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A time series analysis of the Japanese yen

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A TIME SERIES ANALYSIS OF THE JAPANESE YEN

by

JAE-JUNG KWON

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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MASTER OF ARTS

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APRIL, 1988
ABSTRACT

This paper sought to address the question as to whether the exchange rate can be forecasted more accurately by a monetary model of exchange rate determination or the random walk in the case of the Japan-U.S. exchange rate.

The evidence of Meese and Rogoff (1983) on the out-of-sample forecasting performance of structural exchange rate models in comparison to the random walk model portrays a disappointing picture of structural models. I re-considered the issue for the Japanese yen for a more recent period. Besides out-of-sample evidence, within-sample evidence was also examined.

The recent work of Phillips and Perron was employed so as to verify that the exchange rate series is well approximated by a random walk model without drift but with time dependent heteroscedasticity. Having established this benchmark, structural monetary models are constructed to see whether one can obtain better within-sample and/or out-of-sample results. It appeared that the random walk can be beaten.
ACKNOWLEDGEMENTS

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CHAPTER I
INTRODUCTION

One of the main features of the recent float has been the extreme volatility of exchange rates relative to the recent historical past. This contrasts with the prior claims of advocates of flexible exchange rates who argued that speculation would be stabilizing and that this would tend to limit the extent of any exchange rate movements. In particular, the Japanese yen-U.S. dollar exchange rate has substantially appreciated and oscillated. It is at the center of turbulence in the financial markets. Since the era of floating began in March 1973, the yen-dollar exchange rate has undergone rather volatile fluctuations. This paper examines the behavior of the exchange rate between Japan and U.S. since March 1973 to evaluate the forecasting accuracy of monetary and random walk models of the exchange rate.

After strong resistance from Japanese monetary authorities against revaluing and floating the yen, President Nixon's new economic policy of August 1971, called the "Nixon shock" in Japan, drastically changed the course of events in the international monetary sphere. The rules of the game is the world economy changed from the dollar standard system, based on fixed exchanged rates, to the system of floating exchange rates. This meant a fundamental change in the international environment for Japan, which for a long period of its rapid economic growth, had managed to keep the exchange rate at the rigid par value of ¥360 = US$1 that had originated during the occupation in 1949.

The Smithsonian system was in effect from December 1971 to February 1973, and provided a short interlude of fixed exchange rates with a wider band of permissible fluctuation. However, due to the decline in public confidence in the fixity of exchange rates, the exchange market during the Smithsonian system worked quite differently from that in the dollar standard system before Nixon's new economic policy.

We will begin by studying the Japanese experience with floating exchange rates by looking at movements of exchange rates, the degree of intervention by government, the intensity of exchange control, components of the balance of payments, and stances of economic policies. (Figure 1) shows the course of the yen-dollar exchange rates. After the collapse of the Smithsonian system in February 1973, the value of the yen showed a
rapid increase and reached around ¥265 =$1. During the Smithsonian period, the
eyen-dollar exchange rate remained at the lower bounds of the widened band of the
Smithsonian system, that is, around ¥265 = $1. Around these values the Japanese
monetary authorities made attempts to support the dollar until the outbreak of the first oil
crisis in the fall of 1973. The oil shock reversed the course of exchange rates, and
reversed the direction of intervention as well. In spite of heavy operations to sell the
dollar, the value of the yen dropped to the trough value around ¥306 = $1 at the end of
1974. After the beginning of 1975, the yen recovered its value and kept appreciating, with
minor oscillations, until it reached its peak in October 1978 at an exchange rate of almost
¥175 = $1. Thus the exchange rate of the yen in terms of the dollar more than doubled in
seven years after the suspension of the fixed exchange rate.

On November 1, 1978, the United States announced a set of policy measures to keep the
dollar exchange rate from depreciating further. In addition to adopting contractionary
monetary policies, the United States committed itself to intervention in the exchange
market by using funds provided by the swap agreement (reciprocal currency arrangement).
This change reversed the trend of exchange movements. The depreciation of the yen was
then reinforced by the outbreak of the second oil crisis. The dollar continued to appreciate
sharply in the spring of 1980. This was caused partly by high interest rates in the U.S.
and the Eurodollar market that reflected inflationary expectation in the U.S., Germany,
and Switzerland, a series of countermeasures for preventing the depreciation of the yen.

The remainder of the paper is organized as follows. Chapter 2 explains about the
methodology and literature which are employed in this paper. In Chapter 3, the univariate
time series properties of the exchange rate are investigated and a random walk model is
established. The conclusions from this chapter are used to restrict the multivariate analysis
in Chapter 4. Two classes of monetary models are set up to compete with the random
walk benchmark. Chapter 5 offers some conclusions.
CHAPTER II
METHODODOLOGY AND LITERATURE

The empirical exchange rate literature does not give much comfort to any particular exchange rate theory that has been postulated. Any success achieved usually turns out to be episodal and the particular model tends to be no better at out-of-sample forecasting than a random walk.

In fact, a number of authors have argued that the exchange rate follows a random walk. For example, Mussa states as his first empirical regularity that: 'The natural logarithm of the spot exchange rate follows approximately a random walk' (Mussa, 1979, p.10). Meese and Rogoff (1983) compare the out-of-sample forecasting accuracy of various structural and time series exchange rate models to find that a random walk model performs as well as any estimated model for several exchange rates. In addition, several other authors, including Frenkel (1981), Adler and Lehmann (1983) and Hakkio (1986), have argued that the exchange rate - the deviation from purchasing power parity - should, or does, follow a random walk. Many of these authors also discuss the conditions under which the exchange rate follows a random walk.

If a random walk is the case for a exchange rate, inferences from estimates of the parameters of a model containing that series will need to account for the non-stationarity, or else transformations of the series (say, by differencing) are needed to obtain stationarity and the right to use classical distribution theory.

The structural models of exchange rate determination employed here to investigate the behavior of the yen-dollar exchange rate may be termed the "monetary" or "asset" approach to the exchange rate. This point of view has come to dominate the recent literature on the determination of the value of floating exchange rates. Two fundamental elements characterize this approach: exchange rates are viewed as moving to equilibrate the international demand for stocks of assets; and exchange rates are determined by market participant's (rational) current expectations of all the future values of relevant exogenous variables operating upon exchange rates. But within the asset approach there are two different versions which have conflicting implications in particular for the relationship between the exchange rate and the interest rate: (1) the flex-price monetary approach to
the exchange rate; and (2) the fix-price monetary approach to the exchange rate.

The first approach, sometimes called the "Chicago" theory, is in many ways an extension of the purchasing power parity (PPP) view of exchange rates; essentially it appends a theory of the determination of the price level to a PPP equation in order to explain the exchange rate. The use of PPP in a theory of the determination of the exchange rate immediately introduces a problem, for PPP is not well supported empirically. Nevertheless, it is useful to examine the approach as a first stepping stone in the study of more complex asset market models. It has been developed by Frenkel (1976, 1977, 1980), Mussa (1976), and Hodrick (1978).

Purchasing power parity may be a good approximation in the long run, but large deviations seem to appear in the short run. Therefore the second approach which shares many of the features of the first one relaxes the assumption of PPP in the short run. One particularly interesting feature of the fix-price monetary approach is that it offers another explanation of exchange rate volatility in terms of overshooting. This approach originated from Mundell's analysis of perfect capital mobility has been developed by Dornbusch (1976) and Frankel (1979).

Following Meese and Rogoff (1983) and Sheen (1986), this study compares time series and structural models of the exchange rate on the basis of their within-sample and out-of-sample forecasting accuracy.

In doing this, I established monthly models. Except under very special conditions, the minimum available reporting periodicity among all the variables dictates the periodicity of the time series model. A quarterly model of the exchange rate would be called for, since the structural models in this paper include GNP. However, shorter period data provides more degrees of freedom for the estimation process. For this reason, I used industrial production index instead of GNP.

All the competing models are estimated over a monthly data series which states in March 1973, the beginning of the floating rate period, and extends through December 1985. We may have three type of forecasts—ex-post, ex-post extrapolative, and ex-ante. This follows from defining within-sample forecasts as ex-post forecasts, out-of-sample forecasts using realized values of exogenous variables as ex-post extrapolative,
out-of-sample forecasts using expected values of exogenous variables as *ex-ante* forecasts. For their out-of-sample forecasts, Meese and Rogoff use realized values of exogenous variables, i.e., they use *ex-post* extrapolative forecasts. While *ex-ante* forecasts are the true forecasts, use of *ex-post* extrapolative forecasts would be better suited for the forecast comparison of models. This is because while *ex-ante* forecasts errors are related to errors in estimation of exogenous variables and errors in model specification and estimation, *ex-post* extrapolative forecast errors are related only to the latter. For this analysis, we use *ex-post* and *ex-post* extrapolative forecasts, or within-sample and out-of-sample forecasts respectively.

Forecasting accuracy is measured by three statistics: Mean error (ME), mean absolute error (MAE) and root mean square error (RMSE). These are defined as follows:

\[
\text{mean error} = \frac{1}{M} \sum_{s=1}^{M} \frac{[F(t + s) - A(t + s)]}{M}
\]

\[
\text{mean absolute error} = \frac{1}{M} \sum_{s=1}^{M} |F(t + s) - A(t + s)| / M
\]

\[
\text{root mean square error} = \left( \frac{1}{M} \sum_{s=1}^{M} [F(t + s) - A(t + s)]^2 / M \right)^{1/2}
\]

where \(M\) is the total number of forecasts in the projection period for which the actual value \(A(t)\) is known, and \(F(t)\) the forecast value. In these expressions, we have let forecasting begin at period \(t+1\).

Since we are looking at the logarithm of the exchange rate, these statistics are unit-free and in approximate percentage terms. By comparing predictors on the basis of their ability to predict the logarithm of the exchange rate, we also avoid any problems arising from Jensen's inequality.\(^{(1)}\) Due to Jensen's inequality, the best prediction of the level of a
given exchange rate might not be the best prediction of the inverse. However, there isn't an obvious choice for the "numeraire" currency. Root mean square error is the main criterion for comparing forecasts in this paper. However, since RMSE is an inappropriate if exchange rates are governed by a non-normal stable Paretian process with infinite variance, it is important to include mean absolute error. MAE is also a useful criterion when the exchange rate distribution has fat tails, even if the variance is finite. The last criterion, mean error, provides another measure of robustness. By comparing MAE and ME, we can ascertain whether a model is systematically over or underpredicting.
(1) The usefulness of using logarithm transformations in efficiency tests has been demonstrated by Siegel (1972).
CHAPTER III
UNIVARIATE TIME SERIES

1. Model

Analysis of the univariate time series properties of exchange rate data provides a useful starting point, prior to econometric analysis. The requirement that they be consistent with the univariate time series model places restrictions on the set of feasible structural econometric models.

A general consensus has emerged in recent years, that many economic time series, such as GNP, consumption expenditures, disposable income, etc., can be better characterized as non-stationary processes that have no tendency to return to a deterministic path. In particular, Nelson and Plosser (1982) described the property as one of being "difference stationary (DS)" so that the first or higher order difference is a stationary and invertible ARMA process. An alternative "trend stationary (TS)" model, where a process is expressed as a deterministic function of time (i.e. trend) plus a stationary stochastic process with mean zero, has generally been found to be less appropriate.

To see the fundamental difference between the TS and DS classes with reference to exchange rate series, the univariate model has the general representation:

\[ \alpha(L) x_t = \zeta(t) + \beta(L) \epsilon_t \quad t = 1, ..., N \]  \hspace{1cm} (3.1)

where \( x_t \) is the spot exchange rate of U.S. dollar in terms of Japanese yen, \( \epsilon_t \) is zero mean white noise, \( \zeta(t) \) is a polynomial function of time, and \( \alpha(L) \) and \( \beta(L) \) are pth and qth order polynomials in the lag operator. This process is an autoregressive-moving average process of order \( (p, q) \) or ARMA \( (p, q) \). If the AR operator \( \alpha(L) \) of the ARMA process \( x_t \) has unit roots [i.e. \( \alpha(1) = 0 \)], these can be eliminated by differencing. Assuming d unit roots, we get another ARMA process integrated of order d or an ARIMA \( (p, d, q) \) process.
The tendency of economic time series to exhibit variation that increases in mean and dispersion in proportion to absolute level motivates the transformation to natural logs and the assumption that trends are linear in the transformed data. Also, many authors [see Meese and Rogoff, Nelson and Plosser (1982), and Meese and Singleton (1982)] base univariate time series techniques on a long AR which is an unconstrained autoregression where the longest lag considered (p) is a function of sample size (N), p = N/\ln N. A specific case of (3.1) would be:

\[ y_t = \zeta_0 + \zeta_1(t - N/2) + \sum_{s=1}^{p} \alpha_s y_{t-s} + u_t \] (3.2)

where \( y_t \) is the logarithm of the spot exchange rate and \( u_t \) is a (possibly heteroscedastic and/or serially correlated) disturbance term.

The key issue is whether or not the characteristic function of the AR term has any unit roots. Unfortunately, the standard asymptotic theory developed for the model as above is not valid for testing the hypothesis that the polynomial contains a unit root. Much of the work done by the authors cited in the last paragraph assumes independently and identically distributed disturbances, and uses the statistics tabulated in Fuller (1976) and Dickey and Fuller (1981). However, those statistics seem to be inappropriate at first glance because we don't have any evidence that the errors are i.i.d. Rather, there is now a substantial body of documented evidence that many exchange rate and financial market series exhibit time dependent heteroscedasticity; see Engle (1982), McCurdy and Morgan (1983), and Bollerslev (1987).

To get some idea of this problem, the model in this paper considers the simplest version of (3.1), where the null hypothesis is that \( y_t \) is a random walk with or without drift (and the error process is white noise):

\[ y_t = \zeta_0 + \zeta_1(t - N/2) + \alpha y_{t-1} + u_t \] (3.3)
The fact that the exchange rate series, in common with most other economic series, is of a low order in the polynomials could be an excuse for starting from the simple version (3.3). The initial condition \( y_0 \) is assumed to be a known constant.\(^{(1)}\)

Then, tests are undertaken for the following null hypotheses:

\[
\begin{align*}
H_01: & \quad \alpha = 1, \\
H_02: & \quad \zeta_1 = 0 \text{ and } \alpha = 1, \\
H_03: & \quad \zeta_0 = \zeta_1 = 0 \text{ and } \alpha = 1.
\end{align*}
\]

(3.4.a) (3.4.b) (3.4.c)

Under the first null hypothesis, the model would be integrated order of 1. If this hypothesis is accepted, estimation is redone using the appropriately differenced form. The second and third hypotheses are joint ones concerning the existence of the unit root and no linear time trend. In particular, the third one denies that the data exhibit a drift.

If there is to be any credibility in the deduced time series properties, the selected model should be checked for the following criteria.

(A) Uncorrelated Innovations

The modeling process is supposed to account for the relationships between the observations. If it does account for these relationships, the residuals should be unrelated, and hence the autocorrelations of the residuals should be small. It has been found that an effective way to measure the overall adequacy of the tentative model is to examine a quantity that determines whether the first \( k \) autocorrelations of the residuals, considered together, indicate adequacy of the model. This quantity is called the Ljung-Box Chi-Square Statistic is denoted by \( Q \), and is computed using the formula:

\[
Q = N(N+2) \sum_{k=1}^{m} \frac{\gamma_k^2}{(N-k)}
\]
where
\[ \gamma_k = \sum_{k=1}^{N-k} e_t e_{t+k} / \sum_{k=1}^{N} e_k^2 \]

and the \( e_t \) are the residual sequence. \( Q \) would for large \( N \) be distributed as \( \chi^2_m \) since the limiting distribution of \( \gamma = (\gamma_1, ..., \gamma_m) \) is multivariate normal with mean zero vector. This formula has been suggested by Ljung and Box (1978) as yielding a better fit to the asymptotic Chi-square distribution.

The larger \( Q \) is, the larger are the autocorrelations between the residuals, and the more related are the residuals. Thus a large value of \( Q \) indicates that the model is inadequate. It is common practice to accept the adequacy of the model if the calculated value is less than \( \chi^2(m) \), which is defined to be the point on the scale of the chi-square distribution having \( m \) degrees of freedom such that there is an area of 0.5 under the curve of this distribution above this point. The choice of \( m \) is arbitrary, but common practice for monthly data is to compute \( Q \) for \( m = 12 \) (and possibly 18, 24 and 30).

(B) Time Dependent Conditional Heteroscedasticity

The error process should be checked for heteroscedasticity. With time series data, the error term is modeled with some kind of stochastic process, and most of the conventional stochastic processes assume homoscedasticity. However, in the case of heteroscedasticity, the estimate of the variance-covariance matrix of estimates is inconsistent. Heteroscedasticity in an asset price equation is an important and distinct possibility because an influential element in asset choices is relative risk. Risk premia are difficult to specify and generally time varying. Misspecification of risk premia would be expected to be detected as heteroscedasticity of the disturbance term.\(^{(2)}\)

To generalize the traditional econometric assumption of homoscedasticity, Engle (1982) developed a new class of stochastic process called autoregressive conditional heteroscedastic (ARCH) processes. These are mean zero, serially uncorrelated processes with nonconstant variances conditional on the past, but constant unconditional variances.
The ARCH tests are undertaken in this paper because, if rejected, they may help to explain the existence of fat tails in the error distribution.

The ARCH regression model is obtained by assuming that the mean of $y_t$ is given as $x_t \beta$, a linear combination of lagged endogenous and exogenous variables included in the information set $\Omega_{t-1}$ with $\beta$ a vector of unknown parameters:

$$y_t \mid \Omega_{t-1} \sim N(x_t \beta, \mathbf{h}_t)$$

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \cdots + \alpha_p e_{t-p}^2$$

$$\epsilon_t = y_t - x_t \beta$$

where $h_t$ is the variance function.\(^{(3)}\)

To test whether the disturbances do in fact follow an ARCH process, Engle suggests a simplification of the Lagrange Multiplier test procedure. Under the null hypothesis, $\alpha_1 = \alpha_2 = \cdots = \alpha_p = 0$. The test is based on the score under the null and the information matrix under the null. The test procedure is to run the OLS regression and save the residuals. Regress the squared residuals on a constant and $p$ lags. The statistic proposed by Engle is $TR^2$, where $R^2$ is the squared multiple correlation coefficient in the last regression. This statistic has an asymptotic chi-square distribution with the degree of freedom of $p$ under the null hypothesis.

(C) Presence of Unit Roots

One of the critical issues in modeling the autoregressive part of univariate time series models is the test procedure for the presence of unit roots. The work of Phillips and Perron provides tests for unit roots which are robust to a wide variety of serial correlation
and time dependent heteroscedasticity. The tests involve computing the OLS regression of (3.3). Given the equation we can test the null hypotheses $H_01$, $H_02$, and $H_03$ by means of the test statistics $Z(t_\omega)$, $Z(\Phi_3)$ and $Z(\Phi_2)$ respectively.

Following Baillie and Bollerslev (1987), we define the statistics:

\[
Z(t_\omega) = \frac{(s_u / s_{n1}) t_\alpha - (N^3 / 4\sqrt{3}D_X^{1/2} s_{n1})(s_{n1} - s_u)}{2}
\]

(3.6.a)

\[
Z(\Phi_3) = \frac{(s_u^2 / s_{n1}) \Phi_3 - (1/2 s_{n1})(s_{n1}^2 - s_u^2)[N(\alpha - 1)]}{- (N^6 / 48D_X)(s_{n1} - s_u)}
\]

(3.6.b)

\[
Z(\Phi_2) = \frac{(s_u^2 / s_{n1}) \Phi_2 - (1/3 s_{n1})(s_{n1}^2 - s_u^2)[N(\alpha - 1)]}{- (N^6 / 48D_X)(s_{n1} - s_u)}
\]

(3.6.c)

where

\[
t_\alpha = \left(\alpha - \bar{\alpha}\right) / \left(s^2 \ C_3\right)^{1/2}
\]

\[
\Phi_2 = N(s_0^2 - s^2) / 3s
\]

\[
\Phi_3 = N\left\{s_0^2 - (\bar{Y} - \bar{Y}_-)^2 - s^2\right\} / 2s
\]

\[
\bar{\alpha} = \frac{1}{N} \sum_{t=1}^{N} \left\{y_t - \hat{v}_0 - \hat{\xi}_1(t-N/2) - \alpha y_{t-1}\right\}^2
\]
\[ s_0^2 = \frac{1}{N} \sum_{t=1}^{N} (y_t - y_{t-1})^2 \]

\[ \bar{Y}_{-i} = \frac{1}{N} \sum_{t=1}^{N} y_{t-i} \quad \text{(i = 0, 1)} \]

and \( C_i \) is the (i,i) element of the matrix \((XX)^{-1}\). \( D_X \) denotes the determinant of \((XX)\).

In all the above expressions \( s_u^2 \) denotes a consistent estimator of \( \sigma_u^2 = \lim N^{-1} \sum E(u_t^2) \) given by the sample analogue under the appropriate null hypothesis. \( s_{ni}^2 \) is a consistent estimator of \( \sigma^2 = \lim N^{-1} E(s_n^2) \) under the appropriate null hypothesis, where \( s_n = \sum u_t \).

In this paper, the version of \( s_{ni}^2 \) used is

\[ s_{ni}^2 = \frac{1}{N} \sum_{t=1}^{N} u_t^2 + \frac{2}{N} \sum_{t=1}^{N} \sum_{\tau=t+1}^{N} u_t u_{t+\tau} \omega_{\tau} \tag{3.7} \]

where \( \omega_{\tau} = 1 - \tau/(1 + 1) \).

The modification of Phillips and Perron involves a nonparametric estimate of the serial correlation, rather than estimation of nuisance parameters as proposed by Dickey and Fuller. To implement the approach, one must choose a window size and a weighting scheme: I use a 10th-order window and the weighting scheme of Newey and West (1987), but the results are similar for all window sizes between 0 and 10.

Phillips and Perron demonstrate that much of the work on the distribution of the OLS estimate \( \alpha \) and the \( t \) and \( F \) statistics under i.i.d. innovations remains relevant for a very much larger class of models. In fact, it is shown that their tabulations appear to be relevant in almost any time series with a unit root. To test the unit root hypotheses, one
simply computes \( Z(t_2) \), \( Z(\Phi_3) \) and \( Z(\Phi_2) \), and compares these to the relevant critical values given by Evan and Savin (1981) and Fuller (1976). Under the null hypotheses the 1% and 5% critical values of \( Z(t_\omega) \), \( Z(\Phi_3) \) and \( Z(\Phi_2) \) will be -3.96 and -3.47, 6.09 and 4.68, and 8.27 and 6.25 respectively.

2. Empirical Results

In the following analysis we took the monthly spot exchange rate between March, 1973 and December, 1985. The data were obtained from the publication of OECD Main Economic Indicators.

(Table 1) reports the results of the regression (3.3). On the basis of simple t tests, the parameters (apart from the trend coefficient) are significantly different from zero. Similarly, the t value for \( H_0 \), -1.787, indicates that the hypothesis is significantly rejected. The estimate of \( \alpha \), 0.957, is slightly below that of Meese and Singleton (1984)'s estimates for the US dollar against Swiss francs, Canadian dollars and Deutschmarks (respectively 0.999, 0.982, 1.008). Such a result may appear to suggest that the Japanese experience has been somewhat different and does not lend support to the Mussa (1979) contention that the logarithm of spot rates approximately obey a random walk. However, such conclusions are spurious and invalid. The distribution of the autoregressive parameter estimates under the null hypothesis is decidedly skewed to the left of unity, and one should not be surprised to obtain estimates that are significantly less than unity on classical t tests.\(^{(4)}\)

From the Ljung-Box test, the marginal significance levels shown in (Table 2) indicate that we can accept the null of no autocorrelation. On the basis of the ARCH test, we can conclude heteroscedasticity is definitely present and may be consistent with an autoregressive form.

The presence of heteroscedasticity requires the use of the Phillips-Perron test when testing for a unit root. The results of the test are reported in (Table 3). In calculating the test statistics a truncation lag, \( l \), corresponding to the maximum order of nonzero autocorrelation in the disturbances had to be chosen. The statistics were computed for \( l = \)
0, 2, 4, 6, and 10 but were found to be remarkably similar for different values of 1. In (Table 3), there is no evidence against the hypothesis of a unit root in the exchange rate series. $Z(t_{\Phi_1})$, $Z(\Phi_2)$ and $Z(\Phi_2)$ are only -2.137, 2.658 and 1.719, respectively. Therefore none of the null hypotheses can be rejected. This means that non-stationarity of the underlying process likewise can not be rejected. Rather it follows a random walk process without drift.

From the results of this chapter, the following facts have emerged concerning the exchange rate series at hand:

(I) The logarithm of the spot exchange rate is approximately uncorrelated over time.

(II) A Lagrange multiplier test for time varying conditional heteroscedasticity effects was found to be highly significant.

(III) The series is well approximated by a random walk without drift.

A final point on this issue, relevant to the next chapter, is that if the model has a unit root, then first differencing should produce a model with a first order moving average, the parameter of which should reflect the difference of the root from unity.\(^{(5)}\) The results of estimating the ARIMA (1, 1, 1) model are presented in (Table 4).

The estimates of the autoregressive term and the moving average term are quite similar. If their true values are the same, the model cannot be distinguished from the random walk model. Namely, the model \([(1-\alpha L)\Delta y_t = (1-\beta L)e_t]\) would turn out to be the random walk model \([\Delta y_t = e_t]\), since both lag polynomials are canceled out if $\alpha = \beta$. This argument motivates testing the null hypothesis $\alpha = \beta$. The $t$ statistics is 0.9214, and hence the null cannot be rejected.

We conclude that the analysis in the next chapter would at most allow for the possibility of a multivariate ARIMA (1, 1, 1).
CHAPTER III
NOTES

(1) The three alternatives commonly proposed for $y_0$ are (c.f. Phillips (1987)):
   (a) $y_0 = c$, a constant, with probability one;
   (b) $y_0$ has a certain specified distribution;
   (c) $y_0 = y_N$, where $N$ = the sample size.

Equation (c) is a circularity condition, due to Hotelling, that is used mainly as a
mathematical device to simplify distribution theory. (b) is a random initial condition that is
frequently used to achieve stationarity in stable models. In this paper, we shall employ
(a), so that $c$ is taken from the actual data. Therefore, the tests discussed below are not the
most powerful tests against the alternative that $\{y_t\}$ is a stationary autoregressive process.

(2) See Bollerslev (1987).

(3) The variance function can be further generalized to include current and lagged $x$'s as
the explanatory variables. An extension of the analysis (GARCH) is done by Bollerslev

(4) See Granger and Newbold (1977) for a spurious regression of one random walk on
another.

(5) Differencing is not necessarily the best solution to the non-stationarity issue, but could
be a second-best strategy. See Sheen (1986) for the details of this point.
CHAPTER IV
MULTIVARIATE MODELING

In the previous chapter, we were unable to reject the hypothesis that the logarithm of the exchange rate obeys a random walk with a heteroscedastic noise innovation. The next stage of the analysis seeks to reduce the standard error of these residuals by introducing relevant explanatory variables. For the explanatory variables to be consistent with the exchange rate following a random walk, it would appear they would also need to be serially uncorrelated. Possible candidates therefore are unanticipated shocks of various sorts.

In order to explain the variability of exchange rates, international economists have moved away from the flow demand/supply analysis to the asset approach to exchange rate determination. It is suggested that exchange rates should be viewed as prices of durable assets determined in organized markets (like stock and commodity exchanges) in which current prices reflect the market's expectation concerning present and future economic conditions relevant for determining the appropriate values of the durable assets, and in which price changes are largely unpredictable and reflect primarily new information that alters expectations concerning these present and future economic conditions.

This approach has a number of important implications. First, expectations will be important in the determination of the current exchange rates. Since monies are durable, in the sense that they last for a number of periods, expectations about future exchange rates will affect the current exchange rates. Moreover, the fact that the exchange rate is the relative price of two monies implies that one reason why agents may alter their beliefs about the expected exchange rate could be due to a change in the money supply (or the fiscal deficit to the extent that it is financed by printing money) expected to prevail in the future. The importance of expectations in foreign exchange markets should result in a close correspondence between actual exchange rates and the markets' expected future exchange rate.

Secondly, using a monetary framework to analyze the exchange rate is useful since it implies that real factors can affect the exchange rate, but only to the extent that they first
affect the demand for (or supply of) money. Also, since assets are stocks, equilibrium is defined as a situation where the stock demand for money is equal to the stock supply of money. Flows of assets across the foreign exchanges can occur, but such flows are a reflection of disequilibrium between money demand and money supply and must eventually cease. A final implication of regarding the exchange rate as an asset price is that such prices are usually regarded as being determined in efficient markets. Under certain circumstances this implies that exchange rates should behave randomly: they should follow a random walk.(1)

Thus proponents of the asset approach (see, for example, Mussa(1979)) argue that one should use tools normally used for the determination of other asset prices (such as stock and share prices) in analyzing the determination of exchange rates, rather than analyzing exchange rates in terms of flow demands and flow supplies. This general notion of exchange rates as asset prices can be represented in a skeletal model in which the logarithm of the equilibrium exchange rate in period t, denoted by \( y_t \), is determined by

\[
y_t = z_t + \delta [E_t(y_{t+1}) - y_t],
\]

(4.1)

where \( z_t \) represents the fundamental economic conditions that affect the foreign exchange market in period t, \( E_t(y_{t+1}) \) denotes the expected rate at t+1 conditional on information available at t, and the parameter \( \delta \) measures the sensitivity of the current exchange rate to the expected rate of change of the exchange rate between t and t+1.

1. Forecasting the Future Values of Exogenous Variables

In the introductory remarks to this chapter it was argued that regarding the exchange rate as an asset price means that expectation about the future course of its determinants will be important for the current determination of the exchange rate. In this section we consider the way economic agents forecast the movements of the exogenous variables, i.e. \( z_t \) vector in Equation (4.1). (2)
The agents are assumed to predict future values of the exogenous variables making as little error as possible. For this reason, the optimal forecasts are considered to be the forecast which has the minimum mean square forecast error. Since the forecast error is a random walk, we choose the forecast \( z_t(k) \) so that \( E \{ [z_t+k - z_t(k)]^2 \} \) is minimized. Thus it can be shown that if \( \{ z_t \} \) is mean and covariance stationary, the forecast is given by the conditional expectation of \( z_{t+k} \), that is by

\[
z_t(k) = E \left( z_{t+k} | z_t, z_{t-1}, \ldots, z_1 \right) = E_t \left( z_{t+k} | \Omega_t \right) = E_t (z_{t+k}) \quad (4.2)
\]

which is denoted by \( \hat{z}_{t+k} \) for simplicity in notation. So the term \( \hat{z}_{t+k} \) for \( k > 0 \) represents expectations by agents in the market about current and future \( z \) based only on information available up to time \( t \).

Consider the general ARIMA model

\[
\phi(L) \Delta^d z_t = \theta(L) \epsilon_t \quad (4.3)
\]

which is expressed as a purely moving average process of infinite order. Then

\[
z_{t+k} = \psi_0 \epsilon_{t+k} + \psi_1 \epsilon_{t+k-1} + \cdots + \psi_k \epsilon_t + \psi_{k+1} \epsilon_{t-1} + \cdots
\]

\[
= \psi_0 \epsilon_{t+k} + \psi_1 \epsilon_{t+k-1} + \cdots + \psi_{k} \epsilon_{t+1} + \sum_{j=0}^{\infty} \psi_{k+j} \epsilon_{t-j} \quad (4.4)
\]

In this equation, the infinite sum is divided into two parts, the second part beginning with the term \( \psi_k \epsilon_t \), and thus describing information up to and including time period \( t \).

Our objective is to compare the optimal forecast \( \hat{z}_{t+k} \) with the actual value \( z_{t+k} \) as expressed in (4.4). To do so, the forecast is written as a weighted sum of those error
terms which can be estimated, namely, \( e_0, e_{t-1}, \ldots \). Then, the desired forecast is

\[
\hat{z}_{t+k} = \sum_{j=0}^{\infty} \psi_{k+j}^* e_{t-j}
\]

where weights \( \psi^*_{t+j} \) are to be chosen optimally so as to minimize the mean square forecast error.

We can now write an expression for the forecast error, \( e_{t+k} \), using Eqns.(4.4) and (4.5):

\[
e_{t+k} = z_{t+k} - \hat{z}_{t+k} = \psi_0 e_{t+k} + \psi_1 e_{t+k-1} + \cdots + \psi_{k-1} e_{t+k-1} \\
+ \sum_{j=0}^{\infty} (\psi_{k+j}^* - \psi_{k+j}) e_{t-j}
\]

Since by assuming \( E(e_i e_j) = 0 \) for \( i \neq j \), the mean square error is

\[
E[e_{t+k}^2] = (\psi_0 + \psi_1 + \cdots + \psi_{k-1}) \sigma_e^2 + \sum_{j=0}^{\infty} (\psi_{k+j}^* - \psi_{k+j}) \sigma_e^2
\]

Clearly this expression is minimized by setting the "optimum" weights \( \psi_{k+j}^* \) equal to the true weights \( \psi_{k+j} \) for \( j = 0, 1, \ldots \). But then the optimal forecast \( z_{t+k} \) is just the conditional expectation of \( z_{t+k} \). The expected values of \( e_{t+k}, \ldots, e_{t+1} \) are all 0, while the expected values of \( e_t, e_{t-1}, \ldots, \) are just the residuals from the estimated equation which are observable. Thus we have

\[
z_{t+k} = \sum_{j=0}^{\infty} \psi_{k+j}^* e_{t-j} = E(y_{t+k} | y_0, \ldots, y_t)
\]
It is to be noted that minimum mean square error forecast is defined in terms of the conditional expectation, which theoretically requires knowledge of the z's stretching back into the infinite past.

The actual computation of the forecast $z_{t+k}$ can be done recursively using the estimated ARIMA model. This involves first computing a forecast one period ahead, using this forecast to compute a forecast two periods ahead, and continuing until the k-period forecast has been reached.

An observation $z_{t+k}$ generated by the process (4.3) may be expressed in some different ways. Box and Jenkins (1977) provide the explicit forms for the model: (1) Difference equation approach, (2) forecast function in integrated form and (3) forecast as a weighted average of previous observations. To obtain the forecast $z_{t+k}$, one writes down the model for $z_{t+k}$ in any one of the above forms. The forecasts are most easily computed from the difference equation itself. Therefore, in order to arrive at the forecast $z_{t+k}$, let us write the ARIMA $(p,d,q)$ model as

$$w_t = \phi_1 w_{t-1} + \cdots + \phi_p w_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q} + \delta$$

with $w_t = \Delta^d z_t$

For the one-period forecast of $w_t$, $w_{t+1}$, write Eqn. (4.9) with the time period modified:

$$w_{t+1} = \phi_1 w_t + \cdots + \phi_p w_{t-p+1} + \varepsilon_{t+1} - \theta_1 \varepsilon_t - \cdots - \theta_q \varepsilon_{t-q+1} + \delta$$

We then calculate the forecast $w_{t+1}$ by taking the conditional expected value of $w_{t+1}$ in (4.10):

$$w_{t+1} = E(w_{t+1} | w_t, w_{t-1}, \ldots) = \phi_1 w_t + \cdots + \phi_p w_{t-p+1} - \hat{\theta}_1 \varepsilon_t - \cdots$$
- \theta \hat{e}_{t+q+1} + \delta \\
\text{(4.11)}
\[ z_t = a + v_t + u_t \]

where \( u_t \) and \( w_t = v_t - v_{t-1} \) are independently distributed random variables, both serially uncorrelated.

Then the model is \( \Delta z_t = (1-\theta L) e_t \). At time \( t+k \), the model may be written

\[ z_{t+k} = z_{t+k-1} + \varepsilon_{t+k} - \theta e_{t+k-1} \quad (4.14) \]

Taking conditional expectations at \( t \),

\[ tz_{t+1} = z_t - \theta e_t \quad (4.15) \]
\[ tz_{t+k} = tz_{t+k-1} \quad k \geq 2 \]

Hence, for all lead times, the forecasts at origin \( t \) will follow a straight line parallel to the time axis. Using the fact that \( z_t = t_{-1} z_t + \varepsilon_t \), we can write (4.15) in either of two useful forms.

The first of these is

\[ tz_{t+k} = t_{-1} z_t + \lambda e_t \quad (4.16) \]

where \( \lambda = 1- \theta \). This implies that, having seen that our previous forecast \( t_{-1} z_{t+k-1} \) falls short of the realized value by \( \varepsilon_t \), we adjust it by an amount \( \lambda e_t \). It is worth noting that \( \lambda \) measures the proportion of any given shock \( \varepsilon_t \), which is permanently absorbed by the
"level" of the process. Therefore, it is reasonable to increase the forecast by that part $\lambda \varepsilon_t$ of $\varepsilon_t$, which we expected to be absorbed.

The second way of rewriting (4.15) is

$$t z_{t+k} = \lambda z_t + (1-\lambda) t z_{t+k-1} \tag{4.17}$$

This implies that the new forecast is a linear interpolation, with weights $\lambda$ and $1-\lambda$, between the old forecast and the new observation. The form (4.17) makes it clear that if $\lambda$ is very small, we shall be relying principally on a weighted average of past data and heavily discounting the new observation $z_t$. By contrast, if $\lambda = 1$, the evidence of past data is completely ignored, $t z_{t+1} = z_t$, and the forecast for all future time is the current value.

With $\lambda > 1$, we induce an extrapolation rather than an interpolation between $t z_t$ and $z_t$. The forecast error must now be magnified in (4.16) to indicate the change in the forecast.

(B) Exogenous Variable Forecasts

The Box and Jenkins technique was applied to M3 and the nominal and real industrial production of Japan and the U.S. (all deseasonalised). The data were chosen from the same source as the exchange series. The optimal forecasts were obtained using data going back to the beginning of 1970, sequentially adding on data until December, 1985.

It is first necessary to specify the stochastic processes of the underlying exogenous variables. The identification step of the analysis suggested that U.S. real output and Japanese nominal output follow ARIMA (0, 1, 1) processes, U.S. nominal output and Japanese real output follow the random walk, and both countries' money supplies follow ARIMA (0, 2, 1) processes. We therefore have the following suggested specifications:
\[ \Delta Q_t = \delta_1 + \varepsilon_{1t} - \theta_1 \varepsilon_{1t-1} \]
\[ \Delta Q_t^* = \delta_2 + \varepsilon_{2t} \]
\[ \Delta q_t = \delta_3 + \varepsilon_{3t} \]
\[ \Delta q_t^* = \delta_4 + \varepsilon_{4t} - \theta_4 \varepsilon_{4t-1} \]
\[ \Delta^2 m_t = \delta_5 + \varepsilon_{5t} - \theta_5 \varepsilon_{5t-1} \]
\[ \Delta^2 m_t^* = \delta_6 + \varepsilon_{6t} - \theta_6 \varepsilon_{6t-1} \]

The results of the Box-Jenkins estimation of these equations are reported in (Table 5). Also reported are the properties of the forecasts in (Table 6). By comparing the root mean square error (RMSE), we can deduce the benefit from using the Box and Jenkins forecasting technique to that of a pure random walk for z's. Evidently, there is a gain, for all but US money supply. All forecasts errors have means that are not significantly different from zero.

2. A Simple Model

To demonstrate the possibility of multiple solutions under rational expectations, consider the simplest member of (4.2):

\[ y_t = \alpha z_t + \beta y_{t+1} + u_t \quad (4.18) \]

The exchange rate at t depends on the expectation of its value at t+1. With rational expectations, the model then suffers from the problem of multiple solutions.

Let the solution take the form

\[ y_t = C + R_0 u_t + R_1 u_{t-1} + \cdots + K_0 z_t + K_1 z_{t-1} + \cdots \quad (4.19) \]
where the sums are infinite. Taking conditional expectations of the terms in (4.18) given the information $\Omega_t$ up to the beginning of $t$, with $z_t = z_v$ and subtracting the result from (4.18), we have

$$y_t - \gamma_t = u_t$$

Performing the same operations on (4.19) gives $y_t - \gamma_t = R_0 u_t$, implying $R_0 = 1$.

Advancing the time subscripts of (4.18) by 1 yields

$$y_{t+1} = C + u_{t+1} + R_1 u_t + R_2 u_{t-1} + \cdots + K_{0} z_{t+1} + K_{1} z_t + K_{2} z_{t-1} + \cdots$$

and taking expectations given $\Omega_t$ gives

$$\gamma_{t+1} = C + R_2 u_{t+1} + K_0 z_{t+1} + K_1 z_t + K_2 z_{t-1} + \cdots$$

The difference of these two equations is

$$y_{t+1} - \gamma_{t+1} = u_{t+1} + R_1 u_t + K_0 (z_{t+1} - z_{t+1}) \quad (4.20)$$

where $z_{t+1} - z_{t+1}$ is the error in forecasting $z_{t+1}$ one-period ahead, to be denoted by $tFz_{t+1}$.

Using (4.20) to substitute for $\gamma_{t+1}$ in (4.18), we obtain the model

$$y_t = \alpha z_t + \beta y_{t+1} + R_1 u_t + K_0 (tFz_{t+1}) + u_t \quad (4.21)$$

which no longer involves any expectation variables. To show that model (4.21) solves the original model (4.18), we take expectations of (4.21) given $\Omega_t$. 
\[ iy_t = \alpha z_t + \beta y_{t+1} \quad (4.22) \]

Subtract this from (4.18) to yield

\[ y_t - ty_t = u_t. \]

When \( y_t \) in (4.22) is replaced by \( y_t - u_t \), the original model (4.18) results.

Equation (4.21) is a dynamic model explaining \( y_{t+1} \), which is consistent with the original model (4.18) but does not utilize any expectation variables. It has two more parameters \( R_1 \) and \( K_0 \) than the original model. If only model (4.18) is given, one is free to choose the values of these parameters and thus produce many stochastic models in the form of (4.21) which are consistent with the specification (4.18). In other words, model (4.18) has multiple solutions which are generated by different values of parameters \( R_1 \) and \( K_0 \) in (4.21).

There are two strategies for solving the multiplicity of solutions problem. The first utilizes the weak rationality assumption that the unexpected component of the exchange rate arises only on account of information that appeared after the expectation was formed. The second is the standard method of finding the convergent solution for the deterministic difference equation in future expectations.

(A) Weak Rationality

The assumption of weak rationality implies that the conditional forecast error of the exchange rate does not depend on information available at \( t \).

\[ y_{t+1} - ty_{t+1} = \varepsilon_{t+1} \quad (4.23) \]

where
\[ \varepsilon_{t+j} = R_0 u_{t+j} + R_1 u_{t+j-1} + \ldots + R_j u_t \]
\[ + K_0 (z_{t+j} - t z_{t+j}) + K_1 (z_{t+j-1} - t z_{t+j-1}) \]
\[ + \ldots + K_{j-1} (z_{t+1} - t z_{t+1}) \]

The forecast error is conjectured to depend on unexpected events that occur between \( t \) and \( t+j \). As mentioned before, \( R_0 = 1 \). The remaining \( R_1, \ldots, R_j, K_0, \ldots, K_{j-1} \) are free parameters which have to be estimated. To obtain an equation to estimate, consider (4.18) dated at \( t+j \), take expectations as of \( t \), apply the law of iterated expectations, and replace the expected terms from (4.23). This leaves an equation in observables only with a jth order moving average error process.

For \( j = 1 \), this becomes

\[ y_{t+1} = \frac{1}{\beta} y_t - \frac{\alpha}{\beta} z_t + K_0 (z_{t+1} - t z_{t+1}) \]
\[ + u_{t+1} - \left( \frac{1}{\beta} - R_1 \right) u_t \]  

(4.24)

(B) Strong Rationality and Convergent Expectations

In stochastic systems, variables which depend upon their future expectations are the equivalent of non-predetermined variables in deterministic systems. For a unique solution to either system, the number of "unstable" eigenvalues must equal the number of these variables.

Leading (4.18) and taking expectations yields a deterministic ordinary difference equation in expected future exchange rates:
\[ \nu_{t+2} = \frac{1}{\beta} \nu_{t+1} - \frac{\alpha}{\beta} \nu_{t+1} \]

This can easily be solved to give the general solution combining particular and homogeneous elements. One obtains:

\[ \nu_{t+k} = \alpha \sum_{i=0}^{\infty} \beta^i \nu_{t+k+i} + d \beta^{-t} + \sum_{i=t-T}^{t} \beta^{i-t} S_i \]  \hspace{1cm} (4.25)

The first part of the solution is often referred to as the "fundamentals" and is the sum of discounted expected future z's. The existence of the forward sum requires \( \beta < 1 \), unless \( \nu_{t+k+i} \) is declining geometrically. The second term is known as a "deterministic bubble", while the third is a "stochastic bubble". d is an arbitrary constant, while \( S_i \) is a serially uncorrelated random vector which has the critical feature that

\[ \nu_{t+j} = 0 \hspace{1cm} \text{for all} \ j > 0. \]

When one inserts the solution for the expected exchange rate (4.25) into (4.18), one gets

\[ y_t = \alpha \sum_{i=0}^{\infty} \beta^i \nu_{t+i} + d \beta^{-t} + \sum_{i=t-T}^{t} \beta^{i-t} S_i + u_t \]

The strong convergent solution for the exchange rate excludes bubbles of any form, so that \ d and \( S_i \) are always zero, or

\[ y_t = \alpha \sum_{i=0}^{\infty} \beta^i \nu_{t+i} + u_t \]  \hspace{1cm} (4.26)
Computing (4.26) at $t+1$ and subtracting from it, $1/\beta$ of (4.26) at $t$ gives the strong form as

$$y_{t+1} - \frac{1}{\beta} y_t = \alpha \sum_{i=0}^{\infty} \beta^i z_{t+1+i} - \alpha \sum_{i=0}^{\infty} \beta^{i-1} z_{t+i+1} u_{t+1} - \frac{1}{\beta} u_t$$

$$= -\frac{\alpha}{\beta} z_t + \alpha \sum_{i=0}^{\infty} \beta^i (z_{t+i+1} - z_{t+i+1}) + u_{t+1} - \frac{1}{\beta} u_t \quad (4.27)$$

Comparing (4.27) to the weak form solution (4.24), one can see that the two are identical in expectations as of $t$ (because the law of iterated expectations eliminates the forecast error terms). However, the solutions for the actual exchange rate differ because of the existence of $K_0$ and $R_1$ in (4.24) and because updated forecasts of all future $z$ between $t$ and $t+1$ are relevant in (4.27). Fortunately, because of the law of iterated projections, it turns out that (4.27) is a special case, of (4.24) if $R_1=0$.

3. Simple Monetary Model

(a) Model

The simple monetary model predicts that movements in the rate of exchange between two currencies will be determined by current and anticipated future movements in the supplies of, and demands for, the two currencies. The model discussed here is a specific case of the flex-price monetary approach, having the following characteristics. First, the country is assumed to be small and is facing an exogenously determined foreign interest rate, $i^*$, and price level $p^*$. The home country produces a single traded good which is a perfect substitute for the foreign good and thus purchasing power parity holds continuously. The central bank of the home country issues money, which is assumed to be non-traded, and a bond. Since it is further assumed that asset holders can adjust their portfolios immediately
after a disturbance, and thus capital is perfectly mobile, uncovered interest parity must also hold. Secondly, the money supply is assumed to be determined exogenously and money markets are in equilibrium.

Consider the first differenced, two country model satisfying the assumptions above. Then the model becomes:

\[ \Delta m_t - \Delta m_t^* = \Delta p_t - \Delta p_t^* + \alpha_q \Delta q_t - \alpha_q^* \Delta q_t^* - \beta (\Delta i_t - \Delta i_t^*) + v_{1t} \]

\[ \Delta p_t = \Delta p_t^* - \Delta y_t + v_{2t} \]

\[ \Delta i_t = \Delta i_t^* - \Delta (y_{t+1} - y_t) + v_{3t} \]

where \( m \) is the logarithm of the domestic money supply, \( q \) is the logarithm of the real output, \( i \) is the interest rate, \( p \) is the logarithm of the price level, and "*" variables are the foreign counterparts.

Then these equations reduce to:

\[ \Delta y_t = \frac{1}{1 + \beta} \left[ (\Delta m_t - \Delta m_t^*) - \alpha_q \Delta q_t + \alpha_q^* \Delta q_t^* \right] + \frac{\beta}{1 + \beta} \Delta y_{t+1} + u_t \quad (4.28) \]

Applying weak rationality to \( \Delta y_{t+1} \), the equation for \( y \) turns out to be at least an ARIMA (1, 1, 2) Since the time series model seems to indicate, at most, an ARIMA (1, 1, 1) process, it would seem that only the strong convergent solution may be appropriate. In fact, the weak rationality solution does not converge.\(^3\) Applying the assumption of strong rationality and convergent expectations produces the convergent solution\(^4\):

\[ \Delta y_t = (1 + \beta \Delta) [\Delta m_t^* - \Delta m_t^* - \alpha_q \Delta q_t + \alpha_q^* \Delta q_t^*] + u_t \quad (4.29) \]
where the $\bar{\Delta z}$ are expected growth rates as of $t$.

In a stochastic steady state, exchange rate appreciation simply reflects relative expected money growth and output growth weighted by its elasticity. Outside of the steady state, it is a weighted average of current and lagged values of these expected terms. This monetary equation would be attempting to explain the drift as a time-varying phenomenon. An important feature is that an increase in the currently expected domestic (foreign) rate of growth of money leads to a larger depreciation (appreciation).

(b) Empirical Results

In (Table 5), the OLS estimates of the parameters in equation (4.29) are presented.

The estimated sign of $\alpha_q$ is incorrect, all the parameter estimates are insignificant, and sum of squared errors shows no improvement upon the random walk model. The conclusion is that the joint test of the first difference monetary model with convergent rational expectations, and the hypothesized process governing expectations of the money and real output variables makes it sure that we cannot reject the hypothesis of random walk. It is, therefore, not meaningful to examine the forecasting performance of this model.

4. Fix-Price Model

(a) Model

The economy is assumed to be small and facing an exogenously given foreign price and interest rate. The monetary authorities issue domestic money, which is held only in the home country, and a foreign bond. Capital is perfectly mobile. The country is assumed to be operating at full employment and produces a single traded good which is assumed to be an imperfect substitute, at least in the short run, for the foreign produced good. Thus although PPP is assumed to hold in the long run, in the short run deviations from PPP may be allowed and these may be considerable.

Nominal output, $Q_t^*$ and $Q_n$, is also assumed to be the scale flow variable. Net wealth
effects may also influence money demand; and these are assumed to be correlated with past exchange rates and/or money supplies and/or nominal output. In general, consider the following form of an inverted relative money demand equation:

\[ y_{t+1} - y_t = \frac{1}{\beta} \left[ \alpha_0 + \alpha_y(L)y_t + \alpha_m(L)m_t + \alpha_q(L)Q_t - \alpha^*_m(L)m_t^* - \alpha^*_q(L)Q_t^* \right] + u_t \]  

(4.30)

\( \alpha_y(L)y_t \) provides an error correction mechanism which has an important interpretation in terms of equilibrium relationships. If the economy tends always to be at or near the equilibrium, then the variables involved should be influenced by the extent to which the economy is out of equilibrium, and this occurs in (4.30) through the term \( \alpha_y(L)y_t \). The lag polynomial \( \alpha_y(L) \) had a maximum order of 1. If at least one of the first elements in \( \alpha_m(L) \), \( \alpha_m^*(L) \), \( \alpha_y(L) \), or \( \alpha_y^*(L) \) are not restricted to unity, then \( \beta \) cannot be identified in (4.30). For the reason of parsimony, the lag polynomials in money and output of both countries were restricted to a zero-order. Applying weak rationality, (4.30) becomes,

\[ \Delta y_{t+1} = \frac{1}{\beta} \left[ \alpha_0 + \alpha_y y_{t-1} + \alpha_m m_t - \alpha_q Q_t - \alpha^*_m m_t^* - \alpha^*_q Q_t^* \right] + K_m f_{m_{t+1}} - K_q f_{Q_{t+1}} - K_m f_{m_{t+1}}^* - K_q f_{Q_{t+1}}^* + u_{t+1} - (1 - R_j)u_t \]  

(4.31)

where \( f_{z_{t+1}} = z_{t+1} - z_{t+1} \) is the unexpected innovation in \( z \) between \( t \) and \( t+1 \).
(b) Empirical Results

In (Table 8) and (Table 9), six variations of the equation (4.31) are estimated. In the first class, the error correction mechanism is included, while in the second, it is absent. Within each class of models, some parameters are restricted to be unity. We therefore have following models

Model (i): Partial Homogeneity Model With Error Correction
Model (ii): Full Homogeneity Model With Error Correction
Model (iii): Unrestricted Model With Error Correction
Model (iv): Partial Homogeneity Model Without Error Correction
Model (v): Full Homogeneity Model Without Error Correction
Model (vi): Unrestricted Model Without Error Correction

Models (ii) and (v) fix the parameters on money and nominal output at unity, while (i) and (iv) place the same restriction on money alone. All parameters are expected to be positive except $\alpha_y$ and $\alpha_0$ which can take any sign. The K parameters measure the effect of unexpected money and nominal output.

Models (i) and (ii) include the error correction mechanism and place some restrictions on the parameters on money and nominal output. Only the constant terms and the error terms have significant coefficients. Model (iii) gives unrestricted estimates of money and nominal output parameters. Except the constant term and the Japanese money supply, all the parameters have insignificant coefficients. To compare this model with the random walk model without drift, the joint significance of the complete set of explanatory variables is tested by computing F value. We get $F(11, 143) = 9.39$ which is sufficiently large to lead to the rejection of the null hypothesis that none of the explanatory variables is significant. Few of the explanatory variables are found to be significant according to simple t test, and yet, as a whole, the explanatory variables are do in fact explain a significant proportion of the variation in the dependent variable. This indicates that the explanatory variables are so highly intercorrelated that it is not possible to separate out their individual influences (the problem of multicollinearity).
In models (iv) and (v), none of the parameters has a significant coefficient. The results from the model (vi) suggest that the parameters of the constant term, the Japanese money supply, and the unexpected change in the U.S. nominal output are significant at the 10% level. Compared to the random walk without drift, we get $F(10, 144) = 12.70$, so that we can confidently reject the null hypothesis. Therefore, the model yields a significant over the random walk without drift.

The weak rational moving average parameter, $R_1$, was generally insignificant. This is the same result as we found in the previous chapter when investigating the univariate model.

A key test of model capability is obtained with both within-sample and out-of-sample forecasting. For the latter, the actual exchange rate outcome from January, 1986 to September, 1987 was compared with the unconditional forecasts from the following nine models: the random walk with and without drift, univariate ARIMA (1, 1, 1) and the six fix-price models mentioned in the last section (i.e. homogeneity on money alone, homogeneity on money and output, and unrestricted estimates for the models both with and without error correction mechanism). The summary of statistics on forecasting performance (RMSE, MAE, ME) for those models are listed in (Table 10) and (Table 11).

On inspection it is clear that for all the criteria, (iii), (iv), (v) and (vi) are ranked better than the random walk process. In particular, the unrestricted model without error correction mechanism does about three times better than the random walk and it predicted a strengthening of the exchange rate.

This out-of-sample performance might be attributed to the specific sample we chose. In fact, a comparison of the models based on estimates using both within- and out-of-sample data is unlikely to yield as good a result as we obtained in the out-of-sample forecasts. As a result, I am reluctant to draw any firm conclusions.

The structural models would fail to explain and forecast exchange rates when they are misspecified. A crucial source of misspecification is the inadequate modeling of expectations formation in existing structural models. The presence of the expected values of exogenous variables brings difficulties into comparing models on common ground. The other possible causes of the breakdown of empirical exchange rate models may be
volatile time-varying risk premia, volatile long-run real exchange rates, or instability and misspecification of the money demand functions. In particular, modeling time-varying risk premia is likely to be able to describe the time dependent conditional heteroscedasticity which we did not answer when modeling multivariate models. A further point is that current account announcements were not modeled in this study. Some of the recent unexplained dramatic changes in the exchange rate coincided with unanticipated trade balance deficits or surpluses. The simultaneous modeling of the current account and the exchange rate is expected to improve the explanation of the data generation process.

Despite the above mentioned limitations, our results suggest at the least that some structural models might be able to out-perform a simple random walk model.
(1) See, for example, Levich (1979) for the circumstances required to hold for the exchange rate to follow a random walk.

(2) Equation (4.1) implies as one solution

\[(1 + \delta - \delta L^{-1}) E_t y_t = E_t z_t \]

\[\left(1 + \delta \right)^{-1} \left(1 - \frac{\delta}{1 + \delta} L^{-1} \right) E_t y_t = E_t z_t \]

which yields

\[y_t = y_{t-1} + \sum_{i=0}^{\infty} \frac{\delta}{1 + \delta} \left( E_t z_{t+i} - E_t z_{t+i+1} \right) \]

This is not the only solution but it motivates the discussion of exogenous variables forecasting in the existing section.

(3) Applying the weak rationality assumption yields the solution

\[y_{t+1} = \frac{1 + \beta}{\beta} y_t + \frac{1}{\beta} z_t - K_0(z_{t+1} - \bar{z}_{t+1}) + u_{t+1} - \left( \frac{1 + \beta}{\beta} - R_1 \right) u_t \]

Since \( \frac{1 + \beta}{\beta} > 1 \), it does not converge.

(4) Taking expectations of (4.28) at t-1, and writing the vector of explanatory variables and parameters inside the square brackets as \( \Delta z_t \) and \( \alpha \), we get
\[
(1 + \beta (1-L^{-1}))_{t-1} \Delta y_t = \alpha_{t-1} \Delta z_t
\]

which factorizes to give

\[
t_{-1} \Delta y_t = \left( \frac{1}{1+\beta} + \frac{\beta}{(1+\beta)^2} L^{-1} + \frac{\beta^2}{(1+\beta)^3} L^{-2} + \ldots \right) \alpha_{t-1} \Delta z_t \tag{(*)}
\]

When \( z_t \) obeys an ARIMA \((0, 2, 1)\), \( \Delta z_t \) has an ARIMA \((0,1,1)\) process. From (4.15), \( \Delta z_{t+k} \) are the same for any \( k \geq 1 \). It depends only on the information available up to \( t \). This argument also holds for the other exogenous variables whose processes have lower order.

Let \( t_{-1} \Delta z_t = \overline{\Delta z}_{t-1} \). Then \( L^{-j} t_{-1} \Delta z_t = \overline{\Delta z}_{t-1} \).

Hence (*) becomes

\[
\left( \frac{1}{1+\beta} + \frac{\beta}{(1+\beta)^2} \overline{\Delta z}_{t-1} + \frac{\beta^2}{(1+\beta)^3} \overline{\Delta z}_{t-1} + \ldots \right) \alpha \overline{\Delta z}_{t-1} = \alpha \overline{\Delta z}_{t-1}
\]

Then (4.28) is rewritten as:

\[
(1 + \beta) \Delta y_t = \alpha \Delta z_t + \beta (\alpha \Delta y_{t+1} - t_{-1} \Delta y_t + y_t - y_{t-1}) + u_t
\]

\[
= \alpha \overline{\Delta z}_t + \beta [\alpha (\overline{\Delta z_t} - \overline{\Delta z}_{t-1}) + \Delta y_t] + u_t
\]

Therefore,

\[
\Delta y_t = (1 + \beta \Delta) \alpha \overline{\Delta z}_t + u_t
\]
CHAPTER V
CONCLUSIONS

This study re-examined the result of Meese and Rogoff on the forecasting performance of monetary type structural exchange rate models *vis-a-vis* the random walk model. I re-considered the issue for the case of the Japanese yen for the period from March, 1973 to September, 1987.

The series at first appeared not to be a random walk. When account was taken of the appropriate distribution under the null hypothesis of a unit root and of the existence of heteroscedasticity, a random walk without drift could not be rejected.

The results of the multivariate modeling suggest that the estimates of the simple monetary model were not supported by data, while fix-price models were. Using the latter approach, we found that some structural models performs better than the random walk model based on the forecasting accuracy (for both within- and out-of-sample).

In conclusion, it would seem that the explicit incorporation of the hypothesis of rational expectations permits the dynamics of the forces influencing the exchange rate process to be captured more accurately and this study can be considered part of the increasing amount of evidence which concludes that the monetary model of the exchange rate behavior does have empirical content.
Table 1
Estimation of Eqn. (3.3)

| Parameter | Estimate  | Standard Error | T ratio | Prob > |T| |
|-----------|-----------|----------------|---------|--------|---|
| \( \xi_0 \) | 0.233618  | 0.132564       | 1.762   | 0.0800 |
| \( \xi_1 \) | -0.000102 | 0.000069       | -1.474  | 0.1426 |
| \( \alpha \) | 0.957313  | 0.024021       | 39.854  | 0.0001 |

| R\(^2\) | 0.9408 |
| SSE     | 0.1528 |
| ARCH(8) | 19.1884 |
| ARCH(12)| 21.6524 |

Table 2
Ljung-Box Test for Autocorrelation

<table>
<thead>
<tr>
<th>Lag</th>
<th>Q</th>
<th>DF</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5.30</td>
<td>6</td>
<td>0.506</td>
</tr>
<tr>
<td>12</td>
<td>8.32</td>
<td>12</td>
<td>0.760</td>
</tr>
<tr>
<td>18</td>
<td>16.93</td>
<td>18</td>
<td>0.528</td>
</tr>
<tr>
<td>24</td>
<td>26.03</td>
<td>24</td>
<td>0.352</td>
</tr>
<tr>
<td>30</td>
<td>34.56</td>
<td>30</td>
<td>0.259</td>
</tr>
</tbody>
</table>
Table 3

Phillips-Perron Test for a Unit Root

\[ \frac{s^2}{s_0^2} = 0.000992 \quad s_0^2 = 0.001019 \]
\[ C_3 = 0.538469 \quad D_X = 86867003.63 \]
\[ Y = 5.515008 \quad Y_{-1} = 5.516941 \]

H_01: \( \alpha = 1 \)

\[ t_\alpha = -1.846816 \quad s_u^2 = 0.001013 \quad s_{nl}^2 = 0.001374 \]
\[ Z(t_\alpha) = -2.137 \]

H_02: \( \zeta_1 = 0 \) and \( \alpha = 1 \)

\[ \Phi_3 = 2.42994 \quad s_u^2 = 0.001015 \quad s_{nl}^2 = 0.001383 \]
\[ Z(\Phi_3) = 2.658 \]

H_03: \( \zeta_0 = \zeta_1 = 0 \) and \( \alpha = 1 \)

\[ \Phi_2 = 1.382334 \quad s_u^2 = 0.001019 \quad s_{nl}^2 = 0.001406 \]
\[ Z(\Phi_2) = 1.719 \]
Table 4
ARIMA (1, 1, 1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>T ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.019611</td>
<td>0.003269</td>
<td>5.991</td>
</tr>
<tr>
<td>MA 1, 1</td>
<td>0.670397</td>
<td>0.490277</td>
<td>1.367</td>
</tr>
<tr>
<td>AR 1, 1</td>
<td>0.738142</td>
<td>0.449899</td>
<td>1.641</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>Parameter / Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Q_t$</td>
<td>$\delta_1 0.00678(0.00134)$</td>
</tr>
<tr>
<td>$\Delta Q_t^*$</td>
<td>$\delta_2 0.00781(0.00096)$</td>
</tr>
<tr>
<td>$\Delta q_t$</td>
<td>$\delta_3 0.00337(0.00098)$</td>
</tr>
<tr>
<td>$\Delta q_t^*$</td>
<td>$\delta_4 0.00232(0.00098)$</td>
</tr>
<tr>
<td>$\Delta m_t$</td>
<td>$\delta_5 -0.00004(0.00005)$</td>
</tr>
<tr>
<td>$\Delta m_t^*$</td>
<td>$\delta_6 0.00002(0.00009)$</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>----------------</td>
<td>--------</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Nominal Output (Qₚ)</strong></td>
<td></td>
</tr>
<tr>
<td>Rz</td>
<td>-0.0212</td>
</tr>
<tr>
<td>Fz</td>
<td>-0.0014</td>
</tr>
<tr>
<td><strong>Real Output (qₚ)</strong></td>
<td></td>
</tr>
<tr>
<td>Fz</td>
<td>-0.0009</td>
</tr>
<tr>
<td><strong>Money Supply (mₚ)</strong></td>
<td></td>
</tr>
<tr>
<td>Rz</td>
<td>-0.0040</td>
</tr>
<tr>
<td>Fz</td>
<td>-0.0002</td>
</tr>
<tr>
<td><strong>USA</strong></td>
<td></td>
</tr>
<tr>
<td><em><em>Nominal Output (Qₚ</em>)</em>*</td>
<td></td>
</tr>
<tr>
<td>Fz</td>
<td>0.0016</td>
</tr>
<tr>
<td><em><em>Real Output (qₚ</em>)</em>*</td>
<td></td>
</tr>
<tr>
<td>Rz</td>
<td>-0.0006</td>
</tr>
<tr>
<td>Fz</td>
<td>-0.0004</td>
</tr>
<tr>
<td><em><em>Money Supply (mₚ</em>)</em>*</td>
<td></td>
</tr>
<tr>
<td>Rz</td>
<td>-0.0009</td>
</tr>
<tr>
<td>Fz</td>
<td>-0.0003</td>
</tr>
</tbody>
</table>

† Rz denotes the forecast error of the random walk, while Fz denotes that of the specified process in the text.
Table 7

Estimation of Simple Monetary Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>T ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.32619</td>
<td>0.97084</td>
<td>0.34</td>
</tr>
<tr>
<td>$\alpha_q$</td>
<td>0.22918</td>
<td>0.44118</td>
<td>0.52</td>
</tr>
<tr>
<td>$\alpha_q^*$</td>
<td>-0.00534</td>
<td>0.76920</td>
<td>-0.01</td>
</tr>
<tr>
<td>SSE</td>
<td>0.15821</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0120</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 8

Estimation of Fix-Price Models with error correction mechanism

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Partial Homogeneity (i)</th>
<th>Full Homogeneity (ii)</th>
<th>Unrestricted(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>0.571 (0.032)</td>
<td>0.517 (0.246)</td>
<td>0.910 (0.341)</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>-0.060 (0.032)</td>
<td>-0.054 (0.031)</td>
<td>-0.045 (0.033)</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>1</td>
<td>1</td>
<td>-0.153 (0.080)</td>
</tr>
<tr>
<td>$\alpha_m^*$</td>
<td>1</td>
<td>1</td>
<td>-0.074 (0.059)</td>
</tr>
<tr>
<td>$\alpha_q$</td>
<td>0.006 (0.067)</td>
<td>1</td>
<td>-0.006 (0.066)</td>
</tr>
<tr>
<td>$\alpha_q^*$</td>
<td>0.022 (0.057)</td>
<td>1</td>
<td>0.063 (0.075)</td>
</tr>
<tr>
<td>$K_m$</td>
<td>0.607 (0.614)</td>
<td>0.640 (0.601)</td>
<td>0.600 (0.614)</td>
</tr>
<tr>
<td>$K_m^*$</td>
<td>-0.761 (0.580)</td>
<td>-0.764 (0.573)</td>
<td>-0.775 (0.579)</td>
</tr>
<tr>
<td>$K_q$</td>
<td>-0.165 (0.157)</td>
<td>-0.206 (0.151)</td>
<td>-0.180 (0.157)</td>
</tr>
<tr>
<td>$K_q^*$</td>
<td>-0.030 (0.089)</td>
<td>-0.029 (0.088)</td>
<td>-0.018 (0.089)</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.970 (0.084)</td>
<td>0.964 (0.083)</td>
<td>0.988 (0.084)</td>
</tr>
<tr>
<td>$1/\beta$</td>
<td>0.056 (0.032)</td>
<td>0.054 (0.031)</td>
<td></td>
</tr>
</tbody>
</table>
Table 9

Estimation of Fix-Price Models without error correction mechanism

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Partial Homogeneity (iv)</th>
<th>Full Homogeneity (v)</th>
<th>Unrestricted (vi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>0.148 (0.205)</td>
<td>-0.001 (0.063)</td>
<td>0.710 (0.340)</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>1</td>
<td>1</td>
<td>-0.157 (0.089)</td>
</tr>
<tr>
<td>$\alpha_{m}^*$</td>
<td>1</td>
<td>1</td>
<td>-0.057 (0.064)</td>
</tr>
<tr>
<td>$\alpha_q$</td>
<td>0.050 (0.063)</td>
<td>1</td>
<td>0.032 (0.062)</td>
</tr>
<tr>
<td>$\alpha_{q}^*$</td>
<td>0.036 (0.049)</td>
<td>1</td>
<td>0.124 (0.065)</td>
</tr>
<tr>
<td>$K_m$</td>
<td>0.442 (0.608)</td>
<td>0.516 (0.596)</td>
<td>0.475 (0.608)</td>
</tr>
<tr>
<td>$K_m^*$</td>
<td>-0.795 (0.583)</td>
<td>-0.764 (0.575)</td>
<td>-0.804 (0.580)</td>
</tr>
<tr>
<td>$K_q$</td>
<td>-0.148 (0.158)</td>
<td>-0.183 (0.151)</td>
<td>-0.172 (0.157)</td>
</tr>
<tr>
<td>$K_{q}^*$</td>
<td>-0.016 (0.090)</td>
<td>-0.021 (0.088)</td>
<td>-0.005 (0.089)</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.949 (0.083)</td>
<td>0.944 (0.083)</td>
<td>0.980 (0.084)</td>
</tr>
<tr>
<td>$1/\beta$</td>
<td>0.025 (0.064)</td>
<td>0.00003 (0.027)</td>
<td>.</td>
</tr>
</tbody>
</table>

† Standard errors are reported below parameters estimate as (.).
Table 10

Within Sample Forecasting

<table>
<thead>
<tr>
<th></th>
<th>RMSE</th>
<th>MAE</th>
<th>ME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Walk Without Drift</td>
<td>0.1466</td>
<td>0.1239</td>
<td>-0.0676</td>
</tr>
<tr>
<td>Random Walk With Drift</td>
<td>0.1303</td>
<td>0.1125</td>
<td>-0.0734</td>
</tr>
<tr>
<td>ARIMA(1, 1, 1)</td>
<td>0.1358</td>
<td>0.1176</td>
<td>0.0821</td>
</tr>
</tbody>
</table>

With Error Correction

<table>
<thead>
<tr>
<th></th>
<th>RMSE</th>
<th>MAE</th>
<th>ME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial Homogeneity (i)</td>
<td>0.1287</td>
<td>0.1113</td>
<td>0.0567</td>
</tr>
<tr>
<td>Full Homogeneity (ii)</td>
<td>0.1611</td>
<td>0.1366</td>
<td>0.1281</td>
</tr>
<tr>
<td>Unrestricted (iii)</td>
<td>0.1117</td>
<td>0.0952</td>
<td>0.0350</td>
</tr>
</tbody>
</table>

Without Error Correction

<table>
<thead>
<tr>
<th></th>
<th>RMSE</th>
<th>MAE</th>
<th>ME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial Homogeneity (iv)</td>
<td>0.1369</td>
<td>0.1194</td>
<td>0.0627</td>
</tr>
<tr>
<td>Full Homogeneity (v)</td>
<td>0.1410</td>
<td>0.1222</td>
<td>0.0897</td>
</tr>
<tr>
<td>Unrestricted (vi)</td>
<td>0.1065</td>
<td>0.0920</td>
<td>0.0335</td>
</tr>
</tbody>
</table>
### Table 11

**Out of Sample Forecasting**

<table>
<thead>
<tr>
<th></th>
<th>RMSE</th>
<th>MAE</th>
<th>ME</th>
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<td>0.2539</td>
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<td>0.2316</td>
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<td>0.2124</td>
<td>0.2009</td>
<td>-0.2009</td>
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**With Error Correction**

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<td>Unrestricted (iii)</td>
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**Without Error Correction**

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<td>Full Homogeneity (v)</td>
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<td>Unrestricted (vi)</td>
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Figure 2.3 Within-sample Forecasting III

Exchange Rate

Month

0 100 200

51 52 53 54 55 56 57 58
Figure 3.1 Out-of-sample Forecasting 1

Exchange Rate

Month

4.9
5.0
5.1
5.2
5.3
5.4

Actual
ARIMA(1,1,1)
RW (no drift)
RW (drift)
Figure 3.2: Out-of-sample Forecasting II
REFERENCES


