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EFFECTS OF FLUID-STRUCTURE INTERACTION ON THE
DYNAMIC RESPONSE OF SINGLE DEGREE-OF-FREEDOM
SYSTEMS.

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EFFECTS OF FLUID-STRUCTURE INTERACTION ON THE
DYNAMIC RESPONSE OF SINGLE DEGREE-OF-FREEDOM SYSTEMS

by

Marvin Beckmann

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

MASTER OF SCIENCE

APPROVED, THESIS COMMITTEE:

A. S. Veletsos, Brown & Root
Professor, Chairman

W. J. Austin, Chairman Department of Civil Engineering

E. C. Holt, Professor of
Civil Engineering

L. D. Lutes, Professor of
Civil Engineering

HOUSTON, TEXAS

February, 1981
ABSTRACT

EFFECTS OF FLUID-STRUCTURE INTERACTION ON THE DYNAMIC RESPONSE OF SINGLE DEGREE-OF-FREEDOM-SYSTEMS

by

Marvin Beckmann

The objective of this study is to examine the effects of fluid-structure interaction for relatively simple offshore structures responding as single-degree-of-freedom systems. In this study the phrase "fluid-structure interaction" refers to the relative velocity in the drag component of the Morison equation (as opposed to computing the drag forces by neglecting the structural velocity). Due to the nonlinearities this term presents difficulties in a dynamic analysis. For this reason the drag component of the Morison equation was studied extensively. The results were nondimensionalized with the intent to develop their presentation in spectral forms which are similar to those utilized in earthquake engineering.

The regions most affected by fluid-structure interaction are determined and displayed by utilizing a set of derived, dimensionless parameters. In addition, approximations to compensate for fluid-structure interaction (i.e. equivalent viscous damping concepts) are
presented which give acceptable results for design application. It was found that viscous damping and fluid-structure interaction have equivalent effects on the response of a structure subjected to waves.

Several sets of wave histories are used to compute response spectra for single-degree-of-freedom systems. These spectra demonstrate the consequences of fluid-structure interaction and illustrate the accuracy of the approximations considered.
ACKNOWLEDGEMENTS

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I also wish to express my gratitude and appreciation to Drs. W. J. Austin, E. C. Holt, L. D. Lutes, and J. E. Merwin for their suggestions and comments.
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I. INTRODUCTION

1.1 Background

Wave forces exerted on offshore structures are usually computed from current velocity, waveheight, and wave frequency by utilizing the Airy\(^1\) theory or Stokes\(^2\) approximation in connection with the Morison\(^3\) equation. The Morison equation contains two force components; inertia and drag. These force components can be obtained readily for rigid structures.

As structures are extended to greater water depths their flexibility increases, and therefore a dynamic analysis is required. The dynamic analysis is complicated due to the nonlinear relative velocity in the drag component of the Morison equation. If the structural velocity is neglected in the Morison equation then the system response can be computed by direct integration of the equation of motion. A more exact analysis incorporates the relative velocity in the drag term of the Morison equation and then computes the structural response using some type of iterative technique.

The phrase "fluid-structure interaction" is then meant to imply that the relative velocity is used in the Morison equation when computing forces which are exerted on the structure.
The computer solutions that determine maximum values from response histories resulting from wave forces are expensive because they require relatively long running times. Therefore, it would be advantageous to develop simple procedures that give approximate solutions comparable to the maximum response computed from the long time histories. This type of approach has been developed in the field of earthquake engineering. Structural responses resulting from earthquakes are normally characterized by the ground motions. For the offshore structure the response will result from wave forces developed by fluid particle motions. In the latter case, the actual fluid particle kinematics are assumed to be known as a function of time and are used directly to compute the wave force histories.

1.2 Objective and Scope of Study

The objectives of this study are to determine the parameters which govern fluid-structure interaction, obtain solutions using these parameters to demonstrate the areas most affected by the interaction, and to develop simple practical means to include these effects when needed in the design analysis. It was also attempted to explore the applicability of the approximate design concepts used in earthquake engineering to the analysis of structures subjected to wave forces.
1.3 Notation

The letter symbols are defined where they are first introduced in the text, and they are summarized below in alphabetical order. When histories are involved a subscript \( o \) denotes the maximum value of the corresponding history.

\( A \) Cross-sectional area normal to the fluid flow direction.

\( \bar{\rho} \) Equivalent damping factor which represents the effects of fluid-structure interaction.

\( C \) Damping coefficient for a linear system.

\( \bar{C} \) Damping coefficient which represents the effects of fluid-structure interaction.

\( C_M, C_D \) Inertia and drag coefficients, respectively.

\( f \) Systems natural frequency, in Hz.

\( f_w \) Wave frequency, in Hz.

\( F_0, (F_D)_0, (F_I)_0 \) Magnitude of an equivalent static lateral force which produces the same structural response as the absolute maximum response produced by an actual time dependent force \( P(t) \), drag force \( P_D(t) \) or inertial force \( P_I(t) \), respectively.

\( \delta, \gamma \) Dimensionless parameters which govern fluid-structure interaction.

\( h(t) \) Horizontal fluid particle displacement under a wave.

\( I_1(t), I_2(t) \) First and second integral of the force history \( P(t) \).

\( I_{1D}(t), I_{2D}(t) \) First and second integral of the drag force, \( P_D(t) \).

\( I_{1I}(t), I_{2I}(t) \) First and second integral of the inertia force, \( P_I(t) \).
$K$  Structural Stiffness.

$m$  Structural mass.

$m_a$  Added hydrodynamic mass. $= (C_m - 1) \rho \text{ Vol.}$

$M$  $= m + m_a$, Total system mass.

$M_I$  Inertial fluid mass. $= C_m \rho \text{ Vol.}$

$\mu$  Ratio of the maximum inertia to the maximum drag forces, $\mu = 2(C_m/C_D)(\text{Vol. } u_o/Au_o^2)$

$P(t)$  General force history.

$P_D(t), P_I(t)$  Drag and inertia forces exerted on the structure as a function of time, respectively.

$p$  Circular natural frequency of system.

$\rho$  Fluid density.

$T$  Structural period.

$t$  Time.

$t_1$  Characteristic time of the drag force (or velocity) history.

$u, u$  Fluid particle acceleration and velocity, respectively.

$\text{Vel}$  Magnitude of resultant velocities.

$\text{Vol.}$  Displaced fluid volume.

$w$  Wave frequency in rad/sec.

$x, \dot{x}, \ddot{x}$  Structural acceleration, velocity, and displacement, respectively.

$(x_{st})_o$  $(P_D)_o/K$, static deflection resulting from the maximum applied drag force amplitude, $(P_D)_o$.

$\xi$  System damping factor.
II. GOVERNING EQUATIONS

2.1 Equation of Motion

The equation of motion for the forced vibration of a linearly damped single-degree-of-freedom oscillator is:

\[ m\ddot{x} + C\dot{x} + Kx = P(t). \]

When the oscillator is subjected to the environment of ocean wave forces computed via the Morison equation then:

\[ P(t) = \text{Vol.} \rho \dot{u} + (C_m^{-1}) \rho \text{Vol.} \left( \dot{\Delta} \Delta \right) \]

\[ + \frac{1}{2} \rho C_D A(u - \dot{x}) | \text{Vel.}|. \]

In the above equations \( m \), \( C \), and \( K \) are the structural mass, damping and stiffness, respectively. \( x \) is the structural displacement and \( u \) is the fluid velocity, dots over either of these variables denotes differentiation with respect to time. \( \rho \) is the fluid density, \( \text{Vol.} \) is a displaced volume of the object and \( A \) is its cross-sectional area. \( C_m \) and \( C_D \) are an inertia and a drag coefficient which are considered to be constants in this study. The term \( \text{Vel.} \) represents the magnitude of the resultant velocities. For vertical members \( \text{Vel.} = \sqrt{(u - \dot{x})^2} \) and for horizontal members, subjected to two-dimensional flow, is: \( \text{Vel} = \sqrt{(u - \dot{x})^2 + v^2} \) where \( v \) is the vertical velocity component. Rearranging
terms the above equation becomes:

\[ M \ddot{x} + C \dot{x} + Kx = C_m \rho \text{Vol.} \dot{u} + \frac{1}{2} \rho C_D A (u - \dot{x}) | \text{Vol.} | \]

where \( M \) is the sum of the systems mass, \( m \), and the added hydrodynamic mass, \( m_a \) (i.e. \( m_a = (C_m-1) \rho \text{Vol.} \)). By dividing this equation by the mass, \( M \), the following expression is obtained.

\[ \ddot{x} + 2\xi_p \dot{x} + \omega^2 x = \frac{C_m \rho \text{Vol.}}{M} \ddot{u} + \frac{1}{2} \frac{\rho C_D A}{M} (u - \dot{x}) | \text{Vol.} | \]  \hspace{1cm} (2.1)

where \( \omega \) is the circular natural frequency of the system (i.e. \( \omega = \sqrt{\frac{K}{M}} \)) and \( \xi \) is the system damping factor referred to its critical value (i.e. \( \xi = C/2M\rho \)).

Equation 2.1 can be rearranged to:

\[ \ddot{x} + 2\xi_p \dot{x} + \omega^2 x = \frac{1}{2} \frac{\rho C_D A u_o^2}{M} \left\{ 2 \frac{C_m \text{Vol.}}{C_D A u_o^2} \frac{\dot{u}}{u_o} + \left( \frac{u}{u_o} - \frac{\dot{x}}{u_o} \right) \right\} \]

The variables \( \dot{u}_o \) and \( u_o \) represent the maximum fluid particle acceleration and velocity, respectively.

If we now let \( (P_D)_o \) represent the maximum applied drag force (i.e. \( (P_D)_o = 1/2 \rho C_D A u_o^2 \)) and \( \mu \) the ratio of the maximum inertia to drag force as follows:

\[ \mu = 2 \frac{C_m \text{Vol.}}{C_D A u_o^2} \frac{\dot{u}_o}{\frac{1}{2} \rho C_D A u_o^2} = \frac{C_m \rho \text{Vol.} \dot{u}_o}{\frac{1}{2} \rho C_D A u_o^2} \]
Then the above equation reduces to:

\[ \ddot{x} + 2\xi p \dot{x} + p^2 x = F(x, t) \]  \hspace{1cm} (2.2a)

where the value of \( F(t, \dot{x}) \) for vertical members is:

\[ F(x, t) = \frac{(P_d)}{M} \left\{ \mu \left( \frac{\dot{x}}{U_o} + \left( \frac{U}{U_o} - \frac{\dot{x}}{U_o} \right) \right) \right\} \]  \hspace{1cm} (2.2b)

Equation (2.2b) can also be written in the following form:

\[ F(x, t) = \rho^2 (x_{st})_0 \left\{ \mu \left( \frac{\dot{x}}{U_o} + \left( \frac{U}{U_o} - \frac{\dot{x}}{U_o} \right) \right) \right\} \]  \hspace{1cm} (2.2c)

where \((x_{st})_0\) represent the static deflection of the system subjected to the maximum applied drag force.

In equation 2.2 the fluid particle acceleration and velocity are assumed to be known for all values of time. An investigation of this equation reveals that it is nonlinear in terms of the structural and wave velocities. The nonlinearities in structural velocity imply that solutions of equation 2 will have to be obtained by means of some type of iterative technique. If one neglects the effects of fluid-structure interaction, the forcing function in equation (2.2c) becomes:

\[ F(x, t) = (x_{st})_0 \left\{ \mu \left( \frac{\dot{x}}{U_o} + \frac{U}{U_o} \right) \right\} \]  \hspace{1cm} (2.3)

which is nonlinear only with respect to the wave velocity, \( U \).
2.2 Parameters Affecting Fluid-Structure Interaction

Two dimensionless parameters which represent the effects of fluid-structure interaction (i.e. the importance of the structural velocity in equation 2.2) are determined in the following sections. The parameters are obtained by normalizing equation (2.2).

2.2a Interaction Parameter \( \gamma \)

This interaction parameter is obtained by normalizing the time with respect to \( t_1 \) and \( x \) with respect to \( (x_{st})_o \) as follows:

\[
t = t_1 \xi
\]

\[
x = (x_{st})_o Z
\]

where \( t \) represents the time, \( \xi \) and \( z \) are dimensionless functions. The value \( t_1 \) represents a characteristic time of the drag force or velocity time history. For a random wave, \( t_1 \) could be taken equal to one half the significant or average wave period. The value \( (x_{st})_o = (P_D)_o/K \), represents the static displacement of the system due to the maximum amplitude of the applied drag force \( (P_D)_o \), with \( K \) being the structural stiffness. If the above expressions for \( x \) and \( t \) are substituted into equation (2.2 a and c) and the resulting equation is multiplied by \( t_1^2 \) and divided by \( (x_{st})_o \) one obtains:
\[
\frac{d^2 Z}{d \xi^2} + 2 \xi (p t_1) \frac{d Z}{d \xi} + (p t_1)^2 Z = (p t_1)^2 \left\{ \mu \frac{\dot{u}}{u_o} + \left( \frac{u}{u_o} - \frac{\chi_{st}}{u_o \xi} \frac{d Z}{d \xi} \right) \frac{\dot{u}}{u_o} - \frac{(\chi_{st})_o}{u_o \xi} \frac{d Z}{d \xi} \right\}
\]

with:

\[
\gamma' = \frac{(X_{st})_o}{u_o \tau_1}
\]

we have

\[
\frac{d^2 Z}{d \xi^2} + 2 \xi (p t_1) \frac{d Z}{d \xi} + (p t_1)^2 Z = (p t_1)^2 \left\{ \mu \frac{\dot{u}}{u_o} + \left( \frac{u}{u_o} - \gamma \frac{d Z}{d \xi} \right) \frac{\dot{u}}{u_o} - \gamma' \frac{d Z}{d \xi} \right\}
\]

which shows that \( \gamma' \) is a measure of interaction. This equation also shows that for a fixed value of \( \gamma' \) and \( \mu \) its solution is a function of \( p t_1 \) and \( \xi \) for a specified wave history.

The ratio \((x_{st})_o / \tau_1\) in equation (2.4) represents a pseudo-structural velocity, and therefore, \( \gamma' \) may be considered to represent a ratio of a pseudo-structural velocity to the maximum fluid velocity, \( u_o \).

2.2b Interaction Parameter \( \delta \)

This interaction parameter is obtained by normalizing the time with respect to \( p \), the system's circular natural frequency, and \( x \) with respect to \((x_{st})_o\) as follows:
\[ t = (1/p) \xi \]

\[ X = (X_{st})_0 Z \]

where \( t \) again represents time and \( z \) and \( \xi \) are dimensionless functions. If these values are substituted into equation (2.2a and c) and the resulting equation is divided by \( p^2 (X_{st})_0 \) one obtains.

\[ \frac{d^2 Z}{d \xi^2} + 2 \xi \frac{dZ}{d \xi} + Z = \mu \frac{dX}{d \xi} + \left( \frac{U}{U_o} - (X_{st} + \frac{dZ}{d \xi}) \right) \frac{dU}{U_o} \left( \frac{X_{st} + \frac{dZ}{d \xi}}{d \xi} \right) \]

and if we let:

\[ \delta = \frac{(X_{st} + \frac{dZ}{d \xi})}{U_o} \]  

(2.5)

we have

\[ \frac{d^2 Z}{d \xi^2} + 2 \delta \frac{dZ}{d \xi} + Z = \mu \frac{dX}{d \xi} + \left( \frac{U}{U_o} - \delta \frac{dZ}{d \xi} \right) \frac{dU}{U_o} - \delta \frac{dZ}{d \xi} \]

which shows that \( \delta \) is also a measure of fluid-structure interaction. This equation shows that its solution depends only on \( \delta \) and \( \mu \) for a fixed value of \( \xi \) and a specified wave history. The numerator in the expression for \( \delta \) is an approximation of the maximum structural velocity \( \dot{x}_o \), and therefore represents a ratio of maximum structural velocity.
to maximum fluid velocity.

Both of the above parameters represent a velocity ratio, and hence a measure of the importance of fluid-structure interaction. It is noted that as \( \gamma \) or \( \delta \) approached zero equation 2.2 takes the form of equation 2.3 (i.e. the solution with no fluid-structure interaction).

The parameters \( \delta \) and \( \gamma \) are related as follows:

\[
\delta = 2\pi ft, \gamma = pt, \gamma
\]  
(2.6)

where \( f \) is the structural frequency in Hertz. It is believed that \( \gamma \) is a more convenient parameter for design applications. The principal advantage of the parameter \( \delta \) is that it is directly related to the concept of equivalent viscous damping which is developed in the following section.

2.3 Equivalent Damping Concept

Considering that the effects of fluid-structure interaction, like those of viscous damping, reduce the maximum response of the system, it is natural to inquire if the interaction effects can be linearized and approximated by an equivalent viscous damping. That this is possible can be shown by expanding the absolute value in equation (2.2) and approximating the resulting expression with an equivalent averaged damping coefficient. Expanding the absolute value on the right hand side of equation (2.2) results in:
\[ \dddot{x} + 2\xi p \dot{x} + p^2 \dot{x} = p^2(X_{st})_0 \left\{ \mu \frac{\dot{\xi}}{\xi_o} + \left[ \frac{(\dot{\xi})^2}{\xi_o^2} - 2 \frac{\dot{\xi}}{\xi_o} \right] \text{sign}(\xi - \dot{x}) \right\} \] (2.7)

where "sign" keeps track of the sign resulting from the absolute value in equation (2.2). Now the assumption is made that the maximum fluid velocity is much greater than the structural velocity for all times (i.e. \( u_o >> \dot{x} \)). With this assumption the equation above is approximated by dropping the nonlinear term \((\dot{x}/u_o)^2\), and the resulting equation becomes:

\[ \dddot{x} + 2\xi p \dot{x} + p^2 \dot{x} = p^2(X_{st})_0 \left\{ \mu \frac{\dot{\xi}}{\xi_o} + \left[ \frac{(\dot{\xi})^2}{\xi_o^2} - 2 \frac{\dot{\xi}}{\xi_o} \right] \text{sign}(\xi - \dot{x}) \right\} \] (2.8)

We now transpose the terms containing \( \dot{x} \) to the left hand side of the equation and obtain:

\[ \dddot{x} + 2\left\{ \xi + \frac{p(X_{st})_0 \dot{\xi}}{\xi_o} \text{sign}(\xi - \dot{x}) \right\} p \dot{x} + p^2 \dot{x} = p^2(X_{st})_0 \left\{ \mu \frac{\dot{\xi}}{\xi_o} + \left[ \frac{(\dot{\xi})^2}{\xi_o^2} \right] \text{sign}(\xi - \dot{x}) \right\} \]

Examining this equation reveals that \( p(X_{st})_0 / u_o \) is precisely the interaction parameter \( \delta \) derived in section 2.2b. Therefore, this equation can now be written:

\[ \dddot{x} + 2\left\{ \xi + \delta \frac{\dot{\xi}}{\xi_o} \text{sign}(\xi - \dot{x}) \right\} p \dot{x} + p^2 \dot{x} = p^2(X_{st})_0 \left\{ \mu \frac{\dot{\xi}}{\xi_o} + \left[ \frac{(\dot{\xi})^2}{\xi_o^2} \right] \text{sign}(\xi - \dot{x}) \right\} \] (2.9)
This equation shows that a fluid damping term has been found which is the result of fluid-structure interaction, \( \delta \). Its factor can be substituted by the constant, \( \bar{c} \):

\[
\bar{c} = \left[ \frac{\mu}{U_o} \text{Sign}(u - \dot{x}) \right]_{\text{Averaged}}
\]  

(2.10)

The sign \( (u - \dot{x}) \) term can then be replaced by sign \( (u) \) when the expression for \( \bar{c} \) is averaged over each half cycle of the velocity diagram. This averaging over each half cycle is justified since the effect of the damping results in a loss in maximum amplitudes which are considered in this work. Therefore, \( \bar{c} \) represents a ratio of the "effective" fluid velocity to its absolute maximum value, \( U_o \). With \( \bar{c} \) in equation (2.9), we obtain:

\[
\ddot{x} + 2\left\{ \xi + \bar{c} \delta \right\} \rho \dot{x} + \rho^2 x = \rho^2 (x_{e0}) \left[ \mu \frac{\dot{U}}{U_o} + \frac{U^2}{U_o^2} \text{Sign}(u - \dot{x}) \right]
\]  

(2.11)

Therefore the damping factor which represents fluid-structure interaction can now be defined as:

\[
\beta = \bar{c} \delta
\]

The sign \( (U - \dot{x}) \) term on the right hand side of equation (2.11) is also replaced by sign \( (u) \). When this is done we obtain the equivalent damping equation in its final form:

\[
\ddot{x} + 2\left\{ \xi + \beta \right\} \rho \dot{x} + \rho^2 x = \rho^2 (x_{e0}) \left[ \mu \frac{\dot{U}}{U_o} + \frac{U}{U_o} \frac{U}{U_o} \right]
\]  

(2.12)
III. METHOD OF SOLUTION AND PROGRAMS

The response of the single-degree-of-freedom system was evaluated by numerical integration of the governing equations of motion using a modified version of a Brown & Root computer program which had been, in part, developed by the author. The modified program NIT (Numerical Integration Technique) integrates the equation of motion stepwise by assuming a linear variation of the exciting force within each time interval. For the purpose of this study, two additional programs (INTEGRAL) and (WAVE) were written. The program WAVE computes irregular wave kinematic histories utilizing the equations presented in Appendix C. Program (INTEGRAL) numerically integrates the fluid accelerations, fluid velocities, and forcing functions. The programs NIT and INTEGRAL were modified to permit consideration of unequal time intervals. This made it possible to use the program INTEGRAL to more accurately integrate a fluid acceleration diagram that was piecewise linear with slope discontinuities, and thereby improve the accuracy in the values of the associated fluid velocity diagram.

The general structure of the computer program NIT is given in Appendix A.
The program INTEGRAL uses the trapezoid rule to integrate linear variations and Simpson's 1/3 rule for curves. The program WAVE utilizes the equations in Appendix C to compute random wave kinematic histories.
IV. RESULTS

All results were obtained by subjecting single degree-of-freedom systems to specified wave force histories which either included or neglected the effects of fluid-structure interaction. For all the wave histories considered, the system was assumed to be initially at rest.

The maximum responses of single degree-of-freedom systems were computed for a specified wave history and displayed using a response spectrum. Response spectra are a graphical representation of dynamic amplification factors as a function of the systems characteristic frequency or period.

In this study two types of response spectra were utilized. The first type is shown in figure 3.1 and is referred to as a tripartite spectrum. This type of plot is commonly seen in the field of earthquake engineering. A brief description of this plot is given in Appendix B. The appendix basically explains the meaning of the graph when used in connection with earthquakes or ground excited systems. Then these ideas are extended to force excited systems. The data was presented on tripartite response spectra for two reasons: a) to determine if it is practical to graphically represent the maximum response of offshore structures subjected to wave loadings in this form,
and b) it was believed that a better interpretation of the results might be obtainable than with the second type of response spectrum utilized. This type of spectrum is shown in figure 3.2. This conventional plot is more commonly seen in the field of offshore structures. It represents the dynamic amplification factor versus the system's natural period. If the curves in figure 3.2 are turned end for end (i.e. plotted in terms of frequency) then they are represented in figure 3.1 by the diagonal scale which extends upwards from right to left. The subscripted variable F in figure 3 represents the magnitude of an equivalent static lateral force which produces the same structural response as the absolute maximum response produced by an actual time dependent force, P(t). Further descriptions of these plots will be made as the results are presented.

4.1 Pseudo Wave History

The wave particle kinematics considered initially are shown in figure 4. The fluid acceleration is assumed to be represented by the piecewise linear diagram shown in part (c) and the associated fluid velocity and fluid displacement histories are shown in parts (b) and (a), respectively. These fluid kinematics are unrealistic in the case of dispersing gravity waves. Nevertheless, for the sake of demonstration, these fluid kinematic
histories will be utilized to examine the applicability of the tripartite spectrum to waves.

Figure 5c shows the normalized instantaneous drag force plotted against time which is normalized with respect to the initial half cycle of the velocity diagram. In the absence of fluid-structure interaction the drag force is proportional to the square of the fluid particle velocity which is represented in figure 4b. The dragcurve (figure 5c), was integrated once with respect to time to yield $I_{1D}(t)$ (figure 5b), and again integrated with respect to time to obtain, $I_{2D}(t)$, (figure 5a). The maximum values of these curves denoted by $(I_{1D})_O$ and $(I_{2D})_O$ are used to normalize the tripartite spectrum discussed previously (i.e. see figure 3.1b).

The inertia force, $P_I(t)$, is directly proportional to the fluid acceleration and, in this case, its first and second integrals (i.e. $I_{1I}(t)$ and $I_{2I}(t)$) are directly proportional to the fluid velocity and fluid displacement, respectively. These curves are shown in the inset diagram of figure 6a. The maximum values of $I_{1I}(t)$ and $I_{2I}(t)$ (i.e. $(I_{1I})_O$ and $(I_{2I})_O$) are used to normalize the inertia tripartite spectrum (i.e. see figure 3.1a).

The tripartite response spectra of figure 6 represent the maximum response of noninteracting damped systems subjected to the wave force histories on the inset
diagrams. These figures show that the effects of damping, as expected, smooth and reduce the maximum response curves. Figure 6a is the tripartite response spectrum for inertia forces and figure 6b is for drag forces in the absence of fluid-structure interaction.

The limiting values of figure 6a are described as follows. For large values of $f_{t1}$ (i.e. large system frequencies or large wave periods) the maximum value of $(P_I)_{o}$ approaches $(P_I)_{o}$, the maximum applied inertia force. In this region a static analysis is adequate for design purposes.

For small values of $f_{t1}$, the value of $(P_I)_{o}$ approaches $p^2(I_{21})_{o}$, which represents the square of the circular natural frequency multiplied by the maximum value of the second integral of the inertia force history, $P_I(t)$. The value of $(I_{21})_{o}$ can be evaluated from:

$$
(I_{21})_{o} = [I_{21}(t)]_{\max} = \left[ \int_0^t \int_0^t P_I(t) \, dt \, dt \right]_{\max}
$$

where $P_I(t)$ is the inertia force history. Replacing $P_I(t)$ by:

$$
P_I(t) = C_m \rho V_0 \dot{u} = M_I \ddot{u}
$$

where $M_I$ is the inertial mass of the fluid. By inserting
this expression into equation (4.1) one obtains

$$(I_{21})_o = M_1 \left[ \int_0^t \int_0^t \mathcal{W} dt \, dt \right]_{max}$$

The bracketed term can now be evaluated by inspecting figure 4, which gives:

$$(I_{21})_o = M_1 h_o$$

Therefore, for small values of $ft$, the following relationship is realized:

$$(F_1)_o = M_1 p^2 h_o \quad (4.2)$$

Since $(F_1)_o$ represents the static lateral force required to deflect the structure by an amount $X_{max}$, the maximum response resulting from a dynamic analysis, it can also be written:

$$(F_1)_o = K|X_{max}|$$

Using this expression in equation (4.2) gives:

$$K|X_{max}| = M_1 p^2 h_o$$

Now the square of the circular natural frequency, $p$, can be replaced by $K/M$ which results in:

$$|X_{max}| = \frac{M_1}{M} h_o \quad (4.3)$$
This expression infers that for small \( ft \) values, the maximum deformation of the structure is proportional to the maximum horizontal fluid displacement. The proportionality factor being the ratio of the inertial fluid mass to the systems mass.

For the median values of \( ft \), the value of \( \frac{\langle I_{II} \rangle_0}{P(I_{II})_0} \) reaches a plateau with magnitude greater than one. The value of \( (I_{II})_0 \) is obtained from the following relationship:

\[
(I_{II})_0 = (I_{II}(t))_{\text{max}} = \left( \int_0^t P_{I}(t) \, dt \right)_{\text{max}} \tag{4.4}
\]

where \( P_{I}(t) \) represents the inertia force history. The bracketed term can be replaced by:

\[
\int_0^t P_{I}(t) \, dt = C_m p V_0 l \int_0^t \ddot{u} \, dt = M_I \int_0^t \ddot{u} \, dt
\]

where the variables were defined above. With this expression equation (4.4) becomes:

\[
(I_{II})_0 = M_I \left[ \int_0^t \ddot{u} \, dt \right]_{\text{max}}
\]

The bracketted term can again be obtained using figure 4 to be:

\[
(I_{II})_0 = M_I \ddot{u}_0
\]
With this expression the following proportionality for median $f_{t_1}$ values is realized:

\[(F_I)_o \propto M_I (p \ u_o) \]  \hspace{1cm} (4.5)

By multiplying the left side of the equation by $p^2 (M/K)$, (i.e. unity) and recognizing that $|x_{\text{max}}| = (F_I)_o / K$ then equation (4.5) reduces to:

\[p |x_{\text{max}}| \propto \frac{M_3}{M} \ u_o \]

The quantity $p |x_{\text{max}}|$ represents the maximum structural velocity. This relationship shows that this region of the spectrum is characterized by the fluid velocity.

The limiting values of figure 6b (i.e. the tripartite response spectrum for drag forces in the absence of interaction) cannot be given the same physical meanings as above. For small $f_{t_1}$ values the following relationship is realized:

\[(F_D)_o = p^2 (I_{2D})_o \]

where $p$ is the system circular natural frequency, and $(I_{2D})_o$ is the maximum value of the second integral of the drag force history.

As $f_{t_1}$ becomes large then:

\[(F_D)_o = (P_D)_o \]
where \((P_D)_o\) is the maximum applied drag force.

For the median values of \(ft_1\) the following proportionality is obtained:

\[
(F_D)_o \propto \rho (I_{1D})_o
\]

where \((I_{1D})_o\) is the maximum value of the first integral of the drag force history, \(I_{1D}(t)\), the pseudo-momentum.

Throughout the rest of this study equations (2.2), (2.3), and (2.13) are solved with the constant \(\mu\) taken as zero (i.e. a drag dominated system). In this approach the effects of fluid-structure interaction were studied.

To demonstrate the phenomena of fluid-structure interaction clearly, the time histories of figures 7 and 8 were generated for undamped systems with different amounts of the interaction parameter \(\gamma\). These histories were obtained for resonant conditions which dramatize the effects. Part a of the figures is the effectively applied drag force and part b is the corresponding system response. The force histories show that the effects of interaction reduce the magnitude of the applied drag forces. The normalized response histories of part b demonstrate that larger amounts of interaction results in smaller maximum response values. The maximum value of these and many other histories are presented graphically on the response spectra of figures 9a and 9b.
The response spectra of figures 9a and 9b give the same information. Part a is a conventional type spectrum used commonly in structural design and part b represents a tripartite response spectrum of the type used in earthquake engineering. Both spectra are a plot of dynamic amplification factors for undamped systems with different amounts of the interaction parameter \( \gamma \). They show that the maximum undamped structural response corresponding to a fixed value of \( \gamma \) varies as a function of the system's characteristic period or frequency. These spectra also show that the greatest reduction in maximum response for fixed values of \( \gamma \) occurs where resonant conditions prevail. This is expected since this region consists of the larger dynamic amplification factors.

The dashed curves of figure 9b represent the approximate solutions obtained using in equivalent damping for \( \gamma = 0.04 \) and 0.4 (i.e. using equation 2.13). The equivalent damping is obtained through the expression:

\[
\tilde{\beta} = \bar{\zeta} \delta
\]

where \( \bar{\zeta} \) was found to amount to 7/12 by averaging over each half cycle the absolute value of the velocity diagram which was normalized with respect to its maximum value (see figure 4b). The value of \( \delta \) in terms of \( \gamma \) is given by equation 2.6:

\[
\delta = 2\pi ft_1 \gamma
\]
which is a function of the systems frequency. Combining the above two expressions gives an equivalent damping factor in this case as:

$$\bar{\beta} = \frac{7}{6} \pi (ft, ) \gamma$$

where $\gamma$ is a function of $(ft_1)$ itself. Therefore, no direct conclusions can be made from this equation. The interaction parameter $\delta$ is clearly a more suitable choice for presentation of the equivalent damping concepts since it is directly related to the equivalent damping coefficient. The solutions presented for the parameter $\gamma$ demonstrate, however, that either of the two parameters can be utilized to obtain solutions with the equivalent damping concept.

The lower dashed curve of figure 9b is discontinued where the equivalent damping factor becomes greater than unity, which implies that the value of $\gamma = 0.4$ is unrealistic for this region of the spectrum. But it demonstrates that the effects of interaction can be represented effectively, in this case, by an equivalent damping factor for rather large amounts of interaction. It is also noted that the value of $\bar{\beta}$ for the velocity diagram of figure 4b is 7/12 for all half cycles (i.e. the average over the absolute value of the whole record is equal to the average over the absolute value of each individual half cycle), since the amplitude is the same for all half cycles and the curves are proportional to each other.
The limiting values of figure 9b are interpreted as follows. For large values of $ft_1$, the maximum value of $(P_D)_o$ approaches the "static" value which is equal to or slightly less than $(P_D)_o$, the maximum applied drag force. A value less than $(P_D)_o$ results from large amounts of interaction which reduces the effectively applied drag force, as seen in figures 7a and 8a. For small values of $ft_1$ the value of $(P_D)_o$ approaches $p^2(I_{2D})_o$, as was demonstrated previously.

The effects of interaction can also be displayed by using the parameter $\delta$. A response spectrum for this parameter is shown in figure 10 for undamped systems. The dashed curves represent the solution obtained using an equivalent damping factor. The obvious advantage of the parameter $\delta$ is its direct proportionality with the equivalent damping factor discussed previously. The equivalent damping factor, in this case, is given as:

$$\tilde{\beta} = \bar{c} \delta$$

where $\bar{c}$ was found, as before, by averaging the velocity diagram and $\delta$ is as indicated in the figure. This figure shows that for small values of $\delta$ the equivalent damping concepts give acceptable results for design application. Even for the larger values of $\delta$ the solutions are not unreasonable.

The equivalent damping concepts are demonstrated in terms of response time histories in figure 11. The solid
curves correspond to the exact solution (equation 2.2), and the dashed curves represent the solutions obtained using the inscribed equivalent damping factors (equation 2.13). Figure 11 shows that the response computed using an equivalent damping follows the exact solution very closely for all values of time where the fluid velocity is assumed to exist (i.e. \( t/t_1 \leq 4.0 \)). In other words, the equivalent damping representing the effective fluid velocity should be reduced to a value representing the fluid friction for time intervals where the fluid velocity is zero (i.e. \( t/t_1 > 4.0 \) in figure 11).

However, this study was mainly concerned with structures subjected to waves and not structures vibrating in a calm sea and therefore these effects were not considered. It is also noted that all of the solutions computed using the concepts of an equivalent damping resulted in a larger maximum response than the exact solution yielded in this case.

4.2 Extended Pseudo Wave History

The drag force and its first two integrals for the extended wave history are shown in figure 12. These histories are an extension of those shown in figure 5 (i.e. the 1.5 waves considered in figure 5 are followed by another asymmetric 1.5 waves of the same type). By extending the pseudo wave histories in this fashion,
the numerical values of \( I_{1D}^0 \) and \( I_{2D}^0 \) remain unchanged (compare figures 5 and 12). The extended wave history applies the same unrealistic wave patterns as the pseudo wave presented in the previous section. The reason for studying the extended history was to examine the variation of the resulting spectra and compare them to the corresponding spectra resulting from the pseudo wave histories. It also served the purpose of demonstrating that the equivalent damping was accurate for the longer histories, as would be expected in this case.

Figure 13 represents a response spectrum for non-interacting damped systems subjected to the extended histories on the inset diagram. The corresponding spectra for one half of this wave train are given by figure 6b. Comparison of figures 6b and 13 conveys the fact that the amplification factors are increased. However, the general shape is the same except for the additional hump in the central region of figure 13. This additional hump exists since \( I_{2D}(t) \) now consists of two half cycles.

Comparison of the response spectra of figures 9 and 16 for the interaction parameter \( \gamma \) shows that the main differences occur at the resonant frequencies. The limiting values are similar and the intermediate values of the curves better resemble each other for larger values of interaction (i.e. \( \gamma = 0.2 \) and 0.4). The spectra
also agree well in the areas of the secondary peaks (i.e. at $T/t_1 = 1.0$ in figures 9a and 16a) for smaller amounts of interaction. This seems to imply that in this region the maximum response is caused mainly by the pseudo pulse (or the first half of the extended pulse), which is demonstrated by the response history in figure 15b.

The response history of figure 14b corresponds to the main resonant frequency and shows that the response is further increased during the second half cycle of the extended drag force history. However, the increase becomes less significant for larger values of $\gamma$. The effective force history of figure 15a shows that the fluid and structural velocity are not always in phase with each other.

The response spectra for the interaction parameter $\delta$ is shown in figure 17. It shows that the approximate solution yields adequate results for design purposes. The discrepancies between the exact and approximate solution are similar to the discrepancies of figure 10, which gives the corresponding spectra for the shorter wave history.

The time histories of figure 18 which compare the exact and approximate solution are, as before, in excellent agreement for times during which wave action exists (i.e. $t/t_1 \leq 8.0$).
4.3 Irregular Wave History

The irregular wave kinematic histories were generated by superimposing ten sinusoidal waves with different amplitudes, frequencies, and random phase angles. The wavelets were superposed using the equations in Appendix C and a section of the resulting history was extracted which contained a maximum wave as shown in figure 19. The drag force derived from the fluid velocity (figure 19b) is plotted in figure 20 along with its first and second integrals. The motivation behind the investigation of this irregular history with varying amplitudes was to examine the applicability of the equivalent damping concepts. For all previous records the damping factor, $\zeta$, had remained constant for each half cycle of the velocity diagram.

The peculiar variation of $I_{1D}(t)$ and $I_{2D}(t)$ shown in figure 20 is a result of the passage of the maximum wave. The base line shift of $I_{1D}(t)$ and the change in slope of $I_{2D}(t)$ results from an "unbalanced" impulse in the drag force due to the maximum wave.

The only tripartite response spectrum computed for the irregular wave is shown in figure 21. This spectrum represents the maximum response obtained for non-interacting undamped systems subjected to the histories of figure 20. The curve in figure 21 approaches $0.49 \, p \left( I_{1D} \right)_0$. 
asymptotically as $t_1$ approaches 0. The numerical value $0.49$ \( (I_{1D})_0 \) is equal to the final value obtained by $I_{1D}(t)$, the first integral of the drag force history (see figure 20b). This value is acquired as a result of a force unbalance due to the passage of the maximum wave. For offshore structures with periods between 0 and 30 seconds figure 21 shows that the right diagonal scale is a convenient representation of the dynamic amplification factor. Although the tripartite spectra were helpful in interpreting some of the previous results, it is not used to present the results for the irregular wave. Instead, the maximum response values are plotted on conventional response spectra.

The response spectra obtained for undamped systems subjected to different amounts of interaction are shown in figure 22. Part b of the figure utilizes the parameter $\delta$ and part a employs the parameter $\gamma$ multiplied by $t_1$ which amounts to \( (x_{st})_0/u_0 \). Part a was presented in this alternate form (i.e. $\gamma t_1$ instead of $\gamma$) to eliminate the characteristic time $t_1$; since all half cycles of the velocity (or drag force) diagram (see figure 19b or 20c) realize different durations and there is no single value which is representative of $t_1$. The spectra of figure 22 show, as in all previous cases, that the effects of interaction result in a reduction of the maximum response.
Three separate methods were used to compute the approximate solutions. The first method which should provide the most accurate results uses a different damping factor for each half cycle of the velocity diagram. The factor \( \xi \) used for each half cycle was obtained by averaging the absolute value of each half cycle of the velocity diagram normalized with respect to the maximum value. The values obtained are shown in figure 19b below the half cycle they represent. The spectra calculated using this procedure are shown in figure 23 and 24. Figure 23 utilizes the parameter \( \gamma t_1 \) and figure 24 utilizes the parameter \( \delta \). Both of these figures demonstrate that this procedure would result in solutions which are adequate for design application. However, it would be more advantageous to determine a single damping factor that represents the entire velocity history. This later approach is explored in the following paragraphs.

The remaining two methods utilized a single damping factor to represent the entire velocity diagram. The damping values were computed using \( \xi = 0.619 \) and 0.253. The value \( \xi = 0.253 \) corresponds to the averaged absolute value of the entire velocity diagram of figure 19b. The value \( \xi = 0.619 \) corresponds to the averaged absolute value of the peak pulse of the velocity diagram. The response spectra which demonstrate these latter methods of solution are shown in figure 25.
It is interesting to note that the method which used \( \tilde{c} = 0.619 \) to compute the approximate solution gave good results. However, it was not always conservative. Also, only the approximate solution which used \( \tilde{c} = 0.619 \) or a different \( \tilde{c} \) for each half cycle of the velocity diagram were in good agreement with the exact solution for \( \gamma t_1 = 0.4 \). The averaged value, \( \tilde{c} = 0.253 \) turned out to produce overly conservative results.

4.4 Random Wave History

The random wave kinematics were generated from the modified Pierson Moskowitz sea spectrum shown in figure 26. This spectrum is also represented by the following equation

\[
S(f_w) = \frac{H_s^2 T_{av}^{5}}{8 \pi f_w} (f_w T_{av})^{-5} (f_w T_{av})^{-4} \tag{4.6}
\]

with:

- \( H_s = 40 \text{ ft.}, \) significant wave height
- \( T_{av} = 12 \text{ sec.}, \) average wave period.

Using these values, equation 4.6 is representative of a hurricane sea state.

To generate the resulting fluid particle kinematic histories, the sea spectrum was divided into 60 equal frequency bands of 0.005Hz. For each frequency band, \( \Delta f_w \), an individual sine wave was established using that particular
midband frequency, \( f_{w_i} \), and wave amplitude, \( a_i \), given by \(^9\):

\[
a_i = \sqrt{2AF_w S(f_{w_i})}
\]

The 60 wavelets were then superimposed to yield a particular wave train at a 30 foot water depth by using predetermined random phase shifts distributed from 0 to \( 2\pi \) with the equations in Appendix C. The values obtained for each of the 60 waves are shown in Table I.

The frequency increment, \( \Delta f_w \), resulted in the kinematic histories repeating themselves after 200 seconds (1/ \( f_w \)). \(^1^1\) The fluid histories time origin and duration were adjusted so that the first and second integrals of the drag force (\( I_{1D}(t) \) and \( I_{2D}(t) \)) obtained approximate zero values shortly after 200 seconds had passed. This resulted in the wave kinematics shown in figure 27 with a duration of 217.6 seconds. The drag force history derived from the fluid velocity diagram is shown in figure 28 along with its first and second integrals. These integrals, \( I_{1D}(t) \) and \( I_{2D}(t) \), are somewhat balanced about a zero mean value as opposed to the corresponding histories from the irregular wave (figure 20a and 20b) history. The balancing of the curves was accomplished through trial and error by arbitrarily shifting the origin and duration of the time scale. Although these histories balance for this depth, their behavior at other depths,
origins, or random phase angles will result in phenomena similar to those for the irregular wave.

A tripartite spectrum computed for the drag force history is shown in figure 29. This spectrum represents the maximum response obtained for non-interacting undamped systems. Inspection of figure 29 reveals that the right diagonal scale \( (F_D)_o/(F_D)_o \) plotted against the systems frequency or period is the logical choice of coordinates when structures have periods between 0 and 30 seconds. Therefore, the results are presented on conventional type spectra as was the case for the irregular wave train.

The effects of damping are demonstrated in figure 30. This plot is for non-interacting systems with different amounts of damping. The figure shows that even a small amount of damping (2%) smooths and reduces the response curve considerably.

The response spectra computed for undamped interacting systems are shown in figures 31a and 32a. Again, as was the case for the irregular wave, the interaction parameter \( \gamma \) has been presented in a dimensional form: \( (x_{st})_o/u_o \). The explanation for this desired form was discussed in the previous section. As was demonstrated by all previous results, the effects of interaction reduces the maximum response.

The solutions obtained using an equivalent damping factor are shown in figures 31b and 32b. The equivalent
damping factor, $\tilde{\zeta}$, was obtained by averaging the absolute value of the velocity diagram normalized with respect to its maximum value over its peak half cycle. Using this procedure the value of $\tilde{\zeta}$ was found to amount to 0.593. This procedure proved to be the most reasonable approach for the irregular wave train. Inspection of figures 31b and 32b show that this approach gives acceptable results for design application in this case also.
V. CONCLUSIONS

This study has been concerned with the dynamic response of single degree-of-freedom systems subjected to wave forces. Its main goal was to examine the effects of fluid-structure interaction in the drag component of the Morison equation. Two parameters ($\gamma$ and $\delta$) that govern the effects of interaction were derived and employed to display the results. These parameters can be used by the designer of offshore structures to determine the importance of fluid-structure interaction. It was found that the interaction effects can be adequately represented by an equivalent damping factor for design applications. The equivalent damping factor is obtained by averaging the absolute value of the velocity diagram normalized with respect to its maximum value over the dominant half cycle.

All results presented are only applicable to single degree-of-freedom systems at a specified point below the sea surface. The solution for more complicated multi degree-of-freedom systems cannot be solved at this time in connection with the spectra presented herein.
REFERENCES


APPENDIX A

Description of Program NIT

This appendix describes the computer program NIT (Numerical Integration Technique) that was utilized to compute the results presented in this thesis. The description is not intended to give a full documentation. Its purpose is to describe the gross ideology involved. NIT numerically integrates the equations of motion (i.e., equations 2.2, 2.3, and 2.13) to determine the systems maximum response. The main program is described on the next page in flow chart form followed by descriptions of the subroutines it utilizes.
MAIN PROGRAM:

START

Input: Time increment, fluid kinematics under consideration, and other normalizing factors that characterize the force history.

Input: \( f \), the systems frequency, \( NS \), the number of time steps, and other parameters that characterize the system (i.e. parameters such as the amount of damping and the amount of interaction if applicable).

\[ f = 0.0 \]

Yes

No

\[ J = 1, NS \]

Call COEF

Call FORCE

Call MAXIMA

Call DAF

STOP
Subroutine COEF:

This subroutine computes the coefficients that are used to compute the system's response. The coefficients are a function of the time increment and the system's mass, damping, and frequency. Therefore, the subroutine COEF had to be called for each specific system and also when the time increment was changed in the integration process. The coefficients \( C_1 - C_8 \) were evaluated using the equations given in reference 6 and are utilized in the subroutine force.

Subroutine FORCE:

This subroutine computes the fluid forces and the corresponding system responses for the time increment under consideration. It involved different fluid forces depending on what portion of the Morison equation was under consideration (i.e. an inertia dominated system or a drag dominated system which either included or neglected interaction). As an example, the following algorithm was utilized for non-interacting systems subject to drag forces:

\[
F_d(t_j) = K \left| u_j \right| \\
F_d(t_{j+1}) = K \left| u_{j+1} \right|
\]

\[
X(t_{j+1}) = C_1 X(t_j) + C_2 \dot{X}(t_j) + C_3 F(t_j) + C_4 F(t_{j+1}) \\
\dot{X}(t_{j+1}) = C_5 X(t_j) + C_6 \dot{X}(t_j) + C_7 F(t_j) + C_8 F(t_{j+1})
\]
where $K$ is a specified constant, $j$ is the time increment under consideration and the other parameters were described in the text (see page 3). For the inertia dominated system, similar algorithms are used, however in this case the force is directly proportional to the fluid acceleration. When fluid-structure interaction is considered the drag force must be modified to include the structural velocity, $\dot{x}$. In this case, an iterative procedure is used.

**Subroutine MAXIMA:**

This subroutine determines the maximum value of $x(t)$, the systems response and the time at which it occurs.

**Subroutine DAF**

This subroutine normalizes the maximum response so that the results can be plotted in the form of conventional and tripartite response spectra. The amplification factors and time of occurance are printed before returning to the main program.
APPENDIX B

Earthquake Design Concepts

The following section will explain the earthquake response spectrum for ground-excited systems. Then these ideas will be extended to force-excited systems. The final section is devoted to a description of the spectra used to present the data for single-degree-of-freedom systems subjected to wave loadings.

B.1 Ground Excited Systems

Design criteria for land-based structures subjected to earthquakes has been studied extensively. Fundamental to these design practices is the response spectrum, which is defined as a graphical representation of the maximum response of single-degree-of-freedom elastic systems, (see fig. la) with specified damping, subjected to a dynamic ground motion. Although actual response spectra of structures subjected to earthquake motions are quite irregular, they can be approximated by the general shape of the spectrum shown in figure 2a, where the straight line segments represent a maximum response envelope. Procedures to construct these approximate design spectra for lightly damped systems are presented in reference 6 where it is
demonstrated that the gross characteristics of the ground displacement, velocity, and acceleration traces determine the shape of the approximate spectrum. In other words, values for $\alpha_A$, $\alpha_V$, and $\alpha_D$ in figure 2a are determined in reference 6 using simple expressions.

All scales in figure 2a are plotted on logarithmic scales. The pseudo-velocity, $V$, which is a measure of the energy absorption in the "spring" of the system is plotted on a vertical scale. The diagonal scale which extends upward from left to right is the pseudo-displacement, $D$, which is a measure of the deformation in the "spring". The diagonal scale which extends upwards from right to left represents the pseudo-acceleration, $\ddot{A}$, which is a measure of the maximum force in the "spring".

For a specific system frequency, $f$, the relationship between the values of $D$, $V$, and $\ddot{A}$ are defined as follows:

$$V = pD$$

$$\ddot{A} = pV = p^2D$$

where $p$ is the circular natural frequency of the system (i.e. $p = 2\pi f$). These relations show that once $D$, $V$, or $\ddot{A}$ is known, the other two values can be obtained by multiplying or dividing by $p$. Since frequency is the reciprocal of period, the logarithmic scale for period would have exactly the same spacing of points, or in effect, the plot
would be turned end for end.

The extreme parts of the response spectrum in figure 2a are described as follows, on the extreme left which corresponds to low frequency systems, the value of maximum response approaches an asymptote corresponding to the value of maximum ground displacement, $Y_o$. A low frequency system corresponds to a heavy mass or a weak spring. In this region the mass does not have time to respond, and the spring deformation is precisely equal to the value of the ground displacement. Therefore, low frequency systems are affected mainly by the ground displacement. The extreme right of the spectrum corresponds to systems with large frequencies (i.e. systems with stiff springs or light masses). When the ground moves, the stiff spring forces the systems mass to move in the same way, and the acceleration of the system becomes equivalent to the maximum ground acceleration, $\ddot{Y}_o$. Therefore, this part of the spectrum is sensitive to the ground acceleration. The middle part of the spectrum (i.e. medium frequency values) is sensitive mainly to the ground velocity, $\ddot{Y}$.

For the ideas presented above to be applicable to offshore structures, they must be applied to force excited systems. The description of the earthquake response spectrum for ground excited systems in extended to force excited systems in the next section.
B.2 Force Excited Systems

The same procedures used to construct earthquake spectra described in the previous section can be applied to force excited systems with knowledge of the force history, \( P(t) \), and its first two integrals \( (I'_1(t), \text{ and } I'_2(t)) \); where the first integral, \( I'_1(t) \), represents a pseudo-momentum, and \( I'_2(t) \), the second integral of the force history, represents the accumulation of the pseudo-momentum with time. The absolute maximum values of \( P(t), I'_1(t), \) and \( I'_2(t) \) without regards to sign are denoted by \( P'_o, (I'_1)_o, \) and \( (I'_2)_o \), respectively.

A dimensional response spectrum for force excited systems is shown in figure 2b. In this figure, \( F'_o \) represents the necessary static lateral force that produces a structural deflection equivalent to the absolute maximum response of the system subjected to the force history \( P(t) \). The ratio \( F'_o/P'_o \) then represents a dynamic load or amplification factor for the system. \( p \) in the figure represents the systems circular natural frequency. It should be noted that the quantities \( F'_o, F'_o/p, \text{ and } F'_o/p^2 \) of figure 2b are analogous to the quantities \( \bar{X}, V, \text{ and } D \) of figure 2a presented in the previous section.

B.3 Wave Force Excited Systems

The forces due to ocean waves are classified as inertia and drag and for clarity will be represented by
$P_I(t)$ and $P_D(t)$, respectively. So that the response values computed can be displayed in the form of tripartite spectra, the first two integrals of these force histories are required. The first and second integral of the drag force history in the absence of fluid-structure interaction, $P_D(t)$, will be denoted by $I_{1D}(t)$ and $I_{2D}(t)$, respectively, while the corresponding integrals of the inertial force history, $P_I(t)$, are given by $I_{1I}(t)$, $I_{2I}(t)$, respectively.

The non-dimensionalized tripartite response spectra used to display a majority of the results are shown in figure 3.1a and 3.1b. Part a is for systems which are subjected to inertial forces only while figure 3.1b gives the corresponding spectra for systems subjected only to drag force histories. The subscripts ($_o$) used throughout the figures, correspond to maximum values of their associated histories neglecting the effects of fluid structure interaction.
APPENDIX C

Fluid Particle Kinematic History Equations

This appendix presents the equations that were utilized to compute wave trains from a given sea spectrum. The equations represent a deep water Airy wave theory.

Horizontal Fluid Particle Displacement:

\[ h(t_j) = -\sum_{i=1}^{N} a_i e^{K_i Z} \sin(-w_i t_j + \phi_i) \]

Horizontal Fluid Particle Velocity:

\[ U(t_j) = \sum_{i=1}^{N} a_i w_i e^{K_i Z} \cos(-w_i t_j + \phi_i) \]

Horizontal Fluid Particle Acceleration:

\[ U(t_j) = \sum_{i=1}^{N} a_i w_i e^{2K_i Z} \sin(-w_i t_j + \phi_i) \]

where:

- \( N \) - Number of frequency bands.
- \( i \) - Particular frequency band.
- \( j \) - Particular time increment.
- \( a_i = \sqrt{2S(w_i)dw} \)
- \( \Delta w \) - Frequency increment of wave spectrum.
- \( S(w_i) \) = Wave spectrum ordinate.
- \( K_i = 2\pi/\lambda_i \) - Wave number.
- \( \lambda_i \) - Wave length.
- \( Z \) - Water depth (positive up).
$w_i$ - Particular midband wave frequency.
$\theta_i$ - Random phase angle uniformly distributed from 0 to $2\pi$. 
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TABLE I

Values derived from spectrum in figure 26
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Governing Equation: \[ m \dddot{x}_d + C \dot{x}_d + K x_d = -m \ddot{Y} \]

a) For Ground Excited Systems.

Governing Equation: \[ m \ddot{x} + C \dot{x} + K x = F(t) \]

b) For Force Excited Systems.

**FIG. 1 SINGLE DEGREE-OF-FREEDOM SYSTEMS**
a) Design Spectra for Ground Excited Systems.

b) Response Spectra for Force Excited Systems.

FIG. 2 EARTHQUAKE TYPE RESPONSE SPECTRA.

b. Response Spectra for Drag Fluid Forces.

FIG. 3.1 DIMENSIONLESS TRIPARTITE RESPONSE SPECTRA FOR FLUID FORCES.
a. Response Spectrum for Inertia Forces.

b. Response Spectrum for Drag Forces.

FIG. 3.2 CONVENTIONAL RESPONSE SPECTRA FOR FLUID FORCES.
a. Fluid Displacement.

$$h_0 = \frac{7}{2a} \cdot \frac{1}{2} u_0 t_0^2 = \frac{7}{12} u_0 t_0$$

b. Fluid Velocity.

$$\dot{u}_0 = \frac{1}{t} \int_0^t \frac{u(t)}{u_0} \, dt$$

c. Fluid Acceleration.

FIG. 4 HORIZONTAL FLUID PARTICLE KINEMATICS FOR PSEUDO WAVE HISTORY.
FIG. 5  DRAG FORCE AND ITS FIRST TWO INTEGRALS FOR THE PSEUDO WAVE HISTORY.
Fig. 6a
Tripartite response spectra for damped systems subjected to the inertia force history of the inset diagram, \( p(t) \).

\[ \frac{(F_1)_0}{p(t_1)_0} \]

\[ (F_1)_0, p(t_1)_0 \]

\[ (F_1)_0, p(t_1)_0 \]

\[ I_{21}(t) \]

\[ I_{11}(t) \]

\[ p_1(t) \]

\[ 0.0 \]

\[ 2.0 \]

\[ 4.0 \]

\[ 0.0 \]

\[ 2.0 \]

\[ 4.0 \]

\[ 0.0 \]

\[ 2.0 \]

\[ 4.0 \]

Value of \( 2ft_1 \)
FIG. 6b  TRIPARTITE RESPONSE SPECTRA FOR NON-INTERACTING DAMPED SYSTEMS SUBJECTED TO THE FORCE HISTORY OF THE INSET DIAGRAM.
FIG. 7 DRAG FORCE AND RESPONSE HISTORIES FOR UNDAMPED INTERACTING SYSTEMS WITH A CHARACTERISTIC FREQUENCY OF 0.3 (i.e. $f_{t_1} = 0.3$).
a. Drag Force Histories for different amounts of Interaction (ie. for different values of $\gamma$).

b. Corresponding Response Computed using Eqn. 2.2 with $\mu = 0.0$ (ie. drag dominated system).

FIG. 8 DRAG FORCE AND RESPONSE HISTORIES FOR UNDAMPED INTERACTING SYSTEMS WITH A CHARACTERISTIC FREQUENCY OF 1.0 (ie. $f_t = 1.0$).
FIG. 9a  RESPONSE SPECTRA FOR UNDAMPED SYSTEMS SUBJECTED TO DRAG FORCES WITH DIFFERENT AMOUNTS OF THE INTERACTION PARAMETER $\gamma$. 
Fig. 36: Tripartite Response Spectra for Undamped Systems Subjected to Drag Forces with Different Amounts of Interaction, $Y$.

- $Y = (x_{st})_0/\nu_0 t_1$, Interaction Parameter.
- $(x_{st})_0 = (f_D)_0/K$, $K$ is the System Stiffness.
- $\nu_0$ - Maximum Fluid Velocity.
- $t_1$ - Characteristic Time of Drag Force.
- Undamped Systems, i.e., $\zeta = 0$.
- Dashed curves represent solutions obtained using the equivalent damping concept.
FIG. 10 TRIPARTITE RESPONSE SPECTRA FOR UNDAMPED SYSTEMS SUBJECTED TO DRAG FORCES WITH DIFFERENT AMOUNTS OF INTERACTION, $\delta$. 
a. Approximate and Exact solutions for a system with a characteristic frequency of 0.3.

b. Approximate and Exact solutions for a system with a characteristic frequency of 1.0.

NOTE:  
- Solid curves correspond to the exact response with no damping. 
- Dashed curves, approximate solution using equivalent damping.

FIG. 11 COMPARISON OF THE EXACT AND APPROXIMATE SOLUTIONS USING EQNS. 2.2 AND 2.13 RESPECTIVELY, SUBJECT TO THE FORCES OF FIG. 7 & 8.
\[ (I_{2D})_0 = 0.568 \ (P_D)_0 t_1^2 \]

\[ (I_{1D})_0 = 0.463 \ (P_D)_0 t_1 \]


b. First Integral of Drag Force.

c. Drag Force History.

FIG. 12 DRAG FORCE AND ITS FIRST TWO INTEGRALS FOR THE EXTENDED PSEUDO WAVE HISTORY.
FIG. 13 TRIPARTITE RESPONSE SPECTRA FOR NON-INTERACTING DAMPED SYSTEMS SUBJECTED TO THE DRAG FORCE OF THE INSET DIAGRAM.
a. Drag Force Histories for different amounts of Interaction (ie. for different values of $\gamma$).

b. Corresponding Response Computed using Eqn. 2.2 with $\mu = 0.0$ (ie. drag dominated system).

FIG. 14 DRAG FORCE AND RESPONSE HISTORIES FOR UNDAMPED INTERACTING SYSTEMS WITH A CHARACTERISTIC FREQUENCY OF 0.3 (ie. $ft_1 = 0.3$).
a. Drag Force Histories for different amounts of Interaction (ie. for different values of $\gamma$).

b. Corresponding Response Computed using Eqn. 2.2 with $\mu = 0.0$ (ie. drag dominated system).

FIG. 15 DRAG FORCE AND RESPONSE HISTORIES FOR UNDAMPED INTERACTING SYSTEMS WITH A CHARACTERISTIC FREQUENCY OF 1.0 (ie. $f_{t_1} = 1.0$).
FIG. 16a RESPONSE SPECTRA FOR UNDAMPED INTERACTING SYSTEMS SUBJECTED TO THE EXTENDED PSEUDO WAVE HISTORY.
FIG. 16b TRIPARTITE RESPONSE SPECTRA FOR UNDAMPED SYSTEMS SUBJECTED TO DRAG FORCES WITH DIFFERENT AMOUNTS OF INTERACTION, \( \gamma \).
FIG. 17 TRIPARTITE RESPONSE SPECTRA FOR UNDAMPED SYSTEMS SUBJECTED TO DRAG FORCES WITH DIFFERENT AMOUNTS OF INTERACTION, δ.
a. Approximate and Exact solutions for a system with a characteristic frequency of 0.3.

b. Approximate and Exact solutions for a system with a characteristic frequency of 1.0.

NOTE:  - Solid curves correspond to the exact response with no damping.
       - Dashed curves, approximate solution using equivalent damping.

FIG. 18 COMPARISON OF THE EXACT AND APPROXIMATE SOLUTIONS USING EQUATIONS 2.2 AND 2.13 RESPECTIVELY, SUBMITTED TO THE FORCES OF FIG. 14&15.
FIG. 19 HORIZONTAL FLUID PARTICLE KINEMATICS FOR THE IRREGULAR WAVE.
\[ \frac{I_{2D}(t)}{(I_{2D})_0} \]

\[ (I_{2D})_0 = 18.78 \ (P_D)_0 \text{ sec}^2 \]


\[ \frac{I_{1D}(t)}{(I_{1D})_0} \]

\[ (I_{1D})_0 = 1.55 \ (P_D)_0 \text{ sec} \]

b. First Integral of the Drag Force History.

c. Drag Force History (proportional to \( u^2 \)).

Time, \( t \), in seconds.

FIG. 20 DRAG FORCE AND ITS FIRST TWO INTEGRALS FOR THE IRREGULAR WAVE OF FIG. 19.
FIG. 21 TRIPARTITE RESPONSE SPECTRUM FOR NON-INTERACTING UNDAMPED SYSTEMS SUBJECTED TO THE IRREGULAR WAVE HISTORIES OF FIG. 20.
![Graphs showing response spectra for undamped interacting systems.](image)

**FIG. 22** RESPONSE SPECTRA FOR UNDAMPED INTERACTING SYSTEMS.
a. Approximate and Exact Spectra for $(x_{st})_0/u_0 = 0.2$.

b. Approximate and Exact Spectra for $(x_{st})_0/u_0 = 0.4$.

FIG. 23 COMPARISON OF EXACT AND APPROXIMATE SPECTRA FOR UNDAMPED INTERACTING SYSTEMS USING A DIFFERENT EQUIVALENT DAMPING FACTOR FOR EACH HALF CYCLE OF THE VELOCITY DIAGRAM.
FIG. 24 COMPARISON OF EXACT AND APPROXIMATE SPECTRA FOR UNDAMPED INTERACTING SYSTEMS USING A DIFFERENT EQUIVALENT DAMPING FACTOR FOR EACH HALF CYCLE OF THE VELOCITY DIAGRAM.
a. Exact and Approximate Spectra for \( \left( x_{st} \right)_0 / u_0 = 0.2 \).

b. Exact and Approximate Spectra for \( \left( x_{st} \right)_0 / u_0 = 0.4 \).

FIG. 25 EXACT AND APPROXIMATE SPECTRA FOR UNDAMPED INTERACTING SYSTEMS.
Modified Pierson Moskowitz Sea Spectrum:

\[ S(f_w) = \frac{H_s^2 T_{av}}{4 \pi} (f_w T_{av})^{-5} \exp \left( -\frac{1}{\pi} (f_w T_{av})^{-4} \right) \]

with:  \( H_s = 40.0 \) ft., Significant Wave Height.
\( T_{av} = 12.0 \) sec., Average Wave Period.

Wave frequency, \( f_w \), in Hz.
FIG. 27 HORIZONTAL FLUID PARTICLE KINEMATICS AT A DEPTH OF 30 FT. RESULTING FROM THE SPECTRUM IN FIGURE 26.
FIG. 28  DRAG FORCE HISTORY AND IT'S FIRST TWO INTEGRALS AT A DEPTH OF 30 FT. RESULTING FROM SPECTRUM IN FIGURE 26.
FIG. 29 TRIPARTITE RESPONSE SPECTRUM FOR NON-INTERACTING UNDAMPED SYSTEMS SUBJECT TO THE RANDOM DRAG FORCE HISTORIES.
FIG. 30 RESPONSE SPECTRA FOR NON-INTERACTING DAMPED SYSTEMS SUBJECTED TO THE RANDOM DRAG FORCE HISTORY OF FIGURE 28.
FIG. 31a RESPONSE SPECTRA FOR UNDAMPED INTERACTING SYSTEMS SUBJECTED TO THE RANDOM DRAG FORCE HISTORY OF FIGURE 28.
FIG. 31b  COMPARISON OF EXACT AND APPROXIMATE SPECTRA FOR UNDAMPED SYSTEMS USING THE EQUIVALENT DAMPING DERIVED FROM THE LARGEST HALF CYCLE OF THE VELOCITY DIAGRAM.
FIG. 32a RESPONSE SPECTRA FOR UNDAMPED INTERACTING SYSTEMS SUBJECTED TO THE RANDOM DRAG FORCES OF FIGURE 28.
FIG. 32b COMPARISON OF EXACT AND APPROXIMATE SPECTRA FOR UNDAMPED INTERACTING SYSTEMS USING THE EQUIVALENT DAMPING DERIVED FROM THE LARGEST HALF CYCLE OF THE VELOCITY DIAGRAM.