INFORMATION TO USERS

The most advanced technology has been used to photograph and reproduce this manuscript from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book. These are also available as one exposure on a standard 35mm slide or as a 17" x 23" black and white photographic print for an additional charge.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.
Statistical modelling of dynamic auroral fluxes

Shade, John William, M.S.

Rice University, 1989
RICE UNIVERSITY

STATISTICAL MODELLING OF DYNAMIC AURORAL FLUXES

by

JOHN SHADE

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

MASTER OF SCIENCE

APPROVED, THESIS COMMITTEE

Dr. John W. Freeman
Professor of Space Physics and Astronomy,
Director

Dr. Richard A. Wolf
Professor of Space Physics and Astronomy

Dr. Kenneth A. Smith
Associate Research Scientist
Center for Space Physics

Houston, Texas

May, 1989
Statistical Modelling of Dynamic Auroral Fluxes

by John Shade

Abstract

In order to obtain a better understanding of auroral processes, statistical models have been formulated that characterize the spatial dependences of the aurora. These efforts include the Hardy probability model, the Hardy average model, and the Evans average model. Each differs in its technical approach but all three attempt to characterize electron energy fluxes at any given location in the auroral zone.

In an attempt to describe the limitations of each model and perhaps make suggestions on how one can improve them, we reduced a six month sampling of DMSP-F2 electron precipitation data and averaged values over four second time intervals. Then each model generated a flux value for comparison against these satellite measurements so that Chi-square tests could be performed. Also the mean values for the whole six month period were calculated in order to determine the normalcy of the period tested. It is hoped that the knowledge gained through this endeavor will improve the accuracy of the Magnetospheric Specification Model’s forecasts of the fluxes that endanger spacecraft.
## Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>I: Auroral Morphology</td>
<td>5</td>
</tr>
<tr>
<td>II: Average Models</td>
<td>16</td>
</tr>
<tr>
<td>III: Comparison of Real Data with Average Model Values</td>
<td>41</td>
</tr>
<tr>
<td>IV: Summary and Recommendations for Future Work</td>
<td>77</td>
</tr>
<tr>
<td>Bibliography</td>
<td>78</td>
</tr>
</tbody>
</table>
List of Plots

Maps of Hardy Probability Model.................................33-40
Examples of Real Data plotted with Average Values...........53-58
Chi-Square Results....................................................59-76
Introduction

At high latitudes, magnetic field lines converge toward the Earth's poles and guide electrons downward into the dense atmosphere. These particles collisionally excite atmospheric atoms which release photons of characteristic frequencies. Observed from the ground, these emissions form the aurora. Such displays have evoked the talents of writers and musicians alike who wish to capture their ethereal acquiescent beauty. Other people have perceived this light in the sky as being some sort of spiritual apparition. And still others, the physicists, have discovered in the aurora a large number of complicated phenomena that test their old theories and inspire the creation of new ones.

The inaccessibility of space and the aurora's physical complexity initially limited the number of breakthroughs in its scientific study. In the early 1900's Birkeiand first hypothesized that auroral particles originated from the sun and spiraled down magnetic field lines at the poles. Later, Stormer developed detailed theories which described the trajectory of particles in a dipole field (Hess and Mead, 1965). At this point scientists could relate the aurora to some interplay between escaped particles from the sun and the Earth's magnetic field, the magnetosphere. But without in situ observations, scientists had little data to help them develop accurate theories.

When the space age arrived, rocket-borne and satellite-borne instruments acquired high quality data that revealed the detailed structure of the magnetosphere. The availability of this new information in conjunction with theoretical breakthroughs in the field of plasma physics gave birth to Magnetospheric Physics.

Balloons, rockets, and satellites all carry scientific payloads to high altitudes. Each has unique problems and advantages. Balloons travel so slowly that measurements over a single region can be taken over a relatively lengthy period of time, but they cannot reach altitudes where auroral processes occur. This detriment limits their usefulness to the X-ray imaging of the aurora and the measurement of ionospheric electric fields which
can be mapped upward along magnetic field lines to pertinent altitudes. Rockets reach altitudes that make them ideal for the study of the auroral forms which occur below 200 km. However, phenomena that range over hours cannot be measured with fast moving and short lived rocket flights. Satellites fly high indefinitely but move across distances so quickly that their instruments cannot acquire a complete temporal record of transient highly localized activity. For these reasons all three delivery systems complement one another and prove necessary for the thorough study of the aurora.

If one desired a complete picture of high latitude magnetospheric phenomena, he would need a network of spacecraft stationed at fixed points that would measure fields and particles at altitudes from 100 to 15,000 kilometers and at all combinations of longitude and high latitude. Because the aurora is not an isolated phenomenon, these spacecraft would need to probe all regions throughout the magnetosphere and in the solar wind incident to it. Such an array of grid points might give adequate coverage of the enormous spatial expanse of the aurora and provide a temporal record of its frequently severe fluctuations. Of course, such a situation is implausible at best. Instead scientists rely upon real delivery systems and finite budgets and so they only gain information in a piecemeal fashion.

Because of the limitations inherent to satellites, using their data to create a global auroral picture proves challenging. The oval shaped auroral regions surround the polar caps which surround and include the geomagnetic poles. Polar orbiting satellites travel through both sides of the auroral oval and straight across the polar cap giving experimentalists a latitudinal collection of energy spectra along a single meridian. Such data shows that both the polar cap and the auroral regions have radically different compositions and morphology. However, in order to create a circumpolar picture of an auroral configuration, one needs data from either a large number of satellites traveling along different meridians or, if conditions stay constant through numerous orbits, one
satellite precessing through different sectors. Cost limitations and the variable nature of
the aurora, however, preclude such conveniences forcing the attempt of other
approaches. All sky cameras provided the first source of data. Recently photographs of
the aurora taken from high above, using the Dynamics Explorer-1 satellite and the more
recent DMSP satellites have proven successful in determining the location of the more
intense fluxes, but the instruments can give only a limited description of the aurora's
spectra, particle composition, and intensity. Ground based radar stations have succeeded
in creating similar images with similar limitations but can only scan a limited sector of the
night sky at any one time. To circumvent these short comings physicists have developed
statistical models that combine satellite data from different episodes of similar auroral
activity to construct a composite circumpolar view. Such patterns describe the grosser
features of the aurora's morphology.

One method attempted to create such a picture involves grouping energy spectra and
flux values in bins of latitude, local time, and an index such as Kp which describes
magnetospheric activity. The first rigorous quantitative study evaluated the gross
averages of flux bins but had the shortcomings of a small data base (Spiro et al., 1982).
David Hardy of the Air Force Geophysics Laboratory (AFGL) used a much larger
multi-spacecraft data base to develop a similar model with more spatially resolvable
patterns (Hardy et al., 1985). Hardy's model had the additional benefit of increased
portability because he reduced his data set into a realistic analytic expression with only a
modest number of coefficients. Evans of the Space Environment Laboratory of NOAA
performed another average flux study by binning data from the TIROS/NOAA satellites
by P, the auroral power index, instead of Kp, a more ambiguous generalized index
(Evans et al., 1986).

Hardy also determined the probabilities certain energy and number fluxes have of
occurring inside latitude and local time bins analogous to the ones used for his average
model. The coarseness of his energy binning limits the scientific information in the model, but spacecraft operations personnel may find knowing the likelihood of dangerous fluxes useful when determining safe operating modes for satellites.

This report will discuss the auroral morphologies these models hope to describe and give a short history of statistical work in the field. It will then illustrate the shortcomings and advantages of this type of model before going on to describe them more fully. A comparison between real four-second averaged DMSP-F2 flux values and model values will reveal the limitations in using these models to predict fluxes. Finally, in an attempt to recover some of the transient features lost in the averaging process, the results of a chi-square test of the Hardy model versus real data are shown.

By completing this set of tasks, this report should clarify the role such models should play in the operation of the Magnetospheric Specification Model, a software program undergoing development at Rice University that will predict the characteristics of particle fluxes throughout those regions of the magnetosphere important to spacecraft operations. The responsible group hopes that such information will increase the survivability and usefulness of satellites.
I: Auroral Morphology

The highly dynamic nature of the aurora makes its modelling extremely difficult. A representation that accurately approximates fluxes in the aurora at one moment may utterly fail shortly afterwards. Numerous plasma instabilities disrupt the orderly precipitation of particles and cause the large and rapid fluctuations that create a theoretical nightmare and a statistical mess. For this reason simple averaging schemes cannot accurately describe some of the most interesting auroral phenomena. Instead they only give information on the larger more consistent features of the aurora.

If one could scatter a dedicated group of observers throughout one of the polar regions, numerous nights of sky viewing would lead to the discovery of a distinct auroral morphology. Because the aurora's occurrence has a hemispherically symmetric spatial dependence, locations of activity reflect about the equator, enabling one to apply the results of observations from one pole to both. The frequency and shape of the aurora's display depends on latitude and local time. Instruments sensitive to light outside the visible spectrum would detect parts of the aurora not seen in the visible. Traveling into space, observers would find that a multitude of particle compositions, originating from different altitudes and origins, create the variety of images seen. Given some knowledge of a magnetic index, such as AE, the observers could categorize their observations in terms of magnetospheric activity and then create a global map of the aurora's structure.

The above description too simply states the real complexities involved in developing a grand circumpolar scheme because the aurora is neither singular or homogeneous; quite different and largely unrelated processes form its parts. Therefore one must break the phenomenon into two broad classifications, the discrete and the diffuse aurora. The discrete, or structured aurora, the more brilliant of the two, provides observers with the familiar display of green and red light. The diffuse, or continuous, aurora, appears
faintly due to its lower energy density, but covers a much larger extent of the sky. Poleward of the primary auroral region, inside the polar caps, particle fluxes exhibit characteristics similar to both discrete and diffuse aurora. The following discussion will describe both types and illustrate their differences and similarities.

Poleward View of the Aurora

One can categorize discrete aurorae into a few basic groups. The one most often associated with quiet times, arcs, have a very narrow horizontal extent, relative to their great length, and possess a slight curvature. Bands exhibit much greater variability in their curvature and are sometimes highly contorted, but they still have dimensions similar to arcs (they are therefore frequently placed in the same group). A satellite pass measuring total energy fluxes would find a spiked or inverted-V structure at these locations. A two peaked particle spectrum would show that V structures result from particles being accelerated by a potential drop. Patched or surface aurora do not have the same horizontal extent of the bands but have a much broader latitudinal dimension. They
tend to have amorphous shapes that either isolate from or interconnect with similar forms. Veil aurora, the least seen group, uniformly illuminate a very large extent of the night sky.

Terms such as homogeneous, rayed, striated, and diffuse help to describe the internal details of an aurora's structure. Color reveals the level of activity of an aurora and its location in altitude. During normal activity, Oxygen atoms changing from their singlet to doublet state give the aurora its characteristic green color at 5577 Å. The lower altitude edge of this phenomenon occurs at roughly 100 km. Two types of color patterns indicate a higher level of activity. A type A aura is either completely red or has a green lower boundary. During periods of increased activity the photoemitting region creeps upward to between 300 and 400 km where the red oxygen (6300 Å) lines dominate over the green ones. Type B aurora have their colors reversed from type A. Intensified Oxygen and Nitrogen bands produce a lower red boundary usually associated with the most active periods of otherwise normal activity. The intensity or brightness of these colors correspond to the level of geomagnetic activity present throughout the magnetosphere (Hess and Mead, 1965).

To further illustrate an aurora's physical characteristics, one can make a statistical study of its local presence or absence. Most scientists use occurrence, the percentage of time the aurora appears at a location, as a useful value for comparison. A secondary but more informative measurement, incidence, counts the number of auroral forms seen in a given time interval over a given region. All sky cameras measuring occurrence helped to develop the first statistical model that approximate the aurora's spatial dependence.

Suitably named the auroral oval, this simply derived model provides a surprisingly good fit to a large number of real aurora (Feldstein et al., 1967). Even the most recent data analysis studies can add only subtle modifications. Different levels of magnetic activity have associated with them a circle centered about a corrected geomagnetic latitude
of 84.9° and a local time of midnight. The continuous aurora almost exclusively occupies
the dayside; the discrete appears most often on the nightside in addition to a diffuse
component.

Studying the geometries of these two regions, especially in relation to the Earth's
dipole magnetic field, give at least some qualitative insight into the nature of auroral
physics. On the dayside, the auroral oval's latitudinal extent is narrow and generally
lingers at 75° latitude where closed field lines meet open field lines. During active
periods, the dayside aurora moves equatorward. This trend occurs because an increased
number of dayside magnetic field lines merge with those from the sun and sweep
antisunward as open field lines. The similarity between the energy spectra of the solar
wind and the particles found in this region support the contention that the cusps offer a
place of entry into the Earth's magnetosphere.

Spectra of particle populations identify critical differences between the discrete and
continuous aurora. The Maxwellian spectra characterizing the continuous aurora can be
represented on a semi-logarithmic number density versus energy plot as a straight line;
the negative slope being inversely proportional to temperature. The discrete aurora has a
similar spectrum except that a potential drop relocates the location of the slope to higher
energies. Thermalized secondary electrons trapped by an electric field above and merging
magnetic field lines below fill the low energy portion of the spectrum. Numerous
theories exist to explain the source of the potential that accelerates plasmasheet electrons
downward but none is too widely accepted. Good survey papers regarding this topic
include those by Falthammar and by Mozer (e.g. Mozer et al., 1980); both listed in the
bibliography.

Physical processes occurring along day and night side magnetic field lines promote
different particle populations and certain spatial variations. The evening side auroral oval
not only moves equatorward during periods of increased Kp but also grows in latitudinal
width. Many of the particles in this region have been accelerated by poorly understood magnetospheric processes, hence the discrete or structured aurora. The extent and spatial variability of eveningside aurorae indicate that a much larger number of magnetic field lines connect to this region compared to those on the dayside where only field lines closely gathered around an L value of 6.4 make a path for particles to precipitate (L is the value given to a magnetic field line which crosses the equator at an L number of Earth radii). This narrow region of the Earth's dayside magnetic field, called the polar cusp, opens into the magnetosheath, a region just outside the Earth's magnetic field and inside the solar wind's magnetoacoustic shock. Field lines that attach to the eveningside aurora are contiguous to those that pass through the boundary and central plasma sheet, the two reservoirs from which the precipitating particles are drawn.

The equatorward edge of the auroral oval, due to its relative stability, has been the subject of numerous successful statistical studies. Their results include an analytic expression which relates the latitude of the equatorward edge to the magnetic index Q and the local time t (Starkov et al., 1970). The equation is of the form:

\[ \text{lat} = 72° - 0.9°Q - 5.1°\cos (360t/24 - 12°) \]

Because of the difficulty in directly obtaining values of Q, the above equation is frequently rearranged to make Q a function of local time and latitude. A single measured point on the edge of an auroral oval can then give a value for Q and thus describe the rest of the oval.

Precipitating electron data from the DMSP satellites have led to another analytical expression which uses a multitude of coefficients (Gussenhoven et al., 1983). The basic form is rather simple.

\[ \gamma = \gamma(0) + \alpha \times Kp \]

In this model, the latitude of the equatorward boundary increases by the index Kp starting from a constant equal to the latitude of the boundary at Kp = 0. The coefficients \( \gamma(0) \) and
contain the model's dependence on local time and have a fixed value for each hour. The fit this model provides is surprisingly good given the overall erratic behavior of the aurora. However, its approximation of the edge location may vary slightly from the preceding Q-index model. The low energy thresholds of the different models' data acquiring instruments create some of these discrepancies: greater low energy sensitivities move the apparent edge equatorward.

So far only structures dependent on latitude and local time have been considered but the aurora exhibits features that change with altitude as well. These variations are generally placed into three broad regimes, D, E, and F, that correspond to heights of 50 to 90 km, 90 to 160 km, and 160 to 500 km respectively (Whalen, 1984). Precipitating electrons of different kinetic energies ionize and excite atoms at the maximum rate only in certain altitude regimes. For electrons of the lowest energies, on the order of 0.1 keV, this optimum condition occurs in the F region; for energies between 1 and 10 keV, the E region, and for energies greater than 40 keV, the D region.

Because the D, E, and F particle populations originate at various directions and distances from the Earth, different magnetic field lines channel them toward the poles, hence the apparent spatial structure of the aurora, as seen from the ground, evolves from a composite picture of ionizations occurring at different altitudes. The source of the highest energy particles, the radiation belts, have the lowest L values associated with them because they are constrained by the dipolar field lines that neighbor closest to Earth at the equator. The 1 to 10 keV particles originate from the plasma sheet, a magnetospheric region whose closed field lines are stretched many Earth radii tailward on the evening side and, on the dayside, are equatorward of the polar cusp and just overlap the radiation belts. The lowest energy particles move along field lines into the magnetosphere from the magnetosheath, a region populated by solar wind particles heated by the passage through the magnetopause's bow shock. Some workers believe
that these particles are the source of the other two regions but no one fully understands the accelerating mechanisms between them. Particles from the two lower energy ranges may be upgraded to a more energetic level by accelerating through the aurora's electric field (Whalen, 1984).

For a given Kp, the continuous or unstructured aurora within the three different altitude regimes can be characterized as sets of concentric circles which have their centers offset from the geomagnetic poles by solar wind interactions. Because the potential responsible for the discrete aurora acts as a barrier to protons one can identify the discrete aurora by an absence of protons. Ionization in the F region occurs along a region identical to the previously described auroral oval, where the solar wind infiltrates the dayside by way of the polar cusp and where evening side particles filter in from the outer regions of the plasmasheet. E region particles that form a continuous aurora arise from a more deeply buried section of the plasmasheet (a smaller L value) and therefore show up at somewhat lower latitudes. Taking this progression to an extreme, D region particles form the latitudinally lowermost unstructured aurora which appears at 55° latitude during moderate conditions (Whalen, 1984). At this energy level, the auroral oval is no longer offset from the poles because field lines deeply embedded in the magnetosphere are shielded from the solar wind and maintain their symmetric dipolar geometry.

Studies have shown that the behavior and geometry of the continuous aurora are exceedingly predictable (Whalen, 1983). If an ionospheric sounder can measure the location and energy of the maximum flux for one small segment, extrapolation can simply and accurately describe the complete circle. The peak energy flux, magnetically linked to the center of the plasma sheet, uniformly maintains a constant value at all local time meridians, and large temporal variations, simultaneously affecting the whole circle, occur between the values of 0.25 and 7 ergs/cm²-sec. A Gaussian curve with a 3.2° full width at half maximum closely matches the energy flux profile surrounding the peak. The high
latitude region of the continuous aurora, contiguous with the boundary plasma sheet, possesses an energy flux profile that exponentially decreases with increasing latitude.

The discrete aurora at all altitude regimes occupy the previously described auroral oval, suggesting that acceleration occurs exclusively along field lines contiguous with or just equatorward of the boundary plasmashell. Discrete auroral particles from the D region rarely become part the oval. Only near midnight and during anamolies such as substorms can this phenomenon occur. Discrete particles from other energy regimes appear at all local times, although their occurrence, particularly within the day sector, may be close to zero (Whalen, 1984).

Although the occurrence pattern of the discrete aurora may cut a uniform swath around the nightside of the pole, instantaneous fluxes frequently form localized structures which evolve through numerous local time dependent stages. When viewed during dusk or early evening the aurora generally consists of homogeneous and diffuse arcs. During quiet times these forms extend throughout all local times. However, during periods of high activity such as substorms these arcs give way to rayed bands which go through progressively larger contortions until at midnight the so called breakup occurs where the intensity, shape, and location of the aurora varies rapidly. Immediately after breakup, a system of irregular patches remain which sometimes pulsate in intensity. Moving onto morning the aurora once again establishes a banded form which later gives way to well behaved arcs. During daylight hours the aurora forms a diffuse network of interconnected patches (Hess and Mead, 1968).

Substorms, periods of high geomagnetic activity, tend to enhance the luminosity of the aurora and change some of its basic attributes. In order to understand the time evolution of the aurora under substorm conditions one must have some knowledge of the prevailing auroral currents. Interaction with the solar wind bifurcates the polar cap contiguous with open field lines into two convection cells. These Eastward and
Westward currents, called auroral electrojets, both transport particle fluxes from midnight to roughly 10:00 within their respective dawn and dusk perimeters and then transverse the polar cap. At lower L values particles contained by closed field lines gradient and curvature drift.

The discrete aurora occurs between the regions of closed and open field lines where the upward Birkeland currents transport electrons downward into the ionosphere. At the onset of a substorm the equatorward edge of the aurora suddenly brightens and moves equatorward. Soon thereafter a surge of activity moves poleward so that not just one but both edges of the aurora experience an intensity enhancement. From midnight the region of breakup moves both Westward and Eastward converging the numerous bands into patches and degenerating arcs into contorted bands. An increase in a storm's intensity quickens the circulation of energetic particles and enlarges the effective substorm region.

During these active periods the polar caps undergo a slight lessening in both diameter and flux intensities. Interestingly quiet times for the magnetosphere are frequently associated with the most intense fluxes over the polar caps. These periods of low Kp and Northward IMF are frequently characterized by such flux enhancements as polar squalls and polar rain. Also polar arcs, intense peaks in flux, sometime occur. Occasionally a line of enhanced fluxes joins the eveningside and dayside of the auroral oval forming the so called theta aurora.

Through the help of in situ observations, scientists have determined the local time dependence of the aurora's average spectra thereby illuminating some of the physical processes that occur within the oval's different local time sectors (Hardy et al., 1985). At moderate conditions, Kp=3, auroral regions below 65° latitude feature Maxwellian spectra that gradually increase in intensity with progressively higher latitudes. The temperatures of these central plasmasheet particles typically have values on the order of several keV (Whalen, 1983). Above this region the higher energy particles become
sparse creating an intense cool spectra characteristic of the boundary plasmasheet, the source region of most inverted-V events. Spikes of energy flux, narrow in latitude but wide in longitude, are the signature of such inverted-V events. Higher still and one enters the polar cap where the intensity of particle fluxes gradually and uniformly diminish with increasing latitude. Moving from midnight to dawn one finds the maximum intensity dropping by a factor of two and moving poleward from 65° to 72°. This loss in flux can be attributed to particles leaving flux tubes as they convect toward the dayside; the more energetic particles leaving first due to the larger number of bounces they take between poles. At latitudes above peak intensity, harder particle densities diminish but cooler particle densities, rather than remaining constant, actually increase. Some believe that the polar cusp, cleft, or low latitude boundary layer provide the source of these particles (Hardy et al., 1985). These trends continue throughout the dayside as the effects of energetic particle depletion and cool particle injection wear on. At dusk the high temperature low latitude component of the aurora is all but gone finalizing the effects of depletion. At higher latitudes, particles that move directly from the midnight sector through the early evening hours to dusk create a harder spectrum. However, these fluxes are still not as energetic as those found on the morning side because the hottest energy fluxes, injected at midnight, tend to follow the stronger Eastward drift (Hardy et al., 1985).

Because the geometry of the Earth's magnetic field changes as a function of the solar wind's angle of incidence, the auroral oval undergoes diurnal and seasonal variations. The geomagnetic pole, offset from the geographic pole by roughly 11°, precesses with a 24 hour period. This diurnal variation disrupts some of the conjugacy between polar caps by moving one pole closer to the sun every twelve hours continually changing the configuration of the magnetic field (Ching et al., 1979). The angle between the solar ecliptic plane and the rotation axis of the Earth also changes but on a yearly basis instead
of a daily one. This angle change destroys some of the symmetry between the magnetic
field configurations of the poles but it has a more predictable behavior due to its low
frequency. The effects of both of these phenomena together create a second order
variation in the auroral zone's geometry and so are usually neglected in the development
of average models.

The aurora constitutes a very dynamic, complicated system with many interlocking
parts and many poorly understood mechanisms. Modeling the aurora with something as
simple as an averaging scheme may then seem somewhat absurd, but the phenomena just
described reveal themselves in such global patterns. Other characteristics, discussed
later, also become evident when the noise related to the dynamic nature of the aurora is
removed.
II: Average Models

Before exploring the uniqueness of the statistical models evaluated in this report, some common features warrant attention. The basic techniques used to create average models appear identical on the surface. One needs only to amass a very large set of energy or number values, bin them by location and an activity index, and then determine a corresponding set of averages. The size of these bins and the type of activity index chosen allow for some variability between models. Replacing the array values with an analytic function improves the portability of the model and smooths out the details caused by noise. However, such processes may remove some of the structural details hidden in the data base.

During geomagnetic substorms, average models cannot predict the highly dynamic nature of the auroral fluxes. Because such events are represented by such a small portion of the model's data base and involve energy and number density fluctuations that range over orders of magnitude, statistical models based on a simple geomagnetic index cannot hope to accurately describe them. The possibility of mapping average substorm fluxes requires the feasibility of binning data by different stages of substorm development. The scarcity of data for each stage of a substorm's evolution makes this task a difficult undertaking. Perhaps the development of a quasi-theoretical model would have a better chance to describe these erratic flux changes.

Each model employs a smoothing algorithm to remove spurious bumps. Without adequate precautions such a procedure could strip some of the details from the data base which the initial coarseness of binning did not eliminate. A large data base allows for a finer mesh of bins that should minimize the problem.

Average models smear out the latitudinally narrow peaks associated with real auroral fluxes and create patterns that could never actually occur. This trend becomes more evident at high Kp's where discrete auroral arcs have access to a very large range of
latitudes. For this reason statistical models cannot accurately describe the instantaneous flux values experienced by a satellite during any single pass. These models help to determine fluxes into the whole atmosphere or along a satellite pass. They also illustrate large morphological features and their variation with increasing geomagnetic activity. However, when accurate satellite measurements become unavailable, these models may provide the only source of information regarding the distribution of auroral fluxes.

Fine binning provides the spatial resolution necessary to make visible small scale auroral structures. The time resolution of the magnetic activity index plays an important role in providing accurate flux values for a given satellite pass. The geomagnetic indices used to bin data are renewed after a particular time interval. For standard Kp this interval lasts three hours but the auroral power index, P, has a time resolution as small as thirty minutes, or once every polar pass made by a NOAA satellite. AE values can change as often as once every hour. The Air Force derives an independent value for Kp instead of using the time honored one calculated each month in Germany. This capability allows them to have near real time values every one hour. In the future, scientists plan to phase out the use of traditional Kp, replacing it with Air Force Kp, but the rest of this report will use both interchangeably. Models which use an index with a finer time resolution should give patterns more relevant to short lived but critical phenomena such as substorms, which can begin and end in less than thirty minutes.

In addition to a geomagnetic index's time resolution, one must evaluate its immediate relevance to auroral processes in order to make a thorough comparison between the models which use them. The Spiro model (Spiro et al., 1982) bins by both Kp and AE although it prefers the latter because it relates more directly to currents in the auroral region. AE, the auroral electrojet index, quantifies the current induced perturbations of the magnetic field in the polar regions. Kp on the other hand utilizes numerous other sources of information which do not directly influence the auroral region. The integration
of fluxes incident on a satellite at high latitudes allows one to determine the P index, perhaps the most useful parameter. Unfortunately even it cannot provide the wealth of information necessary to encapsulate the multitude of factors necessary to create real instantaneous flux maps. Some have suggested the possibility of binning by two geomagnetic indices such as AE and Dst, a measure of ring current, so as to account for transient substorm effects (Spiro et al., 1982). This extra parameter could increase the size of the necessary data set greatly.

Although reducing average value arrays into simple analytic expressions increases the portability of statistical models, the process may remove some important features. Increasing the number of degrees of freedom may create a better fitting curve, but the increased level of complexity may destroy the whole purpose of replacing the data set with an analytic expression. Hence the chosen functions must strike a balance between simplicity and accuracy.

The average models investigated in this study use coordinate systems that depend on the seasonally variable geometry of the Earth's magnetic field. Before binning measurements, modeled magnetic field lines map the location of satellites downward from a height of 800 km to 110 km, where auroral phenomena occur. In such a manner, Hardy uses the standard IGRF magnetic field model to derive magnetic local time and latitude. The Spiro model bins by the invariant latitude determined from:

\[ \text{inv. lat} = \arccos(L^{-1}) \]

Virtually all of the models discussed in this thesis use magnetic local time as a binning parameter because, like latitude, it does not depend on universal time. When making comparisons between model and real time satellite values one must make certain to have consistent coordinate systems or use a transformation algorithm.

In order to compute the magnetic local time of a specific region one imagines a plane containing both geomagnetic poles and the sun. Another plane containing the poles and
the questioned region intersects the reference plane at an angle. Measured from midnight, this angle, divided by 15, gives the magnetic local time in decimal hours.

This chapter has so far shown the similarities between average models but from here on in will concentrate on their differences. In order to accomplish this task, it presents the characteristics of the data bases used and lists those features apparent in the final mappings. The text will treat each in historical order but will concentrate on the Hardy model because, at least for the purposes of this thesis, it plays the most critical role, one of creating average values to test real data against.

McDiarmid, the first person to derive average values from satellite data, had a small data base to work with (McDiarmid et al. 1975). This problem limited the model's spatial resolution and held the activity binning to moderate levels (Kp<4). Not until the development of the Spiro et al. (1982) model did the scientific community have access to a fully comprehensive statistical model.

I: The Spiro Model

Unfortunately the data set used to generate the Spiro model fails to possess the large size preferred for such statistical work. A set of low energy electron detectors on board the Atmospheric Explorer C and D satellites measured one second energy spectra in the interval between 200 eV and 27 keV. In order to create a more compact data set these spectra were summed into 15 second groupings which wasted some of the instruments' resolution. A simple data reduction algorithm then integrated all sixteen logarithmically spaced channels into a single total energy flux value. A total of 30,407 intervals acquired over a period of 29 months fell into the bins that comprise the global flux maps (Spiro et al., 1982).

The two Atmospheric Explorer satellites (AE-C and AE-D) covered a large portion of the auroral zone due to their different orbital paths and the diurnal precession of the geomagnetic poles. The 11° tilt between the geographic and geomagnetic poles could
change the peak latitude reached by a satellite by as much as 20°. The AE-C and AE-D satellites held orbital inclinations of 90° and 68° respectively which allowed situations where one spacecraft passed through the center of a polar cap while orbiting on a limited range of meridians while the other sampled a large number of local times.

Because of a rather short sampling time, a small number of useable passes resulted, making the coarse binning of data necessary. A total number of 24 local time bins limited the resolution to one full hour. Thirty invariant latitude bins were chosen to distinguish fine scale latitudinal structures. Between 50° and 60° and between 80° and 90° only 5 bins, 2° wide, proved necessary. Twenty 1° bins in the most active region, the auroral oval, completed the coverage.

TheSpiro model binned the data by two activity indices, Kp and AE. The five Kp bins chosen included three intervals for values between 1 and 4, and two ranges for values less than 1 and greater than 4. The AE bins separated AE values less than 100 and greater than 600 gamma and values ranging from 100 to 300 gamma and 300 to 600 gamma. The standard technique used to derive mean values was applied to each bin, that is the sum of all fluxes recorded within each bin were divided by N, the number of flux measurements. A simple smoothing algorithm which used nearest and next nearest neighbors removed spurious irregularities. Because most auroral forms have a narrow latitudinal width but have a large longitudinal extent the smoothing algorithm weighed nearest local time bins more heavily (Spiro et al., 1982).

An assortment of Gaussian curves, fit to the latitude dependent auroral flux profiles, eliminated the binning process's inherent graininess (Simons et al., 1985). Such a format increases the portability of this model but only at the cost of some important details. Average auroral flux peaks do not have the symmetry of Gaussian curves. They rise sharply from the equatorward boundary, attain a maximum, and then drop off more slowly. This behavior arises from the highly variable nature of poleward auroral fluxes
and the ambiguous location of the poleward edge. At midnight, during substorms, the poleward and equatorward edges of the auroral zone have sharp spikes of energy flux. In the original binned model these features average out as small bumps. The best fit Gaussian approximation, however, simply removes them. Another problem involves the Gaussian curves' tendency to exponentially approach a constant value. The binned average flux values approach a small constant value in a far more abrupt fashion, particularly toward the equatorward edge. Also, average energy fluxes have different values poleward and equatorward of the primary flux peak unlike the symmetric Gaussian curves fit to them. Polar rain, the primary source of electron precipitation above the auroral zone, possesses energies below the threshold of the AE-C and AE-D data, making the latitudinal asymmetry invisible. This problem does not exist for the more recent models which use data from instruments with better low energy sensitivity.

As one would expect, the average patterns created by the binning process look very similar to the auroral ovals which describe the spatial variation of occurrence. Although occurrence and average values have different definitions, they both give a quantitative description of the location of the most frequent fluxes. However, the average value plots give better information on the aurora's intensity. For example, a mapping of the dayside occurrence pattern may indicate its continual presence but would not give information on the low intensities found in average plots.

As the activity level increases, the Spiro average patterns show the tendency of the evening side's low latitude edge to move equatorward and the poleward edge to remain relatively stationary. In the highest activity bin, the energy flux of precipitating electrons at midnight attains a value eight times greater than the midnight precipitation occurring within the least active bin. The flux peaks observed at premidnight local times occur at latitudes higher than those found after midnight. The latitude of upward Birkeland currents correspond well to this phenomenon as they too possess an asymmetrical
behavior between the Western and Eastern hemispheres. During particularly high levels of activity these two regions overlap at midnight creating an unusual double crested feature. This pattern is consistent with the equatorward and poleward surges of the aurora observed during substorms.

II: The Hardy Model

As expected, the Hardy or AFGL model (Hardy et al., 1985) possesses features very similar to those of the Spiro model. After all, they both attempt to derive the same quantities. However, the small data base used by Spiro limited the number of flux values available for binning. In the late seventies two programs began which kept a succession of satellites in polar orbits. The American military's need to correctly position its spy satellites over cloudless regions created the need for the Defense Meteorological Satellite Program (DMSP). Since September of 1977, the Air Force has placed DMSP satellites in polar orbits to check cloud cover over the Soviet Union and other places of interest. In addition to their meteorological mission, electron detectors on board these spacecraft made possible the measurement of auroral fluxes. Because these satellites provide continuous coverage of the auroral zone, they can easily create the huge data sets ideal for the types of statistical modeling so far discussed. Hardy's model takes advantage of this situation and improves his statistics over Spiro's thereby resolving finer details. Also Hardy has put a great deal of effort into creating accurate analytic representations of his average flux arrays. In addition, the decreased low energy threshold of the DMSP satellites' detectors allowed for more detailed observations of the polar rain.

The data binned for this study come from the SSJ/3 detectors on board the DMSP-F2, DMSP-F4, and P78-1 satellites. The SSJ/3 detector consists of two electrostatic curved plate analyzers each with eight logarithmically spaced channels. One detector covers the energy range between 50 eV and 1 keV and the other has a complementing range between 1 and 20 keV. In one second, both detectors complete the
taking of a single spectrum, giving the experimentalist excellent time resolution and adequate coverage of the most common auroral particle energies.

The orbits of all three spacecraft provide a composite view of the auroral zone that only leaves low latitude bins in the local time ranges 3:00 and 15:00 empty. The two DMSP satellites use three-axis stabilization which allows the SSJ-3 detectors to measure downward electron precipitation continuously. The spin stabilized P78-1 satellite rotated at roughly 11 rpm. It has a pair of orthogonal SSJ-3 detectors mounted in the spin plane so that electron precipitation measurements can only occur twice per revolution. The DMSP-F2 and F4 satellites initially flew at an altitude of 840 km in sun-synchronous orbits with an inclination of 97.4° along the 6:00 to 18:00 and 10:00 to 22:00 meridians respectively. The F2 satellite later precessed to the 8:00 to 20:00 meridian. The P78-1 satellite attained a height of 600 km and orbited along the noon-midnight meridian.

Orbital coverage used in the Hardy model
A carefully selected 15 months of observations, chosen from four years of available data, provided 13.6 million spectra. Another 11 months of data from the P78-1 satellite produced the .52 million spectra that completed the data base. From there the data divided into 7 Kp groupings, one for each Kp value from 0 to 5 and another covering all Kp values equal to or greater than 6. The most quiet and active groups possessed the lowest number of individual spectra whereas moderate levels, such as those of Kp=3, which alone comprised one fourth of the data base, had the best statistics. A subsequent chapter will discuss the algorithm used to calculate total energy fluxes from each spectrum. After the determination of energy fluxes, a large number of bins sorted the data by latitude and local time. A total of 48 one-half hour magnetic local time bins, five 2° latitude bins between 50° and 60° and between 80° and 90°, and twenty 1° latitude bins between 60° and 80° were chosen to create the global maps. This binning closely assimilates Spiro's except the local time has twice the resolution.

The final data arrays underwent a few reducing steps. Before determining average values, an algorithm removed spurious counts caused by low latitude Van Allen radiation particles. After averaging the flux values in their respective bins, a smoothing algorithm removed the noisiness inherent to auroral electron data. A simple algorithm summed all four average values adjacent to a given bin with three times the bin's own value. Dividing the total sum by 7 produced the bin's average value.

The AFGL group (Hardy et al., 1984) chose an Epstein-Fourier function to describe the pattern of fluxes in the auroral oval. The Epstein portion of the function represents the latitudinal variation whereas the Fourier series accounts for the local time dependence. Constant values represent fluxes outside of the primary peak. The Epstein function has the form,
\[ e(h) = r + s_1(h - h_0) + (s_2 - s_1) \times \ln \left[ \frac{1 - \frac{s_1}{s_2} \left( e^{-(h - h_0)} \right)}{1 - \frac{s_1}{s_2}} \right] \]

where \( e(h) \) is the energy or number flux dependent on the latitude \( h \). \( r \), the peak flux, \( h_0 \), the latitude of the peak flux, and \( s_1 \) and \( s_2 \), the slopes on either side of the peak, all come from a Fourier series that uses numerous coefficients, different for each activity level. The Fourier series has the form,

\[ \alpha(T) = \sum_{n=0}^{6} C_n^\alpha \cos \left( \frac{n\pi T}{12} \right) + S_n^\alpha \sin \left( \frac{n\pi T}{12} \right) \]

where \( \alpha \) is the calculated parameter \( r, h_0, s_1, \) or \( s_2 \). For each Kp interval, this analytic form requires 52 coefficients, represented in the above equation by \( C \) and \( S \). For example, if an energy flux value for a given latitude, \( h \), and local time, \( T \), was desired one would first use the Fourier series to determine \( r, h_0, s_1, \) and \( s_2 \). These four parameters would then go into the Epstein function along with \( h \). In order to calculate number flux, the same process is duplicated except that another set of coefficients would be used.

For some applications 52 numbers may seem somewhat cumbersome but the resolution such an expression provides improves greatly on the previous more straightforward attempts. Epstein functions can represent the auroral flux profile's asymmetry and more accurately assimilates its changes in slope, replacing the Gaussian curve's equatorward exponential tails with an abrupt discontinuity in slope between the auroral oval and its surrounding regions. The energy minimum poleward of the aurora's flux peak has a value of 7 keV/cm²-ster-sec; equatorward of the peak, the flux value falls to 6 keV/cm²-ster-sec (Hardy et al., 1984). The DMSP satellites' detector has the low energy sensitivity necessary to make the energy of the polar rain visible, hence the asymmetry. The use of a Fourier series to represent local time variations instead of
constant values for each half hour of local time dramatically smooths and improves the accuracy of the statistical model. Unfortunately some of the fine scale latitudinal features of the original average flux arrays disappear when transformed into an analytic expression.

The advanced analytical form of Hardy's model along with the greater number of local time and activity bins make for more finely detailed and informative global maps. This improvement comes largely from the availability of a data set two orders of magnitude larger than that used by the Rice group. Also, the larger number of activity bins helps to distinguish patterns associated with more active periods.

![Poleward view of the Hardy model for a low Kp value](image)

By use of this finer resolution, Hardy's model clearly defines the shape of the more obvious auroral structures and allows for the discovery of surprising small scale features. Hardy discusses his results by first dividing his global maps into two energy regimes; one which includes particles with average energies less than 600 eV and another for all
higher energy particles. Because of the better low energy threshold of the DMSP electron detector, his model reveals more structure in the dayside aurora. Particles of lower energy concentrate themselves in a crescent centered roughly upon noon. At the center of this region lies an average energy minimum which Hardy associates with particles injected directly from the solar wind into the polar cusp. Equatorward of this region the gradient of the average energy rises sharply but poleward and to the East and West of the minimum the average energy increases gradually. The whole structure has a latitudinal width of approximately a few degrees and has a longitudinal span of several hours East and West of noon. Particles from the polar cusp and low latitude boundary layer populate these broad intermediary regions. The boundary layer particles' low average energy, comparable to that of the solar wind, increases as magnetic field merging between the IMF and magnetosphere moderately heats them. The energy intensity of particles remains constant throughout increasing levels of activity but covers progressively larger spatial expanses as the apparent cusp moves equatorward from 81° to 78°. Interestingly, a prenoon number flux maximum exists which has no explanation at this time. Low energy particle fluxes on the eveningside of the aurora tend to have lower number densities and higher energies. These particles, most likely associated with the boundary plasma sheet, have the same low temperatures found in the dayside magnetosheath but their average energies differ due to the occasional inverted-V structures.

Previous mappings of the auroral oval, including Spiro et al. (1982) and McDiarmid, looked primarily at the hot or energetic component of the aurora. On the nightside, the Hardy model's high energy structure reveals characteristics very similar to the other models but with important differences. The McDiarmid and Hardy models show a symmetry in the energy flux about a meridian connecting postmidnight and postnoon. Along this meridian some of the most intense fluxes occur. McDiarmid has
suggested that the convection electric field energizes these particles but no one as of yet fully understands the phenomenon. The average energies of these maxima increase with Kp until a value of 3 is reached after which they remain constant. Although hot eveningside fluxes tend to increase in intensity with geomagnetic activity, at values greater than Kp = 2, the energy fluxes at noon actually decrease (Hardy, 1985).

Comparing Hardy's average model with Spiro's, one finds that AFGL energy fluxes have uniformly 20% higher values. This correlation seems good especially in the light that both studies used data from different segments of the sun's twelve year activity cycle. Also the increased low energy sensitivity of the DMSP detector may explain the slight discrepancy between models. An integration over the total hemisphere of AFGL number flux values delivers a value very close to the theoretical one calculated by Hill equal to $10^{26}\text{sec}^{-1}$. One can then conclude that some consistency exists between this model and other efforts (Hardy, 1985).

Recently Hardy has reworked the data base used in this averaging study so as to obtain more accurate coefficients for his analytic expression. The analysis described in the next chapter did not have access to the latest version and cannot attest for its accuracy, although one can presume that the new coefficients provide improved spatial resolution.

III: The Evans Model

The Evans average model (Evans et al., 1986) utilizes satellite data from differently phased but otherwise similar orbits. Like the DMSP satellites, the TIROS/NOAA satellites follow sun-synchronous orbits at an altitude of 830 to 850 km. But where DMSP satellites move southward on the eveningside of their dawn to dusk orbits, NOAA satellites move northward. The NOAA satellites use 3-axis stabilization which keeps their electron detectors continuously looking upward. One pass over the auroral zone requires an average of 30 minutes; the actual time fluctuating with the auroral oval's size and its U.T. dependent precession. A single orbit takes 100 minutes to complete.
The electron detectors on board the NOAA satellites have an energy range of 0.3 to 20 keV. This characteristic eliminates any ability of the instrument to observe low energy phenomena such as the polar rain. A single sweep of the entire electron spectrum takes only one second, but between each electron sweep, the ion detector takes one second to make its own measurements.

The spacecraft used by the NOAA program since 1979 have accumulated a total of 110,000 auroral passes. The acquisition of such a huge data set provides the perfect starting point for the development of another average model. Although the Evans paper (Evans et al., 1986) gives no indication of the number of spectra used, the NOAA data base includes the largest number of individual measurements. Such an attribute allows for the finest binning of any model so far discussed. Evans's global maps have 1° latitude bins from 45° to 90° and local time bins with widths of only eight minutes. He uses a total of ten distinct activity bins instead of the seven used by Hardy and the four used by Spiro. All these factors would seem to give the NOAA model an edge in terms of resolution, although Evans does not discuss any fine scale features unique to his model.

The biggest advantage of the Evans model lies in its unique activity binning. Instead of using ground based magnetometers to determine geomagnetic activity indices such as AE or Kp, Evans uses satellite particle data to directly determine the activity level. In the same spirit of Whalen's continuous aurora model which provides the instantaneous distribution of the aurora given a single ground based observation, Evans defines the aurora's total distribution from the total energy flux measurements of a single satellite pass.

By integrating all the high latitude energy flux data taken by a NOAA satellite pass, Evans obtains a total which identifies the distribution of energy fluxes throughout the polar and auroral regions. An algorithm normalizes the flux's line integral to compensate
for the different trajectories a spacecraft can track through the aurora. The resulting value corresponds to one of ten global maps that Evans has compiled using data from previous passes binned by level of activity. These maps evolved from other global maps which used Kp as the binning parameter. Evans estimated the total hemispherical power input for each Kp level and then interpolated global maps for other input levels, evenly spacing them by total hemispheric power input. The end result consists of ten maps associated with ten power or P levels.

Each satellite pass can update the P value as often as once every 30 minutes. However, should these satellites fail due to storm conditions, it might prove necessary to use another index. Evans has, for this reason, created a conversion table between the Kp and P indices by finding the average Kp for each P (Evans et al., 1986). He notes that the deviation of these averages can range over several integral Kp values making the conversion somewhat unreliable. However, when making comparisons between the Evans and Hardy model without the availability of P values, such a table proves invaluable.

Although models based on both occurrence and average energy flux provide the most useful information for satellite operations, other modeling efforts bin such parameters as number flux, Pederson conductivity, Joule heating, and electric fields to gain a more complete picture of the physics behind the aurora. Such studies may later prove important in developing the Magnetospheric Specification Model but the damaging effects of precipitating electrons give priority to the global energy flux maps.

IV: Hardy Probability Model

Another statistical model that uses satellite measured energy fluxes holds considerable promise in the future attempts at forecasting auroral conditions. Hardy's probability model attempts to combine the advantages of both the occurrence and average types of statistical models. After grouping energy fluxes into one-half hour local time
bins, 2° latitude bins, and four Kp bins, Hardy's probability model subdivides the data base into six ranges of energy flux. The ranges include, in units of keV/cm²-ster-sec, 0 to 4x10⁸, 4x10⁸ to 12x10⁸, 12x10⁸ to 4x10⁹, 4x10⁹ to 12x10⁹, 12x10⁹ to 4x10¹⁰, and all values greater than 4x10¹⁰. Hardy then divides the number of measurements within an energy range by the total number of points within that particular spatial bin. The graphs at the end of this chapter show the resulting probability values for several local times and each Kp bin.

Because of their large number, the statistics of each bin fails to match those used in the average model, but their patterns may have more practical usefulness. Bins containing only several measurements are common, especially within the highest Kp bin. This problem may justify the coarse binning of the energy fluxes: a larger data base may improve the situation at a later date. Nevertheless this model clearly illustrates the same trends visible in the average model and, due to its unique format, gives satellite operators a better assessment of the danger their spacecraft may experience. The average model's tendency to smear out energetic peaks gives auroral fluxes a misleading benign appearance. The probability model keeps high energy flux measurements distinct by not averaging them with low intensity safe values.

By analyzing the plots of Hardy's probability model, shown on the next few pages, one can clearly see many of the trends previously discussed. As the level of activity rises the probability of higher intensities increases, the range of local times accessible to energetic fluxes broadens, the latitude of the oval decreases, and an asymmetry between the dawn and dusk side becomes noticeable. The discrete aurora, represented by the increased probabilities of higher flux values, appears with greater frequency and, particularly on the dawn side, moves closer to noon with increasing activity. Intense fluxes occur at lower latitudes on the dawn side than on the dusk side. This feature corresponds favorably to the location of the two upward Birkeland currents positioned on
either side of midnight. The graphs associated with the highest Kp bin clearly demonstrate the poleward and equatorward peaks that occur at the onset of substorms. Interestingly, the equatorward peak appears the most intense. Usually one associates the more intense discrete fluxes with the boundary plasma sheet which is contiguous with field lines attached to higher latitudes.
THE OFFSET FOR EACH SUCCEEDING LINE IS 0.10

KEY (UNITS=KEV/CM^2*STER*SEC)
0.-4.E3=NO SYMBOL  4.E3-12.E3=CIRCLE
THE OFFSET FOR EACH SUCCEEDING LINE IS 0.10

KEY (UNITS=KEV/1CM**2*STER*SEC)
0.-4.E8=NO SYMBOL 4.E8-12.E8=CIRCLE
THE OFFSET FOR EACH SUCCEEDING LINE IS 0.10

KEY (UNITS=KEV/(CM**2*STER*SEC))
-4.E3=NO SYMBOL  4.E3-12.E3=CIRCLE
THE OFFSET FOR EACH SUCCEEDING LINE IS 0.10

KEY (UNITS=KEV/(CM²*STER*SEC))
III: Comparison of Real Data with Average Model Values

In order to gain a more quantitative understanding of the shortcomings and advantages of average models, this chapter will describe a series of tests applied to them and will discuss the subsequent results. If satellite operations require the use of statistical models to specify real time particle fluxes, such tests may provide supplementary information on the variability of auroral particle energies and densities.

The Air Force Geophysical Lab provided a six month sampling of DMSP electron spectra taken from September 1977 to February 1978. Originally, a polar rain study motivated the acquisition of this data set, but later that effort proved difficult and the study of average models proved more pressing. The incompatibility between the formats of the original data and the average model required the use of a data reduction algorithm which converts counts per channel into integral energy flux values (Hardy et al., 1979).

The equation

\[ J_{TOT} = \sum E_i j(E_i) \frac{(E_{i+1} - E_{i-1})}{2} \]

provides the integral energy flux where \( E_i \) is the central energy of the i-th channel and \( j(E_i) \) is given by

\[ j(E_i) = \frac{C/\Delta T}{G(E_i) \Delta E_i}. \]

\( C_i \) is the number of counts observed by an energy channel over a time interval \( \Delta T \). The geometric factor, \( G_i \), and the energy band width, \( \Delta E_i \), characterize the calibration curves of the detector. Although DMSP data has one second resolution, the program which reduced the data summed counts over a four second interval thus making the size of the data set more manageable.

AFGL did not provide the complex algorithm used to eliminate the spurious counts
caused by penetrating particles or X-rays or direct counts from Van Allen Belt particles. Instead, as done by Hardy in his polar rain study (Hardy, 1985), all channels had subtracted from them the number of counts in the 20 keV channel in the hope that this would reduce the effects of such counts as well as noise. This technique probably works well when the average energy of the particles remains low but one would prefer a more sophisticated approach when measuring fluxes in the auroral zone where average energies can reach 13 keV. However, the energy flux plots shown in this chapter, derived by use of this simple method, compare favorably with those plots published by AFGL, satisfactorily demonstrating the validity of this technique.

In order to compare the qualities of fit provided by the AFGL and NOAA models, a series of plots, showing real energy flux data from a single satellite pass and the values projected by the models, illustrate the similarities and differences of both approaches.

Some simple conversions made the Evans model compatible with the AFGL data. No auroral power index, P, values were available for the time interval used in this study because NOAA only started producing them in 1979. The Kp to P conversion table, described in chapter 3, provided the P value necessary to drive the Evans model. Unfortunately this practice robs much of the appeal of the Evans model, but it should still have some advantage over Hardy's model due to its larger number of activity bins and larger data base. A comparison between models also required changing the NOAA units, ergs/cm^2-sec, into AFGL units, keV/cm^2-ster-sec. The simple conversion between ergs and keV posed no problem but converting between whole angles and an omnidirectional system involved some ambiguity. To treat this problem, the NOAA values had applied to them a factor of 1/π, but this geometric adjustment only approximates the difference between the use of steradians and the integration of fluxes over all angles.

A computer program read in real satellite data and computed the corresponding
average model values. That portion of the program which evokes graphics subroutines requires linking to Dr. Spiro's EZPLOT package. Two variations of this program made possible both a qualitative and quantitative comparison. Figures 1 and 2 show the more sophisticated of the two. Above each of these lies a line identifying the pass and the standard deviation associated with each model. The 'standard deviation' of the Hardy and Evans models are labeled KpEr and PEr respectively. The stepped line with intermittent circular symbols identifies the Evans model; the solid line represents Hardy's model. Calculating this value entailed summing the squares of differences between the log base 10 of the real and model values, dividing the sum by the number of points within the pass, and taking the square root. In the strictest sense the model values differ from the true average so that the term standard deviation is not correctly applied in this context. Instead of achieving mathematical rigor, these values simply provide a convenient measure of the quality of fit for each model. The next three plots lack the pseudo-standard deviation values but otherwise look identical. Such plots took less computer time to produce but still provided the necessary information for analysis. The end of this chapter shows a number of stripped down plots arranged by Kp.

This first plot, Fig. 1, shows a typical pass during a relatively quiet period. A large number of fine scale structures decreases the quality of fit, but the general trends in the data match up well with the two models' curves. The standard deviations of both models linger around a value of 0.8, a typical value. Although Hardy's model gives a marginally better fit, the Evans model fits other data more snugly.

When comparing the two models' curves some interesting details become apparent. At low activity levels the Evans model gives values substantially lower than Hardy's. This discrepancy may evolve from the lack of low energy measurements used to create the NOAA model. Also the peaks of the Evans model seem to occur at higher latitudes than the Hardy model. This characteristic represents the most peculiar difference but
once again the difference in energy spectra may account for it. It may be conjecture that the corrected geomagnetic coordinates used by both models differ, although the literature makes no indication of such a problem. The Hardy model has a smooth appearance due to its analytical form whereas the binned Evans model has bumpy discontinuous features.

Moving onto higher activity levels the two models converge. In Figure 2 the model curves look identical but the data look drastically different. Some kind of error in the ephemeral line may account for this situation. In any case both models drastically fail to predict the flux profile. By looking at other plots, for example those of Figure 3 and
those at the end of the Chapter, one can see further evidence of the differences between the models during periods of low Kp as well as their tendency to converge at Kp values equal to or greater than 3. Note that these plots have latitude on the x-axis. The labelling on these axes should go from 50 to 90 and then back to 50. A problem with the plotting program created this situation. Naturally those values greater than 90 are nonsensical. The plots of Figure VI illustrate the varying quality of fit between the real four second data and the average model values. Because of these similarities and due to the unavailability of accurate P values, no further analysis of the Evans model follows. Instead, a closer scrutinization of the Hardy model is warranted.

Figure 3
The Magnetospheric Specification Model (MSM) requires considerable testing so that various software "knobs" can tweak the magnetic and electric field components to give realistic results. The CDAW-6 (Coordinated Data Analysis Workshop-6) effort brought together data sets from a wide variety of sources to study two magnetic substorms that occurred in 1979. Such conglomerations can give a more complete picture of the different physical processes that comprise a substorm. The MSM, fed data from one time interval and run through a series of time steps, will forecast particle fluxes found in a succeeding period. In the auroral zone, a portion of the code will handle this problem, but should data inputs fail, the Hardy model will provide the only information on particle populations. In order to test the value of Hardy's model in such a role, the following sets of plots (Figure 4), comparing CDAW data with Hardy values were made. The model fits the real data reasonably well within the polar cap, the poleward region of the auroral zone, and the equatorward fluxes. It fails however in the equatorward regions of the auroral zone. The transience of substorm fluxes and the three hour time intervals associated with Kp may explain the discrepancies, although between the two plots there seems to be little variation. For these reasons the Evans model and the P index may provide a better quality of fit but ultimately a more theoretically based model may provide the best predictions.
In order to gain a more quantitative look at the accuracy of the Hardy model, a series of Chi-Square tests determined the deviation of these models from real data in terms of local time, latitude, and Kp. The equation used to perform these tests had the form:

\[ \chi^2 = \sum_{n=1}^{N} \frac{(y_{i} - y(x_i))^2}{N - v} \]

where \( y_{i} \) represents the satellite data; \( y(x_i) \), the average model value, \( N \), the number of points within a bin, and \( v \), the number of free parameters.

Each of the six months tested provided 800 usable satellite passes. As each pass contained roughly 400 four-second intervals, a total of 1.8 million energy flux measurements, divided into 7 Kp ranges and 480 sectors with dimensions of 1 hour local time and 2° latitude, were put through the above summation. Such an arrangement tested the model at all local times and at all latitudes above 50° in both the North and South hemispheres. Although such statistics may sound impressive the large number of bins, coupled with the paucity of data at the extreme Kp ranges, creates some questionable results. Limitations in the coverage provided by the satellite's orbit leave some bins devoid of any measurements. This situation is very unfortunate because the most active
and energetic region near midnight has no data in it at all. Also, as the data comes from
the winter months, some seasonal effects may have influenced the patterns. Nevertheless
some interesting features deserve some discussion.

Ideally, taking the Chi-Square sum would involve dividing each flux measurement
by its experimental error and would not entail dividing the quantity by N-v. The number
of free parameters, the number of points within a bin, and the Chi-Square sum, plugged
into an incomplete gamma function, would then provide an estimate of the goodness of
fit, where a value of 0.01 would indicate a high quality match. Because each flux
measurement has no given error or σ this quantitative rule of thumb has no concise
mathematical meaning. A more rigorous approach to find σ entails taking the square root
of the number of counts in each channel of each bin and modifying them by some
coefficient that reflects the efficiency of the detector at each energy step. This error
would then be propagated through the data reduction algorithm. Instead, this study relies
on a more qualitative assessment of fit where the Chi-Square values have roughly the
same usefulness as standard deviations. The number of free parameters in the Epstein
equation, seven, took the place of v in the summation. In order to have reasonable
statistics for each bin a minimum value for N was set at ten. However, even a value of
ten can give questionable results. Fortunately most bins contain upwards of a thousand
individual measurements.

Initially, a series of plots showed that the averages for certain bins of satellite data
corresponded very unfavorably with the average values provided by the model. Faulty
data, possibly caused by photoemission, had values several orders of magnitude higher
than normal, destroying the quality of statistics in some bins. These measurements were
identified as faulty when it was noticed that the 20 keV channel possessed the highest
number of counts. When making the final Chi-Square plots, however, unusually high
values were filtered out by writing into the Chi-square computer program a test which checked to see if the data exceeded a reasonable value (10^{11} \text{ keV/cm}^2\text{-sec-ster}). Even so, the small size of the data base of some bins and their bias toward the winter season undoubtedly skewed the final results. For this reason only the more obvious features in the three dimensional plots shown at the end of the chapter deserve serious discussion and the more subtle features must be considered to have a somewhat questionable origin. A larger data base with equal seasonal representation could have allowed for more meaningful results.

The end of this chapter displays four different kinds of three dimensional plots. The first of these shows the average flux values as determined by Hardy's analytical function. The second indicates the number of points evaluated in each bin for that Kp value. The next two plots show the results of the Chi-Square test as viewed from two different angles. The final two plots represent a second test where the difference between the real data and the average model values were divided by the model value before the squaring and summing occurred. The deviations appear in units of orders of magnitude squared.

Each set of plots helped to analyze the results of the Chi-Square tests. The "Number in Bin" plots indicate the quality of satellite coverage; the best statistics occur close to or along the dawn to dusk meridian. Toward noon and midnight the coverage falls to zero which explains the gaps seen in the Chi-Square plots. Also, the smoothest regions of the Chi-Square plots are frequently made up of those bins which possess the best statistics. More often than not, the large scale structures seen in the Chi-Square plots closely assimilate those of the average flux model. One would expect the larger fluxes to have larger deviations from the average because they are associated with the highly unpredictable discrete aurora. The Normalized Chi-Square plots reduce this effect, but it still remains.
SASGRAPH's G3D program created the images of these plots. Before graphing, however, a packaged subroutine, also in SASGRAPH, used a spline fit to smooth out some of the spurious small scale features (bumps). The vertical axis gives the Chi-Square values in units of log $10(\text{keV/cm}^2\text{-sec-ster})^2$. The horizontal axes give the corrected geomagnetic latitude and corrected geomagnetic local time of the bin. The Normalized Chi-Square plots have vertical axis units of orders of magnitude squared.

The $Kp = 0$ contour plots found on pages 58 to 61 demonstrate the characteristics of the low intensity fluxes prevalent during quiet activity levels. Because the intensity of these fluxes do not depend greatly on local time these plots have the flattest and most regular surfaces. The lowest deviations occur in the low latitude regions positioned just equatorward of the most energetic fluxes where the RMS of the normalized deviation (the square root of the Chi-square) lies between zero and one order of magnitude. At a specific latitude, most likely the equatorward edge of the aurora, the deviations start to increase until they reach a plateau at the polar cap. In the Normalized Chi-Square $Kp = 0$ plots a dusk side minimum is reached just before the deviations start to increase or at roughly $62^\circ$ latitude. This minimum indicates the region where the real data values most closely resemble the constant value set by the Hardy model. Quite possibly, the real values increase with latitude and surpass the one set by the model at this location.

One of the peculiarities seen at all $Kp$ ranges involves a dawn to dusk asymmetry of the Chi-Square values, especially the normalized ones, evaluated at low latitudes. In each plot, the dusk side deviation reaches values higher than those on the dawn side. In the $Kp = 0$ case the difference in RMS value attains three orders of magnitude. According to the Hardy model both regions should have the same average energy flux value of $10^6 \text{keV/cm}^2\text{-sec-ster}$ but the asymmetry in the deviation shows that either there are greater fluctuations on the dusk side or that there should be two different average values for both
regions. Because only diffuse aurora occur at low latitudes, the fluxes should have uniform and predictable intensities but yet differences in the orders of magnitude exist. In a private communication, Hardy suggested that local time dependent Van Allen radiation fluxes may create the discrepancy.

One would expect that the polar cap would have smaller Chi-Square values because the region lacks the discrete forms of precipitation associated with the auroral oval. Polar rain, a low energy flux of electrons, constitutes the primary energy input into this region. However, so called polar squalls and polar arcs inject energetic particles into the cap, creating a bimodal distribution of flux values. Take for example any one of the satellite passes shown before the Chi-square plots that occurred during Kp values of 2. The polar cap's energy flux profiles are very smooth and remain close to a constant value. In the top Kp = 4 plot, the energy flux profile reaches a peak in the polar cap due to polar squall activity. Note however that the polar cap generally appears most active at the Kp = 1 level when the IMF is northward. This feature is easily seen in the Kp = 1 and Kp = 0 Chi-square plots where the deviations reach their highest value in the polar cap, having values in the range of two to three orders of magnitude, RMS. Fitting a single average value to such diverse populations results in large deviations between model and real values.

Looking at progressively higher levels of activity, other features become visible. Instead of gradually reaching a plateau at the polar cap, plots of higher Kp values show that the normalized deviations achieve a peak located just poleward or at the same location of the average model's maximum. This peak seems most noticeable in the Chi-square plots but even the Normalized plots indicate where the RMS peaks at three orders of magnitude. Take for example the Kp = 1 contour plots. Both the model and the Normalized Chi-Square values appear to reach a maximum at 66°. Perhaps this region
represents that portion of the aurora contiguous with the boundary plasma sheet, the area where most discrete aurora occur. At the highest Kp level the height of this peak diminishes, most probably due to the smearing effect of the averaging process. During active periods the discrete aurora must then occur at a wide range of latitudes.

The deviations of the model's noon values appear to decrease with increasing Kp levels. This trend extends throughout much of the polar cap where the normalized deviation in the regions adjacent to the noon sector become smaller dropping as low as one order of magnitude, RMS, and thereby creating a surface concave in local time. This feature indicates that the variability in the cap decreases during periods of high auroral activity; a result consistent with other polar cap studies. Outside of this region, the polar cap's fluxes share characteristics very similar to those of quiet Kp levels.

Ideally, this study would have used the same data base Hardy developed for his model. Such a large mass of numbers could have improved the Chi-Square statistics markedly and filled some of the data gaps. Nevertheless, some of the conjecture involved in the above analysis may have some value and the deviations indicated could provide an improved guideline for statistically predicting auroral energy fluxes.
Chi-Square

\[ \text{Log}_{10}(\text{Chi-Square}) \]

Latitude

Local Time

\[ \text{Kp}=0 \]
Normalized Chi-Square

Latitude

Local Time

Kp=0
Average Flux

Total Number in Bin

Kp=1
Chi-Square
Normalized Chi-Square
Average Flux

Total Number in Bin

Kp=2
Chi-Square
Normalized Chi-Square

Kp=2
Average Flux

Total Number in Bin

Kp=3
Chi-Square

Kp=3
Normalized Chi-Square

Kp=3
Average Flux

Total Number in Bin

Kp=4
Chi-Square

Kp=4
Normalized Chi-Square

Kp=4
Average Flux

Total Number in Bin

Kp>5+
Chi-Square

\[ \log_{10}(\text{Chi-Square}) \]

Latitude

Local Time

Kp>5+
Normalized Chi-Square
IV: Summary and Recommendations for Future Work

By providing scientists with the average flux intensities associated with different portions of the aurora's morphology, the average models discussed in this thesis make clear improvements over the early occurrence mappings. Although global energy flux maps do not show the altitude dependence of the aurora, they provide a very good representation of the latitude and local time variations that endure through long time scales.

Unfortunately short term fluctuations frequently contain the energy fluxes most hazardous for spacecraft. But if one desires the integrated energy fluxes incident on a satellite during a polar pass so as to determine electrical charging rates or orbital drag these models could prove invaluable. A more theoretical approach may provide the necessary information on the intense fluxes associated with the discrete aurora. Substorms create the most difficult modeling problem because of their highly dynamic nature, but they too may have a solution through a combination of statistical and theoretical systems. Hardy's probability model, improved by finer binning, could also solve the problem in a manner similar to that used by meteorologists. A similar approach would use the average model and some sort of probability envelope around it that would give the chance a value will be within a certain range of the average.

Future work on the Chi-square calculations would require more recent DMSP data so that the Evans model could be tested with real P values. Also a check to see if Air Force Kp would improve the accuracy of the Hardy model might prove interesting.

The same techniques used to model electron fluxes could apply equally well to ion fluxes. The addition of an ion average flux model could reveal more details in the physics of the auroral zone and provide a better warning for spacecraft.
Bibliography

Block, L.P. and C.-G. Falthammar, Mechanisms that may support magnetic field aligned electric fields in the magnetosphere, Royal Institute of Technology, Dept. of Plasma Physics, Stockholm, Sweden, 1975.

Evans, D.S., Global statistical patterns of auroral phenomenon, 1985, Space Environment Laboratory, National Oceanographic and Atmospheric Administration, Boulder, CO 80303.


Mozer, F.S. and M. Temerin, Solitary waves and double layers as the source of parallel electric fields in the auroral acceleration region, Space Science Laboratory, University of California, Berkeley, CA, 94720.


