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A comparative study of the early terrestrial atmospheres with interactive cloud formation

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A COMPARATIVE STUDY OF THE EARLY TERRESTRIAL ATMOSPHERES
WITH INTERACTIVE CLOUD FORMATION

BY

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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MASTER OF SCIENCE

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ABSTRACT

A Comparative Study of the Early Terrestrial Atmospheres
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by

Robert Bradley Schmunk

Due to their formation at about the same time in the same region of the early solar nebula, it is reasonable to assume that the primitive atmospheres of Earth, Mars and Venus were similar and that present-day differences have arisen as a result of their differing masses and incident solar fluxes. Using a radiative-convective model, we determine maximum and minimum carbon dioxide levels for the early atmospheres which are consistent with this assumption and with climatic conditions thought to have existed on the three terrestrial planets 4.0 billion years ago. Rather than employ the cloud-free atmosphere approach of earlier studies, we include an interactive water vapor transport and cloud formation scheme in the model. Due to uncertainties about the direction of cloud cover feedback, we treat cloud cover as fixed. For most cases examined, we set the cloud cover at 50%, but the effect of varying cloud cover is also explored.
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I. INTRODUCTION

Nearly two decades ago, the scientific community was presented with an apparent paradox between the evolutionary increase in the solar luminosity and the evolution of Earth's atmosphere. Some 4.5 billion years before present (BYBP), the Sun would have had a luminosity approximately 75% of its current value. Assuming that Earth's atmospheric composition and planetary albedo had not changed during its long history, the average surface temperature of the planet would not have warmed to above freezing until about 2.3 BYBP. Geologic evidence, however, indicated that there was abundant liquid water as much as 1.5 billion years earlier. In presenting this apparent faint-Sun, warm-Earth paradox, Sagan and Mullen (1972) also suggested a solution, proposing that the early, post-accretion Earth atmosphere contained significantly more ammonia than it does today and consequently had a much more efficient greenhouse effect than at present. Several years later, Hart (1978) presented a more complex version of this solution, outlining an evolutionary path of insolation and atmospheric composition and including numerous feedbacks, such as cloud cover and surface albedo.

Owen et al. (1979) took exception to the ammonia greenhouse theory, citing studies that demonstrated that ammonia does not have the stability for large amounts of the gas to remain in the atmosphere for extended periods, and presented an alternate solution. They used Hart's evolutionary sequence for solar luminosity and CO\(_2\) levels, but omitted ammonia and the cloud and ice-albedo feedbacks (their model did include a fixed cloud at constant altitude). Employing more accurate radiation absorption methods in a radiative-convective model (RCM), they found that increasing the surface partial pressure of carbon dioxide to P(CO\(_2\)) = 0.31 bar (about 1000 times the present value near 0.33 mbars) would have been sufficient to maintain a surface temperature of 310 K for a surface albedo equal
to the modern value ($\approx 0.15$). For a highly reflective albedo of 0.7, they obtained a surface temperature of 285 K, close to the modern global average near 288 K. In a brief aside, they also suggested the possibility that large CO$_2$ abundances in the early Martian atmosphere would result in surface temperatures on that planet that would support liquid water and explain terrain features which appear to have resulted from fluvial erosion.

Rossow et al. (1982) discussed a scheme whereby changes in Earth's atmospheric composition would not have been necessary to explain the paradox, arguing that the earlier models did not sufficiently account for important feedbacks, particularly that of clouds. In this self-admittedly speculative work, they presented a cloud feedback parameterization in which a "cirrus greenhouse" moderates the reduced insolation received by the early Earth. Lower temperatures cause a decrease in low cloud cover and thence in the planetary albedo, resulting in a negative feedback. With only a small increase in the CO$_2$ level of the early atmosphere, on the order of five times the present amount rather than the thousand-fold increase of Owen et al. (1979), they found that the average surface temperature would exceed 274 K. They did not include the positive ice-albedo feedback which would have further cooled the early Earth, suggesting that it may have been secondary to the cloud feedback. Kasting et al. (1984b) have criticized this omission of one feedback in favor of another and also point out that extra heating occurred in the model since Rossow et al. (1982) employed a luminosity value which was several percent higher than used in other models.

Kuhn and Kasting (1983) also adopted Hart's evolutionary sequence of solar luminosity and carbon dioxide levels, but their RCM included an interactive cloud prediction scheme (Hummel and Kuhn, 1981b). Though cloud coverage was fixed at 50%, the model allowed for variations in the vertical structure of the predicted cloud and consequent feedbacks. For a CO$_2$ level 1000 times the present level, they calculated an early Earth surface
temperature of 292 K, about 20 K lower than that found by Owen et al. (1979). Using a
cloud-free RCM, Kasting et al. (1984b) obtained similar results, with a surface tempe-
tature of 289.4 K. The discrepancy between the results of Owen et al. (1979) and Kasting et
al. (1984b) was resolved by Kiehl and Dickinson (1987), who found a surface temperature
for early Earth within two degrees of the result of Kasting et al. (1984b) and determined
that errors in absorption coefficients, particularly in the weak CO₂ bands, had caused Owen
et al. (1979) to overestimate the surface temperature.

Variations of the Kasting et al. (1984b) RCM have been employed in other studies
of the three terrestrial planets: Earth, Mars and Venus. Kasting et al. (1984a) examined the
effect of increased solar flux on the present-day Earth atmosphere, finding that the moist
adiabatic lapse rate used by the model helped moderate the expected increase in the surface
temperature, preventing a runaway greenhouse effect from occurring for solar luminosities
up to 45% above the current level. An analogy to early Venus was drawn, and Kasting et
al. (1984a) determined that it was quite possible for that planet to have lost an Earth ocean
of water during its early history even if a runaway greenhouse condition did not yet exist.
Observing that enough carbon to form 60 bars of CO₂ now resides in the Earth's crust,
Kasting and Ackerman (1986) departed from using the Hart evolutionary path, and
examined the effect of huge amounts of CO₂ being present in the early Earth atmosphere.
Even for the extreme case of P(CO₂) = 100 bar, they found that the atmosphere was stable
against a runaway greenhouse, and that due to the high surface pressure, the boiling point
was high enough that the bulk of the planet's water remained in the ocean. Applying the
model to early Mars, Pollack et al. (1987) sought to determine how much CO₂ need be in
that planet's early atmosphere for the surface temperature to be warm enough to support
liquid water activity — the "faint-Sun, warm-Mars" paradox. They found that a minimum
CO₂ partial pressure between 0.75 and 5 bars was necessary, with the lower estimates
corresponding to the optimal conditions of low surface albedo, perihelion and equatorial latitudes. Kasting (1987) again used the model to examine an evolutionary history of the Earth's atmosphere, with an eye toward establishing upper and lower bounds on its CO₂ content and also on the developing O₂ content. This last study used the maximum initial CO₂ level of 10 bars estimated by Walker (1985) from an analysis of geochemical mass balance.

Further description of the results of the preceding models and other terrestrial atmospheric evolution models may be found in Kasting and Toon (1989). One work not described in that review, however, is that of Durham and Chamberlain (1989), developed from the model of Kasting et al. (1984b). They assumed that the early atmospheres of Earth, Mars and Venus were similar, containing approximately the same proportions of CO₂ and N₂, and that modern differences have arisen as a result of their differing planetary masses and incident solar fluxes. Working from the evidence of surface liquid water activity early in the history of Mars, they estimated a minimum CO₂ partial pressure on that planet of 1.3 bars. Requiring that Venus be able to lose its initial endowment of hydrogen by diffusion-limited escape in less than the planetary lifetime, they set a maximum CO₂ partial pressure of 11.1 bars for that planet. By means of a scaling equation, they equated these values to a CO₂ partial pressure on early Earth lying between 9 and 14 bars, corresponding to surface temperatures between 350 and 363 K.

It should be noted that in the several models developed from Kasting et al. (1984a,b), the radiative-convective schemes have been based on a clear-sky approximation. In other words, clouds are entirely omitted from the atmospheric column. Partial compensation for their absence is made by increasing the surface albedo from the observed modern value near 0.15 to values in the vicinity of 0.23, since these higher values will cause the models to generate a surface temperature near 288 K when modern Earth conditions are
considered. However, this is not necessarily a reasonable approximation of the actual 
reflectivity of a cloud layer at solar flux wavelengths, and this approach also completely 
neglects the effect of clouds in absorbing and re-emitting thermal radiation. These 
competing acts of omission engendered within one approximation are assumed to cancel 
each other. The reason for such a seemingly crude approach is that feedbacks resulting 
from increases in cloud coverage, altitude and thickness have not been well-known. Cess 
(1976), in making an analysis of the importance of cloud feedback, decided that any posi-
tive feedback of increased temperatures increasing the cloud coverage and consequently 
enhancing the thermal opacity would be, within the limits of known data, roughly balanced 
by a negative feedback resulting from the increased cloud coverage raising the effective 
planetary albedo. However, Roads (1978) suggested that increased surface temperatures 
would lead to a decrease in the cloud cover. Even in an RCM study of the modern atmo-
sphere, Lindzen et al. (1982) were reluctant to include fixed clouds of known 
characteristics in their model and employed the cloud-free assumption. Today, confusion 
remains and research into the magnitude and direction of cloud feedbacks continues (Cess 
et al., 1989). Faced with a dilemma of conflicting theories and unable to predict what types 
of clouds might be present, it is not surprising that modelers of early terrestrial climates 
have generally opted not to include clouds at all.

Despite these obstacles, the objective of the research described in this thesis is to 
extend the work of Durham and Chamberlain (1989) by adding to their RCM an interactive 
water vapor transport and cloud prediction scheme. Addition to the model of fixed-cloud 
characteristics, as is often done with RCMs of the modern Earth atmosphere, is unsatisfac-
tory due to the widely varying possible states of the early atmospheres. A cloud inserted at 
a constant pressure level in the modern Earth atmosphere will remain at approximately the 
same altitude for a doubling of CO₂, for instance, but will not do so in the case of an early
terrestrial atmosphere where we may alter the surface pressure by a bar or more. Hence it would be more desirable to join the RCM with a routine that interactively develops clouds. We examine three possible approaches that have appeared in the literature during the past decade.

The RCM of Hummel and Kuhn (1981b) divided the sky into two vertical columns. The first column is clear and uses a well-known constant relative humidity equation to determine its water vapor profile. The second column is potentially cloudy and its water vapor content is found from a vertical transport equation which includes terms for advection and eddy diffusion. Wherever this transport equation calculates a specific humidity that exceeds saturation, a second transport equation is used to calculate a liquid water distribution, in effect creating a cloud. Whereas the liquid water profile within the cloud layers can be affected by rainfall, the two equations are not coupled and do not allow for such microphysical effects as scavenging of water vapor from the lower atmosphere by transient raindrops. Additionally, the area covered by clouds is not interactively determined by this model but is imposed; Hummel and Kuhn set cloud cover at 50%. Other aspects of the model which must be specified by the modeler are the surface relative humidity and the maximum altitude at which the water vapor transport equation is applied. Although the density of liquid water is calculated within the cloud layers, the optical characteristics (and hence the solar flux) through these layers is not dependent on that density (Hummel, 1978). Instead, the effective albedo for the top of the cloud was determined using altitude criteria, and solar flux below the clouds was empirically derived.

Jung and Bach (1985) described two schemes for predicting vertical water vapor distribution and resulting cloud formation. In their first approach, the total water vapor content of the vertical column was found from empirical terms for evaporation and precipitation; a water vapor profile was then determined from the total water vapor. In their second
scheme, a water vapor transport equation similar to that of Hummel and Kuhn (1984b) was employed, except that the advection term was dropped and a parameterized term for precipitation was added. Though both methods predicted relative humidities less than unity, cloud cover in the various atmospheric layers was invoked where the humidity exceeded some threshold value, a method used in some three-dimensional general-circulation models. Where clouds were predicted, the liquid water content was set equal to the saturation density times an empirical constant of about 1%. Despite variations in the cloud profiles generated by the two methods, Jung and Bach found that temperature increases were moderated by cloud feedback, since increases in water vapor density with temperature were not as rapid as increases in the saturation density.

The model of Liou et al. (1985) developed a cloud formation scheme in conjunction with a radiative-turbulent model, which applies vertical transport that is parameterized with a closure scheme for eddy thermal flux, rather than the convective adjustment employed by the numerous RCMs described above. Cloud presence is determined by a coupled set of equations involving thermodynamic energy balance, water vapor transport, latent heat flux transport, and cloud cover. Unlike the method of Hummel and Kuhn (1981b) and like that of Jung and Bach (1985), it does not predict humidities exceeding saturation. Instead, it applies a threshold humidity to the clear-sky region and saturation to the cloudy sky, and it sets the cloud cover so that the weighted average of these values equals the predicted value. The next step of the model is to separate the single continuous cloud predicted thus far into distinct high, middle and low cloud layers. The model uses temperature criteria to set the boundaries between the three cloud zones. Within each zone, the cloud cover is averaged, and its vertical extent is statistically compacted; a single effective cloud cover is then found by assuming random overlap of the three cloud layers. The liquid water contents of the three clouds are determined from observed distributions in appropriate modern Earth cloud
types which correspond to the three cloud layers of the model. Realistic optical characteristics of the clouds are then derived from the liquid water content. Liou et al. (1985) found that the clouds predicted by their model generally exhibited a negative feedback. Temperature increases caused low cloud cover and thickness to increase, raising the albedo, but also caused the high cloud cover and thickness to decrease, lowering their infrared opacity. They did, however, find that the middle cloud layer exhibited a positive feedback due to temperature increases, lowering its cover and thickness, although this effect was outweighed by the low and high clouds.

More recently, Liou and Ou (1989) have extended this last model by including parameterizations of cloud microphysical processes involving condensation, precipitation, and evaporation rates in the group of coupled equations which predict cloud formation and cover. Derivation of these rates requires mean droplet radii, which are determined from observed values. Allowing these radii to vary, as would occur due to a change in the number of available cloud condensation nuclei, they found that for the atmosphere of modern Earth, smaller droplet radii result in less precipitation, allowing the albedo effect to dominate and moderate temperatures (a negative feedback). Larger radii caused more precipitation and the infrared opacity effect of high clouds became more important, accentuating temperature changes (a positive feedback).

In coupling an interactive cloud formation scheme to the model of Durham and Chamberlain (1989), we have decided to employ the method of Hummel and Kuhn (1981b) in conjunction with more realistic cloud optical characteristics. Its more flexible approach allows us to examine atmospheres which are quite different from that of modern Earth, and it avoids the numerous parameterizations of Jung and Bach (1985), whose extension to primitive terrestrial conditions would not be justifiable. While it deprives us of several more realistic features of the Liou et al. (1985) and Liou and Ou (1989) models, it
also avoids the complexity and resulting demands on computer time of that model. Additionally, we do not have to apply a possibly inappropriate cloud compaction scheme to the early atmospheres, and while it is unfortunate that we are left without a cloud cover prediction, it has been noted above that confusion remains on how that factor should be estimated, particularly when applied to RCMs.
II. MODEL

The Radiative-Convective Model

The radiative-convective model used in this study is that of Durham and Chamberlain (1989), which is based on the model of Durham's (1986) doctoral thesis and which in turn was derived from the models of Kasting et al. (1984a, 1984b) and Kasting and Ackerman (1986). The discussion here will be limited to a broad description of the model and to changes from the scheme outlined in Durham (1986); further details may be found in the appendix or in the above-mentioned sources.

The model considers a one-dimensional, plane-parallel atmosphere. The atmosphere is divided into 40 levels at which radiative fluxes are calculated. Level one is situated at the top of the atmosphere at a pressure of 0.01 mbar; level 40 lies at the planet's surface. The independent vertical coordinate is the atmospheric pressure.

Calculation of radiative flux at the various levels is performed in two routines, one for thermal radiation and the other for solar radiation. The thermal spectrum from 2.23 μm longwards is divided into 38 bands, in which absorption by H₂O, CO₂ and O₃ is considered. Net flux in these wavelengths is calculated by numerically evaluating the integral equation of transfer (Kasting et al., 1984a). Solar flux wavelengths from 0.25 μm to 4.0 μm are represented by 26 spectral bands. Rayleigh scattering and absorption by O₂, H₂O, CO₂ and O₃ are included, and the delta-two-stream method is used to solve the differential form of the equation of transfer (Haberle et al., 1985; Toon et al., 1989).

Once thermal and solar fluxes have been determined, a new temperature profile for the upper atmosphere is calculated by the Newton-Raphson method (Kasting et al., 1984a). A convective adjustment is performed to locate the boundary between the upper,
radiative portion of the atmosphere and the lower, convective portion, and the temperature profile for the lower atmosphere is then determined from the moist adiabatic lapse rate (Kasting, 1988). This procedure of determining fluxes and subsequently altering the temperature profile is repeated until equilibrium is obtained.

**Water Vapor Profiles and the Cloud Prediction Routine**

Numerous RCMs have calculated a vertical water vapor profile for the troposphere with variations of an empirical formula first advanced by Manabe and Wetherald (1967):

\[
\frac{q(P)}{q_{\text{sat}}(P)} = r_h(P) = r_h(P_{\text{surf}}) \left[ \frac{P / P_{\text{surf}} - 0.02}{1.0 - 0.02} \right]^{\Omega}
\]  

(2.1)

where \( q(P) \) is the specific humidity (kg kg\(^{-1}\)) of water vapor at pressure level \( P \), \( q_{\text{sat}} \) is the specific humidity for water vapor saturation, \( r_h \) is the relative humidity, \( P_{\text{surf}} \) is the surface pressure, and \( \Omega \) is a parameterized exponent which varies from model to model, having been set equal to unity (Manabe and Wetherald, 1967), made a function of surface temperature (Cess, 1976), or made a function of pressure (Kasting and Ackerman, 1986). The surface relative humidity, \( r_h(P_0) \), is typically set in the range 0.75 to 0.80. When the relative humidity is calculated to be less than zero (when \( P < 0.02 \times P_{\text{surf}} \)), most models instead set the specific humidity equal to a constant value in the vicinity of \( 3.0 \times 10^{-6} \) kg kg\(^{-1}\). However, we follow the Durham (1986) model and set the water vapor mixing ratio in the upper atmosphere equal to the mixing ratio at the boundary between the radiative and convective portions of the atmosphere.

The Manabe-Wetherald formula, however, will not generate a water-vapor profile in which the humidity exceeds saturation. If saturation is used as a necessary condition to consider cloud formation, then another method of determining the vertical water vapor distribution is necessary. For the model used in this study, we determine a water vapor
profile using an approach similar to that described by Hummel (1978) and Hummel and Kuhn (1981b). By this method, the equation of radiative transfer is evaluated separately for two vertical columns, one clear and the other potentially cloudy, and the resulting radiative fluxes are averaged, with weighting as specified by the modeler, in effect explicitly specifying the total cloud cover. In the first, clear-sky column, the water vapor profile is found by the traditional Manabe-Wetherald scheme explained above, using the Kasting and Ackerman (1986) form of $\Omega$. In the potentially cloudy column, a vertical transport equation is used to determine the water vapor distribution for the convective zone of the atmosphere. To derive this transport equation, we begin with conservation of water substance in a one-dimensional atmosphere, which may be expressed (Hummel, 1978) as

$$\frac{\partial \overline{q}}{\partial t} = - \overline{w} \frac{\partial \overline{q}}{\partial z} - \frac{1}{\rho} \frac{\partial}{\partial z} \left( \overline{\rho w q'} \right),$$

(2.2)

where $q$ is again the specific humidity, $w$ is the large-scale vertical velocity, an overbar indicates an average, and a prime indicates a perturbation. First-order closure theory allows us to make the approximation

$$\overline{\rho w q'} = - \rho K_z \frac{\partial \overline{q}}{\partial z},$$

(2.3)

where $K_z$ is an eddy diffusion coefficient (m$^2$ sec$^{-1}$). Consequently, we may write

$$\rho \frac{\partial \overline{q}}{\partial t} = - \rho \overline{w} \frac{\partial \overline{q}}{\partial z} + \frac{\partial}{\partial z} \left( \rho K_z \frac{\partial \overline{q}}{\partial z} \right).$$

(2.4)

The first term on the right-hand side arises from advection, and the second from eddy diffusion. When the steady-state case is considered, equation (2.4) becomes

$$\frac{\partial^2 \overline{q}}{\partial z^2} = \left( \frac{w(z)}{K_z} + \frac{1}{H_p(z)} \right) \frac{\partial \overline{q}}{\partial z}$$

(2.5)
where $H_p$ is the scale height for density,

$$\frac{1}{H_p} = -\frac{1}{\rho} \frac{\partial \rho}{\partial z} .$$

(2.6)

We have followed the method of Hummel (1978) in calculating the large-scale vertical velocity, but rather than employ an empirical determination of the eddy diffusion coefficient, probably inappropriate to the early terrestrial atmospheres, we apply a constant $K_z = 20$ (Warneck, 1988, p. 23). Parameterization methods for the eddy diffusion coefficient vary widely in the literature, and as noted by Hummel, we have also found that the model is relatively insensitive to variations in $K_z$ which are of less than an order of magnitude.

The transport equation is easily solved by numerical differencing and matrix inversion. The two boundary conditions applied are the specific humidities at the top of the potentially cloudy region and at the surface, both of which we have set to the specific humidities at the same altitudes in the clear sky region. The altitude at the top of the potentially cloudy region is set slightly below the tropopause. Above this point, the water vapor profile in the cloudy column is set equal to the content at the same altitudes in the clear-sky column.

Cloud formation is treated as occurring in any layer where the water vapor content exceeds the specific humidity for saturation, $q_{sat}$:

$$r_h = \frac{q}{q_{sat}} = \frac{P(H_2O)}{P_{sat}(H_2O)} > 1.0 .$$

(2.7)

The saturation partial pressure of water, $P_{sat}(H_2O)$, may be calculated from the Clausius-Clapeyron equation:

$$\frac{dP_{sat}(H_2O)}{P_{sat}(H_2O)} = \frac{L}{R(H_2O)} \frac{dT}{T^2} ,$$

(2.8)
where \( L \) is the latent heat (J kg\(^{-1}\)) of the phase change appropriate to the temperature \( T \) and \( R(\text{H}_2\text{O}) \) is the specific gas constant (J kg\(^{-1}\) K\(^{-1}\)) of water. Rather than setting the liquid water content of a cloudy layer exactly equal to the excess water vapor amount, however, we employ a second equation used by Hummel and Kuhn, which they obtained by expanding a less complex equation used by Hess (1976), to determine liquid water transport within the cloud:

\[
\frac{\partial l}{\partial z} = \left( \frac{V}{K_z} + \frac{1}{H_p} \right) l + \left( \frac{w}{K_z} + \frac{1}{H_p} \right) q_{\text{sat}} - \frac{\partial q_{\text{sat}}}{\partial z},
\]

(2.9)

where \( l \) is the mixing ratio of liquid water (kg kg\(^{-1}\)) and \( V \) is the terminal velocity of water droplets. The fourth-order Runge-Kutta method is used to solve equation (2.9) for \( l \), with a boundary condition that \( l = 0 \) at the base of any contiguous group of cloud layers. Latent heat released by the liquid water is determined, and the temperature profile of the cloud layers is adjusted accordingly.

Hummel and Kuhn (1981b) specified the terminal velocity of droplets as a constant, employing a value appropriate to the average droplet radius in modern Earth clouds. We have, however, chosen to let the effective droplet radius, and hence the terminal velocity, vary. Following Betts and Ridgway (1988), we approximate the effective droplet radius by

\[
r_e = A \times \rho_{\text{liq}} + B,
\]

(2.10)

where \( \rho_{\text{liq}} \) is the liquid water concentration (kg m\(^{-3}\)) and the constants \( A \) and \( B \) equal 11\times10^{-3} \text{ m}^2\text{kg}^{-1} \text{ and } 4\times10^{-6} \text{ m}, respectively. The terminal velocity of droplets can then be found from Stokes' law (Fletcher, 1962), which states that

\[
V = -\frac{2}{9} \frac{g \rho_{\text{drop}} r_e^2}{\eta},
\]

(2.11)
where $g$ is gravitational acceleration, $\rho_{\text{drop}}$ is the density of a water droplet, and $\eta$ is the dynamic viscosity of air (kg m$^{-1}$ s$^{-1}$). Viscosity, which is independent of pressure for the atmospheres considered in this study, is given by Hilsenrath et al. (1960) as

$$\eta = \frac{A \tau^{1.5}}{T + B},$$

(2.12)

where the empirical constants $A$ and $B$ equal $1.458 \times 10^{-6}$ kg m$^{-1}$ s$^{-1}$ K$^{-0.5}$ and 110.4 K, respectively.

This formulation of terminal velocity renders values greater than the -0.5 cm s$^{-1}$ (corresponding to $r_e \approx 6.5 \mu$m) employed by Hummel and Kuhn, especially in warmer atmospheres which contain large amounts of water, such as occurs in the case of the warm, early Earth. With greater downwards velocities, more liquid water is removed from the cloud.

**Optical Characteristics of the Cloud**

Optical properties of the cloud for solar flux wavelengths are calculated by a scheme outlined in Slingo (1989). According to this method, the optical depth, single scattering albedo and asymmetry factor of a cloud layer may be found via the formulae

$$\tau_i = \text{LWP} \left( a_i + \frac{b_i}{r_e} \right)$$

$$\bar{\omega}_i = c_i + d_i \times r_e$$

$$g_i = e_i + f_i \times r_e$$

(2.13)

where the subscript $i$ indicates the solar frequency band, LWP is the liquid water path of the layer (g m$^{-2}$), $r_e$ is the effective radius of the liquid water droplet radius distribution, and $a_i$, $b_i$, $c_i$, $d_i$, $e_i$, and $f_i$ are parameterized constants. The values for these constants from the 24 bands of Slingo (1989) have been adjusted to fit the 26 solar bands of the radiative
scheme used in this study. We again apply the effective droplet radius of equation (2.10). At thermal wavelengths, we treat the clouds as blackbodies.

Test-Run to Simulate Modern Earth

Before application of the model is made to the early terrestrial atmospheres, we first examine its reaction to the case of the present Earth atmosphere. The effective cloud cover was set equal to 50%, and we varied the surface albedo until the model found a surface temperature close to the current mean value of 288 K. For a surface albedo of 14%, well within the range of values commonly given for the current surface albedo, we determined a surface temperature of 288.4 K. The resulting vertical temperature profile is shown in Figure 2.1, along with profiles from the Durham and Chamberlain (1989) clear-sky model and the 1976 U.S. standard atmosphere. As expected, our results are quite close to those of Durham and Chamberlain (1989) but differ with those of the standard atmosphere at high altitudes. This latter difference is a result of applying the moist lapse rate to the troposphere. This lapse rate is lower than the standard lapse rate of -6.5 K km\(^{-1}\) near the surface, where the temperature is relatively high, but becomes greater near the tropopause, where the temperature is lower and there is less water vapor present.
Figure 2.1: Vertical temperature profiles calculated for modern Earth conditions.

The results of the present model are presented together with those of Durham and Chamberlain (1989) and the 1976 U.S. Standard Atmosphere. Differences are a result of the differing lapse rate formulations applied within the troposphere.
III. APPLICATIONS

A test application of the model is first made to the early Earth atmosphere of the Hart evolutionary path. A solar flux 78% of the present value (Newman and Rood, 1977) is assumed incident upon an atmosphere whose surface partial pressures are $P(N_2) = 0.69$ bar and $P(CO_2) = 0.31$ bar. A surface albedo of 14% is applied, with a constant cloud cover of 50%. No oxygen, in molecular or ozone form, is present. We find a surface temperature of 290.2 K, lying comfortably amidst the 292 K of Kuhn and Kasting (1983), 289.4 K of Kasting et al. (1984b), and 290.75 K of Kiehl and Dickinson (1987).

In extending the work of Durham and Chamberlain (1989), we follow their initial assumptions about the comparative atmospheric compositions of the three terrestrial planets 4.25 BYBP. Even though less is known about the early climatic conditions which prevailed on Mars and particularly Venus than is known for Earth, it is reasonable to assume that since the three planets formed in the same region of the solar nebula at about the same time, they had similar initial compositions and that with similar outgassing histories, they shared the same major initial atmospheric constituents. These gases were $N_2$, $CO_2$, and $H_2O$; other radiatively active gases were not stable enough (e.g., $NH_3$ and $CH_4$) or had not yet been sufficiently produced ($O_2$ and $O_3$) to be present in significant amounts. Since water could be present in either the primordial oceans or the atmospheres, the amount of water vapor, and cloud liquid water, in the atmosphere is allowed to vary as described by equations (2.1), (2.5) and (2.9). The proportions of $N_2$ and $CO_2$, however, are assumed to have been the same for all three planets, and a scaling function dependent on planetary masses, surface areas and gravities is used to relate their partial pressures on the different planets. In comparing, for instance, the dry surface pressure on Mars to Earth, Durham and Chamberlain employ the relationship
\[ P_m = \frac{M_m}{M_e} \frac{A_e}{A_m} \frac{g_m}{g_e} P_e \]  \hspace{1cm} (3.1)

A similar function is used for Venus. Durham and Chamberlain set the \( N_2 \) partial pressure for early Earth at 1.0 bar, instead of the 0.69 bar used by Hart (1978), reasoning that this larger amount is the total that has been outgassed from Earth's interior and would have all been present in the early atmosphere. According to the scaling function (3.1), this translates to \( P(N_2) = 0.144 \) bar for early Mars and to \( P(N_2) = 0.794 \) bar for early Venus.

We continue using solar fluxes incident at the top of the planetary atmospheres that are 78\% of the present value. We also fix the surface albedo of all three early planets equal to the 14\% value which the model employs for modern Earth. Though application of a constant albedo may be an incorrect assumption due to the extremely different conditions which would have existed for the early planets, it is a procedure which has been adopted by other studies of early Earth (e.g., Owen \textit{et al.}, 1979; Kasting \textit{et al.}, 1984b) and extension to early Mars and Venus should be no less valid. For most model runs, a cloud coverage of 50\% was assumed.

Vertical temperature profiles for selected partial pressures of \( \text{CO}_2 \) are presented in Figure 3.1. An easily discerned tropopause is seen to occur for all of the cases shown at an altitude between 20 and 30 km. The stratospheric bulge in the temperature profile present in the modern Earth profile (Figure 2.1) is not seen in these cases, since stratospheric \( \text{O}_3 \) was absent from the early Earth atmosphere. The surface temperature of the early Earth is seen to increase monotonically for increasing levels of \( \text{CO}_2 \) (Figure 3.2), and even for the extreme case of \( P(\text{CO}_2) = 50 \) bar, we find a surface temperature of 423 K, far below the boiling point for such an excessive pressure, indicating, as did Kasting and Ackerman (1986), that the primitive Earth atmosphere was stable against a runaway greenhouse.
Upward transport of heat by moist convection and the increased planetary albedo caused by Rayleigh scattering contribute to the stability found by these models.

Figure 3.2 also compares the results of this model with those of the cloud-free model of Durham and Chamberlain (1989). With 50% cloud cover, this model tends to find surface temperatures about 10 K higher than did Durham and Chamberlain, except for cases where P(CO₂) < 0.25 bar. The interactive cloud scheme of the present model tends to result in large quantities of water vapor (Figure 3.3) and cloud liquid water in the cloudy column of the atmosphere, particularly for cases where the surface temperature is quite high. The atmosphere becomes warmer due to the increased opacity. The result of variations in the cloud cover are shown in Figure 3.4, and it is seen that 50% cloud cover contributes about 4 K of warming over a cloud-free case with a relatively realistic surface albedo. If cloud cover was much higher, as might be expected for an atmosphere with large amounts of water vapor present, the warming is strengthened even more.

In examining the case of early Mars, we must also consider the effect of variations in the planet's orbital eccentricity, which may attain a maximum of 0.14 (Ward, 1974). For such an eccentricity, the solar flux at perihelion would be 38% greater than its mean annual value. Consequently, in examining the case of early Mars, we perform the calculations for incident solar fluxes of 78% and 1.08% the present mean value.

Figures 3.5 and 3.6 show vertical temperature profiles for selected CO₂ partial pressures in the early Martian atmosphere at the mean orbital radius and at perihelion, respectively. Vertical water vapor profiles for the two cases are presented in Figures 3.7 and 3.8. A comparison of the differing temperature profiles resulting from the two orbital radii is shown in Figure 3.9 for the case of P(CO₂) = 1.5 bar.
For sufficiently high CO\(_2\) levels, the Martian surface temperature would have been warm enough to support surface liquid water activity, as demonstrated in Figure 3.10. Durham and Chamberlain (1989) suggested a minimum temperature of 260 K for such activity to occur, reasoning that this temperature was sufficient for subsurface water activity to occur, and their clear-sky model found that a CO\(_2\) partial pressure of 1.3 bars was necessary to maintain such a temperature at perihelion. At the mean orbital radius, the necessary CO\(_2\) level was about 3 bars. In order to sustain a surface temperature of 273 K, their model required P(CO\(_2\)) = 2 bars for the perihelion case (as did Pollack et al., 1987) and P(CO\(_2\)) = 4 bars for mean annual condition. Due to the higher planetary albedo of the present cloudy model, we find that slightly higher CO\(_2\) levels are necessary in order to maintain these temperatures. With 50% cloud cover, the minimum partial pressure at perihelion which we find will support a surface temperature of 260 K is P(CO\(_2\)) = 1.5 bar. If P(CO\(_2\)) > 2.3 bar, the surface temperature will exceed 273 K, warm enough to support, on average, liquid water activity. For the mean annual condition on Mars, we find that P(CO\(_2\)) = 1.5 bar generates a surface temperature of 237 K, and P(CO\(_2\)) = 2.3 bar gives a surface temperature of 250 K.

Figure 3.11 shows the response of the early Mars atmosphere to changes in cloud cover in the present model. Increases in cloud cover result in rapid decreases in the surface temperature, a result quite different from those described above for the case of the early Earth. The water vapor profile of the clear-sky column is extremely sensitive to the surface pressure of the model (Kasting and Ackerman, 1986) so that higher pressure atmospheres are more heavily laden with water vapor. Additionally, the interactive cloud scheme of this model is quite responsive to the surface temperature. Combining these two factors results in more extensive cloud development for the denser and warmer early Earth atmosphere and the cloud opacity effect dominates over the albedo effect. For the thinner and cooler
early Mars atmosphere, however, the clouds found are less vertically extensive and on average lie at much lower altitudes. In this case, then, the clouds are below much of the upward-traveling thermal radiation and there is less heating of the atmosphere by trapped radiation. Consequently, we find that for early Mars the cloud albedo effect is much more important than the opacity effect, and the surface temperature decreases in response to cloud cover changes.

Vertical temperature profiles for selected CO$_2$ levels on early Venus are presented in Figure 3.12, along with water vapor profiles in Figure 3.13. The surface temperatures found are quite similar to those of Durham and Chamberlain (1989). An examination of the temperature profiles demonstrates that the tropopause was poorly defined, if existent at all. It should be noted that the bulge in the 150-bar profile presented in Figure 3.12 is a result of latent heat released to the atmosphere by the condensation of cloud liquid water, heat which the model does not yet have the capability to transfer to other altitudes. This heating source was not important to the Earth and Mars profiles described above since much less liquid water was present in the clouds formed in those atmospheres.

Due to the ill-defined tropopause, water vapor in the early Venus atmosphere would have been carried by convection to very high altitudes above the bulk of the atmosphere, altitudes where it could be destroyed by photodissociation and the hydrogen freed could escape into space. Durham and Chamberlain (1989) argue that for CO$_2$ levels above 11 bars this effect would have been slow enough that Venus' initial endowment of hydrogen would not have been lost in less than the lifetime of the planet. As a result, they set this value as an upper limit for CO$_2$ in the primitive atmosphere. Ingersoll (1969) argued that the stratospheric cold trap on Venus would have been ineffective if the mixing ratio of water vapor was much more than 10% by volume. We find that this situation occurs in the
present model when the CO\textsubscript{2} partial pressure exceeds 10.0 bars, and consequently we choose to adopt Durham and Chamberlain's value of 11 bars as the upper limit.

By means of the scaling function (3.1), we may use the maximum CO\textsubscript{2} level of early Venus and the minimum CO\textsubscript{2} level of early Mars in order to set maxima and minima for the three early terrestrial planets. Results for an assumption of 50\% cloud cover on all planets are presented in Table 3.1. For comparison, the results of Durham and Chamberlain (1989) are shown in Table 3.2, and the extrema determined by several models may be found in Table 3.3.
<table>
<thead>
<tr>
<th></th>
<th>Mars (perihelion)</th>
<th>Earth</th>
<th>Venus</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(CO₂)</td>
<td>1.5 - 2.3 bar</td>
<td>10.4 - 14.0 bar</td>
<td>8.2 - 11.1 bar</td>
</tr>
<tr>
<td>P (N₂)</td>
<td>0.14 bar</td>
<td>1.0 bar</td>
<td>0.79 bar</td>
</tr>
<tr>
<td>P (H₂O)</td>
<td>= .001 bar</td>
<td>= 0.5 bar</td>
<td>= 2.5 bar</td>
</tr>
<tr>
<td>Total pressure</td>
<td>1.6 - 2.5 bar</td>
<td>11.9 - 15.5 bar</td>
<td>11.5 - 14.4 bar</td>
</tr>
<tr>
<td>Surface temperature</td>
<td>260 - 275 K</td>
<td>362 - 371 K</td>
<td>405 - 415 K</td>
</tr>
</tbody>
</table>

Table 3.1: Constraints on early terrestrial atmospheric compositions from the present model, assuming 50% cloud cover.

<table>
<thead>
<tr>
<th></th>
<th>Mars (perihelion)</th>
<th>Earth</th>
<th>Venus</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(CO₂)</td>
<td>1.3 - 2.3 bar</td>
<td>9.0 - 14.0 bar</td>
<td>7.1 - 11.1 bar</td>
</tr>
<tr>
<td>P (N₂)</td>
<td>0.145 bar</td>
<td>1.0 bar</td>
<td>0.79 bar</td>
</tr>
<tr>
<td>P (H₂O)</td>
<td>= .001 bar</td>
<td>0.3 - 0.6 bar</td>
<td>2.2 - 2.7 bar</td>
</tr>
<tr>
<td>Total pressure</td>
<td>1.4 - 2.5 bar</td>
<td>10.3 - 15.6 bar</td>
<td>10.1 - 14.6 bar</td>
</tr>
<tr>
<td>Surface temperature</td>
<td>260 - 275 K</td>
<td>350 - 363 K</td>
<td>400 - 410 K</td>
</tr>
</tbody>
</table>

Table 3.2: Constraints on early terrestrial atmospheric compositions from the clear-sky model of Durham and Chamberlain (1989).
<table>
<thead>
<tr>
<th></th>
<th>Earth</th>
<th>Mars</th>
<th>Venus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hart track</td>
<td>0.31</td>
<td>0.045</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Minima</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P87*</td>
<td>4.85</td>
<td>0.7</td>
<td>3.87</td>
</tr>
<tr>
<td>DC89*</td>
<td>9.0</td>
<td>1.3</td>
<td>7.15</td>
</tr>
<tr>
<td>This work</td>
<td>10.4</td>
<td>1.5</td>
<td>8.3</td>
</tr>
<tr>
<td>P87, DC89**</td>
<td>13.8</td>
<td>2.0</td>
<td>11.1</td>
</tr>
<tr>
<td><strong>Maxima</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W85, K88</td>
<td>10.0</td>
<td>1.44</td>
<td>7.96</td>
</tr>
<tr>
<td>DC89, this work</td>
<td>14.0</td>
<td>2.0</td>
<td>11.1</td>
</tr>
</tbody>
</table>

Table 3.3: Extrema for early terrestrial atmospheric CO₂ levels from various models.

The values in bold print are those actually determined by the respective models; corresponding values for the other planets are found by applying the scaling equation (3.1).

The various Hart track models (Owen *et al.*, 1979; Kuhn and Kasting, 1983; etc.) all assume that early Earth began with 0.31 bars of CO₂.

P87 is Pollack *et al.* (1987). Minimum CO₂ levels are found by requiring surface liquid water (or T = 273 K) on Mars at perihelion. The entry marked by an asterisk is for the optimal case of low surface albedo and an equatorial latitude.

DC89 is Durham and Chamberlain (1989). Minimum CO₂ levels are found by requiring surface liquid water on Mars at perihelion. The entry marked by an asterisk assumes that T = 260 K is sufficient for liquid water, that with a double asterisk requires T = 273 K. Maximum CO₂ levels are found by the requirement that Venus lose its initial complement of hydrogen in less than that planet's lifetime.

W85 and K88 are Walker (1985) and Kasting (1988), respectively. The maximum CO2 value for Earth is found by an analysis of geochemical mass balance.

All entries for the present model are made assuming a cloud cover of 50%.
Figure 3.1: Vertical temperature profiles for selected CO$_2$ partial pressures on early Earth.

Results of the present model are presented for 50% cloud cover. Note that a well-defined tropopause occurs between 20 and 30 km for all cases.
Figure 3.2: Response of early Earth surface temperature to surface CO$_2$ partial pressure.

Surface temperatures generated by the present model for 50% cloud cover (curve A) are shown with the clear-sky results of Durham and Chamberlain (1989) (curve B). The solar luminosity applied in both cases is $S = 0.78 S_0 = 1060$ W m$^{-2}$.
Figure 3.3: Water vapor profiles for selected CO₂ partial pressures on early Earth

The profiles shown are the average of the clear- and cloudy-sky water vapor profiles. Note that a cold trap is present for all cases.
Figure 3.4: Response of early Earth surface temperature to variations in cloud cover.

Response of the present model to variations in the cloud cover on early Earth is shown for two selected CO₂ partial pressures.
Figure 3.5: Temperature profiles for selected CO$_2$ partial pressures on early Mars at the mean orbital radius.

Note that a well-defined tropopause occurs between 25 and 40 km for all CO$_2$ partial pressures.
Figure 3.6: Temperature profiles for selected CO₂ partial pressures on early Mars at perihelion.

Note that a well-defined tropopause exists for all CO₂ partial pressures.
Figure 3.7: Water vapor profiles for selected CO$_2$ partial pressures on early Mars at the mean orbital radius.

The profiles presented are the average of the clear- and cloudy-sky water vapor profiles. Note that a cold trap is present for all cases.
Figure 3.8: Water vapor profiles for selected CO$_2$ partial pressures on early Mars at perihelion.

The profiles presented are the average of the clear- and cloudy-sky water vapor profiles. Again note the presence of a cold trap in all cases.
Figure 3.9: Comparison of early Mars temperature profiles for $P(\text{CO}_2) = 1.5$ bar
Figure 3.10: Response of early Mars surface temperature to surface CO₂ partial pressure.

The upper two curves (A,B) show surface temperatures realized for the case of perihelion (\(S_{\text{peri}} = .46\) \(S_0 = 625\) W m\(^{-2}\)); the lower two curves (C,D) occur for the mean orbital radius (\(S_{\text{mean}} = .36\) \(S_0 = 490\) W m\(^{-2}\)).

Curves B and D show the results of the present model, in comparison with the results of Durham and Chamberlain (1989) (curves A and C).
Figure 3.11: Response of Mars surface temperature to variations in cloud cover

For \( P(\text{CO}_2) = 1.5 \) bar, the present model never finds \( T > 260 \) K for the mean orbital radius. For \( P(\text{CO}_2) = 2.5 \) bar, the mean annual temperature would exceed 260 K for a cloud cover of 25%.

For \( P(\text{CO}_2) = 1.5 \) bar, the present model never finds \( T > 273 \) K for the case of perihelion, though it exceeds 260 K for a cloud cover of 50%. For \( P(\text{CO}_2) = 2.5 \) bar, cloud cover would have to exceed 85% for temperatures to always remain below 260 K at perihelion.
Figure 3.12: Temperature profiles for selected CO$_2$ partial pressures on early Venus

A well-defined tropopause fails to occur for any of the cases presented.

The large bulge in the curve for P(CO$_2$) = 15.0 bar results from release of latent heat by the cloud liquid water. The present model does not yet have the capability to redistribute this latent heat to other altitudes.
Figure 3.13: Water vapor profiles for selected CO$_2$ partial pressures on early Venus.

Note that a cold trap is not present in any of the cases shown.
IV. DISCUSSION

Potential Flaws in the Model

Despite the interactive nature of the cloud prediction routine employed in this study, there were still several parameters which had to be input. First, the surface relative humidity was fixed at 77%, approximately the average modern Earth value, for all model runs. This value is a significant factor in determining the clear and cloudy-sky water vapor profiles by equations (2.1) and (2.5), but a constant approximation is not necessarily valid for large perturbations from the modern climatic condition. The use of some sort of empirical formula to determine the surface humidity would, however, be inappropriate because this value is a function of numerous factors and more complex parameterizations for use in an RCM would not likely yield a surface humidity value any more valid than the constant approximation. Even making the justifiable assumption that the surfaces of the early terrestrial planets were entirely water-covered, one would still have to allow for such effects as air and water temperature, surface winds, etc., all of which would then have to be globally averaged. Only in more complex three-dimensional models could attempts to predict the surface relative humidity have real hope of success.

We have, in applying the model to early terrestrial atmospheres, also assumed a constant-valued eddy diffusion coefficient. As noted by Hummel and Kuhn (1981b), the cloud prediction scheme is fairly unresponsive to changes in this coefficient, but it remains that this may not be a valid approximation for these cases.

A parameterization of effective liquid water droplet radii applicable to modern Earth was extended to the early terrestrial planets. Due to enhanced aerosol levels in the atmospheres following their accretions, the early terrestrial planets would certainly have had
much higher amounts of cloud condensation nuclei (CCNs) available and the effective radii would have been smaller. Application of smaller radii to equation (2.9), which determines the liquid water profile within any predicted clouds, would result in a less important rainout term. The liquid water density within the clouds would then be much higher, and the upper reaches of the cloud might extend to higher altitudes. Within the confines of this model, this combination of events would not have had much effect on the surface temperatures we found for early Earth and Venus, but it could increase temperatures for early Mars enough that the minimum CO$_2$ partial pressure found in this study would be reduced to a value closer to that of Durham and Chamberlain (1989)

The model does not currently have the ability to consider the transport of the latent heat released by cloud condensation. Though the amount of energy involved was not significant to noticeably alter the temperature profiles calculated for early Earth and Mars, we have noted above that it had a serious effect when high CO$_2$ levels were considered for early Venus. The temperature profile for the convective portion of the atmosphere is currently found by application of a lapse rate formula, with later addition of the latent heat. This could be corrected by actual calculation of heat transport in the lower atmosphere, perhaps by the radiative-turbulent scheme outlined by Liou et al. (1985), but such an effort would be computationally more time-consuming than the present model.

Other than the input of the surface relative humidity, the most disturbing aspect of the current cloud formation scheme is the upper boundary condition applied to the water vapor transport equation (2.5). Hummel and Kuhn (1981b) matched the clear and cloudy-sky water vapor contents at the 0.45-bar level, finding that this altitude gave them a realistic surface temperature for modern Earth and generated a cloud whose vertical extent was of reasonable size. The present model, however, must consider cases where the surface pressure is radically higher than that of modern Earth, and so application of the 0.45-bar
boundary condition would not be feasible. We have instead specified that the clear and cloudy-sky water vapor contents should match at a point slightly below the tropopause. The result is a fairly extensive cloud, and other than a rainout term in the equation for the liquid water profile, there are no dissipative forces such as horizontal winds working to reduce it. Additionally, as mentioned above, the surface relative humidity prescribed for the early terrestrial planets and the empirical nature of the Manabe-Wetherald formula for the clear-sky water vapor profile are not necessarily good parameterizations, and the validity of the boundary conditions which are applied to equation (2.5) is somewhat tenuous.

Unfortunately, due to concern as to the true nature of the feedback between this value and the temperature of the atmosphere, the model does not include real cloud coverage interaction. We have somewhat arbitrarily applied a 50% cloud cover to most of our calculations since this value results in a reasonable surface temperature for modern Earth and is within the range of observed values for the global mean. However, due to the altitudes at which clouds have been found to occur in the early terrestrial atmospheres, we have found that variation in the cloud cover has differing effects on the surface temperatures of the different planets. In the warmer and denser atmospheres of Earth and Venus, clouds occurred at high altitudes and were able to trap much thermal radiation, causing cloud cover increases to result in surface temperature increases. In the cooler and thinner atmosphere of Mars, less thermal radiation was trapped by the lower-lying clouds, and due to the dominance of the cloud albedo effect, increases in cloud cover resulted in surface temperature decreases. Use of a prognostic equation for cloud cover would greatly enhance the results of this model, but until investigation has removed some of the doubt shrouding this issue, this remains a goal currently beyond reach.

One other possibly incorrect parameterization in the present model, not related to the water vapor equations, is that of surface albedo. We have assumed a constant value of
14%, about the observed modern value. For the early terrestrial planets, particularly Mars, this may be a flawed assumption. The early planets probably had less continental crust than present, and their albedos would have been lowered to a value closer to that of open water. Since this would only raise surface temperatures a few degrees over those calculated, it would not have been very important for early Earth and Venus but would be significant in determining a minimum CO₂ level for Mars. More importantly for early Mars, however, would have been the ice-albedo feedback. Temperatures found in this model were not much in excess of 273 K, and so formation of ice cover on that planet could have been extensive, particularly for the case of the mean annual condition. The albedo would consequently have been drastically increased and the surface temperature reduced to levels much lower than the values calculated in this study. Like the surface relative humidity, though, this is an issue which is better addressed by a higher-dimension climate model, rather than a one-dimensional RCM.

Significance of Our Results

While the various problems in the present model appear to be somewhat daunting, the results of this study merit consideration due to their exposure of the differing effect of cloud cover on the early terrestrial planets. On early Earth and Venus, the high temperatures resulted in an expanded troposphere in which water vapor could be convectively transported to high altitudes, and the cloud opacity effect outweighs that of the cloud albedo. With Mars, however, the tropopause is much more compact and water vapor transport is not transported to such high altitudes. Consequently, we find that the cloud albedo effect is here more important and increased cloud cover would cool Mars while it warmed Venus and Earth. Predicting just what the cloud cover, however, is a task beyond the scope of this study.
We have also presented minimum and maximum CO$_2$ levels for the early terrestrial planets, based on an assumption of 50% cloud cover on all three planets. Comparison to the work of Durham and Chamberlain (1989) shows that we require a slightly higher P(CO$_2$) for early Mars (1.5 bars, equivalent to 10.4 bars on early Earth) in order to sustain surface liquid water activity on that planet at perihelion. However, for decreased cloud cover, this minimum value decreases, and for entirely cloud free conditions, we unsurprisingly find agreement with the 0.7-bar minimum set by Pollack et al. (1987) for the case of a low surface albedo and perihelion. Such a very low minimum for Mars corresponds to about 5 bars of CO$_2$ on Earth, well below the 10-bar maximum set by Walker (1985) and above the 0.31-bar minimum of Kuhn and Kasting (1983), Kasting et al. (1984b), etc. Noting that the temperatures for Venus do not significantly differ between the results of this study and those of Durham and Chamberlain (1989), we have accepted their maximum CO$_2$ level of 11 bars for that planet, found by an analysis of the diffusion-limited escape of hydrogen and which corresponds to 14.0 bars on early Earth and 2.3 bars on Mars.
APPENDIX: THE RADIATIVE-CONVECTIVE MODEL

Radiative-convective models (RCMs) are one-dimensional atmospheric models which focus on obtaining a global and annual average of the vertical temperature profile. This averaging is made by selecting an effective solar zenith angle — in this model 60° — representing an average point on the Earth's surface at an average time of year. Although this approach seems limited by its deliberate ignorance of dynamical effects, particularly of horizontal motions, RCMs are valuable in that they allow the user to isolate the effect of a particular radiative forcing, such as a perturbation in gas concentration or solar flux, more clearly than in more complex atmospheric models.

An initial estimate of the probable temperature profile is input, and a series of iterations is performed until radiative equilibrium is reached in the non-convective region of the atmosphere. In each iteration, thermal and solar radiative fluxes are calculated and then a new temperature profile is found. Equilibrium is defined as a balance of the net thermal and solar fluxes at each level to within a fraction of a percent. Since radiation originating from the lower, convective portion of the atmosphere contributes most of the outgoing thermal flux, radiative equilibrium will not be obtained in the upper atmosphere without convective equilibrium also existing in the lower atmosphere.

To calculate radiative fluxes, we begin with the monochromatic equation for radiative transfer at frequency $\nu$ through a plane-parallel atmosphere,

$$\mu \frac{dI_{\nu}(\tau_{\nu},\mu,\phi)}{d\tau_{\nu}} = I_{\nu}(\tau_{\nu},\mu,\phi) - J_{\nu}(\tau_{\nu},\mu,\phi).$$

(A.1)
where \( I_\nu(\tau_\nu, \mu, \phi) \) is the specific intensity of radiation traveling in direction \((\mu, \phi)\) at normal optical depth \( \tau_\nu \), and \( J_\nu \) is a source function. Here \( \mu \) is the cosine of the zenith angle \( \theta \), and \( \phi \) is the azimuthal angle. Normal optical depth is defined by

\[
\tau_\nu(z) = \int_0^\infty k_\nu \rho \, dz', \quad \text{or} \quad d\tau_\nu = -k_\nu \rho \, dz,
\]  
(A.2)

where \( k_\nu \) is the mass extinction coefficient and \( \rho \) is the density of radiatively active gas. We henceforth suppress the subscript \( \nu \).

For thermal flux, the source term arises from the non-directional blackbody radiation of the atmosphere itself, or \( J_T(\tau, \mu, \phi) = B(\tau) \), the Planck function. The equation of transfer for thermal radiation may thus be written

\[
\mu \, dI_T(\tau, \mu, \phi) = I_T(\tau, \mu, \phi) \, d\tau - B(\tau) \, d\tau.
\]  
(A.3)

To obtain the upward intensity at angle \( \mu \) at depth \( \tau \), we consider upward radiation coming from all depths \( \tau' \) below such that \( \tau' \leq \tau \leq \tau^* \), where \( \tau^* \) is the normal optical depth at the planet’s surface. Integrating over this range of \( \tau' \) gives us

\[
I_T(\tau, \mu, \phi) = I_T(\tau^*, \mu, \phi) \exp\left(-\frac{\tau^*-\tau}{\mu}\right) + \frac{1}{\mu} \int_\tau^{\tau^*} B(\tau') \exp\left(-\frac{\tau'-\tau}{\mu}\right) \, d\tau'.
\]  
(A.4)

Similarly, for the downward intensity, we integrate over \( 0 \leq \tau' \leq \tau \). Assuming \( \mu \) is always positive valued so that downward radiation is traveling at angle \(-\mu\), we find that

\[
I_T(\tau, -\mu, \phi) = I_T(0, -\mu, \phi) e^{-\tau/\mu} + \frac{1}{\mu} \int_0^{\tau} B(\tau') \exp\left(-\frac{\tau-\tau'}{\mu}\right) \, d\tau'.
\]  
(A.5)
The boundary conditions for thermal flux are that the planetary surface is radiating as a blackbody, or \( I_T(\tau^*, \mu, \phi) = B(\tau^*) \), and that there is no downward diffuse radiation at the top of the atmosphere, or \( I_T(0, -\mu, \phi) = 0 \). Consequently, the intensities are independent of \( \phi \), and equations (A.4) and (A.5) become

\[
I_T(\tau, \mu) = B(\tau^*) \exp\left\{ -\frac{\tau^* - \tau}{\mu} \right\} + \frac{1}{\mu} \int_{\tau}^{\tau^*} B(\tau') \exp\left\{ -\frac{\tau' - \tau}{\mu} \right\} d\tau' \\
I_T(\tau, -\mu) = \frac{1}{\mu} \int_{0}^{\tau} B(\tau') \exp\left\{ -\frac{\tau - \tau'}{\mu} \right\} d\tau' .
\]

(A.6)

The net upward thermal flux density is obtained by integrating the intensity over all angles;

\[
F_T = \int_{0}^{2\pi} \int_{0}^{1} \{ I_T(\tau, \mu) - I_T(\tau, -\mu) \} \mu \, d\mu \, d\phi
\]

\[
= 2\pi \int_{0}^{1} \left[ B(\tau^*) \exp\left\{ -\frac{\tau^* - \tau}{\mu} \right\} + \frac{1}{\mu} \int_{\tau}^{\tau^*} B(\tau') \exp\left\{ -\frac{\tau' - \tau}{\mu} \right\} d\tau' \\
- \frac{1}{\mu} \int_{0}^{\tau} B(\tau') \exp\left\{ -\frac{\tau - \tau'}{\mu} \right\} d\tau' \right] \mu \, d\mu .
\]

(A.7)

We may simplify this by writing

\[
\int_{0}^{1} e^{-\tau/\mu} \mu \, d\mu = e^{-\beta \tau} ,
\]

(A.8)

where \( \beta \) is known as the "flux diffusivity factor." Possible values for \( \beta \) lie between 1.2 and 2, but the frequently-used value of 1.66 has been found to work to a reasonable degree of accuracy (Kondratyev, 1969, p. 19). With this approximation, equation (A.7) becomes
\[
F_T(\tau) = 2\pi \left[ B(\tau') \exp\{-\beta (\tau' - \tau)\} - \int_\tau^{\tau'} B(\tau') \frac{d}{d\tau'} \exp\{-\beta (\tau' - \tau)\} \ d\tau' \right. \\
\left. + \int_0^\tau B(\tau') \frac{d}{d\tau'} \exp\{-\beta (\tau - \tau')\} \ d\tau' \right].
\]
(A.9)

After partial integration, this may be written as

\[
F_T(\tau) = 2\pi \left[ B(0) e^{-\beta \tau} + \int_\tau^{\tau'} \exp\{-\beta (\tau' - \tau)\} \frac{dB}{d\tau'} \ d\tau' + \int_0^\tau \exp\{-\beta (\tau - \tau')\} \frac{dB}{d\tau'} \ d\tau' \right. \\
\left. = 2\pi \left[ B(0) e^{-\beta \tau} + \int_0^\tau \exp\{-\beta |\tau' - \tau|\} \frac{dB}{d\tau'} \ d\tau' \right] \right]
\]
(A.10)

Since the term \(\frac{dB}{d\tau'}\) can be difficult to calculate, we make the conversion

\[
F_T(\tau) = 2\pi \left[ B(0) e^{-\beta \tau} + \int_0^{\tau'} \exp\{-\beta |\tau' - \tau|\} \frac{dB}{dz'} \frac{dz'}{d\tau'} \ d\tau' \right. \\
\left. = 2\pi \left[ B(0) e^{-\beta \tau} + \int_0^\tau \exp\{-\beta |\tau' - \tau|\} \frac{dB}{dz'} \ d\tau' \right] \right]
\]
\[
= 2\pi \left[ B(0) e^{-\beta \tau} + \int_0^\tau \exp\{-\beta |\tau' - \tau|\} \frac{dB}{dT'} \ dT' \ d\tau' \right.
\left. = 2\pi \left[ B(0) e^{-\beta \tau} + \int_0^\tau \exp\{-\beta |\tau' - \tau|\} \frac{dB}{dT'} \Gamma(z') \ d\tau' \right] \right]
\]
(A.11)

where \(\frac{dB}{dT}\) is the derivative of the Planck function with respect to temperature and \(\Gamma(z')\) is the lapse rate at altitude \(z'\). Temporarily resuming use of the subscript \(v\), we obtain the total thermal flux from the flux density by integrating over all frequencies;
\[
F_T(z) = \int_0^\infty F_{T,v}(\tau_v(z)) \, dv \\
= 2\pi \int_0^\infty [ B_v(0) e^{-\beta \tau_v} + \int_0^\infty \exp\{-\beta |\tau'_v - \tau_v|\} \frac{dB_v}{dT} \Gamma(z') \, dz' ] \, dv \\
= 2\pi \sum_{k=1}^{N_T} [ B_{v_k}(0) e^{-\beta \tau_{v_k}} + \int_0^\infty \exp\{-\beta |\tau'_{v_k} - \tau_{v_k}|\} \frac{dB_{v_k}}{dT} \Gamma(z') \, dz' ] \, \Delta v_k .
\]

(A.12)

We have separated the thermal spectrum into \( N_T = 38 \) bands, each centered at a frequency \( v_k \) and having width \( \Delta v_k \). The spectral bands are selected so that the optical characteristics of the atmosphere are relatively invariant over the frequencies within each band.

The extinction coefficient used to calculate the normal optical depth in equation (A.2) is actually the sum of absorption and scattering coefficients; \( k = k_{\text{ext}} = k_{\text{abs}} + k_{\text{scat}} \). In thermal flux calculations, the scattering coefficient was negligible, but in solar flux calculations, which we now examine, its effect can be significant. We continue to treat the scattered radiation as being removed from the radiation stream but shall return it to the equation of transfer by including it in a source function. For the direction \((\mu, \phi)\) which we are considering, we allow for diffuse radiation which has been scattered from all other directions and for direct sunlight scattered from direction \((-\mu_0, \phi_0)\). The source function is consequently

\[
J_S(\tau, \mu, \phi) = \frac{k_{\text{scat}}}{k_{\text{ext}}} \frac{1}{4\pi} \int_0^2 \int_{-1}^1 I_S(\tau, \mu', \phi') P(\mu, \phi; \mu', \phi') \, d\mu' \, d\phi' \\
+ \frac{k_{\text{scat}}}{k_{\text{ext}}} \frac{1}{4\pi} \pi I_0 \, e^{-\tau/\mu_0} P(\mu, \phi; -\mu_0, \phi_0) ,
\]

(A.13)
where \( P(\mu, \phi; \mu', \phi') \) is a phase function which expresses the probability that radiation traveling toward direction \((\mu', \phi')\) will be scattered toward direction \((\mu, \phi)\), and \( \pi I_0 \) is the intensity (W m\(^{-2}\) m\(^{-1}\)) of direct sunlight at the top of the atmosphere. Since the ratio \( \frac{k_{\text{scat}}}{k_{\text{ext}}} \) is commonly defined as the single scattering albedo, \( \sigma \), we have for the equation of transfer of solar flux

\[
\mu \frac{dI_S(\tau, \mu, \phi)}{d\tau} = I_S(\tau, \mu, \phi) - \frac{\sigma}{4\pi} \int_0^{2\pi} \int_{-1}^{1} I_S(\tau, \mu', \phi') P(\mu, \phi; \mu', \phi') \, d\mu' \, d\phi' \\
- \frac{\sigma}{4\pi} \pi I_0 \, e^{-\tau/\mu_0} P(\mu, \phi; -\mu_0, \phi_0) .
\]  

(A.14)

After expanding the phase function in a series of 2N Legendre polynomials and the intensity function in a Fourier cosine series in \( \phi \), we obtain upon considering the azimuth-independent case,

\[
\mu \frac{dI_S(\tau, \mu)}{d\tau} = I_S(\tau, \mu) - \frac{\sigma}{2} \sum_{l=0}^{N} \omega_l P_l(\mu) \int_{-1}^{1} I_S(\tau, \mu') P_l(\mu') \, d\mu' \\
- \frac{\sigma}{4\pi} \pi I_0 \sum_{l=0}^{N} \omega_l P_l(\mu) P_l(-\mu_0) \, e^{-\tau/\mu_0} ,
\]  

(A.15)

where the \( P_l(\mu) \) are Legendre polynomials and the \( \omega_l \) are coefficients of the Legendre expansion (not to be confused with the single scattering albedo). Gaussian quadrature allows us to eliminate the integral over \( \mu \) by the substitution

\[
\int_{-1}^{1} I_S(\tau, \mu') P_l(\mu') \, d\mu' = \sum_{j=\pm 1}^{\pm n} a_j I_S(\tau, \mu_j) P_l(\mu_j)
\]  

(A.16)

where the \( \mu_j \) are a finite set of quadrature points used to represent the range \( -1 \leq \mu \leq 1 \), \( a_j \) is the weight of angle \( \mu_j \), and \( 2n \) is the number of quadrature points. In the present study,
we use \(2n = 2\) to obtain the so-called "two-stream" solutions. The number of terms in the Legendre expansion is restricted to \(N = 1\). Consequently, for \(i = \pm 1\),

\[
\mu_i \frac{dI_S^i(\tau, \mu_i)}{d\tau} = I_S(\tau, \mu_i) - \frac{\alpha}{2} \sum_{l=0}^{1} \omega_l P_l(\mu_i) \left[ a_{11} I_S(\tau, \mu_{-1}) P_l(\mu_{-1}) + a_1 I_S(\tau, \mu_{1}) P_l(\mu_{1}) \right] 
- \frac{\alpha}{4\pi} \pi j_0 \sum_{l=0}^{1} \omega_l P_l(\mu_i) P_l(-\mu_0) e^{-\tau/\mu_0}.
\]

(A.17)

Writing out the terms of the summations and using the notation

\[
I_S^\pm(\tau) = I_S(\tau, \mu_{\pm 1}) = I_S(\tau, \pm \mu_1),
\]

(A.18)
equation (A.17) may be written for the upward and downward-traveling streams:

\[
\mu_1 \frac{dI_S^+}{d\tau} = I_S^+ - \frac{\alpha}{2} \left[ I_S^+ (\omega_0 - \omega_1 \mu_1^2) + I_S^- (\omega_0 + \omega_1 \mu_1^2) \right] - \frac{\alpha}{4\pi} \pi j_0 (\omega_0 - \omega_1 \mu_1 \mu_0) e^{-\tau/\mu_0},
\]
\[
-\mu_1 \frac{dI_S^-}{d\tau} = I_S^- - \frac{\alpha}{2} \left[ I_S^+ (\omega_0 + \omega_1 \mu_1^2) + I_S^- (\omega_0 - \omega_1 \mu_1^2) \right] - \frac{\alpha}{4\pi} \pi j_0 (\omega_0 + \omega_1 \mu_1 \mu_0) e^{-\tau/\mu_0}.
\]

(A.19)

We note that our selection of solar zenith angle gives \(\mu_0 = \cos 60^\circ\), that two-stream quadrature renders \(\mu_1 = \frac{1}{\sqrt{3}}\), and that using the Henyey-Greenstein form of the phase function (Joseph et al., 1976) results in \(\omega_0 = 1\) and \(\omega_1 = 3\delta\), where \(\delta\) is the asymmetry factor.

As demonstrated in Liou (1980, p. 187), solutions to the equations (A.19) for a single homogeneous layer may be written in the form

\[
I_S^+(\tau) = Ku e^{k\tau} + Hu e^{-k\tau} + \varepsilon e^{-\tau/\mu_0},
\]
\[
I_S^-(\tau) = Ku e^{k\tau} + Hu e^{-k\tau} + \gamma e^{-\tau/\mu_0}.
\]

(A.20)

Values for \(u, v, \varepsilon,\) and \(\gamma\) are easily calculated from \(\alpha, \omega_0, \omega_1, \mu_0, \mu_1,\) and \(\pi j_0,\) but the coefficients \(K\) and \(H\) must be found by application of boundary conditions. Generalization
of this solution to the case of a vertically inhomogeneous atmosphere is made by considering a stack of several homogeneous layers and by requiring continuity of the intensities at the boundaries between the layers (Toon et al., 1989). Two additional boundary conditions, however, are required to close the system; the first is that there is no downward diffuse radiation at the top of the atmosphere, or \( I^+_S(\tau=0) = 0 \), and the other is that any upward diffuse radiation at the surface may only arise from the reflection of downward radiation, or \( F^+_S(\tau=\tau^*) = \Lambda F^-_S(\tau=\tau^*) \), where \( \Lambda \) is the surface albedo. The terms \( F^+_S(\tau) \) and \( F^-_S(\tau) \) are the upward and downward flux densities, respectively, and are defined

\[
F^+_S(\tau) = F^+_{S,diffuse}(\tau) = 2\pi \int_0^1 I(\tau, \mu) \mu \, d\mu = 2\pi I^+_S(\tau) \mu_1
\]

\[
F^-_S(\tau) = F^+_{S,diffuse}(\tau) + F^+_{S,direct}(\tau) = 2\pi \int_0^{-1} I(\tau, \mu) \mu \, d\mu + \mu_0 \pi l_0 \, e^{-\tau/\mu_0}
= 2\pi I^-_S(\tau) \mu_1 + \mu_0 \pi l_0 \, e^{-\tau/\mu_0}.
\]  \( \text{(A.21)} \)

Using this notation, we may write the lower boundary condition as

\[
2\pi \mu_1 I^+_S(\tau^*) = \Lambda \left( 2\pi \mu_1 I^+_S(\tau^*) + \mu_0 \pi l_0 \, e^{-\tau^*/\mu_0} \right).
\]  \( \text{(A.22)} \)

Thus, upward and downward components of the solar flux intensity are determined from application of the boundary conditions and then integrated over their respective hemispheres to find the components of the flux density. Reverting to use of the subscript \( V \) again, we obtain the total solar flux from the flux density by integrating over all frequencies,
\[ F_S(z) = \int_0^{\infty} F_{S,v}(\tau_v(z)) \, dv = \int_0^{\infty} \{ F_{S,v}(\tau_v(z)) - F_{S,v}^+(\tau_v(z)) \} \, dv \]

\[ = \int_0^{\infty} \{ 2\pi I_{S,v}(\tau_v) \mu_1 + \mu_0 \pi I_{0,v} e^{-\tau_v/\mu_0} - 2\pi I_{S,v}^+(\tau_v) \mu_1 \} \, dv \]

\[ = \sum_{k=1}^{N_S} \{ 2\pi I_{S,v_k}(\tau_{v_k}) \mu_1 + \mu_0 \pi I_{0,v_k} e^{-\tau_{v_k}/\mu_0} - 2\pi I_{S,v_k}^+(\tau_{v_k}) \mu_1 \} \Delta v_k \quad (A.23) \]

where the solar spectrum is separated into \( N_S = 26 \) bands, each centered at a frequency \( v_k \) and having width \( \Delta v_k \). Note that under the convention used here, solar flux has been defined as positive downward, but thermal flux was positive upward.

Once the thermal and solar fluxes are calculated, we use the Newton-Raphson method to solve for a new temperature profile. We set up a system of equations of form

\[ \mathbf{J} \Delta \Gamma = - (\mathbf{F}_T - \mathbf{F}_S) = - \mathbf{F} \quad , \quad (A.24) \]

where the vector element \( F_i \) is the net downward radiative flux at level \( i \), \( \Delta \Gamma_j \) is the change to the lapse rate of layer \( j \), and \( \mathbf{J} \) is a Jacobian matrix whose elements are \( \frac{dF_i}{d\Gamma_j} \), the change in net flux at level \( i \) due to a perturbation in the lapse rate of layer \( j \). Further information about construction of this matrix may be found in Kasting et al. (1984a), but we observe here that since the solar flux at each level is relatively independent of atmospheric temperature, we may apply \( \frac{dF_i}{d\Gamma_j} = \frac{dF_{\Gamma_i}}{d\Gamma_j} \). The \( \Delta \Gamma_j \) are calculated by matrix inversion and are added to the lapse rates \( \Gamma_j \). A new temperature profile is then recalculated using the new lapse rates.

Unfortunately, calculation of temperatures for which radiative equilibrium exists at all levels would result in tropospheric lapse rates which exceed the critical lapse rate for convection, and consequently the lower atmosphere would be convectively unstable. We
may correct this by a "convective adjustment." By this method, the radiative lapse rate of each layer is examined, proceeding downwards from the uppermost layer, and when it is found to exceed the critical convective value, we replace it with the critical value and recalculate temperatures for all lower layers using the convective lapse rate. At the boundary level between the upper atmosphere in radiative equilibrium and the lower atmosphere in convective equilibrium, the radiative and convective lapse rates will be equal, and the lapse rate will be continuous throughout the atmosphere.

The first application of convective adjustment (Manabe and Strickler, 1964) employed the well-known average tropospheric lapse rate of ~6.5 K km⁻¹ as the critical value, but subsequent studies have employed variable lapse rate formulae to more closely approximate the observed temperature profile of the troposphere (Hummel and Kuhn, 1981a). In this study, we have employed the moist adiabatic lapse rate (Kasting, 1988):

\[
\frac{d \ln P}{d \ln T} = \frac{L m_v}{R T} \left( 1 + \frac{\alpha_v m_d}{m_v} \right)^{-1} \frac{d \ln \alpha_v}{d \ln T},
\]

\[
\frac{d \ln \alpha_v}{d \ln T} = \frac{m_v L}{m_d T} - \frac{c_{pd} - \alpha_v c_c - \alpha_v ( c_c \frac{L}{T} + \frac{dT}{dT} )}{\frac{L \alpha_v}{T} + \frac{R}{m_d}},
\]

(A.25)

where \( L \) is the latent heat (J kg⁻¹) of sublimation or evaporation for condensed water (the phase appropriate to the temperature \( T \)), \( m_v \) is the molar mass (kg mole⁻¹) of water vapor, \( m_d \) is the molar mass of dry air, \( R \) is the universal gas constant, \( \alpha_v \) is mass of water vapor per unit mass of dry air (kg kg⁻¹), \( \alpha_c \) is the mass of condensed water per unit mass of dry air, \( c_{pd} \) is the specific heat (J kg⁻¹ K⁻¹) at constant pressure for dry air, and \( c_c \) is the specific heat for condensed water. We set \( \alpha_v \) equal to the amount of water vapor which would saturate the atmosphere and \( \alpha_c \) equal to zero. As demonstrated by Stone and Carlson
(1976), the saturation lapse rate more closely approximates the observed lapse rate of the troposphere than does the \(-6.5 \text{ K km}^{-1}\) approximation.
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