alpha \text{tr}(\varepsilon) + \frac{p}{Q} \]

Equation 3 is comprised of two parts. The first part

\[ 2\mu \varepsilon + \lambda \text{tr}(\varepsilon) I \]

is Hooke's law and accounts for the behavior of the soil in the particular case when \( p = 0 \). The second part is the pressure stress in the fluid. (Tensile forces are assumed positive. Care must be taken when considering the pore pressure, \( p \). As specified in these equations an increase in \( p \) will cause an expansion of the porous material.)
Biot defined $\theta$ as the change in volume of fluid per unit volume of soil:

$$\theta = \frac{dV_w}{dV} - \frac{dV_{wo}}{dV_o} \quad [6]$$

He also defined a quantity $Q$ with the sentence, "1/Q is the measure of the amount of fluid which can be forced into the soil under pressure while the volume of soil is kept constant." The quantity $Q$ and its significance will be important in later discussion.

Biot defined $\alpha$ to be

$$\alpha = \frac{E}{3(1-2\nu)H} \quad [7]$$

where "1/H is the measure of the compressibility of the soil for a change in fluid pressure." The exact value and physical significance of $\alpha$ has been a topic of debate. Early experiments showed $\alpha \approx 1$. Various formulas were derived without rigorous proof or data. Recently, using basic principles, Nur and Byerlee\textsuperscript{15} showed that:

$$\alpha = 1 - \frac{K}{K_s} \quad [8]$$
where $K$ is the effective bulk modulus of the soil

$$ K = \frac{E}{3(1-2v)} \quad [9] $$

and $K_s$ is the bulk modulus of the solid grains. They substantiated their formulation with experimental results.

Darcy's law, used in the fourth assumption, states the rate of fluid flow (defined as the volume of fluid passing through a unit area per unit time) is proportional to the gradient of the pore pressure at that point. Mathematically

$$ V = -\frac{\kappa}{\eta} \nabla p \quad [10] $$

where $\eta$ is the viscosity of the fluid, and $\kappa$ is the absolute permeability of the soil in units of square inches. Assumption five and the conservation of mass require:

$$ \frac{\partial \theta}{\partial t} = -\frac{\partial V_x}{\partial x} - \frac{\partial V_y}{\partial y} - \frac{\partial V_z}{\partial z} = -\nabla V \quad [11] $$
Substituting Equation 3 into the equilibrium equations, and differentiating Equation 4 and combining with Equations 10 and 11 yields the differential equations describing the consolidation of soil:

\[ \mu \text{div}(\nabla u) + \frac{\mu}{1 - 2v} \nabla(\text{tr} \varepsilon) - \alpha \nabla p = Q \]  \[ \text{(12)} \]

\[ \kappa \nabla^2 p - \alpha \frac{\partial (\text{tr} \varepsilon)}{\partial t} - \frac{1}{Q} \frac{\partial p}{\partial t} = 0 \]  \[ \text{(13)} \]

Biot later refined his work to include anisotropic materials.\(^5\) His equations have been used with reasonable success in soil consolidation studies. Because \(K_w\), as well as \(K_g\), is far greater than \(K\), both the fluid and solid grains are typically assumed incompressible in soils. As a result \(\alpha = 1\) and \(\theta = \epsilon\). The relationship for most rocks, however, is \(K_g > K > K_w\) with \(K_g\) typically within two orders of magnitude of \(K_w\). Therefore the preceding simplification is not advisable for use in the consolidation of rocks.
Poro-Elasticity for Compressible Materials

The effective stress principle, originated by Terzaghi, states that the total stress in a saturated material is the sum of the effective stress in the porous skeleton and the hydrostatic pressure stress in the fluid. Mathematically:

\[ \sigma = \sigma' - pI \tag{14} \]

Hibbitt, Karlsson, and Sorensen,\(^1\) the developers of ABAQUS, formulated a constitutive model for porous material that takes into account the compressibilities of the solid grains and fluid. Their formulation is based on the strain experienced by the grains and fluid.

The pressure in the pore fluid is \(p\) and the resulting volumetric strain, \(e_w\), is:

\[ e_w = -\frac{p}{K_w} \tag{15} \]

The pressure on the solid grains is:

\[ p - \frac{\sigma'_m}{1-n} \tag{16} \]
where

\[ \sigma'_m = \frac{1}{3} \langle \text{tr} \sigma' \rangle \]  \hspace{1cm} [17]

The factor \(1/(1-n)\) multiplying \(\sigma'_m\) is included because the effective stress, \(\sigma'\), from which it is defined, is averaged over the total volume \(dV\), of which the solid grains only occupy \((1-n)dV\). The resulting volumetric strain, \(e_g\), of the grains is:

\[ e_g = -\frac{1}{K_g} \left( p - \frac{\sigma'_m}{1-n} \right) \]  \hspace{1cm} [18]

Through some simplifying assumptions and algebra (see Appendix 1) \(e_g\) becomes:

\[ e_g = -\frac{1}{K_g} \left( p - \frac{\sigma'_m J}{1-n_o} \right) \]  \hspace{1cm} [19]

where \(J\) is the Jacobian:

\[ J = \left| \frac{dV}{dV_o} \right| \]  \hspace{1cm} [20]

The component \(-p/K_g\) in Equation 19 represents that part of the volume change in the rock caused by the pore fluid pressure acting on the solid grains.
Similar to the total stress, the total strain is the sum of the effective strain in the porous skeleton and the hydrostatic strain due to the pore fluid pressure.

\[
\varepsilon = \varepsilon' - \frac{P}{3K_g} \text{I}
\]  \[21\]

It is assumed that the volumetric strains are small and that the response of the grains and fluid is elastic. The porous skeleton may still exhibit inelastic behavior due to rearrangement and failure of the grains.

The effective stress, \(\sigma'\), is related to the effective strain, \(\varepsilon'\), by Hooke's Law:

\[
\sigma' = 2\mu \varepsilon' + \lambda (\text{tr} \varepsilon') \text{I}
\]  \[22\]

Substituting Equation 21 into Equation 22:

\[
\sigma' = 2\mu \left( \varepsilon' - \frac{P}{3K_g} \text{I} \right) + \lambda \left( \text{tr} \varepsilon' + \frac{P}{K_g} \right) \text{I}
= 2\mu \varepsilon + \lambda (\text{tr} \varepsilon) \text{I} + \frac{pE}{3K_g(1-2\nu)} \text{I}
\]  \[23\]

Substituting Equation 23 into Equation 14:

\[
\sigma = 2\mu \varepsilon + \lambda (\text{tr} \varepsilon) \text{I} - p \left( 1 - \frac{E}{3K_g(1-2\nu)} \right) \text{I}
\]  \[24\]
Substituting Equation 24 into the equilibrium equations

\[ \mu \text{div}(\nabla u) + \frac{\mu}{1 - 2\nu} \nabla (\text{tr} \mathbf{e}) - \left( 1 - \frac{E}{3K_e (1 - 2\nu)} \right) \nabla p = 0 \]  \hspace{1cm} [25]

yields three of the differential equations describing the consolidation of a porous material. It will be shown later that Equation 25 is equivalent to Equation 12. Combining the continuity equation with Darcy's law again yields the fourth differential equation.

**Plane Griffith Criterion for Tensile Failure**

The plane Griffith criterion\(^9,16\) provides a simple, yet generally adequate description of fracture initiation in rock due to excessive tensile stress. Developed by A. A. Griffith, it is based on the assumption that minute, incipient elliptical cracks are distributed randomly throughout the rock. When subjected to compression the cracks propagate from the points of maximum tensile stress on the crack surface. Griffith determined the criteria for fracture initiation to be:

\[
\text{If } \sigma_{\text{max}} - \sigma_{\text{min}} > 0 \quad \text{and} \quad 3\sigma_{\text{max}} + \sigma_{\text{min}} < 0 \quad \text{then}
\]

\[
- \frac{(\sigma_{\text{max}} - \sigma_{\text{min}})^2}{8(\sigma_{\text{max}} + \sigma_{\text{min}})} = T_o
\]  \hspace{1cm} [26]
If \( \sigma_{\text{max}} - \sigma_{\text{min}} > 0 \) and \( 3\sigma_{\text{max}} + \sigma_{\text{min}} > 0 \) then

\[
\sigma_{\text{max}} = T_o \quad [27]
\]

where \( T_o \) is the uniaxial tensile strength of the rock. In the presence of pore pressure the criteria are based on the principal effective stresses \( \sigma'_{\text{max}} \) and \( \sigma'_{\text{min}} \). The criteria may also be expressed in terms of the shear stress, \( \tau \), and the normal stress, \( \sigma \), acting on the plane containing the major axis of the crack.

\[
\tau^2 = 4T_o(\sigma - T_o) \quad [28]
\]

Figure 1A graphically depicts the Griffith failure envelope defined by Equations 26 and 27 in principal stress space and Figure 1B depicts the same failure envelope as defined by Equation 28 in Mohr space.
Figure 1A – Griffith Failure Surface in Principal Effective Stress Space

Figure 1B – Griffith Failure Surface in Mohr Space
PREVIOUS WORK

General Description of Axisymmetric Jet Impingement

The flow field of an axisymmetric fluid jet impinging on a flat surface, as shown in Figure 2, consists of four regions. The first is the transition region. The fluid is assumed to exit the jetting nozzle with a uniform velocity. The velocity difference with the surrounding fluid has the immediate effect of mixing the two fluids. The surrounding fluid is entrained into the jet flow, gradually decelerating the jet. A cone, sometimes called the potential core, in which the velocity is equal to the exit velocity extends from the nozzle exit to the end of the first region.

![Diagram of Axisymmetric Jet Impact](image)

Figure 2 — Diagram of Axisymmetric Jet Impact
The exact length of the potential core is dependent on several parameters, two of which are the fluid properties and the nozzle geometry. For water diffusing in air it was experimentally measured to be 20d to 30d.\textsuperscript{10} For water diffusing into itself it has been analytically determined to be 6.2d\textsuperscript{2} and experimentally measured to be 3d to 9d.\textsuperscript{11,19,26}

The second region is the region of established flow. The velocity profile is fully developed and may be represented by a Gaussian or normal curve with the highest velocity along the nozzle centerline. As no additional energy is being added, the momentum flux of the jet is constant, and by similarity the velocity distribution may be represented by a Gaussian curve at any point along the centerline. The width of the jet increases and the centerline velocity decreases as the distance from the nozzle increases.

The third region is the region of deflection. The bounding surface forces the fluid to divert radially. The velocity of the fluid in the direction of the centerline decreases rapidly to zero with a corresponding increase in pressure. A stagnation point occurs where the centerline intersects the surface and the resulting maximum pressure is called the stagnation pressure.

The fourth region is the region of established radial flow. The flow is similar to a wall jet and the velocity distribution outside of the thin boundary layer may be represented by half of a Gaussian curve. The velocity inside the boundary layer is dependent on the fluid properties and the roughness of the surface.
Jetting a Solid Elastic Material

Leach and Walker\textsuperscript{10} studied the effect of nozzle shape on the reduction of velocity caused by the passage of a water jet through air. One result of their work was the modeling of their experimentally measured pressure distributions at the impingement surface with a cubic polynomial:

\[
P = P_s \left(1 - 3 \left( \frac{r}{C_1 d} \right)^2 + 2 \left( \frac{r}{C_1 d} \right)^3 \right) \quad \text{for } r \leq C_1 d
\]  \quad [29]

\[
P = 0 \quad \text{for } r > C_1 d
\]

where \( P_s \) is the stagnation pressure and \( C_1 \approx 1.25 \). \( P_s \) is equal to the stagnation pressure of the nozzle (the pressure required to stop the flow at the nozzle) less any losses due to nozzle friction and stand-off distance, \( h \).

\[
P_s = \frac{1}{2} \rho U_o^2 - \text{Losses}
\]  \quad [30]

They assumed the shear stress distribution to be negligible based on calculations made by G. Artingstall. It will be shown later in this study that, though small, the shear stress distribution does effect the stress state and should not be neglected.

Powell and Simpson\textsuperscript{20} developed an analytical method for determining the stress state in an elastic, homogeneous, semi-infinite solid struck on its free surface
by an axisymmetric fluid jet. Their method was an extension of the solution by Timoshenko and Goodier for the stresses in a semi-infinite elastic body due to a point load under the assumption of negligible surface shear stress.

Using Leach and Walker's equation for the impact pressure distribution, Equation 29, and ignoring shear stress, they calculated a theoretical stress state for a material impinged by a fluid jet. The plane Griffith criterion for tensile failure was then used to predict the stagnation pressure, otherwise called threshold pressure, at which fracturing would initiate during the initial stage of jet impact. Their predicted threshold pressures were higher than actual, measured values by 50-400 percent.

**Jetting an Elastic Porous Material**

Forman and Secor\(^7\) added the effects of porosity and permeability to the jetting of an elastic material. They solved Biot's system of equations using a finite element code developed by Ghaboussi and Wilson\(^8\) for porous elastic media. Leach and Walker's equation for the impact pressure distribution, Equation 29, was used and shear stress was again ignored. The Griffith failure criterion was used to predict the threshold pressure at which fracturing would initiate. As a means of comparison with the work of Powell and Simpson the same impact pressure distributions were applied to solid elastic materials with the same elastic constants, \(E\) and \(v\). Table 1 contains the physical properties for two of their materials and Table 2 contains their results for these materials.
The predicted threshold pressures for the solid elastic materials were, as Powell and Simpson had found, greater than the measured values. The predicted threshold values for the porous elastic materials were closer than the elastic solid materials, but less than the measured values.

They also showed the effect of porosity empirically through simple experiments. Limestone specimens protected by .0025" and .005" thick copper sheets were able to sustain jetting at stagnation pressures of 7000 and 20,000 psi, respectively, without sustaining damage. Similar jetting produced 1/4" and 3/4" deep holes in unprotected specimens.
### TABLE 1 – MATERIAL PROPERTIES OF INDIANA LIMESTONE AND BARRE GRANITE

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>YOUNG'S MODULUS (PSI×10⁶)</th>
<th>POISSON'S RATIO</th>
<th>TENSILE STRENGTH (PSI)</th>
<th>COMpressive STRENGTH (PSI)</th>
<th>POROSITY</th>
<th>PERMEABILITY VISCOSITY* (IN³/LB-SEC)</th>
<th>BULK MODULUS** OF SOLID GRAINS (PSI×10⁶)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDIANA LIMESTONE</td>
<td>5.00</td>
<td>0.25</td>
<td>750</td>
<td>10,000</td>
<td>0.172</td>
<td>5.72×10⁻⁴</td>
<td>4.623</td>
</tr>
<tr>
<td>BARRE GRANITE</td>
<td>6.41</td>
<td>0.31</td>
<td>1200</td>
<td>30,000</td>
<td>0.0069</td>
<td>5.56×10⁻⁸</td>
<td>5.728</td>
</tr>
</tbody>
</table>

*VISCOSITY OF WATER AT 75°F = 1.375×10⁻⁵ LB-SEC/IN²
**BACK CALCULATED USING EQUATIONS 9 AND 45

### TABLE 2 – FORMAN AND SECOR'S PREDICTED THRESHOLD PRESSURES

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>MEASURED VALUES (PSI)</th>
<th>IMPACT PRESSURE ONLY WITHOUT POROSITY (PSI / %)†</th>
<th>IMPACT PRESSURE ONLY WITH POROSITY (PSI / %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INDIANA LIMESTONE</td>
<td>3500</td>
<td>14,400 / 411</td>
<td>1875 / 54</td>
</tr>
<tr>
<td>BARRE GRANITE</td>
<td>7000</td>
<td>– / –</td>
<td>4200 / 60</td>
</tr>
</tbody>
</table>

†PREDICTED THRESHOLD VALUES ARE EXPRESSED IN PSI AND AS A PERCENTAGE OF THE MEASURED VALUE
Jetting a Rigid Porous Material

Though not pertinent to this study because of its oversimplification of the rock behavior, Rehbinder\textsuperscript{22} took a unique approach to the jet impact problem. He recast Leach and Walker’s impact pressure distribution, Equation 29, in the form of a Gaussian curve

$$P = P_s e^{-2 \left( \frac{r}{a} \right)^2}$$ \hspace{1cm} [31]

and applied it to a rigid rock. He additionally assumed the impinging fluid to be incompressible and that it flowed through the rock according to Darcy’s law. The resulting boundary value problem was solved analytically using a Hankel transform. He hypothesized that failure would be initiated at the point where the force on a surface grain would be directed outward. From his analysis this point was approximately 0.9d from the jet centerline.

Submerged Axisymmetric Jet Impingement

The impact pressure distribution for the submerged jet has been shown analytically\textsuperscript{3,11} to be Gaussian in shape. Experimental results\textsuperscript{3,11,19} have also supported this conclusion. The stagnation pressure, $P_s$, and the pressure distribution from experimental results\textsuperscript{3,19} are given by:

$$P_s = C_2 \rho \left( \frac{U_0 d}{h} \right)^2$$ \hspace{1cm} [32]
\[ P = P_{\infty} e^{-C_3 \left( \frac{r}{h} \right)^2} \]  

where \( C_2 \) and \( C_3 \) were 25 and 114 in Beltaos and Rajaratnam\(^3\) and 30.2 and 117 in Poreh.\(^{19}\)

Equations 32 and 33 are good for \( h \) greater than or equal to the length of the potential core. Ignoring nozzle losses, the stagnation pressure of the nozzle is equal to the stagnation pressure at impact if \( h \) is equal to the length of the potential core. Combining Equations 30 and 32:

\[ C_2 \left( \frac{d}{h} \right)^2 \leq \frac{1}{2} \]  

[34]

And the length of Beltaos and Rajaratnam's and Poreh's potential cores are 7.1d and 7.8d, respectively.

Tsuei et al.\(^{17}\) measured the shear stress in Region IV and Beltaos and Rajaratnam\(^3\) measured the shear stress in Regions III and IV; however, each were impinging on a smooth boundary.
BIOT RESOLVED TO ABAQUS

In order to compare the results of Forman and Secor to the results generated with ABAQUS it is necessary to understand the constitutive parameters used by Biot in terms of the constitutive parameters used by ABAQUS. A harmonization of the two theories is not available in the literature nor from the developers of ABAQUS. The following is derived to relate the two constitutive models.

α Redefined

Substituting Equation 9 into Equation 8, α may be rewritten:

\[ \alpha = 1 - \frac{E}{3K_g (1 - 2 \nu)} \]  \[35\]

Substituting Equation 35 into Biot's first constitutive law, Equation 3,

\[ \alpha = 2\mu \varepsilon + \lambda (tr \varepsilon) - p \left(1 - \frac{E}{3K_g (1 - 2 \nu)}\right) I \]  \[36\]

Which is the same as ABAQUS's constitutive law expressed by Equation 24.

1/Q Redefined

Biot defined the coefficient 1/Q to be "the measure of the amount of fluid which can be forced into the soil under pressure while the volume of soil is kept
constant. He used this coefficient in his fourth constitutive law, Equation 4. As the fluid was assumed incompressible, any change in volume of the fluid was linearly proportional to the change in mass of the fluid. Hence Equation 4, when differentiated, mated conveniently with Darcy's law to satisfy conservation of mass.

If the fluid is compressible $1/Q$ will depend on the compressibility of the fluid, $K_w$, the compressibility of the solid grains, $K_g$, and the porosity, $n$. Consider a unit cube of soil of initial porosity $n_o$. If the volume of the cube is held constant and the solid grains are incompressible, then the amount of fluid added for an increase in pressure $p$ is $pn_o/K_w$. If the solid grains are now compressible, then the amount of additional fluid added is $p(1-n_o)/K_g$. Combining the two:

$$
\frac{1}{Q} = \frac{n_o}{K_w} + \frac{1-n_o}{K_g}
$$

To check the accuracy of Equation 37, consider the case when the rock is stressed but no fluid is allowed to escape. Assuming small strains, $\theta \approx 0$. Rewriting Equation 4 using Equations 8 and 37:

$$
0 = \left(1 - \frac{K}{K_g}\right)(trg) + p\left(\frac{n_o}{K_w} + \frac{1-n_o}{K_g}\right)
$$

[38]
Rewriting Equation 38:

\[ tr e = e = -p \left( \frac{n_o}{K_w} + \frac{1-n_o}{K_g} \right) + \frac{K}{K_g} e \]  

[39]

where \( e \) is commonly referred to as the volumetric strain.

Equation 3 may be rewritten:

\[ e = \frac{1+\nu}{E} \sigma - \frac{\nu}{E} (tr \sigma) I + \frac{p}{3H} I \]  

[40]

Using Equations 9, 14, 17 and 40, the volumetric strain may be written:

\[ e = tr e = \frac{3(1-2\nu)}{E} \sigma_m + \frac{p}{H} \]

\[ = \frac{\sigma_m}{K} + \frac{p}{K} \]

\[ = \frac{\sigma'_m}{K} - \frac{p}{K} + \frac{p}{H} \]  

[41]
Substituting Equation 41 into Equation 39 and using Equation 8, the volumetric strain may be written:

\[ e = -p \left( \frac{n_o}{K_w} + \frac{1-n_o}{K_g} \right) + \frac{K}{K_g} \left( \frac{\sigma_m'}{K} - p \left( \frac{1}{K} - \frac{1}{H} \right) \right) \]

\[ = -p \left( \frac{n_o}{K_w} + \frac{1-n_o}{K_g} \right) + \frac{\sigma_m'}{K_g} - \frac{p}{K_g} (1 - \alpha) \quad [42] \]

\[ = -p \left( \frac{n_o}{K_w} + \frac{1-n_o}{K_g} \right) + \frac{\sigma_m'}{K_g} - \frac{pK}{K_g^2} \]

Using Equations 15 and 19, the volumetric strain from ABAQUS may be written:

\[ e = -\frac{p}{K_w} \frac{dV_w}{dV} - \frac{1}{K_g} \left( p - \frac{\sigma_m'J}{1-n_o} \right) \frac{dV_g}{dV} \quad [43] \]

\[ = -\frac{pn}{K_w} - \frac{1-n}{K_g} \left( p - \frac{\sigma_m'J}{1-n_o} \right) \]

The small volumetric strain assumption implies \( J \approx 1 \) and \( n \approx n_o \). And Equation 43 becomes:

\[ e = -p \left( \frac{n_o}{K_w} + \frac{1-n_o}{K_g} \right) + \frac{\sigma_m'}{K_g} \quad [44] \]
The last term of Equation 42, $-pK/K_g^2$, is negligible compared to $\sigma_m'/K_g$, and Equation 42 may be considered the same as Equation 44. $1/Q$ as expressed by Equation 37 is satisfactory.

**One-Dimensional Consolidation**

In previous work$^{7,8}$ $Q$ was assumed to equal the bulk modulus of the fluid, $K_w$. However, as shown by Equation 37, this is incorrect. To see the error generated by this assumption consider a simple one-dimensional consolidation problem. A rock specimen of unit cross-sectional area ten units tall is restrained and impervious to fluid flow on the sides and at the bottom. A load of 10,000 lbs is applied to the top using a porous piston. The problem is shown graphically with the physical properties in Figure 3.

Using Equation 37 to back calculate $K_w$ for $Q = 300,000$ psi, and $K_g$ and $n$ as shown in Figure 3, yields $K_w = 63,025$ psi. Evaluating the problem using ABAQUS with this $K_w$ yields a pore fluid pressure at time 0+ of 173 psi.

Evaluating the problem correctly with $K_w = 300,000$ psi, the bulk modulus of water, yields a pore fluid pressure at time equal 0+ of 780 psi.

Figure 3 also shows a plot of the pore pressure in the specimen over time. The pore pressure through the entire specimen dissipates to 0 in the same amount of time, 10 sec, for both values of $K_w$. 
\[ F = 1 \times 10^4 \text{ LBS} \]
\[ E = 5 \times 10^6 \text{ PSI} \]
\[ v = 0.25 \]
\[ K_g = 5 \times 10^6 \text{ PSI} \]
\[ \kappa = 0.001 \]

- - - - \[ K_w = 3 \times 10^5 \text{ PSI} \]
- - - - \[ K_w = 6.3 \times 10^4 \text{ PSI} \]

**Figure 3** – One-Dimensional Consolidation with Pore Fluid Dissipation
The primary interest in soil mechanics is the time required for consolidation. Incorrectly specifying $K_w$ does not appear to effect this. The primary interest in rock cutting and fracturing, however, is the state of stress in the rock immediately after being struck by the fluid jet. And as indicated by Equation 14 the pore fluid pressure, $p$, at time $0+$ has direct effect on the stress state of the rock. Due to the incorrect specification of $K_w$ in the previous work, it should be assumed that their failure predictions are in error.
GENERAL PROBLEM DESCRIPTION

Rock Properties

The rocks modeled in this study are Indiana limestone and Barre granite; two of the rocks modeled by Forman and Secor. They were chosen to facilitate comparison with both previous analytical and experimental results. Their physical properties as well as Forman and Secor's results are listed in Tables 1 and 2.

Forman and Secor did not specify the compressibility of the solid grains, $K_g$. These were back calculated using Equation 9 and the following equation used by Forman and Secor:\(^\text{[45]}\):

$$\frac{K}{K_g} = \frac{1}{1 + \frac{3}{2} \left( \frac{1 - v}{1 - 2v} \right) n_o}$$

Finite Element Mesh and Boundary Conditions

Three axisymmetric finite element meshes are used in this study. The first is five nozzle diameters, 5d, square and is shown in Figure 4. The jet impinges on the top boundary and the jet centerline coincides with the left boundary. It is a uniform mesh composed of 400 square elements. In the immediate vicinity of the jet impingement (the area within the 2d square), the mesh has 64 elements. This is the same number of elements as Forman and Secor's\(^\text{[7]}\) mesh within the 2d square.
Figure 4 – 5d Coarse Finite Element Mesh with Displacement Boundary Conditions
Figure 5 - 5d Fine Finite Element Mesh with Displacement Boundary Conditions
The second mesh is also 5d square, but is finer in the 2d square. It has a total of 529 elements with 225 square elements in the 2d square. It is shown in Figure 5.

The third mesh is 20d square in order to more accurately approximate an infinite half-space. It is shown in full in Figure 6 and in detail in Figure 7. The 2d square consists of 225 elements and the area radiating outward from 2d to 5d adds another 360. The total number of elements is 725.

All three meshes are comprised of eight node quadratic elements with reduced integration. The 5d coarse mesh has 1281 nodes, the 5d fine mesh has 1680 nodes, and the 20d mesh has 2194 nodes. Pore pressure is interpolated linearly between the corner nodes of the elements. Forman and Secor used four node linear elements and had a total of 100 elements and 121 nodes.

Displacement is fixed in the R direction along the left (centerline) and right (O.D.) boundaries, and in the Z direction along the bottom boundary.
Figure 6 - 20d Finite Element Mesh with Displacement Boundary Conditions
Figure 7 — Detail of 20d Finite Element Mesh in Area of Interest
Impact Pressure Distribution

Leach and Walker’s\textsuperscript{10} equation for the impact pressure distribution, Equation 29

\[ P = P_s \left( 1 - 3 \left( \frac{r}{C_1 d} \right)^2 + 2 \left( \frac{r}{C_1 d} \right)^3 \right) \quad \text{for } r \leq C_1 d \]  \[ [46] \]

\[ P = 0 \quad \text{for } r > C_1 d \]

is used for a jet diffusing in air with $C_1 = 1.25$.

The stagnation pressure is approximated by Equation 30 with the losses assumed to be negligible.

\[ P_s = \frac{1}{2} \rho U_o^2 \]  \[ [47] \]
Shear Stress Distribution

Assuming that the radial flow is turbulent, an estimate of the shear stress on the bounding rock surface can be made by using Schlichting's interpolation formulae\textsuperscript{23} for the coefficients of skin friction in terms of relative roughness. The coefficient of local skin friction is given by:

\[
\frac{c_f}{\frac{1}{2} \rho U_r^2} = \left( 2.87 + 1.58 \log \frac{r}{k_s} \right)^{-2.5}
\]  \hspace{1cm} [48]

where \( k_s \) is the coefficient of the surface roughness expressed in terms of Nikuradse's sand roughness. In the present case \( k_s \) is a function of the size of the solid grains and the gross roughness of the impinged surface.

Poreh's plot\textsuperscript{19} of the velocity at the boundary layer interface provides the estimate of \( U_r \). Two equations are used to describe the plot.

For \( r/h > .25 \) Poreh's Equation 55 is used:

\[
U_r^2 = \frac{\pi}{4} \left( \frac{dU_o}{h} \right)^2 \left( \frac{h}{r} \right)^2
\]  \hspace{1cm} [49]
For $r/h \leq 0.25$ a seventh order polynomial is fit to the plotted curve:

$$
U_r^2 = \frac{\pi}{4} \left( \frac{dU_o}{h} \right)^2 \left[ \sum a_i \left( \frac{r}{h} \right)^i \right]^2 \tag{50}
$$

where:

- $a_0 = -1.576 \times 10^{-2}$
- $a_1 = 1.232 \times 10^{2}$
- $a_2 = -1.141 \times 10^{2}$
- $a_3 = 5.256 \times 10^{3}$
- $a_4 = -1.411 \times 10^{4}$
- $a_5 = 2.204 \times 10^{4}$
- $a_6 = -1.838 \times 10^{4}$
- $a_7 = 6.298 \times 10^{3}$

The following initial conditions are used to compare the impact pressure (Equation 29) and shear stress (Equations 49 and 50) distributions:

- $d = 0.313$ in
- $h = 1.984$ in
- $\rho = 1.94$ lb sec$^2$/ft$^4$
- $C_1 = 1.25$
- $P_s = 1875$ psi
- $k_s = 0.08$ in

$k_s = 0.08$" is the maximum diameter of a sandstone grain and was chosen to provide a maximum shear stress distribution. Figure 8 provides graphical representation of $P$ and $\tau$ as a function of $r/h$. As can be seen, the maximum value of $\tau$ is about 2 percent of the maximum value of $P$. $\tau$ is greater than $P$ at about $r/h = 0.22$; however, it is less than 1 percent of $P_s$. 
Figure 8 – Example of Impact Pressure and Shear Stress Distributions Resulting from Axisymmetric Fluid Jet Impingement
STUDY OF FLUID JET IMPINGEMENT USING ABAQUS

In this chapter fluid jet impingement of a rock half-space is developed and applied. The analysis is carried out using the commercial finite element analysis software ABAQUS. The predicted results are compared with both previously reported measured and predicted values. The results of the ABAQUS analyses are shown in Table 3.

The first section treats the rock as an elastic solid material loaded only by the impact pressure distribution. This section is included for historical completeness and to introduce the contour plots of fracture initiation.

The second section treats the rock as a porous elastic material loaded only by the impact pressure distribution. Mesh development is discussed in this section.

The third section treats the rock as a porous elastic material loaded by both the impact pressure and shear stress distributions. Two surface roughnesses are studied for each of the two materials.

Solid Elastic Material

The rocks in this section are treated as solid elastic materials and are loaded along their top surfaces by an impact pressure as defined by Equation 46.

Figure 9 shows the contours of fracture initiation for Indiana limestone treated as an elastic solid. The plot is generated by evaluating the maximum and minimum principal stresses using the Griffith failure criterion.
### Table 3 – Predicted Threshold Pressures from ABAQUS Analysis

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Impact Pressure</th>
<th>Shear Stress</th>
<th>Mesh</th>
<th>Indiana Limestone&lt;sup&gt;1&lt;/sup&gt; (PSI / %)</th>
<th>Barre Granite&lt;sup&gt;1&lt;/sup&gt; (PSI / %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Solid</td>
<td>Yes</td>
<td>No</td>
<td>5d Coarse</td>
<td>10,200 / 290</td>
<td>22,200 / 320</td>
</tr>
<tr>
<td>Porous Elastic</td>
<td>Yes</td>
<td>No</td>
<td>5d Coarse</td>
<td>5625 / 161</td>
<td>-- / --</td>
</tr>
<tr>
<td>(Forman &amp; Secor)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Porous Elastic</td>
<td>Yes</td>
<td>No</td>
<td>5d Coarse</td>
<td>5250 / 150</td>
<td>-- / --</td>
</tr>
<tr>
<td>Porous Elastic</td>
<td>Yes</td>
<td>No</td>
<td>5d Fine</td>
<td>5700 / 163</td>
<td>-- / --</td>
</tr>
<tr>
<td>Porous Elastic</td>
<td>Yes</td>
<td>No</td>
<td>20d</td>
<td>6000 / 171</td>
<td>7920 / 113</td>
</tr>
<tr>
<td>Porous Elastic</td>
<td>Yes</td>
<td>$k_s = .08^*$</td>
<td>20d</td>
<td>4875 / 139</td>
<td>6600 / 94</td>
</tr>
<tr>
<td>Porous Elastic</td>
<td>Yes</td>
<td>$k_s = .02^*$</td>
<td>20d</td>
<td>5250 / 150</td>
<td>7200 / 103</td>
</tr>
</tbody>
</table>

<sup>1</sup>The measured threshold pressure for Indiana Limestone is 3500 PSI and for Barre Granite is 7000 PSI

<sup>2</sup>The Predicted threshold pressures are expressed in PSI and as a percentage of the measured values
The contour levels are expressed in multiples of the uniaxial tensile strength of the material and represent the magnitude of the stagnation pressure necessary to initiate failure. The plot indicates that fracture initiation will first occur at a point on the impinged surface just outside the area of impact when the stagnation pressure reaches $13.6T_o$ or 10,200 psi. A local minimum of $17.5T_o$ occurs along the centerline about .5d below the surface.

Figure 10 shows the contours of fracture initiation for Barre granite treated as an elastic solid. The plot indicates that fracture initiation will first occur at a point on the impinged surface just outside the area of impact when the stagnation pressure reaches $18.5T_o$ or 22,200 psi. A local minimum of $19.2T_o$ occurs along the centerline about .6d below the surface.

The actual measured values for the threshold pressures are 3500 psi for Indiana limestone and 7000 psi for Barre granite. The above predicted values are 290 percent and 320 percent greater, respectively, and are consistent with the results of Powell and Simpson, which overpredicted the threshold pressure by 50-400 percent.
Figure 9 – Contours of Fracture Initiation, Indiana Limestone without Porosity, Impact Pressure Only, 5d Coarse Mesh
Figure 10 — Contours of Fracture Initiation, Barre Granite without Porosity, Impact Pressure Only, 5d Coarse Mesh
Elastic Porous Material with Impact Pressure Only

The rocks in this section are treated as porous elastic materials. Both the rock and pore fluid are loaded along the top surface by an impact pressure as defined by Equation 46. The effect of the jet immediately after impact is the primary interest and the results given are for an elapsed time of 10 microseconds.

An attempt was made to duplicate Forman and Secor's results for Indiana limestone. \( K_w \) was back calculated using Equation 37 and their \( Q = 300,000 \) psi to be 54,300 psi. The physical constants for Indiana limestone are shown in Table 1.

Figures 11 and 13 are their plots for the minimum and maximum effective principal stresses, expressed in terms of \( P_s \). Figures 12 and 14 are the comparable plots generated with ABAQUS. Recall that the principal effective stresses are the stresses in the rock skeleton and are also defined by Equation 14.

As can be seen, the maximum principal effective stress plots are in fairly good agreement. The minimum principal effective stress plots, however, do not agree immediately under the area of impact. Figure 11 indicates that the minimum principal effective stress immediately under the area of impact is positive. This is not possible, as shown in the next paragraph.

Consider a volume of material with one surface coincident with the impact surface and another surface coincident with the jet centerline. Its radial dimension is small enough that the impact pressure distribution may be assumed constant and equal to the stagnation pressure, \( P_s \). The \( R, \theta, \) and \( Z \) stresses may then be assumed equivalent to the principal stresses.
Figure 11 - Forman and Secor's 7 Figure 7—Minimum Principal Effective Stress, Indiana Limestone, Impact Pressure Only (© SPE)
Figure 12 – Minimum Principal Effective Stress, Indiana Limestone Using Forman and Secor’s Q, Impact Pressure Only, 5d Coarse Mesh
Figure 13 – Forman and Secor's\textsuperscript{7} Figure 8–Maximum Principal Effective Stress, Indiana Limestone, Impact Pressure Only ($\sigma$SPE)
Figure 14 – Maximum Principal Effective Stress, Indiana Limestone Using Forman and Secor's Q, Impact Pressure Only, 5d Coarse Mesh
Immediately at the impact surface $\sigma_Z = -P_s$ and $p = P_s$. Using Equation 14 $\sigma'_Z = 0$. Therefore, regardless of what $\sigma'_R$ and $\sigma'_\theta$ may be, the minimum principal effective stress can not be greater than 0 in the immediate vicinity of the stagnation pressure.

The apparent error in Figure 11 is probably due to the interpolation functions used and the fact that there is a very steep gradient of stresses across the surface elements, so much so that the maximum and minimum values of the minimum principal effective stress occur in the same element.

Figure 15 is a plot of the contours of fracture initiation for Forman and Secor's Indiana limestone resulting from the principal effective stresses calculated by ABAQUS. Small craters are predicted to form immediately under the load. Using Forman and Secor's criterion of only considering craters a nozzle diameter across indicates a threshold pressure for Indiana limestone of about $7.5T_o$ or 5625 psi or 161 percent of the measured value.

The contours greater than $7.5T_o$ are a convenience for comparison purposes. Obviously material would be removed during the attainment of such pressures, redefining the impinged surface and resultant stress state.

Figure 16 is a plot of the contours of fracture initiation for Indiana limestone using $K_w = 300,000$ psi, the bulk modulus of water. The predicted threshold pressure for a crater of diameter d is now indicated to be about $7T_o$ or 5250 psi or 150 percent of the measured value.
Figure 15 – Contours of Fracture Initiation, Indiana Limestone Using Forman and Secor's Q, Impact Pressure Only, 5d Coarse Mesh
Figure 16 – Contours of Fracture Initiation, Indiana Limestone, Impact Pressure Only, 5d Coarse Mesh
When pore fluid pressure is added (or increased) the pore fluid unloads the rock matrix, making the stress component in the rock matrix less compressive. This is made apparent by the equations relating $\sigma_R$ and $\sigma_Z$ to the principal effective stresses:

$$\sigma_1 = \frac{\sigma_R + \sigma_Z}{2} + \sqrt{\left(\frac{\sigma_R - \sigma_Z}{2}\right)^2 + \tau_{RZ}^2}$$

$$\sigma'_1 = \sigma_1 + p$$

$$\sigma'_1 = \frac{(\sigma_R + p) + (\sigma_Z + p)}{2} + \sqrt{\left(\frac{(\sigma_R + p) - (\sigma_Z + p)}{2}\right)^2 + \tau_{RZ}^2}$$

$$\sigma'_1 = \frac{\sigma_R + \sigma'_Z}{2} + \sqrt{\left(\frac{\sigma'_R - \sigma'_Z}{2}\right)^2 + \tau_{RZ}^2}$$

$$\sigma_2 = \frac{\sigma_R + \sigma_Z}{2} - \sqrt{\left(\frac{\sigma_R - \sigma_Z}{2}\right)^2 + \tau_{RZ}^2}$$

$$\sigma'_2 = \sigma_1 + p$$

$$\sigma'_2 = \frac{(\sigma_R + p) + (\sigma_Z + p)}{2} - \sqrt{\left(\frac{(\sigma_R + p) - (\sigma_Z + p)}{2}\right)^2 + \tau_{RZ}^2}$$

$$\sigma'_2 = \frac{\sigma'_R + \sigma'_Z}{2} - \sqrt{\left(\frac{\sigma'_R - \sigma'_Z}{2}\right)^2 + \tau_{RZ}^2}$$

The stress state is displaced in the direction of the failure envelope and the rock is weakened. Figure 17 shows the effect graphically.
as $p$ increases

Figure 17 – Effect of Pore Pressure on the Effective Stress State
It was shown in a previous section that the effect of incorrectly specifying $Q$ equal to the bulk modulus of the fluid was to lower the pore fluid pressure. Therefore, it is correct that $K_w = 54,300$ psi would predict a greater threshold pressure than $K_w = 300,000$ psi. All subsequent problems use $K_w = 300,000$ psi.

The previous problems were modeled using the 5d coarse mesh because of its similarity to the mesh used by Forman and Secor. Figure 18 is a plot of the contours of fracture initiation for Indiana limestone calculated using the 5d fine mesh. The predicted threshold pressure is $7.6T_o$ or 5700 psi or 163 percent of the measured value.

The increase in predicted threshold pressure is due to improved definition of the impact pressure curve. The 5d coarse mesh provides 11 nodes to define the load curve while the 5d fine mesh provides 20.

Figure 19 is a plot of the contours of fracture initiation for Indiana limestone calculated using the 20d mesh. The predicted threshold pressure is $8T_o$ or 6000 psi or 171 percent of the measured value.

The effect of the larger model is primarily outward from the area of impact. Both meshes see the same amount of input fluid and all input fluid must exit through the top surface. The input fluid in the 5d mesh, however, has both a much smaller total pore volume to move through and less surface to exit through than the 20d mesh. The result is a greater pressure differential across the top surface of the 5d mesh.
There was no significant difference between the 20d mesh and a similar 25d mesh. All subsequent problems use the 20d mesh.

Figure 20 is a plot of the contours of fracture initiation for Barre granite. The predicted threshold pressure is $6.6T_o$ or 7920 psi or 113 percent of the measured value. Similar to the elastic solid, a local minimum threshold pressure of about $18T_o$ occurs along the centerline about .65d below the surface. Unlike the limestone, the effects of the pore fluid are limited to the immediate area of impact due to the low permeability of the granite.

It is clear from this section that the addition of porosity to the elastic material increases the accuracy of the threshold pressure prediction. The predicted threshold pressure for Indiana limestone is 8000 psi or 171 percent of the measured value and the predicted threshold pressure for Barre granite is 7920 psi or 113 percent of the measured value. These predicted values are greater than the measured values and not less than as was indicated by the work of Forman and Secor.
Figure 18 – Contours of Fracture Initiation, Indiana Limestone, Impact Pressure Only, 5d Fine Mesh
Figure 19 – Contours of Fracture Initiation, Indiana Limestone Impact Pressure Only, 20d Mesh
Figure 20 – Contours of Fracture Initiation, Barre Granite, Impact Pressure Only, 20d Mesh
Porous Elastic Material with Both Impact Pressure and Shear Stress

In addition to the impact pressure, the rocks in this section are loaded with a shear stress across the top surface as defined by Equations 49 and 50. The elapsed time is again 10 microseconds.

Figure 21 is a plot of the contours of fracture initiation for Indiana limestone with \( k_s = 0.08'' \). The predicted threshold pressure for a crater of diameter \( d \) is \( 6.5T_o \) or 4875 psi or 139 percent of the measured value. (Though not apparent on the contour plot due to resolution, a .5d crater is predicted by the data at a pressure of \( 4.7T_o \) or 3525 psi or 101 percent of the measured value.)

Figure 22 is a plot of the contours of fracture initiation for Indiana limestone with \( k_s = 0.02'' \). The predicted threshold pressure is \( 7T_o \) or 5250 psi or 150 percent of the measured value.

Figure 23 is a plot of the contours of fracture initiation for Barre granite with \( k_s = 0.08'' \). The predicted threshold pressure is \( 5.5T_o \) or 6600 psi or 94 percent of the measured value.

Figure 24 is a plot of the contours of fracture initiation for Barre granite with \( k_s = 0.02'' \). The predicted threshold pressure is \( 6T_o \) or 7200 psi or 103 percent of the measured value.
Figure 21 - Contours of Fracture Initiation, Indiana Limestone, Impact Pressure and Shear Stress ($k_o = .08'$), 20d Mesh
Figure 22 – Contours of Fracture Initiation, Indiana Limestone, Impact Pressure and Shear Stress \( (k_e=.02') \), 20d Mesh
Figure 23 – Contours of Fracture Initiation, Barre Granite, Impact Pressure and Shear Stress ($k_s = 0.8''$), 20d Mesh
Figure 24 – Contours of Fracture Initiation, Barre Granite, Impact Pressure and Shear Stress ($k_e=.02''$), 20d Mesh
Now it is clear that the addition of shear stress — the very small and neglected in all previous studies shear stress — has the effect of reducing the predicted threshold pressure to more accurate levels.

The effect of shear stress can be explained using Equations 51 and 52 which relate \( \sigma_R \) and \( \sigma_Z \) to the principal stresses. In the immediate area of impact these equations define the maximum and minimum principal effective stresses. The shear stress, \( \tau_{RZ} \), due to the radial flow of fluid along the surface increases the difference between the maximum and minimum principal effective stresses. And with a sufficient increase in shear stress, the stress state is enlarged into the failure surface. Figure 25 shows the effect graphically.

No pretense is made as to the accuracy of the shear stress distribution and roughnesses used here. The exact shear stress distribution and surface roughnesses for Indiana limestone and Barre granite are not known. The shear stress distribution as defined by Equations 49 and 50 in combination with \( k_s = 0.08'' \) for Indiana limestone and \( k_s = 0.02'' \) for Barre granite, however, provide predicted threshold pressures comparable to the measured threshold pressures. And they sufficiently show the importance of including the shear stress due to the radial flow of the deflected jet.
\[
\frac{\sigma'_\text{max} - \sigma'_\text{min}}{2} \text{ increases as } |\tau_{KZ}| \text{ increases}
\]

Figure 25 — Effect of Shear Stress on the Effective Stress State
CONCLUSIONS

• The poro-elastic constitutive model formulated by Biot has been reconciled to a more recent poro-elastic constitutive model implemented in the commercial finite element software ABAQUS. The two were effectively equated after the constitutive parameters in Biot’s model were rewritten in terms of the compressibilities of the solid grains and fluid comprising the rock.

• The effect of porosity is significant. When pore fluid pressure is added (or increased) the pore fluid unloads the rock matrix, making the stress components in the rock matrix less compressive and displacing the Mohr circle in the direction of the failure surface.

• Forman and Secor\textsuperscript{7} showed the importance of porosity in their fluid jet studies. However, they did not properly account for the component compressibilities in their use of Biot’s model and as a result their predicted threshold pressures were incorrect.

• The shear stress resulting from the radial flow of fluid away from the impact area, though small compared to the impact pressure, has significant effect. The addition of this shear increases the difference between the maximum and minimum principal effective stresses, enlarging the Mohr circle toward the failure envelope.
- Accurate predictions of threshold pressures to cause rock failure under an axisymmetric fluid jet were achieved when the solid grain and fluid compressibilities and the shear stress were taken into account.
BIBLIOGRAPHY


APPENDIX

Volumetric strain of the solid grains, $e_g$, in terms of the initial porosity, $n_o$.\(^1\)

**BACKGROUND:**

Porosity is defined as the ratio of the pore volume to the total volume.

$$n = \frac{dV_w}{dV} = \frac{dV - dV_g}{dV} = 1 - \frac{dV_g}{dV} \quad [A1]$$

Using the subscript $o$ to indicate initial values.

$$n = 1 - \frac{dV_g}{dV} \frac{dV_o}{dV_o} = 1 - \frac{dV_g}{dV_g} \frac{dV_o}{dV_o} = 1 - J_g J^{-1}(1-n_o) \quad [A2]$$

where $J$ is the ratio of the porous material's current volume to its initial volume, otherwise known as the Jacobian, and $J_g$ is the ratio of solid grains' current volume to their initial volume.

$$J = \frac{dV}{dV_o} = 1 + e \quad [A3]$$

$$J_g = \frac{dV_g}{dV_g} = 1 + e_g \quad [A4]$$

Manipulating Equation A2

$$\frac{1-n}{1-n_o} = J^{-1} J_g$$

$$= J^{-1}(1 + e_g)$$

$$\frac{J}{1-n_o} \frac{1}{1-n} = \frac{1}{1-n}$$

$$\frac{J}{1-n_o} \frac{e_g}{1-n} = \frac{1}{1-n} \quad [A5]$$
DEVELOPMENT:

The volumetric strain of the solid grains is defined:

\[ e_g = -\frac{1}{K_g} \left( p - \frac{\sigma_m'}{1-n} \right) \]  \hspace{1cm} \text{[18]}

Substituting [A5] into [18]

\[ e_g = -\frac{1}{K_g} \left( p - \sigma_m' \left( \frac{J}{1-n_o} - \frac{e_g}{1-n} \right) \right) \\
\hspace{2cm} = -\frac{\sigma_m e_g}{K_g (1-n)} - \frac{1}{K_g} \left( p - \frac{\sigma_m' J}{1-n_o} \right) \\
\hspace{2cm} e_g \left( 1 + \frac{\sigma_m'}{K_g (1-n)} \right) = -\frac{1}{K_g} \left( p - \frac{\sigma_m' J}{1-n_o} \right) \]

Assuming \( \sigma_m' << K_g \)

\[ e_g = -\frac{1}{K_g} \left( p - \frac{\sigma_m' J}{1-n_o} \right) \]  \hspace{1cm} \text{[19]}