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APPROXIMATE HARMONIC ANALYSIS
OF MARINE RISERS

by

WHEN-YEN TEIN

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

MASTER OF SCIENCE

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April, 1987
ABSTRACT

A model of a marine riser system which is appropriate for the study of its dynamic behavior in deep water conditions is developed. This model is used to estimate the steady-state riser response to harmonic excitation. In this regard, the finite element method yields a linear discrete multi-degree-of-freedom structural model; its stiffness matrix varies with time due to riser top tension fluctuations. Further, Morison's equation is used for estimating the hydrodynamic load on the riser. Due to the nonlinearity of the drag term appearing in this equation, the riser equation of motion becomes nonlinear. An approximate analysis procedure based on the concepts of equivalent linearization and of time averaging leads to efficient determination of the riser maximum stress. A continuous beam model under constant tension which is equal to the average of the top and the bottom riser tension is used to provide an estimate of its natural frequencies. Numerical results from a variety of parameter studies are reported.
ACKNOWLEDGEMENTS

This work was completed thanks to the patient guidance and support provided by Professor Pol D. Spanos; his dedication and personal concern are gratefully acknowledged.

Suggestions made by Dr. Loren Lutes and Dr. John Merwin are gratefully appreciated. The assistance of A. Senthilnathan, B. Subir, Roger Ghanem and Y. C. Chen has proved immensely helpful. My gratitude is hereby expressed to all.
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I. INTRODUCTION

The exploitation of underwater resources becomes increasingly important and offshore technology for deep water drilling systems comes into sharper focus. As the major component of the drilling systems, a marine riser provides a path for the drilling mud and extends from the well bore at the seabed to the floating vessel. Environmental safety and economic feasibility considerations require a complete understanding of the behavior of the riser system under various wave and current loading situations.

Figure 1 shows a layout for a typical drilling riser system. Usually, the system consists of a large diameter steel pipe with varying wall thickness and a length ranging from 150 ft. to 4000 ft., depending on the drilling location. The excitation forces come from ocean currents and waves. The dynamic response of the riser is influenced by the surge, sway, and heave motions of the floating vessel, by its boundary conditions, and by its time-dependent top tension.

Problems in riser design and analysis have been reviewed in a series of articles (Morgan and Peret, 1971-1976). Riser design has often been based on static models (Fischer and Ludwig, 1966). The results obtained are applicable only to shallow water depths, moderate drilling vessel motions, and relatively small riser diameters. These
assumptions are inappropriate for analysis of riser systems to be used in deep water environments.

Rather limited studies concerning dynamic effects on marine risers have been presented in the literature using a variety of models. In some of these the riser system has been modeled as a zero-bending stiffness structure (Kirk et al., 1979), or as a beam-column structure (Burke, 1974). The riser system has also been modeled as having constant top tension and boundary conditions of a ball joint at both ends. To this last model the finite element technique with a linearized damping term has been applied (Spanos and Chen, 1980). Further, nondeterministic analysis based on a finite difference method has been performed (Turker and Murtha, 1973). Finally, large nonlinear riser deformation has also been addressed (Huang and Chruheepsakul, 1985).

Clearly, in these studies it is not feasible to have an exact treatment of the nonlinear hydrodynamic loads applied to the riser system. Consequently, most of the reported results are based on numerical integration schemes for the equations of motion. However, this approach can be extremely costly in terms of the requisite computational time.

In the present study the continuous riser system is modeled by a discrete multi-degree-of freedom system. A time dependent stiffness matrix is considered, which is induced by the heave motion of the floating vessel. An
approximate analytical solution for the single-degree-of-freedom model with time varying stiffness has previously been considered (Spanos and Agarwal, 1984). This approach is extended in the present study and an iteration algorithm is developed for the determination of the steady-state response of the riser model under nonlinear loads which are caused by sea waves and currents. In this regard, Morison's equation is used for estimating the hydrodynamic load. This method of approximate analysis is used to conduct a variety of studies and draw conclusions regarding the dependence of riser response quantities on design parameters such as the tension imposed on the riser top, the current velocity, and the characteristics of the sea wave.

In using the finite element method in structural applications, one of the most crucial tests of the reliability of the associated discrete model involves the prediction of the natural frequencies and modes of the structure. An alternative approach to estimate the first ten riser frequencies and to test the reliability results of the finite element model is presented in Appendix A. This approach relies on modeling the riser as a continuous
beam under constant tension which is equal to the average of the riser top and bottom tensions.

Concluding this introductory section it is pointed out that the prime contributions of this thesis are the development of a response determination technique for nonlinear riser models with time dependent top tension, and the efficient estimation of the riser natural frequencies for various boundary conditions.
II. RISER MODELING

2.1 Equation of Motion

The marine riser, Fig. 1, is modeled mathematically as a discrete multi-degree-freedom dynamic system. For convenience, the riser is divided into \( m \) elements with the same length \( \ell \),

\[
\ell = \frac{L}{m}
\]

where \( L \) is the total length of the riser. The mass of each riser segment is assumed to be lumped at the nodes and connected by massless flexural elements. The nodal points are numbered from the top toward the bottom and are denoted by \( i, i = 1,2,3,\ldots,n = m + 1 \). In Fig. 1 the numbers in parentheses correspond to elements of the discrete riser model. Most of the riser systems in practice are equipped with a telescoping joint which permits the top end of the riser to move freely in the vertical direction. In modeling the riser system, the variation of the top tension has usually been neglected. Clearly, this approximation will cease to give reliable results in severe sea states and if telescoping joints are not used. Therefore, for a more accurate riser analysis, the time varying top tension should be included.

Assume that the heave motion of the vessel influences the stiffness of the system exclusively. Then, the dynamic
motion can be approximately treated as a lateral motion on a plane. Further, the equation of motion for the discretized riser system can be written in the form

\[
\begin{bmatrix}
\mathcal{M}_t & \mathcal{C} \\
0 & \mathcal{K}_r
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathcal{X}}_t \\
\dot{\mathcal{X}}_r
\end{bmatrix}
+ \begin{bmatrix}
\mathcal{K}_t \\
\mathcal{K}_r
\end{bmatrix}
\begin{bmatrix}
\mathcal{X}_t \\
\mathcal{X}_r
\end{bmatrix}
= \begin{bmatrix}
\mathcal{K}_t \\
\mathcal{K}_r
\end{bmatrix}
\begin{bmatrix}
\mathcal{F}_t \\
\mathcal{F}_r
\end{bmatrix}
\]

(2)

where \([\mathcal{M}_t]\) is the mass matrix; \([\mathcal{C}]\) is the stuctural damping matrix; \([\mathcal{K}_t]\) and \([\mathcal{K}_r]\) are the stiffness matrix which will be discussed later; \(\{\mathcal{X}\}_t\), \(\{\mathcal{X}\}_t^\ast\), \(\{\mathcal{X}\}_r\) and \(\{\mathcal{X}\}_r^\ast\) are the nodal displacement and rotation vectors in the lateral direction measured from the vertical and the undeflected positions respectively, and \([\mathcal{F}]\) is the load vector.

2.2 Mass Matrix:

Assume that the riser system has equivalent density \(\rho\), cross-sectional area \(A\), and outside diameter \(D\). Then, each element has the same mass

\[
m_i = \rho A \ell; \quad i = 2, \ldots, n-1
\]

(3)

However, the nodal masses at the two riser system ends, \(i = 1, n\), are equal to \(\rho A \ell / 2\). Therefore, the lumped-mass matrix \([\mathcal{M}_t]\) for this model is
\[
[M_t] = \rho A \ell \begin{bmatrix}
\frac{1}{2} & 1 & 0 \\
\cdot & \cdot & \cdot \\
0 & 1 & \frac{1}{2}
\end{bmatrix} = \rho A \ell [J] 
\] (4)

where \([J]\) is the \(n \times n\) constant matrix. It is may be noted that the lumped-mass matrix used in the present formulation does not represent the riser system inertia as accurately as the consistent-mass matrix could do. However, the present method requires considerably less computational effort since the matrix is diagonal, and allows elimination of the rotational degrees of freedom by using the technique of static condensation in conjunction with the stiffness matrix. (Clough and Penzien, 1975)

2.3 Stiffness Matrix:

In practice, the lateral deflection of the riser system is kept within safe limits by appropriate top tensioning and vessel positioning. For this purpose, the angle of rotation at the riser bottom ball joint is monitored and controlled so that it does not exceed a critical limit, usually within 5 to 10 degrees (Turker and Murtha, 1973). This procedure prevents material overstressing or interference of the pipe with the casing. Because of the limitation of the variability of the bottom angle, the small slope beam theory can be applied in determining the bending stiffness of the riser system.
Bending Stiffness Matrix: There are two degrees of freedom, translation and rotation, associated with each node of an element of the riser system. Relying on the small slope assumption, and using cubic hermitian interpolating polynomials, the following element bending-stiffness matrix is obtained by Clough and Penzien (1975)

\[
[k_B] = \frac{EI}{\ell^3} \begin{bmatrix}
12 & -12 & 6\ell & 6\ell \\
12 & -6\ell & -6\ell & 0 \\
0 & 4\ell^2 & 2\ell^2 & 0 \\
0 & 4\ell^2 & 0 & 0
\end{bmatrix}.
\] (5)

In this equation, $EI$ is the bending rigidity of the element, the subscript $B$ denotes bending, and the superscript $e$ reflects the element reference. Assembling the bending stiffness matrices of all the elements, the system bending stiffness matrix is obtained. This matrix can be put in the form

\[
[K_B] = \frac{EI}{\ell^3} \begin{bmatrix}
[N_B]_{tt} & \ell[N_B]_{tr} \\
\ell[N_B]_{rt} & \ell^2[N_B]_{rr}
\end{bmatrix}.
\] (6)

where the subscripts $t, r$ indicate translation and rotation respectively, and $[N_B]_{tt}, [N_B]_{tr} = [N_B]_{rt}^T$ $[N_B]_{rr}$ are dimensionless constant matrices.
Geometric Stiffness Matrix: The geometric stiffness matrix accounts for the contribution of the top tension to the total stiffness of the riser system. This matrix for an element subjected to a constant axial load has been given by Clough and Penzien (1975). For the present problem, however, due to gravity, the axial tension not only varies with time but also varies linearly with the depth. Consequently, it has been necessary to use the procedure of reference Spanos and Chen (1980) to derive the geometric stiffness matrix for the $i$th element in the form

$$
[k_G] = \frac{T}{\ell} \begin{bmatrix}
\frac{3}{5} [(1-2i)R+2] & -\frac{3}{5} [(1-2i)R+2] & 0 & 0 \\
-\frac{3}{5} [(1-2i)R+2] & \frac{3}{5} [(1-2i)R+2] & 0 & 0 \\
\frac{\ell}{10} [1-iR] & -\frac{\ell}{10} [1-iR] & 0 & 0 \\
\frac{\ell}{10} [1-(i-1)R] & -\frac{\ell}{10} [1-(i-1)R] & 0 & 0 \\
\end{bmatrix}
$$

$$
+ \frac{T}{\ell} \begin{bmatrix}
0 & 0 & \frac{\ell}{10} [1-iR] & \frac{\ell}{10} [1-(i-1)R] \\
0 & 0 & -\frac{\ell}{10} [1-iR] & -\frac{\ell}{10} [1-(i-1)R] \\
0 & \frac{\ell^2}{30} [(3-4i)R+4] & \frac{\ell^2}{30} [(2i-1)R+2] & \frac{\ell^2}{30} [(1-4i)R+4] \\
0 & 0 & \frac{\ell}{60} [(2i-1)R-2] & \frac{\ell}{60} [(1-4i)R+4] \\
\end{bmatrix}
$$

In this matrix $T$ is the tension at the top of the riser system.
\[ T = T_a + T_f \sin(\omega t + \varphi) \]  

(8)

where \( T_a \) is the constant tension, \( T_f \) is the amplitude of the time-dependent tension, \( \omega \) is the frequency of the wave motion and \( \varphi \) is its phase angle.

In equation (7), \( R \) is the ratio of the element weight to mean top tension. That is

\[ R = \frac{g p \Delta \ell}{T_a}. \]  

(9)

By assembling the geometric stiffnesses of all the elements, the geometric stiffness for the entire riser system is obtained. This matrix can be written in a partitioned form and be separated into two parts. One part, \([K_{GC}]\), corresponds to the constant tension and another part, \([K_{Gf}]\), corresponds to the time varying tension. Specifically,

\[ [K_{GC}] = \frac{T_a}{\ell} \begin{bmatrix} [N_G]_{tt} & \ell [N_G]_{tr} \\ \ell [N_G]_{tr} & \ell^2 [N_G]_{rr} \end{bmatrix} \]  

(10)

and

\[ [K_{Gf}] = [K_{GC}] \frac{T_f}{T_a} \sin(\omega t + \varphi) = [K_{GC}](A \sin \omega t + B \cos \omega t). \]  

(11)

where the symbols \([N_G]_{tt}, [N_G]_{tr} = [N_G]_{rt}, [N_G]_{rr}\) denote nondimensional constant matrices of \([K_{GC}]\) and \(A = \frac{T_f}{T_a} \cos \varphi,\)

\(B = \frac{T_f}{T_a} \sin \varphi.\)
Upon determining the bending and the geometric stiffnesses, the riser system combined stiffness matrix is given by the equation

\[ [\tilde{K}_C] = [K_B] + [K_{GC}] + [K_{GR}] \]. \hspace{1cm} (12)

Note that by averaging over one period of the wave, the combined stiffness matrix \([\tilde{K}_C]\), will become in the a time independent matrix \([K_C]\). That is

\[ [K_C] = [K_B] + [K_{GC}] + [K_e] = [K_I] + [K_e] \]. \hspace{1cm} (13)

This time independent matrix accounts for both translational and rotational degrees of freedom. However, since the rotational inertia of the lumped-mass matrix can be neglected, \([M_r] \approx [0]\), the elements of the stiffness matrix corresponding to rotational motion can be eliminated by using the technique of static condensation. This technique is well established and will not be discussed herein in detail. Briefly, it amounts to expressing the vector of rotational displacements \(x^*_r\) in terms of the vector of translational displacements \(x^*_t\) by the equation

\[ x^*_r = -[K_C]^{-1}_{rr} [K_C]_{rt} x^*_t \]. \hspace{1cm} (14)

with
\[ [K_C] = [K_1] + [K_2] = \begin{bmatrix} [K_C]_{tt} & [K_C]_{tr} \\ [K_C]_{rt} & [K_C]_{rr} \end{bmatrix} \] (15)

and \([K_2]\) is an equivalent time dependent term of \([K_{dT}].\)

Equation (15) allows a formulation of the problem by using the following combined stiffness matrix

\[ [K] = [K_C]_{tt} - [K_C]_{tr} [K_C]_{rr}^{-1} [K_C]_{rt}. \] (16)

Obviously, this matrix has the same row numbers and column numbers as the lumped mass-matrix.

2.4 Damping Matrix

The solution method used in this study does not require the existence of classical normal modes for the dynamic model of the riser system. Therefore, a damping matrix could be used that does not satisfy the orthogonality relationship of the normal modes of the structure. However, in view of the unavailability of pertinent data, a special damping matrix in the form

\[ [C] = a_o [K], \] (17)

is assumed. The symbol \(a_o\) represents a constant to be specified. This particular form of the matrix \([C]\) usually represents heavily damped higher modes. This implies that
if modal superposition were used, the response would be dominated by the first few modes. Considering the frequency content of the sea waves and the natural frequencies of typical marine riser systems, this assumption appears to be realistic (Dareign and Huang, 1979). The constant \( a_0 \) can be specified by the equation

\[
a_0 = \frac{2\zeta_1}{\omega_R}
\]  

(18)

where \( \omega_R \) is the fundamental circular frequency of a riser model involving constant tension throughout its length, equal to the average of the top and the bottom riser tensions (see Appendix A). Further, \( \zeta_1 \) is the ratio of critical damping with respect to this frequency. The assumed numerical value of \( \zeta_1 \) is

\[
\zeta_1 = 0.02.
\]  

(19)

2.5 Load Vector

The forces acting on the riser in the lateral direction are the hydrodynamic forces induced by ocean currents and waves. The velocity of the water particle assumed to be a function of time \( t \) and the depth \( z \), is given by the equation

\[
\dot{v}(t,z) = v(z) + \ddot{u}(t,z)
\]  

(20)
The symbol \( V(z) \) represents the current velocity as a function of \( z \). The symbol \( \dot{u}(t,z) \) represents the wave velocity as a function of \( t \) and \( z \). The horizontal wave motion of the water particle is expressed as

\[
    u(t,z) = U(z) \sin(\omega t).
\]  

(21)

In this equation \( U(z) \) is the displacement amplitude at different depths \( z \) expressed by linear wave theory as

\[
    U(z) = \frac{H}{2} e^{-\lambda z}.
\]  

(22)

In this expression, \( H \) is the peak to peak wave height and \( \lambda \) is the wave number. It is commonly assumed by Morgan and Peret (1971-76) that the Morison equation can be modified to model the hydrodynamic force per unit length of a continuous structure by using its kinematics relative to the water. This equation is

\[
    f(t,z) = \rho_f A \ddot{V}(t,z) + C_f \rho_f A [\ddot{V}(t,z) - \dot{X}(t,z)] \\
    + \frac{1}{2} C_D \rho_f B |\dot{V}(t,z) - \dot{X}(t,z)| [\ddot{V}(t,z) - \dot{X}(t,z)],
\]  

(23)

where \( \rho_f \) is the water density, \( C_f \) is the added mass coefficient, and \( C_D \) is the drag coefficient.

For the discrete model used in this study, the
continuously distributed force \( f(t,z) \) must be lumped at the nodal points. In principle the lumping of the force \( f(t,z) \) for each element ought to be based on determining the nodal loads so that they are statically equivalent to the distributed forces. This procedure would require integrations over the element length of functions involving \( f(t,z) \). Clearly, the required integration cannot be performed since \( f \) depends on \( \dot{x} \) and \( \ddot{x} \), which are yet unknown quantities. Alternatively, \( x \), and thus \( \dot{x} \), \( \ddot{x} \), could be expressed in terms of the nodal displacements and the interpolation functions used in determining the bending stiffness matrix. However, the requisite computational effort would far outweigh the accuracy which might be lost by approximating \( f(t,z) \) stepwise. That is, the nodal force \( F_i \) is determined by the equation

\[
F_i \approx f_i(t) \ell = \rho_f A \dot{\ell} \tilde{V}(t,z_i) + C_f \rho_f A \ell [\ddot{\hat{V}}(t,z_i) - \ddot{x}(t,z_i)] \\
+ \frac{1}{2} C_D \rho_f D \ell \dot{V}(t,z_i) - \dot{x}(t,z_i) | \ddot{\hat{V}}(t,z_i) - \ddot{x}(t,z_i)| ;
\]

\[ i = 2, \ldots, n-1 \]  

(24)

2.6 Boundary Conditions

In practice, the riser is connected to the blow-out preventor by a ball joint at the bottom, and to the drilling vessel by a flexible element. This type of connection can accommodate large misalignment due to vessel offset and riser deflection without transmitting excessive
bending moment to the entire riser system.

On the bottom end, the blow out preventor is fixed to the ocean floor where the well head is located. Therefore, the nth node which coincides with the bottom end of the riser system has zero translational displacement, velocity and acceleration. That is,

\[(x_t)_n = (\dot{x}_t)_n = (\ddot{x}_t)_n = 0.\]  \hspace{1cm} (25)

Further, the riser is either free to rotate or clamped at this node as discussed in Appendix B.

On the top end, the ball joint is connected to the drilling vessel. In general, although the motion of a floating vessel has six degrees of freedom; only the sway (perpendicular to the vessel axis) and the surge (parallel to the vessel axis) modes can have a significant effect on the riser system response. The sway and surge motions of a vessel to long-crested simple harmonic waves can be described by sinusoidal functions with a phase lag with respect to the waves (Burke, 1970). In addition to the harmonic response to the waves, the vessel offset due to an existing current should also be considered in specifying the motion of the top end of the riser system. Thus, it is assumed that the motion of the first node which coincides with the top end of the riser, is adequately described by the equation

\[x_1(t) = x_{01} + \dot{x}_1(t) = x_{01} + \dot{x}_1 \sin(\omega t + \gamma_1),\]  \hspace{1cm} (26)
where $x_o$ represents the vessel offset due to the current, and $X_1$, $\varphi_1$ are the amplitude and the phase, respectively, of the vessel dynamic response to the waves. The top motion $x_i(t)$ can be associated with a rigid body rotation of the entire riser system about the bottom node. The two vectors

\[
\begin{bmatrix} \{x\}_t \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \{x\}_r \end{bmatrix}
\]

are related by the equation

\[
\begin{bmatrix} \{x\}_t \end{bmatrix}^* = \begin{bmatrix} \{x\}_t \end{bmatrix} + x_4 \begin{bmatrix} \{-r\} \end{bmatrix}
\]

(27)

where $\begin{bmatrix} \{-r\} \end{bmatrix}$ represents the constant vector caused by the unit top rigid displacement.
III. RESPONSE DETERMINATION

3.1 Solution Form

Upon determining all the variables appearing in equation (2), the system response can be sought. Because of the nonlinearity of the expression for the hydrodynamic loads, it is most unlikely that the exact solution of equation (2) can be determined. In general, for nonlinear dynamic systems, either numerical integration of the equation of motion is performed, or a method of approximate analytical solution is employed. The former approach is usually costly for the determination of the steady-state response, even for simple single-degree-of-freedom dynamic system. In context with the latter approach, the technique of equivalent linearization is applied to the present problem. This technique is well established and will not be discussed herein in detail. Pertinent discussions of its applicability to multi-degree-of-freedom systems can be found in references such as Burke (1970) and Spanos and Iwan (1978).

For convenience in the linearization procedure, the relative displacement vector of the structural response

\[ \{y\} = \{u\} - \{x\}_t \]  \hspace{1cm} (28)

has been used in many references; it exhibits several advantages if the riser top tension remain constant. But
for time varying tension, the relative displacement formulation does not offer any particular advantage. Therefore, the absolute displacement formulation is adopted herein. Specifically equation (2) can be written in the form

\[
[H]\{\ddot{x}\}_t + [C]\{\dot{x}\}_t + \{g(\dot{x})_t\} + [K]\{x\}_t = \{P\} + \{R\}
\] (29)

where

\[
[H] = (\rho + C_f \rho_f) A \ell [J] \tag{30}
\]

is the mass matrix including the added mass. Further,

\[
\{g(\dot{x})_t\} = \frac{1}{2} C_D \rho_f B \ell [J] \{V + \dot{u} - \dot{x}_t \} (V + \dot{u} - \dot{x}_t) \tag{31}
\]

is the vector involving the nonlinear damping. Finally,

\[
\{P\} = (1 + C_f) \rho_f A \ell [J] \{\ddot{u}\} + \frac{EI}{\ell^3} x_i [N] \{r\} \tag{32}
\]

where \([N]\) is the nondimensional form of \([K]\) and
\[
\{R\} = \begin{bmatrix}
R_1 \\
0 \\
\vdots \\
\vdots \\
R_n
\end{bmatrix}
\]

where \( R_1 \) and \( R_n \) are the reaction forces at the top and the bottom.

It is obvious that the elements of the nonlinear term \( \{G\} \) in the equation (29) are nonsymmetric with respect to the origin. The harmonic analysis by equivalent linearization of general nonsymmetric systems has been examined in detail by Spanos and Iwan (1979). Following the developments of this reference the approximate steady-state solution of equation (29) is taken in the form

\[
x_i(t) = (x_0)_i + \hat{x}_i(t) = (x_0)_i + x_i \sin(\omega t + \theta_i) \\
= (x_0)_i + A_i \sin(\omega t) + B_i \cos(\omega t)
\]

where \( i = 1, 2, \ldots, n \), \( \{x_0\} \) represents the time-independent offset component, and \( \{\hat{x}(t)\} \) represents the time-dependent oscillatory component, which can be specified either in terms of the amplitudes vector \( \{X\} \) and the phases vector \( \{\theta\} \), or in terms of the vectors \( \{A\} \) and \( \{B\} \). Furthermore, the following equivalent linear system is constructed

\[
[H](\ddot{x}) + [C](\dot{x}) + [C_e](\ddot{x}) + [K_C](\dot{x}) + [K_e](\ddot{x}) \\
= \{P\} + \{R\}.
\]
In this equation, $[C_e]$ is the equivalent linear damping matrix and $[K_e]$ is the equivalent linear stiffness matrix. The elements of $[C_e]$, and the element matrix $[K_e]_{i}$ of $[K_e]$ are obtained by minimizing the average, over one period of the solution, of an appropriate norm of the vector difference between the original nonlinear system and the equivalent linear system, equation (35).

3.2 Offset equation

To ensure that equation (34) satisfies equation (2) on the average over one period of the wave excitation, the following condition is imposed

$$
\int_{0}^{T} \left[ \tilde{K}_C \begin{bmatrix} x_t \end{bmatrix} \right] \, dt = \int_{0}^{T} \begin{bmatrix} G \end{bmatrix} \, dt = \begin{bmatrix} 0 \end{bmatrix},
$$

(36)

where $\tilde{K}_C$ is defined by equation (12).

Substituting equations (12), (27) and (34) into equation (36) and using dimensionless parameters, yields

$$
\begin{align*}
[H_1]_{rr} \{ \tilde{x} \}_t + [N_1]_{rr} \{ \tilde{x} \}_r &= - \frac{1}{2} \left\{ [N_2]_{rr} \left( \{ \tilde{b} \}_t B_w + \{ \tilde{a} \}_r A_w \right) \\
 &+ [N_2]_{rr} \left( \{ \tilde{b} \}_r B_w + \{ \tilde{a} \}_r A_w \right) \right\} + \frac{1}{2} \left\{ (\tilde{b}_{\ell_0} B_w + \tilde{a}_{\ell_0} A_w) [H_2]_{rr} \{ r \} \right. \\
&\left. + (\tilde{b}_{r_0} B_w + \tilde{a}_{r_0} A_w) [N_2]_{rr} \{ l \} \right\}
\end{align*}
$$

(37)
\[ [N_1]_{t\,t} \{ \bar{x} \}_{t} + [N_2]_{t\,r} \{ \bar{x} \}_{r} = -\frac{1}{2} \left\{ [N_2]_{t\,t} (\{ \bar{B} \}_{t} B_{w} + \{ \bar{A} \}_{t} A_{w}) + \{ \bar{B} \}_{t\,r} (B_{R\,0} + \bar{A}_{R\,0} A_{w}) \right\} \left\{ \{ N_2 \}_{t\,t} \{ 1 \} \right\} + \{ \bar{E} \} \]

(38)

with

\[
\bar{E}_i = \frac{C_D \rho_R}{\kappa \pi^2 (1 + C_T \rho_R)} \begin{cases} 
\pi (2\bar{V}_i^2 + \beta^2 \bar{X}_i^2) & \text{if } \bar{V}_i \geq \beta \bar{X}_i \\
2\bar{V}_i^2 (\pi - 2\alpha_i) + 6\bar{V}_i \bar{X}_i \beta \sin \alpha_i + \beta^2 \bar{X}_i (\pi - 2\alpha_i) & \text{if } \bar{V}_i < \beta \bar{X}_i
\end{cases}
\]

(39)

where \( i = 2, 3, 4, \ldots, n-1 \).

\[
\kappa = \left( \frac{m}{\pi} \right)^i
\]

(40)

\[
\beta = \frac{\omega}{\omega_1}, \quad \rho_R = \frac{\rho_f}{\rho}
\]

(41)

\[
\omega_1 = \frac{\pi^2}{L} \left[ \frac{EI}{(\rho + C_T \rho_f) A} \right]^{\frac{1}{2}}
\]

(42)

\[
\alpha_i = \cos^{-1} \left( \frac{\bar{V}_i}{\beta \bar{X}_i} \right)
\]

(43)

and the nondimensional form

\[
\{ \bar{x} \}_{t} = \{ x \}_{t} / D; \quad \{ \bar{A} \}_{t} = \{ A \}_{t} / D; \quad \{ \bar{B} \}_{t} = \{ B \}_{t} / D;
\]

\[
\bar{V}_i = \frac{V_i}{\omega_1 D}; \quad \bar{X}_i = \frac{[A_i^2 + (B_i - U_i)^2]^{\frac{1}{2}}}{D}
\]

(44)
3.3 Equivalent Damping Matrix

The elements in \([C_e]\) are obtained by minimizing the average of the square of the Euclidian norm of the difference between the original nonlinear system, \(G(\{\ddot{x}\})\), and its equivalent linear system, \([C_e]\{\ddot{x}\}\), over one period of the solution. That is, for

\[
\{\varepsilon\} = \{G(\{\ddot{x}\})\} - [C_e]\{\ddot{x}\}
\]

(45)

the criterion

\[
\int_0^T \{\varepsilon\} \{\varepsilon\} dt = \text{minimum}
\]

(46)

is satisfied; in which the prime defines the transpose operation on a vector or a matrix. Note that the nonlinear elements in the force vector \(G\) of the system (31) are all uncoupled. This property makes \([C_e]\) diagonal.

Implementing the minimization criterion defined by equation (46) leads to the formulas

\[
C_{ei} = \frac{\int_0^T G_i(\ddot{x}_i) \dddot{x}_i dt}{\int_0^T \dddot{x}_i^2 dt}
\]

(47)

where \(T = 2\pi/\omega\). Using the sinusoidal expression for \(x\) and integrating both the numerator and the denominator yields
\[
C_{ei} = \begin{cases} 
C_D \rho_f B \frac{\tilde{x}}{X_i} \cos(\phi_i) V_i & \text{for } V_i \geq \omega \tilde{x} \\
\frac{C_D \rho_f B}{2 \pi} \cos(\phi_i) \frac{\tilde{x}}{X_i} \left[ \frac{4}{3} \omega X_i \sin \alpha_i \left( 2 + \cos^2 \alpha_i \right) - V_i \left( \alpha_i - 2 \pi \right) \right] & \text{for } V_i < \omega \tilde{x}
\end{cases}
\]

(48)

where

\[
X_i^2 = A_i^2 + B_i^2
\]

(49)

\[
\tilde{x}_i = A_i^2 + (B_i - U_i)^2
\]

(50)

\[
\phi_i = \tan^{-1} \left( -\frac{B_i}{A_i} \right) - \tan^{-1} \left( -\frac{B_i - U_i}{A_i} \right)
\]

(51)

3.4 Equivalent Stiffness Matrix

According to Spanos and Iwan (1978), the vector space \( \mathbb{R} \) of the approximation solutions is the set

\[ T = \{ \sin \omega t, \cos \omega t \} \]

Therefore, a unique solution will exist for the linearization procedure only if \( m = 1 \). If \( m > 1 \), it represents an equivalent linear system which is as good as any other system in the sense of the equation difference minimization. Hence, another optimization criterion could be satisfied. Specifically, it is reasonable to assume that the time dependent stiffness \( [K_2]^{(i)} \sin(\omega t + \theta) \), of the \( i^{th} \) element, can be replaced according to the rule

\[
[K_2]^{(i)} \sin(\omega t + \theta) \rightarrow [K_\theta]^{(i)} \equiv S_i [K_2]^{(i)}
\]

(52)
where, similarly to equation (36), the minimization of the average of the square of the Euclidian norm of the difference between the two sides of the equation yields

\[
S_i = \frac{\sum_k \sum_j X_{ik} (A_j + B_j)}{\sum_k \sum_j (A_k A_j + B_k B_j)}
\]  \hspace{1cm} (53)

In this equation \( k, j = i, i+1, i+n, i+n+1 \) are the numbers of the two translations and the two rotations which correspond to the \( i \)th element.

3.5 Response Parameters Equation

The number of equations in (29) is \( n \). Two sets of boundary conditions are considered in Appendix B. For the hinge-hinge case, the vector equation of motion (29) can be rewritten in following form of dimension \( n-2 \)

\[
[M]_{n-2} \ddot{x}_{n-2} + [C]_{n-2} \dot{x}_{n-2} + [K]_{n-2} x_{n-2} = \{P\}_{n-2} + x_{10} [K]_{n-2} \dot{r}_{n-2} - \{BC\}_{n-2}
\]  \hspace{1cm} (54)

The matrices on the left hand side of equation (54) are obtained by deleting the first row, the first column, the last row, and the last column of their corresponding matrices in equation (29). The vector \( \{BC\} \), representing the quantities associated with the boundary motion, is given by the equation
\[ \{BC\} = \dot{X}_i \{C_j\} \]

(55)

where \( \{C_j\} \) is the first column of the damping matrix \([C]\), \(j=2,3,\ldots,n-1\). Substituting equation (26) into equation (54) yields

\[
-\omega^2[H]\{A\}_t \cos \omega t + \{B\}_t \sin \omega t + \omega\{[C] + [C_0]\}\{-\{A\}_t \sin \omega t + \{B\}_t \cos \omega t\} + [K]\{\{A\}_t \cos \omega t + \{B\}_t \sin \omega t\} = -\omega^2[H']\{U\} \sin \omega t
+ (A_0 \cos \omega t + B_0 \sin \omega t)[K]\{r\} + \omega(A_0 \sin \omega t - B_0 \cos \omega t)\{C_{j_1}\}
\]

(56)

Separating the coefficients of \( \sin \omega t \) and \( \cos \omega t \) in equation (56) yields

\[
(-\omega^2[H] + [K])\{A\}_t + \omega \left( \frac{2 \zeta}{\omega_T} [K] + [C_0] \right)\{B\}_t = A_0 [K]\{r\} - \omega B_0 \frac{2 \zeta}{\omega_T} \{K\}_j, \quad \text{(57)}
\]

\[
-\omega \left( \frac{2 \zeta}{\omega_T} [K] + [C_0] \right)\{A\}_t + (-\omega^2[H] + [K])\{B\}_t = B_0 [K]\{r\} + \omega A_0 \frac{2 \zeta}{\omega_T} \{K\}_j, - \omega^2[H']\{U\} \quad \text{'(58)}
\]

Define

\[
\{\bar{A}\}_t = \{A\}_t / D \quad \text{(59)}
\]

\[
\{\bar{B}\}_t = \{B\}_t / D \quad \text{(60)}
\]

\[
\{\bar{U}\} = \frac{N}{2} e^{-\lambda Z} / D \quad \text{(61)}
\]

\[
\beta_n = \frac{\omega}{\omega_T} \quad \text{(62)}
\]
\[ \omega_1 = \frac{\pi^2}{L^2} \sqrt{\frac{EI}{(\rho + C_T \rho_f)A}} \]

\[ [C_e] = \frac{1}{(\rho + C_T \rho_f)A\omega_1^2} [C_e]. \] (64)

Then, equation (57) is rewritten as

\[ (-\beta^2[I] + \kappa[N_1])\{\vec{A}\}_t + (2\zeta_1 \beta \rho \kappa[N_1] + \beta[C_e])\{\vec{B}\}_t \]

\[ = \kappa[N_1]\{r\}\vec{A}_0 - 2\zeta_1 \beta \rho \kappa[N_1]j_1 \vec{B}_0 \] (65)

and equation (58) is rewritten as

\[ -(2\zeta_1 \beta \rho \kappa[N_1] + \beta[C_e])\{\vec{A}\}_t + (-\beta^2[I] + \kappa[N_1])\{\vec{B}\}_t \]

\[ = \kappa[N_1]\{r\}\vec{B}_0 + 2\zeta_1 \beta \rho \kappa[N_1]j_1 \vec{A}_0 - \beta^2 \frac{(1+C_T)}{1+C_T \rho} \{U\} \] (66)

The unknowns for each node are the offset, \( \vec{x}_i \), the amplitude of \( \cos \omega t \), \( \vec{B}_i \), and the amplitude of \( \sin \omega t \), \( \vec{A}_i \).

Clearly, equations (14), (37), (38), (65), and (66) are coupled. Thus, an iterative scheme is adopted for their solution. First, estimate \( \vec{A}_i^{(0)} \), \( \vec{B}_i^{(0)} \), and \( \vec{x}_i^{(0)} \), and substitute these values into equation (39) and (45) to predict the equivalent damping and stiffness coefficients. Then, solve the linear equations (65), (66), and (14) to derive a new set \( \vec{A}_i^{(1)} \), \( \vec{B}_i^{(1)} \). Use these values in equation (37) and (38) to derive a new offset \( \vec{x}_i^{(1)} \). Repeat this procedure to update \( \vec{A}_i^{(r)} \), \( \vec{B}_i^{(r)} \), and \( \vec{x}_i^{(r)} \), where \( (r) \) is the iteration number, until the difference between the \( (r) \) and
(r+1) estimates is within a desirable tolerance margin.

3.6 Stress Calculation

From a design point of view, the most important parameter of any dynamic analysis of the riser system is the variation of the stress along the length of the riser. Clearly, the overall normal stress at a section consists of the tensile stress induced by the top tension and of the bending stress due to the lateral deflection.

The tensile stress at the node $i$ is

$$\sigma = \frac{T}{A_S} [1 - (i - 1)R],$$

(67)

where $A_S$ is the cross-sectional area of the riser pipe.

In order to calculate the bending stress at a node, it is necessary to determine the applied bending moment. The moment, however, depends not only on the translational displacements, but on the rotational displacements, as well. The nodal translation, can be obtained by using equation (26), while the corresponding rotations can be determined by equation (14). Upon determining both the translational and the rotational displacements, the deflection $x^{(i)}(z)$ within the $i$th element can be expressed in terms of the nodal displacements and the interpolation functions by the equation
\[ x^{(i)}(\eta) = x_i^* g_1(\eta) + x_{i+1}^* g_2(\eta) + \theta_i^* g_3(\eta) \]
\[ + \theta_{i+1}^* g_4(\eta); \quad 0 < \eta < \xi. \]  

(68)

In this equation, \( x_i^* \), \( x_{i+1}^* \), \( \theta_i^* \), \( \theta_{i+1}^* \) are the translational and the rotational displacements with respect to the undeflected position at nodes \( i \) and \( i+1 \), respectively. Further \( g_i, i=1,2,3,4 \), are the interpolating functions used in developing the bending stiffness matrix. That is

\[ g_1(\eta) = 1 - 3(\frac{\eta}{\xi})^2 + 2(\frac{\eta}{\xi})^3 \]  

(69)

\[ g_2(\eta) = 3(\frac{\eta}{\xi})^2 - 2(\frac{\eta}{\xi})^3 \]  

(70)

\[ g_3(\eta) = \eta(1 - \frac{\eta}{\xi})^2 \]  

(71)

\[ g_4(\eta) = \frac{\eta^2}{\xi} (\frac{\eta}{\xi} - 1) \]  

(72)

Using equation (68), the bending moment within the element \( (i) \) can be obtained as

\[ M^{(i)}(\eta) = EI \; x^{(i)(i)} (\eta) \]
\[ = x_i^* g_1''(\eta) + x_{i+1}^* g_2''(\eta) + \theta_i^* g_3''(\eta) + \theta_{i+1}^* g_4''(\eta) \]  

(73)

The (') denotes the derivative with respect to \( \eta \). The maximum bending stress occurs at the outmost fiber of the riser and is given by the formula
\[
\sigma^{(i)}(\eta) = \left| \frac{M^{(i)}(\eta) \text{D}}{2I} \right| = \left| \frac{\text{Ex}^{(i)}(\eta) \text{D}}{2} \right| \tag{74}
\]

The bending stress calculated by equation (74) is continuous within each element since the interpolation functions have continuous second derivatives. However, the bending stress is not continuous across two adjacent elements. Hence, the bending stress at node \(i\) is calculated as the average of the values corresponding to two adjacent elements. That is

\[
\sigma_i = \frac{\sigma^{(i-1)} + \sigma^{(i)}}{2}, \quad i = 2, 3, \ldots, n-1 \tag{75}
\]
IV. NUMERICAL RESULTS

4.1 Reliability of the Discrete Model

Partly, the reliability of the present approach depends on whether or not the discrete model used represents the riser system adequately. For the purpose of investigating the adequacy of the discrete model, the mass, the combined stiffness matrices, and a standard eigensolver subroutine EIGRS from IMSL (International Mathematical and Statistical Libraries, Inc.) is applied to determine natural frequencies of a specific riser system with length and other design parameters given in Tables 1 and 2. Another estimating method is presented in Appendix A. The results are compared in Table 2. Examining Table 2., it can be seen that there exists only a minor discrepancy between the values of the first and the second natural frequencies obtained by either of the above methods. However the discrepancy of the discrete models increases with increasing order of the natural frequencies. On the basis of Table 2 it can be concluded that the first and the second natural frequencies obtained by a six-element discrete system, or estimated by the method of Appendix A are accurate. Furthermore, the ten-element discrete system predicts reliably the first five (5) or six (6) natural frequencies.
4.2 Reliability of the Solution Method

The reliability of the technique of equivalent linearization, typically within 0 percent to 15 percent relative error, has been established by several studies. Pertinent information for asymmetric multi-degree-of-freedom systems has been given by Spanos and Iwan (1979). For the present problem a study has been conducted regarding the steady-state solutions for the nodal displacements of a riser system with length and other parameters specified in Table 3. Also shown are the corresponding values of the response estimated by numerical integration of the equation of motion. Examining closely Table 3, it is seen that the two solutions compare quite well. Not only the right trends are observed but also the actual numerical values are in a close agreement. Furthermore, it was found that computation time required for the determination of the steady-state system response by the linearization approach was approximately 10% of that required by the numerical integration method.

The nonlinear differential equation solver DGEAR of IMSL, based on the implicit Adam's method, provides a steady-state response of equations (29) to (32). The offset and the amplitude of the oscillatory component is obtained by the following procedure. Let \( \bar{x}_{\text{max}} \) and \( \bar{x}_{\text{min}} \), respectively, denote the maximum and the minimum values of
\( \ddot{x} \) over one period after the numerical solution has reached the steady state. Then, the offset \( \dddot{x}_0 \) and the amplitude \( \dddot{x} \) are given by the equations

\[
\dddot{x} = ( \dddot{x}_{\text{max}} - \dddot{x}_{\text{min}} ) / 2 \\
\dddot{x}_0 = \dddot{x} + \dddot{x}_{\text{min}} = ( \dddot{x}_{\text{max}} + \dddot{x}_{\text{min}} ) / 2
\]

(76)  (77)

4.3 Parameter study

Numerical results of parameter studies conducted by using the present formulation are examined in this section. Attention is given to the investigation of the dependence of the riser response to certain environmental variables. In this regard, it is generally accepted that the maximum bending stress is the most significant response measure. The amplitudes of the bending stress at the nodal points of a riser system for four different ratios of \( T_i / T_d \) are shown in Fig. 22. It is noticed that the maximum bending stress occurs at locations close to either of the ends. Similar results, obtained for constant top tension, have previously been reported Spanos and Chen (1980). The maximum bending stress increases with increasing values of the ratio of \( T_i / T_d \).

There are various profiles that can be assumed in describing the variation of the ocean current velocity along the ocean depth. The most commonly used profiles are the uniform, the triangular, the trapezoidal, and the stepped. For simplicity, the triangular distribution
current velocity profile is selected in these examples. Surface current velocity equal to 2 knots is considered, as it is usually encountered in the actual sea states.

The tension force applied at the top supports the riser system. For convenience, the constant term of the top tension value has been specified in terms of its ratio with respect to the riser's weight. The reported studies investigate the top tension effect using values for this ratio ranging from 1.0 to 1.6. The ratio of the time varying tension with respect to the constant tension ranges from 0.0 to 0.3.

The range of the wave period considered in this study is from 6 sec. to 20 sec. reflecting actual wave periods encountered in the ocean. The waves considered have a peak-to-peak wave height from 5 ft. to 20 ft. at the surface.

It has been mentioned previously that the vessel position consists, in general, of two components, an offset and a time-dependent motion. The offset of the drilling vessel as a certain percentage of the riser length is approximately the same for a wide range of ocean depths if the mooring lines are sufficiently long (Kirk and Cooper, 1979). In the present study, a value of three percent (3%) of the riser length is assumed for the vessel static offset. The vessel dynamic motion is caused by sea waves and can be directly related to their dynamic
characteristiccs. Specifically, the sway or the surge motion of the vessel due to harmonic waves can be expressed in terms of sinusoidal functions as shown in equation (26). The amplitude of the vessel motion depends on the wave period and wave height (Burke, 1970). Hence, the vessel motion can be defined by specifying the wave condition. In this study, only the sway motion effect of the vessel is considered.

Clearly, the accuracy of the numerical results depends on the number of elements used in the discrete model. Thus, it is necessary to determine the appropriate number of elements which have to be used in order to obtain reasonable results. For this purpose, a comparison of riser nodal displacements obtained by using different number of elements in the model has been conducted. Specifically, it has been found that for a 1200 ft. long riser, using a 10-element model gives all nodal displacements having an error less then two percent (2%) with respect to corresponding values obtained by using a 20-element model. Therefore, the 10-element model was considered adequate for the 1200 ft. long riser. All the subsequent parameter studies for riser systems of this length have been obtained using a 10-element model. However, for risers of larger length more elements should be used.

The effect of riser design parameters on its maximum (over time and length) bending stresses has been examined
using a 1200 ft. long riser. The variation of the maximum bending stress versus the wave period $T$ is shown in Figs. 3 through 10. The values of the wave height $WH$ and the top tension ratio $T_d/W$ and $T_f/T_d$ have been used as parameters.

Examining Fig. 3 through 6 it is seen that the maximum bending stress increases with increasing $WH$ and $T_f/T_d$.

Examining Fig. 7 through 10 it is observed that the maximum bending stress increases with increasing values of the ratio of $T_f/T_d$ but not necessarily with decreasing values of the ratio of $T_d/W$.

It has been found through the study of this particular riser system that five to ten iterations are sufficient to generate solutions with less than ten percent (10%) error using the linearization approach, see Table 3. Furthermore, the number of iterations required does not depend on the number of elements used or the contributions of the bending rigidity and of the tension force to the combined stiffness of the riser system.
V. CONCLUDING REMARKS

A discrete MDOF dynamic system has been used to model the steady-state motions of a marine riser system. The methodology for determining the mass, stiffness, and damping matrices, and the nodal loads of the discrete node has been outlined. Further, the fluctuation of the top tension is considered to have the same frequency as the sea wave. This is a reasonable assumption for the steady state behavior.

A linearization-averaging scheme which leads to the determination of the approximate steady state response of the riser system has been developed. It involves solving iteratively a set of algebraic equations for the amplitude and the phase of the oscillatory components of the response. These response parameters are coupled with the offsets of the nodal points through a condition which ensures that the approximate solution satisfies the riser equation of motion on the average.

Comparisons of solutions obtained by numerical integration of the equation of motion with corresponding solutions derived by the linearization-averaging scheme have shown that the latter is reliable within an acceptable engineering error. Furthermore, it requires approximately one tenth (1/10) of the requisite computation time of the numerical integration procedure. Furthermore, the parameter
studies have indicated that the magnitude of the maximum bending stresses depend strongly on the top tension, the amplitude and period of the wave, and the boundary conditions.

Finally, it should be emphasized that the simplified mathematical model of this study does not capture all the intrinsic features of the complex marine riser structure. However, it offers a quite meaningful and efficient approach to estimating reliably the magnitudes of the primary design parameters of the marine riser.
BIBLIOGRAPHY


APPENDIX - A

NATURAL FREQUENCIES - MODES FORMULATION FOR LONG RISER
WITH CONSTANT TENSION

Solution Form

Consider the equation of motion of a riser model shown in the figure

\[
\begin{align*}
\uparrow & \quad T & \quad \uparrow & \quad T \\
0 & \quad x = 0 & \quad 0 & \quad x = 0 \\
| & \quad \downarrow & \quad | & \quad \downarrow \\
\downarrow & \quad T & \quad \downarrow & \quad T \\
0 & \quad x = L & \quad 0 & \quad x = L
\end{align*}
\]

For this introduce the bending rigidity EI, the top tension \( T \), the length \( L \), the effective inertia term \( \rho A \), and the lateral deflection \( Y(x,t) \). Then, the equation of motion can be written as

\[
EI \frac{\partial^4 Y}{\partial x^4} - T \frac{\partial^2 Y}{\partial x^2} = -\rho A \frac{\partial^2 Y}{\partial t^2}
\]  \hspace{1cm} (A1)

Note that in the real riser structure the tension has a linear distribution as shown in the figure

\[
\begin{align*}
\uparrow & \quad T_{\text{max}} & \quad \uparrow & \quad T \\
\downarrow & \quad T & \quad \downarrow & \quad T \\
\downarrow & \quad T_{\text{min}} & \quad \downarrow & \quad T \\
0 & \quad x = 0 & \quad 0 & \quad x = L
\end{align*}
\]
Clearly, a logical value to take for $T$ in the riser model is

\[ T = \frac{T_{\text{max}} + T_{\text{min}}}{2} \]  \hspace{1cm} (A2)

To derive the modes associated with either of the two sets of boundary conditions, clamped or hinged, set

\[ Y(x,t) = X(x)e^{i\omega t}. \]  \hspace{1cm} (A3)

Substituting equation (A3) into equation (A1), and manipulating yields

\[ EIX'''(x) - TX''(x) - \rho A\omega^2 X(x) = 0, \]  \hspace{1cm} (A4)

where the symbol \( (\cdot)' \) denotes differentiation with respect to $x$.

The characteristic algebraic equation associated with equation (A4) is

\[ EI r^4 - Tr^2 - \rho A\omega^2 = 0 \]  \hspace{1cm} (A5)

The roots of equation (A5) can be expressed as
\[ r_{1,2}^2 = \frac{T}{2EI} + \left[ \left( \frac{T}{2EI} \right)^2 + \frac{\rho A}{EI} \omega^2 \right]^{\frac{1}{2}} \] (A6)

and

\[ r_{3,4}^2 = \frac{T}{2EI} - \left[ \left( \frac{T}{2EI} \right)^2 + \frac{\rho A}{EI} \omega^2 \right]^{\frac{1}{2}} \] (A7)

Define the quantities

\[ m = \left[ \frac{T}{2EI} + \left( \frac{T}{2EI} \right)^2 + \frac{\rho A}{EI} \omega^2 \right]^{\frac{1}{2}} \] (A8)

\[ n = \left[ - \frac{T}{2EI} + \left( \frac{T}{2EI} \right)^2 + \frac{\rho A}{EI} \omega^2 \right]^{\frac{1}{2}} \] (A9)

Then, considering equations (A4)-(A9), \( X(x) \) can be expressed in the following form

\[ X(x) = c_1 \sinh(mx) + c_2 \cosh(mx) + c_3 \sin(nx) + c_4 \cos(nx) \] (A10)

Further

\[ X'(x) = c_1 m \cosh(mx) + c_2 m \sinh(mx) + c_3 n \cos(nx) - c_4 n \sin(nx) \] (A11)

and
$$X''(x) = c_1 m^2 \sinh(mx) + c_2 m^2 \cosh(mx) - c_3 n^2 \sin(nx) - c_4 n^2 \cos(nx)$$  \hspace{1cm} (A12)

**Hinge-Hinge Case**

For the case of hinged-hinged design the following boundary conditions apply

$$X(0) = X''(0) = 0$$  \hspace{1cm} (A13)

and

$$X(L) = X''(L) = 0$$  \hspace{1cm} (A14)

Using equation (A10), (A12), (A13) and (A14) it is a trivial matter to prove that the natural frequencies are

$$\omega_i = \frac{i^2 \pi^2}{2} \sqrt{\frac{EI}{L^2}} \sqrt{1 + \frac{TL^2}{i^2 EI \pi^2}}, \quad i = 1, 2, \ldots$$  \hspace{1cm} (A15)

while the natural modes are given by the equation

$$X_i(x) = \sin \frac{i \pi x}{L}$$  \hspace{1cm} (A16)

Note that for the case of a long riser it is logical to assume, for several values of $i$, that
Thus, equation (A15) can be approximated by the well known string equation

\[ \omega_i = \frac{i\pi}{L} \left( \frac{T}{\rho A} \right)^{\frac{1}{2}} \]  

(A18)

**Clamp-Clamp Case**

**Natural Frequencies:** For the case of clamp-clamp beam the analysis becomes considerably more complicated. The associated boundary conditions are

\[ X(0) = X'(0) = 0 \]  

(A19)

and

\[ X(L) = X'(L) = 0 \]  

(A20)

Combining equations (A10), (A11) and (A19) yields

\[ c_1(0) + c_2(L) + c_3(0) + c_4(L) = 0 \]  

(A21)

\[ c_1m(L) + c_2m(0) + c_3n(L) - c_4n(0) = 0 \]  

(A22)

Combining equations (A10), (A11) and (A20) yields
\[(\sinh mL)c_1 + (\cosh mL)c_2 + (\sin nL)c_3 + (\cos nL)c_4 = 0\]

(A23)

and

\[(m \cosh mL)c_1 + (m \sinh mL)c_2 + (n \cos nL)c_3 - (n \sin nL)c_4 = 0\]

(A24)

The set of homogeneous equations (A21)-(A24) accept a non-trivial solution if the determinant equation

\[
\begin{vmatrix}
0 & 1 & 0 & 1 \\
m & 0 & n & 0 \\
\sinh(mL) & \cosh(mL) & \sin(nL) & \cos(nL) \\
m \cosh(mL) & m \sinh(mL) & n \cos(nL) & -n \sin(nL)
\end{vmatrix} = 0
\]

(A25)

is satisfied.

Equation (A25) can be further expanded as

\[
\begin{vmatrix}
m & n & 0 \\
\sinh(mL) & \sin(nL) & \cos(nL) \\
m \cosh(mL) & n \cos(nL) & -n \sin(nL)
\end{vmatrix} + \begin{vmatrix}
m & 0 & n \\
\sinh(mL) & \cosh(mL) & \sin(nL) \\
m \cosh(mL) & m \sinh(mL) & n \cos(nL)
\end{vmatrix} = 0
\]

(A26)

Further expansion of equation (A26) yields
\[-2mn + (n^2 - m^2) \sinh(mL) \sin(nL) + 2mn \cosh(mL) \cos(nL) = 0 \]

(A27)

Using equations (A8) and (A9) it is found that

\[ mn = \left[ \frac{T}{2EI} \right]^2 + \frac{\rho A \omega^2}{EI} - \left( \frac{T}{2EI} \right)^2 \] \(\frac{1}{2} = \left( \frac{\rho A \omega^2}{EI} \right) \frac{1}{2} \quad (A28) \)

and

\[ n^2 - m^2 = - \frac{T}{2EI} + \left[ \frac{T}{2EI} \right]^2 + \frac{\rho A \omega^2}{EI} - \frac{T}{2EI} - \left[ \frac{T}{2EI} \right]^2 + \frac{\rho A \omega^2}{EI} = - \frac{T}{EI} \quad (A29) \]

Introduce the parameters

\[ \alpha = \left( \frac{EI}{\rho A} \right)^{\frac{1}{2}} \quad (A30) \]

\[ U = \frac{TL^2}{2EI} \quad (A31) \]

and

\[ \Omega = \frac{\omega L^2}{\alpha} = L^2 \omega \left( \frac{\rho A}{EI} \right)^{\frac{1}{2}} \quad (A32) \]

Then

\[ mn = \frac{\omega}{\alpha} \quad (A33) \]

\[ n^2 - m^2 = - \frac{2U}{L^2} \quad (A34) \]
\[ mL = L \left[ \frac{T}{2EI} + \sqrt{\left(\frac{T}{2EI}\right)^2 + \frac{\rho \lambda \omega^2}{EI}} \right]^{\frac{1}{2}} = \left[ U + \sqrt{\frac{U^2 + \Omega^2}{U^2}} \right]^{\frac{1}{2}} \]

(A35)

and

\[ nL = L \left[ -\frac{T}{2EI} + \sqrt{\left(\frac{T}{2EI}\right)^2 + \frac{\rho \lambda \omega^2}{EI}} \right]^{\frac{1}{2}} = \left[ -U + \sqrt{\frac{U^2 + \Omega^2}{U^2}} \right]^{\frac{1}{2}} \]

(A36)

Thus, the characteristic equation (27) becomes

\[ \Omega + U \sinh\left(\left[U + \sqrt{\frac{U^2 + \Omega^2}{U^2}}\right]^{\frac{1}{2}}\right) \sin\left(\left[-U + \sqrt{\frac{U^2 + \Omega^2}{U^2}}\right]^{\frac{1}{2}}\right) \]

\[ = \Omega \cosh\left(\left[U + \sqrt{\frac{U^2 + \Omega^2}{U^2}}\right]^{\frac{1}{2}}\right) \cos\left(\left[-U + \sqrt{\frac{U^2 + \Omega^2}{U^2}}\right]^{\frac{1}{2}}\right) \]

(A37)

This is the characteristic equation involving non-dimensional parameters.

Observe that for \( U = 0 \) equation (A37) reduces to

\[ \Omega - \Omega \cosh(\sqrt{\Omega}) \cos(\sqrt{\Omega}) = 0. \]

(A38)

which is the classical equation for the natural frequencies of a clamp-clamp beam without tension.

Equation (A37) is applicable for both short and long risers. However, for long risers it can be further
simplified. In non-dimensional terms the long riser assumption is reflected by taking

\[ e^{\sqrt{\frac{U}{1}}} \to \infty \]  
(A39)

In this case the following approximation can be made

\[ \sinh\left((U+\left(U^{2}+\Omega^{2}\right)^{1/2}\right) \approx \frac{1}{2} e^{(U+\left(U^{2}+\Omega^{2}\right)^{1/2}}} \]  
(A40)

\[ \cosh\left((U+\left(U^{2}+\Omega^{2}\right)^{1/2}\right) \approx \frac{1}{2} e^{(U+\left(U^{2}+\Omega^{2}\right)^{1/2}}} \]  
(A41)

and

\[ \frac{2}{\Omega \cosh\left((U+\left(U^{2}+\Omega^{2}\right)^{1/2}\right) \cos\left((-U+\left(U^{2}+\Omega^{2}\right)^{1/2}\right) \approx 0 \]  
(A42)

Thus, equation (A37) becomes

\[ \frac{\Omega}{U} = \tan\left[U\left(-1 + \sqrt{1 + \frac{\Omega^{2}}{U^{2}}} \right)\right]^{1/2} \]  
(A43)

Given \( U \), equation (A43) can be solved by using an algorithm for nonlinear algebraic equations. A quite simple algorithm is
\[
\left(\frac{\nu}{U}\right)_{k+1} = \tan\left[U\left(-1 + \frac{1 + \left(\frac{\nu}{U}\right)^2_k}{\sqrt{1 + \left(\frac{\nu}{U}\right)^2_k}}\right)\right]^{\frac{1}{2}}
\]  
(A44)

which relates the \((k+1)^{th}\) estimate in terms of the \(k^{th}\) estimate of \(\left(\frac{\nu}{U}\right)\). It should be noted that both equations (A37) and (A44) yield an infinite set of solutions which leads to the infinite set of the non-dimensional natural frequencies \(\Omega_i, i=1,2,...\)

**Natural Modes:** Next proceed with the determination of the natural modes.

Without loss of generality assume that

\[
C_1 = 1
\]
(A45)

Then equation (A22) yields

\[
C_3 = -\frac{m}{n}
\]
(A46)

Further, equation (A21) yields

\[
C_2 = -C_4
\]
(A47)

and equation (A23) gives

\[
C_2 = \frac{m}{n} \frac{\sin(nL) - \sinh(ml)}{\cosh(ml) - \cos(nL)}
\]
(A48)

Therefore, the modal shape is
\[ X(x) = \sinh(mx) + \frac{(m/n)\sin(nL) - \sinh(mL)}{\cosh(mL) - \cos(nL)} \cosh(mx) - \]
\[ - \frac{m}{n} \sin(nx) - \frac{(m/n)\sin(nL) - \sinh(nL)}{\cosh(mL) - \cos(nL)} \cos(nx) \quad (A49) \]

For long risers equation (A49) can be further simplified by noting that

\[
\sinh(mL) \gg \sin(nL), \quad e^{\sqrt{U}} \to \infty \quad (A50)
\]

and

\[
\cosh(mL) \gg \cos(nL), \quad e^{\sqrt{U}} \to \infty \quad (A51)
\]

Further,

\[
\sinh(mL) \approx \cosh(mL), \quad e^{\sqrt{mL}} \to \infty \quad (A52)
\]

Thus,

\[
C_2 \approx -1 + \frac{m \sin(nL)}{n \cosh(mL)} \quad (A53)
\]

and the modal shape will be

\[
X(x) = \sinh(mx) - [1 - \frac{m}{n} \frac{\sin(nL)}{\cosh(mL)} \cosh(mL) - \frac{m}{n} \sin(nx)] + \]
\[ + [1 - \frac{m}{n} \frac{\sin(nL)}{\cosh(mL)} \cos(nx)] \quad (A54)\]

where

\[
0 \leq x \leq L \quad (A55)
\]

Define

\[
\varepsilon = \frac{x}{L} \quad (A56)
\]
\[ M = \left[ U + \sqrt{U^2 + \Omega^2} \right]^{\frac{1}{2}} = mL \quad \text{(A57)} \]

and

\[ N = \left[ -U + \sqrt{U^2 + \Omega^2} \right]^{\frac{1}{2}} = nL \quad \text{(A58)} \]

Then, the mode will be given by the expressions

\[
X(\xi) = \sinh(M\xi) - \left[ 1 - M \frac{\sin N}{N \cosh M} \right] \cosh(M\xi) - M \frac{\sin(N\xi)}{N} \\
+ \left[ 1 - M \frac{\sin N}{N \cosh M} \right] \cos(N\xi) \quad \text{(A59)}
\]

or

\[
X(\xi) = -e^{-M\xi} + \frac{M}{N} \sin(N)e^{-M(1-\xi)} - M \frac{\sin(N\xi)}{N} + \cos(N\xi) \quad \text{(A60)}
\]

and

\[
X(\xi) = \sin N\xi - \frac{N}{M} \cos N\xi + \frac{Ne^{-M\xi}}{M} + \left[ \frac{N}{M} \cos N - \sin N \right] e^{-M(1-\xi)} \quad \text{(A61)}
\]

This form satisfies the boundary conditions

\[
X(0) = \frac{N}{M} - \frac{N}{M} = 0 \quad \text{(A62)}
\]

\[
X'(0) = -N + N = 0 \quad \text{(A63)}
\]

\[
X(1) = \sin N - \frac{N}{M} \cos N - \sin N + \frac{N}{M} \cos N = 0 \quad \text{(A64)}
\]
\[ X'(1) = -M\sin N + N\cos N + N\cos N + \frac{N^2}{M} \sin N = (N^2 - M^2) \sin N + 2MN\cos N \]  

(A65)

because of the characteristic equation for \( \Omega \).

**Initial Estimates**

Returning to equation (A49) for the case of \( e^M \rightarrow \infty \) a simpler equation can be determined. Namely using equations (A40), (A41) and the approximations

\[ \cos(nL) \ll \cosh(mL) \]  

(A66)

and

\[ n \sin(nL) \ll m \cosh(mL) \]  

(A67)

\[ \sinh(mL) - \frac{m}{n} \sin(nL) = \cosh(mL) \]  

(A68)

which gives

\[ \sin(nL) = 0 \]  

(A69)

Thus

\[ N = \left[ -U + \sqrt{U^2 + \Omega_i^2} \right]^{\frac{1}{2}} = i\pi; \quad i = 1, 2, \ldots \]  

(A70)

Equation (A70) gives

\[ \Omega_i^2 = \left[ i^2\pi^2 + U \right]^2 - U^2 = i^2\pi^2 \left[ i^2\pi^2 + 2U \right] \]  

(A71)

That is, the equation for the hinge-hinge boundary conditions is recovered! Clearly, equations (A15) and (A18)
can be used as initial estimates of the solution for the clamp-clamp case. Then, equation (A37) or (A43) can be used to determine more accurate values for $\omega_k$. 
APPENDIX - B

STATIC CONDENSATION PROCEDURE

The equations of motion for the hinge-hinge case are

\[
\begin{bmatrix}
[M]_t & [0] \\
[0] & [0]
\end{bmatrix}
\begin{bmatrix}
\{x\}_t \\
\{x\}_r
\end{bmatrix}
+ \begin{bmatrix}
[K]_{tt} & [K]_{tr} \\
[K]_{rt} & [K]_{rr}
\end{bmatrix}
\begin{bmatrix}
\{x\}_t \\
\{x\}_r
\end{bmatrix}
= \begin{bmatrix}
\{F\}_t \\
\{0\}
\end{bmatrix}
\]

(B1)

and for the clamp-clamp case are

\[
\begin{bmatrix}
[M]_t & [0] \\
[0] & [0]
\end{bmatrix}
\begin{bmatrix}
\{x\}_t \\
\{x\}_r
\end{bmatrix}
+ \begin{bmatrix}
[K]_{tt} & [K]_{tr} \\
[K]_{rt} & [K]_{rr}
\end{bmatrix}
\begin{bmatrix}
\{x\}_t \\
\{x\}_r
\end{bmatrix}
= \begin{bmatrix}
\{F\}_t \\
\{0\}
\end{bmatrix}
\]

(B2)

where \( t \) denotes translation, \( r \) denotes rotation, and \( N = M + 1 \) where \( M \) is the number of elements of the modes.

Clearly, the vectors \( \{x\}_t \) and \( \{x\}_r \) are \( N \times 1 \) in equation (B1) while they are \( (N - 1) \times 1 \) in equation (B2) (without bottom node).

From equation (B1)

\[
\{x\}_r = -[Kc]_{rr} [Kc]_{rt} \{x\}_t
\]

(B3)

while from Equation (B2)
\[
\{x\}_r = -[Kc]_{rr}[Kc]_{rt}\{x\}_t \quad \text{where the symbol } (\ )^{\dagger} \text{ denotes lack of top rotation.}
\]

Therefore, \(\{x\}_r\) is \((N-2)\times1\) vector where the scalar \(x_{r_1}\) represents the top rotation. Substituting equation (B3) into Equation (B1) yields

\[
[M]_t\{\dot{x}\}_t + [K]\{\dot{x}\}_r = \{F\}_t \qquad \text{(B5)}
\]

Further, due to equation (B4) where

\[
[K] = [Kc]_{tt} - [Kc]_{tr}[Kc]_{rr}[Kc]_{rt} \quad \text{(B6)}
\]

Equation (B2) can be rewritten as

\[
\begin{bmatrix}
[M]_t & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\{\dot{x}\}_t \\
\dot{x}_{r_1}
\end{bmatrix} + [K]
\begin{bmatrix}
\{x\}_t \\
X_{r_1}
\end{bmatrix} = \begin{bmatrix}
\{F\}_t \\
M
\end{bmatrix} \quad \text{(B7)}
\]

where

\[
[K] = \begin{bmatrix}
[Kc]_{tt} & (K_{tr})_t \\
(K_{rt})_t & (K_{rr})_t
\end{bmatrix} - [Kc]_{tr}[Kc]_{rr}[Kc]_{rt} \quad \text{(B8)}
\]

and \((\ )^{\dagger}\) means stiffness related to top rotation.

Consider the boundary conditions for the hinge-hinge case. From equation (5), the equation of motion becomes
\[
[M]_{n-2}\{\ddot{X}_t\}_{n-2} + [C]_{n-2}\{\dot{X}_t\}_{n-2} + [K]_{n-2}\{X\}_{n-2} \\
= \{F\}_{n-2} + [K]_{n-2}\{X_r\}_{n-2} - \ddot{X}_{r_1}\{C_{j_1}\} \tag{B9}
\]

where \(n-2\) accounts for the unknown nodal displacement.

Similarly, for the case of clamp-clamp Equation (B6) yields

\[
[M]_{n-2}\{\ddot{X}_t\}_{n-2} + [C]_{n-2}\{\dot{X}_t\}_{n-2} + [K]_{n-2}\{X_t\}_{n-2} \\
= \{F\}_{n-2} + [K]_{n-2}\{X_r\}_{n-2} - \ddot{X}_{r_1}\{C_{j_1}\} \tag{B10}
\]

where \(X_{t_1}, X_{r_1}\) are the top translation and rotation.
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<th>4,000</th>
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<td>(including the contribution</td>
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<td>from added mass)</td>
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Table 1. Design Parameters of the Riser Systems
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<tr>
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<td>10</td>
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</table>

Table 2. Natural Frequencies (Hz) of Riser for Different Lengths and Boundary Conditions; Hinge-Hinge (H-H); Clamp-Clamp (C-C); Riser Parameters as in Table 1.
<table>
<thead>
<tr>
<th>Node</th>
<th>$T_f/T_a = 0$</th>
<th>$T_f/T_a = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude, $X_i/D$</td>
<td>Offset, $X_0i/D$</td>
</tr>
<tr>
<td>1</td>
<td>4.20</td>
<td>4.20</td>
</tr>
<tr>
<td>2</td>
<td>2.81</td>
<td>2.93</td>
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<tr>
<td>3</td>
<td>2.21</td>
<td>2.44</td>
</tr>
<tr>
<td>4</td>
<td>1.80</td>
<td>2.07</td>
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<tr>
<td>5</td>
<td>1.28</td>
<td>1.46</td>
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<tr>
<td>6</td>
<td>1.16</td>
<td>1.33</td>
</tr>
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<td>7</td>
<td>1.29</td>
<td>1.57</td>
</tr>
<tr>
<td>8</td>
<td>0.76</td>
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<td>0.71</td>
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<tr>
<td>10</td>
<td>1.03</td>
<td>1.31</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3 Values of amplitudes and offsets of nodal displacement determined by equivalent linearization (EL) and by numerical integration (NI); $L=1200'$ and remaining parameters as in Table 1.
Figure 1. Riser layout and modeling
Fig. 2. Temporal Maximum Bending Stress along the Riser; L= 1200';
Remaining Parameter as in Table 1.
Fig. 3. Riser Maximum Bending Stress versus Wave Period; L= 1200' and Remaining Parameters as in Table 1.
Fig. 4. Riser Maximum Bending Stress versus Wave Period; $L = 1200'$ and Remaining Parameters as in Table 1.
Fig. 5. Riser Maximum Bending Stress versus Wave Period; L = 1200' and Remaining Parameters as in Table 1.
Fig. 6. Riser Maximum Bending Stress versus Wave Period; L = 1200', and Remaining Parameters as in Table 1.
Fig. 7. Riser Maximum Bending Stress versus Wave Period; L = 1200' and Remaining Parameters as in Table 1.
Fig. 8. Riser Maximum Bending Stress versus Wave Period; L= 1200' and Remaining Parameters as in Table 1.
Fig. 9. Riser Maximum Bending Stress versus Wave Period; L = 1200' and Remaining Parameters as in Table 1.
Fig. 10. Riser Maximum Bending Stress versus Wave Period; L= 1200' and Remaining Parameters as in Table 1.